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Online Appendix

O.1. Analytical Details on Model Extensions

O.1.1. Preemptive EMA When Farms are Averse to Quality Uncertainty

To model this extension, we subtract a penalty, $\lambda\mathbb{E}[\mathbb{I}\{n_L > \gamma m\}]$, from the farm's expected payoff function formulated in §3.1. This penalty captures the farm's aversion to producing too many low-quality units, which can threaten the livelihood of his family. The next two theorems characterize farms' equilibrium adulteration behavior under perfect and imperfect testing when they are averse to quality uncertainty.

THEOREM O.1. *For preemptive EMA with perfect testing and farms aversive to quality uncertainty, the total number of adulterating farms in any NE of the game is characterized by $n_a^{RA^*} = \min\{n, \lceil T^{RA} - 1 \rceil\}$, where*

$$T^{RA} \equiv n^2 \frac{(r_H - r_L)(p_L^{\max} - p_L^{\min}) + (\lambda/m)\left(\overline{F}(\gamma, m, p_L^{\max}) - \overline{F}(\gamma, m, p_L^{\min})\right)}{cqt},$$

and $\overline{F}(\gamma, m, h(x)) \equiv 1 - \int_0^{\gamma m} f(x, m, h(x))\mathrm{d}x$. We have the following three cases.

(i) If $T^{RA} \le 1$, then no farm adulterates.

(ii) If $T^{RA} > n$, then all farms adulterate.

(iii) If $T^{RA} \in (1, n]$, then any subset of $n_a^{RA^*}$ farms adulterating while the rest $(n - n_a^{RA^*})$ farms not adulterating constitutes a NE of the game.

THEOREM O.2. *Define $T(x) \equiv -h'(x) \dfrac{(r_H - r_L) + \lambda f(\gamma m, m, h(x))\left(\frac{\gamma(1-2h(x)) + h(x)}{2h(x)(1-h(x))}\right)}{cq(t/n)((n+1)/n)}$. For preemptive EMA with imperfect testing and farms aversive to quality uncertainty, if $\gamma \le \dfrac{p_L^{\min}}{1 - 2p_L^{\min}(1 - p_L^{\min})}$, then there exists a unique symmetric NE of the game in which $x_{RA}^{PV^*}$ is determined as follows.*

(a) If $c < T(1)$, then all farms adulterate to the maximum level; i.e., $x_{RA}^{PV^*} = 1$.

(b) If $c \ge T(1)$, then the farms adulterate to some extent; i.e., $x_{RA}^{PV^*} \in (0,1]$ and is the solution to the equation: $x = T(x)$.

We observe that the farms' equilibrium adulteration behavior under either testing scenario follows a very similar pattern as in Theorems 1 and 2 in §3.1. The condition on γ in Theorem O.2 means that a farm should begin to exhibit aversion when the number of his low-quality units is not too large. This condition is reasonable given our focus on smallholder farms. Our next result shows that the risk of preemptive EMA in the supply chain is higher when farms are averse to quality uncertainty than when they are expected-profit maximizers, under both perfect and imperfect testing.

PROPOSITION O.1. *$n_a^{RA^*} \ge n_a^*$ and $x_{RA}^{PV^*} \ge x^{PV^*}$.*

With respect to the effect of supply chain dispersion on preemptive EMA risk in the supply chain, we show as in Proposition 2 that greater dispersion leads to higher risk under imperfect testing.

PROPOSITION O.2. *If $\gamma \le \dfrac{p_L^{\min}}{1 - 2p_L^{\min}(1 - p_L^{\min})}$, then $\dfrac{\partial x_{RA}^{PV^*}}{\partial n} \ge 0$.*[17]

For the case of perfect testing, we cannot characterize the effect of dispersion on risk analytically. Therefore, we perform extensive numerical simulation and observe that in a total of 32,000,000 numerical instances we run, greater dispersion always leads to a higher risk.[18]

[17] We treat n as a continuous variable in this analysis for tractability.

[18] We use the following parameter values in the numerical simulation: $k = 100000$, 100 m values in $\{100, \ldots, 1000\}$, 20 $p_L^{\max}$ values in $\{0.1, \ldots, 0.9\}$, $p_L^{\min} = 0.1$, $c = 19.07$, 20 q values in $\{0.1, \ldots, 0.9\}$, $r_H = 19.07$, $r_L = 0$, $t = n$, $\lambda = \{1, 2, \ldots, 40\}$, and 20 γ values in $\{0.1, \ldots, 0.9\}$.

O.1.2. EMA Risk When the Quality of All Units from the Same Farm is Perfectly Correlated

In this extension we analyze a setting where *all* units of the same farm are of low (high) quality with probability $p_L^{max}(1-p_L^{max})$. This setup captures scenarios where quality of units of the same farm are highly correlated (in contrast to being independent as in our base model). We again analyze a setting where farms decide whether or not to adulterate low quality units. We first note that our results for preemptive EMA in §3.1 will not change in this setup. This is because farms' expected payoff does not change in either perfect or imperfect testing with this updated distrbution of low quality units. Next, we characterize farms' equilibrium adulteration behavior under both perfect and inperfect testing for reactive EMA.

Theorem O.3. *(i) For reactive EMA with perfect testing,*

(a) if $r_H - r_L \leq q(t/n)c$, then none of the farms adulterate low quality units.

(b) If $r_H - r_L > q(t/n)c$, then all the farms always adulterate low qualtiy units.

(ii) For reactive EMA with imperfect testing,

(a) if $q(t/n)c \leq \dfrac{(r_H-r_L)n}{p_L^{max}(n-1)+1}$, then all the farms always adulterate low quality units.

(b) If $cq(t/n) \in (\dfrac{(r_H-r_L)n}{p_L^{max}(n-1)+1}, (r_H-r_L)n)$, then a mixed strategy Nash equilibrium exists where all farms adulterate low quality units with probability $p_{ad} \in (0,1)$ and $p_{ad} = \dfrac{(r_H-r_L)n^2}{cqt(n-1)p_L^{max}} - \dfrac{1}{(n-1)p_L^{max}}$.

(c) if $(r_H-r_L)n \leq q(t/n)c$, then farms never adulterate low quality units.

Theorem O.3 shows that under perfect testing, farms always (never) adulterate low quality units if the expected per unit penalty is smaller (larger) than the expected per unit revenue gain from adulteration. In contrast, since expected penalty increases with total amount of adulterants under imperfect testing, a symmetric mixed strategy Nash equilbirum that randomizes between adulteration and non adulteration balances the payoffs. Our next proposition shows that similar to Proposition 2B, the risk of reactive EMA as measured by the total expected number of adulterated output in the supply chain is again increasing in supply chain dispersion.

Proposition O.3. *For reactive EMA with imperfect testing where all units of the same farm are of low quality with probability p_L^{max}, $\dfrac{\partial E_n}{\partial n} \geq 0$.*

O.1.3. Reactive EMA with Decision on How Much to Adulterate

In §3.2.2, we focus on the setup where farms adulterate either all or none of the realized low-quality units. An alternative setup is for farms to decide how many of the realized low-quality units to adulterate. This setup can capture scenarios in which a farm adulterates to fake the overall quality of his output to a desirable level. We first note that our results for perfect testing will not change under this setup. This is because under perfect testing, any amount of adulteration induces the same level of expected penalty, whereas the revenue gain increases in the number of units being adulterated. Hence, a farm would always adulterate all of his low-quality units if he decides to adulterate. Theorem O.4 below characterizes the farms' equilibrium adulteration strategy under imperfect testing, where $a^*(n_{L,i})$ denotes the number of realized low-quality units that farm i adulterates in equilibrium.

Theorem O.4. *For reactive EMA with imperfect testing where a farm can choose how many of the realized low-quality units to adulterate, there exists a unique symmetric BNE of the game in which a farm's adulteration strategy is a threshold strategy: $a^*(n_{L,i}) = n_{L,i}$ if $n_{L,i} \in [0, \beta_f^{RV}]$ and $a^*(n_{L,i}) = \beta_f^{RV}$ if $n_{L,i} \in (\beta_f^{RV}, m]$, for all i. The threshold β_f^{RV} is unique and determined as follows:*

(a) If $cq\left(\dfrac{t}{n}\right) \geq \dfrac{n(r_H-r_L)}{2+(n-1)p_L}$, then $\beta_f^{RV} \in (0,m)$ and is the solution to the equation:

$$2\beta = \frac{nk(r_H-r_L)}{cqt} - (n-1)\Big(\textstyle\int_0^\beta x f(x, \frac{k}{n}, p_L)\mathrm{d}x + \int_\beta^m \beta f(x, \frac{k}{n}, p_L)\mathrm{d}x\Big).$$

(b) If $cq\left(\dfrac{t}{n}\right) < \dfrac{n(r_H-r_L)}{2+(n-1)p_L}$, then $\beta_f^{RV} = m$.

Theorem O.4 shows that when farms can choose to adulterate a fraction of his low-quality units, then they adulterate all low-quality units up to a threshold, after which they adulterate a constant number of low-quality units. This structure is very similar to that in Theorem 4. The only difference is that in the current setup, when a farm has many low-quality units, the farm would adulterate just enough to make the marginal revenue gain from adulteration equal to the marginal penalty, i.e., adulterating β_f^{RV} units (as opposed to not adulterating at all in Theorem 4). Similar to Proposition 2B, we show that the risk of reactive EMA as measured by the total expected number of adulterated output in the supply chain is increasing in supply chain dispersion.

PROPOSITION O.4. *For reactive EMA with imperfect testing where a farm can choose how many of the realized low-quality units to adulterate,* $\dfrac{\partial E_n}{\partial n} \geq 0$.

O.1.4. Alternative Models of Imperfect Testing Sensitivity

In this extension, we analyze settings of imperfect testing to model scenarios where detection probability is not linearly increasing in the relative amount of adulterated output in the total supply chain output (as in §3.1.2 and §3.2.2). In particular, we examine three alternative models where the detection probability is (i) convex increasing in the relative amount of adulterated output, (ii) convex increasing in the relative amount of adulterated output and reaches 1 in the interior of the (0, 1) interval, and (iii) linearly increasing in the relative amount of adulterated output and reaches 1 in the interior of the (0, 1) interval. The last two alternatives capture scenarios in which detection probability increases quickly as a small amount of adulterants are added.

Formally, let the detection probability $S_1 : [0,1] \to [0,1]$ be a convex increasing function such that $S_1(0) = 0$ and $S_1(1) = 1$ under scenario (i). That is, the detection probability should be 0 (1) if none (all) of the output is adulterated. Under imperfect testing, if farm i adulterates with x_i, then the chance that the manufacturer detects adulteration when testing the aggregated supply is equal to $\dfrac{S_1(x_i) + \sum_{-i} S_1(x_{-i})}{n}$. If the manufacturer further tests the individual sample of farm i, then she detects adulteration in the sample with probability $S_1(x_i)$. Since the manufacturer tests the aggregated supply with probability q, the ultimate probability for farm i to be caught if he adulterates is equal to

$$\gamma_i(x_i, x_{-i})) \equiv q\left(\frac{t}{n}\right)\left(\frac{S_1(x_i) + \sum_{-i} S_1(x_{-i})}{n}\right) S_1(x_i).$$

Similarly, the probability for farm i to be caught if he adulterates under imperfect testing is equal to

$$\gamma_i(n_{L,i}, a_{-i}(n_{L,-i})) \equiv q\left(\frac{t}{n}\right) S_1\left(\frac{n_{L,i}}{m}\right) \mathbb{E}_{n_{L,-i}}\left[S_1\left(\frac{n_{L,i} + \sum_{-i} n_{L,-i} a_{-i}(n_{L,-i})}{k}\right)\right].$$

Under model (ii), let the detection probability $S_2 : [0,1] \to [0,1]$ be such that $S_2(0) = 0$, $S_2(a)$ is convex increasing in a for $a \in [0, \tau)$, and $S_2(a) = 1$ for $a \in [\tau, 1]$, for some $\tau \in (0,1)$. Lastly, let the detection probability $S_3 : [0,1] \to [0,1]$ under model (iii) be a piecewise increasing function such that $S_3(x) = min(\alpha x, 1)$ for some $\alpha \geq 1$. Note that α captures the detection level of testing. Higher the α, higher is the range in which detection of adulterants happens with perfect accuracy. Under preemptive case, if farm i adulterates with x_i, then the chance that the manufacturer detects the adulteration when testing the aggregated supply is equal to $min(\dfrac{\alpha(x_i + \sum_{-i} x_{-i})}{n}, 1)$. If the manufacturer further tests the individual sample of farm i, then she detects the adulterants in the sample with probability $min(\alpha x_i, 1)$.

We characterize farms equilibrium adulteration behavior under both preemptive and reactive EMA for the three models. Under preemptive EMA, the equilibrium adulteration behavior (Theorem O.5) again follows the same structure as in Theorem 2 with updated thresholds. In particular, if penalty is smaller than a threshold, then all farms adulterate upto the maximum level and if it is large enough then farms adulterate to some extent but not to the maximum level.

THEOREM O.5. *(i) For preemptive EMA with imperfect testing and convex increasing testing sensitivity modeled by $S_1(\cdot)$, there exists a unique symmetric NE in which x^{PV^*} is determined as follows.*

(a) If $c < -h'(1)(r_H - r_L)/[S_1'(1)q(t/n)((n+1)/n)]$, then all farms adulterate to the maximum level; i.e., $x^{PV^*} = 1$.

(b) If $c \geq [-h'(1)(r_H - r_L)/[S_1'(1)q(t/n)((n+1)/n)]$, then the farms adulterate to some extent; i.e., $x^{PV^*} \in (0,1)$ and is the solution to the following equation: $-h'(x) = S_1(x)S_1'(x)q(t/n)((n+1)/n)c/(r_H - r_L)$.

(ii) For preemptive EMA with imperfect testing and convex increasing testing sensitivity modeled by $S_2(\cdot)$, there exists a unique symmetric NE in which x^{PV^*} is determined as follows.

(a) If $c < -h'(\tau)(r_H - r_L)/[S_2(\tau)S_2'(\tau)q(t/n)((n+1)/n)]$, then all farms adulterate to the maximum level; i.e., $x^{PV^*} = 1$.

(b) If $c \geq -h'(\tau)(r_H - r_L)/[S_2(\tau)S_2'(\tau)q(t/n)((n+1)/n)]$, let $x^* \in (0,\tau)$ be the solution to the equation: $-h'(x) = S_2(x)S_2'(x)q(t/n)((n+1)/n)c/(r_H - r_L)$. There are two cases:

i. *If $c < h(x^*)/[q(t/n)(1 - S_2(x^*)^2)]$, then all farms adulterate to the maximum level; i.e., $x^{PV^*}=1$.*

ii. *If $c \geq h(x^*)/[q(t/n)(1 - S_2(x^*)^2)]$, then all farms adulterate to some extent; i.e., $x^{PV^*} = x^*$.*

(iii) For preemptive EMA with imperfect testing and testing sensitivity modeled by S_3, there exists a unique symmetric NE of the game in which x^{PV^*} is determined as follows.

(a) *If $c < -h'(1/\alpha)(r_H - r_L)/[\alpha q(t/n)((n+1)/n)]$, then all farms adulterate to the maximum level; i.e. $x^{PV^*} = 1$.*

(b) *If $c \leq -h'(1/\alpha)(r_H - r_L)/[\alpha q(t/n)((n+1)/n)]$, then let $x^* \in (0, 1/\alpha)$ be the solution to the following equation: $-h'(x)/x = q(t/n)((n+1)/n)c\alpha^2/(r_H - r_L)$.*

i. *If $c < (h(x^*) - h(1))(r_H - r_L)/[\alpha^2 q(t/n)(1 - x^{*2})]$ then $x^{PV^*} = 1$*

ii. *If $c \geq (h(x^*) - h(1))(r_H - r_L)/[\alpha^2 q(t/n)(1 - x^{*2})]$ then $x^{PV^*} = x^*$*

Under reactive EMA, we again analyze a setting where a farm adulterates either all or none of his realized low-quality units; i.e., the adulteration strategy can be characterized by the mapping $a_i(n_{L,i}) : \{1, \ldots, m\} \to \{0, 1\}$.

THEOREM O.6. *(i) For reactive EMA with imperfect testing and convex increasing testing sensitivity modeled by $S_1(\cdot)$, there exists a unique symmetric BNE of the game in which a farm's adulteration strategy is a threshold strategy: $a^*(n_{L,i}) = 1$ if $n_{L,i} \in [0, \beta^S)$ and $a^*(n_{L,i}) = 0$ if $n_{L,i} \in [\beta^S, m]$, for all i. The threshold β^S is unique and determined as follows:*

(a) If $cq\left(\frac{t}{n}\right) \geq \frac{(r_H - r_L)}{\mathbb{E}_{n_{L,-i}}\left[S_1\left(\frac{m+\sum_{-i} n_{L,-i}}{k}\right)\right]}$, then $\beta^S \in (0, m)$ and is the solution to the equation:

$$\beta^S(r_H - r_L) = q\left(\tfrac{t}{n}\right) S_1\left(\tfrac{\beta^S}{m}\right) \mathbb{E}_{n_{L,-i}}\left[S_1\left(\tfrac{\beta^S + \sum_{-i} n_{L,-i}\mathbb{I}\{n_{L,-i} < \beta^S\}}{k}\right)\right] cm,$$

where $\mathbb{I}\{\cdot\}$ is an indicator function whose value is 1 if the argument is true and 0 otherwise.

(b) If $cq\left(\frac{t}{n}\right) < \frac{(r_H - r_L)}{\mathbb{E}_{n_{L,-i}}\left[S_1\left(\frac{m+\sum_{-i} n_{L,-i}}{k}\right)\right]}$, then $\beta^S = m$.

(ii) For reactive EMA with imperfect testing and convex increasing testing sensitivity modeled by $S_2(\cdot)$, in any BNE of the game, there exist thresholds β^l and β^u such that $\beta^l < \beta^u$ and the following must hold:

(a) If $cq\left(\frac{t}{n}\right) \geq \frac{(r_H - r_L)}{\mathbb{E}_{n_{L,-i}}\left[S_2\left(\frac{m+\sum_{-i} n_{L,-i}}{k}\right)\right]}$, then in equilibrium $a^*(n_{L,i}) = 1$ if $n_{L,i} < \beta^l$ or if $n_{L,i} > \beta^u$; $a^*(n_{L,i}) = 0$ if $n_{L,i} \in [\beta^l, \beta^u]$. Furthermore, we must have $\beta^u \geq \tau m$.

(b) If $cq\left(\frac{t}{n}\right) < \frac{(r_H - r_L)}{\mathbb{E}_{n_{L,-i}}\left[S_2\left(\frac{m+\sum_{-i} n_{L,-i}}{k}\right)\right]}$, then all farms always adulterate.

(iii) For reactive EMA with imperfect testing and testing sensitivity modeled by S_3, there exists a symmetric BNE of the game in which a farm's adulteration strategy is a threhold strategy: $a^*(n_{L,i}) = 1$ if $n_{L,i} \in [0, \beta^S]$ or if $n_{L,i} \in [\beta^U, m]$.

(a) *If $cq(t/n) \leq max\Big(\frac{r_H - r_L}{\alpha}, \frac{k(r_H - r_L)}{\alpha(p\alpha(k-m)+m)}\Big)$ then $\beta^S = \beta^U = m$*

(b) *If $cq(t/n) > max\Big(\frac{r_H - r_L}{\alpha}, \frac{k(r_H - r_L)}{\alpha(p\alpha(k-m)+m)}\Big)$ then*

$$\beta^S = max\Big(0, \frac{k(r_H - r_L)}{cq\alpha^2} - (\frac{k}{m} - 1)(\textstyle\int_0^{\beta^S} xf(x,m,p)dx + \int_{\beta^U}^m xf(x,m,p)dx)\Big) \text{ and}$$

$$\beta^U = min\Big(m, \frac{cqm(t/n)}{r_H - r_L}, \frac{(\frac{k}{m} - 1)(\int_0^{\beta^S} xf(x,m,p)dx + \int_{\beta^U}^m xf(x,m,p)dx)}{\frac{k(r_H - r_L)}{cqm\alpha} - 1}\Big)$$

Theorems O.5 and O.6 below show that the farms' equilibrium adulteration behavior under scenario (i) follows a very similar structure as in Theorems 4 in and §3.2.2. Under scenario (ii) and (iii), the structure of the equilibrium strategy in Theorem O.6

can be viewed as a combination of the equilibrium strategies described in Theorems 3 and 4 in §3.2. In particular, the equilbrium strategy here combines the adulteration strategies identified in perfect and imperfect testing. If the number of low quality units are very low, then the testing sensitivity is similar to the imperfect testing case and farms adulterate when their low quality units are less than a threshold. In contrast, if low quality units are greater than a thereshold, then testing sensitivity is similar to perfect testing and farms adulterate when their low quality units are greater than a threshold.

O.1.5. Alternative Penalty Structures

In this extension, we first analyze cases in which per unit penalty is linearly increasing in the amount of adulterants. This section is valuable in capturing scenarios where harmful effects of adulterants are increasing in the amount of adulterants. We model such case by assuming that penalty is linearly increasing in the amount of adulterants. Thus for a farm that adds x_i adulterants per unit under preemptive testing and is caught, the penalty is $cm(mx)$. We again find that the equilibrium structure under all the scenarios is similar to the ones identified in Section§3.2 and §3.1 except in one case: For penalty alternative (i) and preemptive EMA with perfect testing, adulterating farms use an amount of adulterants that balances the revenue gain with the penalty from adulterating. The additional tradeoff arise because the penalty on being caught adulterating is no longer constabt but increaisng in the amount of adulterants.

Theorem O.7. *(i) For preemptive EMA with perfect testing, there exists a unique symmetric NE in which x^{PP^*} is determined as follows.*

(a) If $c \geq -h'(0)(r_H - r_L)n/[mq(t/n)(n+1)]$, then

$$n_a^* = \left\lceil \frac{-h'(0)(r_H - r_L)n^2}{mcqt} - 1 \right\rceil \tag{O.1}$$

(b) If $c < -h'(0)(r_H - r_L)n/[mq(t/n)(n+1)]$, then $n_a^* = n$

Let x^* be the solution to the following equation:$-h'(x)(r_H - r_L) = mcq(t/n)(n_a^*/n)$. Then $x_a^{PP^*} = min(x^*, 1)$ and $x_a^{PP^*} \in (0,1)$.

(ii) For preemptive EMA with imperfect testing, there exists a unique symmetric NE in which x^{PV^*} is determined as follows.

(a) If $c < -h'(1)(r_H - r_L)/[mq(t/n)((2n+1)/n)]$, then all farms adulterate to the maximum level; i.e., $x^{PV^*} = 1$.

(b) If $c \geq [-h'(1)(r_H - r_L)/[mq(t/n)((2n+1)/n)]$, then the farms adulterate to some extent; i.e., $x^{PV^*} \in (0,1)$ and is the solution to the following equation: $-h'(x)/x^2 = q(t/n)((2n+1)/n)c/(r_H - r_L)$.

(iii) For reactive EMA with perfect testing,

(a) if $r_H - r_L \leq q(t/n)cm$, then none of the farms adulterate; i.e. $\beta^{RP^*} = 0$.

(b) If $r_H - r_L > q(t/n)cm$, then all the farms adulterate to the maximum level; i.e., $\beta^{RP^*} = m$

(iv) For reactive EMA with imperfect testing, there exists a unique symmetric BNE of the game in which a farm's adulteration strategy is a threshold strategy: $a^*(n_{L,i}) = 1$ if $n_{L,i} \in [0, \beta^{RV})$ and $a^*(n_{L,i}) = 0$ if $n_{L,i} \in [\beta^{RV}, m]$, for all i. The threshold β^{RV} is unique and determined as follows:

(a) If $cq\left(\frac{t}{n}\right) \geq \frac{k(r_H - r_L)}{m(m+(n-1)mp)}$ then $\beta^{RV} \in (0,m)$ and is the solution to the equation: $\beta^2 + \beta(n-1)\int_0^\beta xf(x, \frac{k}{n}, p_L)\mathrm{d}x = \frac{nk(r_H - r_L)}{cqt}$.

(b) If $cq\left(\frac{t}{n}\right) < \frac{k(r_H - r_L)}{m(m+(n-1)mp)}$, then $\beta^{RV} = m$.

Next, we find qualitative evidence that larger companies who are caught adulterating face more severe penalty than smaller ones. For example, they face longer jail terms and are fined more heavily (Yan 2017). This observation can be captured by modeling the total penalty from adulterating as convex increasing in m. We find that all of our results continue to hold in this alternative setup with updated threshold values.

Proposition O.5. *All of our results in §3.1 and §3.2 continue to hold if the penalty that a farm incurred for adulterating is convex increasing in the total number of units, m, supplied by the farm.*

O.2. Investing in Traceability and Testing Frequency to Mitigate EMA Risk

Given a supply network of farms, the manufacturer has two levers to mitigate the risk of EMA in the supply chain: increasing supply chain traceability and the frequency of testing the aggregated supply. Developing these capabilities can be costly. For example, it is very difficult to trace and inspect every individual farm in a supply chain sourcing from thousands of farms (Nestlé 2015). Therefore, the manufacturer needs to balance between the cost of investing in these capabilities and the benefit of reducing EMA risk in the supply chain. To address this tradeoff, we develop an optimization model from the manufacturer's perspective, where the objective is to minimize total investment costs while satisfying a constraint that the resulting risk of EMA in the supply chain cannot exceed a certain level.

First consider preemptive EMA. In this setting, the overall risk of EMA in the supply chain is measured by n_a^*/n under perfect testing and x^{PV^*} under imperfect testing (see §4). Define $l(q)$ and $g(t)$ as the manufacturer's investment costs for increasing testing frequency and traceability, both of which are convex and increasing functions. The manufacturer's optimization problem under preemptive EMA can be characterized as follows.

$$\Pi^{PP}(q,t) \equiv \min_{q,t}\{l(q)+g(t)\,|n_a^*/n \le \alpha,\ q\in[0,1],\ t\in[0,n]\}, \tag{O.2}$$

$$\Pi^{PV}(q,t) \equiv \min_{q,t}\left\{l(q)+g(t)\,\middle|x^{PV^*} \le \alpha,\ q\in[0,1],\ t\in[0,n]\right\}, \tag{O.3}$$

where n_a^* and x^{PV^*} are defined in Theorems 1 and 2, and α is the maximum level of risk allowed. For reactive EMA, the manufacturer's optimization problem can be modeled similarly as follows.

$$\Pi^{RI}(q,t) \equiv \min_{q,t}\{l(q)+g(t)\,|P_n \le \alpha,\ q\in[0,1],\ t\in[0,n]\}, \tag{O.4}$$

$$\Pi^{RE}(q,t) \equiv \min_{q,t}\{l(q)+g(t)\,|E_n \le \alpha,\ q\in[0,1],\ t\in[0,n]\}, \tag{O.5}$$

where P_n and E_n are defined in §4 given the farms' optimal adulteration strategies under perfect and imperfect testing, characterized in Theorems 3 and 4. The key difference is that we measure the risk of reactive EMA in the supply chain in two ways: the probability of an individual farm adulterating (i.e., P_n as in Model (O.4)) and the expected total amount of adulterated output in the supply chain (i.e., E_n as in Model (O.5)).

Before characterizing the manufacturer's optimal investment strategy under each of these model scenarios, we first define the following useful constants.

(i) For Model (O.2):

$$u^{PP} \equiv \frac{(r_H-r_L)(p_L^{\max}-p_L^{\min})n^2}{c(1+\lfloor n\alpha\rfloor)}. \tag{O.6}$$

(ii) For Model (O.3):

$$u^{PV} \equiv \frac{-h'(\alpha)(r_H-r_L)n}{\alpha c(n+1)/n}. \tag{O.7}$$

(iii) For Model (O.4) under perfect testing:

$$u^{RP} \equiv \left(\frac{n^2(r_H-r_L)}{ck}\right)\left(\frac{p_Lk}{n}+\phi^{-1}(1-\alpha)\sqrt{\frac{kp_L(1-p_L)}{n}}\right). \tag{O.8}$$

(iv) For Model (O.4) under imperfect testing:

$$u^{RV} \equiv \left(\frac{kn(r_H-r_L)}{c}\right)\left(\frac{p_Lk}{n}+\phi^{-1}(\alpha)\sqrt{\frac{kp_L(1-p_L)}{n}}+(n-1)\int_0^{\frac{p_Lk}{n}+\phi^{-1}(\alpha)\sqrt{\frac{kp_L(1-p_L)}{n}}} xf(x,k/n,p_L)dx\right)^{-1}. \tag{O.9}$$

The notation ϕ represents the PDF of the standard normal distribution. These constants are the values of qt when the risk constraint in the corresponding models indicated is binding. Note that in the optimal solution to these models, the risk constraint must be binding because n_a^*, x^{PV^*}, and P_n are all decreasing in q and t, whereas the investment costs are increasing in q and t. The following theorem summarizes the manufacturer's optimal investment strategy for Models (O.2), (O.3), and (O.4) under perfect and imperfect testing.

Theorem O.8. *Given the constants u^j with $j \in \{PP, PV, RP, RV\}$ defined in Equations* (O.6)–(O.9)*, we have the following results for $\alpha \leq 0.5$.*

(i) If $u^j > n$, then the corresponding manufacturer problem is infeasible.

(ii) If $u^j \leq n$, then the optimal solution to the corresponding manufacturer problem (q^*, t^*) can be characterized as follows.

(a) If $l'(1) \leq u^j g'(u^j)$, then $(q^*, t^*) = (1, u^j)$.

(b) If $g'(n) \leq \frac{u^j}{n^2} l'\left(\frac{u^j}{n}\right)$, then $(q^*, t^*) = \left(\frac{u^j}{n}, n\right)$.

(c) If $l'(1) > u^j g'(u^j)$ and $g'(n) > \frac{u^j}{n^2} l'\left(\frac{u^j}{n}\right)$, then $(q^*, t^*) \in (0,1) \times (0, n)$ and satisfy the following first-order conditions:

$$q^* = \sqrt{\frac{u^j g'(u^j/q^*)}{l'(q^*)}} \text{ and } t^* = \frac{u^j}{q^*}.$$

Theorem O.8 part (i) suggests that if the manufacturer cannot satisfy the risk constraint even when the supply chain is fully traceable and she always tests the aggregated supply, then additional levers are necessary to meet the risk constraint. Given our earlier discussions in §4, one possible solution is to reduce supply chain dispersion. Theorem O.8 part (ii) shows that when a feasible solution exists, the manufacturer always chooses the solution with the best cost-effectiveness. If the marginal cost at maximum testing frequency is lower than the marginal cost at the minimum necessary traceability to satisfy the risk constraint (i.e., increasing testing frequency is in general more cost effective than increasing traceability; Theorem O.8 part (ii-a)), then it is optimal for the manufacturer to always test the aggregated supply and build just enough traceability given the risk constraint. Conversely, if increasing traceability is in general more cost effective than increasing testing frequency (Theorem O.8 part (ii-b))), then it is optimal for the manufacturer to build full traceability in the supply chain and test just enough given the risk constraint. If neither of the above is true (Theorem O.8 part (ii-c))), then the optimal investment is an interior solution that achieves the best cost balance between investing in the two levers.

Our next proposition characterizes how the optimal investment solution (q^*, t^*) described in Theorem O.8 and the resulting optimal cost change with supply chain dispersion.

Proposition O.6. *Let SC_H and SC_L be two supply chains such that the supply chain dispersion in SC_H is greater than that in SC_L (i.e., $n_H > n_L$). Consider each of the manufacturer's optimization problems formulated in Models* (O.2)*,* (O.3)*, and* (O.4) *under perfect and imperfect testing. We have the following results for $\alpha \leq 0.5$.*

(i) If the manufacturer's problem is infeasible for SC_L, then it is also infeasible for SC_H.

(ii) If the manufacturer's problem is feasible for SC_H, then it is also feasible for SC_L.

(iii) Assume that the manufacturer's problem is feasible for SC_H. Let (q_H^*, t_H^*) and (q_L^*, t_L^*) be the optimal solution for SC_H and SC_L respectively. Then, $q_L^* \leq q_H^*$, $t_L^* \leq t_H^*$, and the resulting optimal cost for the manufacturer is lower in SC_L than in SC_H.

Proposition O.6 highlights two results. First, given a desirable risk constraint, it is always more difficult for a manufacturer with a more dispersed supply chain to satisfy the constraint (parts (i) and (ii)). Second, conditional on being able to satisfy the risk constraint, it is always more costly for a manufacturer with a more dispersed supply chain to do so (part (iii)). Therefore, higher supply chain dispersion results in greater challenges for a manufacturer to manage and mitigate the risk of individual farms adulterating, from both feasibility and financial standpoints.

Finally, we consider the manufacturer's problem formulated in Model (O.5). The key difference in this model versus the others is that the risk constraint is imposed on E_n, the expected total amount of adulterated output in the supply chain. Since E_n aggregates all farms' adulteration decisions, we cannot derive the manufacturer's optimal decisions analytically. Nevertheless, consistent with Proposition O.6, we show that higher supply chain dispersion again makes it more costly for the manufacturer to satisfy a desirable risk constraint, regardless of testing sensitivity (perfect or imperfect testing).

Proposition O.7. *Let SC_H and SC_L be two supply chains such that the supply chain dispersion in SC_H is greater than that in SC_L (i.e., $n_H > n_L$). Consider the manufacturer's optimization problem formulated in Model* (O.5) *and assume that it is feasible for SC_H. If $\alpha \leq n \int_{mp/3} x f(x, m, p) \mathrm{d}x$, then for both perfect and imperfect testing, the optimal cost for the manufacturer is lower in SC_L than in SC_H.*

Table O.1 Total EMA Risk When Farms Engage in Both Preemptive and Reactive EMA

	Reactive EMA, perfect testing	Reactive EMA, imperfect testing
Preemptive EMA, perfect testing	$\lambda m n_a^* + (n-n_a^*)\int_{\beta^{RP}}^{m} x f(x,m,p_L^{\max})dx$	$\lambda m n_a^* + (n-n_a^*)\int_{0}^{\beta^{RV}} x f(x,m,p_L^{\max})dx$
Preemptive EMA, imperfect testing	$\lambda k x^{PV^*} + n\int_{\beta^{RP}}^{m} x f(x,m,h(x^{PV^*}))dx$	$\lambda k x^{PV^*} + n\int_{0}^{\beta^{RV}} x f(x,m,h(x^{PV^*}))dx$

O.3. Investing in Testing Capabilities to Mitigate EMA Risk

We examine the effect of the manufacturer investing in perfect testing on mitigating EMA risk in the supply chain. To this end, we allow farms to engage in both preemptive and reactive EMA in response to the manufacturer's testing capability. The model dynamics are similar to those described in §2 with the first two steps revised as follows. (i) The manufacturer chooses whether or not to adopt perfect testing for preemptive or reactive EMA respectively. The farms observe the manufacturer's choice. (ii) Each farm simultaneously and individually decides the amount of adulterants to add to reduce the likelihood of producing low-quality output from $p_L^{\max}$ to some $p_L \le p_L^{\max}$ (preemptive EMA). (iii) The uncertain quality of each unit of output is realized. (iv) Each farm simultaneously and individually decides whether or not to adulterate all of the realized low quality units n_L to create fake high-quality ones (reactive EMA). The remaining steps are exactly the same as in Figure 1. We are interested in analyzing how the total EMA risk, accounting for both preemptive and reactive EMA, is affected by the manufacturer's testing capability for either type of EMA. In particular, we analyze whether adopting perfect testing always reduces EMA risk in the supply chain. In this analysis, we take the farms to be short-term oriented (see footnote 8), and thus, they do not account for every possible realization of n_L and the corresponding reactive EMA decision when making their preemptive EMA decision. To simplify exposition, we also assume that when the farms adulterate preemptively with the maximum dosage, p_L becomes 0. Our results remain qualitatively the same without this simplifying assumption. Table O.1 summarizes the total EMA risk in the supply chain for the four different scenarios we analyze in §3. For example, the top left cell shows the total EMA risk if the manufacturer invests in perfect testing for both preemptive and reactive EMA. By Theorem 1 we know that under perfect testing, a subset of n_a^* farms adulterate preemptively with the maximum dosage while the remaining do not adulterate at all. Thus, the expected total amount of adulterants added preemptively is mn_a^*. In the reactive EMA stage, we again know from Theorem 3 that under perfect testing, farms adulterate when their low-quality units are greater than β^{RP}. Hence, the total EMA risk in the supply chain is equal to $\lambda m n_a^* + (n-n_a^*)\int_{\beta^{RP}}^{m} x f(x,m,p_L^{\max})dx$. Note that λ here measures the importance of preemptive EMA relative to reactive EMA when the manufacturer evaluates the total EMA risk. We can similarly characterize the total EMA risk in the supply chain for the other three scenarios.

By Proposition 1, we know that reactive EMA risk is always lower when the manufacturer adopts perfect testing. Thus, we only need to compare the total EMA risk in the left two cells in Table O.1. We first focus on comparing the preemptive EMA risk between these two scenarios. Our results are summarized in the next proposition.

PROPOSITION O.8. *Let $R_p^P \equiv mn_a^*$ and $R_{ip}^P \equiv kx^{PV^*}$ denote the preemptive EMA risk under perfect and imperfect testing respectively . Then,*

(i) If $c < -h'(1)(r_H - r_L)/[q(t/n)(n+1)/n]$, then $R_p^P = k$ and $R_{ip}^P = k$. Thus, all farms adulterate to the maximum level under both perfect and imperfect testing cases.

(ii) If $c \in [-h'(1)(r_H - r_L)/[q(t/n)(n+1)/n, (r_H - r_L)(p_L^{max} - p_L^{min})/[q(t/n)]]$, then $x^{PV^*} \in (0,1)$ and $n_a^* = n$. Thus, $R_p^P \ge R_{ip}^P$, i.e., preemptive EMA risk is higher under perfect testing than under imperfect testing.

(iii) If $c \ge (r_H - r_L)(p_L^{max} - p_L^{min})/[q(t/n)]]$, and

(a) *If $c < (r_H - r_L)(p_L^{max} - p_L^{min})/[q(t/n)h^{-1}((p_L^{min} - p_L^{max})(1+1/n))]$, then $R_p^P \ge R_{ip}^P$, i.e., preemptive EMA risk is higher under perfect testing than under imperfect testing.*

(b) *If $c \ge (r_H - r_L)(p_L^{max} - p_L^{min})/[q(t/n)h^{-1}((p_L^{min} - p_L^{max})(1+1/n))]$ then $R_{ip}^P \ge R_p^P$, i.e., preemptive EMA risk is higher under imperfect testing than under perfect testing.*

Proposition O.8 shows that when the per-unit penalty is not high enough, adopting perfect testing can in fact backfire and result in *higher* preemptive EMA risk. This is because under perfect testing, adulterating farms all adulterate with the maximum dosage, while under imperfect testing, they adulterate at a lower level to trade off revenue gain with the expected penalty. When the penalty is not high enough, more farms adulterate to the maximum dosage under perfect testing, therefore leading to a higher risk. Our next result shows that this observation remains true when considering the total EMA risk that also accounts for reactive EMA.

THEOREM O.9. *Let R_p^T denote the total EMA risk under perfect testing for both preemptive and reactive EMA, and R_{ip}^T the total EMA risk under imperfect testing for preemptive EMA and perfect testing for reactive EMA. We have the following results.*

(i) If $c < -h'(1)(r_H - r_L)/[q(t/n)(n+1)/n]$, then $R_p^T = R_{ip}^T$.

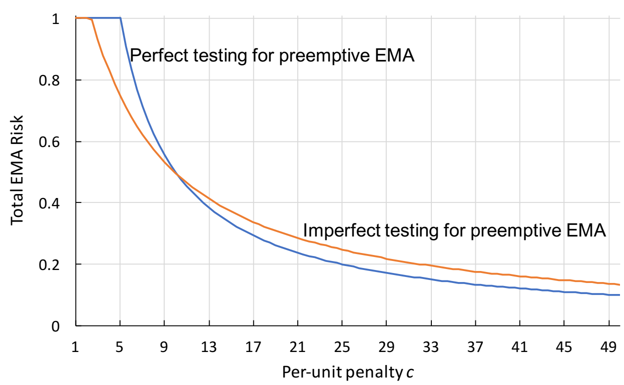
(ii) If $c \geq (r_H - r_L)/[q(t/n)]$, and

(a) *If $c < (r_H - r_L)(p_L^{max})/[q(t/n)h^{-1}(-p_L^{max}(1+1/n))])$, then $R_p^T \geq R_{ip}^T$.*

(b) *If $c \geq (r_H - r_L)(p_L^{max})/[q(t/n)h^{-1}(-p_L^{max}(1+1/n))])$, then $R_{ip}^T \geq R_p^T$.*

We complement Theorem O.9 with extensive numerical simulation for the range of c values that we cannot characterize the total risk analytically. Figure O.1 presents a representative pattern of how the total EMA risk changes with c under either perfect or imperfect testing for preemptive EMA and perfect testing for reactive EMA. Observe that adopting perfect testing for preemptive EMA in fact leads to higher total EMA risk inadvertently when c is not sufficiently high (for $c < r_H$ in this example).

Figure O.1 Total EMA Risk under Perfect or Imperfect Testing for Preemptive EMA and Perfect Testing for Reactive EMA



Note. We use the following parameters in this example: $k = 100,000$, $m = 1,000$, $q = 1$, $r_H = 10$, $r_L = 0$, $t = n$, $c \in \{1, 1.5, \dots, 50\}$, $p_L^{\max} = 0.5$, and $p_L^{\min} = 0$.

O.4. Proofs

Proof of Theorem 1

Let us assume that a subset n_a out of a total of n farms are adulterating. Consider a farm i that is adulterating at equilbrium. Then, revenue from adulteration (π_a^i) should be strictly greater than revenue from non adulteration (π_{na}^i). $\pi_a^i = r_H m(1 - p_L^{min}) + r_L m p_L^{min} - q\frac{n_a}{n}(t/n)cm$ and $\pi_{na}^i = r_H m(1 - p_L^{max}) + r_L m p_L^{max}$. Now consider a farmer i' who is not adulterating at equilbrium. Then, $\pi_{na}^{i'} \geq \pi_a^{i'}$. $\pi_a^{i'} = r_H m(1 - p_L^{min}) + r_L m p_L^{min} - q\frac{n_a+1}{n}(t/n)cm$ and $\pi_{na}^{i'} = r_H m(1 - p_L^{max}) + r_L m p_L^{max}$. Simplfying the two conditions we get, $\frac{n^2(r_H - r_L)(p_L^{max} - p_L^{min})}{cqt} > n_a \geq \frac{n^2(r_H - r_L)(p_L^{max} - p_L^{min})}{cqt} - 1$. Thus we get the desired result using the fact that n_a has to lie between 0 and n.

Proof of Theorem 2

We first define a Nash equilibrium in the game.

DEFINITION O.1. The strategy profile $\{x_i^{PV^*}, i = 1, \dots, n\}$ constitutes a Nash equilibrium of the game if $x_i^{PV^*} \in \arg\max_{x_i \in [0,1]} \pi^{PV}(x_i, x_{-i}^{PV^*})$, where $\pi^{PV}(\cdot,\cdot)$ is defined in Equation (2).

At an NE, farm i chooses x_i^* so as to maximize his expected payoff in Equation (2) given the choices of all other farms. Note that $\frac{\partial \pi^{PV}(x_i,x_{-i})}{\partial x_i} = -mh'(x)(r_H - r_L) - q\frac{t}{n}cm(\frac{\sum_{-i} x_{-i} + 2x_i}{n})$ and $\frac{\partial^2 \pi^{PV}(x_i,x_{-i})}{\partial x_i^2} = -mh''(x)(r_H - r_L) - q\frac{t}{n}cm(\frac{2}{n}) < 0$. Further, since we consider only symmetric equilibrium, we have $x_{-i}^* = x_i^* = x_{PV^*}$. Substituting this, we get $\frac{\partial \pi^{PV}(x_i,x_{-i})}{\partial x_i} = -h'(x^*)(r_H - r_L) - q\frac{t}{n}c\frac{n+1}{n}x^*$. Since $\pi^{PV}(x_i,x_{-i})$ is a concave function, optimal solution is achieved either at the first order condition or at the boundary:

(a) If $c < -h'(1)(r_H - r_L)/[q(t/n)((n+1)/n)]$, $x^{PV^*} = 1$.

(b) If $c \geq -h'(1)(r_H - r_L)/[q(t/n)((n+1)/n)]$, then $x^{PV^*} \in (0,1)$ and is the solution to the FOC: $-h'(x)/x = q(t/n)((n+1)/n)c/(r_H - r_L)$.

Proof of Theorem 3

Farmer's revenue is $\pi^{RP}(n_L) = \max\{r_H(m - n_L) + r_L n_L, r_H m - q(t/n)cm\}$ under perfect testing for reactive cases. Thus, he adulterates if $r_H m - q(t/n)cm > r_H(m - n_L) + r_L n_L$. This simplifies to $n_L > \frac{q(t/n)cm}{r_H - r_L} \equiv \beta^{RP}$. Observe that $\frac{\partial \beta^{RP}}{\partial (r_H - r_L)} = -\frac{q(t/n)cm}{(r_H - r_L)^2} \leq 0$, $\frac{\partial \beta^{RP}}{\partial q} = \frac{(t/n)cm}{(r_H - r_L)} \geq 0$, $\frac{\partial \beta^{RP}}{\partial t} = \frac{q(1/n)cm}{(r_H - r_L)} \geq 0$ and $\frac{\partial \beta^{RP}}{\partial c} = \frac{q(t/n)m}{(r_H - r_L)} \geq 0$.

Proof of Theorem 4

We first formally define a Bayesian Nash equilibrium in the game as follows.

Definition O.2. The strategy profile $\{a_i^*(n_{L,i}), i = 1, \ldots, n\}$ constitutes a Bayesian Nash equilibrium (BNE) of the reactive adulteration game if $a_i^*(n_{L,i}) \in \arg\max_{a \in \{0,1\}} \mathbb{E}_{n_{L,-i}}\left[\pi_i^{RV}(a, n_{L,i}, a_{-i}^*(n_{L,-i}))\right]$, where

$$\mathbb{E}_{n_{L,-i}}\left[\pi_i^{RV}(a, n_{L,i}, a_{-i}^*(n_{L,-i}))\right] = \begin{cases} r_H(m - n_{L,i}) + r_L n_{L,i}, & \text{if } a = 0, \\ r_H m - \gamma_i(n_{L,i}, a_{-i}^*(n_{L,-i}))cm, & \text{if } a = 1, \end{cases} \tag{O.10}$$

and $\gamma_i(\cdot,\cdot)$ is defined in Equation (3).

First we show that any optimal strategy in this game is a threshold policy. Farmer i's revenue is $\max\{r_H(m - n_L) + r_L n_L, r_H m - \gamma_i(n_{L,i}, a_{-i}(n_{L,-i}))cm\}$, where $\gamma_i(n_{L,i}, a_{-i}(n_{L,-i})) \equiv q\left(\frac{t}{n}\right)\left(\frac{n_{L,i}}{m}\right)\left(\frac{n_{L,i} + \mathbb{E}_{n_{L,-i}}\left[\sum_{-i} n_{L,-i}a_{-i}(n_{L,-i})\right]}{k}\right)$ from equation 3. Thus he adulterates if $r_H m - \gamma_i(n_{L,i}, a_{-i}(n_{L,-i}))cm > r_H(m - n_{L,i}) + r_L n_{L,i}$. This simplifies to $\frac{kn(r_H - r_L)}{cqt} - \mathbb{E}_{n_{L,-i}}\left[\sum_{-i} n_{L,-i}a_{-i}(n_{L,-i})\right] \geq n_{L,i}$. Let $\beta \equiv \frac{kn(r_H - r_L)}{cqt} - \mathrm{E}(\sum_{j=1\setminus i}^{n} a_{n_{L,j}} b_j)$. Under any BNE, farm i will adulterate only when his low quality units are less than the threshold β. Since all the farms behave symmetrically and they are all identical, all farms adulterate only when their realized low quality units are less than β. Thus, $a^*(n_{L,i}) = 1$ if $n_{L,i} \in [0, \beta)$ and $a^*(n_{L,i}) = 0$ if $n_{L,i} \in (\beta, m]$, for all i. Next, we characterize β and show that it is unique.

Using the fact that all farms have the same threshold β, we have that $\mathbb{E}_{n_{L,-i}}\left[\sum_{-i} n_{L,-i}a_{-i}(n_{L,-i})\right] = (n-1)\int_0^{\beta^{RV}} xf(x,m,p_L)dx$. Note that $max_\beta\left\{\int_0^\beta xf(x,m,p_L)dx\right\} = mp_L$ and it is attained when $\beta \geq m$. If $\beta = m$ is a symmetric BNE, then given that $\beta_{-i} = m$, we should get that $\beta_i = m$ as well. This happens when $\beta_i \equiv \frac{kn(r_H - r_L)}{cqt} - (n-1)mp_L \geq m$, because it implies that, given that all other farms have $\beta_{-i} = m$, $\beta_i = m$ is optimal for farm i. This condition simplifies to $\frac{kn^2(r_H - r_L)}{cqt} - (n-1)kp_L - k \geq 0$, which is a quadratic in n and is satisfied when $n > 2(1-p_L) \Big/ \left(\sqrt{p_L^2 + \frac{4(1-p_L)(r_H - r_L)}{cqt}} - p_L\right)$. This proves part (b) of the proposition. If $n \leq 2(1-p_L) \Big/ \left(\sqrt{p_L^2 + \frac{4(1-p_L)(r_H - r_L)}{cqt}} - p_L\right)$, we will use the Intermediate Value Theorem to show the existence of β^{RV}. Define $F(\beta) \equiv \beta - \frac{kn(r_H - r_L)}{cqt} + (n-1)\int_0^\beta xf(x,m,p_L)dx$ for $\beta \in$ [0,m]. Note that $F(0) = -\frac{kn(r_H - r_L)}{cqt} < 0$ and $F(m) = m - \frac{kn(r_H - r_L)}{cqt} + (n-1)mp_L > 0$. Since $F(.)$ is continuous on $[0,m]$ and $F(0) < 0 < F(m)$, there exists $\beta^{RV} \in$ (0,m) s.t. $\beta^{RV} = \frac{kn(r_H - r_L)}{cqt} - (n-1)\int_0^{\beta^{RV}} xf(x,m,p_L)dx$. Finally, since the RHS in the equation is monotonically decreasing in β^{RV}, the fixed point is also unique.

Using the Implicit function theorem, we have $\frac{\partial \beta^{RV}}{\partial (r_H - r_L)} = \frac{kn}{cqt(1 + (n-1)\beta f(\beta,m,p_L))} > 0$ and $\frac{\partial \beta^{RV}}{\partial c} = -\frac{kn(r_H - r_L)}{c^2qt(1 + (n-1)\beta f(\beta,m,p_L))} < 0$ and $\frac{\partial \beta^{RV}}{\partial q} = -\frac{kn(r_H - r_L)}{cq^2t(1 + (n-1)\beta f(\beta,m,p_L))} < 0$ and $\frac{\partial \beta^{RV}}{\partial t} = -\frac{kn(r_H - r_L)}{cqt^2(1 + (n-1)\beta f(\beta,m,p_L))} < 0$.

Proof of Proposition 1

Note that $\beta^{RP} = \frac{cqmt}{n(r_H - r_L)}$ from Theorem 3. First, we will show that if $\beta^{RP} \in (0,m)$ than $\beta^{RV} = m$. Note that $\beta^{RP} < m$ is equivalent to $\dfrac{cqt}{n(r_H - r_L)} < 1$. Further, we already know from theorem 4 that $\beta^{RV} = m$ if $\dfrac{kn(r_H - r_L)}{cqt} - (n-1)mp_L \geq m$. This condition simplifies to $\dfrac{n}{(n-1)p+1} > \dfrac{cqt}{n(r_H - r_L)}$. Since $p < 1$, $\dfrac{n}{(n-1)p+1} > 1 > \dfrac{cqt}{n(r_H - r_L)}$. Similarly if $\beta^{RV} \in (0,m)$, then $m < \dfrac{nm}{(n-1)p+1} < \dfrac{cqtm}{n(r_H - r_L)}$ and this implies $\beta^{RP} = m$

Proof of Corollary 1

We have shown above that $\beta^{RV} = m$ if $\beta^{RP} \in (0,m)$. Since EMA risk is maximum when $\beta^{RV} = m$, we have that EMA risk under imperfect testing is always greater than that under perfect testing. Next, note that if $\beta^{RP} = m$, then EMA risk under perfect testing is 0 and is again always smaller than that under imperfect testing.

Next, we prove some lemmas that will be useful in proving the dispersion and quality uncretainty results in Proposition 2 and 3 under the reactive adulteration case. Again note that $f(x,m,p_L)$ is the probability density function of a normal distribution with mean mp and variance $mp(1-p)$ evaluated at x. Since normal distribution is a good approximation in our case, we will assume $\int_0^m f(x,m,p_L)dx = 1$. Define, $\beta' \equiv \frac{\partial \beta}{\partial m}$

Lemma O.1. $\int_0^\beta xf(x,m,p_L)dx = \int_0^\beta \int_t^\beta f(x,m,p_L)dxdt$

Using interchange in order of integration,

$$\int_0^\beta xf(x,m,p_L)dx = \int_0^\beta \int_0^x f(x,m,p_L)dtdx = \int_0^\beta \int_t^\beta f(x,m,p_L)dxdt$$

Lemma O.2. $\int_\beta^\infty xf(x,m,p_L)dx = \int_0^\beta \int_\beta^\infty f(x,m,p_L)dxdt + \int_\beta^\infty \int_t^\infty f(x,m,p_L)dxdt$

Using interchange in order of integration,

$$\begin{aligned}\int_\beta^\infty xf(x,m,p_L)dx &= \int_\beta^\infty \Big(\int_0^\beta f(x,m,p_L)dt + \int_\beta^x f(x,m,p_L)dt\Big)dx \\ &= \int_\beta^\infty \int_0^\beta f(x,m,p_L)dtdx + \int_\beta^\infty \int_\beta^x f(x,m,p_L)dtdx \\ &= \int_0^\beta \int_\beta^\infty f(x,m,p_L)dxdt + \int_\beta^\infty \int_t^\infty f(x,m,p_L)dxdt\end{aligned}$$

Lemma O.3. $\frac{\partial}{\partial m}(\int_0^\beta xf(x,m,p_L)dx) = \beta\frac{\partial}{\partial m}(\int_0^\beta f(x,m,p_L)dx) + \int_0^\beta \frac{(t+mp)f(t,m,p)}{2m}dt$

Using lemma O.1,

$$\begin{aligned}&\frac{\partial}{\partial m}\int_0^\beta xf(x,m,p_L)dx = \frac{\partial}{\partial m}\Big(\int_0^\beta \int_t^\beta f(x,m,p_L)dxdt\Big) \\ &= \frac{\partial}{\partial m}\Big(\int_0^\beta \int_0^\beta f(x,m,p_L)dxdt - \int_0^\beta \int_0^t f(x,m,p_L)dxdt\Big) \\ &= \frac{\partial}{\partial m}\Big(\beta\int_0^\beta f(x,m,p_L)dx - \int_0^\beta \int_0^t f(x,m,p_L)dxdt\Big) \\ &= \beta'\int_0^\beta f(x,m,p_L)dx + \beta\frac{\partial}{\partial m}\int_0^\beta f(x,m,p_L)dx - \int_0^\beta \frac{\partial}{\partial m}\int_0^t f(x,m,p_L)dxdt - \beta'\int_0^\beta f(x,m,p_L)dx \\ &= \beta\frac{\partial}{\partial m}\Big(\int_0^\beta f(x,m,p_L)dx\Big) - \int_0^\beta \frac{\partial}{\partial m}\Big(\int_0^{\frac{t-mp}{\sqrt{mp(1-p)}}} \frac{e^{-(x^2/2)}}{\sqrt{2\pi}}dx\Big)dt \\ &= \beta\frac{\partial}{\partial m}\Big(\int_0^\beta f(x,m,p_L)dx\Big) + \int_0^\beta \frac{(t+mp)f(t,m,p)}{2m}dt\end{aligned}$$

Lemma O.4. $\frac{\partial}{\partial m}(\int_\beta^\infty xf(x,m,p_L)dx) = \beta\frac{\partial}{\partial m}\Big(\int_\beta^\infty f(x,m,p_L)dx\Big) + \int_\beta^\infty \frac{t+mp)f(t,m,p)}{2m}dt$

Using lemma O.2,

$$\begin{aligned}&\frac{\partial}{\partial m}\int_\beta^\infty xf(x,m,p_L)dx = \frac{\partial}{\partial m}\Big(\int_0^\beta \int_\beta^\infty f(x,m,p_L)dxdt + \int_\beta^\infty \int_t^\infty f(x,m,p_L)dxdt\Big) \\ &= \frac{\partial}{\partial m}\Big(\beta\int_\beta^\infty f(x,m,p_L)dx + \int_\beta^\infty \int_t^\infty f(x,m,p_L)dxdt\Big) \\ &= \beta'\int_\beta^\infty f(x,m,p_L)dx + \beta\frac{\partial}{\partial m}\int_\beta^\infty f(x,m,p_L)dx + \int_\beta^\infty \frac{\partial}{\partial m}\int_t^\infty f(x,m,p_L)dxdt - \beta'\int_\beta^\infty f(x,m,p_L)dx \\ &= \beta\frac{\partial}{\partial m}\Big(\int_\beta^\infty f(x,m,p_L)dx\Big) + \int_\beta^\infty \frac{\partial}{\partial m}\Big(\int_{\frac{t-mp}{\sqrt{mp(1-p)}}}^\infty \frac{e^{-(x^2/2)}}{\sqrt{2\pi}}dx\Big)dt \\ &= \beta\frac{\partial}{\partial m}\Big(\int_\beta^\infty f(x,m,p_L)dx\Big) + \int_\beta^\infty \frac{(t+mp)f(t,m,p)}{2m}dt\end{aligned}$$

LEMMA O.5. *Let $P_m \equiv P(n_L \leq \beta_m)$. Then, $\frac{\partial P_m}{\partial m} \leq 0$ if $\frac{\partial \beta_m}{\partial m} \leq \frac{\beta + mp}{2m}$*

$$\begin{aligned}
P_m &= P(n_L \leq \beta_m) \\
&= P\Big(\mathcal{N}(0,1) \leq \frac{\beta_m - mp}{\sqrt{mp(1-p)}}\Big) \\
\frac{\partial P_m}{\partial m} &= f(\beta, m, p)(\frac{\partial \beta_m}{\partial m} - \frac{\beta + mp}{2m})
\end{aligned} \tag{O.11}$$

Thus, if $\frac{\partial \beta}{\partial m} \leq \frac{\beta + mp}{2m}$ then $\frac{\partial P_m}{\partial m} \leq 0$ as well.

LEMMA O.6. *Let $P_m \equiv P(n_L \leq \beta_m)$. Then, $\frac{\partial P_m}{\partial p} \leq 0$ if $\frac{\partial \beta}{\partial p} \leq \frac{\beta(1-2p)+mp}{2p(1-p)}$*

$$\begin{aligned}
P_m = P(n_L \leq \beta) &= P\Big(\mathcal{N}(0,1) \leq \frac{\beta_m - mp}{\sqrt{mp(1-p)}}\Big) \\
\frac{\partial P_m}{\partial p} &= f(\beta, m, p)(\frac{\partial \beta}{\partial p} - \frac{\beta(1-2p) + mp}{2p(1-p)})
\end{aligned} \tag{O.12}$$

Thus, if $\frac{\partial \beta}{\partial p} \leq \frac{\beta(1-2p)+mp}{2p(1-p)}$ then $\frac{\partial P_m}{\partial p} \leq 0$ as well.

Proof of Proposition 2A

First we show the results for the preemptive case under both perfect and imperfect testing. First we prove the result for perfect testing. From Theorem 1, we have that if $t = n$ (full traceability) $n_a^* = \min\left\{n, \left\lceil \frac{(r_H - r_L)(p_L^{\max} - p_L^{\min})n}{cq} - 1\right\rceil\right\}$ and if $t < n$ (partial traceability) $n_a^* = \min\left\{n, \left\lceil \frac{(r_H - r_L)(p_L^{\max} - p_L^{\min})n^2}{cqt} - 1\right\rceil\right\}$. Thus, it is straightforward to see that n_a^*/n increases with n under partial traceability and remains constant under full traceability..

Next, we prove the results for reactive case. First we prove the results for P_n for both fully $(t = n)$ and partially $(t < n)$ traceable supply chains. From Theorem 3 we have that $\beta^{RP} = \dfrac{qcm(t/n)}{r_H - r_L}$ and $P_n = Prob(n_L > \beta^{RP})$. If $t = n$, $\beta^{RV} = \dfrac{qcm}{r_H - r_L}$ and $\dfrac{\partial P_n}{\partial n} = (\frac{k}{n^2}) f(\frac{cqk}{n(r_H - r_L)}, \frac{k}{n}, p_L)(\frac{cq - p_L(r_H - r_L)}{2(r_H - r_L)})$. If $t < n$, $\beta^{RP} = \dfrac{qcm(t/n)}{r_H - r_L}$ and $\frac{\partial P_n}{\partial n} = (\frac{k}{n^2}) f(\frac{tcqk}{n^2(r_H - r_L)}, \frac{k}{n}, p_L)(\frac{3cqt - np(r_H - r_L)}{2n(r_H - r_L)})$

Next, we prove the results for E_n. We will prove this result by showing that $\frac{\partial \mathbf{E_m}}{\partial m} \leq 0$. Since $m = k/n$, this implies that $\frac{\partial \mathbf{E_n}}{\partial n} = \frac{\partial m}{\partial n}\frac{\partial \mathbf{E_m}}{\partial m} = -\frac{k}{n^2}\frac{\partial \mathbf{E_n}}{\partial m} \geq 0$ in all the cases. Let X_i^m be the amount of supply adulterated by farm i when he supplies a total of m units. Then total supply that is adulterated is just $\sum_{i=1}^n X_i^m$. For fully traceable networks we have $\beta^{RP} = \frac{cqm}{r_H - r_L}$. Let $\alpha \equiv \frac{cq}{r_H - r_L}$. Using lemma O.4, if $c \geq \frac{p_L(r_H - r_L)}{q}$,

$$\begin{aligned}
\mathbf{E_m} &= \mathbf{E}[\sum_{i=1}^n X_i^m] = \sum_{i=1}^n \mathbf{E}[X_i^m] = n\mathbf{E}[X_i^m] = \frac{k}{m}\int_\beta^\infty x f(x, m, p_L)dx \\
\frac{\partial \mathbf{E_m}}{\partial m} &= \frac{k}{m}\frac{\partial}{\partial m}\Big(\int_\beta^\infty x f(x, m, p_L)dx\Big) - \frac{k}{m^2}\int_\beta^\infty x f(x, m, p_L)dx \\
&= \frac{k}{m}\Big(\beta\frac{\partial}{\partial m}\Big(\int_\beta^\infty f(x, m, p_L)dx\Big) + \int_\beta^\infty \frac{(x + mp_L)f(x, m, p_L)}{2m}dx\Big) - \frac{k}{m^2}\int_\beta^\infty x f(x, m, p_L)dx \\
&= k\alpha\frac{\partial P_m}{\partial m} - \frac{k}{m^2}\int_\beta^\infty x f(x, m, p_L)dx + \frac{k}{m}\int_\beta^\infty \frac{(mp_L + x)}{2m} f(t, m, p_L)dt \\
&= k\alpha\frac{\partial P_m}{\partial m} + \frac{k}{2m}\int_\beta^\infty (p_L - \frac{x}{m}) f(x, m, p_L)dx \\
&\leq k\alpha\frac{\partial P_m}{\partial m} + \frac{k}{2m}\int_\beta^\infty (p_L - \frac{\beta}{m}) f(x, m, p_L)dx \\
&= k\alpha\frac{\partial P_m}{\partial m} + \frac{k}{2m}(p_L - \frac{cq}{r_H - r_L})\int_\beta^\infty f(x, m, p_L)dx \\
&\leq 0
\end{aligned}$$

The last inequality follows because $\frac{\partial P_m}{\partial m} \leq 0$ when $c \geq \frac{p_L(r_H - r_L)}{q}$ from the fact that P_m is decreasing in m in this range.

Now we consider partially traceable networks. If $qc \geq \dfrac{p_L(r_H - r_L)}{3(t/n)q}$ then this implies $m \geq \dfrac{kp(r_H - r_L)}{3cqt}$. We will prove this result by considering two cases. Let $\alpha \equiv \dfrac{cqt}{k(r_H - r_L)}$. Here, $\beta^{RP} = \dfrac{qcm(t/n)}{r_H - r_L} = \dfrac{cqm^2 t}{k(r_H - r_L)} = \alpha m^2$.

Case 1. If $\dfrac{kp_L(r_H - r_L)}{3cqt} < m < \dfrac{kP_L(r_H - r_L)}{cqt}$

$$\mathbf{E_m} = \mathbf{E}[\sum_{i=1}^n X_i^m] = \sum_{i=1}^n \mathbf{E}[X_i^m] = n\mathbf{E}[X_i^m] = \frac{k}{m}\int_\beta^\infty x f(x, m, p_L)dx = kp - \frac{k}{m}\int_0^\beta x f(x, m, p_L)dx$$

$$\begin{aligned}
\frac{\partial \mathbf{E_m}}{\partial m} &= \frac{k}{m^2}\int_0^\beta xf(x,m,p_L)dx - \frac{k}{m}\frac{\partial\Big(\int_0^\beta xf(x,m,p_L)dx\Big)}{\partial m}\\
&= \frac{k}{m^2}\int_0^\beta xf(x,m,p_L)dx - \frac{k}{m}\Big(\beta\frac{\partial}{\partial m}(\int_0^\beta f(x,m,p_L)dx) + \int_0^\beta \frac{(y+mp)f(y,m,p_L)}{2m}dy\Big)\\
&= -\frac{k}{m}\beta\frac{\partial(1-P_m)}{\partial m} + \frac{k}{2m}\int_0^\beta(\frac{x}{m}-p_L)f(x,m,p_L)dx\\
&\le -\frac{k}{m}\beta\frac{\partial(1-P_m)}{\partial m} + \frac{k}{2m}(\frac{\beta}{m}-p_L)\int_0^\beta f(x,m,p_L)dx\\
&= -\frac{k}{m}\beta\frac{\partial(1-P_m)}{\partial m} + \frac{k}{2m}(\frac{cqtm}{k(r_H-r_L)}-p_L)\int_0^\beta f(t,m,p_L)dt\\
&\le 0
\end{aligned}$$

The second equality follows from lemma O.3. The last inequality follows from the assumption that $\frac{kp_L(r_H-r_L)}{3cqt} < m < \frac{kp_L(r_H-r_L)}{cqt}$ and the fact that P_m is decreasing in m in this range.

Case 2. If $m \ge \dfrac{kp_L(r_H-r_L)}{cqt}$ Following the same procedure as above, we get

$$\begin{aligned}
\frac{\partial \mathbf{E_m}}{\partial m} &= k\beta\frac{\partial P_m}{\partial m} + \frac{k}{2m}\int_\beta^\infty(p_L-\frac{x}{m})f(x,m,p_L)dx\\
&\le k\beta\frac{\partial P_m}{\partial m} + \frac{k}{2m}\int_\beta^\infty(p_L-\frac{\beta}{m})f(x,m,p_L)dx\\
&= k\beta\frac{\partial P_m}{\partial m} + \frac{k}{2m}(p_L-\frac{cqmt}{k(r_H-r_L)})\int_\beta^\infty f(x,m,p_L)dx\\
&\le 0
\end{aligned}$$

The last inequality follows from the assumption that $m > \dfrac{kp(r_H-r_L)}{cqt}$ and the fact that P_m is decreasing in m in this range.

Proof of Proposition 2B

Under imperfect testing with preemptive EMA, we have from Theorem 2 that x^{PV^*} can either be the solution to the first order equation or at the boundaries. We will show that $x_{n+1}^{PV^*} \ge x_n^{PV^*}$ for all n in all these cases.

Consider the case when $x_n^{PV^*}$ is the solution to the FOC and $t=n$. Then from theorem 2, we have $-h'(x_n^{PV^*})/x_n^{PV^*} = q((n+1)/n)c/(r_H-r_L)$. If $x_{n+1}^{PV^*}$ is a solution to the FOC then $-h'(x_{n+1})/x_{n+1} = q((n+2)/(n+1))c/(r_H-r_L)$. Let $F(x,n) \equiv xq((n+1)/(n))c/(r_H-r_L)+h'(x)$. Note that $F(x_n^{PV^*},n)=0$ and $F(x_{n+1}^{PV^*},n+1)=0$ and $F(0,n)<0$ since h is a decreasing function. Since h is a convex function, $F(x,n)$ is montonically increasing in x. Further, $F(x_{n+1}^{PV^*},n) = c/(r_H-r_L)x_{n+1}^{PV^*}q((n+1/n)-(n+2)/(n+1)) = c/(r_H-r_L)x_{n+1}q(1/(n(n+1)))>0$ using the definition of F and the fact that $F(x_{n+1}^{PV^*},n+1)=0$. Since $F(x,n)$ is montonically increasing in x, and $F(x_n^{PV^*},n)=0<F(x_{n+1},n)$, we have $x_{n+1}^{PV^*}\ge x_n^{PV^*}$ in this case as well. If $t<n$ then letting $F(x,n)\equiv xq(t/n)((n+1)/n)c/(r_H-r_L)+h'(x)$ and redoing the analysis yields the same result. If $x_{n+1}^{PV^*}=1$ then since $x_n^{PV^*}\le 1$, we again have $x_{n+1}^{PV^*}\ge x_n^{PV^*}$. Finally, if $x_n^{PV^*}=1$ then since $-h'(1)(r_H-r_L)/[q(t/n)((n+1)/n)]$ is increasing in n, $x_{n+1}^{PV^*}$ will be 1 for $n+1$ as well.

Next we show the results for reactive EMA. We prove that $\dfrac{\partial P_n}{\partial n}\ge 0$ by showing that $\dfrac{\partial P_m}{\partial m}\le 0$. From lemma O.5 we only need to show that $\frac{\partial\beta_m}{\partial m}\le\frac{\beta+mp_L}{2m}$ in order to show $\dfrac{\partial P_m}{\partial m}\le 0$.

For fully traceable supply chains (t=n) and $\beta^{RV} = \frac{k(r_H-r_L)}{cq} - (\frac{k}{m}-1)\int_0^{\beta^{RV}} xf(x,m,p_L)dx$

Case 1: Assume $\beta^{RV}\le mp_L$.

$$\text{Let } g(\beta^{RV},m) = \frac{k(r_H-r_L)}{cq} - (\frac{k}{m}-1)\int_0^{\beta^{RV}} xf(x,m,p_L)dx - \beta^{RV}$$

$$\begin{aligned}
\frac{\partial g}{\partial m} &= \frac{k}{m^2}\int_0^{\beta^{RV}} xf(x,m,p_L)dx - (\frac{k}{m}-1)\frac{\partial}{\partial m}\Big(\int_0^{\beta^{RV}} xf(x,m,p_L)dx\Big)\\
&= \frac{k}{m^2}\int_0^{\beta^{RV}} xf(x,m,p_L)dx - \Big(\frac{k}{m}-1\Big)\Big(\beta^{RV}\frac{\partial}{\partial m}(\int_0^{\beta^{RV}} f(x,m,p_L)dx) + \int_0^{\beta^{RV}}\frac{(t+mp_L)f(t,m,p_L)}{2m}dt\Big)\\
&= (\frac{k}{m}-1)\beta^{RV}f(\beta^{RV},m,p_L)(\frac{\beta^{RV}+mp_L}{2m}) + \frac{k}{m^2}\int_0^{\beta^{RV}} tf(t,m,p_L)dt - (\frac{k}{m}-1)\int_0^{\beta^{RV}} f(t,m,p_L)(\frac{t+mp_L}{2m})dt
\end{aligned}$$

$$= (\frac{k}{m}-1)\beta^{RV} f(\beta^{RV},m,p_L)(\frac{\beta^{RV}+mp_L}{2m}) + \frac{k}{2m^2}\int_0^{\beta^{RV}} (t-mp)f(t,m,p_L)dt + \int_0^{\beta^{RV}} f(t,m,p_L)(\frac{t+mp_L}{2m})dt$$

$$\frac{\partial g}{\partial \beta^{RV}} = -1-(\frac{k}{m}-1)\beta^{RV} f(\beta^{RV},m,p_L)$$

$$\frac{\partial \beta^{RV}}{\partial m} = \frac{(\frac{k}{m}-1)\beta^{RV} f(\beta^{RV},m,p_L)(\frac{\beta^{RV}+mp_L}{2m}) + \frac{k}{2m^2}\int_0^{\beta^{RV}} (t-mp)f(t,m,p_L)dt + \int_0^{\beta^{RV}} f(t,m,p_L)(\frac{t+mp_L}{2m})dt}{1+(\frac{k}{m}-1)\beta^{RV} f(\beta^{RV},m,p_L)}$$

Since $\beta^{RV} \leq mp_L$ from assumption, we have

$$\frac{k}{2m^2}\int_0^{\beta^{RV}} (t-mp)f(t,m,p)dt + \int_0^{\beta^{RV}} f(t,m,p)(\frac{t+mp_L}{2m})dt \leq \int_0^{\beta^{RV}} f(t,m,p_L)(\frac{t+mp_L}{2m})dt$$

$$\leq (\frac{\beta^{RV}+mp_L}{2m})\int_0^{\beta^{RV}} f(t,m,p_L)dt \leq (\frac{\beta^{RV}+mp_L}{2m})$$

Adding $(\frac{k}{m}-1)\beta^{RV} f(\beta^{RV},m,p_L)(\frac{\beta^{RV}+mp_L}{2m})$ on both sides, we get

$$= (\frac{k}{m}-1)\beta^{RV} f(\beta^{RV},m,p_L)(\frac{\beta^{RV}+mp_L}{2m}) + \frac{k}{2m^2}\int_0^{\beta^{RV}} (t-mp_L)f(t,m,p_L)dt + \int_0^{\beta^{RV}} f(t,m,p_L)(\frac{t+mp_L}{2m})dt$$

$$\leq \frac{\beta^{RV}+mp_L}{2m}(1+(\frac{k}{m}-1)\beta^{RV} f(\beta^{RV},m,p_L))$$

$$= \frac{(\frac{k}{m}-1)\beta^{RV} f(\beta^{RV},m,p_L)(\frac{\beta^{RV}+mp_L}{2m}) + \frac{k}{2m^2}\int_0^{\beta^{RV}} (t-mp_L)f(t,m,p_L)dt + \int_0^{\beta^{RV}} f(t,m,p_L)(\frac{t+mp_L}{2m})dt}{1+(\frac{k}{m}-1)\beta^{RV} f(\beta^{RV},m,p_L)} \leq \frac{\beta^{RV}+mp_L}{2m}$$

This implies $\frac{\partial \beta^{RV}}{\partial m} \leq \frac{\beta^{RV}+mp}{2m}$. From lemma O.5 we get that since $\frac{\partial \beta^{RV}}{\partial m} \leq \frac{\beta^{RV}+mp}{2m}$, $\frac{\partial P_m}{\partial m} \leq 0$ as well.

Case 2: Assume $\beta^{RV} > mp_L$.

$$\beta^{RV} = \frac{k(r_H-r_L)}{cq} - (\frac{k}{m}-1)\int_0^{\beta^{RV}} xf(x,m,p_L)dx$$

$$= \frac{k(r_H-r_L)}{cq} - kp + mp_L + (\frac{k}{m}-1)(\frac{k}{m}-1)(\int_{\beta^{RV}}^{\infty} xf(x,m,p_L)dx)$$

$$g(\beta^{RV},m) = \frac{k(r_H-r_L)}{cq} - kp_L + mp_L + (\frac{k}{m}-1)(\int_{\beta^{RV}}^{\infty} xf(x,m,p_L)dx) - \beta^{RV}$$

$$\frac{\partial g}{\partial m} = p_L + (\frac{k}{m}-1)\frac{\partial}{\partial m}\Big(\int_{\beta^{RV}}^{\infty} xf(x,m,p_L)dx\Big) - \frac{k}{m^2}(\int_{\beta^{RV}}^{\infty} xf(x,m,p_L)dx)$$

$$= p_L + (\frac{k}{m}-1)\Big(\beta^{RV}\frac{\partial}{\partial m}\Big(\int_{\beta^{RV}}^{\infty} f(x,m,p_L)dx\Big) + \int_{\beta^{RV}}^{\infty} \frac{t+mp_L)f(t,m,p_L)}{2m}dt\Big) - \frac{k}{m^2}(\int_{\beta^{RV}}^{\infty} xf(x,m,p_L)dx)$$

$$= p_L + (\frac{k}{m}-1)\beta^{RV} f(\beta^{RV},m,p_L)(\frac{\beta^{RV}+mp}{2m}) - \frac{k}{m^2}\int_{\beta^{RV}}^{\infty} tf(t,m,p_L)dt + (\frac{k}{m}-1)\int_{\beta^{RV}}^{\infty} f(t,m,p_L)(\frac{t+mp_L}{2m})dt$$

$$= p_L + (\frac{k}{m}-1)\beta^{RV} f(\beta^{RV},m,p_L)(\frac{\beta^{RV}+mp_L}{2m}) + \frac{k}{2m^2}\int_{\beta^{RV}}^{\infty} (mp_L-t)f(t,m,p_L)dt - \int_{\beta^{RV}}^{\infty} f(t,m,p_L)(\frac{t+mp_L}{2m})dt$$

$$\frac{\partial g}{\partial \beta^{RV}} = -1-(\frac{k}{m}-1)\beta^{RV} f(\beta^{RV},m,p_L)$$

$$\frac{\partial \beta^{RV}}{\partial m} = \frac{p_L + (\frac{k}{m}-1)\beta^{RV} f(\beta^{RV},m,p_L)(\frac{\beta^{RV}+mp_L}{2m}) + \frac{k}{2m^2}\int_{\beta^{RV}}^{\infty} (mp_L-t)f(t,m,p_L)dt - \int_{\beta^{RV}}^{\infty} f(t,m,p_L)(\frac{t+mp_L}{2m})dt}{1+(\frac{k}{m}-1)\beta^{RV} f(\beta^{RV},m,p_L)}$$

Since $\beta^{RV} > mp$ from assumption, we have

$$p_L + \frac{k}{2m^2}\int_{\beta^{RV}}^{\infty} (mp_L-t)f(t,m,p_L)dt - \int_{\beta^{RV}}^{\infty} f(t,m,p_L)(\frac{t+mp_L}{2m})dt \leq p_L = \frac{mp_L+mp_L}{2m} < \frac{\beta^{RV}+mp_L}{2m}$$

Adding $(\frac{k}{m}-1)\beta^{RV} f(\beta^{RV},m,p_L)(\frac{\beta^{RV}+mp_L}{2m})$ on both sides, we get

$$= p_L + (\frac{k}{m}-1)\beta^{RV} f(\beta^{RV},m,p_L)(\frac{\beta^{RV}+mp_L}{2m}) + \frac{k}{2m^2}\int_{\beta^{RV}}^{\infty} (mp_L-t)f(t,m,p_L)dt - \int_{\beta^{RV}}^{\infty} f(t,m,p_L)(\frac{t+mp_L}{2m})dt$$

$$\leq \frac{\beta^{RV}+mp_L}{2m}(1+(\frac{k}{m}-1)\beta^{RV} f(\beta^{RV},m,p_L))$$

$$= \frac{p_L + (\frac{k}{m}-1)\beta^{RV} f(\beta^{RV},m,p_L)(\frac{\beta^{RV}+mp_L}{2m}) + \frac{k}{2m^2}\int_{\beta^{RV}}^{\infty} (mp_L-t)f(t,m,p_L)dt - \int_{\beta^{RV}}^{\infty} f(t,m,p_L)(\frac{t+mp_L}{2m})dt}{1+(\frac{k}{m}-1)\beta^{RV} f(\beta^{RV},m,p_L)} \leq \frac{\beta^{RV}+mp_L}{2m}$$

This implies $\frac{\partial \beta^{RV}}{\partial m} \leq \frac{\beta^{RV}+mp_L}{2m}$. From lemma O.5 we get that since $\frac{\partial \beta^{RV}}{\partial m} \leq \frac{\beta^{RV}+mp_L}{2m}$, $\frac{\partial P_m}{\partial m} \leq 0$ as well.

Let β_f^{RV} be the threshold for fully traceable systems. For partially traceable systems, $\beta_p^{RV} = \frac{k^2(r_H-r_L)}{cqmt} - (\frac{k}{m}-1)\int_0^{\beta^{RV}} xf(x,m,p_L)dx = \frac{k^2(r_H-r_L)}{cqmt} + \beta_f^{RV} - \frac{k(r_H-r_L)}{cq}$. Then $\frac{\partial \beta_p^{RV}}{\partial m} = -\frac{k^2(r_H-r_L)}{cqm^2t} + \frac{\partial \beta_f^{RV}}{\partial m} < \frac{\beta^{RV}+mp_L}{2m}$. Thus for both partially and fully traceable systems P_n increases as n increases if β^{RV} is a solution to the fixed point equation.

Next, we will show that $\frac{\partial E_m}{\partial m} \leq 0$ to prove that $\frac{\partial E_n}{\partial n} \geq 0$. Let X_i^m be the total supply adulterated by supplier i=1,..,n when each supplier supplies m units. For fully traceable systems, $\beta = \frac{k(r_H-r_L)}{cq} - (\frac{k}{m}-1)\int_0^{\beta} xf(x,m,p_L)dx$. Let $\alpha = \frac{k(r_H-r_L)}{cq}$. Then $\int_0^{\beta} xf(x,m,p_L)dx = \frac{\alpha-\beta}{n-1}$ Then for fully traceable systems,

$$\begin{aligned}
&E_m = E[\sum_{i=1}^n X_i^m] = \sum_{i=1}^n E[X_i^m] = nE[X_i^m] = \frac{k}{m}\int_0^{\beta} xf(x,m,p_L)dx \\
&\frac{\partial E_m}{\partial m} = -\frac{k}{m^2}\int_0^{\beta} xf(x,m,p_L)dx + \frac{k}{m}\Big(\beta\frac{\partial}{\partial m}(\int_0^{\beta} f(x,m,p_L)dx) + \int_0^{\beta}\frac{(t+mp_L)f(t,m,p_L)}{2m}\Big) \\
&= -\frac{k}{m^2}\int_0^{\beta} xf(x,m,p_L)dx + \frac{k}{m}\beta\frac{\partial P_m}{\partial m} + \frac{k}{m}\int_0^{\beta} f(t,m,p)(\frac{t+mp_L}{2m})dt \\
&= -\frac{k}{2m^2}\int_0^{\beta} xf(x,m,p_L)dx + \frac{k}{m}\beta\frac{\partial P_m}{\partial m} + \frac{kp_L}{2m}\int_0^{\beta} f(t,m,p_L)dt \\
&= -\frac{k}{2m^2}\int_0^{\beta} xf(x,m,p_L)dx + \frac{k}{m}\beta\frac{\partial P_m}{\partial m} + \frac{kp_L}{2m}\int_0^{\beta} f(t,m,p_L)dt
\end{aligned}$$

Replacing the value of $\frac{\partial P_m}{\partial m}$ and $\frac{\partial \beta}{\partial m}$, we get that showing $\frac{\partial E_m}{\partial m} \leq 0$ is equivalent to showing:

$$0 \leq \beta^2 f(\beta,m,p_L) + mp\Big(\beta f(\beta,m,p_L) - \int_0^{\beta} f(x,m,p_L)dx\Big) + \Big(1-2\beta f(\beta,m,p_L)\Big)\int_0^{\beta} xf(x,m,p_L)dx \tag{O.13}$$

Note that $\int_0^{\beta} xf(x,m,p_L)dx = mp_L(1-p_L)f(\beta,m,p_L) + mp_L\int_0^{\beta} f(x,m,p_L)dx$ using integration by parts and the assumption that $\int_{-\infty}^{0} xf(x,m,p_L)dx = 0$. Replacing the value in equation O.13, and after some algebraic simplifications, we get that $\frac{\partial E_m}{\partial m} \leq 0$ is equivalent to showing:

$$0 \leq \beta\Big(\beta + mp_L - 2mp_L\int_0^{\beta} f(x,m,p_L)dx\Big) + mp_L(1-p_L)\Big(1-2\beta f(\beta,m,p_L)\Big) \tag{O.14}$$

$$\Leftrightarrow 0 \leq \beta + mp_L - 2mp_L\int_0^{\beta} f(x,m,p_L)dx + mp_L(1-p_L)\Big(\frac{1}{\beta} - 2f(\beta,m,p_L)\Big) \tag{O.15}$$

We will show that the inequalities hold by considering different cases:

Case 1: Assume $\beta \leq mp_L$.

(i) If $1-2\beta f(\beta,m,p_L) > 0$, then second term in equation O.14 is non negative. Since $\beta \leq mp_L$, $\int_0^{\beta} f(x,m,p_L)dx \leq 1/2$ and the first term is also non negative because $\beta + mp_L - 2mp_L\int_0^{\beta} xf(x,m,p_L)dx \geq \beta + mp_L - mp_L \geq 0$.

(ii) If $1-2\beta f(\beta,m,p_L) \leq 0$, then $1-2mp_L f(\beta,m,p_L) \leq 1-2\beta f(\beta,m,p_L) \leq 0$. Differentiating the RHS in equation O.15 w.r.t. β, we get $1-2mp_L f(\beta,m,p_L) - mp_L(1-p_L)\Big(\frac{1}{\beta^2} + 2f(\beta,m,p_L)\frac{(mp_L-\beta)}{mp_L(1-p_L)}\Big)$. Since $1-2mp_L f(\beta,m,p_L) < 0$ and $\beta \leq mp_L$, the differential is always negative and the expression is decreasing in β. Thus the smallest value of the RHS is when $\beta = min(mp,\beta^*)$ where β^* is the largest β s.t. $1-2\beta f(\beta,m,p_L) \leq 0$. Since $\beta f(\beta,m,p_L)$ is increasing in β for $\beta \leq mp_L$ and $mp_L > 5$ from assumption, it is easy to check that $1-2mp_L f(mp_L,m,p_L) > 0$ and $min(mp_L,\beta^*) = \beta^*$. Note that at β^*, $1-2\beta^* f(\beta^*,m,p_L) = 0$ since $\beta f(\beta,m,p_L)$ is increasing in β and we want the largest β s.t. $1-2\beta f(\beta,m,p_L) \leq 0$. Evaluating at β^*, the second term in equation O.15 is 0 while the first term is again non negative. Finally, since the RHS is non negative at β^*, it must be non negative for all $\beta < \beta^*$.

Case 2: Assume $\beta > mp_L$.

(i) If $1-2\beta f(\beta,m,p_L) \geq 0$, then the second term equation O.15 is non negative and the first term is increasing in β (this is because $1-2mp_L f(\beta,m,p_L) \geq 1-2\beta f(\beta,m,p_L) \geq 0$). Since the first term evaluated at $\beta = mp_L$ is positive, it must be positive for all $\beta > mp_L$.

(ii) If $1-2\beta f(\beta,m,p_L) < 0$, then since $\int_0^{\beta} xf(x,m,p_L)dx \leq mp_L$ and $\int_0^{\beta} f(x,m,p_L)dx \leq 1$, from equation O.13 we have that $\beta^2 f(\beta,m,p_L) + mp\Big(\beta f(\beta,m,p_L) - \int_0^{\beta} f(x,m,p_L)dx\Big) + \Big(1-2\beta f(\beta,m,p_L)\Big)\int_0^{\beta} xf(x,m,p_L)dx > \beta^2 f(\beta,m,p_L) + mp_L\Big(\beta f(\beta,m,p_L) - 1\Big) + \Big(1-2\beta f(\beta,m,p_L)\Big)mp_L = \beta f(\beta,m,p_L)(\beta - mp_L) \geq 0$.

For partially traceable systems let $\alpha = \frac{k^2(r_H - r_L)}{cqmt}$. Note that α is decreasing in m and since we have already shown that $\frac{\partial E_m}{\partial m} \le 0$ under full traceability (with a constant α), E_m must be decreasing in m in the partial traceability case as well.

Proof of Proposition 3

Under preemptive EMA with perfect testing, as $p_L^{\max}$ increases, it is straightforward to see that the threshold $n_a^* = \min\left\{n, \left\lceil \frac{(r_H - r_L)(p_L^{\max} - p_L^{\min})n^2}{cqt} - 1 \right\rceil\right\}$ also increases. Under imperfect testing, since $h(1) = p_L^{\min}$ stays constant and $h(x)$ is a convex decreasing function, this implies that $\dfrac{\partial h'(x)}{\partial p_L^{\max}} \le 0$ for all x. Thus, $-h'(1)(r_H - r_L)/[q(t/n)((n+1)/n)]$ increases as $p_L^{\max}$ increases and the range in which $x^{PV^*} = 1$ also increases.

Under reactive EMA, we will first prove the results for P_n. Note that we will use p and p_L interchngably in all the proofs that follow. Under perfect testing, $\beta^{RP} = \frac{cqtm^2}{k(r_H - r_L)}$. Thus $\frac{\partial \beta^{RP}}{\partial p} = 0$. Using lemma O.6, we have that $\frac{\partial(1-P_m)}{\partial p} \le 0$ if $\frac{\beta(1-2p)+mp}{2p(1-p)} \ge \frac{\partial \beta}{\partial p} = 0$. We will show that this is indeed the case for both fully and partially traceable networks. Under fully traceable networks, if $cq > (r_H - r_L)$ then $P_m = 0$ as $\beta^{RP} = m$. If $cq \le (r_H - r_L)$, $\frac{\beta(1-2p)+mp}{2p(1-p)} = \frac{m(cq(1-2p)+p(r_H - r_L))}{2p(1-p)(r_H - r_L)}$. If $p \le 0.5$ then $cq(1-2p) + p(r_H - r_L)$ is always non-negative. If $p > 0.5$ then $\frac{p(r_H - r_L)}{2p-1} - cq$ is again non-negative since $\frac{p}{2p-1} \ge 1$ and $r_H - r_L \ge cq$ from assumption.

For partially traceable networks, if $m > \frac{k(r_H - r_L)}{cqt}$ then $\beta^{RP} = m$ and $P_m = 0$. If $m \le \frac{k(r_H - r_L)}{cqt}$, $\frac{\beta(1-2p)+mp}{2p(1-p)} = \frac{m(cqmt(1-2p)+pk(r_H - r_L))}{2kp(1-p)(r_H - r_L)}$. If $p \le 0.5$ then $m(cqmt(1-2p) + pk(r_H - r_L))$ is always non-negative. If $p > 0.5$ then $\frac{kp(r_H - r_L)}{2p-1} - cqmt$ is again non-negative since $\frac{p}{2p-1} \ge 1$ and $k(r_H - r_L) \ge cqmt$ from assumption.

Under imperfect testing, the threshold is the solution to the following fixed point equation: $\beta^{RV} = \frac{k(r_H - r_L)}{cq} - (\frac{k}{m} - 1)\int_0^\beta x f(x, m, p_L)dx$ Let $g(\beta, p) = \frac{k(r_H - r_L)}{cq} - (\frac{k}{m} - 1)\int_0^\beta x f(x, m, p_L)dx - \beta$ for fully traceable systems and $g(\beta, p) = \frac{k^2(r_H - r_L)}{cqmt} - (\frac{k}{m} - 1)\int_0^\beta x f(x, m, p_L)dx - \beta$ for partially traceable systems.

$$
\begin{aligned}
\frac{\partial g}{\partial p} &= -(\frac{k}{m} - 1)\frac{\partial\Big(\int_0^\beta x f(x, m, p_L)dx\Big)}{\partial p} \\
&= -(\frac{k}{m} - 1)\frac{\partial\Big(\beta\int_0^\beta f(x, m, p_L)dx\Big)}{\partial p} - \frac{\partial\Big(\int_0^\beta\int_0^t f(x, m, p_L)dxdt\Big)}{\partial p} \\
&= (\frac{k}{m} - 1)\Big(\frac{\beta f(\beta, m, p)(\beta(1-2p) + mp)}{2p(1-p)} - \frac{\int_0^\beta (t - 2tp + mp) f(t, m, p)dt}{2p(1-p)}\Big) \\
\frac{\partial g}{\partial \beta} &= -1 - (\frac{k}{m} - 1)\beta f(\beta, m, p) \\
\frac{\partial \beta}{\partial p} &= \frac{(\frac{k}{m} - 1)\Big(\frac{\beta f(\beta, m, p)(\beta(1-2p)+mp)}{2p(1-p)} - \frac{\int_0^\beta (t-2tp+mp) f(t,m,p)dt}{2p(1-p)}\Big)}{1 + (\frac{k}{m} - 1)\beta f(\beta, m, p)}
\end{aligned}
$$

Note that we have $-(\frac{k}{m} - 1)\Big(\int_0^\beta (t - 2tp + mp) f(t, m, p)dt\Big) \le \beta(1-2p) + mp$. This is because if $0 < p < 0.5$ then RHS is always positive while LHS is always negative and the inequality is satisfied. If $1 > p \ge 0.5$ then

$$
\begin{aligned}
& -(\frac{k}{m} - 1)\Big(\int_0^\beta (t - 2tp + mp) f(t, m, p)dt\Big) = (\frac{k}{m} - 1)\Big(\int_0^\beta (t(2p-1) - mp) f(t, m, p)dt\Big) \\
& \le (\frac{k}{m} - 1)\Big((\beta(2p-1) - mp)\int_0^\beta f(t, m, p)dt\Big) \le (\frac{k}{m} - 1)\Big(\beta(2p-1) - mp\Big) \\
& \le 0 \le mp + \beta(1-2p)
\end{aligned}
$$

Adding $(\frac{k}{m} - 1)\frac{\beta f(\beta, m, p)(\beta(1-2p)+mp)}{2p(1-p)}$ on both sides, we get

$$
\begin{aligned}
& (\frac{k}{m} - 1)\Big(\frac{\beta f(\beta, m, p)(\beta(1-2p) + mp)}{2p(1-p)} - \frac{\int_0^\beta (t - 2tp + mp) f(t, m, p)dt}{2p(1-p)}\Big) \\
& \le \frac{\beta(1-2p) + mp}{2p(1-p)}(1 + (\frac{k}{m} - 1)\beta f(\beta, m, p)) \\
& = \frac{(\frac{k}{m} - 1)\Big(\frac{\beta f(\beta, m, p)(\beta(1-2p)+mp)}{2p(1-p)} - \frac{\int_0^\beta (t-2tp+mp) f(t,m,p)dt}{2p(1-p)}\Big)}{1 + (\frac{k}{m} - 1)\beta f(\beta, m, p)} \le \frac{\beta(1-2p) + mp}{2p(1-p)}
\end{aligned}
$$

Using lemma O.6, we have that $\frac{\partial P_m}{\partial p} \le 0$ since $\frac{\beta(1-2p)+mp}{2p(1-p)} \ge \frac{\partial \beta}{\partial p}$.

Next we prove results for E_n. Under perfect testing, $\frac{\partial E_m}{\partial p} = \frac{k}{m}\frac{\partial \int_{\beta RP}^{\infty} xf(x,m,p_L)dx}{\partial p}$.

$$\begin{aligned}
\frac{\partial \int_{\beta RP}^{\infty} xf(x,m,p_L)dx}{\partial p} &= \frac{\partial\Big(\int_0^{\beta}\int_{\beta}^{\infty} f(x,m,p_L)dxdt + \int_{\beta}^{\infty}\int_t^{\infty} f(x,m,p_L)dxdt\Big)}{\partial p} \\
&= \int_0^{\beta} \frac{\partial\Big(\int_{\beta}^{\infty} f(x,m,p_L)dx\Big)}{\partial p}dt + \int_{\beta}^{\infty} \frac{\partial\Big(\int_t^{\infty} f(x,m,p_L)dx\Big)}{\partial p}dt \\
&= \int_0^{\beta} f(\beta,m,p)\frac{\beta(1-2p)+mp}{2p(1-p)}dt + \int_{\beta}^{\infty} f(t,m,p)\frac{t(1-2p)+mp}{2p(1-p)}dt \\
&= \beta f(\beta,m,p)\frac{\beta(1-2p)+mp}{2p(1-p)} + \int_{\beta}^{m} f(t,m,p)\frac{t(1-2p)+mp}{2p(1-p)}dt
\end{aligned}$$

If $p<.5$ then both the terms in the last equation are positive and we have that $\frac{\partial E_n}{\partial n} \geq 0$. If $p>0.5$, then the first term is positive since we have already shown $\frac{\partial P_n}{\partial p} = f(\beta,m,p)\frac{\beta(1-2p)+mp}{2p(1-p)} \geq 0$ under perfect testing. Further, $\int_{\beta}^{m} f(t,m,p)\frac{t(1-2p)+mp}{2p(1-p)}dt \geq \frac{-m(2p-1)+mp}{2p(1-p)}\int_{\beta}^{m} f(t,m,p)dt = \frac{m}{2p}\int_{\beta}^{m} f(t,m,p)dt \geq 0$ and again we have $\frac{\partial E_n}{\partial p} \geq 0$.

Proof of Theorem O.1

Let us assume that a subset n_a out of a total of n farms are adulterating. Consider a farm i that is adulterating at equilbrium. Then, revenue from adulteration (π_a^i) should be strictly greater than revenue from non adulteration (π_{na}^i). $\pi_a^i = r_H m(1-p_L^{min}) + r_L m p_L^{min} - q\frac{n_a}{n}(t/n)cm - \lambda\overline{F}(\gamma,m,p_L^{\min})$ and $\pi_{na}^i = r_H m(1-p_L^{max}) + r_L m p_L^{max} - \lambda\overline{F}(\gamma,m,p_L^{\max})$. Now consider a farmer i' who is not adulterating at equilbrium. Then, $\pi_{na}^{i'} \geq \pi_a^{i'}$. $\pi_a^{i'} = r_H m(1-p_L^{min}) + r_L m p_L^{min} - q\frac{n_a+1}{n}(t/n)cm - \lambda\overline{F}(\gamma,m,p_L^{\min})$ and $\pi_{na}^{i'} = r_H m(1-p_L^{max}) + r_L m p_L^{max} - \lambda\overline{F}(\gamma,m,p_L^{\max})$. Simplfying the two conditions we get the desired result in the theorem.

Proof of Theorem O.2

Differentiating the revenue function for risk averse farmers w.r.t x, we get:

$$\frac{\partial \pi_L^{PP}(x)}{\partial x} = -mh'(x)(r_H-r_L) - q\frac{t}{n}cm(\frac{\sum_{-i} x_{-i}^* + 2x_i}{n}) - h'(x_i)\frac{\lambda m f(\gamma m,m,p)(\gamma(1-2h(x_i))+h(x_i))}{2h(x_i)(1-h(x_i))}$$

$$\begin{aligned}
\frac{\partial^2 \pi_L^{PV}(x_i,x_{-i})}{\partial x_i^2} = {}& -mh''(x)(r_H-r_L) - q\frac{t}{n}cm(\frac{2}{n}) - h''(x_i)\frac{\lambda m f(\gamma m,m,p)(\gamma(1-2h(x_i))+h(x_i))}{2h(x_i)(1-h(x_i))} \\
& - h'(x_i)^2\lambda m f(\gamma m,m,p)\frac{-\gamma+h(x)(2\gamma+h(x)(1-2\gamma))}{2(1-h(x))^2h(x)^2}
\end{aligned}$$

Note that the first three terms are less than 0 because of the convexity of h(x), and the last term is again less than 0 because $-\gamma+h(x)(2\gamma+h(x)(1-2\gamma) \geq -\gamma+p_L^{\min}(2\gamma+p_L^{\min}(1-2\gamma) \geq 0$ from the assumption that $\gamma \leq \dfrac{p_L^{\min}}{1-2p_L^{\min}(1-p_L^{\min})}$. Since $\pi_L^{PV}(x_i,x_{-i})$ is a concave function and we are considering symmetric equilibrium, replacing $x_-^*i = x*i = x_L^{PV^*}$ gives us the desired result.

Proof of Proposition O.1

Let $T^O = n^2\frac{(R_H-r_L)(p_L^{max}-p_L^{min})}{cqt}$. From Theorem 1 we have that $n_a^* = \min\{n, \lceil T^O - 1\rceil\}$. From lemma O.6 we have that $\overline{F}(\gamma,m,p)$ is increasing in p since $\gamma m(1-2p)+mp$ is positive for all p . This implies that $\lambda(\overline{F}(\gamma,m,p_L^{\max}) - \overline{F}(\gamma,m,p_L^{\min}))$ is positive and $T^{RA} > T$. Since $n_a^{RA^*} = \min\{n, \lceil T^{RA}-1\rceil\}$ and $T^{RA} > T$ we have that $n_a^{RA^*} > n_a^*$. Similarly, since $(\gamma(1-2h(x))+h(x))$ is positive for all x, we again have that $\lambda f(\gamma m,m,h(x))\frac{(\gamma(1-2h(x))+h(x))}{2h(x)(1-h(x))}$ is positive. Following the same logic as above, we have that $x_{RA}^{PV^*} > x^{PV^*}$.

Proof of Proposition O.2

Define $F(x) \equiv x + h'(x)\dfrac{(r_H-r_L)+\lambda f(\gamma m,m,h(x))\frac{(\gamma(1-2h(x))+h(x))}{2h(x)(1-h(x))}}{[cq(t/n)((n+1)/n)]}$. Since $\gamma \leq \dfrac{p_L^{\min}}{1-2p_L^{\min}(1-p_L^{\min})}$ by assumption, the function is concave and we have that $\dfrac{\partial F(x)}{\partial n} \leq 0$. Note that $[cq(t/n)((n+1)/n)]$ is decreasing in n and $\dfrac{\partial f(\gamma m,m,h(x))}{\partial m} = -f(\gamma m,m,p)\dfrac{m(\gamma-p)^2+p(1-p)}{2mp(1-p)} < 0$. This implies that $\dfrac{\partial F(x)}{\partial n} > 0$. By implicit function theorem, $\dfrac{\partial x_{RA}^{pv^*}}{\partial n} = -\dfrac{(\partial F(x))/(\partial n)}{(\partial F(x))/(\partial x_{RA}^{pv^*})} \geq 0$.

Proof of Theorem O.3

For part (i), it is easy to check that farms' payoff from adulteration is $r_H m - q(t/n)cm$ and non adulteration is $r_L m$. Comparing the two payoffs gives us the desired result in the theorem.

For part (ii), note that expected payoff for a farm from adulteration is $mr_H - q(t/n)\dfrac{(m+m(n-1)p_L^{max}p_{ad})}{k}cm$ and from non adulteration is mr_L. For part (a), It is easy to check that if $q(t/n)c \leq \dfrac{(r_H-r_L)n}{p_L^{max}(n-1)+1}$, then even if all other farms always adulterate (i.e, $p_{ad}=1$), it is optimal for the given farm to also always adulterate. Similarly, for part (c), it is easy to check that if $(r_H - r_L)n \le q(t/n)c$, then even if none of the other farms adulterate (i.e., $p_{ad}=0$), it is optimal for the given farm to also not adulterate. For part (b), it is easy to find p_{ad} using the fact that the expected payoff from adulteration should be equal to that of non adultration at equilibrium.

Proof of Proposition O.3

Checking that E_n is increasing in n is equivalent to checking that p_{ad} is increasing in n because $E_n = nmp_L^{max}p_{ad} = kp_L^{max}p_{ad}$. First, consider the case when $t=n$. Consider n_1 and n_2 such that $n_2 > n_1$. We will show that $p_{ad}^{n_2} > p_{ad}^{n_1}$ for all n_1 and n_2. If $(r_H - r_L)n \le qc$ then both $p_{ad}^{n_1} = p_{ad}^{n_2} = 0$ from theorem O.3. Next, if both n_1 and n_2 are such that $cq \in [\dfrac{(r_H - r_L)n_i}{p_L^{max}(n_i-1)+1}, (r_H - r_L)n_i]$ for $i=1,2$, then it is easy to check that $\dfrac{\partial p_{ad}}{\partial n} \ge 0$. Since $n_2 > n_1$, this implies that $p_{ad}^{n_2} \ge p_{ad}^{n_1}$. Finally, note that since $\dfrac{(r_H - r_L)n}{p_L^{max}(n-1)+1}$ is increasing in n, either $qc \le \dfrac{(r_H - r_L)n_i}{p_L^{max}(n_i-1)+1}$ for both $i=1,2$ or $\dfrac{(r_H - r_L)n_1}{p_L^{max}(n_1-1)+1} \le qc \le \dfrac{(r_H - r_L)n_2}{p_L^{max}(n_2-1)+1}$. In both the cases it is easy to check that $p_{ad}^{n_2} \ge p_{ad}^{n_1}$. The proof for partial traceability follows similarly and we avoid redoing it here for the sake of brevity.

Proof of Theorem O.4

Let x_{n_L} be the fraction of n_L low quality units adulterated at equilibrium and let $a_i(n_{L,i}) : \{1,\dots,m\}$ be farm i's adulteration strategy. then revenue for a farm with n_L low quality units is given by $\pi_F^{RV} = (m - n_L + n_L x_{n_L})r_H + (n_L - x_{n_L})r_L - \gamma_i(n_{L,i}, a_{-i}(n_{L,-i}))cm$, where $\gamma_i(n_{L,i}, a_{-i}(n_{L,-i})) \equiv q\left(\frac{t}{n}\right)\left(\frac{x_{n_{L,i}}}{m}\right)\left(\frac{x_{n_{L,i}} + \mathbb{E}_{n_{L,-i}}\left[\sum_{-i} n_{L,-i}a_{-i}(n_{L,-i})\right]}{k}\right)$. Note that $\dfrac{\partial \pi_F^{RV}}{\partial x_{n_l}} = n_L(r_H - r_L) - cq(t/n)(2n_L^2 x_{n_L} + n_L \mathbb{E}_{n_{L,-i}}\left[\sum_{-i} n_{L,-i}a_{-i}(n_{L,-i})\right])$ and $\dfrac{\partial \pi_F^{RV}}{\partial^2 x_{n_l}^2} = -cq(t/n)(2n_L^2) \le 0$. Since the revenue function is concave, the optimal value of $x_{n_L}^*$ is either at the boundary points or at the FOC. Thus $x_{n_L}^* = max\{0, min\{1, \dfrac{1}{2n_L}\Big(\dfrac{kn(r_H - r_L)}{cqt} - \mathbb{E}_{n_{L,-i}}\left[\sum_{-i} n_{L,-i}a_{-i}(n_{L,-i})\right]\Big)\Big)\}\}$. Further, note that the optimal value of $x_{n_L}^* n_L$ is a constant if the optimal solution is not at the boundary. Rewriting the condition, we get that if $2n_L \le \Big(\dfrac{kn(r_H - r_L)}{cqt} - \mathbb{E}_{n_{L,-i}}\left[\sum_{-i} n_{L,-i}a_{-i}(n_{L,-i})\right]\Big)\Big)$ then $x^*(n_L) = 1$ and otherwise it is $\dfrac{1}{2n_L}\Big(\dfrac{kn(r_H - r_L)}{cqt} - \mathbb{E}_{n_{L,-i}}\left[\sum_{-i} n_{L,-i}a_{-i}(n_{L,-i})\right]\Big)\Big)$ as long as $\dfrac{kn(r_H - r_L)}{cqt} \ge \mathbb{E}_{n_{L,-i}}\left[\sum_{-i} n_{L,-i}a_{-i}(n_{L,-i})\right]\Big)$. Since we are considering symmetric BNE, we can rewrite the above FOC to determine the threshold β as $2\beta = \dfrac{nk(r_H - r_L)}{cqt} - (n-1)\Big(\int_0^\beta x f(x, \frac{k}{n}, p_L)dx + \int_\beta^m \beta f(x, \frac{k}{n}, p_L)dx\Big)$. Note that $max_\beta \left\{\int_0^\beta x f(x, \frac{k}{n}, p_L)dx + \int_\beta^m \beta f(x, \frac{k}{n}, p_L)dx\right\} = mp$ and it is attained when $\beta \ge m$. If $\beta = m$ is a symmetric BNE, then given that $\beta_{-i} = m$, we should get that $\beta_i = m$ as well. If $\dfrac{1}{2}\Big(\dfrac{nk(r_H - r_L)}{cqt} - (n-1)mp\Big) \ge m$ then this implies that $\beta_i = m$. This proves part (b) of the theorem. If $\dfrac{1}{2}\Big(\dfrac{nk(r_H - r_L)}{cqt} - (n-1)mp\Big) < m$ then we can use the intermediate value theorem and the fact that the fixed point equation is monotonic in β to show the existence and uniqueness of β as we did in Theorem 4.

Proof of Proposition O.4

Proceeding in the same way as in proposition 2 first for fully traceable systems, we get that proving $\dfrac{\partial E_m}{\partial m} \le 0$ is equivalent to showing the following:

$$0 \le \int_0^\beta f(x,m,p)dx\Big(\int_0^\beta f(x,m,p)dx + \int_\beta^m \beta f(x,m,p)dx - mp\Big) + \int_\beta^m \beta f(x,m,p)dx \tag{O.16}$$

$$\Leftrightarrow 0 \le \Big(\int_0^\beta f(x,m,p)dx\Big)\Big(\int_\beta^m (\beta - x)f(x,m,p_L)dx\Big) + \int_\beta^m \beta f(x,m,p)dx \tag{O.17}$$

Differentiating the RHS w.r.t to β, we get:

$$-f(\beta,m,p)\Big(\int_\beta^m (x-\beta)f(x,m,p_L)dx + \beta\Big) + 1 - \Big(\int_0^\beta f(x,m,p)dx\Big)^2 \tag{O.18}$$

Differentiating equation O.18, we get:

$$-f(\beta,m,p)\Big(2\int_0^\beta f(x,m,p)dx + \int_0^\beta f(x,m,p)dx\Big) - \Big(\int_\beta^m (x-\beta)f(x,m,p_L)dx + \beta\Big)f(\beta,m,p)\frac{mp-\beta}{mp(1-p)} \tag{O.19}$$

We will again consider two cases:

Case 1:Assume $\beta \le mp$: We wish to show that the minimum value of the RHS in equation O.17 for any $\beta \le mp$ is greater than 0. Note that the first term in equation O.18 is always non positive. Further, since $\int_\beta^m (x-\beta)f(x,m,p_L)dx + \beta$ is increasing in β and it is non-negative at $\beta=0$ it must always be non negative for $\beta \geq 0$. If $\beta \leq mp$ then $\dfrac{mp-\beta}{mp(1-p)}$ is also non negative and the second term is also non positive. Thus since the double differential of the RHS is negative the function is concave in β. Since we are trying to find the minimum value of a concave function, we only need to check the boundary points. It is easy to check that equation O.17 is non negative when evaluated at $\beta=0$ and $\beta=mp$. Thus it must be non negative at all $\beta \in (0,mp)$.

Case 2: Assume $\beta > mp$: Replacing the value of $\int_0^\beta xf(x,m,p)dx = mp(1-p)f(\beta,m,p) + mp\int_0^\beta f(x,m,p)dx$ in equation O.16, and after some algebraic simplifications, we get that $\frac{\partial E_m}{\partial m} \leq 0$ is equivalent to showing:

$$0 \leq \Big(\beta - mp\Big)\int_0^\beta f(x,m,p)dx \int_\beta^m f(x,m,p)dx + \beta\int_\beta^m f(x,m,p)dx + mp(1-p)f(\beta,m,p)dx\int_0^\beta f(x,m,p)dx \tag{O.20}$$

Note that the second and the third term in equation O.20 are always non negative. If $\beta > mp$ then the first term is also non negative and thus RHS must be non negative when $\beta > mp$.

For partially traceable systems, since the $\frac{\partial E_m}{\partial m} \leq 0$ under full traceability, E_m must be decreasing in m in the partial traceability case as well. This is because partial traceability can only increase risk since traceability factor decreases as n increases.

Proof of Theorem O.5

At NE, farm i chooses x_i^* so as to maximize his expected payoff given the choices of all other farms. Note that $\frac{\partial \pi^{PV}(x_i,x_{-i})}{\partial x_i} = -mh'(x)(r_H - r_L) - q\dfrac{t}{n}cmS'(x_i)\left(\dfrac{\sum_{-i} x_{-i} + 2S(x_i)}{n}\right)$. It is easy to check that this is a concave function. Further, since we consider only symmetric equilibrium, we have $x_{-i}^* = x_i^* = x_{PV^*}$. Substituting this, and noting that x cannot be greater than 1 gives us the desired result.

Note that in the second case there are two regions of the S_2 function. We will compare the optimal solutions in both the regions and pick the one which is optimal. For the first region i.e. $x \leq \tau$, differentiating farmers' revenue w.r.t. x gives us a first order condition equivalent to one in theorem O.5. Since it is again a concave function, the optimal solution is obtained either at the first order condition or at the boundary. If $x > \tau$, revenue function increases linearly since the detection probability stays constant while the revenue from adulteration increases linearly. This implies that if the optimal solution in the first region is at the boundary, the revenue is always increasing and $x^{PV^*} = 1$ is optimal. This proves the first part. Otherwise we have two points to compare: x^* of the first order condition and $x^* = 1$. Using the symmetry condition, we get that if $r_H m - cmq(t/n) > mr_H(1-h(x^*) + mr_L h(x^*) - cmq(t/n)S_2(x^*)^2$, $x^{PV^*} = 1$ is optimal, otherwise $x^{PV^*} = 1$ is optimal. This proves the second part of the equation.

Because of the piecewise linear nature of the testing sensitivity in S_3, we need to consider four scenarios depending on where the adulterants in mixed and individual supply lie respectively. For example, if $\sum x_j/n \leq 1/\alpha$ then detection probability in the aggregated supply is $\alpha \sum x_j/n$ and it is 1 otherwise. Similarly, for farmer i, if $x_i \leq 1/\alpha$ then detection probability for individual sample is $x_i\alpha$ and it is 1 otherwise. Since we are considering symmetric equilibrium, $x_j = x^*$ for all j at equilibrium. Thus, either both $\sum x^*/n$ and x^* are less than $1/\alpha$ or they are both greater than $1/\alpha$. The revenue for farmer i if $x_i \leq 1/\alpha$ is given by $\pi_i^{ip} = mr_H - mh(x)(r_H - r_L) - cmq(t/n)\frac{\sum_{-i} x_{-i} + x_i}{n} x_i \alpha^2$. Similarly if $x_i > 1/\alpha$, then $\pi_i^p = mr_H - mh(x)(r_H - r_L) - cmq(t/n)$. It is easy to check that π_i^{ip} is concave in x_i. Using the fact that $x_i = x_{-i}$ for a symmetric equilibrium, we get that the optimal soultion is either at the first order condition or at the boundary points. Checking that the value of $\frac{\partial \pi_i^{ip}}{\partial x_i}$ at $1/\alpha$ is positive simplifies to $c < -h'(1/\alpha)(r_H - r_L)/[\alpha q(t/n)((n+1)/n)]$. Next, note that π_i^p is increasing in x thus $x^* = 1$ is a local optimal in this region. Thus while the revenue function is concave in the first region it is monotonically increasing in the second region. Comparing the optimal solution in the first and the second region gives us the desired result in the theorem.

Proof of Theorem O.6

We prove the first part when detection probability is modeled as S_1. It is optimal for a farm to adulterate only if revenue with adulteration is greater than revenue without adulteration i.e.

$$mr_H - q\left(\frac{t}{n}\right)S_1\left(\frac{n_L}{m}\right)\mathbb{E}_{n_{L,-i}}\left[S_1\left(\frac{n_L + \sum_{-i} n_{L,-i}a_{-i}(n_{L,-i})}{k}\right)\right]cm \geq mr_H - n_L(r_H - r_L) \tag{O.21}$$

$$\Leftrightarrow \frac{n(r_H - r_L)}{qtcm} \geq \frac{1}{n_L}S_1\left(\frac{n_L}{m}\right)\mathbb{E}_{n_{L,-i}}\left[S_1\left(\frac{n_L + \sum_{-i} n_{L,-i}a_{-i}(n_{L,-i})}{k}\right)\right] \tag{O.22}$$

Notice that the LHS in equation O.22 does not depend on n_L. If we show that the RHS is increasing in n_L, then the LHS and RHS can only be equal at a unique point. Next, we will show that the RHS is indeed increasing in n_L.

$$\frac{\partial\left(\frac{1}{n_L}S_1\left(\frac{n_L}{m}\right)\right)}{\partial n_L} \geq 0 \Leftrightarrow \frac{n_L}{m}S_1'\left(\frac{n_L}{m}\right) - S\left(\frac{n_L}{m}\right) \geq 0 \tag{O.23}$$

$$\frac{\partial\left(n_L m S_1'\left(\frac{n_L}{m}\right) - S_1'\left(\frac{n_L}{m}\right)\right)}{\partial n_L} = \frac{n_L}{m^2}S_1''\left(\frac{n_L}{m}\right) \geq 0 \tag{O.24}$$

The last inequality in equation O.24 follows from the convexity of S. Since equation O.24 is always non negative, it implies that the LHS in O.23 is increasing in n_L and we only need to evaluate the smallest n_L to check that it is always non negative. Since $S(0)=0$ and S is an increasing function, LHS in equation O.23 evaluated at $n_L=0$ is indeed non negative. Thus we have that $\frac{1}{n_L}S\left(\frac{n_L}{m}\right)$ is increasing in n_L. Finally note that since S(.) is an increasing function and $\frac{n_{L,i}+\sum_{-i}n_{L,-i}a_{-i}(n_{L,-i})}{k}$ increases as n_L increases, $\mathbb{E}_{n_{L,-i}}\left[S\left(\frac{n_L+\sum_{-i}n_{L,-i}a_{-i}(n_{L,-i})}{k}\right]\right)$ is also increasing in n_L. Since both the terms in the RHS of equation O.22 are increasing in n_L, their product must also increase as n_L increases. Thus, since there is a unique point at which the LHS is equal to RHS, and the RHS is increasing in n_L, farm's adulteration strategy is indeed a threshold strategy with farms adulterating only when there low quality units are less than a threshold. We characterize the threshold under a symmetric BNE. If $\beta^S = m$, under a symmetric BNE, then given $\beta^S_{-i}=m$, $\beta^S=m$ should be optimal. This implies equation O.22 evaluated at $\beta^S=m$ should hold true i.e. $\dfrac{n(r_H-r_L)}{qtcm} \geq \dfrac{1}{m}\mathbb{E}_{n_{L,-i}}\left[S\left(\frac{m+\sum_{-i}n_{L,-i}}{k}\right]\right)$. Simplifying this gives us the desired result in part (b) of the theorem. If $\dfrac{n(r_H-r_L)}{qtcm} < \dfrac{1}{m}\mathbb{E}_{n_{L,-i}}\left[S\left(\frac{m+\sum_{-i}n_{L,-i}}{k}\right]\right)$, then using the fact that all farms have the same threshold at which condition in O.22 is satisfied at equality under the symmetric case, we can rewrite the condition in O.22 as follows: $\dfrac{n(r_H-r_L)}{qtcm} = \frac{1}{\beta^S}S\left(\frac{\beta_S}{m}\right)\mathbb{E}_{n_{L,-i}}\left[S\left(\frac{\beta_S+\sum_{-i}n_{L,-i}\mathbb{I}\{n_{L,-i}<\beta_S\}}{k}\right]\right)cm$. We will use the Intermediate Value Theorem to show the existence of β^S. Define $F(\beta)\equiv \frac{1}{\beta^S}S\left(\frac{\beta_S}{m}\right)\mathbb{E}_{n_{L,-i}}\left[S\left(\frac{\beta_S+\sum_{-i}n_{L,-i}\mathbb{I}\{n_{L,-i}<\beta_S\}}{k}\right]\right)cm - \dfrac{n(r_H-r_L)}{qtcm}$ for $\beta\in$ (0,m). Note that $F(0) = -\frac{n(r_H-r_L)}{cqtm} < 0$ and $F(m) = \dfrac{1}{m}\mathbb{E}_{n_{L,-i}}\left[S\left(\frac{m+\sum_{-i}n_{L,-i}}{k}\right]\right) - \dfrac{n(r_H-r_L)}{qtcm} > 0$ from assumption. Since $F(.)$ is continuous and $F(0)<0<F(m)$, there exists $\beta^{RV}\in$ (0,m) s.t. the condition is satisfied at equality. Finally, since the RHS in the equation is monotonic in β^S, the fixed point is also unique.

Next, we characterize the optimal solution in the case when detection probability is modeled by S_2. It is optimal for a farmer to adulterate only if revenue with adulteration is greater than revenue without adulteration i.e.

$$mr_H - q\left(\frac{t}{n}\right)S_2\left(\frac{n_L}{m}\right)\mathbb{E}_{n_{L,-i}}\left[S_2\left(\frac{n_L+\sum_{-i}n_{L,-i}a_{-i}(n_{L,-i})}{k}\right]\right)cm \geq mr_H - n_L(r_H-r_L) \tag{O.25}$$

$$\Leftrightarrow n_L\frac{n(r_H-r_L)}{qtcm} \geq S_2\left(\frac{n_L}{m}\right)\mathbb{E}_{n_{L,-i}}\left[S_2\left(\frac{n_L+\sum_{-i}n_{L,-i}a_{-i}(n_{L,-i})}{k}\right]\right) \tag{O.26}$$

We characterize the threshold under a symmetric BNE. If $\beta^S=m$, under a symmetric BNE, then given $\beta^S_{-i}=m$, $\beta^S=m$ should be optimal. This implies equation O.26 evaluated at $\beta^S=m$ should hold true i.e. $\dfrac{n(r_H-r_L)}{qtcm} \geq \dfrac{1}{m}\mathbb{E}_{n_{L,-i}}\left[S_2\left(\frac{m+\sum_{-i}n_{L,-i}}{k}\right]\right)$. Simplifying this gives us the desired result in part (b) of the theorem.

If $\dfrac{n(r_H-r_L)}{qtcm} < \dfrac{1}{m}\mathbb{E}_{n_{L,-i}}\left[S_2\left(\frac{m+\sum_{-i}n_{L,-i}}{k}\right]\right)$: Using the fact that RHS is convex increasing initially and a constant finally and LHS is linearly increasing, we have that the LHS in equation O.26 can be equal to RHS only at most two points (the first point is when LHS is convex increasing and the second point is when LHS is a constant). We will show that both β^l and β^u cannot be less than τm using contradiction. Assume it is indeed the case β^l and β^u are less than τm. Using equation O.26, this implies that $\dfrac{n(r_H-r_L)}{qtcm} = \dfrac{S_2\left(\frac{\beta^l}{m}\right)\mathbb{E}_{n_{L,-i}}\left[S_2\left(\frac{\beta^l+\sum_{-i}n_{L,-i}a_{-i}(n_{L,-i})}{k}\right]\right)}{\beta^l} = \dfrac{S_2\left(\frac{\beta^u}{m}\right)\mathbb{E}_{n_{L,-i}}\left[S_2\left(\frac{\beta^l+\sum_{-i}n_{L,-i}a_{-i}(n_{L,-i})}{k}\right]\right)}{\beta^u}$. For all $n_l\in(\beta^l,\tau m)$, note that $S_2(\frac{n_L}{m})$ is convex increasing and $\mathbb{E}_{n_{L,-i}}[.]$ is also increasing. This implies that $\dfrac{S_2\left(\frac{n_l}{m}\right)\mathbb{E}_{n_{L,-i}}\left[S_2\left(\frac{n^l+\sum_{-i}n_{L,-i}a_{-i}(n_{L,-i})}{k}\right]\right)}{n^l}$ is increasing for all $n_l\in(\beta^l,\tau m)$ and

$\dfrac{S_2\left(\frac{\beta_l}{m}\right)\mathbb{E}_{n_{L,-i}}\left[S_2\left(\frac{\beta^l+\sum_{-i}n_{L,-i}a_{-i}(n_{L,-i})}{k}\right)\right]}{\beta^l} < \dfrac{S_2\left(\frac{\beta_u}{m}\right)\mathbb{E}_{n_{L,-i}}\left[S_2\left(\frac{\beta^u+\sum_{-i}n_{L,-i}a_{-i}(n_{L,-i})}{k}\right)\right]}{\beta^u}$. This is a contradiction and thus we have shown that both β^l and β^u cannot be less than τm simultaneously.

Finally, we consider the case when detection probability is modeled as S_3. We will first show that when $cq \leq max\Big(\dfrac{r_H-r_L}{\alpha}, \dfrac{k(r_H-r_L)}{\alpha(p\alpha(k-m)+m)}\Big)$ then always adulterating is indeed an equilibirum. Let us first consider the case when $\dfrac{r_H-r_L}{\alpha} < \dfrac{k(r_H-r_L)}{\alpha(p\alpha(k-m)+m)}$. This implies $\dfrac{m}{\alpha} \leq \dfrac{k}{\alpha} - (\dfrac{k}{m}-1)mp$ because $\alpha p \leq 1$. If $n_L \leq \dfrac{m}{\alpha}$, then the probability of detection in both mixed (p_m) (in expectation) and individual supply (p_i) is less than 1. This is because $p_i = \dfrac{n_L\alpha}{m} \leq 1$ and $p_m = \dfrac{\alpha(n_L+(\frac{k}{m}-1)mp)}{k} \leq 1$ for all $n_L \leq \dfrac{m}{\alpha}$. It is easy to check that revenue from adulteration is more than revenue from non-adulteration when $n_L = \dfrac{m}{\alpha}$. Further, note that the adulteration strategy when $n_L \leq \dfrac{m}{\alpha}$ is to adulterate below a threshold as already proved in theorem 4. Thus, if it is optimal to adulterate when $n_L = \dfrac{m}{\alpha}$ then it should indeed be optimal to adulterate for all $n_L \leq \dfrac{m}{\alpha}$. Next, if $\dfrac{m}{\alpha} < n_L < \dfrac{k}{\alpha} - (\dfrac{k}{m}-1)mp$, then while $p_i = 1$, $p_m = \dfrac{\alpha(n_L+(\frac{k}{m}-1)mp)}{k}$. It is easy to check that in this range, it is optimal to adulterate only when the number of low quality units are greater than threshold. However, since we have already shown that it is optimal to adulterate at $n_L = \dfrac{m}{\alpha}$, then it should indeed be optimal to adulterate for all $\dfrac{m}{\alpha} < n_L \leq \dfrac{k}{\alpha} - (\dfrac{k}{m}-1)mp$. Finally, note that if $n_L > \dfrac{k}{\alpha} - (\dfrac{k}{m}-1)mp$, then $p_i = 1$ and $p_m = 1$. This is similar to the prefect testing case we have already analyzed and the adulteration strategy in perfect testing case is to adulterate above a threshold. Since adulteration is indeed optimal at $\dfrac{k}{\alpha} - (\dfrac{k}{m}-1)mp$, it should be optimal for all $n_L \geq \dfrac{k}{\alpha} - (\dfrac{k}{m}-1)mp$. Thus we have shown that it in indeed optimal for this farm to always adulterate when all other farms are adulterating. The proof for the other case when $\dfrac{r_H-r_L}{\alpha} > \dfrac{k(r_H-r_L)}{\alpha(p\alpha(k-m)+m)}$ follows similarly. Next, note that if $cq \in \Big(max(\dfrac{r_H-r_L}{\alpha}, \dfrac{k(r_H-r_L)}{\alpha(p\alpha(k-m)+m)}), max(\dfrac{k(r_H-r_L)}{m\alpha}, r_H-r_L)\Big)$, then as in the previous case, it is optimal to adulterate till a threshold (β^S) when $n_L \leq tr \equiv min(\dfrac{k}{\alpha} - \mathbb{E}_{n_{L,-i}}\left[\sum_{-i} n_{L,-i}a_{-i}(n_{L,-i})\right], \dfrac{m}{\alpha})$ as the detection probability in this range is similar to the imperfect tetsting case we have already analyzed. It is again easy to check that if $n_L > tr$, it is optimal to adulterate above a threshold (β^U). Further, note that β^U has to be smaller than $\dfrac{cqm(t/n)}{r_H-r_L}$ because we already know from theorem 3 that even under perfect testing, which has the highest risk of being penalized, farms still adulterate when $n_L \geq \dfrac{cqm(t/n)}{r_H-r_L}$. The two conditions imply that in any symmetric BNE, equilibrium adulteration strategy is to adulterate below a threshold (β^S) and then adulterate above a threshold (β^U). The exact values of the thresholds in the theorem follow from the symmetric BNE assumption and the constraints that $\beta^S \geq 0$ and $\beta^U \leq min(m, \dfrac{cqm(t/n)}{r_H-r_L})$.

Proof of Theorem O.7

First, we prove the result for part (i). Farm i chooses x_i^* so as to maximize his expected payoff $\pi^{PP} = mr_H h(x) + mr_L(1-h(x)) - cm^2xq(t/n)$. Let us assume that a subset n_a out of a total of n farms are adulterating. Consider a farm i that is adulterating at equlibrium. Let x^* be the optimal amount of adulterants for farms that are adulterating. Then, revenue from adulteration (π_a^i) should be strictly greater than revenue from non adulteration (π_{na}^i). $\pi_a^i = r_H m - mh(x^*)(r_H-r_L) - q\frac{n_a}{n}(t/n)cm^2x^*$ and $\pi_{na}^i = r_H m - mh(0)(r_H-r_L)$. This is equivalent to checking that $\frac{\partial \pi_a^{i'}}{\partial x}$ at $x=0$ is greater than 0. Now consider a farmer i' who is not adulterating at equilibrium. Then, $\pi_{na}^{i'} \geq \pi_a^{i'}$. $\pi_a^{i'} = r_H m - mh(x^*)(r_H-r_L) - q\frac{n_a+1}{n}(t/n)cm^2x^*$ and $\pi_{na}^{i'} = r_H m - mh(0)(r_H-r_L)$. This is equivalent to checking that $\frac{\partial \pi_{na}^{i'}}{\partial x}$ at $x=0$ is less than 0. Simplfying the two conditions we get, $\frac{nh'(0)(r_H-r_L)}{cmq(t/n)} - 1 \leq n_a < \frac{nh'(0)(r_H-r_L)}{cmq(t/n)}$. Next, we also know that $n_a^* \leq n$. Thus we need to have $\frac{-h'(0)(r_H-r_L)n^2}{mcqt} - 1 < n$. Simplifying this condition gives us the condition on c for part (a). Condition for part (b) follows similarly from the condition on n_a^*. Finally, in both the cases optimal x^{PP^*} can be calculated using the fact that $\pi_a = mr_H - mh(x)(r_H-r_L) - cm^2q(t/n)x$ is concave in x, and the optimal x^* is either the solution to the first order condition or at the boundary point.

The proof for the second part follows exactly in the same way as in theorem 2 with cm replaced with cm^2x. At an NE, farm i chooses x_i^* so as to maximize his expected payoff in Equation (2) given the choices of all other farms. Note that $\frac{\partial \pi^{PV}(x_i,x_{-i})}{\partial x_i} = -mh'(x)(r_H-r_L) - q\dfrac{t}{n}cm(\dfrac{2x_i\sum_{-i}x_{-i}+3x_i^2}{n})$ and $\frac{\partial^2 \pi^{PV}(x_i,x_{-i})}{\partial x_i^2} = -mh''(x)(r_H-r_L) - q\dfrac{t}{n}cm(\dfrac{2\sum_{-i}x_{-i}+6x_i}{n}) < 0$. Further, since we consider only symmetric equilibrium, we have $x_{-i}^* = x_i^* = x_{PV*}$. Substituting this, we get that $\pi^{PV}(x_i, x_{-i})$ is a concave function and the optimal solution is achieved either at the first order condition or at the boundary.

For the third part, farms adulterate if $r_H m - q(t/n)cmn_L > r_H m - n_L(r_H - r_L)$. This simplifies to $r_H - r_L > q(t/n)cm$.

For the last part, the proof for showing that any optimal strategy in this game is a threshold policy follows exactly as in Theorem 4 and we omit it here for brevity. Using the fact that all farms have the same threshold β, we have that $\mathbb{E}_{n_{L,-i}}\left[\sum_{-i} n_{L,-i}a_{-i}(n_{L,-i})\right] = (n-1)\int_0^{\beta^{RV}} xf(x,m,p_L)dx$. Note that $max_\beta \left\{\int_0^\beta xf(x,m,p_L)dx\right\} = mp_L$ and it is attained when $\beta \geq m$. If $\beta = m$ is a symmetric BNE, then given that $\beta_{-i} = m$, we should get that $\beta_i = m$ as well. This happens when $\pi(a, \beta_{-i} = m) > \pi(na, \beta_{-i} = m)$, because it implies that, given that all other farms have $\beta_{-i} = m$, $\beta_i = m$ is optimal for farm i. This condition simplifies to $cq\left(\frac{t}{n}\right) < \frac{k(r_H - r_L)}{m(m + (n-1)mp)}$, . This proves part (b). If $cq\left(\frac{t}{n}\right) \geq \frac{k(r_H - r_L)}{m(m + (n-1)mp)}$, it is again easy to show the existence of β^{RV} using the Intermediate Value Theorem and the fact that the threshold should be the same for all farms in a symmetric equilibrium as in Theorem 4 .

Proof of Proposition O.5

It is easy to check that all the results for both reactive and preemptive adulteration follow exactly as in the above proofs with cm replaced by $cf(m)$ where $f(m)$ is a convex increasing function. We omit the proofs here for brevity.

Proof of Theorem O.8

We will first write the risk constraint in terms of qt for all the problems. For instance, the risk constraint in the original problem for reactive scenarios under perfect testing can be written as $P_n \equiv \int_0^{\frac{\beta^{RP} - mp}{\sqrt{mp(1-p)}}} f(x,0,1)dx \leq \alpha$. This constraint can be equivalently written as $qt \geq u^{RP} \equiv \left(\frac{n^2(r_H - r_L)}{ck}\right)\left(\frac{pk}{n} + \phi^{-1}(1-\alpha)\sqrt{\frac{kp(1-p)}{n}}\right)$ using the fact that $\beta^{RP} = \frac{cqtm^2}{k(r_H - r_L)}$. In all the cases, we can similarly rewrite the risk constraint in terms of qt. Since $q \leq 1$ and $t \leq n$, qt cannot be greater than n. Thus if $u^j > n$ in any of the cases, then the problem becomes infeasible. This proves the first part of the proposition. Now if $u^j \leq n$, then note that since cost is increasing in both q and t and P_m decreases as q or t increases, the risk constraint will be strict in all the cases and $qt = u^j$ at optimality. We can thus replace t in the problem as $\frac{u^j}{q}$ and rewrite the problem as

$$\Pi^j(q,t) \equiv \min_q \left\{ l(q) + g(\frac{u^j}{q}) \,\middle|\, q \in [0, \frac{u_j}{n}], \right\}, \tag{O.27}$$

Note that $\frac{\partial(l(q)+g(\frac{u_a}{q}))}{\partial q} = l'(q) - \frac{u_j^j}{q^2}g'(\frac{u^j}{q})$ and $\frac{\partial^2(l(q)+g(\frac{u_j}{q}))}{\partial q^2} = l''(q) + \frac{u_j^2}{q^4}g''(\frac{u_j}{q}) + 2\frac{u_j}{q^3}g'(\frac{u_j}{q}) > 0$ since l and g are convex increasing functions. This problem is just a constrained convex optimization problem. Thus the optimal solution is either at the point where $\frac{\partial(l(q)+g(\frac{u_a}{q}))}{\partial q} = l'(q) - \frac{u_j^j}{q^2}g'(\frac{u^j}{q}) = 0$ or at the boundary points i.e. $q = 1$ or $q = \frac{mu_1}{k}$. There are three cases:

(i) Case 1: If $l'(1) - u^j g'(u^j) < 0$ then setting q to 1 is optimal since cost is always decreasing in $q \in [\frac{u^j}{n}, 1]$. Thus $(q^*, t^*) = (1, u_1)$

(ii) Case 2: If $g'(n) \leq \frac{u^j}{n^2}l'\left(\frac{u^j}{n}\right)$, then setting q to be minimum is optimal since cost is always increasing in $q \in [\frac{u^j}{n}, 1]$. Thus $(q^*, t^*) = (\frac{u^j}{n}, n)$

(iii) Case 3: If $l'(1) > g'(u^j)$ and $g'(n) > \frac{u^j}{n^2}l'\left(\frac{u^j}{n}\right)$ there exists q where $f'(q) - \frac{u_1}{q^2}g'(\frac{u_1}{q}) = 0$ and that q is optimal. Simplifying, we get $q^* = \sqrt{\frac{u^j g'(u^j/q^*)}{l'(q^*)}}$ and $t^* = \frac{u^j}{q^*}$.

Proof of Proposition O.6

From Theorem O.8, we know that if $\frac{u^j}{n} > 1$, then the manufacturer's problem with n suppliers is infeasible. It is easy to check that $\frac{u^j}{n}$ is increasing in n for all cases and we present the proof only for $\frac{u^{RV}}{n}$ here for the sake of brevity. Define $d = cmp + c\phi^{-1}(\alpha)\sqrt{mp(1-p)} + (\frac{k}{m} - 1)c\int_0^\beta xf(x,m,p_L)dx$. Then $\frac{u^{RV}}{n} = \frac{k(r_H - r_L)}{d}$ using the fact that $m = \frac{k}{n}$. Define $h = p + \phi^{-1}(\alpha)\frac{\sqrt{p(1-p)}}{2\sqrt{m}}$We will show that $\frac{\partial d}{\partial m} \geq 0$.

$$\begin{aligned}
\frac{\partial d}{\partial m} &= p + \phi^{-1}(\alpha)\frac{\sqrt{p(1-p)}}{2\sqrt{m}} - \frac{k}{m^2}\int_0^\beta xf(x,m,p_L)dx + (\frac{k}{m} - 1)\frac{\partial}{\partial m}\Big(\int_0^\beta xf(x,m,p_L)dx\Big) \\
&= h - \frac{k}{m^2}\int_0^\beta xf(x,m,p_L)dx + \Big(\frac{k}{m} - 1\Big)\Big(\beta\frac{\partial}{\partial m}(\int_0^\beta f(x,m,p_L)dx) + \int_0^\beta \frac{(t+mp)f(t,m,p)}{2m}dt\Big) \\
&= h - \frac{k}{m^2}\int_0^\beta xf(x,m,p_L)dx + \Big(\frac{k}{m} - 1\Big)\Big(\beta\frac{\partial \alpha}{\partial m} + \int_0^\beta \frac{(t+mp)f(t,m,p)}{2m}dt\Big)
\end{aligned}$$

$$
\begin{aligned}
&= h - \frac{k}{m^2}\int_0^{\beta} x f(x,m,p_L)dx + \Big(\frac{k}{m}-1\Big)\Big(\int_0^{\beta}\frac{(t+mp)f(t,m,p)}{2m}dt\Big)\\
&= h + \frac{k}{2m^2}\int_0^{\beta}(mp-x)f(x,m,p_L)dx - \int_0^{\beta}\frac{(t+mp)f(t,m,p)}{2m}dt\\
&\geq h - \frac{\beta+mp}{2m} + \frac{k}{2m^2}\int_0^{\beta}(mp-x)f(x,m,p_L)dx\\
&= h - \frac{\phi^{-1}(\alpha)\sqrt{mp(1-p)}+2mp}{2m} + \frac{k}{2m^2}\int_0^{\beta}(mp-x)f(x,m,p_L)dx\\
&= p + \phi^{-1}(\alpha)\frac{\sqrt{p(1-p}}{2\sqrt{m}} - \frac{\phi^{-1}(\alpha)\sqrt{mp(1-p)}+2mp}{2m} + \frac{k}{2m^2}\int_0^{\beta}(mp-x)f(x,m,p_L)dx\\
&= \frac{k}{2m^2}\int_0^{\beta}(mp-x)f(x,m,p_L)dx\\
&\geq 0
\end{aligned}
$$

The last inequality follows because $\phi^{-1}(\alpha) < 0$ for $\alpha < 0.5$ and thus $\beta \leq mp$. Finally, since d is increasing in m, d is decreasing in n.

Now let us assume that the manufacturer's problem is infeasible for SC_L, then $\frac{u^j_{n_L}}{n_L} > 1$ by Theorem O.8. But since $n_H > n_L$ and $\frac{u^j_n}{n}$ is increasing in n, we have that $\frac{u^j_{n_H}}{n_H} \geq \frac{u^j_{n_L}}{n_L} > 1$. This proves the first part of the proposition. Similarly if we assume that the manufacturer's problem is feasible for SC_H, then $\frac{u^j_{n_H}}{n_H} < 1$ by Theorem O.8 and we have that $\frac{u^j_{n_L}}{n_L} \leq \frac{u^j_{n_H}}{n_H} < 1$. This proves the second part of the proposition.

Next, we prove the third part of the proposition. From Theorem O.8 we also know that for any value of u^j_n there are three possibilities for optimal solutions. We will consider n_H and n_L and compare optimal solutions for all the nine possible cases. Let u_{n_H} and u_{n_L} be the value of the constant u^j_n for n_H and n_L respectively. Note that $u_{n_L} < u_{n_H}$ since $\frac{\partial u^j}{\partial n} \leq 0$. We will show that in all the cases $q^*_{n_L} \leq q^*_{n_H}$ and $t^*_{n_L} \leq t^*_{n_H}$.

Let's first assume that $(q^*_{n_H}, t^*_{n_H}) = (1, u_{n_H})$. There are three possible solutions for n_L.

(i) Case 1: $(q^*_{n_L}, t^*_{n_L}) = (1, u_{n_L})$ then since $u_{n_L} < u_{n_H}$ we have $q^*_{n_L} = q^*_{n_H}$ and $t^*_{n_L} < t^*_{n_H}$.

(ii) Case 2: $(q^*_{n_L}, t^*_{n_L}) = (\frac{u_{n_L}}{n_L}, n_L)$ If $u_{n_H} \geq n_L$ then we are done since $q^*_{n_L} \leq q^*_{n_H}$ and $t^*_{n_L} \leq t^*_{n_H}$. We will show that $u_{n_H} < n_L$ is not possible by contradiction. Assume that $u_{n_H} < n_L$. If $(q^*_{n_L}, t^*_{n_L}) = (\frac{u_{n_L}}{n_L}, n_L)$ then this implies $l'(\frac{u_{n_L}}{n_L}) - \frac{n_L^2}{u_{n_L}} g'(n_L) > 0$ from Theorem O.8. Also since l is convex and $\frac{u_{n_L}}{n_L} \leq 1$ from feasibility conditions, we have $l'(1) \geq l'(\frac{u_{n_L}}{n_L})$. Similarly since g is also convex, $u_{n_H} < n_L$ and $1 \leq \frac{n_L}{u_{n_L}}$, we have $\frac{n_L^2}{u_{n_L}} g'(n_L) > u_{n_H} g'(u_{n_H})$. This implies $l'(1) - u_{n_H} g'(u_{n_H}) > l'(\frac{u_{n_L}}{n_L}) - \frac{n_L^2}{u_{n_L}} g'(n_L) > 0$. This is a contradiction because we have $l'(1) - u_{n_H} g'(u_{n_H}) < 0$ from the assumption that $(q^*_m, t^*_m) = (1, u_{n_H})$.

(iii) Case 3: $(q^*_{n_L}, t^*_{n_L}) = (\sqrt{\frac{u_{n_L} g'(u_{n_L}/q^*)}{l'(q^*)}}, \frac{u_{n_L}}{q^*})$. We have that $q^*_{n_L} \leq q^*_{n_H} = 1$. We need to check if $t^*_{n_L} = \frac{u_{n_L}}{q^*} \leq u_{n_H}$. Let $h = \frac{u_{n_L}}{u_{n_H}}$. Note that if we show $l'(h) - \frac{u_{n_L}}{h^2} g'(\frac{u_{n_L}}{h}) \leq 0$ then $q^*_{n_L} \geq h$ because we are minimizing a convex function whose derivative is less than 0 at h. We have $l'(h) - \frac{u_{n_L}}{h^2} g'(\frac{u_{n_L}}{h}) \leq 0$ because $l'(h) - \frac{u_{n_L}}{h^2} g'(\frac{u_{n_L}}{h}) = l'(\frac{u_{n_L}}{u_{n_H}}) - \frac{u^2_{n_H}}{u_{n_L}} g'(u_{n_H}) < l'(1) - u_{n_H} g'(u_{n_H}) < 0$. The first inequality follows because $h < 1$ and the second inequality follows from the assumption that $(q^*_m, t^*_m) = (1, u_{n_H})$.

Now let's assume that $(q^*_{n_H}, t^*_{n_H}) = (\frac{u_{n_H}}{n_H}, n_H)$. We will consider all the three possible optimal solutions for n_L.

(i) Case 1: Let's assume $(q^*_{n_L}, t^*_{n_L}) = (1, u_{n_L})$, then $l'(1) - u_{n_L} g'(u_{n_L}) < 0 < l'(\frac{u_{n_H}}{n_H}) - \frac{n_H^2}{u_{n_H}} g'(n_H)$. Also note that $u_{n_H} \leq n_H$ and $u_{n_L} \leq n_L < n_H$ from feasibility conditions. This implies $l'(1) \geq l'(\frac{u_{n_H}}{n_H})$ and $u_{n_L} g'(u_{n_L}) < \frac{n_H^2}{u_{n_H}} g'(n_H)$. Thus $l'(1) - u_{n_L} g'(u_{n_L}) > l'(\frac{u_{n_H}}{n_H}) - \frac{n_H^2}{u_{n_H}} g'(n_H)$ which is a contradiction.

(ii) Case 2: $(q^*_{n_L}, t^*_{n_L}) = (\frac{m' u_{n_L}}{k}, n_L)$. We have already shown that $\frac{\partial \frac{u^j_n}{n}}{\partial n} \geq 0$ and this implies $\frac{u_{n_L}}{n_L} \leq \frac{u_{n_H}}{n_H}$. We also have $n_L < n_H$ by construction.

(iii) Case 3: $(q^*_{n_L}, t^*_{n_L}) = (\sqrt{\frac{u_{n_L} g'(u_{n_L}/q^*)}{l'(q^*)}}, \frac{u_{n_L}}{q^*})$. We need to show $q^*_{n_L} \leq \frac{u_{n_H}}{n_H}$ and $t^*_{n_L} \leq n_H$. We already have $t^*_{n_L} \leq n_L < n_H$. We also have $l'(\frac{u_{n_H}}{n_H}) - \frac{n_H^2}{u_{n_H}} g'(n_H) > 0$ because $(q^*_m, t^*_m) = (\frac{u_{n_H}}{n_H}, n_H)$. Let $h = \frac{u_{n_H}}{n_H}$. Then $h \leq 1$ because the problem is feasible for m. Since $n_H \geq 1$ and $u_{n_L} < u_{n_H}$ we have that $\frac{u_{n_L}}{h^2} g'(\frac{u_{n_L}}{h}) \leq \frac{n_H^2}{u_{n_H}} g'(n_H)$. This implies $l'(h) - \frac{u_{n_L}}{h^2} g'(\frac{u_{n_L}}{h}) > l'(\frac{u_{n_H}}{n_H}) - \frac{n_H^2}{u_{n_H}} g'(n_H) > 0$ and thus $q^*_{n_L} < \frac{u_{n_H}}{n_H}$.

Now let's assume that $(q^*_{n_H}, t^*_{n_H}) = (\sqrt{\frac{u_{n_H} g'(u_{n_H}/q^*)}{l'(q^*)}}, \frac{u_{n_H}}{q^*})$. . We will consider all three possible optimal solutions for m'.

(i) Case 1: Let's assume $(q^*_{n_L}, t^*_{n_L}) = (1, u_{n_L})$ then $l'(1) - u_{n_L} g'(u_{n_L}) < 0$. Further, we also have that $l'(q^*_{n_H}) - \frac{u_{n_H}}{q^*_{n_H}} g'(\frac{u_{n_H}}{q^*_{n_H}}) = 0$, $q^*_{n_H} \le 1$ and $u_{n_L} < u_{n_H}$. This implies $l'(1) \ge l'(q^*_{n_H})$ and $u_{n_L} g'(u_{n_L}) < \frac{u_{n_H}}{q^*_{n_H}} g'(\frac{u_{n_H}}{q^*_{n_H}})$. Combining the two we get that $l'(q^*_{n_H}) - \frac{u_{n_H}}{q^*_{n_H}} g'(\frac{u_{n_H}}{q^*_{n_H}}) < l'(1) - u_{n_L} g'(u_{n_L}) < 0$. This is a contradiction because we have that $l'(q^*_{n_H}) - \frac{u_{n_H}}{q^*_{n_H}} g'(\frac{u_{n_H}}{q^*_{n_H}}) = 0$. Thus this case is not possible.

(ii) Case 2: Let's assume $(q^*_{n_L}, t^*_{n_L}) = (\frac{u_{n_L}}{n_L}, n_L)$. We will prove that $q^*_{n_H} \ge \frac{u_{n_L}}{n_L}$ and $\frac{u_{n_H}}{q^*} \ge n_L$. We have already shown that $\frac{u_{n_L}}{n_L} \le \frac{u_{n_H}}{n_H}$. By feasibility, $q^*_{n_H} \ge \frac{u_{n_H}}{n_H} \ge \frac{u_{n_L}}{n_L}$. Further note that since $u_{n_H} > u_{n_L}$ we have that $l'(\frac{u_{n_H}}{n_L}) > l'(\frac{u_{n_L}}{n_L})$ and $\frac{n_L^2}{u_{n_H}} g'(n_L) < \frac{n_L^2}{u_{n_L}} g'(n_L)$. This implies that $l'(\frac{u_{n_H}}{n_L}) - \frac{n_L^2}{u_{n_H}} g'(n_L) > l'(\frac{u_{n_L}}{n_L}) - \frac{n_L^2}{u_{n_L}} g'(n_L) > 0$. The last inequality follows because we have assumed that $(q^*_{n_L}, t^*_{n_L}) = (\frac{u_{n_L}}{n_L}, n_L)$. Thus we have shown that $q^*_{n_H} < \frac{u_{n_H}}{n_L}$.This implies that $t^*_{n_H} = \frac{u_{n_H}}{q^*} \ge n_L$ as well.

(iii) Case 3: $(q^*_{n_L}, t^*_{n_L}) = (\sqrt{\frac{u_{n_L} g'(u_{n_L}/q^*)}{l'(q^*)}}, \frac{u_{n_L}}{q^*})$. Using implicit function theorem,

$$\frac{\partial q}{\partial m} = \frac{\frac{1}{q^2} \frac{\partial u^j}{\partial m} [\frac{u^j}{q} g''(\frac{u^j}{q}) + g'(\frac{u^j}{q})]}{l''(q) + 2\frac{u^j}{q^3} g'(\frac{u^j}{q}) + \frac{u^{j2}}{q^4} g'(\frac{u^j}{q})}$$

Since l and g are convex functions all the terms in this expression are positive except $\frac{\partial u^j}{\partial m}$ which we have already shown is non positive. Thus $\frac{\partial q}{\partial m} < 0$ and this implies that $q^*_{n_L} < q^*_{n_H}$. Next we will show that $\frac{u_{n_L}}{q^*_{n_L}} < \frac{u_{n_H}}{q^*_m}$. Since $u_{n_H} < u_{n_L}$ we have that $l'(q^*_m) < l'(\frac{u_{n_L} q^*_m}{u_{n_H}})$ and $\frac{u_{n_H}}{q^{*2}_m} g'(\frac{u_{n_H}}{q^*_m}) < \frac{u^2_{n_H}}{u_{n_L} q^{*2}_m} g'(\frac{u_{n_H}}{q^*_m})$. This implies that $l'(\frac{u_{n_L} q^*_m}{u_{n_H}}) - \frac{u^2_{n_H}}{u_{n_L} q^{*2}_m} g'(\frac{u_{n_H}}{q^*_m}) < l'(q^*_m) - \frac{u_{n_H}}{q^{*2}_m} g'(\frac{u_{n_H}}{q^*_m}) = 0$ Thus $q^*_{n_L} > \frac{u_{n_L} q^*_m}{u_{n_H}}$ which proves that $t^*_{n_L} < t^*_{n_H}$.

Thus we have shown that in all possible cases optimal q and t increases as n increases.

Proof of Proposition O.7

We will consider perfect testing case first. Let $(q^*_{n_H}, t^*_{n_H})$ be the optimal solutions for SC_{n_H} and $(q^*_{n_L}, t^*_{n_L})$ be the optimal solutions for SC_L. Note that $E_m = \frac{k}{m} \int_\beta^x f(x, m, p_L) dx$ and $\beta = \frac{q(t/n)cm}{(r_H - r_L)}$. There are two cases to consider:

(i) If $t^*_{n_H} \le n_L$ then $q^*_{n_H} \le 1$ then $(q^*_{n_H}, t^*_{n_H})$ is a feasible solution for SC_L as long as $E_{n_L}(q^*_{n_H}, t^*_{n_H}) \le \alpha$. Since $\alpha \le n \int_{\frac{mp}{3}} x f(x, m, p_L) dx$ we have that $\frac{q(t/n_L)cm}{(r_H - r_L)} \ge mp/3$. From proposition 2, this implies that $\frac{\partial E_n}{n} \ge 0$. Thus $E_{n_L}(q^*_H, t^*_H) \le E_{n_H}(q^*_{n_H}, t^*_{n_H}) \le \alpha$ and $(q^*_{n_H}, t^*_{n_H})$ is a feasible solution for SC_L. Since $(q^*_{n_L}, t^*_{n_L})$ is the optimal solution, this implies $\pi^{PP}_{n_L}(q^*_{n_L}, t^*_{n_L}) \le \pi^{PP}_{n_L}(q^*_{n_H}, t^*_{n_H}) \le \pi^{PP}_{n_H}(t^*_{n_H}, q^*_{n_H})$.

(ii) If $n_H \ge t^*_{n_H} > n_L$ then we have that $(q^*_{n_H}, t^*_{n_H})$ is infeasible for SC_L. We will prove by contradiction that we can find a $t' \le n_L$ s.t. $(q^*_{n_H}, t')$ is feasible for SC_L. Assume there does not exist a $t' < n_L$ s.t $(q^*_{n_H}, t')$ is feasible. This implies that $n_L \int_{\frac{ckq^*_{n_H}}{n_L(r_H - r_L)}} x f(x, k/n_L, p) dx > \alpha$ since $(q^*_{n_H}, n_L)$ is also infeasible. Further since $t^*_{n_H} \le n_H$ we also have that $n_H \int_{\frac{ckq^*_{n_H}}{n_H(r_H - r_L)}} x f(x, m, p_L) dx \le \alpha$. This implies that $E_{n_H} = n_H \int_{\frac{ckq^*_{n_H}}{n_H(r_H - r_L)}} x f(x, k/n_H, p_L) dx < n_L \int_{\frac{ckq^*_{n_H}}{n_L(r_H - r_L)}} x f(x, k/n_L, p) dx = E_{n_L}$. This is a contradiction since by Proposition 2 we know that $\frac{\partial E_n}{\partial n} \ge 0$. Thus we have that a feasible $(q^*_{n_H}, t')$ for SC_L exists. This implies $\pi_{n_L} \le l(q^*_{n_H}) + g(t') < f(q^*_{n_H}) + g(t^*_m) = \pi_{n_H}$.

The proof follows similarly for imperfect testing scenarios since we have already proved that $\frac{\partial E_n}{\partial n} \ge 0$ in proposition 2.

Proof of Proposition O.8

From theorem 1 and theorem 2 we already know farms' equilibrium adulteration strategy. Note that n^*_a can be equivalently interpreted as a mixed strategy equilibrium with each farm adulterating with probability n^*_a/n and not adulterating otherwise. The total (expected) amount of adulterants in the two cases are just kn^*_a/n and kx^{PV^*}. If $c < -h'(1)(r_H - r_L)/[q(t/n)(n+1)/n]$, then we have that $n^*_a = n$ and $x^{PV^*} = 1$ from theorem 1 and theorem 2. Thus all farms adulterate to the maximum level. The result for the remaining parts in the proposition follows similarly by comparing the two values under different values for c.

Proof of Theorem O.9

If $c < -h'(1)(r_H - r_L)/[q(t/n)(n+1)/n]$, then we have that all farms adulterate to the maximum level from theorem 2. This implies that the realized low quality likelihood for all farms is p_L^{min}. Since we have assumed $p_L^{min} = 0$, none of the farms will have any low quality units and the reactive EMA risk is 0. Thus $R^T_p = R^T_{ip}$. Next, note that if $c \ge (r_H - r_L)/[q(t/n)]\beta^{RP} = \dfrac{cqm(t/n)}{r_H - r_L} > m$ and reactive EMA risk is again 0 from theorem 3. The result in the theorem then follows directly from proposition O.8.