Online Appendix - Financing Capacity with Stealing and Shirking

(Authors' names blinded for peer review)

Appendix A: Proof of Lemma 1

For all $q \geq x$, objective (2)'s first-order derivative with respect to q is $-c(1-\lambda)$, which is negative. Hence $x = q$ at an optimum. For $x = q$, the project's value $\pi_e(\cdot)$ is as in the standard newsvendor model and the optimal capacity is $\bar{F}_e^{-1}(c/r)$. Condition (3) implies that $e = 1$ is optimal. ■

Appendix B: Proof of Lemma 2

Point (i): Objective (5) is strictly increasing in I. The only upper bound on I is set by condition (6), which must thus be binding.

Point (ii): The firm makes zero profit if the project is abandoned ($q = 0$) and must thus make a non-negative profit at an optimum. To prove that $e = 0$ cannot be optimal, we show that if $e = 0$, the firm's objective (5) is strictly negative. For $e = 0$, it is

$$\mathbb{E}\left[P_{0,x,q} - R(P_{0,x,q})\right] + \lambda c(q-x) + I - cq - \kappa_0$$

which is equal to

$$\left(\mathbb{E}\left[P_{0,x,q}\right] - \mathbb{E}\left[P_{0,x,x}\right]\right) + \mathbb{E}\left[P_{0,x,x}\right] - \mathbb{E}\left[R(P_{0,x,q})\right] + \lambda c(q-x) + I - (cq - cx) - cx - \kappa_0$$

or, noting further that $P_{0,x,q} = P_{0,x,x}$,

$$\left(\mathbb{E}\left[P_{0,x,x}\right] - cx - \kappa_0\right) + \left(I - \mathbb{E}\left[R(P_{0,x,q})\right]\right) - c(1-\lambda)(q-x)$$

The first term is $\pi_0(x) - \kappa_0$ which is strictly negative from (3). The second term is negative due to (6). The third term is negative from $\lambda < 1$. Hence objective (5) is strictly negative, which implies that $q = 0$ dominates $q > 0$ and $e = 0$.

Point (iii): Suppose contract (q, I, R) satisfying constraints (6)-(9) implies diversion $(x < q)$. We show that it is strictly dominated by another contract with no diversion. Consider contract (x, I', R') with investment $I' = I - c(q-x)$ and claim $R'(\cdot)$ defined for all p as, $R'(p) = R(p) - c(q-x)$.

First, we check that contract (x, I', R') is feasible, i.e., that claim $R'(\cdot)$ is feasible. $R'(\cdot)$ inherits $R(\cdot)$'s monotonicity and, since $P_{0,x,x} = P_{0,x,q}$, it also inherits its satisfying limited liability:

$$R(P_{0,x,q}) \leq P_{0,x,q} \Rightarrow R(P_{0,x,q}) - c(q-x) \leq P_{0,x,q} \Rightarrow R'(P_{0,x,x}) \leq P_{0,x,x}$$

We now check that (x, I', R') satisfies conditions (6)-(9) given (q, I, R) does. Condition (6) holds:

$$\mathbb{E}[R(P_{e,x,q})] \geq I \Rightarrow \mathbb{E}[R(P_{e,x,x}) - c(q-x)] \geq I - c(q-x) \Rightarrow \mathbb{E}[R'(P_{e,x,x})] \geq I'$$

Next, for all $\epsilon \in \{0,1\}$ and $y \in [0,x]$, define $\Pi_{\epsilon,y,q} \equiv \mathbb{E}[P_{\epsilon,y,q} - R(P_{\epsilon,y,q})] + \lambda c(q-y) + I - cq - \kappa_\epsilon$ and $\Pi'_{\epsilon,y,q} \equiv \mathbb{E}[P_{\epsilon,y,q} - R'(P_{\epsilon,y,q})] + \lambda c(q-y) + I' - cq - \kappa_\epsilon$. We have:

$$\begin{aligned}
\Pi_{\epsilon,y,q} &= \mathbb{E}[P_{\epsilon,y,q} - R(P_{\epsilon,y,q})] + \lambda c(q-y) + I - cq - \kappa_\epsilon \\
&= \mathbb{E}[P_{\epsilon,y,x} - R(P_{\epsilon,y,x})] + \lambda c(q-y) + I' - cx - \kappa_\epsilon \\
&= \mathbb{E}[P_{\epsilon,y,x} - R'(P_{\epsilon,y,x})] - c(q-x) + \lambda c(q-y-x+x) + I' - cx - \kappa_\epsilon \\
&= \mathbb{E}[P_{\epsilon,y,x} - R'(P_{\epsilon,y,x})] - c(1-\lambda)(q-x) + \lambda c(x-y) + I' - cx - \kappa_\epsilon
\end{aligned}$$

Hence:

$$\Pi_{\epsilon,y,q} = \Pi'_{\epsilon,y,x} - c(1-\lambda)(q-x) \tag{1}$$

Condition (7) holds as using (1) we have:

$$\text{if } q > 0, \quad \Pi_{e,y,q} \geq W \Rightarrow \text{if } x > 0, \quad \Pi'_{e,y,x} \geq c(1-\lambda)(q-x) \geq 0$$

Condition (8) holds:

$$I \geq cq \Rightarrow (I - c(q-x)) \geq cq - c(q-x) \Rightarrow I' \geq cx$$

Condition (9) holds as using (1) we have:

$$\forall y \in [0,q],\ \forall \epsilon \in \{0,1\},\ \Pi_{e,x,q} \geq \Pi_{\epsilon,y,q} \Rightarrow \Pi'_{e,x,x} - c(1-\lambda)(q-x) \geq \Pi'_{\epsilon,y,x} - c(1-\lambda)(q-x) \Rightarrow \Pi'_{e,x,x} \geq \Pi'_{\epsilon,y,x}$$

Last, objective (5) is strictly larger for (x, I', R') than for (q, I, R) as (1) implies:

$$\Pi_{e,x,q} = \Pi'_{e,x,x} - c(1-\lambda)(q-x) < \Pi'_{e,x,x}$$

Thus the optimal contract must induce no diversion, i.e., $x = q$. ■

Appendix C: Proof of Theorem 1

For $0 \le x \le q$ and $e \in \{0,1\}$, define

$$\bar{r}_e(q,x) \equiv \mathbb{E}\left[P_{e,x,q} - R(P_{e,x,q})\right] + \lambda c(q-x). \tag{2}$$

For contract $(q,R) \in \mathbb{R}^+ \times \mathcal{C}$, denote the firm's optimal decisions as $e^*_{q,R}$ and $x^*_{q,R}$. For a given $x \in [0,q]$ and $e = 0,1$ define $e^*_{q,R}(x) = \arg\max_{e=0,1} \bar{r}_e(q,x)$ and $x^*_{q,R}(e) = \arg\max_{x\in[0,q]} \bar{r}_e(q,x)$. Further, for all $x \le q$, denote the cashflow implied by demand realization $u \ge 0$ as $p_{x,q}(u) \equiv r(u \wedge x)$.

We prove the theorem by showing that if optimal contracts with $q > 0$ exist, they include one with a debt claim (Lemma 1) raising the needed funds cq (Lemma 2).

Lemma 1 *If an optimal contract $(q,R) \in \mathbb{R}^*_+ \times \mathcal{C}$ exists, then (i) a debt claim $L \in \mathcal{L}$ exists such that $\mathbb{E}[L(P_{1,q,q})] = \mathbb{E}[R(P_{1,q,q})]$, and (ii) it is such that contract (q,L) is also optimal.*

Proof. Let (q,R) be an optimal contract with $q > 0$. We have $e^*_{q,R} = 1$ and $x^*_{q,R} = q$ (Lemma 2). Point (i): For $L_K \in \mathcal{L}$ with face value $K \in [0, rq]$ and given q. We have:

$$\mathbb{E}[L_K(P_{1,q,q})] = \int_0^{K/r} p_{q,q}(u) f_1(u) du + K \cdot \bar{F}_1(K/r)$$

For $K = 0$, $\mathbb{E}[D_0(P_{1,q,q})] = \int_0^0 p_{q,q}(u) f_1(u) du + 0 \cdot \bar{F}_1(0) = 0$. That is, debt with zero face value has zero value. For $K = rq$,

$$\mathbb{E}[L_{rq}(P_{1,q,q})] = \int_0^q (p_{q,q}(u) \wedge rq) f_1(u) du = \int_0^q p_{q,q}(u) f_1(u) du = \mathbb{E}[P_{1,q,q}].$$

That is, if debt's face value rq exceeds the highest cashflow realization possible, the investor receives the whole cashflow. Moreover, the expression is continuous and strictly increasing in K over $[0, rq]$, taking values from 0 to $\mathbb{E}[P_{1,q,q}]$ since

$$\frac{\partial \mathbb{E}[L_K(P_{1,q,q})]}{\partial K} = \frac{1}{r} \cdot p_{q,q}(K/r) f_1(K/r) + K \cdot \frac{1}{r} \cdot (-f_1(K/r)) + \bar{F}_1(K/r).$$

Given that $p_{q,q}(K/r) = r(\frac{K}{r} \wedge q) = K$, we have $\partial \mathbb{E}[L_K(P_{1,q,q})]/\partial K = \bar{F}_1(K/r) > 0$.

Finally, we have $\mathbb{E}[R(P_{1,q,q})] \in [0, rq]$. Indeed, constraint (12) implies $\mathbb{E}[R(P_{1,q,q})] \ge cq \ge 0$, limited liability implies $R(P_{1,q,q}) \le P_{1,q,q}$ and thus $\mathbb{E}[R(P_{1,q,q})] \le \mathbb{E}[P_{1,q,q}]$.

Point (ii): To show that contract (q,L) is optimal, it suffices to show that $e^*_{q,L} = 1$ and $x^*_{q,L} = q$. To do so, we show that the fact that optimal contract (q,R) satisfies condition (13) implies that contract (q,L) satisfies it as well, i.e., $\forall \epsilon \in \{0,1\},\ \forall y \in [0,q]$

$$\mathbb{E}[P_{1,q,q} - R(P_{1,q,q})] - \kappa_1 \ge \mathbb{E}[P_{\epsilon,y,q} - R(P_{\epsilon,y,q})] - \lambda c(q-y) - \kappa_\epsilon$$

$$\Rightarrow \mathbb{E}[P_{1,q,q} - L(P_{1,q,q})] - \kappa_1 \geq \mathbb{E}[P_{\epsilon,y,q} - L(P_{\epsilon,y,q})] - \lambda c(q-y) - \kappa_\epsilon$$

After simplification, this amounts to showing that

$$\mathbb{E}[P_{1,q,q} - R(P_{1,q,q})] - \mathbb{E}[P_{\epsilon,y,q} - R(P_{\epsilon,y,q})] \geq \kappa_1 - \kappa_\epsilon - \lambda c(q-y)$$

$$\Rightarrow \mathbb{E}[P_{1,q,q} - L(P_{1,q,q})] - \mathbb{E}[P_{\epsilon,y,q} - L(P_{\epsilon,y,q})] \geq \kappa_1 - \kappa_\epsilon - \lambda c(q-y)$$

which, both inequalities' right-hand sides being identical, amounts to showing

$$\mathbb{E}[P_{1,q,q} - L(P_{1,q,q})] - \mathbb{E}[P_{\epsilon,y,q} - L(P_{\epsilon,y,q})] \geq \mathbb{E}[P_{1,q,q} - R(P_{1,q,q})] - \mathbb{E}[P_{\epsilon,y,q} - R(P_{\epsilon,y,q})]$$

Denoting $\Delta(\cdot) \equiv L(\cdot) - R(\cdot)$ and given that $\mathbb{E}[L(P_{1,q,q})] = \mathbb{E}[R(P_{1,q,q})]$, the condition amounts to

$$\forall \epsilon \in \{0,1\}, \forall y \in [0,q], \quad \mathbb{E}[\Delta(P_{\epsilon,y,q})] \geq 0 \tag{3}$$

We complete the proof in two steps, first showing that for all $y \in [0,q]$, condition (3) holds for $\epsilon = 1$ and then showing that it holds for $\epsilon = 0$. We start with the case $\epsilon = 1$. We have for all $u \in \mathbb{R}^+$,

$$\begin{aligned} p_{y,q}(u) &= r(u \wedge y) = r(u \wedge y \wedge q) \quad \text{(because } y \leq q\text{)} \\ &= r[(u \wedge q) \wedge (y \wedge q)] = [r(u \wedge q)] \wedge [r(y \wedge q)] = p_{q,q}(u) \wedge p_{q,q}(y) \end{aligned}$$

Thus, denoting debt claim L's face value as K, we have

$$L\left(p_{y,q}\left(u\right)\right) = p_{y,q}\left(u\right) \wedge K = p_{q,q}(u) \wedge p_{q,q}(y) \wedge K.$$

Case 1: $y \geq \hat{d}(K)$ which amounts to $p_{q,q}(y) \geq K$. In that case, we have:

$$\forall u \in \mathbb{R}^+, \quad L(p_{y,q}(u)) = p_{q,q}(u) \wedge p_{q,q}(y) \wedge K = p_{q,q}(u) \wedge K = L(p_{q,q}(u))$$

Hence $\mathbb{E}[\Delta(P_{\epsilon,y,q})] \equiv \mathbb{E}[L(P_{1,y,q})] - \mathbb{E}[R(P_{1,y,q})] = \mathbb{E}[L(P_{1,q,q})] - \mathbb{E}[R(P_{1,y,q})]$. Moreover, $R(\cdot)$ being non-decreasing, $p_{y,q}(u) \leq p_{q,q}(u)$ implies $R(p_{y,q}(u)) \leq R(p_{q,q}(u))$ which implies, by definition of $L(\cdot)$,

$$\mathbb{E}[\Delta(P_{1,y,q})] \geq \mathbb{E}[L(P_{1,q,q})] - \mathbb{E}[R(P_{1,q,q})] \geq 0.$$

Case 2: $y < \hat{d}(K)$ which amounts to $p_{q,q}(y) < K$. In that case, we have:

$$\forall u \in \mathbb{R}^+, \quad p_{y,q}(u) \leq p_{y,q}(y) \leq p_{q,q}(y) < K \tag{4}$$

$$\begin{aligned} \text{Therefore,} \quad \forall u \in \mathbb{R}^+, \quad L(p_{y,q}(u)) - R(p_{y,q}(u)) &= p_{y,q}(u) \wedge K - R(p_{y,q}(u)) \\ &= p_{y,q}(u) R(p_{y,q}(u)) \quad \text{from condition (4)} \\ &\geq 0 \quad \text{due to limited liability} \end{aligned}$$

which implies $\mathbb{E}[\Delta(P_{1,y,q})] \equiv \mathbb{E}[L(P_{1,y,q})] - \mathbb{E}[R(P_{1,y,q})] \geq 0$.

Now consider the case $\epsilon = 0$. First, there exists $p^* \in [0, rq]$ such that $\Delta(p) \geq 0$ for all $p \in [0, p^*)$ and $\Delta(p) \leq 0$ for all $p \in (p^*, rq]$. (This is because $\partial L(p)/\partial p$ is as high as possible for $p < K$ and as low as possible for $p > K$.) Define $g_{e,x}(\cdot)$ as the distribution of cashflows p given effort e and effective capacity x, i.e., $g_{e,x}(p) = f_e\left(P^{-1}_{e,x,q}(p)\right)$ for $p \in [0, rx)$ and $g_{e,x}(rx) = \bar{F}_e(x)$. For all $y \in [0, q]$, we have

$$\mathbb{E}[\Delta(P_{0,y,q})] = \int_0^{p_{y,q}(y)} \Delta(p) g_{0,y}(p) dp.$$

Case 1: $p^* \leq p_{y,q}(y)$. In that case, we have:

$$\begin{aligned}
\mathbb{E}[\Delta(P_{0,y,q})] &= \int_0^{p^*} \Delta(p) g_{0,y}(p) dp + \int_{p^*}^{p_{y,q}(y)} \Delta(p) g_{0,y}(p) dp \\
&= \int_0^{p^*} \Delta(p) g_{1,y}(p) \frac{g_{0,y}(p)}{g_{1,y}(p)} dp + \int_{p^*}^{p_{y,q}(y)} \Delta(p) g_{1,x}(p) \frac{g_{0,y}(p)}{g_{1,y}(p)} dp
\end{aligned}$$

It can be shown easily that g_0 and g_1 inherit the MLRP from f_0 and f_1, i.e., $g_{0,y}(p)/g_{1,y}(p)$ increases strictly over $[0, p_{y,q}(y)]$ (see de Véricourt and Gromb (2018)).

$$\mathbb{E}[\Delta(P_{0,y,q})] \geq \frac{g_{0,y}(p^*)}{g_{1,y}(p^*)} \int_0^{p^*} \Delta(p) g_{1,y}(p) dp + \frac{g_{0,y}(p^*)}{g_{1,y}(p^*)} \int_{p^*}^{p_{y,q}(y)} \Delta(p) g_{1,y}(p) dp \geq \frac{g_{0,y}(p^*)}{g_{1,y}(p^*)} \cdot \mathbb{E}[\Delta(P_{1,y,q})].$$

Since we have shown that $\mathbb{E}\Delta(P_{1,y,q})] \geq 0$, this implies $\mathbb{E}[\Delta(P_{0,y,q})] \geq 0$.

Case 2: $p^* > p_{y,q}(y)$, which amounts to $\forall p \in [0, p_{y,q}(y)],\ \Delta(p) > 0$. In that case, we have:

$$\mathbb{E}[\Delta(P_{0,y,q})] = \int_0^{p_{y,q}(y)} \Delta(p) g_{0,y}(p) dp > 0.$$

Last we show (q, L) to be an optimal contract. We have shown that $e^*_{q,L} = 1$ and $x^*_{q,L} = q$. Hence (q, L) is optimal because objective (10) and condition (11) only depend on e, q and x, which are the same as for (q, R), and $\mathbb{E}\left[L\left(P_{1,q^*,q^*}\right)\right] = \mathbb{E}\left[R\left(P_{1,q^*,q^*}\right)\right]$ ensures condition (12) holds. ■

Lemma 2 *If contract $(q, L) \in \mathbb{R}^*_+ \times \mathcal{L}$ is optimal, a unique $L^* \in \mathcal{L}$ exists such that $E\left[L^*\left(P_{1,q,q}\right)\right] = cq$ and it is such that (q, L^*) is also optimal.*

Proof. For all $e \in \{0, 1\}$, $q \in \mathbb{R}^*_+$ and $x \in [0, q]$, the firm's objective (5) given debt with face value K is, using a slight abuse of notations,

$$\Pi(K, e, x, q) = r\left(\int_{K/r}^{x} \bar{F}_e(u)\, du\right)^+ + \lambda c(q - y) - \kappa_e.$$

We have,

$$\frac{\partial \Pi}{\partial K}(K,e,x,q) = -\mathbf{1}_{\{x\geq K/r(K,q)>0\}}\cdot \bar{F}_e\left(K/r\left(K,q\right)\right)\leq 0 \tag{5}$$

which also implies

$$\frac{\partial \Pi}{\partial K}(K,1,q,q) - \frac{\partial \Pi}{\partial K}(K,e,x,q) - \mathbf{1}_{\{q\geq K/r\}}\bar{F}_1\left(K/r>0\right) + \mathbf{1}_{\{x\geq K/r>0\}}\cdot \bar{F}_e\left(K/r\right)\leq 0, \tag{6}$$

where the inequality holds because $x\leq q$ and $\bar{F}_1(\cdot)\geq \bar{F}_0(\cdot)$ from MLRP. Now consider contract (q,L) as in the lemma, with L's face value K. By Lemma 2, $e_L^*=1$, $x_L^*=q$. Since $\mathbb{E}[L(P_{1,q,q})]\geq cq$, $L^*\in\mathcal{L}$ with face value $K^*\leq K$ exists such that $\mathbb{E}[L^*(P_{1,q,q})]=cq$ as per previous arguments. By (6), we have $e^*_{L^*}=1$ and $x^*_{L^*}=q$. Moreover, given (5), the firm's objective (5) decreases with K and so (6) holds for K^* as it does for K. Hence, (q,L^*) is also optimal. ■

Appendix D: Counter-example

We provide an elementary example with two moral hazards satisfying the MLRP in which debt is suboptimal. Consider a wealthless firm with a project requiring an investment I and generating a random cashflow. The simplest way of capturing our idea is to consider a three point cashflow support: $\{p_0,p_1,p_2\}$ with $p_2>p_1>p_0$. The firm chooses two efforts $(e_1,e_2)\in\{0,1\}^2$ at cost $\kappa_{i,j}\geq 0$ for $e_1=i$ and $e_2=j$. Efforts (e_1,e_2) determine the probability distribution of the cashflow P_{e_1,e_2}, with $f_{i,j}(p_k)\equiv \Pr\left[P_{i,j}=p_k \mid e_1=i,e_2=j\right]$. Thus, if the firm undertakes the project and exerts efforts $(e_1,e_2)=(i,j)$, its expected profit gross of effort cost is: $\pi_{i,j}\equiv E\left[P_{i,j}\right]=\sum_{k=0,1,2} f_{i,j}(p_k)r_k - I$.

We assume that the first best is to undertake the project and exert $(e_1,e_2)=(1,0)$, i.e.,

$$\pi_{1,0}-\kappa_{1,0}>0 \quad \text{and} \quad \forall (i,j)\in\{0,1\}^2 \;\; \pi_{1,0}-\kappa_{1,0}>\pi_{i,j}-\kappa_{i,j} \tag{7}$$

We assume that for a given e_i, the probability distributions satisfy the MLRP for $e_{j\neq i}$, i.e.,

$$\forall j=0,1, \qquad \frac{f_{1,j}(p_2)}{f_{0,j}(p_2)}>\frac{f_{1,j}(p_1)}{f_{0,j}(p_1)}>\frac{f_{1,j}(p_0)}{f_{0,j}(p_0)} \quad \text{and} \quad \forall i=0,1, \qquad \frac{f_{i,1}(p_2)}{f_{i,0}(p_2)}>\frac{f_{i,1}(p_1)}{f_{i,0}(p_1)}>\frac{f_{i,1}(p_0)}{f_{i,0}(p_0)} \tag{8}$$

The firm can raise funds from a competitive investor against a claim with repayments $R(p)$. For $(e_1,e_2)=(1,0)$, the project can be financed with a given claim $R(\cdot)$ if and only if

$$E\left[R(P_{1,0})\right]\equiv \sum_{k=0,1,2} f_{1,0}(p_k)R(p_k)\geq I \tag{9}$$

Denoting $\bar{r}_{i,j}\equiv E\left[P_{i,j}-R(P_{i,j})\right]$, the firm chooses $(e_1,e_2)=(i,j)$ to maximize

$$\bar{r}_{i,j}-\kappa_{i,j}=\sum_{k=0,1,2} f_{i,j}(p_k)R(p_k)-\kappa_{i,j} \tag{10}$$

and the first-best is implemented if and only if

$$\forall (i,j) \in \{0,1\}^2 \qquad \bar{r}_{1,0} - \kappa_{1,0} \geq \bar{r}_{i,j} - \kappa_{i,j} \tag{11}$$

We now exhibit an example in which the first-best cannot be implemented with debt but can be implemented with non-debt claims, for example, equity, implying that debt is strictly suboptimal.

Lemma 3 *For the funding problem defined in (9), (10) and (11), parameters exist such that debt financing cannot achieve first best, while equity financing does, implying that debt is suboptimal.*

Assume the project's investment cost is $I = 99$, the possible cashflows are $(p_0, p_1, p_2) = (0, 100, 156)$, the effort costs are $(\kappa_{0,0}, \kappa_{1,0}, \kappa_{0,1}, \kappa_{1,1}) = (0, 10, 10, 20)$, and the probability distributions are

$$\begin{aligned}
\left(p^2_{0,0}, p^1_{0,0}, p^0_{0,0}\right) &= (0.01,\ 0.05\ , 0.94) \qquad \left(p^2_{1,0}, p^1_{1,0}, p^0_{1,0}\right) = (0.19\ , 0.80,\ 0.01) \\
\left(p^2_{0,1}, p^1_{0,1}, p^0_{0,1}\right) &= (0.20,\ 0.60,\ 0.20) \qquad \left(p^2_{1,1}, p^1_{1,1}, p^0_{1,1}\right) = (0.35,\ 0.65,\ 0.00)
\end{aligned} \tag{12}$$

First, we check that the first best is to undertake the project and exert $(e_1, e_2) = (1, 0)$. We have $\pi_{0,0} - \kappa_{0,0} = -92.44$, $\pi_{1,0} - \kappa_{1,0} = 0.64$, $\pi_{0,1} - \kappa_{0,1} = -17.80$, and $\pi_{1,1} - \kappa_{1,1} = 0.60$. Hence (7) holds. Second, we can check that for a given $e_i, i = 0, 1$, the MLRP holds for e_{1-i}, i.e., that (8) holds:

$$\begin{aligned}
&\frac{f_{1,0}(p_2)}{f_{0,0}(p_2)} = 19 > \frac{f_{1,0}(p_1)}{f_{0,j}(p_1)} = 16 > \frac{f_{1,0}(p_0)}{f_{0,0}(p_0)} = 0 \qquad \frac{f_{0,1}(p_2)}{f_{0,0}(p_2)} = 20 > \frac{f_{0,1}(p_1)}{f_{0,0}(p_1)} = 12 > \frac{f_{0,1}(p_0)}{f_{0,0}(p_0)} = 0 \\
&\frac{f_{1,1}(p_2)}{f_{1,0}(p_2)} = 2 > \frac{f_{1,1}(p_1)}{f_{1,0}(p_1)} = 1 > \frac{f_{1,1}(p_0)}{f_{1,0}(p_0)} = 0 \qquad \frac{f_{1,1}(p_2)}{f_{0,1}(p_2)} = 2 > \frac{f_{1,1}(p_1)}{f_{0,1}(p_1)} = 1 > \frac{f_{1,1}(p_0)}{f_{0,1}(p_0)} = 0
\end{aligned}$$

We then show that the first-best cannot be implemented with debt, i.e., that for all $K \geq 0$, if $R(p) = p \wedge K$ satisfies (9), it violates (11). For debt with face value K, (9) is written $0.19 \times (156 \wedge K) + 0.80 \times (100 \wedge K) \geq 99$ which holds if and only if $K \geq 100$. For all $K \geq 100$, we have

$$\bar{r}_{1,0} - \kappa_{1,0} - (\bar{r}_{0,1} - \kappa_{0,1}) = -0.01 \times (156 - K)^+ \quad \text{and} \quad \bar{r}_{1,0} - \kappa_{1,0} - (\bar{r}_{0,0} - \kappa_{0,0}) = 0.18 \times (156 - K) - 10$$

Hence, no debt contract implements the first-best. Indeed, for $K \in [100, 156)$, we have $\Pi_{1,0}(K) < \Pi_{1,0}(K)$ and for $K \geq 156$, we have $\Pi_{1,0}(K) < \Pi_{0,0}(K)$.

Last, we show that the first-best can be implemented with equity, i.e., that there exists $\alpha \in [0, 1]$ such that $R(p) = \alpha \cdot p$ satisfies (9) and (11). For a claim on a fraction α of equity, (9) is written

$$\alpha\,(0.19 \times 156 + 0.80 \times 100 + 0.01 \times 0) \geq 99 \quad \text{or} \quad \alpha \geq 0.902956 \tag{13}$$

We consider $\alpha = 0.90296$. In that case, the firm's expected profit for $(e_1, e-2) = (1, 0)$ is

$$\bar{r}_{1,0} - \kappa_{1,0} = 0.90296 \times (0.19 \times 156 + 0.80 \times 100 + 0.01 \times 0) - 10 = 0.640,$$

and similarly $\bar{r}_{0,0} - \kappa_{0,0} = 0.637$, $\bar{r}_{0,1} - \kappa_{0,1} = -1.150$ and $\bar{r}_{1,1} - \kappa_{1,1} = -8.393$. Thus the firm optimally chooses $(e_1, e_2) = (1, 0)$ and the first-best is implemented. This proves that in our example, debt financing is strictly suboptimal despite the MLRP. ■

The intuition for debt being suboptimal in our example is as follows. Under the MLRP, debt maximizes the incentive to exert effort in each moral hazard separately. Thus debt also maximizes the incentive to choose $(e_1, e_2) = (i, j)$ over $(e_1, e_2) = (i', j')$ with $i \geq i'$ and $j \geq j'$. However, MLRP does not imply that debt maximizes the incentive to choose $(e_1, e_2) = (i, j)$ over $(e_1, e_2) = (i', j')$ with $i < i'$ and $j > j'$ or $i > i'$ and $j < j'$. And indeed this is the binding constraint in our example. Looking at the probability distribution in (12), going from $(e_1, e_2) = (1, 0)$ to $(e_1, e_2) = (0, 1)$ increases the variance of cashflows and decreasing their average, and debt motivates the firm to engage in such risk-shifting whereas equity nullifies those incentives (Jensen and Meckling 1976). By contrast, in our model, the complementarity between diversion and shirking ensures that mitigating one moral hazard with debt also mitigates the other, hence the optimality of debt for both moral hazards.

Appendix E: Proof of Theorems 2 and 3 and Corollaries

E.1. Preliminaries

Recall the definition of $\bar{r}_e(q, x)$ in (2). For all $q \leq \bar{q}_1$, denote $\hat{d}(q) \equiv K(q)/r \leq q$ the lowest demand realization allowing the firm to repay the optimal face value $K(q)$ of Theorem 1. For q and x and effort e, the firm's objective (5) under the debt claim of Theorem 1 is

$$\bar{r}_e(q, x) = r \left(\int_{\hat{d}(q)}^{x} \bar{F}_e(u)\, du \right)^+ + \lambda c(q - x). \tag{14}$$

Lemma 4 *We have, $\partial \hat{d}(q)/\partial q = \bar{F}_1(q^{FB})/\bar{F}_1\left(\hat{d}(q)\right)$ and $\hat{d}(q)$ increases with q.*

Proof. Taking the first-order derivative of (14) with respect to q yields

$$\frac{\partial K(q)}{\partial q} - r \frac{\partial \hat{d}(q)}{\partial q} F_1\left(\frac{K}{r}\right) = c$$

and since $\partial \hat{d}(q)/\partial q = \partial K(q)/\partial q/r$, this can be rewritten as

$$\frac{\partial \hat{d}(q)}{\partial q} = \frac{c}{r} \frac{1}{1 - F_1\left(\hat{d}(q)\right)} = \frac{\bar{F}_1(q^{FB})}{\bar{F}_1\left(\hat{d}(q)\right)}.$$

■

E.2. Problem reformulation

We first derive properties of optimal effective capacity x, given capacity q and effort e. For $e=0,1$, define $q_e^{FB} \equiv \bar{F}_e^{-1}(c/r)$, the standard newsvendor quantity for demand D_e.

Lemma 5 *For $e \in \{0,1\}$, define $q_e^{\lambda\star} \equiv \bar{F}_e^{-1}(\lambda c/r)$ for $\lambda \in (0,1]$ and $q_e^{0\star} \equiv +\infty$. We have: (i) $q_e^{\lambda\star}$ is decreasing in λ over $[0,1]$ from $+\infty$ to q_e^{FB}, and (ii) for all $q \le \bar{q}_1$, $\arg\max_{x\in[0,q]} \bar{r}_e(q,x) \in \{0,\, q\wedge q_e^{\lambda\star}\}$ with $\bar{r}_e(q,q) \le \bar{r}_e(q, q\wedge q_e^{\lambda\star})$.*

Proof. From (14), we have for $x \in [\hat{d}(q), q]$, $d\bar{r}_e(q,x)/dx = r\bar{F}_e(x) - \lambda c$ which is decreasing in x, equal to zero if $x = q_e^{\lambda\star} \ge \hat{d}(q)$ and thus maximum at $x = q\wedge q_e^{\lambda\star}$ if $q_e^{\lambda\star} \ge \hat{d}(q)$ and at $x = \hat{d}(q)$ otherwise. For $x \in [0, \hat{d}(q)]$, $\bar{r}_e(q,x) = \lambda c(q-x)$, which being decreasing in x, is maximum at $x=0$. Last, $q_e^{\lambda\star}$ is decreasing in λ as $F_e(\cdot)$ is a cdf. Further, $q_e^{\lambda\star} = \bar{F}_e^{-1}(c/r) = q^{FB}$ and thus $q_e^{FB} < q_e^{\lambda\star}$. ■

From Lemma 5, constraint (13) is rewritten as

$$\bar{r}_1(q,q) - \lambda c q \ge 0, \tag{15}$$

$$\bar{r}_1(q,q) - \bar{r}_1(q,\, q\wedge q_1^{\lambda\star}) \ge 0, \tag{16}$$

$$\Phi_1^\lambda(q,\Delta\kappa) \equiv \bar{r}_1(q,q) - \lambda c q - \Delta\kappa \ge 0, \tag{17}$$

$$\Phi_2^\lambda(q,\Delta\kappa) \equiv \bar{r}_1(q,q) - \bar{r}_0(q,\, q\wedge q_0^{\lambda\star}) - \Delta\kappa \ge 0. \tag{18}$$

(17) implies (15), i.e., if the firm diverts the full capacity ($x=0$), it will not exert effort. Further from Lemma 5, if $q > q_1^{\lambda\star}$, $\arg\max_{x\in[0,q]} \bar{r}_e(q,x) < q$ and so (15) or (16) is violated. Hence, $q \le q_1^{\lambda\star}$ must hold, in which case (16) holds.

E.3. Properties of Φ_1^λ and Φ_2^λ

Lemma 6 *(i) For all $q \in [0,\bar{q}_1]$, a unique $\hat{\lambda}_1(q) \in (0,1]$ exists such that $\lambda \in [0,\hat{\lambda}_1(q)] \Leftrightarrow \Phi_1^\lambda(q,\Delta\kappa) \ge \Phi_2^\lambda(q,\Delta\kappa)$, with equality if and only if $\lambda = \hat{\lambda}_1(q)$. (ii) For all $q \ge q^{FB}$, $\hat{\lambda}_1(q)$ is decreasing in q.*

Proof. Point (i): Define $\hat{\lambda}_1(q) \equiv \max\{\lambda \in (0,1],\, s.t.\, \Delta\Phi(q,\lambda) \ge 0\}$, where using (14), we have,

$$\Delta\Phi(q,\lambda) \equiv \Phi_1^\lambda(q,\Delta\kappa) - \Phi_2^\lambda(q,\Delta\kappa) = \bar{r}_0(q,\, q\wedge q_0^{\lambda\star}) - \lambda c q = r\left(\int_{\hat{d}(q)}^{q\wedge q_0^{\lambda\star}} \bar{F}_0(u)\,du\right)^+ - \lambda c\left(q\wedge q_0^{\lambda\star}\right).$$

For $q_0^{\lambda\star} \ge q$, $\Delta\Phi(q,\lambda) = r\int_{\hat{d}(q)}^{q} \bar{F}_0(u)\,du - \lambda c q$, which is decreasing in λ. So too for $\hat{d}(q) \le q_0^{\lambda\star} < q$, as

$$\frac{\partial\Delta\Phi}{\partial\lambda}(q,\lambda) = r\bar{F}_0(q_0^{\lambda\star})\frac{\partial q_0^{\lambda\star}}{\partial\lambda} - \lambda c\frac{\partial q_0^{\lambda\star}}{\partial\lambda} - c q_0^{\lambda\star} = -c q_0^{\lambda\star} < 0.$$

For $q_0^{\lambda\star} < \hat{d}(q)$, $\Delta\Phi(q,\lambda) = -c\lambda q_0^{\lambda\star} < 0$. Hence, $q_0^{\lambda\star}$ being decreasing in λ (Lemma 5(i)) and $\Delta\Phi(q,\cdot)$ is continuous, $\Delta\Phi(q,\cdot)$ is decreasing for all λ such that $q_0^{\lambda\star} \ge \hat{d}(q)$ and negative otherwise. Thus $\hat{\lambda}_1(q)$ exists and is unique. Further $\Delta\Phi(q,0) > 0$ implies $\hat{\lambda}_1(q) > 0$.

Point (ii): Assume $q \geq q^{FB}$. For $q \leq q_0^{\lambda\star}$, and using Lemma 5(i), we have

$$\frac{\partial \Delta\Phi}{\partial q}(q,\lambda) = r\left[\bar{F}_0(q) - \bar{F}_0(\hat{d}(q))\frac{\partial \hat{d}}{\partial q}(q)\right] = r\left[\bar{F}_0(q) - \bar{F}_1\left(q^{FB}\right)\frac{\bar{F}_0\left(\hat{d}(q)\right)}{\bar{F}_1\left(\hat{d}(q)\right)}\right] - \lambda c$$
$$< r\left[\bar{F}_0(q) - \bar{F}_1\left(q^{FB}\right)\frac{\bar{F}_0(q)}{\bar{F}_1(q)}\right] < 0$$

where the first inequality holds from the MLRP and the last one from $q \geq q^{FB}$. For $q \geq q_0^{\lambda\star}$, $\Delta\Phi(q,\lambda) < 0$ if $q_0^{\lambda\star} < \hat{d}(q)$ and $\Delta\Phi(q,\lambda) = r\int_{\hat{d}(q)}^{q_0^{\lambda\star}} \bar{F}_0(u)\,du - \lambda c q_0^{\lambda\star}$ otherwise, which is decreasing in q. Hence $\Delta\Phi(q,\lambda)$ is decreasing in q when $q \geq q^{FB}$ and $\Delta\Phi(q,\lambda) \geq 0$, and $\Delta\Phi(q,\lambda)$ being decreasing in λ, $\hat{\lambda}_1(q)$ is decreasing in q. ■

Lemma 7 *$\Phi_1^\lambda(q,\Delta\kappa)$ is strictly concave in q for $q \in [0,\bar{q}_1]$ and decreasing in q for $q \geq q^{FB}$.*

Proof. From Lemma 4, the first order derivative of Φ_1^λ with respect to q yields,

$$\frac{\partial \Phi_1^\lambda}{\partial q}(q,\Delta\kappa) = r\left[\bar{F}_1(q) - \bar{F}_1(\hat{d}(q))\frac{d\hat{d}}{\partial q}(q)\right] - \lambda c = r\left[\bar{F}_1(q) - \bar{F}_1(q^{FB})\right] - \lambda c,$$

which is strictly decreasing in q over $[0,\bar{q}_1]$ and negative for $q \geq q^{FB}$. ■

Lemma 8 *$\Phi_2^\lambda(q,\Delta\kappa)$ is constant in λ for $q \in [0,q_0^{\lambda*}]$ and strictly decreasing otherwise. Further, a unique threshold $\hat{\lambda}_2 \in (0,1)$ exists such that (i) If $\lambda < \hat{\lambda}_2$, $\Phi_2^\lambda(q,\Delta\kappa)$ is increasing in q for all $q \leq q^{FB}$, (ii) If $\lambda > \hat{\lambda}_2$, $\Phi_2^\lambda(q,\Delta\kappa)$ is decreasing in q for all $q \geq q^{FB}$, (iii) If $\lambda = \hat{\lambda}_2$, $\Phi_2^\lambda(q,\Delta\kappa) \leq \Phi_2^\lambda(q^{FB},\Delta\kappa)$ for all $q \in [0,\bar{q}_1]$.*

Proof. From (18) we have,

$$\Phi_2^\lambda(q,\Delta\kappa) = r\int_{\hat{d}(q)}^{q} \bar{F}_1(y)\,dy - r\left(\int_{\hat{d}(q)}^{q\wedge q_0^{\lambda\star}} \bar{F}_0(u)\,du\right)^+ - \lambda c(q - q_0^{\lambda\star})^+.$$

If $q \in [0,q_0^{\lambda\star}]$, $d\Phi_2^\lambda(q,\Delta\kappa)/d\lambda = 0$. Otherwise, and given $\bar{F}_0(q_0^{\lambda\star}) = c\lambda/r$, we have

$$d\Phi_2^\lambda(q,\Delta\kappa)/d\lambda = -c(q - q_0^{\lambda\star}) - \left[r\bar{F}_0(q_0^{\lambda\star}) - \lambda c\right] dq_0^{\lambda\star}/d\lambda = -c(q - q_0^{\lambda\star}) < 0.$$

For Points (i), (ii) and (iii), $\Phi_2^\lambda(q)$ is piece-wise differentiable and, $\mathbb{1}_{\{.\}}$ being the indicator function,

$$\begin{aligned}
\frac{1}{r}\frac{\partial \Phi_2^\lambda}{\partial q}(q,\Delta\kappa) &= \bar{F}_1(q) - \bar{F}_1(\hat{d}(q))\frac{\partial \hat{d}}{\partial q}(q) - \bar{F}_0(q)\mathbb{1}_{\{q \leq q_0^{\lambda\star}\}} + \bar{F}_0\left(\hat{d}(q)\right)\frac{\partial \hat{d}}{\partial q}(q)\mathbb{1}_{\{q_0^{\lambda\star} > \hat{d}(q)\}} - \lambda c\mathbb{1}_{\{q > q_0^{\lambda\star}\}} \\
&= \bar{F}_1(q) - \bar{F}_1(q^{FB}) - \bar{F}_0(q)\mathbb{1}_{\{q \leq q_0^{\lambda\star}\}} - \bar{F}_0\left(q_0^{\lambda\star}\right)\mathbb{1}_{\{q > q_0^{\lambda\star}\}} + \bar{F}_1\left(q^{FB}\right)\frac{\bar{F}_0\left(\hat{d}(q)\right)}{\bar{F}_1\left(\hat{d}(q)\right)}\mathbb{1}_{\{q_0^{\lambda\star} > \hat{d}(q)\}} \\
&= \bar{F}_1(q) - \bar{F}_1(q^{FB}) - \bar{F}_0\left(q \wedge q_0^{\lambda\star}\right) + \bar{F}_1\left(q^{FB}\right)\frac{\bar{F}_0\left(\hat{d}(q)\right)}{\bar{F}_1\left(\hat{d}(q)\right)}\mathbb{1}_{\{q_0^{\lambda\star} > \hat{d}(q)\}},
\end{aligned} \tag{19}$$

where the second equality holds from Lemma 4. From Lemma 5(i), a unique $\hat{\lambda}_0 \in (0,1)$ exists such that $q^{FB} \leq q_0^{\lambda\star}$ if and only if $\lambda \leq \hat{\lambda}_0$.

Case 1: $\lambda \leq \hat{\lambda}_0$. For $q \leq q_0^{\lambda *}$, (19) is rewritten

$$\frac{1}{r}\frac{\partial \Phi_2^\lambda}{\partial q}(q,\Delta\kappa) = \bar{F}_1(q) - \bar{F}_0(q) - \bar{F}_1\left(q^{FB}\right)\left(1 - \frac{\bar{F}_0\left(\hat{d}\left(q\right)\right)}{\bar{F}_1\left(\hat{d}\left(q\right)\right)}\right) \tag{20}$$

In particular, for $q \leq q^{FB}$, we have $\bar{F}_1\left(q\right) \geq \bar{F}_1\left(q^{FB}\right)$ and so

$$\frac{1}{r}\frac{d\Phi_2^\lambda}{dq}(q,\Delta\kappa) \,\geq\, \bar{F}_1(q) - \bar{F}_0(q) - \bar{F}_1\left(q\right)\left(1 - \frac{\bar{F}_0\left(\hat{d}\left(q\right)\right)}{\bar{F}_1\left(\hat{d}\left(q\right)\right)}\right) = \bar{F}_1\left(q\right)\left[\frac{\bar{F}_0\left(\hat{d}\left(q\right)\right)}{\bar{F}_1\left(\hat{d}\left(q\right)\right)} - \frac{\bar{F}_0\left(q\right)}{\bar{F}_1\left(q\right)}\right] > 0$$

where the last inequality holds from the MLRP. Hence, $\Phi_2^\lambda(\cdot,\Delta\kappa)$ increases for $q \leq q^{FB}$ when $\lambda \leq \hat{\lambda}_0$.

Case 2: $\lambda > \hat{\lambda}_0$. When $q \leq q_0^{\lambda\star}$ and hence $q \leq q^{FB}$, $\partial\Phi_2^\lambda(q,\Delta\kappa)/\partial q$ reduces to (20) which is positive. When $q > q_0^{\lambda\star}$,

$$\frac{1}{r}\frac{\partial \Phi_2^\lambda}{\partial q}(q,\Delta\kappa) = \bar{F}_1(q) - \bar{F}_1(q^{FB}) - \bar{F}_0\left(q_0^{\lambda\star}\right) + \bar{F}_1\left(q^{FB}\right)\frac{\bar{F}_0\left(\hat{d}\left(q\right)\right)}{\bar{F}_1\left(\hat{d}\left(q\right)\right)}\mathbb{1}_{\left\{q_0^{\lambda\star} > \hat{d}(q)\right\}},$$

which is decreasing in q since $\bar{F}_1(q)$ is, while $\bar{F}_0(x)/\bar{F}_1(x)$ and $\mathbb{1}_{\left\{q_0^{\lambda\star} > x\right\}}$ are both non-negative non-increasing in x and $\hat{d}(q)$ is increasing. Thus, $\Phi_2^\lambda(q,\Delta\kappa)$ is concave in q if $q \geq q_0^{\lambda\star}$. Hence for $\lambda > \hat{\lambda}_0$, $\Phi_2^\lambda(\cdot,\Delta\kappa)$ is unimodal, $q_2^{\lambda\max} \equiv argmax_{0\leq q\leq \bar{q}_1}\Phi_2^\lambda(q,\Delta\kappa)$ is unique and $q_2^{\lambda\max} > q_0^{\lambda\star}$.

Further, $\partial\Phi_2^\lambda(q)/\partial q$ in (19) is decreasing in λ since $q_0^{\lambda\star}$ is non-increasing and hence $\bar{F}_0\left(q \wedge q_0^{\lambda\star}\right)$ and $\mathbb{1}_{\left\{q_0^{\lambda\star} > \hat{d}(q)\right\}}$ are non-decreasing and non-increasing in λ, respectively. Thus $\Phi_2^\lambda(q)$ is concave and submodular in (q,λ) for $\lambda \geq \hat{\lambda}_0$ and $q_2^{\lambda\max}$ is non-increasing in λ.

For $\lambda = \hat{\lambda}_0$, $\partial\Phi_2^\lambda(q^{FB})/\partial q > 0$ from (20) and thus $q_2^{\lambda\max} > q^{FB}$. Further $\bar{F}_0(q_0^{1\star}) = c/r = \bar{F}_1(q^{FB})$ and

$$\frac{1}{r}\frac{\partial \Phi_2^1}{\partial q}(q^{FB},\Delta\kappa) = -\bar{F}_1\left(q^{FB}\right)\left(1 - \frac{\bar{F}_0\left(\hat{d}\left(q\right)\right)}{\bar{F}_1\left(\hat{d}\left(q\right)\right)}\mathbb{1}_{\left\{q_0^{\lambda\star} > \hat{d}(q)\right\}}\right) < 0,$$

where the inequality holds with $\bar{F}_0(\cdot) < \bar{F}_1(\cdot)$ from the MLRP. Thus, a unique $\hat{\lambda}_2 \in (\hat{\lambda}_0, 1)$ exists such that $q_2^{\lambda\max} \geq q^{FB}$ if and only if $\lambda \leq \hat{\lambda}_2$ and where the equality holds only for $\lambda = \hat{\lambda}_2$. Points (i), (ii) and (iii) hold because $\Phi_2^\lambda(\cdot)$ is unimodal. ■

Defining $\Phi^\lambda(q,\Delta\kappa) \equiv \Phi_1^\lambda(q,\Delta\kappa) \wedge \Phi_2^\lambda(q,\Delta\kappa)$, problem (10) rewrites

$$\max_{q\in\left[\underline{q}_1,\bar{q}_1\wedge q_1^{\lambda\star}\right]} \pi_1(q) \;\text{ s.t. }\; \Phi^\lambda(q,\Delta\kappa) \geq 0. \tag{21}$$

We characterize next the monotonicity of $\Phi^\lambda(q,\Delta\kappa)$ in q.

Lemma 9 *Define capacity* $q^{\lambda\max} \equiv argmax_{q\in[\underline{q}_1,\bar{q}_1\wedge q_1^{\lambda\star}]}\Phi^\lambda(q,\Delta\kappa)$*. A unique threshold* $\hat{\lambda}\in[0,1)$ *exists such that: (i) If* $\lambda<\hat{\lambda}$*,* $q^{\lambda\max}>q^{FB}$ *and* $\Phi^\lambda(q,\Delta\kappa)$ *is strictly increasing in* q *for all* $q\le q^{FB}$*; If* $\lambda>\hat{\lambda}$*,* $q^{\lambda\max}<q^{FB}$ *and* $\Phi^\lambda(q,\Delta\kappa)$ *is strictly decreasing in* q *for all* $q\ge q^{FB}$*; If* $\lambda=\hat{\lambda}$*,* $q^{\lambda\max}=q^{FB}$*.*

Proof. Define $\hat{\lambda}\equiv\hat{\lambda}_1(q^{FB})\wedge\hat{\lambda}_2$. Lemma 8 implies $\hat{\lambda}\le\hat{\lambda}_2<1$.

Point (i): For $\lambda<\hat{\lambda}.$, $\Phi_1^\lambda(\cdot,\Delta\kappa)$ is strictly concave (Lemma 7) and $\Phi_2^\lambda(q,\Delta\kappa)$ increasing for $q\le q^{FB}$ (Lemma 8 with $\lambda<\hat{\lambda}_2$). Further $\Phi_2^\lambda(q^{FB},\Delta\kappa)<\Phi_1^\lambda(q^{FB},\Delta\kappa)$ (Lemma 6 with $\lambda<\hat{\lambda}_1(q^{FB})$). Thus, $\Phi^\lambda(q,\Delta\kappa)$ is increasing for $q\le q^{FB}$. Indeed, denote $\tilde{q}$ the unique maximand of $\Phi_1^\lambda(\cdot,\Delta\kappa)$ over $[0,q^{FB}]$. When $q<\tilde{q}$, $\Phi_1^\lambda(q,\Delta\kappa)$ and hence $\Phi_1^\lambda(q,\Delta\kappa)\wedge\Phi_2^\lambda(q,\Delta\kappa)$ increase in q. When $\tilde{q}\le q\le q^{FB}$, if $\Phi_1^\lambda(q,\Delta\kappa)\le\Phi_2^\lambda(q,\Delta\kappa)$ then $\Phi_2^\lambda(q^{FB},\Delta\kappa)>\Phi_1^\lambda(q^{FB},\Delta\kappa)$ since $\Phi_1^\lambda(\cdot,\Delta\kappa)$ and $\Phi_2^\lambda(\cdot,\Delta\kappa)$ are decreasing and increasing, respectively. This yields a contradiction and thus $\Phi^\lambda(q,\Delta\kappa)=\Phi_2^\lambda(q,\Delta\kappa)$ for $q\in[\tilde{q},q^{FB}]$, which is also increasing. Finally, $q^{FB}<q_1^{\lambda\star}$ for $\lambda<\hat{\lambda}$ (Lemma 5) and hence $q^{FB}<\bar{q}_1\wedge q_1^{\lambda\star}$. It follows that $q^{FB}<q^{\lambda\max}$.

Point (ii): For $\lambda>\hat{\lambda}$, $\Phi_1^\lambda(\cdot,\Delta\kappa)$ is decreasing for $q\ge q^{FB}$ (Lemma 7). If $\lambda>\hat{\lambda}_2$, $\Phi_2^\lambda(q,\Delta\kappa)$ is decreasing for $q\ge q^{FB}$ (Lemma 8(ii)), and the result holds. If $\lambda>\hat{\lambda}_1(q^{FB})$, $\Phi^\lambda(q,\Delta\kappa)=\Phi_1^\lambda(q,\Delta\kappa)$ for $q\ge q^{FB}$ from Lemma 6 so that $\Phi^\lambda(q,\Delta\kappa)$ is still strictly decreasing, which yields the result.

Point (iii) stems from $\Phi^\lambda(q,\Delta\kappa)$'s continuity in λ, which holds from that of $\Phi_i^\lambda(q,\Delta\kappa)$, $i=1,2$. ■

Lemma 10 *For all* $q\in[0,\bar{q}_1]$*, a unique* $\tilde{\lambda}(q)\in[0,\hat{\lambda}_1(q)]$ *exists such that* $\Phi^\lambda(q,\Delta\kappa)$ *is constant in* λ *for* $\lambda\le\tilde{\lambda}(q)$ *and strictly decreasing otherwise. Further,* $\tilde{\lambda}(q)$ *is decreasing in* q *for* $q\ge q^{FB}$*.*

Proof. Define $\tilde{\lambda}(q)\equiv\max\{\lambda\in[0,\hat{\lambda}_1(q)], s.t.\, q_0^{\lambda*}\ge q\}$. For $\lambda\le\tilde{\lambda}(q)$, $q\le q_0^{\lambda*}$ since $q_0^{\lambda*}$ is decreasing in λ from Lemma 5. Further, by construction, $\tilde{\lambda}(q)\le\hat{\lambda}_1(q)$ and thus Lemma 6 implies $\Phi^\lambda(q,\Delta\kappa)=\Phi_2^\lambda(q,\Delta\kappa)$, which is constant in λ for $q<q_0^{\lambda*}$. For $\lambda>\tilde{\lambda}(q)$, we have either $\Phi^\lambda(q,\Delta\kappa)=\Phi_2^\lambda(q,\Delta\kappa)$ with $q<q_0^{\lambda*}$, which is decreasing in λ (Lemma 8), or $\Phi^\lambda(q,\Delta\kappa)=\Phi_1^\lambda(q,\Delta\kappa)$, which is decreasing in λ (Lemma 7). and thus $\Phi^\lambda(q,\Delta\kappa)$ is decreasing for $\lambda>\tilde{\lambda}(q)$. Further, $\tilde{\lambda}(q)$ is decreasing for $q\ge q^{FB}$ since $\hat{\lambda}_1(q)$ and $q_0^{\lambda*}$ are decreasing for $q\ge q^{FB}$ from Lemmas 6(ii) and 5, respectively. ■

E.4. Proof of Theorem 2

For $\lambda\in[0,1)$, define $\Delta\bar{\kappa}(\lambda)$ and $\Delta\underline{\kappa}(\lambda)$ by $\Phi^\lambda(q^{\lambda\max},\Delta\bar{\kappa}(\lambda))=0$ and $\Phi^\lambda(q^{FB},\Delta\underline{\kappa}(\lambda))=0$. Both are uniquely defined and non-increasing in λ as $\Phi^\lambda(q,\Delta\kappa)$ is continuous, non-increasing in λ and decreasing in $\Delta\kappa$. Further, for all λ and $\Delta\kappa$, $\Phi^\lambda(q^{FB},\Delta\kappa)\le\Phi^\lambda(q^{\lambda\max},\Delta\kappa)$ by definition of $q^{\lambda\max}$, with equality if and only if $\lambda=\hat{\lambda}$ (Lemma 9), and thus $\Delta\underline{\kappa}(\lambda)\le\Delta\bar{\kappa}(\lambda)$ with equality if and only if $\lambda=\hat{\lambda}$.

Point (i): For $\Delta\kappa \leq \Delta\underline{\kappa}(\lambda)$, we have, $\Phi^{\lambda}(q^{FB}, \Delta\kappa) \geq \Phi^{\lambda}(q^{FB}, \Delta\underline{\kappa}) = 0$, since $\Phi^{\lambda}(q, \cdot)$ is decreasing. Hence, q^{FB} satisfies the constraints of problem (21), and thus $q^* = q^{FB}$.

Point (ii): Assume $\Delta\underline{\kappa}(\lambda) < \Delta\kappa \leq \Delta\bar{\kappa}(\lambda)$ (and thus $\lambda \neq \hat{\lambda}$).

For $\lambda < \hat{\lambda}$, $q^{FB} < q^{\lambda\max}$ (Lemma 9). Because $\Delta\kappa \leq \Delta\bar{\kappa}(\lambda)$, $\Phi^{\lambda}(q^{\lambda\max}, \Delta\kappa) \geq 0$ and $\underline{S} \equiv \{q \leq q^{\lambda\max}\ s.t.\ \Phi^{\lambda}(q, \Delta\kappa) \geq 0\}$ is non-empty. For $q \leq q^{FB}$, $\Phi^{\lambda}(q, \Delta\kappa) \leq \Phi^{\lambda}(q^{FB}, \Delta\kappa) < \Phi^{\lambda}(q^{FB}, \Delta\underline{\kappa}(\lambda)) = 0$ (Lemma 9). Thus for $q \in \underline{S}$, $q^{FB} < q$ and (21) is decreasing in q for $q \in \underline{S}$. Hence $q^* = \min\{q\ s.t.\ q \in \underline{S}\}$ and $q^* > q^{FB}$.

For $\lambda > \hat{\lambda}$, $q^{FB} > q^{\lambda\max}$. Since $\Delta\kappa < \Delta\bar{\kappa}(\lambda)$, $\Phi^{\lambda}(q^{\lambda\max}, \Delta\kappa) > \Phi^{\lambda}(q^{\lambda\max}, \Delta\bar{\kappa}) = 0$ and $\bar{S} \equiv \{q \geq q^{\lambda\max}\ s.t.\ \Phi^{\lambda}(q, \Delta\kappa) \geq 0\}$ is non-empty. For $q \geq q^{FB}$, $\Phi^{\lambda}(q, \Delta\kappa) \leq \Phi^{\lambda}(q^{FB}, \Delta\kappa)$ (Lemma 9). Thus, for $q \in \bar{S}$, $q^{FB} > q$ and (21) is increasing in q. Thus $q^* = \max\{q\ s.t.\ q \in \bar{S}\}$ and $q^{\lambda\max} \leq q^* < q^{FB}$.

Point (iii): For $\Delta\kappa > \Delta\bar{\kappa}(\lambda)$, we have for all q, $\Phi^{\lambda}(q, \Delta\kappa) \leq \Phi^{\lambda}(q^{\lambda\max}, \Delta\kappa) < \Phi^{\lambda}(q^{\lambda\max}, \Delta\bar{\kappa}(\lambda)) = 0$, where the first inequality holds by definition of $q^{\lambda\max}$, the second one since $\Phi^{\lambda}(q, \cdot)$ is decreasing and the last equality by definition of $\Delta\bar{\kappa}(\lambda)$. Thus, Problem (21) is not feasible and $q^* = 0$.

E.5. Proof of Theorem 3

Monotonicity in $\Delta\kappa$. For $\lambda > \hat{\lambda}$ and $\Delta\underline{\kappa}(\lambda) < \Delta\kappa < \Delta\bar{\kappa}(\lambda)$, Theorem 2 implies $q^* = \max\{q \geq q^{\lambda\max}\ s.t.\ \Phi^{\lambda}(q, \Delta\kappa) = 0\}$ with $q^* < q^{FB}$. From Lemma 9 with $\lambda < \hat{\lambda}$ and $q^* < q^{FB}$, $\Phi^{\lambda}(q, \Delta\kappa)$ is decreasing q, and since it is also decreasing in $\Delta\kappa$, q^* is decreasing as well. Similarly, when $\lambda \leq \hat{\lambda}$ and $\Delta\kappa \in (\Delta\underline{\kappa}(\lambda), \Delta\bar{\kappa}(\lambda))$, we have $q^* = \max\{q\ s.t.\ q^{FB} \leq q \leq q^{\lambda\max},\ \Phi^{\lambda}(q, \Delta\kappa) = 0\}$. Since $\Phi^{\lambda}(q, \Delta\kappa)$ is decreasing and increasing in $\Delta\kappa$ and q, respectively (from Lemma 9 with $\lambda \leq \hat{\lambda}$ and $q^* \geq q^{FB}$), q^* is increasing in $\Delta\kappa$.

Monotonicity in λ. For the sake of simplicity, the following notations omit the dependence of the different parameters with $\Delta\kappa$.

Case 1: $\lambda > \hat{\lambda}$ and $\Delta\underline{\kappa}(\lambda) < \Delta\kappa < \Delta\bar{\kappa}(\lambda)$. Theorem 2 implies $q^{\lambda\max} \leq q^{\lambda*} < q^{FB}$, where $q^{\lambda*} = \max\{q \geq q^{\lambda\max}\ s.t.\ \Phi^{\lambda}(q, \Delta\kappa) = 0\}$. $\Phi^{\lambda}(q)$ being non-increasing in λ (Lemma 10) and q (Lemma 9 with $\lambda > \hat{\lambda}$ and $q^* < q^{FB}$), $q^{\lambda*}$ is also non-increasing. We show next that $q^{\lambda*}$ is decreasing in λ.

For $\lambda = 0$, $\Phi^0(q) = \Phi_2^0(q)$ which is increasing in $q \leq q^{FB}$ (Lemma 9 with $0 < \hat{\lambda}$). Hence $\underline{q}^0 \equiv \min\{q < q^{FB}\ s.t.\ \Phi^0(q) \geq 0\}$ is uniquely defined with $\Phi^0(q) < 0$ if $q < \underline{q}^0$ and $\Phi^0(q) \geq 0$ if $\underline{q}^0 < q \leq q^{FB}$.

Assume then that $\Phi^{\lambda}(q^{\lambda*}) = \Phi_2^{\lambda}(q^{\lambda*})$. We show by contradiction that we must have $q^{\lambda*} > q_0^{\lambda*}$ in this case. Indeed, if $q^{\lambda*} \leq q_0^{\lambda*}$, from Lemma 8 and the fact that $q_0^{\lambda*}$ is decreasing in λ (see Lemma 5), we have $\Phi_2^{\lambda}(q^{\lambda*}) = \Phi_2^0(q^{\lambda*})$. If $q^{\lambda*} \leq \underline{q}^0$, then $0 \geq \Phi_2^0(q^{\lambda*}) = \Phi^{\lambda}(q^{\lambda*})$ which is impossible when $\Delta\kappa < \Delta\bar{\kappa}(\lambda)$. If $\underline{q}^0 < q \leq q^{FB}$, then $0 < \Phi_2^0(q^{\lambda*}) = \Phi^{\lambda}(q^{\lambda*})$ which is impossible when $\Delta\underline{\kappa}(\lambda) < \Delta\kappa$.

Thus, when $\Phi^\lambda(q^{\lambda *}) = \Phi_2^\lambda(q^{\lambda *})$, we must have $q^{\lambda *} > q_0^{\lambda *}$, in which case $\Phi_2^\lambda(q^{\lambda *})$ is decreasing in λ (Lemma 8). $\Phi^\lambda(q^{\lambda *})$ is then decreasing in λ since $\Phi_1^\lambda(q)$ is decreasing in λ and thus $q^{\lambda *} = \max\{q \geq q^{\lambda \max}\; s.t.\; \Phi^\lambda(q, \Delta\kappa) = 0\}$ is also strictly decreasing.

Case 2: $\lambda \leq \hat{\lambda}$ and $\Delta\underline{\kappa}(\lambda) < \Delta\kappa < \Delta\bar{\kappa}(\lambda)$. Theorem 2 implies that $q^{\lambda \max} \geq q^{\lambda *} \geq q^{FB}$, where $q^{\lambda *} = \min\{q \leq q^{\lambda \max}\; s.t.\; \Phi^\lambda(q) = 0\}$. In particular, given $\Delta\kappa$, optimal capacity q^{0*} for $\lambda = 0$ is such that $q^{0*} \geq q^{FB}$, and is decreasing in $\Delta\kappa$ from the previous argument. Lemma 10 implies $\tilde{\lambda}\,(q^{0*}) \in (\alpha, \beta)$ is decreasing in $\Delta\kappa \in (\Delta\underline{\kappa}(0), \Delta\bar{\kappa}(0))$, where α and β are equal to $\tilde{\lambda}\,(q^{0*})$ for $\Delta\kappa = \Delta\underline{\kappa}(0)$ and $\Delta\kappa = \Delta\bar{\kappa}(0)$, respectively. Hence, a unique threshold $\Delta\tilde{\kappa}(\lambda)$ exists such that for $\Delta\kappa \in (\Delta\underline{\kappa}(\lambda), \Delta\bar{\kappa}(\lambda))$, $\Delta\kappa \leq \Delta\tilde{\kappa}(\lambda)$ if and only if $\lambda \leq \tilde{\lambda}\,(q^{0*})$ with equality only for $\Delta\kappa = \Delta\tilde{\kappa}(\lambda)$, and where $\Delta\tilde{\kappa}(\lambda) = \Delta\underline{\kappa}(\lambda)$ if $\lambda \leq \alpha$ and $\Delta\tilde{\kappa}(\lambda) = \Delta\bar{\kappa}(\lambda)$ if $\lambda \geq \beta$.

Take first $\Delta\kappa > \Delta\tilde{\kappa}(\lambda)$ and hence $\lambda > \tilde{\lambda}(q^{0*})$. For all $q \geq q^{0*} \geq q^{FB}$, $\tilde{\lambda}(q) \leq \tilde{\lambda}(q^{0*}) < \lambda$ where the first inequality holds since $\tilde{\lambda}(\cdot)$ is decreasing for $q \geq q^{FB}$ (Lemma 10). And since $\lambda > \tilde{\lambda}(q^{0*}) > \tilde{\lambda}(q)$, Lemma 10 also implies that $\Phi^\lambda(q)$ is decreasing in λ. It follows that $q^{\lambda *}$ is increasing in λ for $\lambda > \hat{\lambda}$ since $\Phi^\lambda(q)$ is increasing $q \geq q^{FB}$ from Lemma 9.

Take now $\Delta\kappa \leq \Delta\tilde{\kappa}(\lambda)$ and hence $\lambda \leq \tilde{\lambda}(q^{0*})$. From Lemma 10, $\Phi^\lambda(q^{0*}) = \Phi^0(q^{0*}) \geq 0$, where the inequality holds from the definition of q^{0*}, and $\Phi^\lambda(q) \leq \Phi^0(q) < 0$ for all $q < q^{0*}$, where the first inequality holds since $\Phi^\lambda(q)$ is non-increasing in λ (Lemma 10) and the second one from the definition of q^{0*}. Hence, $q^{\lambda *} = q^{0*}$.

References

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