Bad Greenwashing, Good Greenwashing: Corporate Social Responsibility and Information Transparency Online Appendix

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Hybrid Equilibria

While in the paper we focus on pure-strategy equilibria (as in Mailath et al. 1993 and Mezzetti and Tsoulouhas 2000), below we briefly discuss mixed-strategy equilibria. Mixed strategies introduce another type of equilibria in addition to separating and pooling, namely hybrid equilibria, where at least one type of firm randomizes between pooling and separating. We will show that the socially responsible firm always prefers the pure-strategy undefeated equilibrium characterized in the basic model to any hybrid PBE. Note that whenever a firm randomizes between pooling and separating, she is indifferent between the pooling payoff and the separating payoff.

First, when both types of firms randomize between pooling and separating, the socially responsible firm can obtain a profit that is equal to at most her profit in the pure-strategy separating equilibrium (see Lemma A.1), because she needs to ensure that the profit maximizer is unwilling to deviate from putting positive probability on separating without mimicking.

Second, when only the socially responsible firm randomizes, the pooling payoff she receives is lower than her profit in the pure-strategy pooling equilibrium (see Lemma A.3).

Third, when only the profit maximizer randomizes, she is indifferent between the pooling payoff and the separating payoff $(V_L - U_0)^+$, and the socially responsible firm always chooses pooling. In this case, the expected consumer reward is $\phi' R$ upon pooling, where $\phi' \in (\phi, 1)$ is the conditional probability that the firm is socially responsible upon pooling. It can be shown that the socially responsible firm strictly prefers the pure-strategy separating equilibrium (see Lemma A.1) to the pooling payoff she receives in the hybrid equilibrium.

Proof of Proposition 3

To have a complete picture, we restrict our attention to the non-trivial case wherein the equilibrium is a cheap signaling equilibrium when the budget constraint is not binding (otherwise, no cheap signaling equilibrium exists). As such, our proof focuses on $R \leq \alpha c(z_{\text{pe}})$. The proof for $R > \alpha c(z_{\text{pe}})$ is very similar and is thus omitted.

Before analyzing the equilibrium outcome, we first characterize the complete/full-information benchmark. Since in this extension we have assumed that $U_0 = 0$, the optimal prices are $p_H = V_H$ and $p_L = V_L$. Because $\alpha x + (1-\alpha)y$ (subject to $\alpha c(x) + (1-\alpha)c(y) = B$) is maximized at $x = y = c^{-1}(B)$ for $B < c(z_{\text{pe}})$, we have the following lemma.

Lemma W.1. *Under complete information, for $B \geq c(z_{\text{pe}})$, the socially responsible firm's optimal strategy is the same as that in the basic model; for $B < c(z_{\text{pe}})$, the optimal investment level of the socially responsible firm is $x = y = c^{-1}(B)$ in both observed and unobserved activities. The profit maximizer does not spend anything on CSR.*

Under incomplete information, in any pooling equilibrium, the two firms charge the same price, $p = \phi V_H + (1-\phi)V_L - U_0$; in any separating equilibrium, firm H sets $p_H = V_H - U_0$ and firm L sets $p_L = (V_L - U_0)^+$. We first find the candidate undefeated separating equilibrium in Lemma W.2.

Lemma W.2. *There exists a separating equilibrium iff B is sufficiently large. In the equilibrium that is undefeated by any other separating equilibrium, $x_H = x_H^{\text{sep}} \equiv \max\{\min\{z_{\text{pe}}, c^{-1}(B)\}, c^{-1}(R/\alpha)\}$, $y_H = y_H^{\text{sep}} \equiv \min\left\{z_{\text{pe}}, c^{-1}\left(\frac{B-\alpha c(x_H^{\text{sep}})}{1-\alpha}\right)\right\}$, $x_L = x_L^{\text{sep}} \equiv 0$, and $y_L = y_L^{\text{sep}} \equiv 0$. The out-of-equilibrium belief can be specified as $\psi(x) = 0$ for any off-equilibrium action x.*

Proof. It can be verified that, if $B < R$, then no separating equilibrium exists, because the budget B is too small to prevent the profit maximizer from mimicking the socially responsible firm. In the rest of this proof, we suppose that $B \geq R$.

Given that the equilibrium is a cheap signaling equilibrium without the budget constraint, we have $R \leq \alpha c(z_{\text{pe}})$. As such, there are three cases for the stated x_H^{sep}: (I) when $B \geq c(z_{\text{pe}})$, $x_H^{\text{sep}} = z_{\text{pe}}$ (cheap signaling); (II) when $R/\alpha \equiv \widehat{B} \leq B < c(z_{\text{pe}})$, $x_H^{\text{sep}} = c^{-1}(B)$ (cheap signaling); (III) when $B < \widehat{B} \equiv R/\alpha$, $x_H^{\text{sep}} = c^{-1}(R/\alpha)$ (costly signaling).

We first show that for the equilibrium strategies stated in the lemma, the profit maximizer has no incentive to deviate. Similar to the basic model, it can be verified that the first two cases refer to the cheap signaling outcome where the profit maximizer prefers x_L^{sep} to x_H^{sep}, whereas the third case refers to the costly signaling outcome in which the profit maximizer is indifferent between x_H^{sep}

and x_L^{sep}. In the cheap signaling outcome, the socially responsible firm would never deviate to any off-equilibrium strategy. In the costly signaling outcome ($B < R/\alpha$), the socially responsible firm would not deviate to her optimal strategy under the pessimistic belief $\psi = 0$ (which coincides with the complete-information optimal) iff $M \equiv R + \gamma \left[\alpha c^{-1}\left(\frac{R}{\alpha}\right) + (1-\alpha)c^{-1}\left(\frac{B-R}{1-\alpha}\right) - c^{-1}(B)\right] \geq 0$. Because $\frac{\partial M}{\partial B} = \gamma \left(\frac{1}{c'(c^{-1}(\frac{B-R}{1-\alpha}))} - \frac{1}{c'(c^{-1}(B))} \right) > 0$ (since $B < R/\alpha$ implies that $\frac{B-R}{1-\alpha} < B$), $M \geq 0$ iff B is sufficiently large.

Given that B is sufficiently large such that $M \geq 0$, we now prove that the equilibrium strategy x_H^{sep} stated in the lemma is the socially responsible firm's favorite, and so the equilibrium sustains and cannot be defeated by any other separating equilibrium. Cases (I) and (III) are similar to those in the basic model. For case (II), $x_H^{\text{sep}} = c^{-1}(B)$. Given the budget $B < c(z_{\text{pe}})$, x_H^{sep} is the high-type firm's optimal strategy under complete information, and so it is the socially responsible firm's favorite separating equilibrium strategy. □

Note that in Lemma W.2, for $B \geq R/\alpha$ the equilibrium is a cheap signaling equilibrium where $x_H = x_H^{\text{sep}} \equiv \min\{z_{\text{pe}}, c^{-1}(B)\}$, which is undefeated by any other equilibrium (including any pooling equilibrium); for $B < R/\alpha$, the equilibrium is a costly signaling equilibrium where $x_H = x_H^{\text{sep}} \equiv c^{-1}(R/\alpha)$, which is undefeated by any other separating equilibrium. We next prove that the costly signaling equilibrium is defeated by a pooling equilibrium iff B is sufficiently small. In the rest of the proof, we can restrict our attention to $B < R/\alpha$.

The following lemma provides the candidate pooling equilibrium and echoes Lemma A.3.

Lemma W.3. *There exists a pooling equilibrium iff B is sufficiently small, where $x_H = x_L = x^{\text{pool}} \equiv \min\{c^{-1}(\phi R/\alpha), c^{-1}(B)\}$, $y_H = y_H^{\text{pool}} \equiv \min\left\{z_{\text{pe}}, c^{-1}\left(\frac{B-\alpha c(x^{\text{pool}})}{1-\alpha}\right)\right\}$, and $y_L = y_L^{\text{pool}} \equiv 0$. The out-of-equilibrium belief can be specified as $\psi(x) = 0$ for any off-equilibrium action x. The socially responsible firm prefers this pooling equilibrium to any other pooling equilibrium.*

Proof. In order for the profit maximizer to prefer the pooling equilibrium strategy to her optimal strategy under complete information, we need $x^{\text{pool}} \leq c^{-1}(\phi R/\alpha)$. We next examine the socially responsible firm's deviation incentive. There are three cases for the stated equilibrium strategies: (I) when $B > \phi R/\alpha$ and $B \geq \phi R + (1-\alpha)c(z_{\text{pe}})$, $x^{\text{pool}} = c^{-1}(\phi R/\alpha)$ and $y_H^{\text{pool}} = z_{\text{pe}}$; (II) when $\phi R/\alpha < B < \phi R + (1-\alpha)c(z_{\text{pe}})$, $x^{\text{pool}} = c^{-1}(\phi R/\alpha)$ and $y_H^{\text{pool}} = c^{-1}\left(\frac{B-\phi R}{1-\alpha}\right)$;

(III) when $B \leq \phi R/\alpha$, $x^{\text{pool}} = y_H^{\text{pool}} = c^{-1}(B)$. In case (III), the socially responsible firm cannot make a profitable deviation to any off-equilibrium strategy. In case (I), the socially responsible firm does not want to deviate to her optimal strategy under complete information iff $M_{\text{I}} \equiv \alpha\gamma c^{-1}(\phi R/\alpha) + (1-\alpha)(\gamma z_{\text{pe}} - c(z_{\text{pe}})) - (\gamma c^{-1}(B) - B) \geq 0$. Since $(\gamma c^{-1}(B) - B)$ is increasing in B (because $B < R/\alpha \leq c(z_{\text{pe}})$), $M_{\text{I}} \geq 0$ iff B is sufficiently small. In case (II), the socially responsible firm would not deviate to her complete-information optimal strategy iff $M_{\text{II}} \equiv \phi R + \gamma\left[\alpha c^{-1}\left(\frac{\phi R}{\alpha}\right) + (1-\alpha)c^{-1}\left(\frac{B-\phi R}{1-\alpha}\right) - c^{-1}(B)\right] \geq 0$. Since $\frac{\partial M_{\text{II}}}{\partial B} = \gamma\left(\frac{1}{c'(c^{-1}(\frac{B-\phi R}{1-\alpha}))} - \frac{1}{c'(c^{-1}(B))}\right) < 0$ (because $B > \phi R/\alpha$ implies that $\frac{B-\phi R}{1-\alpha} > B$), $M_{\text{II}} \geq 0$ iff B is sufficiently small. In sum, the pooling equilibrium stated in the lemma sustains iff B is sufficiently small. It can be verified that the equilibrium strategy stated in the lemma is the socially responsible firm's favorite, and so this pooling equilibrium is the one that most easily satisfies the socially responsible firm's incentive compatibility constraint. □

Under certain parameter settings, for very small ϕ, there may be a small interval $(\subset (\phi R/\alpha, R/\alpha))$ for B, within which no pure-strategy equilibrium exists: In the proof of Lemma W.3, it can be shown that both M_{I} and M_{II} are increasing in ϕ. In the proof of Lemma W.2, M does not depend on ϕ. As such, for very small ϕ, under certain parameter settings there can be some B such that both M_{I} (or M_{II}) and M are negative and no pure-strategy equilibrium exists. In this case, only hybrid equilibria exist. Similar to the discussion of hybrid equilibria for the basic model, when both types of firms randomize between pooling and separating, the socially responsible firm has incentive to deviate, because $M < 0$; when only the socially responsible firm randomizes, the socially responsible firm has an incentive to deviate, because $M_{\text{I}} < 0$ for case (I) or $M_{\text{II}} < 0$ for case (II). Therefore, when no pure-strategy equilibrium exists, in any hybrid equilibrium only the profit maximizer randomizes between pooling and separating.

Now we are ready to compare the pooling equilibrium with the separating equilibrium.

Lemma W.4. *There exists an undefeated separating equilibrium iff B is sufficiently large; the equilibrium strategies are unique (see Lemma W.2 for equilibrium strategies). There exists an undefeated pooling equilibrium iff B is sufficiently small; under SUE, the equilibrium strategies are unique (see Lemma W.3 for equilibrium strategies).*

Proof. We need to show only that the socially responsible firm prefers the costly signaling equilibrium with $x_H^{\text{sep}} = c^{-1}(R/\alpha)$ and $y_H^{\text{sep}} = c^{-1}\left(\frac{B-R}{1-\alpha}\right)$ to the pooling equilibrium stated in Lemma W.3 iff B is sufficiently large. Since the socially responsible firm's profit is continuous in the pooling equilibrium, we need to show only that this preference holds in each of the three cases discussed in the proof of Lemma W.3.

In case (I), the socially responsible firm prefers the separating equilibrium iff $G_{\text{I}} \equiv R + \gamma\left[\alpha c^{-1}\left(\frac{R}{\alpha}\right) + (1-\alpha)c^{-1}\left(\frac{B-R}{1-\alpha}\right)\right] - B - [\alpha\gamma c^{-1}(\phi R/\alpha) + (1-\alpha)(\gamma z_{\text{pe}} - c(z_{\text{pe}}))] \geq 0$. Note that $\frac{\partial G_{\text{I}}}{\partial B} = \frac{\gamma}{c'(c^{-1}(\frac{B-R}{1-\alpha}))} - 1$. Since $B < R/\alpha$ implies that $\frac{B-R}{1-\alpha} < B < c(z_{\text{pe}})$, we have $\frac{\partial G_{\text{I}}}{\partial B} > 0$, and so $G_{\text{I}} \geq 0$ iff B is sufficiently large. In case (II), the socially responsible firm prefers the separating equilibrium iff $G_{\text{II}} \equiv R + \gamma\left[\alpha c^{-1}\left(\frac{R}{\alpha}\right) + (1-\alpha)c^{-1}\left(\frac{B-R}{1-\alpha}\right)\right] - \phi R - \gamma\left[\alpha c^{-1}\left(\frac{\phi R}{\alpha}\right) + (1-\alpha)c^{-1}\left(\frac{B-\phi R}{1-\alpha}\right)\right] \geq 0$. Since $\frac{\partial G_{\text{II}}}{\partial B} = \gamma\left(\frac{1}{c'(c^{-1}(\frac{B-R}{1-\alpha}))} - \frac{1}{c'(c^{-1}(\frac{B-\phi R}{1-\alpha}))}\right) > 0$, $G_{\text{II}} \geq 0$ iff B is sufficiently large. In case (III), the socially responsible firm prefers the separating equilibrium iff $G_{\text{III}} \equiv (1-\phi)R + \gamma\left[\alpha c^{-1}\left(\frac{R}{\alpha}\right) + (1-\alpha)c^{-1}\left(\frac{B-R}{1-\alpha}\right) - c^{-1}(B)\right] \geq 0$. Since $\frac{\partial G_{\text{III}}}{\partial B} = \gamma\left(\frac{1}{c'(c^{-1}(\frac{B-R}{1-\alpha}))} - \frac{1}{c'(c^{-1}(B))}\right) > 0$ (because $B < R/\alpha$ implies that $\frac{B-R}{1-\alpha} < B$), $G_{\text{III}} \geq 0$ iff B is sufficiently large. In summary, the socially responsible firm prefers the separating equilibrium iff $B > \widetilde{B}$, where $\widetilde{B}$ is determined by $G_k = 0$ for each case (k), $k = \text{I}, \text{II}, \text{III}$. At $B = \widetilde{B}$, if $M_{\text{I}} < 0$ for case (I) or $M_{\text{II}} < 0$ for case (II) (iff $M < 0$), then $\widetilde{B}^{\text{sep}}$ is determined by $M = 0$ (see M in the proof of Lemma W.2) and $\widetilde{B}^{\text{pool}}$ is determined by $M_{\text{I}} = 0$ for case (I) and by $M_{\text{II}} = 0$ for case (II) (where M_{I} and M_{II} are as in the proof of Lemma W.3); otherwise ($M_{\text{I}} \geq 0$ for case (I), or $M_{\text{II}} \geq 0$ for case (II), or in case (III)), $\widetilde{B}^{\text{sep}} = \widetilde{B}^{\text{pool}} = \widetilde{B}$. Note that $\widetilde{B}^{\text{pool}}$ is always less than or equal to $\widetilde{B}^{\text{sep}}$. □

Thus far, we have characterized the equilibrium with respect to B. According to Lemmas W.2 and W.3, as the equilibrium shifts from the pooling equilibrium to the separating equilibrium, the socially responsible firm shifts a portion of her CSR spending from the unobserved activity to the observed one: $x_H^{\text{sep}} > c^{-1}(B) \geq x_H^{\text{pool}}$ and $y_H^{\text{sep}} < c^{-1}(B) \leq y_H^{\text{pool}}$. Furthermore, the investment allocation becomes less socially efficient for sure in cases (II) and (III) discussed in the proof of Lemma W.3 and leads to lower welfare.

Proof of Proposition 4

With bargaining power, in any pooling equilibrium, the two types of firms charge the same price, $p = (1-\delta)[\phi V_H + (1-\phi)V_L - U_0]$; in any separating equilibrium, firm H sets $p_H = (1-\delta)(V_H - U_0)$ and firm L sets $p_L \equiv (1-\delta)(V_L - U_0)^+$.

Table W.1: Equilibrium Characterization

Equilibrium outcome	**Pooling equilibrium**	**Separating equilibrium**	
		Costly signaling	**Cheap signaling**
Existence condition	$\delta < \widetilde{\delta}$	$\widetilde{\delta} < \delta < \widehat{\delta}$	$\delta \geq \widehat{\delta}$
Observed investment			
Socially responsible firm x_H	$x^{\text{pool}} \equiv z_{\text{pe}}$	$x_H^{\text{sep}} \equiv c^{-1}\left(\frac{V_H - \max\{V_L, U_0\}}{\alpha}\right) > z_{\text{pe}}$	$x_H^{\text{sep}} \equiv z_{\text{pe}}$
Profit maximizer x_L		$x_L^{\text{sep}} \equiv 0$	$x_L^{\text{sep}} \equiv 0$
Unobserved investment			
Socially responsible firm y_H	z_{pe}		
Profit maximizer y_L	0		

• $\delta = \widetilde{\delta}$ is uniquely determined by
$\gamma\alpha h\left(c^{-1}\left(\frac{(1-\delta)(V_H - \max\{V_L, U_0\})}{\alpha}\right)\right) - (1-\delta)[\phi V_H + (1-\phi)V_L - \max\{V_L, U_0\}] - \alpha(\gamma h(z_{\text{pe}}) - c(z_{\text{pe}})) = 0.$

• $\widehat{\delta} = 1 - \alpha c(z_{\text{pe}})/(V_H - \max\{V_L, U_0\})$.

We can show that the unique undefeated equilibrium can be characterized in Table W.1. The proof largely follows the proof of Lemma 2. We highlight the proof of the existence condition: It can be shown that the equilibrium is a pooling equilibrium iff $\alpha c(z_{\text{pe}}) < (1-\delta)[\phi V_H + (1-\phi)V_L - \max\{V_L, U_0\}]$ ($\iff \delta < 1 - \alpha c(z_{\text{pe}})/[\phi V_H + (1-\phi)V_L - \max\{V_L, U_0\}]$) and $G \leq 0$, where $G = \gamma\alpha c^{-1}\left(\frac{(1-\delta)(V_H - \max\{V_L, U_0\})}{\alpha}\right) - (1-\delta)[\phi V_H + (1-\phi)V_L - \max\{V_L, U_0\}] - \alpha(\gamma h(z_{\text{pe}}) - c(z_{\text{pe}}))$. We have

$$\frac{\partial G}{\partial \delta} = -\frac{\gamma(V_H - \max\{V_L, U_0\})}{c'\left(c^{-1}\left(\frac{(1-\delta)(V_H - \max\{V_L, U_0\})}{\alpha}\right)\right)} + \phi V_H + (1-\phi)V_L - \max\{V_L, U_0\},$$

which is decreasing in δ, and so G is concave in δ. At $\delta = 1 - \alpha c(z_{\text{pe}})/[\phi V_H + (1-\phi)V_L - \max\{V_L, U_0\}]$, $G = \gamma\alpha\left(c^{-1}\left(\frac{V_H - \max\{V_L, U_0\}}{\phi V_H + (1-\phi)V_L - \max\{V_L, U_0\}} \cdot c(z_{\text{pe}})\right) - z_{\text{pe}}\right) > 0$. Therefore, there exists a pooling equilibrium iff $\delta < \widetilde{\delta}$, where $\widetilde{\delta} < 1 - \alpha c(z_{\text{pe}})/[\phi V_H + (1-\phi)V_L - \max\{V_L, U_0\}]$ is the root of G (as a function of δ). Note that if the equilibrium is a costly signaling equilibrium in the basic model (i.e., if $\widetilde{\alpha} < \alpha < \widehat{\alpha}$), then $\widetilde{\delta} \leq 0 < \widehat{\delta}$, and thus the pooling equilibrium does not exist for any $\delta \in (0, 1)$. If the equilibrium is a cheap signaling equilibrium in the basic model (i.e., if

$\alpha \geq \widehat{\alpha}$), then $\widetilde{\delta}, \widehat{\delta} \leq 0$, which implies that the equilibrium is the cheap signaling equilibrium for any $\delta \in (0, 1)$.

The proof of Lemma 3 implies that the welfare can decrease as the equilibrium outcome shifts from a pooling equilibrium to a separating equilibrium. The expected consumer surplus is $\delta(\phi V_H + (1-\phi)V_L - U_0)$ in the pooling equilibrium, and it is $\delta(\phi V_H + (1-\phi)\max\{V_L, U_0\} - U_0)$ in the separating equilibrium. As a result, the consumer surplus always increases with δ.

Generalized Formulation of Consumer Reward

In our basic model, the consumer rewards a firm's motive, that is, her type. One may argue that the consumer should also reward a firm's actual action, that is, her investment in CSR. Put differently, the consumer would pay a premium as long as the firm invests in CSR, no matter what her motive is. To address this possibility, we generalize our basic model by allowing the consumer to reward firm motives as well as CSR activities (see Baron 2001, 2009). Suppose that the consumer reward for CSR activities is proportional to the expected CSR benefit a firm generates. We introduce a parameter $r \in (0, 1]$ to capture the magnitude of such reward. It turns out that the main insights from the basic model are preserved: Propositions 1 and 2 can be generalized. More importantly, the rationale behind these results is still valid. In the following discussion, we first formally introduce the generalized formulation of consumer reward. We then examine how the extra consumer reward for CSR activities affects the firm's decisions under complete information. We finally characterize the equilibrium outcome under incomplete information and compare the reward for firm motives with the reward for firm investments.

Generalized Formulation

For the observed activity, the consumer reward $r \cdot \alpha h(x)$ is based on the actual investment x.[20] For the unobserved activity, the consumer has to make an inference, which in turn depends on his belief about a firm's intrinsic motive. Since the CSR investment is not observed by the consumer, the unobserved investment itself does not affect the belief. Therefore, for the unobservable activity, firms have no incentive to deviate from their respective optimal strategies under complete infor-

[20]We focus on $r \leq 1 - \gamma$ to rule out the anomaly where under complete information the socially responsible firm's optimal investment is beyond the socially efficient one.

mation. Hence, the socially responsible firm invests the privately efficient level z_{pe}; at the same time, the profit maximizer does not invest anything in the unobserved activity. Knowing each type of firm's optimal unobservable actions, the consumer would pay a premium of $r \cdot (1-\alpha)h(z_{\text{pe}}) \cdot \psi$ for the unobserved activity, where ψ refers to the perceived probability that the firm is socially responsible. Formally, the consumer can derive a value $V_L + r \cdot \alpha h(x_L)$ from the profit maximizer's product and obtain a value $V_H + r \cdot [\alpha h(x_H) + (1-\alpha)h(z_{\text{pe}})]$ (in which $V_H = V_L + R$) from the socially responsible firm's product. For tractability, we assume $U_0 = 0$ and $c(z) \equiv \beta z^2/2$, where $\beta > 0$ measures the cost of investment. This generalized formulation is related to the multitask principal-agent model. While in such a model (e.g., Holmstrom and Milgrom 1991) the principal maximizes its profit and usually seeks the optimal contract for the agents, in our model the consumer is given a take-it-or-leave-it offer and the reward "contract" is fixed. This simplified incentive scheme allows us to focus on investment decisions and provide welfare implications in the presence of asymmetric information.

Benchmark: Complete Information

In this extended model, since the consumer's willingness to pay depends not only on the firm's perceived motive but also on the firm's actual investment, Lemma 1 no longer applies. Put differently, both types of firms would over-invest in their observable activity (relative to the complete-information benchmark). We present the result under complete information in Lemma W.5.

Lemma W.5. *Under complete information, both the socially responsible firm and the profit maximizer over-invest relative to the complete-information benchmark; that is, the investment level in the observed activity is higher than that in the unobserved activity.*

In this generalized formulation, the firm's strategy for the observed activity is different from that for the unobserved one under complete information. Furthermore, the observed over-investment is different from that in the basic model under incomplete information. Here, the over-investment is made by both types of firms to obtain the extra reward the consumer is giving for the observed CSR investment, whereas in the basic model the over-investment is made by the socially responsible firm under incomplete information to separate from the profit maximizer.

Incomplete Information

With incomplete information, firms always make over-investment, because the consumer would reward a firm's CSR investment regardless of the firm's motive. The key finding is presented in Proposition W.1.

Proposition W.1.

- *Greenwashing and over-investment (see Table W.2):*
    - ◇ *The profit maximizer greenwashes to mimic the socially responsible firm when information transparency is low.*
    - ◇ *The socially responsible firm over-invests in the observed activity to separate from the profit maximizer when information transparency is moderate.*
    - ◇ *The two types of firms adopt their respective optimal strategies under complete information when information transparency is high.*
    - ◇ *For each equilibrium outcome (pooling, costly signaling, or cheap signaling), the investment in the observed activity is higher than that in the basic model.*
- *Welfare implication:*
    - ◇ *Information transparency has a non-monotonic impact on welfare: The welfare can decrease with information transparency when the equilibrium outcome moves from a pooling equilibrium to a separating equilibrium.*

Table W.2: Generalized Formulation of Consumer Reward

Equilibrium outcome	**Pooling equilibrium**	**Separating equilibrium**	
		Costly signaling	**Cheap signaling**
	Greenwashing (profit maximizer)	Over-investment (socially responsible firm)	No greenwashing or over-investment
Existence condition	$\alpha < \widetilde{\alpha}$	$\widetilde{\alpha} < \alpha < \widehat{\alpha}$	$\alpha \geq \widehat{\alpha}$
Observed investment	higher than in the basic model (i.e., where $r \to 0$, see Table 1)		
Unobserved investment	the same as in the basic model		
• The thresholds $\widetilde{\alpha}$ and $\widehat{\alpha}$ generalize those in the basic model. See the proof of Proposition W.1.			

Consistent with Lemma 2 and Proposition 1 for the basic model, in the pooling equilibrium the profit maximizer engages in greenwashing, whereas in the costly signaling equilibrium the socially responsible firm has a strong incentive to over-invest. As in the basic model, Proposition W.1 implies that the separating equilibrium arises iff the CSR information is sufficiently transparent. Yet this outcome appears to be surprising in our extended model: Compared with the basic model, as more information is available to the consumer, the profit maximizer has a greater incentive to invest more in the observed activity, because the consumer appreciates CSR effort *per se*. This incentive seems to compensate for the cost of the profit maximizer to mimic the socially responsible firm along the observed activity. As such, the pooling equilibrium should be more likely to occur. However, high information transparency still leads to a separating outcome, because, with higher α, the socially responsible firm also has a higher incentive to invest in the observed activity. Since both firms become more motivated to invest in the observed activity, the possibility of separation does not diminish. The welfare implication is similar to that in the basic model. Note that, similar to the basic model, for very large R, the separating equilibrium may disappear (i.e., $\widetilde{\alpha}$ and $\widehat{\alpha}$ can be greater than 1).

It is worthwhile to compare two special cases of the generalized formulation. Case (I) *reward for firm motives*: The extra reward for CSR activity is negligible (i.e., $r \to 0$). This case reduces to the basic model, where the consumer pays a premium only for the firm's motives. Case (II) *reward for investments*: The consumer rewards only the firm's actions (i.e., $R \to 0$). At first glance, the results in case (II) are expected to be totally different from those in case (I), since the consumer rewards the firm differently in the two cases. However, Proposition W.1 (which holds for $R \to 0$) implies that these two cases have many similarities. When the premium is given according to firm investments (case (II)), the consumer rewards the firm based not only on the observed investment but also on the expected unobserved investment. The reward for the unobserved spending, $r \cdot (1-\alpha)h(z_{\text{pe}}) \cdot \psi$, hinges on the perceived probability ψ that the firm is socially responsible. Hence, similar to R, the premium $r \cdot (1-\alpha)h(z_{\text{pe}})$ can be treated as a reward for the firm's motive. On the other hand, in case (I) the premium R can be interpreted as a reward for other unobserved actions of the firm that are not explicitly modeled. This explains why the extended model (including the special case (II)) and the basic model (case (I)) have the same set of key results qualitatively. The main difference is that the reward $r \cdot (1-\alpha)h(z_{\text{pe}})$ for firm motives in the extended model decreases with information transparency α. As a result, in case (II) higher transparency facilitates the separation outcome not only through the direct impact on the profit maximizer's cost of mimicking but also via an indirect effect by lowering the reward for firm motives.

Proof of Lemma W.5

We need to prove only the following lemma.

Lemma W.6. *Under complete information,*

(i) *the socially responsible firm invests x_H^{ci} in the observed activity and z_{pe} in the unobserved activity, where $c'(x_H^{\text{ci}}) = r + \gamma$ and $x_H^{\text{ci}} > z_{\text{pe}}$;*[21]

(ii) *the profit maximizer invests x_L^{ci} in the observed activity and zero in the unobserved activity, where $c'(x_L^{\text{ci}}) = r$.*

Proof. We prove this lemma by rewriting the firms' objective functions. The extra reward for the unobserved CSR activity is $r(1-\alpha)z_{\text{pe}}$. It should be clear that the reward for the expected unobserved investment is not so different from the reward for firm motives. The main qualitative difference is that, the former depends on information transparency but the latter does not. Adding up the consumer reward for firm motives (R), the objectives of the two types of firms can be written as follows.

$$\pi_L(x) = V_L + \alpha(rx - c(x)); \tag{W.1}$$

$$\pi_H(x) = V_H + (1-\alpha)rz_{\text{pe}} + \alpha[(r+\gamma)x - c(x)] + (1-\alpha)(\gamma z_{\text{pe}} - c(z_{\text{pe}})). \tag{W.2}$$

Note that, when $r \to 0$, the objectives reduce to those in our basic model. From firms' objectives given above, the optimal observed investments are x_H^{ci} and x_L^{ci} for the high-type firm and the low-type firm, respectively, where $c'(x_H^{\text{ci}}) = r + \gamma$ and $c'(x_L^{\text{ci}}) = r$. □

Proof of Proposition W.1

Similar to the basic model, the firm charges a price to appropriate as much consumer surplus as possible in equilibrium. In any pooling equilibrium, the two firms charge the same price $p = \phi[V_H + r\alpha x_H^{\text{ci}} + r(1-\alpha)z_{\text{pe}}] + (1-\phi)(V_L + r\alpha x_H^{\text{ci}})$. In any separating equilibrium, firm H sets $p_H = V_H + r\alpha x_H^{\text{sep}} + r(1-\alpha)z_{\text{pe}}$ and firm L sets $p_L = V_L + r\alpha x_L^{\text{ci}}$. The welfare implication is implied by Proposition 2. We need to prove only the following lemma.

Lemma W.7. *Suppose $c(z) = \beta z^2/2$. The unique undefeated equilibrium is shown in the following table.*

[21] The superscript "ci" signifies complete information.

<table>
<tr><th rowspan="2">Equilibrium outcome</th><th rowspan="2">Pooling equilibrium</th><th colspan="2">Separating equilibrium</th></tr>
<tr><th>Costly signaling</th><th>Cheap signaling</th></tr>
<tr><td>Existence condition</td><td>$\alpha < \widetilde{\alpha}$</td><td>$\widetilde{\alpha} < \alpha < \widehat{\alpha}$</td><td>$\alpha \geq \widehat{\alpha}$</td></tr>
<tr><td>Observed investment
Socially responsible firm x_H
Profit maximizer x_L</td><td>$x^{\text{pool}} \equiv x_H^{\text{ci}}$</td><td>$x_H^{\text{sep}} \equiv x_{\text{indiff}}^{\text{sep}}$
$x_L^{\text{sep}} \equiv x_L^{\text{ci}}$</td><td>$x_H^{\text{sep}} \equiv x_H^{\text{ci}}$
$x_L^{\text{sep}} \equiv x_L^{\text{ci}}$</td></tr>
<tr><td>Unobserved investment
Socially responsible firm y_H
Profit maximizer y_L</td><td colspan="3">z_{pe}
0</td></tr>
</table>

- The thresholds $\widetilde{\alpha}$ and $\widehat{\alpha}$ generalize those in the basic model.
- In the separating equilibrium $x_H^{\text{ci}} \geq x_{\text{indiff}}^{\text{sep}}$ iff $\alpha \geq \widehat{\alpha}$, where $x_{\text{indiff}}^{\text{sep}}$ will be defined in Lemma W.8.
- The pooling equilibrium is unique under SUE.
  The separating equilibrium is unique under both SUE and UE.
- The out-of-equilibrium belief can be specified as $\psi(\cdot) = 0$.

The proof is similar to (but more complicated than) that of Lemma 2. As such, we highlight the differences between them and simplify the parts that are similar. We first provide a candidate undefeated separating equilibrium.

Lemma W.8. *There exists a separating equilibrium with* $x_H = x_H^{\text{sep}} \equiv \max\{x_H^{\text{ci}}, x_{\text{indiff}}^{\text{sep}}\}$ *and* $x_L = x_L^{\text{sep}} \equiv x_L^{\text{ci}}$*, where* $x_{\text{indiff}}^{\text{sep}} \in (x_L^{\text{ci}}, +\infty)$ *is uniquely determined by*

$$\alpha(rx_L^{\text{ci}} - c(x_L^{\text{ci}})) = R + (1-\alpha)rz_{\text{pe}} + \alpha(rx_{\text{indiff}}^{\text{sep}} - c(x_{\text{indiff}}^{\text{sep}})). \tag{W.3}$$

The out-of-equilibrium belief can be specified as $\psi(x) = 0$ *for any off-equilibrium action* x*. This equilibrium cannot be defeated by any other separating equilibrium. There exists a threshold* $\widehat{\alpha}$ *such that* $x_H^{\text{ci}} \geq x_{\text{indiff}}^{\text{sep}}$ *iff* $\alpha \geq \widehat{\alpha}$*.*

Proof. Note that $x_{\text{indiff}}^{\text{sep}} \to c^{-1}(R/\alpha)$ when $r \to 0$. This extreme case reduces to Lemma A.1. Now let us first verify that $x_H^{\text{sep}} = \max\{x_H^{\text{ci}}, x_{\text{indiff}}^{\text{sep}}\}$ and $x_L^{\text{sep}} = x_L^{\text{ci}}$ constitute a separating equilibrium (in which the low-type firm charges $p_L = V_L + r\alpha x_L^{\text{ci}}$). The pessimistic out-of-equilibrium belief guarantees that the low-type firm has no incentive to deviate to any out-of-equilibrium strategy. She would not deviate to the high-type firm's equilibrium strategy either, for the following reason: If $x_H^{\text{sep}} = x_{\text{indiff}}^{\text{sep}} \geq x_H^{\text{ci}}$, then the low-type firm is indifferent to a deviation.[22] Otherwise ($x_H^{\text{sep}} = x_H^{\text{ci}} > x_{\text{indiff}}^{\text{sep}}$), because $rh(\cdot) - c(\cdot)$ is strictly decreasing over $(x_L^{\text{ci}}, +\infty)$, we have $R + (1-\alpha)rz_{\text{pe}} + \alpha(rx_H^{\text{ci}} - c(x_H^{\text{ci}})) < R + (1-\alpha)rz_{\text{pe}} + \alpha(rx_{\text{indiff}}^{\text{sep}} - c(x_{\text{indiff}}^{\text{sep}})) = \alpha(rx_L^{\text{ci}} - c(x_L^{\text{ci}}))$. The above inequality

[22] We use the subscript "indiff" to represent the *indifference*.

implies that $\pi_L(x_H^{\text{sep}}) < \pi_L(x_L^{\text{sep}})$, and so the low-type firm has no incentive to mimic the high-type firm's equilibrium strategy.

We next examine the high-type firm's incentive to deviate to x_L^{sep} or any out-of-equilibrium strategy under the pessimistic belief $\psi(\cdot) = 0$. If $x_H^{\text{sep}} = x_H^{\text{ci}} \geq x_{\text{indiff}}^{\text{sep}}$, then she achieves her best possible payoff in equilibrium, and so she is not willing to deviate to any other strategy. Otherwise ($x_H^{\text{sep}} = x_{\text{indiff}}^{\text{sep}} > x_H^{\text{ci}}$), there are two cases pertaining to a deviation strategy x: (a) $x < x_{\text{indiff}}^{\text{sep}}$ and (b) $x > x_{\text{indiff}}^{\text{sep}}$. For case (a), the high-type firm can earn directly from the observed activity a payoff $\alpha[(r+\gamma)x - c(x)] < \alpha(\gamma x_H^{\text{sep}} + r x_L^{\text{ci}} - c(x_L^{\text{ci}}))$, because $x < x_{\text{indiff}}^{\text{sep}} = x_H^{\text{sep}}$ and x_L^{ci} maximizes $rh(\cdot) - c(\cdot)$. According to Equation (W.3), $\alpha(\gamma x_H^{\text{sep}} + r x_L^{\text{ci}} - c(x_L^{\text{ci}})) = R + (1-\alpha) r z_{\text{pe}} + \alpha[(r+\gamma) x_H^{\text{sep}} - c(x_H^{\text{sep}})]$, and so $\pi_H(x) < \pi_H(x_H^{\text{sep}})$. For case (b), $x > x_H^{\text{sep}} > x_H^{\text{ci}}$. Since $(r+\gamma)h(\cdot) - c(\cdot)$ is strictly decreasing over $(x_H^{\text{ci}}, +\infty)$, we have $\alpha[(r+\gamma)x - c(x)] < \alpha[(r+\gamma)x_H^{\text{sep}} - c(x_H^{\text{sep}})] < R + (1-\alpha) r z_{\text{pe}} + \alpha[(r+\gamma)x_H^{\text{sep}} - c(x_H^{\text{sep}})] \Longrightarrow \pi_H(x) < \pi_H(x_H^{\text{sep}})$.

Similarly to the proof of Lemma A.1, it can be verified that no other separating equilibrium can defeat the above separating equilibrium.

Finally, we derive the threshold $\widehat{\alpha}$ at which $x_H^{\text{ci}} \geq x_{\text{indiff}}^{\text{sep}}$ iff $\alpha \geq \widehat{\alpha}$: Note that the inequality $R + (1-\alpha) r z_{\text{pe}} + \alpha(r x_H^{\text{ci}} - c(x_H^{\text{ci}})) \leq \alpha(r x_L^{\text{ci}} - c(x_L^{\text{ci}}))$ is equivalent to $R + (1-\alpha) r z_{\text{pe}} + \alpha(r x_H^{\text{ci}} - c(x_H^{\text{ci}}) \leq \alpha(r x_L^{\text{ci}} - c(x_L^{\text{ci}}))$. Since x_H^{ci} and x_L^{ci} are invariant in α, $x_H^{\text{ci}} \geq x_{\text{indiff}}^{\text{sep}}$ iff $\alpha \geq \widehat{\alpha} \equiv (R + r z_{\text{pe}})/[r x_L^{\text{ci}} - c(x_L^{\text{ci}}) - (r x_H^{\text{ci}} - c(x_H^{\text{ci}})) + r z_{\text{pe}}]$. □

The following lemma echoes Lemma A.2.

Lemma W.9. *If*

$$\alpha(r x_L^{\text{ci}} - c(x_L^{\text{ci}})) \geq \phi[R + (1-\alpha) r z_{\text{pe}}] + \alpha(r x_H^{\text{ci}} - c(x_H^{\text{ci}})), \tag{W.4}$$

then there is no pooling equilibrium that can defeat the separating equilibrium stated in Lemma W.8.

Proof. Note that for any $x > x_L^{\text{ci}}$, there exists a unique $x_{\text{indiff}}^{\text{pool}} > x_L^{\text{ci}}$ such that

$$\alpha(r x_L^{\text{ci}} - c(x_L^{\text{ci}})) \gtreqqless \phi[R + (1-\alpha) r z_{\text{pe}}] + \alpha(r x - c(x)) \iff x \gtreqqless x_{\text{indiff}}^{\text{pool}},$$

$$\text{where} \quad \alpha(r x_L^{\text{ci}} - c(x_L^{\text{ci}})) = \phi[R + (1-\alpha) r z_{\text{pe}}] + \alpha(r x_{\text{indiff}}^{\text{pool}} - c(x_{\text{indiff}}^{\text{pool}})). \tag{W.5}$$

No investment level higher than $x_{\text{indiff}}^{\text{pool}}$ can constitute a pooling equilibrium, because otherwise $\pi_L(x_L^{\text{ci}}) >$

$\pi_L(x^{\text{pool}})$. Inequality (W.4) implies that $x^{\text{pool}}_{\text{indiff}} \leq x^{\text{ci}}_H$. Thus, the favorite pooling equilibrium investment for the high-type firm is $x^{\text{pool}}_{\text{indiff}}$.

Comparing Equations (W.5) and (W.3), we can see that $x^{\text{pool}}_{\text{indiff}} < x^{\text{sep}}_{\text{indiff}}$. Recall that in the separating equilibrium, $x^{\text{sep}}_H = \max\{x^{\text{ci}}_H, x^{\text{sep}}_{\text{indiff}}\}$ (see Lemma W.8). If $x^{\text{sep}}_H = x^{\text{sep}}_{\text{indiff}}$, then

$$\begin{aligned}
&\phi[R + (1-\alpha)rz_{\text{pe}}] + V_L + \alpha[(r+\gamma)x^{\text{pool}}_{\text{indiff}} - c(x^{\text{pool}}_{\text{indiff}})] \\
&= \alpha(\gamma x^{\text{pool}}_{\text{indiff}} + rx^{\text{ci}}_L - c(x^{\text{ci}}_L)) + V_L \\
&< \alpha(\gamma x^{\text{sep}}_H + rx^{\text{ci}}_L - c(x^{\text{ci}}_L)) + V_L \\
&= V_H + (1-\alpha)rz_{\text{pe}} + \alpha[(r+\gamma)x^{\text{sep}}_H - c(x^{\text{sep}}_H)].
\end{aligned}$$

Otherwise ($x^{\text{sep}}_H = x^{\text{ci}}_H$),

$$\begin{aligned}
&\phi[R + (1-\alpha)rz_{\text{pe}}] + V_L + \alpha[(r+\gamma)x^{\text{pool}}_{\text{indiff}} - c(x^{\text{pool}}_{\text{indiff}})] \\
&< V_H + (1-\alpha)rz_{\text{pe}} + \alpha[(r+\gamma)x^{\text{sep}}_H - c(x^{\text{sep}}_H)].
\end{aligned}$$

Therefore, the high-type firm always strictly prefers the separating equilibrium outcome. Thus, no pooling equilibrium can defeat the separating equilibrium. □

Similarly to Lemma A.3, it can be verified that the following result holds.

Lemma W.10. *If*

$$\alpha(rx^{\text{ci}}_L - c(x^{\text{ci}}_L)) < \phi[R + (1-\alpha)rz_{\text{pe}}] + \alpha(rx^{\text{ci}}_H - c(x^{\text{ci}}_H)), \tag{W.6}$$

then there exists a pooling equilibrium with $x^{\text{pool}} = x^{\text{ci}}_H$. *The out-of-equilibrium belief can be specified as* $\psi(x) = 0$ *for any off-equilibrium action* x. *The socially responsible firm prefers this pooling equilibrium to any other pooling equilibrium.*

Now we are ready to characterize the equilibrium.

Proof of **Lemma W.7**. According to Lemmas W.8, W.9, and W.10, the separating equilibrium stated in the lemma is undefeated iff one of the following two conditions is satisfied: (a) Inequality (W.4) holds; (b) Inequality (W.6) (the opposite of Inequality (W.4)) holds, and the separating equilibrium is undefeated by the pooling equilibrium stated in Lemma W.10.

In this lemma, we utilize the functional form of the CSR cost $c(z) = \beta z^2/2$. We have $z_{\text{pe}} = \gamma/\beta$, $x_H^{\text{ci}} = (r+\gamma)/\beta$, and $x_L^{\text{ci}} = r/\beta$. Inequality (W.6) can then be written as

$$\frac{\alpha r^2}{2\beta} < \phi\left[R + \frac{(1-\alpha)r\gamma}{\beta}\right] + \frac{\alpha r(r+\gamma)}{\beta} - \frac{\alpha(r+\gamma)^2}{2\beta} \iff \alpha < \frac{2\phi(\beta R + r\gamma)}{\gamma(\gamma+2\phi r)}. \tag{W.7}$$

Thus, to characterize the existence of the separating equilibrium, we need to show only that for $\alpha < \frac{2\phi(\beta R+r\gamma)}{\gamma(\gamma+2\phi r)}$ the socially responsible firm prefers the separating equilibrium outcome iff α is sufficiently large.

According to Equation (W.3), since $x_{\text{indiff}}^{\text{sep}} > x_L^{\text{ci}}$,

$$R + \frac{(1-\alpha)r\gamma}{\beta} + \alpha r x_{\text{indiff}}^{\text{sep}} - \frac{\alpha\beta {x_{\text{indiff}}^{\text{sep}}}^2}{2} = \frac{\alpha r^2}{2\beta} \iff x_{\text{indiff}}^{\text{sep}} = \frac{\alpha r + \sqrt{2\alpha\beta\left[R + \frac{(1-\alpha)r\gamma}{\beta}\right]}}{\alpha\beta}.$$

Under Inequality (W.7) (equivalent to Inequality (W.6)), $\alpha < \frac{2\phi(\beta R+r\gamma)}{\gamma(\gamma+2\phi r)} < \frac{2(\beta R+r\gamma)}{\gamma(\gamma+2r)}$. It can be verified that $\alpha < \frac{2(\beta R+r\gamma)}{\gamma(\gamma+2r)} \iff x_{\text{indiff}}^{\text{sep}} > x_H^{\text{ci}}$. Therefore, for $\alpha < \frac{2\phi(\beta R+r\gamma)}{\gamma(\gamma+2\phi r)}$, $x_H^{\text{sep}} = x_{\text{indiff}}^{\text{sep}} > x_H^{\text{ci}}$. Thus, the socially responsible firm prefers the separating equilibrium outcome iff

$$\begin{aligned} g(\alpha) \equiv R &+ (1-\alpha)rz_{\text{pe}} + \alpha[(r+\gamma)x_{\text{indiff}}^{\text{sep}} - c(x_{\text{indiff}}^{\text{sep}})] \\ &- \phi[R + (1-\alpha)rz_{\text{pe}}] - \alpha[(r+\gamma)x_H^{\text{ci}} - c(x_H^{\text{ci}})] > 0. \end{aligned}$$

To prove that the high-type firm prefers the separating equilibrium outcome iff α is large enough, we need to show only that, when Inequality (W.6) holds ($\alpha < \frac{2\phi(\beta R+r\gamma)}{\gamma(\gamma+2\phi r)}$), there exists a threshold $\widetilde{\alpha}$ such that $g(\alpha) \geqq 0$ iff $\alpha \geqq \widetilde{\alpha}$. For $g(\cdot)$ to cross zero exactly once in going from negative to positive, it is sufficient to show that $g(0) < 0$, $g\left(\frac{2(\beta R+r\gamma)}{\gamma(\gamma+2r)}\right) \geq 0$, and $g''(\alpha) < 0$.

Equation (W.3) implies that

$$\begin{aligned} g(\alpha) &= \alpha(\gamma x_{\text{indiff}}^{\text{sep}} + r x_L^{\text{ci}} - c(x_L^{\text{ci}})) - \phi[R + (1-\alpha)rz_{\text{pe}}] - \alpha[(r+\gamma)x_H^{\text{ci}} - c(x_H^{\text{ci}})] \\ &= \alpha\gamma x_{\text{indiff}}^{\text{sep}} + \frac{\alpha r^2}{2\beta} - \phi\left[R + \frac{(1-\alpha)r\gamma}{\beta}\right] - \frac{\alpha(r+\gamma)^2}{2\beta}. \end{aligned}$$

Thus, we have $g(0) = -\phi\left(R + \frac{r\gamma}{\beta}\right) < 0$, and

$$\begin{aligned} g(\alpha)\big|_{\alpha = \frac{2(\beta R+r\gamma)}{\gamma(\gamma+2r)}} &= \frac{\alpha\gamma(r+\gamma)}{\beta} + \frac{\alpha r^2}{2\beta} - \frac{\phi\left[(\gamma+2r)R + \frac{r\gamma^2 - 2r\beta R}{\beta}\right]}{\gamma+2r} - \frac{\alpha(r+\gamma)^2}{2\beta} \\ &= \frac{\alpha\gamma(r+\gamma)}{\beta} + \frac{\alpha r^2}{2\beta} - \frac{\alpha\phi\gamma^2}{2\beta} - \frac{\alpha(r+\gamma)^2}{2\beta} \\ &= \frac{\alpha(1-\phi)\gamma^2}{2\beta} \geq 0. \end{aligned}$$

The second derivative is

$$
\begin{aligned}
g''(\alpha) = \gamma \cdot \frac{d^2(\alpha x^{\text{sep}}_{\text{indiff}})}{d\alpha^2} &= \frac{\sqrt{2}\gamma}{\beta} \cdot \frac{d^2\left\{\sqrt{\alpha[\beta R + (1-\alpha)r\gamma]}\right\}}{d\alpha^2} \\
&= \frac{\gamma}{\sqrt{2}\beta} \cdot \frac{d\left\{\frac{\beta R + r\gamma - 2\alpha r\gamma}{\sqrt{\alpha[\beta R + (1-\alpha)r\gamma]}}\right\}}{d\alpha} \\
&= \frac{\gamma}{\sqrt{2}\beta} \cdot \frac{-2r\gamma\sqrt{\alpha[\beta R + (1-\alpha)r\gamma]} - \frac{(\beta R + r\gamma - 2\alpha r\gamma)^2}{2\sqrt{\alpha[\beta R + (1-\alpha)r\gamma]}}}{\alpha[\beta R + (1-\alpha)r\gamma]} \\
&= -\frac{\gamma}{\sqrt{2}\beta} \cdot \frac{(\beta R + r\gamma)^2}{2\left\{\alpha[\beta R + (1-\alpha)r\gamma]\right\}^{3/2}} < 0.
\end{aligned}
$$

Therefore, there exists a unique $\widetilde{\alpha}$ such that $g(\alpha) \gtreqless 0$ iff $\alpha \gtreqless \widetilde{\alpha}$, where $\widetilde{\alpha}$ is determined by $g(\widetilde{\alpha}) = 0$.

The proof with regard to the pooling equilibrium is very similar to that of Lemma A.5, and so it is omitted. □