Online Appendix: How fast should trades settle?

Abstract

Recent regulatory and industry initiatives aim to streamline post-trade infrastructures. Does faster settlement benefit markets? We build a model of intermediated trading with imperfectly competitive securities lending. Faster settlement benefits impatient traders but increases borrowing needs. We find that flexible failure-to-deliver penalties reduce this tension, disciplining security lender competition and allowing for real-time settlement. Optimal penalties resemble put options on the lending market: They protect traders against high settlement costs, but do not eliminate failures-to-deliver. Mandating automatic security borrowing to prevent failures-to-deliver triggers a toxic settlement rat race to lock in low borrowing costs.

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JEL Codes: D43, D47, G10, G20

B Proofs

Proposition 1

Proof. The proof follows immediately from the discussion in the main text. For (i), equation (12) pins down the competitive price for intermediaries.

The optimal settlement time in (ii) solves the first order condition

$$\frac{\partial \mathbb{E}\left[U_{\mathbf{B}}\right]}{\partial \tau} = 0, \tag{B.1}$$

where $\mathbb{E}\left[U_{\mathbf{B}}\right]$ is given in equation (13). It follows that:

$$\frac{\partial \mathbb{E}\left[U_{\mathbf{B}}\right]}{\partial \tau} = e^{-\tau(\delta+\lambda)}\left[\frac{\Gamma}{N+1}\left(R-1\right)\sigma^2\left(\delta+\lambda\right) - \delta\theta e^{\lambda\tau}\right], \tag{B.2}$$

and therefore $\tau^\star = \max\left\{0, \frac{1}{\lambda}\log\left[\frac{(\delta+\lambda)}{\theta\delta}\frac{\Gamma}{N+1}\sigma^2\left(R-1\right)\right]\right\}$, that is the expression in equation (14).

The security lenders competitive prices in (iii), that is $\ell_i = \frac{\gamma_i}{2}\sigma^2$, follow from the optimal bidding strategies in second price sealed-bid auctions, a standard result in, e.g., Myerson (1981). Finally, by assumption, the penalty is high enough such that the intermediary always borrows the asset at $\tau^\star$ if she does not own it. ☐

Corollary 1

Proof. Let's introduce an auxiliary variable

$$\tilde{\tau} = \frac{1}{\lambda}\log\left[\frac{(\delta+\lambda)}{\theta\delta}\frac{\Gamma}{N+1}\sigma^2\left(R-1\right)\right] \tag{B.3}$$

so that $\tau^\star = \max\left\{0, \tilde{\tau}\right\}$. We take the partial derivatives of $\tilde{\tau}$ with respect to all the parameters and sign them to establish monotonicity, that is

$$\left(\frac{\partial\tilde{\tau}}{\partial\Gamma}, \frac{\partial\tilde{\tau}}{\partial\sigma}, \frac{\partial\tilde{\tau}}{\partial R}, \frac{\partial\tilde{\tau}}{\partial\delta}, \frac{\partial\tilde{\tau}}{\partial\theta}, \frac{\partial\tilde{\tau}}{\partial N}\right) = \Big(\underbrace{\frac{1}{\Gamma\lambda}}_{>0}, \underbrace{\frac{2}{\sigma\lambda}}_{>0}, \underbrace{\frac{1}{\lambda\left(R-1\right)}}_{>0}, \underbrace{-\frac{1}{\delta\left(\delta+\lambda\right)}}_{<0}, \underbrace{-\frac{1}{\theta\lambda}}_{<0}, \underbrace{-\frac{1}{\lambda\left(N+1\right)}}_{<0}\Big). \tag{B.4}$$

The partial derivative of $\tilde{\tau}$ with respect to the large seller's arrival rate λ is

$$\frac{\partial\tilde{\tau}}{\partial\lambda} = \frac{1}{\lambda^2}\left[\frac{\lambda}{\delta+\lambda} - \log\left(\frac{\Gamma(R-1)\sigma^2(\delta+\lambda)}{(N+1)\delta\theta}\right)\right]. \tag{B.5}$$

Define a function f as

$$f\left(\lambda,\cdot\right) = \frac{\lambda}{\delta+\lambda} - \log\left(\frac{\Gamma(R-1)\sigma^2(\delta+\lambda)}{(N+1)\delta\theta}\right). \tag{B.6}$$

Since $\frac{1}{\lambda^2}$ is positive, it follows that the sign of $\frac{\partial\tilde{\tau}}{\partial\lambda}$ is the same as the sign of $f\left(\lambda,\cdot\right)$. First, we note that $f(\lambda,\cdot)$ decreases in λ since

$$\frac{\partial f(\lambda,\cdot)}{\partial\lambda} = -\frac{\lambda}{(\lambda+\delta)^2} < 0. \tag{B.7}$$

Second, we compute the limits of $f(\lambda,\cdot)$, that is:

$$\lim_{\lambda\to 0} f(\lambda,\cdot) = \log(\theta) - \log\left(\frac{\Gamma}{N+1}\sigma^2(R-1)\right) \text{ and} \tag{B.8}$$

$$\lim_{\lambda\to\infty} f(\lambda,\cdot) = -\infty. \tag{B.9}$$

Since $\theta > \frac{\Gamma}{2}\sigma^2(R-1)$, $\lim_{\lambda\to 0} f(\lambda,\cdot) > 0$. Then since $f(\lambda,\cdot)$ is monotonous it follows that there exists a unique $\Lambda_0 > 0$ such that $\tilde{\tau}$ increases in λ for $\lambda \le \Lambda_0$ and $\tilde{\tau}$ decreases in λ for $\lambda > \Lambda_0$. □

Lemma 1

Proof. First, we state the intermediary's expected utility under imperfect competition on the security lending market:

$$\mathbb{E}\left[U_{\mathbf{I}_j} \mid \tau_j\right] = p(\tau_j) - e^{-\lambda\tau_j}c_N, \tag{B.10}$$

where c_N is the cost-at-settlement faced by the intermediary.

Since intermediaries are competitive, from (B.10) it follows that the equilibrium price schedule is

$$p(\tau_j) = e^{-\lambda\tau_j}c_N. \tag{B.11}$$

From the definition of c_N, we note that the intermediary's borrowing decision depends on the sum between the price and the penalty, and we define $x(\tau) = p(\tau) + z(\tau)$. Let $\alpha_N(\tau) = \mathbb{P}_N(b \le x(\tau))$, that is the probability that each intermediary borrows at settlement, conditional on a given N. Note that the intermediary's cost-at-settlement can be rewritten as

$$c_N = \alpha_N(\tau)\,\mathbb{E}_N[b \mid b \le x(\tau)] + (1-\alpha_N(\tau))\,(p(\tau)+z(\tau)). \tag{B.12}$$

The expected utility of the buyer depends on the borrowing probability α_N, which is a function of N, that is

$$\mathbb{E}[U_{\mathbf{B}}] = e^{-\delta\tau}\left(1-e^{-\lambda\tau}\right)(\theta - p(\tau)) \tag{B.13}$$

$$+\, e^{-(\delta+\lambda)\tau}\left[\alpha_N(\tau)(\theta-p(\tau)) + (1-\alpha_N(\tau))\left(z(\tau) + \frac{\lambda}{\lambda+\delta}\theta\right)\right]. \tag{B.14}$$

Substituting in the competitive price from (B.11) and the intermediary's cost-at-settlement from (B.12), leads to a simplified expression for the buyer's utility, which we denote as $V_N(\cdot)$:

$$V_N(\tau) = e^{-\delta\tau}\left[\left(1-e^{-\lambda\tau}\right)\theta + e^{-\lambda\tau}\left(\frac{\lambda}{\lambda+\delta}\theta + \alpha_N(\tau)\left(\frac{\delta}{\lambda+\delta}\theta - \mathbb{E}_N[b \mid b \le x(\tau)]\right)\right)\right]. \tag{B.15}$$

For a given τ we maximize $\mathbb{E}[U_{\mathbf{B}}]$ with respect to $x(\tau)$. To prove the Lemma, it suffices to show that $V_N(\tau)$ reaches its maximum if $x(\tau) = \frac{\delta}{\lambda+\delta}\theta$, for any N and τ. Dropping the argument τ for exposition purposes, we want to maximize:

$$x^{\star}=\arg\max_{x}\mathbb{P}_{N}\left(b\leq x\right)\left\{\frac{\delta}{\lambda+\delta}\theta-\mathbb{E}\left[b\mid b\leq x\right]\right\}. \tag{B.16}$$

Denote the cumulative distribution of b, conditional on N as $H_{N}\left(\cdot\right):\left[\underline{b}_{N},\bar{b}_{N}\right]\mapsto\left[0,1\right]$ and let $h_{N}\left(\cdot\right)$ be the density function. We can therefore rewrite (B.16) as

$$x^{\star}=\arg\max_{x}\frac{\delta}{\lambda+\delta}\theta H_{N}\left(x\right)-\int_{\underline{b}_{N}}^{x}y\,\mathrm{d}H_{N}\left(y\right). \tag{B.17}$$

From the first order condition, it follows that $x^{\star}$ solves

$$\left(\frac{\delta}{\lambda+\delta}\theta-x^{\star}\right)h_{N}\left(x^{\star}\right)=0, \tag{B.18}$$

and $x^{\star}=\frac{\delta}{\lambda+\delta}\theta$ since $h_{N}\left(x\right)>0$ on the support. Consequently, $p\left(\tau\right)+z\left(\tau\right)=\frac{\delta}{\lambda+\delta}\theta$. □

Lemma 2

Proof. It is enough to show that the lender rent increases in y. First, we show that the sum of the two inner integrals,

$$\Psi\left(\gamma^{(1)},y\right)=\int_{\gamma^{(1)}}^{y}\left(\gamma^{(2)}-\gamma^{(1)}\right)\mathrm{d}F_{2}\left(\gamma^{(2)}\right)+\int_{y}^{\Gamma}\left(y-\gamma^{(1)}\right)\mathrm{d}F_{2}\left(\gamma^{(2)}\right), \tag{B.19}$$

increases in y. Note that we can rewrite $\Psi\left(\gamma^{(1)},y\right)$ as

$$\Psi\left(\gamma^{(1)},y\right)=\int_{\gamma^{(1)}}^{\Gamma}\left(\gamma^{(2)}-\gamma^{(1)}\right)\mathrm{d}F_{2}\left(\gamma^{(2)}\right)-\int_{y}^{\Gamma}\left(\gamma^{(2)}-y\right)\mathrm{d}F_{2}\left(\gamma^{(2)}\right), \tag{B.20}$$

where the first term does not depend on y. The second term decreases in y: From the Leibniz rule,

$$\frac{\mathrm{d}}{\mathrm{d}y}\int_{y}^{\Gamma}\left(\gamma^{(2)}-y\right)\mathrm{d}F_{2}\left(\gamma^{(2)}\right)=-\int_{y}^{\Gamma}\mathrm{d}F_{2}\left(\gamma^{(2)}\right)<0. \tag{B.21}$$

Therefore, $\Psi\left(\gamma^{(1)},y\right)$ increases in y. Next, we write the lender rent as

$$\text{Lender rent}=\frac{\sigma^{2}}{2}\epsilon\times\int_{0}^{y}\Psi\left(\gamma^{(1)},y\right)\mathrm{d}F_{1}\left(\gamma^{(1)}\right). \tag{B.22}$$

Again, from the Leibniz rule, it follows that

$$\frac{\mathrm{d}}{\mathrm{d}y}\int_{0}^{y}\Psi\left(\gamma^{(1)},y\right)\mathrm{d}F_{1}\left(\gamma^{(1)}\right)=\int_{0}^{y}\underbrace{\frac{\partial\Psi\left(\gamma^{(1)},y\right)}{\partial y}}_{>0}\mathrm{d}F_{2}\left(\gamma^{(2)}\right)>0, \tag{B.23}$$

and consequently the lender rent increases in y. Since y increases in δ and θ and decreases in λ, the proof is complete. □

Lemma 3

Proof. Let $y = \frac{2\rho}{\sigma^2(R-1)}$.We compute the partial derivative of c_N with respect to N:

$$\frac{\partial c_N}{\partial N} = -\frac{\sigma^2}{2}(R-1)\frac{2\Gamma + \left(1-\frac{y}{\Gamma}\right)^N \left((N+1)(2\Gamma+(N-1)y)\log\left(1-\frac{y}{\Gamma}\right) + 2(y-\Gamma)\right)}{(N+1)^2}. \tag{B.24}$$

It is enough to show that the numerator in (B.24),

$$2\Gamma + \underbrace{\left(1-\frac{y}{\Gamma}\right)^N \left((N+1)(2\Gamma+(N-1)y)\log\left(1-\frac{y}{\Gamma}\right) + 2(y-\Gamma)\right)}_{\equiv h_0(N)} \tag{B.25}$$

is positive for any $N \geq 2$. First, we show that $h_0(N)$ increases in N by computing its partial derivative,

$$\frac{\partial h_0(N)}{\partial N} = (N+1)\left(1-\frac{y}{\Gamma}\right)^N \log\left(1-\frac{y}{\Gamma}\right)\left((2\Gamma+(N-1)y)\log\left(1-\frac{y}{\Gamma}\right) + 2y\right), \tag{B.26}$$

which is positive if and only if

$$h_1(N) = (2\Gamma+(N-1)y)\log\left(1-\frac{y}{\Gamma}\right) + 2y$$

is negative. Indeed, we show $h_1(N) < 0$ since it decreases in N and is negative for $N = 1$, that is,

$$\begin{aligned} \frac{\partial h_1(N)}{\partial N} &= y\log\left(1-\frac{y}{\Gamma}\right) < 0, \text{ and} \\ h_1(1) &= 2\Gamma\left(\frac{y}{\Gamma} + \log\left(1-\frac{y}{\Gamma}\right)\right) < 0. \end{aligned} \tag{B.27}$$

Therefore $h_0(N)$ increases in N. It is enough to show that $2\Gamma + h_0(1) > 0$ since

$$2\Gamma + h_0(N) > 2\Gamma + h_0(1) > 0, \forall N > 1. \tag{B.28}$$

Let $h_2(y) \equiv 2\Gamma + h_0(1)$, that is

$$h_2(y) = 2y\left(2-\frac{y}{\Gamma}\right) + 4(\Gamma-y)\log\left(1-\frac{y}{\Gamma}\right). \tag{B.29}$$

It follows immediately that $h_2(0) = 0$ and, further, $h_2(y)$ increases in y since

$$\frac{\partial h_2(y)}{\partial y} = -4\left(\log\left(1-\frac{y}{\Gamma}\right) + \frac{y}{\Gamma}\right) > 0. \tag{B.30}$$

Therefore $h_2 > 0$ for any y, and it follows that $2\Gamma + h_0(N) > 0$ for any N, which concludes the proof. □

Proposition 2

Proof. From equation (7) and Lemma 1 we can rewrite the intermediaries' expected utility as:

$$\mathbb{E}\left[U_{\mathbf{I}} \mid \mathbf{I} \text{ survives}\right] = p\left(\tau\right) - e^{-\lambda\tau}\mathbb{E}\left[\min\left\{b, \frac{\delta}{\delta+\lambda}\theta\right\}\right]. \tag{B.31}$$

Since intermediaries are competitive, it follows that the equilibrium price schedule posted by intermediary j is

$$p\left(\tau_j\right) = e^{-\lambda\tau_j}c_N. \tag{B.32}$$

Further, from Lemma 1, if the optimal penalty schedule is implemented, the buyers' expected utility in (4) and (6) is the same irrespective of whether the intermediary borrows at settlement or not, that is:

$$\mathbb{E}\left[U_{\mathbf{B}}\right] = e^{-\delta\tau}\left(\theta - p\left(\tau\right)\right) = e^{-\delta\tau}\left(\theta - e^{-\lambda\tau_j}c_N\right). \tag{B.33}$$

It follows that, since $c_N = \mathbb{E}_N\left[\min\left\{b, \frac{\delta}{\delta+\lambda}\theta\right\}\right]$, the optimal settlement time is

$$\tau_N = \max\left\{0, \frac{1}{\lambda}\log\left(\frac{(\delta+\lambda)}{\theta\delta}\mathbb{E}_N\left[\min\left\{b, \frac{\delta}{\delta+\lambda}\theta\right\}\right]\right)\right\}. \tag{B.34}$$

Since $\mathbb{E}\left[\min\left\{b, \frac{\delta}{\delta+\lambda}\theta\right\}\right] \leq \frac{\delta}{\delta+\lambda}\theta$, it follows that $\tau_N = 0$ for any N.

Using this result in equation (B.32) together with Lemma 1 gives us the competitive price and penalty schedules of the intermediaries as

$$p\left(\tau\right) = e^{-\lambda\tau}c_N \text{ and } z(\tau) = \frac{\delta}{\delta+\lambda}\theta - p\left(\tau\right) \tag{B.35}$$

Repo prices. From Proposition 4 in Riley and Samuelson (1981) (pp. 381), securities lenders bid repo prices that match the value of the hedge, that is, their reservation value: $\ell_i = \frac{\gamma_i}{2}\sigma^2$. □

C A settlement rat race

In this Appendix, we consider a market with two buyers, two intermediaries, and mandatory borrowing. For simplicity of exposition, we assume each buyer trades with a different intermediary at $t = 0$. This is an extension of the benchmark model of Section 4 that allows for strategic interaction between buyers' choice of contracts. Importantly, unlike in Section 5, mandatory borrowing or a "high enough" failure-to-deliver penalty implies that intermediaries always borrow at the settlement time if they do not have the asset. We show that the inelastic supply of lendable securities generates a settlement rat race in which buyers choose sub-optimally short settlement delays.

Since intermediaries always borrow rather than fail to deliver, it follows from equation (7) that the expected utility of intermediary j depends not only on the settlement time of her trade, τ_j, but also on that of her competitor, τ_{-j}:

$$\mathbb{E}\left[U_{\mathbf{I}_j} \mid \tau_j, \tau_{-j},\ \mathbf{I}_j \text{ survives}\right] = \begin{cases} p\left(\tau_j\right) - e^{-\lambda \tau_j} c_N, & \text{if } \tau_j < \tau_{-j} \\ p\left(\tau_j\right) - e^{-\lambda \tau_j} c_{N-1}, & \text{if } \tau_j > \tau_{-j} \\ p\left(\tau_j\right) - e^{-\lambda \tau_j} \frac{c_N + c_{N-1}}{2}, & \text{if } \tau_j = \tau_{-j}, \end{cases} \tag{C.1}$$

where c_N is the cost-at-settlement faced by the first intermediary to settle and c_{N-1} is the cost-at-settlement of the second intermediary. If both intermediaries settle at the same time (i.e., $\tau_j = \tau_{-j}$) they are equally likely to approach security lenders first. Intermediaries' cost-at-settlement (equal to the expected borrowing cost in this case since borrowing is mandatory) depends on the order in which they access the repo market. As established in Lemma 3, repo market rents decrease in the number of security lenders available, that is, $c_N < c_{N-1}$. Therefore, for intermediaries and buyers, settling the trade before anyone else becomes valuable.

Let us denote by $G(\cdot)$ the symmetric equilibrium cumulative distribution function of the settlement times chosen by the two buyers. Since $G(\cdot)$ could be discontinuous if buyers put positive probability on a particular time-to-settlement, let $\tau^- = \lim_{\varepsilon \downarrow 0} (\tau - \varepsilon)$ such that $G(\tau) - G(\tau^-) = \text{Prob}(\tau_j = \tau)$. Since intermediaries are competitive, it follows that the equilibrium price schedule is

$$p\left(\tau_j\right) = e^{-\lambda \tau_j} \left[G\left(\tau_j^-\right) c_{N-1} + \left(G\left(\tau_j\right) - G\left(\tau_j^-\right)\right) \frac{c_N + c_{N-1}}{2} + \left(1 - G\left(\tau_j\right)\right) c_N \right]. \tag{C.2}$$

The competitive price is equal to the expected borrowing cost on the repo market. With probability $G\left(\tau_j^-\right)$, $\tau_j > \tau_{-j}$ and intermediary j is the second to borrow, that is she borrows at cost c_{N-1}. With probability $1 - G\left(\tau_j\right)$, $\tau_j < \tau_{-j}$ and intermediary j is the first to borrow, that is she borrows at cost c_N. With the complementary probability, $\tau_{-j} = \tau_j$ and each intermediary is equally likely to borrow at c_N or c_{N-1}.

Definition C.1. (Benchmark utilities.) Let $V_N(\tau)$ denote the expected utility of each buyer if the repo borrowing cost for competitive intermediaries is fixed at c_N, and let τ_N be the optimal settlement time that maximizes $V_N(\tau)$:

$$V_N(\tau) \equiv \mathbb{E}\left[U_{\mathbf{B}} \mid \text{borrowing cost is } c_N\right] = e^{-\delta \tau}\left(\theta - e^{-\lambda \tau} c_N\right) \text{ and}$$

$$\tau_N \equiv \arg\max_{\tau} V_N(\tau) = \max\left\{0, \frac{1}{\lambda} \log\left[\frac{(\delta + \lambda)}{\theta \delta} c_N\right]\right\}. \tag{C.3}$$

The benchmark utilities and corresponding optimal settlement delays are equivalent to those in Section 4 for $N > 0$. It immediately follows from Lemma 3 that $V_N(\tau) > V_{N-1}(\tau)$ for all $\tau \geq 0$ and that $\tau_N \leq \tau_{N-1}$, with equality only if $\tau_N = \tau_{N-1} = 0$. The intuition is the same as in the benchmark equilibrium: buyers are overall better off, and would optimally settle faster, when borrowing costs for the intermediary are lower.

From equations (4), (C.2), and using Definition C.1, we can write the expected buyer utility as a linear combination of benchmark utilities, that is

$$\mathbb{E}\left[U_{\mathbf{B}_j}\right] = G\left(\tau_j^-\right) V_{N-1}(\tau) + \left(G(\tau_j) - G\left(\tau_j^-\right)\right) \frac{V_N(\tau) + V_{N-1}(\tau)}{2} + \left(1 - G(\tau_j)\right) V_N(\tau). \tag{C.4}$$

Lemma C.1. (Buyers' strategy) *If $V_{N-1}(0) + V_N(0) \geq 2V_{N-1}(\tau_{N-1})$, both buyers choose immediate settlement with probability $\pi = 1$.*

If $V_{N-1}(0) + V_N(0) < 2V_{N-1}(\tau_{N-1})$, buyers choose immediate settlement with probability $\pi < 1$, where

$$\pi = \max\left\{0, 2\frac{V_N(0) - V_{N-1}(\tau_{N-1})}{V_N(0) - V_{N-1}(0)}\right\}. \tag{C.5}$$

With probability $1-\pi$, each buyer chooses a random settlement time in $[\underline{\tau}, \tau_{N-1}]$ where $\underline{\tau} < \tau_N$ is pinned down by:

$$V_{N-1}(\tau_{N-1}) = (1-\pi) V_N(\underline{\tau}) + \pi V_{N-1}(\underline{\tau}), \tag{C.6}$$

such that the buyer's expected utility is $V_{N-1}(\tau_{N-1})$.

The proof of Lemma C.1 follows at the end of this Appendix.

From Lemma C.1, two types of equilibria emerge depending on the model parameters: an immediate-settlement equilibrium and a delayed-settlement equilibrium. In the immediate-settlement equilibrium, buyers choose the pure strategy $\tau^\star = 0$ and intermediaries always enter repo contracts with security lenders. From equation (C.2), the price $p(0)$ reflects the average cost of borrowing, that is,

$$p(0) = \frac{1}{2}(c_N + c_{N-1}). \tag{C.7}$$

In the delayed-settlement equilibrium, buyers randomize over possible times-to-settlement, using the cumulative distribution function

$$G(\tau) = \begin{cases} \pi, & \text{if } \tau \in [0, \underline{\tau}) \\ \frac{V_N(\tau) - V_{N-1}(\tau_{N-1})}{V_N(\tau) - V_{N-1}(\tau)}, & \text{if } \tau \in [\underline{\tau}, \tau_{N-1}] \\ 1, & \text{if } \tau > \tau_{N-1}. \end{cases} \tag{C.8}$$

A settlement "rat race" emerges in which each buyer chooses a short settlement delay, so that his trade is more likely to be settled first. Note that, from equation (C.3), a settlement delay choice $\tau > \tau_{N-1}$ is always sub-optimal for buyers. Therefore, by choosing τ_{N-1} a buyer is sure to be the second to settle his trade. This pins down the equilibrium utility: In any mixed strategy (i.e., delayed settlement) equilibrium, buyers earn $V_{N-1}(\tau_{N-1})$.

The expected utility for buyers, across both the immediate- and delayed-settlement equilibria is therefore

$$\mathbb{E}\left[U_{\mathbf{B}}\right] = \max\left\{ \underbrace{V_{N-1}\left(\tau_{N-1}\right)}_{\substack{\text{delayed-settlement}\\ \text{equilibrium}}}, \underbrace{\frac{1}{2}\left[V_N\left(0\right) + V_{N-1}\left(0\right)\right]}_{\substack{\text{immediate-settlement}\\ \text{equilibrium}}} \right\}. \tag{C.9}$$

Proposition C.1 formally states the equilibrium of the trading game for $N \geq 2$.

> **Proposition C.1.** (Rat race equilibrium) *In a model with two buyers, two intermediaries, and if the late-delivery penalty z is sufficiently large such that* (**BC**) *is always true, the following strategies form an equilibrium:*
>
> *(i) Intermediaries post $\{\tau, p(\tau)\}$ contract schedules with $\tau \geq 0$ and $p(\tau)$ as defined in equation* (C.2)*, where the equilibrium $G(\tau)$ is given in equation* (C.8)*.*
>
> *(ii) If $V_{N-1}(0) + V_N(0) \geq 2V_{N-1}(\tau_{N-1})$, buyers choose immediate settlement ($\tau^\star = 0$). Otherwise, buyers choose a random time-to-settlement from the equilibrium cumulative distribution $G(\tau)$ defined in equation* (C.8)*.*
>
> *(iii) Securities lenders bid repo prices $\ell_i = \frac{\gamma_i}{2}\sigma^2, \forall i \in \{1, 2, \ldots, N\}$.*
>
> *(iv) Intermediaries always borrow the asset at settlement time if the large seller did not arrive before settlement.*

The proof of Proposition C.1 follows immediately from the discussion above. First, equation (C.2), evaluated at the equilibrium buyer's strategy $G(\tau)$ given in (C.8), implies that the intermediaries earn zero expected profit if they rationally anticipate the buyers' and security lenders' equilibrium behaviour. Second, Lemma C.1 solves for the equilibrium buyers' strategy given competitive intermediaries and the equilibrium lenders' behaviour. Third, the equilibrium behaviour of security lenders is described in Section 5.1. Finally, we assume intermediaries always borrow the asset at settlement if the large seller did not arrive.

We can measure the cost of the settlement rat race for buyers by comparing their equilibrium expected utility in (C.9) with a benchmark utility. The rat race emerges either if buyers trade simultaneously at $t = 0$, or they cannot observe each other's choices, or both. Therefore, a natural utility benchmark is to consider the optimal choices buyers would make if they arrive sequentially rather than simultaneously, that is τ_N and τ_{N-1}. Let

$$\mathcal{C} = \underbrace{V_N\left(\tau_N\right) + V_{N-1}\left(\tau_{N-1}\right)}_{\text{benchmark utility}} - \underbrace{\max\left\{2V_{N-1}\left(\tau_{N-1}\right), V_N\left(0\right) + V_{N-1}\left(0\right)\right\}}_{\text{equilibrium buyers' utility}} \tag{C.10}$$

be a measure of the settlement rat race cost for the buyers.

The settlement rat race leads to buyers choosing immediate settlement in equilibrium even for relatively low trading urgency, for which they would optimally choose delayed settlement in a perfect information setting (where optimal settlement delay choices are τ_N and τ_{N-1}). Therefore, there is excessive demand for immediate settlement in equilibrium.

Robustness. In this section, we assume both buyers arrive to the market and contact intermediaries at $t = 0$. However, the settlement rat race also obtains under less restrictive assumptions. Opacity, for example, is a natural assumption in OTC markets: If the buyers' arrival times are drawn from the same distribution, such that each is equally likely to trade first, and if they have imperfect information about whether the other already traded, then from Sublemmas C.2 through C.4, it follows immediate settlement still emerges as the unique pure strategy equilibrium if $\frac{1}{2}\left[V_N\left(0\right)+V_{N-1}\left(0\right)\right] \geq V_{N-1}\left(\tau_{N-1}\right)$.

Further, a rat race can also emerge if buyers arrive sequentially to the market (i.e, in a Stackelberg-type game). To illustrate, we sketch the proof for the case where $V_N\left(0\right) > V_{N-1}\left(\tau_{N-1}\right)$. Without loss of generality, let the early buyer's arrival time be $t_1 = 0$ and the late buyer's arrival time be $t_2 > 0$. If the early buyer chooses $\tau' > 0$, then a late buyer arriving one instant τ' optimally chooses immediate settlement and earns $V_N\left(0\right)$, which is larger than any profit he could have earned by settling second. Indeed, if the late buyer arrives before the first trade is settled, he is always better off by jumping ahead of the early buyer. The early buyer, in turn, prefers a lower τ' to reduce the probability that $t_2 < \tau'$, that is, that the late buyer will settle first.

Proof of Lemma C.1

Proof.

Sublemma C.1. *Buyers never choose* $\tau > \tau_{N-1}$.

Proof. Since $\tau_{N-1} > \tau_N$, $V_N\left(\tau_{N-1}\right) > V_N\left(\tau\right)$ for all $\tau > \tau_{N-1}$. Therefore, from the definition of τ_{N-1}, for any $\alpha \in [0,1]$,

$$\alpha V_N\left(\tau_{N-1}\right) + \left(1-\alpha\right)V_{N-1}\left(\tau_{N-1}\right) > \alpha V_N\left(\tau\right) + \left(1-\alpha\right)V_{N-1}\left(\tau\right), \forall \tau > \tau_{N-1}. \tag{C.11}$$

It follows that deviations from $\tau > \tau_{N-1}$ to τ_{N-1} are profitable for any probability of being the first to settle the trade. □

Sublemma C.2. *If* $\frac{1}{2}\left[V_N\left(0\right)+V_{N-1}\left(0\right)\right] \geq V_{N-1}\left(\tau_{N-1}\right)$, *then* $\left(\tau_j^\star, \tau_{-j}^\star\right) = (0,0)$ *is a pure strategy equilibrium.*

Proof. If both buyers choose immediate settlement, they are equally likely to settle first or second, and have expected utility $\frac{1}{2}\left(V_N\left(0\right)+V_{N-1}\left(0\right)\right)$. If any buyer unilaterally deviates to $\tau > 0$, then he is sure to settle after his competitor and can earn at most $V_{N-1}\left(\tau_{N-1}\right)$. Therefore, no deviation from $\tau = 0$ is profitable. □

Sublemma C.3. *Buyers never choose a specific* $\tau > 0$ *with positive probability.*

Proof. Assume buyers choose $\tau' > 0$ with positive probability η. With probability η^2, both buyers select τ' and earn $\frac{\eta^2}{2}\left(V_N\left(\tau'\right)+V_{N-1}\left(\tau'\right)\right)$. A buyer can increase his expected profit by putting zero weight on τ' and choosing instead $\tau' - \varepsilon \geq 0$ with positive probability, for a sufficiently small ε. Such deviation increases the expected profit to $\eta^2 V_N\left(\tau'-\varepsilon\right)$. □

Sublemma C.4. *The upper bound of the support for any equilibrium mixed strategy of* $\boldsymbol{B}$, *if such a strategy exists, is* τ_{N-1}.

Proof. Assume there exists an equilibrium mixed strategy with cumulative distribution function $G(\tau)$. Further, assume $\overline{\tau} \neq \tau_{N-1}$ is the smallest time-to-settlement such that $G(\overline{\tau}) = 1$, that is the upper bound of the mixed strategy support. From Sublemma C.1 it must be that $\overline{\tau} < \tau_{N-1}$. If a buyer chooses $\overline{\tau}$, he is sure to be the second to settle and obtain $V_{N-1}(\overline{\tau})$. If the buyer instead deviates to τ_{N-1}, since $G(\tau_{N-1}) = 1$, he also is the second to settle but obtains the higher utility level $V_{N-1}(\tau_{N-1}) > V_{N-1}(\overline{\tau})$ (by the definition of τ_{N-1}). Therefore, it must be that $\overline{\tau} = \tau_{N-1}$. □

Corollary C.1. *If there exists an equilibrium mixed strategy for* $\boldsymbol{B}$*, it yields expected utility* $V_{N-1}(\tau_{N-1})$.

Proof. From Sublemma C.4, if an equilibrium mixed strategy exists, then τ_{N-1} is in the mixing support and further $G(\tau_{N-1}) = 1$. Therefore, by choosing τ_{N-1} a buyer is guaranteed to be second to settle his trade and earns expected utility $V_{N-1}(\tau_{N-1})$, which is necessarily the expected utility of any other time-to-settlement in the mixed strategy support. □

Sublemma C.5. *If* $\frac{1}{2}[V_{N-1}(0) + V_N(0)] \geq V_{N-1}(\tau_{N-1})$*, then* $\left(\tau_j^\star, \tau_{-j}^\star\right) = (0,0)$ *is the unique equilibrium.*

Proof. From Sublemma C.2, $\left(\tau_j^\star, \tau_{-j}^\star\right) = (0,0)$ is a pure strategy equilibrium yielding expected utility $\frac{1}{2}[V_{N-1}(0) + V_N(0)]$. Sublemma C.3 further establishes that $\left(\tau_j^\star, \tau_{-j}^\star\right) = (0,0)$ is the unique pure strategy equilibrium. It remains to be shown there are no mixed strategy equilibria. If there is one, from Corollary C.1 it yields utility $V_{N-1}(\tau_{N-1})$, which is weakly lower than the utility of the pure strategy equilibrium, that is $\frac{1}{2}[V_{N-1}(0) + V_N(0)]$. Therefore, there can be no mixed strategy equilibrium. □

Sublemma C.5 completes the proof for the first part of Lemma C.1. In what follows, we discuss the buyers' equilibrium strategies if $V_{N-1}(\tau_{N-1}) > \frac{1}{2}[V_{N-1}(0) + V_N(0)]$.

Sublemma C.6. *If* $V_{N-1}(\tau_{N-1}) > V_N(0)$*, then the unique equilibrium is in mixed strategies: buyers randomize between* $\underline{\tau}$ *and* τ_{N-1}*, where* $\underline{\tau}$ *solves*

$$V_N(\underline{\tau}) = V_{N-1}(\tau_{N-1}). \tag{C.12}$$

Proof. First off, if $V_{N-1}(\tau_{N-1}) > V_N(0)$ then since (i) $V_N(\tau_N) > V_{N-1}(\tau_{N-1})$, and (ii) $V_N(\tau)$ increases on $[0, \tau_N]$, it follows that there exists a strictly positive $\underline{\tau} < \tau_N$ such that $V_N(\underline{\tau}) = V_{N-1}(\tau_{N-1})$.

Second, $\underline{\tau}$ must be the lower bound of any equilibrium mixed strategy support. Assume instead $\underline{\tau}^- < \underline{\tau}$ is the lower bound. If the buyer chooses $\underline{\tau}^-$, then he is the first to settle and obtains $V_N(\underline{\tau}^-)$. Since $\underline{\tau} < \tau_N$, it follows that $V_N(\underline{\tau}^-) < V_N(\underline{\tau}) = V_{N-1}(\tau_{N-1})$, which is the expected utility of a mixed strategy equilibrium, from Corollary C.1. Assume now $\underline{\tau}^+ > \underline{\tau}$ is the lower bound. A buyer can profitably deviate by choosing $\underline{\tau} + \varepsilon < \underline{\tau}^+$, then he is the first to settle and obtains $V_N(\underline{\tau} + \varepsilon) > V_N(\underline{\tau}) = V_{N-1}(\tau_{N-1})$.

Third, no buyer can profitably deviate to immediate settlement. Choosing $\tau = 0$ ensures that a buyer is first to settle and gain $V_N(0) < V_{N-1}(\tau_{N-1})$. □

If $\frac{1}{2}[V_{N-1}(0) + V_N(0)] < V_{N-1}(\tau_{N-1}) \leq V_N(0)$ then:

1. Buyers are better off by playing a mixed strategy yielding expected utility $V_{N-1}(\tau_{N-1})$ than by choosing immediate settlement in pure strategies.
2. However, if the competitor plays such a mixed strategy and chooses a random time to settlement between $\underline{\tau} > 0$ and τ_{N-1}, then immediate settlement is a profitable deviation as it guarantees being first to settle the trade and yields expected utility $V_N(0)$.

It follows that buyers choose immediate settlement with some positive probability $\pi < 1$. The expected utility of $\mathbf{B}_j$ from choosing $\tau = 0$ is:

$$\mathbb{E}\left[U_{\mathbf{B}_j}(\tau = 0)\right] = \underbrace{\frac{\pi}{2}\left[V_{N-1}(0) + V_N(0)\right]}_{\mathbf{B}_{-j}\text{ chooses }\tau=0} + \underbrace{(1-\pi)V_N(0)}_{\mathbf{B}_{-j}\text{ chooses }\tau>0}. \tag{C.13}$$

The expected utility from choosing immediate settlement needs to be equal, in equilibrium, with the expected utility of any other settlement time choice in the mixed distribution support. Therefore, π is pinned down by

$$\frac{\pi}{2}\left[V_{N-1}(0) + V_N(0)\right] + (1-\pi)V_N(0) = V_{N-1}(\tau_{N-1}). \tag{C.14}$$

If a buyer chooses the lower bound of the mixed strategy distribution, $\underline{\tau}$, then he settles first with probability $1-\pi$. Therefore, since all points in the mixed strategy support yield the same expected utility, $\underline{\tau}$ solves

$$(1-\pi)V_N(\underline{\tau}) + \pi V_{N-1}(\underline{\tau}) = V_{N-1}(\tau_{N-1}). \tag{C.15}$$

Note that the two mixed strategy cases, that is $\frac{1}{2}\left[V_{N-1}(0) + V_N(0)\right] < V_{N-1}(\tau_{N-1}) \leq V_N(0)$ and $V_{N-1}(\tau_{N-1}) > V_N(0)$, can be put together if define π as

$$\pi = \max\left\{0, 2\frac{V_N(0) - V_{N-1}(\tau_{N-1})}{V_N(0) - V_{N-1}(0)}\right\}. \tag{C.16}$$

$\square$

D Endogenous borrowing with two buyers

In this Appendix, we solve for the endogenous borrowing equilibrium (as in Section 5.2) for a market with two buyers and two intermediaries. First, we prove that the result in Lemma 1 is unchanged. Second, we state and prove the equivalent of Proposition 2. We find that real-time settlement is still optimal.

Lemma D.1. (Optimal borrowing decision) *Buyers optimally offer incentives for intermediaries to borrow if and only if* $b \leq \frac{\delta}{\delta+\lambda}\theta$. *Furthermore, in any contract implementing the borrowing decision that maximizes the buyers' expected utility, it is the case that*

$$p\left(\tau\right) + z\left(\tau\right) = \frac{\delta}{\delta+\lambda}\theta. \tag{D.1}$$

Proof. The expected utility of intermediary j depends not only on the settlement time of her trade, τ_j, but also on that of her competitor, τ_{-j}:

$$\mathbb{E}\left[U_{\mathbf{I}_j} \mid \tau_j, \tau_{-j},\ \mathbf{I}_j \text{ survives}\right] = \begin{cases} p\left(\tau_j\right) - e^{-\lambda\tau_j}c_N, & \text{if } \tau_j < \tau_{-j} \\ p\left(\tau_j\right) - e^{-\lambda\tau_j}c_{N-1}, & \text{if } \tau_j > \tau_{-j} \\ p\left(\tau_j\right) - e^{-\lambda\tau_j}\frac{c_N + c_{N-1}}{2}, & \text{if } \tau_j = \tau_{-j}, \end{cases} \tag{D.2}$$

where c_N is the cost-at-settlement faced by the first intermediary to settle and c_{N-1} is the cost-at-settlement of the second intermediary. We assume that if both intermediaries settle at the same time (i.e., $\tau_j = \tau_{-j}$) they are equally likely to approach security lenders first.

To better manipulate the intermediary's expected utility, we need to conjecture a distribution for the settlement delays. Without loss of generality, let $G\left(\cdot\right)$ denote the symmetric equilibrium cumulative distribution function of the settlement times chosen by the two buyers. Since $G\left(\cdot\right)$ could be discontinuous if buyers put positive probability on a particular time-to-settlement, let $\tau^{-} = \lim_{\varepsilon\downarrow 0}\left(\tau - \varepsilon\right)$ such that $G\left(\tau\right) - G\left(\tau^{-}\right) = \text{Prob}\left(\tau_j = \tau\right)$. Since intermediaries are competitive, from (D.2) it follows that the equilibrium price schedule is

$$p\left(\tau_j\right) = e^{-\lambda\tau_j}\left[G\left(\tau_j^{-}\right)c_{N-1} + \left(G\left(\tau_j\right) - G\left(\tau_j^{-}\right)\right)\frac{c_N + c_{N-1}}{2} + \left(1 - G\left(\tau_j\right)\right)c_N\right]. \tag{D.3}$$

From the definition of c_N, we note that the intermediary's borrowing decision depends on the sum between the price and the penalty, and we define $x\left(\tau\right) = p\left(\tau\right) + z\left(\tau\right)$. Let $\alpha_N\left(\tau\right) = \mathbb{P}_N\left(b \leq x\left(\tau\right)\right)$, that is the probability that each intermediary borrows at settlement, conditional on a given N. Note that the intermediary's cost-at-settlement can be rewritten as

$$c_N = \alpha_N\left(\tau\right)\mathbb{E}_N\left[b \mid b \leq x\left(\tau\right)\right] + \left(1 - \alpha_N(\tau)\right)\left(p(\tau) + z(\tau)\right). \tag{D.4}$$

The expected utility of the buyer depends on the borrowing probability α_N, which is a function of N. In turn, the maximum number of security lenders at the market, N, is a function of the settlement delay order, and therefore of $G(\tau)$. It follows that the expected utility of the buyer is

$$\begin{aligned}\mathbb{E}\left[U_{\mathbf{B}}\right] &= e^{-\delta\tau}\left(1-e^{-\lambda\tau}\right)\left(\theta-p\left(\tau\right)\right) \qquad\qquad \text{(D.5)}\\ &+ e^{-(\delta+\lambda)\tau}\Big\{G\left(\tau^{-}\right)\left[\alpha_{N-1}\left(\tau\right)\left(\theta-p\left(\tau\right)\right)+\left(1-\alpha_{N-1}\left(\tau\right)\right)\left(z\left(\tau\right)+\frac{\lambda}{\lambda+\delta}\theta\right)\right]\\ &+ \left(1-G\left(\tau\right)\right)\left[\alpha_{N}\left(\tau\right)\left(\theta-p\left(\tau\right)\right)+\left(1-\alpha_{N}\left(\tau\right)\right)\left(z\left(\tau\right)+\frac{\lambda}{\lambda+\delta}\theta\right)\right]\\ &+ \frac{1}{2}\left(G\left(\tau\right)-G\left(\tau^{-}\right)\right)\Big\{\left[\alpha_{N}\left(\tau\right)\left(\theta-p\left(\tau\right)\right)+\left(1-\alpha_{N}\left(\tau\right)\right)\left(z\left(\tau\right)+\frac{\lambda}{\lambda+\delta}\theta\right)\right]\\ &+ \left[\alpha_{N-1}\left(\tau\right)\left(\theta-p\left(\tau\right)\right)+\left(1-\alpha_{N-1}\left(\tau\right)\right)\left(z\left(\tau\right)+\frac{\lambda}{\lambda+\delta}\theta\right)\right]\Big\}\Big\}.\end{aligned}$$

Substituting in the competitive price from (D.3) and the intermediary's cost-at-settlement from (D.4), leads to a simplified expression for the buyer's utility:

$$\mathbb{E}\left[U_{\mathbf{B}}\right] = G\left(\tau^{-}\right)V_{N-1}\left(\tau\right)+\left(G\left(\tau_{j}\right)-G\left(\tau^{-}\right)\right)\frac{V_{N}\left(\tau\right)+V_{N-1}\left(\tau\right)}{2}+\left(1-G\left(\tau\right)\right)V_{N}\left(\tau\right), \tag{D.6}$$

where $V_N(\cdot)$ denotes

$$V_{N}\left(\tau\right)=e^{-\delta\tau}\left[\left(1-e^{-\lambda\tau}\right)\theta+e^{-\lambda\tau}\left(\frac{\lambda}{\lambda+\delta}\theta+\alpha_{N}\left(\tau\right)\left(\frac{\delta}{\lambda+\delta}\theta-\mathbb{E}_{N}\left[b\mid b\leq x\left(\tau\right)\right]\right)\right)\right]. \tag{D.7}$$

For a given τ we maximize $\mathbb{E}\left[U_{\mathbf{B}}\right]$ with respect to $x(\tau)$. To prove the Lemma, it suffices to show that $V_N(\tau)$ reaches its maximum if $x(\tau)=\frac{\delta}{\lambda+\delta}\theta$, for any N and τ. The argument proceeds as in the proof of Lemma D.1. ☐

Proposition D.1. (Equilibrium, endogenous penalty.) *If buyers and intermediaries can contract on failure-to-deliver penalties, then the following strategies form an equilibrium:*

(i) Intermediaries offer payment and penalty schedules such that

$$p\left(\tau\right)=\begin{cases}\frac{1}{2}\left(c_{N}+c_{N-1}\right), & \text{if } \tau=0,\\ e^{-\lambda\tau}c_{N-1}, & \text{if } \tau>0\end{cases} \quad \text{and } z(\tau)=\frac{\delta}{\delta+\lambda}\theta-p\left(\tau\right), \tag{D.8}$$

where $c_N=\mathbb{E}_N\left[\min\left\{b,\frac{\delta}{\delta+\lambda}\theta\right\}\right]$ and the closed form expression is given in Lemma 3.

(ii) Buyers choose immediate (real-time) settlement.

(iii) Securities lenders bid repo prices $\ell_i=\frac{\gamma_i}{2}\sigma^2, \forall i\in\{1,2,\ldots,N\}$.

(iv) Intermediaries only borrow the asset if $b\leq\dfrac{\delta}{\delta+\lambda}\theta$.

Proof. From equation (7) and Lemma D.1 we can rewrite the intermediaries' expected utility as:

$$\mathbb{E}\left[U_{\mathbf{I}} \mid \mathbf{I} \text{ survives}\right] = p\left(\tau\right) - e^{-\lambda\tau}\mathbb{E}\left[\min\left\{b, \frac{\delta}{\delta+\lambda}\theta\right\}\right]. \tag{D.9}$$

Let $\mathbb{P}\left(\tilde{\tau} \leq \tau\right)$ be the conjectured symmetric equilibrium cumulative distribution of the settlement times chosen by the two buyers. This distribution could be discontinuous if buyers put positive probability on a particular time-to-settlement. With probability $\mathbb{P}(\tilde{\tau}_{-j} < \tau_j)$, intermediary j is the second to borrow (after her competitor $-j$), that is she faces cost-at-settlement c_{N-1}, as introduced in Definition 1. With probability $\mathbb{P}(\tilde{\tau}_{-j} > \tau_j)$, intermediary j is the first to borrow, that is she faces cost-at-settlement c_N. With the complementary probability, $\tilde{\tau}_{-j} = \tau_j$ and each intermediary is equally likely to be first or second with cost-at-settlement c_N or c_{N-1}.

Since intermediaries are competitive, it follows that the equilibrium price schedule posted by intermediary j is

$$p\left(\tau_j\right) = e^{-\lambda\tau_j}\left[\mathbb{P}(\tilde{\tau}_{-j} < \tau_j)c_{N-1} + \mathbb{P}(\tilde{\tau}_{-j} = \tau_j)\frac{c_N + c_{N-1}}{2} + \mathbb{P}(\tilde{\tau}_{-j} > \tau_j)c_N\right]. \tag{D.10}$$

Further, from Lemma D.1, if the optimal penalty schedule is implemented, the buyers' expected utility in (4) and (6) is the same irrespective of whether the intermediary borrows at settlement or not, that is:

$$\mathbb{E}\left[U_{\mathbf{B}}\right] = e^{-\delta\tau}\left(\theta - p\left(\tau\right)\right). \tag{D.11}$$

From equations (D.10) and (D.11), the buyer's expected utility can be written as follows

$$\mathbb{E}\left[U_{\mathbf{B}}\right] = e^{-\delta\tau}\Big(\mathbb{P}(\tilde{\tau} < \tau)(\theta - e^{-\lambda\tau}c_{N-1}) + \mathbb{P}(\tilde{\tau} = \tau)(\theta - e^{-\lambda\tau}\frac{c_N + c_{N-1}}{2}) + \mathbb{P}(\tilde{\tau} > \tau)(\theta - e^{-\lambda\tau}c_N)\Big). \tag{D.12}$$

Let $V_N\left(\tau\right) = e^{-\delta\tau}\left(\theta - e^{-\lambda\tau}c_N\right)$.[1] It follows that we can rewrite the expected utility more simply as:

$$\mathbb{E}\left[U_{\mathbf{B}}\right] = \mathbb{P}(\tilde{\tau} < \tau)V_{N-1}\left(\tau\right) + \mathbb{P}(\tilde{\tau} > \tau)V_N\left(\tau\right) + \mathbb{P}(\tilde{\tau} = \tau)\frac{V_{N-1}\left(\tau\right) + V_N\left(\tau\right)}{2}. \tag{D.13}$$

We note two properties of $V_N\left(\tau\right)$. First, from Lemma 3, since the cost-at-settlement satisfies $c_N < c_{N-1}$, it follows that $V_N\left(\tau\right) > V_{N-1}\left(\tau\right)$. Second, from the definitions of c_N and V_N, the settlement time τ'_N that maximizes V_N is

$$\tau'_N = \max\left\{0, \frac{1}{\lambda}\log\left(\frac{(\delta+\lambda)}{\theta\delta}\mathbb{E}_N\left[\min\left\{b, \frac{\delta}{\delta+\lambda}\theta\right\}\right]\right)\right\}. \tag{D.14}$$

Since $\mathbb{E}\left[\min\left\{b, \frac{\delta}{\delta+\lambda}\theta\right\}\right] \leq \frac{\delta}{\delta+\lambda}\theta$, it follows that $\tau'_N = 0$ for any N.

Real-time settlement is the unique equilibrium. We show next that real-time settlement ($\tau^\star = 0$) is a pure-strategy equilibrium. The buyers' utility from real-time settlement is, from equation (D.13),

$$\mathbb{E}\left[U_{\mathbf{B}} \mid (\tau_j, \tau_{-j}) = (0, 0)\right] = \frac{V_{N-1}\left(0\right) + V_N\left(0\right)}{2}. \tag{D.15}$$

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[1] We will use this notation again in Appendix C.

If buyer j deviates and sets $\tau_j > 0$, then he earns $V_{N-1}(\tau_j) < V_{N-1}(0)$. Since $V_N(0) > V_{N-1}(0)$, such a deviation is not profitable. It follows that $\tau^\star = 0$ is a pure strategy equilibrium for buyers.

We turn next to prove the uniqueness of the equilibrium time-to-settlement choice. First, let us note that buyers never choose a specific $\tau > 0$ with positive probability. If buyers choose $\tau' > 0$ with positive probability η, then with probability η^2, both buyers select τ' and earn $\frac{\eta^2}{2}\left(V_N(\tau') + V_{N-1}(\tau')\right)$. A buyer can increase his expected profit by putting zero weight on τ' and choosing instead $\tau' - \varepsilon \geq 0$ with positive probability, for a sufficiently small ε. Such deviation increases the expected profit to $\eta^2 V_N(\tau' - \varepsilon)$. Therefore, the expected utility of the buyer for any $\tau > 0$ becomes:

$$\mathbb{E}[U_{\mathbf{B}}] = \mathbb{P}(\tilde{\tau} < \tau)V_{N-1}(\tau) + \mathbb{P}(\tilde{\tau} > \tau)V_N(\tau), \text{ for } \tau > 0 \tag{D.16}$$

The expected utility of buyer j from choosing $\tau = 0$, conjecturing a general mixing distribution for the buyer $-j$, is:

$$\begin{aligned}\mathbb{E}[U_{\mathbf{B}}] &= \underbrace{\mathbb{P}(\tilde{\tau} < 0)V_{N-1}(0)}_{=0} + \mathbb{P}(\tilde{\tau} > 0)V_N(0) + \mathbb{P}(\tilde{\tau} = 0)\frac{V_{N-1}(0) + V_N(0)}{2} \\ &= \left[\mathbb{P}(\tilde{\tau} > 0) + \frac{1}{2}\mathbb{P}(\tilde{\tau} = 0)\right] V_N(0) + \frac{1}{2}\mathbb{P}(\tilde{\tau} = 0)V_{N-1}(0). \end{aligned} \tag{D.17}$$

Since $V_N(\tau) > V_{N-1}(\tau)$, $V_{N-1}(0) > V_{N-1}(\tau)$, $V_N(0) > V_{N-1}(0)$, and $\mathbb{P}(\tilde{\tau} > 0) > \mathbb{P}(\tilde{\tau} > \tau)$ for any $\tau > 0$, it follows that it is optimal for a buyer to deviate to $\tau = 0$ regardless of his competitor's strategy. Therefore, real-time settlement is the unique equilibrium.

Price and penalty schedules. Buyers follow a pure strategy and thus the equilibrium cumulative distribution function of the settlement times is of the form

$$\mathbb{P}(\tilde{\tau} \leq \tau) = \begin{cases} 1, & \tau \geq 0, \\ 0, & \text{elsewhere.} \end{cases} \tag{D.18}$$

Using this result in equation (D.10) together with Lemma D.1 gives us the competitive price and penalty schedules of the intermediaries as

$$p(\tau) = \begin{cases} \frac{1}{2}(c_N + c_{N-1}), & \text{if } \tau = 0, \\ e^{-\lambda\tau}c_{N-1}, & \text{if } \tau > 0 \end{cases} \quad \text{and } z(\tau) = \frac{\delta}{\delta + \lambda}\theta - p(\tau) \tag{D.19}$$

Repo prices. From Proposition 4 in Riley and Samuelson (1981) (pp. 381), securities lenders bid repo prices that match the value of the hedge, that is, their reservation value: $\ell_i = \frac{\gamma_i}{2}\sigma^2$. $\square$

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