# Appendix

## A Asymmetric Manufacturing

The complete list of all scenarios of recycling structures are shown as follows.

| products 1, 2, and 3 | product 1 | product 2 | product 3 | Note |
|---|---|---|---|---|
| recycler $z$ | $\lambda c_1$ | $\lambda c_2$ | $\lambda c_3$ | all-inclusive |

| products 1 and 2 | product 3 | product 1 | product 2 | product 3 | Note |
|---|---|---|---|---|---|
| recycler $x$ | recycler $y$ | $c_1$ | $c_2$ | $c_3$ | market-based |
| recycler $z$ | recycler $y$ | $\lambda c_1$ | $\lambda c_2$ | $c_3$ | inferior to market-based |
| recycler $x$ | recycler $z$ | $c_1$ | $c_2$ | $\lambda c_3$ | inferior to market-based |
| recycler $z1$ | recycler $z2$ | $\lambda c_1$ | $\lambda c_2$ | $\lambda c_3$ | inferior to market-based |

| product 1 | products 2 and 3 | product 1 | product 2 | product 3 | Note |
|---|---|---|---|---|---|
| recycler $x$ | recycler $z$ | $c_1$ | $\lambda c_2$ | $\lambda c_3$ | firm-based |
| recycler $z1$ | recycler $z2$ | $\lambda c_1$ | $\lambda c_2$ | $\lambda c_3$ | inferior to firm-based |

| products 1 and 3 | product 2 | product 1 | product 2 | product 3 | Note |
|---|---|---|---|---|---|
| recycler $z$ | recycler $x$ | $\lambda c_1$ | $c_2$ | $\lambda c_3$ | cross-market/firm |
| recycler $z1$ | recycler $z2$ | $\lambda c_1$ | $\lambda c_2$ | $\lambda c_3$ | inferior to cross-market/firm |

| product 1 | product 2 | product 3 | product 1 | product 2 | product 3 | Note |
|---|---|---|---|---|---|---|
| recycler $x1$ | recycler $x2$ | recycler $y$ | $c_1$ | $c_2$ | $c_3$ | product-based |
| recycler $z$ | recycler $x$ | recycler $y$ | $\lambda c_1$ | $c_2$ | $c_3$ | inferior to product-based |
| recycler $x$ | recycler $z$ | recycler $y$ | $c_1$ | $\lambda c_2$ | $c_3$ | inferior to product-based |
| recycler $x1$ | recycler $x2$ | recycler $z$ | $c_1$ | $c_2$ | $\lambda c_3$ | inferior to product-based |
| recycler $z1$ | recycler $z2$ | recycler $y$ | $\lambda c_1$ | $\lambda c_2$ | $c_3$ | inferior to product-based |
| recycler $z1$ | recycler $x$ | recycler $z2$ | $\lambda c_1$ | $c_2$ | $\lambda c_3$ | inferior to product-based |
| recycler $x$ | recycler $z1$ | recycler $z2$ | $c_1$ | $\lambda c_2$ | $\lambda c_3$ | inferior to product-based |
| recycler $z1$ | recycler $z2$ | recycler $z3$ | $\lambda c_1$ | $\lambda c_2$ | $\lambda c_3$ | inferior to product-based |

In the asymmetric model, let $\alpha_1 - c_1 = M_1$, $\alpha_2 - c_2 = M_2$, $\alpha_3 - c_3 = M_3$, $\alpha_1 - \lambda c_1 = P_1$, $\alpha_2 - \lambda c_2 = P_2$, $\alpha_3 - \lambda c_3 = P_3$, $1 - \kappa = X$, and $\kappa - \gamma = Y$. From the first order conditions, the equilibrium quantities are as follows:

- $q_1^{\{123\}} = [2(1-2\kappa)P_1 + (\kappa - \gamma + \kappa\gamma)P_2 + \kappa(1-\gamma)P_3]/[4 - 12\kappa + 6\kappa^2 + 2\kappa\gamma - (1-\kappa)\gamma^2]$, $q_2^{\{123\}} = [2(\kappa - \gamma + \kappa\gamma)P_1 + (4 - 8\kappa + 3\kappa^2)P_2 + \kappa(4 - 3\kappa - \gamma)P_3]/\{2[4 - 12\kappa + 6\kappa^2 + 2\kappa\gamma - (1-\kappa)\gamma^2]\}$, and $q_3^{\{123\}} = \{2\kappa(1-\gamma)P_1 + \kappa(4 - 3\kappa - \gamma)P_2 + [4(1-\kappa)^2 - (\kappa-\gamma)^2]P_3\}/\{2\left[4 - 12\kappa + 6\kappa^2 + 2\kappa\gamma - (1-\kappa)\gamma^2\right]\}$;
- $q_1^{\{12\}\{3\}} = [2(1-\kappa)M_1 + (\kappa-\gamma)M_2]/[4(1-\kappa)^2 - (\kappa-\gamma)^2]$, $q_2^{\{12\}\{3\}} = [(\kappa-\gamma)M_1 + 2(1-\kappa)M_2]/[4(1-\kappa)^2 - (\kappa-\gamma)^2]$, and $q_3^{\{12\}\{3\}} = M_3/[2(1-\kappa)]$;

- $q_1^{\{1\}\{23\}} = [2(1-2\kappa)M_1 - (1-\kappa)\gamma P_2 - \kappa\gamma P_3]/[(1-\kappa)(4-8\kappa-\gamma^2)]$, $q_2^{\{1\}\{23\}} = [-\gamma M_1 + 2(1-\kappa)P_2 + 2\kappa P_3]/[4-8\kappa-\gamma^2]$, and $q_3^{\{1\}\{23\}} = \{-2\kappa\gamma M_1 + 4\kappa(1-\kappa)P_2 + [4(1-\kappa)^2-\gamma^2]P_3\}/[2(1-\kappa)(4-8\kappa-\gamma^2)]$;
- $q_1^{\{13\}\{2\}} = [2(1-\kappa)P_1 - \gamma M_2 + \kappa P_3]/[4-8\kappa+3\kappa^2-\gamma^2]$, $q_2^{\{13\}\{2\}} = [-2(1-\kappa)\gamma P_1 + (4-8\kappa+3\kappa^2)M_2 - \kappa\gamma P_3]/[2(1-\kappa)(4-8\kappa+3\kappa^2-\gamma^2)]$, and $q_3^{\{13\}\{2\}} = \{2\kappa(1-\kappa)P_1 - \kappa\gamma M_2 + [4(1-\kappa)^2-\gamma^2]P_3\}/[2(1-\kappa)(4-8\kappa+3\kappa^2-\gamma^2)]$;
- $q_1^{\{1\}\{2\}\{3\}} = [2(1-\kappa)M_1 - \gamma M_2]/[4(1-\kappa)^2-\gamma^2]$, $q_2^{\{1\}\{2\}\{3\}} = [-\gamma M_1 + 2(1-\kappa)M_2]/[4(1-\kappa)^2-\gamma^2]$, and $q_3^{\{1\}\{2\}\{3\}} = M_3/[2(1-\kappa)]$.

The equilibrium profits are:

- $\Pi_A^{\{123\}} = (1-\kappa)[2(1-2\kappa)P_1 + (\kappa-\gamma+\kappa\gamma)P_2 + \kappa(1-\gamma)P_3]^2/[4-12\kappa+6\kappa^2+2\kappa\gamma-(1-\kappa)\gamma^2]^2$ and $\Pi_B^{\{123\}} = \{(1-\kappa)[2(\kappa-\gamma+\kappa\gamma)P_1 + (4-8\kappa+3\kappa^2)P_2 + \kappa(4-3\kappa-\gamma)P_3]^2 + (1-\kappa)\{2\kappa(1-\gamma)P_1 + \kappa(4-3\kappa-\gamma)P_2 + [4(1-\kappa)^2-(\kappa-\gamma)^2]P_3\}^2 - 2\kappa[2(\kappa-\gamma+\kappa\gamma)P_1 + (4-8\kappa+3\kappa^2)P_2 + \kappa(4-3\kappa-\gamma)P_3]\{2\kappa(1-\gamma)P_1 + \kappa(4-3\kappa-\gamma)P_2 + [4(1-\kappa)^2-(\kappa-\gamma)^2]P_3\}\}/4/[4-12\kappa+6\kappa^2+2\kappa\gamma-(1-\kappa)\gamma^2]^2$;
- $\Pi_A^{\{12\}\{3\}} = (1-\kappa)[2(1-\kappa)M_1 + (\kappa-\gamma)M_2]^2/[4(1-\kappa)^2-(\kappa-\gamma)^2]^2$ and $\Pi_B^{\{12\}\{3\}} = (1-\kappa)[(\kappa-\gamma)M_1 + 2(1-\kappa)M_2]^2/[4(1-\kappa)^2-(\kappa-\gamma)^2]^2 + M_3^2/4/(1-\kappa)$;
- $\Pi_A^{\{1\}\{23\}} = [2(1-2\kappa)M_1 - (1-\kappa)\gamma P_2 - \kappa\gamma P_3]^2/(1-\kappa)/(4-8\kappa-\gamma^2)^2$ and $\Pi_B^{\{1\}\{23\}} = \{4(1-\kappa)^2[-\gamma M_1 + 2(1-\kappa)P_2 + 2\kappa P_3]^2 + \{-2\kappa\gamma M_1 + 4\kappa(1-\kappa)P_2 + [4(1-\kappa)^2-\gamma^2]P_3\}^2 - 4\kappa[-\gamma M_1 + 2(1-\kappa)P_2 + 2\kappa P_3]\{-2\kappa\gamma M_1 + 4\kappa(1-\kappa)P_2 + [4(1-\kappa)^2-\gamma^2]P_3\}\}/4/(1-\kappa)/(4-8\kappa-\gamma^2)^2$;
- $\Pi_A^{\{13\}\{2\}} = (1-\kappa)[2(1-\kappa)P_1 - \gamma M_2 + \kappa P_3]^2/(4-8\kappa+3\kappa^2-\gamma^2)^2$ and $\Pi_B^{\{13\}\{2\}} = \{[-2(1-\kappa)\gamma P_1 + (4-8\kappa+3\kappa^2)M_2 - \kappa\gamma P_3]^2 + \{2\kappa(1-\kappa)P_1 - \kappa\gamma M_2 + [4(1-\kappa)^2-\gamma^2]P_3\}^2\}/4/(1-\kappa)/(4-8\kappa+3\kappa^2-\gamma^2)^2$;
- $\Pi_A^{\{1\}\{2\}\{3\}} = (1-\kappa)[2(1-\kappa)M_1 - \gamma M_2]^2/[4(1-\kappa)^2-\gamma^2]^2$ and $\Pi_B^{\{1\}\{2\}\{3\}} = (1-\kappa)[-\gamma M_1 + 2(1-\kappa)M_2]^2/[4(1-\kappa)^2-\gamma^2]^2 + M_3^2/4/(1-\kappa)$.

**Proof of Proposition 1:** With equilibrium quantities $q_i^X$, $i = 1, 2, 3$, shown above, we calculate and compare $W^X(q_1^X, q_2^X, q_3^X)$ (given in equation 3) for all $X \in \mathbf{X}$, and then obtain the optimal recycling structure with the highest $W^X(q_1^X, q_2^X, q_3^X)$. ■

**Proof of Proposition 2:** We first find expressions for payoffs when $\kappa = 0$:

$$\Pi_A^{\{1\}\{2\}\{3\}} = \left(\frac{2M_1 - \gamma M_2}{4-\gamma^2}\right)^2, \qquad \Pi_B^{\{1\}\{2\}\{3\}} = \left(\frac{-\gamma M_1 + 2M_2}{4-\gamma^2}\right)^2 + \left(\frac{M_3}{2}\right)^2;$$
$$\Pi_A^{\{13\}\{2\}} = \left(\frac{2P_1 - \gamma M_2}{4-\gamma^2}\right)^2, \qquad \Pi_B^{\{13\}\{2\}} = \left(\frac{-\gamma P_1 + 2M_2}{4-\gamma^2}\right)^2 + \left(\frac{P_3}{2}\right)^2;$$
$$\Pi_A^{\{1\}\{23\}} = \left(\frac{2M_1 - \gamma P_2}{4-\gamma^2}\right)^2, \qquad \Pi_B^{\{1\}\{23\}} = \left(\frac{-\gamma M_1 + 2P_2}{4-\gamma^2}\right)^2 + \left(\frac{P_3}{2}\right)^2;$$

$$\Pi_A^{\{12\}\{3\}} = \left(\frac{2M_1 - \gamma M_2}{4-\gamma^2}\right)^2, \qquad \Pi_B^{\{12\}\{3\}} = \left(\frac{-\gamma M_1 + 2M_2}{4-\gamma^2}\right)^2 + \left(\frac{M_3}{2}\right)^2;$$
$$\Pi_A^{\{123\}} = \left(\frac{2P_1 - \gamma P_2}{4-\gamma^2}\right)^2, \qquad \Pi_B^{\{123\}} = \left(\frac{-\gamma P_1 + 2P_2}{4-\gamma^2}\right)^2 + \left(\frac{P_3}{2}\right)^2.$$

Under our assumption that $c_1 = c_2$ and $\gamma < 1$, it is easy to verify that $\Pi_A^{\{13\}\{2\}} < \Pi_A^{\{123\}} < \Pi_A^{\{1\}\{2\}\{3\}} = \Pi_A^{\{12\}\{3\}} < \Pi_A^{\{1\}\{23\}}$, and that $\Pi_B^{\{1\}\{23\}} < \Pi_B^{\{123\}} < \Pi_B^{\{2\}\{13\}} < \Pi_B^{\{1\}\{2\}\{3\}} = \Pi_B^{\{12\}\{3\}}$. As mentioned in the body of the document, whenever product-based structure generates highest profit for $B$, it is uniquely stable. ■

**Proof of Proposition 3:** As mentioned above, when $\{1\}\{2\}\{3\}$ is strictly preferred by firm $B$ to all other structures, it is uniquely stable. This is the most common outcome under different scenarios. When this does not hold, we can have examples like the ones below:

- $\Pi_A^{\{1\}\{23\}} > \Pi_A^{\{1\}\{2\}\{3\}} > \ldots$ and $\Pi_B^{\{2\}\{13\}} > \Pi_B^{\{1\}\{2\}\{3\}} > \ldots$ Under this scenario, firm $A$ can defect from $\{2\}\{13\}$, $\{2\}\{13\} \rightharpoonup_A \{1\}\{2\}\{3\}$, to product-based structure, which is its second-most-preferred structure. As neither $A$ nor $B$ can unilaterally defect from this outcome to their most preferred outcome, product-based structure is stable. This can happen when market size of product 2 dominates that of product 1 and products are either moderately substitutable, or have low substitutability with high diseconomies of scale. In addition, this can occur when products are highly substitutable with similar market sizes, high diseconomies of scale and low $\lambda$.

- $\Pi_A^{\{1\}\{23\}} > \Pi_A^{\{1\}\{2\}\{3\}} > \ldots$ and $\Pi_B^{\{2\}\{13\}} > \Pi_B^{\{3\}\{12\}} > \Pi_B^{\{1\}\{2\}\{3\}} > \ldots$. Under this scenario, firm $A$ can defect from $\{2\}\{13\}$, $\{2\}\{13\} \rightharpoonup_A \{1\}\{2\}\{3\}$, to product-based structure, which is its second-most-preferred structure. As neither $A$ nor $B$ can unilaterally defect from this outcome to the outcomes they prefer more, product-based structure is stable. This happens when market size of product 2 dominates that of product 1 and products are highly substitutable with high diseconomies of scale.

- When market size of product 1 dominates that of product 2 and products are highly substitutable with high diseconomies of scale, both firms prefer firm-based recycling to all other outcomes and it is uniquely stable.

When the market size of product 3 is low compared with the other products, we can have cases in which $\Pi_A^{\{1\}\{23\}} > \Pi_A^{\{1\}\{2\}\{3\}} > \ldots$ and $\Pi_B^{\{2\}\{13\}} > \Pi_B^{\{1\}\{23\}} > \Pi_B^{\{1\}\{2\}\{3\}} > \ldots$ or $\Pi_B^{\{2\}\{13\}} > \Pi_B^{\{123\}} > \Pi_B^{\{1\}\{23\}} > \Pi_B^{\{1\}\{2\}\{3\}} > \ldots$. The first case occurs, say, when $\alpha_1 = \alpha_2 = 300, \alpha_3 = 100, c_1 = c_2 = 2, c_2 = 5, \gamma = 0.5, \lambda = 1.5, \kappa = -0.1$, while the second case occurs, say, when $\alpha_1 = 150, \alpha_2 = 300, \alpha_3 = 50, c_1 = c_2 = 2, c_2 = 5, \gamma = 0.7, \lambda = 1.5, \kappa = -0.05$. Firm $B$ would prefer cross market/firm recycling, which is preferred by $A$ less than either product-based or firm-based recycling. Thus, $A$ would defect from $\{2\}\{13\}$ to $\{1\}\{2\}\{3\}$, and the only joint defection that $A$ would join is to $\{1\}\{23\}$. Thus, it is easy to see that firm-based recycling is the only stable outcome. ■

**Proof of Proposition 4:**

1. When both firms prefer all-inclusive recycling to all other outcomes, it is trivially the only stable outcome. This happens when product substitutability is low, or when product substitutability is moderate and market sizes of product 2 dominates market size of product 1.

   When products 1 and 2 have similar market sizes and product substitutability is moderate or high, we can observe $\Pi_A^{\{13\}\{2\}} > \Pi_A^{\{123\}} > \ldots$ and $\Pi_B^{\{123\}} > \Pi_A^{\{23\}\{2\}} > \ldots$. Then, $\{13\}\{2\}$ cannot be stable as $B$ can always defect to $\{23\}\{1\}$, which it prefers more, and $A$ cannot unilaterally change that outcome. Thus, all-inclusive recycling is uniquely stable again. The same is true when market size of product 1 dominates market size of product 2, product substitutability is moderate, and economies of scale are moderate to high.

   Under most scenarios with low economies of scale and moderate to high substitution level, the preferred outcome for firm $B$ is market-based recycling, followed by the product-based recycling, while firm $A$ favors all-inclusive recycling, but prefers market-based to product-based recycling. Under such scenario, the only possible defection by firm $A$ from market based recycling leads to product-based recycling, which it prefers even less, and market-based recycling is stable.

2. When market size of product 2 dominates that of product 1, and market size of product 3 dominates that of product 2, economies of scale are low, cost increase is low, and products are highly substitutable, firm $B$ prefers firm-based recycling to all other outcomes. As firm $A$ cannot unilaterally change this outcome, firm-based recycling is uniquely stable.

3. When market size of product 1 dominates that of product 2, products are highly substitutable, and economies of scale are moderate to high, both firms prefer cross-market recycling to all other outcomes, and it is the unique stable outcome.

4. When market size of one product (1 or 2) dominates that of the other product, products are highly substitutable, and economies of scale are low, product-based recycling is either the most-preferred outcome of firm $B$, or it is its second favorite, after market-based recycling. At the same time, firm $A$ prefers product-based recycling to both all-inclusive and market-based. As a result, neither firm can unilaterally move from product-based recycling to the outcome it prefers to it, and product-based recycling is uniquely stable. ■

**Proof of Proposition 5:**

1. Consider the case $\alpha_1 = \alpha_2 = 300, \alpha_3 = 100, c_1 = c_2 = 2, c_3 = 5, \gamma = 0.9, \lambda = 2, \kappa = 0.1$. We then have $\Pi_A^{\{12\}\{3\}} > \Pi_A^{\{123\}} > \ldots$ and $\Pi_B^{\{123\}} > \Pi_B^{\{12\}\{3\}} > \ldots$ It is then easy to evaluate that both all-inclusive and market-based recycling are stable. Similar preference ordering holds, for instance, when $\alpha_1 = \alpha_2 = 300, \alpha_3 = 100, c_1 = c_2 = 2, c_3 = 5, \gamma = 0.1, \lambda = 1.2, \kappa = 0.02$.

2. When market size of product 1 dominates that of product 2 and they both dominate market size of product 3, products are moderately substitutable, and economies of scale are moderate, one can evaluate that $\Pi_A^{\{13\}\{2\}} > \Pi_A^{\{123\}} > \Pi_A^{\{1\}\{2\}\{3\}} > \ldots$ and $\Pi_B^{\{123\}} > \Pi_B^{\{12\}\{3\}} > \Pi_B^{\{13\}\{2\}} > \ldots$ We can see that both all-inclusive and cross-market recycling are stable by considering following sequences: $\{13\}\{2\} \rightharpoonup_B \{23\}\{1\} \rightharpoonup_{A,B} \{13\}\{2\}$, $\{13\}\{2\} \rightharpoonup_B \{1\}\{2\}\{3\} \rightharpoonup_{A,B} \{13\}\{2\}$, $\{123\} \rightharpoonup_A \{23\}\{1\} \rightharpoonup_{A,B} \{123\}$. ■

# B Symmetric Manufacturing

In the symmetric model, let $\alpha_1 - c_1 = M_1$, $\alpha_2 - c_2 = M_2$, $\alpha_3 - c_3 = M_3$, $\alpha_3 - c_3 = M_4$, $\alpha_1 - \lambda c_1 = P_1$, $\alpha_2 - \lambda c_2 = P_2$, $\alpha_3 - \lambda c_3 = P_3$, $\alpha_4 - \lambda c_4 = P_4$, $1 - \kappa = X$, and $\kappa - \gamma = Y$. From the first order conditions, equilibrium quantities and payoffs under different recycling structures are:

- $q_1^{\{1234\}} = \{8P_1 - 4P_4\kappa\gamma + 4P_3\kappa - 24P_1\kappa + 8P_4\kappa + 4\kappa P_2 - 4\gamma P_2 - 8\gamma P_3\kappa + 8\gamma\kappa P_2 + 6\kappa^2\gamma P_3 + 4\kappa\gamma P_1 - 6\kappa^2\gamma P_2 - 3\kappa\gamma^2 P_2 + 2\kappa\gamma^2 P_1 + \gamma^2 P_3\kappa + 2P_4\kappa\gamma^2 - 2P_1\gamma^2 + \gamma^3 P_2 - 12P_4\kappa^2 + 12P_1\kappa^2\}/\{-8\gamma^2 + 16\kappa\gamma + 48\kappa^2 - 64\kappa + 16\kappa\gamma^2 + 16 + \gamma^4 - 12\kappa^2\gamma^2 - 4\kappa\gamma^3\}$, $q_2^{\{1234\}} = \{\gamma^3 P_1 - 2\gamma^2 P_2 + 2\kappa\gamma^2 P_2 - 3\kappa\gamma^2 P_1 + 2\gamma^2 P_3\kappa + P_4\kappa\gamma^2 + 6\gamma P_4\kappa^2 + 8\kappa\gamma P_1 - 4\gamma P_3\kappa - 6\gamma P_1\kappa^2 - 8P_4\kappa\gamma - 4P_1\gamma + 4\gamma\kappa P_2 + 8P_3\kappa + 4P_1\kappa + 4P_4\kappa - 12\kappa^2 P_3 - 24\kappa P_2 + 8P_2 + 12\kappa^2 P_2\}/\{-8\gamma^2 + 16\kappa\gamma + 48\kappa^2 - 64\kappa + 16\kappa\gamma^2 + 16 + \gamma^4 - 12\kappa^2\gamma^2 - 4\kappa\gamma^3\}$, $q_3^{\{1234\}} = \{-4P_4\gamma + 8P_3 + 4\gamma P_3\kappa - 4\gamma\kappa P_2 - 2\gamma^2 P_3 + 4P_4\kappa + 4P_1\kappa - 24P_3\kappa + 8\kappa P_2 + 12\kappa^2 P_3 - 12\kappa^2 P_2 + 2\gamma^2 P_3\kappa + 2\kappa\gamma^2 P_2 + P_4\gamma^3 + 8P_4\kappa\gamma - 8\kappa\gamma P_1 - 3P_4\kappa\gamma^2 + \kappa\gamma^2 P_1 - 6\gamma P_4\kappa^2 + 6\gamma P_1\kappa^2\}/\{-8\gamma^2 + 16\kappa\gamma + 48\kappa^2 - 64\kappa + 16\kappa\gamma^2 + 16 + \gamma^4 - 12\kappa^2\gamma^2 - 4\kappa\gamma^3\}$, and $q_4^{\{1234\}} = \{6\kappa^2\gamma P_2 + 8P_4 - 6\kappa^2\gamma P_3 - 4\gamma P_3 + 8\gamma P_3\kappa - 8\gamma\kappa P_2 - 2P_4\gamma^2 - 24P_4\kappa + 8P_1\kappa + 4P_3\kappa + 4\kappa P_2 + 12P_4\kappa^2 - 12P_1\kappa^2 - 3\gamma^2 P_3\kappa + \kappa\gamma^2 P_2 + \gamma^3 P_3 + 4P_4\kappa\gamma - 4\kappa\gamma P_1 + 2P_4\kappa\gamma^2 + 2\kappa\gamma^2 P_1\}/\{-8\gamma^2 + 16\kappa\gamma + 48\kappa^2 - 64\kappa + 16\kappa\gamma^2 + 16 + \gamma^4 - 12\kappa^2\gamma^2 - 4\kappa\gamma^3\}$.

$$\Pi_A^{\{1234\}} = q_1(P_1 - q_1 - \gamma q_2) + q_4(P_4 - q_4 - \gamma q_3) + \frac{q_1 + q_4}{q_1 + q_2 + q_3 + q_4}\kappa(q_1 + q_2 + q_3 + q_4)^2;$$
$$\Pi_B^{\{1234\}} = q_2(P_2 - q_2 - \gamma q_1) + q_3(P_3 - q_3 - \gamma q_4) + \frac{q_2 + q_3}{q_1 + q_2 + q_3 + q_4}\kappa(q_1 + q_2 + q_3 + q_4)^2.$$

- $q_1^{\{12\}\{34\}} = \{-M_2\kappa + 2M_1\kappa + \gamma M_2 - 2M_1\}/\{\gamma^2 - 2\kappa\gamma - 3\kappa^2 - 4 + 8\kappa\}$, $q_2^{\{12\}\{34\}} = \{-2M_2 + 2M_2\kappa + M_1\gamma - M_1\kappa\}/\{\gamma^2 - 2\kappa\gamma - 3\kappa^2 - 4 + 8\kappa\}$, $q_3^{\{12\}\{34\}} = \{-M_4\kappa + 2M_3\kappa + \gamma M_4 - 2M_3\}/\{\gamma^2 - 2\kappa\gamma - 3\kappa^2 - 4 + 8\kappa\}$, and $q_4^{\{12\}\{34\}} = \{-2M_4 + 2M_4\kappa + \gamma M_3 - M_3\kappa\}/\{\gamma^2 - 2\kappa\gamma - 3\kappa^2 - 4 + 8\kappa\}$.

$$\Pi_A^{\{12\}\{34\}} = q_1(M_1 - q_1 - \gamma q_2) + q_4(M_4 - q_4 - \gamma q_3) + \frac{q_1}{q_1 + q_2}\kappa(q_1 + q_2)^2 + \frac{q_4}{q_3 + q_4}\kappa(q_3 + q_4)^2;$$
$$\Pi_B^{\{12\}\{34\}} = q_2(M_2 - q_2 - \gamma q_1) + q_3(M_3 - q_3 - \gamma q_4) + \frac{q_2}{q_1 + q_2}\kappa(q_1 + q_2)^2 + \frac{q_3}{q_3 + q_4}\kappa(q_3 + q_4)^2.$$

- $q_1^{\{13\}\{24\}} = \{3\kappa^3 P_3 + 22P_1\kappa^2 - 6\kappa^3 P_1 - 5\kappa^2\gamma P_2 + 4P_3\kappa + 8P_1 - 24P_1\kappa - 4\gamma P_2 - 8\kappa^2 P_3 + 8\gamma\kappa P_2 + \gamma^2 P_3\kappa - 2P_1\gamma^2 + 2\kappa\gamma^2 P_1 + \gamma^3 P_2 - 4P_4\kappa\gamma + 4\gamma P_4\kappa^2\}/\{-8\gamma^2 + 16\kappa\gamma^2 - 10\kappa^2\gamma^2 + 9\kappa^4 + 88\kappa^2 - 48\kappa^3 + 16 - 64\kappa + \gamma^4\}$, $q_2^{\{13\}\{24\}} = \{3\kappa^3 P_4 - 6\kappa^3 P_2 + 4\kappa^2\gamma P_3 - 5\gamma P_1\kappa^2 - 8P_4\kappa^2 + 22\kappa^2 P_2 - 24\kappa P_2 + 2\kappa\gamma^2 P_2 + P_4\kappa\gamma^2 - 4\gamma P_3\kappa + 8\kappa\gamma P_1 + 4P_4\kappa + 8P_2 - 4P_1\gamma + \gamma^3 P_1 - 2\gamma^2 P_2\}/\{-8\gamma^2 + 16\kappa\gamma^2 - 10\kappa^2\gamma^2 + 9\kappa^4 + 88\kappa^2 - 48\kappa^3 + 16 - 64\kappa + \gamma^4\}$, $q_3^{\{13\}\{24\}} = \{4\kappa^2\gamma P_2 + \kappa\gamma^2 P_1 - 5\gamma P_4\kappa^2 + 2\gamma^2 P_3\kappa + P_4\gamma^3 - 4\gamma\kappa P_2 - 2\gamma^2 P_3 - 4P_4\gamma + 8P_4\kappa\gamma + 8P_3 + 22\kappa^2 P_3 +$

$4P_1\kappa-8P_1\kappa^2-24P_3\kappa+3\kappa^3P_1-6\kappa^3P_3\}/\{-8\gamma^2+16\kappa\gamma^2-10\kappa^2\gamma^2+9\kappa^4+88\kappa^2-48\kappa^3+16-64\kappa+\gamma^4\}$, and $q_4^{\{13\}\{24\}}=\{-5\kappa^2\gamma P_3-8\kappa^2P_2+3\kappa^3P_2-4\kappa\gamma P_1+4\gamma P_1\kappa^2+\kappa\gamma^2P_2+22P_4\kappa^2-6\kappa^3P_4+8\gamma P_3\kappa-2P_4\gamma^2+2P_4\kappa\gamma^2-4\gamma P_3+\gamma^3P_3+8P_4-24P_4\kappa+4\kappa P_2\}/\{-8\gamma^2+16\kappa\gamma^2-10\kappa^2\gamma^2+9\kappa^4+88\kappa^2-48\kappa^3+16-64\kappa+\gamma^4\}$.

$$\Pi_A^{\{13\}\{24\}}=q_1(P_1-q_1-\gamma q_2)+q_4(P_4-q_4-\gamma q_3)+\frac{q_1}{q_1+q_3}\kappa(q_1+q_3)^2+\frac{q_4}{q_2+q_4}\kappa(q_2+q_4)^2;$$
$$\Pi_B^{\{13\}\{24\}}=q_2(P_2-q_2-\gamma q_1)+q_3(P_3-q_3-\gamma q_4)+\frac{q_2}{q_2+q_4}\kappa(q_2+q_4)^2+\frac{q_3}{q_1+q_3}\kappa(q_1+q_3)^2.$$

- $q_1^{\{14\}\{23\}}=\{-4\gamma P_2-16P_4\kappa^2+16P_1\kappa^2+8P_1+2P_4\kappa\gamma^2+2\kappa\gamma^2P_1+8\kappa^2\gamma P_3-8\kappa^2\gamma P_2+\gamma^3P_2-2P_1\gamma^2-8\gamma P_3\kappa+8\gamma\kappa P_2+8P_4\kappa-24P_1\kappa\}/\{\gamma^4+16-64\kappa+64\kappa^2-16\kappa^2\gamma^2-8\gamma^2+16\kappa\gamma^2\}$, $q_2^{\{14\}\{23\}}=\{-24\kappa P_2-4P_1\gamma+8P_2+2\gamma^2P_3\kappa+8\gamma P_4\kappa^2-8\gamma P_1\kappa^2+2\kappa\gamma^2P_2-8P_4\kappa\gamma+8\kappa\gamma P_1+8P_3\kappa-16\kappa^2P_3+16\kappa^2P_2+\gamma^3P_1-2\gamma^2P_2\}/\{\gamma^4+16-64\kappa+64\kappa^2-16\kappa^2\gamma^2-8\gamma^2+16\kappa\gamma^2\}$, $q_3^{\{14\}\{23\}}=\{-8\gamma P_4\kappa^2+8\gamma P_1\kappa^2+16\kappa^2P_3-16\kappa^2P_2-8\kappa\gamma P_1+8P_4\kappa\gamma-24P_3\kappa+2\gamma^2P_3\kappa+2\kappa\gamma^2P_2+8\kappa P_2+P_4\gamma^3-2\gamma^2P_3+8P_3-4P_4\gamma\}/\{\gamma^4+16-64\kappa+64\kappa^2-16\kappa^2\gamma^2-8\gamma^2+16\kappa\gamma^2\}$, and $q_4^{\{14\}\{23\}}=\{-8\gamma\kappa P_2+\gamma^3P_3+8P_4+8P_1\kappa-4\gamma P_3+2P_4\kappa\gamma^2+2\kappa\gamma^2P_1+8\gamma P_3\kappa-8\kappa^2\gamma P_3+8\kappa^2\gamma P_2-2P_4\gamma^2-24P_4\kappa+16P_4\kappa^2-16P_1\kappa^2\}/\{\gamma^4+16-64\kappa+64\kappa^2-16\kappa^2\gamma^2-8\gamma^2+16\kappa\gamma^2\}$.

$$\Pi_A^{\{14\}\{23\}}=q_1(P_1-q_1-\gamma q_2)+q_4(P_4-q_4-\gamma q_3)+\kappa(q_1+q_4)^2;$$
$$\Pi_B^{\{14\}\{23\}}=q_2(P_2-q_2-\gamma q_1)+q_3(P_3-q_3-\gamma q_4)+\kappa(q_2+q_3)^2.$$

- $q_1^{\{123\}\{4\}}=\{4\kappa P_2+8P_1-4\gamma P_2+4P_3\kappa+16P_1\kappa^2-4\kappa^2P_3-2\gamma\kappa M_4-4\kappa^2P_2+4\kappa^2\gamma P_3-4\kappa^2\gamma P_2-\kappa\gamma^2P_2-2P_1\gamma^2+\gamma^3P_2-24P_1\kappa-4\gamma P_3\kappa+8\gamma\kappa P_2+2\kappa\gamma^2P_1+2\gamma^2\kappa M_4\}/\{16-64\kappa+\gamma^4+16\kappa\gamma^2-7\kappa^2\gamma^2-8\gamma^2+72\kappa^2-2\kappa\gamma^3-8\kappa^2\gamma+8\kappa\gamma-24\kappa^3\}$, $q_2^{\{123\}\{4\}}=\{\gamma^3P_1+\gamma^2\kappa M_4-\kappa\gamma^2P_1+2\kappa\gamma^2P_2-2\gamma^2P_2+8\kappa\gamma P_1-4\gamma\kappa M_4+3\gamma\kappa^2M_4+2\kappa^2\gamma P_3-4\gamma P_1\kappa^2-4P_1\gamma-2\gamma P_3\kappa+8P_2+4P_1\kappa-24\kappa P_2+22\kappa^2P_2+6\kappa^3P_3+8P_3\kappa-4P_1\kappa^2-6\kappa^3P_2-14\kappa^2P_3\}/\{16-64\kappa+\gamma^4+16\kappa\gamma^2-7\kappa^2\gamma^2-8\gamma^2+72\kappa^2-2\kappa\gamma^3-8\kappa^2\gamma+8\kappa\gamma-24\kappa^3\}$, $q_3^{\{123\}\{4\}}=\{-14\kappa^2P_2+4P_1\kappa-24P_3\kappa+8P_3-2\gamma^2P_3-4\gamma M_4+8\kappa P_2-4\kappa\gamma P_1+4\gamma P_3\kappa-2\gamma\kappa P_2+8\gamma\kappa M_4+22\kappa^2P_3+\gamma^3M_4-6\kappa^3P_3-3\gamma\kappa^2M_4+6\kappa^3P_2+2\gamma^2P_3\kappa-2\gamma^2\kappa M_4-4P_1\kappa^2+4\gamma P_1\kappa^2-4\kappa^2\gamma P_3+2\kappa^2\gamma P_2\}/\{16-64\kappa+\gamma^4+16\kappa\gamma^2-7\kappa^2\gamma^2-8\gamma^2+72\kappa^2-2\kappa\gamma^3-8\kappa^2\gamma+8\kappa\gamma-24\kappa^3\}$, and $q_4^{\{123\}\{4\}}=\{3\kappa^2\gamma P_2+8M_4-24M_4\kappa-2\kappa\gamma P_1+8\gamma P_3\kappa-4\gamma P_3+12\kappa^2M_4+\gamma^3P_3-2\gamma^2M_4-4\gamma\kappa P_2+4\gamma\kappa M_4+2\kappa\gamma^2P_1-2\gamma^2P_3\kappa+\kappa\gamma^2P_2+2\gamma^2\kappa M_4-3\kappa^2\gamma P_3\}/\{16-64\kappa+\gamma^4+16\kappa\gamma^2-7\kappa^2\gamma^2-8\gamma^2+72\kappa^2-2\kappa\gamma^3-8\kappa^2\gamma+8\kappa\gamma-24\kappa^3\}$.

$$\Pi_A^{\{123\}\{4\}}=q_1(P_1-q_1-\gamma q_2)+q_4(M_4-q_4-\gamma q_3)+\frac{q_1}{q_1+q_2+q_3}\kappa(q_1+q_2+q_3)^2+\kappa q_4^2;$$
$$\Pi_B^{\{123\}\{4\}}=q_2(P_2-q_2-\gamma q_1)+q_3(P_3-q_3-\gamma q_4)+\frac{q_2+q_3}{q_1+q_2+q_3}\kappa(q_1+q_2+q_3)^2.$$

- $q_1^{\{124\}\{3\}}=\{\gamma^3P_2+\gamma^2\kappa M_3-\kappa\gamma^2P_2+2\kappa\gamma^2P_1-2P_1\gamma^2-4\gamma P_2-2P_4\kappa\gamma+3\gamma\kappa^2M_3+8\gamma\kappa P_2-4\gamma\kappa M_3-4\kappa^2\gamma P_2+2\gamma P_4\kappa^2-24P_1\kappa+8P_1-6\kappa^3P_1-14P_4\kappa^2-4\kappa^2P_2+22P_1\kappa^2+8P_4\kappa+6\kappa^3P_4+4\kappa P_2\}/\{16-64\kappa+\gamma^4+16\kappa\gamma^2-7\kappa^2\gamma^2-8\gamma^2+72\kappa^2-2\kappa\gamma^3-8\kappa^2\gamma+8\kappa\gamma-24\kappa^3\}$, $q_2^{\{124\}\{3\}}=\{4P_1\kappa-4P_1\gamma+4P_4\kappa+8P_2+4\gamma P_4\kappa^2-4\gamma P_1\kappa^2+16\kappa^2P_2-4P_4\kappa^2-4P_1\kappa^2-2\gamma\kappa M_3+2\kappa\gamma^2P_2-\kappa\gamma^2P_1+2\gamma^2\kappa M_3-4P_4\kappa\gamma+8\kappa\gamma P_1-24\kappa P_2+\gamma^3P_1-2\gamma^2P_2\}/\{16-64\kappa+\gamma^4+16\kappa\gamma^2-7\kappa^2\gamma^2-8\gamma^2+72\kappa^2-2\kappa\gamma^3-8\kappa^2\gamma+8\kappa\gamma-24\kappa^3\}$,

$q_3^{\{124\}\{3\}} = \{2\kappa\gamma^2 P_2 - 2P_4\kappa\gamma^2 + 3\gamma P_1\kappa^2 + \kappa\gamma^2 P_1 - 3\gamma P_4\kappa^2 + 2\gamma^2\kappa M_3 - 4P_4\gamma + 12\kappa^2 M_3 - 2\gamma\kappa P_2 + 8P_4\kappa\gamma - 4\kappa\gamma P_1 + 4\gamma\kappa M_3 + 8M_3 + P_4\gamma^3 - 2\gamma^2 M_3 - 24M_3\kappa\}/\{16 - 64\kappa + \gamma^4 + 16\kappa\gamma^2 - 7\kappa^2\gamma^2 - 8\gamma^2 + 72\kappa^2 - 2\kappa\gamma^3 - 8\kappa^2\gamma + 8\kappa\gamma - 24\kappa^3\}$, and $q_4^{\{124\}\{3\}} = \{-4\gamma\kappa P_2 + 4P_4\kappa\gamma - 14P_1\kappa^2 - 2\kappa\gamma P_1 + 22P_4\kappa^2 + 8\gamma\kappa M_3 + 8P_4 + 4\kappa P_2 - 24P_4\kappa + 8P_1\kappa + \gamma^3 M_3 - 2P_4\gamma^2 - 4\gamma M_3 - 6\kappa^3 P_4 + 6\kappa^3 P_1 - 3\gamma\kappa^2 M_3 + 2P_4\kappa\gamma^2 + 4\kappa^2\gamma P_2 - 4\gamma P_4\kappa^2 + 2\gamma P_1\kappa^2 - 4\kappa^2 P_2 - 2\gamma^2\kappa M_3\}/\{16 - 64\kappa + \gamma^4 + 16\kappa\gamma^2 - 7\kappa^2\gamma^2 - 8\gamma^2 + 72\kappa^2 - 2\kappa\gamma^3 - 8\kappa^2\gamma + 8\kappa\gamma - 24\kappa^3\}$.

$$\Pi_A^{\{124\}\{3\}} = q_1(P_1 - q_1 - \gamma q_2) + q_4(P_4 - q_4 - \gamma q_3) + \frac{q_1 + q_4}{q_1 + q_2 + q_4}\kappa(q_1 + q_2 + q_4)^2;$$
$$\Pi_B^{\{124\}\{3\}} = q_2(P_2 - q_2 - \gamma q_1) + q_3(M_3 - q_3 - \gamma q_4) + \frac{q_2}{q_1 + q_2 + q_4}\kappa(q_1 + q_2 + q_4)^2 + \kappa q_3^2.$$

- $q_1^{\{134\}\{2\}} = \{8P_4\kappa - 4\gamma P_3\kappa + 4\kappa\gamma P_1 - 14P_4\kappa^2 - 2P_4\kappa\gamma + 8\gamma\kappa M_2 + 22P_1\kappa^2 + 6\kappa^3 P_4 - 6\kappa^3 P_1 - 3\gamma\kappa^2 M_2 + 4\kappa^2\gamma P_3 + 8P_1 + 4P_3\kappa - 24P_1\kappa - 2P_1\gamma^2 - 4\gamma M_2 + \gamma^3 M_2 - 4\gamma P_1\kappa^2 + 2\gamma P_4\kappa^2 - 4\kappa^2 P_3 + 2\kappa\gamma^2 P_1 - 2\gamma^2\kappa M_2\}/\{16 - 64\kappa + \gamma^4 + 16\kappa\gamma^2 - 7\kappa^2\gamma^2 - 8\gamma^2 + 72\kappa^2 - 2\kappa\gamma^3 - 8\kappa^2\gamma + 8\kappa\gamma - 24\kappa^3\}$, $q_2^{\{134\}\{2\}} = \{12\kappa^2 M_2 - 4P_4\kappa\gamma + 4\gamma\kappa M_2 + 2\gamma^2 P_3\kappa - 2\kappa\gamma^2 P_1 + 3\gamma P_4\kappa^2 + P_4\kappa\gamma^2 + 2\gamma^2\kappa M_2 - 3\gamma P_1\kappa^2 - 24M_2\kappa - 4P_1\gamma - 2\gamma P_3\kappa + 8\kappa\gamma P_1 + \gamma^3 P_1 - 2\gamma^2 M_2 + 8M_2\}/\{16 - 64\kappa + \gamma^4 + 16\kappa\gamma^2 - 7\kappa^2\gamma^2 - 8\gamma^2 + 72\kappa^2 - 2\kappa\gamma^3 - 8\kappa^2\gamma + 8\kappa\gamma - 24\kappa^3\}$, $q_3^{\{134\}\{2\}} = \{4P_4\kappa - 4P_4\gamma + 4P_1\kappa - 4P_4\kappa^2 + 4\gamma P_1\kappa^2 - 4\gamma P_4\kappa^2 + 16\kappa^2 P_3 - 4P_1\kappa^2 - 2\gamma\kappa M_2 + 8P_3 - 4\kappa\gamma P_1 + 8P_4\kappa\gamma - 24P_3\kappa - P_4\kappa\gamma^2 + 2\gamma^2\kappa M_2 + 2\gamma^2 P_3\kappa + P_4\gamma^3 - 2\gamma^2 P_3\}/\{16 - 64\kappa + \gamma^4 + 16\kappa\gamma^2 - 7\kappa^2\gamma^2 - 8\gamma^2 + 72\kappa^2 - 2\kappa\gamma^3 - 8\kappa^2\gamma + 8\kappa\gamma - 24\kappa^3\}$, and $q_4^{\{134\}\{2\}} = \{\gamma^3 P_3 - \gamma^2 P_3\kappa + 2P_4\kappa\gamma^2 - 2P_4\gamma^2 + \gamma^2\kappa M_2 + 8\gamma P_3\kappa - 2\kappa\gamma P_1 - 4\kappa^2\gamma P_3 - 4\gamma P_3 + 3\gamma\kappa^2 M_2 + 2\gamma P_1\kappa^2 - 4\gamma\kappa M_2 - 24P_4\kappa + 4P_3\kappa + 22P_4\kappa^2 + 6\kappa^3 P_1 + 8P_1\kappa - 6\kappa^3 P_4 + 8P_4 - 4\kappa^2 P_3 - 14P_1\kappa^2\}/\{16 - 64\kappa + \gamma^4 + 16\kappa\gamma^2 - 7\kappa^2\gamma^2 - 8\gamma^2 + 72\kappa^2 - 2\kappa\gamma^3 - 8\kappa^2\gamma + 8\kappa\gamma - 24\kappa^3\}$.

$$\Pi_A^{\{134\}\{2\}} = q_1(P_1 - q_1 - \gamma q_2) + q_4(P_4 - q_4 - \gamma q_3) + \frac{q_1 + q_4}{q_1 + q_3 + q_4}\kappa(q_1 + q_3 + q_4)^2;$$
$$\Pi_B^{\{134\}\{2\}} = q_2(M_2 - q_2 - \gamma q_1) + q_3(P_3 - q_3 - \gamma q_4) + \kappa q_2^2 + \frac{q_3}{q_1 + q_3 + q_4}\kappa(q_1 + q_3 + q_4)^2.$$

- $q_1^{\{1\}\{234\}} = \{8M_1 - 4\gamma P_3\kappa + 4\gamma\kappa M_1 + 2P_4\kappa\gamma^2 - 2\kappa\gamma^2 P_2 + \gamma^2 P_3\kappa - 3\kappa^2\gamma P_2 + 2\gamma^2\kappa M_1 + \gamma^3 P_2 - 2\gamma^2 M_1 + 3\kappa^2\gamma P_3 - 24M_1\kappa - 2P_4\kappa\gamma + 8\gamma\kappa P_2 - 4\gamma P_2 + 12\kappa^2 M_1\}/\{16 - 64\kappa + \gamma^4 + 16\kappa\gamma^2 - 7\kappa^2\gamma^2 - 8\gamma^2 + 72\kappa^2 - 2\kappa\gamma^3 - 8\kappa^2\gamma + 8\kappa\gamma - 24\kappa^3\}$, $q_2^{\{1\}\{234\}} = \{8P_3\kappa - 4P_4\kappa\gamma + 4\gamma\kappa P_2 - 2\gamma P_3\kappa + 22\kappa^2 P_2 + 8\gamma\kappa M_1 - 2\gamma^2 P_2 - 4M_1\gamma + \gamma^3 M_1 - 14\kappa^2 P_3 + 4P_4\kappa - 24\kappa P_2 + 6\kappa^3 P_3 - 3\gamma\kappa^2 M_1 - 6\kappa^3 P_2 + 4\gamma P_4\kappa^2 - 4\kappa^2\gamma P_2 + 2\kappa^2\gamma P_3 + 2\kappa\gamma^2 P_2 - 4P_4\kappa^2 - 2\gamma^2\kappa M_1 + 8P_2\}/\{16 - 64\kappa + \gamma^4 + 16\kappa\gamma^2 - 7\kappa^2\gamma^2 - 8\gamma^2 + 72\kappa^2 - 2\kappa\gamma^3 - 8\kappa^2\gamma + 8\kappa\gamma - 24\kappa^3\}$, $q_3^{\{1\}\{234\}} = \{P_4\gamma^3 - P_4\kappa\gamma^2 + 2\gamma^2 P_3\kappa + \gamma^2\kappa M_1 - 2\gamma^2 P_3 + 8P_4\kappa\gamma - 4P_4\gamma + 3\gamma\kappa^2 M_1 - 2\gamma\kappa P_2 - 4\gamma P_4\kappa^2 + 2\kappa^2\gamma P_2 - 4\gamma\kappa M_1 - 14\kappa^2 P_2 - 24P_3\kappa - 6\kappa^3 P_3 + 4P_4\kappa + 22\kappa^2 P_3 + 8P_3 + 8\kappa P_2 + 6\kappa^3 P_2 - 4P_4\kappa^2\}/\{16 - 64\kappa + \gamma^4 + 16\kappa\gamma^2 - 7\kappa^2\gamma^2 - 8\gamma^2 + 72\kappa^2 - 2\kappa\gamma^3 - 8\kappa^2\gamma + 8\kappa\gamma - 24\kappa^3\}$, and $q_4^{\{1\}\{234\}} = \{-4\gamma P_3 + 4\kappa P_2 + 4P_3\kappa + 8P_4 - 4\kappa^2 P_3 + 4\kappa^2\gamma P_2 - 4\kappa^2\gamma P_3 - 2\gamma\kappa M_1 + 16P_4\kappa^2 - 4\kappa^2 P_2 - 4\gamma\kappa P_2 + 8\gamma P_3\kappa - 24P_4\kappa + \gamma^3 P_3 - \gamma^2 P_3\kappa - 2P_4\gamma^2 + 2\gamma^2\kappa M_1 + 2P_4\kappa\gamma^2\}/\{16 - 64\kappa + \gamma^4 + 16\kappa\gamma^2 - 7\kappa^2\gamma^2 - 8\gamma^2 + 72\kappa^2 - 2\kappa\gamma^3 - 8\kappa^2\gamma + 8\kappa\gamma - 24\kappa^3\}$.

$$\Pi_A^{\{1\}\{234\}} = q_1(M_1 - q_1 - \gamma q_2) + q_4(P_4 - q_4 - \gamma q_3) + \kappa q_1^2 + \frac{q_4}{q_2 + q_3 + q_4}\kappa(q_2 + q_3 + q_4)^2;$$
$$\Pi_B^{\{1\}\{234\}} = q_2(P_2 - q_2 - \gamma q_1) + q_3(P_3 - q_3 - \gamma q_4) + \frac{q_2 + q_3}{q_2 + q_3 + q_4}\kappa(q_2 + q_3 + q_4)^2.$$

- $q_1^{\{12\}\{3\}\{4\}} = \{-M_2\kappa + 2M_1\kappa + \gamma M_2 - 2M_1\}/\{\gamma^2 - 2\kappa\gamma - 3\kappa^2 - 4 + 8\kappa\}$, $q_2^{\{12\}\{3\}\{4\}} = \{-2M_2 + 2M_2\kappa + M_1\gamma - M_1\kappa\}/\{\gamma^2 - 2\kappa\gamma - 3\kappa^2 - 4 + 8\kappa\}$, $q_3^{\{12\}\{3\}\{4\}} = \{\gamma M_4 - 2M_3 + 2M_3\kappa\}/\{\gamma^2 - 4 + 8\kappa - 4\kappa^2\}$, and $q_4^{\{12\}\{3\}\{4\}} = \{-2M_4 + 2M_4\kappa + \gamma M_3\}/\{\gamma^2 - 4 + 8\kappa - 4\kappa^2\}$.

$$\Pi_A^{\{12\}\{3\}\{4\}} = q_1(M_1 - q_1 - \gamma q_2) + q_4(M_4 - q_4 - \gamma q_3) + \frac{q_1}{q_1 + q_2}\kappa(q_1 + q_2)^2 + \kappa q_4^2;$$
$$\Pi_B^{\{12\}\{3\}\{4\}} = q_2(M_2 - q_2 - \gamma q_1) + q_3(M_3 - q_3 - \gamma q_4) + \frac{q_2}{q_1 + q_2}\kappa(q_1 + q_2)^2 + \kappa q_3^2.$$

- $q_1^{\{13\}\{2\}\{4\}} = \{8\gamma\kappa M_2 - 2\gamma\kappa M_4 - 8\kappa^2 P_3 + 4P_3\kappa + 24P_1\kappa^2 + 8P_1 - 24P_1\kappa + 2\gamma\kappa^2 M_4 + 2\kappa\gamma^2 P_1 - 4\gamma\kappa^2 M_2 + 4\kappa^3 P_3 - 8\kappa^3 P_1 - 4\gamma M_2 - 2P_1\gamma^2 + \gamma^3 M_2\}/\{92\kappa^2 - 56\kappa^3 + 12\kappa^4 + \gamma^4 - 8\gamma^2 + 16\kappa\gamma^2 - 8\kappa^2\gamma^2 - 64\kappa + 16\}$, $q_2^{\{13\}\{2\}\{4\}} = \{-6\kappa^3 M_2 + 22\kappa^2 M_2 + 2\kappa^2\gamma P_3 - 4\gamma P_1\kappa^2 + \gamma^2\kappa M_4 - 2\gamma P_3\kappa + 8\kappa\gamma P_1 - 24M_2\kappa + 2\gamma^2\kappa M_2 - 4P_1\gamma + 8M_2 - 2\gamma^2 M_2 + \gamma^3 P_1\}/\{92\kappa^2 - 56\kappa^3 + 12\kappa^4 + \gamma^4 - 8\gamma^2 + 16\kappa\gamma^2 - 8\kappa^2\gamma^2 - 64\kappa + 16\}$, $q_3^{\{13\}\{2\}\{4\}} = \{-2\gamma\kappa M_2 + 8\gamma\kappa M_4 + 24\kappa^2 P_3 - 24P_3\kappa - 8P_1\kappa^2 + 8P_3 - 2\gamma^2 P_3 - 4\gamma M_4 + \gamma^3 M_4 + 4P_1\kappa - 4\gamma\kappa^2 M_4 + 2\gamma^2 P_3\kappa + 2\gamma\kappa^2 M_2 - 8\kappa^3 P_3 + 4\kappa^3 P_1\}/\{92\kappa^2 - 56\kappa^3 + 12\kappa^4 + \gamma^4 - 8\gamma^2 + 16\kappa\gamma^2 - 8\kappa^2\gamma^2 - 64\kappa + 16\}$, and $q_4^{\{13\}\{2\}\{4\}} = \{\gamma^3 P_3 + \gamma^2\kappa M_2 - 2\gamma^2 M_4 + 2\gamma^2\kappa M_4 - 4\kappa^2\gamma P_3 + 22\kappa^2 M_4 - 6\kappa^3 M_4 - 4\gamma P_3 + 8M_4 - 24M_4\kappa + 8\gamma P_3\kappa - 2\kappa\gamma P_1 + 2\gamma P_1\kappa^2\}/\{92\kappa^2 - 56\kappa^3 + 12\kappa^4 + \gamma^4 - 8\gamma^2 + 16\kappa\gamma^2 - 8\kappa^2\gamma^2 - 64\kappa + 16\}$.

$$\Pi_A^{\{13\}\{2\}\{4\}} = q_1(P_1 - q_1 - \gamma q_2) + q_4(M_4 - q_4 - \gamma q_3) + \frac{q_1}{q_1 + q_3}\kappa(q_1 + q_3)^2 + \kappa q_4^2;$$
$$\Pi_B^{\{13\}\{2\}\{4\}} = q_2(M_2 - q_2 - \gamma q_1) + q_3(P_3 - q_3 - \gamma q_4) + \kappa q_2^2 + \frac{q_3}{q_1 + q_3}\kappa(q_1 + q_3)^2.$$

- $q_1^{\{14\}\{2\}\{3\}} = \{\gamma^3 M_2 - 24P_1\kappa + 8P_1 - 16P_4\kappa^2 + 8P_4\kappa + 24P_1\kappa^2 + 8\gamma\kappa M_2 - 4\gamma\kappa M_3 + 8\kappa^3 P_4 - 8\kappa^3 P_1 - 4\gamma\kappa^2 M_2 + 4\gamma\kappa^2 M_3 + 2\kappa\gamma^2 P_1 - 4\gamma M_2 - 2P_1\gamma^2\}/\{80\kappa^2 - 32\kappa^3 - 64\kappa + 16\kappa\gamma^2 + 16 - 8\gamma^2 - 8\kappa^2\gamma^2 + \gamma^4\}$, $q_2^{\{14\}\{2\}\{3\}} = \{4\gamma P_4\kappa^2 + 16\kappa^2 M_2 - 4\gamma P_1\kappa^2 - 4P_4\kappa\gamma - 24M_2\kappa + 8\kappa\gamma P_1 + 2\gamma^2\kappa M_2 + 2\gamma^2\kappa M_3 + 8M_2 + \gamma^3 P_1 - 4P_1\gamma - 2\gamma^2 M_2\}/\{80\kappa^2 - 32\kappa^3 - 64\kappa + 16\kappa\gamma^2 + 16 - 8\gamma^2 - 8\kappa^2\gamma^2 + \gamma^4\}$, $q_3^{\{14\}\{2\}\{3\}} = \{-4\gamma P_4\kappa^2 + 16\kappa^2 M_3 - 4P_4\gamma + 8M_3 - 24M_3\kappa + 8P_4\kappa\gamma - 4\kappa\gamma P_1 + 4\gamma P_1\kappa^2 + P_4\gamma^3 + 2\gamma^2\kappa M_2 - 2\gamma^2 M_3 + 2\gamma^2\kappa M_3\}/\{80\kappa^2 - 32\kappa^3 - 64\kappa + 16\kappa\gamma^2 + 16 - 8\gamma^2 - 8\kappa^2\gamma^2 + \gamma^4\}$, and $q_4^{\{14\}\{2\}\{3\}} = \{8P_1\kappa - 2P_4\gamma^2 - 4\gamma M_3 + \gamma^3 M_3 + 8P_4 + 24P_4\kappa^2 - 24P_4\kappa - 16P_1\kappa^2 - 4\gamma\kappa M_2 + 8\gamma\kappa M_3 - 8\kappa^3 P_4 + 8\kappa^3 P_1 + 2P_4\kappa\gamma^2 + 4\gamma\kappa^2 M_2 - 4\gamma\kappa^2 M_3\}/\{80\kappa^2 - 32\kappa^3 - 64\kappa + 16\kappa\gamma^2 + 16 - 8\gamma^2 - 8\kappa^2\gamma^2 + \gamma^4\}$.

$$\Pi_A^{\{14\}\{2\}\{3\}} = q_1(P_1 - q_1 - \gamma q_2) + q_4(P_4 - q_4 - \gamma q_3) + \kappa(q_1 + q_4)^2;$$
$$\Pi_B^{\{14\}\{2\}\{3\}} = q_2(M_2 - q_2 - \gamma q_1) + q_3(M_3 - q_3 - \gamma q_4) + \kappa q_2^2 + \kappa q_3^2.$$

- $q_1^{\{1\}\{23\}\{4\}} = \{16\kappa^2 M_1 + 4\kappa^2\gamma P_3 - 4\kappa^2\gamma P_2 - 4\gamma P_3\kappa + 8\gamma\kappa P_2 - 24M_1\kappa + 2\gamma^2\kappa M_1 + 2\gamma^2\kappa M_4 - 4\gamma P_2 - 2\gamma^2 M_1 + 8M_1 + \gamma^3 P_2\}/\{80\kappa^2 - 32\kappa^3 - 64\kappa + 16\kappa\gamma^2 + 16 - 8\gamma^2 - 8\kappa^2\gamma^2 + \gamma^4\}$, $q_2^{\{1\}\{23\}\{4\}} = \{8\gamma\kappa M_1 - 4\gamma\kappa M_4 - 16\kappa^2 P_3 + 8P_3\kappa + 24\kappa^2 P_2 + 4\gamma\kappa^2 M_4 + 2\kappa\gamma^2 P_2 - 4\gamma\kappa^2 M_1 + 8\kappa^3 P_3 - 8\kappa^3 P_2 - 2\gamma^2 P_2 + \gamma^3 M_1 - 4M_1\gamma - 24\kappa P_2 + 8P_2\}/\{80\kappa^2 - 32\kappa^3 - 64\kappa + 16\kappa\gamma^2 + 16 - 8\gamma^2 - 8\kappa^2\gamma^2 + \gamma^4\}$, $q_3^{\{1\}\{23\}\{4\}} = \{-2\gamma^2 P_3 - 4\gamma M_4 + \gamma^3 M_4 - 4\gamma\kappa M_1 + 8\gamma\kappa M_4 + 24\kappa^2 P_3 - 24P_3\kappa - 16\kappa^2 P_2 - 4\gamma\kappa^2 M_4 + 2\gamma^2 P_3\kappa + 4\gamma\kappa^2 M_1 - 8\kappa^3 P_3 + 8\kappa^3 P_2 + 8\kappa P_2 + 8P_3\}/\{80\kappa^2 - 32\kappa^3 - 64\kappa + 16\kappa\gamma^2 + 16 - 8\gamma^2 - 8\kappa^2\gamma^2 + \gamma^4\}$, and $q_4^{\{1\}\{23\}\{4\}} =$

$\{\gamma^3 P_3 + 2\gamma^2\kappa M_1 - 2\gamma^2 M_4 + 2\gamma^2\kappa M_4 - 4\kappa^2\gamma P_3 + 16\kappa^2 M_4 - 4\gamma P_3 + 8M_4 - 24M_4\kappa + 8\gamma P_3\kappa - 4\gamma\kappa P_2 + 4\kappa^2\gamma P_2\}/\{80\kappa^2 - 32\kappa^3 - 64\kappa + 16\kappa\gamma^2 + 16 - 8\gamma^2 - 8\kappa^2\gamma^2 + \gamma^4\}$.

$$\Pi_A^{\{1\}\{23\}\{4\}} = q_1(M_1 - q_1 - \gamma q_2) + q_4(M_4 - q_4 - \gamma q_3) + \kappa q_1^2 + \kappa q_4^2;$$
$$\Pi_B^{\{1\}\{23\}\{4\}} = q_2(P_2 - q_2 - \gamma q_1) + q_3(P_3 - q_3 - \gamma q_4) + \kappa(q_2 + q_3)^2.$$

- $q_1^{\{1\}\{24\}\{3\}} = \{-6\kappa^3 M_1 + 2\gamma P_4\kappa^2 - 4\kappa^2\gamma P_2 + 22\kappa^2 M_1 - 2P_4\kappa\gamma + 8\gamma\kappa P_2 + 2\gamma^2\kappa M_1 + \gamma^2\kappa M_3 - 24M_1\kappa + 8M_1 - 2\gamma^2 M_1 - 4\gamma P_2 + \gamma^3 P_2\}/\{92\kappa^2 - 56\kappa^3 + 12\kappa^4 + \gamma^4 - 8\gamma^2 + 16\kappa\gamma^2 - 8\kappa^2\gamma^2 - 64\kappa + 16\}$, $q_2^{\{1\}\{24\}\{3\}} = \{8P_2 + 2\gamma\kappa^2 M_3 + 2\kappa\gamma^2 P_2 - 4\gamma\kappa^2 M_1 + 4\kappa^3 P_4 - 8\kappa^3 P_2 - 2\gamma^2 P_2 - 4M_1\gamma + 8\gamma\kappa M_1 - 2\gamma\kappa M_3 + \gamma^3 M_1 - 24\kappa P_2 - 8P_4\kappa^2 + 4P_4\kappa + 24\kappa^2 P_2\}/\{92\kappa^2 - 56\kappa^3 + 12\kappa^4 + \gamma^4 - 8\gamma^2 + 16\kappa\gamma^2 - 8\kappa^2\gamma^2 - 64\kappa + 16\}$, $q_3^{\{1\}\{24\}\{3\}} = \{P_4\gamma^3 + \gamma^2\kappa M_1 - 2\gamma^2 M_3 + 2\gamma^2\kappa M_3 - 4\gamma P_4\kappa^2 + 22\kappa^2 M_3 - 6\kappa^3 M_3 - 4P_4\gamma + 8M_3 - 24M_3\kappa + 8P_4\kappa\gamma - 2\gamma\kappa P_2 + 2\kappa^2\gamma P_2\}/\{92\kappa^2 - 56\kappa^3 + 12\kappa^4 + \gamma^4 - 8\gamma^2 + 16\kappa\gamma^2 - 8\kappa^2\gamma^2 - 64\kappa + 16\}$, and $q_4^{\{1\}\{24\}\{3\}} = \{8P_4 - 4\gamma\kappa^2 M_3 + 2P_4\kappa\gamma^2 + 2\gamma\kappa^2 M_1 - 8\kappa^3 P_4 + 4\kappa^3 P_2 - 2\gamma\kappa M_1 + 8\gamma\kappa M_3 + 4\kappa P_2 + 24P_4\kappa^2 - 24P_4\kappa - 8\kappa^2 P_2 - 2P_4\gamma^2 - 4\gamma M_3 + \gamma^3 M_3\}/\{92\kappa^2 - 56\kappa^3 + 12\kappa^4 + \gamma^4 - 8\gamma^2 + 16\kappa\gamma^2 - 8\kappa^2\gamma^2 - 64\kappa + 16\}$.

$$\Pi_A^{\{1\}\{24\}\{3\}} = q_1(M_1 - q_1 - \gamma q_2) + q_4(P_4 - q_4 - \gamma q_3) + \kappa q_1^2 + \frac{q_4}{q_2 + q_4}\kappa(q_2 + q_4)^2;$$
$$\Pi_B^{\{1\}\{24\}\{3\}} = q_2(P_2 - q_2 - \gamma q_1) + q_3(M_3 - q_3 - \gamma q_4) + \frac{q_2}{q_2 + q_4}\kappa(q_2 + q_4)^2 + \kappa q_3^2.$$

- $q_1^{\{1\}\{2\}\{34\}} = \{\gamma M_2 - 2M_1 + 2M_1\kappa\}/\{\gamma^2 - 4 + 8\kappa - 4\kappa^2\}$, $q_2^{\{1\}\{2\}\{34\}} = \{-2M_2 + 2M_2\kappa + M_1\gamma\}/\{\gamma^2 - 4 + 8\kappa - 4\kappa^2\}$, $q_3^{\{1\}\{2\}\{34\}} = \{-M_4\kappa + 2M_3\kappa + \gamma M_4 - 2M_3\}/\{\gamma^2 - 2\kappa\gamma - 3\kappa^2 - 4 + 8\kappa\}$, and $q_4^{\{1\}\{2\}\{34\}} = \{-2M_4 + 2M_4\kappa + \gamma M_3 - M_3\kappa\}/\{\gamma^2 - 2\kappa\gamma - 3\kappa^2 - 4 + 8\kappa\}$.

$$\Pi_A^{\{1\}\{2\}\{34\}} = q_1(M_1 - q_1 - \gamma q_2) + q_4(M_4 - q_4 - \gamma q_3) + \kappa q_1^2 + \frac{q_4}{q_3 + q_4}\kappa(q_3 + q_4)^2;$$
$$\Pi_B^{\{1\}\{2\}\{34\}} = q_2(M_2 - q_2 - \gamma q_1) + q_3(M_3 - q_3 - \gamma q_4) + \kappa q_2^2 + \frac{q_3}{q_3 + q_4}\kappa(q_3 + q_4)^2.$$

- $q_1^{\{1\}\{2\}\{3\}\{4\}} = \{\gamma M_2 - 2M_1 + 2M_1\kappa\}/\{\gamma^2 - 4 + 8\kappa - 4\kappa^2\}$, $q_1^{\{1\}\{2\}\{3\}\{4\}} = \{-2M_2 + 2M_2\kappa + M_1\gamma\}/\{\gamma^2 - 4 + 8\kappa - 4\kappa^2\}$, $q_1^{\{1\}\{2\}\{3\}\{4\}} = \{\gamma M_4 - 2M_3 + 2M_3\kappa\}/\{\gamma^2 - 4 + 8\kappa - 4\kappa^2\}$, and $q_1^{\{1\}\{2\}\{3\}\{4\}} = \{-2M_4 + 2M_4\kappa + \gamma M_3\}/\{\gamma^2 - 4 + 8\kappa - 4\kappa^2\}$.

$$\Pi_A^{\{1\}\{2\}\{3\}\{4\}} = q_1(M_1 - q_1 - \gamma q_2) + q_4(M_4 - q_4 - \gamma q_3) + \kappa q_1^2 + \kappa q_4^2;$$
$$\Pi_B^{\{1\}\{2\}\{3\}\{4\}} = q_2(M_2 - q_2 - \gamma q_1) + q_3(M_3 - q_3 - \gamma q_4) + \kappa q_2^2 + \kappa q_3^2.$$

**Proof of Proposition 7:** With equilibrium quantities $q_i^X$, $i = 1, 2, 3, 4$, shown above, we calculate and compare $W^X(q_1^X, q_2^X, q_3^X, q_4^X)$ (given in equation 7) for all $X \in \mathbf{X}$, and then obtain the optimal recycling structure with the highest $W^X(q_1^X, q_2^X, q_3^X, q_4^X)$. ■

**Proof of Proposition 8:** Under our assumption that $c_1 = c_2, c_3 = c_4$, and $\gamma < 1$, it is easy to verify that, in most scenarios, outcomes that one player prefers more than product-based recycling, $\{1\}\{2\}\{3\}\{4\}$, are favored less then $\{1\}\{2\}\{3\}\{4\}$ by the other player; consequently, $\{1\}\{2\}\{3\}\{4\}$ emerges as stable. ■

**Proof of Proposition 9:** As mentioned above, under our assumption that $c_1 = c_2, c_3 = c_4$, and $\gamma < 1$, it is easy to verify that, in most scenarios, outcomes that one player prefers more than product-based recycling, $\{1\}\{2\}\{3\}\{4\}$, are favored less then $\{1\}\{2\}\{3\}\{4\}$ by the other player; consequently, $\{1\}\{2\}\{3\}\{4\}$ emerges as stable. ■

**Proof of Proposition 10:**

1. In most of the possible scenarios, for different parameter combinations, all-inclusive recycling dominates other structures for both firms, or firm $A$ prefers $\{134\}\{2\}$ or $\{124\}\{3\}$ to all-inclusive recycling, and/or firm $B$ prefers $\{123\}\{4\}$ or $\{234\}\{1\}$ to all-inclusive recycling. Under these scenarios, all-inclusive recycling is uniquely stable.

   Under the condition that economies of scale are low and product substitutability is high, a similar statement holds for market-based recycling.

2. Next, we consider firm-based recycling. Suppose that $\alpha_1 = 115, \alpha_2 = 200, \alpha_3 = 285, \alpha_4 = 150, c_1 = c_2 = 2, c_3 = c_4 = 5, \gamma = 0.7, \lambda = 1.2,\ kappa = 0.09$. Then, we have $\Pi_A^{\{1234\}} > \Pi_A^{\{134\}\{2\}} > \Pi_A^{\{124\}\{3\}} > \Pi_A^{\{12\}\{34\}} > \Pi_A^{\{13\}\{24\}} > \Pi_A^{\{13\}\{2\}\{4\}} > \Pi_A^{\{1\}\{2\}\{34\}} > \Pi_A^{\{12\}\{3\}\{4\}} > \Pi_A^{\{1\}\{24\}\{3\}} > \Pi_A^{\{1\}\{234\}} > \Pi_A^{\{123\}\{4\}} > \Pi_A^{\{14\}\{2\}\{3\}} > \Pi_A^{\{1\}\{2\}\{3\}\{4\}} > \Pi_A^{\{14\}\{23\}} > \Pi_A^{\{1\}\{23\}\{4\}}$ and $\Pi_B^{\{1\}\{23\}\{4\}} > \Pi_B^{\{14\}\{23\}} > \Pi_B^{\{123\}\{4\}} > \Pi_B^{\{1\}\{234\}} > \Pi_B^{\{1234\}} > \Pi_B^{\{1\}\{2\}\{3\}\{4\}} > \Pi_B^{\{12\}\{3\}\{4\}} > \Pi_B^{\{13\}\{2\}\{4\}} > \Pi_B^{\{1\}\{24\}\{3\}} > \Pi_B^{\{1\}\{2\}\{34\}} > \Pi_B^{\{14\}\{2\}\{3\}} > \Pi_B^{\{12\}\{34\}} > \Pi_B^{\{13\}\{24\}} > \Pi_B^{\{124\}\{3\}} > \Pi_B^{\{134\}\{2\}}$. If we consider $B$'s most preferred outcome, $\{1\}\{23\}\{4\}$, $A$ can defect unilaterally only to $\{14\}\{23\}$, which improves its payoff and is the second most preferred outcome for $B$. Any unilateral defection from firm-based recycling by $B$ would only reduce its profit, and the same goes for $A$. In addition, any joint defection would reduce $B$'s profit, hence firm-based recycling is stable. Now, if we consider $\{1234\}$, $A$'s most preferred outcome, $B$ can unilaterally defect to $\{14\}\{23\}$, which is stable. it is least preferred by $B$ and any defection would improve $B$'s profit. If we consider $\{134\}\{2\}$, $A$'s second most preferred outcome, it is least preferred by $B$ and any defection would improve $B$'s profit. We can similarly see that for all other structures preferred to firm-based recycling by $A$, $B$ can unilaterally defect to either $\{14\}\{23\}$, or to $\{1\}\{23\}\{4\}$, which will be followed by $A$'s move to $\{14\}\{23\}$, hence firm-based recycling is the only stable outcome.

   When recycling becomes more costly, half-firm based recycling becomes stable. Suppose that $\alpha_1 = 78, \alpha_2 = 100, \alpha_3 = 300, \alpha_4 = 155, c_1 = c_2 = 2, c_3 = c_4 = 5, \gamma = 0.8, \lambda = 2, \kappa = 0.1$. Then, we have $\Pi_A^{\{134\}\{2\}} > \Pi_A^{\{1234\}} > \Pi_A^{\{13\}\{24\}} > \Pi_A^{\{13\}\{2\}\{4\}} > \Pi_A^{\{12\}\{34\}} > \Pi_A^{\{1\}\{2\}\{34\}} > \Pi_A^{\{124\}\{3\}} > \Pi_A^{\{123\}\{4\}} > \Pi_A^{\{12\}\{3\}\{4\}} > \Pi_A^{\{1\}\{234\}} > \Pi_A^{\{1\}\{24\}\{3\}} > \Pi_A^{\{1\}\{2\}\{3\}\{4\}} > \Pi_A^{\{14\}\{2\}\{3\}} > \Pi_A^{\{1\}\{23\}\{4\}} > \Pi_A^{\{14\}\{23\}}$ and $\Pi_B^{\{14\}\{23\}} > \Pi_B^{\{1\}\{23\}\{4\}} > \Pi_B^{\{123\}\{4\}} > \Pi_B^{\{1\}\{234\}} > \Pi_B^{\{14\}\{2\}\{3\}} > \Pi_B^{\{1234\}} > \Pi_B^{\{1\}\{2\}\{3\}\{4\}} > \Pi_B^{\{12\}\{3\}\{4\}} > \Pi_B^{\{1\}\{24\}\{3\}} > \Pi_B^{\{124\}\{3\}} > \Pi_B^{\{13\}\{24\}} > \Pi_B^{\{13\}\{2\}\{4\}} > \Pi_B^{\{1\}\{2\}\{34\}} > \Pi_B^{\{12\}\{34\}} > \Pi_B^{\{134\}\{2\}}$. Similarly as above, we can show that in this case half firm-based recycling, $\{1\}\{23\}\{4\}$, is the only stable outcome, as $A$ will always defect from its least preferred outcome, $\{14\}\{23\}$.

3. We now look at the $i$-inclusive recycling. Consider the case $\Pi_A^{\{134\}\{2\}} > \Pi_A^{\{1234\}} > \Pi_A^{\{13\}\{2\}\{4\}} > \Pi_A^{\{13\}\{24\}} > \Pi_A^{\{12\}\{34\}} > \Pi_A^{\{1\}\{2\}\{34\}} > \Pi_A^{\{124\}\{3\}} > \Pi_A^{\{123\}\{4\}} > \Pi_A^{\{12\}\{3\}\{4\}} > \Pi_A^{\{1\}\{2\}\{3\}\{4\}} > \Pi_A^{\{14\}\{2\}\{3\}} > \Pi_A^{\{1\}\{24\}\{3\}} > \Pi_A^{\{1\}\{234\}} > \Pi_A^{\{1\}\{23\}\{4\}} > \Pi_A^{\{14\}\{23\}}$ and $\Pi_B^{\{123\}\{4\}} > \Pi_B^{\{1\}\{23\}\{4\}} > \Pi_B^{\{14\}\{23\}} > \Pi_B^{\{1\}\{234\}} > \Pi_B^{\{12\}\{3\}\{4\}} > \Pi_B^{\{1234\}} > \Pi_B^{\{1\}\{24\}\{3\}} > \Pi_B^{\{1\}\{2\}\{3\}\{4\}} > \Pi_B^{\{14\}\{2\}\{3\}} > \Pi_B^{\{13\}\{24\}} > \Pi_B^{\{124\}\{3\}} > \Pi_B^{\{13\}\{2\}\{4\}} > \Pi_B^{\{12\}\{34\}} > \Pi_B^{\{1\}\{2\}\{34\}} > \Pi_B^{\{134\}\{2\}}$, which occurs, for instance, when $\alpha_1 = \alpha_2 = 100, \alpha_3 = 300, \alpha_4 = 150, c_1 = c_2 = 2, c_3 = c_4 = 5, \gamma = 0.75, \lambda = 1.2, \kappa = 0.02$. We can verify that in this case $\{123\}\{4\}$, the outcome most preferred by firm $B$, is the only stable outcome. The only possible defections from this outcome by firm $A$ lead to $\{14\}\{23\}$ or $\{23\}\{1\}\{4\}$, the outcomes least preferred by $A$, so $A$ does not want to defect. Consider, for instance, outcome most preferred by firm $A$, $\{134\}\{2\}$. This outcome is the least preferred by firm $B$ and any defection can only improve its profit, so it cannot be stable. Next, consider firm-based recycling, $\{14\}\{23\}$. Both firms prefer $\{123\}\{4\}$ to $\{14\}\{23\}$ and it can be verified that firm-based recycling is not stable. Similarly, one can verify that all-inclusive recycling cannot be stable as firm $B$ can unilaterally defect to $\{14\}\{23\}$. If the current outcome is $\{13\}\{24\}$, firm $B$ can unilaterally defect to $\{1\}\{23\}\{4\}$, and firm $A$ then benefits by defecting to $\{14\}\{23\}$. We can do the analysis for all the remaining possible defections.

4. Finally, we look at the half-market-based recycling. Consider, for example, case when $\alpha_1 = \alpha_2 = 100, \alpha_3 = 300, \alpha_4 = 150, c_1 = c_2 = 2, c_3 = c_4 = 5, \gamma = 0.66, \lambda = 2, \kappa = 0.02$. Then, $\Pi_A^{\{12\}\{34\}} > \Pi_A^{\{13\}\{2\}\{4\}} > \Pi_A^{\{1\}\{2\}\{34\}} > \Pi_A^{\{134\}\{2\}} > \Pi_A^{\{123\}\{4\}} > \Pi_A^{\{12\}\{3\}\{4\}} > \Pi_A^{\{1234\}} > \Pi_A^{\{13\}\{24\}} > \Pi_A^{\{1\}\{2\}\{3\}\{4\}} > \Pi_A^{\{1\}\{23\}\{4\}} > \Pi_A^{\{1\}\{24\}\{3\}} > \Pi_A^{\{1\}\{234\}} > \Pi_A^{\{124\}\{3\}} > \Pi_A^{\{14\}\{2\}\{3\}} > \Pi_A^{\{14\}\{23\}}$ and $\Pi_B^{\{14\}\{2\}\{3\}} > \Pi_B^{\{1\}\{24\}\{3\}} > \Pi_B^{\{124\}\{3\}} > \Pi_B^{\{12\}\{3\}\{4\}} > \Pi_B^{\{1\}\{2\}\{3\}\{4\}} > \Pi_B^{\{12\}\{34\}} > \Pi_B^{\{1\}\{2\}\{34\}} > \Pi_B^{\{14\}\{23\}} > \Pi_B^{\{1234\}} > \Pi_B^{\{1\}\{234\}} > \Pi_B^{\{123\}\{4\}} > \Pi_B^{\{13\}\{24\}} > \Pi_B^{\{134\}\{2\}} > \Pi_B^{\{1\}\{23\}\{4\}} > \Pi_B^{\{13\}\{2\}\{4\}}$. We can verify that $\{12\}\{3\}\{4\}$, the outcome not most favored by either firm, is the only stable outcome in this case. It is easy to see that firm $A$ can unilaterally defect from any of the three outcomes most preferred by firm $B$ to product-based recycling, which it prefers to $\{14\}\{2\}\{3\}$, $\{1\}\{24\}\{3\}$ and $\{124\}\{3\}$. The only possible unilateral defection from product-based recycling by firm $B$ leads to $\{1\}\{23\}\{4\}$, which is among least preferred outcome for $B$; hence, neither of these three structures is stable. A similar statement is true for four out of five outcomes most preferred by firm $A$ and possible unilateral defection by firm $B$ to product-based recycling. The one exception is $\{134\}\{2\}$, from which $B$ can defect to $\{14\}\{2\}\{3\}$; as this is $B$'s most preferred outcome and among $A$'s least preferred outcome, $A$ can defect from $\{14\}\{2\}\{3\}$ to product-based recycling. This leads us to $\{12\}\{3\}\{4\}$ as an outcome from which no firm can start a sequence of defection that would ultimately lead to a stable outcome that the defecting firm prefers to $\{12\}\{3\}\{4\}$; thus, $\{12\}\{3\}\{4\}$ is stable. Similar relationship hold in other cases with low economies of scale and high substitutability level in which products in one market have similar market sizes, while in the other market one product's market size dominates the other. ∎

**Proof of Proposition 11:**

1. Consider the case $\Pi_A^{\{12\}\{34\}} > \Pi_A^{\{1\}\{2\}\{34\}} > \Pi_A^{\{1234\}} > \Pi_A^{\{123\}\{4\}} > \Pi_A^{\{12\}\{3\}\{4\}} > \Pi_A^{\{13\}\{2\}\{4\}} > \Pi_A^{\{134\}\{2\}} > \Pi_A^{\{1\}\{2\}\{3\}\{4\}} > \Pi_A^{\{1\}\{23\}\{4\}} > \Pi_A^{\{1\}\{234\}} > \Pi_A^{\{124\}\{3\}} > \Pi_A^{\{13\}\{24\}} > \Pi_A^{\{1\}\{24\}\{3\}} > \Pi_A^{\{14\}\{2\}\{3\}} > \Pi_A^{\{14\}\{23\}}$ and $\Pi_B^{\{1234\}} > \Pi_B^{\{12\}\{34\}} > \Pi_B^{\{1\}\{2\}\{34\}} > \Pi_B^{\{1\}\{234\}} > \Pi_B^{\{124\}\{3\}} > \Pi_B^{\{1\}\{24\}\{3\}} > \Pi_B^{\{12\}\{3\}\{4\}} > \Pi_B^{\{14\}\{2\}\{3\}} > \Pi_B^{\{1\}\{2\}\{3\}\{4\}} > \Pi_B^{\{123\}\{4\}} > \Pi_B^{\{134\}\{2\}} > \Pi_B^{\{14\}\{23\}} > \Pi_B^{\{1\}\{23\}\{4\}} > \Pi_B^{\{13\}\{24\}} > \Pi_B^{\{13\}\{2\}\{4\}}$, which occurs, for instance, when $\alpha_1 = 100, \alpha_2 = 200, \alpha_3 = \alpha_4 = 300, c_1 = c_2 = 2, c_3 = c_4 = 5, \gamma = 0.1, \lambda = 2, \kappa = 0.02$. We can verify that in this case $\{1234\}$, the outcome most preferred by firm $B$, is stable. The only possible defections from this outcome by firm $A$ lead to $\{14\}\{23\}$ or $\{23\}\{1\}\{4\}$, the outcomes with low preference by $A$, so $A$ does not want to defect. Now, consider the outcome most preferred by firm $A$, $\{12\}\{34\}$. This outcome is the second-most preferred by firm $B$ and any defection can only reduce its profit, so it emerges as stable as well. Thus, both all-inclusive and market-based recycling are stable in this case.

2. Next, consider the case $\Pi_A^{\{124\}\{3\}} > \Pi_A^{\{1234\}} > \Pi_A^{\{134\}\{2\}} > \Pi_A^{\{13\}\{2\}\{4\}} > \Pi_A^{\{12\}\{34\}} > \Pi_A^{\{1\}\{2\}\{34\}} > \Pi_A^{\{12\}\{3\}\{4\}} > \Pi_A^{\{14\}\{2\}\{3\}} > \Pi_A^{\{13\}\{24\}} > \Pi_A^{\{1\}\{2\}\{3\}\{4\}} > \Pi_A^{\{1\}\{24\}\{3\}} > \Pi_A^{\{123\}\{4\}} > \Pi_A^{\{14\}\{23\}} > \Pi_A^{\{1\}\{23\}\{4\}} > \Pi_A^{\{1\}\{234\}}$ and $\Pi_B^{\{1\}\{234\}} > \Pi_B^{\{1234\}} > \Pi_B^{\{12\}\{34\}} > \Pi_B^{\{1\}\{2\}\{34\}} > \Pi_B^{\{1\}\{24\}\{3\}} > \Pi_B^{\{13\}\{24\}} > \Pi_B^{\{124\}\{3\}} > \Pi_B^{\{134\}\{2\}} > \Pi_B^{\{123\}\{4\}} > \Pi_B^{\{1\}\{23\}\{4\}} > \Pi_B^{\{12\}\{3\}\{4\}} > \Pi_B^{\{1\}\{2\}\{3\}\{4\}} > \Pi_B^{\{13\}\{2\}\{4\}} > \Pi_B^{\{14\}\{23\}} > \Pi_B^{\{14\}\{2\}\{3\}}$, which occurs, for instance, when $\alpha_1 = 70, \alpha_2 = 100, \alpha_3 = 200, \alpha_4 = 400, c_1 = c_2 = 2, c_3 = c_4 = 5, \gamma = 0.5, \lambda = 1.2, \kappa = 0.02$. $\{124\}\{3\}$ is the most preferred outcome by firm $A$. Any possible sole defection by $B$ results in one of the $B$'s two least preferred outcomes, and $\{124\}\{3\}$ is stable. $\{1\}\{234\}$ is the most preferred outcome by firm $B$; $A$ can defect to $\{14\}\{23\}$, which it prefers to $\{1\}\{234\}$, and if $B$ unilaterally defects to $\{14\}\{2\}\{3\}$, it improves $A$'s profit further, so $\{1\}\{234\}$ cannot be stable. $\{1234\}$ is the second most-preferred outcome by both firms. If $A$ defects unilaterally, it leads to outcomes it prefers less, and $B$ would not want to defect as it is cannot improve its profits. Hence, $\{1234\}$ is stable as well.

   Now, suppose that $\Pi_A^{\{134\}\{2\}} > \Pi_A^{\{1234\}} > \Pi_A^{\{124\}\{3\}} > \Pi_A^{\{12\}\{34\}} > \Pi_A^{\{1\}\{2\}\{34\}} > \Pi_A^{\{14\}\{2\}\{3\}} > \Pi_A^{\{13\}\{2\}\{4\}} > \Pi_A^{\{13\}\{24\}} > \Pi_A^{\{12\}\{3\}\{4\}} > \Pi_A^{\{1\}\{2\}\{3\}\{4\}} > \Pi_A^{\{1\}\{234\}} > \Pi_A^{\{14\}\{23\}} > \Pi_A^{\{1\}\{24\}\{3\}} > \Pi_A^{\{123\}\{4\}} > \Pi_A^{\{1\}\{23\}\{4\}}$ and $\Pi_B^{\{1\}\{234\}} > \Pi_B^{\{1234\}} > \Pi_B^{\{123\}\{4\}} > \Pi_B^{\{12\}\{34\}} > \Pi_B^{\{1\}\{2\}\{34\}} > \Pi_B^{\{1\}\{23\}\{4\}} > \Pi_B^{\{1\}\{24\}\{3\}} > \Pi_B^{\{13\}\{24\}} > \Pi_B^{\{12\}\{3\}\{4\}} > \Pi_B^{\{1\}\{2\}\{3\}\{4\}} > \Pi_B^{\{134\}\{2\}} > \Pi_B^{\{14\}\{23\}} > \Pi_B^{\{13\}\{2\}\{4\}} > \Pi_B^{\{124\}\{3\}} > \Pi_B^{\{14\}\{2\}\{3\}}$, which occurs, for instance, when $\alpha_1 = \alpha_2 = 100, \alpha_3 = \alpha_4 = 300, c_1 = c_2 = 2, c_3 = c_4 = 5, \gamma = 0.9, \lambda = 2, \kappa = 0.1$. We can show stability of $\{1234\}$ similarly as above. $\{1\}\{234\}$ is most preferred outcome for $B$; any possible sole defection by $A$ results in outcomes that both $A$ and $B$ prefer less than $\{123\}\{4\}$, and $\{123\}\{4\}$ is stable. Similar analysis holds for stability of $\{134\}\{2\}$, the favorite outcome of firm $A$. Thus, $\{1234\}, \{134\}\{2\}$ and $\{1\}\{234\}$ are all in the LCS.

3. We now look at the case $\Pi_A^{\{134\}\{2\}} > \Pi_A^{\{13\}\{2\}\{4\}} > \Pi_A^{\{13\}\{24\}} > \Pi_A^{\{1234\}} > \Pi_A^{\{12\}\{34\}} > \Pi_A^{\{1\}\{2\}\{34\}} > \Pi_A^{\{124\}\{3\}} > \Pi_A^{\{14\}\{2\}\{3\}} > \Pi_A^{\{123\}\{4\}} > \Pi_A^{\{12\}\{3\}\{4\}} > \Pi_A^{\{1\}\{2\}\{3\}\{4\}} > \Pi_A^{\{1\}\{24\}\{3\}} > \Pi_A^{\{1\}\{234\}} >$

$\Pi_A^{\{14\}\{23\}} > \Pi_A^{\{1\}\{23\}\{4\}}$ and $\Pi_B^{\{123\}\{4\}} > \Pi_B^{\{1\}\{23\}\{4\}} > \Pi_B^{\{13\}\{2\}\{4\}} > \Pi_B^{\{13\}\{24\}} > \Pi_B^{\{14\}\{23\}} > \Pi_B^{\{1234\}} > \Pi_B^{\{1\}\{234\}} > \Pi_B^{\{12\}\{3\}\{4\}} > \Pi_B^{\{1\}\{2\}\{3\}\{4\}} > \Pi_B^{\{1\}\{24\}\{3\}} > \Pi_B^{\{12\}\{34\}} > \Pi_B^{\{14\}\{2\}\{3\}} > \Pi_B^{\{124\}\{3\}} > \Pi_B^{\{1\}\{2\}\{34\}} > \Pi_B^{\{134\}\{2\}}$, which occurs, for instance, when $\alpha_1 = 100, \alpha_2 = 70, \alpha_3 = 300, \alpha_4 = 150, c_1 = c_2 = 2, c_3 = c_4 = 5, \gamma = 0.75, \lambda = 1.2, \kappa = 0.07$. The outcome most preferred by $A$, $\{134\}\{2\}$, cannot be stable as a defection by $B$ to $\{14\}\{23\}$ increases its payoff, and a further sole defection by $A$ can only make $B$ better off. If $B$ defects from $\{13\}\{2\}\{4\}$ to $\{1\}\{23\}\{4\}$, which is its second most-preferred outcome, $A$ would defect to $\{14\}\{23\}$, which it prefers most but, at the same time, makes $B$ worse off. Thus, $\{13\}\{2\}\{4\}$ is stable. If $A$ defects from $\{123\}\{4\}$, which is most preferred by $B$, it leads to one of its two least preferred outcomes, so $\{123\}\{4\}$ is stable as well.

If the cost increase in the above example changes from $\lambda = 1.2$ to $\lambda = 2$, we have $\Pi_A^{\{134\}\{2\}} > \Pi_A^{\{13\}\{2\}\{4\}} > \Pi_A^{\{13\}\{24\}} > \Pi_A^{\{1234\}} > \Pi_A^{\{12\}\{34\}} > \Pi_A^{\{1\}\{2\}\{34\}} > \Pi_A^{\{123\}\{4\}} > \Pi_A^{\{12\}\{3\}\{4\}} > \Pi_A^{\{124\}\{3\}} > \Pi_A^{\{1\}\{2\}\{3\}\{4\}} > \Pi_A^{\{1\}\{24\}\{3\}} > \Pi_A^{\{14\}\{2\}\{3\}} > \Pi_A^{\{1\}\{234\}} > \Pi_A^{\{1\}\{23\}\{4\}} > \Pi_A^{\{14\}\{23\}}$ and $\Pi_B^{\{123\}\{4\}} > \Pi_B^{\{1\}\{24\}\{3\}} > \Pi_B^{\{12\}\{3\}\{4\}} > \Pi_B^{\{1\}\{2\}\{3\}\{4\}} > \Pi_B^{\{13\}\{24\}} > \Pi_B^{\{1\}\{23\}\{4\}} > \Pi_B^{\{14\}\{2\}\{3\}} > \Pi_B^{\{124\}\{3\}} > \Pi_B^{\{14\}\{23\}} > \Pi_B^{\{13\}\{2\}\{4\}} > \Pi_B^{\{1234\}} > \Pi_B^{\{1\}\{234\}} > \Pi_B^{\{12\}\{34\}} > \Pi_B^{\{1\}\{2\}\{34\}} > \Pi_B^{\{134\}\{2\}}$. We can show stability of $\{123\}\{4\}$ and instability of $\{134\}\{2\}$ similarly as above. If we consider $\{13\}\{2\}\{4\}$, $B$ can defect to $\{1\}\{4\}\{23\}$, which it prefers more than $\{13\}\{2\}\{4\}$. $A$ may chose to move from $\{1\}\{4\}\{23\}$ to $\{14\}\{23\}$, which it prefers to $\{1\}\{4\}\{23\}$ , and $B$ would still be better off than in $\{13\}\{2\}\{4\}$. Consequently, $\{13\}\{2\}\{4\}$ is not stable in this case. If we now look at $\{13\}\{24\}$, we can consider the same sequence of defections: $B$ moving to $\{1\}\{4\}\{23\}$, and $A$ moving to $\{14\}\{23\}$. However, this sequence would make $B$ worse off compared to $\{13\}\{24\}$, so $\{13\}\{24\}$ is stable, too. ■