

Online Appendix for  
Optimal accounting rules, private benefits of  
control, and efficient liquidation  
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**A1 General security design framework**

We present a security design framework to show that our results of accounting rule design are robust to general security designs.

Instead of restricting the entrepreneur's choice to debt securities, we allow the entrepreneur to issue a general security characterized by a triple  $(c, s, M_I)$ . As in the main model,  $c : X \rightarrow \{E, I\}$  is a covenant that allocates the control right of the project at date 1 according to the signal realization  $x$ . The entrepreneur obtains the control right to decide whether to continue or terminate the project if  $c(x) = E$ , and the investor has the control right if  $c(x) = I$ . Note that the terminology "covenant" does not necessarily refer to a debt covenant. Even with equity securities, it is possible that equityholders may want to liquidate the firm (e.g., Chapter 7) when the accounting signal is sufficiently bad. It should be understood as a general rule that allocates the control right in the intermediate stage. If the project continues to the end, it generates a cash flow  $\theta$ , in which  $s(\theta)$  goes to the investor and  $\theta - s(\theta)$  remains with the entrepreneur. If the project is terminated in the intermediate stage, the investor receives  $M_I$ , and the entrepreneur receives  $M - M_I$ . The entrepreneur's security design is subject to the limited liability constraint. That is, the entrepreneur can only choose  $s$  and  $M_I$  from

$$S = \{s : [\underline{\theta}, \bar{\theta}] \rightarrow [0, \bar{\theta}] \mid s(\theta) \in [0, \theta] \text{ for all } \theta\},$$

and  $[0, M]$ , respectively.

Accordingly, an equilibrium is a set of rules  $\{(s, c, M_I), h, a\}$  such that

i) for any  $x \in X$ ,

$$a(x) \in \arg \max_{a \in \{0,1\}} a \cdot \left( \mathbf{E}^h [s(\theta) | x] - M_I \right) \quad \text{if } c(x) = I,$$

and

$$a(x) \in \arg \max_{a \in \{0,1\}} a \cdot \left[ \mathbf{E}^h [\theta - s(\theta) | x] + B - (M - M_I) \right] \quad \text{if } c(x) = E,$$

where  $\mathbf{E}^h [\cdot]$  denotes the expectation operator under accounting rule  $h$ ;

ii) given the accounting rule  $h$ ,  $\{(s, c, M_I), a\}$  maximizes

$$\mathbf{E}[a(x) \cdot \left( \mathbf{E}^h(\theta - s(\theta)) | x \right) + B] + (1 - a(x)) \cdot (M - M_I) \quad (\text{A.1})$$

subject to  $s \in S$ ,  $M_I \in [0, M]$ , Condition i) and

$$v_I^h(s, M_I, a) \geq \underline{v}_I,$$

where

$$v_I^h(s, M_I, a) = \mathbf{E}[a(x) \cdot \mathbf{E}^h(s(\theta) | x) + (1 - a(x)) \cdot M_I]$$

is the investor's expected payoff given  $s, M_I$  and the decision rule  $a$ ;

iii) the accounting rule  $h$  maximizes

$$\mathbf{E}[a(x) \cdot \mathbf{E}^h(\theta | x) + (1 - a(x)) \cdot M], \quad (\text{A.2})$$

subject to Conditions i) and ii).

We first show that Proposition 1 in the main text holds for arbitrary securities by stating the more general version, Proposition 6. The proof of Proposition 1 in the main text is omitted as it is contained in the proof of the more general Proposition 6.

Let  $SM^h$  denote the set of all pairs of  $(s, M_I)$  that make the investor just break even when  $a_h$  is implemented. That is,

$$SM^h = \left\{ (s, M_I) \mid s \in S, M_I \in [0, M], v_I^h(s, M_I, a_h) = \underline{v}_I \right\}.$$

The following proposition shows that  $SM^h$  is non-empty. It contains popular contracts such as debt and equity.

**Proposition 5** *There exist debt securities and equity securities such that the investor breaks even when  $a_h$  is implemented.*

**Proof.** Note that, by definition, set  $DM^h$ , which consists of all debt securities that make the investor break even if  $a_h$  is implemented, belongs to  $SM^h$ . Now we show that  $SM^h$  also contains equity securities. Let  $s_\tau(\theta) = \tau \cdot \theta$  for  $\tau \in [0, 1]$ . It is straightforward to see that  $v_I^h(s_\tau, 0, a_h) = 0 \leq \underline{v}_I$  when  $\tau = 0$ . When  $\tau = 1$ ,

$$\begin{aligned} & v_I^h(s_\tau, M - B, a_h) \\ &= \mathbf{E}[\max(\mathbf{E}(\theta|x), M - B)] \\ &\geq \mathbf{E}[\theta] \\ &\geq \underline{v}_I. \end{aligned}$$

Since  $v_I^h(s_\tau, M_I, a_h)$  is continuous in  $(\tau, M_I)$  and the domain  $[0, 1] \times [0, M]$  is connected, there must exist a pair  $(\tau, M_I)$  such that  $v_I^h(s_\tau, M_I, a_h) = \underline{v}_I$ . Moreover, note that, since  $v_I^h(s_\tau, M_I, a_h)$  increases in both  $\tau$  and  $M_I$ , when  $v_I^h(s_\tau, M_I, a_h) = \underline{v}_I$ , we can choose some  $\Delta_1$  and  $\Delta_2$  appropriately such that  $v_I^h(s_{\tau+\Delta_1}, M_I - \Delta_2, a_h)$  remains  $\underline{v}_I$ . Thus set  $SM^h$  consists of multiple equity contracts  $(s_\tau, M_I)$ . ■

This proposition shows the richness of set  $SM^h$ . Since  $SM^h$  contains all debt contracts in  $DM^h$ , Proposition 1 in the main text is a special case of the following proposition.

**Proposition 6** *For any given accounting rule  $h$ , (i) for any pair  $(s, M_I) \in SM^h$ , there exists a covenant  $c$  such that  $(s, M_I, c)$  implements  $a_h$ ; ii) it is optimal for the entrepreneur to choose  $(s, M_I) \in SM^h$  together with the corresponding covenant  $c$  to implement  $a_h$ .*

**Proof.** We first show that  $DM^h$  is not empty. Intuitively,  $M_I$  can be set

between 0 and  $M - B$  such that the investor just breaks even.<sup>14</sup> Since  $v_I^h(D, M_I, a_h)$  is continuous in  $(D, M_I)$  and the domain  $\mathbb{R}_+ \times [0, M]$  is connected, there must exist a pair  $(D, M_I)$  such that  $v_I^h(D, M_I, a_h) = \underline{v}_I$ .<sup>15</sup> If the entrepreneur chooses a pair  $(D, M_I) \in DM^h$  and implements the decision rule  $a_h$ , he simultaneously maximizes the total surplus (from his perspective) and minimizes the investor's expected payoff subject to her participation constraint and thus obtains the highest possible expected payoff. In addition, for any pair  $(D, M_I) \in DM^h$ , the entrepreneur can always find a covenant to implement the decision rule  $a_h$ . To see this, note that, for any  $x \in X$  such that  $a_h(x) = 1$ , we have

$$\begin{aligned} & \mathbf{E}^h [\max(\theta - D, 0)|x] + B - (M - M_I) + \mathbf{E}^h [\min(D, \theta)|x] - M_I \\ = & \mathbf{E}^h [\theta|x] + B - M > 0. \end{aligned}$$

Note that  $\mathbf{E}^h [\max(\theta - D, 0)|x] + B - (M - M_I)$  and  $\mathbf{E}^h [\min(D, \theta)|x] - M_I$  are the entrepreneur's and investor's payoff gains from continuation over termination, respectively. Hence there must be a party preferring to continue, and assigning the control right to this party implements  $a_h(x) = 1$ . The same argument applies to  $x \in X$  such that  $a_h(x) = 0$ . We now prove the proposition.

i) For  $x \in X$  s.t.  $a_h(x) = 1$ , we have

$$\begin{aligned} & \mathbf{E}[\theta - s(\theta)|x] + B - (M - M_I) + \mathbf{E}[s(\theta)|x] - M_I \\ = & \mathbf{E}[\theta|x] + B - M > 0. \end{aligned}$$

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<sup>14</sup>Technically,  $v_I^h(D, 0, a_h) = 0 \leq \underline{v}_I$  if  $D = 0$ , and

$$\begin{aligned} & v_I^h(D, M - B, a_h) \\ = & \mathbf{E}^h \left[ \max \left( \mathbf{E}^h(\theta|x), M - B \right) \right] \\ \geq & \mathbf{E}[\theta] \\ \geq & \underline{v}_I, \end{aligned}$$

if  $D = \bar{\theta}$ .

<sup>15</sup>Note that, since  $v_I^h(D, M_I, a_h)$  increases in both  $D$  and  $M_I$ , when  $v_I^h(D, M_I, a_h) = \underline{v}_I$  we can choose some  $\Delta_1$  and  $\Delta_2$  appropriately such that  $v_I^h(D + \Delta_1, M_I - \Delta_2, a_h)$  remains  $\underline{v}_I$ . Thus set  $DM^h$  consists of multiple pairs of  $(D, M_I)$ .

Note that  $\mathbf{E}[\theta - s(\theta)|x] + B - (M - M_I)$  and  $\mathbf{E}[s(\theta)|x] - M_I$  are the entrepreneur's and investor's payoff gains from continuation over termination, respectively. Thus at least one of the two parties prefers to continue. In this case, the control right shall be given to the party who wants to continue.

For  $x \in X$  s.t.  $a_h(x) = 0$ , we have

$$\begin{aligned} & \mathbf{E}[\theta - s(\theta)|X] + B - (M - M_I) + \mathbf{E}[s(\theta)|X] - M_I \\ = & \mathbf{E}[\theta|x] + B - M \leq 0. \end{aligned}$$

Thus at least one of the two parties (weakly) prefers termination. In this case, the control right shall be given to the party who wants to terminate.

Hence  $a_h$  can be implemented at  $t = 1$ .

In addition, since  $(s, M_I) \in SM^h$  and  $a_h$  is implemented, the investor's expected payoff is  $v_I^h(s, M_I, a_h) = \underline{v}_I$ . Hence the investor's participation constraint at  $t = 0$  is satisfied.

ii) By choosing  $(s, M_I)$  and the corresponding covenant  $c$ , the entrepreneur implements  $a_h$  that maximizes the total surplus and makes the investor just break even. Thus the entrepreneur reaches his highest possible expected payoff. ■

## A2 Details of how more downside risk affects $\hat{\theta}$

In this section we characterize the impact of more general (non-parametric) belief changes on  $\hat{\theta}$ . As mentioned in the main text, to capture a belief change, we perturb the density of the prior,  $p(\theta)$ , by  $\alpha \cdot \Delta p(\theta)$ , where  $\alpha$  is the magnitude of the perturbation. We require  $\int_{\underline{\theta}}^{\bar{\theta}} \Delta p(\theta) d\theta = 0$  so that the perturbed prior  $p(\theta) + \alpha \cdot \Delta p(\theta)$  remains a probability density. Intuitively, what it means is that it shifts the density from certain parts of the prior distribution into other parts so that we can discuss the notion of “more downside risk”.

Our purpose is to consider how  $\hat{\theta}(\alpha)$ , the threshold of optimal accounting

rule, varies under the perturbed belief  $p + \alpha \cdot \Delta p$ . The marginal effect is captured by  $\left. \frac{\partial \hat{\theta}}{\partial \alpha} \right|_{\alpha=0}$ . Proposition 2 immediately implies that  $\left. \frac{\partial \hat{\theta}}{\partial \alpha} \right|_{\alpha=0} = 0$  when  $B \leq E(M - \theta | \theta \leq M)$ . The following proposition provides the comparative statics for the more interesting case of  $B > E(M - \theta | \theta \leq M)$ .

**Proposition 7** *When  $B > E(M - \theta | \theta \leq M)$ ,  $\left. \frac{\partial \hat{\theta}}{\partial \alpha} \right|_{\alpha=0} > 0$  if and only if*

$$\int_{\underline{\theta}}^{\hat{\theta}} \Delta p(\theta)(M - B - \theta)d\theta > 0. \quad (\text{A.3})$$

To obtain some intuition for this result, note that  $M - B - \theta$  is positive over  $[\underline{\theta}, M - B)$  and negative over  $(M - B, \hat{\theta}]$ . Since  $\Delta p(\theta)$  is the difference between the original prior and the perturbed prior,  $\int_{\underline{\theta}}^{\hat{\theta}} \Delta p(\theta)d\theta = 0$ . Hence

$\int_{\underline{\theta}}^{\hat{\theta}} \Delta p(\theta)(M - B - \theta)d\theta$  tends to be positive (negative) if  $\Delta p(\theta)$  is mainly positive (negative) in  $[\underline{\theta}, M - B)$  and negative (positive) in  $(M - B, \hat{\theta}]$ . In other words, shifting probabilities in the prior distribution from  $(M - B, \hat{\theta}]$  to  $[\underline{\theta}, M - B)$  results in  $\int_{\underline{\theta}}^{\hat{\theta}} \Delta p(\theta)(M - B - \theta)d\theta > 0$  and thus  $\left. \frac{\partial \hat{\theta}}{\partial \alpha} \right|_{\alpha=0} > 0$ . This is consistent with our intuition that deteriorating belief about the cash flow  $\theta$  calls for a higher reporting threshold  $\hat{\theta}$ , that is, more conservative accounting rules.

Proposition 7 states that accounting is more conservative if there is more downside risk. Intuitively, when there is more downside risk, continuing is more likely to generate a lower payoff. It is thus easier to convince the entrepreneur to terminate. As a result,  $\hat{\theta}$  increases and the accounting rule becomes more conservative. The “more downside risk” notion is different from both first order stochastic dominance (FOSD) and monotone likelihood ratio property (MLRP), which are commonly used in the accounting literature, because while FOSD and MLRP implies a uniform way of shift-

ing density from high realizations to low realizations, the shifting of density in “more downside risk” does not have to be uniform. For example, you can shift density from high realizations to low realizations and intermediate realizations.

**Proof of Proposition 7:**

**Proof.** When  $\hat{\theta} < M$ , it is defined by

$$\mathbf{E}[\theta | \theta \leq \hat{\theta}] = M - B, \quad (\text{A.4})$$

By the Bayes’ rule, we rewrite equation (A.4) as

$$\int_{\underline{\theta}}^{\hat{\theta}} \theta p(\theta) d\theta = (M - B) \int_{\underline{\theta}}^{\hat{\theta}} p(\theta) d\theta. \quad (\text{A.5})$$

Consider a perturbation to the prior distribution,  $\Delta p(\theta)$  s.t.  $\int_{\underline{\theta}}^{\bar{\theta}} \Delta p(\theta) d\theta = 0$ , that is, change  $p(\theta)$  to  $p(\theta) + \alpha \Delta p(\theta)$  and look at how  $\hat{\theta}$  varies with respect to  $\alpha$  at  $\alpha = 0$ . Replacing  $p(\theta)$  by  $p(\theta) + \alpha \Delta p(\theta)$  in equation (A.5) leads to

$$\int_{\underline{\theta}}^{\hat{\theta}} \theta [p(\theta) + \alpha \Delta p(\theta)] d\theta = (M - B) \int_{\underline{\theta}}^{\hat{\theta}} [p(\theta) + \alpha \Delta p(\theta)] d\theta.$$

Taking derivative with respect to  $\alpha$  at  $\alpha = 0$  results in

$$\hat{\theta} p(\hat{\theta}) \frac{\partial \hat{\theta}}{\partial \alpha} + \int_{\underline{\theta}}^{\hat{\theta}} \theta \Delta p(\theta) d\theta = (M - B) [p(\hat{\theta}) \frac{\partial \hat{\theta}}{\partial \alpha} + \int_{\underline{\theta}}^{\hat{\theta}} \Delta p(\theta) d\theta],$$

that is,

$$[\hat{\theta} - (M - B)] \cdot p(\hat{\theta}) \frac{\partial \hat{\theta}}{\partial \alpha} = \int_{\underline{\theta}}^{\hat{\theta}} \Delta p(\theta) (M - B - \theta) d\theta. \quad (\text{A.6})$$

Since  $\hat{\theta} - (M - B) > 0$  from Proposition 2, the sign of  $\frac{\partial \hat{\theta}}{\partial \alpha}$  is the same as

that of  $\int_{\underline{\theta}}^{\widehat{\theta}} \Delta p(\theta)(M - B - \theta)d\theta$ . This completes the proof. ■

We now provide some intuitive discussion of Proposition 7 as the proof itself may be too abstract and do not convey much intuition.

Proposition 7 states that  $\widehat{\theta}$  increases if and only if the belief perturbation  $\Delta p$  satisfies inequality (A.3), which is a sufficient and necessary condition that applies to all kinds of belief perturbations, whether parametric or not. For any family of prior distributions  $\{p(\theta|\mu)\}$  parameterized by  $\mu$ , it immediately follows the proposition that  $\frac{\partial \widehat{\theta}}{\partial \mu} > 0$  if and only if

$$\int_{\underline{\theta}}^{\widehat{\theta}} p_{\mu}(\theta|\mu)(M - B - \theta)d\theta > 0.$$

Note that the monotone likelihood ratio property (MLRP) of  $\{p(\theta|\mu)\}$  implies neither this inequality nor its complement. To see this, rewrite the left hand side of the above inequality as

$$\int_{\underline{\theta}}^{\widehat{\theta}} \frac{p_{\mu}(\theta|\mu)}{p(\theta|\mu)} p(\theta|\mu)(M - B - \theta)d\theta.$$

MLRP is equivalent to that  $\frac{p_{\mu}(\theta|\mu)}{p(\theta|\mu)}$  increases in  $\theta$ , which does not determine the sign of the above integral.

We now go beyond the parametric forms to study in general how  $\widehat{\theta}$  responds to FOSD type of belief perturbations. Note that, if the perturbed belief,  $p + \alpha \Delta p$ , is first order stochastically dominated by the original belief  $p$ , then  $p + \alpha \Delta p$  can be obtained from  $p$  by a sequence of operations, each of which shifts a tiny mass from high values of  $\theta$  to low values. The way the distribution is shifted is crucial for whether  $\widehat{\theta}$  increases, decreases, or remains unchanged, as illustrated in Figure 2. In Figure 2, the straight solid line represents  $M - B - \theta$ , whereas the three arrows represent three typical operations of shifting a small mass from higher to lower values of  $\theta$ . They all

shift the distribution to the left so that the perturbed belief is first order stochastically dominated by the original one. Operation 1 shifts a small mass from relatively higher values above  $\hat{\theta}$  to relatively lower values above  $\hat{\theta}$ . This operation has no marginal impact on  $\hat{\theta}$  as  $\int_{\underline{\theta}}^{\hat{\theta}} \Delta p(\theta)(M-B-\theta)d\theta = 0$ . Operation 2 shifts the mass from above  $\hat{\theta}$  to an intermediate state between  $M-B$  and  $\hat{\theta}$ , resulting in a decrease in  $\hat{\theta}$  as  $\int_{\underline{\theta}}^{\hat{\theta}} \Delta p(\theta)(M-B-\theta)d\theta < 0$ . Operation 3 shifts the mass from the intermediate values of  $\theta$  to the lowest range (lower than  $M-B$ ), resulting in an increase in  $\hat{\theta}$  as  $\int_{\underline{\theta}}^{\hat{\theta}} \Delta p(\theta)(M-B-\theta)d\theta > 0$ . It is straightforward to see that the impact of FOSD type of belief perturbations on  $\hat{\theta}$  is ambiguous.

To appreciate the intuition, take operation 3 as an example. Operation 3 increases the downside risk about  $\theta$  so that the bad news of  $\theta \leq \hat{\theta}$  is more pessimistic. This reduces  $\mathbf{E}[\theta|\theta \leq \hat{\theta}]$ , the expected cash flow conditional on continuation, calling for a higher  $\hat{\theta}$  to restore the entrepreneur's indifference between continuation and termination. Operation 2, meanwhile, results in a relatively more optimistic belief of the bad news of  $\theta \leq \hat{\theta}$ , as captured by a greater  $\mathbf{E}[\theta|\theta \leq \hat{\theta}]$  if  $\hat{\theta}$  is fixed at the original level. This is because Operation 2 increases the probability mass in  $[M-B, \hat{\theta}]$  while originally  $\mathbf{E}[\theta|\theta \leq \hat{\theta}] = M-B$ . Hence a lower  $\hat{\theta}$  is required to restore the entrepreneur's indifference condition.

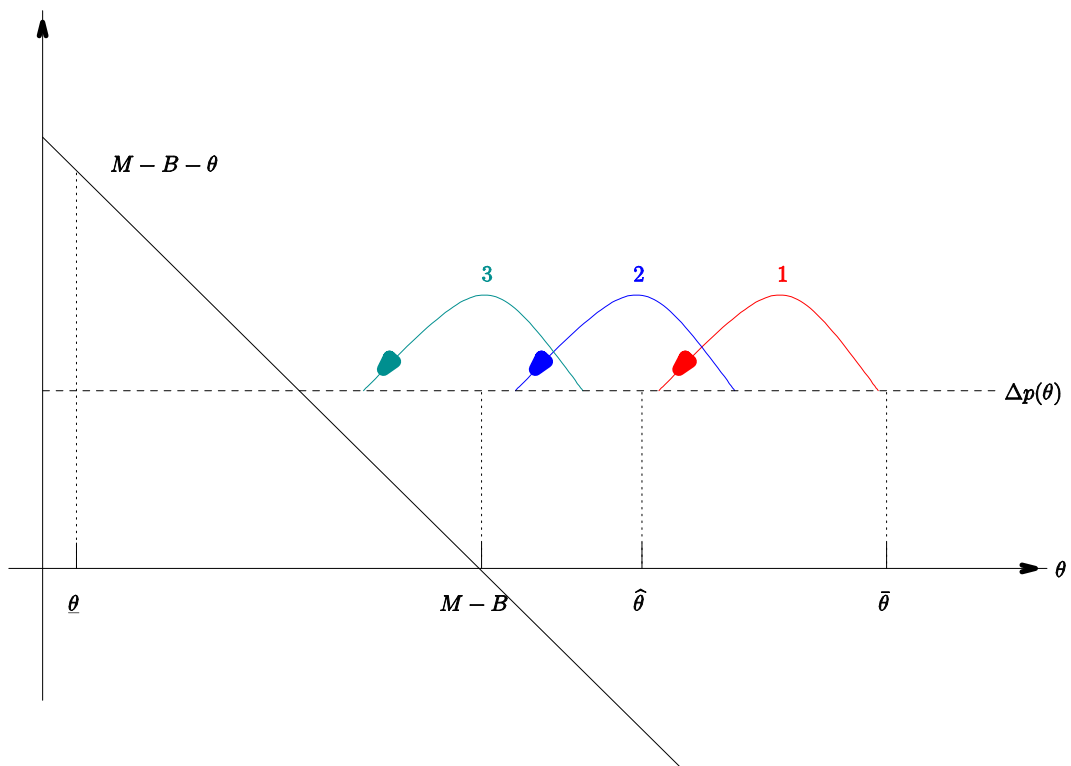


Figure 2: A graphical illustration of shifting distribution to the left in the sense of FOSD.