# Electronic Companions to "The Economics of Line-Sitting"

## EC.1. Auxiliary Results

PROPOSITION EC.1.1. *In the FIFO benchmark, customers join the system if and only if their hourly waiting cost $c$ is less than a cost threshold $c^{FIFO}$, i.e., $c \le c^{FIFO}$, where*

$$c^{FIFO} = \begin{cases} \overline{c}, & \text{if } \rho \le \frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}; \\ \frac{\overline{c}(\overline{R}-\overline{B})}{(\overline{R}-\overline{B})\rho+1}, & \text{if } \rho > \frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}. \end{cases}$$

*The service provider's revenue $\Pi^{FIFO}$ is given by*

$$\Pi^{FIFO} = \begin{cases} \overline{c}\rho\overline{B} = \Lambda B, & \text{if } \rho \le \frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}; \\ \overline{c}\rho\overline{B} \cdot \frac{\overline{R}-\overline{B}}{(\overline{R}-\overline{B})\rho+1}, & \text{if } \rho > \frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}; \end{cases}$$

*and customer welfare $CW^{FIFO}$ is given by*

$$CW^{FIFO} = \begin{cases} \overline{c}\rho\left(\overline{R}-\overline{B}-\frac{1}{2(1-\rho)}\right), & \text{if } \rho \le \frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}; \\ \overline{c}\rho \cdot \frac{(\overline{R}-\overline{B})^2}{2[(\overline{R}-\overline{B})\rho+1]}, & \text{if } \rho > \frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}. \end{cases}$$

LEMMA EC.1.1. *Given the hourly line-sitting rate $r \in (0, \overline{c})$, the customer equilibrium is characterized by $c^{LS}$ as follows:*
*(1) if $\rho > \frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}$: if $r < c^{FIFO}$, line-sitting activities exist and*

$$c^{LS} = \min\left\{\overline{c}\left(\frac{1+(\frac{r}{\overline{c}}+\overline{R}-\overline{B})\rho-\sqrt{[1+(\frac{r}{\overline{c}}+\overline{R}-\overline{B})\rho]^2-4(\overline{R}-\overline{B})\rho}}{2\rho}\right), \overline{c}\right\} > c^{FIFO};$$

*otherwise, line-sitting activities do not exist and some customers balk, and $c^{LS} = c^{FIFO}$;*
*(2) if $\rho \le \frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}$: there are no balking customers and $c^{LS} = c^{FIFO} = \overline{c}$.*

PROPOSITION EC.1.2. *In the line-sitting model, under the optimal hourly rate $r^*$ of the line-sitting firm, the service provider's revenue $\Pi^{LS}$ and customer welfare $CW^{LS}$ are as follows:*
*(1) if $\rho < 1$,*

$$\Pi^{LS} = \begin{cases} \overline{c}\rho\overline{B} = \Lambda B, & \text{if } \rho \le \frac{(\overline{R}-\overline{B})^2-1}{\overline{R}-\overline{B}}; \\ \overline{c}\rho\overline{B} \cdot \frac{\overline{R}-\overline{B}}{\sqrt{(\overline{R}-\overline{B})\rho+1}}, & \text{if } \rho > \frac{(\overline{R}-\overline{B})^2-1}{\overline{R}-\overline{B}}. \end{cases}$$

$$CW^{LS} = \begin{cases} \overline{c}\rho\left(\overline{R}-\overline{B}-\frac{4-\rho}{8(1-\rho)}\right), & \text{if } \rho \le \frac{2(\overline{R}-\overline{B}-1)}{2(\overline{R}-\overline{B})-1}; \\ \frac{1}{2}\overline{c}\left[(\overline{R}-\overline{B}-1)^2(1-\rho)+\rho\right], & \text{if } \frac{2(\overline{R}-\overline{B}-1)}{2(\overline{R}-\overline{B})-1} < \rho \le \frac{(\overline{R}-\overline{B})^2-1}{\overline{R}-\overline{B}}; \\ \overline{c}\rho\frac{(\overline{R}-\overline{B})[\sqrt{(\overline{R}-\overline{B})\rho+1}-1]}{\sqrt{(\overline{R}-\overline{B})\rho+1}} \cdot \frac{2\sqrt{(\overline{R}-\overline{B})\rho+1}-(\overline{R}-\overline{B})\rho}{2\rho}, & \text{if } \rho > \frac{(\overline{R}-\overline{B})^2-1}{\overline{R}-\overline{B}}. \end{cases}$$

*(2) if $\rho \geq 1$,*

$$\Pi^{LS} = \begin{cases} \overline{c}\rho\overline{B}\cdot\frac{\overline{R}-\overline{B}}{\sqrt{(\overline{R}-\overline{B})\rho+1}}, & \text{if } \rho \leq \frac{\sqrt{5}+1}{2(\overline{R}-\overline{B})}; \\ \overline{c}\overline{B} = \mu B, & \text{if } \rho > \frac{\sqrt{5}+1}{2(\overline{R}-\overline{B})}. \end{cases}$$

$$CW^{LS} = \begin{cases} \overline{c}\rho\frac{(\overline{R}-\overline{B})[\sqrt{(\overline{R}-\overline{B})\rho+1}-1]}{\sqrt{(\overline{R}-\overline{B})\rho+1}}\cdot\frac{2\sqrt{(\overline{R}-\overline{B})\rho+1}-(\overline{R}-\overline{B})\rho}{2\rho}, & \text{if } \rho \leq \frac{\sqrt{5}+1}{2(\overline{R}-\overline{B})}; \\ \frac{1}{2}\overline{c}/\rho, & \text{if } \rho > \frac{\sqrt{5}+1}{2(\overline{R}-\overline{B})}. \end{cases}$$

Lemma EC.1.2. *Given priority premium $P$ (and $\overline{P} \triangleq \mu P/\overline{c}$), the customer equilibrium is characterized by $(c_a^{PRI}, c_b^{PRI})$ as follows:*

*(1) if $\rho > \frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}$: if $P < \frac{\overline{c}\rho^2(\overline{R}-\overline{B})^3}{\mu[(\overline{R}-\overline{B})\rho+1]^2}$, $c_a^{PRI} = \frac{\overline{c}\overline{P}}{\rho[(\overline{R}-\overline{B}-\overline{P})(\overline{R}-\overline{B})\rho-\overline{P}]}$ and $c_b^{PRI} = \frac{\overline{c}(\overline{R}-\overline{B})}{(\overline{R}-\overline{B})\rho+1}$; otherwise, $c_a^{PRI} = c_b^{PRI} = c^{FIFO}$, i.e., the system degenerates to the FIFO case;*

*(2) if $\rho \leq \frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}$: if $P < \frac{\overline{c}\rho^2}{\mu-\Lambda}$, $c_a^{PRI} = \frac{\overline{c}\overline{P}(1-\rho)^2}{\rho[\rho-\overline{P}(1-\rho)]}$ and $c_b^{PRI} = \overline{c}$; otherwise, $c_a^{PRI} = c_b^{PRI} = c^{FIFO} = \overline{c}$, i.e., the system degenerates to the FIFO case without balking.*

Proposition EC.1.3. *In the priority purchasing scheme, the unique priority premium that maximizes the service provider's revenue is given by*

$$P^* = \begin{cases} \frac{\overline{c}\rho(1-\sqrt{1-\rho})}{\mu(1-\rho)}, & \text{if } \rho \leq \frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}; \\ \overline{c}\rho(\overline{R}-\overline{B})^2\left(\frac{(\overline{R}-\overline{B})\rho+1-\sqrt{(\overline{R}-\overline{B})\rho+1}}{\mu[(\overline{R}-\overline{B})\rho+1]^2}\right), & \text{if } \rho > \frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}. \end{cases}$$

Proposition EC.1.4. *In the priority model, under the service provider's optimal priority premium $P^*$, the service provider's revenue is given by*

$$\Pi^{PRI} = \begin{cases} \overline{c}\rho\cdot\left(\frac{(1-\sqrt{1-\rho})^2}{1-\rho}+\overline{B}\right), & \text{if } \rho \leq \frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}; \\ \overline{c}\rho(\overline{R}-\overline{B})\cdot\frac{(\overline{R}-\overline{B})+[(\overline{R}-\overline{B})\rho+1]\overline{R}-2(\overline{R}-\overline{B})\sqrt{(\overline{R}-\overline{B})\rho+1}}{[(\overline{R}-\overline{B})\rho+1]^2}, & \text{if } \rho > \frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}; \end{cases}$$

*and customer welfare is given by*

$$CW^{PRI} = \begin{cases} \overline{c}\rho\left(\overline{R}-\overline{B}-\frac{(1-\sqrt{1-\rho})^2}{1-\rho}-\frac{2-\sqrt{1-\rho}}{2\sqrt{1-\rho}}\right), & \text{if } \rho \leq \frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}; \\ \overline{c}\rho(\overline{R}-\overline{B})^2\cdot\frac{\sqrt{(\overline{R}-\overline{B})\rho+1}-1/2}{[(\overline{R}-\overline{B})\rho+1]^2}, & \text{if } \rho > \frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}. \end{cases}$$

Proposition EC.1.5. *The optimal basic fees for the different schemes are given as follows:*

$$B^{FIFO} = \begin{cases} \overline{c}\left(\overline{R}-\frac{\sqrt{\overline{R}\rho+1}-1}{\rho}\right)/\mu, & \text{if } (1-\rho)\sqrt{\overline{R}\rho+1} \leq 1; \\ \overline{c}\left(\overline{R}-\frac{1}{1-\rho}\right)/\mu, & \text{if } (1-\rho)\sqrt{\overline{R}\rho+1} > 1. \end{cases}$$

$$B^{LS} = \begin{cases} \overline{c}\left(\overline{R}-\frac{\sqrt{5}+1}{2\rho}\right)/\mu, & \text{if } \rho \geq 1 \text{ and } \overline{R}-\overline{B}_a \geq \frac{\sqrt{5}+1}{2\rho}; \\ \overline{c}\left(\overline{R}-\frac{\rho+\sqrt{\rho^2+4}}{2}\right)/\mu, & \text{if } \rho < 1 \text{ and } \overline{R}-\overline{B}_a \geq \frac{\sqrt{\rho^2+4}+\rho}{2}; \\ \overline{c}\cdot\min\{\overline{B}_a, \overline{R}-1\}/\mu, & \text{otherwise.} \end{cases}$$

$$B^{PRI} = \begin{cases} \bar{c}\left(\overline{R} - \frac{1}{1-\rho}\right)/\mu, & \text{if } \rho < 1 \text{ and } \overline{R} - \overline{B}_b \geq \frac{1}{1-\rho}; \\ \bar{c} \cdot \min\{\overline{B}_b, \overline{R} - 1\}/\mu, & \text{otherwise} \end{cases}$$

*where* $\overline{B}_a = \frac{4+5\overline{R}\rho - \sqrt{16+16\overline{R}\rho + \overline{R}^2\rho^2}}{6\rho}$ *and* $\overline{B}_b$ *is the unique solution of* $\overline{B}$ *to the following equation*

$$(\overline{R}-\overline{B})\sqrt{(\overline{R}-\overline{B})\rho+1}[(\overline{R}-\overline{B})\rho+4] = \overline{R} + (\overline{R}\rho+2)(\overline{R}-\overline{B}).$$

## EC.2. Proofs

We define additional notation $\overline{R}_1, \overline{R}_2, \overline{R}_3$ in Table EC.2.1 that will be used throughout the proofs. It is easy to verify $\overline{R}_1 \leq \overline{R}_2 \leq \overline{R}_3$.

**Table EC.2.1 Additional Notation Used in the Proofs.**

| Symbol | Description | Value |
|---|---|---|
| $\overline{R}_1$ | solves $\rho = \frac{\overline{R}_1 - \overline{B} - 1}{\overline{R}_1 - \overline{B}}$ | $\overline{R}_1 = \overline{B} + \frac{\rho + \sqrt{\rho^2+4}}{2}$ |
| $\overline{R}_2$ | solves $\rho = \frac{2(\overline{R}_2 - \overline{B} - 1)}{2(\overline{R}_2 - \overline{B}) - 1}$ | $\overline{R}_2 = \overline{B} + \frac{2-\rho}{2(1-\rho)}$ |
| $\overline{R}_3$ | solves $\rho = \frac{(\overline{R}_3 - \overline{B})^2 - 1}{\overline{R}_3 - \overline{B}}$ | $\overline{R}_3 = \overline{B} + \frac{1}{1-\rho}$ |

*Proof of Proposition EC.1.1* It is clear from (1) that the equilibrium is a threshold type, i.e., there exists a $c^{FIFO} \leq \bar{c}$ so that customers join the system if and only if $c \leq c^{FIFO}$. Furthermore, $c^{FIFO}$ is uniquely determined by $\frac{c^{FIFO}}{\mu - \Lambda c^{FIFO}/\bar{c}} = R - B$ subject to $c^{FIFO} \leq \bar{c}$. It follows that if $\rho > \frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}$, we have $c^{FIFO} = \frac{\bar{c}(\overline{R}-\overline{B})}{(\overline{R}-\overline{B})\rho+1}$. Otherwise, $c^{FIFO} = \bar{c}$, i.e., all customers will join the system. The service provider's revenue is given by $\Pi^{FIFO} = B\lambda_e^{FIFO} = B\Lambda c^{FIFO}/\bar{c}$, and customer welfare by

$$CW^{FIFO} = \Lambda \int_0^{c^{FIFO}} U^{FIFO}(c)/\bar{c}\, dc = \Lambda c^{FIFO}/\bar{c}\left(R - B - \frac{1}{\mu - \Lambda c^{FIFO}/\bar{c}} \cdot \frac{c^{FIFO}}{2}\right).$$

The results follow from plugging $c^{FIFO}$ into the expressions of $\Pi^{FIFO}$ and $CW^{FIFO}$. □

*Proof of Lemma EC.1.1* For a given $r$, we consider the following two cases.

(1) If $\rho > \frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}$: if $r \geq c^{FIFO} = \frac{\bar{c}(\overline{R}-\overline{B})}{(\overline{R}-\overline{B})\rho+1}$, there will be no customers using a line-sitter in equilibrium, and the system degenerates to the FIFO case, so $c^{LS} = c^{FIFO}$. If $r < c^{FIFO}$, a fraction of customers will use a line-sitter. In particular, if $\rho < 1$ and $U^{LS}(\bar{c}) > 0$, all customers will join the system, so $c^{LS} = \bar{c} > c^{FIFO}$. Otherwise, $c^{LS}$ is uniquely determined by $U^{LS}(c^{LS}) = 0$, which leads to $c^{LS} = \bar{c}\left(\frac{1+(\frac{r}{\bar{c}}+\overline{R}-\overline{B})\rho - \sqrt{[1+(\frac{r}{\bar{c}}+\overline{R}-\overline{B})\rho]^2 - 4(\overline{R}-\overline{B})\rho}}{2\rho}\right)$. Note that $c^{LS}$ can be rewritten as $\bar{c}\left(\frac{2(\overline{R}-\overline{B})\bar{c}}{1+(\frac{r}{\bar{c}}+\overline{R}-\overline{B})\rho + \sqrt{[1+(\frac{r}{\bar{c}}+\overline{R}-\overline{B})\rho]^2 - 4(\overline{R}-\overline{B})\rho}}\right)$, which is strictly decreasing in $r < c^{FIFO} = \frac{\bar{c}(\overline{R}-\overline{B})}{(\overline{R}-\overline{B})\rho+1}$. By plugging $r = c^{FIFO}$ into $c^{LS}$, we have $c^{LS}|_{r=c^{FIFO}} = c^{FIFO}$, which implies that $c^{LS} > c^{FIFO}$ for all $r < c^{FIFO}$.

(2) If $\rho \leq \frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}$, we have $U^{LS}(\bar{c}) > 0$ for all $r \in (0, \bar{c})$, and all customers will join the system, i.e., $c^{LS} = \bar{c}$. □

*Proof of Proposition 1* First, consider the case where there exist, in equilibrium, both customers who purchase line-sitting and customers who balk. By Lemma EC.1.1, we have $c^{LS}(r) = \overline{c}\left(\frac{1+(\frac{r}{\overline{c}}+\overline{R}-\overline{B})\rho-\sqrt{[1+(\frac{r}{\overline{c}}+\overline{R}-\overline{B})\rho]^2-4(\overline{R}-\overline{B})\rho}}{2\rho}\right)$, which is decreasing in $r$. Because $c^{LS}$ needs to be greater than $r$, we must have $r \le \overline{r}$ for some unique threshold $\overline{r}$ which is the solution of $r$ to $c^{LS}(r) = r$. It follows that $\overline{r} = \frac{(\overline{R}-\overline{B})\overline{c}}{(\overline{R}-\overline{B})\rho+1}$.

Recall from (3) that the revenue (rate) of the line-sitting firm in equilibrium is $\frac{r\Lambda(c^{LS}-r)}{\overline{c}} \frac{c^{LS}\Lambda/\overline{c}}{\mu(\mu-c^{LS}\Lambda/\overline{c})} = \frac{\Lambda^2 c^{LS}(c^{LS}-r)r}{\overline{c}^2\mu[\mu-(c^{LS}/\overline{c})\Lambda]}$, so the firm's objective is equivalent to maximizing $S^{LS}(r) := \frac{c^{LS}(r)(c^{LS}(r)-r)r}{[1-(c^{LS}(r)/\overline{c})\rho]}$ over $r \in [0, \overline{r}]$ with $c^{LS}(r) = \overline{c}\left(\frac{1+(\frac{r}{\overline{c}}+\overline{R}-\overline{B})\rho-\sqrt{[1+(\frac{r}{\overline{c}}+\overline{R}-\overline{B})\rho]^2-4(\overline{R}-\overline{B})\rho}}{2\rho}\right)$, that is, $\max_{r\in[0,\overline{r}]} S^{LS}(r)$ with

$$S^{LS}(r) = \frac{r\overline{c}^2}{2\rho^2} \cdot \frac{\left(1+(\frac{r}{\overline{c}}+\overline{R}-\overline{B})\rho-\sqrt{[1+(\frac{r}{\overline{c}}+\overline{R}-\overline{B})\rho]^2-4(\overline{R}-\overline{B})\rho}\right)\left(1+(\overline{R}-\overline{B}-\frac{r}{\overline{c}})\rho-\sqrt{[1+(\frac{r}{\overline{c}}+\overline{R}-\overline{B})\rho]^2-4(\overline{R}-\overline{B})\rho}\right)}{1-(\frac{r}{\overline{c}}+\overline{R}-\overline{B})\rho+\sqrt{[1+(\frac{r}{\overline{c}}+\overline{R}-\overline{B})\rho]^2-4(\overline{R}-\overline{B})\rho}}.$$

Notice that in equilibrium, we must have $1-(c^{LS}(r)/\overline{c})\rho > 0$ to ensure system stability. $r < c^{LS}(r)$ ($\Leftrightarrow r < \overline{r}$) further gives $S^{LS}(r) > 0$ for all $r \in (0, \overline{r})$. Because $S^{LS}(0) = S^{LS}(\overline{r}) = 0$ and $S^{LS}(r)$ is continuous, we can solve for the unique maximizer $\widehat{r} \in (0, \overline{r})$ by setting the first-order condition (FOC) of $S^{LS}(r)$ with respect to $r$ equal to zero. We obtain three roots: $r_1 = 0$, $r_2 = \overline{c}\left(\frac{[2(\overline{R}-\overline{B})\rho+1]\sqrt{(\overline{R}-\overline{B})\rho+1}-[(\overline{R}-\overline{B})\rho+1]^2}{[(\overline{R}-\overline{B})\rho+1]\rho}\right)$ and $r_3 = -\overline{c}\left(\frac{[2(\overline{R}-\overline{B})\rho+1]\sqrt{(\overline{R}-\overline{B})\rho+1}+[(\overline{R}-\overline{B})\rho+1]^2}{[(\overline{R}-\overline{B})\rho+1]\rho}\right) < 0$. Only $r_2$ is valid because $r_1$ and $r_3$ are not in $(0, \overline{r})$, and thus can be omitted. Thus, it follows that

$$\widehat{r} = \overline{c}\left(\frac{[2(\overline{R}-\overline{B})\rho+1]\sqrt{(\overline{R}-\overline{B})\rho+1}-[(\overline{R}-\overline{B})\rho+1]^2}{[(\overline{R}-\overline{B})\rho+1]\rho}\right) \text{ and } c^{LS} = \frac{\overline{c}(\overline{R}-\overline{B})}{\sqrt{(\overline{R}-\overline{B})\rho+1}}. \quad \text{(EC.2.1)}$$

Let $x = \sqrt{(\overline{R}-\overline{B})\rho+1} \ge 1$, we have $\widehat{r} \le c^{LS} \Leftrightarrow \frac{(2x^2-1)x-x^4}{x^2\rho} \le \frac{x^2-1}{x\rho} \Leftrightarrow 2x^2-1-x^3 \le x^2 \Leftrightarrow x \ge 1$. Thus we have $\widehat{r} \le c^{LS}$ for all $\sqrt{(\overline{R}-\overline{B})\rho+1} \ge 1$. On the other hand, Notice that $\hat{r} \ge 0 \Leftrightarrow (2x^2-1)x-x^4 \ge 0 \Leftrightarrow x^2-x^3 \ge 1-x^2 \Leftrightarrow x^2-x-1 \le 0 \Leftrightarrow x \le \frac{\sqrt{5}+1}{2}$. That is, $\hat{r} \ge 0$ if and only if $\overline{R}-\overline{B} \le \frac{\sqrt{5}+1}{2\rho}$. Therefore, for $c^{LS}(r) = \overline{c}\left(\frac{1+(\frac{r}{\overline{c}}+\overline{R}-\overline{B})\rho-\sqrt{[1+(\frac{r}{\overline{c}}+\overline{R}-\overline{B})\rho]^2-4(\overline{R}-\overline{B})\rho}}{2\rho}\right)$ and $r \in [0, \overline{r}]$, the maximum of $S^{LS}(r)$ can be attained at $\widehat{r}$ when $\overline{R}-\overline{B} \le \frac{\sqrt{5}+1}{2\rho}$. That is, $S^{LS}(r)$ increases in $r \le \widehat{r}$ and decreases in $r \ge \widehat{r}$. Building on the above properties, we will prove the different cases in Proposition 1 as follows.

When $\rho < 1$,

- Case (1):

(i) If $1/(1-\rho) \le \overline{R}-\overline{B}$, i.e., $\rho \le \frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}$, all customers will join the system in equilibrium even if the line-sitting service is not provided, i.e., $c^{LS} = \overline{c}$ and $\lambda_e^{LS} = \Lambda$. The revenue of the line-sitting firm can be simplified to $S^{LS}(r) = \frac{\overline{c}(\overline{c}-r)r}{1-\rho}$, which is maximized at $r^* = \overline{c}/2$.

(ii) If $(\rho/2)/(1-\rho)+1 \le \overline{R}-\overline{B} < 1/(1-\rho)$, i.e., $\frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}} < \rho \le \frac{2(\overline{R}-\overline{B}-1)}{2(\overline{R}-\overline{B})-1}$, all customers will join the system in equilibrium if the hourly line-sitting rate $r$ is set at $\frac{\overline{c}}{2}$. As such, $r^* = \overline{c}/2$ continues to be optimal because $S^{LS}(r)=\frac{c^{LS}(c^{LS}-r)r}{[1-(c^{LS}/\overline{c})\rho]}$ obtains its theoretical upper bound when $r=\overline{c}/2$ and $\lambda_e=\Lambda$ (i.e., maximum $\rho$). Combining (i) and (ii), we can conclude that for $\rho\le\frac{2(\overline{R}-\overline{B}-1)}{2(\overline{R}-\overline{B})-1}$, we always have $r^*=\overline{c}/2$ and $c^{LS}=\overline{c}$.

• Case (2): If $\rho\widehat{r}/[\overline{c}(1-\rho)]+1<\overline{R}-\overline{B}<(\rho/2)/(1-\rho)+1$, i.e., $\frac{2(\overline{R}-\overline{B}-1)}{2(\overline{R}-\overline{B})-1}<\rho<\min\{\frac{(\overline{R}-\overline{B})^2-1}{\overline{R}-\overline{B}},1\}$, which combines both Case (2)-a and Case (2)-b, the feasible region is nonempty because $\frac{2(\overline{R}-\overline{B}-1)}{2(\overline{R}-\overline{B})-1}<1$ and $\overline{R}-\overline{B}>1\Rightarrow 2(\overline{R}-\overline{B})^2>\overline{R}-\overline{B}+1\Leftrightarrow\frac{2}{2(\overline{R}-\overline{B})-1}<\frac{\overline{R}-\overline{B}+1}{\overline{R}-\overline{B}}\Leftrightarrow\frac{2(\overline{R}-\overline{B}-1)}{2(\overline{R}-\overline{B})-1}<\frac{(\overline{R}-\overline{B})^2-1}{\overline{R}-\overline{B}}$. Because all customers will join the system with $\widehat{r}$ as $\rho\widehat{r}/[\overline{c}(1-\rho)]+1<\overline{R}-\overline{B}$, the optimal price $r^*$ cannot be smaller than $\widehat{r}$ (this follows the same argument of the earlier proof for Case (1)-(ii)). Under the condition of Case (2), it can be shown that all customers join the system in equilibrium if and only if $r\le\frac{\overline{c}(\overline{R}-\overline{B}-1)(1-\rho)}{\rho}$ based on solving $\rho r/[\overline{c}(1-\rho)]+1=\overline{R}-\overline{B}$. Furthermore, we have $\widehat{r}<\frac{\overline{c}(\overline{R}-\overline{B}-1)(1-\rho)}{\rho}<\overline{c}/2$. When $r\in(0,\frac{\overline{c}(\overline{R}-\overline{B}-1)(1-\rho)}{\rho}]$, all customers join the system, so $S^{LS}(r)$ reduces to $\frac{\overline{c}(\overline{c}-r)r}{1-\rho}$ which is clearly increasing in $r$. When $r>\frac{\overline{c}(\overline{R}-\overline{B}-1)(1-\rho)}{\rho}$, we have already shown that $S^{LS}(r)$ decreases in $r\in(\widehat{r},\overline{c})$ when there are balking and line-sitting customers. Therefore, the optimal $r^*$ is attained at $r^*=\frac{\overline{c}(\overline{R}-\overline{B}-1)(1-\rho)}{\rho}$.

• Case (3)-a-(i): If $\rho\widehat{r}/[\overline{c}(1-\rho)]+1\ge\overline{R}-\overline{B}$, i.e., $\frac{(\overline{R}-\overline{B})^2-1}{\overline{R}-\overline{B}}\le\rho<1$, which implies $R-B\le\frac{\sqrt{5}+1}{2}$ because $(\overline{R}-\overline{B})^2-(\overline{R}-\overline{B})-1\le 0$, then we must have $\frac{(\overline{R}-\overline{B})^2-1}{\overline{R}-\overline{B}}\le\rho<1\le\frac{\sqrt{5}+1}{2(\overline{R}-\overline{B})}$. In this case, customer equilibrium can be described by two thresholds $0<\widehat{r}<c^{LS}<\overline{c}$ such that customers join without line-sitting, or join with line-sitting, or balk when $c\le\widehat{r}$, or $c\in(\widehat{r},c^{LS}]$, or $c>c^{LS}$ respectively, where $\widehat{r}$ and $c^{LS}$ are given in (EC.2.1). We have $r^*=\widehat{r}$ and $\lambda_e=\frac{\Lambda(\overline{R}-\overline{B})}{\sqrt{(\overline{R}-\overline{B})\rho+1}}$.
When $\rho\ge 1$, there must exist some balking customers in equilibrium.

• Case (3)-a-(ii): If $\frac{\Lambda(\overline{R}-\overline{B})}{\sqrt{(\overline{R}-\overline{B})\rho+1}}\le\mu$, i.e., $1\le\rho\le\frac{\sqrt{5}+1}{2(\overline{R}-\overline{B})}$, which implies $R-B\le\frac{\sqrt{5}+1}{2}$, then we have $\frac{(\overline{R}-\overline{B})^2-1}{\overline{R}-\overline{B}}\le 1<\rho\le\frac{\sqrt{5}+1}{2(\overline{R}-\overline{B})}$. From (EC.2.1), $r^*=\hat{r}$ is indeed the optimal solution.

Combining Case (3)-a-(i) and Case (3)-a-(ii) gives Case (3)-a in Proposition 1. In particular, when $\overline{R}=\overline{B}+\frac{\sqrt{5}+1}{2\rho}$, i.e., $\rho=\frac{\sqrt{5}+1}{2(\overline{R}-\overline{B})}$, $r^*$ reaches zero.

• Case (4): If $\frac{\Lambda(\overline{R}-\overline{B})}{\sqrt{(\overline{R}-\overline{B})\rho+1}}>\mu$, i.e., $\rho>\max\{\frac{\sqrt{5}+1}{2(\overline{R}-\overline{B})},1\}$, which combines both Case (4)-a and Case (4)-b in Proposition 1, we continue to have $r^*=0$ because a negative rate is certainly not optimal. In equilibrium, we must have $\lim_{r\to 0}\frac{(c^{LS}/\overline{c})\rho r}{\overline{c}(1-\rho(c^{LS}/\overline{c}))}+\frac{c^{LS}}{\overline{c}}=\overline{R}-\overline{B}\Leftrightarrow\lim_{r\to 0}\frac{(c^{LS}/\overline{c})\rho r}{\overline{c}(1-\rho(c^{LS}/\overline{c}))}=\overline{R}-\overline{B}-\frac{c^{LS}}{\overline{c}}$. Since $\overline{R}-\overline{B}>1>\frac{c^{LS}}{\overline{c}}$, for the right-hand-side (RHS) to have a finite positive number, the denominator of the left-hand-side (LHS) must approach zero as $r\to 0$, which implies that $c^{LS}=\overline{c}/\rho$. It follows that $\lambda_e^{LS}=c^{LS}\Lambda/\overline{c}=\mu$. □

*Proof of Corollary 1* We consider three cases based on the different forms of $r^*$ in Table 2 of Proposition 1.

Case (2) of Proposition 1: if $\rho\in\left(\frac{2(\overline{R}-\overline{B}-1)}{2(\overline{R}-\overline{B})-1},\min\left\{1,\frac{(\overline{R}-\overline{B})^2-1}{(\overline{R}-\overline{B})}\right\}\right]$, we have $dr^*/d\rho=\overline{c}[1-(\overline{R}-\overline{B})]/\rho^2\le 0$ for $\overline{R}-\overline{B}\ge 1$. On the other hand, we have $d((c^{LS}-r^*)/c^{LS})/d\rho=(\overline{R}-\overline{B}-1)/\rho^2\ge 0$ for $\overline{R}-\overline{B}\ge 1$.

Case (3) of Proposition 1: if $\rho\left(\frac{(\overline{R}-\overline{B})^2-1}{(\overline{R}-\overline{B})}, \frac{\sqrt{5}+1}{2(\overline{R}-\overline{B})}\right]$, we have $dr^*/d\rho = -\frac{(x-1)^2(2x^2+2x+1)}{2\rho^2((\overline{R}-\overline{B})\rho+1)^2}$, where $x=\sqrt{(\overline{R}-\overline{B})\rho+1}\geq 1$. It is immediate that $dr^*/d\rho\leq 0$ for all $x\geq 1$. Also, we have $d((c^{LS}-r^*)/c^{LS})/d\rho = \frac{(\overline{R}-\overline{B})(x+2)}{2(x+1)^2}>0$, where $x=\sqrt{(\overline{R}-\overline{B})\rho+1}\geq 1$.

Other cases of Proposition 1: $r^*$ and $(c^{LS}-r^*)/c^{LS}$ are both constant in $\rho$.

Therefore, $r^*$ is weakly decreasing in $\rho\in(0,\infty)$, and $(c^{LS}-r^*)/c^{LS}$ is weakly increasing in $\rho\in(0,\infty)$. □

*Proof of Proposition EC.1.2* Given $\overline{B}$, the service provider's revenue is given by $\Pi^{LS}=\lambda_e^{LS}B = (\Lambda c^{LS}/\bar{c})\cdot(\overline{B}\bar{c}/\mu)=\Lambda c^{LS}\overline{B}/\mu$, and customer welfare by

$$\begin{aligned} CW^{LS} &= \Lambda\left[\int_0^r U^{FIFO}(c)/\bar{c}\,dc + \int_r^{c^{LS}} U^{LS}(c)/\bar{c}\,dc\right] \\ &= \frac{\Lambda c^{LS}}{\bar{c}}(R-B) - \frac{\Lambda c^{LS}}{\bar{c}}\left[\left(\frac{r\Lambda c^{LS}}{\mu(\bar{c}\mu-\Lambda c^{LS})}+\frac{c^{LS}+r}{2\mu}\right)\frac{c^{LS}-r}{c^{LS}}+\frac{r^2}{2c^{LS}\left(\mu-\frac{\Lambda c^{LS}}{\bar{c}}\right)}\right] \\ &= c^{LS}\rho\left[(\overline{R}-\overline{B})+\frac{\left[(\frac{c^{LS}}{\bar{c}})^2+(\frac{r}{\bar{c}})^2\right]\rho-(\frac{c^{LS}}{\bar{c}})(1+\frac{2r}{\bar{c}}\rho)}{2(1-(\frac{c^{LS}}{\bar{c}})\rho)}\right]. \end{aligned}$$

The results of Proposition EC.1.2 follow from plugging $r^*$ and $c^{LS}$ from Proposition 1 into $\Pi^{LS}$ and $CW^{LS}$. □

*Proof of Theorem 1* Based on the expressions of $\Pi^{FIFO}$ and $\Pi^{LS}$ in Propositions EC.1.1 and EC.1.2, we consider the following five cases.

- When $\rho<1$,
  (1) If $\rho\leq\frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}$, we have $\Pi^{FIFO}=\Pi^{LS}=\bar{c}\rho\overline{B}$.
  (2) If $\frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}<\rho\leq\frac{(\overline{R}-\overline{B})^2-1}{\overline{R}-\overline{B}}$, we have $\Pi^{FIFO}=\bar{c}\rho\overline{B}\cdot\frac{(\overline{R}-\overline{B})}{(\overline{R}-\overline{B})\rho+1}<\bar{c}\rho\overline{B}=\Pi^{LS}$.
  (3) If $\frac{(\overline{R}-\overline{B})^2-1}{\overline{R}-\overline{B}}<\rho$, we have $\Pi^{FIFO}=\bar{c}\rho\overline{B}\cdot\frac{(\overline{R}-\overline{B})}{(\overline{R}-\overline{B})\rho+1}<\bar{c}\rho\overline{B}\cdot\frac{(\overline{R}-\overline{B})}{\sqrt{(\overline{R}-\overline{B})\rho+1}}=\Pi^{LS}$.
- When $\rho\geq 1$,
  (4) If $\rho\leq\frac{\sqrt{5}+1}{2(\overline{R}-\overline{B})}$, we have $\Pi^{FIFO}=\bar{c}\rho\overline{B}\cdot\frac{(\overline{R}-\overline{B})}{(\overline{R}-\overline{B})\rho+1}<\frac{\bar{c}\rho\overline{B}(\overline{R}-\overline{B})}{\sqrt{(\overline{R}-\overline{B})\rho+1}}=\Pi^{LS}$.
  (5) If $\rho>\frac{\sqrt{5}+1}{2(\overline{R}-\overline{B})}$, we have $\Pi^{FIFO}=\bar{c}\rho\overline{B}\cdot\frac{(\overline{R}-\overline{B})}{(\overline{R}-\overline{B})\rho+1}<\bar{c}\overline{B}$. □

*Proof of Theorem 2* Based on the expressions of $CW^{FIFO}$ and $CW^{LS}$ in Propositions EC.1.1 and EC.1.2, we consider six cases in total, four for $\rho<1$ and two cases for $\rho\geq 1$.

- When $\rho<1$,
  (1) If $\rho\leq\frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}$, i.e., $\overline{R}\geq\overline{R}_3$, we have $CW^{FIFO}=\bar{c}\rho\left(\overline{R}-\overline{B}-\frac{1}{2(1-\rho)}\right)<\bar{c}\rho\left(\overline{R}-\overline{B}-\frac{4-\rho}{8(1-\rho)}\right)=CW^{LS}$.
  (2) If $\frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}<\rho\leq\frac{2(\overline{R}-\overline{B}-1)}{2(\overline{R}-\overline{B})-1}$, i.e., $\overline{R}_2\leq\overline{R}<\overline{R}_3$, we have

$$\frac{\partial(CW^{FIFO}-CW^{LS})}{\partial\overline{R}}=\bar{c}\rho\left(\frac{[(\overline{R}-\overline{B})\rho+2](1-2\rho)-2}{2[(\overline{R}-\overline{B})\rho+1]^2}\right).$$

(i) If $2\rho-1\geq 0$, we have $[(\overline{R}-\overline{B})\rho+2](1-2\rho)-2<0$, which implies that $\frac{\partial(CW^{FIFO}-CW^{LS})}{\partial \overline{R}}<0$.

(ii) If $2\rho-1<0$, because $\overline{R}-\overline{B}<1/(1-\rho)$, we have $[(\overline{R}-\overline{B})\rho+2](1-2\rho)-2<(\frac{\rho}{1-\rho}+2)(1-2\rho)-2=\frac{(2\rho-3)\rho}{1-\rho}<0$ which also implies $\frac{\partial(CW^{FIFO}-CW^{LS})}{\partial \overline{R}}<0$. Therefore, for case (2), $CW^{FIFO}-CW^{LS}$ is decreasing in $\overline{R}\in[\overline{R}_2,\overline{R}_3)$. Furthermore, at $\overline{R}=\overline{R}_3$, we have $CW^{LS}|_{\overline{R}=\overline{R}_3}=\frac{\overline{c}\rho(4+\rho)}{8(1-\rho)}>\frac{4\overline{c}\rho}{8(1-\rho)}=CW^{FIFO}|_{\overline{R}=\overline{R}_3}$. At $\overline{R}=\overline{R}_2$, $CW^{LS}|_{\overline{R}=\overline{R}_2}>CW^{FIFO}|_{\overline{R}=\overline{R}_2}\Leftrightarrow \overline{c}\rho^2\frac{3\rho^2-6\rho+2}{8(1-\rho)(\rho^2-2)}<0\Leftrightarrow 3(\rho-1)^2<1\Leftrightarrow \rho<\frac{3-\sqrt{3}}{3}$. Therefore, if $\rho<\frac{3-\sqrt{3}}{3}$, we always have $CW^{LS}>CW^{FIFO}$.

If $\rho\geq\frac{3-\sqrt{3}}{3}$, let

$$\overline{R}_b\triangleq\overline{B}+\frac{\rho^2-12\rho+8-\sqrt{\rho^4+8\rho(\rho^2-2\rho+2)}}{8(\rho-1)(2\rho-1)}\in[\overline{R}_2,\overline{R}_3)$$

denote the unique $\overline{R}$ value such that $CW^{LS}|_{\overline{R}=\overline{R}_b}=CW^{FIFO}|_{\overline{R}=\overline{R}_b}$. Thus, $CW^{FIFO}<CW^{LS}$ if and only if $\overline{R}>\overline{R}_b$.

(3) If $\frac{2(\overline{R}-\overline{B}-1)}{2(\overline{R}-\overline{B})-1}<\rho\leq\frac{(\overline{R}-\overline{B})^2-1}{\overline{R}-\overline{B}}$, i.e., $\overline{R}_1\leq\overline{R}<\overline{R}_2$,

$$\frac{\partial(CW^{FIFO}-CW^{LS})}{\partial\overline{R}}=\overline{c}\rho\left(\frac{(\overline{R}-\overline{B})^2}{2[(\overline{R}-\overline{B})\rho+1]}-\frac{(\overline{R}-\overline{B}-1)^2(1-\rho)+\rho}{2\rho}\right)$$

and $\frac{\partial^2(CW^{FIFO}-CW^{LS})}{\partial\overline{R}^2}=\overline{c}\rho\left(\frac{(\overline{R}-\overline{B})\rho[(\overline{R}-\overline{B})^2\rho^2+3(\overline{R}-\overline{B})\rho+3](\rho-1)+(2\rho-1)}{\rho[(\overline{R}-\overline{B})\rho+1]^3}\right)$. Because the numerator $(\overline{R}-\overline{B})\rho((\overline{R}-\overline{B})^2\rho^2+3(\overline{R}-\overline{B})\rho+3)(\rho-1)+(2\rho-1)$ is decreasing in $\overline{R}$ and the denominator $\rho[(\overline{R}-\overline{B})\rho+1]^3$ is positive, we know that $\frac{\partial(CW^{FIFO}-CW^{LS})}{\partial\overline{R}}$ is unimodal in $\overline{R}\in[\overline{R}_1,\overline{R}_2)$. However, $\frac{\partial(CW^{FIFO}-CW^{LS})}{\partial\overline{R}}|_{\overline{R}=\overline{R}_1}=\overline{c}\rho\frac{2(\overline{R}_1-\overline{B})^5-2(\overline{R}_1-\overline{B})^4-(\overline{R}_1-\overline{B})^3+\overline{R}_1-\overline{B}+1}{2(\overline{R}_1-\overline{B})^3(1+\overline{R}_1-\overline{B})}>0$ and $\frac{\partial(CW^{FIFO}-CW^{LS})}{\partial\overline{R}}|_{\overline{R}=\overline{R}_2}=\overline{c}\rho\frac{2(\overline{R}_2-\overline{B})^3-2(\overline{R}_2-\overline{B})^2+2(\overline{R}_2-\overline{B})-1}{2(2(\overline{R}_2-\overline{B})^2-1)}>0$ because $2x_1^5-2x_1^4-x_1^3+x_1+1$ for $x_1:=\overline{R}_1-\overline{B}>1$ and $2x_2^3-2x_2^2+2x_2-1$ for $x_2:=\overline{R}_2-\overline{B}>1$. Therefore, $\frac{\partial(CW^{FIFO}-CW^{LS})}{\partial\overline{R}}>0$ for $\overline{R}\in[\overline{R}_1,\overline{R}_2)$. Furthermore, recall that $(\overline{R}_1-\overline{B})^2-(\overline{R}_1-\overline{B})\rho-1=0$, i.e., $\overline{R}_1=\frac{\sqrt{\rho^2+4}+\rho}{2}+\overline{B}$, so $(CW^{FIFO}-CW^{LS})|_{\overline{R}=\overline{R}_1}=\frac{\overline{c}(\overline{R}_1-\overline{B}-1)^2[(\overline{R}_1-\overline{B})^2-(\overline{R}_1-\overline{B})-1]}{2(\overline{R}_1-\overline{B})}<\frac{\overline{c}(\overline{R}_1-\overline{B}-1)^2[(\overline{R}_1-\overline{B})^2-(\overline{R}_1-\overline{B})\rho-1]}{2(\overline{R}_1-\overline{B})}=0$, which implies $CW^{FIFO}|_{\overline{R}=\overline{R}_1}<CW^{LS}|_{\overline{R}=\overline{R}_1}$. We still can have $CW^{FIFO}|_{\overline{R}=\overline{R}_2}<CW^{LS}|_{\overline{R}=\overline{R}_2}$ or $CW^{FIFO}|_{\overline{R}=\overline{R}_2}\geq CW^{LS}|_{\overline{R}=\overline{R}_2}$. It can be shown (similar to Case (2)) that if $\rho<\frac{3-\sqrt{3}}{3}$, we always have $CW^{FIFO}|_{\overline{R}=\overline{R}_2}<CW^{LS}|_{\overline{R}=\overline{R}_2}$ and in this case $CW^{FIFO}<CW^{LS}$ for $\overline{R}\in[\overline{R}_1,\overline{R}_2)$. If $\rho\geq\frac{3-\sqrt{3}}{3}$, let

$$\overline{R}_a\triangleq\overline{B}+\frac{2\rho-1+\sqrt{4\rho^2+1}}{2\rho}\in(\overline{R}_1,\overline{R}_2]$$

denote the unique $\overline{R}$ value such that $CW^{FIFO}|_{\overline{R}=\overline{R}_a}=CW^{LS}|_{\overline{R}=\overline{R}_a}$. Thus, $CW^{LS}>CW^{FIFO}$ if and only if $\overline{R}<\overline{R}_a$.

(4) If $\frac{(\overline{R}-\overline{B})^2-1}{\overline{R}-\overline{B}}<\rho<1$, i.e., $\overline{R}<\overline{R}_1$, in this case, we have

$$\frac{\partial^2(CW^{FIFO}-CW^{LS})}{\partial\overline{R}^2}=\overline{c}\rho\left(\frac{3(\overline{R}-\overline{B})\rho^2[(\overline{R}-\overline{B})\rho+2]\sqrt{(\overline{R}-\overline{B})\rho+1}+4(1-2\rho)}{4[(\overline{R}-\overline{B})\rho+1]^3}\right).$$

Because the numerator $3(\overline{R}-\overline{B})\rho^2[(\overline{R}-\overline{B})\rho+2]\sqrt{(\overline{R}-\overline{B})\rho+1}+4(1-2\rho)>2(3\rho^2-4\rho+2)>0$ is positive and so is the denominator, we know $\frac{\partial^2(CW^{FIFO}-CW^{LS})}{\partial\overline{R}^2}>0$. That is, $CW^{FIFO}-CW^{LS}$ is convex in $\overline{R}\in[\overline{B}+1,\overline{R}_1)$. Recall from Case (3) that $(CW^{FIFO}-CW^{LS})|_{\overline{R}=\overline{R}_1}<0$. In the meantime, we can show $(CW^{FIFO}-CW^{LS})|_{\overline{R}=\overline{B}+1}<0$ due to the following. Let $x:=\sqrt{1+\rho}\in(1,\sqrt{2})$, we have that $(CW^{FIFO}-CW^{LS})|_{\overline{R}=\overline{B}+1}=\overline{c}\rho\left(\frac{1}{2(1+\rho)}-\frac{\sqrt{1+\rho}-1}{\sqrt{1+\rho}}\frac{2\sqrt{1+\rho}-\rho}{2\rho}\right)=\frac{\overline{c}(x^2-1)}{2x}\left(\frac{1}{x}-\frac{-x^2+2x+1}{x+1}\right)=\frac{\overline{c}(x^2-1)}{2x}\left(\frac{(1-x)(-x^2+x+1)}{x(1+x)}\right)<0$ because $1-x<0$ and $-x^2+x+1>0$ for $x\in(1,\sqrt{2})$. Therefore, $CW^{LS}>CW^{FIFO}$ for all $\overline{R}\in[\overline{B}+1,\overline{R}_1)$.

• When $\rho\geq 1$,

(5) If $\rho\leq\frac{\sqrt{5}+1}{2(\overline{R}-\overline{B})}$, i.e., $\overline{R}-\overline{B}\leq\frac{\sqrt{5}+1}{2\rho}$, we have $CW^{LS}>CW^{FIFO}$ for $\overline{R}\in[\overline{B}+1,\overline{B}+\frac{\sqrt{5}+1}{2\rho})$ and the proof is identical to that of Case (4).

(6) If $\rho>\frac{\sqrt{5}+1}{2(\overline{R}-\overline{B})}$, i.e., $\overline{R}-\overline{B}>\frac{\sqrt{5}+1}{2\rho}$, we have $CW^{FIFO}=\overline{c}\cdot\frac{(\overline{R}-\overline{B})^2\rho^2}{2\rho[(\overline{R}-\overline{B})\rho+1]}>\overline{c}/(2\rho)=CW^{LS}$ because $(\overline{R}-\overline{B})\rho>\frac{\sqrt{5}+1}{2}\Rightarrow[(\overline{R}-\overline{B})\rho]^2-(\overline{R}-\overline{B})\rho-1>0\Rightarrow[(\overline{R}-\overline{B})\rho]^2>(\overline{R}-\overline{B})\rho+1$. Let $\overline{R}_c\triangleq\overline{B}+\frac{\sqrt{5}+1}{2\rho}$. Thus, $CW^{LS}>CW^{FIFO}$ if and only if $\overline{R}<\overline{R}_c$. In particular, when $\rho>\frac{\sqrt{5}+1}{2}$, $\overline{R}_c=\overline{B}+\frac{\sqrt{5}+1}{2\rho}<\overline{B}+1$. Since $\overline{R}\geq\overline{B}+1$ by Assumption 1, we have $CW^{LS}<CW^{FIFO}$ for all $\rho>\frac{\sqrt{5}+1}{2}$. □

*Proof of Lemma EC.1.2* Recall that $w_1(\lambda_e,q)=\frac{\lambda_e}{(\mu-\lambda_e)(\mu-\lambda_e q)}+\frac{1}{\mu}$ and $w_2(\lambda_e,q)=\frac{\lambda_e}{\mu(\mu-\lambda_e q)}+\frac{1}{\mu}$. At any equilibrium in which some customers purchase priority, we must have $P=(w_1-w_2)c_a^{PRI}$ and $R-B-P=w_2c_b^{PRI}$ which can be transferred into

$$\frac{\mu P}{c_a^{PRI}}=\frac{\rho^2(c_b^{PRI})^2}{(\overline{c}-\rho c_b^{PRI})[\overline{c}-\rho(c_b^{PRI}-c_a^{PRI})]},\tag{EC.2.2}$$

$$\frac{\mu(R-B-P)}{c_b^{PRI}}=\frac{\overline{c}+\rho c_b^{PRI}(1-q)}{\overline{c}-\rho(c_b^{PRI}-c_a^{PRI})}.\tag{EC.2.3}$$

By solving (EC.2.2)-(EC.2.3), we obtain $c_a^{PRI}=\frac{\overline{c}\overline{P}}{\rho[(\overline{R}-\overline{B}-\overline{P})(\overline{R}-\overline{B})\rho-\overline{P}]}$ and $c_b^{PRI}=\frac{\overline{c}(\overline{R}-\overline{B})}{(\overline{R}-\overline{B})\rho+1}$. It follows that $c_b^{PRI}<\overline{c}\Leftrightarrow\rho>\frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}$, otherwise $c_b^{PRI}=\overline{c}$. (1) When $\rho>\frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}$, we have $\frac{\overline{c}\overline{P}}{\rho[(\overline{R}-\overline{B}-\overline{P})(\overline{R}-\overline{B})\rho-\overline{P}]}$ and $c_b=\frac{\overline{c}(\overline{R}-\overline{B})}{(\overline{R}-\overline{B})\rho+1}$. To have a nonzero amount of priority purchasing, we also require $c_a^{PRI}<c_b^{PRI}\Leftrightarrow\overline{P}<\frac{(\overline{R}-\overline{B})^3\rho^2}{[(\overline{R}-\overline{B})\rho+1]^2}$. Therefore, when $\overline{P}<\frac{(\overline{R}-\overline{B})^3\rho^2}{[(\overline{R}-\overline{B})\rho+1]^2}$, customers will purchase priority if and only if $c\in(c_a^{PRI},c_b^{PRI}]$, otherwise (when $\overline{P}\geq\frac{(\overline{R}-\overline{B})^3\rho^2}{[(\overline{R}-\overline{B})\rho+1]^2}$), there will be no priority customers. (2) When $\rho\leq\frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}$, we have $c_b^{PRI}=\overline{c}$ and all customers will join the system. By solving $(w_1-w_2)c_a^{PRI}=P$ for $c_a^{PRI}$, we have $c_a^{PRI}=\frac{\overline{c}\overline{P}(1-\rho)^2}{\rho[\rho-\overline{P}(1-\rho)]}$. When $c_a^{PRI}<\overline{c}\Leftrightarrow\overline{P}<\frac{\rho^2}{1-\rho}$, customers will purchase priority if and only if $c>c_a^{PRI}$, otherwise (i.e., when $\overline{P}\geq\frac{\rho^2}{1-\rho}$) no customers purchase priority. □

*Proofs of Propositions EC.1.3 and EC.1.4* The service provider's revenue is $\Pi^{PRI}=\lambda_e(B+qP)=(\Lambda c_b^{PRI}/\overline{c})(\overline{B}+q\overline{P})$ which, after algebraic manipulation, can be written as

$$\Pi^{PRI}(\overline{P})=\begin{cases}\overline{c}\rho\left(\frac{(\overline{P}+\overline{B})(\overline{R}-\overline{B})}{(\overline{R}-\overline{B})\rho+1}-\frac{\overline{P}^2}{\rho[(\overline{R}-\overline{B}-\overline{P})(\overline{R}-\overline{B})\rho-\overline{P}]}\right), & \text{if } \rho>\frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}};\\ \overline{c}\rho\left(\overline{B}+\frac{\overline{P}(\rho^2-\overline{P}(1-\rho))}{\rho[\rho-\overline{P}(1-\rho)]}\right), & \text{if } \rho\leq\frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}.\end{cases}$$

Taking the derivative of $\Pi^{PRI}$ with respect to $\overline{P}$ gives

$$\frac{\partial \Pi^{PRI}(\overline{P})}{\partial \overline{P}} = \begin{cases} \overline{c}\rho\left(-\frac{2\overline{P}}{\rho[(\overline{R}-\overline{B})(\overline{R}-\overline{B}-\overline{P})\rho-\overline{P}]}+\frac{\overline{R}-\overline{B}}{1-\overline{B}\rho+\overline{R}\rho}+\frac{\overline{P}^2(-1+\overline{B}\rho-\overline{R}\rho)}{\rho\left[(\overline{R}-\overline{B})^2\rho+\overline{P}(-1+\overline{B}\rho-\overline{R}\rho)\right]^2}\right), & \text{if } \rho > \frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}; \\ \overline{c}\rho\left(\frac{\overline{P}^2(1-\rho)^2-2\overline{P}(1-\rho)\rho+\rho^3}{\rho[\rho-\overline{P}(1-\rho)]^2}\right), & \text{if } \rho \le \frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}. \end{cases}$$

$$\frac{\partial^2 \Pi^{PRI}(\overline{P})}{\partial \overline{P}^2} = \begin{cases} -\overline{c}\rho\left(\frac{2(\overline{R}-\overline{B})^4\rho}{\left[(\overline{R}-\overline{B})^2\rho-\overline{\overline{P}}((\overline{R}-\overline{B})\rho+1)\right]^3}\right) < 0, & \text{if } \rho > \frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}; \\ \overline{c}\rho\left(\frac{2(1-\rho)^2\rho}{[P(1-\rho)-\rho]^3}\right) < 0, & \text{if } \rho \le \frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}. \end{cases}$$

Based on Lemma EC.1.2, to have priority purchase, the service provider must consider $\overline{P} < \frac{(\overline{R}-\overline{B})^3\rho^2}{[(\overline{R}-\overline{B})\rho+1]^2}$ when $\rho > \frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}$ and $\overline{P} < \frac{\rho^2}{1-\rho}$ when $\rho \le \frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}$. For each of these cases, there exists a unique $\overline{P}^* \in (0, \overline{R}-\overline{B})$ that satisfies the FOC. $\overline{P}^*$ is given as follows.

$$\overline{P}^* = \begin{cases} (\overline{R}-\overline{B})^2\rho\left(\frac{(\overline{R}-\overline{B})\rho+1-\sqrt{(\overline{R}-\overline{B})\rho+1}}{[(\overline{R}-\overline{B})\rho+1]^2}\right), & \text{if } \rho > \frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}; \\ \frac{\rho(1-\sqrt{1-\rho})}{1-\rho}, & \text{if } \rho \le \frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}. \end{cases}$$

$P^*$ and $\Pi^{PRI}$ follow from plugging $\overline{P}^*$ into $\overline{P}^*\overline{c}/\mu$ and $\Pi^{PRI}(\overline{P})$, respectively. On the other hand, customer welfare is given by

$$\begin{aligned} CW^{PRI} &= \Lambda\left[\int_0^{c_a^{PRI}} U^{REG}(c)/\overline{c}dc + \int_{c_a^{PRI}}^{c_b^{PRI}} U^{PRI}(c)/\overline{c}dc\right] \\ &= \lambda_e(R-B) - \lambda_e\left[\frac{w_1 c_a^{PRI}}{2}\cdot c_a^{PRI}/c_b^{PRI} + \left(\frac{w_2(c_b^{PRI}+c_a^{PRI})}{2}+P^*\right)\cdot(c_b^{PRI}-c_a^{PRI})/c_b^{PRI}\right] \\ &= c_b^{PRI}\rho\left(\overline{R}-\overline{B}-\frac{w_1\mu(c_a^{PRI})^2}{2c_b^{PRI}\overline{c}} - \frac{w_2\mu((c_b^{PRI})^2-(c_a^{PRI})^2)}{2c_b^{PRI}\overline{c}} - \frac{\overline{P}^*(c_b^{PRI}-c_a^{PRI})}{c_b^{PRI}}\right). \end{aligned}$$

The results follow from plugging $c_a^{PRI}$, $c_b^{PRI}$ from Lemma EC.1.2 into $\overline{P}^*$. □

*Proof of Theorem 3* Based on the expressions of $\Pi^{FIFO}$, $\Pi^{PRI}$, $CW^{FIFO}$ and $CW^{PRI}$ in Propositions EC.1.1 and EC.1.4, we consider the following five cases, we consider the following two cases.

(1) If $\rho \le \frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}$, we have $\Pi^{FIFO} - \Pi^{PRI} = -\overline{c}\rho\cdot\frac{(1-\sqrt{1-\rho})^2}{1-\rho} < 0$ and $CW^{FIFO} - CW^{PRI} = \frac{\overline{c}\rho(1-\sqrt{1-\rho})^2}{2(1-\rho)} > 0$.

(2) If $\rho > \frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}$, we have $\Pi^{FIFO} - \Pi^{PRI} = -\overline{c}\rho\frac{(\overline{R}-\overline{B})^2(\sqrt{(\overline{R}-\overline{B})\rho+1}-1)^2}{[(\overline{R}-\overline{B})\rho+1]^2} < 0$ and $CW^{FIFO} - CW^{PRI} = \overline{c}\rho\frac{(\overline{R}-\overline{B})^2(\sqrt{(\overline{R}-\overline{B})\rho+1}-1)^2}{2[(\overline{R}-\overline{B})\rho+1]^2} > 0$, which completes this proof. □

*Proof of Theorem 4* Based on the expressions of $\Pi^{LS}$ and $\Pi^{PRI}$ in Propositions EC.1.2 and EC.1.4, we consider the following five cases.

- When $\rho < 1$,

(1) If $\rho \le \frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}$, i.e., $\overline{R} \ge \overline{R}_3$, we have $\Pi^{PRI} = \overline{c}\rho\cdot\left(\frac{(1-\sqrt{1-\rho})^2}{1-\rho}+\overline{B}\right) > \overline{c}\rho\overline{B} = \Pi^{LS}$.

(2) If $\frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}} < \rho \le \frac{(\overline{R}-\overline{B})^2-1}{\overline{R}-\overline{B}}$, i.e., $\overline{R}_1 \le \overline{R} < \overline{R}_3$, we have

$$\begin{aligned}\Pi^{PRI} > \Pi^{LS} &\Leftrightarrow (\overline{R}-\overline{B}) \cdot \frac{(\overline{R}-\overline{B}) + [(\overline{R}-\overline{B})\rho+1]\overline{R} - 2(\overline{R}-\overline{B})\sqrt{(\overline{R}-\overline{B})\rho+1}}{[(\overline{R}-\overline{B})\rho+1]^2} > \overline{B} \\ &\Leftrightarrow (\sqrt{(\overline{R}-\overline{B})\rho+1}-1)^2(\overline{R}-\overline{B})^2 > \overline{B}[(\overline{R}-\overline{B})\rho+1]((\overline{R}-\overline{B})\rho+1-(\overline{R}-\overline{B})) \\ &\Leftrightarrow \left(1-\frac{1}{\sqrt{(\overline{R}-\overline{B})\rho+1}}\right)^2 \cdot \frac{(\overline{R}-\overline{B})^2}{(\overline{R}-\overline{B})\rho+1-(\overline{R}-\overline{B})} > \overline{B}.\end{aligned}$$

Note that the LHS of the last inequality is increasing in $\overline{R}$. By substituting $\overline{R}=\overline{R}_1$ into the LHS of this inequality, we have

$$\left(1-\frac{1}{\sqrt{(\overline{R}-\overline{B})\rho+1}}\right)^2 \cdot \frac{(\overline{R}-\overline{B})^2}{(\overline{R}-\overline{B})\rho+1-(\overline{R}-\overline{B})}\Big|_{\overline{R}=\overline{R}_1} = 1-\frac{2}{\rho+\sqrt{4+\rho^2}}.$$

Therefore, when $\overline{B} < 1-\frac{2}{\rho+\sqrt{4+\rho^2}}$, we always have $\Pi^{PRI} > \Pi^{LS}$. Otherwise, i.e., when $\overline{B} \ge 1-\frac{2}{\rho+\sqrt{4+\rho^2}}$, there exists a threshold $\overline{R}^{(1)}$ such that $\Pi^{LS} > \Pi^{PRI}$ if and only if $\overline{R} < \overline{R}^{(1)}$ (and $\overline{R}_1 \le \overline{R} < \overline{R}_3$) where $\overline{R}^{(1)}$ is the unique solution of $\overline{R}$ to the equation

$$F_1(R,\rho) \equiv \left(1-\frac{1}{\sqrt{(\overline{R}-\overline{B})\rho+1}}\right)^2 \cdot \frac{(\overline{R}-\overline{B})^2}{(\overline{R}-\overline{B})\rho+1-(\overline{R}-\overline{B})} - \overline{B} = 0.$$

Because $1-\frac{2}{\rho+\sqrt{4+\rho^2}} > \frac{\sqrt{1+\rho}-1}{1+\rho}$ for $\rho \in (0,1)$[EC.2.1], then we always have $\Pi^{PRI} > \Pi^{LS}$ when $\overline{B} \le \frac{\sqrt{1+\rho}-1}{1+\rho}$.

Next, we prove that $\overline{R}^{(1)}$ is increasing in $\rho \in \left(\frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}, \frac{(\overline{R}-\overline{B})^2-1}{\overline{R}-\overline{B}}\right)$. We need to show that $\frac{d\overline{R}^{(1)}}{d\rho} = -\frac{\partial F_1(\overline{R}^{(1)},\rho)/\partial\rho}{\partial F_1(\overline{R}^{(1)},\rho)/\partial\overline{R}^{(1)}} > 0$. First, $\partial F_1(\overline{R}^{(1)},\rho)/\partial\overline{R}^{(1)} > 0$ because $\left(1-\frac{1}{\sqrt{(\overline{R}-\overline{B})\rho+1}}\right)^2 \cdot \frac{(\overline{R}-\overline{B})^2}{(\overline{R}-\overline{B})\rho+1-(\overline{R}-\overline{B})}$ is increasing in $\overline{R}$. Next, let $x=\sqrt{(\overline{R}-\overline{B})\rho+1}$, we have $d\left(\frac{(1-1/x)^2}{x^2-(\overline{R}-\overline{B})}\right)/dx = -\frac{2(x-1)\left(\overline{R}-\overline{B}+(x-2)x^2\right)}{x^3\left(\overline{R}-\overline{B}-x^2\right)^2} < 0$ because $\overline{R}-\overline{B}+(x-2)x^2 \ge x+(x-2)x^2 = x(x-1)^2 \ge 0$. Thus, $\partial F_1(\overline{R}^{(1)},\rho)/\partial\rho < 0$. Therefore, $\overline{R}^{(1)}$ is increasing in $\rho \in \left[\frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}, \frac{(\overline{R}-\overline{B})^2-1}{\overline{R}-\overline{B}}\right]$.

(3) If $\rho \in \left(\frac{(\overline{R}-\overline{B})^2-1}{\overline{R}-\overline{B}}, 1\right)$, i.e., $\overline{R} < \overline{R}_1$, we have

$$\begin{aligned}\Pi^{PRI} > \Pi^{LS} &\Leftrightarrow \frac{(\overline{R}-\overline{B}) + [(\overline{R}-\overline{B})\rho+1]\overline{R} - 2(\overline{R}-\overline{B})\sqrt{(\overline{R}-\overline{B})\rho+1}}{[(\overline{R}-\overline{B})\rho+1]^2} > \frac{\overline{B}}{\sqrt{(\overline{R}-\overline{B})\rho+1}} \\ &\Leftrightarrow \frac{\left(\sqrt{(\overline{R}-\overline{B})\rho+1}-1\right)(\overline{R}-\overline{B})}{(\overline{R}-\overline{B})\rho+1} > \overline{B}.\end{aligned}$$

[EC.2.1] Define $\Phi(\rho) = 1-\frac{2}{\rho+\sqrt{4+\rho^2}} - \frac{\sqrt{1+\rho}-1}{1+\rho}$. We have $d\Phi(\rho)/d\rho = \frac{1}{2}\left(1-\frac{\rho}{\sqrt{4+\rho^2}}+\frac{1}{(1+\rho)^{3/2}}\right) > 0$ and $\Phi(0)=0$, which gives $\Phi(\rho) > 0$ for all $\rho \in (0,1)$.

Note that $\frac{\left(\sqrt{(\overline{R}-\overline{B})\rho+1}-1\right)(\overline{R}-\overline{B})}{(\overline{R}-\overline{B})\rho+1}$ is increasing in $\overline{R}$. Then if $\frac{\left(\sqrt{(\overline{R}-\overline{B})\rho+1}-1\right)(\overline{R}-\overline{B})}{(\overline{R}-\overline{B})\rho+1}|_{\overline{R}=\overline{R}_1}<\overline{B}$ (or equivalently, $\overline{B}>1-\frac{2}{\rho+\sqrt{4+\rho^2}}$), we have $\Pi^{LS}>\Pi^{PRI}$ for all $\overline{R}<\overline{R}_1$. Otherwise, i.e., when $\frac{\left(\sqrt{(\overline{R}-\overline{B})\rho+1}-1\right)(\overline{R}-\overline{B})}{(\overline{R}-\overline{B})\rho+1}|_{\overline{R}=\overline{R}_1}\geq\overline{B}$, we have $(\Pi^{PRI}-\Pi^{LS})|_{\overline{R}=\overline{R}_1}>0$.

(i) If $(\Pi^{PRI}-\Pi^{LS})|_{\overline{R}=\overline{B}+1}\geq 0$, i.e., $\overline{B}\leq\frac{\sqrt{1+\rho}-1}{1+\rho}$, we have $\Pi^{PRI}>\Pi^{LS}$ for all $\overline{R}\in[\overline{B}+1,\overline{R}_1)$.

(ii) If $(\Pi^{PRI}-\Pi^{LS})|_{\overline{R}=\overline{B}+1}<0$, i.e., $\overline{B}>\frac{\sqrt{1+\rho}-1}{1+\rho}$, there exists a unique threshold $\overline{R}^{(2)}$ such that $\Pi^{LS}>\Pi^{PRI}$ if and only if $\overline{R}<\overline{R}^{(2)}$ where $\overline{R}^{(2)}$ is the unique solution of $\overline{R}$ to the equation

$$F_2(R,\rho)\equiv\frac{\left(\sqrt{(\overline{R}-\overline{B})\rho+1}-1\right)(\overline{R}-\overline{B})}{(\overline{R}-\overline{B})\rho+1}-\overline{B}=0.$$

Next we prove that $\overline{R}^{(2)}$ is decreasing in $\rho\in\left(\frac{(\overline{R}-\overline{B})^2-1}{\overline{R}-\overline{B}},1\right)$. We need to show that $\frac{d\overline{R}^{(2)}}{d\rho}=-\frac{\partial F_2(\overline{R}^{(2)},\rho)/\partial\rho}{\partial F_2(\overline{R}^{(2)},\rho)/\partial\overline{R}^{(2)}}<0$. First, we have $\partial F_2(\overline{R}^{(2)},\rho)/\partial\overline{R}^{(2)}>0$ because $\frac{\left(\sqrt{(\overline{R}-\overline{B})\rho+1}-1\right)(\overline{R}-\overline{B})}{(\overline{R}-\overline{B})\rho+1}$ is increasing in $\overline{R}$. Next, let $x=\sqrt{(\overline{R}-\overline{B})\rho+1}$. We have $d\left(\frac{x-1}{x^2}\right)/dx=\frac{2-x}{x^3}>0\Leftrightarrow x<2\Leftrightarrow\rho<\frac{3}{\overline{R}-\overline{B}}$. Notice that $\frac{(\overline{R}-\overline{B})^2-1}{\overline{R}-\overline{B}}<1\Leftrightarrow\overline{R}-\overline{B}<\frac{\sqrt{5}+1}{2}<2$. It follows that $\rho<1<\frac{3}{\overline{R}-\overline{B}}$. Thus, we obtain $\partial F_2(\overline{R}^{(2)},\rho)/\partial\rho>0$, which implies that $\overline{R}^{(2)}$ is decreasing in $\rho\in\left(\frac{(\overline{R}-\overline{B})^2-1}{\overline{R}-\overline{B}},1\right)$.

- When $\rho\geq 1$,

(4) If $\rho\leq\frac{\sqrt{5}+1}{2(\overline{R}-\overline{B})}$, i.e., $\overline{R}\leq\overline{R}_c$ ($\overline{R}_c$ is defined in Theorem 2), similar to Case (3) ($\rho<1$), we have $\Pi^{PRI}>\Pi^{LS}\Leftrightarrow\frac{\left(\sqrt{(\overline{R}-\overline{B})\rho+1}-1\right)(\overline{R}-\overline{B})}{(\overline{R}-\overline{B})\rho+1}>\overline{B}$. Note that $(\Pi^{PRI}-\Pi^{LS})|_{\overline{R}=\overline{R}_c}>0\Leftrightarrow\overline{B}<\frac{\sqrt{1+\frac{(\sqrt{5}+1)\rho}{2}}-1}{1+\frac{(\sqrt{5}+1)\rho}{2}}\cdot\frac{\sqrt{5}+1}{2}$ and $(\Pi^{PRI}-\Pi^{LS})|_{\overline{R}=\overline{B}+1}>0\Leftrightarrow\overline{B}<\frac{\sqrt{1+\rho}-1}{1+\rho}$. Therefore, $\Pi^{LS}>\Pi^{PRI}$ if and only if

$$\overline{B}>\max\left\{\frac{\sqrt{1+\rho}-1}{1+\rho},\frac{\sqrt{1+\frac{(\sqrt{5}+1)\rho}{2}}-1}{1+\frac{(\sqrt{5}+1)\rho}{2}}\cdot\frac{\sqrt{5}+1}{2}\right\}=\frac{\sqrt{1+\frac{(\sqrt{5}+1)\rho}{2}}-1}{1+\frac{(\sqrt{5}+1)\rho}{2}}\cdot\frac{\sqrt{5}+1}{2}.$$

Otherwise, (i) if $\overline{B}\leq\frac{\sqrt{1+\rho}-1}{1+\rho}$, we have $\Pi^{PRI}\geq\Pi^{LS}$, or (ii) if $\frac{\sqrt{1+\rho}-1}{1+\rho}<\overline{B}\leq\frac{\sqrt{1+\frac{(\sqrt{5}+1)\rho}{2}}-1}{1+\frac{(\sqrt{5}+1)\rho}{2}}\cdot\frac{\sqrt{5}+1}{2}$, we have $\Pi^{LS}>\Pi^{PRI}$ if and only if $\overline{R}\leq\overline{R}^{(3)}$, where $\overline{R}^{(3)}=\overline{B}+\frac{\sqrt{5}+1}{2\rho}$ and it is immediate that it is decreasing in $\rho$.

(5) If $\rho>\frac{\sqrt{5}+1}{2(\overline{R}-\overline{B})}$, i.e., $\overline{R}>\overline{R}_c$, we have

$$\Pi^{PRI}>\Pi^{LS}\Leftrightarrow(\overline{R}-\overline{B})\cdot\frac{(\overline{R}-\overline{B})+[(\overline{R}-\overline{B})\rho+1]\overline{R}-2(\overline{R}-\overline{B})\sqrt{(\overline{R}-\overline{B})\rho+1}}{[(\overline{R}-\overline{B})\rho+1]^2}>\frac{\overline{B}}{\rho}$$
$$\Leftrightarrow\left(1-\frac{1}{\sqrt{(\overline{R}-\overline{B})\rho+1}}\right)^2(\overline{R}-\overline{B})^2\rho>\overline{B}.$$

Note that the LHS of the last inequality is increasing in $\overline{R}$. Therefore, if $(\Pi^{PRI}-\Pi^{LS})|_{\overline{R}=\overline{R}_c}\geq 0$ (if $\overline{B}\leq\frac{\sqrt{1+\frac{(\sqrt{5}+1)\rho}{2}}-1}{1+\frac{(\sqrt{5}+1)\rho}{2}}\cdot\frac{\sqrt{5}+1}{2}$), we always have $\Pi^{PRI}>\Pi^{LS}$. Otherwise, (if $\overline{B}>\frac{\sqrt{1+\frac{(\sqrt{5}+1)\rho}{2}}-1}{1+\frac{(\sqrt{5}+1)\rho}{2}}\cdot\frac{\sqrt{5}+1}{2}$), there exists a unique $\overline{R}^{(4)}$ such that $\Pi^{LS} > \Pi^{PRI}$ if and only if $\overline{R} < \overline{R}^{(4)}$ where $\overline{R}^{(4)}$ is the unique solution of $\overline{R}$ to the equation

$$F_3(R,\rho) \equiv \left(1 - \frac{1}{\sqrt{(\overline{R}-\overline{B})\rho+1}}\right)^2 (\overline{R}-\overline{B})^2\rho - \overline{B} = 0.$$

Because $\frac{\sqrt{1+\frac{(\sqrt{5}+1)\rho}{2}}-1}{1+\frac{(\sqrt{5}+1)\rho}{2}} \cdot \frac{\sqrt{5}+1}{2} > \frac{\sqrt{1+\rho}-1}{1+\rho}$ for $\rho \geq 1$[EC.2.2], we always have $\underline{\Pi^{PRI} > \Pi^{LS} \text{ when } \overline{B} \leq \frac{\sqrt{1+\rho}-1}{1+\rho}}$.

Next, we prove that $\overline{R}^{(4)}$ is decreasing in $\rho$. We need to show that $\frac{d\overline{R}^{(4)}}{d\rho} = -\frac{\partial F_3(\overline{R}^{(4)},\rho)/\partial\rho}{\partial F_3(\overline{R}^{(4)},\rho)/\partial \overline{R}^{(4)}} < 0$. Let $x = \sqrt{(\overline{R}-\overline{B})\rho+1}$, we have $d((1-1/x)^2)(x^2-1)/dx = \frac{2(x-1)^2(x^2+x+1)}{x^3} > 0$, which implies $\partial F_3(\overline{R}^{(4)},\rho)/\partial\rho > 0$. By inspection, it is obvious that $\partial F_3(\overline{R}^{(4)},\rho)/\partial\overline{R}^{(4)} > 0$. Hence, $\overline{R}^{(4)}$ is decreasing in $\rho$.

Combining the underlined parts in cases (1)-(5), if $\overline{B} \leq \frac{\sqrt{1+\rho}-1}{1+\rho}$, we have $\Pi^{PRI} > \Pi^{LS}$ for all $\overline{R} \geq \overline{B}+1$ and for all $\rho$. Otherwise, define $\overline{R}'$ for the mutually exclusive and collectively exhaustive cases for $\overline{B} > \frac{\sqrt{1+\rho}-1}{1+\rho}$. We have

$$\overline{R}' = \begin{cases} \overline{R}^{(1)} \uparrow \text{ in } \rho, \text{ if } \overline{B} > 1 - \frac{2}{\rho+\sqrt{4+\rho^2}} \text{ and } \rho < 1; \\ \overline{R}^{(2)} \downarrow \text{ in } \rho, \text{ if } \overline{B} \leq 1 - \frac{2}{\rho+\sqrt{4+\rho^2}} \text{ and } \rho < 1; \\ \overline{R}^{(3)} \downarrow \text{ in } \rho, \text{ if } \overline{B} \leq \frac{\sqrt{1+\frac{(\sqrt{5}+1)\rho}{2}}-1}{1+\frac{(\sqrt{5}+1)\rho}{2}} \cdot \frac{\sqrt{5}+1}{2} \text{ and } \rho \geq 1; \\ \overline{R}^{(4)} \downarrow \text{ in } \rho, \text{ if } \overline{B} > \frac{\sqrt{1+\frac{(\sqrt{5}+1)\rho}{2}}-1}{1+\frac{(\sqrt{5}+1)\rho}{2}} \cdot \frac{\sqrt{5}+1}{2} \text{ and } \rho \geq 1, \end{cases} \Leftrightarrow \overline{R}' = \begin{cases} \overline{R}^{(1)} \uparrow \text{ in } \rho, \text{ if } 0 \leq \rho < \rho_a; \\ \overline{R}^{(2)} \downarrow \text{ in } \rho, \text{ if } \rho_a \leq \rho < 1; \\ \overline{R}^{(3)} \downarrow \text{ in } \rho, \text{ if } \rho_b \leq \rho < \rho_2; \\ \overline{R}^{(4)} \downarrow \text{ in } \rho, \text{ if } 1 \leq \rho < \rho_b \text{ or } \rho \geq \rho_2, \end{cases} \tag{EC.2.4}$$

where $\rho_1 = \frac{1+\sqrt{5}-4\overline{B}2\left(\sqrt{5}-1\right)\overline{B}^2 - \sqrt{2}\sqrt{3+\sqrt{5}-4\left(1+\sqrt{5}\right)\overline{B}}}{4\overline{B}^2}$, $\rho_2 = \frac{1+\sqrt{5}-4\overline{B}2\left(\sqrt{5}-1\right)\overline{B}^2 + \sqrt{2}\sqrt{3+\sqrt{5}-4\left(1+\sqrt{5}\right)\overline{B}}}{4\overline{B}^2}$, $\rho_a = 1 \cdot \mathbb{I}_{\{\overline{B} \geq \frac{3-\sqrt{5}}{2}\}} + \frac{(\overline{B}-2)\overline{B}}{\overline{B}-1} \cdot \mathbb{I}_{\{\overline{B} < \frac{3-\sqrt{5}}{2}\}}$ and $\rho_b = \max\{1,\rho_1\} \cdot \mathbb{I}_{\{\overline{B} \leq \frac{3+\sqrt{5}}{4(1+\sqrt{5})}\}} + \rho_2 \cdot \mathbb{I}_{\{\overline{B} > \frac{3+\sqrt{5}}{4(1+\sqrt{5})}\}}$. Thus we have $\Pi^{LS} > \Pi^{PRI}$ if and only if $\overline{R} < \overline{R}'$. Moreover, it is not difficult to verify from (EC.2.4) that $\overline{R}'$ is first increasing and then decreasing in $\rho > 0$. □

*Proof of Theorem 5* Based on the expressions of $CW^{LS}$ and $CW^{PRI}$ in Propositions EC.1.2 and EC.1.4, we consider the following six cases.

- When $\rho < 1$,

(1) If $\rho \leq \frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}$, i.e., $\overline{R} \geq \overline{R}_3$, based on the proofs of Theorem 2 and Theorem 3, we have $CW^{PRI} < CW^{FIFO} < CW^{LS}$.

(2) If $\frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}} < \rho \leq \frac{2(\overline{R}-\overline{B}-1)}{2(\overline{R}-\overline{B})-1}$, i.e., $\overline{R}_2 \leq \overline{R} < \overline{R}_3$, we have $\frac{\partial(CW^{PRI}-CW^{LS})}{\partial\overline{R}} = \frac{\partial(CW^{FIFO}-CW^{LS})}{\partial\overline{R}} - \frac{\partial(CW^{FIFO}-CW^{PRI})}{\partial\overline{R}}$. Because $\frac{\partial(CW^{FIFO}-CW^{LS})}{\partial\overline{R}} < 0$ (see the proof of Theorem 2) and

[EC.2.2] Define $\Psi(x) = x\frac{\sqrt{1+x\rho}-1}{1+x\rho}$ for $\rho \geq 1$ and $x \geq 1$. We have $\Psi(1) = \frac{\sqrt{1+\rho}-1}{1+\rho}$ and $d\Psi(x)/dx = \frac{\sqrt{x\rho+1}(x\rho+2)-2}{2(x\rho+1)^2} > 0$, which implies that $\frac{\sqrt{1+\frac{(\sqrt{5}+1)\rho}{2}}-1}{1+\frac{(\sqrt{5}+1)\rho}{2}} \cdot \frac{\sqrt{5}+1}{2} > \frac{\sqrt{1+\rho}-1}{1+\rho}$ for $\rho \geq 1$.

$\frac{\partial(CW^{FIFO}-CW^{PRI})}{\partial\overline{R}} = \overline{c}\frac{(\overline{R}-\overline{B})\rho([(\overline{R}-\overline{B})\rho]^2+3[(\overline{R}-\overline{B})\rho]+4-\sqrt{(\overline{R}-\overline{B})\rho+1}[(\overline{R}-\overline{B})\rho+4])}{2[(\overline{R}-\overline{B})\rho+1]^3} > 0$, we know that $\frac{\partial(CW^{PRI}-CW^{LS})}{\partial\overline{R}} < 0$. Therefore, if $(CW^{PRI}-CW^{LS})|_{\overline{R}=\overline{R}_2} < 0$, we have $CW^{PRI} < CW^{LS}$ for all $\overline{R} \in [\overline{R}_2, \overline{R}_3)$. Otherwise, there exists an $\overline{R}_b' \in (\overline{R}_2, \overline{R}_3]$ such that $CW^{LS} > CW^{PRI}$ if and only if $\overline{R} \in (\overline{R}_b', \overline{R}_3)$ (note that this can be an empty set).

(3) If $\frac{2(\overline{R}-\overline{B}-1)}{2(\overline{R}-\overline{B})-1} < \rho \leq \frac{(\overline{R}-\overline{B})^2-1}{\overline{R}-\overline{B}}$, i.e., $\overline{R}_1 \leq \overline{R} < \overline{R}_2$, we have $\frac{\partial(CW^{LS}-CW^{PRI})}{\partial\overline{R}} = (1-\rho)(\overline{R}-\overline{B}-1) - \frac{(\overline{R}-\overline{B})\rho([(\overline{R}-\overline{B})\rho+4]\sqrt{(\overline{R}-\overline{B})\rho+1}-2)}{2[(\overline{R}-\overline{B})\rho+1]^3}$. Also, by letting $x := \sqrt{(\overline{R}-\overline{B})\rho+1}$, we have $\frac{\partial^2(CW^{LS}-CW^{PRI})}{\partial\overline{R}^2} = [4x^8(1-\rho)+\rho\cdot(x^5+6x^3-8x^2-15x+12)]/(4x^8)$. For any fixed $x$, we have $d[4x^8(1-\rho)+\rho\cdot(x^5+6x^3-8x^2-15x+12)]/d\rho = -4x^8+x^5+6x^3-8x^2-15x+12 < 0$ for $x \geq 1$. Thus, $4x^8(1-\rho)+\rho\cdot(x^5+6x^3-8x^2-15x+12)$ is decreasing in $\rho$ and is minimized at $\rho = \min\{1, \frac{(\overline{R}-\overline{B})^2-1}{\overline{R}-\overline{B}}\}$.

(i) If $\overline{R}-\overline{B} > \frac{\sqrt{5}+1}{2}$, we have $\rho = 1$ and $(\overline{R}-\overline{B})\rho > \frac{\sqrt{5}+1}{2}$, which implies that $x > \frac{\sqrt{5}+1}{2}$. We can verify that $\frac{\partial^2(CW^{LS}-CW^{PRI})}{\partial\overline{R}^2} > 0$ because $x^5+6x^3-8x^2-15x+12 > 0$ for $x > \frac{\sqrt{5}+1}{2}$.

(ii) Otherwise, if $\overline{R}-\overline{B} \leq \frac{\sqrt{5}+1}{2}$, by plugging $\rho = \frac{(\overline{R}-\overline{B})^2-1}{\overline{R}-\overline{B}}$ and $x = \sqrt{(\overline{R}-\overline{B})\rho+1}$ into $4x^8(1-\rho)+\rho\cdot(x^5+6x^3-8x^2-15x+12)$, we can verify that $-4(\overline{R}-\overline{B})^8[(\overline{R}-\overline{B})^2-(\overline{R}-\overline{B})-1]+[(\overline{R}-\overline{B})^2-1][(\overline{R}-\overline{B})^5+6(\overline{R}-\overline{B})^3-8(\overline{R}-\overline{B})^2-15(\overline{R}-\overline{B})+12] > 0$ for $1 \leq \overline{R}-\overline{B} \leq \frac{\sqrt{5}+1}{2}$[EC.2.3]. Then we have $\frac{\partial^2(CW^{LS}-CW^{PRI})}{\partial\overline{R}^2} > 0$ for $1 \leq \overline{R}-\overline{B} \leq \frac{\sqrt{5}+1}{2}$. Therefore, $(CW^{LS}-CW^{PRI})$ is convex in $\overline{R}_1 \leq \overline{R} < \overline{R}_2$. When $\overline{R} = \overline{R}_1$, from the proofs of Theorem 2 and Theorem 3, we have $(CW^{LS}-CW^{PRI})|_{\overline{R}=\overline{R}_1} > (CW^{LS}-CW^{FIFO})|_{\overline{R}=\overline{R}_1} > 0$. Also, from Case (1) of this proof, we know that $(CW^{LS}-CW^{FIFO})|_{\overline{R}=\overline{R}_3} > 0$. Together with Case (2) of this proof, it follows that if there exists an $\overline{R} \in (\overline{R}_1, \overline{R}_2)$ such that $CW^{LS} < CW^{PRI} \Leftrightarrow CW^{LS}-CW^{PRI} < 0$, there must exist two thresholds $\overline{R}_1 < \overline{R}_a' < \overline{R}_b' < \overline{R}_3$ such that $CW^{LS} < CW^{PRI}$ if and only if $\overline{R} \in (\overline{R}_a', \overline{R}_b')$, i.e., $CW^{LS} > CW^{PRI}$ if and only if $\overline{R} < \overline{R}_a'$ or $\overline{R} > \overline{R}_b'$. Otherwise, we have $CW^{LS} > CW^{PRI}$ for all $\overline{R} \in [\overline{R}_1, \overline{R}_3)$.

(4) If $\rho > \frac{(\overline{R}-\overline{B})^2-1}{\overline{R}-\overline{B}}$, i.e., $\overline{R} < \overline{R}_1$, from the proofs of Theorem 2 and Theorem 3, we have $CW^{LS} > CW^{FIFO} > CW^{PRI}$ for $\overline{R} < \overline{R}_1$.

Because $CW^{LS} > CW^{FIFO} > CW^{PRI}$ for all $\rho < 1-\sqrt{3}/3$ (by Theorems 2 and 3), there must exist $\rho' > 1-\sqrt{3}/3$ such that $CW^{LS} > CW^{PRI}$ for all $\rho < \rho'$. On the other hand, if $\rho \in [\rho', 1)$, there exist two thresholds $\overline{R}_a' \leq \overline{R}_b'$ such that $CW^{LS} \geq CW^{PRI}$ if and only if $\overline{R} \leq \overline{R}_a'$ or $\overline{R} \geq \overline{R}_b'$.

- When $\rho \geq 1$,

(5) If $\rho \leq \frac{\sqrt{5}+1}{2(\overline{R}-\overline{B})}$, i.e., $\overline{R} \leq \overline{B} + \frac{\sqrt{5}+1}{2\rho}$, using a similar argument to that for Case (4) ($\rho < 1$), we have $CW^{LS} > CW^{FIFO}$ for $\overline{R} \in [\overline{B}+1, \overline{B}+\frac{\sqrt{5}+1}{2\rho})$.

[EC.2.3] Define $\Upsilon(x) = -4x^8(x^2-x-1)+(x^2-1)(x^5+6x^3-8x^2-15x+12)$ for $x \in [1, \frac{\sqrt{5}+1}{2}]$, we have $d^9\Upsilon(x)/dx^9 = -4\cdot 9!(10x-1) < 0$, and we can verify that $d^k\Upsilon(x)/dx^k < 0$ for $4 \leq k \leq 8$ sequentially by noticing that $(d^{k+1}\Upsilon(x)/dx^{k+1})|_{x=1} < 0$ for $4 \leq k \leq 8$. Because $(d^3\Upsilon(x)/dx^3)|_{x=1} > 0$ and $(d^3\Upsilon(x)/dx^3)|_{x=(\sqrt{5}+1)/2} < 0$, we have $d^2\Upsilon(x)/dx^2$ is unimodal in $x \in [1, \frac{\sqrt{5}+1}{2}]$. Because $(d^2\Upsilon(x)/dx^2)|_{x=1} > 0$ and $(d^2\Upsilon(x)/dx^2)|_{x=(\sqrt{5}+1)/2} < 0$, we have $d\Upsilon(x)/dx$ is unimodal in $x \in [1, \frac{\sqrt{5}+1}{2}]$. Also, it follows from $(d\Upsilon(x)/dx)|_{x=1} > 0$ and $(d\Upsilon(x)/dx)|_{x=(\sqrt{5}+1)/2} < 0$ that $\Upsilon(x)$ is unimodal in $x \in [1, \frac{\sqrt{5}+1}{2}]$. It implies that $\Upsilon(x) > 0$ because $\Upsilon(x)|_{x=1} > 0$ and $\Upsilon(x)|_{x=(\sqrt{5}+1)/2} > 0$.

(6) If $\rho > \frac{\sqrt{5}+1}{2(\overline{R}-\overline{B})}$, i.e., $\overline{R} > \overline{B} + \frac{\sqrt{5}+1}{2\rho}$, we have $CW^{LS} > CW^{PRI} \Leftrightarrow \frac{2\sqrt{(\overline{R}-\overline{B})\rho+1}-1}{[(\overline{R}-\overline{B})\rho+1]^2} \cdot ((\overline{R}-\overline{B})\rho)^2 < 1 \Leftrightarrow \frac{2\sqrt{(\overline{R}-\overline{B})\rho+1}-1}{(1+1/((\overline{R}-\overline{B})\rho))^2} < 1$. Because $\frac{2\sqrt{(\overline{R}-\overline{B})\rho+1}-1}{(1+1/((\overline{R}-\overline{B})\rho))^2}$ is increasing in $\overline{R}$, there must exist an $\overline{R}_c'$ such that $CW^{LS} = \overline{c}/(2\rho) < CW^{PRI}$ if and only if $\overline{R} < \overline{R}_c'$, which completes this proof. □

*Proof of Proposition EC.1.5* Define $\overline{B}^s = B^s\mu/\overline{c}$ for $s \in \{FIFO, LS, PRI\}$. From Proposition EC.1.1, we have

$$\frac{\partial \Pi^{FIFO}(\overline{B})}{\partial \overline{B}} = \begin{cases} \overline{c}\rho, & \text{if } \rho < 1 \text{ and } \overline{B} \leq \overline{R} - \frac{1}{1-\rho}; \\ \overline{c}\rho \frac{\overline{B}^2\rho + (\overline{R}-2\overline{B})(1+\overline{R}\rho)}{[(\overline{R}-\overline{B})\rho+1]^2}, & \text{otherwise}; \end{cases}$$

$$\frac{\partial^2 \Pi^{FIFO}(\overline{B})}{\partial \overline{B}^2} = \begin{cases} 0, & \text{if } \rho < 1 \text{ and } \overline{B} \leq \overline{R} - \frac{1}{1-\rho}; \\ -\overline{c}\rho \frac{1+\overline{R}\rho}{[(\overline{R}-\overline{B})\rho+1]^3}, & \text{otherwise.} \end{cases}$$

Thus, $\Pi^{FIFO}(\overline{B})$ is quasi-concave. In the region of $\rho < 1$ and $\overline{B} \leq \overline{R} - \frac{1}{1-\rho}$, the maximizer of $\Pi^{FIFO}(\overline{B})$ is $\overline{B} = \overline{R} - \frac{1}{1-\rho}$. Otherwise, the maximizer is $\overline{B} = \overline{R} - \frac{\sqrt{\overline{R}\rho+1}-1}{\rho}$, which is obtained by solving $\partial \Pi^{FIFO}(\overline{B})/\partial \overline{B} = 0$, or $\overline{c}\rho \frac{\overline{B}^2\rho + (\overline{R}-2\overline{B})(1+\overline{R}\rho)}{[(\overline{R}-\overline{B})\rho+1]^2} = 0$. It can be verified that $\overline{B} = \overline{R} - \frac{\sqrt{\overline{R}\rho+1}-1}{\rho}$ is contained in the region of $\rho < 1$ and $\overline{B} \leq \overline{R} - \frac{1}{1-\rho}$ if and only if $\frac{\sqrt{\overline{R}\rho+1}-1}{\rho} > \frac{1}{1-\rho}$, i.e., $(1-\rho)\sqrt{\overline{R}\rho+1} > 1$, in which case the optimal $\overline{B}$ is determined by the boundary condition that $\overline{B}^{FIFO} = \overline{R} - \frac{1}{1-\rho}$. Therefore, $\overline{B}^{FIFO} = \overline{R} - \frac{1}{1-\rho}$ if and only if $(1-\rho)\sqrt{\overline{R}\rho+1} > 1$. Otherwise, $\overline{B}^{FIFO} = \overline{R} - \frac{\sqrt{\overline{R}\rho+1}-1}{\rho}$.

Similarly, from Proposition EC.1.2, we have

$$\frac{\partial \Pi^{LS}(\overline{B})}{\partial \overline{B}} = \begin{cases} \overline{c}\rho, & \text{if } \rho < 1 \text{ and } \overline{B} \leq \overline{R} - \frac{\sqrt{\rho^2+4}+\rho}{2}; \\ \overline{c}\rho \frac{3\overline{B}^2\rho + 2\overline{R}(1+\overline{R}\rho) - \overline{B}(4+5\overline{R}\rho)}{2[(\overline{R}-\overline{B})\rho+1]^{3/2}}, & \text{if } \rho < 1 \text{ and } \overline{B} > \overline{R} - \frac{\sqrt{\rho^2+4}+\rho}{2} \text{ or, if } \rho \geq 1 \text{ and } \overline{B} > \overline{R} - \frac{\sqrt{5}+1}{2\rho}; \\ \overline{c}, & \text{if } \rho \geq 1 \text{ and } \overline{B} \leq \overline{R} - \frac{\sqrt{5}+1}{2\rho}. \end{cases}$$

$$\frac{\partial^2 \Pi^{LS}(\overline{B})}{\partial \overline{B}^2} = \begin{cases} 0, & \text{if } \rho < 1 \text{ and } \overline{B} \leq \overline{R} - \frac{\sqrt{\rho^2+4}+\rho}{2}; \\ \overline{c}\rho \frac{(\overline{R}-\overline{B})(3\overline{B}-4\overline{R})\rho^2 + (8\overline{B}-12\overline{R})\rho - 8}{4[(\overline{R}-\overline{B})\rho+1]^{5/2}}, & \text{if } \rho < 1 \ \overline{B} > \overline{R} - \frac{\sqrt{\rho^2+4}+\rho}{2} \text{ or, if } \rho \geq 1 \text{ and } \overline{B} > \overline{R} - \frac{\sqrt{5}+1}{2\rho}; \\ 0, & \text{if } \rho \geq 1 \text{ and } \overline{B} \leq \overline{R} - \frac{\sqrt{5}+1}{2\rho}. \end{cases}$$

Because $\overline{B} < \overline{R}$, we have $\frac{\partial^2 \Pi^{LS}(\overline{B})}{\partial \overline{B}^2} \leq 0$, i.e., $\Pi^{LS}(\overline{B})$ is quasi-concave. The $\overline{B}$ value that solves $\overline{c}\rho \frac{3\overline{B}^2\rho + 2\overline{R}(1+\overline{R}\rho) - \overline{B}(4+5\overline{R}\rho)}{2[(\overline{R}-\overline{B})\rho+1]^{3/2}} = 0$ is $\overline{B}_a = \frac{4+5\overline{R}\rho - \sqrt{16+16\overline{R}\rho+\overline{R}^2\rho^2}}{6\rho}$. When $\rho < 1$, the maximizer of $\Pi^{LS}(\overline{B})$ is attained at $\overline{B}^{LS} = \overline{R} - \frac{\sqrt{\rho^2+4}+\rho}{2}$ if and only if $\overline{B}_a \leq \overline{R} - \frac{\sqrt{\rho^2+4}+\rho}{2}$. When $\rho \geq 1$, the maximizer is attained at $\overline{B}^{LS} = \overline{R} - \frac{\sqrt{5}+1}{2\rho}$ if and only if $\overline{B}_a \leq \overline{R} - \frac{\sqrt{5}+1}{2\rho}$. Otherwise, the maximizer is attained at $\min\{\overline{B}_a, \overline{R}-1\}$.

Finally, from Proposition EC.1.4, we have

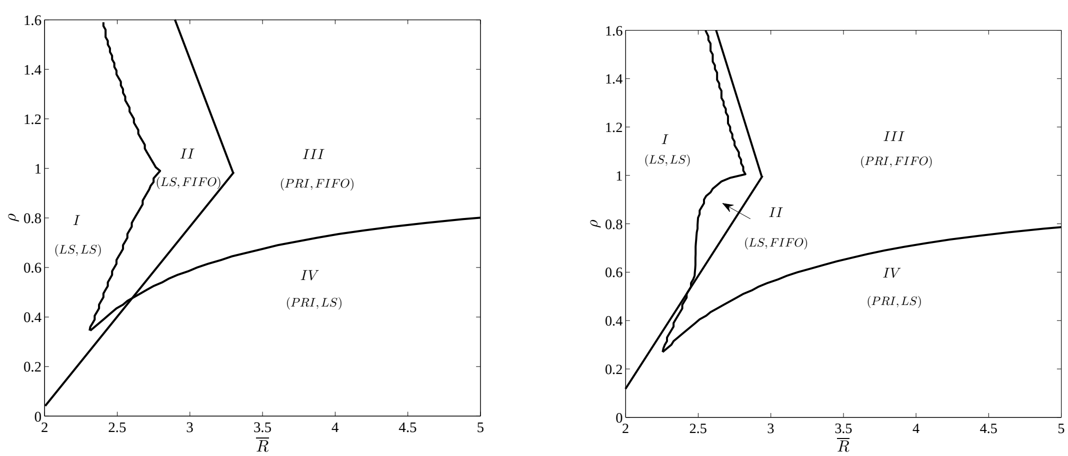
$$\frac{\partial \Pi^{PRI}(\overline{B})}{\partial \overline{B}} = \begin{cases} \overline{c}\rho, & \text{if } \rho < 1 \text{ and } \overline{B} \leq \overline{R} - \frac{1}{1-\rho}; \\ \overline{c}\rho \frac{\sqrt{(\overline{R}-\overline{B})\rho+1}[(\overline{R}-\overline{B})\rho+4] - \frac{\overline{R}}{\overline{R}-\overline{B}} - (\overline{R}\rho+2)}{[(\overline{R}-\overline{B})\rho+1]^3/(\overline{R}-\overline{B})}, & \text{otherwise.} \end{cases}$$

Note that the numerator of the "otherwise" case, $\sqrt{(\overline{R}-\overline{B})\rho+1}[(\overline{R}-\overline{B})\rho+4]-\frac{\overline{R}}{\overline{R}-\overline{B}}-(\overline{R}\rho+2)$, decreases in $\overline{B}$. Therefore, $\Pi^{PRI}(\overline{B})$ is unimodal in $\overline{B}$. Because $\frac{\partial \Pi^{PRI}(\overline{B})}{\partial \overline{B}}|_{\overline{B}\to 0^+}>0$ and $\frac{\partial \Pi^{PRI}(\overline{B})}{\partial \overline{B}}|_{\overline{B}\to \overline{R}^-}<0$, there exists a $\overline{B}_b\in(0,\overline{R})$ which is the unique solution of $\overline{B}$ to the FOC $\frac{\partial \Pi^{PRI}(\overline{B})}{\partial \overline{B}}=0 \Leftrightarrow (\overline{R}-\overline{B})\sqrt{(\overline{R}-\overline{B})\rho+1}[(\overline{R}-\overline{B})\rho+4]=\overline{R}+(\overline{R}\rho+2)(\overline{R}-\overline{B})$. Therefore, the maximizer of $\Pi^{PRI}(\overline{B})$ is attained at $\overline{B}^{PRI}=\overline{R}-\frac{1}{1-\rho}$ if and only if $\rho<1$ and $\overline{R}-\overline{B}_b\geq \frac{1}{1-\rho}$. Otherwise, we have $\overline{B}^{PRI}=\min\{\overline{B}_b,\overline{R}-1\}$, which completes this proof. $\square$

## EC.3. More Details on the Extensions in §5

### EC.3.1. Other Waiting-Cost Distributions

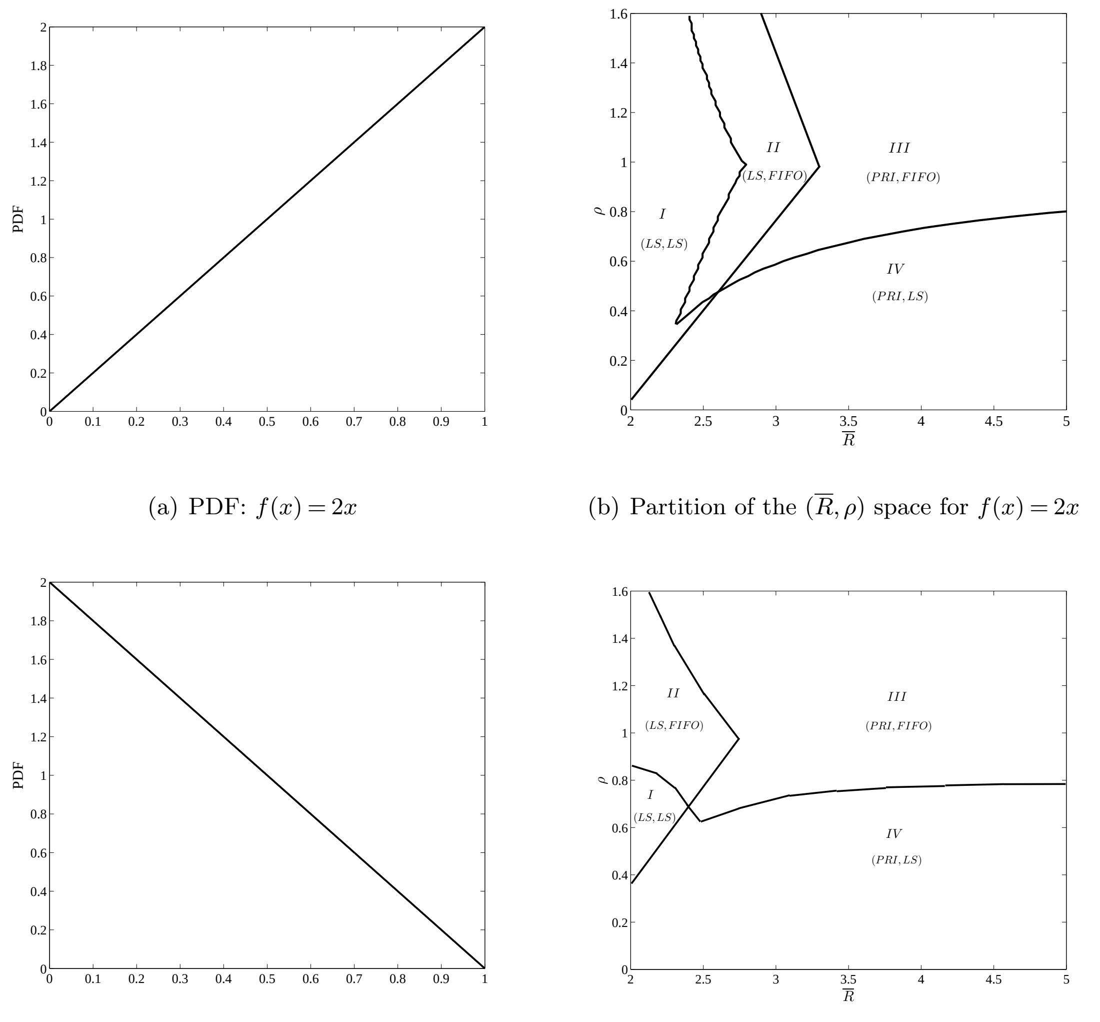
**EC.3.1.1. Power Distribution** Figure EC.3.1 reproduces Figure 4-(a) by replacing the uniform-distribution assumption $U(0,\bar{c})$ with the assumption that customers' hourly waiting cost follows a power distribution with the cumulative distribution function (CDF) $F(x)=x^\alpha, x\in(0,1)$. In particular, Figures EC.3.1-(a) and -(b) present the cases of $\alpha=2$ and $\alpha=3$, respectively. We observe that the qualitative insights from Figure 4-(a) (which considers a uniform distribution) are preserved under these power distributions. Note that as $\alpha$ gets large, the distribution would be more concentrated around 1, making customers more homogeneous vis-à-vis their waiting costs, in which case, the three schemes (FIFO, line-sitting, priority) would yield similar equilibrium outcomes.




(a) $\alpha=2$ (b) $\alpha=3$

**Figure EC.3.1** **Illustration of** $(\arg\max_s \Pi^s, \arg\max_s CW^s)$ **for** $s\in\{FIFO,LS,PRI\}$ **when customer waiting-cost rates follow a power distribution with CDF** $F(x)=x^\alpha$**.** $\mu=1$**,** $B=1$**.**

**EC.3.1.2. Triangular Distribution** Figure EC.3.2 reproduces Figure 4-(a) by replacing the uniform-distribution assumption $U(0,\bar{c})$ with the assumption that customers' hourly waiting cost follows a triangular distribution. In particular, Figure EC.3.2-(a) (-(c)) presents the probability




(a) PDF: $f(x) = 2x$

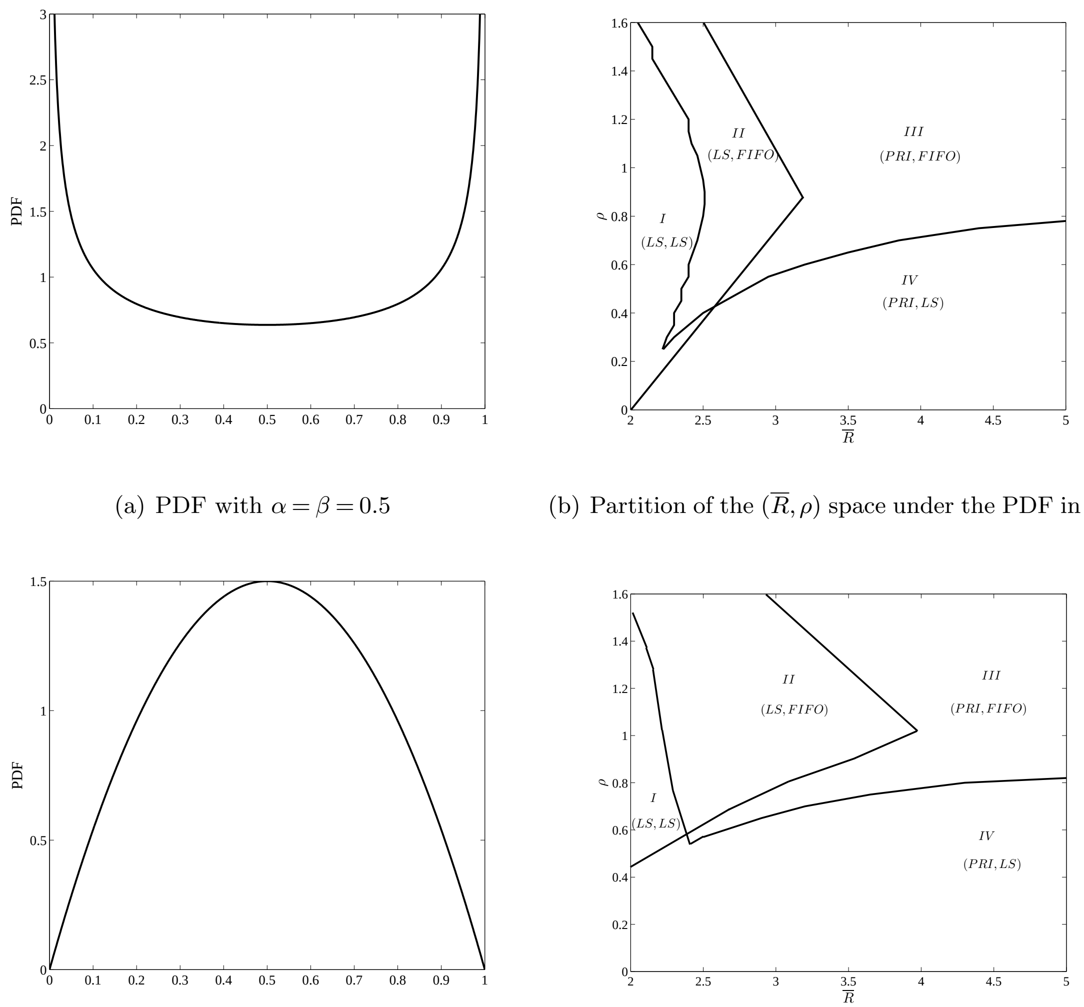
(b) Partition of the $(\overline{R}, \rho)$ space for $f(x) = 2x$

(c) PDF $f(x) = 2(1-x)$

(d) Partition of the $(\overline{R}, \rho)$ space for $f(x) = 2(1-x)$

**Figure EC.3.2** **The illustrations of PDF and** $(\arg\max_s \Pi^s, \arg\max_s CW^s)$ **for** $s \in \{FIFO, LS, PRI\}$ **when customer waiting-cost rates follow a triangular distribution.** $\mu = 1$, $B = 1$.

density function (PDF) $f(x) = 2x$ ($f(x) = 2(1-x)$) and Figure EC.3.2-(b) (-(d)) presents the corresponding partition of the $(\overline{R}, \rho)$ space. We observe that the qualitative insights from Figure 4-(a) (which considers a uniform distribution) are preserved under these triangular distributions. Another observation is that for the triangular distribution with PDF $f(x) = 2(1-x)$, the win-win region $(LS, LS)$, despite its existence, is notably small (see EC.3.2-(d)). Under this "top-heavy" triangular distribution, the majority of customers have low waiting costs and few customers have high waiting costs. The former exacerbates the negative congestion externalities that jeopardize customer welfare because the many low waiting-cost customers are the ones who tend to wait themselves are thus the most affected by the increased congestion. The latter dampens the demand expansion effect that boosts the service provider's revenue because the few high waiting-cost customers are the ones who join as a result of line-sitting but would not otherwise. Therefore, this "top-heavy" triangular distribution limits both the service provider's revenue increase and customer welfare improvement.




(a) PDF with $\alpha = \beta = 0.5$

(b) Partition of the $(\overline{R}, \rho)$ space under the PDF in (a)

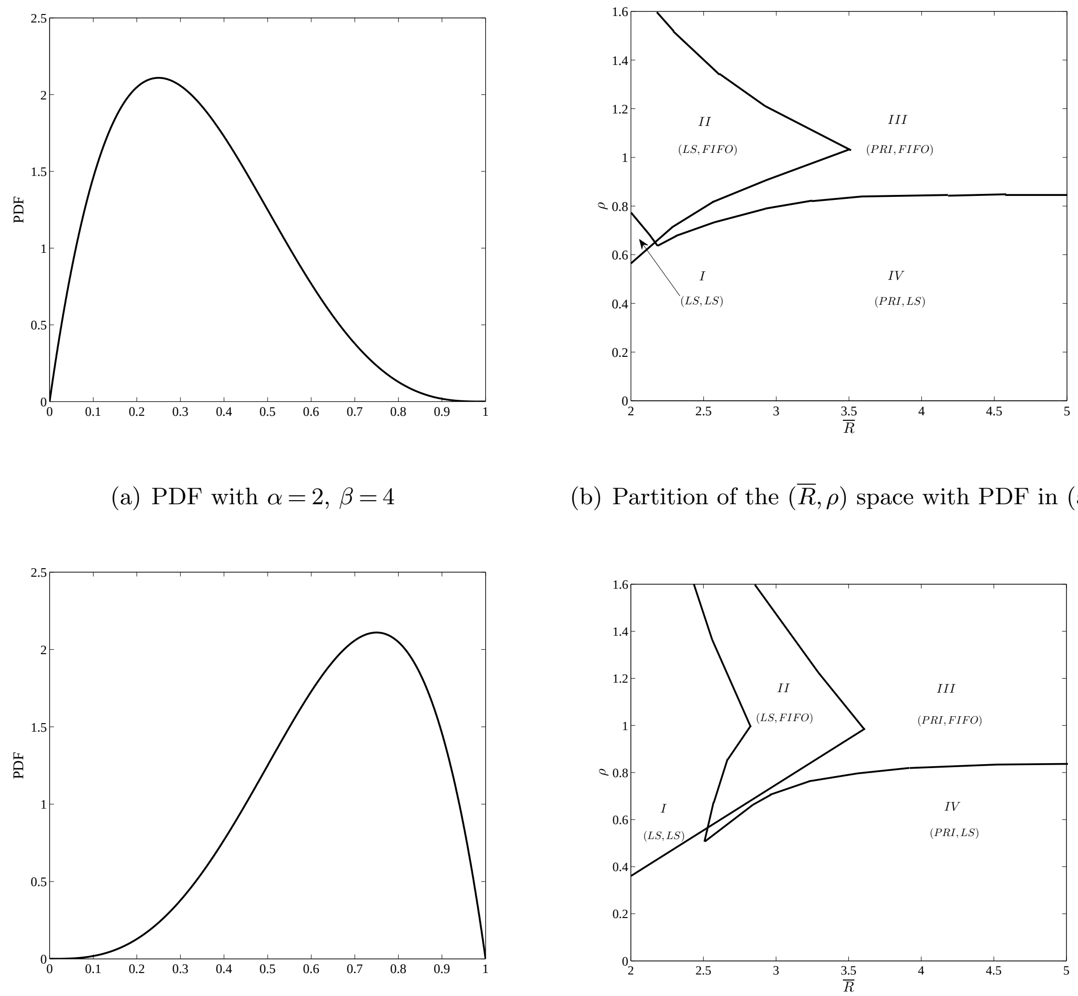
(c) PDF with $\alpha = \beta = 2$

(d) Partition of the $(\overline{R}, \rho)$ space under the PDF in (c)

**Figure EC.3.3** **The illustrations of PDF and** $(\arg\max_s \Pi^s, \arg\max_s CW^s)$ **for** $s \in \{FIFO, LS, PRI\}$ **when customer waiting-cost rates follow a Beta distribution.** $\mu = 1$, $B = 1$.

**EC.3.1.3. Beta Distribution** We consider Beta distributions with probability density function $f(x;\alpha,\beta) = \frac{1}{B(\alpha,\beta)} x^{\alpha}(1-x)^{\beta-1}$, for $x \in (0,1)$ and $\alpha, \beta > 0$. Note that the uniform, triangular and power distributions we studied are all special cases of the Beta distribution. Figures EC.3.3 and EC.3.4 investigate Beta distributions with functional shapes not studied previously. The left panels plot the PDFs with the specified $(\alpha, \beta)$ values, the right panels plot the corresponding partitions of the $(\overline{R}, \rho)$ space. Figure EC.3.3 considers two cases: (1) $\alpha = \beta = 0.5$, (2) $\alpha = \beta = 2$; Figure

EC.3.4 consider another two cases: (1) $\alpha = 2$, $\beta = 4$ and (2) $\alpha = 4$, $\beta = 2$. We observe that the qualitative insights from Figure 4-(a) (which considers a uniform distribution) are preserved under these Beta distributions. In particular, for the Beta distribution with $\alpha = 2, \beta = 4$, the win-win region $(LS, LS)$, despite its existence, is notably small (see Figure EC.3.4-(b)). Given that this is also a "top-heavy" distribution (Figure EC.3.4-(a)), the explanation for this observation is similar to the triangular distribution with PDF $f(x) = 2(1-x)$ (see Figures EC.3.2-(c) and -(d)).




(a) PDF with $\alpha = 2$, $\beta = 4$

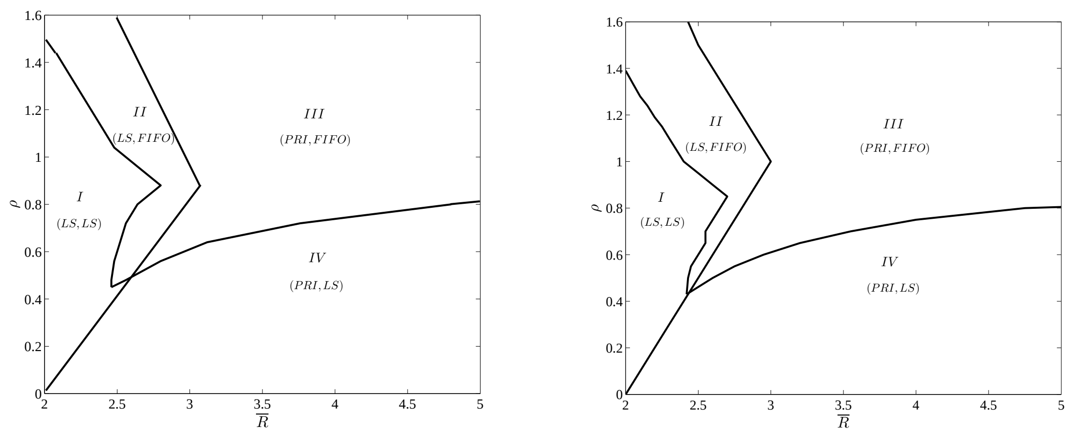
(b) Partition of the $(\overline{R}, \rho)$ space with PDF in (a)

(c) PDF with $\alpha = 4$, $\beta = 2$

(d) Partition of the $(\overline{R}, \rho)$ space with PDF in (c)

**Figure EC.3.4** **The illustrations of PDF and** $(\arg\max_s \Pi^s, \arg\max_s CW^s)$ **for** $s \in \{FIFO, LS, PRI\}$ **when customer waiting-cost rates follow a Beta distribution.** $\mu = 1$, $B = 1$.

**EC.3.1.4. Uniform Distribution with a Positive Lower Bound** Figure EC.3.5 reproduces Figure 4 by replacing the uniform-distribution assumption $U(0,\bar{c})$ with the assumption that customers' hourly waiting cost follows a uniform distribution $U(\underline{c},1)$ with $\underline{c} \in (0,1)$. The case of $\underline{c}=0.05$ is plotted in Figure EC.3.5-(a), and the case of $\underline{c}=0.1$, in Figure EC.3.5-(b). We observe that the qualitative insights from Figure 4-(a) (which sets $\underline{c}=0$) are preserved when $\underline{c}$ takes these positive values.




(a) $\underline{c}=0.05$ (b) $\underline{c}=0.1$

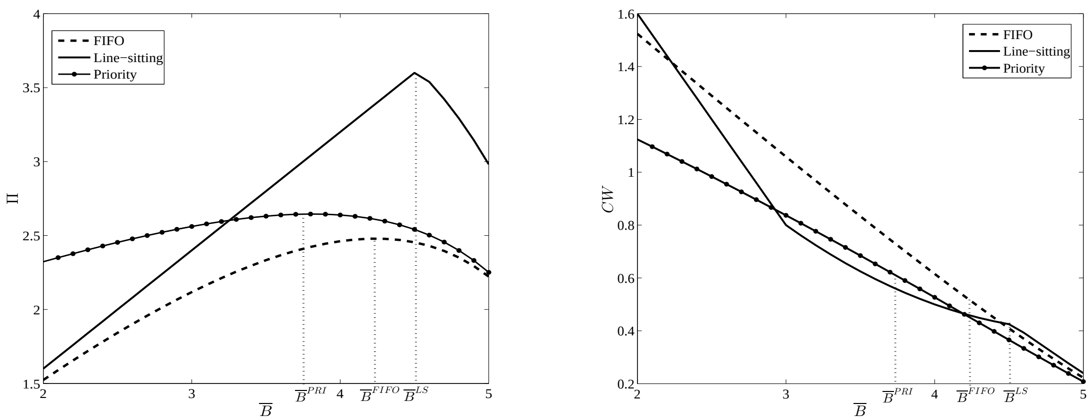
**Figure EC.3.5** **Illustration of** $(\arg\max_s \Pi^s, \arg\max_s CW^s)$ **for** $s \in \{FIFO, LS, PRI\}$ **when customer waiting-cost rates follow a uniform distribution** $U(\underline{c},1)$**.** $\mu=1$**,** $B=1$**.**

## EC.3.2. Endogenizing Service Fee $B$

Our numerical study suggests that for the service provider who endogenizes the base service fee, accommodating line-sitting always raises more revenue than selling priority. To provide intuition for this observation, we plot the service provider's revenue under FIFO, line-sitting and priority as a function of normalized service fee $\overline{B}=B\mu/\bar{c}$ in Figure EC.3.6-(a), with the optimal (normalized) service fee for each scheme labeled.

It is evident that the revenue-maximizing service provider raises the service fee with line-sitting (i.e., $\overline{B}^{LS} > \overline{B}^{FIFO}$) but lowers it with priority (i.e., $\overline{B}^{FIFO} > \overline{B}^{PRI}$). This is because demand expansion of line-sitting is most helpful for revenue improvement of the service provider when the service fee is reasonably high (otherwise the revenue would be too minuscule), whereas price discrimination of the priority purchasing scheme works best when the service fee is relatively low (otherwise the pricing policy would be close to uniform pricing). As such, with a fixed service fee $B$, a sufficiently high service reward $R$ would favor the priority purchasing scheme over the line-sitting scheme. However, when $B$ is endogenized, the service provider can always raise the service fee to align with a high service reward, realizing the full potential of line-sitting, which,

**Figure EC.3.6** **The service provider's revenue and customer welfare as a function of the normalized service fee $\overline{B}$.**




(a) Service provider's revenue. (b) Customer welfare.

*Note.* $\rho = 0.8$ and $\overline{R} = 6$.

in our case, always dominates that of priority purchasing. As a consequence of the high service charge, customer welfare of the line-sitting case, on the other hand, becomes the lowest among the three schemes; see Figure EC.3.6-(b). This explains why the line-sitting practice in Figure 5 is no longer welfare-maximizing for customers when $\overline{R}$ is considerably large (cf. Figure 4). By contrast, the optimal service fee charged by the service provider in the priority case is the lowest among the three schemes, and under the right circumstances the priority purchasing scheme can now become welfare-maximizing for customers.

## EC.3.3. Minimum Line-Sitting Time or Hourly Rate

**EC.3.3.1. Minimum Line-Sitting Time** We continue the derivation in the main text of the paper. Let $T$ be the waiting time in the queue conditioned on the service provider being busy. In an $M/M/1$ queue, $T$ is exponentially distributed with rate $(\mu - \lambda_e)$. Hence, the expected line-sitting payment is

$$(1-\lambda_e/\mu)\, r\theta + \frac{\lambda_e}{\mu} r \int_0^{\infty} \max\{\theta, T\}(\mu-\lambda_e)e^{-(\mu-\lambda_e)T} dT = r\left[\frac{\lambda_e}{\mu}\frac{e^{-(\mu-\lambda_e)\theta}}{\mu-\lambda_e} + \theta\right]. \tag{EC.3.1}$$

As in the base model, the expected waiting cost reduction due to line-sitting is $c\lambda_e/[\mu(\mu-\lambda_e)]$ for a customer with waiting cost rate $c$. Thus, a joining customer purchases the line-sitting service if and only if her expected waiting cost reduction is at least as large as the expected line-sitting payment given in (EC.3.1). The condition can be simplified to

$$c \geq r[e^{-(\mu-\lambda_e)\theta} + \theta\mu(\mu-\lambda_e)/\lambda_e]. \tag{EC.3.2}$$

One can show that the right-hand side of (EC.3.2) is increasing in $\theta \geq 0$, which implies that (for fixed line-sitting hourly rate $r$ and system throughput $\lambda_e$) a larger minimum wait requirement makes line-sitting less attractive to purchase in the sense that only more delay-sensitive customers would buy. In particular, recall from the base model which corresponds to $\theta = 0$, joining customers purchase line-sitting as long as hiring a line-sitter is less costly than waiting on their own, i.e., $c \geq r$; when $\theta$ is positive, however, customers with a waiting cost rate $c$ marginally higher than $r$ may be reluctant to recruit a line-sitter because they could end up paying for longer line-sitting time than necessary due to the minimum wait requirement.

Given $\theta$, the line-sitting firm sets the hourly rate $r$ to maximize total revenue:

$$\max_{r\in(0,\bar{c})} \quad \underbrace{r\left[\frac{\lambda_e}{\mu}\frac{e^{-(\mu-\lambda_e)\theta}}{\mu-\lambda_e}+\theta\right]}_{\text{the expected line-sitting payment per customer}} \quad \cdot \underbrace{\frac{\Lambda\left(c^{LS}-r[e^{-(\mu-\lambda_e)\theta}+\theta\mu(\mu-\lambda_e)/\lambda_e]\right)}{\bar{c}}}_{\text{the expected number of line-sitting users per hour}}, \tag{EC.3.3}$$

$$R-B-\frac{c^{LS}}{\mu}-r\left[\theta+\rho_e\frac{e^{-(\mu-\lambda_e)\theta}}{\mu-\lambda_e}\right]\geq 0, \text{ and } \lambda_e=\Lambda c^{LS}.$$

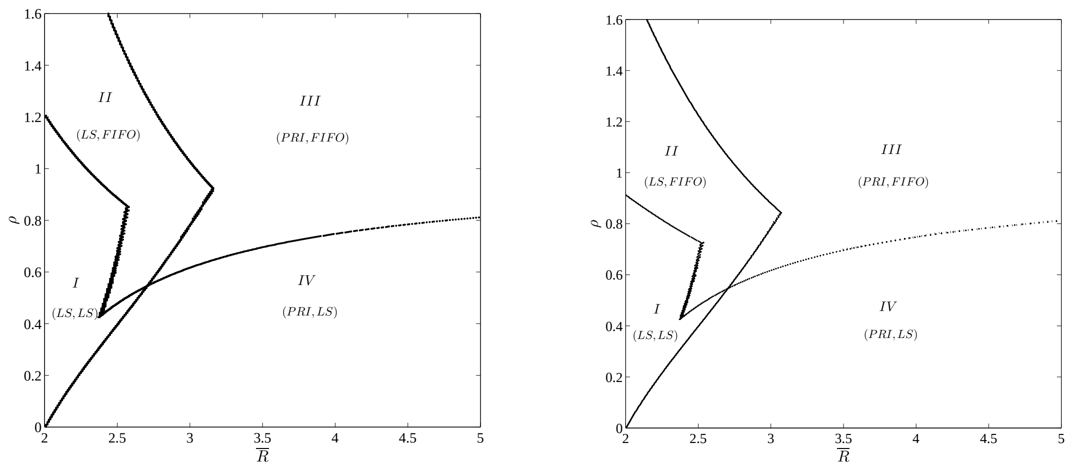
Through a change of variables, i.e., letting $x = r[e^{-(\mu-\lambda_e)\theta} + \theta(\mu-\lambda_e)\mu/\lambda_e] \in (0,\bar{c})$, the line-sitting firm's revenue-maximization problem becomes

$$\max_{x\in(0,\bar{c})} \quad x\cdot\frac{\Lambda(c^{LS}-x)}{\bar{c}}\frac{\lambda_e}{\mu(\mu-\lambda_e)},$$

$$R-B-\frac{c^{LS}}{\mu}-\frac{x\lambda_e}{\mu(\mu-\lambda_e)}\geq 0, \quad \lambda_e=\Lambda c^{LS}.$$

The new optimization problem over $(x, c^{LS}, \lambda_e)$ is equivalent to the original optimization problem in (3) over $(r, c^{LS}, \lambda_e)$. Hence, the maximum revenue of the new problem is attained at $x = r^*$, where $r^*$ is given in Table 2 of Proposition 1. That is, as a response to the minimum wait $\theta$, the revenue-maximizing line-sitting firm would optimally adjust the hourly rate of line-sitting such that its revenue remains unchanged. Moreover, the equilibrium outcome characterized by $c^{LS}$ and $\lambda_e^{LS}$ would also not be affected; neither would customer welfare nor the service provider's revenue. All our results regarding these two metrics carry over.

**EC.3.3.2. Minimum Hourly Rate** Figure EC.3.7 reproduces Figure 4-(a) by imposing a price floor $\underline{r}$ on the line-sitting hourly rate $r$, i.e., $r \geq \underline{r}$. (In Figure 4-(a), $\underline{r} = 0$.) The case of $\underline{r} = 0.1$ is plotted in Figure EC.3.7-(a), and the case of $\underline{r} = 0.2$, in Figure EC.3.7-(b). We observe that the qualitative insights from Figure 4-(a) (which sets $\underline{r} = 0$) are preserved when $r$ is subject to these price floors. Further, by comparing Figure 4-(a) and Figures EC.3.7-(a) and (b), we observe that a higher price floor $\underline{r}$ would detract from the win-win case $(LS, LS)$ of line-sitting. This is because the line-sitting firm must set a higher hourly rate with the increase of the price floor, which results in less line-sitting purchase, possibly decreasing both the service provider's revenue and customer welfare. Understandably, the line-sitting business would not be viable if the price floor is too high.




(c) $\underline{r} = 0.1$ (d) $\underline{r} = 0.2$

**Figure EC.3.7** **Illustration of** $(\arg\max_s \Pi^s, \arg\max_s CW^s)$ **for** $s \in \{FIFO, LS, PRI\}$ **when the hourly rate** $r$ **is subject to a price floor** $\underline{r}$**, i.e.,** $r \geq \underline{r}$**.** $\bar{c} = 1$**,** $\mu = 1$**,** $B = 1$**.**

### EC.3.4. Finitely Many Line-Sitters

Figure EC.3.8 reproduces Figure 4-(a) by replacing the assumption of an infinite supply of line-sitters with the assumption that only $N$ line-sitters are available. The case of $N = 1$ is plotted in Figure EC.3.8-(a), and the case of $N = 5$, in Figure EC.3.8-(b). We observe that our qualitative insights from Figure 4-(a) (which sets $N = \infty$) are preserved even when only a single line-sitter is available ($N = 1$). With five line-sitters, the directional impact of line-sitting on the service provider's revenue and customer welfare is already reasonably similar to the infinite supply case as in the base model, judging from the similarity between Figure EC.3.8-(b) and Figure 4-(a).

## EC.4. More Comparative Statics Results

### EC.4.1. For the Line-Sitting Model

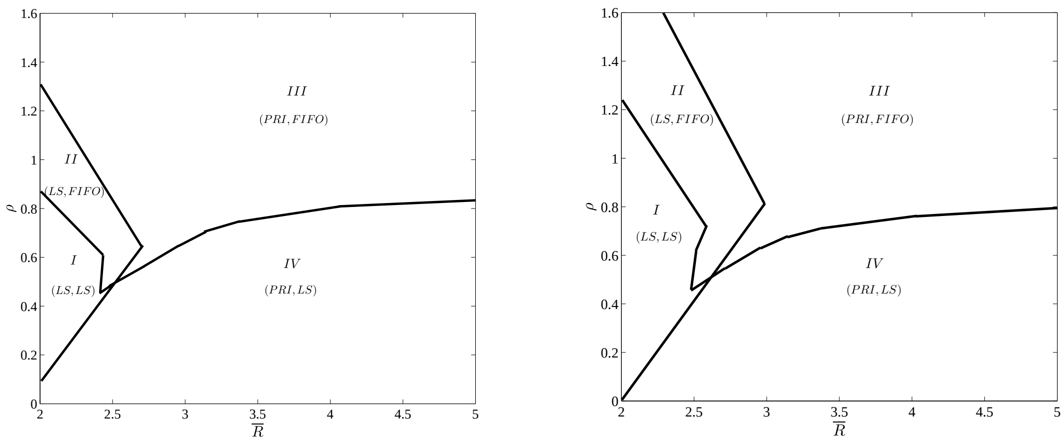
Corollary EC.4.1. *The effective joining rate* $\lambda_e^{LS}$ *(or* $c^{LS} = \lambda_e^{LS}/\Lambda$*) is weakly increasing in the normalized service reward* $\overline{R}$*. However, the optimal hourly rate for the line-sitting firm is not. In particular,*

*(1) if* $\rho \leq \underline{\rho}$*,* $r^*$ *is increasing in* $\overline{R} \in [\overline{B}+1, \overline{B}+\frac{2-\rho}{2(1-\rho)})$ *and constant in* $\overline{R} \in [\overline{B}+\frac{2-\rho}{2(1-\rho)}, \infty)$*;*

*(2) if* $\underline{\rho} < \rho \leq \overline{\rho}$*,* $r^*$ *is unimodal in* $\overline{R} \in [\overline{B}+1, \overline{B}+\frac{\sqrt{\rho^2+4}+\rho}{2})$*, increasing in* $\overline{R} \in [\overline{B}+\frac{\sqrt{\rho^2+4}+\rho}{2}, \overline{B}+\frac{2-\rho}{2(1-\rho)})$ *and constant in* $\overline{R} \in [\overline{B}+\frac{2-\rho}{2(1-\rho)}, \infty)$*;*

*(3) if* $\overline{\rho} < \rho < 1$*,* $r^*$ *is decreasing in* $\overline{R} \in [\overline{B}+1, \overline{B}+\frac{\sqrt{\rho^2+4}+\rho}{2})$*, increasing in* $\overline{R} \in [\overline{B}+\frac{\sqrt{\rho^2+4}+\rho}{2}, \overline{B}+\frac{2-\rho}{2(1-\rho)})$ *and constant in* $\overline{R} \in [\overline{B}+\frac{2-\rho}{2(1-\rho)}, \infty)$*;*

*(4) if* $\rho \geq 1$*,* $r^*$ *is decreasing in* $\overline{R} \in [\overline{B}+1, \overline{B}+\frac{\sqrt{5}+1}{2\rho})$ *and constant in* $\overline{R} \in [\overline{B}+\frac{\sqrt{5}+1}{2\rho}, \infty)$*.*




(a) $N=1$ (b) $N=5$

**Figure EC.3.8** **Illustration of** $(\arg\max_s \Pi^s, \arg\max_s CW^s)$ **for** $s \in \{FIFO, LS, PRI\}$ **under a finite** $N$ **number of line-sitters.** $\overline{B}=1$**.**

Corollary EC.4.2. *The number of customers who hire a line-sitter per hour,* $\Lambda(c^{LS}-r^*)/\bar{c}$*, and the fraction of joining customers who use line-sitting,* $(c^{LS}-r^*)/c^{LS}$*, change with* $\overline{R}$ *as follows:*
*(1) if* $\rho<1$*, they are increasing in* $\overline{R} \in [\overline{B}+1, \overline{B}+\frac{\sqrt{\rho^2+4}+\rho}{2})$*, decreasing in* $\overline{R} \in [\overline{B}+\frac{\sqrt{\rho^2+4}+\rho}{2}, \overline{B}+\frac{2-\rho}{2(1-\rho)})$ *and constant in* $\overline{R} \in [\overline{B}+\frac{2-\rho}{2(1-\rho)}, \infty)$*;*
*(2) if* $\rho \geq 1$*, they are increasing in* $\overline{R} \in [\overline{B}+1, \overline{B}+\frac{\sqrt{5}+1}{2\rho})$ *and constant in* $\overline{R} \in [\overline{B}+\frac{\sqrt{5}+1}{2\rho}, \infty)$*.*

We give the exact values of $\underline{\rho}$ and $\overline{\rho}$ in Corollary 2 in its proof. The key takeaway from Corollary EC.4.1 is that the line-sitting firm does not necessarily find it optimal to increase the hourly rate of line-sitting $r^*$ when the service reward $R$ increases. To understand this result, first consider a system whose potential workload $\rho$ is small enough that all customers choose to join when they experience a service need. In this case, it is indeed revenue-maximizing for the line-sitting firm to increase $r^*$ as $R$ increases, because doing so does not reduce the expected in-line waiting time per customer, which is the same as the expected line-sitting time for customers who hire a line-sitter. Increasing $r^*$ will reduce the number of line-sitting adopters as evidenced by the decreasing segment shown in Corollary EC.4.2. Next, consider the case when $\rho$ is sufficiently large. The expected in-line waiting time per customer becomes sensitive to the hourly rate of the line-sitting service. In fact, as $R$ increases, the line-sitting firm would be better off reducing the hourly rate, as opposed to increasing it, to take advantage of a greater number of line-sitting adopters, each paying for longer line-sitting time.

## EC.4.2. For the Priority Model

Corollary EC.4.3. *When* $\rho<1$*, the optimal priority premium* $P^*$*, the effective arrival rate* $\lambda_e^{PRI}$ *(or* $c_b^{PRI}=\lambda_e^{PRI}/\Lambda$*), the number of priority buyers per hour* $\Lambda(c_b^{PRI}-c_a^{PRI})/\bar{c}$*, and the fraction* *of joining customers who purchase priority* $q=(c_b^{PRI}-c_a^{PRI})/c_b^{PRI}$ *are all increasing in* $\overline{R}\in[\overline{B}+1,\overline{B}+\frac{1}{1-\rho})$ *and constant in* $\overline{R}\in[\overline{B}+\frac{1}{1-\rho},\infty)$*. When* $\rho\geq 1$*, all of these quantities,* $P^*$*,* $\lambda_e^{PRI}$*,* $\Lambda(c_b^{PRI}-c_a^{PRI})/\overline{c}$ *and* $q$*, are increasing in* $\overline{R}\in[\overline{B}+1,\infty)$*.*

COROLLARY EC.4.4. *The threshold* $c_a^{PRI}$ *is non-monotone in* $\overline{R}$*. In particular,*

*(1) if* $\rho\leq 3/4$*,* $c_a^{PRI}$ *is increasing in* $\overline{R}\in[\overline{B}+1,\overline{B}+\frac{1}{1-\rho})$ *and constant in* $\overline{R}\in[\overline{B}+\frac{1}{1-\rho},\infty)$*;*

*(2) if* $\rho\in(3/4,1)$*,* $c_a^{PRI}$ *is increasing in* $\overline{R}\in[\overline{B}+1,\overline{B}+3/\rho)$*, decreasing in* $\overline{R}\in[\overline{B}+3/\rho,\overline{B}+1/(1-\rho))$ *and constant in* $\overline{R}\in[\overline{B}+1/(1-\rho),\infty)$*;*

*(3) if* $\rho\geq 1$*,* $c_a^{PRI}$ *is increasing in* $\overline{R}\in[\overline{B}+1,\overline{B}+3/\rho)$*, and decreasing in* $\overline{R}\in[\overline{B}+3/\rho,\infty)$*.*

By comparing Corollaries EC.4.3 and EC.4.4 of the priority model with Corollaries EC.4.1 and EC.4.2 of the line-sitting model, we discern several differences between the two schemes. For example, while the number of priority buyers in the priority scheme weakly increases in the service reward $R$, the number of line-sitting adopters in the line-sitting scheme is non-monotone in $R$. This is due to the different pricing strategies of the two models. In the priority model, the total revenue contributed by priority purchasing is the priority premium times the number of adopters. As $R$ increases, it is optimal for the service provider to raise the priority premium, because if some customers purchase priority, it motivates even more customers to imitate them so as to avoid being overtaken by future arrivals (i.e., priority adoption is a form of "follow-the-crowd" behavior according to Hassin and Haviv 2003). By contrast, as we have explained in Corollary EC.4.2, the optimal hourly line-sitting price is non-monotone in $R$ because the line-sitting firm's total revenue is determined by not only the hourly rate and the number of adopters, but also by the line-sitting time. Consequently, the fraction of line-sitting users among the joining customers is also non-monotone in $R$.

### EC.4.3. Proofs

*Proofs of Corollaries EC.4.1 and EC.4.2* First, we give exact and numeric values of $\underline{\rho}$ and $\overline{\rho}$.

$$\underline{\rho}=\frac{\left[2+\left(62-6\sqrt{105}\right)^{1/3}+\left(62+6\sqrt{105}\right)^{1/3}\right]^2-36}{6\left[2+\left(62-6\sqrt{105}\right)^{1/3}+\left(62+6\sqrt{105}\right)^{1/3}\right]}\approx 0.526,$$

$$\overline{\rho}=\frac{\left[2+\left(62-6\sqrt{105}\right)^{1/3}+\left(62+6\sqrt{105}\right)^{1/3}\right]^2-36}{36}\approx 0.683.$$

Next, we prove the corollaries. We consider the following two cases.

- When $\rho<1$,

If $\overline{R}\geq\overline{R}_2$, i.e., $\rho\leq\frac{2(\overline{R}-\overline{B}-1)}{2(\overline{R}-\overline{B})-1}$, we have $r^*=\overline{c}/2$ and $c^{LS}=\overline{c}$ (or $\lambda_e^{LS}=\Lambda$). Therefore, $(c^{LS}-r^*)/r^*$ and $\Lambda(c^{LS}-r^*)/\overline{c}$ are both constant in $\overline{R}$. If $\overline{R}_1<\overline{R}<\overline{R}_2$, i.e., $\frac{2(\overline{R}-\overline{B}-1)}{2(\overline{R}-\overline{B})-1}<\rho<\frac{(\overline{R}-\overline{B})^2-1}{\overline{R}-\overline{B}}$, we have

$r^* = \frac{\overline{c}(\overline{R}-\overline{B}-1)(1-\rho)}{\rho}$ which increases in $\overline{R}$ and $c^{LS} = \overline{c}$ (or $\lambda_e^{LS} = \Lambda$). It follows that $(c^{LS} - r^*)/r^*$ and $\Lambda(c^{LS} - r^*)/\overline{c}$ are both decreasing in $\overline{R} \in (\overline{R}_1, \overline{R}_2)$. If $\overline{R} \leq \overline{R}_1$, i.e., $\rho \geq \frac{(\overline{R}-\overline{B})^2-1}{\overline{R}-\overline{B}}$, we have $\lambda_e^{LS} = \frac{\Lambda(\overline{R}-\overline{B})}{\sqrt{(\overline{R}-\overline{B})\rho+1}}$. Therefore, $\lambda_e^{LS}$ is weakly increasing in $\overline{R} \in [\overline{B}+1, \infty)$. Taking the derivatives of $(c^{LS} - r^*)/r^*$ and $r^*$ with respect to $\overline{R}$ yields

$$\frac{\partial(c^{LS}-r^*)/r^*}{\partial\overline{R}} = \frac{\left(\sqrt{(\overline{R}-\overline{B})\rho+1}+(\overline{R}-\overline{B})\rho-1\right)\left(\sqrt{(\overline{R}-\overline{B})\rho+1}-1\right)\rho}{2[(\overline{R}-\overline{B})\rho]^2} > 0; \quad \text{(EC.4.1)}$$

$$\frac{\partial r^*(\overline{R})}{\partial\overline{R}} = \overline{c}\cdot\frac{2(\overline{R}-\overline{B})\rho+3-2[(\overline{R}-\overline{B})\rho+1]^{3/2}}{2\rho[(\overline{R}-\overline{B})\rho+1]^{3/2}}; \quad \text{(EC.4.2)}$$

$$\frac{\partial^2 r^*(\overline{R})}{\partial\overline{R}^2} = -\overline{c}\cdot\frac{\rho^2[5+2(\overline{R}-\overline{B})\rho]}{4\rho[(\overline{R}-\overline{B})\rho+1]^{5/2}} < 0. \quad \text{(EC.4.3)}$$

First, (EC.4.1) implies $(c^{LS} - r^*)/r^*$ is increasing in $\overline{R} \in [\overline{B}+1, \overline{R}_1)$. Because $c^{LS}$ ($\lambda_e^{LS}$) weakly increases in $\overline{R}$, we know $\Lambda(c^{LS} - r^*)/\overline{c}$ is also increasing in $\overline{R} \in [\overline{B}+1, \overline{R}_1)$. Second, (EC.4.3) implies that $\frac{\partial r^*(\overline{R})}{\partial\overline{R}}$ is decreasing in $\overline{R} \in [\overline{B}+1, \overline{R}_1)$. Therefore, if $\frac{\partial r^*(\overline{R})}{\partial\overline{R}}|_{\overline{R}=\overline{R}_1} \geq 0 \Leftrightarrow 2(\overline{R}_1 - \overline{B})^2 + 1 \geq 2(\overline{R}_1 - \overline{B})^3 \Leftrightarrow 0 < \rho \leq \underline{\rho} = \frac{\left[2+\left(62-6\sqrt{105}\right)^{1/3}+\left(62+6\sqrt{105}\right)^{1/3}\right]^2-36}{6\left[2+\left(62-6\sqrt{105}\right)^{1/3}+\left(62+6\sqrt{105}\right)^{1/3}\right]} \approx 0.526$, we must have that $r^*$ increases in $\overline{R} \in [\overline{B}+1, \overline{R}_1)$. Otherwise, if $\rho > \underline{\rho}$, we have $\frac{\partial r^*(\overline{R})}{\partial\overline{R}}|_{\overline{R}=\overline{B}+1} \geq 0 \Leftrightarrow \rho \leq \overline{\rho} = \frac{\left[2+\left(62-6\sqrt{105}\right)^{1/3}+\left(62+6\sqrt{105}\right)^{1/3}\right]^2-36}{36} \approx 0.683$. Therefore, when $\rho \in (\underline{\rho}, \overline{\rho}]$, $r^*$ first increases then decreases in $\overline{R}$ (i.e., $r^*$ is unimodal). Solving for FOC $\frac{\partial r^*(\overline{R})}{\partial\overline{R}} = 0$ reveals that maximum $r^*(\overline{R})$ occurs at $\overline{R} = \overline{B} + \overline{\rho}/\rho$. Thus, $r^*$ increases in $\overline{R} \in [\overline{B}+1, \overline{B}+\overline{\rho}/\rho]$ and decreases in $\overline{R} \in \left(\overline{B}+\overline{\rho}/\rho, \overline{R}_1\right)$. Finally, when $\rho > \overline{\rho}$, we have that $r^*$ decreases in $\overline{R} \in [\overline{B}+1, \overline{R}_1)$.

- When $\rho \geq 1$, it is easy to verify using a similar argument to the one for the case $\rho < 1$ and $\overline{R} < \overline{R}_1$ above that $r^*$ decreases while $(c^{LS} - r^*)/c^{LS}$ and $\Lambda(c^{LS} - r^*)/\overline{c}$ increase in $\overline{R} \in [\overline{B}+1, \overline{B}+\frac{\sqrt{5}+1}{2\rho})$. On the other hand, when $\overline{R} \geq \overline{B} + \frac{\sqrt{5}+1}{2\rho}$, we have $r^* = 0$ and $c^{LS} = \overline{c}/\rho$. □

*Proof of Corollary EC.4.3* First, consider the impact of $\overline{R}$ on $\lambda_e^{PRI}$. When $\overline{R} < \overline{R}_3$, i.e., $\rho > \frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}$, we have $\lambda_e^{PRI} = \frac{(\overline{R}-\overline{B})\Lambda}{(\overline{R}-\overline{B})\rho+1}$. When $\overline{R} \geq \overline{R}_3$, i.e., $\rho \leq \frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}$, we have $\lambda_e^{PRI} = \Lambda$. Therefore, $\lambda_e^{PRI}$ is weakly increasing in $\overline{R}$. Next, we consider two cases for $P^*$ and $(c_b^{PRI} - c_a^{PRI})/c_b^{PRI}$.

- When $\rho < 1$,

(1) If $\overline{R} \in [\overline{B}+1, \overline{B}+\frac{1}{1-\rho})$, we have $P^* = \overline{c}\rho(\overline{R}-\overline{B})^2\left(\frac{(\overline{R}-\overline{B})\rho+1-\sqrt{(\overline{R}-\overline{B})\rho+1}}{\mu[(\overline{R}-\overline{B})\rho+1]^2}\right)$, and it follows that

$$\frac{\partial P^*}{\partial\overline{R}} = \frac{(\overline{R}-\overline{B})\rho([4+2(\overline{R}-\overline{B})\rho]\sqrt{(\overline{R}-\overline{B})\rho+1}-4-(\overline{R}-\overline{B})\rho)}{2[(\overline{R}-\overline{B})\rho+1]^{5/2}} > 0,$$

$$\frac{\partial(c_b^{PRI}-c_a^{PRI})/c_b^{PRI}}{\partial\overline{R}} = \frac{(\sqrt{(\overline{R}-\overline{B})\rho+1}-1)^2}{2[(\overline{R}-\overline{B})\rho]^2\sqrt{(\overline{R}-\overline{B})\rho+1}} > 0.$$

Thus, $P^*$ and $(c_b^{PRI} - c_a^{PRI})/c_b^{PRI}$ are increasing in $\overline{R} \in [\overline{B}+1, \overline{B}+\frac{1}{1-\rho})$.

(2) If $\overline{R} \in [\overline{B}+\frac{1}{1-\rho}, \infty)$, we have $P^* = \frac{\overline{c}\rho(1-\sqrt{1-\rho})}{\mu(1-\rho)}$ and $(c_b^{PRI} - c_a^{PRI})/c_b^{PRI} = \frac{1-\sqrt{1-\rho}}{\rho}$, both of which are constant in $\overline{R}$.

- When $\rho \geq 1$, $P^*$ and $(c_b^{PRI} - c_a^{PRI})/c_b^{PRI}$ have the same expressions as those when $\rho < 1$ and (1), thus they are increasing in $\overline{R} \in [\overline{B}+1, \infty)$.

Finally, for all $\rho > 0$, because $c_b^{PRI}$ is weakly increasing in $\overline{R}$, it follows that $\Lambda(c_b^{PRI} - c_a^{PRI})/\overline{c}$ is also weakly increasing in $\overline{R}$. □

*Proof of Corollary EC.4.4* Recall that $c_a^{PRI}$ is a constant in $\overline{R} \geq \overline{R}_3$, i.e., when $\rho \leq \frac{\overline{R}-\overline{B}-1}{\overline{R}-\overline{B}}$. Consider the following three cases.

(1) If $\rho \geq 1$, $c_a^{PRI} = \overline{c}\frac{\sqrt{(\overline{R}-\overline{B})\rho+1}-1}{\rho[(\overline{R}-\overline{B})\rho+1]}$. Taking the derivative of $c_a^{PRI}$ with respect to $\overline{R}$ gives $\frac{\partial c_a^{PRI}}{\partial \overline{R}} = \frac{[2-\sqrt{(\overline{R}-\overline{B})\rho+1}]\overline{c}}{2[(\overline{R}-\overline{B})\rho+1]^2}$ which implies that $\frac{\partial c_a^{PRI}}{\partial \overline{R}} > 0$ if and only if $\overline{R} \in [\overline{B}+1, \overline{B}+3/\rho)$. Therefore, $c_a^{PRI}$ is increasing in $\overline{R} \in [\overline{B}+1, \overline{B}+3/\rho)$ and decreasing in $\overline{R} \in [\overline{B}+3/\rho, \infty)$.

(2) If $\rho \in (3/4, 1)$, i.e., $\overline{R}_3 > \overline{B}+3/\rho$, we have that $c_a^{PRI}$ increases in $\overline{R} \in [\overline{B}+1, \overline{B}+3/\rho)$, decreases in $\overline{R} \in [\overline{B}+3/\rho, \overline{R}_3)$ and is constant in $\overline{R} \in [\overline{R}_3, \infty)$

(3) If $\rho \in (0, 3/4]$, i.e., $\overline{R}_3 \leq \overline{B}+3/\rho$, we have that $c_a^{PRI}$ increases in $\overline{R} \in [\overline{B}+1, \overline{R}_3)$ and is constant in $\overline{R} \in [\overline{R}_3, \infty)$. □

## EC.5. Observable Queues

In this section, we consider an observable queueing model in which all arriving customers observe the queue length when their service request arises, and decide whether to join or balk (or join by hiring a line-sitter in the case of the line-sitting model) based on the queue length. We assume for expositional convenience that customers' hourly waiting costs follow a uniform distribution over $(0, 1)$. We denote $V \triangleq R - B$ for simplicity. We compare the system throughput and customer welfare between the FIFO benchmark and the line-sitting model. We demonstrate the robustness of our findings from the main model of an unobservable queue.

**FIFO without line-sitting.** A customer with waiting cost rate $c$ (referred to as customer $c$ for simplicity below) who sees $n$ customers (excluding herself) in the system upon arrival joins if and only if

$$V - \frac{c(n+1)}{\mu} \geq 0.$$

Thus, customer $c$ joins if and only if $c \leq \frac{V\mu}{n+1}$.

Let $N^F(t)$ be the number of customers at time $t$ in the FIFO system. $N^F(t)$ is a birth-death process. The birth rate in state $n$ is $\lambda_n^F = \Lambda \min\left\{\frac{V\mu}{n+1}, 1\right\}$; the death rate, $\mu$. The steady-state distribution $\{\pi_n^F\}_{n=0}^{\infty}$ is a variant of Poisson distribution:

$$\pi_n^F = (\Lambda/\mu)\pi_{n-1}^F, \quad n = 1, 2, \ldots, \lfloor V\mu \rfloor.$$

$$\pi_n^F = \frac{V\mu}{n}(\Lambda/\mu)\pi_{n-1}^F, \quad n = \lfloor V\mu \rfloor + 1, \ldots$$

Let $\bar{n} \triangleq \lfloor V\mu \rfloor$ .

$$\pi_0^F = \left[1 + \sum_{n=1}^{\bar{n}} (\Lambda/\mu)^n + \bar{n}!/(V\mu)^{\bar{n}} \cdot \sum_{n=\bar{n}+1}^{\infty} (\Lambda V)^n/n!\right]^{-1}.$$

The FIFO system throughput is

$$\lambda^{FIFO} = \mu(1 - \pi_0^F).$$

Customer welfare is

$$\begin{aligned} CW^{FIFO} =& \Lambda \left[\sum_{n=0}^{\bar{n}-1} \pi_n^F \int_0^1 \left(V - \frac{c(n+1)}{\mu}\right) dc + \sum_{n=\bar{n}}^{\infty} \pi_n^F \int_0^{\frac{V\mu}{n+1}} \left(V - \frac{c(n+1)}{\mu}\right) dc\right] \\ =& \Lambda \left[\sum_{n=0}^{\bar{n}-1} \pi_n^F \left(V - \frac{n+1}{2\mu}\right) + \frac{V^2\mu}{2} \sum_{n=\bar{n}}^{\infty} \frac{\pi_n^F}{(n+1)}\right]. \end{aligned}$$

**Line-Sitting.** Given an hourly rate $r$ of line-sitting, a joining customer with $c \geq r$ uses line-sitting, and this customer $c$'s expected utility for using line-sitting is

$$V - \frac{rn}{\mu} - \frac{c}{\mu}.$$

Customer $c$ joins by hiring a line-sitter if $c \geq r$ and $V - \frac{rn}{\mu} - \frac{c}{\mu} \geq 0$, or $c \leq V\mu - rn$. If $V\mu - rn < r$, or $n > V\mu/r - 1$, then no joining customers hire a line-sitter, and customers join if and only if $c \leq \frac{V\mu}{n+1}$ as in the FIFO case.

Let $N^L(t)$ be the number of customers at time $t$ in the line-sitting system. $N^L(t)$ is a birth-death process. The birth rate in state $n$ is $\lambda_n^L = \Lambda \min\left\{\max\left\{\frac{V\mu}{n+1}, V\mu - rn\right\}, 1\right\}$; the death rate, $\mu$.

If customers see an idle server upon arrival, none would purchase line-sitting. If the queue is short, a fraction of delay sensitive customers purchase the line-sitting service; if the queue is long, i.e., $n \geq \lfloor V\mu/r \rfloor$, then no customers purchase line-sitting.

Thus, the line-sitting system' arrival rates at state $\{1, \ldots, \lfloor V\mu/r \rfloor - 1\}$ are different (larger) than the arrival rates at the corresponding states in the FIFO system, whereas the arrival rates to the other states are the equal across the two systems.

Generically, the steady state distribution $\{\pi_n^L\}_{n=0}^{\infty}$ satisfy

$$\pi_{n+1}^L = \pi_n^L (\Lambda/\mu) \min\left\{\max\left\{\frac{V\mu}{n+1}, V\mu - rn\right\}, 1\right\}, \quad n = 0, 1, \ldots$$

Specifically,

$$\pi_n^L = \pi_{n-1}^L (\Lambda/\mu), \quad n = 1, \ldots, \lfloor (V\mu - 1)/r \rfloor + 1;$$

$$\pi_n^L = \pi_{n-1}^L (\Lambda/\mu)(V\mu - r(n-1)), \quad n = \lfloor (V\mu - 1)/r \rfloor + 2, \ldots, \lfloor V\mu/r \rfloor;$$

$$\pi_n^L = \pi_{n-1}^L (\Lambda/\mu) \frac{V\mu}{n}, \quad n = \lfloor V\mu/r \rfloor + 1, \ldots$$

Denote $n_1 = \lfloor (V\mu - 1)/r \rfloor + 1$ and $n_2 = \lfloor V\mu/r \rfloor$ for later use.

System throughput is

$$\lambda^{LS} = \mu(1 - \pi_0^L).$$

PROPOSITION EC.5.1. *For any $r \in (0, \min\{1, V\mu/2\}]$, $\lambda^{LS} > \lambda^{FIFO}$. Otherwise, no customers use line-sitting and $\lambda^{LS} = \lambda^{FIFO}$.*

*Proof of Proposition EC.5.1* At any given state, the line-sitting's arrival rate is always weakly higher than that of FIFO, i.e., $\lambda_n^L \geq \lambda_n^F$ for $n = 0, 1, \ldots$. When $r \in (0, \min\{1, V\mu/2\}]$, the inequality is strict for states $\{1, \ldots n_2 - 1\}$.

$$\pi_0^F = \frac{1}{1 + \sum_{n=1}^{\infty} \Pi_{i=0}^{n-1} \lambda_i^F/\mu^n}, \quad \pi_0^L = \frac{1}{1 + \sum_{n=1}^{\infty} \Pi_{i=0}^{n-1} \lambda_i^L/\mu^n}.$$

Since $\lambda_n^L \geq \lambda_n^F$ for $n = 0, 1, \ldots$, we have $\pi_0^F \geq \pi_0^L$. Since $\lambda^{LS} = \mu(1 - \pi_0^L)$ and $\lambda^{FIFO} = \mu(1 - \pi_0^F)$, we have $\lambda^{LS} \geq \lambda^{FIFO}$ with inequality being strict when $\lambda_n^L > \lambda_n^F$ for some state $n$. □

Proposition EC.5.1 shows that for any nontrivial hourly rate $r$, the line-sitting system's throughput is strictly higher than the FIFO throughput, which implies that line-sitting *strictly* improves the service provider's revenue. Note that this is a slightly stronger result than what we obtained with the unobservable queue setting in which line-sitting could lead to the same service provider's revenue as FIFO. Here, line-sitting is strictly better because there are always some customers who balk upon seeing long queues (even when $\Lambda$ is small) and line-sitting is able to bring some customers back due to its demand expansion effect, whereas in an unobservable queue, it may happen that all arriving customers join the FIFO system, in which case, there would be a tie between line-sitting and FIFO from the perspective of the service provider's revenue.

The line-sitting firm's revenue is given by

$$r\Lambda \left[ (1 - r) \sum_{n=0}^{n_1 - 1} \frac{n}{\mu} \pi_n^L + \sum_{n=n_1}^{n_2 - 1} \frac{n}{\mu} \pi_n^L (V\mu - rn - r) \right], \tag{EC.5.1}$$

To parse the line-sitting firm's revenue, note that for an arrival that sees queue length $n$, the expected line-sitting payment for each line-sitting user is $rn/\mu$; if $n$ is less than $n_1$, the probability of purchasing line-sitting is $(1 - r)$ (the probability that the customer's waiting cost rate is greater than $r$); if $n$ is between $n_1$ and $n_2 - 1$, the probability of purchasing line-sitting is $(V\mu - rn - r)$.

Customer welfare is given by

$$\Lambda \left[ \sum_{n=0}^{n_1 - 1} \pi_n \bar{U}_1(n) + \sum_{n=n_1}^{n_2 - 1} \pi_n (V\mu - rn) \bar{U}_2(n) + \sum_{n=n_2}^{\infty} \pi_n \frac{V\mu}{n+1} \left( V - \frac{n+1}{2\mu} \frac{V\mu}{n+1} \right) \right],$$

where

$$\bar{U}_1(n) = \int_0^r \left( V - \frac{c(n+1)}{\mu} \right) dc + \int_r^1 \left( V - \frac{rn + c}{\mu} \right) dc = V - \frac{1 + n(2 - r)r}{2\mu}$$

$$\bar{U}_2(n) = \left[\int_0^r \left(V - \frac{c(n+1)}{\mu}\right) dc + \int_r^{V\mu - rn} \left(V - \frac{rn+c}{\mu}\right) dc\right] / (V\mu - rn)$$
$$= \frac{n^2r^2 + \mu^2V^2 + nr(r - 2\mu V)}{2\mu} / (V\mu - rn).$$

Hence, customer welfare simplifies to

$$CW^{LS} = \Lambda \left[\sum_{n=0}^{n_1-1} \pi_n \left(V - \frac{1+n(2-r)r}{2\mu}\right) + \sum_{n=n_1}^{n_2-1} \pi_n \frac{n^2r^2 + \mu^2V^2 + nr(r-2\mu V)}{2\mu} + \frac{V^2\mu}{2} \sum_{n=n_2}^{\infty} \frac{\pi_n}{(n+1)}\right].$$

We conduct a numerical study to compare customer welfare in the FIFO system, $CW^{FIFO}$ and that under line-sitting, $CW^{LS}$, when the line-sitting firm optimizes hourly rate $r$ to maximize its revenue as expressed in (EC.5.1). The results are reported in Table EC.5.1 on the next page. We observe that when the potential system workload is small (or large), line-sitting benefits (or harms) customer welfare, which is largely consistent with our findings from the unobservable queueing model.

**Table EC.5.1 Comparison of customer welfare under FIFO and line-sitting.**

| $\rho$ | $V=1$ | $V=1.5$ | $V=2$ | $V=2.5$ | $V=3$ | $V=3.5$ |
|---|---|---|---|---|---|---|
| 1.4 | - | - | - | - | - | - |
| 1.2 | + | - | - | - | - | - |
| 1 | + | + | + | + | - | - |
| 0.8 | + | + | + | + | + | + |
| 0.6 | + | + | + | + | + | + |
| 0.4 | + | + | + | + | + | + |
| 0.2 | + | + | + | + | + | + |

"+" means $CW^{LS} > CW^{FIFO}$ and "-" means $CW^{LS} < CW^{FIFO}$. $\mu = 1$.

Here is the intuition. For a customer seeing the same queue length, her expected utility is always weakly higher under line-sitting than under FIFO. However, customers are likely to see a longer queue upon arrival due to the demand expansion effect (a higher throughput) which carries over from the unobservable queueing model. Demand expansion could generate value by increasing access to service but could also introduce excessive congestion negative externalities, making the overall impact of customer welfare unclear a priori. Thus, the driving mechanism for the impact of line-sitting on customer welfare in the observable queue is quite similar to the one identified in the unobservable queueing model in the paper.