Online Appendix for "Green Technology Development and Adoption: Competition, Regulation, and Uncertainty – A Global Game Approach"

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A Detailed Proofs

Proof of Lemma 1. We prove that a switching strategy around $x^{(2)}$ is an equilibrium by showing that if all firms but firm i follow this switching strategy, then firm i's best response is to follow this switching strategy. (In Lemma O1 of Online Appendix, we show how we derive this strategy using the argument of higher-order beliefs.)

We first derive firm i's expected gain $u_i(x_i)$ as a function of x_i, and then show that $u_i(x_i) \geq 0$ if and only if $x_i \geq x^{(2)}$. For notational convenience, define $\eta_i \equiv \frac{x_i - x^{(2)}}{2\epsilon}$. Given that every other firm follows a switching strategy around $x^{(2)}$, we can use get the expressions of α:

$$\alpha = \begin{cases} 0 & \text{if } \theta < x^{(2)} - \epsilon; \\ \frac{\theta + \epsilon - x^{(2)}}{2\epsilon} & \text{if } x^{(2)} - \epsilon \leq \theta \leq x^{(2)} + \epsilon; \\ 1 & \text{if } \theta > x^{(2)} + \epsilon. \end{cases} \tag{4}$$

We then use (4) to integrate $u_i(\theta, \alpha)$ in (1) to get the expression of $u_i(x_i)$. When $x_i < x^{(2)} - 2\epsilon$, we get $\alpha = 0$ and $u_i(x_i) = x_i - (1-q)(f_H - m_L) + f_H - f_L$. When $x^{(2)} - 2\epsilon \leq x_i \leq x^{(2)}$, we obtain the following expression of $u_i(x_i)$:

$$\begin{aligned} u_i(x_i) &= \int_{x_i-\epsilon}^{x_i+\epsilon} \frac{\theta}{2\epsilon} d\theta - (1-q)(f_H - m_L) + f_H - f_L \\ & \frac{1}{2\epsilon} \int_{x^{(2)}-\epsilon}^{x_i+\epsilon} \left\{ (1-q)(f_H - m_L) \left(\frac{\theta + \epsilon - x^{(2)}}{2\epsilon} \right)^r - b \frac{\theta + \epsilon - x^{(2)}}{2\epsilon} \right\} d\theta \\ &= x_i - (1-q)(f_H - m_L) + f_H - f_L + \int_0^{\frac{x_1 + 2\epsilon - x^{(2)}}{2\epsilon}} \left\{ (1-q)(f_H - m_L) \alpha^r - b\alpha \right\} d\alpha, \end{aligned} \tag{5}$$

where the equality is obtained by changing the integration variable from θ to $\alpha = \frac{\theta + \epsilon - x^{(2)}}{2\epsilon}$. By using $\eta_i = \frac{x_i - x^{(2)}}{2\epsilon}$ (which is between -1 and 0 because $x^{(2)} - 2\epsilon \leq x_i \leq x^{(2)}$), we rewrite $u_i(x_i)$ in (5) as:

$$u_i\left(\eta_i\right) = x^{(2)} + 2\epsilon\eta_i - (1-q)\left(f_H - m_L\right) + f_H - f_L + \int_0^{1+\eta_i}\left\{-b\alpha + (1-q)\left(f_H - m_L\right)\alpha^r\right\}d\alpha.$$

Similarly, when $x^{(2)} < x_i \le x^{(2)} + 2\epsilon$, we obtain
$u_i\left(\eta_i\right) = x^{(2)} + 2\epsilon\eta_i - (1-q)\left(f_H - m_L\right) + f_H - f_L$
$+ \int_{\eta_i}^1\left\{-b\alpha + (1-q)\left(f_H - m_L\right)\alpha^r\right\}d\alpha + \eta_i\left\{(1-q)\left(f_H - m_L\right) - b\right\}.$
When $x_i > x^{(2)} + 2\epsilon$, we obtain $u_i\left(x_i\right) = x_i - b + f_H - f_L$.

Next, we prove $u_i\left(x_i\right) \ge 0$ if and only if $x_i \ge x^{(2)}$ by showing: (i) $u_i\left(x^{(2)}\right) = 0$, and (ii) if $b \le b^{(2)}$, then $u_i\left(x_i\right)$ is nondecreasing with x_i. To prove (i), from (5), we obtain $u_i\left(x^{(2)}\right) = x^{(2)} - (1-q)\left(f_H - m_L\right) + f_H - f_L - \frac{b}{2} + \frac{(1-q)\left(f_H - m_L\right)}{r+1}$. By substituting $x^{(2)} = \frac{b}{2} + f_L - f_H + \frac{r}{r+1}(1-q)\left(f_H - m_L\right)$ into $u_i\left(x^{(2)}\right)$, we get $u_i\left(x^{(2)}\right) = 0$.

To prove (ii), we derive conditions for $du_i\left(x_i\right)/dx_i \ge 0$ in each interval of x_i, and express those conditions as $b \le b^{(2)}$. When $x_i < x^{(2)} - 2\epsilon$ or $x_i > x^{(2)} + 2\epsilon$, it is easy to show $du_i\left(x_i\right)/dx_i = 1 > 0$. When $x^{(2)} - 2\epsilon \le x_i \le x^{(2)}$, we analyze the sign of $du_i\left(\eta_i\right)/d\eta_i$ because $du_i\left(x_i\right)/dx_i = 2\epsilon du_i\left(\eta_i\right)/d\eta_i$ and $\epsilon > 0$. In preparation, we compute $\frac{du_i}{d\eta_i} = 2\epsilon - b\left(1+\eta_i\right) + (1-q)\left(f_H - m_L\right)\left(1+\eta_i\right)^r$, $\frac{d^2u_i}{d\eta_i^2} = -b + r(1-q)\left(f_H - m_L\right)\left(1+\eta_i\right)^{r-1}$, and $\frac{d^3u_i}{d\eta_i^3} = r(r-1)(1-q)\left(f_H - m_L\right)\left(1+\eta_i\right)^{r-2}$. First, consider the case when $r \le 1$. Then $\frac{d^3u_i}{d\eta_i^3} \le 0$ and $\frac{du_i}{d\eta_i}$ is concave. So if $\left.\frac{du_i}{d\eta_i}\right|_{\eta_i=-1} \ge 0$ and $\left.\frac{du_i}{d\eta_i}\right|_{\eta_i=0} \ge 0$, then $\frac{du_i}{d\eta_i} \ge 0$ for any $-1 \le \eta_i \le 0$. Since $\left.\frac{du_i}{d\eta_i}\right|_{\eta_i=-1} = 2\epsilon$ and $\left.\frac{du_i}{d\eta_i}\right|_{\eta_i=0} = 2\epsilon + (1-q)\left(f_H - m_L\right) - b$, $\frac{\partial u_i}{\partial\eta_i} \ge 0$ if $b \le 2\epsilon + (1-q)\left(f_H - m_L\right)$. Second, consider the case when $r > 1$. Then $\frac{d^3u_i}{d\eta_i^3} > 0$, and $\frac{du_i}{d\eta_i}$ achieves its minimum value of $\left(\frac{du_i}{d\eta_i}\right)^{\min} = 2\epsilon + \left\{\frac{b}{r(1-q)\left(f_H - m_L\right)}\right\}^{\frac{1}{r-1}} b\left\{\frac{1}{r} - 1\right\}$ at $\eta_i^{\min} = \left\{\frac{b}{r(1-q)\left(f_H - m_L\right)}\right\}^{\frac{1}{r-1}} - 1$. If $r \ge \frac{b}{(1-q)\left(f_H - m_L\right)}$, then $-1 \le \eta_i^{\min} \le 0$ and $\frac{du_i}{d\eta_i} \ge 0$ if $\left(\frac{du_i}{d\eta_i}\right)^{\min} \ge 0$, which simplifies to $b \le r\left(\frac{2\epsilon}{r-1}\right)^{1-\frac{1}{r}}\left\{(1-q)\left(f_H - m_L\right)\right\}^{\frac{1}{r}}$; otherwise, $\frac{du_i}{d\eta_i}$ is decreasing with $\eta_i \in [-1, 0]$, so $\frac{du_i}{d\eta_i} \ge 0$ if $\left.\frac{du_i}{d\eta_i}\right|_{\eta_i=0} \ge 0$, which simpilifies to $b \le 2\epsilon + (1-q)\left(f_H - m_L\right)$. Following the same procedure as above, when $x^{(2)} < x_i \le x^{(2)} + 2\epsilon$, we can obtain the condition for $\frac{du_i}{dx_i} \ge 0$ as follows: $b \le 2\epsilon + (1-q)\left(f_H - m_L\right)$ if $r \ge \min\left\{1, \frac{b}{\left(f_H - m_L\right)(1-q)}\right\}$, and

$$\frac{1-r}{\left\{(1-q)\left(f_H - m_L\right)\right\}^{\frac{1}{r-1}}}\left(\frac{b}{r}\right)^{\frac{r}{r-1}} + b \le 2\epsilon + (1-q)\left(f_H - m_L\right), \tag{6}$$

if $r < \min\left\{1, \frac{b}{(1-q)\left(f_H - m_L\right)}\right\}$. We can show that the left-hand side of (6) is increasing with b if $r < \min\left\{1, \frac{b}{(1-q)\left(f_H - m_L\right)}\right\}$ (see Lemma O2 for the proof). Thus, we can rewrite the condition in (6) as follows: $b \le b^{(2)}$, where $b^{(2)}$ is the unique solution of $\frac{1-r}{\left\{(1-q)\left(f_H - m_L\right)\right\}^{\frac{1}{r-1}}}\left(\frac{b}{r}\right)^{\frac{r}{r-1}} + b = 2\epsilon + (1-q)\left(f_H - m_L\right)$. Finally, by combining the conditions for different intervals of x_i, we have

$\frac{du_i}{dx_i} \geq 0$ for any x_i if $b \leq b^{(2)}$, where

$$b^{(2)} = \begin{cases} \begin{array}{l} \text{the solution of} \\ \frac{1-r}{\{(1-q)(f_H - m_L)\}^{\frac{1}{r-1}}} \left(\frac{b}{r}\right)^{\frac{r}{r-1}} + b \\ = 2\epsilon + (1-q)(f_H - m_L) \end{array} & \text{if } r < \min\left\{1, \frac{b}{(1-q)(f_H - m_L)}\right\}; \\ 2\epsilon + (1-q)(f_H - m_L) & \text{if } \min\left\{1, \frac{b}{(1-q)(f_H - m_L)}\right\} \leq r \leq \max\left\{1, \frac{b}{(1-q)(f_H - m_L)}\right\}; \\ r\left(\frac{2\epsilon}{r-1}\right)^{1-\frac{1}{r}} \{(1-q)(f_H - m_L)\}^{\frac{1}{r}} & \text{if } r > \max\left\{1, \frac{b}{(1-q)(f_H - m_L)}\right\}. \end{cases} \tag{7}$$

We show that $b^{(2)}$ in (7) is increasing with both ϵ and $f_H - m_L$ in Lemma O2. $\square$

Lemma O1 *If $r \leq 1$ and $b \leq r(1-q)(f_H - m_L)$, then the equilibrium described in Lemma 1 is unique.*

Proof. Before we proceed to the proof, we first prove $u_i(\theta, \alpha)$ in (1) is increasing with θ and nondecreasing with α. From (1), $\partial u_i(\theta, \alpha)/\partial\theta = 1$ and thus $u_i(\theta, \alpha)$ is increasing with θ. In addition, we can get $\partial u_i(\theta, \alpha)/\partial\alpha = -b + r(1-q)(f_H - m_L)\alpha^{r-1}$. Since $0 \leq \alpha \leq 1$, $r \leq 1$, and $b \leq r(1-q)(f_H - m_L)$, we have $\partial u_i(\theta, \alpha)/\partial\alpha \geq 0$ and thus $u_i(\theta, \alpha)$ is nondecreasing with α.

We prove the argument in 3 steps. In step 1, we will prove by induction that a strategy survives n rounds of iterated deletion of strictly dominated strategies if and only if

$$a_i(x_i) = \begin{cases} 0 & \text{if } x_i < \underline{x}^{(n)}; \\ 1 & \text{if } x_i > \overline{x}^{(n)}. \end{cases} \tag{8}$$

In step 2, we prove $\underline{x}^{(n+1)} \geq \underline{x}^{(n)}$ and $\overline{x}^{(n+1)} \leq \overline{x}^{(n)}$ such that as $n \to \infty$, $\underline{x}^{(n)} \to \underline{x}$ and $\overline{x}^{(n)} \to \overline{x}$. In step 3, we finally prove $\underline{x} = \overline{x} = x^{(2)}$ such that the switching strategy around $x^{(2)}$ is the only strategy that survives iterated deletion of strictly dominated strategy.

Step 1: We first prove the statement holds for $\underline{x}^{(0)}$ and $\overline{x}^{(0)}$. Let $\overline{x}^{(0)} = b + (1-q)(f_H - m_L) - f_H + f_L + \epsilon$. Since $x_i = \theta + \widetilde{\varepsilon}_i$ and $\widetilde{\varepsilon}_i$ is uniformly distributed on $[-\epsilon, \epsilon]$, for any $x_i > \overline{x}^{(0)}$, $\theta > b + (1-q)(f_H - m_L) - f_H + f_L$ and from (1) $u_i(\theta, \alpha) > \alpha^r(1-q)(f_H - m_L) \geq 0$. So $a_i = 1$ for $x_i > \overline{x}^{(0)}$. Similarly, we let $\underline{x}^{(0)} = f_L - f_H - \epsilon$ and get the following: $a_i = 0$ if $x_i < \underline{x}^{(0)}$.

We next prove that a strategy that survives $n+1$ rounds of iterated deletion of strictly dominated strategies is in the form of (8); i.e., if firm i knows that other firms follow a strategy in the form of (8) with thresholds $\underline{x}^{(n)}$ and $\overline{x}^{(n)}$, then firm i's best response should be in the form of (8) with thresholds $\underline{x}^{(n+1)}$ and $\overline{x}^{(n+1)}$. Note that firm i expects $a_j = 1$ ($j \neq i$) for $x_j > \overline{x}^{(n)}$, and $a_j = 0$ ($j \neq i$) for $x_j < \underline{x}^{(n)}$. Firm i is not sure firm j's strategy for $\underline{x}^{(n)} \leq x_j \leq \overline{x}^{(n)}$. So the lowest value of α is achieved if $a_j = 0$ for any $x_j \in (\underline{x}^{(n)}, \overline{x}^{(n)})$. Since $u_i(\theta, \alpha)$ is nondecreasing with α, firm i expects the lowest gain in this case. It is easy to see that in this case other firms follow a switching strategy around $\overline{x}^{(n)}$. Define $u_i^*(x_i, x)$ the expected value of $u_i(\theta, \alpha)$ given that firm i observes x_i and any other firm will follow a switching strategy around x. Then $u_i^*\left(x_i, \overline{x}^{(n)}\right)$ is the lower bound of firm i's expected gain. If $u_i^*\left(x_i, \overline{x}^{(n)}\right) > 0$, then firm i chooses $a_i = 1$ if observing x_i. Let $\overline{x}^{(n+1)}$ be the solution of $u^*\left(x_i, \overline{x}^{(n)}\right) = 0$. Following a similar procedure to that in the proof of Lemma 1, we can show that $u^*\left(x_i, \overline{x}^{(n)}\right)$ is increasing with x_i. So $a_i = 1$ for any $x_i > \overline{x}^{(n+1)}$. Similarly, $u_i^*\left(x_i, \underline{x}^{(n)}\right)$ is the upper bound of firm i's expected gain. Let $\underline{x}^{(n+1)}$ be the solution of $u^*\left(x_i, \underline{x}^{(n)}\right) = 0$. Then $a_i = 0$ for any $x_i < \underline{x}^{(n+1)}$.

Step 2: We prove $\underline{x}^{(n+1)} \geq \underline{x}^{(n)}$ and $\overline{x}^{(n+1)} \leq \overline{x}^{(n)}$ by induction. Since we have shown that $u^*(x_i, x)$ increases with x_i and $u^*\left(\overline{x}^{(1)}, \overline{x}^{(0)}\right) = 0$, we need to show $u^*\left(\overline{x}^{(0)}, \overline{x}^{(0)}\right) \geq 0$ to prove $\overline{x}^{(1)} \leq \overline{x}^{(0)}$. Following a similar procedure to that in the proof of Lemma 1, we get

$$u^*(x, x) = x - \frac{b}{2} - f_L + f_H - \frac{r}{r+1}(1-q)(f_H - m_L). \tag{9}$$

Using $\overline{x}^{(0)} = b + (1-q)(f_H - m_L) - f_H + f_L + \epsilon$ we get $u^*\left(\overline{x}^{(0)}, \overline{x}^{(0)}\right) = \frac{1}{r+1}(1-q)(f_H - m_L) + \frac{b}{2} + \epsilon > 0$. Similarly, we get $\underline{x}^{(1)} \geq \underline{x}^{(0)}$.

We next show $\overline{x}^{(n+1)} \leq \overline{x}^{(n)}$ given that $\overline{x}^{(n)} \leq \overline{x}^{(n-1)}$. Following a similar procedure to that in the proof of Lemma 1, we can show that $u^*(x_i, x)$ is decreasing with x. Since $u^*\left(\overline{x}^{(n)}, \overline{x}^{(n-1)}\right) = 0$ and $\overline{x}^{(n)} \leq \overline{x}^{(n-1)}$, we get $u^*\left(\overline{x}^{(n)}, \overline{x}^{(n)}\right) \geq 0$. Since $u^*(x_i, x)$ increases with x_i, $u^*\left(\overline{x}^{(n+1)}, \overline{x}^{(n)}\right) = 0$, and $u^*\left(\overline{x}^{(n)}, \overline{x}^{(n)}\right) \geq 0$, we get $\overline{x}^{(n+1)} \leq \overline{x}^{(n)}$. Similarly, we obtain the following: $\underline{x}^{(n+1)} \geq \underline{x}^{(n)}$ given that $\underline{x}^{(n)} \geq \underline{x}^{(n-1)}$.

Step 3: As $n \to \infty$, $\underline{x}^{(n)} \to \underline{x}$ and $\overline{x}^{(n)} \to \overline{x}$, where $u^*(\underline{x}, \underline{x}) = 0$ and $u^*(\overline{x}, \overline{x}) = 0$. From (9) we can show that $x^{(2)}$ is the unique solution to $u^*(x, x) = 0$. Thus $\underline{x} = \overline{x} = x^{(2)}$. □

Remark 1. The proof of Lemma O1 uses an argument of iterated deletion of strictly dominated strategies. This argument was used in Morris and Shin (2003) to derive a unique equilibrium when only strategic complementarity among firms is present. This process can also be viewed as a process in which firm i considers its higher-order beliefs to eliminate possible strategies. First, firm i considers its own signal and gets a strategy specified by (8) with thresholds $\underline{x}^{(0)}$ and $\overline{x}^{(0)}$. Then by considering its belief that all other firms follow a strategy specified by (8) with thresholds $\underline{x}^{(0)}$ and $\overline{x}^{(0)}$, firm i refines its strategy and gets a strategy specified by (8) with thresholds $\underline{x}^{(1)}$ and $\overline{x}^{(1)}$. Next by considering its belief about other firms' strategy with $\underline{x}^{(1)}$ and $\overline{x}^{(1)}$, firm i gets a strategy with thresholds $\underline{x}^{(1)}$ and $\overline{x}^{(1)}$. This process continues and finally firm i gets a switching strategy with a threshold $x^{(2)}$.

We use an argument of iterated deletion of strictly dominated strategies to derive the equilibrium strategy in Lemma 1 (a switching strategy around $x^{(2)}$) and prove the uniqueness of this equilibrium under the condition $r \leq 1$ and $b \leq r(1-q)(f_H - m_L)$. We show in Lemma 1 that this switching strategy continues to be an equilibrium strategy for every firm under a more general condition. Similarly, Karp et al. (2007) proved that every firm following a switching strategy is an equilibrium in a one-period setting with both strategic complementarity and substitutability.

Remark 2. In the proof of Lemma O1, we used the property that $u_i(\theta,\alpha)$ is nondecreasing with α under the conditions $r \leq 1$ and $b \leq r(1-q)(f_H - m_L)$. This property enables us to use the argument of iterated deletion of strictly dominated strategies: In every iteration we can obtain new thresholds $\underline{x}^{(n+1)}$ and $\overline{x}^{(n+1)}$ from previous thresholds $\underline{x}^{(n)}$ and $\overline{x}^{(n)}$, where $\underline{x}^{(n+1)} \geq \underline{x}^{(n)}$ and $\overline{x}^{(n+1)} \leq \overline{x}^{(n)}$. However, if the conditions $r \leq 1$ and $b \leq r(1-q)(f_H - m_L)$ are not satisfied, this property does not hold and we cannot use the same argument. Since the action space is not continuous, other standard approach such as the contraction mapping method cannot be used, either.

It is well-known that finding equilibria in games of incomplete information with finite actions and continuous types is quite difficult, let alone proving the uniqueness (Rabinovich et al. 2013). A common approach in literature (e.g., Karp et al. 2007, Rabinovich et al. 2013) is to use a numerical analysis to verify uniqueness. Similarly, we conduct a numerical experiment to find all possible equilibria. Since there are an infinite number of possible asymmetric equilibria among a continuum of firms, we focus on symmetric equilibria in our numerical experiment. From (1), we can show that firm i does not adopt the technology regardless of other firms' decisions if $\theta < f_L - f_H$, and that firm i adopts the technology regardless of other firms' decisions if $\theta > b + f_L - m_L$. Therefore, we focus on $\theta \in [f_L - f_H, b + f_L - m_L]$. To ensure the existence of an equilibrium, we set $b \leq b^{(2)}$, which is the existence condition provided in Lemma 1. Following Karp et al. (2007), we use a finite-state approximation to obtain possible equilibria in period 2, including pure strategy and mixed strategy equilibria. To approximate the continuous state of θ, we assume that θ can take N possible values θ_1, θ_2, ..., θ_N, where $\theta_j = f_L - f_H + \frac{j-1}{N-1}(b + f_H - m_L)$. The signal x_i can take any of the M values above and below a θ value with equal probability, where $M = \lceil \frac{\epsilon}{b+f_H-m_L} N \rceil$ reflects the magnitude of noise.

We next describe the algorithm to find all possible equilibria. Let p_j denote the probability that a firm adopts the technology if the firm observes a signal with the value θ_j. Define $\mathbf{p} \equiv (p_1, p_2, ..., p_N)$. The vector $\mathbf{p}$ specifies a strategy for a firm. Let $u(\mathbf{p},\theta_j)$ denote a firm's expected gain of adopting the technology when the firm observes a signal with value θ_j and all firms follow the strategy $\mathbf{p}$. The strategy $\mathbf{p}$ is an equilibrium if and only if the following statement holds: If $p_j = 0$, then $u(\mathbf{p},\theta_j) \leq 0$; if $0 < p_j < 1$, then $u(\mathbf{p},\theta_j) = 0$ (the condition $0 < p_j < 1$ implies a mixed strategy, which is possible only if $u(\mathbf{p},\theta_j) = 0$); or if $p_j = 1$, then $u(\mathbf{p},\theta_j) \geq 0$. This statement can be rewritten as the following nonlinear complementarity problem: Find a vector $\mathbf{p}$ (where $p_j \in [0,1]$) such that the following conditions are met: $p_j < 1 \Rightarrow u(\mathbf{p},\theta_j) \leq 0$, and $p_j > 0 \Rightarrow u(\mathbf{p},\theta_j) \geq 0$. We solve this nonlinear complementarity problem using the Matlab toolbox provided by Miranda and Fackler (2002). We refer interested readers to Miranda and Fackler (2002) for details about the complementarity problem and the toolbox.

We conduct a numerical experiment using the following parameter values: $r \in \{0.3, 0.5, 1, 2, 3\}$, $m_L \in \{-1, -2, -3\}$, $f_H \in \{1, 2, 3\}$, $f_L \in \{0.2f_H, 0.5f_H, 0.8f_H\}$, $\epsilon \in \{0.1(f_H - m_L), 0.2(f_H - m_L), 0.3(f_H - m_L)\}$, $b \in \{0.1b^{(2)}, 0.4b^{(2)}, 0.7b^{(2)}, b^{(2)}\}$, $q \in \{0, 0.2, 0.4\}$, and $N = 50$. We use $N = 50$ to achieve a balance between computational speed and accuracy. We do not include λ and f_1 because they do not appear in (1) and they do not affect the equilibrium in period 2. For each instance, we use a random generated vector $\mathbf{p}$ as the starting value for the solver. If the solver does not converge, we generate another random vector and start again. In all the 4860 instances, we only find one equilibrium in which firms follow a threshold strategy. Therefore, we numerically verify that if the existence condition in Lemma 1 is satisfied, there is a unique symmetric equilibrium.

Lemma O2 *The threshold $b^{(2)}$ is increasing with ϵ and $(f_H - m_L)$.*

Proof. From (7) it is straight forward that $b^{(2)}$ is increasing with ϵ and $(f_H - m_L)$ if $r > \max\left\{1, \frac{b}{(1-q)(f_H-m_L)}\right\}$ or $\min\left\{1, \frac{b}{(1-q)(f_H-m_L)}\right\} \leq r \leq \max\left\{1, \frac{b}{(1-q)(f_H-m_L)}\right\}$. For the case in which $r < \min\left\{1, \frac{b}{(1-q)(f_H-m_L)}\right\}$, $b^{(2)}$ is defined as the solution to $g(b) = 0$, where $g(b) = \frac{1-r}{\{(1-q)(f_H-m_L)\}^{\frac{1}{r-1}}}\left(\frac{b}{r}\right)^{\frac{r}{r-1}} + b - 2\epsilon - (1-q)(f_H - m_L)$. Using the implicit function theorem, we have that $\frac{db^{(2)}}{d\epsilon} = -\frac{\partial g/\partial\epsilon}{\partial g/\partial b}$ and $\frac{db^{(2)}}{d(f_H-m_L)} = -\frac{\partial g/\partial(f_H-m_L)}{\partial g/\partial b}$. We next show that $\partial g/\partial\epsilon < 0$, $\partial g/\partial b > 0$, and $\partial g/\partial(f_H - m_L) < 0$ such that $b^{(2)}$ is increasing with ϵ and $(f_H - m_L)$. It is easy to show $\partial g/\partial\epsilon = -2 < 0$. We can calculate $\partial g/\partial b$ as follow: $\frac{\partial g}{\partial b} = -\left\{\frac{b}{r(1-q)(f_H-m_L)}\right\}^{\frac{1}{r-1}} + 1$. Since $r < \min\left\{1, \frac{b}{(1-q)(f_H-m_L)}\right\}$, we have $\frac{b}{r(1-q)(f_H-m_L)} > 1$ and $\frac{1}{r-1} < 0$. So $\left\{\frac{b}{(1-q)(f_H-m_L)r}\right\}^{\frac{1}{r-1}} < 1$ and $\frac{\partial g}{\partial b} = -\left\{\frac{b}{(1-q)(f_H-m_L)r}\right\}^{\frac{1}{r-1}} + 1 > 0$. Finally, we obtain the expression of $\partial g/\partial(f_H - m_L)$: $\frac{\partial g}{\partial(f_H-m_L)} = (1-q)\left\{\frac{b}{r(1-q)(f_H-m_L)}\right\}^{\frac{r}{r-1}} - (1-q)$. Similar to the proof of $\partial g/\partial b > 0$ we can show that $\partial g/\partial(f_H - m_L) < 0$. $\square$

Proof of Proposition 1. Since $x_1 = \theta + \widetilde{\varepsilon}_1$, for any given x_1, the posterior distribution of θ is a uniform distribution on $[x_1 - \epsilon, x_1 + \epsilon]$. Since $\widetilde{\varepsilon}_i$ is independent of $\widetilde{\varepsilon}_1$, it is also independent of θ. Thus, $x_i = \theta + \widetilde{\varepsilon}_i$ is a sum of two uniformly distributed random variables that are independent. Using convolution, we can show that the posterior distribution of x_i for any given x_1 is a symmetric triangular distribution on $[x_1 - 2\epsilon, x_1 + 2\epsilon]$. Using this result, we next prove (a) and (b).

(a) Following the same procedure as in the proof of Lemma 1, we obtain the expression of $u_1(x_1)$ as follows:

$$u_1(x_1) = \begin{cases} \lambda - f_1 + m_L + q(m_H - m_L) + x_1 & \text{if } x_1 \leq x^{(2)} - 2\epsilon; \\ \lambda - f_1 + m_L + q(m_H - m_L) + x_1 - \frac{b}{2}\left(\frac{x_1 - x^{(2)} + 2\epsilon}{2\epsilon}\right)^2 & \text{if } x^{(2)} - 2\epsilon < x_1 < x^{(2)}; \\ \lambda - f_1 + m_L + q(m_H - m_L) + x_1 & \\ \quad -\frac{b}{2}\left\{1 - \left(\frac{x_1 - x^{(2)}}{2\epsilon}\right)^2\right\} - b\left(\frac{x_1 - x^{(2)}}{2\epsilon}\right) & \text{if } x^{(2)} \leq x_1 < x^{(2)} + 2\epsilon; \\ \lambda - f_1 + m_H + x_1 - b & \text{if } x_1 \geq x^{(2)} + 2\epsilon. \end{cases} \tag{10}$$

Using (10), we obtain the expression of $\frac{\partial u_1}{\partial x_1}$:

$$\frac{du_1(x_1)}{dx_1} = \begin{cases} 1 & \text{if } x_1 \leq x^{(2)} - 2\epsilon; \\ 1 - \frac{b}{2\epsilon}\left(\frac{x_1 - x^{(2)} + 2\epsilon}{2\epsilon}\right) & \text{if } x^{(2)} - 2\epsilon < x_1 < x^{(2)}; \\ 1 - \frac{b}{2\epsilon}\left(\frac{2\epsilon - x_1 + x^{(2)}}{2\epsilon}\right) & \text{if } x^{(2)} \leq x_1 < x^{(2)} + 2\epsilon; \\ 1 & \text{if } x_1 \geq x^{(2)} + 2\epsilon. \end{cases} \tag{11}$$

We next consider two cases: $\epsilon \geq b/2$ and $\epsilon < b/2$.

For the case of $\epsilon \geq b/2$, from (11) $\frac{\partial u_1}{\partial x_1} \geq 0$. So u_1 crosses zero only once. Let $x_{subs}^{(1)}$ be the solution of $u_1(x_1)$. Then $u_1(x_1) > 0$ if and only if $x_1 > x_{subs}^{(1)}$.

For the case of $\epsilon < b/2$, we will first prove that u_1 increases with x_1, then decreases with x_1, and finally increases with x_1 again. We then compare the local maximum and minimum of u_1 with 0 to determine firm 1's decision. First, we show the shape of u_1 on $\left[x^{(2)} - 2\epsilon, x^{(2)}\right]$. From (11), $\left.\frac{\partial u_1}{\partial x_1}\right|_{x_1 = x^{(2)} - 2\epsilon} > 0$, $\left.\frac{\partial u_1}{\partial x_1}\right|_{x_1 = x^{(2)}} < 0$, and $\frac{\partial^2 u_1}{\partial x_1^2} = -\frac{b}{(2\epsilon)^2} < 0$ when $x^{(2)} - 2\epsilon \leq x_1 < x^{(2)}$. Therefore, There exists $x_1^M \in \left(x^{(2)} - 2\epsilon, x^{(2)}\right)$ such that at $x_1 = x_1^M$, $\frac{\partial u_1}{\partial x_1} = 0$ and $u_1(x_1)$ achieves its local maximum $u_1(x_1^M)$. Solving $\frac{\partial u_1}{\partial x_1} = 0$ in this case yields $x_1^M = x^{(2)} - 2\epsilon + \frac{(2\epsilon)^2}{b}$. Substituting this expression to (10) we obtain $u_1(x_1^M) = \lambda - f_1 + m_L + q(m_H - m_L) + x^{(2)} - 2\epsilon + \frac{2\epsilon^2}{b}$. Similarly, from (11) we get $\left.\frac{\partial u_1}{\partial x_1}\right|_{x_1 = x^{(2)}} < 0$, $\left.\frac{\partial u_1}{\partial x_1}\right|_{x_1 = x^{(2)} + 2\epsilon} > 0$, and $\frac{\partial^2 u_1}{\partial x_1^2} = \frac{b}{(2\epsilon)^2} > 0$ when $x^{(2)} \leq x_1 \leq x^{(2)} + 2\epsilon$. So there exists $x_1^m \in \left(x^{(2)}, x^{(2)} + 2\epsilon\right)$ such that at $x_1 = x_1^m$, $\frac{\partial u_1}{\partial x_1} = 0$ and $u_1(x_1)$ achieves its local minimum value $u_1(x_1^m) = \lambda - f_1 + m_L + q(m_H - m_L) + x^{(2)} + 2\epsilon - b - \frac{2\epsilon^2}{b}$.

We next analyze three cases: (Case I) $u_1(x_1^M) < 0$, (Case II) $u_1(x_1^m) > 0$, and (Case III) $u_1(x_1^m) \leq 0$ and $u_1(x_1^M) \geq 0$.

(Case I) From the expression of $u_1(x_1^M)$, $u_1(x_1^M) < 0$ when $\lambda < -x^{(2)} + 2\epsilon - \frac{2\epsilon^2}{b} + f_1 - m_L - q(m_H - m_L)$. Let $\underline{\lambda}_{subs} = -x^{(2)} + 2\epsilon - \frac{2\epsilon^2}{b} + f_1 - m_L - q(m_H - m_L)$. Then $u_1(x_1^M) < 0$ if and only if $\lambda < \underline{\lambda}_{subs}$. In this case, given that the local maximum $u_1(x_1^M) < 0$, $u_1(x_1)$ can cross zero once in the interval (x_1^m, ∞). Denote by $x_{subs}^{(1)}$ the solution of $u_1(x_1) = 0$. Then $u_1(x_1) > 0$ if and only if $x_1 > x_{subs}^{(1)}$.

(Case II) Similar to (Case I), we can show $u_1\left(x_1^m\right) > 0$ when $\lambda > \overline{\lambda}_{subs}$, where $\overline{\lambda}_{subs} = -x^{(2)} - 2\epsilon + b + \frac{2\epsilon^2}{b} + f_1 - m_L - q\left(m_H - m_L\right)$. There exists $x_{subs}^{(1)} < x_1^m$ such that $u_1\left(x_1\right) > 0$ if and only if $x_1 > x_{subs}^{(1)}$.

(Case III) Similarly, $u_1\left(x_1^m\right) \leq 0$ and $u_1(x_1^M) \geq 0$ when $\underline{\lambda}_{subs} \leq \lambda \leq \overline{\lambda}_{subs}$. We can prove that there exists $x_{subs}^{(1)}$, $y_{subs}^{(1)}$, and $z_{subs}^{(1)}$ ($x_{subs}^{(1)} \leq x_1^M \leq y_{subs}^{(1)} \leq x_1^m \leq z_{subs}^{(1)}$) such that $u_1\left(x_1\right) \geq 0$ if and only if $x_1 \in [x_{subs}^{(1)}, y_{subs}^{(1)}] \cup [z_{subs}^{(1)}, \infty)$.

(b) In order to compute $\frac{dx_{subs}^{(1)}}{dr}$, we apply the implicit function theorem to the equation $u_1\left(x_1\right) = 0$ and obtain the following: $\frac{dx_{sub}^{(1)}}{dr} = -\left.\frac{\partial u_1/\partial r}{\partial u_1/\partial x_1}\right|_{x_1 = x_{subs}^{(1)}}$. From the proof of part (a), $\left.\frac{\partial u_1}{\partial x_1}\right|_{x_1 = x_{subs}^{(1)}} > 0$ and $\frac{dx^{(1)}}{dr}$ has the same sign as $-\left.\frac{\partial u_1}{\partial r}\right|_{x_1 = x_{subs}^{(1)}}$. We examine the sign of $\left.\frac{\partial u_1}{\partial r}\right|_{x_1 = x_{subs}^{(1)}}$ in the following four cases: (Case I) $x_{subs}^{(1)} < x^{(2)} - 2\epsilon$, (Case II) $x^{(2)} - 2\epsilon \leq x_{subs}^{(1)} \leq x^{(2)}$, (Case III) $x^{(2)} < x_{subs}^{(1)} \leq x^{(2)} + 2\epsilon$, and (Case IV) $x_{subs}^{(1)} > x^{(2)} + 2\epsilon$.

(Cases I and IV) From (10), we obtain $\frac{\partial u_1}{\partial r} = 0$. So $\frac{dx_{sub}^{(1)}}{dr} = 0$ and $x_{sub}^{(1)}$ is independent of r.

(Case II) From (10) and $x^{(2)} = \frac{b}{2} + f_L - f_H + \frac{r}{r+1}\left(1-q\right)\left(f_H - m_L\right)$ in Lemma 1, we obtain $\frac{\partial u_1}{\partial r} = \frac{\partial u_1}{\partial x^{(2)}}\frac{\partial x^{(2)}}{\partial r} = \frac{b}{2\epsilon}\left(\frac{x_1 - x^{(2)} + 2\epsilon}{2\epsilon}\right)\frac{(1-q)(f_H - m_L)}{(r+1)^2} > 0$. So $\frac{dx_{sub}^{(1)}}{dr} < 0$.

(Case III) Similar to Case II, we obtain $\frac{\partial u_1}{\partial r} = \frac{b}{2\epsilon}\left\{1 - \left(\frac{x_1 - x^{(2)}}{2\epsilon}\right)\right\}\frac{(1-q)(f_H - m_L)}{(r+1)^2} > 0$. So $\frac{dx_{sub}^{(1)}}{dr} < 0$. Following the same procedure, we can prove the results for $y_{subs}^{(1)}$ and $z_{subs}^{(1)}$.

(c) Similar to part (b), the sign of $\frac{dx_{subs}^{(1)}}{d\epsilon}$ is the same as $-\left.\frac{\partial u_1}{\partial \epsilon}\right|_{x_1 = x_{subs}^{(1)}}$. We check the same four cases as in part (b).

(Cases I and IV) From (10), we obtain $\frac{\partial u_1}{\partial \epsilon} = 0$. So $x_{sub}^{(1)}$ is independent of ϵ.

(Case II) From (10) we obtain $\frac{\partial u_1}{\partial \epsilon} = b\left(\frac{x_1 - x^{(2)}}{2\epsilon} + 1\right)\left(\frac{x_1 - x^{(2)}}{2\epsilon^2}\right) \leq 0$ when $x_1 = x_{subs}^{(1)} \in [x^{(2)} - 2\epsilon, x^{(2)})$. So $\frac{dx_{sub}^{(1)}}{d\epsilon} \geq 0$.

(Case III) Similar to Case II, we obtain $\frac{\partial u_1}{\partial \epsilon} = b\left(\frac{x_1 - x^{(2)}}{2\epsilon^2}\right)\left(1 - \frac{x_1 - x^{(2)}}{2\epsilon}\right) \geq 0$ when $x_1 = x_{subs}^{(1)} \in [x^{(2)}, x^{(2)} + 2\epsilon]$. So $\frac{dx_{sub}^{(1)}}{d\epsilon} \leq 0$.

Following the same procedure, we can prove the results for $y_{subs}^{(1)}$ and $z_{subs}^{(1)}$. $\square$

Proof of Proposition 2. Following the same procedure as in the proof of Lemma 1, we obtain the expression of $\frac{\partial u_1}{\partial x_1}$ as follows:

$$\frac{du_1\left(x_1\right)}{dx_1} = \begin{cases} 1 & \text{if } x_1 < x^{(2)} - 2\epsilon; \\ 1 + \frac{(1-q)(m_H - m_L)}{2\epsilon}\left(\frac{x_1 - x^{(2)} + 2\epsilon}{2\epsilon}\right)^r & \text{if } x^{(2)} - 2\epsilon \leq x_1 < x^{(2)}; \\ 1 + \frac{(1-q)(m_H - m_L)}{2\epsilon}\left\{1 - \left(\frac{x_1 - x^{(2)}}{2\epsilon}\right)^r\right\} & \text{if } x^{(2)} \leq x_1 \leq x^{(2)} + 2\epsilon; \\ 1 & \text{if } x_1 > x^{(2)} + 2\epsilon. \end{cases} \tag{12}$$

We can verify that $\frac{\partial u_1}{\partial x_1} > 0$ in all intervals. So u_1 crosses zero only once. Let $x_{comp}^{(1)}$ be the solution of $u_1(x_1)=0$. Then $u_1(x_1)>0$ if and only if $x_1>x_{comp}^{(1)}$.

(b) and (c) The proof follows the same procedure as in the proof of Proposition 1(b) and (c), respectively. $\square$

Proof of Proposition 3. (a) Similar to the proof of Proposition 1(a), we can show that there are two cases. In the first case, $u_1(x_1)$ is nondecreasing with x_1, and there exists threshold $x_{aggr}^{(1)}$ such that $u_1(x_1)>0$ if and only if $x_1>x_{aggr}^{(1)}$. In the second case, $u_1(x_1)$ first increases with x_1, then decreases with x_1, and finally increases with x_1 again. In this case, there exists $x_1^M<x_1^m$ such that $u_1(x_1)$ achieves a local maximum at x_1^M and a local minimum at x_1^m. If $u_1\left(x_1^M\right)<0$ or $u_1(x_1^m)>0$, then there exists threshold $x_{aggr}^{(1)}$ such that $u_1(x_1)>0$ if and only if $x_1>x_{aggr}^{(1)}$. If $u_1\left(x_1^M\right)\geq 0$ and $u_1(x_1^m)\leq 0$, then there exists $x_{aggr}^{(1)}\leq x_1^M\leq y_{aggr}^{(1)}\leq x_1^m\leq z_{aggr}^{(1)}$ such that $u_1(x_1)\geq 0$ if and only if $x_1\in[x_{aggr}^{(1)},y_{aggr}^{(1)}]\cup[z_{aggr}^{(1)},\infty)$. We let $\underline{\lambda}_{aggr}$ be the value of λ such that $u_1\left(x_1^M\right)=0$ if $\lambda=\underline{\lambda}_{aggr}$, and $\overline{\lambda}_{aggr}$ be the value of λ such that $u_1(x_1^m)=0$ if $\lambda=\overline{\lambda}_{aggr}$. Then $u_1\left(x_1^M\right)\geq 0$ and $u_1(x_1^m)\leq 0$ if and only if $\underline{\lambda}_{aggr}\leq\lambda\leq\overline{\lambda}_{aggr}$.

Finally, we show in Lemma O3 that the first case (respectively, the second case) occurs if $b\leq b^{(1)}$ (respectively, $b>b^{(1)}$), where

$$b^{(1)}=\begin{cases}\text{the solution of} & \\ \frac{1-r}{\{(1-q)(m_H-m_L)\}^{\frac{1}{r-1}}}\left(\frac{b}{r}\right)^{\frac{r}{r-1}}+b & \text{if } r<\min\left\{1,\frac{b}{(1-q)(m_H-m_L)}\right\};\\ =2\epsilon+(1-q)(m_H-m_L) & \\ 2\epsilon+(1-q)(m_H-m_L) & \text{if } \min\left\{1,\frac{b}{(1-q)(m_H-m_L)}\right\}\leq r\leq\max\left\{1,\frac{b}{(1-q)(m_H-m_L)}\right\};\\ r\left(\frac{2\epsilon}{r-1}\right)^{1-\frac{1}{r}}\{(1-q)(m_H-m_L)\}^{\frac{1}{r}} & \text{if } r>\max\left\{1,\frac{b}{(1-q)(m_H-m_L)}\right\}.\end{cases}\tag{13}$$

Similar to Lemma 1, we can show that $b^{(1)}$ is increasing with both ϵ and m_H-m_L.

(b) As in the proof of Proposition 1(b), we can show that $\frac{dx_{aggr}^{(1)}}{dr}$ has the same sign as $-\left.\frac{\partial u_1}{\partial r}\right|_{x_1=x_{aggr}^{(1)}}$. We examine the sign of $\left.\frac{\partial u_1}{\partial r}\right|_{x_1=x_{aggr}^{(1)}}$ in the following four cases: (Case I) $x_{aggr}^{(1)}<x^{(2)}-2\epsilon$, (Case II) $x^{(2)}-2\epsilon\leq x_{aggr}^{(1)}\leq x^{(2)}$, (Case III) $x^{(2)}<x_{aggr}^{(1)}\leq x^{(2)}+2\epsilon$, and (Case IV) $x_{aggr}^{(1)}>x^{(2)}+2\epsilon$. To do so, we write the expression of $u_1(x_1)$ as follows: Following the same procedure as in the proof of Lemma 1, we obtain the expression of $u_1(x_1)$ as follows:

$$u_1(x_1) = \begin{cases} \lambda - f_1 + m_L + q(m_H - m_L) + x_1 & \text{if } x_1 < x^{(2)} - 2\epsilon; \\ \begin{array}{l} \lambda - f_1 + m_L + q(m_H - m_L) + x_1 - \frac{b}{2}(1+\eta_1)^2 \\ + \frac{(1-q)(m_H - m_L)}{r+1}(1+\eta_1)^{r+1} \end{array} & \text{if } x^{(2)} - 2\epsilon \le x_1 \le x^{(2)}; \\ \begin{array}{l} \lambda - f_1 + m_L + q(m_H - m_L) + x_1 - \frac{b}{2}\left(1-\eta_1^2\right) \\ + \frac{(1-q)(m_H - m_L)}{r+1}\left(1-\eta_1^{r+1}\right) + \{(1-q)(m_H - m_L) - b\}\eta_1 \end{array} & \text{if } x^{(2)} \le x_1 \le x^{(2)} + 2\epsilon; \\ \lambda - f_1 + m_H + x_1 - b, & \text{if } x_1 > x^{(2)} + 2\epsilon, \end{cases} \tag{14}$$

where $\eta_1 = \frac{x_1 - x^{(2)}}{2\epsilon}$. We next check the four cases.

(Cases I and IV) From (14), $\frac{\partial u_1}{\partial r} = 0$. So $\frac{dx^{(1)}_{aggr}}{dr} = 0$ and $x^{(1)}_{aggr}$ is independent of r.

(Case II) From (15) and $x^{(2)} = \frac{b}{2} + f_L - f_H + \frac{r}{r+1}(1-q)(f_H - m_L)$ in Lemma 1, we obtain the following:

$$\frac{du_1}{dr} = \frac{\partial u_1}{\partial r} + \frac{\partial u_1}{\partial x^{(2)}}\frac{\partial x^{(2)}}{\partial r} + \frac{\partial u_1}{\partial \eta_1}\frac{\partial \eta_1}{\partial x^{(2)}}\frac{\partial x^{(2)}}{\partial r}$$
$$= \frac{(1-q)(1+\eta_1)}{r+1}\left[-(m_H - m_L)(1+\eta_1)^{r-1}\left\{\frac{1+\eta_1}{r+1} - (1+\eta_1)\ln(1+\eta_1) + \frac{(f_H - m_L)(1-q)}{2\epsilon(r+1)}\right\} + \frac{(f_H - m_L)b}{2\epsilon(r+1)}\right].$$

Let $b^*_x = (m_H - m_L)\left(1+\eta_{aggr}\right)^{r-1}\left\{\frac{2\epsilon\left(1+\eta_{aggr}\right)}{f_H - m_L} - \frac{2\epsilon(r+1)\left(1+\eta_{aggr}\right)}{f_H - m_L}\ln\left(1+\eta_{aggr}\right) + 1 - q\right\}$, where $\eta_{aggr} = \frac{x^{(1)}_{aggr} - x^{(2)}}{2\epsilon}$. Finally, from the above equation, we get $\frac{du_1}{dr} > 0$, and thus $\frac{dx^{(1)}_{aggr}}{dr} < 0$ if and only if $b > b^*_x$.

(Case III) Similar to (Case II), we can show that $\frac{dx^{(1)}_{aggr}}{dr} < 0$ if and only if $b > b^*_x$, where $b^*_x = \frac{m_H - m_L}{1-\eta_{aggr}}\left\{\frac{2\epsilon\left(1-\eta^{r+1}_{aggr}\right)}{f_H - m_L} + \frac{2\epsilon(r+1)\eta^{r+1}_{aggr}}{f_H - m_L}\ln\eta_{aggr} + (1-q)\left(1-\eta^{r}_{aggr}\right)\right\}$.

Following the same procedure, we can prove the results for $y^{(1)}_{aggr}$ and $z^{(1)}_{aggr}$.

(c) Similar to part (b), we can show that $\frac{dx^{(1)}_{aggr}}{d\epsilon}$ has the same sign as $-\left.\frac{\partial u_1}{\partial \epsilon}\right|_{x_1 = x^{(1)}_{aggr}}$. We examine the sign of $\left.\frac{\partial u_1}{\partial \epsilon}\right|_{x_1 = x^{(1)}_{aggr}}$ in the same four cases:

(Cases I and IV) From (14), $\frac{\partial u_1}{\partial \epsilon} = 0$ and $x^{(1)}_{aggr}$ is independent of ϵ.

(Case II) From (14) we obtain the following

$$\frac{du_1}{d\epsilon} = \frac{du_1}{d\eta_1}\frac{\partial \eta_1}{\partial \epsilon} = \left\{-b(1+\eta_1) + (1-q)(m_H - m_L)(1+\eta_1)^r\right\}\left(\frac{x^{(2)} - x_1}{2\epsilon^2}\right).$$

Let $b^{**}_x = (1-q)(m_H - m_L)\left(1+\eta_{aggr}\right)^{r-1}$. From the above equation, we get $\frac{du_1}{d\epsilon} \le 0$, and thus $\frac{dx^{(1)}_{aggr}}{d\epsilon} \ge 0$ if and only if $b > b^{**}_x$.

(Case III) Similar to (Case II), we can show that $\frac{dx^{(1)}_{aggr}}{d\epsilon} < 0$ if and only if $b > b^{**}_x$, where $b^{**}_x = (1-q)(m_H - m_L)\frac{1-\eta^r_{aggr}}{1-\eta_{aggr}}$.

Following the same procedure, we can prove the results for $y^{(1)}_{aggr}$ and $z^{(1)}_{aggr}$. □

Lemma O3 *If $b \le b^{(1)}$, where $b^{(1)}$ is given in (13), then $u_1(x_1)$ increases with x_1. Otherwise $u_1(x_1)$ first increases with x_1, then decreases with x_1, and finally increases with x_1 again.*

Proof. From (14), it is easy to observe that $u_1(x_1)$ is increasing with x_1 in the first and last intervals of x_1. We focus our analysis on the two middle intervals. Let $\eta_1 = \frac{x_1 - x^{(2)}}{2\epsilon}$ and

$$u_1(\eta_1) = \begin{cases} \lambda - f_1 + m_L + q(m_H - m_L) + x^{(2)} + 2\epsilon\eta_1 - \frac{b}{2}(1+\eta_1)^2 \\ + \frac{(1-q)(m_H - m_L)}{r+1}(1+\eta_1)^{r+1} & \text{if } \eta_1 \leq 0; \\ \lambda - f_1 + m_L + q(m_H - m_L) + x^{(2)} + 2\epsilon\eta_1 - \frac{b}{2}\left(1-\eta_1^2\right) \\ + \frac{(1-q)(m_H - m_L)}{r+1}\left(1-\eta_1^{r+1}\right) - \{b - (1-q)(m_H - m_L)\}\eta_1 & \text{if } \eta_1 > 0. \end{cases} \tag{15}$$

Then from (14) we get $u_1(x_1) = u_1(\eta_1)$ and $\frac{du_1}{dx_1} = \frac{1}{2\epsilon}\frac{du_1(\eta_1)}{d\eta_1}$ for $x^{(2)} - 2\epsilon \leq x_1 \leq x^{(2)} + 2\epsilon$ (corresponding to $-1 \leq \eta_1 \leq 1$). We calculate the first, second, and third derivatives of $u_1(\eta_1)$ as follows:

$$\frac{du_1(\eta_1)}{d\eta_1} = \begin{cases} 2\epsilon - b(1+\eta_1) + (1-q)(m_H - m_L)(1+\eta_1)^r & \text{if } \eta_1 \leq 0; \\ 2\epsilon - b(1-\eta_1) + (1-q)(m_H - m_L)\left(1-\eta_1^r\right) & \text{if } \eta_1 > 0; \end{cases} \tag{16}$$

$$\frac{d^2u_1(\eta_1)}{d\eta_1^2} = \begin{cases} -b + r(1-q)(m_H - m_L)(1+\eta_1)^{r-1} & \text{if } \eta_1 \leq 0; \\ b - r(1-q)(m_H - m_L)\eta_1^{r-1} & \text{if } \eta_1 > 0; \end{cases} \tag{17}$$

$$\frac{d^3u_1(\eta_1)}{d\eta_1^3} = \begin{cases} r(r-1)(1-q)(m_H - m_L)(1+\eta_1)^{r-2} & \text{if } \eta_1 \leq 0; \\ -r(r-1)(1-q)(m_H - m_L)\eta_1^{r-2} & \text{if } \eta_1 > 0. \end{cases} \tag{18}$$

We next discuss three cases: (Case I) $r = 1$, (Case II) $r > 1$, and (Case III) $r < 1$.

(Case I) From (16), $\frac{du_1(\eta_1)}{d\eta_1}$ is linear in η_1. In addition, $\frac{du_1(-1)}{d\eta_1} = \frac{du_1(1)}{d\eta_1} = 2\epsilon > 0$ and $\frac{du_1(0)}{d\eta_1} = 2\epsilon - b + (1-q)(m_H - m_L)$. There are two cases regarding the sign of $\frac{du_1(0)}{d\eta_1}$. First, if $\epsilon \geq \frac{b-(1-q)(m_H - m_L)}{2}$ (which implies $2\epsilon - b + (1-q)(m_H - m_L) \geq 0$), then $\frac{du_1(\eta_1)}{d\eta_1} \geq 0$ for any $\eta_1 \in [-1, 1]$, and $\frac{du_1(x_1)}{dx_1} \geq 0$ in this case. Second, if $\epsilon < \frac{b-(m_H - m_L)}{2}$, then $\frac{du_1(0)}{d\eta_1} < 0$, and $\frac{du_1(\eta_1)}{d\eta_1}$ decreases from positive to negative, and then increases from negative to positive. In this case, $u_1(x_1)$ first increases with x_1, then decreases with x_1, and finally increases with x_1 again. The condition for the monotonic change of $u_1(x_1)$ is $\epsilon \geq \frac{b-(1-q)(m_H - m_L)}{2}$.

(Case II) We need to compare the minimum value of $\frac{du_1(\eta_1)}{d\eta_1}$ with 0 to determine the shape of $u_1(x_1)$. From (18), $\frac{d^3u_1(\eta_1)}{d\eta_1^3} > 0$ for $\eta_1 \leq 0$ and $\frac{d^3u_1(\eta_1)}{d\eta_1^3} < 0$ for $\eta_1 > 0$. So $\frac{du_1(\eta_1)}{d\eta_1}$ is convex in η_1 for $\eta_1 \leq 0$ and concave in η_1 for $\eta_1 > 0$. We can show that there exists a minimum value of $\frac{du_1(\eta_1)}{d\eta_1}$ on $(-\infty, 0]$. To get this minimum value, we solve $\frac{d^2u_1^-(\eta_1)}{d\eta_1^2} = 0$ from (17) and get $\eta_1^{(1)} = \left\{\frac{b}{r(1-q)(m_H - m_L)}\right\}^{\frac{1}{r-1}} - 1$.

If $\eta_1^{(1)} \leq 0$ (which happens when $\frac{b}{r(1-q)(m_H - m_L)} \leq 1$), then $\frac{du_1(\eta_1)}{d\eta_1}$ achieves its minimum value $\frac{du_1\left(\eta_1^{(1)}\right)}{d\eta_1} = 2\epsilon - \left\{\frac{b}{r(1-q)(m_H - m_L)}\right\}^{\frac{1}{r-1}} b\left(1 - \frac{1}{r}\right)$. If $\epsilon \geq \frac{1}{2}\left\{\frac{b}{r(1-q)(m_H - m_L)}\right\}^{\frac{1}{r-1}} b\left(1 - \frac{1}{r}\right)$ (which implies $\frac{du_1\left(\eta_1^{(1)}\right)}{d\eta_1} \geq 0$), then $\frac{du_1(\eta_1)}{d\eta_1} \geq 0$ for any $\eta_1 \in [-1, 1]$ and $u_1(x_1)$ increases with x_1. If $\epsilon < \frac{1}{2}\left\{\frac{b}{r(1-q)(m_H - m_L)}\right\}^{\frac{1}{r-1}} b\left(1 - \frac{1}{r}\right)$ (which implies $\frac{du_1\left(\eta_1^{(1)}\right)}{d\eta_1} < 0$), we can show that $\frac{du_1(\eta_1)}{d\eta_1}$ changes from positive to negative, and then from negative to positive for $\eta_1 \in [-1, 1]$ using $\frac{du_1(-1)}{d\eta_1} = \frac{du_1(1)}{d\eta_1} =$

$2\epsilon > 0$ and the fact that $\frac{du_1(\eta_1)}{d\eta_1}$ is convex in η_1 for $\eta_1 \leq 0$ and concave in η_1 for $\eta_1 > 0$. In this case, $u_1(x_1)$ first increases with x_1, then decreases with x_1, and finally increases with x_1 again.

If $\eta_1^{(1)} > 0$ (which happens when $\frac{b}{r(1-q)(m_H-m_L)} > 1$), then $\frac{du_1(\eta_1)}{d\eta_1}$ achieves its minimum value $\frac{du_1(0)}{d\eta_1} = 2\epsilon - b + (1-q)(m_H - m_L)$. So we get the condition for $\frac{du_1(\eta_1)}{d\eta_1} \geq 0$ is $\epsilon \geq \frac{b-(1-q)(m_H-m_L)}{2}$. The rest of the proof is similar to case in which $\eta_1^{(1)} \leq 0$.

(Case III) Similar to (Case II), we get the conditions for the monotonic change of $u_1(x_1)$ as follows: $\frac{b}{r(1-q)(m_H-m_L)} \geq 1$ and $\epsilon \geq \frac{b-(1-q)(m_H-m_L)}{2} - \frac{1}{2}\left\{\frac{b}{r(1-q)(m_H-m_L)}\right\}^{\frac{1}{r-1}} b\left(1-\frac{1}{r}\right)$, or $\frac{b}{r(1-q)(m_H-m_L)} < 1$ and $\epsilon \geq \frac{b-(1-q)(m_H-m_L)}{2}$.

Finally, by combining all the conditions in (Case I), (Case II), and (Case III), we can prove that if $b \leq b^{(1)}$, $u_1(x_1)$ increases with x_1. $\square$

Lemma O4 *(a) If* $r \geq 1$*, then there exists* $x^{(b)} > x^{(2)} - 2\epsilon$ *such that the following holds:* $b_x^* < b^{(2)}$ *if* $x_{aggr}^{(1)} < x^{(b)}$*. If* $0 < r < 1$*, then there exists* $x^{(b)} < x^{(2)} + 2\epsilon$ *such that the following holds:* $b_x^* < b^{(2)}$ *if* $x_{aggr}^{(1)} > x^{(b)}$*.*

(b) If $r > 1$*, then there exists* $x^{(\epsilon)} > x^{(2)} - 2\epsilon$ *such that the following holds:* $b_x^{**} < b^{(2)}$ *if* $x_{aggr}^{(1)} < x^{(\epsilon)}$*. If* $0 < r < 1$*, then there exists* $x^{(\epsilon)} < x^{(2)} + 2\epsilon$ *such that the following holds:* $b_x^{**} < b^{(2)}$ *if* $x_{aggr}^{(1)} > x^{(b)}$*. If* $r = 1$*, then* $b_x^{**} < b^{(2)}$*.*

Proof. **(a)** We will first prove the case of $r \geq 1$. Note that from the proof of Proposition 3 $b_x^* = (m_H - m_L)\left(1+\eta_{aggr}\right)^{r-1}\left\{\frac{2\epsilon\left(1+\eta_{aggr}\right)}{f_H-m_L} - \frac{2\epsilon(r+1)\left(1+\eta_{aggr}\right)}{f_H-m_L}\ln\left(1+\eta_{aggr}\right) + 1 - q\right\}$ for $-1 \leq \eta_{aggr} \leq 0$, where $\eta_{aggr} = \frac{x_{aggr}^{(1)}-x^{(2)}}{2\epsilon}$. As $x_{aggr}^{(1)} \to x^{(2)} - 2\epsilon$, $\eta_{aggr} \to -1$ and $b_x^* \to 0$, which is smaller than $b^{(2)}$. By continuity, for the given value $b^{(2)}$, there exists $x^{(b)} > x^{(2)} - 2\epsilon$ such that $b_x^* < b^{(2)}$ if $x_{aggr}^{(1)} \in [x^{(2)} - 2\epsilon, x^{(b)})$. In addition, if $x_{aggr}^{(1)} < x^{(2)} - 2\epsilon$, in the proof of Proposition 3 we have shown that $x_{aggr}^{(1)}$ does not change with r and $b_x^* = 0$. Combining all these conditions we can prove the case of $r \geq 1$.

We next prove the case of $0 < r < 1$. Note that from the proof of Proposition 3 $b_x^* = \frac{m_H-m_L}{1-\eta_{aggr}}\left\{\frac{2\epsilon\left(1-\eta_{aggr}^{r+1}\right)}{f_H-m_L} + \frac{2\epsilon(r+1)\eta_{aggr}^{r+1}}{f_H-m_L}\ln\eta_{aggr} + (1-q)\left(1-\eta_{aggr}^{r}\right)\right\}$ for $0 \leq \eta_{aggr} \leq 1$. Using L'Hospital's Rule we get $\lim_{\eta_{aggr}\to 1} b_x^* = r(1-q)(m_H - m_L)$. Similar to the case of $r \geq 1$, we need to prove $\lim_{\eta_{aggr}\to 1} b_x^* = r(1-q)(m_H - m_L) < b^{(2)}$. From (7), if $\min\left\{1, \frac{b}{(1-q)(f_H-m_L)}\right\} \leq r \leq 1$, $b^{(2)} = 2\epsilon + (1-q)(f_H - m_L)$, which is larger than $\lim_{\eta_{aggr}\to 1} b_x^* = r(1-q)(m_H - m_L)$ because $f_H > m_H$ and $r < 1$. If $r < \min\left\{1, \frac{b}{(1-q)(f_H-m_L)}\right\}$, from (7) $b^{(2)}$ is the maximum solution of $g(x) = \frac{1-r}{\{(1-q)(f_H-m_L)\}^{\frac{1}{r-1}}}\left(\frac{x}{r}\right)^{\frac{r}{r-1}} + x - 2\epsilon - (1-q)(f_H - m_L) = 0$. To show $r(1-q)(m_H - m_L) < b^{(2)}$, we need to prove the following statements: $g(x)$ achieves its minimum value -2ϵ at $x = r(1-q)(f_H - m_L)$, and it is increasing with x if and only if $x > r(1-q)(f_H - m_L)$. From these two statements, $b^{(2)}$ must be on the right side of $r\left(1-q\right)\left(f_H - m_L\right)$ because $b^{(2)}$ is the maximum solution of $g\left(x\right) = 0$ and $g\left(x\right) > 0$ for sufficiently large x. So $b^{(2)} > r\left(1-q\right)\left(f_H - m_L\right) > r\left(1-q\right)\left(m_H - m_L\right)$. To prove the statements, we calculate $\frac{dg(x)}{dx} = 1 - \left\{\frac{x}{r(1-q)(f_H - m_L)}\right\}^{\frac{1}{r-1}}$, which has a root at $x = r\left(1-q\right)\left(f_H - m_L\right)$. Note that $\frac{1}{r-1} < 0$ because $r < 1$. We can show $\frac{dg(x)}{dx} > 0$ if and only if $x > r\left(1-q\right)\left(f_H - m_L\right)$. The function $g\left(x\right)$ achieves its minimum value $g\left(r\left(1-q\right)\left(f_H - m_L\right)\right) = -2\epsilon < 0$.

(b) The proofs for the cases of $r > 1$ and $0 < r < 1$ are similar to part (a). We focus on the case of $r = 1$. From the proof of Proposition 3, $b_x^{**} = \left(1-q\right)\left(m_H - m_L\right)$ if $r = 1$. From (7) $b^{(2)} = 2\epsilon + \left(1-q\right)\left(f_H - m_L\right)$ if $r = 1$. Since $f_H > m_H$, we can prove that $b_x^{**} < b^{(2)}$. □

Proof of Lemma 2. (a) Following the same procedure as in the proof of Lemma 1, we show that if all firms but firm i follow a switching strategy around $\widehat{x}^{(2)}$, then firm i's best response is to follow this switching strategy as well. Similar to Lemma 1, we first derive the expected gain $\widehat{u}_i\left(x_i\right)$ when other firms follow a switching strategy around $\widehat{x}^{(2)}$. We can show that if $b \le 2\epsilon$, then $\widehat{u}_i\left(\widehat{x}^{(2)}\right) = 0$, and $\widehat{u}_i\left(x_i\right)$ is nondecreasing with x_i. Thus $\widehat{u}_i\left(x_i\right) > 0$ if and only if $x_i > \widehat{x}^{(2)}$.

(b) Following the same procedure as in the proof of Lemma 1, we obtain the expression of $\widehat{u}_1\left(x_1\right)$ as follows:

$$\widehat{u}_1\left(x_1\right) = \begin{cases} \lambda - f_1 + m_L + \widehat{q}\left(m_H - m_L\right) + x_1 & \text{if } x_1 < \widehat{x}^{(2)} - 2\epsilon; \\ \lambda - f_1 + m_L + \widehat{q}\left(m_H - m_L\right) + x_1 - \frac{b}{2}\left(\frac{x_1 - \widehat{x}^{(2)} + 2\epsilon}{2\epsilon}\right)^2 & \text{if } \widehat{x}^{(2)} - 2\epsilon \le x_1 < \widehat{x}^{(2)}; \\ \begin{array}{l} \lambda - f_1 + m_L + \widehat{q}\left(m_H - m_L\right) + x_1 \\ -\frac{b}{2}\left\{1 - \left(\frac{x_1 - \widehat{x}^{(2)}}{2\epsilon}\right)^2\right\} - b\left(\frac{x_1 - \widehat{x}^{(2)}}{2\epsilon}\right) \end{array} & \text{if } \widehat{x}^{(2)} \le x_1 \le \widehat{x}^{(2)} + 2\epsilon; \\ \lambda - f_1 + m_L + \widehat{q}\left(m_H - m_L\right) + x_1 - b & \text{if } x_1 > \widehat{x}^{(2)} + 2\epsilon. \end{cases} \quad (19)$$

The rest of the proof follows the same procedure as in the proof of Proposition 1. □

Proof of Proposition 4. We first solve for the value of r that results in $x^{(2)} = \widehat{x}^{(2)}$. Noting that $x^{(2)} = \frac{b}{2} + f_L - f_H + \frac{r}{r+1}\left(1-q\right)\left(f_H - m_L\right)$ from Lemma 1 and $\widehat{x}^{(2)} = \frac{1}{2}b + f_L - \left(1-\widehat{q}\right)m_L - \widehat{q}f_H$ from Lemma 2(a), we solve $x^{(2)} = \widehat{x}^{(2)}$ and obtain $r = \frac{1-q}{\widehat{q}-q} - 1$. From the condition $r > 0$, we obtain $\frac{1-q}{\widehat{q}-q} > 0$ and $\widehat{q} > q$.

To compare $x_{aggr}^{(1)}$ and $\widehat{x}^{(1)}$, we compare $u_1\left(x_1\right)$ in (14) and $\widehat{u}_1\left(x_1\right)$ in (19) for $x_1 = \widehat{x}^{(1)}$ given that $r = \frac{1-q}{\widehat{q}-q} - 1$. If $u_1(\widehat{x}^{(1)}) \ge \widehat{u}_1(\widehat{x}^{(1)}) = 0$, then $x_{aggr}^{(1)} \le \widehat{x}^{(1)}$ because $u_1\left(x_1\right) \ge 0$ for $x_1 \ge x_{aggr}^{(1)}$. To compare $u_1(\widehat{x}^{(1)})$ and $\widehat{u}_1(\widehat{x}^{(1)})$, we compute $u_1(\widehat{x}^{(1)}) - \widehat{u}_1(\widehat{x}^{(1)})$ using (19) and the expression of

$u_1(x_1)$ in the proof of Proposition 3:

$$u_1(\widehat{x}^{(1)})-\widehat{u}_1(\widehat{x}^{(1)}) = \begin{cases} (q-\widehat{q})(m_H-m_L) & \text{if } \widehat{x}^{(1)} < \widehat{x}^{(2)}-2\epsilon; \\ (\widehat{q}-q)(m_H-m_L)\left\{\left(1+\frac{\widehat{x}^{(1)}-\widehat{x}^{(2)}}{2\epsilon}\right)^{r+1}-1\right\} & \text{if } \widehat{x}^{(2)}-2\epsilon \le \widehat{x}^{(1)} < \widehat{x}^{(2)}; \\ (m_H-m_L)\left(\frac{\widehat{x}^{(1)}-\widehat{x}^{(2)}}{2\epsilon}\right)\left\{1-q-(\widehat{q}-q)\left(\frac{\widehat{x}^{(1)}-\widehat{x}^{(2)}}{2\epsilon}\right)^{r}\right\} & \text{if } \widehat{x}^{(2)} \le \widehat{x}^{(1)} \le \widehat{x}^{(2)}+2\epsilon; \\ (1-\widehat{q})(m_H-m_L) & \text{if } \widehat{x}^{(1)} > \widehat{x}^{(2)}+2\epsilon. \end{cases} \tag{20}$$

From (20) $u_1(\widehat{x}^{(1)})-\widehat{u}_1(\widehat{x}^{(1)}) = 0$ if $m_H = m_L$. In this case $x_{aggr}^{(1)} = \widehat{x}^{(1)}$. We next examine the case in which $m_H > m_L$. From (20) $u_1(\widehat{x}^{(1)})-\widehat{u}_1(\widehat{x}^{(1)}) < 0$ if $\widehat{x}^{(1)} < \widehat{x}^{(2)}-2\epsilon$, and $u_1(\widehat{x}^{(1)})-\widehat{u}_1(\widehat{x}^{(1)}) > 0$ if $\widehat{x}^{(1)} > \widehat{x}^{(2)}+2\epsilon$ (note that we have shown $\widehat{q} > q$). We next show that $d\left\{u_1\left(\widehat{x}^{(1)}\right)-\widehat{u}_1\left(\widehat{x}^{(1)}\right)\right\}/d\widehat{x}^{(1)} \ge 0$ such that there exists $x^{(q)}$ such that $u_1(\widehat{x}^{(1)})-\widehat{u}_1(\widehat{x}^{(1)}) \ge 0$ if and only if $\widehat{x}^{(1)} \ge x^{(q)}$. From (20) we get the following:

$$\frac{d\left\{u_1\left(\widehat{x}^{(1)}\right)-\widehat{u}_1\left(\widehat{x}^{(1)}\right)\right\}}{d\widehat{x}^{(1)}} = \begin{cases} 0 & \text{if } \widehat{x}^{(1)} < \widehat{x}^{(2)}-2\epsilon; \\ \frac{(1-q)(m_H-m_L)}{2\epsilon}\left(1+\frac{\widehat{x}^{(1)}-\widehat{x}^{(2)}}{2\epsilon}\right)^{r} & \text{if } \widehat{x}^{(2)}-2\epsilon \le \widehat{x}^{(1)} < \widehat{x}^{(2)}; \\ \frac{(1-q)(m_H-m_L)}{2\epsilon}\left\{1-\left(\frac{\widehat{x}^{(1)}-\widehat{x}^{(2)}}{2\epsilon}\right)^{r}\right\} & \text{if } \widehat{x}^{(2)} \le \widehat{x}^{(1)} \le \widehat{x}^{(2)}+2\epsilon; \\ 0 & \text{if } \widehat{x}^{(1)} > \widehat{x}^{(2)}+2\epsilon. \end{cases}$$

We can verify $d\left\{u_1\left(\widehat{x}^{(1)}\right)-\widehat{u}_1\left(\widehat{x}^{(1)}\right)\right\}/d\widehat{x}^{(1)} \ge 0$ for any $\widehat{x}^{(1)}$ because $r > 0$.

We finally change the condition $\widehat{x}^{(1)} \ge x^{(q)}$ to a condition on λ using the implicit function theorem. Note that $\widehat{x}^{(1)}$ is the solution of $\widehat{u}_1(x_1) = 0$ and $\widehat{u}_1(x_1)$ increases linearly with λ from (19). By the implicit function theorem, $\frac{d\widehat{x}_1}{d\lambda} = -\left.\frac{\partial\widehat{u}_1/\partial\lambda}{\partial\widehat{u}_1/\partial x_1}\right|_{x_1=\widehat{x}^{(1)}} < 0$ because $\widehat{u}_1(x_1)$ increases with x_1 at $x_1 = \widehat{x}^{(1)}$. So there exists $\widehat{\lambda}$ such that $\widehat{x}^{(1)} \ge x^{(q)}$ if and only if $\lambda \le \widehat{\lambda}$. $\square$

Proposition O1 *(a) Under the regulation that considers industry capability, the following results hold: The threshold $x^{(2)}$ is decreasing with q. If $x_{aggr}^{(1)} < x^{(2)}-2\epsilon$, then $x_{aggr}^{(1)}$ is decreasing with q. If $x^{(2)}-2\epsilon \le x_{aggr}^{(1)} \le x^{(2)}+2\epsilon$, then there exists a threshold b_x^{***} such that $x_{aggr}^{(1)}$ is decreasing with q if and only if $b < b_x^{***}$. If $x_{aggr}^{(1)} > x^{(2)}+2\epsilon$, then $x_{aggr}^{(1)}$ is independent of q.*
(b) Under the regulation that does not consider industry capability, the following results hold: The threshold $\widehat{x}^{(2)}$ is decreasing with $\widehat{q}$. If $\widehat{x}^{(1)} < \widehat{x}^{(2)}-2\epsilon$ or $\widehat{x}^{(1)} > \widehat{x}^{(2)}+2\epsilon$, then $\widehat{x}^{(1)}$ is decreasing with $\widehat{q}$. If $\widehat{x}^{(2)}-2\epsilon \le \widehat{x}^{(1)} \le \widehat{x}^{(2)}+2\epsilon$, then there exists a threshold $\widehat{b}_x^{***}$ such that $\widehat{x}^{(1)}$ is decreasing with $\widehat{q}$ if and only if $b < \widehat{b}_x^{***}$.

Proof. **(a)** The result that the threshold $x^{(2)}$ is decreasing with q follows directly from $x^{(2)} = \frac{b}{2}+f_L-f_H+\frac{r}{r+1}(1-q)(f_H-m_L)$ in Lemma 1 and $f_H > m_L$.

As in the proof of Proposition 3(b), we can show that $\frac{dx_{aggr}^{(1)}}{dq}$ has the same sign as $-\left.\frac{\partial u_1}{\partial q}\right|_{x_1=x_{aggr}^{(1)}}$. We examine the sign of $\left.\frac{\partial u_1}{\partial q}\right|_{x_1=x_{aggr}^{(1)}}$ in the following four cases: (Case I) $x_{aggr}^{(1)} < x^{(2)}-2\epsilon$, (Case

II) $x^{(2)} - 2\epsilon \leq x_{aggr}^{(1)} \leq x^{(2)}$, (Case III) $x^{(2)} < x_{aggr}^{(1)} \leq x^{(2)} + 2\epsilon$, and (Case IV) $x_{aggr}^{(1)} > x^{(2)} + 2\epsilon$.
(Cases I) From (14) we obtain $\frac{\partial u_1}{\partial q} = m_H - m_L > 0$. So $x_{aggr}^{(1)}$ is decreasing with q.
(Case II) Note that $\frac{du_1}{dq} = \frac{\partial u_1}{\partial q} + \frac{\partial u_1}{\partial \eta_1}\frac{d\eta_1}{dx^{(2)}}\frac{dx^{(2)}}{dq}$, where $\frac{d\eta_1}{dx^{(2)}} = -\frac{1}{2\epsilon}$ and $\frac{dx^{(2)}}{dq} = -\frac{r}{r+1}\left(f_H - m_L\right)$ from Lemma 1. Using this expression and (14), we obtain the following:

$$\frac{du_1}{dq} = \left(m_H - m_L\right)\left\{1 - \frac{\left(1+\eta_1\right)^{r+1}}{r+1}\right\} + \left\{\left(1-q\right)\left(m_H - m_L\right)\left(1+\eta_1\right)^r - b\left(1+\eta_1\right)\right\}\frac{r\left(f_H - m_L\right)}{2\epsilon\left(r+1\right)}.$$

Let $b_x^{***} = \frac{m_H - m_L}{1+\eta_{aggr}}[\frac{2\epsilon(r+1)}{r(f_H - m_L)}\{1 - \frac{\left(1+\eta_{aggr}\right)^{r+1}}{r+1}\} + (1-q)\left(1+\eta_{aggr}\right)^r]$, where $\eta_{aggr} = \frac{x_{aggr}^{(1)} - x^{(2)}}{2\epsilon}$. We can show $b_x^{***} \geq 0$ for $-1 \leq \eta_{aggr} \leq 0$. Then from the above equation, we get $\frac{du_1}{dq} > 0$ and thus $\frac{dx_{aggr}^{(1)}}{dq} < 0$ if and only if $b < b_x^{***}$.
(Case III) Similar to Case II, we obtain

$$\frac{du_1}{dq} = \left(m_H - m_L\right)\left(1 - \eta_1 - \frac{1 - \eta_1^{r+1}}{r+1}\right) + \left\{\left(1-q\right)\left(m_H - m_L\right)\left(1 - \eta_1^r\right) - b\left(1 - \eta_1\right)\right\}\frac{r\left(f_H - m_L\right)}{2\epsilon\left(r+1\right)}.$$

Let $b_x^{***} = \frac{m_H - m_L}{1-\eta_1}\{\frac{2\epsilon(r+1)}{r(f_H - m_L)}(1 - \eta_{aggr} - \frac{1-\eta_{aggr}^{r+1}}{r+1}) + (1-q)\left(1 - \eta_{aggr}^r\right)\}$. We can show $b_x^{***} \geq 0$ for $0 \leq \eta_{aggr} \leq 1$. Then we get $\frac{du_1}{dq} > 0$ and thus $\frac{dx_{aggr}^{(1)}}{dq} < 0$ if and only if $b < b_x^{***}$.
(Case IV) From (14) we obtain $\frac{du_1}{dq} = 0$. So $x_{aggr}^{(1)}$ is independent of q.
(b) The result that the threshold $\widehat{x}^{(2)}$ is decreasing with $\widehat{q}$ follows directly from $\widehat{x}^{(2)} = \frac{1}{2}b + f_L - (1-\widehat{q})\, m_L - \widehat{q} f_H$ in Lemma 2 and $f_H > m_L$.

Similar to (a), we can show that $\frac{d\widehat{x}^{(1)}}{d\widehat{q}}$ has the same sign as $-\left.\frac{\partial \widehat{u}_1}{\partial \widehat{q}}\right|_{x_1 = \widehat{x}^{(1)}}$. We examine the sign of $\left.\frac{\partial \widehat{u}_1}{\partial \widehat{q}}\right|_{x_1 = \widehat{x}^{(1)}}$ in the following four cases: (Case I) $\widehat{x}^{(1)} < \widehat{x}^{(2)} - 2\epsilon$, (Case II) $\widehat{x}^{(2)} - 2\epsilon \leq \widehat{x}^{(1)} \leq \widehat{x}^{(2)}$, (Case III) $\widehat{x}^{(2)} < \widehat{x}^{(1)} \leq \widehat{x}^{(2)} + 2\epsilon$, and (Case IV) $\widehat{x}^{(1)} > \widehat{x}^{(2)} + 2\epsilon$.
(Cases I and IV) From (14) we obtain $\frac{\partial \widehat{u}_1}{\partial \widehat{q}} = m_H - m_L > 0$. So $\widehat{x}^{(1)}$ is decreasing with $\widehat{q}$.
(Case II) From (14) and $\widehat{x}^{(2)} = \frac{1}{2}b + f_L - (1-\widehat{q})\, m_L - \widehat{q} f_H$ in Lemma 2 we obtain

$$\frac{d\widehat{u}_1}{d\widehat{q}} = \left(m_H - m_L\right) - \frac{b\left(f_H - m_L\right)}{2\epsilon}\left(\frac{\widehat{x}^{(1)} - \widehat{x}^{(2)} + 2\epsilon}{2\epsilon}\right).$$

Let $\widehat{b}_x^{***} = \frac{4\epsilon^2(m_H - m_L)}{(f_H - m_L)\left(\widehat{x}^{(1)} - \widehat{x}^{(2)} + 2\epsilon\right)}$. From the above equation, we get $\frac{d\widehat{u}_1}{d\widehat{q}} > 0$ and thus $\frac{d\widehat{x}^{(1)}}{d\widehat{q}} < 0$ if and only if $b < \widehat{b}_x^{***}$.
(Case III) Similar to Case II, we obtain

$$\frac{d\widehat{u}_1}{d\widehat{q}} = \left(m_H - m_L\right) - \frac{b\left(f_H - m_L\right)\left\{2\epsilon - \left(\widehat{x}^{(1)} - \widehat{x}^{(2)}\right)\right\}}{4\epsilon^2}.$$

Let $\widehat{b}_x^{***} = \frac{4\epsilon^2(m_H - m_L)}{(f_H - m_L)(2\epsilon - \widehat{x}^{(1)} + \widehat{x}^{(2)})}$. Then $\frac{d\widehat{u}_1}{d\widehat{q}} > 0$ and thus $\frac{d\widehat{x}^{(1)}}{d\widehat{q}} < 0$ if and only if $b < \widehat{b}_x^{***}$. $\square$

B Additional Analysis on the Existence of Equilibrium

We have shown that an equilibrium in period 2 exists if b is sufficiently small. We have also provided a sufficient condition in Lemma 1 for the existence of an equilibrium: $b \le b^{(2)}$. In Proposition 3(b), we have shown that $x_{aggr}^{(1)}$ is nonincreasing with r if and only if $b > b_x^*$; i.e., this result requires b to be sufficiently large. These two conditions (an equilibrium in period 2 exists and $b > b_x^*$) pose constraints on b on different directions. We have analytically proven in Lemma O4 that there always exists an interval of $x_{aggr}^{(1)}$ in which $b_x^* < b^{(2)}$ and these two conditions can hold simultaneously. In this section, we numerically analyze the regions in which both constraints are satisfied. We find that these regions comprise significant portions of the parameter space in which $x_{aggr}^{(1)}$ is dependent on r. We also obtain similar results for the regions in which $b > b_x^{**}$ and an equilibrium exists.

We perform our numerical experiment as follows. Notice that in Lemma O4 we have shown that there always exists an interval of $x_{aggr}^{(1)}$ in which $b_x^* < b^{(2)}$. So we use $x_{aggr}^{(1)}$ as the x-axis and b as the y-axis to show the region in which $b > b_x^*$ and an equilibrium exists. Since the equilibrium is more likely to exist when the complementarity effect is strong, we normalize the x-axis and y-axis with respect to $(f_H - m_L)$, which is the coefficient of α^r in firm i's utility function in equation (1) and affects the complementarity effect. Since the existence of an equilibrium is strongly affected by the magnitude of uncertainty, we plot figures with two uncertainty values: $\epsilon = 0.1\,(f_H - m_L)$ (small uncertainty) and $\epsilon = 0.3\,(f_H - m_L)$ (moderate uncertainty). We also plot figures with two values of r to reflect the strength of regulation: $r = 0.3$ (strong regulation) and $r = 3$ (moderate regulation). Finally, since the coefficient of α^r in firm i's utility function in equation (1) is $(f_H - m_L)$ and the coefficient in firm 1's utility function in equation (2) is $(m_H - m_L)$, we check the following values of $(m_H - m_L)$: $m_H - m_L = (f_H - m_L)/4$, $m_H - m_L = (f_H - m_L)/2$ and $m_H - m_L = f_H - m_L$. We do not include different values of λ and f_1 because $x_{aggr}^{(1)}$ decreases with $(\lambda - f_1)$; different values of $x_{aggr}^{(1)}$ on the x-axis already cover possible values of λ and f_1. We focus on the case in which $q = 0$ because the cases with $q > 0$ are just rescales of the axis without changing the shape of different regions.

We plot Figure 4 with $m_H - m_L = (f_H - m_L)/4$. Figures 4(a)-(d) correspond to different values of ϵ and r. The value $\bar{b}$ is a *numerically computed* upper bound on b that guarantees the existence of an equilibrium. The value $b^{(2)}$ is the bound specified by the sufficient condition in Lemma 1. Besides $\bar{b}$ and $b^{(2)}$, two lines and one curve separate a figure into four regions. In the left and right regions $x_{aggr}^{(1)}$ is independent of r because in these regions firm 1 expects $\alpha = 0$ or $\alpha = 1$

and α is not affected by r. In the lower region, $x_{aggr}^{(1)}$ is strictly increasing with r, and in the upper region $x_{aggr}^{(1)}$ is strictly decreasing with r. The curve that separates the upper region and the lower region is b_x^*. We can see that in all figures the upper region, in which $x_{aggr}^{(1)}$ is strictly increasing with r, takes more than half of the middle region, in which $x_{aggr}^{(1)}$ is dependent on r.

Figure 5 corresponds to $m_H - m_L = (f_H - m_L)/2$, indicating the complementarity effect for firm 1 is larger than that in Figure 4. As a result, larger values of b are required to observe that $x_{aggr}^{(1)}$ is decreasing with r because this result happens when the substitutability effect dominates the complementarity effect, yielding smaller regions in which $x_{aggr}^{(1)}$ is strictly increasing with r than Figure 4. Nevertheless, the upper region still takes about one third of the figure's middle region.

Figure 6 corresponds to the maximum possible value of $(m_H - m_L)$: $m_H - m_L = f_H - m_L$ (note that $f_H \geq 0$ and $m_H \leq 0$). Figure 6 shows that even in this extreme case, an upper region in a figure still takes a considerable portion of the figure's middle region.

To sum up, Figures 4, 5, and 6 show that the region in which $b > b_x^*$ and an equilibrium exists (such that $x_{aggr}^{(1)}$ is decreasing with r) takes a significant portion of the region in which an equilibrium exists and $x_{aggr}^{(1)}$ is affected by r. So our result that $x_{aggr}^{(1)}$ is decreasing with r when b is sufficiently large is quite robust.

Similarly, Figures 7, 8, and 9 show the region in which $b > b_x^{**}$ and an equilibrium exists (such that $x_{aggr}^{(1)}$ is decreasing with ϵ if $x_{aggr}^{(1)} > x^{(2)}$, and increasing with ϵ if $x_{aggr}^{(1)} < x^{(2)}$) also takes a significant portion of the region in which an equilibrium exists and $x_{aggr}^{(1)}$ is affected by ϵ. Our result that uncertainty may encourage innovation when b is sufficiently large is also quite robust.

C Extension: Discovery of a New Technology by a Non-Leading Firm

In the base model, if the leading firm does not develop the technology, then there will be no technology available to reduce the pollutant. In this section we study the following case: If the leading firm does not develop the technology, then there is a probability $q_d < 1$ that another firm will discover and implement a similar technology.

We solve this problem backward. First, we analyze firm 1's decision if the technology has been discovered by another firm. In this case, the leading firm is similar to a following firm in the base model. Following the same procedure as in §4.1.2 of the main body, we obtain firm 1's decision as follows: adopt the technology if $x_1 \geq x^{(2)}$, where $x^{(2)} = \frac{b}{2} + f_L - f_H + \frac{r}{r+1}(1-q)(f_H - m_L)$ is the same as that in Lemma 1.

Next, we study firm 1's decision in the first period. If firm 1 decides to develop the technology, then its expected payoff for any given θ and α is $\pi_1^{(1)}(1;\theta,\alpha) = \lambda - f_1 + \pi_1^{(2)}(\theta,\alpha)$, where $\pi_1^{(2)}(\theta,\alpha)$

is the payoff received after period 1 with the following expression:

$$\pi_1^{(2)}(\theta, \alpha) = m_L + q(m_H - m_L) + \theta - b\alpha + \alpha^r (1-q)(m_H - m_L).$$

Here we use the superscript "(1)" to denote the first period, and "(2)" to denote the time after period 1. If firm 1 decides not to develop the technology, in the base model the expected payoff is zero. However, in this case there is a probability q_d that another firm will discover the technology. If $x_1 \geq x^{(2)}$, then firm 1 will adopt the technology after the discovery of the technology, and its expected payoff from adopting the technology is $\pi_1^{(1)}\left(0; \theta, \alpha | x_1 \geq x^{(2)}\right) = -f_L + \pi_1^{(2)}(\theta, \alpha)$. If $x_1 < x^{(2)}$, then firm 1 will not adopt the technology after the discovery of the technology by the other firm, and its expected payoff from not adopting the technology is $\pi_1^{(1)}\left(0; \theta, \alpha | x_1 < x^{(2)}\right) = \{q + (1-q)\alpha^r\}(m_H - f_H)$. Using the expressions of $\pi_1^{(1)}(1; \theta, \alpha)$, $\pi_1^{(1)}\left(0; \theta, \alpha | x_1 \geq x^{(2)}\right)$, and $\pi_1^{(1)}\left(0; \theta, \alpha | x_1 < x^{(2)}\right)$, we obtain the expected gain of developing the technology in period 1:

$$\begin{aligned} u_1(\theta, \alpha) &\equiv \begin{cases} \pi_1^{(1)}(1; \theta, \alpha) - q_d \pi_1^{(1)}\left(0; \theta, \alpha | x_1 < x^{(2)}\right) & \text{if } x_1 < x^{(2)}; \\ \pi_1^{(1)}(1; \theta, \alpha) - q_d \pi_1^{(1)}\left(0; \theta, \alpha | x_1 \geq x^{(2)}\right) & \text{if } x_1 \geq x^{(2)}; \end{cases} \\ &= \begin{cases} \begin{array}{l} \lambda - f_1 + m_L + q\{(m_H - m_L) - q_d(m_H - f_H)\} + \theta - b\alpha + \\ \alpha^r(1-q)\{(m_H - m_L) - q_d(m_H - f_H)\} \end{array} & \text{if } x_1 < x^{(2)}; \\ \begin{array}{l} \lambda - f_1 + q_d f_L + (1-q_d) \\ \{m_L + q(m_H - m_L) + \theta - b\alpha + \alpha^r(1-q)(m_H - m_L)\} \end{array} & \text{if } x_1 \geq x^{(2)}. \end{cases} \end{aligned} \tag{21}$$

By comparing (21) with equation (2) of the main body (the expression of the expected gain in the base model), we can see that they have the same functional forms except some differences in the coefficients of θ, α, α^r, and constant terms.[19] As a result, although the specific values of thresholds now depend on the new parameter q_d, all the qualitative insights obtained in §4 and §5 continue to hold.

D Extension: Firm 1 May Not Implement the Technology

In the base model, if the leading firm develops the technology, it always implements the technology. In this section, we study the following case: If the leading firm develops the technology, then there is a probability $q_f < 1$ that the technology fails to meet expectations and the leading firm does not implement it.

If firm 1 develops the technology but it does not implement it, then it incurs a development

---

[19] The term $(\theta - b\alpha)$ in equation (2) of the main body remains the same if $x < x^{(2)}$, or changes to $(1-q_d)(\theta - b\alpha)$ if $x \geq x^{(2)}$. The term $\alpha^r(1-q)(m_H - m_L)$ in equation (2) of the main body changes to $\alpha^r(1-q)\{(m_H - m_L) - q_d(m_H - f_H)\}$ if $x < x^{(2)}$ or $(1-q_d)\alpha^r(1-q)(m_H - m_L)$ if $x \geq x^{(2)}$. These changes do not alter the signs of the coefficients.

cost f_1, while receiving zero benefits. If firm 1 develops the technology and implements it, then its expected gain is given in equation (2) in §3. Combining both cases, we obtain firm 1's expected gain for any given θ and α:

$$\begin{aligned} u_1(\theta,\alpha) &= (1-q_f)\left\{\lambda - f_1 + m_L + q\left(m_H - m_L\right) + \theta - b\alpha + \alpha^r\left(1-q\right)\left(m_H - m_L\right)\right\} - f_1 q_f \\ &= (1-q_f)\left\{\lambda + m_L + q\left(m_H - m_L\right) + \theta - b\alpha + \alpha^r\left(1-q\right)\left(m_H - m_L\right)\right\} - f_1. \qquad (22) \end{aligned}$$

By comparing (22) with equation (2) in §3 (the expression of the expected gain in the base model), we can see that they have the same functional forms except that every term but f_1 is multiplied by $(1-q_f)$. As a result, although the specific values of thresholds now depend on the new parameter q_f, all the qualitative insights obtained in §4 and §5 continue to hold.

E Extension: Credits of Voluntary Adopters

In the base model, we use the condition $m_L \leq m_H$ to capture the positive benefit new regulation generates for a voluntary adopter by eliminating its cost disadvantage. In this extension, we focus on another potential benefit from early adoption: compliance credits accumulated before new regulation is enforced. For example, when the NHTSA announced a new corporate average fuel economy standard, Toyota gained a significant amount of fuel economy credits that could be used to reduce compliance to future standards, because Toyota's average fuel economy met the new standard long before the enforcement of the standard (NHTSA 2009).

The model is similar to the base model, except that the payoffs in period 3 are slightly different. As in the base model, the firms that have adopted the technology before period 3 receive m_L in period 3 if new regulation is not enforced, and receive m_H if new regulation is enforced. We assume $m_L \leq 0$ to reflect the fact that the green technology tends to be more expensive to use, and we use $m_H - m_L \geq 0$ to model the benefit of credits under new regulation. Those firms that have not adopted the technology before period 3 receive 0 in period 3 if new regulation is not enforced, and receive m_L if new regulation is enforced. Since they have not used the green technology before period 3, they cannot accumulate any credits. The two differences between this model and the base model are the following. First, in this model we do not restrict m_H to be nonnegative as in the base model, such that the benefit of accumulating credits is not constrained. Second, a firm that is forced to adopt the new technology in period 3 receives m_L in this model instead of m_H as in the base model. We make these two changes because in this model the early adopters receive benefits relative to those who are forced to adopt, in the form of accumulated credits. This benefit does not exist in the base model, where the benefit is the elimination of cost disadvantage.

Similar to the base model, for any given θ and α, if firm i ($\in [0,1)$) adopts the new green technology in period 2 (i.e., $a_i = 1$ in period 2), then its total expected payoff is given as $\pi_i(1;\theta,\alpha) \equiv -f_L + \theta - b\alpha + \{q + (1-q)\alpha^r\} m_H + \{1 - q - (1-q)\alpha^r\} m_L$; and if firm i chooses $a_i = 0$, its total expected payoff is $\pi_i(0;\theta,\alpha) \equiv \{q + (1-q)\alpha^r\}(m_L - f_H)$. Thus, the expected gain from adopting the technology, $u_i(\theta,\alpha)$, is

$$\begin{aligned} u_i(\theta,\alpha) &\equiv \pi_i(1;\theta,\alpha) - \pi_i(0;\theta,\alpha) \\ &= -f_L + m_L + q(f_H - 2m_L + m_H) + \theta - b\alpha + \alpha^r (1-q)(f_H - 2m_L + m_H). \quad (23) \end{aligned}$$

Similarly, the expected gain of firm 1 is

$$u_1(\theta,\alpha) = \lambda - f_1 + m_L + q(m_H - m_L) + \theta - b\alpha + \alpha^r (1-q)(m_H - m_L).$$

The above expression of $u_1(\theta,\alpha)$ is exactly the same as that in (2). The expression of $u_i(\theta,\alpha)$ in (23) is similar to that in (1), except that there is an additional positive term $(m_H - m_L)$ in the coefficient for α^r and in the coefficient for q. Therefore, except that the expression of $x^{(2)}$ and the specific values of thresholds will be slightly different from the base model, all the qualitative insights obtained in §4 and §5 continue to hold.

F Extension: Welfare Maximizing Government

In our base model, we use $q + (1-q)\alpha^r$ (where $r \in (0,\infty)$) to model regulation probability that is increasing with the voluntary adoption level. In this section we study a case in which the government agency maximizes social welfare. We show that if the government agency's regulation cost is a convex function of the number of firms, and if the regulation cost is the agency's private information, then from a firm's point of view the regulation probability is increasing with the voluntary adoption level. We further show that our insights from the main body hold through numerical studies. For notational convenience, we use subscript "wm" (welfare maximizing) to denote thresholds in this case.

The games in periods 1 and 2 are the same as in the base model. In period 3, the government agency decides whether to enforce a stricter new standard to maximize social welfare. We normalize social welfare in the absence of the new regulation to 0. If the agency enforces new regulation, social welfare consists of four parts. The first part is positive externality from pollution reduction. Denote by e the unit externality generated if one firm implements the new green technology. If the new regulation is enforced, $(1-\alpha)$ firms are forced to implement the technology. The total externality is $e(1-\alpha)$ (noting that firms are uniformly distributed on $[0,1)$ with unit density). The second part is the payoff of those firms that have not implemented the technology. A firm has to pay f_H to implement the technology, and it will receive m_H in period 3. The total payoff is $-(f_H - m_H)(1-\alpha)$. The third part is the payoff of those firms that have already implemented the technology. In the absence of the new regulation, each of these firms' payoff is m_L. With the new regulation, the payoff is m_H. So the total difference is $(m_H - m_L)\alpha$. The last part is the government agency's cost. The government agency receives a fine $(f_H - f_L)(1-\alpha)$ from those firms that have not implemented the technology. The government agency also incurs a regulation cost $f_r(1-\alpha)^k$, where $f_r > 0$ is the coefficient of the cost. We assume $k > 1$ such that the cost is a convex function of the number of firms that will be forced to implement the technology. The convex cost function indicates that it is increasingly more difficult to force firms to implement the green technology as the number of firms increases. Thus, the total welfare is

$$w(\alpha) = e(1-\alpha) - (f_H - m_H)(1-\alpha) + (m_H - m_L)\alpha + (f_H - f_L)(1-\alpha) - f_r(1-\alpha)^k. \quad (24)$$

The government agency enforces the new regulation if $w(\alpha) > 0$.

Since the regulation cost is usually not known to firms, we assume that firms have a belief that the cost coefficient is a random variable $\widetilde{f_r}$ with support on $(0,\infty)$. The probability of regulation is the probability of $w(\alpha) > 0$. Using (24) we obtain

$$\Pr(w(\alpha) > 0) = \Pr\left(\widetilde{f_r} < \{e - (f_L - m_L)\}(1-\alpha)^{1-k} + (m_H - m_L)(1-\alpha)^{-k}\right). \quad (25)$$

Since $\widetilde{f_r}$ is distributed on $(0,\infty)$, we assume $\{e - (f_L - m_L)\}(1-\alpha)^{1-k} + (m_H - m_L)(1-\alpha)^{-k} > 0$ such that $\Pr(w(\alpha) > 0)$ is positive; otherwise the problem is trivial. The condition $\{e - (f_L - m_L)\}$ $(1-\alpha)^{1-k} + (m_H - m_L)(1-\alpha)^{-k} > 0$ can be simplified to $e - (f_L - m_L) + \frac{m_H - m_L}{1-\alpha} > 0$. We next use this condition to show that $\{e - (f_L - m_L)\}(1-\alpha)^{1-k} + (m_H - m_L)(1-\alpha)^{-k}$ in (25) increases with α and k, and thus the probability of regulation $\Pr(w(\alpha) > 0)$ also increases with α and k. To do so, we differentiate $\{e - (f_L - m_L)\}(1-\alpha)^{1-k} + (m_H - m_L)(1-\alpha)^{-k}$ with respect to α and obtain the following:

$$
\begin{aligned}
& \frac{\partial}{\partial\alpha}\left[\left\{e-\left(f_L-m_L\right)\right\}(1-\alpha)^{1-k}+\left(m_H-m_L\right)(1-\alpha)^{-k}\right] \\
= \;& (1-\alpha)^{-k}\left[(k-1)\left\{e-\left(f_L-m_L\right)\right\}+k\frac{m_H-m_L}{1-\alpha}\right] \\
> \;& (1-\alpha)^{-k}\left[(k-1)\left\{e-\left(f_L-m_L\right)\right\}+(k-1)\frac{m_H-m_L}{1-\alpha}\right]>0,
\end{aligned}
$$

where the first inequality is due to $m_H-m_L>0$ and $1-\alpha>0$, and the last inequality is due to $e-(f_L-m_L)+\frac{m_H-m_L}{1-\alpha}>0$. We differentiate $\left\{e-\left(f_L-m_L\right)\right\}(1-\alpha)^{1-k}+\left(m_H-m_L\right)(1-\alpha)^{-k}$ with respect to k and obtain the following:

$$
\begin{aligned}
& \frac{\partial}{\partial k}\left[\left\{e-\left(f_L-m_L\right)\right\}(1-\alpha)^{1-k}+\left(m_H-m_L\right)(1-\alpha)^{-k}\right] \\
= \;& -\ln(1-\alpha)\left[\left\{e-\left(f_L-m_L\right)\right\}(1-\alpha)^{1-k}+\left(m_H-m_L\right)(1-\alpha)^{-k}\right]>0,
\end{aligned}
$$

where the inequality is due to $\ln(1-\alpha)<0$ and $\left\{e-\left(f_L-m_L\right)\right\}(1-\alpha)^{1-k}+\left(m_H-m_L\right)(1-\alpha)^{-k}$ >0. It is intuitive that the probability of regulation $\Pr(w(\alpha)>0)$ increases with k: As k increases, the regulation cost $f_r(1-\alpha)^k$ decreases (noting that $1-\alpha<1$). Define $\widehat{r}\equiv 1/k$, where $\widehat{r}\in(0,1)$ since $k>1$. Then the regulation cost is $f_r(1-\alpha)^{\frac{1}{\widehat{r}}}$. The probability of regulation $\Pr(w(\alpha)>0)$ decreases with $\widehat{r}$ and increases with α in a manner similar to our base model in which the regulation probability decreases with r and increases with α.

To sum up, if the regulation cost is a convex function $f_r(1-\alpha)^{\frac{1}{\widehat{r}}}$, where $f_r>0$ and $0<\widehat{r}<1$, and f_r is private information to the government agency, then in a firm's belief the probability of regulation increases with the voluntary level α and decreases with $\widehat{r}$.

We next conduct a numerical study to show that our intuitions in the main body still hold in this case. Figure 11 shows that firm 1's expected gain from developing the technology is nonmonotonic with x_1: The function $u_1(x_1)$ can cross 0 three times and there are two intervals of x_1 in which $u_1(x_1)>0$ ($\left[x_{wm}^{(1)},y_{wm}^{(1)}\right]$ and $[z_{wm}^{(1)},\infty)$), confirming our intuitions from Proposition 3(a) still hold in this case. Figure 12 shows how an increase of $\widehat{r}$ affects $x_{wm}^{(1)}$, the threshold of x_1 at which $u_1(x_1)$ crosses 0. When b is sufficiently large, Figure 12(a) shows that $x_{wm}^{(1)}$ decreases with $\widehat{r}$. When b is sufficiently small, Figure 12(b) shows that $x_{wm}^{(1)}$ increases with $\widehat{r}$. We can see that $\widehat{r}$ affects the threshold in the same direction as shown in Proposition 3(b). Figure 13 shows how an increase of ϵ affects the threshold $x_{wm}^{(1)}$. Figure 13(a) shows that larger uncertainty decreases $x_{wm}^{(1)}$ when b is sufficiently large, and Figure 13(b) shows that larger uncertainty increases $x_{wm}^{(1)}$ when b is sufficiently small (in both figures $x_{wm}^{(1)}>x_{wm}^{(2)}$). We can see that ϵ affects the threshold in the same direction as shown in Proposition 3(c).

G Extension: Information Advantage of the Leading Firm

In this section, we study a case in which firm 1's signal is more accurate than others; i.e., $\epsilon_1 \leq \epsilon$. Firm 1 will develop the new green technology if and only if the expected gain from this technology $u_1(x_1) \geq 0$. Since $u_1(x_1)$ depends on the voluntary adoption level α, we first derive the expression for α using Lemma 1. From Lemma 1, any other firm i $(\in [0,1))$ will adopt the technology when it observes its signal x_i $(= \theta + \widetilde{\varepsilon}_i)$ higher than $x^{(2)}$. Thus, for any given θ, we can express α as follows:

$$\alpha = \begin{cases} 0 & \text{if } \theta < x^{(2)} - \epsilon; \\ \frac{\theta+\epsilon-x^{(2)}}{2\epsilon} & \text{if } x^{(2)} - \epsilon \leq \theta \leq x^{(2)} + \epsilon; \\ 1 & \text{if } \theta > x^{(2)} + \epsilon. \end{cases}$$

Since firm 1's signal is $x_1 = \theta + \widetilde{\varepsilon}_1$, where $\widetilde{\varepsilon}_1$ is uniformly distributed on $[-\epsilon_1, \epsilon_1]$, the posterior distribution of the fundamental θ is uniformly distributed on $[x_1 - \epsilon_1, x_1 + \epsilon_1]$ for any given x_1. Using this property, we can derive the following expression for $u_1(x_1)$:

$$u_1(x_1) = \begin{cases} \lambda - f_1 + m_L + q(m_H - m_L) + x_1, & \text{if } x_1 < x^{(2)} - \epsilon - \epsilon_1; \\ \begin{array}{l} \lambda - f_1 + m_L + q(m_H - m_L) + x_1 \\ -\frac{\epsilon}{\epsilon_1}\frac{b}{2}\left(\frac{\epsilon_1+\epsilon}{2\epsilon} + \frac{x_1-x^{(2)}}{2\epsilon}\right)^2 \\ +\frac{\epsilon}{\epsilon_1}\frac{(1-q)(m_H-m_L)}{r+1}\left(\frac{\epsilon_1+\epsilon}{2\epsilon} + \frac{x_1-x^{(2)}}{2\epsilon}\right)^{r+1}, \end{array} & \begin{array}{l} \text{if } x^{(2)} - \epsilon - \epsilon_1 \leq x_1 \\ \leq x^{(2)} - \epsilon + \epsilon_1; \end{array} \\ \begin{array}{l} \lambda - f_1 + m_L + q(m_H - m_L) + x_1 + \frac{(1-q)(m_H-m_L)\epsilon}{(r+1)\epsilon_1} \cdot \\ \left\{\left(\frac{x_1+\epsilon_1+\epsilon-x^{(2)}}{2\epsilon}\right)^{r+1} - \left(\frac{x_1-\epsilon_1+\epsilon-x^{(2)}}{2\epsilon}\right)^{r+1}\right\} \\ -\frac{b\epsilon}{2\epsilon_1}\left\{\left(\frac{x_1+\epsilon_1+\epsilon-x^{(2)}}{2\epsilon}\right)^2 - \left(\frac{x_1-\epsilon_1+\epsilon-x^{(2)}}{2\epsilon}\right)^2\right\}, \end{array} & \begin{array}{l} \text{if } x^{(2)} - \epsilon + \epsilon_1 < x_1 \\ < x^{(2)} + \epsilon - \epsilon_1; \end{array} \\ \begin{array}{l} \lambda - f_1 + m_L + q(m_H - m_L) + x_1 \\ -\frac{\epsilon}{\epsilon_1}\frac{b}{2}\left\{1 - \left(\frac{x_1-x^{(2)}-\epsilon_1+\epsilon}{2\epsilon}\right)^2\right\} \\ +\frac{\epsilon}{\epsilon_1}\frac{(1-q)(m_H-m_L)}{r+1}\left\{1 - \left(\frac{x_1-x^{(2)}-\epsilon_1+\epsilon}{2\epsilon}\right)^{r+1}\right\} \\ +\{(1-q)(m_H - m_L) - b\}\frac{x_1-x^{(2)}-\epsilon+\epsilon_1}{2\epsilon_s}, \end{array} & \begin{array}{l} \text{if } x^{(2)} + \epsilon - \epsilon_1 \leq x_1 \\ \leq x^{(2)} + \epsilon + \epsilon_1; \end{array} \\ \lambda - f_1 + x_1 + m_H - b, & \text{if } x_1 > x^{(2)} + \epsilon + \epsilon_1. \end{cases} \tag{26}$$

We can see that the expression of $u_1(x_1)$ is quite similar to equation (14).[20] Thus, by following the same procedures as in the proofs of the results in the main body, we can obtain similar results as well.

H Extension: Payoff Reduction of Voluntary Adopters after New Regulation

In the base model, we assume that new regulation does not reduce a voluntary adopter's market benefit from better reputation. Alternatively, one might argue that some environmentally conscious consumers do not care about a firm's voluntary adoption; instead they care only about whether a firm uses the green technology or not. In this case, new regulation might reduce a voluntary adopter's payoff because a firm that is forced to adopt the green technology by new regulation might steal some environmentally conscious consumers from voluntary adopters.

In this case, to model diffusion of consumer sentiment, we assume that β ($\in [0,1]$) portion of the market benefit from adopting a new green technology realizes in period 2, and $(1-\beta)$ portion realizes in period 3. In period 2, a firm that adopts the technology voluntarily receives $\beta(\theta - b\alpha)$, and a firm that does not adopt the technology receives zero. If new regulation is not enforced in period 3, then the proportion of adopters is still α. In this case, a firm that has adopted the technology receives $(1-\beta)(\theta - b\alpha)$, and a firm that has not adopted the technology receives zero in period 3. If new regulation is enforced in period 3, then all firms are forced to adopt the technology and the proportion of adopters is 1. Therefore, all firms receive $(1-\beta)(\theta - b)$ in period 3. If a firm adopts the technology in period 2, its payoff is $\pi_i(1;\theta,\alpha) = -f_L + \beta(\theta - b\alpha) + \{q + (1-q)\alpha^r\}\{(1-\beta)(\theta - b) + m_H\} + (1 - q - (1-q)\alpha^r)\{(1-\beta)(\theta - b\alpha) + m_L\}$. If a firm does not adopt the technology in period 2, its payoff is $\pi_i(0;\theta,\alpha) = \{q + (1-q)\alpha^r\}\{(1-\beta)(\theta - b) + m_H - f_H\}$. We can write $u_i(\theta,\alpha) = \pi_i(1;\theta,\alpha) - \pi_i(0;\theta,\alpha)$ as follows:

$$\begin{aligned} u_i(\theta,\alpha) &= -f_L + m_L + q(f_H - m_L) + (\theta - b\alpha)\{1 - q(1-\beta)\} \\ &\quad + (1-q)\alpha^r\{f_H - m_L - (1-\beta)(\theta - b\alpha)\}. \end{aligned}$$

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[20] If $x_1 \le x^{(2)} - \epsilon - \epsilon_1$ or $x_1 \ge x^{(2)} + \epsilon + \epsilon_1$, the expression of $u_1(x_1)$ is the same as $u_1(x_1)$ in (14). If $x^{(2)} - \epsilon - \epsilon_1 \le x_1 \le x^{(2)} - \epsilon + \epsilon_1$, there is an additional factor of $\frac{\epsilon}{\epsilon_1}$ for the terms that contain $\frac{x_1 - x^{(2)}}{2\epsilon}$ in (26), and the constant 1 in (14) is replaced by $\frac{\epsilon_1 + \epsilon}{2\epsilon}$ in (26). If $x^{(2)} + \epsilon - \epsilon_1 \le x_1 \le x^{(2)} + \epsilon + \epsilon_1$, the term $\frac{x_1 - x^{(2)}}{2\epsilon}$ in (14) is replaced by $\frac{x_1 - x^{(2)} - \epsilon_1 + \epsilon}{2\epsilon}$ in (26), and there is also an additional factor $\frac{\epsilon}{\epsilon_1}$ for the terms that contain $\frac{x_1 - x^{(2)} - \epsilon_1 + \epsilon}{2\epsilon}$. These differences only change the magnitudes of affected terms, not their signs. If $x^{(2)} - \epsilon + \epsilon_1 \le x_1 \le x^{(2)} + \epsilon - \epsilon_1$, the expression of $u_1(x_1)$ in (26) is different from (14). However, the substitutability effect modeled by the negative term $-\frac{b\epsilon}{2\epsilon_1}\{(\frac{x_1 + \epsilon_1 + \epsilon - x^{(2)}}{2\epsilon})^2 - (\frac{x_1 - \epsilon_1 + \epsilon - x^{(2)}}{2\epsilon})^2\}$ and the complementarity effect modeled by the positive term $\frac{(1-q)(m_H - m_L)\epsilon}{(r+1)\epsilon_1}\{(\frac{x_1 + \epsilon_1 + \epsilon - x^{(2)}}{2\epsilon})^{r+1} - (\frac{x_1 - \epsilon_1 + \epsilon - x^{(2)}}{2\epsilon})^{r+1}\}$ still share a lot of similarities with those in (14) and the qualitative intuitions are similar.

Similarly, we obtain the expression of $u_1(\theta,\alpha)$:

$$\begin{aligned}u_1(\theta,\alpha) &= \lambda - f_1 + m_L + q\left\{m_H - m_L - b\left(1-\beta\right)\right\} + \theta - \left\{1 - q\left(1-\beta\right)\right\} b\alpha \\ &\quad + \alpha^r\left(1-q\right)\left\{m_H - m_L - b\left(1-\beta\right)\left(1-\alpha\right)\right\}.\end{aligned}$$

The above two expressions are similar to the expressions of $u_i(\theta,\alpha)$ and $u_1(\theta,\alpha)$ in (1) and (2) of the main body, except that the coefficients of a few terms are slightly different. Specifically, there is an additional factor $\left\{1 - q\left(1-\beta\right)\right\} > 0$ for the coefficients of α in the above two expressions. This additional factor does not change the signs of the coefficients. The coefficients for $\alpha^r\left(1-q\right)$ in the above two expressions become $\left\{f_H - m_L - \left(1-\beta\right)\left(\theta - b\alpha\right)\right\}$ in $u_i(\theta,\alpha)$ and $\left\{m_H - m_L - b\left(1-\beta\right)\left(1-\alpha\right)\right\}$ in $u_1(\theta,\alpha)$. As long as $\left(1-\beta\right)$ is sufficiently small, the coefficients for $\alpha^r\left(1-q\right)$ remain positive. As a result, all the qualitative insights obtained in §4 and §5 continue to hold if $\left(1-\beta\right)$ is sufficiently small (i.e., a firm that is forced to adopt a new green technology does not attract many environmentally conscious consumers), although the specific values of thresholds are different in this case.

I Extension: Single Representative Firm in Period 2

In this section, we use a single representative firm to model the whole industry. Let firm 2 denote the representative firm. To be consistent with the base model, we assume that the size of firm 1 is very small such that the voluntary adoption level is determined by the adoption in period 2. If firm 2 adopts the technology, then $\alpha = 1$. In this case, the government will enforce the new standard with certainty in period 3. If firm 2 does not adopt the technology, then $\alpha = 0$ and the government will not enforce the new standard in period 3. Firm 2 adopts the technology if and only if $x_2 \geq x^{(2)}$. To be consistent with the base model, we assume that $x^{(2)}$ increases with r. The following lemma shows how firm 1's threshold changes with r.

Lemma O7 *Firm 1's threshold* $x_{aggr}^{(1)}$ is nonincreasing with r if and only if $b > \left(1-q\right)\left(m_H - m_L\right)$.

Proof. We first derive the expression of firm 1's expected gain for a given signal x_1. We then use the expression to complete the proof.

Denote by β the probability that firm 2 will adopt the technology. If firm 2 adopts the technology, firm 1's gain for developing the technology for a given θ is $u_1\left(\theta\right) = \lambda - f_1 + m_H + \theta - b$. If firm 2 does not adopt the technology, firm 1's gain is $u_1\left(\theta\right) = \lambda - f_1 + m_L + q\left(m_H - m_L\right) + \theta$. So firm 1's expected gain for given θ and β is

$$u_1(\theta, \beta) = \lambda - f_1 + m_L + q(m_H - m_L) + \theta - \{b - (1-q)(m_H - m_L)\}\beta.$$

We next use this expression to derive $u_1(x_1)$, firm 1's expected gain for a given signal x_1.

For a given θ, $x_2 = \theta + \widetilde{\varepsilon}_2$ is uniformly distributed on $[\theta - \epsilon, \theta + \epsilon]$. Since firm 2 adopts the technology if $x_2 > x^{(2)}$, the expression for β is as follows:

$$\beta = \begin{cases} 0 & \text{if } \theta < x^{(2)} - \epsilon; \\ \frac{\theta + \epsilon - x^{(2)}}{2\epsilon} & \text{if } x^{(2)} - \epsilon \le \theta \le x^{(2)} + \epsilon; \\ 1 & \text{if } \theta > x^{(2)} + \epsilon. \end{cases}$$

For a given x_1, firm 1's posterior belief of θ is a uniform distribution on $[x_1 - \epsilon, x_1 + \epsilon]$. By following the same procedure as in the proof of Proposition 1(a), we can obtain $u_1(x_1)$ as follows:

$$u_1(x_1) = \begin{cases} \lambda - f_1 + m_L + q(m_H - m_L) + x_1 & \text{if } x_1 \le x^{(2)} - 2\epsilon; \\ \quad \lambda - f_1 + m_L + q(m_H - m_L) + x_1 \\ \quad - \frac{b - (1-q)(m_H - m_L)}{2} \left(\frac{x_1 - x^{(2)} + 2\epsilon}{2\epsilon} \right)^2 & \text{if } x^{(2)} - 2\epsilon < x_1 < x^{(2)}; \\ \quad \lambda - f_1 + m_L + q(m_H - m_L) + x_1 \\ \quad - \frac{b - (1-q)(m_H - m_L)}{2} \left\{ 1 - \left(\frac{x_1 - x^{(2)}}{2\epsilon} \right)^2 \right\} & \text{if } x^{(2)} \le x_1 < x^{(2)} + 2\epsilon; \\ \quad - \{b - (1-q)(m_H - m_L)\} \left(\frac{x_1 - x^{(2)}}{2\epsilon} \right) \\ -f_1 + \lambda + m_H + x_1 - b & \text{if } x_1 \ge x^{(2)} + 2\epsilon. \end{cases}$$

By following the same procedure as in the proof of Proposition 1(b), we can prove that $x_{aggr}^{(1)}$ is nonincreasing with r if and only if $b > (1-q)(m_H - m_L)$. $\square$

Lemma O7 suggests that a smaller r discourages firm 1 from developing the technology if and only if $b > (1-q)(m_H - m_L)$, where $(1-q)(m_H - m_L)$ is a constant and does not change with r. Once the value of b is given, more aggressive regulation (smaller r) either always discourages or always encourages innovation; there is no way the government can change that. However, in the main body, Proposition 3 shows that a smaller r discourages firm 1 from developing the technology if and only if $b > b_x^*$, where b_x^* is a function of r. In this case, it is possible for the government to change how more aggressive regulation affects innovation. For example, consider a given b that is just slightly larger than $(1-q)(m_H - m_L)$ in Figure 10. In the single firm case, more aggressive regulation always discourages innovation because $b > (1-q)(m_H - m_L)$. However, in our base model case, if r is sufficiently small, then $b < b_x^*$ and more aggressive regulation encourages innovation; if r is sufficiently large, then $b > b_x^*$ and more aggressive regulation discourages innovation.

J Extension: Heterogeneous Technology Payoffs

In the main body we study a case in which firm i observes a noisy signal x_i of the maximum payoff θ. Since x_i is the sum of θ and a random noise $\widetilde{\varepsilon}_i$, all firms' signals are correlated. In this section we study a case in which firms have different maximum payoffs. Firm i's payoff from voluntary adoption is given as $\theta_i - b\alpha$. We assume θ_i is uniformly distributed on $\left[0, \overline{\theta}\right]$. The distribution of θ_i is known to every firm. To get a closed-form solution, we focus on the case with $r = 1$. We can obtain similar results for $r \neq 1$, but the analysis is much more complicated. We show the equilibrium in period 2 in the following lemma.

Lemma O5 *The equilibrium in period 2 is as follows:*
(i) If $b - (f_H - f_L) < 0$ *and* $q(f_L - f_H) + (1-q)(f_L - m_L) > \overline{\theta}$*, then there are three equilibria: All firms adopt the technology, no firms adopt the technology, or firms with* $\theta_i \geq \frac{b - f_H + f_L}{\overline{\theta} + b - (1-q)(f_H - m_L)}\overline{\theta}$ *adopt the technology.*
(ii) If $b - (f_H - f_L) \leq 0$ *and* $q(f_L - f_H) + (1-q)(f_L - m_L) \leq \overline{\theta}$*, then there is one equilibrium: All firms adopt the technology.*
(iii) If $b - (f_H - f_L) \geq 0$ *and* $q(f_L - f_H) + (1-q)(f_L - m_L) \geq \overline{\theta}$*, then there is one equilibrium: No firms adopt the technology.*
(iv) If $b - (f_H - f_L) > 0$ *and* $q(f_L - f_H) + (1-q)(f_L - m_L) < \overline{\theta}$*, then there is one equilibrium: Firms with* $\theta_i \geq \frac{b - f_H + f_L}{\overline{\theta} + b - (1-q)(f_H - m_L)}\overline{\theta}$ *adopt the technology.*

Proof. Following the same procedure as in §3 of the main body, we obtain the expected gain of firm i:

$$u_i(\theta_i, \alpha) = \theta_i - \alpha\{b - (1-q)(f_H - m_L)\} + q(f_H - m_L) - (f_L - m_L). \tag{27}$$

We next use this expression to prove the four cases in Lemma O5.
(i) Firm i adopts the technology if its expected gain is nonnegative. Using (27) we get $u_i(\theta_i, 1) = \theta_i + f_H - f_L - b$. So $u_i(\theta_i, 1) \geq 0$ simplifies to $\theta_i \geq b - (f_H - f_L)$. Since $b - (f_H - f_L) < 0$ and $\theta_i \in \left[0, \overline{\theta}\right]$, $u_i(\theta_i, 1) \geq 0$ for any θ_i. Firm i's expected gain is nonnegative if all other firms adopt. Thus all firms adopt the technology is an equilibrium.

Firm i does not adopt the technology if its expected gain is nonpositive. Using (27) we get $u_i(\theta_i, 0) = \theta_i + q(f_H - m_L) - (f_L - m_L)$. So $u_i(\theta_i, 0) < 0$ simplifies to $\theta_i < f_L - m_L - q(f_H - m_L) = q(f_L - f_H) + (1-q)(f_L - m_L)$. Since $q(b - f_H + f_L) + (1-q)(f_L - m_L) > \overline{\theta}$ and $\theta_i \in \left[0, \overline{\theta}\right]$, $u_i(\theta_i, 0) \leq 0$ for any θ_i. Firm i's expected gain is nonpositive if no other firms adopt. Thus no firms adopt the technology is an equilibrium.

A possible equilibrium is that a firm will adopt the technology if and only if $\theta_i \geq \theta^{(2)}$. In this case $\alpha = \left(\overline{\theta} - \theta^{(2)}\right)/\overline{\theta}$. Solving $u_i(\theta^{(2)}, \alpha) = 0$ yields $\theta^{(2)} = \frac{b - f_H + f_L}{\overline{\theta} + b - (1-q)(f_H - m_L)}\overline{\theta}$. We can show that the constraint $0 < \theta^{(2)} < \overline{\theta}$ hold if one the following two conditions are satisfied: (I) $b - (f_H - f_L) > 0$ and $q(f_L - f_H) + (1-q)(f_L - m_L) < \overline{\theta}$; (II) $b - (f_H - f_L) < 0$ and $q(f_L - f_H) + (1-q)(f_L - m_L) > \overline{\theta}$. Condition (II) is satisfied in this case.

We can prove (ii), (iii), and (iv) following the same procedure as in the proof of part (i). $\square$

In case (i), there are multiple equilibria in period 2. It is quite similar to the complete information case with multiple equilibria. Multiple equilibria have weak predictive power because we do not know which equilibrium might be reached. We next show the equilibrium in period 1 in the following Lemma.

Lemma O6 *The equilibrium in period 1 is as follows:*
(i) If $b - (f_H - f_L) < 0$ *and* $q(f_L - f_H) + (1-q)(f_L - m_L) > \overline{\theta}$*, then there are three equilibria: If firm 1 believes that all firms will adopt the technology, it develops the technology if and only if* $\theta_1 \geq f_1 - \lambda - m_H + b$*; if firm 1 believes that no firms will adopt the technology, then it develops the technology if and only if* $\theta_1 \geq f_1 - \lambda - m_L - q(m_H - m_L)$*; if firm 1 believes only a proportion of firms will adopt the technology, then it develops the technology if and only if* $\theta_1 \geq f_1 - \{(1-q)(m_H - m_L) - b\}\frac{\overline{\theta} - q(f_L - f_H) - (1-q)(f_L - m_L)}{\overline{\theta} + b - (1-q)(f_H - m_L)} - \lambda - m_L - q(m_H - m_L)$*.*
(ii) If $b - (f_H - f_L) \leq 0$ *and* $q(f_L - f_H) + (1-q)(f_L - m_L) \leq \overline{\theta}$*, then firm 1 develops the technology if and only if* $\theta_1 \geq f_1 - \lambda - m_H + b$*.*
(iii) If $b - (f_H - f_L) \geq 0$ *and* $q(f_L - f_H) + (1-q)(f_L - m_L) \geq \overline{\theta}$*, then firm 1 develops the technology if and only if* $\theta_1 \geq f_1 - \lambda - m_L - q(m_H - m_L)$*.*
(iv) If $b - (f_H - f_L) > 0$ *and* $q(f_L - f_H) + (1-q)(f_L - m_L) < \overline{\theta}$*, then firm 1 develops the technology if and only if* $\theta_1 \geq f_1 - \{(1-q)(m_H - m_L) - b\}\frac{\overline{\theta} - q(f_L - f_H) - (1-q)(f_L - m_L)}{\overline{\theta} + b - (1-q)(f_H - m_L)} - \lambda - m_L - q(m_H - m_L)$*.*

Proof. Following the same procedure as in §3 of the main body, we obtain the expected gain of firm 1:

$$u_1(\theta_1, \alpha) = \lambda - f_1 + m_L + q(m_H - m_L) + \theta - b\alpha + \alpha(1-q)(m_H - m_L). \tag{28}$$

We next use this expression to prove the four cases in Lemma O6.
(i) If firm 1 believes that all firms will adopt the technology, then $\alpha = 1$. Using (28) we get $u_1(\theta_1, 1) = \lambda - f_1 + m_H + \theta_1 - b$. So $u_1(\theta_1, 1) \geq 0$ simplifies to $\theta_1 \geq f_1 - \lambda - m_H + b$. Similarly, we get $u_1(\theta_1, 0) = \lambda - f_1 + m_L + q(m_H - m_L) + \theta$. And $u_1(\theta_i, 0) \leq 0$ simplifies to $\theta_i \leq f_1 - \lambda - m_L - q(m_H - m_L)$.

If firm 1 believes that firms with $\theta_i \geq \frac{b - f_H + f_L}{\overline{\theta} + b - (1-q)(f_H - m_L)}\overline{\theta}$ will adopt the technology, then

$$\alpha = \left(\overline{\theta} - \tfrac{b - f_H + f_L}{\overline{\theta} + b - (1-q)(f_H - m_L)} \overline{\theta}\right) / \overline{\theta} = \tfrac{\overline{\theta} - q(f_L - f_H) - (1-q)(f_L - m_L)}{\overline{\theta} + b - (1-q)(f_H - m_L)},$$

$$u_1(\theta_1, \alpha) = \lambda - f_1 + m_L + q\left(m_H - m_L\right) + \theta + \left\{\left(1-q\right)\left(m_H - m_L\right) - b\right\} \tfrac{\overline{\theta} - q(f_L - f_H) - (1-q)(f_L - m_L)}{\overline{\theta} + b - (1-q)(f_H - m_L)},$$

and the solution of $u_1(\theta_1, \alpha) > 0$ is

$$\theta_1 \geq f_1 - \left\{\left(1-q\right)\left(m_H - m_L\right) - b\right\} \tfrac{\overline{\theta} - q(f_L - f_H) - (1-q)(f_L - m_L)}{\overline{\theta} + b - (1-q)(f_H - m_L)} - \lambda - m_L - q\left(m_H - m_L\right).$$

Following a similar procedure as in the proof of part (i), we can prove parts (ii)-(iv). □

Lemma O6 suggests that in the case of uncorrelated payoffs among firms, the equilibrium in period 1 is different from that in the case of correlated payoffs. With correlated payoffs, it is possible that the expected gain decreases with the payoff signal x_1. As a result, firm 1 may develop the technology if x_1 is relatively small, but choose not to develop the technology if x_1 is relatively large. This happens because firm 1 updates its beliefs on the expected value of α based on x_1: If x_1 increases, then firm 1 expects that more firms will adopt the technology and there will be more competing firms, and a larger number of competing firm can discourage firm 1 from developing the technology. However, in the case of uncorrelated payoffs, the expected gain increases monotonically with θ_1. There is only one threshold, and firm 1 develops the technology if its payoff is higher than this threshold. Since the distribution of θ_i is known to firm 1 and it does not change as θ_1 changes, firm 1's belief on the expected value of α does not change with θ_1. The complementarity and substitutability effects are constant for firm 1.

Additional Reference

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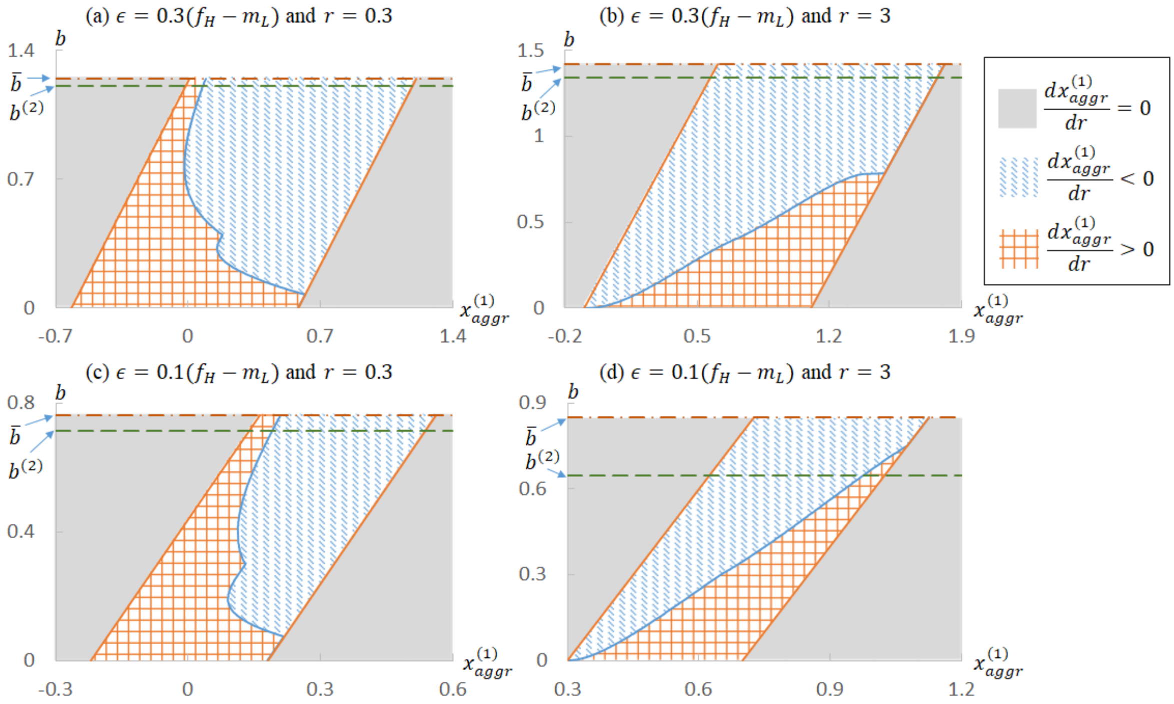


Figure 4: The Impact of Regulation Aggressiveness (r) on Firm 1's Threshold ($x^{(1)}_{aggr}$)
Notes. Other parameter values: $f_H = 2$, $f_L = 1$, $m_H = -1$, $m_L = -2$.

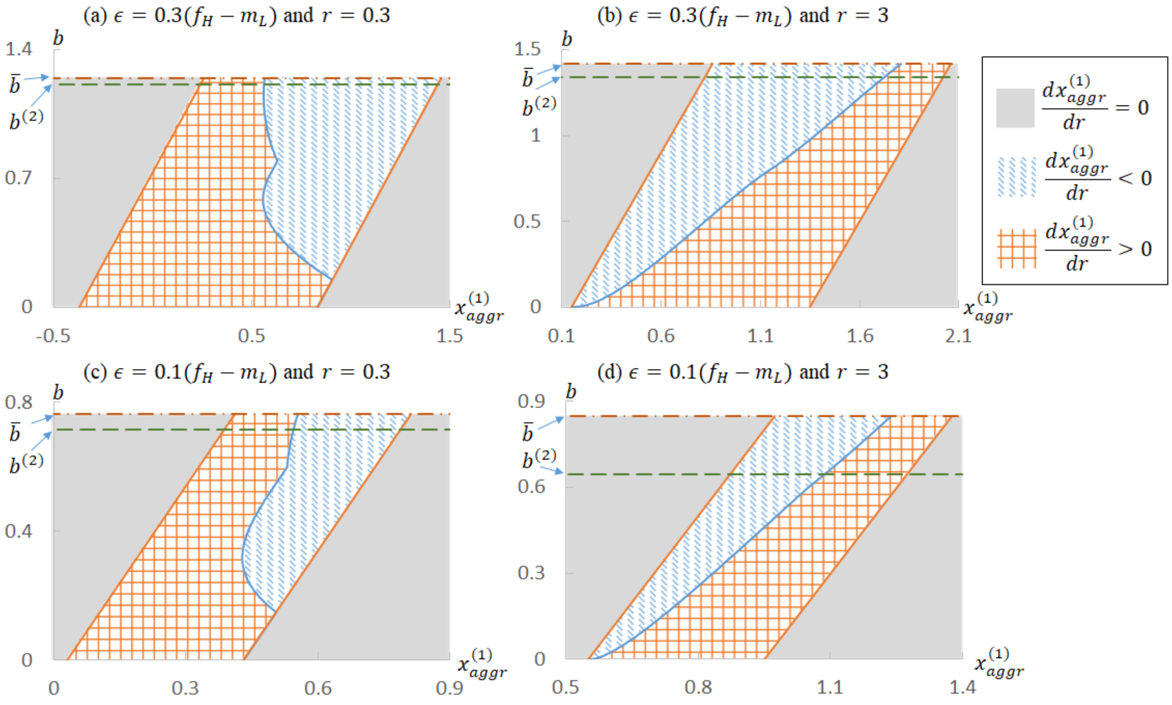


Figure 5: The Impact of Regulation Aggressiveness (r) on Firm 1's Threshold ($x^{(1)}_{aggr}$)
Notes. Other parameter values: $f_H = 2$, $f_L = 0$, $m_H = 0$, $m_L = -2$.

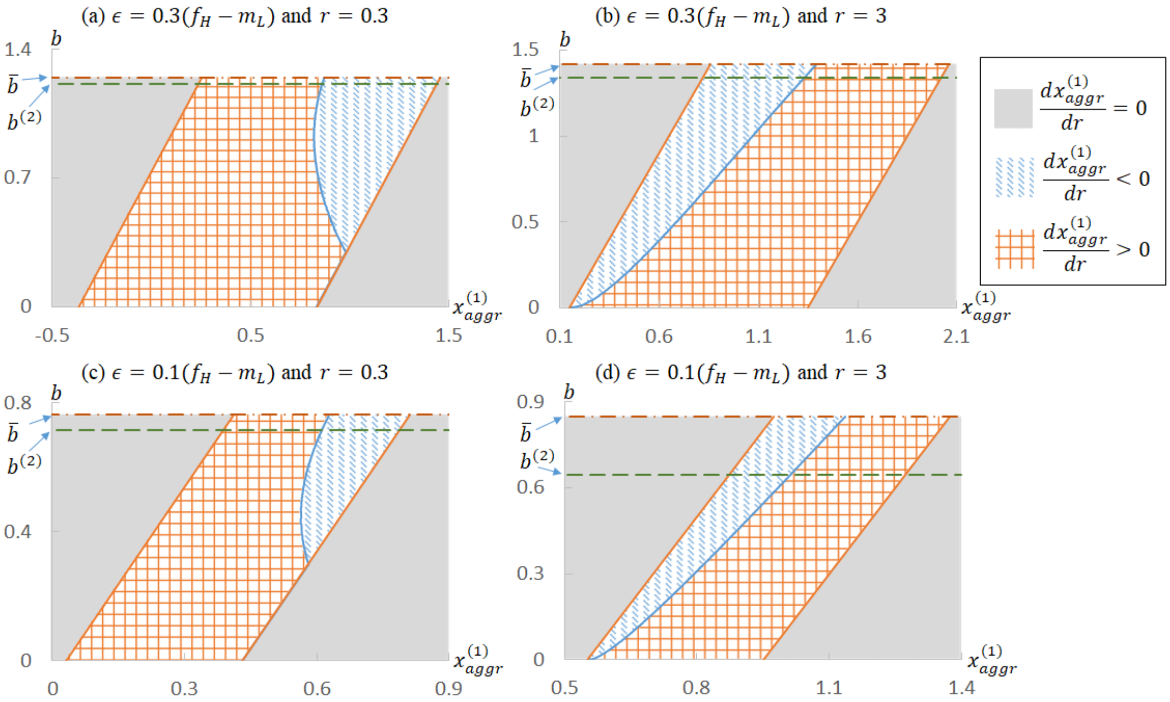


Figure 6: The Impact of Regulation Aggressiveness (r) on Firm 1's Threshold ($x_{aggr}^{(1)}$)

Notes. Other parameter values: $f_H = 0$, $f_L = 0$, $m_H = 0$, $m_L = -2$.

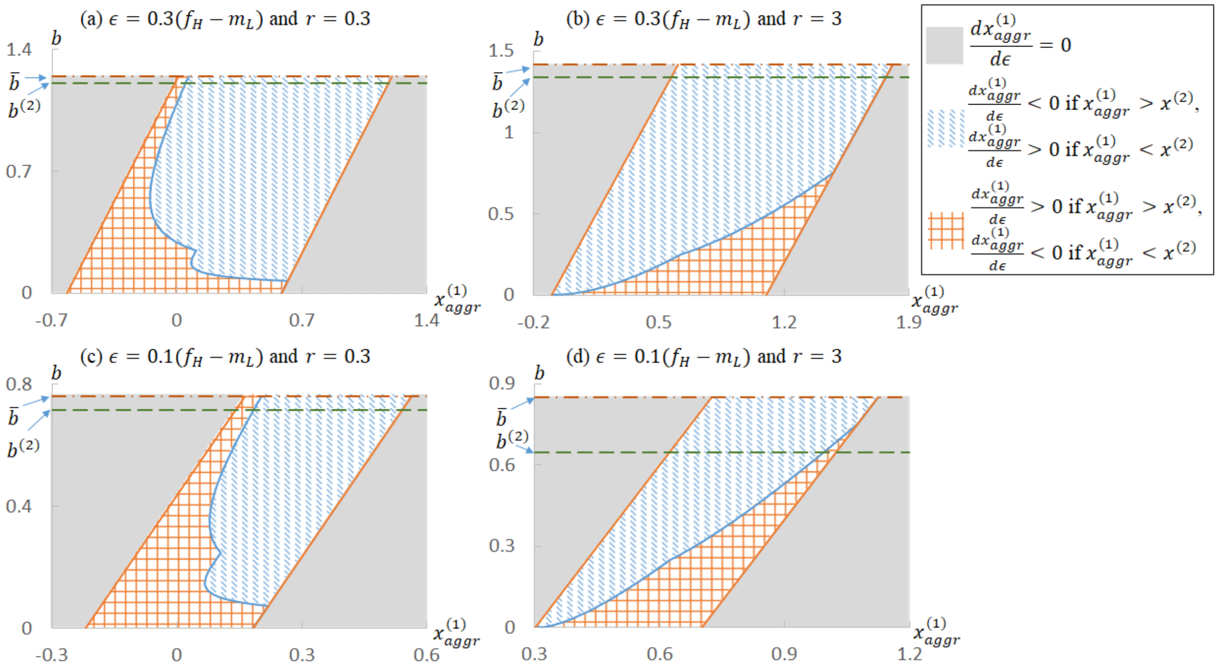


Figure 7: The Impact of the Level of Uncertainty (ϵ) on Firm 1's Threshold ($x_{aggr}^{(1)}$)

Notes. Other parameter values: $f_H = 2$, $f_L = 1$, $m_H = -1$, $m_L = -2$.

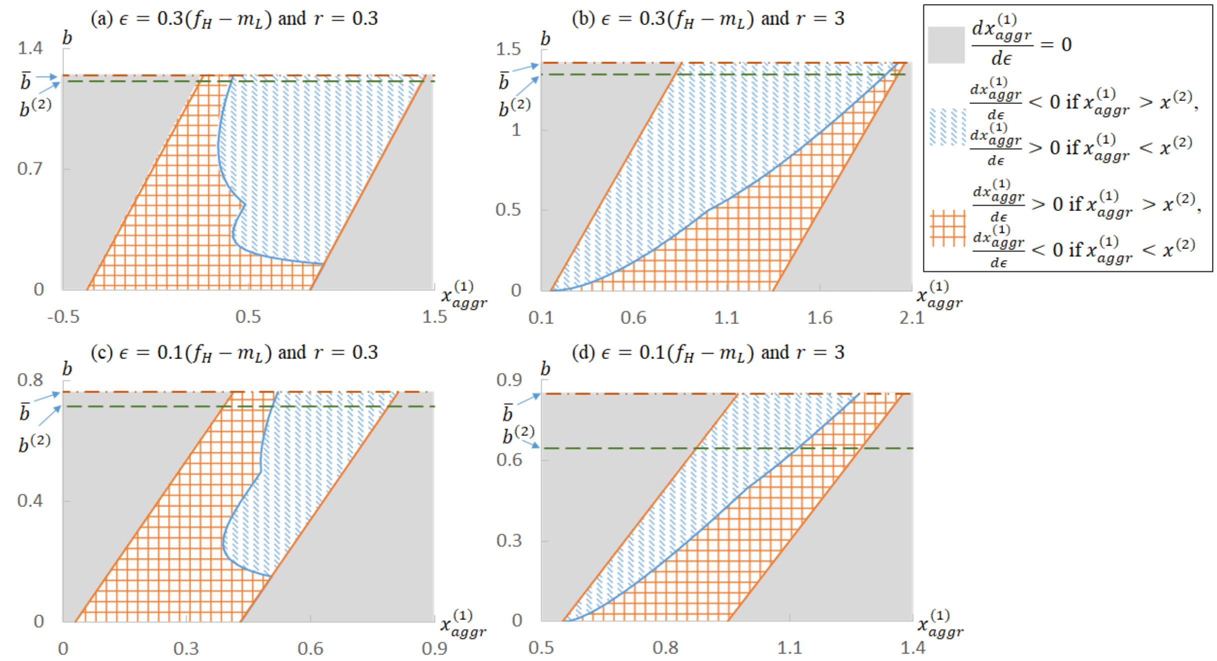


Figure 8: The Impact of the Level of Uncertainty (ϵ) on Firm 1's Threshold ($x^{(1)}_{aggr}$)
Notes. Other parameter values: $f_H = 2$, $f_L = 0$, $m_H = 0$, $m_L = -2$.

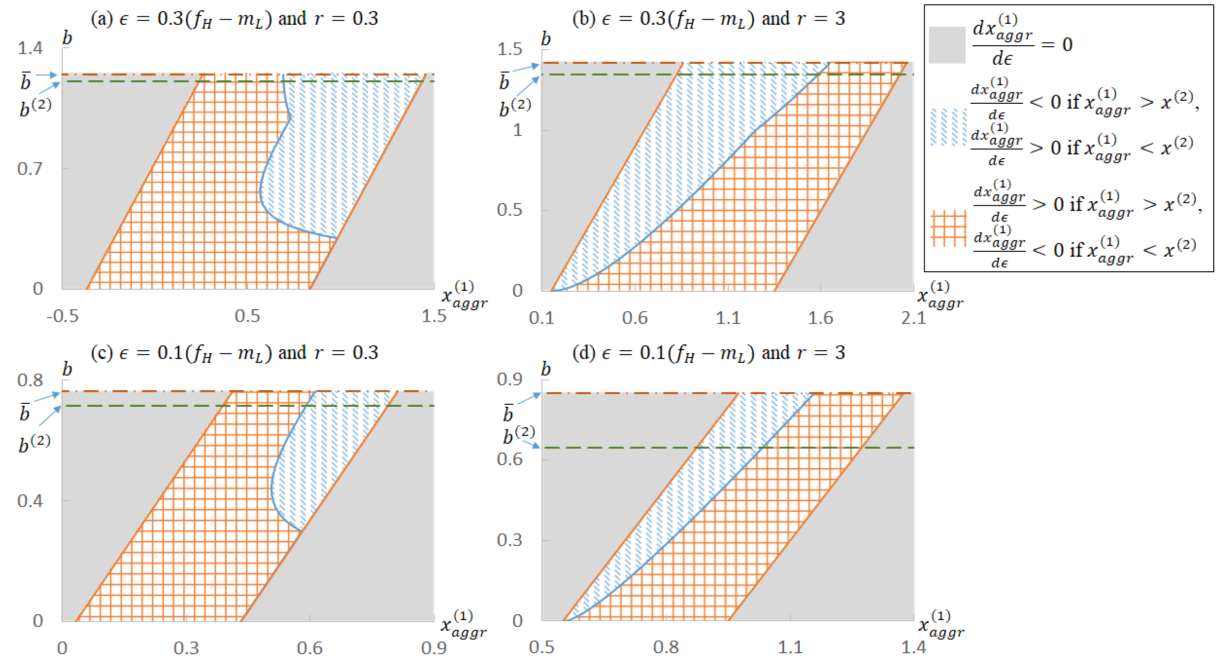


Figure 9: The Impact of the Level of Uncertainty (ϵ) on Firm 1's Threshold ($x^{(1)}_{aggr}$)
Notes. Other parameter values: $f_H = 0$, $f_L = 0$, $m_H = 0$, $m_L = -2$.

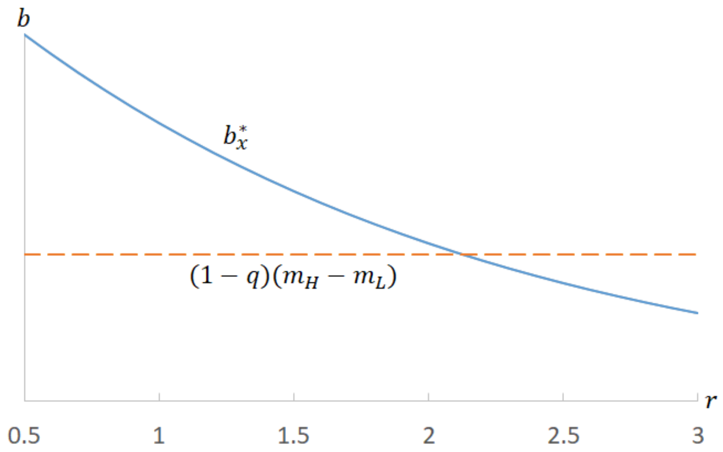


Figure 10: The Thresholds Above Which More Aggressive Regulation Discourages Innovation

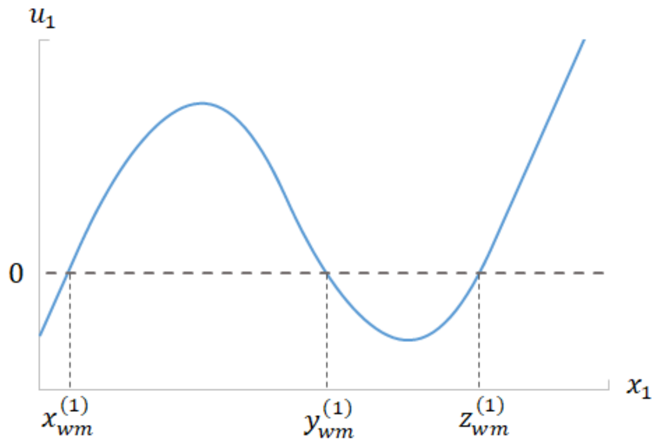


Figure 11: Firm 1's Expected Gain (u_1) When the Government Maximizes Welfare
Notes. Parameter values: $b = 2$, $\widehat{r} = 0.5$, $\lambda = 3$, $f_1 = 1.2$, $f_L = 1$, $f_H = 2$, $m_H = -1$, $m_L = -2$, $e = 6$, $\epsilon = 0.4$, $\widetilde{f}_r$ follows a gamma distribution with both shape and scale parameters equal to 2.

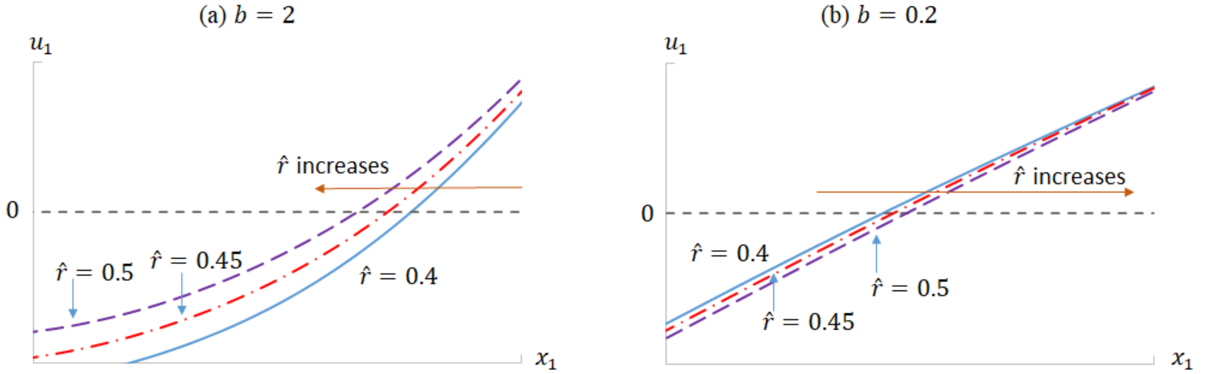
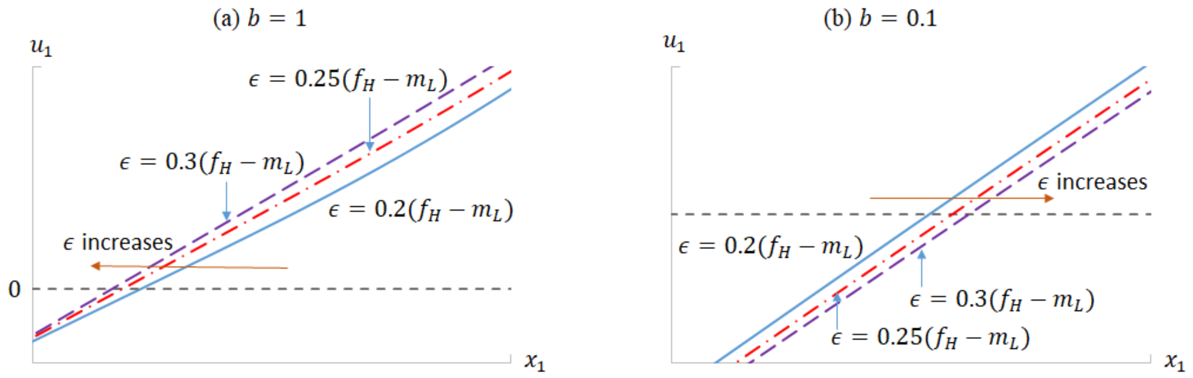


Figure 12: The Impact of Regulation Aggressiveness ($\widehat{r}$) on Firm 1's Threshold When the Government Maximizes Welfare
Notes. Other parameter values: the same as Figure 11 except $f_1 = 1.5$ and $\epsilon = 0.8$.


(a) $b = 1$

(b) $b = 0.1$

Figure 13: The Impact of the Level of Uncertainty (ϵ) on Firm 1's Threshold When the Government Maximizes Welfare

Notes. Other parameter values: the same as Figure 11 except $f_1 = 1.5$ and $\widehat{r} = 0.4$.