A Web Appendix (Proofs)

The following observation proves useful in ruling out preferences that induce actors' giving decisions to corner above:

Remark 1 *We rule out utility functions that would induce the planner to optimally choose allocations, which left shareholders with no profits from the firm. It immediately follows that neither the manager, nor shareholders would choose such allocations either.*

Lemma 1 *Given any belief $\tilde{\alpha}$ the voluntary contributions game among shareholders has a unique, symmetric equilibrium.*

Proof. First, we show that any equilibrium is symmetric. Suppose to the contrary that shareholders i, j give different amounts $0 \leq \beta_i < \beta_j$. Then, since these are optimizing, it follows that the optimality condition for shareholder i satisfies:

$$u_c\left(\frac{\pi - \tilde{\alpha}}{n} - \beta_i, \tilde{\alpha} + \sum_{k=1}^{n} \beta_k\right) \geq u_g\left(\frac{\pi - \tilde{\alpha}}{n} - \beta_i, \tilde{\alpha} + \sum_{k=1}^{n} \beta_k\right)$$

and likewise for j

$$u_c\left(\frac{\pi - \tilde{\alpha}}{n} - \beta_j, \tilde{\alpha} + \sum_{k=1}^{n} \beta_k\right) = u_g\left(\frac{\pi - \tilde{\alpha}}{n} - \beta_j, \tilde{\alpha} + \sum_{k=1}^{n} \beta_k\right)$$

But since $\beta_j > \beta_i$ and since $u_{cc} < 0$ then

$$u_c\left(\frac{\pi - \tilde{\alpha}}{n} - \beta_j, \tilde{\alpha} + \sum_{k=1}^{n} \beta_k\right) > u_c\left(\frac{\pi - \tilde{\alpha}}{n} - \beta_i, \tilde{\alpha} + \sum_{k=1}^{n} \beta_k\right) \geq u_g\left(\frac{\pi - \tilde{\alpha}}{n} - \beta_i, \tilde{\alpha} + \sum_{k=1}^{n} \beta_k\right)$$

But this is a contradiction—the level of public goods is identical for i and j, so with concave utility they cannot both equalize their marginal utilities of consumption to the marginal utilities of public goods at different consumption levels. Therefore $\beta_i = \beta_j$ for all i, j in any equilibrium.

Now, we show uniqueness. Suppose to the contrary that there are two equilibrium giving levels, β and β' such that $\beta \neq \beta'$, where β (β') denotes the equilibrium contribution of every shareholder.

When $\beta, \beta' > 0$, then for each to comprise an equilibrium requires that

$$u_c\left(\frac{\pi-\tilde{\alpha}}{n}-\beta,\tilde{\alpha}+n\beta\right)=u_g\left(\frac{\pi-\tilde{\alpha}}{n}-\beta,\tilde{\alpha}+n\beta\right)$$

and similarly for β'. Treating β as a parameter and differentiating $u_c - u_g$, we obtain

$$\begin{aligned}\frac{\partial\left(u_c-u_g\right)}{\partial\beta}&=-u_{cc}+\left(n+1\right)u_{cg}-nu_{gg}\\&=-\left(u_{cc}-u_{cg}\right)-n\left(u_{gg}-u_{cg}\right)\end{aligned}\tag{10}$$

Since $\beta^i > 0$, the Lemma 5 shows that $u_{cc} - u_{cg} < 0$ and $u_{gg} - u_{cg} < 0$. This implies that that (10) is positive: in the neighborhood of any equilibrium, $u_c - u_g$ is negative for slightly lower values of β and positive for higher values of β. Were there to be multiple equilibria, then at least one such equilibria must have the reverse sign in the neighborhood of the equilibrium point, but this is impossible.

It remains to deal with the case where one of the possible equilibrium points occurs at $\beta = 0$. If $\beta = 0$ comprises an equilibrium, then it must be that

$$u_c\left(\frac{\pi-\tilde{\alpha}}{n}-\beta,\alpha+n\beta\right)\bigg|_{\beta=0}-u_g\left(\frac{\pi-\tilde{\alpha}}{n}-\beta,\tilde{\alpha}+n\beta\right)\bigg|_{\beta=0}>0$$

From our previous analysis, we know that, at any interior equilibrium point, $u_c - u_g$ is negative for slightly lower values of β and positive for higher values of β. Since $u_c - u_g > 0$ at $\beta = 0$, then at the smallest $\beta > 0$ comprising an equilibrium, it must be that $u_c - u_g$ is positive for slightly lower values of β and negative for slightly higher values. But this is a contradiction, because $\frac{\partial(u_c-u_g)}{\partial\beta} > 0$. Therefore, if $\beta = 0$ is an equilibrium, it is the unique equilibrium, and similarly, if $\beta > 0$ is an equilibrium, then $\beta = 0$ is not an equilibrium. ■

Theorem 1 *When shareholders are identical: (i) No shareholder contributes privately—all contributions are delegated to the manager. (ii) If shareholders would (not) contribute positively absent manager contributions, then this delegation strictly (weakly) raises overall public goods provisioning and shareholder welfare.*

Proof. (i) Suppose to the contrary that every shareholder contributes $\beta' > 0$ to the public good while the the manager optimally contributes $\alpha' \geq 0$. Since shareholders contribute positive amounts, their FOC must hold:

$$u_c\left(\frac{\pi-\alpha'}{n}-\beta',\alpha'+n\beta'\right)=u_g\left(\frac{\pi-\alpha'}{n}-\beta',\alpha'+n\beta'\right) \tag{11}$$

Since the manager's contribution is non-negative, the derivative of the representative shareholder's utility (*i.e.* the manager's objective) with respect to α must be non-positive:

$$-\frac{1}{n}u_c\left(\frac{\pi-\alpha'}{n}-\beta',\alpha'+n\beta'\right)+u_g\left(\frac{\pi-\alpha'}{n}-\beta',\alpha'+n\beta'\right)\leq 0$$

Since this can be reorganized

$$u_c\left(\frac{\pi-\alpha'}{n}-\beta',\alpha'+n\beta'\right)\geq nu_g\left(\frac{\pi-\alpha'}{n}-\beta',\alpha'+n\beta'\right)$$

this clearly contradicts (11) for all $n > 1$. Said differently, optimality for the manager precludes interiority for shareholders.

(ii) Case 1 ($\beta' > 0$, $\alpha' = 0$): We know from Lemma 2 that if the manager were (publicly) restricted to contribute $\tilde{\alpha} = n\beta'$ that shareholders would contribute nothing and enjoy the same utility as if he were restricted to contribute some lower amount. In particular, shareholders FOCs would be satisfied: $u_c = u_g$. If the restriction on the manager were lifted, he could increase α and cause an increase in utility (for every shareholder) of nu_g at a cost of u_c, a strict increase in utility for all.

Case 2 ($\beta' = 0$, $\alpha' = 0$): When shareholders are strictly cornered below, $u_c > u_g$. An increase in α by the manager will still cause an increase of nu_g for a cost of u_c.The manager will implement such an increase iff $u_c < nu_g$, which may or may not be true. If it is, delegation raises public goods an makes shareholders strictly better off. If it is not, then delegation does not change public goods levels or shareholder welfare. ∎

Proposition 1 *In all trembling hand perfect equilibria, the venture is funded with positive probability.*

Proof. From Lemma 7, there is no trembling hand perfect PSE in which strictly less than n citizens subscribe to the IPO. From Lemma 6, the PSE in which exactly n citizens subscribe to the IPO is trembling hand perfect. There is no PSE in which strictly more than n citizens subscribe—it is always a profitable deviation for a putative subscriber to buy the bond. Any completely MSE (including the symmetric MSE characterized in Lemma 8) is trembling hand perfect and has a positive probability of funding the venture. The last remaining type of equilibrium, which needs to be ruled out, is one in which x individuals always subscribe, and y individuals mix, where $x+y < n$. The steps of Lemma 7 may be applied to show such an equilibrium is not trembling hand perfect. If a trembling hand equilibrium in which $x + y \geq n$, then it conforms to the proposition, because the venture is also funded with positive probability. ■

Proposition 4 *The manager will always produce strictly less than the profit maximizing quantity.*

Proof. Suppose the firm produced the profit maximizing quantity $\widehat{q}$. A slight production decrease produces a first order gain in shareholder welfare due to increased public good (*i.e.* $\psi'(\widehat{q}) > 0$), but no first order loss in profits (*i.e.* $\pi'(\widehat{q}) = 0$). Thus, some $q < \widehat{q}$ is optimal. ■

Lemma 4 *The socially optimal production level, q^*, is the unique value of q solving $\pi'(q^*) = \psi'(q^*)$.*

Proof. First, consider the case where there is an interior solution for both q and α. In that case, the first order condition of the planner's problem (4) for α is

$$u_c = nu_g + \sum_{i=1}^{N-n} v_g^i \tag{12}$$

where v_g^i denotes the derivative of the i^{th} non-shareholding citizen's utility with respect to its argument, the public goods level. The first order condition of the planner's problem for q is

$$\pi'(q)\, u_c = \psi'(q) \left(nu_g + \sum_{i=1}^{N-n} v_g^i \right) \tag{13}$$

Together, equations (12) and (13) imply that the socially optimal production quantity, q^*, satisfies $\pi'(q) = \psi'(q)$.

We have ruled out utility functions such that the planner would optimally contribute all positive profits to the public good. Since individual fundability holds $u_c(0,0) < u_g(0,0)$, it must be that

$$u_c(\pi(q), -\psi(q)) < nu_g(\pi(q), -\psi(q)) + \sum_{i=1}^{N-n} v_g^i(-\psi(q))$$

for any $q \geq 0$. Thus, the planner's choice of α is interior for any positive q.

It only remains to show that $\langle q = 0, \alpha = 0 \rangle$ is never optimal. Suppose, instead that the planner produced an arbitrarily small amount, $q = \varepsilon$, and used (almost) the entire proceeds for the public good, *i.e.* $\alpha \approx \pi(\varepsilon)$ then welfare would be approximately

$$nu(0, \pi(\varepsilon) - \psi(\varepsilon)) + \sum_{i=1}^{N-n} v^i(\pi(\varepsilon) - \psi(\varepsilon)) > nu(0,0) + \sum_{i=1}^{N-n} v^i(0)$$

where the inequality follows from the fact that $\pi'(0) > \psi'(0)$. The right-hand side of the above inequality is welfare under zero production and contribution; thus, $q = 0$ is never optimal, and equations (12) and (13) always hold with equality.

Hence, the socially optimal production quantity satisfies $\pi'(q) = \psi'(q)$. Strict concavity of $\pi(\cdot)$ and $\psi(\cdot)$ following the curvature condition of Lemma 12 (convexity suffices) imply that this solution is unique. ■

Theorem 2 *(i) The manager chooses the socially optimal quantity q^* iff the public good is corporately fundable; otherwise the firm overproduces (i.e. $q \in (q^*, \widehat{q})$). (ii) Furthermore, the manager provisions strictly positive amounts of the public good iff the inequality in equation (7) is strict.*

Proof. Let q' and α' be the manager's optimal production quantity and public goods contribution decisions respectively. The (unconstrained) first order condition of the manager's problem (2) for α is

$$u_c = nu_g \tag{14}$$

and the (unconstrained) first order condition of the manager's problem for q is

$$\pi'(q')\, u_c = \psi'(q')\, n u_g \tag{15}$$

When the corporate fundability condition holds, there exists $\alpha' \geq 0$, given that $q = q^*$, such that equation (14) holds. Substituting this equation into the above implies that $q' = q^*$ is optimal.

When corporate fundability does not hold, then at $q = q^*$, LHS of equation (14) is strictly greater that RHS evaluated at $\alpha = 0$. Hence, substituting in equation (15) at $(\alpha = 0,\ q = q^*)$ we have

$$\pi'(q')\, u_c > \psi'(q')\, n u_g$$

Hence q^* is not optimal. Furthermore, since $\pi'(\cdot)$ is decreasing and $\psi'(\cdot)$ increasing in q, then optimal production is given by $q' > q^*$.

Remark 1 rules out utility functions, which cause α to corner above. ■

Proposition 5 *Suppose that individual fundability holds and the manager cannot divert profits. The manager optimally distorts output downwards compared to the social optimum. The more widely held is the firm, the larger is the distortion. Formally, let q_n be the manager's optimal output when there are n shareholders, then $\frac{d(q^* - q_n)}{dn} > 0$, while $q_1 = q^*$.*

Proof. Individual fundability implies that shareholders make positive contributions at all production levels. Thus, in equilibrium $u_c = u_g$. The manager's FOC with respect to production q is $\frac{\pi'(q)}{n} u_c = \psi'(q)\, u_g$. Thus, the manager chooses q_n satisfying $\frac{\pi'(q_n)}{n} = \psi'(q_n)$. From the IFT

$$\frac{dq_n}{dn} = \frac{\psi'(q_n)}{\pi''(q_n) - n\psi''(q_n)} < 0$$

where the inequality follows because the denominator is the SOC and negative and the externality (i.e. the numerator) strictly increases in production. Observe that when $n = 1$, $q_n = q^*$. ■

Proposition 6 *Suppose the corporate fundability condition holds. Then firm output is identical under the cleaner or cheaper technology. Furthermore, total public goods are identical under the two technology improvements.*

Proof. Since corporate fundability holds, marginal profits equals marginal externality. With the cleaner technology, there is a unique q solving

$$\hat{\psi}'(q) = \pi'(q) \tag{16}$$

while with the cheaper technology, there is a unique q' solving

$$\psi'\left(q'\right) = \hat{\pi}'\left(q'\right) \tag{17}$$

We will show that $q = q'$. Rewriting equation (12), we have

$$\pi'(q) - \hat{\psi}'(q) = \hat{\pi}'(q) - \psi'(q)$$

Next, notice from equation (16) that $\pi'(q) - \hat{\psi}'(q) = 0$; therefore, for this same value of q, it must be that $\hat{\pi}'(q) - \psi'(q) = 0$, which is the solution to equation (17). Hence, we may conclude that $q' = q$.

Next, notice that following the production decision, the manager uses α to balance consumption, c, and public goods provisioning, g. One can think of this as the manager's budget set. The trade-off between these is unaffected by the technology, so the slope of the manager's budget set is constant and identical under the two technology improvements.

Next, we will show that the budget sets themselves are identical. To see this, we will show that the two technology improvements lead to budget sets that share a common point. Specifically, notice that the maximum total public goods under the cleaner technology is $\pi(q^*) - \hat{\psi}(q^*)$ while the maximum under the cheaper technology is $\hat{\pi}(q^*) - \psi(q^*)$, where q^* solve equations (16) and (17). We claim that

$$\pi(q^*) - \hat{\psi}(q^*) = \hat{\pi}(q^*) - \psi(q^*)$$

Recall that

$$\pi(q^*) - \hat{\psi}(q^*) = \pi(0) + \int_0^{q^*} \pi'(q) - \hat{\psi}'(q)\, dq$$

while

$$\hat{\pi}(q^*) - \psi(q^*) = \hat{\pi}(0) + \int_0^{q^*} \hat{\pi}'(q) - \psi'(q)\, dq$$

Differencing the two expressions, we obtain

$$\int_0^{q^*} \left(\hat{\pi}'(q) - \pi'(q)\right) - \left(\psi'(q) - \hat{\psi}'(q)\right) dq = 0$$

where the equality follows from equation (9). Since the budget sets are identical under the two technology improvements, the optimal choice must likewise be identical. Hence, total public goods are *identical* under the two technology improvements. ■

The following are useful technical lemmas which support the above results that are stated in the main text:

Lemma 5 *If an actor's contribution is strictly positive, then* $u_{cc} < ku_{cg}$ *and* $ku_{gg} < u_{cg}$*.*

Proof. For future reference, the slope of an actor's indifference curve is, as usual

$$\frac{dg}{dc} = -\frac{u_c}{u_g} \tag{18}$$

Next, we construct the relationship between c and g under optimization. By assumption, public goods contributions are strictly positive. The assumption that the planner would not leave shareholders with zero consumption implies that neither would the manager nor a shareholder herself. Hence, an agent's optimal contribution under the premise of the Lemma is interior and satisfies the following FOC: $u_c(c, g) = ku_g(c, g)$. Implicitly differentiating both sides

$$u_{cc}(c, g)dc + u_{cg}(c, g)dg = ku_{cg}(c, g)dc + ku_{gg}(c, g)dg$$

and rearranging yields

$$\frac{dg^*}{dc^*} = \frac{u_{cc}(c, g) - ku_{cg}(c, g)}{ku_{gg}(c, g) - u_{cg}(c, g)} \tag{19}$$

where the stars denote the fact that this slope only holds for optimal bundles. Since public goods are assumed to be strictly normal goods, then $dg^*/dc^* > 0$. Equivalently the numerator and denominator of the RHS must have the same sign. ■

Lemma 6 *There exists a pure strategy equilibrium where exactly* n *individuals subscribe to the IPO, such that the venture is funded with probability* 1*. It is trembling hand perfect.*

Proof. Consider the PSE in which n individuals contribute and $N-n$ do not. WLOG let the equilibrium IPO subscribers (except a focal individual) be numbered $1, ..., n-1$, equilibrium non-subscribers be numbered $n, ..., N-1$, and N be a focal individual who subscribes in equilibrium. Define $\varepsilon = \langle \varepsilon_1, ..., \varepsilon_{N-1} \rangle$ as the vector of probabilities that each individual, except the focal one, trembles. Then $\mathbf{p} = \langle p_1, ..., p_{N-1} \rangle$ is the vector of probabilities that each individual, except the focal one, actually subscribes, where

$$p_i = \begin{cases} 1-\varepsilon_i & \text{if} \quad i \in \{1, ..., n-1\} \\ \varepsilon_i & \text{if} \quad i \in \{n, ..., N-1\} \end{cases}$$

If the focal individual follows the putative strategy and subscribes, his payoff will be

$$\sum_{j=1}^{n-2} f(j; \mathbf{p}) U_\emptyset + f(n-1; \mathbf{p}) U_S + \sum_{j=n}^{N-1} f(j; \mathbf{p}) \left(\frac{n}{j+1} U_S + \left(1 - \frac{n}{j+1}\right) U_B \right) \tag{20}$$

where $f(x; \mathbf{p})$ is the pmf of Poisson's Binomial Distribution as defined in Wang (1993).[26] The first term denotes the probability that there are not enough other subscribers and the IPO fails, the second term denotes the probability that exactly a sufficient number co-subscribe, and the last term denotes the probability of over subscription and the possibility of being selected for the IPO or not. If the focal individual deviates to not subscribe, his payoff will be

$$\sum_{j=1}^{n-2} f(j; \mathbf{p}) U_\emptyset + f(n-1; \mathbf{p}) U_\emptyset + \sum_{j=n}^{N-1} f(j; \mathbf{p}) U_B \tag{21}$$

Here the first term denotes the probability that there are too few subscribers for the focal individual to even be pivotal and the IPO fails. The second term denotes the probability that the focal individual is pivotal and his deviation causes the IPO to fail. The third term denotes the probability that the IPO succeeds without the subscription of the focal individual. Thus, differencing the two equations (*i.e.* (20) minus (21)) we can compute the net payoff Δ from subscribing to the IPO (the putative strategy):

$$\Delta = f(n-1; \mathbf{p}) (U_S - U_\emptyset) + (U_S - U_B) \sum_{j=n}^{N-1} f(j; \mathbf{p}) \frac{n}{j+1} \tag{22}$$

[26] http://www.ressources-actuarielles.net/EXT/ISFA/1226.nsf/0/069d7ef52c771fd0c1257e1d002a65cc/$FILE/A3n23.pdf

The first term (positive) represents the net payoff when $n-1$ other individuals contribute (*i.e.* the focal individual is pivotal), and the second (negative) the net payoff when the IPO is already oversubscribed.

We will show that sufficiently far along the limit path deviation to non-subscription is never profitable: $\lim_{\varepsilon\to 0}\Delta > 0$.[27] Note that $\Delta > 0$ iff $\Delta/f(n-1;\mathbf{p}) > 0$, so that it suffices to show that $\lim_{\varepsilon\to 0}\Delta/f(n-1;\mathbf{p}) > 0$.

$$\begin{aligned}
\lim_{\varepsilon\to 0}\frac{\Delta}{f(n-1;\mathbf{p})} &= U_S - U_\emptyset + (U_S-U_B)\sum_{j=n}^{N-1}\left[\lim_{\varepsilon\to 0}\frac{f(j;\mathbf{p})}{f(n-1;\mathbf{p})}\right]\frac{n}{j+1} \\
&= U_S - U_\emptyset + (U_S-U_B)\sum_{j=n}^{N-1}\left[\lim_{\varepsilon\to 0}\frac{f(j;\mathbf{p})}{f(j-1;\mathbf{p})}\frac{f(j-1;\mathbf{p})}{f(j-2;\mathbf{p})}\cdots\frac{f(n;\mathbf{p})}{f(n-1;\mathbf{p})}\right]\frac{n}{j+1} \\
&= U_S - U_\emptyset + (U_S-U_B)\sum_{j=n}^{N-1}\left[\lim_{\varepsilon\to 0}\left\{\prod_{i=n}^{j}\frac{f(i;\mathbf{p})}{f(i-1;\mathbf{p})}\right\}\right]\frac{n}{j+1} \qquad (23)
\end{aligned}$$

From Theorem 2 of Wang (1993) we know that

$$\frac{f(i;\mathbf{p})}{f(i-1;\mathbf{p})} > C(i)\,\frac{f(i+1;\mathbf{p})}{f(i;\mathbf{p})}$$

where

$$C(i) = \max\left\{\frac{i+1}{i}, \frac{(N-1)-i+1}{(N-1)-i}\right\} > 1$$

Hence

$$\prod_{i=n}^{j}\frac{f(i;\mathbf{p})}{f(i-1;\mathbf{p})} < \left(\frac{f(n;\mathbf{p})}{f(n-1;\mathbf{p})}\right)^{j-n} < \left(\frac{f(q;\mathbf{p})}{f(q-1;\mathbf{p})}\right)^{j-n} \qquad (24)$$

for any $q < n$ (Note: the final inequality is only used in the proof of Lemma 7). Observe that $\lim_{\varepsilon\to 0} f(n-1;\mathbf{p}) = 1$ and $\lim_{\varepsilon\to 0} f(n;\mathbf{p}) = 0$. Thus, from the first inequality in (24), the expression in curly braces in equation (23) goes to 0 as the vector $\varepsilon \to 0$. Hence, we conclude

$$\lim_{\varepsilon\to 0}\frac{\Delta}{f(n-1;\mathbf{p})} = U_S - U_\emptyset > 0 \qquad (25)$$

Therefore, always contributing is a best response sufficiently far along the limit path. The equilibrium in which exactly n individuals always subscribe and $N-n$ do not is trembling hand perfect. ■

[27] Here, by $\lim_{\varepsilon\to 0}$, we mean that each element of the vector ε approaches 0, but not necessarily at the same rate, *i.e.* for i, j, $\lim_{\varepsilon\to 0}\frac{p_i}{p_j}$ need not converge.

Lemma 7 *No pure strategy trembling hand equilibrium in which strictly fewer than n individuals invest exists.*

Proof. This proof follows the logic of the proof of Lemma 6 closely. Consider the PSE in which $k \in \{0, ..., n-1\}$ individuals contribute and $N-k$ do not. WLOG let equilibrium IPO subscribers be numbered $1, ..., k$, equilibrium non-subscribers be numbered $k+1, ..., N-1$ and N be a focal individual who does not subscribe in equilibrium. Define $\varepsilon = \langle \varepsilon_1, ..., \varepsilon_{N-1} \rangle$ as the vector of probabilities that each individual, except the focal one, trembles. Then $\mathbf{p} = \langle p_1, ..., p_{N-1} \rangle$ is the vector of probabilities that each individual, except the focal one, actually subscribes, where

$$p_i = \begin{cases} 1-\varepsilon_i & \text{if} \quad i \in \{1, ..., k\} \\ \varepsilon_i & \text{if} \quad i \in \{k+1, ..., N-1\} \end{cases}$$

If the focal individual deviates to subscribe, his payoff will be exactly as in (20), and if he follows the putative strategy and abstains his payoff will be exactly as in (21), except that $\mathbf{p}$ is redefined as immediately above. Hence, net payoff Δ from subscribing to the IPO (now, a deviation) is exactly as in (22). We will show that sufficiently far along the limit path subscribing is always a profitable deviation: again, $\lim_{\varepsilon \to 0} \Delta > 0$.[28] Again it suffices to show $\lim_{\varepsilon \to 0} \Delta / f(n-1; \mathbf{p}) > 0$ where $\Delta / f(n-1; \mathbf{p})$ is defined as in (23). Setting $q = k$ in inequality (24), and observing that $\lim_{\varepsilon \to 0} f(k-1; \mathbf{p}) = 1$ and $\lim_{\varepsilon \to 0} f(k; \mathbf{p}) = 0$, the expression in curly braces in equation (23) goes to 0 as the vector $\varepsilon \to 0$. Hence inequality (25) again holds, this time indicating the direction of the net benefit of deviating to contribute. Therefore, always contributing is a best response sufficiently far along the limit path, which shows that free-riding, such that the venture is never formed, is not a trembling hand perfect equilibrium. ■

Lemma 8 *There exists mixed strategy equilibria (including a symmetric one) in which n or more individuals subscribe, such that the venture is funded with positive probability. (Furthermore, all completely mixed strategies are trembling hand perfect.)*

[28] Here, by $\lim_{\varepsilon \to 0}$, we mean that each element of the vector ε approaches 0, but not necessarily at the same rate, *i.e.* for i, j, $\lim_{\varepsilon \to 0} \frac{p_i}{p_j}$ need not converge.

Proof. We show that a symmetric MSE exists. Other MSE may exist. In any MSE, the venture is funded with positive probability.

Let p^* be the probability that each of $N-1$ citizens subscribe to the IPO. If the focal individual subscribes his expected payoff is

$$U(1) = \sum_{i=0}^{n-2} \binom{N-1}{i} (p^*)^i (1-p^*)^{N-1-i} U_\emptyset + \binom{N-1}{n-1} (p^*)^{n-1} (1-p^*)^{N-1-(n-1)} U_S$$
$$+ \sum_{i=n}^{N-1} \binom{N-1}{i} (p^*)^i (1-p^*)^{N-1-i} \left(\frac{n}{i+1} U_S + \left(1 - \frac{n}{i+1}\right) U_B \right)$$

If he does not subscribe his expected payoff is

$$U(0) = \sum_{i=0}^{n-2} \binom{N-1}{i} (p^*)^i (1-p^*)^{N-1-i} U_\emptyset + \binom{N-1}{n-1} (p^*)^{n-1} (1-p^*)^{N-1-(n-1)} U_\emptyset$$
$$+ \sum_{i=n}^{N-1} \binom{N-1}{i} (p^*)^i (1-p^*)^{N-1-i} U_B$$

Thus, he is indifferent iff there exists p^* such that $U(1) - U(0) = 0$ or equivalently

$$(U_S - U_\emptyset) \binom{N-1}{n-1} (p^*)^{n-1} (1-p^*)^{N-1-(n-1)} + (U_S - U_B) \sum_{i=n}^{N-1} \binom{N-1}{i} (p^*)^i (1-p^*)^{N-1-i} \frac{n}{i+1} = 0$$

After some simplification this becomes

$$\frac{U_S - U_\emptyset}{U_B - U_S} \binom{N-1}{n-1} = \sum_{i=n}^{N-1} \binom{N-1}{i} \left(\frac{p^*}{1-p^*} \right)^{i-(n-1)} \frac{n}{i+1}$$

The LHS is a strictly positive, finite number while $\lim_{p^* \to 0} RHS = 0$ and $\lim_{p^* \to 1} RHS = +\infty$. Thus, by continuity there exists a p^* which makes the focal individual indifferent between subscribing and not. Hence the symmetric MSE exists. ■

Lemma 9 *When the manager cannot divert profits and shareholders contribute after the manager chooses production, then the equilibrium increase in collective shareholder contributions (when interior) due to a production increase always lies between the increase in damage to the public goods and the increase in profits. Formally,*

$$\psi'(q) \le n \frac{d\beta}{dq} \le \pi'(q) \iff \psi'(q) \le \pi'(q) \iff q \le q^*$$

Proof. The representative shareholder's FOC with respect to β, given equilibrium beliefs $\tilde{q}$ about production is

$$\frac{du}{d\beta} = -u_c\left(\frac{\pi(q)}{n} - \beta, (n-1)\bar{\beta} + \beta - \psi(q)\right) + u_g\left(\frac{\pi(q)}{n} - \beta, (n-1)\bar{\beta} + \beta - \psi(q)\right) = 0$$

where $\bar{\beta}$ is the equilibrium contribution of each other shareholder. From the IFT, when β is interior

$$\frac{d\beta}{dq} = -\frac{-\frac{\pi'(q)}{n}(u_{cc} - u_{cg}) + \left((n-1)\frac{d\bar{\beta}}{dq} - \psi'(q)\right)(u_{gg} - u_{cg})}{u_{cc} - u_{cg} + u_{gg} - u_{cg}}$$

By symmetry $\bar{\beta} = \beta$. Thus substituting $\frac{d\bar{\beta}}{dq} = \frac{d\beta}{dq}$ and solving for $\frac{d\beta}{dq}$ yields

$$\frac{d\beta}{dq} = \frac{\frac{\pi'(q)}{n}(u_{cc} - u_{cg}) + \psi'(q)(u_{gg} - u_{cg})}{(u_{cc} - u_{cg}) + n(u_{gg} - u_{cg})}$$

Hence, $\psi'(q) \leq n\frac{d\beta}{dq}$ iff

$$\psi'(q) \leq \frac{\pi'(q)(u_{cc} - u_{cg}) + \psi'(q)\, n\,(u_{gg} - u_{cg})}{(u_{cc} - u_{cg}) + n(u_{gg} - u_{cg})}$$

By Lemma 5 $u_{cc} < u_{cg}$ and $u_{gg} < u_{cg}$, and hence this inequality holds iff $\psi'(q) \leq \pi'(q)$. Following analogous steps, $n\frac{d\beta}{dq} \leq \pi'(q)$ iff $\psi'(q) \leq \pi'(q)$. ■

B Web Appendix (Extensions)

B.1 Heterogeneous Shareholders

Assuming shareholders are identical considerably simplifies the analysis, but this is obviously unrealistic. In this section, we investigate the extent to which our conclusions about socially responsible firms change when shareholders differ. We find that with suitable modification of the corporate fundability condition, shareholders continue to delegate some (not necessarily all) contribution to the manager and he continues to choose the socially optimal output level.

With respect to this finding, a natural question arises as to whether the shareholder differences make the corporate fundability condition more stringent. It turns out that there is no general answer to this question. Depending on the form of the heterogeneity, corporate fundability can be harder, easier, or equally difficult to satisfy than when shareholders are homogenous. We demonstrate this using a quasilinear utility specification with heterogeneous, exogenous taste parameters such that the generalized mean of these parameters coincides with the parameter in the identical case. Depending on which mean (*e.g.* geometric, arithmetic or quadratic) one introduces heterogeneity around, the stringency of the corporate fundability condition can either increase, decrease, or stay the same.

Before proceeding, some preliminaries are in order. When shareholders are identical, the results of the first stage negotiations as to the manager's objective function are unambiguous. When shareholders differ, this is no longer the case. The manager's objective function will depend on the rules used by shareholders for collective decision making. A rule where shareholders weights are proportional to ownership shares might produce a share-weighted utilitarian objective function. A voting process might produce an objective function corresponding to the *median* shareholder.[29] And so on.

Rather than modeling the exact process by which shareholders arrive at the manager's contract, we study a general, flexible form nesting many approaches. Fix an increasing and concave aggregator function f, and suppose that shareholder i is entitled to a positive fraction λ_i (where $\sum \lambda_i \leq 1$) of net profits. The manager solves:

$$\max_{0\leq q, 0\leq \alpha \leq \pi(q)} W = f\left(u^1\left(c^1\left(q,\alpha\right), g\left(q,\alpha\right)\right), ..., u^n\left(c^n\left(q,\alpha\right), g\left(q,\alpha\right)\right)\right) \tag{26}$$

where

$$c^i\left(q,\alpha\right) = \lambda_i\left(\pi\left(q\right) - \alpha\right) - \beta^{i*}$$
$$g\left(q,\alpha\right) = -\psi\left(q\right) + \alpha + \sum_{j=1}^{n} \beta^{j*}$$

Here, β^{i*} is the equilibrium individual contribution of shareholder i in voluntary contribution game when shareholders believe that the manager chooses output $\tilde{q}$ and contributes $\tilde{\alpha}$ to the public good,

[29] Even this is problematic since, in the context of multidimensional preferences, the median voter theorem is not generally valid. See, e.g., McKelvey 1976

and beliefs $\tilde{q}$ and $\tilde{\alpha}$ are confirmed in equilibrium. For the manager's problem to be well specified requires that we delineate the subsequent voluntary contributions that arise following any beliefs $(\tilde{q}, \tilde{\alpha})$. The following lemma shows that there is a unique equilibrium following each such choice; hence, there is no ambiguity in the manager's problem.

Lemma 10 *Following any beliefs* $(\tilde{q}, \tilde{\alpha})$, *there exists a unique equilibrium in the voluntary contributions game.*

Proof. First we show that conditional on any set of beliefs about equilibrium behavior by the other shareholders (and manager), a shareholder's optimal response exists and is unique. Then we show that only this equilibrium is the only possible one.

Since shareholder i's contribution is optimizing it must satisfy the following complementary slackness condition:

$$\beta_i \left[-u_c^i \left(\lambda_i \left(\pi\left(\tilde{q}\right) - \tilde{\alpha}\right) - \beta_i, \tilde{\alpha} + \beta_i + \sum_{k=1;k\neq i}^{n} \tilde{\beta}_k \right) + u_g^i \left(\lambda_i \left(\pi\left(\tilde{q}\right) - \tilde{\alpha}\right) - \beta_i, \tilde{\alpha} + \beta_i + \sum_{k=1;k\neq i}^{n} \tilde{\beta}_k \right) \right] = 0 \tag{27}$$

where $\tilde{\beta}_k$ is shareholder i's beliefs about shareholder k's contribution, which is confirmed in equilibrium. Observe that derivative of square bracketed factor with respect to β_i is $u_{cc}^i - u_{cg}^i + u_{gg}^i - u_{cg}^i < 0$, where the inequality follows from a version of Lemma 5 for heterogeneous shareholders (omitted but following the same steps). Thus, if (under equilibrium beliefs) shareholder i's marginal utility of public goods exceeds her marginal utility of consumption, at zero contribution; *i.e.* the square bracket factor of eqn. (27) is positive when $\beta_i = 0$, then by Remark 1 a unique $\beta_i > 0$ solving $u_c^i = u_g^i$ exists. Otherwise, *i.e.* the square bracket factor of eqn. (27) is non-positive when $\beta_i = 0$, $\beta_i = 0$ is the unique non-negative solution. Since, this is true for arbitrary equilibrium beliefs, at least one solution exists.

It remains to rule out multiple valid equilibrium beliefs of other shareholders' contributions. To do so we show that each shareholder's response to an increase in beliefs about another's giving (equivalently the aggregate of the other shareholder's giving) is *partial crowd out*: $\frac{d\beta_i}{d\tilde{\beta}_j} \in (-1, 0)$.

From the IFT, when $\beta_i > 0$

$$\frac{d\beta_i}{d\tilde{\beta}_j} = -\frac{u^i_{gg} - u^i_{cg}}{u^i_{cc} - u^i_{cg} + u^i_{gg} - u^i_{cg}} \in (-1, 0) \tag{28}$$

where the bounds follow because $u^i_{cc} - u^i_{cg} < 0$ and $u^i_{gg} - u^i_{cg} < 0$ and (due to a version of Lemma 5 for heterogeneous shareholders, omitted for brevity). To see that this suffices, define reaction curves $\beta_i(\beta_j)$ and $\beta_j(\beta_i)$ and let them intersect at solution $\left\langle \beta_i(\beta_j) = \beta'_i, \beta_j(\beta_i) = \beta'_j \right\rangle$. Since $\beta'_j(\beta_i) \in (-1, 0)$ we may write its inverse $B_j(\beta) = \beta_j^{-1}(\beta)$, where $B_j(\beta) \in (-\infty, -1)$. Thus, for all $\beta_j < \beta'_j$, $B_j(\beta_j) > \beta_i(\beta_j)$ and for all $\beta_j > \beta'_j$, $B_j(\beta_j) < \beta_i(\beta_j)$, implying single crossing. ■

We now examine the choice of α for a given output, q. When shareholders are identical, they optimally delegate *all* public goods provisioning to the manager. When shareholders differ, delegation is only partial: shareholders who place a relatively high value on public goods may continue to privately contribute. The broader intuition that the firm plays an important delegation role remains intact: The manager optimally increases the overall provisioning of the public good and leaves all shareholders better off than when they contribute only individually. Formally,

Theorem 3 (Delegation (Heterogeneous)) *When shareholders are heterogeneous: (i) At least one shareholder delegates all contributions to the manager. (ii) If all shareholders would (not) contribute positively absent manager contributions, then this delegation strictly (weakly) raises overall public goods provisioning and aggregate shareholder welfare.*

Proof. (i) Suppose not: $\beta_i > 0$ for all $i \in \{1, ..., n\}$. Either the manager is cornered below or he contributes positively ($\alpha \geq 0$)—the derivative of the aggregate shareholder welfare function with respect to α is non-positive:

$$\frac{dW}{d\alpha} = \sum_{i=1}^{n} \left(-\lambda_i u^i_c + u^i_g\right) f_i \leq 0 \tag{29}$$

Since all shareholders contribute, then all of their FOCs hold: $u^i_c = u^i_g$ for all $i \in \{1, ..., n\}$. Further, since $\lambda_i < 1$, the parenthesized term in eqn. (29) is strictly positive and $f_i > 0$ for all i, and hence a contradiction.

(ii) First, we will show that if all shareholders would contribute absent manager contributions, then the manager will contribute positively (completely crowding out private contributions of at least one shareholder as argued in part (i)). Next we will show that, if all shareholders would not contribute absent manager contributions, then the manager will contribute iff doing so increases aggregate shareholder welfare (and any cornered shareholders will remain so). Finally, we show that whenever any shareholders' individual giving is cornered, then an increase in managerial contributions increases the level of the public good.

Case 1 (absent manager contribution all shareholders would contribute): Let $\beta_i' > 0$ for all $i \in \{1, ..., n\}$ be the amount each shareholder would contribute absent managerial contribution. Rank shareholders in ascending order of share weighted contribution: $\beta_1'/\lambda_1 \leq ... \leq \beta_n'/\lambda_n$. Now assume the manager contributed $\alpha'' = \beta_1'/\lambda_1$. Then from Lemma 2, each shareholder will contribute $\beta_i'' = \beta_i' - \lambda_i \alpha''$, such that all shareholders' giving remains interior (i.e. $\beta_i'' > 0$, for all $i \in \{2, ..., n\}$ and $\beta_1'' = 0$, but $u_c^i = u_g^i$ for all $i \in \{1, ..., n\}$). This level of delegation results in no change to the public good. But observe that each parenthesized term in eqn. (29) is strictly positive, implying that the manager would strictly increase the his contributions if allowed, because it strictly increases aggregate shareholder welfare. (Note that this implies that every shareholder is better off when α is increased slightly from α'' but not necessarily that every shareholder is better off when α is optimal from the aggregate shareholder welfare perspective.)

Case 2 (absent manager contribution some shareholder(s) would not contribute): In this case, it could be that when the manager is barred from contribution that $u_c^i > u_g^i$ for some i. In this case, the manager would contribute positively iff $\frac{dW}{d\alpha}\big|_{\alpha=0} > 0$, meaning that aggregate shareholder welfare would be increased (Note that so long as infinitessimal managerial giving does not corner additional shareholders, already cornered shareholders are strictly worse off, but this does not that mean either aggregate welfare is lower or that cornered shareholders are worse off at the optimal α).

Claim 1: For any given (and known) level of managerial contribution α, if one or more shareholders' optimal giving is cornered, then an increase in managerial giving to $\alpha' > \alpha$ strictly increases public goods levels.

Proof of claim: Suppose not: public goods weakly decrease. Let $\beta_i(\alpha)$ be optimal giving under beliefs α. Since all shareholders' individual giving responds only to (beliefs of) aggregate public goods levels, the optimal giving of all interior shareholders moves in the same direction in response to a change in managerial giving. In particular, since some shareholders are cornered, in order for public goods to weakly fall, all interior shareholders must strictly decrease their giving. Thus, the marginal utility of consumption decreases for all interior shareholders (they are richer), while the marginal utility of public goods falls for all shareholders (public goods levels fall). Formally,

$$u_c^i\left(\alpha' + \sum \beta_j(\alpha')\right) < u_c^i\left(\alpha + \sum \beta_j(\alpha)\right) = u_g^i\left(\alpha + \sum \beta_j(\alpha)\right) < u_g^i\left(\alpha' + \sum \beta_j(\alpha')\right)$$

where we have abused notation slightly to omit non-varying parameters. But this is a contradiction, since optimal individual giving requires $u_c^i(\alpha' + \sum \beta_j(\alpha')) = u_g^i(\alpha' + \sum \beta_j(\alpha'))$ for all giving positive amounts. ■

The proof resembles that of the homogeneous shareholder analog: Theorem 1. Limited delegation is a consequence of imperfect alignment between the manager's objectives and those of an *individual* shareholder. While the manager optimizes for some expression of collective preferences, an individual may care sufficiently about the public good that she continues to contribute privately. Likewise, when shareholders are heterogeneous, although the manager contributes if and only if doing so increases *aggregate* shareholder welfare, there is no guarantee that this centralized giving benefits every single shareholder.

We now turn to the production decisions. Since the arguments in Proposition 4 did not rely on shareholders being identical, it follows immediately that,

Remark 2 *With heterogeneous shareholders, the firm produces strictly less than the profit maximizing quantity.*

But how much production does the firm undertake? It is intuitive (and may be readily verified) that the addition of shareholder heterogeneity leaves the socially optimal production level unchanged; thus, Lemma 4 continues to hold. We previously showed that the firm optimally chose the socially optimal level of production provided that the corporate fundability condition held. An analogous condition that accounts for shareholder differences also implies that the firm produces at the socially optimal level. As in the homogenous shareholder setting with production, *corporate fundability* is satisfied if, at production level q^*, the manager's public goods contributions are not cornered at zero.

Condition 4 (Corporate Fundability: Heterogeneous) *Shareholders enjoy public goods enough, in aggregate, at the socially optimal level of production to direct the manager to sacrifice some dividends for them. Formally,*

$$\sum_{i=1}^{n} \lambda_i u_c^i \left(\lambda_i \pi' \left(q^* \right) - \beta_i^0, \sum_{j=1}^{n} \beta_j^0 - \psi' \left(q^* \right) \right) f_i \leq \sum_{i=1}^{n} u_g^i \left(\lambda_i \pi' \left(q^* \right) - \beta_i^0, \sum_{j=1}^{n} \beta_j^0 - \psi' \left(q^* \right) \right) f_i \quad (30)$$

where f_i, and β_i^0 are evaluated at $\langle q, \alpha \rangle = \langle q^*, 0 \rangle$ for all i (i.e. as if the the manager were barred from contributing).

Thus, as when shareholders are identical, this condition implies that the manager will choose to provide public goods both through production abatement and direct contribution. As in the identical shareholder case, he chose abatement right up to the level where it is the most efficient—until production is socially optimal—and then will choose direct contribution. Theorem 4 states this formally.

Theorem 4 (Efficient Abatement (Heterogeneous)) *When shareholders are heterogeneous, the manager chooses the socially optimal quantity q^* iff the public good is* corporately fundable *(in the sense of Condition 4) otherwise the firm overproduces (i.e. $q \in (q^*, \widehat{q})$). Furthermore, the manager provisions strictly positive amounts of the public good iff the inequality in equation (30) is strict.*

Proof. Let q' and α' be the manager's optimal production quantity and public goods contribution decisions respectively, and let β_i' be shareholder i's optimal public goods contribution decision in equilibrium. The (unconstrained) first order condition of the manager's problem (26) for α is

$$\sum_{i=1}^{n} \lambda_i u_c^i f_i = \sum_{i=1}^{n} u_g^i f_i \tag{31}$$

and the (unconstrained) first order condition of the manager's problem for q is

$$\pi'(q') \sum_{i=1}^{n} \lambda_i u_c^i f_i = \psi'(q') \sum_{i=1}^{n} u_g^i f_i \tag{32}$$

When the corporate fundability condition holds, there exists $\alpha' \geq 0$, given that $q = q^*$, such that equation (31) holds. Substituting (31) into (32) implies that q' solving $\pi'(q') = \psi'(q')$, equivalently $q' = q^*$, is optimal.

When corporate fundability does not hold, then at $q = q^*$, LHS of equation (31) is strictly greater that RHS evaluated at $\alpha = 0$. Hence, substituting in equation (32) at $(\alpha = 0,\ q = q^*)$ we have

$$\pi'(q') \sum_{i=1}^{n} \lambda_i u_c^i f_i > \psi'(q') \sum_{i=1}^{n} u_g^i f_i$$

Hence q^* is not optimal. Furthermore, since $\pi'(\cdot)$ is decreasing and $\psi'(\cdot)$ increasing in q, then optimal production is given by $q' > q^*$.

Remark 1 rules out utility functions, which cause α to corner above. ■

We now compare the stringency of the corporate fundability condition compared to the identical case. Suppose that preferences are of the quasi-linear form $U_i = c_i + \theta_i h(g)$, where h is a strictly increasing and strictly concave function, and θ_i is an individual specific parameter capturing differing tastes for the public good. This permits a simple comparison between a group of heterogeneous shareholders with an analogous set of identical shareholders, all of whom have a θ parameter equal to the (generalized) mean of the θ_i parameters under heterogeneity. Recall that the generalized p-mean of an equally weighted list $\{\theta_i\}_{i=1}^n$ is simply

$$\bar{\theta}_p = \left(\sum \frac{1}{n} (\theta_i)^p\right)^{\frac{1}{p}}$$

It is well-known that $\bar{\theta}_p$ strictly increases in p. Setting $p = 1$ produces the usual arithmetic mean while other values of p produce geometric, harmonic and other means.[30]

We can now compare the corporate fundability condition under differing specifications of heterogeneity. When the manager is a utilitarian, the corporate fundability condition for identical shareholders is simply

$$nh'(-\psi(q^*))\,\bar{\theta}_p \geq 1 \tag{33}$$

Introducing a small amount of heterogeneity around $\bar{\theta}_p$, such that no shareholder wishes to contribute privately, the analogous condition is

$$h'(-\psi(q^*)) \sum \theta_i \geq 1 \tag{34}$$

Comparing equations (33) and (34), notice that the stringency of the corporate fundability condition amounts to comparing the arithmetic mean to the p-mean. When $p < 1$, the arithmetic mean is larger and hence a *less* stringent condition. The reverse is true when $p > 1$. The boundary case occurs where heterogeneity is introduced around the arithmetic mean, in which case the two conditions are identical. Thus, we have shown:

Proposition 7 *When shareholder have quasi-linear preferences with generalized mean $\bar{\theta}_p$ then:*

(i) When $p > 1$, corporate fundability is less likely to be satisfied under heterogeneity.

(ii) When $p < 1$, corporate fundability is more likely to be satisfied under heterogeneity.

(iii) When $p = 1$, corporate fundability is satisfied under heterogeneity iff it is satisfied under homogeneity.

There is no particular justification for favoring any particular mean as being the "right" way to introduce heterogeneity. While the arithmetic mean is the most familiar form, it is purely arbitrary; thus, we can conclude that heterogeneity has no systematic effect on corporate fundability. Using numerical methods, the result can be extended for global changes in heterogeneity as well.

[30] The geometric mean is the limit as $p \to 0$.

B.2 Contributions by Non-shareholders

For the sake of parsimoniously showing the key forces at work, our base model ignored any contributions by non-shareholding citizens. Here we show that is without loss of generality. Suppose that the sum of all contributions to the public good by any source other than the firm is given by G^*, where G^* itself is a function of equilibrium beliefs of firm choices. The manager maximizes the welfare of one of his n identical constituent shareholders:

$$\max_{0 \leq q, 0 \leq \alpha \leq \pi(q)} u\left(\frac{\pi(q) - \alpha}{n}, \alpha - \psi(q) + G^*\right)$$

The manager's FOC with respect to α is $u_c = nu_g$, and his FOC with respect to q is $u_c\pi' = nu_g\psi'$, exactly as in the base model. Thus, substituting the FOC with respect to α into the FOC with respect to q we find that $\pi' = \psi'$. Thus, the manager produces q^* if corporate fundability holds. Of course, the core intuition is unchanged by this extension—given that shareholders feel the public good is sufficiently worthwhile, the manager still wishes to provide it as efficiently as possible.

Note that although the analysis does not fundamentally change with the inclusion of active citizens, the corporate fundability condition could fail if non-shareholding citizens cared for the public good so much more than shareholders that their contributions completely crowded out those of the firm. Or perhaps, even more likely the crowding out by government agencies could cause the corporate fundability condition to fail.

B.3 Microfoundations of Public Goods Production

In the main analysis we summarized the level of public goods by the dollar denominated quantity $x - \psi(q)$, where x denotes the net expenditures on the public good. Here we explain why this simple form suffices to capture the broad class of public goods production functions that depend only on the current level of public goods.

Firm output reduces the stock of the public good according to some increasing function $h(q)$, denominated in units of the public good. That is, the more output a firm generates, the more air/water it pollutes. We permit the scale of this impact to be arbitrary: production can cause diminishing, increasing, or constant damage to the public good. If the initial stock of the public good is g_0 and the firm produces output q, then, absent any remediation, the net public goods amount is $g = g_0 - h(q)$.

Apart from indirectly influencing the public good through its production, the firm may also directly convert profits, denominated in units of the consumption good, into public goods production. Shareholders can also directly provision public goods through private contributions. The cost to add a unit of the public good, in terms of foregone consumption/profits depends only on the current level of the public good—the instantaneous cost of cleaning up the air/water can vary with the current air/water quality, but not how the air/water got that dirty or who is doing the cleaning *per se*. Formally, the marginal cost of the public good, when the level of public goods is g, is given by positive $k(g)$, so the cost of increasing public goods from level g_0 to g_1 is $K(g_0, g_1) = \int_{g_0}^{g_1} k(g)\, dg$. As with firm pollution, the scale economies for converting consumption into public goods are also arbitrary, so, over the relevant region, each sacrificed unit of consumption can have diminishing, increasing, or constant marginal effect on the creation of public good. Equivalently, we may write the inverse of expenditure $K(g_0, g_1)$ given an initial level of the public good g_0 as $g_1 = \Gamma(K(g_0, g_1); g_0)$.

Since impacts of various activities depend only the current level of the public good at each point in which they are undertaken, we refer to this setting as the *instantaneous cost model of public goods*.[31] Most standard models of the private provision of public goods fall into this class—all that is required is that public goods production depends only on the sum of individual contributions.

Adding in production, it then follows that the amount of foregone consumption required to produce an ending stock g_1 of the public good is $x = K(g_0 - h(q), g_1)$. Thus, to leave the ending stock of the public good unchanged from its initial level g_0 requires expenditures $\psi(q) =$

[31] One might imagine a model of public goods production where adding a unit depended on, say, how much money had been spent on public goods provision already. Then the cost of public goods would depend not only on their level, but *how* that level was achieved. We specifically rule out such path dependencies.

$K(g_0 - h(q), g_0)$. As g_0 does not vary in our analyses, we suppress this dependence hereafter. The expression $\psi(q)$ may be thought of as the impact of production denominated in foregone consumption, so the final level of the public good depends only on the net expenditures, $x - \psi(q)$. The following lemma formalizes this idea:

Lemma 11 *Under the instantaneous cost model of public goods, the level of public goods is a monotone transformation of net expenditures:*

$$g_1 = \Gamma(x - \psi(q))$$

Proof. When there is no production, it requires consumption expenditures $K(g_0, g_1)$ to attain public goods level g_1. Adding and subtracting ψ to this expression yields

$$\int_{g_0}^{g_1} k(g)\,dg + \int_{g_0 - h(q)}^{g_0} k(g)\,dg - \psi(q)$$

$$= K(g_0 - h(q), g_1) - \psi(q)$$

$$= x - \psi(q)$$

Inverting this yields the form of the lemma. ■

The instantaneous cost model suits situations where the public good represents the stock of an environmental resource. For example, the type of technology employed in, and hence marginal cost of, cleaning up wastewater (or sequestering carbon) depends on the concentration of toxins (CO_2) it contains. This is due, in part, to nature's remarkable ability to clean up limited concentrations of industrial waste without intervention. Thus, it naturally follows that the costs of cleanup should vary with the the current level of the public good.

Much of the economic intuition we develop in the analysis involves the trade-off between consuming private and public goods. In most public goods models, these are the only two goods, and hence the choice to denominate them either in units of the numeraire or the public good is arbitrary. Our model, however, introduces a third good, production output, which influences consumption through profits and public goods through pollution. In this setting the accounting is much simpler when performed in units of the numeraire. Furthermore, since public goods can be expressed as an increasing function of the numeraire, we simply subsume Γ into the utility function for the analysis. This is, of course, precisely analogous to how consumption is typically modelled—since the quality and size of the basket of goods consumed is an increasing function of the amount spent on it, we simply subsume the vector of prices into the utility function and let utility increase in the amount of numeraire dedicated to consumption.

In our three stage game, shareholders first (passively) determine the manager's contract. Second, the manager simultaneously chooses the production quantity q and an amount α to contribute to the public good. The remaining profits are distributed equally among the shareholders. Third, each shareholder simultaneously contributes an amount, β_i, to the public good. So, the ending amount of the public good g_1 solves the following equality expressed in units of the numeraire:

$$\int_{g_0}^{g_1} k(g)\,dg + \int_{g_0-h(q)}^{g_0} k(g)\,dg = \alpha + \sum_{j=1}^{n} \beta_j$$

or equivalently

$$\int_{g_0}^{g_1} k(g)\,dg = \alpha + \sum_{j=1}^{n} \beta_j - \psi(q)$$

Thus,

$$g_1 = \Gamma\left(\alpha + \sum \beta_j - \psi(q)\right) \tag{35}$$

After contributing to the public goods, all remaining cash is consumed by the shareholder. Thus, we can write each shareholder's utility as function of q, α, and β_i

$$u\left(\frac{\pi(q)-\alpha}{n} - \beta_i, \alpha + \sum_{j=1}^{n} \beta_j - \psi(q)\right)$$

where the function $\Gamma(\cdot)$ given in expression (35) is subsumed in the utility function. As described in the main text, the manager selects q and α to maximize the utility of a representative shareholder. Thus, the manager's objective function is

$$\max_{0\le q, 0\le\alpha\le\pi(q)} u\left(\frac{\pi(q)-\alpha}{n} - \beta_i^*(q,\alpha), \alpha + \sum_{j=1}^{n} \beta_j^*(q,\alpha) - \psi(q)\right)$$

where $\beta_i^* (q, \alpha)$ denotes equilibrium contribution of shareholders to the public good, under output q and manager contribution α.

That the manager's solution to his FOC is unique and satisfies the SOC depends on the relative curvatures of the profits function π and the cost of remediation function ψ. One measure of the curvature (concavity) of a function is the Arrow-Pratt measure of risk aversion, $\rho = -\frac{u''}{u'}$, which represents the elasticity of the slope of a function. More generally, consider some function $f(x)$ and define the curvature measure $\rho_f(x) = -f''(x)/f'(x)$.

Definition 1 *Consider two functions f and g. We say that f is more concave than g on domain X iff for all $x \in X$, $\rho_f(x) > \rho_g(x)$.*

Of course, this measure of curvature inherits all of the properties of the Arrow-Pratt measure. For instance, if g is a concavification of f, then it is more concave than f according to our measure, and so on. The following condition suffices to guarantee that the manager's problem is well-behaved:

Lemma 12 *If π is more concave than ψ according to ρ (*i.e.* $\rho_\pi > \rho_\psi$), then there is a unique solution to the manager's problem.*

Proof. The manager's problem is

$$\max_{0 \leq q, 0 \leq \alpha \leq \pi(q)} u\left(\frac{\pi(q) - \alpha}{n}, -\psi(q) + \alpha\right)$$

Let q' and α' be the manager's optimal production quantity and public goods contribution decisions respectively. The (unconstrained) FOC of the manager's problem (2) for α is

$$u_c = n u_g$$

and the (unconstrained) FOC of the manager's problem for q is

$$\pi'(q') u_c = \psi'(q') n u_g$$

After some rearrangement, the SOC, evaluated at an output satisfying the FOC requires

$$(\pi''(q') - \psi''(q')) u_c + (\pi'(q'))^2 \left(\frac{1}{n} u_{cc} - u_{cg} + n u_{gg} - u_{cg}\right) < 0 \tag{36}$$

From the claim of Lemma 4 and the fact that $\pi'(q) > 0$ over the relevant domain implies that the second term of the above expression is negative. Thus, it suffices to show that the first term is also negative to ensure a unique solution to the manager's problem. We may rewrite the left-hand side of equation (36) as

$$\left(\frac{\pi''(q')}{\pi'(q')} - \frac{\psi''(q')}{\psi'(q')}\right) u_c + \left(\pi'(q')\right)\left(\frac{1}{n} u_{cc} - u_{cg} + n u_{gg} - u_{cg}\right)$$

where we have used the fact that $\pi'(q') = \psi'(q')$ at any point satisfying the first-order condition.

Notice that the first term in this expression is simply $\rho_\psi(q') - \rho_\pi(q')$, our measure of curvature. Therefore, it immediately follows that if π is more concave than ψ, the SOC holds at any critical point. Therefore, the critical point is unique (and optimal). ■

Recall that the cost of remediation ψ itself depends on the returns to scale properties of public goods production k relative to the scale properties of public goods depletion h: decreasing $k(g)$ implies public goods production exhibits increasing returns to scale, whereas increasing $k(g)$ implies decreasing returns to scale. In terms of these more primitive functions, the predicate of Lemma 12 is equivalent to

$$\rho_\pi > \rho_\psi \iff -\frac{\pi''(q)}{\pi'(q)} > -\frac{\psi''(q)}{\psi'(q)} = -\left(\frac{h''(q)}{h'(q)} + \frac{K_{11}(g_0 - h(q), g_0)}{K_1(g_0 - h(q), g_0)} h'(q)\right)$$

Obviously, convex ψ suffices to satisfy the predicate of Lemma 12. This automatically holds when output damages the public good at an increasing rate (h is convex) and when the clean up technology exhibits decreasing returns to scale (K is convex). However, even in circumstances where public goods production exhibits increasing returns to scale (K is concave), the manager's problem may still be well-behaved if either profits are sufficiently concave (according to the ρ measure) or output damages the public good at a sufficiently increasing rate, *i.e.* h is sufficiently convex.

Example 1 *If demand is given by linear $p(q) = a - bq$ and production costs by quadratic $C(q) = f + \frac{v}{2}q^2$, then profits are given by*

$$\pi(q) = (a - bq)q - \left(f + \frac{v}{2}q^2\right)$$

with curvature

$$\rho_\pi = -\frac{\pi''(q)}{\pi'(q)} = \frac{2b+v}{a-(2b+v)q}$$

Let production damage public goods according to quadratic $h(q) = \frac{h}{2}q^2$ *and public goods be provisioned at flexible cost* $K(g) = \frac{k}{x}g^x$. *Here* x *measures the returns to scale in provisioning public goods:* $x > 1$ *implies decreasing returns to scale, while* $x < 1$ *implies increasing returns to scale. Then the costs of remediating damage from* q *units of production will have curvature*

$$\rho_\psi = -\frac{\psi''(q)}{\psi'(q)} = -\left(\frac{h}{hq} + \frac{(x-1)k\left(\frac{h}{2}q^2\right)^{x-2}}{k\left(\frac{h}{2}q^2\right)^{x-1}}hq\right) = -\frac{2x-1}{q}$$

Thus, after simplifying, $\rho_\pi > \rho_\psi$ *iff*

$$1 - \frac{a}{2\pi'(q)} < x \tag{37}$$

Clearly, were the firm a strict profit maximizer, this condition would be satisfied by arbitrary returns to scale, since $\pi'(\widehat{q}) = 0$. *Although we show (in Proposition 4) that when a manager maximizes shareholder welfare solutions to the FOCs will have the property that* $\pi'(q) > 0$, *condition (37) illustrates that public goods provision can still exhibit increasing returns to scale but leave the manager's problem well behaved.*

Assuming $\rho_\pi > \rho_\psi$ is analogous to assuming that the revenue function is more concave than the production cost function in the familiar profit maximization setting, in order to guarantee that the first order approach is valid.

B.4 Production Choice under No Delegation and Sequential Timing

Proposition 5 showed that the alignment of private and social preferences regarding output depends crucially on the tools available to the manager. Our next result, fixes the set of tools but varies the timing of decision. Consider a timing change in the above game where direct manager contributions are prohibited—instead of the manager and shareholders acting simultaneously, let shareholders observe the manager's chosen production level and then *subsequently* choose contributions. So long as individual fundability holds (at q^*), this linkage again leads the manager to choose the socially optimal level of production. Formally,

Proposition 8 *Let timing be sequential—the manager chooses production, and afterwards shareholders contribute. When the manager cannot divert profits, then*

(i) If individual fundability holds (at the socially optimal level of production), the manager will produce at socially optimal level.[32]

(ii) If (strict) corporate fundability holds, but individual fundability (at the socially optimal level of production) does not, the manager will produce (strictly) less than the socially optimal level.

(iii) If corporate fundability does not hold at the socially optimal level of production, the manager will produce strictly more than the socially optimal level.

Proof. The manager's problem is

$$\max_q W = u\left(\frac{\pi(q)}{n} - \beta, n\beta - \psi(q)\right)$$

and each shareholder's problem is

$$\max_\beta U = u\left(\frac{\pi(q)}{n} - \beta, (n-1)\bar{\beta} + \beta - \psi(q)\right)$$

Differentiating repulsively by q and β

$$\frac{\partial W}{\partial q} = \frac{\pi'(q)}{n}u_c - \psi'(q)\,u_g$$
$$\frac{\partial U}{\partial \beta} = -u_c + u_g$$

Differentiating with respect to q

$$\frac{dW}{dq} = \left(\frac{\pi'(q)}{n} - \frac{d\beta}{dq}\right)u_c + \left(-\psi'(q) + n\frac{d\beta}{dq}\right)u_g$$

[32] Again, individual fundability at any positve level of production, including the socially optimal one, is a weaker condition than Condition 1, because of the diminishing marginal utilities of consumption and public goods.

where $\frac{d\beta}{dq}$ denotes the change in the equilibrium contributions of shareholders to the manager's choice of q. Evaluated at the socially optimal level

$$\left.\frac{dW}{dq}\right|_{q=q^*} = \left(\frac{\pi'(q^*)}{n} - \beta'(q^*)\right)(u_c - nu_g)$$

(i). If individual fundability holds at q^*, then $u_c = u_g$, and from Lemma 9, $\beta'(q^*) = \frac{\pi'(q^*)}{n}$, implying that $\left.\frac{dW}{dq}\right|_{q=q^*} = 0$, and q^* is optimal.

(ii) If individual fundability does not hold at q^*, then $\beta(q^*) = \beta'(q^*) = 0$. If strict corporate fundability holds, then $u_c < nu_g$. Thus, $\left.\frac{dW}{dq}\right|_{q=q^*} < 0$, and some $q < q^*$ is optimal. When weak corporate fundability holds, $u_c = nu_g$, $\left.\frac{dW}{dq}\right|_{q=q^*} = 0$, and q^* is optimal.

(iii) If corporate fundability fails, so does individual fundability at q^*, implying respectively that $u_c > nu_g$ and $\beta'(q^*) = 0$. Thus, $\left.\frac{dW}{dq}\right|_{q=q^*} > 0$, and some $q > q^*$ is optimal. ■

Intuitively, although the manager here cannot contribute directly, he can rely on shareholders to provide at least some portion of the public good through decentralized giving when individual fundability holds. When increasing production from any point below the profit maximizing quantity the manager increases dividends and destroys the public good. Shareholders respond by diverting some of their increased dividends to the public good: $\frac{d\beta}{dq} > 0$. When the increase in production occurs from a point below the socially optimal quantity, total profits increase faster than destruction of the public goods. Since public goods are normal, shareholders exploit this boon and increase their giving such that both consumption and public goods increase: $\psi'(q) < n\frac{d\beta}{dq} < \pi'(q)$. For production increases above the socially optimal level (but below $\hat{q}$), though, shareholders still increase giving, but in such a way that both consumption and public goods fall: $\pi'(q) < n\frac{d\beta}{dq} < \psi'(q)$. Although the manager would prefer a higher level of public goods, abating production from q^* would induce a reduction in shareholder giving that reduces both their consumption and total public goods, something that the manager strictly dislikes. So, unlike in the simultaneous game, the manager does not attempt to inefficiently use the production lever as a way to increase public goods—it will not work.

When individual fundability fails but corporate fundability holds, the manager desires to divert profits when producing at q^*, but derives no help from individual shareholders in providing public goods. Knowing that the ideal public goods level, from the collective shareholder perspective, is higher than the provision under q^*, the manager relies on the only lever available, production, to increase the provision of public goods. This case is similar to the simultaneous timing case, in that cornered shareholders cannot counter abatement by the manager. There is no crowd-out, so an increase in abatement increases public goods at the cost of consumption, a worthwhile trade-off, given the lack of alternatives.

References

[1] Wang, Y. H. (1993). "On the number of successes in independent trials." *Statistica Sinca,* 3, 295-312.