For Online Publication
Appendix for 'Real and Nominal Equilibrium Yield Curves

1 U.S. and U.K. Inflation-Linked Bonds and Macroeconomic Data

We use quarterly data from January 1985 to September 2008 for the U.S. and the U.K., and report statistics for the periods 1985-2008 and 1999-2008. The data sample periods are motivated by two reasons. First, TIPS data in the U.S. and inflation-linked gilts data in the U.K. are only available since 1999 and 1985, respectively.[1] Second, the period September-December 2008 coincides with the collapse of Lehman Brothers that drove short-term interest rates close to zero, and triggered a switch to unconventional monetary policies. The period after September 2008 is then not covered to focus on the effects on bond yields of a (conventional) monetary policy conducted using an interest-rate rule.

The consumption growth and inflation series for the U.S. are constructed using quarterly data from the Bureau of Economic Analysis, following the methodology in Piazzesi and Schneider (2007). These series capture only consumption of non-durables and services and its related inflation, and then consistent with the model variables. Wages are real wages per hour of non-farm business from the Federal Reserve Economic Data (FRED) database from the Federal Reserve Bank of St. Louis. The source for labor hours is non-farm business hours worked per capita from the BLS. The real yields from the Federal Reserve Bank of Cleveland can be found here: https://www.clevelandfed.org/our-research/indicators-and-data/inflation-expectations.aspx. Please see Haubrich, Pennacchi and Ritchken (2011) for the methodology. The data on U.S. zero-coupon nominal bond and TIPS yields are constructed following the procedure in Gurkaynak, Sack and Wright (2006, 2008), respectively. These data are obtained from the Federal Reserve website.[2] The short-term nominal interest rate is the 3-month T-bill rate from the Fama risk-free rates database. Dividends and stock market returns correspond to the market portfolio obtained from the Center for Research

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[1] Results using comparable monthly data are very similar. We present results for quarterly data to be consistent with the model estimation. The same macroeconomic and term structure data for the United States are used to estimate the model, for the longer period January 1982 to September 2008.

[2] https://www.federalreserve.gov/pubs/feds/2006/200628/200628abs.html and https://www.federalreserve.gov/pubs/feds/2008/200805/200805abs.html, respectively.

in Security Prices (CRSP). For the U.K., consumption growth and inflation are obtained directly from the FRED database. The historical yields for U.K. real and nominal bonds are taken from the Bank of England website.[3] The bond yields under study correspond to maturities from 2 to 10 years. The long end of the curves has been excluded for comparison purposes across countries. Greenwood and Vayanos (2010) document a significant effect on long-term inflation-linked bond yields in the U.K, resulting from the increased demand from pension funds to meet the Minimum Funding Requirements. Table 1 summarizes the empirical evidence.

Finally, to calibrate the model, the series we use in the estimation for the real yield curve comes from Chernov and Mueller (2012) until 2004 and TIPS data from the Federal Reserve Board since 2004.

2 Model

2.1 The pricing kernel

$$V_t = (1-\beta)U(C_{h,t}, N_t^s)^{1-\varphi} + \beta \mathbb{E}_t \left[V_{t+1}^{\frac{1-\gamma}{1-\varphi}} \right]^{\frac{1-\varphi}{1-\gamma}},$$

where

$$U(C_{h,t}, N_t^s) = \left(\frac{C_{h,t}^{1-\varphi}}{1-\varphi} + \kappa_t \frac{\left(\bar{L} - N_t^s\right)^{1-\omega}}{1-\omega} \right)^{\frac{1}{1-\varphi}},$$

The intertemporal marginal rate of substitution, i.e., the real pricing kernel is given by

$$M_{t,t+1} = \frac{\partial V_t / \partial C_{t+1}}{\partial V_t / \partial C_t},$$

where

$$\begin{aligned}
\frac{\partial V_t}{\partial C_t} &= (1-\beta)U'(C_t) \\
\frac{\partial V_t}{\partial C_{t+1}} &= \frac{\partial V_t}{\partial V_{t+1}} \frac{\partial V_{t+1}}{\partial C_{t+1}} \\
&= \beta(1-\beta)U'(C_{t+1}) \mathbb{E}_t \left[V_{t+1}^{\frac{1-\gamma}{1-\varphi}} \right]^{\frac{\gamma-\varphi}{1-\gamma}} V_{t+1}^{\frac{\varphi-\gamma}{1-\varphi}} \\
&= \beta_t(1-\beta)U'(C_{t+1}) \left(\mathbb{E}_t \left[V_{t+1}^{(1-\gamma)/(1-\psi)} \right]^{1/(1-\gamma)} \right)^{\psi-\gamma} V_{t+1}^{(\psi-\gamma)/(1-\psi)}.
\end{aligned}$$

It thus can be easily shown that

$$M_{t,t+1} \equiv \beta_t \frac{U'(C_{t+1})}{U'(C_t)} \left(\frac{V_{t+1}^{\frac{1}{1-\psi}}}{\left(\mathbb{E}_t \left[V_{t+1}^{\frac{1-\gamma}{1-\psi}} \right] \right)^{\frac{1}{1-\gamma}}} \right)^{\psi-\gamma}$$

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[3] https://www.bankofengland.co.uk/statistics/yield-curves.

2.2 Optimal wages

Let $W_t^*(k)$ be the optimal price set by the household for labor type k.

$$\frac{\partial V_t}{\partial W_t^*(k)} = (1-\beta)U_t^{-\varphi}\frac{\partial U_t}{\partial W_t^*(k)} + \beta\frac{\partial}{\partial W_t^*(k)}\left[\mathbb{E}_t\left[V_{t+1}^{\frac{1-\gamma}{1-\varphi}}\right]^{\frac{1-\varphi}{1-\gamma}}\right].$$

$$\begin{aligned}\frac{\partial U_{t+s}}{\partial W_t^*(k)} &= U_{t+s}^{\varphi}\kappa_{t+s}(\bar{L}-N_{t+s}^s)^{-\omega}(-1)\frac{\partial N_{t+s}^s}{\partial W_t^*(k)} \\ &= -U_{t+s}^{\varphi}\kappa_{t+s}(\bar{L}-N_{t+s}^s)^{-\omega}\frac{\partial N_{t+s}^s}{\partial W_t^*(k)}\end{aligned}$$

$$\begin{aligned}\beta\frac{\partial}{\partial W_t^*(k)}\left[\mathbb{E}_{t+s}\left[V_{t+s+1}^{\frac{1-\gamma}{1-\varphi}}\right]^{\frac{1-\varphi}{1-\gamma}}\right] &= \left(\frac{1-\varphi}{1-\gamma}\right)\beta\mathbb{E}_{t+s}\left[V_{t+s+1}^{\frac{1-\gamma}{1-\varphi}}\right]^{\frac{1-\varphi}{1-\gamma}-1}\mathbb{E}_{t+s}\left[\left(\frac{1-\gamma}{1-\varphi}\right)V_{t+s+1}^{\frac{\varphi-\gamma}{1-\varphi}}\frac{\partial V_{t+s+1}}{W_t^*(k)}\right] \\ &= \mathbb{E}_{t+s}\left[\beta\left(\frac{V_{t+s+1}^{\frac{1}{1-\varphi}}}{\mathbb{E}_{t+s}\left[V_{t+s+1}^{\frac{1-\gamma}{1-\varphi}}\right]^{\frac{1}{1-\gamma}}}\right)^{\varphi-\gamma}\frac{\partial V_{t+s+1}}{W_t^*(k)}\right]\end{aligned}$$

For $s=1$:

$$\frac{\partial V_{t+1}}{\partial W_t^*(k)} = (1-\beta)U_{t+1}^{-\varphi}\frac{\partial U_{t+1}}{\partial W_t^*(k)} + \beta\frac{\partial}{\partial W_t^*(k)}\left[\mathbb{E}_{t+1}\left[V_{t+2}^{\frac{1-\gamma}{1-\varphi}}\right]^{\frac{1-\varphi}{1-\gamma}}\right],$$

such that

$$\begin{aligned}\frac{\partial V_t}{\partial W_t^*(k)} &= (1-\beta)U_t^{-\varphi}\frac{\partial U_t}{\partial W_t^*(k)} + \mathbb{E}_t\left[\beta\left(\frac{V_{t+1}^{\frac{1}{1-\varphi}}}{\mathbb{E}_t\left[V_{t+1}^{\frac{1-\gamma}{1-\varphi}}\right]^{\frac{1}{1-\gamma}}}\right)^{\varphi-\gamma}\left[(1-\beta)U_{t+1}^{-\varphi}\frac{\partial U_{t+1}}{\partial W_t^*(k)}+\right.\right. \\ &\quad \left.\left.\mathbb{E}_{t+1}\left[\beta\left(\frac{V_{t+2}^{\frac{1}{1-\varphi}}}{\mathbb{E}_{t+1}\left[V_{t+2}^{\frac{1-\gamma}{1-\varphi}}\right]^{\frac{1}{1-\gamma}}}\right)^{\varphi-\gamma}\left[(1-\beta)U_{t+2}^{-\varphi}\frac{\partial U_{t+2}}{\partial W_t^*(k)}+\ldots\right]\right]\right]\right] \\ &= -(1-\beta)\sum_{s=0}^{\infty}\mathbb{E}_t\left[\prod_{i=o}^{s}\left\{\beta\left(\frac{V_{t+i}^{\frac{1}{1-\varphi}}}{\mathbb{E}_{t+i}\left[V_{t+i+1}^{\frac{1-\gamma}{1-\varphi}}\right]^{\frac{1}{1-\gamma}}}\right)^{\varphi-\gamma}\right\}\right. \\ &\quad \left.\kappa_{t+s}(\bar{L}-N_{t+s}^s)^{-\omega}\frac{\partial N_{t+s}^s}{\partial W_t^*(k)}\right] \\ &= -(1-\beta)\sum_{s=0}^{\infty}\mathbb{E}_t\left[M_{t,t+s}^{\$}\left(\frac{C_{h,t+s}}{C_{h,t}}\right)^{\varphi}\left(\frac{P_{t+s}}{P_t}\right)\kappa_{t+s}\right. \\ &\quad \left.(\bar{L}-N_{t+s}^s)^{-\omega}\frac{\partial N_{t+s}^s}{\partial W_t^*(k)}\right].\end{aligned}$$

Let the Lagrangian be

$$\mathfrak{L}_t = V_t + \lambda \mathbb{E}_t \left[\sum_{s=0}^{\infty} M_{t,t+s}^{\$} P_{t+s} (LI_{t+s} + D_{t+s} + C_{h,t+s}) \right].$$

then

$$\frac{\partial \mathfrak{L}_t}{\partial W_t^*(k)} = \frac{\partial V_t}{\partial W_t^*(k)} + \lambda \mathbb{E}_t \left[\sum_{s=0}^{\infty} M_{t,t+s}^{\$} P_{t+s} \frac{\partial LI_{t+s}}{\partial W_t^*(k)} \right].$$

Define

$$N_{t+s}^s = \int_0^1 N_{t+s}^s(k) dk = N_{t+s}^d \int_0^1 \left(\frac{W_{t+s}(k)}{W_{t+s}} \right)^{-\theta_w} dk,$$

and

$$LI_{t+s} = \int_0^1 \frac{W_{t+s}(k)}{P_{t+s}} N_{t+s}(k) dk = N_{t+s}^d \frac{W_{t+s}}{P_{t+s}} \int_0^1 \left(\frac{W_{t+s}(k)}{W_{t+s}} \right)^{1-\theta_w} dk.$$

Furthermore, $Prob(W_{t+s}(k) = W_t^*(k) \Lambda_{w,t,t+s}) = (1-\alpha_w)\alpha_w^s$, such that

$$\frac{\partial N_{t+s}^s}{\partial W_t^*(k)} = N_{t+s}^d (1-\alpha_w) \alpha_w^s \frac{-\theta_w}{W_t^*(k)} \left(\frac{W_t^*(k) \Lambda_{w,t,t+s}}{W_{t+s}} \right)^{-\theta_w},$$

and

$$\frac{\partial LI_{t+s}}{\partial W_t^*(k)} = \frac{N_{t+s}^d}{P_{t+s}} (1-\alpha_w) \alpha_w^s (1-\theta_w) \left(\frac{W_t^*(k) \Lambda_{w,t,t+s}}{W_{t+s}} \right)^{-\theta_w} \Lambda_{w,t,t+s}.$$

Setting the F.O.C. of the Lagrangian to zero and replace the derivatives:

$$\begin{aligned}
& -(1-\beta) \sum_{s=0}^{\infty} \mathbb{E}_t \left[M_{t,t+s}^{\$} \left(\frac{C_{h,t+s}}{C_{h,t}} \right)^{\varphi} \left(\frac{P_{t+s}}{P_t} \right) \kappa_{t+s} \right. \\
& \left. (\bar{L} - N_{t+s}^s)^{-\omega} N_{t+s}^d (1-\alpha_w) \alpha_w^s \frac{-\theta_w}{W_t^*(k)} \left(\frac{W_t^*(k) \Lambda_{w,t,t+s}}{W_{t+s}} \right)^{-\theta_w} \right] \\
= \quad & -\lambda \mathbb{E}_t \left[\sum_{s=0}^{\infty} M_{t,t+s}^{\$} P_{t+s} \frac{N_{t+s}^d}{P_{t+s}} (1-\alpha_w) \alpha_w^s (1-\theta_w) \left(\frac{W_t^*(k) \Lambda_{w,t,t+s}}{W_{t+s}} \right)^{-\theta_w} \Lambda_{w,t,t+s} \right].
\end{aligned}$$

Since $\lambda_t = (1-\beta) \frac{1}{P_t} C_{h,t}^{-\varphi}$ and $\mu_w = \frac{\theta_w}{\theta_w - 1}$, we can rewrite the F.O.C. to

$$\begin{aligned}
& \frac{1}{P_t} C_{h,t}^{-\varphi} W_t^*(k) \mathbb{E}_t \left[\sum_{s=0}^{\infty} M_{t,t+s}^{\$} \alpha_w^s \Lambda_{w,t,t+s}^{-\theta_w} N_{t+s}^d W_{t+s}^{\theta_w} \right] \\
= \quad & \mu_w \mathbb{E}_t \left[\sum_{s=0}^{\infty} M_{t,t+s}^{\$} \alpha_w^s \Lambda_{w,t,t+s}^{1-\theta_w} W_{t+s}^{\theta_w} \left(\frac{C_{h,t+s}}{C_{h,t}} \right)^{\varphi} \left(\frac{P_{t+s}}{P_t} \right) \kappa_{t+s} (\bar{L} - N_{t+s}^s)^{-\omega} N_{t+s}^d \right].
\end{aligned}$$

In equilibrium, $W_t^*(k) = W_t^*$, therefore,

$$L.H.S. = \frac{W_t^*}{P_t} C_{h,t}^{-\varphi} W_t^{\theta_w} N_t^d H_{w,t},$$

where

$$H_{w,t} = 1 + \alpha_w \mathbb{E}_t \left[M^{\$}_{t,t+1} \Lambda^{-\theta_w}_{w,t,t+1} \frac{N^d_{t+1}}{N^d_t} \left(\frac{W_{t+1}}{W_t} \right)^{\theta_w} H_{w,t+1} \right].$$

As well as,

$$R.H.S. = \mu_w W_t^{\theta_w} \kappa_t^{1-\varphi} (1 - N^s_t)^{-\omega} N^d_t G_{w,t},$$

where

$$\begin{aligned} G_{w,t} &= 1 + \alpha_w \mathbb{E}_t \left[M^{\$}_{t,t+1} \Lambda^{1-\theta_w}_{w,t,t+1} \frac{N^d_{t+1}}{N^d_t} \left(\frac{W_{t+1}}{W_t} \right)^{\theta_w} \left(\frac{C_{h,t+1}}{C_{h,t}} \right)^{\varphi} \left(\frac{P_{t+1}}{P_t} \right) \left(\frac{\kappa_{t+1}}{\kappa_t} \right) \right. \\ & \left. \left(\frac{\bar{L} - N^s_{t+1}}{\bar{L} - N^s_t} \right)^{-\omega} G_{w,t+1} \right]. \end{aligned}$$

Together, the equilibrium condition is

$$\frac{W^*_t}{P_t} = \mu_w \kappa_t \frac{C^{\varphi}_{h,t}}{(\bar{L} - N^s_t)^{\omega}} \frac{G_{w,t}}{H_{w,t}},$$

2.3 Endogenous Growth

The production function for firm i is

$$Y_t = \left(K^{\alpha}_{t-1} \left(Z_t N_t \right)^{1-\alpha} \right)^{1-\xi} J^{\xi}_t,$$

where

$$J_t = \left[\int_0^{A_t} X^{g\ \nu}_{i,t} di \right]^{\frac{1}{\nu}}.$$

Optimizing output over $X^g_{i,t}$:

$$\frac{\partial Y_t}{\partial X^g_{i,t}} = P^g_{i,t} = \left(K^{\alpha}_{t-1} \left(Z_t N_t \right)^{1-\alpha} \right)^{1-\xi} \frac{\xi}{\nu} \left[\int_0^{A_t} X^{g\ \nu}_{i,t} di \right]^{\frac{\xi}{\nu}-1} \nu X^{g\ \nu-1}_{i,t}.$$

Under symmetric equilibrium, $X^g_{i,t} = X^g_t$ and $P^g_{i,t} = P^g_t = \frac{1}{\nu}$ such that,

$$\frac{1}{\nu} = \left(K^{\alpha}_{t-1} \left(Z_t N_t \right)^{1-\alpha} \right)^{1-\xi} \xi A_t^{\frac{\xi}{\nu}-1} X^{g\,\xi-\nu}_t X^{g\,\nu-1}_t \Rightarrow X^g_t = \left[\xi\nu \left(K^{\alpha}_{t-1} \left(Z_t N_t \right)^{1-\alpha} \right)^{1-\xi} A_t^{\frac{\xi}{\nu}-1} \right]^{\frac{1}{1-\xi}}.$$

As a result, the production function becomes

$$\begin{aligned} Y_t &= \left(K^{\alpha}_{t-1} \left(Z_t N_t \right)^{1-\alpha} \right)^{1-\xi} A_t^{\frac{\xi}{\nu}} X^{g\,\xi}_t \\ &= \left(K^{\alpha}_{t-1} \left(Z_t N_t \right)^{1-\alpha} \right)^{1-\xi} A_t^{\frac{\xi}{\nu}} \left[\xi\nu \left(K^{\alpha}_{t-1} \left(Z_t N_t \right)^{1-\alpha} \right)^{1-\xi} A_t^{\frac{\xi}{\nu}-1} \right]^{\frac{\xi}{1-\xi}} \\ &= (\xi\nu)^{\frac{\xi}{1-\xi}} K^{\alpha}_{t-1} \left(Z_t N_t \right)^{1-\alpha} A_t^{\frac{\xi/\nu-\xi}{1-\xi}} \\ &= (\xi\nu)^{\frac{\xi}{1-\xi}} K^{\alpha}_{t-1} \left(Z_t A_t N_t \right)^{1-\alpha}, \end{aligned}$$

where we restrict $1-\alpha = \frac{\xi/\nu-\xi}{1-\xi}$.

The law of motion for the stock of intangible capital follows:

$$A_{t+1} = (1-\delta_a)A_t + \vartheta_t S_t,$$

where

$$\vartheta_t = \frac{\chi A_t}{S_t^{1-\eta} A_t^{\eta}}.$$

Under profit maximization, the value of patent i is

$$V_{i,t} = \Pi_{i,t} + (1-\delta_a)\mathbb{E}_t\left[M_{t+1}V_{i,t+1}\right] \Rightarrow V_t = \Pi_t + (1-\delta_a)\mathbb{E}_t\left[M_{t+1}V_{t+1}\right],$$

such that

$$\Pi_{i,t} = \Pi_t = \left(\frac{1}{\nu} - 1\right) X_t^g.$$

Finally, the optimality condition for R&D investment is

$$S_t = \mathbb{E}_t\left[M_{t+1}V_{t+1}\right]\left(A_{t+1} - (1-\delta_a)A_t\right),$$

which can be explained by equating marginal cost with marginal benefit of investment in intangible capital $\frac{1}{\vartheta_t} = \mathbb{E}_t\left[M_{t+1}V_{t+1}\right]$.

2.4 Optimal prices

The production of differentiated goods is characterized by monopolistic competition and price rigidities in a continuum of firms. Firms set the price of their differentiated goods in a Calvo (1983) staggered price setting: At each time t, with probability α_p, a firm sets the price of the good as the previous period price adjusted by the price indexation factor $\Lambda_{p,t-1,t}$. The specific functional form of this factor is presented in Section 4 of the paper. With probability $1-\alpha_p$, the firm sets the product price to maximize the present value of profits. The maximization problem for firm k can be written as

$$\max_{\{P_t^*\}} \mathbb{E}_t\left\{\sum_{s=0}^{\infty} \alpha_p^s M_{t,t+s}^{\$}\left[\Lambda_{p,t,t+s}P_t^* Y_{t+s}(k) - P_{w,t+s}Y_{t+s}\right]\right\}, \tag{1}$$

subject to the production function

$$Y_{t+s}(k) = Y_{t+s}, \tag{2}$$

and the demand function

$$Y_{t+s}(k) = \left(\frac{P_t^*\Lambda_{p,t,t+s}}{P_{t+s}}\right)^{-\theta_p} Y_{t+s}^{final}. \tag{3}$$

The output $Y_{t+s}(k)$ is the production of firm k at time $t+s$. The production problem takes into account the probability of not being able to adjust the price optimally in the future, and the corresponding indexation $\Lambda_{p,t,t+s}$.

The first order condition can be derived after taking partial derivative with respect to P_t^*:

$$\mathbb{E}_t\left\{\sum_{s=0}^{\infty}\alpha_p^s M_{t,t+s}^{\$}\left[\Lambda_{p,t,t+s}Y_{t+s}(k)+P_{t+s}^*(-\theta_p)\Lambda_{p,t,t+s}\frac{Y_{t+s}}{P_{t+s}^*}\right]\right\} \tag{4}$$

$$= \quad \mathbb{E}_t\left\{\sum_{s=0}^{\infty}\alpha_p^s M_{t,t+s}^{\$}\left[P_{w,t+s}(-\theta_p)\frac{Y_{t+s}}{P_{t+s}^*}\right]\right\} \tag{5}$$

$$\Rightarrow \qquad P_{t+s}^*\mathbb{E}_t\left\{\sum_{s=0}^{\infty}\alpha_p^s M_{t,t+s}^{\$}\Lambda_{p,t,t+s}Y_{t+s}(k)\right\} \tag{6}$$

$$= \quad \underbrace{\frac{\theta_p}{\theta_p-1}}_{\mu_p}\mathbb{E}_t\left\{\sum_{s=0}^{\infty}\alpha_p^s M_{t,t+s}^{\$}P_{w,t+s}Y_{t+s}\right\} \tag{7}$$

The final equilibrium condition is

$$\left(\frac{P_t^*}{P_t}\right)H_{p,t} \quad = \quad \mu_p\frac{P_{w,t}}{P_t}G_{p,t}, \tag{8}$$

where $\mu_p = \frac{\theta_p}{\theta_p-1}$. The recursive equations for $H_{p,t}$ and $G_{p,t}$ are written explicitly in Appendix 2.6. Equation (8) can be interpreted as follows: In the absence of price rigidities, the product price is the markup-adjusted marginal cost of production, with optimal markup μ_p. Price rigidities generate the time-varying markup $\mu_p\frac{G_{p,t}}{H_{p,t}}$, since some firms do not adjust their prices optimally.

2.5 Bond Risk Premia

Consider the no arbitrage equation for the n-period real bond:

$$B_t^{c,(n)} \quad = \quad e^{-nr_t^{(n)}} = \mathbb{E}_t\left[M_{t,t+1}B_{t+1}^{c,(n-1)}\right] = \mathbb{E}_t\left[e^{m_{t,t+1}-(n-1)r_{t+1}^{(n-1)}}\right],$$

where $m_{t,t+1} \equiv \log M_{t,t+1}$. Assuming normality and homoskedasticity for the log-pricing kernel and bond yields, it follows that

$$e^{-nr_t^{(n)}} \quad = \quad \mathbb{E}_t\left[e^{m_{t,t+1}}\right]\mathbb{E}_t\left[e^{-(n-1)r_{t+1}^{(n-1)}}\right]e^{-\mathrm{cov}_t(m_{t,t+1},(n-1)r_{t+1}^{(n-1)})}.$$

The equation above also implies

$$nr_t^{(n)} = r_t - \frac{1}{2}\mathrm{var}_t\left((n-1)r_{t+1}^{(n-1)}\right) + rTP_t^{(n)} + \mathbb{E}_t\left[(n-1)r_{t+1}^{(n-1)}\right].$$

Solving for the last term iteratively and applying unconditional expectations, we get

$$rTP_t^{(n)} = \mathrm{cov}_t\left(\log M_{t,t+1},(n-1)r_{t+1}^{(n-1)}\right). \tag{9}$$

Consider the inflation risk premium in equation (11) for $n=1$,

$$\pi TP_t^{(1)} = \mathrm{cov}_t(m_{t,t+1},\pi_{t+1}) = i_t - r_t + \log\mathbb{E}_t[\exp(-\pi_{t,t+1})]. \tag{10}$$

In general, the inflation risk premium in equation (11) can be written in terms of bond yields as

$$\begin{aligned}\pi TP_t^{(n)} &= n(i_t^{(n)} - r_t^{(n)}) + \log \mathbb{E}_t \left[e^{\left(-(n-1) i_{t+1}^{(n-1)}\right)} \right] - \log \mathbb{E}_t \left[e^{\left(-(n-1) r_{t+1}^{(n-1)}\right)} \right] \\ &+ \log \mathbb{E}_t [e^{(-\pi_{t,t+1})}] + \mathrm{cov}_t \left((n-1) i_{t+1}^{(n-1)}, \pi_{t+1} \right).\end{aligned}$$

From equation (10), the recursive bond pricing equation

$$e^{-n i_t^{(n)}} = e^{-i_t} \mathbb{E}_t \left[e^{-(n-1) i_{t+1}^{(n-1)}} \right] e^{-\mathrm{cov}_t \left(m_{t,t+1}^{\$}, (n-1) i_{t+1}^{(n-1)} \right)},$$

where $m_{t,t+1}^{\$} \equiv \log M_{t,t+1}^{\$}$, and a similar equation for the comparable real bond yield, it follows that

$$\begin{aligned}\pi TP_t^{(n)} &= \pi TP_t^{(1)} + \mathrm{cov}_t \left(m_{t,t+1}^{\$}, (n-1) i_{t+1}^{(n-1)} \right) - \mathrm{cov}_t \left(m_{t,t+1}, (n-1) i_{t+1}^{(n-1)} \right) \\ &\quad + \mathrm{cov}_t \left(\pi_{t+1}, (n-1) i_{t+1}^{(n-1)} \right) \\ &= \pi TP_t^{(1)} + \mathrm{cov}_t \left(m_{t,t+1}^{\$}, (n-1) \left(i_{t+1}^{(n-1)} - r_{t+1}^{(n-1)} \right) \right),\end{aligned}$$

where the second equality follows from $m_{t,t+1} = m_{t,t+1}^{\$} + \pi_{t+1}$. Realizing that under log-normality and homoskedasticity assumptions the nominal-real bond spread is

$$(n-1) \left(i_{t+1}^{(n-1)} - r_{t+1}^{(n-1)} \right) = \sum_{s=1}^{n-1} \mathbb{E}_t[\pi_{t+s}] - \frac{1}{2} \mathrm{var}_t \left(\sum_{s=1}^{n-1} \pi_{t+s} \right) - \mathrm{cov}_t \left(\sum_{s=1}^{n-1} m_{t,t+s}, \sum_{s=1}^{n-1} \pi_{t+s} \right).$$

Since the variance and covariance terms are constant, it follows that

$$\pi TP_t^{(n)} = \mathrm{cov}_t \left(m_{t,t+1}, \sum_{s=1}^{n-1} \pi_{t+s} \right).$$

Computing the unconditional expectation of the nominal-real bond spread above and replacing the covariance terms for the one-period inflation risk premia, we get

$$\pi TP_t^{(n)} \equiv \log \frac{\mathbb{E}_t \left[\exp(-\pi_{t,t+n}) \right]}{B_t^{\$,(n)}} - \log \frac{1}{B_t^{(n)}} = \mathrm{cov}_t \left(m_{t,t+n}, \pi_{t,t+n} \right), \tag{11}$$

2.6 Equilibrium

$$\begin{aligned}X_t^g &= (\xi \nu)^{\frac{1}{1-\xi}} K_{t-1}^{\alpha} \left(Z_t N_t \right)^{1-\alpha} A_t^{\frac{\xi/\nu - 1}{1-\xi}}. \\ A_{t+1} &= \left[(1-\delta_a) + \chi \left(\frac{S_t}{A_t} \right)^{\eta} \right] A_t. \\ V_t &= \left(\frac{1}{\nu} - 1 \right) X_t^g + (1-\delta_a) \mathbb{E}_t \left[M_{t+1} V_{t+1} \right]. \\ \left(\frac{S_t}{A_t} \right)^{1-\eta} &= \mathbb{E}_t \left[M_{t+1} V_{t+1} \right] \chi.\end{aligned}$$

Capital accumulation

$$K_t = (1-\delta)K_{t-1} + \Phi\left(\frac{I_t}{K_{t-1}}\right)K_{t-1}.$$

Adjustment cost

$$\Phi\left(\frac{I_t}{K_{t-1}}\right) = b_1 + \frac{b_2}{1-1/\zeta}\left(\frac{I_t}{K_{t-1}}\right)^{1-1/\zeta}.$$

Marginal product of labor

$$L_t = (1-\xi)\frac{(1-\alpha)Y_t}{W_t N_t^d}.$$

Return on Investment

$$\begin{aligned}
1 &= \mathbb{E}_t[M_{t,t+1}R_{t+1}^I] \\
R_t^I Q_{t-1} &= (1-\xi)\frac{\alpha Y_t}{L_t K_{t-1}} + Q_t\left(1-\delta+\Phi_t-\Phi_t'\frac{I_t}{K_{t-1}}\right) \\
1 &= Q_t\Phi_t'
\end{aligned}$$

Wage setting

$$\begin{aligned}
\frac{W_t^*}{P_t} &= \mu_w\kappa_t\left(\bar{L}-N_t^s\right)^{-\omega}C_{h,t}^{\varphi}\frac{G_{w,t}}{H_{w,t}}, \\
H_{w,t} &= 1+\alpha_w\mathbb{E}_t\left[M_{t,t+1}^{\$}\Lambda_{w,t,t+1}^{-\theta_w}\left(\frac{N_{t+1}^d}{N_t^d}\right)\left(\frac{W_t}{W_{t+1}}\right)^{-\theta_w}H_{w,t+1}\right], \\
G_{w,t} &= 1+\alpha_w\mathbb{E}_t\left[M_{t,t+1}^{\$}\Lambda_{w,t,t+1}^{-\theta_w}\left(\frac{P_{t+1}}{P_t}\right)\left(\frac{C_{h,t+1}}{C_{h,t}}\right)^{\varphi}\left(\frac{N_{t+1}^d}{N_t^d}\right)\left(\frac{\kappa_{t+1}}{\kappa_t}\right)\left(\frac{\bar{L}-N_{t+1}^s}{\bar{L}-N_t^s}\right)^{-\omega}\left(\frac{W_t}{W_{t+1}}\right)^{-\theta_w}G_{w,t+1}\right]
\end{aligned}$$

Price dispersion

$$F_{p,t} = \int_0^1\left(\frac{P_t(j)}{P_t}\right)^{-\theta_p}dj = (1-\alpha_p)\left(\frac{P_t^*}{P_t}\right)^{-\theta_p} + \alpha_p\Lambda_{p,t-1,t}^{-\theta_p}\left(\frac{P_{t-1}}{P_t}\right)^{-\theta_p}F_{p,t-1}.$$

Wage dispersion

$$F_{w,t} = \int_0^1\left(\frac{W_t(k)}{W_t}\right)^{-\theta_w}dk = (1-\alpha_w)\left(\frac{W_t^*}{W_t}\right)^{-\theta_w} + \alpha_w\Lambda_{w,t-1,t}^{-\theta_w}\left(\frac{W_{t-1}}{W_t}\right)^{-\theta_w}F_{w,t-1}.$$

Wage aggregator

$$\left(\frac{W_t}{P_t}\right)^{1-\theta_w} = \int_0^1\left(\frac{W_t(k)}{P_t}\right)^{1-\theta_w}dk = (1-\alpha_w)\left(\frac{W_t^*}{P_t}\right)^{1-\theta_w} + \alpha_w\Lambda_{w,t-1,t}^{1-\theta_w}\left(\frac{P_{t-1}}{P_t}\right)^{1-\theta_w}\left(\frac{W_{t-1}}{P_{t-1}}\right)^{1-\theta_w},$$

Price setting

$$\begin{aligned}
\left(\frac{P_t^*}{P_t}\right)H_{p,t} &= \mu_p\frac{P_{w,t}}{P_t}G_{p,t}, \\
H_{p,t} &= 1+\alpha_p\mathbb{E}_t\left[M_{t,t+1}^{\$}\Lambda_{p,t,t+1}^{1-\theta_p}\left(\frac{Y_{t+1}^{final}}{Y_t^{final}}\right)\left(\frac{P_t}{P_{t+1}}\right)^{-\theta_p}H_{p,t+1}\right], \\
G_{p,t} &= 1+\alpha_p\mathbb{E}_t\left[M_{t,t+1}^{\$}\Lambda_{p,t,t+1}^{-\theta_p}\left(\frac{Y_{t+1}^{final}}{Y_t^{final}}\right)\left(\frac{P_t}{P_{t+1}}\right)^{-\theta_p}\left(\frac{P_{w,t+1}}{P_{w,t}}\right)G_{p,t+1}\right].
\end{aligned}$$

Price aggregator

$$1 \quad = \quad (1-\alpha_p)\left(\frac{P_t^*}{P_t}\right)^{1-\theta_p} + \alpha_p \Lambda_{p,t-1,t}^{1-\theta_p}\left(\frac{P_{t-1}}{P_t}\right)^{1-\theta}.$$

Production

$$Y_t \quad = \quad (\xi\nu)^{\frac{\xi}{1-\xi}} K_{t-1}^{\alpha}\left(Z_t A_t N_t\right)^{1-\alpha}.$$

Aggregate labor supply and demand

$$N_t^s = F_{w,t} N_t^d, \qquad Y_t = Y_t^{final} F_{p,t}.$$

Pricing kernel

$$M_{t,t+1} \quad = \quad \left[\beta\left(\frac{C_{h,t+1}}{C_{h,t}}\right)^{-\varphi}\right]^{\frac{1-\gamma}{1-\varphi}}\left(\frac{1}{R_{Q,t+1}}\right)^{1-\frac{1-\gamma}{1-\varphi}},$$

Indexation

$$\log \Lambda_{p,t,t+1} \quad = \quad \pi_t^{\star}, \quad \log \Lambda_{w,t,t+1} = g_a + \pi_t^{\star}.$$

Policy rule

$$r_t = \rho r_{t-1} + (1-\rho)\left[\bar{\imath} + \imath_\pi(\pi_t - \pi_{t-1}^{\star}) + \imath_y(\tilde{y}_t - \tilde{y}_{ss})\right] + u_t.$$

Goods market clearing

$$Y_t^{final} = C_t + I_t + S_t + A_t X_t.$$

Habit

$$C_{h,t} \quad = \quad C_t - b_h C_{t-1},$$

3 Estimation of models with different rigidity specifications

In this section, we provide the estimated models with four different specifications of nominal rigidities: both rigidities (Baseline), wage rigidities only (WR), price rigidities only (PR), and no rigidities (NR). Table 7 in the main body of the paper reports moments where all parameter values are kept as in the baseline calibration except for the corresponding column rigidity specification. Table 7 presents moments of recalibrations where parameter values are chosen according to the calibration procedure in Section 4.1. That is, the second approach gives a better chance to match selected moments under the specific rigidity configuration. The specific parameter values for these calibrations are reported in Table 1.

Panel C of Table 2 show that all four calibrations match the average 5-year real and nominal bond spreads in the data. Table 1 shows that all calibrations require similar values for the habit parameter b_h, between 0.5 and 0.6 to obtain a positive average spread, assuming the same risk aversion parameter $\gamma = 40$ across calibrations. While the baseline calibration imply a significant degree of price rigidity $\alpha_p \approx 0.65$ and a low wage rigidity $\alpha_w \approx 0.27$ the calibrations where only one rigidity is active at a time imply a higher degree of wage rigidity, $\alpha_w = 0.8$ and a lower degree of price rigidity, $\alpha_p \approx 0.38$. Also, relative to the baseline calibration, the autocorrelation parameter for productivity shocks, ϕ_z is higher in the absence of rigidities or in the case of only wage rigidities, but lower in the case of only price rigidities. In terms of macroeconomic performance, the only price and only wage rigidity specifications show that both price and wage rigidities help the model in different dimensions. For instance, including only price or only wage rigidities helps to match better the autocorrelations of consumption growth and inflation, and only wage rigidities increase the volatility of labor growth to levels closer to those in the data. However, the calibrations with only wage and only price rigidities require a very negative correlation between consumption growth and inflation, relative to the milder negative autocorrelation of -0.15 in the data, in order to generate enough inflation risk premia to match the average 5-year nominal spread. On the other hand, the baseline model with both rigidities is able to simultaneously match the this spread and the correlation of consumption growth and inflation in the data. In sum, calibrations with no rigidities or only price or only wage rigidities have the ability to generate the positive average spreads in real and nominal long-term bonds in models with habits. However, including both types of rigidities improve the model performance matching macroeconomic moments.

Table 1: **Specific Parameter Values - Calibrations with Different Nominal Rigidities**

The table reports specific parameter values for the model for calibrations with external habit formation ($b_h > 0$) and different specifications for nominal rigidities. “Baseline” refers to the benchmark calibration with both price and wage rigidities. “WR” indicates no price rigidities ($\alpha_p = 0$). “PR” indicates no wage rigidities (α_w=0). “NR” indicates no price and wage rigidities ($\alpha_p = \alpha_w = 0$). The remaining parameter values are reported in Table 2 in the paper.

Parameter	Description	Baseline	WR	PR	NR
β	Subjective discount factor	0.98886	0.9944	0.9946	0.985
γ	Risk aversion parameter	40	40	40	40
b_h	External habit parameter	0.5568	0.5716	0.5285	0.599
ω	Inverse of Frisch labor elasticity	1.8771	1.9810	1.9673	2.0229
κ	Leisure preference parameter	1.1504	1.1588	1.2461	1.8318
α_p	Price rigidity parameter	0.6477	0	0.3836	0
α_w	Wage rigidity parameter	0.2717	0.8000	0	0
ϕ_z	Autocorrelation of productivity shocks	0.915	0.9358	0.8893	0.9596
$\sigma_z \times 10^2$	Conditional volatility of productivity shocks	1.36668	1.0293	1.6074	1.25
$\imath_y$	Response to detrended output in the policy rule	0.0175	0.0997	0.0730	0.0322
$\pi^\star \times 10^2$	Inflation target	1.953	0.984	1.127	2.693
ϕ_u	Autocorrelation of policy shocks	0.5243	0.0220	0.2263	0.8963
$\sigma_u \times 10^2$	Conditional volatility of policy shocks	0.0995	0.01	0.01	0.03

Table 2: **Data and Model Implied Statistics - Calibrations with Different Nominal Rigidities**

The data statistics are for the 1982:Q1 to 2008:Q3 period. The parameter values of the baseline model are reported in Table 2 in the paper and Table 1 above. The operators $\mathbb{E}[\cdot]$, $\sigma(\cdot)$, and AC$(\cdot)$ denote the unconditional mean, volatility, and first-order autocorrelation, respectively. "Baseline" refers to the benchmark calibration with both price and wage rigidities. "WR" indicates no price rigidities ($\alpha_p = 0$). "PR" indicates no wage rigidities (α_w=0). "NR" indicates no price and wage rigidities ($\alpha_p = \alpha_w = 0$). The baseline model statistic corresponds to the closed-form average of the second-order approximation of the solution. Volatilities and yields are in percentage terms. The inflation rate, yields, excess returns, and risk premia are annualized. The data statistics related to the real rate r are obtained from the estimated real rate. Values not reported are not available.

	(1)	(2)	(3)	(4)	(5)
			Model		
Statistic	Data	Baseline	WR	PR	NR
Panel A: Mean Growth Rates					
$\mathbb{E}[\Delta c]$	0.46	0.46	0.44	0.45	0.44
$\mathbb{E}[\pi]$	3.26	3.52	3.11	3.22	3.64
Panel B: Standard Deviation and Correlations					
$\sigma(\Delta c)$	0.38	0.45	0.39	0.46	0.42
$\sigma(\pi)$	1.36	1.36	1.91	1.61	1.72
$\sigma(\Delta w)$	0.66	0.69	0.44	0.74	0.73
$\sigma(\Delta n)$	0.71	0.72	0.74	0.26	0.24
$\sigma(\Delta y)$	0.65	1.04	1.13	1.20	0.96
$\sigma(\Delta i)$	2.45	2.03	1.78	1.85	2.33
$AC(\Delta c)$	0.42	0.45	0.42	0.46	0.51
$AC(\Delta c_h)$		-0.07	-0.10	-0.04	-0.10
$AC(\pi)$	0.42	0.50	0.38	0.44	0.51
$corr(\Delta c, \pi)$	-0.15	-0.15	-0.90	-0.89	-0.40
$corr(\Delta c_h, \pi)$		-0.15	-0.81	-0.80	-0.39
$corr(\Delta c, \Delta n)$	0.36	0.51	0.75	0.93	0.88
Panel C: Term Structure Moments and Mean Equity Return					
$\mathbb{E}[r^{\$}]$	5.20	4.52	5.58	5.27	4.32
$\mathbb{E}[r^{\$,(20)}]$	6.58	5.92	7.01	6.69	5.72
$\sigma(r^{\$})$	2.59	0.48	1.13	1.01	1.04
$\sigma(r^{\$,(20)})$	2.63	0.2	0.46	0.34	0.48
$\mathbb{E}[r^{(20)} - r]$	0.27	0.28	0.28	0.28	0.24
$\mathbb{E}[r^{\$,(20)} - r^{\$}]$	1.38	1.40	1.43	1.42	1.40
$\mathbb{E}[R_d - r]$		2.71	1.42	1.56	4.22

References

Calvo, Guillermo. 1983. “Staggered Prices in a Utility-Maximizing Framework.” *Journal of Monetary Economics* 12:383–398.

Chernov, Mikhail and Philippe Mueller. 2012. “The term structure of inflation expectations.” *Journal of Financial Economics, Elsevier* 106(2):367–394.

Greenwood, Robin and Dimitri Vayanos. 2010. “Price Pressure in the Government Bond Market.” *American Economic Review* 100(2):585–90.

Gurkaynak, Refet S., Brian Sack and Jonathan H. Wright. 2006. The U.S. Treasury yield curve: 1961 to the present. Technical report.

Gurkaynak, Refet S., Brian Sack and Jonathan H. Wright. 2008. The TIPS yield curve and inflation compensation. Technical report.

Haubrich, Joseph G., George G. Pennacchi and Peter Ritchken. 2011. “Inflation Expectations, Real Rates, and Risk Premia: Evidence from Inflation Swaps.” *Review of Financial Studies* 25(5):1588–1629.

Piazzesi, Monika and Martin Schneider. 2007. “Equilibrium Yield Curves.” pp. 389–442. NBER Macroeconomics Annual 2006, MIT Press.