Appendix C: Core price computation (ONLINE ONLY)

Let $W \subseteq I$ denote the set of winners, $S^* = (S_1^*, \dots, S_{|I|}^*)$ the allocation that maximizes social welfare, $p^{VCG} = (p_1^{VCG}, \dots, p_{|I|}^{VCG})$ the VCG payment vector, $p^\tau = (p_1^\tau, \dots, p_{|I|}^\tau)$ the payment vector, and $C^\tau \subseteq I$ the blocking coalition of bidders in iteration τ.

Following Day and Raghavan (2007), we define the core separation problem, which yields the most violated core constraint, if any. S_i^* denotes the set of items that is allocated to bidder i from the set of winners $W \subseteq I$. To calculate equitable bidder-Pareto optimal (EBPO) core payments iteratively, the procedure is as follows. Solve the core separation problem (SEP$^\tau$):

$$z(p^\tau) = \max \sum_{i\in I}\sum_{S\neq\emptyset} v_i(S)x_{iS}^\tau - \sum_{i\in W}(v_i(S_i^*) - p_i^\tau)\gamma_{iS}^\tau \tag{SEP$^\tau$}$$

$$\text{s. t.} \sum_{i\in I}\sum_{S:r_{kt}\in S} x_{iS}^\tau \leq c_{kt} \qquad \forall k\in K, t\in T, S\neq\emptyset, \tag{4a}$$

$$\sum_{S\neq\emptyset} x_{iS}^\tau \leq 1 \qquad \forall i \in I\setminus W, \tag{4b}$$

$$\sum_{S\neq\emptyset} x_{iS}^\tau \leq \gamma_{iS}^\tau \qquad \forall i\in W, \tag{4c}$$

$$x_{iS}^\tau \in \{0,1\} \qquad \forall i\in I, S\neq\emptyset, \tag{4d}$$

$$\gamma_{iS}^\tau \in \{0,1\} \qquad \forall i\in W, S\neq\emptyset. \tag{4e}$$

SEP$^\tau$ yields the most violated core constraint, if any. That is, it finds coalitions of bidders C^τ that block the current outcome (who "would pay more" than the current payments, i.e., if $z(p^\tau) > \sum_{i\in W} p_i^\tau$, then there is a blocking coalition $C^\tau = \{i | x_{iS}^\tau = 1\}$). γ_{iS}^τ is a binary variable, which ensures that any winning bidder will be compensated his opportunity cost if selected as part of the optimal solution to SEP$^\tau$. Let $p^{core,\tau} = (p_1^{core,\tau}, \dots, p_{|I|}^{core,\tau})$ denote the (temporary) core payment vector in iteration τ. Then, the minimal payments in the core satisfying the core constraints found (EBPOCORE) are calculated in EBPO$^\tau$:

$$\theta^\tau(\epsilon) = \min \sum_{i\in W} p_i^{core,\tau} + \epsilon m^\tau \tag{EBPO$^\tau$}$$

$$\text{s. t.} \sum_{i\in W\setminus C^{\tau'}} p_i^{core,\tau} \geq z(p^{\tau'}) - \sum_{i\in W\cap C^{\tau'}} p_i^{\tau'} \qquad \forall \tau' \leq \tau, \tag{EBPOCORE}$$

$$p_i^{core,\tau} - m^\tau \leq p_i^{VCG} \qquad \forall i\in W, \tag{5a}$$

$$p_i^{core,\tau} \leq v_i(S_i^*) \qquad \forall i\in W, \tag{5b}$$

$$p_i^{core,\tau} \geq p_i^{VCG} \qquad \forall i\in W. \tag{5c}$$

The minimal payments minimize potential gains from deviation and EBPO minimizes the maximum deviation from VCG payments as a secondary objective. Using a sufficiently small value of ϵ, the calculated sum of payments is minimal in the core. This procedure is repeated until no further core constraint violation is found using (SEP$^\tau$), i.e., it is repeated while $z(p^\tau) > \theta^{\tau-1}(\epsilon)$ with $\theta^0(\epsilon) := \sum_i p_i^{\text{VCG}}$.

Now, Algorithm 1 shows how core payments are computed in pseudo code. First, the winner determination problem is solved ($\omega(I)$) and the VCG prices are calculated for the winners. Next, the core separation problem is solved to find the most violated core constraint. Then, the minimal payments in the core satisfying the core constraints found are calculated. This procedure is repeated until no further core constraint violation is found.

Algorithm 1: Core constraints generation (following Day and Raghavan (2007, p. 1398))

$W, S^* \leftarrow$ solve the winner determination problem $\omega(I)$
foreach $i \in W$ **do**
  $p_i^{\mathrm{VCG}} \leftarrow$ compute the VCG price $v_i(S_i^*) - \big(\omega(I) - \omega(I_{-i})\big)$
$p^1 \leftarrow p^{\mathrm{VCG}}$
$\theta^0(\epsilon) \leftarrow \sum_i p_i^{\mathrm{VCG}}$
$\tau \leftarrow 1$
while *true* **do**
  $C^\tau \leftarrow$ solve the separation problem SEP^τ
  if $z(p^\tau) > \theta^{\tau-1}(\epsilon)$ **then**
    add constraint $\sum_{i \in W \setminus C^\tau} p_i^{core,\tau} \geq z(p^\tau) - \sum_{i \in W \cap C^\tau} p_i^\tau$ to EBPO^τ and solve
    $p^{\tau+1} \leftarrow p^{core,\tau}$ from EBPO^τ
  else
    $p \leftarrow p^\tau$
    break
  $\tau \leftarrow \tau + 1$

Core payments suffer from sub-optimal computations of the WDP. For example, it might well be that $\omega(I_{-i}) > \omega(I)$, which would then lead to $p_i^{VCG} > v_i(S_i^*)$. The algorithms by Goetzendorff et al. (2015) trim payments or update the winning allocation, in cases a better one is found during the computation of payments. It was proposed for markets where it is possible to compute near-optimal allocations quickly (measured by the integrality gap of the MIP solver), but where proving optimality turns out to be intractable. We use the TRIM algorithm by Goetzendorff et al. (2015) in our experiments to deal with near-optimal allocations.

Appendix D: Detailed Results (ONLINE ONLY)

Each line in Table 11 describes a treatment combination consisting of the number of carriers, the warehouse capacity, and mechanism. We average over number of warehouses per tour. Computation times, seller revenue, winning bidder payments, and the numbers of submitted and winning bids are reported in a similar way in Tables 12 and 13.

no. of carriers	warehouse capacity	mechanism	mean waiting time per tour	mean RTT
20	1	no coord.	113.09	464.69
20	1	OPT	89.98	449.09
20	1	Core0.1	90.64	449.25
20	1	RaR B&C0.1	99.55	456.33
20	1	RaR B&C1.0	103.02	458.65
20	1	RaR	111.90	463.48
20	2	no coord.	18.27	369.88
20	2	OPT	4.50	361.58
20	2	Core0.1	5.48	362.35
20	2	RaR B&C0.1	8.64	363.80
20	2	RaR B&C1.0	12.31	366.06
20	2	RaR	17.40	369.00
30	1	no coord.	225.86	577.31
30	1	Core0.1	198.48	554.41
30	1	OPT	195.72	552.10
30	1	RaR B&C0.1	206.49	561.92
30	1	RaR B&C1.0	211.33	565.98
30	1	RaR	227.42	578.90
30	2	no coord.	45.64	397.09
30	2	OPT	22.06	382.96
30	2	Core0.1	26.14	387.17
30	2	RaR B&C0.1	29.55	389.14
30	2	RaR B&C1.0	36.55	393.09
30	2	RaR	45.99	397.57
40	1	no coord.	348.13	700.32
40	1	OPT	301.97	657.97
40	1	Core0.1	304.95	660.83
40	1	RaR B&C0.1	315.01	670.67
40	1	RaR B&C1.0	321.32	676.37
40	1	RaR	347.29	699.48
40	2	no coord.	89.82	442.01
40	2	OPT	64.71	426.12
40	2	Core0.1	67.03	426.91
40	2	RaR B&C0.1	68.97	428.56
40	2	RaR B&C1.0	77.52	434.41
40	2	RaR	88.95	441.24

Table 11 **Mean waiting and round trip times per tour (in minutes).**

no. of carriers	warehouse capacity	mechanism	mean comp. time	max. comp. time	mean revenue	mean payment
20	1	Pay-as-bid	0.60	1.78	140.68	11.63
20	1	VCG	7.28	23.42	89.60	7.40
20	1	Core	15.85	76.66	123.86	10.24
20	1	Core0.1	12.11	62.15	124.08	10.52
20	1	RaR B&C0.1	7681.99	18608.53	68.10	8.00
20	1	RaR B&C1.0	24.73	74.78	55.45	8.30
20	1	RaR	0.33	0.58	0.44	0.44
20	2	Pay-as-bid	0.09	0.27	236.74	11.84
20	2	VCG	1.14	2.74	5.62	0.28
20	2	Core	2.23	7.05	6.75	0.34
20	2	Core0.1	0.89	1.88	65.60	3.36
20	2	RaR B&C0.1	2805.83	7836.91	3.29	0.20
20	2	RaR B&C1.0	7.62	22.83	1.93	0.17
20	2	RaR	0.22	0.49	0.05	0.03
30	1	Pay-as-bid	6.04	20.62	152.28	11.62
30	1	VCG	77.60	257.68	104.12	7.95
30	1	Core	121.68	439.96	139.93	10.68
30	1	Core0.1	96.28	400.61	141.22	11.03
30	1	RaR B&C0.1	12188.96	17611.05	78.29	8.11
30	1	RaR B&C1.0	78.94	202.53	62.77	8.22
30	1	RaR	0.65	1.19	0.51	0.51
30	2	Pay-as-bid	8.65	46.60	313.73	11.71
30	2	VCG	236.35	1453.47	180.72	6.74
30	2	Core	501.92	2677.93	246.75	9.21
30	2	Core0.1	23.54	172.51	259.03	10.12
30	2	RaR B&C0.1	10826.56	27630.60	159.25	7.23
30	2	RaR B&C1.0	55.46	118.26	101.41	7.33
30	2	RaR	0.60	1.10	2.52	2.40
40	1	Pay-as-bid	18.92	108.09	160.43	11.63
40	1	VCG	196.06	702.52	138.89	10.06
40	1	Core	275.63	858.31	153.77	11.14
40	1	Core0.1	220.46	712.80	153.93	11.24
40	1	RaR B&C0.1	17666.27	37750.68	88.40	8.51
40	1	RaR B&C1.0	194.83	428.78	72.31	8.64
40	1	RaR	1.18	2.17	0.59	0.59
40	2	Pay-as-bid	174.49	914.03	345.35	11.59
40	2	VCG	5704.92	27836.04	215.12	7.22
40	2	Core	17823.52	103834.17	283.29	9.51
40	2	Core0.1	166.48	814.86	298.57	10.59
40	2	RaR B&C0.1	22345.39	50618.55	185.83	7.54
40	2	RaR B&C1.0	120.84	220.31	117.35	7.57
40	2	RaR	1.16	2.14	2.98	2.87

Table 12 Computation times (in seconds), auctioneer revenue, and winning bidder payments for non-empty allocations (in monetary units). "Pay-as-bid" describes the values for the winner determination based on true valuations.

no. of carriers	warehouse capacity	mechanism	mean no. bids	mean no. win. bids	mean no. bids p. carrier	mean no. win. bids p. carrier
20	1	OPT	170.20	12.10	8.51	0.60
20	1	Core0.1	170.20	11.80	8.51	0.59
20	1	RaR B&C0.1	170.20	8.51	8.51	0.43
20	1	RaR B&C1.0	170.20	6.68	8.51	0.33
20	1	RaR	170.20	0.06	8.51	0.00
20	2	OPT	170.00	20.00	8.50	1.00
20	2	Core0.1	170.00	19.50	8.50	0.97
20	2	RaR B&C0.1	170.00	16.45	8.50	0.82
20	2	RaR B&C1.0	170.00	10.00	8.50	0.50
20	2	RaR	170.00	0.25	8.50	0.01
30	1	OPT	255.20	13.10	8.51	0.44
30	1	Core0.1	255.20	12.80	8.51	0.43
30	1	RaR B&C0.1	255.20	9.65	8.51	0.32
30	1	RaR B&C1.0	255.20	7.64	8.51	0.25
30	1	RaR	255.20	0.06	8.51	0.00
30	2	OPT	255.30	26.80	8.51	0.89
30	2	Core0.1	255.30	25.60	8.51	0.85
30	2	RaR B&C0.1	255.30	22.02	8.51	0.73
30	2	RaR B&C1.0	255.30	13.84	8.51	0.46
30	2	RaR	255.30	0.35	8.51	0.01
40	1	OPT	342.40	13.80	8.56	0.35
40	1	Core0.1	342.40	13.70	8.56	0.34
40	1	RaR B&C0.1	342.40	10.39	8.56	0.26
40	1	RaR B&C1.0	342.40	8.37	8.56	0.21
40	1	RaR	342.40	0.07	8.56	0.00
40	2	OPT	342.20	29.80	8.56	0.74
40	2	Core0.1	342.20	28.20	8.56	0.70
40	2	RaR B&C0.1	342.20	24.65	8.56	0.62
40	2	RaR B&C1.0	342.20	15.51	8.56	0.39
40	2	RaR	342.20	0.39	8.56	0.01

Table 13 Numbers of (winning) bids.