Online Appendix of "Competition and Opacity in the Financial System"

1 Introduction

This appendix provides a set of analysis to supplement the main analysis of the paper. Section 2 shows the uniqueness of the equilibrium characterized in Lemma 1 and Proposition 3 among all possible equilibria. Section 3 provides additional numerical analysis of the total welfare. Section 4 and 5 extend the main model by considering the cases of low disclosure cost and high rollover risk.

2 Uniqueness of Equilibrium in Lemma 1 and Proposition 3

I follow the higher-order-belief approach outlined in Morris and Shin (2002) and show that the linear equilibrium characterized in Lemma 1 and Proposition 3 is indeed the unique equilibrium of the investment game. The proof proceeds in two steps. In Step 1, I characterize the hierarchy of the average higher-order beliefs for the group of investors. In Step 2, I show that the linear equilibrium derived in Lemma 1 and Proposition 3 is the unique equilibrium.

Step 1: I show that the k-th order average expectation of the fundamentals θ_i by the group of investors takes the following functional form,

$$\bar{E}^k[\theta_i] = \delta_{ik}\, z_i + l_{ik}\, \theta_i + r_{ik}\, \bar{\theta}, \tag{1}$$

where

$$\begin{aligned}\delta_{ik} &= \frac{m_i}{m_i+q}\left[1-\left(\frac{n}{m_i+n+q}\right)^k\right], \qquad (2)\\ l_{ik} &= \left(\frac{n}{m_i+n+q}\right)^k,\\ r_{ik} &= \frac{q}{m_i+q}\left[1-\left(\frac{n}{m_i+n+q}\right)^k\right].\end{aligned}$$

This can be shown by induction. At $k=1$, an individual j's expectation of θ_i is:

$$E_j(\theta_i) = \frac{m_i z_i + n x_{ij} + q\bar{\theta}}{m_i+n+q}. \qquad (3)$$

Therefore, the average expectation of θ_i by the group of investors becomes

$$\bar{E}[\theta_i] = \int_0^1 E_j(\theta_i)di = \frac{m_i z_i + n\theta_i + q\bar{\theta}}{m_i+n+q}. \qquad (4)$$

Now suppose (1) holds for $k-1$. Then

$$\begin{aligned}E_j[\bar{E}^{k-1}[\theta_i]] &= E_j[\delta_{ik-1}\, z_i + l_{ik-1}\,\theta_i + r_{ik-1}\,\bar{\theta}] \qquad (5)\\ &= \delta_{ik-1}\, z_i + r_{ik-1}\,\bar{\theta} + l_{ik-1}\, E_j[\theta_i]\\ &= \delta_{ik-1}\, z_i + r_{ik-1}\,\bar{\theta} + \frac{m_i z_i + n x_{ij} + q\bar{\theta}}{m_i+n+q}.\end{aligned}$$

Thus the average expectation is given by

$$\bar{E}^k[\theta_i] = \delta_{ik-1}\, z_i + r_{ik-1}\,\bar{\theta} + \frac{m_i z_i + n\theta_i + q\bar{\theta}}{m_i+n+q}. \qquad (6)$$

After a few simplifying steps, $\bar{E}^k[\theta_i]$ becomes

$$\bar{E}^k[\theta_i] = \delta_{ik}\, z_i + l_{ik}\, \theta_i + r_{ik}\, \bar{\theta}, \tag{7}$$

which concludes the proof on the linear form of $\bar{E}^k[\theta_i]$.

Step 2: I verify that the linear equilibrium derived in Lemma 1 and Proposition 3 is indeed the unique equilibrium. Notice that the individual optimal investments can be rewritten as:

$$\boldsymbol{k}_j = \boldsymbol{A}\, \boldsymbol{E}_j[\boldsymbol{\theta}] + \boldsymbol{B}\, \boldsymbol{E}_j[\boldsymbol{K}], \tag{8}$$

where

$$\boldsymbol{k}_j = \begin{bmatrix} k_{1j} \\ k_{2j} \end{bmatrix}, \boldsymbol{E}_j[\boldsymbol{\theta}] = \begin{bmatrix} E_j[\theta_1] \\ E_j[\theta_2] \end{bmatrix}, \boldsymbol{E}_j[\boldsymbol{K}] = \begin{bmatrix} E_j[K_1] \\ E_j[K_2] \end{bmatrix}, \boldsymbol{A} = \begin{bmatrix} 1 & -b \\ -b & 1 \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} a & -ba \\ -ba & a \end{bmatrix}. \tag{9}$$

The aggregate investment can be similarly written as:

$$\boldsymbol{K} = \boldsymbol{A}\, \bar{\boldsymbol{E}}[\boldsymbol{\theta}] + \boldsymbol{B}\, \bar{\boldsymbol{E}}[\boldsymbol{K}]. \tag{10}$$

Substituting the aggregate investment into the individual investment gives

$$\begin{aligned} \boldsymbol{k}_j &= \boldsymbol{A}\, \boldsymbol{E}_j[\boldsymbol{\theta}] + \boldsymbol{B}\, \boldsymbol{A}\, \boldsymbol{E}_j \bar{\boldsymbol{E}}[[\boldsymbol{\theta}]] + \boldsymbol{B}^2\, \boldsymbol{A}\, \boldsymbol{E}_j\, [\bar{\boldsymbol{E}}^{\boldsymbol{2}}[\boldsymbol{\theta}]] + \, ... \qquad (11) \\ &= \sum_{k=0}^{\infty} \boldsymbol{B}^k\, \boldsymbol{A}\, \boldsymbol{E}_j[\bar{\boldsymbol{E}}^{\boldsymbol{k}}[\boldsymbol{\theta}]], \end{aligned}$$

where I have shown that

$$\bar{\boldsymbol{E}}^k\, [\boldsymbol{\theta}] = \begin{bmatrix} \delta_{1k}\, z_1 + l_{1k}\, \theta_1 + r_{1k}\, \bar{\theta} \\ \delta_{2k}\, z_2 + l_{2k}\, \theta_2 + r_{2k}\, \bar{\theta} \end{bmatrix}. \tag{12}$$

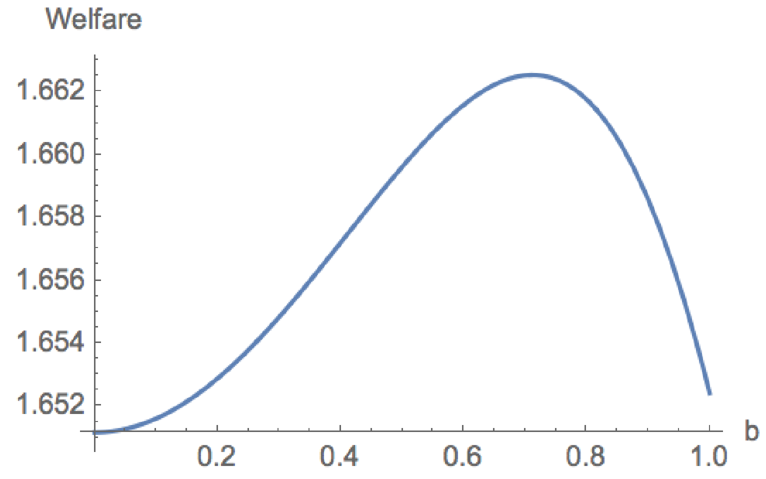


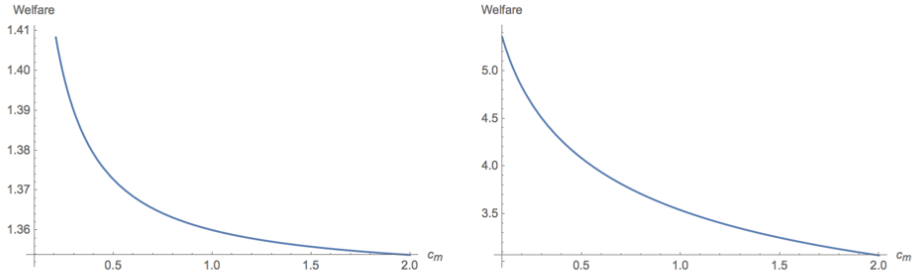
Figure 1: Effect of competition intensity on the total welfare. The following parameter values are used in the plot: $a = 0.21$, $n = 1$, $q = 1$ and $c_m = 1$.

Given $0 < a < \frac{1}{2}$, and $0 < b < 1$, the eigenvalues of $\boldsymbol{B}$ are $a\,(1-b)$ and $a\,(1+b)$, both of which are between 0 and 1. Therefore, the sum $\sum_{k=0}^{\infty} \boldsymbol{B}^k\, \boldsymbol{A}\, \boldsymbol{E}_j[\bar{\boldsymbol{E}}^{\boldsymbol{k}}[\boldsymbol{\theta}]]$ converges. After a few simplifying steps, this sum reduces to the exact linear forms as in Lemma 1 and Proposition 3 and hence I have verified the uniqueness of the equilibrium.

3 Additional Numerical Analysis of Social Welfare

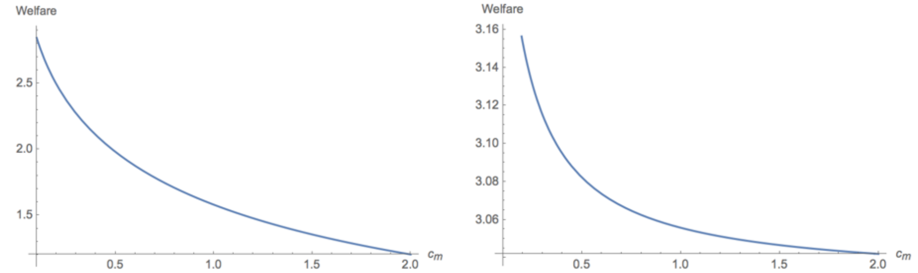
In the main text of the paper, I find that the welfare effect of competition depends on the level of rollover risk. Competition reduces welfare if the rollover risk is sufficiently low whereas it improves welfare if the rollover risk is sufficiently high. One may conjecture whether the welfare effect of competition can be non-monotonic, especially at some intermediate levels of rollover risk. Numerical analysis indeed suggests the existence of such possibility. In Figure 1, when the rollover risk takes an intermediate value, the total welfare is hump-shaped in the competition intensity. In other words, when the rollover risk is intermediate, there exists an optimal level of competition intensity that maximizes the total welfare.

Another finding regarding welfare is that the total welfare is strictly decreasing in the disclosure



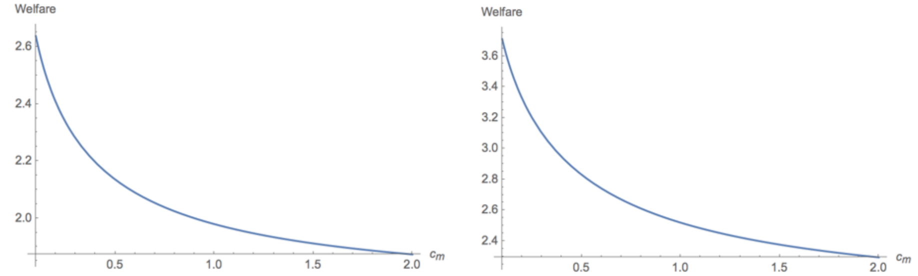
Effect of disclosure cost under low rollover risk

Effect of disclosure cost under high rollover risk



Effect of disclosure cost under low private information precision

Effect of disclosure cost under high private information precision



Effect of disclosure cost under low competition intensity

Effect of disclosure cost under high competition intensity

Figure 2: Effect of disclosure cost on the total welfare. In Panel I, $n = 1$, $a = 0.1$, $q = 1$ and $b = 0.5$. In Panel II, $n = 1$, $a = 0.4$, $q = 1$ and $b = 0.5$. In Panel III, $n = 0.1$, $a = 0.3$, $q = 1$ and $b = 0.5$. In Panel IV, $n = 3$, $a = 0.3$, $q = 1$ and $b = 0.5$. In Panel V, $n = 1$, $a = 0.3$, $q = 1$ and $b = 0.1$. In Panel VI, $n = 1$, $a = 0.3$, $q = 1$ and $b = 0.9$.

cost. To assess the robustness of this finding, I provide in Figure 2 additional numerical plots of the relationship between the welfare and the disclosure cost. The Figure shows that the total welfare is strictly decreasing in the disclosure cost at various parameter values (e.g., low/high rollover risk, low/high private information precision, and low/high competition intensity). In fact, I cannot obtain a numerical plot in which the welfare is increasing in the disclosure cost.

4 Extension I: Small Disclosure Cost

In the main model, I assume that the disclosure cost c_m is sufficiently large to ensure that the FI's payoff is globally concave in its disclosure precision. In this extension, I relax this assumption and examine the FIs' disclosure decisions when the disclosure cost is small. I first solve a variation of the main model in which the disclosure cost is zero.

Proposition 1 *With zero disclosure cost ($c_m = 0$), full-disclosure (the disclosure precision equals infinity) is the unique equilibrium in both the no-competition case ($b = 0$) and the competition case with b sufficiently large ($b = 1$).*

Proposition 1 suggests that when disclosure is costless, there exists a unique equilibrium in which the FIs make full disclosure either when there is no competitive interaction ($b = 0$) or when the competitive interaction is sufficiently strong ($b = 1$). For some interior $b \in (0, 1)$, the equilibrium disclosure decisions cannot be solved in closed form due to the complexity of the modeling structure. Nonetheless, numerical analyses suggest that full-disclosure remains the unique equilibrium. These results thereby suggest that even though competition may change the net benefit of disclosure, it is unable to shift the equilibrium away from full disclosure. The benefit of full disclosure remains maximal even with competition.

Next, I consider an extension in which the disclosure cost is positive but sufficiently small. I summarize the equilibrium disclosure decisions in the proposition below.

Proposition 2 *When the disclosure cost c_m is sufficiently small,*

1. *partial-disclosure (the disclosure precision is below infinity) is the unique equilibrium in both the no-competition case ($b = 0$) and the competition case with b sufficiently large ($b = 1$);*
2. *with rollover risk, $a > 0$, the FIs choose a lower disclosure precision with sufficiently intense*

competitive interactions ($b = 1$) than without competitive interactions ($b = 0$) if and only if the private information precision is sufficiently high, i.e., there exists a unique threshold $\hat{n} > 0$ such that $m^* < m^{NC}$ if and only if the private information precision $n > \hat{n}$.

Proposition 2 shows that, with some disclosure cost, the equilibrium shifts from full-disclosure to partial-disclosure. This equilibrium shift, in turn, allows competition to play a role in changing the FIs' disclosure decisions. I find that the effect of competition on disclosure is similar to that in the main model (Proposition 4 in the main text). Competition reduces disclosure if and only if the private information is sufficiently precise.

5 Extension II: High Rollover Risk

I now examine another extension of my main model in which the rollover risk $a > \frac{1}{2}$. Following similar steps to the proof of Proposition 3, I can show that there exists a unique linear equilibrium in which investors choose

$$k_{ij}^* = \beta_i^* x_{ij} + \gamma_i^* z_i - b(\lambda_i^* x_{-ij} + \omega_i^* z_{-i}) + h_i^*, \tag{13}$$

where the weights are the same as in the main model (Proposition 3 of the main text).

Analyzing the equilibrium investment shows that, with $a > \frac{1}{2}$, the equilibrium may exhibit some highly unrealistic features that go against the common intuition about the effects of information and competition. The first unrealistic feature is that, the equilibrium investment in a FI may either *decrease* in the FI's own disclosure or *decrease* in investors' private signals about the FI's fundamentals. To see this, consider the case that the competition is sufficiently intense with $b = 1$. In this case, the weight β_i^* on the private signal x_{ij} and the weight γ_i^* on the public signal z_i reduce into:

$$\begin{aligned} \beta_i^* &= \frac{n}{q + m_i + (1-2a)n}, \\ \gamma_i^* &= \frac{m_i}{(1-2a)[q + m_i + (1-2a)n]}. \end{aligned} \tag{14}$$

Therefore, if $\beta_i^* > 0$, then $q + m_i + (1-2a)n > 0$. This, in turn, suggests that $\gamma_i^* < 0$ given $a > \frac{1}{2}$. In other words, the weights β_i^* and γ_i^* cannot be positive at the same time.

The other unrealistic feature is that, the equilibrium investment in a FI may either *increase* in the other competing FI's disclosure or *increase* in investors' private signals about the other FI's fundamentals. To see this, consider, again, the case that the competition is sufficiently intense with $b = 1$. In this case, the weight λ_i^* on the private signal x_{-ij} and the weight ω_i^* on the public signal z_{-i} reduce into:

$$\begin{aligned} \lambda_i^* &= \frac{n}{q + m_{-i} + (1-2a)n}, \\ \omega_i^* &= \frac{m_{-i}}{(1-2a)[q + m_{-i} + (1-2a)n]}. \end{aligned} \tag{15}$$

Therefore, if $\lambda_i^* > 0$, then $q + m_{-i} + (1-2a)n > 0$. This, in turn, suggests that $\omega_i^* < 0$ given $a > \frac{1}{2}$. In other words, the weights λ_i^* and ω_i^* cannot be positive at the same time.

In summary, the analysis above suggests that, with $a > \frac{1}{2}$, the equilibrium features seem highly unrealistic and thus less relevant. Yet, in environments with strong strategic complementarities (i.e., high a), these unrealistic features could sustain in equilibrium because of self-fulfilling prophecies. In other words, when investors are extremely concerned about others' actions, they could act in an abnormal way purely because they believe others will do the same and vice versa. In this light, assuming $a \leq \frac{1}{2}$ in the main model helps to ensure that the equilibrium investment in a FI is strictly increasing in the signals about the FI itself and decreasing in the signals about the other FI. These standard features of investments are essential for my main analysis of optimal disclosure decisions.

References

[1] Morris, S., and H. Shin. "Social Value of Public Information." *The American Economic Review* 92 (2002): 1521-1534.

6 Proofs

Proof. of Proposition 1: From the proof of Proposition 2 in the main text, given $b = 0$ and $c_m = 0$, the first-order condition on m^{NC} is given by

$$\frac{\partial \Pi_1^{NC}}{\partial m_1} = \frac{m_1 + q - (1-a)(1-4a)n}{2(1-a)^2 [q + m_1 + (1-a)n]^3}. \tag{16}$$

If $q - (1-a)(1-4a)n > 0$ $(a > \frac{5}{8} - \frac{1}{8}\sqrt{\frac{16q}{n} + 9})$, $\frac{\partial \Pi_1^{NC}}{\partial m_1} > 0$ for all $m_1 > 0$. Therefore, $m^{NC} = \infty$ and the unique equilibrium is full disclosure. If $a \leq \frac{5}{8} - \frac{1}{8}\sqrt{\frac{16q}{n} + 9}$, there exists a unique solution of $\hat{m}^{NC} = (1-a)(1-4a)n - q > 0$ that solves $\frac{\partial \Pi_1^{NC}}{\partial m_1} = 0$. At $m_1 = \hat{m}^{NC}$, the second-order condition is given by

$$\begin{aligned} \frac{\partial^2 \Pi_1^{NC}}{\partial m_1^2} &= -\frac{\hat{m}^{NC} + q - 2(1-a)(1-3a)n}{(1-a)^2 [q + \hat{m}^{NC} + (1-a)n]^4} \\ &= \frac{(1-2a)n}{(1-a)[q + \hat{m}^{NC} + (1-a)n]^4} > 0. \end{aligned} \tag{17}$$

Therefore, $\hat{m}^{NC}$ is a global minimum of Π_1^{NC}. In other words, Π_1^{NC} increases in m_1 if and only if $m_1 > \hat{m}^{NC}$. This implies that the equilibrium is either full disclosure or no disclosure. To determine which of the two cases sustains in equilibrium, I compare the value of Π_1^{NC} at $m_1 = 0$

and $m_1 = \infty$:

$$\lim_{m_1 \to \infty} \Pi_1^{NC} - \lim_{m_1 \to 0} \Pi_1^{NC} = \frac{1}{2} \left[\frac{q + 2(1-a)an}{(1-a)^2 [q + (1-a)n]^2} \right] > 0. \tag{18}$$

Therefore, at $b = 0$ and $c_m = 0$, full disclosure is the unique equilibrium.

At $b = 1$ and $c_m = 0$, from the proof of Proposition 4 in the main text, the first-order condition on m^* is given by

$$\frac{\partial \Pi_i^*}{\partial m_i} = \frac{m_i + q - (1-2a)n}{2(1-2a)[q + m_i + (1-2a)n]^3}. \tag{19}$$

If $q - (1-2a)n > 0$ ($a > \frac{1}{2} - \frac{q}{2n}$), $\frac{\partial \Pi_i^*}{\partial m_i} > 0$ for all $m_i > 0$. Therefore, $m^* = \infty$ and the unique equilibrium is full disclosure. If $a \leq \frac{1}{2} - \frac{q}{2n}$, there exists a unique solution of $\hat{m}^C = (1-2a)n - q > 0$ that solves $\frac{\partial \Pi_i^*}{\partial m_i} = 0$. At $m_i = \hat{m}^C$, the second-order condition is given by

$$\frac{\partial^2 \Pi_i^*}{\partial m_i^2} = \frac{1}{16(1-2a)^4 n^3} > 0. \tag{20}$$

Therefore, the equilibrium is either full disclosure or no disclosure. To determine which of the two cases sustains in equilibrium, I compare the value of Π_i^* at $m_i = 0$ and $m_i = \infty$:

$$\lim_{m_i \to \infty} \Pi_i^* - \lim_{m_i \to 0} \Pi_i^* = \frac{q}{2(1-2a)[q + (1-2a)n]^2} > 0. \tag{21}$$

Therefore, at $b = 1$ and $c_m = 0$, full disclosure is also the unique equilibrium. ■

Proof. of Proposition 2: The proof contains three steps. In Step 1, I solve for the FIs' disclosure decisions with no competition, m^{NC}. In Step 2, I solve for the FIs' disclosure decisions with sufficiently intense competition ($b = 1$), m^*. In Step 3, I compare m^{NC} with m^*.

Step 1: I first solve for m^{NC}. Recall from the proof of Proposition 2 in the main text, that at $m_1 = \infty$, $\lim_{m_1 \to \infty} \frac{\partial \Pi_1^{NC}}{\partial m_1} = -\infty < 0$. At $m_1 = 0$, $\lim_{m_1 \to 0} \frac{\partial \Pi_1^{NC}}{\partial m_1} = \frac{q-(1-a)(1-4a)n}{2(1-a)^2[q+(1-a)n]^3}$ which is positive if and only if $q-(1-a)(1-4a)n>0$ ($a>\frac{5}{8}-\frac{1}{8}\sqrt{\frac{16q}{n}+9}$). I now solve for m^{NC} in two cases: 1) $a>\frac{5}{8}-\frac{1}{8}\sqrt{\frac{16q}{n}+9}$ and 2) $a\leq\frac{5}{8}-\frac{1}{8}\sqrt{\frac{16q}{n}+9}$.

Case 1: When $a>\frac{5}{8}-\frac{1}{8}\sqrt{\frac{16q}{n}+9}$, by the intermediate value theorem, there always exists an interior solution of m_1 to $\frac{\partial\Pi_1^{NC}}{\partial m_1}=0$. In addition, it can be verified that $\frac{\partial\Pi_1^{NC}}{\partial m_1}=0$ is a fourth-order polynomial of m_1 with 1 sign change in its coefficients, given $a>\frac{5}{8}-\frac{1}{8}\sqrt{\frac{16q}{n}+9}$. Therefore, by the Descartes rule of signs, $\frac{\partial\Pi_1^{NC}}{\partial m_1}=0$ has a unique positive solution for any $c_m>0$.

Case 2: Consider now the case in which $a\leq\frac{5}{8}-\frac{1}{8}\sqrt{\frac{16q}{n}+9}$ such that $\lim_{m_1\to 0}\frac{\partial\Pi_1^{NC}}{\partial m_1}<0$. Note that

$$\frac{\partial\Pi_1^{NC}}{\partial m_1}=\frac{m_1+q-(1-a)(1-4a)n}{2(1-a)^2[q+m_1+(1-a)n]^3}-c_m m_1, \tag{22}$$

which is negative if and only if

$$c_m>\frac{m_1+q-(1-a)(1-4a)n}{2(1-a)^2 m_1[q+m_1+(1-a)n]^3}. \tag{23}$$

It can be verified that the RHS of (23) is hump-shaped in m_1 and reaches its maximum at $m_1=\bar{m}^{NC}>0$, where

$$\bar{m}^{NC}=\frac{2}{3}((1-a)(1-4a)n-q)+\frac{\sqrt{((1-a)(7-16a)n-q)((1-a)(1-4a)n-q)}}{3}. \tag{24}$$

I consider two subcases of Case 2: 1) $c_m\geq\frac{\bar{m}^{NC}+q-(1-a)(1-4a)n}{2(1-a)^2\bar{m}^{NC}[q+\bar{m}^{NC}+(1-a)n]^3}$ and 2) $c_m<\frac{\bar{m}^{NC}+q-(1-a)(1-4a)n}{2(1-a)^2\bar{m}^{NC}[q+\bar{m}^{NC}+(1-a)n]^3}$.

Subcase 1: If $c_m\geq\frac{\bar{m}^{NC}+q-(1-a)(1-4a)n}{2(1-a)^2\bar{m}^{NC}[q+\bar{m}^{NC}+(1-a)n]^3}$, $\frac{\partial\Pi_1^{NC}}{\partial m_1}\leq 0$ for all $m_1\geq 0$. This in turn implies that the unique equilibrium is $m^{NC}=0$.

Subcase 2: Consider next that $c_m<\frac{\bar{m}^{NC}+q-(1-a)(1-4a)n}{2(1-a)^2\bar{m}^{NC}[q+\bar{m}^{NC}+(1-a)n]^3}$. Therefore, at $m_1=\bar{m}^{NC}$, $\frac{\partial\Pi_1^{NC}}{\partial m_1}>0$. Combined with $\lim_{m_1\to\infty}\frac{\partial\Pi_1^{NC}}{\partial m_1}=-\infty<0$, $\lim_{m_1\to 0}\frac{\partial\Pi_1^{NC}}{\partial m_1}<0$ and by the intermediate value theorem, there exist two positive roots of $\frac{\partial\Pi_1^{NC}}{\partial m_1}=0$. Denote the two roots as m_s^{NC} and m_l^{NC}, such that $m_l^{NC} > m_s^{NC}$. Since $\lim_{m_1 \to 0} \frac{\partial \Pi_1^{NC}}{\partial m_1} < 0$, $\lim_{m_1 \to \infty} \frac{\partial \Pi_1^{NC}}{\partial m_1} < 0$ and $\frac{\partial \Pi_1^{NC}}{\partial m_1}|_{m_1 = \bar{m}^{NC}} > 0$, it must be the case that $\frac{\partial^2 \Pi_1^{NC}}{\partial m_1^2}|_{m_1 = m_s^{NC}} > 0$ and $\frac{\partial^2 \Pi_1^{NC}}{\partial m_1^2}|_{m_1 = m_l^{NC}} < 0$. That is, m_s^{NC} is a local minimum of Π_1^{NC} while m_l^{NC} is a local maximum. For the local maximum m_l^{NC} to be a global maximum and thus an equilibrium, I need the FI to attain a higher payoff at $m_1 = m_l^{NC}$ than at $m_1 = 0$. I show this holds when the cost c_m is smaller than a threshold $\hat{c}_m^{NC}$. To see this, consider first the trivial case that for all $c_m < \frac{\bar{m}^{NC} + q - (1-a)(1-4a)n}{2(1-a)^2 \bar{m}^{NC} [q + \bar{m}^{NC} + (1-a)n]^3}$, $\Pi_1^{NC}\left(m_l^{NC}\right) > \Pi_1^{NC}(0)$. This implies that $\hat{c}_m^{NC} = \frac{\bar{m}^{NC} + q - (1-a)(1-4a)n}{2(1-a)^2 \bar{m}^{NC} [q + \bar{m}^{NC} + (1-a)n]^3}$. Next, suppose there exists some c_m such that $\Pi_1^{NC}\left(m_l^{NC}\right) < \Pi_1^{NC}(0)$. Now consider an extreme in which c_m is sufficiently small and close to 0. Note that, at $c_m = 0$,

$$\frac{\partial \Pi_1^{NC}}{\partial m_1} = \frac{m_1 + q - (1-a)(1-4a)n}{2(1-a)^2 [q + m_1 + (1-a)n]^3} = 0, \tag{25}$$

has two roots, a local minimum of Π_1^{NC}, $m_1 = (1-a)(1-4a)n - q$ and a local maximum $m_1 = \infty$. This implies that

$$\lim_{c_m \to 0} m_s^{NC} = (1-a)(1-4a)n - q, \lim_{c_m \to 0} m_l^{NC} = \infty. \tag{26}$$

At $m_1 = \lim_{c_m \to 0} m_l^{NC}$, the FI's payoff is:

$$\lim_{c_m \to 0} \Pi_1^{NC}\left(m_l^{NC}\right) = \frac{1 + q\bar{\theta}^2}{2q(1-a)^2} - \lim_{c_m \to 0} \frac{c_m}{2}\left(m_l^{NC}\right)^2. \tag{27}$$

The second term,

$$\begin{aligned} \lim_{c_m \to 0} \frac{c_m}{2}\left(m_l^{NC}\right)^2 &= \lim_{c_m \to 0} \frac{m_l^{NC}}{2} \frac{m_l^{NC} + q - (1-a)(1-4a)n}{2(1-a)^2 \left[q + m_l^{NC} + (1-a)n\right]^3} \\ &= \lim_{m_1^l \to \infty} \frac{m_l^{NC}}{2} \frac{m_l^{NC} + q - (1-a)(1-4a)n}{2(1-a)^2 \left[q + m_l^{NC} + (1-a)n\right]^3} = 0. \end{aligned} \tag{28}$$

The first equality uses the first-order condition $c_m m_l^{NC} = \frac{m_l^{NC} + q - (1-a)(1-4a)n}{2(1-a)^2 \left[q + m_l^{NC} + (1-a)n\right]^3}$. The second equality uses (26). In sum,

$$\lim_{c_m \to 0} \Pi_1^{NC}\left(m_l^{NC}\right) = \frac{1+q\bar{\theta}^2}{2q\left(1-a\right)^2}. \tag{29}$$

At $m_1 = 0$, the FI's payoff is:

$$\Pi_1^{NC}\left(0\right) = \frac{q^3\bar{\theta}^2 + \left(1-a\right)n\left[2q\left(1-a+q\bar{\theta}^2\right) + \left(1+q\bar{\theta}^2\right)\left(1-a\right)n\right]}{2q\left(1-a\right)^2\left(q+\left(1-a\right)n\right)^2}. \tag{30}$$

With some algebras, it can verified that $\Pi_1^{NC}\left(0\right) < \frac{1+q\bar{\theta}^2}{2q(1-a)^2}$. Therefore, for c_m close to 0, $\lim_{c_m \to 0} \Pi_1^{NC}\left(m_l^{NC}\right) > \Pi_1^{NC}\left(0\right)$. By the intermediate value theorem, there exists a threshold $\hat{c}_m^{NC} \in \left(0, \frac{\bar{m}^{NC}+q-(1-a)(1-4a)n}{2(1-a)^2\bar{m}^{NC}[q+\bar{m}^{NC}+(1-a)n]^3}\right)$ that solves $\Pi_1^{NC}\left(m_l^{NC}\right) = \Pi_1^{NC}\left(0\right)$ such that for $c_m < \hat{c}_m^{NC}$, $\Pi_1^{NC}\left(m_l^{NC}\right) > \Pi_1^{NC}\left(0\right)$ and $m_1 = m_l^{NC}$ is the unique equilibrium. Such a threshold $\hat{c}_m^{NC}$ is also unique because by the envelope theorem,

$$\frac{d\Pi_1^{NC}\left(m_l^{NC}\right)}{dc_m} = \frac{\partial\Pi_1^{NC}\left(m_l^{NC}\right)}{\partial c_m} = -\frac{\left(m_l^{NC}\right)^2}{2} < 0. \tag{31}$$

Thus $\Pi_1^{NC}\left(m_l^{NC}\right)$ is strictly decreasing in c_m whereas $\Pi_1^{NC}\left(0\right)$ is independent of c_m, which implies that $\hat{c}_m^{NC}$ is unique.

In summary, if $a > \frac{5}{8} - \frac{1}{8}\sqrt{\frac{16q}{n}+9}$ or $c_m < \hat{c}_m^{NC}$, m^{NC} is either the larger root or the unique root that solves $\frac{\partial\Pi_1^{NC}}{\partial m_1} = 0$. Otherwise, $m^{NC} = 0$.

Step 2: Next, I solve for m^*. Recall from the proof of Proposition 4 in the main text that at $m_i = \infty$, $\lim_{m_i \to \infty} \frac{\partial\Pi_i^*}{\partial m_i} = -\infty < 0$. At $m_i = 0$, $\lim_{m_i \to 0} \frac{\partial\Pi_i^*}{\partial m_i} = \frac{q-(1-2a)n}{2(1-2a)[q+(1-2a)n]^3}$, which is positive if and only if $q - \left(1-2a\right)n > 0$ ($a > \frac{1}{2} - \frac{q}{2n}$). I now solve for m^* in two cases: 1) $a > \frac{1}{2} - \frac{q}{2n}$ and 2) $a \leq \frac{1}{2} - \frac{q}{2n}$.

Case 1: When $a > \frac{1}{2} - \frac{q}{2n}$, by the intermediate value theorem, there always exists an interior solution of m_i to $\frac{\partial\Pi_i^*}{\partial m_i} = 0$. In addition, it can be verified that $\frac{\partial\Pi_i^*}{\partial m_i} = 0$ is a fourth-order polynomial of m_i with 1 sign change in its coefficients, given $a > \frac{1}{2} - \frac{q}{2n}$. Therefore, by the Descartes rule of signs, $\frac{\partial \Pi_i^*}{\partial m_i} = 0$ has a unique positive solution for any $c_m > 0$.

Case 2: Consider now the case in which $a < \frac{1}{2} - \frac{q}{2n}$ such that $\lim_{m_i \to 0} \frac{\partial \Pi_i^*}{\partial m_i} < 0$. Note that

$$\frac{\partial \Pi_i^*}{\partial m_i} = \frac{m_i + q - (1-2a)\, n}{2\,(1-2a)\left[q + m_i + (1-2a)\, n\right]^3} - c_m m_i, \tag{32}$$

which is negative if and only if

$$c_m > \frac{m_i + q - (1-2a)\, n}{2\,(1-2a)\, m_i \left[q + m_i + (1-2a)\, n\right]^3}. \tag{33}$$

It can be verified that that the RHS of (33) is hump-shaped in m_i and reaches its maximum at $m_i = \bar{m}^C > 0$, where

$$\bar{m}^C = \frac{2}{3}\left((1-2a)\, n - q\right) + \frac{\sqrt{\left(7\,(1-2a)\, n - q\right)\left((1-2a)\, n - q\right)}}{3}. \tag{34}$$

I consider two subcases of Case 2: 1) $c_m \geq \frac{\bar{m}^C + q - (1-2a)n}{2(1-2a)\bar{m}^C[q+\bar{m}^C+(1-2a)n]^3}$ and 2) $c_m < \frac{\bar{m}^C + q - (1-2a)n}{2(1-2a)\bar{m}^C[q+\bar{m}^C+(1-2a)n]^3}$.

Subcase 1: If $c_m \geq \frac{\bar{m}^C + q - (1-2a)n}{2(1-2a)\bar{m}^C[q+\bar{m}^C+(1-2a)n]^3}$, $\frac{\partial \Pi_i^*}{\partial m_i} \leq 0$ for all $m_i \geq 0$. This in turn implies that the unique equilibrium is $m_i^* = 0$.

Subcase 2: Consider next the other case that $c_m < \frac{\bar{m}^C + q - (1-2a)n}{2(1-2a)\bar{m}^C[q+\bar{m}^C+(1-2a)n]^3}$. Therefore, at $m_i = \bar{m}^C$, $\frac{\partial \Pi_i^*}{\partial m_i} > 0$. By the intermediate value theorem, there exist two positive roots of $\frac{\partial \Pi_i^*}{\partial m_i} = 0$. Denote the two roots as m_s^C and m_l^C, such that $m_l^C > m_s^C$. Following similar steps as in Subcase 2 of the analysis of m^{NC}, I show that there exists a unique threshold $\hat{c}_m^C \in \left(0, \frac{\bar{m}^C + q - (1-2a)n}{2(1-2a)\bar{m}^C[q+\bar{m}^C+(1-2a)n]^3}\right)$ that solves $\Pi_i^*\left(m_l^C\right) = \Pi_i^*(0)$, such that for $c_m < \hat{c}_m^C$, m_l^C is the unique equilibrium.

In summary, if $a > \frac{1}{2} - \frac{q}{2n}$ or $c_m < \hat{c}_m^C$, m^* is either the larger root or the unique root that solves $\frac{\partial \Pi_i^*}{\partial m_i} = 0$. Otherwise $m^* = 0$.

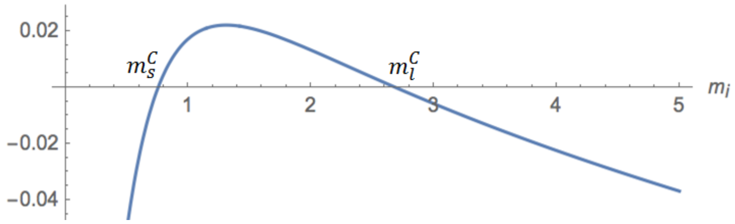


Figure 3: First-order condition $\frac{\partial \Pi_i^*}{\partial m_i}$.

Step 3: I compare m^{NC} and m^* when the disclosure cost is sufficiently small and close to 0. Note for c_m sufficiently small, I have $c_m < \min\left\{\hat{c}_m^{NC}, \hat{c}_m^{C}\right\}$. This implies that in both the case of $b = 0$ and the case of $b = 1$, the equilibrium disclosure decision is either the larger root or the unique root that solves the first-order condition. Consider now the two cases separately.

Case 1: If m^* is the unique root that solves $\frac{\partial \Pi_i^*}{\partial m_i} = 0$, then since $\frac{\partial^2 \Pi_i^*}{\partial m_i^2} < 0$, $\frac{\partial \Pi_i^*}{\partial m_i}|_{m_i = m^{NC}} < 0$ if and only if $m^{NC} > m^*$. Thus

$$m^{NC} > m^* \Leftrightarrow \frac{\partial \Pi_i^*}{\partial m_i}|_{m_i = m^{NC}} < 0 \Leftrightarrow n > \hat{n}. \tag{35}$$

The second step holds because, recall from the proof of Proposition 4 in the main text, $\frac{\partial \Pi_i^*}{\partial m_i}|_{m_i = m^{NC}} < 0$ if and only if $n > \hat{n}$. Therefore, $m^{NC} > m^*$ if and only if $n > \hat{n}$.

Case 2: Consider now the other case that $m^* = m_l^C$, i.e., m^* is the larger root that solves $\frac{\partial \Pi_i^*}{\partial m_i} = 0$. Recall from (26), that for c_m sufficiently small, m^{NC} approaches infinity and must be greater than m_s^C, the smaller root that solves $\frac{\partial \Pi_i^*}{\partial m_i} = 0$. In this case, given $m^{NC} > m_s^C$, $\frac{\partial \Pi_i^*}{\partial m_i}|_{m_i = m^{NC}} < 0$ if and only if $m^{NC} > m_l^C = m^*$. Figure 3 provides a graphical illustration. Thus

$$m^{NC} > m^* \Leftrightarrow \frac{\partial \Pi_i^*}{\partial m_i}|_{m_i = m^{NC}} < 0 \Leftrightarrow n > \hat{n}. \tag{36}$$

Therefore, $m^{NC} > m^*$ if and only if $n > \hat{n}$. ■