Online appendix for "Capturing the Benefits of Autonomous Vehicles in Ride-Hailing: The Role of Market Configuration"

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A. Technical lemmas and proofs

We start with a number of technical lemmas and their proofs, which the rest of the proofs are based on. We start with Lemma 1, which formally introduces the definitions and computation of important functions such as $\underline{n}(d)$, $\bar{d}(n)$, and $\bar{u}(d)$, which appear throughout the paper and the appendix. Lemma 2 next analyzes the property of $\bar{u}(d)$. Finally, Lemma 3 introduces a specific function, $f(d;n)$, which represents the hour earnings per HV (or AV) when the residual demand for HVs (or AVs) is $(d-\bar{d}(n))$. This function is useful in analyzing the market equilibrium when both AVs and HVs are present. Furthermore, properties of $f(d;n)$ are also discussed in Lemma 3.

A.1. Optimal level of open cars

LEMMA 1. *Given the ETA function $t_1(s)=as^{-r}$, for any demand level d, there exists a unique minimal supply level $\underline{n}(d)$, for which the following holds:*

1. *$\underline{n}(d)$ has a closed-form expression, given by*

$$\underline{n}(d)=\tilde{a}d^{\frac{1}{r+1}}+t_2 d$$

where $\tilde{a}=r^{\frac{1}{r+1}}a^{\frac{1}{r+1}}+r^{-\frac{r}{r+1}}a^{\frac{1}{r+1}}$.

2. *$\underline{n}(d)$ has an inverse function, denoted as $\bar{d}(n)$. $\bar{d}(n)$ represents the highest demand rate that a fleet size of n vehicles can supply.*
3. *When using $\underline{n}(d)$ to serve the demand level d, the utilization of the HV fleet is maximized. Denote this utilization as $\bar{u}(d)$. Then its expression is given by*

$$\bar{u}(d)=\frac{dt_2}{\underline{n}(d)}=\frac{dt_2}{\tilde{a}d^{1/(r+1)}+t_2 d} \tag{16}$$

Proof. We provide proof for each item.

Proof for Item 1 As discussed in Section 3, to serve a demand level of d, the number of vehicles required is given by

$$n=s+d(t_1(s)+t_2)$$

where $t_1(s)=as^{-r}$. Take the first-order derivative of n over s gives

$$\frac{\partial n}{\partial s}=1-ra\cdot s^{-r-1}\cdot d$$

$$\frac{\partial^2 n}{\partial s^2}=r(r+1)a\cdot s^{-r-2}\cdot d>0$$

Thus, the minimum of n for a given d can be derived by setting $\frac{\partial n}{\partial s}=0$, which gives

$$s=(ra)^{\frac{1}{r+1}}d^{\frac{1}{r+1}}$$

Therefore, the minimal supply level to serve the demand level d is given by

$$\begin{aligned}\underline{n}(d)&=(ra)^{\frac{1}{r+1}}d^{\frac{1}{r+1}}+d\cdot(t_1((ra)^{\frac{1}{r+1}}d^{\frac{1}{r+1}})+t_2)\\&=(r^{\frac{1}{r+1}}a^{\frac{1}{r+1}}+r^{-\frac{r}{r+1}}a^{\frac{1}{r+1}})d^{\frac{1}{r+1}}+t_2 d\\&=\tilde{a}d^{\frac{1}{r+1}}+t_2 d\end{aligned}$$

Proof for Item 2 The function $\underline{n}(d)$ is a polynomial in d of degree $\frac{1}{r+1}$ for the first term and degree 1 for the second term, and the coefficients are strictly positive. Therefore, $\underline{n}(d)$ is strictly increasing in d. Thus, by the inverse function theorem, its inverse function $\bar{d}(n)$ exists and is strictly increasing in n.

Proof for Item 3 Since the utilization is defined as the quotient of the demand hours served, dt_2, and the level of supply, n, the highest utilization is achieved when the same level of demand dt_2 is served with the lowest possible n. Therefore, the maximum utilization rate given the demand level d is given by

$$\bar{u} = \frac{dt_2}{\underline{n}(d)}$$

This concludes our proof. □

A.2. Utilization

We first introduce Lemma 2 that discuss the characteristics of utilization function $\bar{u}(d)$.

LEMMA 2 **(Utilization)**. *The maximum utilization $\bar{u}(d)$ (16) defined in Lemma 1 is increasing and strictly concave in d. Moreover, $\bar{u}(0) = 0$, $\lim_{d\to\infty} \bar{u}(d) = 1$.*

In other words, with a higher level of demand rate, the fleet can have higher utilization.

Proof. By Lemma 1,

$$\bar{u}(d) = \frac{dt_2}{\tilde{a}d^{1/(r+1)} + t_2 d} = \frac{1}{\tilde{a}d^{-r/(r+1)} + 1},$$

where $\tilde{a} = r^{\frac{1}{r+1}} a^{\frac{1}{r+1}} + r^{-\frac{r}{r+1}} a^{\frac{1}{r+1}}$. The limit can be easily checked from the expression.

Then

$$\begin{aligned}\bar{u}'(d) &= -(\tilde{a}d^{-r/(r+1)} + 1)^{-2}\tilde{a}(-\frac{r}{r+1})d^{-r/(r+1)-1} \\ &= \frac{\tilde{a}r}{r+1}\frac{1}{(\tilde{a}d^{-r/(r+1)} + 1)(\tilde{a}d + d^{r/(r+1)+1})} > 0\end{aligned}$$

Then

$$\begin{aligned}\bar{u}''(d) &= -\frac{\tilde{a}r}{r+1}\frac{\tilde{a}(-r/(r+1))d^{-r/(r+1)-1}(\tilde{a}d + d^{r/(r+1)+1}) + (\tilde{a}d^{-r/(r+1)} + 1)(\tilde{a} + (r/(r+1)+1)d^{r/(r+1)})}{((\tilde{a}d^{-r/(r+1)} + 1)(\tilde{a}d + d^{r/(r+1)+1}))^2} \\ &= -\frac{\tilde{a}r}{r+1}\frac{(\tilde{a}d^{-r/(r+1)} + 1)(\tilde{a}/(r+1) + (r/(r+1)+1)d^{r/(r+1)})}{((\tilde{a}d^{-r/(r+1)} + 1)(\tilde{a}d + d^{r/(r+1)+1}))^2} < 0\end{aligned}$$

Thus, the concavity and monotonicity of $\bar{u}(d)$ is confirmed. □

A.3. Hourly earnings per vehicle (AV, HV)

The next lemma is about the hourly earnings that a vehicle (either AV or HV) may make given the demand level and the market configuration. In particular, we define the following function:

$$f(d; n) = p_i(d)\bar{u}(d - \bar{d}(n))$$

When $n = 0$, $f(d;n)$ is the product of the market price $p_i(d)$ and the utilization $\bar{u}(d)$, which represents the per vehicle hourly earnings for a market with just one type of supply (HVs or AVs) to satisfy the demand rate d. It is also the per-vehicle hourly earnings under a common platform market when AVs and HVs are supplying the demand together, because the utilization rate is shared among vehicles on the same dispatch platform.

When $n > 0$, $f(d;n)$ represents the hourly earnings per AV (HV) as a function of the total demand served when a fleet size of n HVs (AVs) are providing service in an independent platform market. We show that all else being equal, as the fleet size of HVs (AVs) n increases, the residual demand $(d - \bar{d}(n))$ for AVs decreases, which leads to a lower utilization rate and reduced hourly earnings per AV for AVs.

LEMMA 3 **(Hourly earnings per vehicle)**. *Let n be a fleet size that satisfies $\bar{d}(n) < m_i$. For a given n, define function $f(d;n) = p_i(d)\bar{u}(d - \bar{d}(n))$, $\bar{d}(n) \le d \le m_i$, which represents the hourly earnings per vehicle at demand rate d when the residual demand for this type of vehicle is $(d - \bar{d}(n))$. Then $f(d;n)$ has the following properties:*

1. *Fixing n, $f(d;n)$ is strictly concave in d.*
2. *Fixing n, $f(d;n)$ is unimodal with a maximizer $d^\dagger$ that can be uniquely determined by the first-order condition $f'(d^\dagger;n) = 0$.*
3. *Define $g(n) = \max_{\bar{d}(n) \le d \le m_i} f(d;n)$, which represents the highest per vehicle hourly earnings for HVs (AVs) when n AVs (HVs) are operating on an independent platform platform. Moreover, $g(n)$ is continuous and decreasing in n; $g(\underline{n}(m_i)) = 0$.*
4. *Under Assumption 2, when $n = 0$, it must hold that*

$$g(0) = \max_{0 \le d \le m_i} p_i(d)\bar{u}(d) > w_0$$

That is, the highest hourly earnings per vehicle must be strictly higher than w_0 when there is only one type of supply.

Proof. We prove each property one by one.

- Item 1: The first-order derivative is given by

$$f'(d;n) = p_i'(d)\bar{u}(d - \bar{d}(n_a)) + p_i(d)\bar{u}'(d - \bar{d}(n_a))$$

The second-order derivative is given by

$$f''(d;n) = p_i''(d)\bar{u}(d - \bar{d}(n)) + 2p_i'(d)\bar{u}'(d - \bar{d}(n)) + p_i(d)u''(d - \bar{d}(n))$$

Moreover, since $p_i(d)$ is linear and decreasing in d,

$$p_i''(d) = 0, p_i'(d) < 0$$

By Lemma 2, $\bar{u}(\cdot)$ is increasing and strictly concave; hence,

$$\bar{u}'(d-\bar{d}(n))>0, u''(d-\bar{d}(n))<0$$

Therefore, $f''(d)<0$ for $d\geq\bar{d}(n)$.

- Item 2: When $d=\bar{d}(n)$, $f'(d)\to\infty$; when $d=m_i$, $f'(d)<0$. Since $f'(d)$ is decreasing in d, and $[\bar{d}(n),m_i]$ is a compact set, by the intermediate value theorem, $f'(d)=0$ has a unique solution on $[\bar{d}(n),m_i]$. Moreover, $f'(d)>0$ for $d<d^\dagger$ and $f'(d)<0$ for $d>d^\dagger$.
- Item 3: The continuity of g is implied by the continuity of $\bar{u}$ and p_i. By the envelop theorem,

$$g'(n)=\frac{\partial f}{\partial n}|_{d=d^\dagger}=p_i(d^\dagger)\bar{u}'(d^\dagger-\bar{d}(n))(-\bar{d}'(n))<0$$

  In addition, one can easily check that $f(d;\underline{n}(m_i))=0$ for any d. Thus, $g(\underline{n}(m_i))=0$.
- Item 4: By definition, $g(0)$ is the highest hourly earnings per vehicle when only one type of supply (AVs or HVs) serve the market. By Assumption 2, the scenarios in consideration are those in which HVs provide service under a pure-HV market. Thus, it must be true that there exists a demand d such that the hourly earnings per HV (i.e. $f(d;0)$) are strictly above the lowest possible HV reservation earnings w_0, otherwise no HV provides services. Since $g(0)\geq f(d;0)$ for all d, it must also be true that $g(0)\geq w_0$. □

B. Pure-HV market

In this section, we provide the mathematical details for our benchmark case.

B.1. Proof of Proposition 1

B.1.1. Existence and uniqueness of equilibrium First, we restate the loose labor market assumption (Assumption 2) here. That is, the revenue curve $r_i(d_{HV})$ intersects with the cost curve $w(\underline{n}(d_{HV}))\underline{n}(d_{HV})$, on its right branch. This condition is equivalent to

$$r_i(m_i/2)>w(\underline{n}(m_i/2))\underline{n}(m_i/2) \tag{17}$$

which translates into a lower bound on the maximum trip value V. That is,

$$V>4w(\underline{n}(m_i/2))\underline{n}(m_i/2)/(t_2m_i)$$

We prove that under Eq. (17), there is precisely one solution that satisfies equilibrium condition (9) in the interval $d_{HV}\in[m_i/2,m_i]$; moreover, it is a stable equilibrium (Definition 1). Therefore, this solution is the unique maximal stable equilibrium.

For convenience, we repeat the equilibrium condition (9) here:

$$r_i(d_{HV})=w(\underline{n}(d_{HV}))\underline{n}(d_{HV})$$

At $d_{HV} = m_i/2$, by Eq. (17), the revenue $r_i(d_{HV})$ is strictly higher than the cost $w(\underline{n}(d_{HV}))\underline{n}(d_{HV})$. At $d_{HV} = m_i$, the revenue is given by

$$r_i(m_i) = p_i(m_i)m_it_2 = 0$$

The cost is given by

$$w(\underline{n}(m_i))\underline{n}(m_i) > 0$$

Then it must be true that

$$r_i(m_i/2) - w(\underline{n}(m_i/2))\underline{n}(m_i/2) > 0, r_i(m_i) - w(\underline{n}(m_i))\underline{n}(m_i) < 0$$

By continuity, there must exist $d_{HV} \in (m_i/2, m_i)$ such that

$$r_i(d_{HV}) - w(\underline{n}(d_{HV}))\underline{n}(d_{HV}) = 0$$

This confirms the existence of an equilibrium.

Furthermore, such a d_{HV} must be unique. To see why, note that $\underline{n}(d_{HV})$ and $w(\underline{n}(d_{HV}))$ are both strictly increasing in d_{HV}; in the meantime, $r_i(d_{HV})$ is strictly decreasing in d_{HV} for $d_{HV} \in [m_i/2, m_i]$. Therefore, the function $r_i(d_{HV}) - w(\underline{n}(d_{HV}))\underline{n}(d_{HV})$ is strictly decreasing for $d_{HV} \in [m_i/2, m_i]$, implying that there is only one $d_{HV} \in [m_i/2, m_i]$ such that $r_i(d_{HV}) - w(\underline{n}(d_{HV}))\underline{n}(d_{HV}) = 0$. This confirms the uniqueness of the equilibrium.

We can also verify that this unique solution on $(m_i/2, m_i)$ is a stable equilibrium. Since $r_i(d_{HV}) - w(\underline{n}(d_{HV}))\underline{n}(d_{HV})$ is strictly decreasing, it must hold that for a small disturbance $\epsilon > 0$,

$$r_i(d_{HV} - \epsilon) - w(\underline{n}(d_{HV} - \epsilon))\underline{n}(d_{HV} - \epsilon) > 0$$

and

$$r_i(d_{HV} + \epsilon) - w(\underline{n}(d_{HV} + \epsilon))\underline{n}(d_{HV} - \epsilon) < 0$$

which satisfies Definition 1. This confirms that the equilibrium in consideration is stable.

Since this solution is the only stable equilibrium on $(m_i/2, m_i)$, it must also be the largest solution of Eq. (9). Therefore, we have proved the uniqueness and existence of a maximal stable equilibrium.

B.1.2. The expressions of the equilibrium demand and HV fleet size (Item 1) The equilibrium demand definition is just a repetition of the equilibrium condition Eq. (9). The expression of the HV fleet size $n^*_{HV,i}$ is given by Lemma 1.

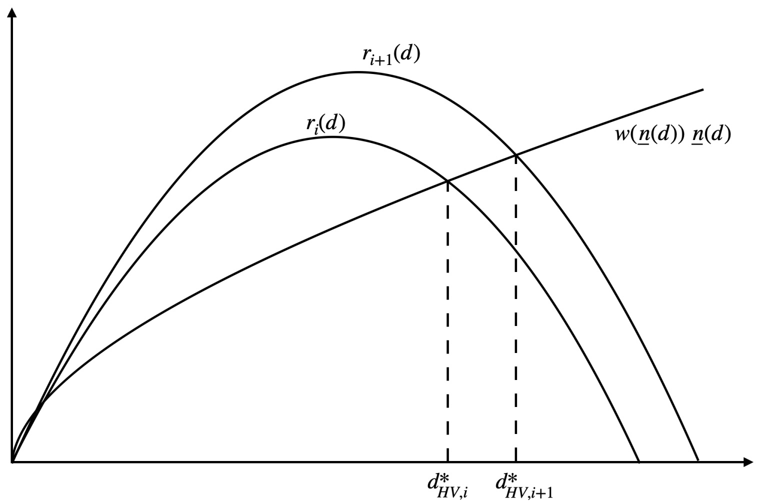


Figure 4 **Illustration of the comparison between $d^*_{HV,i}$ and $d^*_{HV,i+1}$**

B.1.3. The monotonicity of the equilibrium demand $d^*_{HV,i}$ (Item 2) In this section, we prove that the equilibrium demand $d^*_{HV,i}$ is increasing in the potential demand mass m_i. We first visually illustrate the comparison with Fig. 4 and then formally prove the monotonicity of $d_{HV,i}$ in i. Consider the equilibrium point $d = d^*_{HV,i}$. Now suppose the potential demand mass increases from m_i to m_{i+1}. Then the revenue immediately increases due to higher prices from the increased demand mass. That is,

$$r_{i+1}(d^*_{HV,i}) > w(n^*_{HV,i})n^*_{HV,i} \tag{18}$$

where $n^*_{HV,i} = \underline{n}(d^*_{HV,i})$. Thus, more HVs enter the market, which increases the right-hand side of Eq. (18), until the point where the revenue exactly covers the cost of the HV supply. That is, the demand level at which the following holds:

$$r_{i+1}(d^*_{HV,i+1}) = w(n^*_{HV,i+1})n^*_{HV,i+1}$$

where $n^*_{HV,i+1} = \underline{n}(d^*_{HV,i+1})$.

B.1.4. The monotonocity of the equilibrium utilization rate $u^*_{HV,i}$ (Item 3) In this section, we show that the equilibrium utilization rate $u^*_{HV,i}$ is increasing in i, providing the ETA elasticity $r > 0$; when $r = 0$, $u^*_{HV,i}$ is constant in i. The proof of this item is based on proving that the utilization function $\bar{u}(d_{HV})$ is decreasing in d_{HV}, which is defined as

$$\bar{u}(d_{HV}) = d_{HV}t_2/\underline{n}(d_{HV}), \text{ where } \underline{n}(d_{HV}) = \tilde{a}d_{HV}^{1/(r+1)} + t_2 d_{HV} \tag{19}$$

Once we have shown this, together with Item 2, it directly follows that in a scenario with a higher potential demand mass, the equilibrium utilization rate is also higher.

The utilization rate can be directly written as

$$\bar{u}(d_{HV}) = \frac{d_{HV} t_2}{\tilde{a} d_{HV}^{1/(r+1)} + t_2 d_{HV}} = \frac{t_2}{\tilde{a} d_{HV}^{-r/(r+1)} + t_2} \tag{20}$$

When $r > 0$, the denominator, $(\tilde{a} d_{HV}^{-r/(r+1)} + t_2)$, is strictly decreasing in d_{HV}. Thus, $\bar{u}(d_{HV})$ is strictly increasing in d_{HV}. Consider two scenarios, i and $i+1$, with $m_i < m_{i+1}$. Then by Item 2,

$$d^*_{HV,i} < d^*_{HV,i+1}$$

which implies that

$$u^*_{HV,i} = \bar{u}(d^*_{HV,i}) < \bar{u}(d^*_{HV,i+1}) = u^*_{HV,i+1}, \text{ when } r > 0$$

When $r = 0$, the denominator in Eq. (20), $(\tilde{a} d_{HV}^{-r/(r+1)} + t_2)$, is a constant and equal to $(\tilde{a} + t_2)$. Thus, both the numerator and denominator of Eq. (20) are constants, implying that the utilization is also a constant. That is,

$$u^*_{HV,i} = u^*_{HV,i+1} = \frac{t_2}{\tilde{a} + t_2}, \text{ when } r = 0$$

B.1.5. The monotonicity of the equilibrium HV fleet size $n^*_{HV,i}$ (Item 4) In this section, we show that the equilibrium HV fleet size $n^*_{HV,i}$ is also increasing in m_i. Similarly, we prove this item by showing that the fleet size $\underline{n}(d_{HV})$ is strictly increasing in d_{HV}. By Eq. (19), $\underline{n}(d_{HV})$ is a positive power function of d_{HV}. (when $r = 0$, it is linear in d_{HV}.) Thus, $\underline{n}(d_{HV})$ is strictly increasing in d_{HV}. Then, by Item 2, $d^*_{HV,i} < d^*_{HV,i+1}$, and it must hold that

$$n^*_{HV,i} = \underline{n}(d^*_{HV,i}) < \underline{n}(d^*_{HV,i+1}) = n^*_{HV,i+1}$$

B.1.6. Driving forces in the equilibrium price $p^*_{HV,i}$ (Item 5) In this section, we show that the equilibrium price $p^*_{HV,i}$ may be increasing, decreasing, or non-monotonic in m_i. For this item, we would like to discuss the monotonicity of $p^*_{HV,i}$, defined as

$$p^*_{HV,i} = p_i(d^*_{HV,i}) = \frac{w(\underline{n}(d^*_{HV,i})}{\bar{u}(d^*_{HV,i})}$$

As stated in Proposition 1, $p^*_{HV,i}$ can be either increasing, decreasing, or non-monotonic in m_i. This can be seen from the two forces that determine $p^*_{HV,i}$:

The numerator, $w(\underline{n}(d^*_{HV,i}))$, represents the HV reservation wage when the market is in equilibrium in scenario i. In a scenario with a higher demand mass, say scenario $i+1$ with mass m_{i+1}, the equilibrium demand is higher, requiring a larger HV fleet to serve the demand. That is $n_{HV,i+1} > n_{HV,i}$. If HVs have an upward-sloping supply curve, then it becomes more expensive to acquire HVs in scenario $i+1$ compared with scenario i. Such an increased cost of labor requires the price to be higher to cover the cost.

On the other hand, the denominator, $\bar{u}(d^*_{HV,i})$, represents the equilibrium utilization rate of the HV fleet in scenario i. In Item 3, we have shown that the equilibrium utilization is strictly increasing in the potential demand mass when $r > 0$. Therefore, in a scenario with a higher demand mass, there is higher utilization of the HV fleet, which means it requires fewer units of labor to complete each unit of demand.

The joint impact of both factors thus depends on the value of the parameters. For example, when the density elasticity $r = 0$ and the supply elasticity is finite ($\alpha > 0$), the utilization rate is constant across scenarios, but the reservation wage is still increasing in the HV fleet size. In this case, there only exists the supply effect of labor, but no technical efficiency from density. The equilibrium price is thus strictly increasing in m_i.

Another special example is the setting with a positive density elasticity $r > 0$ and perfect supply elasticity ($\alpha = 0$). In this case, the utilization rate is increasing as the demand gets denser, but the reservation wage remains constant across scenarios. In other words, there is technical efficiency from density, but no supply effect of labor. The equilibrium price is thus strictly decreasing in m_i.

B.2. Extension of the equilibrium concept to a market with both AVs and HVs

As a preparation for the analysis of markets with AVs, we extend the equilibrium concept in Definition 1 to a market with both AVs and HVs. Similarly, the market reaches an equilibrium when the total revenue (expected earnings) for HVs equals the total cost (reservation earnings) of HVs. That is,

Definition 2 (Equilibrium with AVs). Consider a market with both AVs and HVs serving the demand. Suppose AVs serve a demand level of d_{AV} and AVs serve a demand level of d_{HV}. Moreover, the HV fleet size is n_{HV}. Then the market is in equilibrium if and only if the revenue of HVs exactly equals the cost of HVs:

$$R_i(d_{HV}; d_{AV}) = w(n_{HV})n_{HV} \tag{21}$$

where $R_i(d_{HV}; d_{AV}) = d_{HV} \cdot p_i(d_{HV} + d_{AV}) \cdot t_2$. When $n_{HV} > 0$, Eq. (21) is equivalent to

$$p_i(d_{HV} + d_{AV})\bar{u}(d_{HV}) = w(n_{HV}) \tag{22}$$

Compared with the equilibrium condition Eq. (9) for a pure-HV market, Definition 2 differs in that the price is impacted by the number of AVs in the market. Moreover, the number of HVs n_{HV} needed to supply a demand level of d_{HV} depends on the dispatch platform design, which we formally discuss in Appendix F, Appendix E, and Appendix D. Here, for completeness, we preview the results here:

1. Under a common platform market,

$$n_{HV} + n_{AV} = \underline{n}(d_{HV} + d_{AV})$$

   where $n_{HV}/n_{AV} = d_{HV}/d_{AV}$.

2. Under an independent platform market,

$$n_{HV} = \underline{n}(d_{HV})$$

Furthermore, we can define the stable equilibrium with AVs similar to Definition 1 for the pure-HV market:

Definition 3 (Stable equilibrium with AVs). Suppose AVs serve a demand level of d_{AV}. Then a market is not stable if $R_i(d_{HV} - \epsilon; d_{AV}) < w(n_{HV} - \epsilon)(n_{HV} - \epsilon)$ and $R_i(d_{HV} + \epsilon; d_{AV}) > w(n_{HV} + \epsilon)(n_{HV} + \epsilon)$ for a small disturbance $\epsilon > 0$. If not, then the market is in a stable equilibrium.

Similar to the pure-HV market, we call the stable equilibrium with the largest HV demand level d_{HV} as the *maximal stable equilibrium.* In the main body, for brevity, we use the term "equilibrium", "stable equilibrium", and "maximal stable equilibrium" interchangeably unless otherwise stated.

C. Conditions for AVs and HVs to coexist (Proof of Proposition 2)

Proof. Here, we present sufficient conditions under which HVs will participate in at least one scenario for each market configuration. Since Proposition 2 is a summary result for the four market configurations analyzed in Section 6, as well as Appendix F, Appendix E, and Appendix D, the proof in this section may depend on propositions and lemmas that appear subsequent to Proposition 2 and Appendix C.

Note that the conditions under which HVs participate in each market configuration under "perfect supply elasticity and density effect" and "finite supply elasticity and no density effect" are analytically derived and presented in Appendix C.1 and Appendix C.2, respectively. In the setting of "finite supply elasticity and density effect", the analytical results are intractable, but the HV participation can be validated through the numerical example 9 and 11 in Appendix H.5 in the sense that both charts have a positive average driver participation across the four market configurations.

The condition is defined as an upper bound on the probability mass of the highest demand scenario, denoted by $P(m_I)$. The intuition behind this is that when the demand distribution exhibits peaks with both high mass and low probability of occurrence, it becomes uneconomical for AV suppliers to maintain a capacity so high that it can supply the peak demand independently. The criteria for what constitutes low probability and high demand mass for each configuration are illustrated below in Table 5, for a market with perfect supply elasticity and positive density elasticity, and in Table 6, for a market with finite supply elasticity and zero density elasticity.

Table 5 Conditions for HV to participate in at least one scenario under $\alpha = 0$, $r > 0$

	Common Platform	**Independent Platform**
Monopoly	$P(m_I) < \frac{c_f}{w_0 - c_v}$	$P(m_I) < \frac{c_f(1-\hat{n}_{AV,(I-1)})}{p_I(d^{\dagger}_{AV,I})\bar{u}(d^{\dagger}_{AV,I}) - c_v}$ and $n^{\dagger}_{AV,I} > \tilde{n}_{AV,I}$
Competitive	$P(m_I) < \frac{c_f - \sum_1^{I-1} P(m_i)\left(p_i(d^*_{HV,I})\bar{u}(d^*_{HV,I}) - c_v\right)^+}{w_0 - c_v}$	$P(m_I) < \frac{c_f - \sum_{i=1}^{I-1} P(m_i)\left(p_i(d^{\dagger}_{AV,I}).\bar{u}(d^{\dagger}_{AV,I}) - c_v\right)^+}{p_I(d^{\dagger}_{AV,I})\bar{u}(d^{\dagger}_{AV,I}) - c_v}$, $n^{\dagger}_{AV,I} > \tilde{n}_{AV,I}$, and $p_I(d^{\dagger}_{AV,I})\bar{u}(d^{\dagger}_{AV,I}) > c_v$

Note: The content presented in this table and its proof are based on analysis and definitions that are detailed further in the subsequent text Section 6, as well as Appendix F, Appendix E, and Appendix D. The paragraph below offers an overview of the definitions and notations necessary for understanding the conditions in this table.

C.1. Perfect supply elasticity and positive density elasticity ($\alpha = 0$, $r > 0$)

First, we formally introduce and revisit some definitions for notations that appear in Table 5.

1. $d^*_{HV,i}$ and $n^*_{HV,i}$: The equilibrium demand and HV fleet size in scenario i under the pure-HV market. Proposition 1 gives full definition of them.
2. $d^{\dagger}_{AV,i}$ and $n^{\dagger}_{AV,i}$: The maximum demand served by AVs and AV fleet size that allow HVs to participate in scenario i under an independent platform market. The definitions of $n^{\dagger}_{AV,i}$ and $d^{\dagger}_{AV,i}$ are given by Eq. (104) and Eq. (102), respectively.
3. $\tilde{d}_{AV,i}$ and $\tilde{n}_{AV,i}$: The optimal demand rate and the AV fleet size that maximizes the total variable profit of a monopoly AV supplier in scenario i when AVs serve the market alone (Eq. (90)). In other words,

$$\tilde{d}_{AV,i} = \arg\max_d \{p_i(d)dt_2 - c_v \underline{n}(d)\} \tag{23}$$

   The fleet size $\tilde{n}_{AV,i}$ is the corresponding supply level that supports $\tilde{d}_{AV,i}$, i.e. $\tilde{n}_{AV,i} = \underline{n}(\tilde{d}_{AV,i})$.
4. $\hat{n}_{AV,i}$: the maximum of $\tilde{n}_{AV,i}$ and $n^{\dagger}_{AV,i}$. In other words,

$$\hat{n}_{AV,i} = \max\{\tilde{n}_{AV,i}, n^{\dagger}_{AV,i}\}$$

In the monopoly, common platform market, regardless of how high the potential demand mass is, as long as $P(m_I)$ is below the threshold, AVs will share the market with HVs in scenario i. In the rest three cases, the demand mass m_I also plays a role. Furthermore, under a common platform market, the threshold for the competitive case is lower than that for the monopoly case. In other words, all else being equal, AVs are more likely to serve the market alone when they are perfectly competitive in a common platform market; The difference between the two thresholds, $\sum_1^{I-1} P(m_i)\left(p_i(d^*_{HV,I})\bar{u}(d^*_{HV,I}) - c_v\right)^+$ (normalized by $(w_0 - c_v)$), is the aggregate variable profit an AV extract from all but the highest demand scenario under the competitive setting. In other words, AVs will participate and squeeze out HVs as long as they can still break even; in contrast, the monopoly supplier will strategically coexist with HVs to maximize its total profit.

Next, we provide details on how we derive conditions for each of the four market configurations under various supply and density elasticities.

C.1.1. Common platform market, monopoly. Proposition 8 shows the structure and the optimal choice of the AV capacity. It can be easily verified that, when $P(m_I) < \frac{c_f}{w_0 - c_v}$, the optimal capacity N^* will be lower than the total equilibrium fleet size $n^*_{HV,I}$ in scenario I, implying that HVs will participate at least in the highest demand scenario.

C.1.2. Common platform market, competition. We start from the intuition of the proof. A sufficient condition for the HV participation is that, at any AV capacity that can drive HVs out of the market in all scenarios, AVs cannot break even (i.e. the aggregate variable profit per vehicle is below c_f).

Recall that under a common platform market, AVs can supply the market alone in scenario i if and only if the AV fleet size is no less than $n^*_{HV,i}$; thus, the condition is equivalent to:

$$\sum_{i=1}^{I} P(m_i)(p_i(d)\bar{u}(d) - c_v)^+ < c_f, \text{ for all } d \geq d^*_{HV,I} \tag{24}$$

This condition can be simplified as

$$\sum_{i=1}^{I} P(m_i)(p_i(d^*_{HV,I})\bar{u}(d^*_{HV,I}) - c_v)^+ < c_f \tag{25}$$

To see why, we show that the left-hand side of Eq. (24), which represents the variable profit per vehicle at demand rate d, is decreasing in d for $d \geq d^*_{HV,I}$. Essentially, we show that in any scenario i, the variable profit per vehicle, $((p_i(d)\bar{u}(d) - c_v)^+$, is decreasing for $d \geq d^*_{HV,I}$.

Consider two functions, $r_i(d)$ (defined in Section 4), and the variable revenue per vehicle $p_i(d)\bar{u}(d)$. Both functions are strictly concave and unimodal. ($r_i(d)$ is just a quadratic function of d; Lemma 3 shows the characteristics of $p_i(d)\bar{u}(d)$) By Assumption 2, it is true that $d^*_{HV,i} \geq \arg\max_d r_i(d)$. By definition, $r_i(d) = p_i(d)dt_2$, which can further be written as

$$r_i(d) = (p_i(d)\bar{u}(d))\underline{n}(d)t_2$$

Then $\arg\max_d r_i(d) > \arg\max_d p_i(d)\bar{u}(d)$ must hold; to see why, taking the first order derivative of $r_i(d)$ gives the following:

$$r_i'(d) = (p_i(d)\bar{u}(d))'\underline{n}(d)t_2 + (p_i(d)\bar{u}(d))\underline{n}'(d)t_2$$

when the revenue per vehicle $p_i(d)\bar{u}(d)$ reaches its maximum (i.e. $(p_i(d)\bar{u}(d))' = 0$), $r_i'(d) > 0$, meaning that $r_i(d)$ is still increasing. Thus, we have

$$d^*_{HV,I} \geq d^*_{HV,i} \geq \arg\max_d r_i(d) > \arg\max_d p_i(d)\bar{u}(d)$$

for all $i = 1, ..., I$. The leftmost inequality is by the monotonicity of $d^*_{HV,i}$ in Proposition 1. Since $p_i(d)\bar{u}(d)$ is unimodal, it is strictly decreasing once d is over the peak. Therefore, $p_i(d^*_{HV,i})\bar{u}(d^*_{HV,i}) >$

$p_i(d)\bar{u}(d)$ for any $d > d^*_{HV,i}$. Since this is true for every i, this completes the proof for the equivalence between Eq. (24) and Eq. (25).

Therefore, Eq. (25) is a sufficient condition for the HV participation. Next, we rewrite it to make it more readable. By definition of $d^*_{HV,i}$, $p_i(d^*_{HV,i})\bar{u}(d^*_{HV,i}) = w_0$. Thus, Eq. (25) can be rewritten as the following:

$$P(m_I)(w_0 - c_v) + \sum_{i=1}^{I-1} P(m_i)(p_i(d^*_{HV,I})\bar{u}(d^*_{HV,I}) - c_v)^+ < c_f$$

By rearranging the terms, we have an inequality about $P(m_I)$ with the same expression as the expression in Table 5 under "Competitive, Common Platform".

C.1.3. Independent platform market, monopoly. Our proof contains three steps. First, we compute an upper bound for the net profit of a monopoly supplier (Eq. (4)), subject to the constraint that AVs serve the demand alone in every demand scenario. That is, we require the AV capacity N to satisfy

$$N \geq \max_i \{n^\dagger_{AV,i}\} = n^\dagger_{AV,I}$$

and the AV fleet size in each scenario to satisfy

$$n_{AV,i} \geq n^\dagger_{AV,i}$$

Next, we compute a lower bound for the net profit of a monopoly supplier (Eq. (4)), subject to the constraint that HVs participate in at least one demand scenario. That is, we require the AV capacity N' to satisfy

$$N' < n^\dagger_{AV,I}$$

Thus, under the AV capacity N', HVs at least will participate in scenario I.

Last, we compare the net profit of the AV supplier under N and N' and prove that the net profit (4) is strictly higher under N' than under N, when the condition for "independent platform market, monopoly" in Table 5 holds. Hence, any AV capacity that leads to AVs serving the market alone in every scenario is a dominated strategy, and thus cannot be the optimal capacity for the monopoly supplier.

Step 1: The monopoly supplier's net profit (4) *when AVs serve the market alone in every scenario.* When $N \geq \max_i\{n^\dagger_{AV,i}\} = n^\dagger_{AV,I}$, the profit of AVs only depends on its own fleet sizing decision. The monopoly supplier's profit (4) can be simplified as

$$\sum_1^I P(m_i) \max_{d^\dagger_{AV,i} \leq d_{AV} \leq \bar{d}(N)} (p_i(d_{AV})d_{AV}t_2 - c_v\underline{n}(d_{AV})) - c_f N \tag{26}$$

The variable profit term, $(p_i(d_{AV})d_{AV}t_2 - c_v\underline{n}(d_{AV}))$, is formally proved in Eq. (111).

Now recall the definition of $\tilde{d}_{AV,i}$ and $\tilde{n}_{AV,i}$ in Eq. (23). The term $\tilde{d}_{AV,i}$ is the maximize of the variable profit term, $(p_i(d_{AV})d_{AV}t_2 - c_v\underline{n}(d_{AV}))$, when there is no constraint on d_{AV}. Thus, the variable profit in scenario i is strictly decreasing when $d_{AV} > \tilde{d}_{AV,i}$.

Moreover, under the condition in Table 5, it is required that

$$n^{\dagger}_{AV,I} > \tilde{n}_{AV,I}$$

which is equivalent to

$$d^{\dagger}_{AV,I} > \tilde{d}_{AV,I}$$

Thus, we have the following inequality for Eq. (26) for any $N \geq n^{\dagger}_{AV,I}$:

$$\begin{aligned}
&\sum_{1}^{I} P(m_i) \max_{d^{\dagger}_{AV,i} \leq d_{AV} \leq \bar{d}(N)} (p_i(d_{AV})d_{AV}t_2 - c_v\underline{n}(d_{AV})) - c_f N \\
\leq &\sum_{1}^{I} P(m_i) \max_{d^{\dagger}_{AV,i} \leq d_{AV} \leq d^{\dagger}_{AV,I}} (p_i(d_{AV})d_{AV}t_2 - c_v\underline{n}(d_{AV})) - c_f n^{\dagger}_{AV,I}
\end{aligned} \tag{27}$$

That is, the capacity N will not be greater than $n^{\dagger}_{AV,I}$.

Define the following

$$\hat{d}_{AV,i} = \max\{\tilde{d}_{AV,i}, d^{\dagger}_{AV,i}\} \tag{28}$$

which represents the optimal demand rate in scenario i that maximizes the variable profit $(p_i(d_{AV})d_{AV}t_2 - c_v\underline{n}(d_{AV}))$ while keeping HVs out of the market. Let $\hat{n}_i = \underline{n}(\hat{d}_{AV,i})$. Then Eq. (27) can be further written as

$$\begin{aligned}
&\sum_{1}^{I} P(m_i) \max_{d^{\dagger}_{AV,i} \leq d_{AV} \leq d^{\dagger}_{AV,I}} (p_i(d_{AV})d_{AV}t_2 - c_v\underline{n}(d_{AV})) - c_f n^{\dagger}_{AV,I} \\
= &P(m_I)(p_I(d^{\dagger}_{AV,I})d^{\dagger}_{AV,I}t_2 - c_v\underline{n}(d^{\dagger}_{AV,I})) + \sum_{1}^{I-1} P(m_i)(p_i(\hat{d}_{AV,i})\hat{d}_{AV,i}t_2 - c_v\underline{n}(\hat{d}_{AV,i})) - c_f n^{\dagger}_{AV,I}
\end{aligned} \tag{29}$$

Eq. (29) is then an upper bound for the net profit Eq. (26) when AVs serve the market alone in every scenario. This upper bound is tight and is achieved at $N = n^{\dagger}_{AV,I}$.

Step 2: A lower bound of the monopoly supplier's net profit Eq. (4) *when HVs participate in at least one scenario.* Next, we compute a lower bound of the net profit Eq. (4) when HVs serve the market in some scenarios. In particular, we choose the AV capacity N' such that

$$N' = \hat{n}_{AV,I-1} = \max\{\tilde{n}_{AV,I-1}, n^{\dagger}_{AV,I-1}\}$$

In other words, capacity N' equals the AV fleet size that maximizes the variable profit $(p_i(d_{AV})d_{AV}t_2 - c_v\underline{n}(d_{AV})$ while exactly keeping HVs out of the market in scenario $I-1$.

Note that $N' < n^{\dagger}_{AV,I}$ must hold. This is because, by the monotonicity of $n^{\dagger}_{AV,i}$ and $\tilde{n}_{AV,i}$,

$$n^{\dagger}_{AV,I-1} < n^{\dagger}_{AV,I}$$

$$\tilde{n}_{AV,I-1} < \tilde{n}_{AV,I} < n^{\dagger}_{AV,I}$$

The last inequality is by the condition in Table 5. Thus, with an AV capacity of N', HVs must participate in scenario I.

Now we derive a lower bound for the Eq. (4) when the AV capacity is N'. Let the AV fleet size be $n_{AV,i} = \hat{n}_{AV,i}$ for all scenarios $i = 1, ..., I-1$ except for scenario I. Since $\hat{n}_{AV,i} \geq n^{\dagger}_{AV,i}$ by its definition in Eq. (28), in all scenarios except for scenario I, AVs serve the market alone. Therefore, the profit Eq. (4) can be written as

$$P(m_I) \max_{n_{AV,I} \leq N'} \pi^I_I(n_{AV,I}) n_{AV,I} + \sum_1^{N-1} P(m_i)(p_i(\hat{d}_{AV,i})\hat{d}_{AV,i}t_2 - c_v \underline{n}(\hat{d}_{AV,i})) - c_f \hat{n}_{AV,I-1}$$

which must be no less than

$$\sum_1^{N-1} P(m_i)(p_i(\hat{d}_{AV,i})\hat{d}_{AV,i}t_2 - c_v \underline{n}(\hat{d}_{AV,i})) - c_f \hat{n}_{AV,I-1} \tag{30}$$

Therefore, Eq. (30) is a lower bound for the monopoly supplier's net profit Eq. (4) when HVs participate in some scenarios.

Comparison Now we show that the lower bound Eq. (30) with HV participation is strictly higher than the upper bound with no HV participation Eq. (29), under the condition in Table 5. Deducting the lower bound Eq. (30) by the upper bound Eq. (29), we have

$$\left\{ \sum_1^{N-1} P(m_i)(p_i(\hat{d}_{AV,i})\hat{d}_{AV,i}t_2 - c_v \underline{n}(\hat{d}_{AV,i})) - c_f \hat{n}_{AV,I-1} \right\}$$
$$- \left\{ P(m_I)(p_I(d^{\dagger}_{AV,I})d^{\dagger}_{AV,I}t_2 - c_v \underline{n}(d^{\dagger}_{AV,I})) + \sum_1^{I-1} P(m_i)(p_i(\hat{d}_{AV,i})\hat{d}_{AV,i}t_2 - c_v \underline{n}(\hat{d}_{AV,i})) - c_f n^{\dagger}_{AV,I} \right\}$$

which equals

$$\begin{aligned} & - c_f \hat{n}_{AV,I-1} - P(m_I)(p_I(d^{\dagger}_{AV,I})d^{\dagger}_{AV,I}t_2 - c_v \underline{n}(d^{\dagger}_{AV,I})) + c_f n^{\dagger}_{AV,I} \\ = & c_f(n^{\dagger}_{AV,I} - \hat{n}_{AV,I-1}) - P(m_I)(p_I(d^{\dagger}_{AV,I})d^{\dagger}_{AV,I}t_2 - c_v n^{\dagger}_{AV,I}) \end{aligned} \tag{31}$$

By Table 5,

$$P(m_I) < \frac{c_f(1 - \hat{n}_{AV,I-1})}{p_I(d^{\dagger}_{AV,I})\bar{u}(d^{\dagger}_{AV,I}) - c_v} = \frac{c_f(n^{\dagger}_{AV,I} - c_f \hat{n}_{AV,I-1})}{p_I(d^{\dagger}_{AV,I})d^{\dagger}_{AV,I}t_2 - c_v n^{\dagger}_{AV,I}}$$

Therefore, Eq. (31) is strictly above zero, implying that the lower bound Eq. (30) is strictly higher than the upper bound Eq. (29).

C.1.4. Independent platform market, competition. Consider an arbitrary AV capacity N that allows AVs to serve the market alone in every demand scenario. Then by the analysis of the independent platform market, N must satisfy that i.e.

$$N \geq \max_i \{n^\dagger_{AV,i}\} = n^\dagger_{AV,I}, \tag{32}$$

Recall that $d^\dagger_{AV,i}$ and $n^\dagger_{AV,i}$ are defined as the maximum demand served by AVs and AV fleet size that allow HVs to participate in scenario i under an independent platform market. The definitions of $n^\dagger_{AV,i}$ and $d^\dagger_{AV,i}$ are given by Eq. (104) and Eq. (102), respectively.

Suppose we can show that for any N that satisfies Eq. (32), the aggregate variable profit per AV (5) is strictly lower than the AV fixed cost c_f, i.e. the net profit per AV is strictly negative. Then this is a sufficient condition for the statement that HVs participate in at least one demand scenario (otherwise AVs can never break even).

Thus, in the rest of the proof, we show that the condition in Table 5 under "Competitive, Independent Platform" implies that the net profit per AV is strictly negative for any AV capacity N that satisfies Eq. (32). More precisely, we prove that

$$\sum_{i=1}^{I} P(m_i)(p_i(\bar{d}(N))\bar{u}(\bar{d}(N)) - c_v)^+ < c_f, \text{ for any } N \geq n^\dagger_{AV,I}. \tag{33}$$

Our proof takes two steps. We prove that under the condition in Table 5, the following two things hold true:

1. The variable profit per AV in scenario i, $p_i(d_{AV})\bar{u}(d_{AV}) - c_v$, is strictly decreasing for $d \geq d^\dagger_{AV,I}$ in all scenarios. This guarantees that the right-hand side of Eq. (33),

$$\sum_{i=1}^{I} P(m_i)(p_i(\bar{d}(N))\bar{u}(\bar{d}(N)) - c_v)^+, \tag{34}$$

   is maximized at $N = n^\dagger_{AV,I}$.
2. At capacity $N = n^\dagger_{AV,I}$, Eq. (34) is strictly lower than c_f. This guarantees that for any $N > n^\dagger_{AV,I}$, Eq. (34) is also strictly lower than c_f.

Combining the two statements, we prove that Eq. (33) holds under the conditions in Table 5. Next, we prove each statement, respectively.

Proof of Item 1: First, we show that the variable profit per AV, $p_i(d_{AV})\bar{u}(d_{AV}) - c_v$, is strictly decreasing for $d \geq d^\dagger_{AV,I}$ in scenario I. The reason is the following:

By the condition in Table 5, in scenario I, it holds that

$$d^\dagger_{AV,I} > \tilde{d}_{AV,I}$$

Recall that $\tilde{d}_{AV,I}$ maximizes the total variable profit

$$p_I(d_{AV})d_{AV}t_2 - c_v\underline{n}(d_{AV}) = (p_I(d_{AV})\bar{u}(d_{AV}) - c_v) \cdot \underline{n}(d_{AV}) \tag{35}$$

in scenario I. Then it must be true that

$$d^{\dagger}_{AV,I} > \tilde{d}_{AV,I} > \arg\max_{d_{AV}}(p_I(d_{AV})\bar{u}(d_{AV})) \tag{36}$$

In other words, $d^{\dagger}_{AV,I}$ must be greater than the demand rate that maximizes the variable revenue per AV. To see why, we take the first-order derivative of the total variable profit (35), which gives:

$$(p_I(d_{AV})\bar{u}(d_{AV}))' \cdot \underline{n}(d_{AV}) + (p_I(d_{AV})\bar{u}(d_{AV}) - c_v) \cdot \underline{n}'(d_{AV}) \tag{37}$$

At $d_{AV} = d^{\dagger}_{AV,I}$, Eq. (37) is zero.

Now consider the value of Eq. (37) when $d_{AV} = \arg\max_{d_{AV}}(p_I(d_{AV})\bar{u}(d_{AV}))$. Since the function $p_I(d_{AV})\bar{u}(d_{AV})$ is strictly concave (Lemma 3), it must be true that when $d_{AV} = \arg\max_{d_{AV}}(p_I(d_{AV})\bar{u}(d_{AV}))$, the derivative

$$(p_I(d_{AV})\bar{u}(d_{AV}))' = 0$$

Furthermore, regarding the second term of Eq. (37), Then there are two possibilities:

1. $\max_{d_{AV}} p_I(d_{AV})\bar{u}(d_{AV}) \leq c_v$. Then AVs cannot break even in scenario I, which implies that AVs also cannot break even in any other scenario ($p_1(d_{AV}) \leq p_2(d_{AV}) \leq ... \leq p_I(d_{AV})$ for any d_{AV}). Then in this setting, AVs can never make a non-negative net profit and will never enter the ride-hailing market. This is not the setting that we are interested in as implied by Assumption 2.
2. $\max_{d_{AV}} p_I(d_{AV})\bar{u}(d_{AV}) > c_v$. Then in this case, at $d = \arg\max_{d_{AV}}(p_I(d_{AV})\bar{u}(d_{AV}))$, it must hold that the first-order derivative (37) is strictly positive, i.e.

$$(p_I(d_{AV})\bar{u}(d_{AV}))' \cdot \underline{n}(d_{AV}) + (p_I(d_{AV})\bar{u}(d_{AV}) - c_v) \cdot \underline{n}'(d_{AV}) > 0$$

   In other words, at $d_{AV} = \arg\max_{d_{AV}}(p_I(d_{AV})\bar{u}(d_{AV}))$, the total variable profit Eq. (35) is strictly increasing. Then it must be true that the maximizer of Eq. (35), $\tilde{d}_{AV,I}$, is greater than $\arg\max_{d_{AV}}(p_I(d_{AV})\bar{u}(d_{AV}))$. This verifies Eq. (36).

With Eq. (36), by concavity, it must be true that the variable profit per AV, $p_i(d_{AV})\bar{u}(d_{AV}) - c_v$, is strictly decreasing for $d \geq d^{\dagger}_{AV,I}$ in scenario I.

Next, we show that the variable profit per AV, $p_i(d_{AV})\bar{u}(d_{AV}) - c_v$, is strictly decreasing for $d \geq d^{\dagger}_{AV,I}$ in scenario $i = 1, ..., I-1$. The reasoning is similar to that for $i = I$. We leverage the proof for $i = I$, and additionally, we show that

$$\arg\max_{d_{AV}} p_1(d_{AV})\bar{u}(d_{AV}) < \arg\max_{d_{AV}} p_2(d_{AV})\bar{u}(d_{AV}) < ... < \arg\max_{d_{AV}} p_I(d_{AV})\bar{u}(d_{AV}) \tag{38}$$

In other words, the demand rate that maximizes the variable profit per AV, $p_i(d_{AV})\bar{u}(d_{AV})$, is strictly increasing in the potential demand mass (and therefore the scenario index i). Once we have shown Eq. (38), it is directly implied by Eq. (36) that

$$d^{\dagger}_{AV,I} > \arg\max_{d_{AV}}(p_I(d_{AV})\bar{u}(d_{AV})) \geq \arg\max_{d_{AV}}(p_i(d_{AV})\bar{u}(d_{AV})) \text{ for all } i$$

which will conclude our proof that the variable profit per AV, $p_i(d_{AV})\bar{u}(d_{AV}) - c_v$, is strictly decreasing for $d \geq d^{\dagger}_{AV,I}$ in scenario $i = 1, 2, ..., I-1$.

Thus, in the rest of the proof for this step, we focus on proving Eq. (38). To do this, we compute the first-order derivative of $p_i(d_{AV})\bar{u}(d_{AV})$ over d_{AV} and examine its relationship with the potential demand mass m_i. The first-order derivative is given by

$$p'_i(d_{AV})\bar{u}(d_{AV}) + p_i(d_{AV})\bar{u}'(d_{AV})$$

Setting the above term to zero and rearranging the terms gives

$$-\frac{p'_i(d_{AV})}{p_i(d_{AV})} = \frac{\bar{u}'(d_{AV})}{\bar{u}(d_{AV})} \tag{39}$$

Given the same d_{AV}, the right-hand side of Eq. (39) is the same regardless of i. The left-hand side of Eq. (39) can be further simplified by plugging in the expression for $p_i(d)$:

$$-\frac{p'_i(d_{AV})}{p_i(d_{AV})} = -\frac{-V/m_i}{V(1 - d_{AV}/m_i)} = \frac{1}{m_i - d_{AV}}$$

Then the first-order condition Eq. (39) is equivalent to

$$\frac{1}{m_i - d_{AV}} = \frac{\bar{u}'(d_{AV})}{\bar{u}(d_{AV})} \Leftrightarrow m_i = d_{AV} + \frac{\bar{u}(d_{AV})}{\bar{u}'(d_{AV})}$$

Note that the function $\bar{u}(d_{AV})$ is strictly increasing and concave (Lemma 2), implying that the term on the right-hand side, $\frac{\bar{u}(d_{AV})}{\bar{u}'(d_{AV})}$, is strictly increasing in d_{AV}. Thus, the entire right-hand side,

$$d_{AV} + \frac{\bar{u}(d_{AV})}{\bar{u}'(d_{AV})},$$

is strictly increasing in d_{AV}. Therefore, as the potential demand mass m_i (and the corresponding scenario index i) increases, the maximizer $\arg\max_{d_{AV}} p_i(d_{AV})\bar{u}(d_{AV})$ is strictly increasing. This completes the proof for Eq. (38).

To summarize, we have proved that $p_i(d_{AV})\bar{u}(d_{AV})$ is strictly decreasing in d_{AV} for $d_{AV} \geq d^{\dagger}_{AV,I}$ for all i. As a result, the net profit per AV (34) is maximized at $N = n^{\dagger}_{AV,I}$. That is, the maximum of Eq. (34) is given by

$$\sum_{i=1}^{I} P(m_i)(p_I(\bar{d}(n^{\dagger}_{AV,I}))\bar{u}(\bar{d}(n^{\dagger}_{AV,I})) - c_v)^+ = \sum_{i=1}^{I} P(m_i)(p_I(d^{\dagger}_{AV,I})\bar{u}(d^{\dagger}_{AV,I}) - c_v)^+ \tag{40}$$

Proof of Item 2: We now prove that under the condition in Table 5, the maximum of the net profit per AV, Eq. (40), is less than the fixed cost c_f. By Table 5, we have that

$$P(m_I) < \frac{c_f - \sum_{i=1}^{I-1} P(m_i)\left(p_I(d^\dagger_{AV,I}).\bar{u}(d^\dagger_{AV,I}) - c_v\right)^+}{p_I(d^\dagger_{AV,I})\bar{u}(d^\dagger_{AV,I}) - c_v} \tag{41}$$

By Table 5, it also holds that $p_I(d^\dagger_{AV,I})\bar{u}(d^\dagger_{AV,I}) - c_v > 0$. Then Eq. (41) is equivalent to

$$P(m_I)(p_I(d^\dagger_{AV,I})\bar{u}(d^\dagger_{AV,I}) - c_v) + \sum_{i=1}^{I-1} P(m_i)\left(p_I(d^\dagger_{AV,I}).\bar{u}(d^\dagger_{AV,I}) - c_v\right)^+ < c_f$$

which can be further written as

$$\sum_{i=1}^{I} P(m_i)\left(p_I(d^\dagger_{AV,I}).\bar{u}(d^\dagger_{AV,I}) - c_v\right)^+ < c_f$$

Note that the left-hand side is just Eq. (40). This concludes our proof for Item 2.

C.2. Finite supply elasticity and zero density elasticity ($\alpha > 0$, $r = 0$)

Table 6 Conditions for HV to participate in at least one scenario under $\alpha > 0$, $r = 0$ under a two-scenario demand distribution

Monopoly	$P(m_2) < \frac{c_f(\frac{(\bar{u}V - w_0)t_2}{\bar{u}^2 V} - \frac{\underline{n}(m_1)}{m_2})}{Vt_2/4}$, $m_2 > \frac{\underline{n}(m_1)}{(\bar{u}V - w_0)t_2/(\bar{u}^2 V)}$
Competitive	$P(m_2) < \frac{c_f \frac{(\bar{u}V - w_0)t_2}{\bar{u}^2 V} - \frac{Vt_2}{4}\frac{m_1}{m_2}}{(1 - m_1/m_2)Vt_2/4}$, $m_2 > \frac{m_1}{c_f \frac{(\bar{u}V - w_0)t_2}{\bar{u}^2 V}/(\frac{Vt_2}{4})}$

By Proposition 10, when the density elasticity $r = 0$, there is no differentiation between common and independent platform markets. Therefore, in this setting, we only need to derive the conditions for the two levels of AV competition. Table 6 summarizes the sufficient conditions under which HVs participate in at least one demand scenario under monopoly and perfectly competitive AVs. Appendix C.2.1 and Appendix C.2.2 provide formal proofs for the two settings, respectively.

C.2.1. Monopoly Consider two scenarios, $i = 1, 2$, with $m_1 < m_2$. We prove that the following condition is a sufficient condition under which HVs participate at least in scenario 2:

$$P(m_2) < \frac{c_f(\frac{(\bar{u}V - w_0)t_2}{\bar{u}^2 V} - \frac{\underline{n}(m_1)}{m_2})}{Vt_2/4},\ m_2 > \frac{\underline{n}(m_1)}{(\bar{u}V - w_0)t_2/(\bar{u}^2 V)} \tag{42}$$

We prove the statement by contradiction:

Suppose not. Then under Eq. (42), HVs do not participate in any scenario, i.e. $n_{HV,i} = 0$. Denote the AV fleet sizes in scenario 1 and 2 as $n_{AV,1}$ and $n_{AV,2}$, respectively. By Eq. (119) in Appendix F, the necessary and sufficient condition for $n_{HV,i} > 0$ is given by

$$n_{AV,i} < \frac{(\bar{u}V - w_0)t_2}{\bar{u}^2 V} m_i, \tag{43}$$

Therefore, if HVs do not participate in any scenario, it holds that

$$n_{AV,i} \geq \frac{(\bar{u}V - w_0)t_2}{\bar{u}^2 V} m_i, \ i = 1, 2 \tag{44}$$

Moreover, the total profit of the monopoly AV supplier is given by

$$\sum_i P(m_i)(\pi_i(n_{AV,i})n_{AV,i}) - c_f N$$

where $\pi_i(n_{AV,i})$ is the variable profit per AV in scenario i and N is the AV capacity. The capacity N must be no less than the AV fleet size in any scenario. That is,

$$N \geq n_{AV,1}, N \geq n_{AV,2}$$

Thus, it must be true that the total profit

$$\begin{aligned} &\sum_i P(m_i)(\pi_i(n_{AV,i})n_{AV,i}) - c_f N & (45)\\ \leq &\sum_i P(m_i)(\pi_i(n_{AV,i})n_{AV,i}) - c_f n_{AV,2} \\ \leq &\sum_i P(m_i)(\pi_i(n_{AV,i})n_{AV,i}) - c_f (\frac{(\bar{u}V - w_0)t_2}{\bar{u}^2 V} m_2) & (46) \end{aligned}$$

The last inequality is by Eq. (44). Moreover, since AVs serve the market alone, it must also hold that the variable profit in scenario 2 satisfies:

$$\begin{aligned} \pi_2(n_{AV,2})n_{AV,2} = &p_i(d_{AV,2})d_{AV,2}t_2 - c_v n_{AV,2} \\ < &\max_{d_{AV,2}} p_i(d_{AV,2}) \cdot d_{AV,2} \cdot t_2 \\ = &\frac{V}{2} \cdot \frac{m_2}{2} \cdot t_2 & (47) \end{aligned}$$

Combining Eq. (46) with Eq. (47), we have the following upper bound on the total profit when HVs do not participate in any scenario:

$$\sum_i P(m_i)(\pi_i(n_{AV,i})n_{AV,i}) - c_f N < P(m_1)(\pi_1(n_{AV,1})n_{AV,1}) + P(m_2) \cdot \frac{V}{2} \cdot \frac{m_2}{2} \cdot t_2 - c_f (\frac{(\bar{u}V - w_0)t_2}{\bar{u}^2 V} m_2) \tag{48}$$

Now consider a smaller AV capacity $N' < N$, where $N' = n_{AV,1}$. That is, let the AV capacity be the AV fleet size in scenario 1. We prove that at N', HVs will participate in scenario 2, and the resulting total profit for the AV supplier is strictly higher than under AV capacity N.

Claim 1: Under AV capacity $N' = n_{AV,1}$, HVs participate in scenario 2. The proof is the following. First, it must be true that

$$n_{AV,1} \leq \underline{n}(m_1) \tag{49}$$

Recall that $\underline{n}(m_i)$ is the number of AVs needed to satisfy all the potential demand m_i. When $n_{AV,1} = \underline{n}(m_i)$, the price $p_i(m_i) = 0$, leading to zero revenue and a negative variable profit. Therefore, a monopoly supplier never extends the AV fleet size in scenario i beyond $\underline{n}(m_i)$. Furthermore, Eq. (42) implies that

$$\underline{n}(m_1) < \frac{(\bar{u}V - w_0)t_2}{\bar{u}^2 V} m_2$$

which combined with Eq. (49) leads to

$$N' = n_{AV,1} < \frac{(\bar{u}V - w_0)t_2}{\bar{u}^2 V} m_2 \leq n_{AV,2}$$

Since the largest possible AV fleet size in scenario 2 is the capacity N', by Eq. (43), HVs must participate in scenario 2.

Claim 2: Under AV capacity $N' = n_{AV,1}$, the total profit of the monopoly AV supplier is strictly higher than the total profit Eq. (45) when HVs do not participate in any scenario. The proof is the following. Denote the AV fleet size in scenario 1 and 2 as $n'_{AV,1}$ and $n'_{AV,2}$, respectively. It must be true that

$$n'_{AV,1} = n_{AV,1}, \tag{50}$$

and

$$n'_{AV,2} \leq n_{AV,1} \tag{51}$$

Eq. (50) holds because the $n_{AV,1}$ is the AV fleet size that maximizes the variable profit in scenario 1 when the capacity is $N > N'$, i.e. when there are more choices for the AV fleet size. Eq. (51) is simply the capacity constraint. Therefore, we have that the total profit at $N' = n_{AV,1}$ is given by

$$P(m_1)(\pi_1(n_{AV,1})n_{AV,1})) + P(m_2)(\pi_2(n'_{AV,2})n'_{AV,2})) - c_f n_{AV,1}$$

Since the variable profit in scenario 2 must be non-negative, the total profit at $N' = n_{AV,1}$ must satisfy:

$$\begin{aligned} &P(m_1)(\pi_1(n_{AV,1})n_{AV,1})) + P(m_2)(\pi_2(n'_{AV,2})n'_{AV,2})) - c_f n_{AV,1} \\ \geq &P(m_1)(\pi_1(n_{AV,1})n_{AV,1})) - c_f n_{AV,1} \\ \geq &P(m_1)(\pi_1(n_{AV,1})n_{AV,1})) - c_f \underline{n}(m_1) \end{aligned} \tag{52}$$

The last inequality is by Eq. (49). Moreover, by Eq. (42),

$$\underline{n}(m_1) < m_2\left(\frac{(\bar{u}V - w_0)t_2}{\bar{u}^2 V} - P(m_2)\frac{V t_2}{4c_f}\right)$$

Combined with Eq. (52), we have

$$\begin{aligned}
&P(m_1)(\pi_1(n_{AV,1})n_{AV,1})) + P(m_2)(\pi_2(n'_{AV,2})n'_{AV,2})) - c_f n_{AV,1} \\
\geq& P(m_1)(\pi_1(n_{AV,1})n_{AV,1})) - c_f \underline{n}(m_1) \\
>& P(m_1)(\pi_1(n_{AV,1})n_{AV,1})) - c_f m_2(\frac{(\bar{u}V - w_0)t_2}{\bar{u}^2 V} - P(m_2)\frac{Vt_2}{4c_f}) \\
=& P(m_1)(\pi_1(n_{AV,1})n_{AV,1})) + P(m_2)\cdot\frac{V}{2}\cdot\frac{m_2}{2}\cdot t_2 - c_f(\frac{(\bar{u}V - w_0)t_2}{\bar{u}^2 V}m_2) \qquad (53)
\end{aligned}$$

But Eq. (53) is just the upper bound Eq. (48) of the total profit when HVs do not participate in any scenario. Therefore, we have proved that under Eq. (42), the monopoly AV supplier will never choose AV fleet sizes and an AV capacity to keep the HVs out in every scenario.

C.2.2. Perfect competition Similar to Appendix C.2.1, consider two scenarios $i = 1, 2$, with $m_1 < m_2$. We prove that the following condition is a sufficient condition under which HVs must participate in at least one scenario under perfect AV competition:

$$P(m_2) < \frac{c_f \frac{(\bar{u}V - w_0)t_2}{\bar{u}^2 V} - \frac{Vt_2}{4} m_1/m_2}{\frac{Vt_2}{4}(1 - m_1/m_2)},\ m_2 > \frac{m_1}{c_f \frac{(\bar{u}V - w_0)t_2}{\bar{u}^2 V} / (\frac{Vt_2}{4})} \qquad (54)$$

The intuition of the proof is the following:

Item 2 of Proposition 10 states that AVs always enter the ride-hailing market under perfect AV competition. In other words, there exists a strictly positive AV capacity N^* satisfies the AV equilibrium condition Eq. (6); that is,

$$\sum_i P(m_i)\max\{\pi_i(N^*), 0\} = c_f$$

However, we show that under Eq. (54), for any AV capacity N such that HVs do not participate in any scenario, AVs make a strictly negative profit. That is, for any N such that $n_{HV,i} = 0$ for all i,

$$\sum_i P(m_i)\max\{\pi_i(N), 0\} < c_f \qquad (55)$$

Thus, in equilibrium, under the AV capacity N^*, it must be true that HVs participate in at least one scenario.

Next, we provide the formal proof. Denote the AV fleet size under capacity N as $n_{AV,1}$ and $n_{AV,2}$ in scenario 1 and 2, respectively. As we have discussed in Appendix C.2.1, HVs participate in scenario i if and only if Eq. (43) holds. Therefore,

$$n_{HV,i} = 0 \Leftrightarrow n_{AV,i} \geq \frac{(\bar{u}V - w_0)t_2}{\bar{u}^2 V} m_i,\ i = 1, 2$$

Since the AV fleet size cannot go above the AV capacity, it must hold that

$$N \geq \max_i\{n_{AV,i}\} \geq \max_i\{\frac{(\bar{u}V - w_0)t_2}{\bar{u}^2 V} m_i\} = \frac{(\bar{u}V - w_0)t_2}{\bar{u}^2 V} m_2 \qquad (56)$$

Now consider the left-hand side of Eq. (55):

$$\sum_i P(m_i)\max\{\pi_i(N),0\} = 1/N\sum_i P(m_i)\max\{\pi_i(N)N,0\}$$

The term $\pi_i(N)N$ is just the total variable profit of all AVs in scenario i. Therefore, it must be not greater than the total revenue of all AVs in scenario i. That is,

$$\pi_i(N)N = p_i(d_{AV,i})d_{AV,i}t_2 - c_v N < p_i(d_{AV,i})d_{AV,i}t_2 \le (\frac{V}{2})(\frac{m_i}{2})t_2$$

Therefore, we can derive the following bound of the left-hand of Eq. (55):

$$\begin{aligned}
&\sum_i P(m_i)\max\{\pi_i(N),0\}\\
<&1/N\sum_i P(m_i)\max\{(\frac{V}{2})(\frac{m_i}{2})t_2,0\}\\
=&1/N\sum_i P(m_i)(\frac{V}{2})(\frac{m_i}{2})t_2
\end{aligned}$$

Then by Eq. (56),

$$\begin{aligned}
&\sum_i P(m_i)\max\{\pi_i(N),0\}\\
<&1/N\sum_i P(m_i)(\frac{V}{2})(\frac{m_i}{2})t_2\\
\le&\frac{\sum_i P(m_i)(\frac{V}{2})(\frac{m_i}{2})t_2}{\frac{(\bar{u}V-w_0)t_2}{\bar{u}^2V}m_2}\\
=&\frac{(1-P(m_2))(\frac{V}{2})(\frac{m_1}{2})t_2 + P(m_2)(\frac{V}{2})(\frac{m_2}{2})t_2}{\frac{(\bar{u}V-w_0)t_2}{\bar{u}^2V}m_2}\\
=&\frac{(\frac{Vt_2}{4})m_1/m_2 + P(m_2)(\frac{Vt_2}{4})(1-m_1/m_2)}{\frac{(\bar{u}V-w_0)t_2}{\bar{u}^2V}}
\end{aligned}$$

But one can check that by applying the upper bound of $P(m_2)$ in condition Eq. (54), it must be true that

$$\frac{(\frac{Vt_2}{4})m_1/m_2 + P(m_2)(\frac{Vt_2}{4})(1-m_1/m_2)}{\frac{(\bar{u}V-w_0)t_2}{\bar{u}^2V}} < c_f$$

Therefore, we have proved that under Eq. (54),

$$\sum_i P(m_i)\max\{\pi_i(N),0\} < c_f$$

if HVs do not participate in any scenario under AV capacity N. Thus, in equilibrium, it must be true that HVs participate in at least one scenario. □

D. Finite supply elasticity and some density effect

In this section, we prove each item in Proposition 3 and Proposition 4, respectively.

D.1. Common platform market (Proof of Proposition 3, Item 1)

In this section, we examine the market outcomes when the supply of HVs is finitely elastic ($\alpha > 0$) and the dispatch market has positive density elasticity ($r > 0$), under a common dispatch market between AVs and HVs. In particular, in the first part of this proof, we want to show that the price is strictly lower under a common platform market compared to a pure-HV market. The high-level idea of the proof is that, after introducing AVs, the total demand served by AVs and HVs in any scenario is higher than that served by HVs in a pure-HV market; thus, the price is strictly lower with AVs; in the second part of this proof, we show that prices under perfectly competitive AVs are always strictly lower than that under monopoly.

D.1.1. Part 1. AVs lead to strictly lower prices. Consider scenario i. We use the notation of $d_{AV,CPM}$ and $d_{HV,CPM}$ to represent the demand served by HVs and AVs, respectively, and omit the subscript i since we are focusing on one particular demand scenario. When $d_{HV,CPM} > 0$, it means that HVs serve some demand in scenario i. Since HVs and AVs share the dispatch platform, they also have the same utilization rate, determined by the total demand served in the market. That is:

$$u_{HV,CPM} = u_{AV,CPM} = \bar{u}(d_{HV,CPM} + d_{AV,CPM})$$

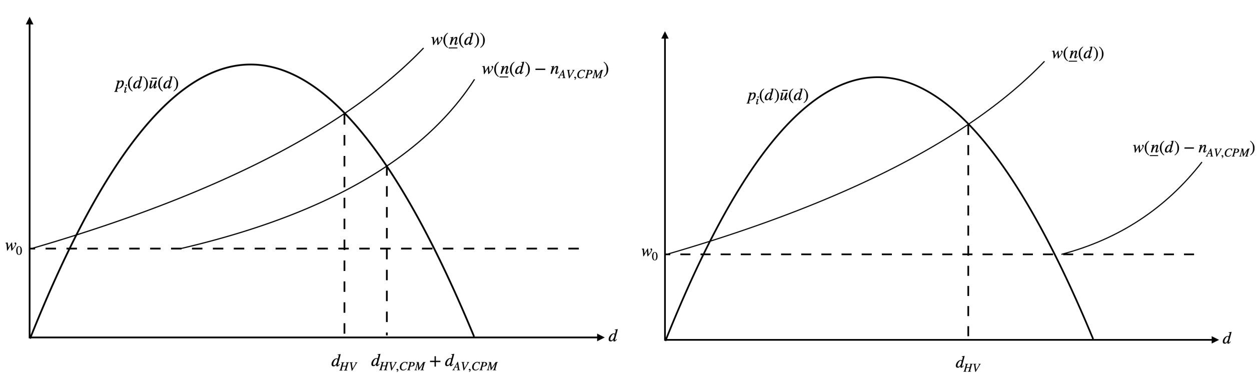
The AV and HV fleet sizes are then given by

$$n_{AV,CPM} = \frac{d_{AV,CPM} t_2}{u_{AV,CPM}} = \frac{d_{AV,CPM} t_2}{\bar{u}(d_{HV,CPM} + d_{AV,CPM})},$$

and

$$n_{HV,CPM} = \frac{d_{HV,CPM} t_2}{u_{HV,CPM}} = \frac{d_{HV,CPM} t_2}{\bar{u}(d_{HV,CPM} + d_{AV,CPM})},$$

respectively. Then there are two cases:



(a) Small $n_{AV,CPM}$ (b) Large $n_{AV,CPM}$

Figure 5 Comparison of the equilibrium demand under a pure-HV market and a common platform market with an AV fleet size of $n_{AV,CPM}$

1. $n_{HV,CPM} > 0$. That is, HVs participate in the ride-hailing market. In this case, it must be true that the marginal HV's reservation earnings are equal to the price discounted by utilization. That is,

$$w(n_{HV,CPM}) = p_i(d_{CPM})\bar{u}(d_{CPM}) \tag{57}$$

where d_{CPM} is the total demand served by AVs and HVs. That is, $d_{CPM} = d_{AV,CPM} + d_{HV,CPM}$. On the other hand, under a pure HV market, it should also hold that in equilibrium the reservation earnings of the marginal HV should equal the price discounted by utilization:

$$w(n^*_{HV,i}) = p_i(d^*_{HV,i})\bar{u}(d^*_{HV,i})$$

In other words, $d^*_{HV,i}$ is the root of the following equation:

$$w(\underline{n}(d)) = p_i(d)\bar{u}(d) \tag{58}$$

To compare $d^*_{HV,i}$ and d_{CPM}, we also rewrite Eq. (57) in a similar format:

$$w(\underline{n}(d) - n_{AV,CPM}) = p_i(d)\bar{u}(d) \tag{59}$$

That is, given an AV fleet size $n_{AV,CPM}$, the total demand d_{CPM} is a root to Eq. (59).

Fig. 5a visually compares the two roots under Eq. (58) and Eq. (59). It must hold that there is a root d_{CPM} that satisfies Eq. (59) and $d_{CPM} > d^*_{HV,i}$. This can be proved by the Intermediate Value Theorem: Consider the interval $d \in [d^*_{HV,i}, m_i]$. Define the following function

$$l(d) = w(\underline{n}(d) - n_{AV,CPM}) - p_i(d)\bar{u}(d) \tag{60}$$

which is simply the left-hand side of Eq. (59) deducting the right-hand side of Eq. (59). $l(d)$ is a continuous function because $w(n)$, $\underline{n}(d)$, $p_i(d)$ and $\bar{u}(d)$ are all continuous functions. Then at $d = d_{HV}$, we have

$$\begin{aligned} l(d_{HV}) &= w(\underline{n}(d_{HV}) - n_{AV,CPM}) - (p_i(d_{HV})\bar{u}(d_{HV})) \\ &< w(\underline{n}(d_{HV})) - (p_i(d_{HV})\bar{u}(d_{HV})) = 0 \end{aligned}$$

The last equality is by Eq. (58). At $d = m_i$, we have

$$\begin{aligned} l(m_i) &= w(\underline{n}(m_i) - n_{AV,CPM}) - (p_i(m_i)\bar{u}(m_i)) \\ &= w(\underline{n}(m_i) - n_{AV,CPM}) > 0 \end{aligned}$$

The last equality was by the definition of the price function that at $d = m_i$, the price goes down to zero. Therefore, by the intermediate value theorem, there must exist a root $d \in (d_{HV}, m_i)$, such that

$$l(d) = w(\underline{n}(d) - n_{AV,CPM}) - p_i(d)\bar{u}(d) = 0$$

This root is d_{CPM} defined in Eq. (59). Therefore, we have proved that when HVs participate in scenario i, the equilibrium demand d_{CPM} under a common platform market is strictly higher than d_{HV} in a pure-HV market, which further implies that the price is strictly lower under a common platform market than a pure-HV market.

2. $n_{HV,CPM} = 0$. That is, the price is so low, that $p_i(d_{CPM})\bar{u}(d_{CPM})$ is below w_0 (which is the lowest possible reservation earnings for HVs), and HVs do not participate in scenario i. This case happens when the AV fleet size $n_{AV,CPM}$ is high, which drives down the price and makes it impossible for HVs to join. Fig. 5b visually illustrates this case. (Note that the market outcome, in this case, depends on the AV supplier(s) fleet size decision and no longer depends on the HV supply curve. Therefore, Fig. 5b does not contain an equilibrium point for the common platform market.) Clearly, the total demand served under a common platform market must be strictly greater than d_{HV}. We prove this rigorously below:

   Suppose not. Then $d^*_{HV,i} \geq d_{AV,CPM}$. Consider the function $l(d)$ defined by Eq. (60). Then the analysis of $l(d)$ on the interval $d \in [d^*_{HV,i}, m_i]$ continues to hold here, and it remains true that $l(d_{HV}) < 0$, $l(m_i) > 0$; therefore, it must be true that there exists $d \in (d^*_{HV,i}, m_i)$ such that $l(d) = 0$. In other words, if $d^*_{HV,i} \geq d_{AV,CPM}$, then there must exist a demand rate $d > d^*_{HV,i}$ that allows a positive number of HVs to participate. This contradicts the premise that $n_{HV,CPM} = 0$. Contradiction.

Therefore, combining both cases, we conclude that under a common platform market, the total demand served must be strictly higher than that under a pure-HV market; consequently, the price is strictly lower than that under a pure-HV market.

D.1.2. Part 2. Perfectly competitive AVs lead to strictly lower prices than monopoly. Next, we prove the prices under perfectly competitive AVs are strictly lower than that under a monopoly AV supplier in all scenarios. The structure of our proof is the following:

1. Denote the optimal AV capacity under a monopoly supplier as N^*. Prove that the equilibrium AV capacity under perfect competition, N', must be strictly higher than N^*
2. Compare the price in every scenario under capacity N' with that under capacity N^*.

Proof of Item 1: We prove this item by showing that, when the AV capacity under perfect competition, $N' = N^*$, the variable profit per AV, Eq. (5), is strictly higher than the fixed cost c_f. That is,

$$\sum_1^I P(m_i) \max\{\pi_i^C(N^*), 0\} > c_f \tag{61}$$

where $\pi_i^C(N^*)$ represents the variable profit per AV at the AV fleet size $n_{AV} = N^*$ in scenario i. If this holds true, then more than N^* AVs must enter ride-hailing in the zero-profit equilibrium under perfect competition, which implies that $N' > N^*$.

We start with characterizing some properties of N^*. We leverage the fact that N^* is an optimal solution to the monopoly supplier's profit maximization problem (4). We then use these properties to prove Eq. (61).

First, note that N^* is the optimal solution to the monopoly supplier's profit (4). That is,

$$N^* = \arg\max_N \sum_1^I P(m_i)\left\{\max_{n_{AV,i}\leq N} \pi_i^C(n_{AV,i})n_{AV,i}\right\} - c_f N$$

Let $n_{AV,CPM,i}$ denote the optimal AV fleet size in each scenario i, under the optimal AV capacity N^*. Then we can write the profit function in the neighborhood of $N = N^*$ as below:

$$\begin{aligned}&\sum_1^I P(m_i)\left\{\pi_i^C(n_{AV,CPM,i})n_{AV,CPM,i}\right\} - c_f N \\ =&\sum_{i,n_{AV,CPM,i}<N}\left\{\pi_i^C(n_{AV,CPM,i})n_{AV,CPM,i}\right\} + \sum_{j,n_{AV,CPM,j}=N}\left\{\pi_j^C(N)N\right\} - c_f N \end{aligned} \tag{62}$$

Since N^* maximizes the profit of the monopoly supplier, it satisfies that the derivative of Eq. (62) over N is zero at $N = N^*$. That is,

$$\sum_{j,n_{AV,CPM,j}=N^*}\left\{\frac{\partial(\pi_j^C(N)N)}{\partial N}|_{N=N^*}\right\} - c_f = 0 \tag{63}$$

We can further expand the derivative in side the bracket for scenarios that satisfy $n_{AV,CPM,j} = N^*$:

$$\frac{\partial(\pi_j^C(N)N)}{\partial N}|_{N=N^*} = \frac{\partial \pi_j^C(N)}{\partial N}|_{N=N^*}N^* + \pi_j^C(N^*)$$

Furthermore, for these scenarios, it must be true that

$$\frac{\partial \pi_j^C(N)}{\partial N}|_{N=N^*} < 0$$

In other words, the variable profit per AV, $\pi_j^C(n_{AV})$, is strictly decreasing in the AV fleet size n_{AV}, when $n_{AV} = N^*$. In fact, we can show that this holds in general. In other words, under a common platform market, the variable profit per AV $\pi_j^C(n_{AV})$ is always strictly decreasing in the AV fleet size n_{AV}. To keep the flow of the proof, We leave the proof of this item at the end of this subsection, and for now take it as given. Then we have that for all j such that $n_{AV,CPM,j} = N^*$,

$$\frac{\partial(\pi_j^C(N)N)}{\partial N}|_{N=N^*} < \pi_j^C(N^*)$$

Then by Eq. (63), it must be true that

$$\sum_{j,n_{AV,CPM,j}=N^*}\pi_j^C(N^*) > \sum_{j,n_{AV,CPM,j}=N^*}\left\{\frac{\partial(\pi_j^C(N)N)}{\partial N}|_{N=N^*}\right\} = c_f$$

Now consider the variable profit per AV under perfect competition, which is the left-hand side of Eq. (61). It must hold that

$$\sum_1^I P(m_i)\max\{\pi_i^C(N^*),0\} \geq \sum_{j,n_{AV,CPM,j}=N^*} \pi_j^C(N^*) > c_f$$

Thus, we have verified Eq. (61). Therefore, at the same AV capacity N^* as the monopoly supplier, under perfect competition, AVs make strictly positive profits; this cannot happen in the perfectly competitive equilibrium, becuase competition will drive more potential suppliers to purchase AVs, increasing the AV capacity.

Finally, we prove that under a common platform market and finite supply elasticity, the variable profit per AV $\pi_j^C(n_{AV})$ is always strictly decreasing in the AV fleet size n_{AV}. There are two cases:

1. n_{AV} is small. AVs and HVs serve the market together. Then in equilibrium, it must be true that the earnings per AV is the same as the earnings per HV. This is because AVs and HVs face the same price and utilization under a common platform market. In this case, when HVs reach equilibrium, it holds that

$$p_i(d)\bar{u}(d) = w(\underline{n}(d_{HV}))$$

where d represents the total demand served by AVs and HVs. $w(\underline{n}(d_{HV}))$ is then the earnings for each AV and HV in the market. The variable profit per AV is then equal to

$$\pi_i^C(n_{AV}) = w(\underline{n}(d_{HV})) - c_v$$

where d_{HV} represents the demand served by HVs when HVs provide service together with n_{AV} AVs. When n_{AV} increases, it must be true that its corresponding HV equilibrium demand d_{HV} decreases and the corresponding earnings $w(\underline{n}(d_{HV}))$ also decreases; otherwise, more HVs can enter in equilibrium. Therefore, $\pi_i^C(n_{AV})$ is strictly decreasing in n_{AV}, when AVs and HVs serve the market together.

2. n_{AV} is large. AVs serve the market alone. Then the variable profit per AV is given by

$$\pi_i^C(n_{AV}) = p_i(d_{AV})\bar{u}(d_{AV}) - c_v$$

where $d_{AV} = \bar{d}(n_{AV})$. Function $\bar{d}(n)$ is the inverse function of $\underline{n}(d)$ and is the maximum demand level satisfied by a fleet size of n. This case happens when n_{AV} is so large, that

$$p_i(d_{AV})\bar{u}(d_{AV}) \leq w_0 \tag{64}$$

and there does not exist any $d_{HV} > 0$ where $p_i(d_{AV}+d_{HV})\bar{u}(d_{AV}+d_{HV}) = w(\underline{n}(d_{HV}))$ can hold. Then it must be true that $p_i(d_{AV})\bar{u}(d_{AV})$ is also strictly decrasing in d_{AV} in this case, otherwise by the continuity of $p_i(d_{AV})\bar{u}(d_{AV})$ (proved in Lemma 3), Eq. (64) cannot hold for all d_{AV} in this region.

Therefore, we have proved that the variable profit per AV, $\pi_i^C(n_{AV})$, is strictly decreasing in the AV fleet size n_{AV} in general. This concludes our proof for Item 1 that the AV capacity under perfect competition must be strictly higher than that under monopoly.

Proof of Item 2: For this item, we compare the price in each scenario under perfect competition and monopoly. Recall that we denote the optimal capacity of the monopoly supplier as N^* and the equilibrium capacity of perfectly competitive AVs as N'. In Item 1, we have proved that $N' > N^*$.

We discuss each scenario from the perspective of the monopoly supplier. Again, denote the optimal AV fleet size chosen by the monopoly supplier in scenario i as $n_{AV,CPM,i}$. That is, $n_{AV,CPM,i} = \arg\max_{n_{AV}} \pi_i^C(n_{AV}) n_{AV}$.

Then it must be true that $\pi_i^C(n_{AV,CPM,i}) \geq 0$, since the monopoly supplier has no incentive to choose a fleet size that leads to negative variable profit in scenario i. In the mean time, in scenario i, under perfect competition, there are two possibilities:

1. All N' AVs participate. That is, perfectly competitive AVs run out all of their capacity. Thus, it must hold that $N' > N^* \geq n_{AV,CPM,i}$. At the beginning of the proof, we have shown that the total demand served is a strictly increasing function of the number of AVs participating in a scenario. Thus, it must be true that AVs supply more demand and the price is strictly lower under perfect competition.
2. A fraction of the N' AVs participate. That is, perfectly competitive AVs particpate to the point where the variable profit per AV is zero. Since a monopoly supplier never has the incentive to choose a demand that leads to zero profit, the demand served is also strictly higher and the price is strictly lower under perfect competition.

This concludes our proof of Item 2.

D.2. Independent platform market

D.2.1. Proof of Proposition 3, Item 2a In this section, we prove that under the condition Eq. (13), the equilibrium price under a monopoly independent platform market is no less than that under a pure-HV market, and strictly higher than that under a pure-HV market in at least one demand scenario. We also discuss the price when AVs and HVs coexist, when AVs serve the market alone, and when HVs serve the market alone.

Our proof takes two steps. First, we show that in any demand scenario, for any arbitrary AV capacity $N > 0$, under Eq. (13), a monopoly AV supplier never chooses an AV fleet size n_{AV} that leads to a lower price than in the pure-HV market. Then we show that there must exist at least one demand scenario under which the price is strictly higher than in the pure-HV market.

Step 1. Consider an arbitrary AV capacity $N > 0$ and demand scenario i. Then there are three possibilities:

1. AVs and HVs serve the market together.
2. AVs serve the market alone.
3. HVs serve the market alone.

We show that if the monopoly supplier chooses an AV fleet size that leads to case 1 and case 2, then the price is strictly higher than that under a pure-HV market. If the monopoly supplier chooses an AV fleet size that leads to case 3, then the price is identical to that under a pure-HV market. Below, we analyze each case, respectively.

1. *AVs and HVs serve the market together.* **We start with an exogenously given AV fleet size n_{AV}.** The corresponding demand served by n_{AV} is then given by $d_{AV} = \bar{d}(n_{AV})$. Note that the function $\bar{d}(n)$ is defined in Lemma 1 and is the inverse function of $\underline{n}(d)$. We analyze the equilibrium outcome when AVs serve a demand level of d_{AV}.

   We first characterize the conditions under which both AVs and HVs participate, which are given by

$$p_i(d_{HV} + d_{AV})\bar{u}(d_{HV}) = w(n_{HV}) = w(\underline{n}(d_{HV})) \tag{65}$$

   and

$$p_i(d_{HV} + d_{AV})\bar{u}(d_{AV}) \geq c_v \tag{66}$$

   Eq. (65) is the equilibrium condition (22). The left-hand side of Eq. (66) is the earnings per AV, which is required to be at least the variable cost c_v. When Eq. (65) has a solution with $n_{HV} > 0$ and $n_{AV} > 0$, and Eq. (66) holds, then in equilibrium, AVs and HVs serve the market together.

   Then given d_{AV}, the exogenous demand level served by AVs, in equilibrium, the price, denoted as $p_{IPM,i}$, and the demand served by HVs, denoted as $d_{HV,IPM,i}$,[11] satisfies the following relationship:

$$p_{IPM,i} = p_i(d_{HV,IPM,i} + d_{AV}) = \frac{w(\underline{n}(d_{HV,IPM,i}))}{\bar{u}(d_{HV,IPM,i})} \tag{67}$$

   Recall that a similar relationship holds for the pure-HV market between its equilibrium price $p^*_{HV,i}$ and equilibrium demand $d^*_{HV,i}$:

$$p^*_{HV,i} = \frac{w(\underline{n}(d^*_{HV,i}))}{\bar{u}(d^*_{HV,i})} \tag{68}$$

   We prove the following two statements:

   (a) The equilibrium demand served by HVs reduces after AV participation for any $d_{AV} > 0$. That is,

$$d_{HV,IPM,i} < d^*_{HV,i} \tag{69}$$

   Moreover, $d_{HV,IPM,i}$, defined by Eq. (67), is strictly decreasing in d_{AV}. That is, for $d_{AV} < d'_{AV}$,

$$d_{HV,IPM,i} > d'_{HV,IPM,i}$$

[11] We do not add the superscript * for these two quantities because d_{AV} is exogenously given and not a choice of the monopoly supplier.

(b) Under the condition Eq. (13), the function

$$\frac{w(\underline{n}(d))}{\bar{u}(d)} \tag{70}$$

is strictly decreasing in d for $0 < d \leq d^*_{HV,I}$.

These two statements, combined with Eq. (67) and Eq. (68), gives

$$p^*_{HV,i} = \frac{w(\underline{n}(d^*_{HV,i}))}{\bar{u}(d^*_{HV,i})} < \frac{w(\underline{n}(d_{HV,IPM,i}))}{\bar{u}(d_{HV,IPM,i})} = p_{IPM,i}$$

for any $d_{AV} > 0$. Moreover, the equilibrium price, $p_{IPM,i}$, is strictly increasing in d_{AV}. That is, for $d_{AV} < d'_{AV}$,

$$p_{IPM,i} = \frac{w(\underline{n}(d_{HV,IPM,i}))}{\bar{u}(d_{HV,IPM,i})} < \frac{w(\underline{n}(d'_{HV,IPM,i}))}{\bar{u}(d'_{HV,IPM,i})} = p'_{IPM,i}$$

This proves that the price when AVs and HVs serve the market together under an independent platform market is strictly higher than that under a pure-HV market for any exogenously given AV fleet size n_{AV}, and the price is strictly increasing in n_{AV}. Next, we prove the two statements, respectively.

Proof of Item 1a: Suppose not. Then

$$d_{HV,IPM,i} \geq d^*_{HV,i}$$

Then since $d^*_{HV,i}$ is an equilibrium point, for any $d > d^*_{HV,i}$, it must hold that

$$p_i(d)\bar{u}(d) < w(\underline{n}(d))$$

Since $d_{HV,IPM,i} \geq d^*_{HV,i}$, it means that

$$p_i(d_{HV,IPM,i})\bar{u}(d_{HV,IPM,i}) \leq w(\underline{n}(d^*_{HV,IPM,i}))$$

Then for $d_{AV} > 0$, by the monotonicity of the price function $p_i(d)$, it must be true that

$$p_i(d_{AV} + d^*_{HV,IPM,i})\bar{u}(d^*_{HV,IPM,i}) < p_i(d^*_{HV,IPM,i})\bar{u}(d^*_{HV,IPM,i}) \leq w(\underline{n}(d^*_{HV,IPM,i})) \tag{71}$$

But this contradicts the definition of $d^*_{HV,IPM,i}$ in Eq. (67), which requires the left-hand side and the right-hand side of Eq. (71) are equal. This concludes our proof for Eq. (69).

In fact, this proof can be extended to prove that the equilibrium demand served by HVs, $d_{HV,IPM,i}$, is strictly decreasing in d_{AV}. Let $d'_{HV,IPM,i}$ be the equilibrium demand served by HVs when the demand served by AVs is d'_{AV}. Let $d'_{AV} > d_{AV}$. Again, we prove the statement by contradiction:

Suppose not. Then $d'_{HV,IPM,i} \geq d_{HV,IPM,i}$. Then since $d_{HV,IPM,i}$ is an equilibrium point, it must be true that for any $d > d_{HV,IPM,i}$, it holds that

$$p_i(d_{AV}+d)\bar{u}(d) < w(\underline{n}(d))$$

Then by $d'_{HV,IPM,i} \geq d_{HV,IPM,i}$, we must have that

$$p_i(d_{AV}+d'_{HV,IPM,i})\bar{u}(d'_{HV,IPM,i}) \leq w(\underline{n}(d'_{HV,IPM,i}))$$

Furthermore, since $d_{AV} < d'_{AV}$, we have that

$$p_i(d'_{AV}+d'_{HV,IPM,i})\bar{u}(d'_{HV,IPM,i}) < p_i(d_{AV}+d'_{HV,IPM,i})\bar{u}(d'_{HV,IPM,i}) \leq w(\underline{n}(d'_{HV,IPM,i}))$$

This contradicts the definition for $d'_{HV,IPM,i}$. Therefore, it must be true that $d_{HV,IPM,i}$, defined by Eq. (65), is strictly decreasing in d_{AV}.

Proof of Item 1b We take the first-order derivative of the function in Eq. (70) and show that it is strictly negative under condition Eq. (13).

The first-order derivative is then given by

$$\frac{\frac{\partial w}{\partial d}\bar{u}(d) - w(\underline{n}(d))\frac{\partial \bar{u}}{\partial d}}{\bar{u}(d)^2} \tag{72}$$

Thus, the sign of the first-order derivative of Eq. (70) is the same as that for the numerator

$$\begin{aligned} &\frac{\partial w}{\partial d}\bar{u}(d) - w(\underline{n}(d))\frac{\partial \bar{u}}{\partial d} \\ =&\frac{\partial w}{\partial \underline{n}}\frac{\partial \underline{n}}{\partial d}\bar{u}(d) - w(\underline{n}(d))\frac{\partial \bar{u}}{\partial d} \\ =&\alpha\underline{n}'(d)\bar{u}(d) - w(\underline{n}(d))\bar{u}'(d) \\ =&\bar{u}(d)\cdot w(\underline{n}(d))\cdot\left(\frac{\alpha\underline{n}'(d)}{w(\underline{n}(d))} - \frac{\bar{u}'(d)}{\bar{u}(d)}\right) \end{aligned} \tag{73}$$

Thus, the sign of the numerator Eq. (73) is the same as the term

$$\frac{\alpha\underline{n}'(d)}{w(\underline{n}(d))} - \frac{\bar{u}'(d)}{\bar{u}(d)} \tag{74}$$

We then compute the sign of the term Eq. (74). We first restate the definition of $\underline{n}(d)$ and $\bar{u}(d)$ in Eq. (8) and Eq. (16), respectively:

$$\underline{n}(d) = \tilde{a}d^{1/(r+1)} + t_2 d$$

$$\bar{u}(d) = \frac{dt_2}{\underline{n}(d)} = \frac{dt_2}{\tilde{a}d^{1/(r+1)} + t_2 d}$$

Thus,

$$\bar{u}'(d) = \frac{t_2\cdot\underline{n}(d) - dt_2\cdot\underline{n}'(d)}{\underline{n}(d)^2}$$

which implies that

$$\frac{\bar{u}'(d)}{\bar{u}(d)} = \frac{t_2 \cdot \underline{n}(d) - dt_2 \cdot \underline{n}'(d)}{dt_2 \cdot \underline{n}(d)} = \frac{1}{d} - \frac{\underline{n}'(d)}{\underline{n}(d)}$$

Then the term Eq. (74) can be written as

$$\begin{aligned}
&\frac{\alpha \underline{n}'(d)}{w(\underline{n}(d))} - \frac{\bar{u}'(d)}{\bar{u}(d)} \\
=&\frac{\alpha \underline{n}'(d)}{w(\underline{n}(d))} - \left(\frac{1}{d} - \frac{\underline{n}'(d)}{\underline{n}(d)}\right) \\
=&\frac{\alpha \underline{n}'(d)}{w_0 + \alpha \underline{n}(d)} - \frac{1}{d} + \frac{\underline{n}'(d)}{\underline{n}(d)} \\
=&\frac{\underline{n}'(d)}{w_0/\alpha + \underline{n}(d)} - \frac{1}{d} + \frac{\underline{n}'(d)}{\underline{n}(d)}
\end{aligned} \tag{75}$$

Since $\underline{n}(d)$ is an increasing function in d, Eq. (75) is strictly increasing in α. Thus, the inequality

$$\frac{\alpha \underline{n}'(d)}{w(\underline{n}(d))} - \frac{\bar{u}'(d)}{\bar{u}(d)} < 0$$

is equivalent to an upper bound for α. More specifically,

$$\begin{aligned}
&\frac{\alpha \underline{n}'(d)}{w(\underline{n}(d))} - \frac{\bar{u}'(d)}{\bar{u}(d)} < 0 \\
\Leftrightarrow &\frac{\underline{n}'(d)}{w_0/\alpha + \underline{n}(d)} < \frac{1}{d} - \frac{\underline{n}'(d)}{\underline{n}(d)}
\end{aligned} \tag{76}$$

For now, we will assume that

$$\frac{1}{2d} < \frac{\underline{n}'(d)}{\underline{n}(d)} < \frac{1}{d} \tag{77}$$

Later we will formally prove that Eq. (77) indeed holds. Then by Eq. (77), the inequality Eq. (76) is equivalent to

$$\begin{aligned}
&\frac{\underline{n}'(d)}{\frac{1}{d} - \frac{\underline{n}'(d)}{\underline{n}(d)}} < \frac{w_0}{\alpha} + \underline{n}(d) \\
\Leftrightarrow &\frac{\underline{n}'(d) - \frac{\underline{n}(d)}{d} + \underline{n}'(d)}{\frac{1}{d} - \frac{\underline{n}'(d)}{\underline{n}(d)}} < \frac{w_0}{\alpha} \\
\Leftrightarrow &\frac{(2\frac{\underline{n}'(d)}{\underline{n}(d)} - \frac{1}{d}) \cdot \underline{n}(d)}{\frac{1}{d} - \frac{\underline{n}'(d)}{\underline{n}(d)}} < \frac{w_0}{\alpha}
\end{aligned}$$

Again, by Eq. (77), we can rearrange the terms and obtain the following inequality:

$$\alpha < \frac{w_0(\frac{1}{d} - \frac{\underline{n}'(d)}{\underline{n}(d)})}{2\underline{n}(d)(\frac{\underline{n}'(d)}{\underline{n}(d)} - \frac{1}{2d})} \tag{78}$$

Eq. (78) is then a sufficient and necessary condition for making the first-order derivative Eq. (72) strictly negative.

We now verify Eq. (77). This requires the calculation of $\underline{n}'(d)$, which is given by

$$\underline{n}'(d) = \tilde{a}\frac{1}{r+1}d^{\frac{1}{r+1}-1} + t_2$$

Thus,

$$\begin{aligned}
&\underline{n}(d) - d\cdot\underline{n}'(d) \\
=&\tilde{a}d^{\frac{1}{r+1}} + t_2 d - \tilde{a}\frac{1}{r+1}d^{\frac{1}{r+1}} - dt_2 \\
=&\tilde{a}\frac{r}{r+1}d^{\frac{1}{r+1}} > 0
\end{aligned} \tag{79}$$

Similarly,

$$\begin{aligned}
&\underline{n}(d) - 2d\cdot\underline{n}'(d) \\
=&\tilde{a}d^{\frac{1}{r+1}} + t_2 d - 2\tilde{a}\frac{1}{r+1}d^{\frac{1}{r+1}} - 2dt_2 \\
=&\tilde{a}\frac{r-1}{r+1}d^{\frac{1}{r+1}} - t_2 d \\
=&-\tilde{a}\frac{1-r}{r+1}d^{\frac{1}{r+1}} - t_2 d < 0
\end{aligned} \tag{80}$$

The last inequality is by $r < 1$, which is required when we first introduce r in Eq. (3). Dividing both sides of the inequality Eq. (79) and Eq. (80) by $\underline{n}(d)$, we verify both inequalities in Eq. (77).

With Eq. (79) and Eq. (80), we can further expand Eq. (78) as a function of the demand d:

$$\begin{aligned}
\frac{w_0\left(\frac{1}{d} - \frac{\underline{n}'(d)}{\underline{n}(d)}\right)}{2\underline{n}(d)\left(\frac{\underline{n}'(d)}{\underline{n}(d)} - \frac{1}{2d}\right)} &= \frac{w_0}{\underline{n}(d)}\cdot\frac{\tilde{a}\frac{r}{r+1}}{\tilde{a}\frac{1-r}{r+1} + t_2 d^{\frac{r}{r+1}}} \\
&= \frac{w_0}{\underline{n}(d)}\cdot\frac{\frac{\tilde{a}r}{t_2(r+1)}}{\frac{\tilde{a}(1-r)}{t_2(1+r)} + d^{\frac{r}{r+1}}}
\end{aligned}$$

That is, if the following inequality holds, then the function Eq. (70), $w(\underline{n}(d))/\bar{u}(d)$, is strictly decreasing in d:

$$\alpha < \frac{w_0}{\underline{n}(d)}\cdot\frac{\frac{\tilde{a}r}{t_2(r+1)}}{\frac{\tilde{a}(1-r)}{t_2(1+r)} + d^{\frac{r}{r+1}}} \tag{81}$$

Note that the right-hand side of Eq. (81) is strictly decreasing in d. Thus, Eq. (81) can also be viewed as an upper bound for the demand d to keep $w(\underline{n}(d))/\bar{u}(d)$ strictly decreasing.

Now consider Eq. (13). Under Eq. (13), we have that

$$\alpha < \frac{w_0}{\underline{n}(d^*_{HV,i})}\frac{\frac{\tilde{a}r}{t_2(r+1)}}{\frac{\tilde{a}(1-r)}{t_2(r+1)} + (d^*_{HV,i})^{\frac{r}{r+1}}},\ \text{for all } i$$

Since the right-hand side is strictly decreasing in $d^*_{HV,i}$, it is equivalent to

$$\alpha < \frac{w_0}{\underline{n}(d^*_{HV,I})} \frac{\frac{\tilde{a}r}{t_2(r+1)}}{\frac{\tilde{a}(1-r)}{t_2(r+1)} + (d^*_{HV,I})^{\frac{r}{r+1}}}$$

Thus, by Eq. (81), when Eq. (13) holds, it implies that the function $w(\underline{n}(d))/\bar{u}(d)$ is strictly decreasing for $0 < d \leq d^*_{HV,I}$. This concludes our proof for the second statement.

Therefore, under Eq. (13), the price under a monopoly, independent platform market, is strictly higher than that under a pure-HV market, as long as AVs and HVs serve the market together.

Now we consider the monopoly supplier's decision for the AV fleet size n_{AV} (and correspondingly, d_{AV}), given an AV capacity N. Given capacity N, in scenario i, the monopoly supplier solves the following problem:

$$\max_{d_{AV} \leq \bar{d}(N)} (p_{IPM,i} \cdot \bar{u}(d_{AV}) - c_v) \cdot \underline{n}(d_{AV}) \tag{82}$$

Now we have proved that under Eq. (65) and Eq. (66), the price term $p_{IPM,i}$ is strictly increasing in n_{AV}. Then the condition Eq. (66) can then be written as

$$p_{IPM,i} \cdot \bar{u}(d_{AV}) \geq c_v$$

which holds when d_{AV} is above a certain threshold.

Now we go back to the objective function Eq. (82). Then as long as the HV equilibrium condition (65) has a solution with $d_{HV} > 0$, we have the monopoly's objective Eq. (82) being strictly increasing in the AV fleet size n_{AV}. This is because, both the variable profit margin, $(p_{IPM,i} \cdot \bar{u}(d_{AV}) - c_v)$, and the quantity, n_{AV}, are strictly increasing in n_{AV}, when AVs and HVs coexist.

Therefore, when a monopoly supplier chooses an AV fleet size in scenario i while AVs and HVs coexist, the optimal solution is either the capacity N, or the highest AV fleet size that keeps AVs and HVs serving the market together (i.e. Eq. (65) has a solution). In either case, the resulting price is strictly higher than the pure-HV equilibrium price $p^*_{HV,i}$. That is,

$$p^*_{IPM,i} > p^*_{HV,i}$$

if the optimal AV fleet size falls into the range where AVs and HVs serve the market together.

2. *AVs serve the market alone.* In this case, the HV equilibrium condition (65) does not have a solution with $d_{HV} > 0$, but AVs can make a non-negative variable profit (i.e. Eq. (66) holds).

   Therefore, in this case, given capacity N, the monopoly AV solves the following problem:

$$\max_{d_{AV} \leq \bar{d}(N)} (p_i(d_{AV}) \cdot \bar{u}(d_{AV}) - c_v) \cdot \underline{n}(d_{AV}) = \max_{d_{AV} \leq \bar{d}(N)} p_i(d_{AV}) d_{AV} t_2 - c_v \underline{n}(d_{AV})$$

subject to the constrain that HVs do not participate:

$$p_i(d_{HV}+d_{AV})\bar{u}(d_{HV}) \le w(\underline{n}(d_{HV})) \text{ for any } d_{HV} \tag{83}$$

All else being equal, the left-hand side of Eq. (83) is strictly decreasing d_{AV}. Thus, Eq. (83) is a lower bound of d_{AV}. Then there are two possibilities:

(a) $p_i(d_{HV}+\bar{d}(N))\bar{u}(d_{HV}) > w(\underline{n}(d_{HV}))$ for some d_{HV}. In other words, in scenario i, even when the monopoly supplier uses up all of its capacity N, Eq. (83) still does not hold, and HVs will participate. Then this setting with AVs serving the market alone does not exist.

(b) $p_i(d_{HV}+\bar{d}(N))\bar{u}(d_{HV}) \le w(\underline{n}(d_{HV}))$ for all d_{HV}. In other words, in scenario i, AV can serve the market alone at least when AVs use up all of the capacity N. We show that in this case, the monopoly supplier never extends the demand served, d_{AV}, at or above the pure-HV equilibrium demand $d^*_{HV,i}$, which implies that the corresponding price is strictly lower than the pure-HV equilibrium price $p^*_{HV,i}$.

We prove this by leveraging Lemma 4 in the appendix. Lemma 4 states that the total variable profit function, $p_i(d_{AV})d_{AV}t_2 - c_v\underline{n}(d_{AV})$, is strictly decreasing in d_{AV} for $d_{AV} \ge d^*_{HV,i}$, where $d^*_{HV,i}$ is the equilibrium demand in a pure-HV market in scenario i. Then there are three possibilities:

i. $N < n^*_{HV,i}$. Then even if all AVs are dispatched, it must be true that the price is strictly lower than $p^*_{HV,i}$, because $d_{AV} < \bar{d}(N) < \bar{d}(n^*_{HV,i}) = d^*_{HV,i}$.

ii. $N \ge n^*_{HV,i}$, and $p_i(d_{HV}+\bar{d}(n^*_{HV,i}))\bar{u}(d_{HV}) \le w(\underline{n}(d_{HV}))$ for all d_{HV}. Then by Lemma 4, the monopoly supplier must choose an AV fleet size that satisfies:

$$n_{AV} < n^*_{HV,i}$$

This implies that

$$d_{AV} < d^*_{HV,i}$$

and

$$p_i(d_{AV}) > p_i(d^*_{HV,i}) = p^*_{HV,i}$$

iii. $N \ge n^*_{HV,i}$, and $p_i(d_{HV}+\bar{d}(n^*_{HV,i}))\bar{u}(d_{HV}) > w(\underline{n}(d_{HV}))$ for some d_{HV}. In this case, the total variable profit (82) is strictly decreasing for the entire support where AVs serve the market alone. Thus, the monopoly supplier chooses the AV fleet size at the boundary that exactly keeps the HVs out of the market, i.e. the AV fleet size $n_{AV,IPM,i}$ satisfies:

$$p_i(d_{HV}+\bar{d}(n_{AV,IPM,i}))\bar{u}(d_{HV}) = w(\underline{n}(d_{HV}))$$

and for any $n_{AV} > n_{AV,IPM,i}$, it holds that

$$p_i(d_{HV} + \bar{d}(n_{AV}))\bar{u}(d_{HV}) < w(\underline{n}(d_{HV})) \text{ for all } d_{HV}$$

for which the analysis in the coexisting case will apply. Thus, the price is also strictly higher than that in a pure-HV market.

Therefore, when AVs serve the market alone, under a monopoly, independent platform market, it must be true that the price $p^*_{IPM,i} > p^*_{HV,i}$ if the optimal fleet size falls into the region where AVs serve the market alone.

3. *HVs serve the market alone.* This is the only setting where the price is the same as in the pure-HV market. This happens when AVs cannot break even (i.e., the condition Eq. (66) cannot hold), while the HV equilibrium condition (65) has a solution with $d_{HV} > 0$. In this case, the monopoly AV supplier chooses not to dispatch any AV, i.e. $n_{AV,IPM,i} = 0$. The total variable profit for AVs is zero. And since AVs do not participate, the price is just identical to that under a pure-HV market, i.e. $p^*_{IPM,i} = p^*_{HV,i}$

This concludes our proof that in any scenario, the price under the monopoly independent platform market is no lower than that under a pure-HV market.

Step 2. Next, we show that there exists at least one scenario under which the price is strictly higher than the pure-HV market. We prove it by contradiction:

Suppose not. Then it must be true that in every demand scenario, $p^*_{HV,i} = p^*_{IPM,i}$. Among all the cases analyzed, the only setting where this can happen is Item 3, when AVs do not participate and HVs serve the market alone. If this is true for all scenarios, i.e.

$$n^*_{AV,IPM,i} = 0, \text{ for all } i$$

then the total aggregate net profit for the monopoly supplier with capacity N is just $(-c_f N)$, which is strictly negative. Contradiction. This concludes our proof that $p^*_{HV,i} < p^*_{IPM,i}$ holds true for at least some i.

D.2.2. Proof of Proposition 3, Item 2b This item states that when the AVs are perfectly competitive under an independent platform market and if Eq. (13) holds for all scenarios, then the price is higher than under the pure-HV market when AVs and HVs coexist. This is directly implied by the proof in Appendix D.2.1, Item 1.

The high-level idea is that when the condition Eq. (13) holds, the function $w(\underline{n}(d))/\bar{u}(d)$ is strictly decreasing in d. And when AVs and HVs coexist, the equilibrium price must be exactly $w(\underline{n}(d_{HV}))/\bar{u}(d_{HV})$, where d_{HV} is the equilibrium HV fleet size. Since the equilibrium HV fleet size d_{HV} decreases in the presence of an AV fleet, the price increases after the introduction of AVs.

D.3. Proof of Proposition 4

Proof. In this section, we prove the statement that in demand scenario i, under an independent platform market, if AVs and HVs serve the market together, then the price is strictly higher than that under a pure-HV market, providing that the function $w(\underline{n}(d_{HV}))/\bar{u}(d_{HV})$ is strictly decreasing in d_{HV} for $d \le d^*_{HV,i}$ (Item 1); moreover, the aforementioned condition is equivalent to

$$\alpha < \frac{w_0}{\underline{n}(d^*_{HV,i})} \frac{\frac{\tilde{a}r}{t_2(r+1)}}{\frac{\tilde{a}(1-r)}{t_2(r+1)} + (d^*_{HV,i})^{\frac{r}{r+1}}} \tag{84}$$

where $d^*_{HV,i}$ represents the equilibrium demand in a pure-HV market in scenario i (Item 2).

We then prove the two items, respectively. Note that a large portion of the proof also appears in the proof for Proposition 3 because Proposition 4 is an intermediate step for establishing the results in Proposition 3.

D.3.1. Proof of Proposition 4, Item 1 We first show that $d_{HV,IPM,i}$, which represents the equilibrium demand served by HVs under the independent platform market when AVs and HVs serve the market together, is strictly lower than $d^*_{HV,i}$, which represents the equilibrium demand served by HVs under a pure-HV market. Then we compare the prices under the two settings.

First, by definition, the prices in the two settings are given by

$$p_{IPM,i} = p_i(d_{HV,IPM,i} + d_{AV}) = \frac{w(\underline{n}(d_{HV,IPM,i}))}{\bar{u}(d_{HV,IPM,i})}$$

for the independent platform market when AVs and HVs coexist, and

$$p^*_{HV,i} = \frac{w(\underline{n}(d^*_{HV,i}))}{\bar{u}(d^*_{HV,i})}$$

for the pure-HV market.

Next, we prove the statement that $d_{HV,IPM,i} < d^*_{HV,i}$ by contradiction: Suppose not. Then

$$d_{HV,IPM,i} \ge d^*_{HV,i}$$

Then since $d^*_{HV,i}$ is an equilibrium point, for any $d > d^*_{HV,i}$, it must hold that

$$p_i(d)\bar{u}(d) < w(\underline{n}(d))$$

Since $d_{HV,IPM,i} \ge d^*_{HV,i}$, it means that

$$p_i(d_{HV,IPM,i})\bar{u}(d_{HV,IPM,i}) \le w(\underline{n}(d^*_{HV,IPM,i}))$$

Then for $d_{AV} > 0$, by the monotonicity of the price function $p_i(d)$, it must be true that

$$p_i(d_{AV} + d^*_{HV,IPM,i})\bar{u}(d^*_{HV,IPM,i}) < p_i(d^*_{HV,IPM,i})\bar{u}(d^*_{HV,IPM,i}) \le w(\underline{n}(d^*_{HV,IPM,i})) \tag{85}$$

But this contradicts the definition of $d^*_{HV,IPM,i}$, which requires the left-hand side and the right-hand side of Eq. (85) to be equal. This concludes our proof for $d_{HV,IPM,i} < d^*_{HV,i}$.

Now assume that $w(\underline{n}(d_{HV}))/\bar{u}(d_{HV})$ is strictly decreasing in d_{HV} when $d_{HV} \leq d^*_{HV,i}$. Then since $d_{HV,IPM,i} < d^*_{HV,i}$, it must be true that

$$\frac{w(\underline{n}(d_{HV,IPM,i}))}{\bar{u}(d_{HV,IPM,i})} > \frac{w(\underline{n}(d^*_{HV,i}))}{\bar{u}(d^*_{HV,i})}$$

which by the definition of $d_{HV,IPM,i}$ and $d^*_{HV,i}$ is equivalent to

$$p_{IPM,i} > p^*_{HV,i}$$

This concludes our proof of Item 1.

D.3.2. Proof of Proposition 4, Item 2 We take the first-order derivative of the function $w(\underline{n}(d_{HV}))/\bar{u}(d_{HV})$ and show that it is strictly negative for $d \leq d^*_{HV,i}$ if and only if

$$\alpha < \frac{w_0}{\underline{n}(d^*_{HV,i})} \frac{\frac{\tilde{a}r}{t_2(r+1)}}{\frac{\tilde{a}(1-r)}{t_2(r+1)} + (d^*_{HV,i})^{\frac{r}{r+1}}}$$

The first-order derivative is given by

$$\frac{\frac{\partial w}{\partial d}\bar{u}(d) - w(\underline{n}(d))\frac{\partial \bar{u}}{\partial d}}{\bar{u}(d)^2}$$

Thus, the sign of the first-order derivative is the same as its numerator:

$$\frac{\frac{\partial w}{\partial d}\bar{u}(d) - w(\underline{n}(d))\frac{\partial \bar{u}}{\partial d}}{\bar{u}(d)^2} < 0 \Leftrightarrow \frac{\partial w}{\partial d}\bar{u}(d) - w(\underline{n}(d))\frac{\partial \bar{u}}{\partial d} < 0$$

The numerator can further be expanded as

$$\begin{aligned}
&\frac{\partial w}{\partial d}\bar{u}(d) - w(\underline{n}(d))\frac{\partial \bar{u}}{\partial d} \\
=&\frac{\partial w}{\partial \underline{n}}\frac{\partial \underline{n}}{\partial d}\bar{u}(d) - w(\underline{n}(d))\frac{\partial \bar{u}}{\partial d} \\
=&\alpha \underline{n}'(d)\bar{u}(d) - w(\underline{n}(d))\bar{u}'(d) \\
=&\bar{u}(d) \cdot w(\underline{n}(d)) \cdot \left(\frac{\alpha \underline{n}'(d)}{w(\underline{n}(d))} - \frac{\bar{u}'(d)}{\bar{u}(d)}\right)
\end{aligned}$$

Thus,

$$\frac{\frac{\partial w}{\partial d}\bar{u}(d) - w(\underline{n}(d))\frac{\partial \bar{u}}{\partial d}}{\bar{u}(d)^2} < 0 \Leftrightarrow \frac{\alpha \underline{n}'(d)}{w(\underline{n}(d))} - \frac{\bar{u}'(d)}{\bar{u}(d)} < 0$$

We can further compute the term $(\frac{\alpha \underline{n}'(d)}{w(\underline{n}(d))} - \frac{\bar{u}'(d)}{\bar{u}(d)})$. We first restate the definition of $\underline{n}(d)$ and $\bar{u}(d)$ in Eq. (8) and Eq. (16), respectively:

$$\underline{n}(d) = \tilde{a}d^{1/(r+1)} + t_2 d$$

$$\bar{u}(d) = \frac{dt_2}{\underline{n}(d)} = \frac{dt_2}{\tilde{a}d^{1/(r+1)} + t_2 d}$$

Thus,

$$\bar{u}'(d) = \frac{t_2 \cdot \underline{n}(d) - dt_2 \cdot \underline{n}'(d)}{\underline{n}(d)^2}$$

which implies that

$$\frac{\bar{u}'(d)}{\bar{u}(d)} = \frac{t_2 \cdot \underline{n}(d) - dt_2 \cdot \underline{n}'(d)}{dt_2 \cdot \underline{n}(d)} = \frac{1}{d} - \frac{\underline{n}'(d)}{\underline{n}(d)}$$

Therefore,

$$\frac{\frac{\partial w}{\partial d}\bar{u}(d) - w(\underline{n}(d))\frac{\partial \bar{u}}{\partial d}}{\bar{u}(d)^2} < 0 \Leftrightarrow \frac{\alpha \underline{n}'(d)}{w(\underline{n}(d))} - \frac{\bar{u}'(d)}{\bar{u}(d)} < 0 \Leftrightarrow \frac{\alpha \underline{n}'(d)}{w(\underline{n}(d))} - (\frac{1}{d} - \frac{\underline{n}'(d)}{\underline{n}(d)}) < 0$$

We can further simplify the terms in the rightmost inequality:

$$\begin{aligned}
&\frac{\alpha \underline{n}'(d)}{w(\underline{n}(d))} - \frac{\bar{u}'(d)}{\bar{u}(d)} \\
=&\frac{\alpha \underline{n}'(d)}{w(\underline{n}(d))} - (\frac{1}{d} - \frac{\underline{n}'(d)}{\underline{n}(d)}) \\
=&\frac{\alpha \underline{n}'(d)}{w_0 + \alpha \underline{n}(d)} - \frac{1}{d} + \frac{\underline{n}'(d)}{\underline{n}(d)} \\
=&\frac{\underline{n}'(d)}{w_0/\alpha + \underline{n}(d)} - \frac{1}{d} + \frac{\underline{n}'(d)}{\underline{n}(d)}
\end{aligned}$$

Therefore,

$$\frac{\frac{\partial w}{\partial d}\bar{u}(d) - w(\underline{n}(d))\frac{\partial \bar{u}}{\partial d}}{\bar{u}(d)^2} < 0 \Leftrightarrow \frac{\underline{n}'(d)}{w_0/\alpha + \underline{n}(d)} < \frac{1}{d} - \frac{\underline{n}'(d)}{\underline{n}(d)} \tag{86}$$

For now, we will assume that

$$\frac{1}{2d} < \frac{\underline{n}'(d)}{\underline{n}(d)} < \frac{1}{d} \tag{87}$$

Later we will formally prove that the above inequality indeed holds. Then we can write Eq. (86) as

$$\begin{aligned}
&\frac{\underline{n}'(d)}{w_0/\alpha + \underline{n}(d)} < \frac{1}{d} - \frac{\underline{n}'(d)}{\underline{n}(d)} \\
\Leftrightarrow&\frac{\underline{n}'(d)}{\frac{1}{d} - \frac{\underline{n}'(d)}{\underline{n}(d)}} < \frac{w_0}{\alpha} + \underline{n}(d) \\
\Leftrightarrow&\frac{\underline{n}'(d) - \frac{\underline{n}(d)}{d} + \underline{n}'(d)}{\frac{1}{d} - \frac{\underline{n}'(d)}{\underline{n}(d)}} < \frac{w_0}{\alpha} \\
\Leftrightarrow&\frac{(2\frac{\underline{n}'(d)}{\underline{n}(d)} - \frac{1}{d}) \cdot \underline{n}(d)}{\frac{1}{d} - \frac{\underline{n}'(d)}{\underline{n}(d)}} < \frac{w_0}{\alpha}
\end{aligned}$$

We can then rearrange the terms and obtain

$$\frac{\alpha \underline{n}'(d)}{w(\underline{n}(d))} - \frac{\bar{u}'(d)}{\bar{u}(d)} \Leftrightarrow \alpha < \frac{w_0(\frac{1}{d} - \frac{\underline{n}'(d)}{\underline{n}(d)})}{2\underline{n}(d)(\frac{\underline{n}'(d)}{\underline{n}(d)} - \frac{1}{2d})}$$

Thus, the condition that $\alpha < \frac{w_0(\frac{1}{d} - \frac{\underline{n}'(d)}{\underline{n}(d)})}{2\underline{n}(d)(\frac{\underline{n}'(d)}{\underline{n}(d)} - \frac{1}{2d})}$ is a sufficient and necessary condition for the function $w(\underline{n}(d_{HV}))/\bar{u}(d_{HV})$ being strictly decreasing in d_{HV}. We now verify Eq. (87). This requires the calculation of $\underline{n}'(d)$, which is given by

$$\underline{n}'(d) = \tilde{a}\frac{1}{r+1}d^{\frac{1}{r+1}-1} + t_2$$

Thus,

$$\begin{aligned}
&\underline{n}(d) - d\cdot\underline{n}'(d) \\
=&\tilde{a}d^{\frac{1}{r+1}} + t_2 d - \tilde{a}\frac{1}{r+1}d^{\frac{1}{r+1}} - dt_2 \\
=&\tilde{a}\frac{r}{r+1}d^{\frac{1}{r+1}} > 0
\end{aligned}$$

Similarly,

$$\begin{aligned}
&\underline{n}(d) - 2d\cdot\underline{n}'(d) \\
=&\tilde{a}d^{\frac{1}{r+1}} + t_2 d - 2\tilde{a}\frac{1}{r+1}d^{\frac{1}{r+1}} - 2dt_2 \\
=&\tilde{a}\frac{r-1}{r+1}d^{\frac{1}{r+1}} - t_2 d \\
=&-\tilde{a}\frac{1-r}{r+1}d^{\frac{1}{r+1}} - t_2 d < 0
\end{aligned}$$

The last inequality is by $r < 1$, which is required when we first introduce r in Eq. (3). Dividing both sides of the above two inequalities by $\underline{n}(d)$, we verify both inequalities in Eq. (87).

We can then further expand the sufficient and necessary condition:

$$\begin{aligned}
\alpha < \frac{w_0(\frac{1}{d} - \frac{\underline{n}'(d)}{\underline{n}(d)})}{2\underline{n}(d)(\frac{\underline{n}'(d)}{\underline{n}(d)} - \frac{1}{2d})} &\Leftrightarrow \alpha < \frac{w_0}{\underline{n}(d)}\cdot\frac{\tilde{a}\frac{r}{r+1}}{\tilde{a}\frac{1-r}{r+1} + t_2 d^{\frac{r}{r+1}}} \\
&\Leftrightarrow \alpha < \frac{w_0}{\underline{n}(d)}\cdot\frac{\frac{\tilde{a}r}{t_2(r+1)}}{\frac{\tilde{a}(1-r)}{t_2(1+r)} + d^{\frac{r}{r+1}}}
\end{aligned}$$

Thus, the function $w(\underline{n}(d_{HV}))/\bar{u}(d_{HV})$ being strictly decreasing at d_{HV} is equivalent to

$$\alpha < \frac{w_0}{\underline{n}(d)}\cdot\frac{\frac{\tilde{a}r}{t_2(r+1)}}{\frac{\tilde{a}(1-r)}{t_2(1+r)} + d^{\frac{r}{r+1}}}$$

Moreover, the right-hand side of the above inequality is strictly decreasing in d_{HV}. Thus, when the above inequality holds at $d_{HV} = d^*_{HV,i}$, it must be true that it also holds for all $d_{HV} \le d^*_{HV,i}$. This proves the sufficiency of the condition Eq. (84). Moreover, this condition must also be necessary, otherwise the function $w(\underline{n}(d_{HV}))/\bar{u}(d_{HV})$ will not be strictly decreasing at $d_{HV} = d^*_{HV,i}$. $\square$

E. Perfect supply elasticity and some density effect (Proof of Proposition 5)

Proposition 5 describes how the prices under the two dispatch platform designs (common, independent) and two levels of AV competition (monopoly, perfect competition) compare with a pure-HV market. We prove Proposition 5 by analyzing the market outcomes in each market configuration. Since the proof is quite lengthy, we break it down into several parts.

Appendix E.1 contains the analysis of the common platform market. Appendix E.1.1 contains an intermediate analysis of the equilibrium outcomes (prices, utilization rates, earnings) as a function of an exogenous AV fleet size, denoted by n_{AV}. Appendix E.1.2 analyzes the AV fleet size choice in a demand scenario for a monopoly supplier. Appendix E.1.3 analyzes the AV capacity choice of a monopoly supplier. The above propositions fully characterize the optimal decision of a monopoly supplier and show that the prices are identical to those under a pure-HV market. Appendix E.1.4 leverages this result and compares the equilibrium prices under perfectly competitive AVs with a monopoly AV to show that perfectly competitive AVs lead to equal or lower prices than a pure-HV market.

Appendix E.2 contains the analysis of the independent platform market. Similar to the common platform market setting, we start with Appendix E.2.1, which analyzes the market outcomes for an exogenously given AV fleet size n_{AV}. Appendix E.2.2 and Appendix E.2.3 analyzes the market outcome of the fleet sizing and capacity decisions made by a monopoly supplier and by perfectly competitive AVs, respectively.

Combining the analysis of all four cases, we conclude the proof of Proposition 5.

E.1. Common platform market

In this section, we analyze the equilibrium outcomes under a common platform market.

E.1.1. Exogenous AV fleet size (Proof of Proposition 6) Consider scenario i and an AV fleet size n_{AV}. Recall that $n^*_{HV,i}$ represents the equilibrium HV fleet size under a pure-HV market. Then, there are two possibilities:

1. $n_{AV} < n^*_{HV,i}$. Suppose HVs do not participate. Then the expected earnings per AV solely depend on n_{AV}, which is given by the product of the price and utilization:

$$p_i(d_{AV})\bar{u}(d_{AV}), \text{ where } \underline{n}(d_{AV}) = n_{AV}$$

Then it must be true that

$$p_i(d_{AV})\bar{u}(d_{AV}) > p_i(d^*_{HV,i})\bar{u}(d^*_{HV,i}) = w_0, \text{ where } \underline{n}(d^*_{HV,i}) = n^*_{HV,i}$$

Otherwise, in a pure-HV market, HVs will stop entering the market before the HV fleet size reaches $n^*_{HV,i}$. Therefore, when AVs serve the market alone and the AV fleet size $n_{AV} < n^*_{HV,i}$, the expected earnings of a marginal HV to participate in the market is strictly above w_0, which will incentivize HVs to participate.

Thus, it must be true that HVs participate and serve the demand together with AVs. Moreover, in equilibrium, the demand served by AVs and HVs, d_{AV} and d_{HV}, must satisfy that

$$p_i(d_{HV}+d_{AV})\cdot u_{HV}=w_0 \tag{88}$$

where u_{HV} represents the HV fleet utilization. Since AVs and HVs share the same dispatch platform, the utilization is a result of the total demand served by AVs and HVs. That is,

$$u_{HV}=\bar{u}(d_{HV}+d_{AV}) \tag{89}$$

Combining Eq. (88) and Eq. (89), the following must hold in equilibrium:

$$p_i(d_{HV}+d_{AV})\cdot\bar{u}(d_{HV}+d_{AV})=w_0$$

But this is the same as the pure-HV equilibrium condition (10) by replacing the demand served by HVs (d_{HV}) with the total demand served by HVs and AVs ($d_{HV}+d_{AV}$). Therefore, the total equilibrium fleet size, demand, utilization, and price are identical to that under a pure-HV market.

Moreover, the variable profit per AV is then given by

$$p_i(d_{AV}+d_{HV})\bar{u}(d_{AV}+d_{HV})-c_v=w_0-c_v$$

This concludes our proof of Item 1 in Proposition 6.

2. $n_{AV}\geq n^*_{HV,i}$. Suppose HVs do not participate. Then the expected earnings per AV is again the product of the price and utilization and satisfies:

$$p_i(d_{AV})\bar{u}(d_{AV})\leq p_i(d^*_{HV,i})\bar{u}(d^*_{HV,i})=w_0$$

In other words, HVs cannot make w_0 by participating in the ride-hailing market and therefore do not participate. Moreover, the variable profit per AV is a function of the demand served by AVs. That is,

$$p_i(d_{AV})\bar{u}(d_{AV})-c_v\leq w_0-c_v$$

where $\underline{n}(d_{AV})=n_{AV}$. The total variable profit of a monopoly AV supplier is then given by

$$(p_i(d_{AV})\bar{u}(d_{AV})-c_v)n_{AV}=p_i(d_{AV})d_{AV}t_2-c_vn_{AV} \tag{90}$$

where $\underline{n}(d_{AV})=n_{AV}$. By Item 2 of Lemma 1, we can also write d_{AV} based on the inverse function of $\underline{n}(\cdot)$, that is, $d_{AV}=\bar{d}(n_{AV})$, which makes Eq. (90) a function of n_{AV}. This concludes our proof for Item 2 of Proposition 6.

As a summary, in scenario i, for a common platform market, the variable profit per AV is given by

$$\pi_i^C(n_{AV})=\begin{cases} w_0-c_v, & n_{AV}<n^*_{HV,i} \\ p_i(\bar{d}(n_{AV}))\bar{u}(\bar{d}(n_{AV}))-c_v, & n_{AV}\geq n^*_{HV,i}\end{cases} \tag{91}$$

E.1.2. Optimal AV fleet size for a monopoly supplier (Proof of Proposition 7) In this section, we consider a monopoly AV supplier. Given scenario i and an exogenous AV capacity N, we analyze the optimal AV fleet size that maximizes the monopoly supplier's total variable profit in scenario i. Recall that $n^*_{HV,i}$ represents the equilibrium HV fleet size in scenario i in a pure-HV market. Then there are two cases:

1. $N < n^*_{HV,i}$. Here we leverage the results in Proposition 6. Since the AV fleet size n_{AV} cannot go above the AV capacity, any choice of n_{AV} falls into Item 1 of Proposition 6. Thus, the variable profit per AV is constant at $(w_0 - c_v)$. The total variable profit of all AVs is then given by

$$(w_0 - c_v)n_{AV}$$

which is strictly increasing in the AV fleet size n_{AV}. Thus, it is optimal to let the AV fleet size as large as possible. That is, the monopoly supplier will choose $n_{AV} = N$. Furthermore, since N does not exceed $n^*_{HV,i}$, there will be leftover demand for HVs; HVs will continue to join until the total fleet size of HVs and AVs is exactly at $n^*_{HV,i}$, as analyzed in the proof of Item 1 of Proposition 6. This concludes our proof for Item 1 of Proposition 7.

2. $N \geq n^*_{HV,i}$. In this case, the monopoly supplier has two types of strategies in choosing the AV fleet size n_{AV}:
   (a) $n_{AV} \leq n^*_{HV,i}$. As shown in Item 1, the dominating strategy is $n_{AV} = n^*_{HV,i}$;
   (b) $n_{AV} > n^*_{HV,i}$. By Proposition 6, AVs serve the market alone, and the monopoly supplier's total variable profit is a function of the AV fleet size n_{AV} (Eq. (90)).

   We show that under Assumption 2, the total variable profit of the monopoly AV supplier, Eq. (90), is strictly decreasing in the AV fleet size n_{AV} when $n_{AV} > n^*_{HV,i}$. We formally present this result in Lemma 4 and provide its proof right after the statement of the lemma.

Lemma 4. *Suppose Assumption 2 holds. Consider the total variable profit of a monopoly AV supplier* (90)*,* $p_i(d_{AV})d_{AV}t_2 - c_v n_{AV}$. *In scenario* i*, if* $n_{AV} \geq n^*_{HV,i}$*, then Eq.* (90) *is a strictly decreasing function in the AV fleet size* n_{AV}.

Proof. First, we rewrite the total variable profit (90) as a function of the demand served by AVs, d_{AV}, and prove the monotonicity for $d_{AV} \geq d^*_{HV,i}$, where $d^*_{HV,i}$ is the equilibrium demand in the pure-HV market defined in Proposition 1 and $n^*_{HV,i} = \underline{n}(d^*_{HV,i})$. This is equivalent to the statement in Lemma 4 because when AVs serve the market alone, there is a one-one correspondence between the demand d_{AV} and the AV fleet size n_{AV}, given by $n_{AV} = \underline{n}(d_{AV})$. Eq. (90) is then equivalent to

$$p_i(d_{AV})d_{AV}t_2 - c_v\underline{n}(d_{AV})$$

which can be further written as

$$r_i(d_{AV}) - c_v \underline{n}(d_{AV}) \tag{92}$$

Then the first-order derivative of Eq. (92) is given by

$$r_i'(d_{AV}) - c_v \underline{n}'(d_{AV})$$

Note that the revenue function, $r_i(d_{AV})$, is strictly concave in d_{AV}. Thus,

$$d_{AV} > d^*_{HV,i} \Leftrightarrow r_i'(d_{AV}) < r_i'(d^*_{HV,i}) \tag{93}$$

Furthermore, Assumption 2 requires that

$$p^*_{HV,i} \leq \arg\max_p r_i(d_i(p))$$

which is equivalent to

$$d^*_{HV,i} \geq \arg\max_d r_i(d)$$

Combined with Eq. (93), it implies that

$$d_{AV} > d^*_{HV,i} \geq \arg\max_d r_i(d)$$

and therefore

$$r_i'(d_{AV}) < r_i'(d^*_{HV,i}) \leq 0$$

The last inequality is because the revenue-maximizing demand, $\arg\max_d r_i(d)$, must satisfy $r_i'(d) = 0$. Since $\underline{n}(d_{AV})$ is a strictly increasing function in d_{AV} (Lemma 1), $\underline{n}'(d_{AV}) > 0$. Hence, when $d_{AV} = d^*_{HV,i}$, we have

$$r_i'(d^*_{HV,i}) - c_v \underline{n}'(d^*_{HV,i}) < 0$$

which by concavity implies that for $d_{AV} > d^*_{HV,i}$

$$r_i'(d_{AV}) - c_v \underline{n}'(d_{AV}) < 0$$

which confirms that the total variable profit Eq. (90) is strictly decreasing in d_{AV} when $d_{AV} \geq d^*_{HV,i}$. □

With Lemma 4, we confirm that the monopoly supplier does not have an incentive to choose an AV fleet size $n_{AV} > n^*_{HV,i}$. Thus, in the case of the capacity $N > n^*_{HV,i}$, it is optimal for the AV supplier to choose the AV fleet size $n_{AV} = n^*_{HV,i}$. This concludes our proof for Item 2 of Proposition 7.

E.1.3. Optimal AV capacity for a monopoly supplier (Proof of Proposition 8) In this section, we prove that the optimal solution N^* to the monopoly supplier's capacity choice problem Eq. (14) is given by Eq. (15) in Proposition 8. The proof takes two steps.

First, we show that the optimal AV capacity N^* must be one of the equilibrium HV fleet size in the pure-HV market, i.e. $N^* = n^*_{HV,K}$ for some K in $\{1,2,...,I\}$. The reason is the following. Consider any arbitrary AV capacity N. Then there are three possibilities:

1. $N < n^*_{HV,1}$. In other words, the AV capacity N is smaller than the equilibrium HV fleet size in the pure-HV market in any scenario, i.e. $N < n^*_{HV,i}$ for all i. As shown by Proposition 7, in this case, the monopoly supplier chooses to dispatch all N AVs in every demand scenario and the variable profit per AV is a constant, $(w_0 - c_v)$. Therefore, the monopoly supplier's total net profit (14) is given by

$$\Pi(N) = \sum_{1}^{I} P(m_i)(w_0 - c_v)N - c_f N = (w_0 - c_v - c_f)N. \tag{94}$$

where we write the net profit Eq. (14) as a function of the AV capacity N and denote it as $\Pi(N)$. Since $w_0 > c_v + c_f$ (Assumption 1), Eq. (94) is strictly increasing in N. Therefore, any AV capacity $N < n^*_{HV,1}$ is strictly dominated by $n^*_{HV,1}$ and cannot be the optimal AV capacity.

2. $N > n^*_{HV,N}$. In other words, the AV capacity N is larger than the equilibrium HV fleet size in the pure-HV market in any scenario, i.e. $N > n^*_{HV,i}$ for all i. Then again, as shown by Proposition 7, in this case, the monopoly supplier chooses to dispatch exactly $n^*_{HV,i}$ in scenario i. Therefore, the monopoly supplier's total net profit (14) is given by

$$\Pi(N) = \sum_{1}^{I} P(m_i)(w_0 - c_v)n^*_{HV,i} - c_f N \tag{95}$$

Eq. (95) is strictly decreasing in N. Therefore, any AV capacity $N > n^*_{HV,N}$ is strictly dominated by $n^*_{HV,N}$ and cannot be the optimal AV capacity.

3. $n^*_{HV,1} \le N \le n^*_{HV,I}$. Then there must exist an index K, $1 \le K \le I-1$, such that

$$n^*_{HV,K} \le N < n_{HV,K+1}$$

Then the monopoly supplier's total net profit (14) is given by

$$\Pi(N) = \sum_{1}^{K} P(m_i)(w_0 - c_v)n^*_{HV,i} + \sum_{K+1}^{I} P(m_i)(w_0 - c_v)N - c_f N \tag{96}$$

Eq. (96) is linear in the AV capacity N. Thus, it is maximized at the endpoints of the interval, i.e. either $n^*_{HV,K}$ or $n^*_{HV,K+1}$.

Then finding the optimal AV capacity N^* that maximizes Eq. (14) is equivalent to finding the optimal K. Let $N = n^*_{HV,K}$ and we can further write Eq. (96) as the following:

$$\Pi(n^*_{HV,K}) = \sum_{1}^{K} P(m_i)(w_0 - c_v)n^*_{HV,i} + \sum_{K+1}^{I} P(m_i)(w_0 - c_v)n^*_{HV,K} - c_f n^*_{HV,K} \tag{97}$$

Similarly, let $N = n^*_{HV,K+1}$ and Eq. (96) is given by

$$\Pi(n^*_{HV,K+1}) = \sum_{1}^{K+1} P(m_i)(w_0 - c_v)n^*_{HV,i} + \sum_{K+2}^{I} P(m_i)(w_0 - c_v)n^*_{HV,K+1} - c_f n^*_{HV,K+1} \tag{98}$$

Deducting Eq. (98) by Eq. (97) gives

$$P(m_{K+1})(w_0 - c_v)n^*_{HV,K+1} + \sum_{K+1}^{I} P(m_i)(w_0 - c_v)(n^*_{HV,K+1} - n^*_{HV,K}) \\ - P(m_{K+1})(w_0 - c_v)n^*_{HV,K+1} - c_f(n^*_{HV,K+1} - n^*_{HV,K})$$

which can be consolidated into a function of $(n^*_{HV,K+1} - n^*_{HV,K})$:

$$\Pi(n^*_{HV,K+1}) - \Pi(n^*_{HV,K}) = (\sum_{K+1}^{I} P(m_i)(w_0 - c_v) - c_f) \cdot (n^*_{HV,K+1} - n^*_{HV,K})$$

Note that $n^*_{HV,K+1} - n^*_{HV,K} > 0$ holds for all K (Item 4, Proposition 1). Thus, if the optimal AV capacity $N^* = n^*_{HV,K}$, one necessary condition is that

$$\Pi(n^*_{HV,K+1}) - \Pi(n^*_{HV,K}) < 0 \Leftrightarrow \sum_{K+1}^{I} P(m_i)(w_0 - c_v) - c_f < 0$$

Another necessary condition is that

$$\Pi(n^*_{HV,K}) - \Pi(n^*_{HV,K-1}) \geq 0 \Leftrightarrow \sum_{K}^{I} P(m_i)(w_0 - c_v) - c_f \geq 0$$

Combining the two conditions gives

$$\sum_{K+1}^{I} P(m_i) < \frac{c_f}{w_0 - c_v} \leq \sum_{K}^{I} P(m_i) \tag{99}$$

It turns out Eq. (99) is also a sufficient condition. This is because the term, $\sum_K^I P(m_i)$, is strictly decreasing in K. Thus, if there is a K that satisfies Eq. (99), it must be unique.

Moreover, a K that satisfies Eq. (99) must always exist. This is because the critical fractile, $c_f/(w_0 - c_v)$, by Assumption 1, is between 0 and 1, which means

$$\sum_{K=I}^{I} P(m_i) = 0 < \frac{c_f}{w_0 - c_v} < 1 = \sum_{K=1}^{I} P(m_i)$$

Thus, $(\sum_{K+1}^I P(m_i), \sum_K^I P(m_i)]$, $K = 1, ..., I-1$, covers all possible values of $c_f/(w_0 - c_v)$.

Hence, the optimal AV capacity is $n^*_{HV,K}$, where K is the smallest index i such that $\sum_{K+1}^I < c_f/(w_0 - c_v)$, which confirms Eq. (15).

E.1.4. Equilibrium price for perfectly competitive AVs (Proof of Proposition 9) In this section, we prove that under a common platform market and perfectly competitive AVs, the price in any scenario is no higher than under a monopoly AV supplier, and there exists at least one scenario where the price is strictly lower than under a monopoly supplier. That is, if we denote the equilibrium price in scenario i under a monopoly AV supplier and perfectly competitive AVs as $p^*_{mono,i}$ and $p^*_{comp,i}$, respectively, and recall that $p^*_{HV,i}$ represents the equilibrium price under a pure-HV market, then it must hold that

$$p^*_{HV,i} = p^*_{mono,i} \geq p^*_{comp,i} \text{ for all } i$$

and

$$p^*_{HV,i} = p^*_{mono,i} > p^*_{comp,i} \text{ for at least some } i$$

We first characterize the equilibrium condition under this market configuration. Denote the equilibrium AV capacity under perfect competition as N'. Then it must satisfies the equilibriuim condition Eq. (6), which we restate here:

$$\sum_{1}^{I} P(m_i) \max\{\pi_i^C(N'), 0\} = c_f$$

The function $\pi_i^C(\cdot)$, which is the variable profit per AV when an AV fleet size of n_{AV} AVs participate in scenario i, is given by Eq. (91), which we also restate here:

$$\pi_i^C(n_{AV}) = \begin{cases} w_0 - c_v, & n_{AV} < n^*_{HV,i} \\ p_i(\bar{d}(n_{AV}))\bar{u}(\bar{d}(n_{AV})) - c_v, & n_{AV} \geq n^*_{HV,i} \end{cases} \tag{100}$$

Next, given the equilibrium AV capacity N', for any scenario i, there are two possibilities:

1. $N' \leq n^*_{HV,i}$. Then by Proposition 6, all N' AVs participate and $(n^*_{HV,i} - N')$ HVs participate. The equilibrium price is identical to the pure-HV market. That is, $p^*_{comp,i} = p^*_{HV,i}$.
2. $N' > n^*_{HV,i}$. In this case, AVs must serve the market alone (i.e. the AV fleet size $n_{AV} > n^*_{HV,i}$); otherwise, the variable profit per AV, $(w_0 - c_v)$, is strictly positive, which will attract more AVs to participate. As suggested by Eq. (100), in this case, the variable profit per AV is then $(p_i(\bar{d}(n_{AV}))\bar{u}(\bar{d}(n_{AV})) - c_v)$ with an AV fleet size of n_{AV}. Within this setting, there are two subcases:
   (a) $p_i(\bar{d}(N'))\bar{u}(\bar{d}(N')) \geq c_v$. That is, the variable profit per AV, $\pi_i^C(N')$, is non-negative, when all N' AVs participate. Thus, all AVs participate, i.e., $n_{AV} = N'$. Moreover,

   $$c_v \leq p_i(\bar{d}(N'))\bar{u}(\bar{d}(N')) < w_0$$

   (b) $p_i(\bar{d}(N'))\bar{u}(\bar{d}(N')) < c_v$. That is, the variable profit per AV, $\pi_i^C(N')$, is strictly negative, when all N' AVs participate. Then AVs participate up to the point where the variable profit per AV is exactly zero. That is, the AV fleet size n_{AV} satisfies

   $$p_i(\bar{d}(n_{AV}))\bar{u}(\bar{d}(n_{AV})) = c_v < w_0$$

In both cases, the AV fleet size n_{AV} is strictly higher than $n^*_{HV,i}$, implying strictly more demand served and lower prices. Therefore, $p^*_{comp,i} < p^*_{HV,i}$.

Combining Item 1 and Item 2, we have shown that $p^*_{comp,i} \leq p^*_{HV,i}$ for all i. It remains to be shown that strict inequality holds for at least some scenarios. In other words, we need to show that the case of Item 2 above always exists. We prove it by contradiction:

Suppose not. Then the equilibrium AV capacity $N' \leq n^*_{HV,i}$ for all i. But this will imply that the variable profit per AV, $\pi_i^C(N') = w_0 - c_v > 0$, for all i. This further implies that the aggregate variable profit per AV satisfies:

$$\sum_{1}^{I} P(m_i)(w_0 - c_v) = w_0 - c_v > c_f$$

which contradicts the equilibrium condition (6).

Therefore, there must exist at least one scenario i, such that $N' > n^*_{HV,i}$. In this demand scenario, $p^*_{comp,i} < p^*_{HV,i}$. This concludes our proof.

E.2. Independent platform market

In this section, we consider the independent platform market. Similar to the analysis for the common platform market in Appendix E.1, before we analyze the capacity choice, we first analyze the market outcome under an exogenously given AV fleet size n_{AV} and demand scenario i.

E.2.1. Exogenous AV fleet size Consider scenario i. Suppose HVs serve a demand level of d_{HV} and AVs serve a demand level of d_{AV}. The total ride-hours supplied by AVs and HVs is just $(d_{HV} + d_{AV})$. The market price is then a function of d_{AV} and d_{HV}, given by

$$p = p_i(d_{HV} + d_{AV})$$

Thus, the total HV revenue is given by

$$R_i(d_{HV}; d_{AV}) = p_i(d_{HV} + d_{AV})d_{HV}t_2$$

$R_i(d_{HV}; d_{AV})$ is defined in Definition 2 and represents the HV revenue curve when AVs serve d_{AV} level of demand.

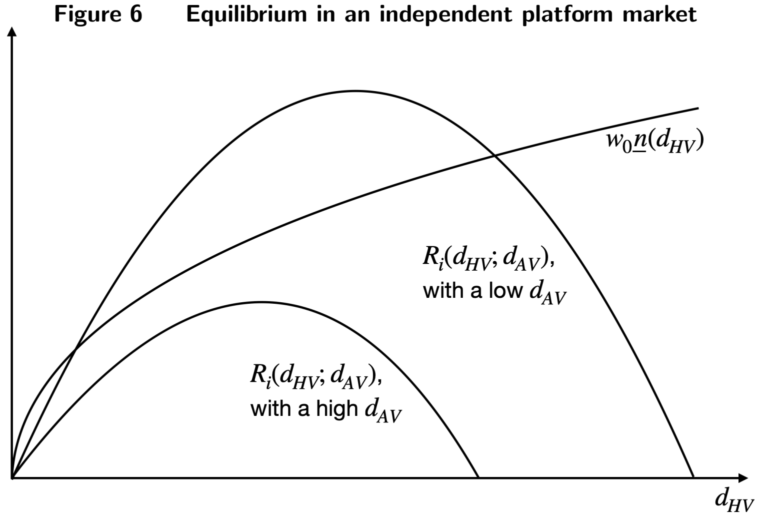
Moreover, the number of HVs required to supply d_{HV} is just $\underline{n}(d_{HV})$. Thus, the total HV cost is given by

$$w_0\underline{n}(d_{HV})$$

By Definition 2, in equilibrium, it must hold that

$$R_i(d_{HV}; d_{AV}) = w_0\underline{n}(d_{HV}) \Leftrightarrow p_i(d_{HV} + d_{AV})d_{HV}t_2 = w_0\underline{n}(d_{HV})$$

Figure 6 Equilibrium in an independent platform market



which is equivalent to

$$p_i(d_{HV} + d_{AV})\bar{u}(d_{HV}) = w_0 \tag{101}$$

when $d_{HV} > 0$. Moreover, the equilibrium demand d_{HV} must be the largest root that satisfies Eq. (101) such that it is a stable equilibrium (Definition 3).

Fig. 6 illustrates the equilibria at different AV fleet sizes. All else being equal, the HV revenue $R_i(d_{HV}; d_{AV})$ strictly decreases as the demand served by AVs, d_{AV}, increases. When the AV fleet size is sufficiently large and AVs serve a high demand level (the curve with "$R_i(d_{HV}; d_{AV})$ with a high d_{AV}"), the residual demand for HVs is so low that the revenue no longer covers the cost. In this case, the only equilibrium is $d_{HV} = 0$, meaning that AVs serve the market alone. When the AV fleet size is smaller and AVs serve a lower demand level (the curve with "$R_i(d_{HV}; d_{AV})$ with a low d_{AV}"), the residual demand for HVs is sufficient to generate enough revenue for HVs to cover the cost. In this case, there exists a positive equilibrium point for d_{HV}.

Thus, we define a threshold, $d^{\dagger}_{AV,i}$, to denote the highest level of demand served by AVs that allows $R_i(d_{HV}; d_{AV})$ to intersect with $w_0\underline{n}(d_{HV})$ at $d_{HV} > 0$. In other words, $d^{\dagger}_{AV,i}$ can be determined by solving Eq. (102) below:

$$\max_{d_{HV}} \{R_i(d_{HV}; d^{\dagger}_{AV,i}) - w_0\underline{n}(d_{HV})\} = 0 \tag{102}$$

Dividing both sizes of Eq. (102) by $\underline{n}(d^{\dagger}_{AV,i})$ yields an equilvalent definition of $d^{\dagger}_{AV,i}$:

$$\max_{d_{HV}} \{p_i(d_{HV} + d^{\dagger}_{AV,i})\bar{u}(d_{HV})\} = w_0 \tag{103}$$

That is, $d^{\dagger}_{AV,i}$ is the highest demand by AVs that still allows HVs to participate in scenario i. Denote the corresponding AV fleet size as $n^{\dagger}_{AV,i}$, i.e.

$$n^{\dagger}_{AV,i} = \underline{n}(d^{\dagger}_{AV,i}) \tag{104}$$

We then introduce Proposition 11, which describes the equilibrium price in scenario i for an exogenously given AV fleet size n_{AV}, using $n^{\dagger}_{AV,i}$ as the threshold that separates two different cases. The proof of Proposition 11 is given right after the statement.

PROPOSITION 11. *Consider scenario i in an independent platform market. Given an AV fleet size n_{AV}, in equilibrium, there are only two possibilities:*

1. *AVs and HVs serve the market together. The equilibrium price is given by*

$$p^*_{IPM,i} = p_i(d^*_{HV,IPM,i} + \bar{d}(n_{AV})) = \frac{w_0}{\bar{u}(d^*_{HV,IPM,i})}, \tag{105}$$

where $d^*_{HV,IPM}$ is equilibrium HV demand rate determined by Eq. (101)*, with the demand served by AVs being $\bar{d}(n_{AV})$. Moreover, all else being equal, the equilibrium price $p^*_{IPM,i}$ is strictly increasing in n_{AV}. This case happens when $n_{AV} \leq n^{\dagger}_{AV,i}$.*

2. *AVs serve the entire market alone. The equilibrium price is given by $p_i(\bar{d}(n_{AV}))$. This happens when $n_{AV} > n^{\dagger}_{AV,i}$.*

Proof. Given an AV fleet size n_{AV} and a scenario i, there are two possibilities:

1. $n_{AV} > n^{\dagger}_{AV,i}$. By definition of $n^{\dagger}_{AV,i}$ in Eq. (103), there does not exist $d_{HV} > 0$ such that the expected earnings of an HV meet w_0. Therefore, AVs serve the market alone. The price is then a function of the demand served by AVs, which is $\bar{d}(n_{AV})$.
2. $n_{AV} \leq n^{\dagger}_{AV,i}$. By definition of $n^{\dagger}_{AV,i}$ in Eq. (103), there exists d_{HV} such that the expected earnings of an HV are at least w_0. Thus, in equilibrium, AVs and HVs serve the market together. Moreover, the demand level served by HVs, denoted as $d^*_{HV,IPM,i}$, satisfies the equilibrium condition Eq. (101):

$$p_i(d^*_{HV,IPM,i} + d_{AV})\bar{u}(d^*_{HV,IPM,i}) = w_0 \tag{106}$$

where $d_{AV} = \bar{d}(n_{AV})$. Rearranging Eq. (106) gives Eq. (105).

Our last step is to prove that the equilibrium price $p^*_{IPM,i}$ is strictly increasing in the exogenous AV fleet size n_{AV}. We prove it by contradiction:

Suppose not. Then there exists $n_{AV} > n'_{AV}$, such that $p^*_{IPM,i} \leq {p^*_{IPM,i}}'$, where $p^*_{IPM,i}$ is the equilibrium price under an AV fleet size of n_{AV} and ${p^*_{IPM,i}}'$ is the equilibrium price under an AV fleet size of $n_{AV'}$. Then by the definition of the equilibrium price in Eq. (105), it must be true that

$$p^*_{IPM,i} = \frac{w_0}{\bar{u}(d^*_{HV,IPM,i})} \leq {p^*_{IPM,i}}' = \frac{w_0}{\bar{u}({d^*_{HV,IPM,i}}')}$$

where $d^*_{HV,IPM,i}$ and ${d^*_{HV,IPM,i}}'$ represent the equilibrium demand served by HVs under n_{AV} and n'_{AV}, respectively. By the monotonicity of the utilization $\bar{u}(d)$ (Lemma 2), we must have

$$d^*_{HV,IPM,i} \geq {d^*_{HV,IPM,i}}'$$

In other words, with a larger AV fleet size n_{AV}, there is more HV participation. But this cannot be true. To see why, consider the equilibrium definition Eq. (101). It must hold that:

$$p_i(d^*_{HV,IPM,i} + \bar{d}(n_{AV}))\bar{u}(d^*_{HV,IPM,i}) = w_0 \tag{107}$$

and

$$p_i({d^*_{HV,IPM,i}}' + \bar{d}(n'_{AV}))\bar{u}({d^*_{HV,IPM,i}}') = w_0 \tag{108}$$

By Eq. (108), it must be true that for any $d_{HV} \geq {d^*_{HV,IPM,i}}'$, it holds that

$$p_i(d_{HV} + \bar{d}(n'_{AV}))\bar{u}(d_{HV}) \leq p_i({d^*_{HV,IPM,i}}' + \bar{d}(n'_{AV}))\bar{u}({d^*_{HV,IPM,i}}') = w_0$$

Otherwise, ${d^*_{HV,IPM,i}}'$ cannot be an equilibrium. Furthermore, by our assumption that $n_{AV} > n'_{AV}$, we have

$$p_i(d_{HV} + \bar{d}(n'_{AV}))\bar{u}(d_{HV}) > p_i(d_{HV} + \bar{d}(n_{AV}))\bar{u}(d_{HV}) \tag{109}$$

This is by the monotonicity of the price function $p_i(d)$ and the inverse function $\bar{d}(n)$. Thus, it must be true that for any $d_{HV} > {d^*_{HV,IPM,i}}'$, the right-hand side of Eq. (109) satisifies

$$p_i(d_{HV} + \bar{d}(n_{AV}))\bar{u}(d_{HV}) < w_0$$

But this contradicts the definition of $d^*_{HV,IPM,i}$ in Eq. (107).

Therefore, we have proved both items in Proposition 11. □

One thing to note is that, as the AV fleet size n_{AV} approaches zero, the equilibrium price $p^*_{IPM,i}$ in Item 1 of Proposition 11 converges to the equilibrium price in the pure HV market $p^*_{HV,i}$. That is,

$$\lim_{n_{AV}\to 0} p^*_{IPM,i} = p^*_{HV,i}$$

Combined with the result that $p^*_{IPM,i}$ is strictly increasing in n_{AV}, we immediately obtain the following: for any AV fleet size $n_{AV} > 0$ that allows HVs to coexist with AVs ($n_{AV} \leq n^\dagger_{AV,i}$), it holds that

$$p^*_{IPM,i} > p^*_{HV,i} \tag{110}$$

We formally present this result in Corollary 1:

Corollary 1. *For an independent platform market, in scenario i, if the AV fleet size n_{AV} satisfies $0 < n_{AV} < n^\dagger_{AV,i}$, then AVs and HVs serve the market together, and the equilibrium price is strictly higher than that in a pure HV market (i.e. Inequality* (110) *holds).*

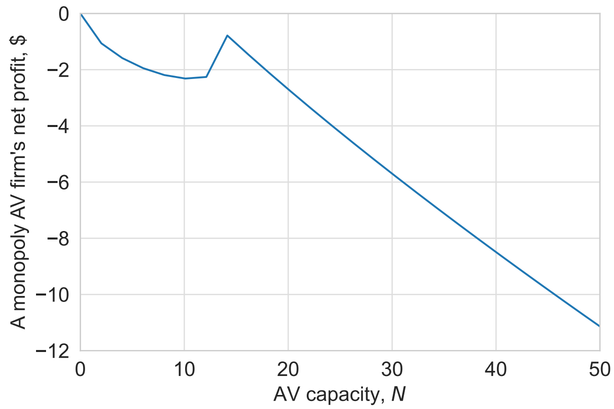
Corollary 1 plays a key role in proving that the prices are strictly higher under an independent platform market than under a pure-HV market.

Therefore, in an independent platform market, in scenario i, given an AV fleet size of n_{AV}, the variable profit rate per AV is given by:

$$\pi^I_i(n_{AV}) = \begin{cases} p_i(\bar{d}(n_{AV}) + d^*_{HV,IPM,i}) \cdot \bar{u}(\bar{d}(n_{AV})) - c_v, & n_{AV} \leq n^\dagger_{AV,i} \\ p_i(\bar{d}(n_{AV})) \cdot \bar{u}(\bar{d}(n_{AV})) - c_v, & n_{AV} > n^\dagger_{AV,i} \end{cases} \tag{111}$$

where $d^*_{HV,IPM,i}$ is the equilibrium demand served by HVs given by Item 1 in Proposition 11 and $n^\dagger_{AV,i}$ is the threshold value defined by Eq. (103) and Eq. (104).

Figure 7 An example of an independent platform market in which a monopoly supplier only generates negative profits



Note. Parameters: Demand distribution: $M = \{m_1 = 15 \text{ requests/hour}, m_2 = 1,000,000 \text{ requests/hour}\}$, $P(m_1) = 0.2, P(m_2) = 0.8$; cost and price parameters: $w_0 = 5$, $c_v = 3.85$, $c_f = 1$, $V = 11$; trip parameters: $a = 0.1$, $t_2 = 2$, $r = 0.4$

E.2.2. Monopoly A monopoly AV supplier in an independent platform market faces a similar tradeoff as in a common platform market: if the capacity is too high, the supplier incurs high fixed costs and a lot of idleness of AVs; if the capacity is too low, the supplier incurs high opportunity costs from lost demand. However, the difference is that, under an independent platform market, the trade-off is more extreme. When the AV capacity is too low, not only does the supplier lose the opportunity to serve more demand, but the AV fleet also becomes less efficient, making AVs less competitive than HVs. In certain demand scenarios where an HV can still make hourly earnings of w_0, an AV may not even be able to make c_v if its capacity is too low to achieve a break-even utilization. (See more on this in Appendix E.2.) With some demand distributions, this nonviable regime leads to surprising results, as shown in Fig. 7 where there exist cases where any AV capacity choice produces negative profit. In effect, the market may be “too thin” to support AV technology.

We summarize this observation in the following proposition:

Proposition 12. *In an independent platform market, even though AVs have lower total costs, there exist demand distributions under which AVs cannot make positive aggregate profits even when they are supplied by a monopoly.*

Proof. The numerical example in Figure 7 proves the existence of a market in which AVs cannot make a positive profit in a monopoly, independent platform market. A sketch for an analytical proof: consider two demand scenarios, Low (L) and High (H), with $m_L \ll m_H$. Then the optimal AV fleet size under the two scenarios will be significantly different. We denote the optimal AV fleet sizes under the two scenarios as $n^*_{AV,IPM,L}$ and $n^*_{AV,IPM,H}$, respectively. If the monopoly AV supplier chooses a capacity closer to $n^*_{AV,IPM,L}$, then under the high scenario, AVs may make zero profit because the fleet size is too low, which leads to low AV utilization and AVs cannot break even. If the monopoly AV supplier chooses a capacity closer to $n^*_{AV,IPM,H}$, then under the low scenario, a high proportion of the AV capacity remains idle. We can construct the distribution of the potential demand mass in these scenarios, such that in the former case, a monopoly supplier only makes revenue in the low scenario, which is too low to support the fixed cost; and in the latter case, a monopoly supplier makes revenue in both scenarios, but the fixed cost is too high. The result is that there does not exist an AV capacity that generates enough revenue to cover the costs. □

This is particularly surprising given that AVs have strictly lower costs than HVs and are managed by a profit-maximizing monopoly supplier. The driving force behind this result is the loss of technical efficiency from independent dispatch platforms. Separating the dispatch network and the rider pool for two platforms means AVs and HVs need to "compete" for density and scale. As AVs require pre-committed capacity investment but HVs do not, AVs can be disadvantaged in this competition.

However, when the demand distribution and cost parameters allow AVs to profit, the monopoly supplier can exploit the density effect. If AVs are profitable at a reasonably large capacity, this leaves little residual demand for HVs, which lowers the HV utilization and drives up the equilibrium HV price. A monopoly AV supplier recognizes this price effect of crowding out HVs and chooses a large capacity in order to drive up prices. From Proposition 11 in Appendix E.2.1, we know that when AVs and HVs coexist in the market, the price is strictly *increasing* in the AV fleet size, which consequently implies that the price is strictly higher than in a pure-HV market. This combined with the monopoly AV's incentives gives us the following proposition:

PROPOSITION 13. *Conditional on AVs being viable, in a monopoly independent platform market, the equilibrium price will be no less than that in a pure-HV market for each scenario, and strictly higher than in a pure-HV market in at least one scenario.*

Proof. Our proof takes two steps. First, we show that given an arbitrary AV capacity N and any demand scenario i, a monopoly AV supplier never chooses an AV fleet size n_{AV} that leads to a lower price than in the pure-HV market. Then we show that there must exist at least one demand scenario such that the price is strictly higher than in the pure-HV market.

Consider an arbitrary AV capacity N and demand scenario i. Then by Proposition 11, there are two possibilities:

1. $N \le n^{\dagger}_{AV,i}$. In this case, with any AV fleet size $n_{AV} \le N$, HVs can break even and will participate, and the equilibrium price $p^*_{IPM,i}$ is a strictly increasing function of the AV fleet size n_{AV}. By Eq. (111), the total variable profit for the AV fleet is given by

$$\pi^I_i(n_{AV})n_{AV} = (p^*_{HV,i} \cdot \bar{u}(\bar{d}(n_{AV})) - c_v)n_{AV} \tag{112}$$

The total variable profit Eq. (112) is strictly increasing in n_{AV}, providing that $p^*_{HV,i} \cdot \bar{u}(\bar{d}(n_{AV}) > c_v$. Therefore, there are two cases:

(a) $p^*_{HV,i} \cdot \bar{u}(\bar{d}(N)) \geq c_v$. That is, if the monopoly supplier dispatches all of its capacity N, the variable profit is non-negative. In this case, the optimal AV fleet size is just $n_{AV} = N$. Moreover, AVs and HVs serve the market together, and the equilibrium price $p^*_{IPM,i}$ is strictly higher than the pure-HV equilibrium price $p^*_{HV,i}$ (Corollary 1).

(b) $p^*_{HV,i} \cdot \bar{u}(\bar{d}(N)) < c_v$. That is, if the monopoly supplier dispatches all of its capacity N, the variable profit is negative. Given that $p^*_{HV,i} \cdot \bar{u}(\bar{d}(N))$ is the highest earnings that an AV can make in this scenario, this means that AVs cannot break even at any fleet size in this scenario. In this case, the monopoly supplier is better off not providing service, i.e. $n_{AV} = 0$. Thus, HVs serve the market alone. As a result, the price is identical to that under a pure-HV market, $p^*_{HV,i}$.

2. $N > n^\dagger_{AV,i}$. In this case, the supplier has two types of choices for the AV fleet size n_{AV}:

(a) $n_{AV} \leq n^\dagger_{AV,i}$. This has been analyzed in Item 1. If $p^*_{HV,i} \cdot \bar{u}(\bar{d}(n^\dagger_{AV,i})) \geq c_v$, then the optimal AV fleet size in this region is $n_{AV} = n^\dagger_{AV,i}$. Otherwise, $n_{AV} = 0$. In the former case, $p^*_{IPM,i} > p^*_{HV,i}$; in the latter case, $p^*_{IPM,i} = p^*_{HV,i}$.

(b) $n_{AV} > n^\dagger_{AV,i}$. By Proposition 11, AVs serve the market alone. By Eq. (112), the total variable profit for the supplier is then given by

$$p_i(\bar{d}(n_{AV})) \cdot \bar{u}(\bar{d}(n_{AV})) \cdot n_{AV} - c_v n_{AV} = p_i(\bar{d}(n_{AV})) \cdot \bar{d}(n_{AV}) \cdot t_2 - c_v n_{AV} \tag{113}$$

Note that Eq. (113) is just the same variable profit function as Eq. (90) in Lemma 4. Therefore, Lemma 4 applies here, which states that the variable profit Eq. (113) is strictly decreasing in n_{AV} for $n_{AV} \geq n^*_{HV,i}$. Therefore, the largest AV fleet size that the monopoly supplier may possibly choose is $n^*_{HV,i}$. In order to discuss the ordering of $n^*_{HV,i}$, $n^\dagger_{AV,i}$, we present the following lemma, with its proof immediately following the statement

LEMMA 5. *In any scenario, it must hold that*

$$n^\dagger_{AV,i} < n^*_{HV,i} \text{ for all } i$$

In other words, given the same AV fleet size n_{AV}, if HVs cannot break even under a common platform market, then they also cannot break even under an independent platform market.

Proof. We prove Lemma 5 by contradiction:

Suppose not. Then there exists scenario i such that

$$n^\dagger_{AV,i} \geq n^*_{HV,i} \tag{114}$$

Recall that $d^{\dagger}_{AV,i}$ and $n^{\dagger}_{AV,i}$ are defined as the maximum demand served by AVs and AV fleet size that allow HVs to participate in scenario i under an independent platform market. The definitions of $n^{\dagger}_{AV,i}$ and $d^{\dagger}_{AV,i}$ are given by Eq. (104) and Eq. (102), respectively. Thus, for any $n_{AV} \le n^{\dagger}_{AV,i}$, it holds that

$$p_i(d_{HV} + \bar{d}(n_{AV}))\bar{u}(d_{HV}) = w_0$$

has a solution for some $d_{HV} > 0$. Then by Eq. (114), it must also be true that

$$p_i(d_{HV} + \bar{d}(n^*_{HV,i}))\bar{u}(d_{HV}) = w_0$$

has a solution for some $d_{HV} > 0$. Since the utilization $\bar{u}(d)$ is strictly increasing in d (Lemma 2), it must be true that

$$p_i(d_{HV} + \bar{d}(n^*_{HV,i}))\bar{u}(d_{HV} + \bar{d}(n^*_{HV,i})) > p_i(d_{HV} + \bar{d}(n^*_{HV,i}))\bar{u}(d_{HV}) = w_0$$

In other words, there exists $d = d_{HV} + \bar{d}(n^*_{HV,i}) > \bar{d}(n^*_{HV,i})$, such that

$$p_i(d)\bar{u}(d) > w_0$$

But this contradicts the equilibrium definition of $n^*_{HV,i}$ that $n^*_{HV,i}$ is the largest root of $p_i(d)\bar{u}(d) = w_0$ and that for any $d > n^*_{HV,i}$, it holds that $p_i(d)\bar{u}(d) < w_0$. Contradiction. □

That is, the optimal AV fleet size in this case must satisfy $n^{\dagger}_{AV,i} < n_{AV} \le n^*_{HV,i}$. One can check that the inequality is, in fact, strict; in other words, a monopoly supplier will not choose $n_{AV} = n^*_{HV,i}$ in this case, because the total variable profit Eq. (113) is strictly decreasing at $n_{AV} = n^*_{HV,i}$. Therefore, it must be true that

$$p_i(\bar{d}(n_{AV})) > p_i(\bar{d}(n^*_{HV,i})) = p^*_{HV,i}$$

Thus, we have verified that the price is strictly higher than the pure-HV market if the supplier chooses an AV fleet size $n_{AV} > n^{\dagger}_{AV,i}$.

Therefore, we have confirmed the first part of the proof. Now consider the second part of the proof. We want to show that there exists at least one demand scenario under which the price is strictly higher than the pure-HV equilibrium price. We prove it by contradiction:

Suppose not. Then it must be true that under the optimal capacity N^* chosen by the monopoly supplier, the prices are identical to the pure-HV equilibrium price in any demand scenario, i.e. $p^*_{IPM,i} = p^*_{HV,i}$ for all i. In the first part of the proof, we have shown that the only case where $p^*_{IPM,i} = p^*_{HV,i}$ holds is under Item 1b when AVs cannot break even in scenario i. Therefore, the statement that $p^*_{IPM,i} = p^*_{HV,i}$ for all i implies that AVs do not participate in any demand scenario, which leads to zero variable profit and negative aggregate total profit. The AV capacity N^* thus cannot be an optimal capacity. Contradiction. □

The rationale is that in scenarios in which AVs and HVs coexist, the equilibrium price is strictly higher than the pure-HV price due to the efficiency loss for HVs. In scenarios where AVs serve the market alone, a monopoly AV supplier does not benefit from extending supply above the HV equilibrium supply level; hence, prices are no less than that in the pure-HV case.

In conclusion, a monopoly supplier in an independent platform market faces extreme situations: it is either nonviable or capable of extracting higher prices than in a pure HV market. In either case, consumers do not stand to benefit.

E.2.3. Perfect competition With perfectly competitive AVs, suppliers no longer jointly optimize profit. In the absence of a central decision-maker to coordinate the dispatch decision, perfectly competitive AVs are more likely to be nonviable than a monopoly supplier. Therefore, we have the following corollary of Proposition 12:

COROLLARY 2. *In a perfectly competitive independent platform market, even though AVs have lower total costs, there exist demand distributions where AVs cannot make positive aggregate profits.*

Furthermore, the results in Proposition 11 from Appendix E.2.1 continue to apply here:

PROPOSITION 14. *For a perfectly competitive independent platform market, the equilibrium price is strictly higher than that in a pure-HV market when AVs and HVs serve the market together.*

Proof. To prove this, we leverage the result from Proposition 11. Proposition 11 and its Corollary 1 show that, whenever in a scenario where AVs and HVs coexist, the price must be strictly higher than the pure-HV equilibrium price. Then, it is sufficient to show that, in a perfectly competitive independent platform market, there exist demand scenarios under which AVs and HVs coexist.

In Appendix C, we provide sufficient conditions under which HVs will supply the market along with AVs at least in the scenario with the highest potential demand. Then under this set of conditions, at least in the highest demand scenario, AVs and HVs coexist, and the price must be higher than the pure-HV equilibrium price. Numerical example Fig. 3b also exhibits a wide range of potential demand m_i under which AVs and HVs coexist under a perfectly competition common platform market (the region with a strictly decreasing market price of the dashed red curve). □

That is, under an independent platform market, even when AVs are perfectly competitive, the prices are still strictly higher than that in the pure-HV market, as long as HVs participate. In other words, the welfare gain from AVs can only be realized when HVs are completely out of the market. This result underscores the significant impact of the efficiency loss when separating the dispatch platform between AVs and HVs.

To summarize, in a perfectly competitive, independent platform market, the equilibrium prices are not uniformly lower than those in the pure-HV market.

F. Finite supply elasticity and no density effect (Proof of Proposition 10)

Proof. We start with a benchmark analysis of the pure-HV market when the density elasticity $r=0$, which is the result from Proposition 1 under the specific parameter $r=0$. Then we prove each statement in Proposition 10 one by one.

F.1. Baseline Analysis: a Pure-HV Market

We start by characterizing the baseline price in the pure-HV market. Eq. (11) in Proposition 1 characterizes the implicit function that determines the equilibrium price $p^*_{HV,i}$; for convenience, we repeat the result here: in a pure-HV market, under scenario i, the equilibrium price $p^*_{HV,i}=p_i(d)$ where d can be solved by

$$p_i(d)\ \bar{u}(d)=w(\underline{n}(d))$$

where $\underline{n}(d)$ is the minimal HV fleet size that can satisfy a demand level of d, and $\bar{u}(d)=dt_2/\underline{n}(d)$ is the highest utilization rate that can be achieved at a demand level of d. The closed-form expressions of $\underline{n}(d)$ and $\bar{u}(d)$ are also given in Item 1 in Proposition 1. It can be easily verified that when the density elasticity $r=0$, the minimal supply level $\underline{n}(d)$ is a linear function of d:

$$\underline{n}(d)=\tilde{a}d^{1/(r+1)}+t_2 d=(\tilde{a}+t_2)d$$

As a result, the utilization $\bar{u}(d)$ is a constant in d. That is,

$$\bar{u}(d)=\frac{t_2 d}{\underline{n}(d)}=\frac{t_2 d}{\tilde{a}d+t_2 d}=\frac{t_2}{\tilde{a}+t_2},$$

when $d>0$. Thus, with a slight abuse of notation, we use $\bar{u}$ to denote the utilization in the case of $r=0$ to reflect the fact that the utilization rate is constant regardless of the demand level. $\bar{u}\equiv t_2/(\tilde{a}+t_2)$. Therefore, under a pure-HV market, the equilibrium price satisfies

$$p^*_{HV,i}=p_i(d),\ \text{where } p_i(d)=w(\underline{n}(d))/\bar{u} \tag{115}$$

Closed-form solutions of the equilibrium price and demand can thus be derived by solving Eq. (115):

$$d^*_{HV,i}=\frac{V-w_0/\bar{u}}{V/m_i+\alpha t_2/\bar{u}^2}$$

and

$$p^*_{HV,i}=V(1-d^*_{HV,i}/m_i)$$

providing that $(V-w_0/\bar{u})>0$. If not, then the price does not cover the reservation earnings, and the equilibrium demand $d^*_{HV,i}=0$. Nonetheless, such a case will not happen under the scenarios in consideration, as implied by Assumption 2.

F.2. Equivalence between common and independent platform markets (Item 1)

We start with the common platform market; then we show that the analysis for the independent platform market is identical, both when HVs participate and when they do not participate.

F.2.1. Common platform market. Recall that we denote the HV fleet size as n_{HV} and the AV fleet size as n_{AV}. We also denote the demand served by HVs and AVs as d_{HV} and d_{AV}, respectively.

Case 1: HVs participate ($n_{HV} > 0$). Then in equilibrium, it must be true that the marginal HV's reservation earnings are equal to the price times the utilization rate. That is,

$$w(n_{HV}) = p_i(d_{AV} + d_{HV})\bar{u} \tag{116}$$

where $\bar{u} = t_2/(\tilde{a} + t_2)$ is the constant utilization rate under $r = 0$. Furthermore, since AVs and HVs are dispatched from a common platform, they must also share the same utilization $\bar{u}$, which means the following equations must also hold in equilibrium:

$$n_{HV} = d_{HV}t_2/\bar{u},\ n_{AV} = d_{AV}t_2/\bar{u} \tag{117}$$

That is, the ratio between the demand hours served and the fleet size for AVs and HVs is the same and equal to $\bar{u}$. Therefore, given an AV fleet size n_{AV}, the rest terms d_{AV}, d_{HV} and n_{HV} can be uniquely determined by combining Eq. (116) and Eq. (117). One can check that n_{HV} is in fact a linear function in n_{AV}:

$$n_{HV} = \frac{(\bar{u}V - w_0) - \frac{\bar{u}^2 V}{m_i t_2} n_{AV}}{\alpha + \frac{\bar{u}^2 V}{m_i t_2}} \tag{118}$$

by which $n_{HV} \geq 0$ is equivalent to:

$$n_{AV} \leq \frac{(\bar{u}V - w_0) m_i t_2}{\bar{u}^2 V} \tag{119}$$

Case 2: HVs do not participate ($n_{HV} = 0$). This is a trivial case; without HVs, AVs are the only type of supply; thus, there is only one dispatch platform, making the two dispatch platform designs (independent & common platforms between AVs and HVs) identical.

F.2.2. Independent platform market. Now consider an independent platform market. Eq. (116) remains to hold; Eq. (117) in general does not hold because HVs and AVs operate on separate platforms and typically do not serve the same level of demand, and thus will have different utilization rates; however, in the case of $r = 0$, the utilization rate is constant regardless of the density of demand. Therefore, Eq. (117) also holds for an independent platform market. Therefore, given the same AV fleet size n_{AV}, the two dispatch platforms yield the exact same outcome.

F.3. AVs always enter the ride-hailing market (Item 2)

We provide proof for a monopoly AV supplier and perfectly competitive AVs, respectively.

F.3.1. Monopoly. First, we restate the aggregate function for the monopoly AV supplier in Eq. (4):

$$\max_N \sum_1^I P(m_i) \left\{ \max_{n_{AV,i} \leq N} \pi_i(n_{AV,i}) n_{AV,i} \right\} - c_f N \tag{120}$$

Since the dispatch platform design does not make a difference when $r = 0$ as proved in Item 1, we omit the superscript on the total variable profit function $\pi_i(n_{AV,i})$. Instead of deriving the optimal capacity decision, we instead prove that there exists an AV capacity N, such that the aggregate profit in Eq. (120) is strictly positive. Since the optimal capacity decision can do at least as good, this is sufficient to show that the monopoly supplier will enter the market.

We start by considering an AV capacity sufficiently small. That is, let $N = \epsilon$, where $\epsilon > 0$ and is infinitely close to zero. Then we solve the AV dispatch problem in each demand scenario by maximizing the total variable profit $(\pi_i(n_{AV,i})n_{AV,i})$, subject to the capacity constraint that $n_{AV,i} \leq \epsilon$. Since the AV capacity is infinitely small, HVs will remain in the market even if all AVs participate. Thus, each AV must make the same earnings as each HV, since they face the same price and utilization. Therefore, the variable profit per AV, $\pi_i(n_{AV,i})$, should satisfy:

$$\pi_i(n_{AV,i}) = p_i(d_{AV,i} + d_{HV,i})\bar{u} - c_v = w(n_{HV,i}) - c_v > w_0 - c_v \tag{121}$$

That is, for each AV that is dispatched in scenario i, they must make at least $(w_0 - c_v)$ per hour per AV in this scenario.

Furthermore, it can be shown that if the AV capacity ϵ is sufficiently small, the AV supplier will choose to dispatch all of its capacity in every demand scenario. The high-level intuition is that the choice of how many AVs to dispatch in each scenario depends on the trade-off between the variable profit margin $\pi_i(n_{AV,i})$ and the demand served by AVs; when the AV fleet size is extremely low, the latter dominates the former, and it is beneficial to increase the demand served by AVs at the cost of slightly reducing the profit margin. This statement can be formally proved by taking the first-order derivative of the total variable profit $\pi_i(n_{AV,i})n_{AV,i}$ over the AV fleet size $n_{AV,i}$, which is given by

$$\frac{\partial \pi_i}{\partial n_{AV,i}} n_{AV,i} + \pi_i(n_{AV,i}) \tag{122}$$

Moreover, the partial derivative can be computed from Eq. (121) by the chain rule:

$$\frac{\partial \pi_i}{\partial n_{AV,i}} = \frac{\partial w}{\partial n_{HV,i}} \cdot \frac{\partial n_{HV,i}}{\partial n_{AV,i}} = \alpha \cdot \frac{\partial n_{HV,i}}{\partial n_{AV,i}} < 0 \tag{123}$$

In fact, we can show that $\partial \pi_i / \partial n_{AV,i}$ is not only strictly negative but also a constant. This is because $\frac{\partial n_{HV,i}}{\partial n_{AV,i}}$ is strictly negative and a constant, which can be directly computed from Eq. (118).

Here we also provide an alternative proof using the chain rule, which may give the readers a better idea of the multiple forces that drive the direction of change in $n_{HV,i}$.

The proof is as follows: the partial derivative $\frac{\partial n_{HV,i}}{\partial n_{AV,i}}$ relies on the relationship between $n_{HV,i}$ and $n_{AV,i}$, which can be jointly determined by Eq. (116) and Eq. (117):

$$w(n_{HV,i}) = p_i(d)\bar{u},\ d = \frac{(n_{AV,i} + n_{HV,i})\bar{u}}{t_2}$$

Then again, by the chain rule, we can compute the partial derivative of $n_{HV,i}$ over $n_{AV,i}$, which givens

$$\frac{\partial n_{HV,i}}{\partial n_{AV,i}} = \frac{\frac{\partial p_i}{\partial d}\frac{\bar{u}^2}{t_2}}{\frac{\partial w}{\partial n_{HV,i}} - \frac{\partial p_i}{\partial d}\frac{\bar{u}^2}{t_2}}$$

It happens that the partial derivatives on the right-hand side above are all constants:

$$\frac{\partial p_i}{\partial d} = -V/m_i < 0,\ \frac{\partial w}{\partial n_{HV,i}} = \alpha > 0$$

Thus, $\frac{\partial n_{HV,i}}{\partial n_{AV,i}}$ is also a constant and is strictly negative.

Now if we go back to Eq. (122), the first-order derivative of the total variable profit is strictly positive if and only if

$$n_{AV,i} < \frac{\pi_i(n_{AV,i})}{-\partial \pi_i/\partial n_{AV,i}} = \frac{\pi_i(n_{AV,i})}{\alpha \cdot (-\partial n_{HV,i}/\partial n_{AV,i})}$$

It can be verified that this condition holds as long as $n_{AV,i}$ is sufficiently small.

Therefore, in any demand scenario i, there exists an AV fleet size $\bar{n}_{AV,i} > 0$ such that the total variable profit of AVs in this scenario is strictly increasing in the AV fleet size as long as $n_{AV,i} \le \bar{n}_{AV,i}$. Let the AV capacity $\epsilon \le \min_i\{\bar{n}_{AV,i}\}$. Hence, in any scenario, a monopoly supplier will choose to dispatch all ϵ AVs in every demand scenario. Combined with Eq. (121), it must also be true that every AV earns at least $(w_0 - c_v)$ per hour per AV in every scenario. The aggregate profit of the AV supplier must be no less than the following:

$$\sum_1^I P(m_i)\{(w_0 - c_v)\epsilon\} - c_f\epsilon = (w_0 - c_v - c_f)\epsilon > 0$$

which means at an AV capacity of ϵ, the monopoly supplier can make a positive profit. Since the supplier's optimal capacity decision should lead to at least as good as this outcome, the AV supplier will always choose to enter the market with a positive AV capacity.

F.3.2. Perfect competition Similarly, we start by repeating Eq. (6) that determines the equilibrium AV capacity N for the perfectly competitive setting:

$$\sum_1^I P(m_i)\max\{\pi_i(N), 0\} = c_f \tag{124}$$

Note that we omit the superscript D because the dispatch platform design does not make a difference under $r = 0$. Eq. (124) computes the AV fleet size N such that the aggregate variable profit per AV exactly equals the fixed cost c_f, which is a result of the free entry of AVs in the perfectly competitive market. We show that the example constructed above for the monopoly AV setting also applies here.

Consider an AV capacity $\epsilon \leq \min_i\{\bar{n}_{AV,i}\}$, where $\bar{n}_{AV,i}$ represents an AV fleet size at or below which the total variable profit of AVs is strictly increasing in the AV fleet size (see details on this in the bullet point above for the monopoly case.) That is, in any scenario i, if the AV fleet size $n_{AV,i} \leq \bar{n}_{AV,i}$, the total variable profit, $(\pi_i(n_{AV,i})n_{AV,i})$, is strictly increasing in $n_{AV,i}$. Thus, it must be true that $\pi_i(n_{AV,i}) > 0$ for $n_{AV,i} \leq \bar{n}_{AV,i}$. Indeed, as shown in Eq. (121), the variable profit per AV is not only strictly positive but in fact strictly higher than $(w_0 - c_v)$. That is,

$$\pi_i(n_{AV,i}) > (w_0 - c_v), \text{ for } n_{AV,i} \leq \bar{n}_{AV,i}$$

Hence, with an AV capacity ϵ, in all scenarios, all of the AVs will choose to be dispatched because the variable profit per AV is strictly positive and above $(w_0 - c_v)$. The aggregate variable profit per AV on the left-hand side of Eq. (124) can then be simplified as

$$\begin{aligned}\sum_1^I P(m_i)\max\{\pi_i(\epsilon), 0\} &= \sum_1^I P(m_i)\pi_i(\epsilon) \\ &> \sum_1^I P(m_i)(w_0 - c_v) \\ &= (w_0 - c_v) > c_f\end{aligned} \tag{125}$$

In other words, at an AV capacity of $N = \epsilon$, the aggregate variable profit per AV is strictly higher than the fixed cost of purchasing the AV. Therefore, more AVs will be purchased, and the equilibrium AV capacity will be strictly higher than ϵ.

Before we conclude the proof for the perfect competition case, the final step is to prove that the equilibrium AV capacity exists. In other words, we need to show that there exists a capacity N such that Eq. (124) does hold true (instead of having the aggregate variable profit per AV to be above c_f all the time and leading to infinite AV capacity). This can be proved by verifying the following characteristics of the aggregate variable profit per AV, $\sum_1^I P(m_i)\max\{\pi_i(N), 0\}$:

- *It is continuous and non-decreasing in N.* By Eq. (123), $\pi_i(N)$ is strictly decreasing in N for any scenario i. Thus, the maximum of $\pi_i(N)$ and 0 is then a non-increasing function of N; $\sum_1^I P(m_i)\max\{\pi_i(N), 0\}$ is then a linear combination of I non-decreasing functions. And its continuity is implied by the fact that it is differentiable.

- *Its value spans from above c_f to below c_f.* We have shown that at $N = \epsilon$, the aggregate variable profit per AV is strictly above c_f (Eq. (125)); it can be verified that with N sufficiently large, $\pi_i(N) < 0$ by the fact that $\pi_i(N)$ is strictly decreasing in N. Thus, with a sufficiently large N, $\sum_1^I P(m_i)\max\{\pi_i(N), 0\}$ will eventually equals to zero.

Combining the two characteristics, by the intermediate value theorem, it must be true that $\sum_1^I P(m_i)\max\{\pi_i(N), 0\} = c_f$ will hold for some AV capacity $N > 0$.

To conclude, we have proved that under perfect competition among AVs, there will be a positive number of AVs being purchased and dispatched in equilibrium.

F.4. AVs lead to strictly lower prices in all scenarios. (Item 3)

For this item, we compare the equilibrium price when an AV fleet size of $n_{AV} > 0$ is dispatched and when only HVs serve the market (the pure-HV market).

Consider scenario i. Again, denote the demand served by AVs and HVs as d_{AV} and d_{HV}, respectively. Denote the HV fleet size as n_{HV}. Then in equilibrium, there are two cases:

a. $n_{HV} > 0$. That is, HVs participate in the ride-hailing market. In this case, it must be true that the marginal HV's reservation earnings are equal to the price discounted by utilization. That is,

$$w(n_{HV}) = p_i(d_{AV} + d_{HV})\bar{u}$$

which implies the equilibrium price satisfies

$$p_i(d_{AV} + d_{HV}) = \frac{w(n_{HV})}{\bar{u}} \tag{126}$$

In contrast, the equilibrium price in scenario i under a pure-HV market is given by:

$$p^*_{HV,i} = p_i(d), \text{ where } p_i(d) = w(\underline{n}(d))/\bar{u} \tag{127}$$

This is the same as Eq. (115) in the pure-HV market analysis at the beginning of this proof, which we just repeat here for convenience.

Comparing Eq. (126) with Eq. (127), it must be true that

$$d < d_{AV} + d_{HV},\ p_i(d) > p_i(d_{AV} + d_{HV})$$

If not, then $d \geq d_{AV} + d_{HV}$, which implies that $\underline{n}(d) \geq n_{AV} + n_{HV}$, which further implies $\underline{n}(d) > n_{HV}$; in other words, the HV fleet size under the pure-HV market is larger than that under the common platform market. Since the HV supply curve is upward-sloping, this implies that the reservation earnings $w(\underline{n}(d)) > w(n_{HV})$, which implies

$$p_i(d) = \frac{w(\underline{n}(d))}{\bar{u}} > \frac{w(n_{HV})}{\bar{u}} = p_i(d_{AV} + d_{HV}) \tag{128}$$

Since price is a strictly decreasing function of demand, Eq. (128) implies that $d < d_{AV} + d_{HV}$, leading to a contradiction.

Therefore, in summary, it must be true that introducing AVs under a common platform market strictly increases the total demand served and decreases the price in equilibrium, providing that HVs still participate in the ride-hailing market after the introduction of AVs. Next, we analyze the second case where HVs do not participate.

b. $n_{HV} = 0$. That is, HVs no longer participate in the ride-hailing market after the introduction of AVs. Then it must be true that

$$p_i(d_{AV})\bar{u} \leq w(0) = w_0 \Leftrightarrow p_i(d_{AV}) \leq \frac{w_0}{\bar{u}}$$

which means

$$p_i(d_{AV}) < \frac{w(\underline{n}(d))}{\bar{u}}$$

In other words, the price, in this case, must be lower than any price that can support a positive HV fleet size, which includes the price under a pure-HV market.

F.5. Perfectly competitive AVs lead to strictly lower prices than the monopoly AV supplier in all scenarios. (Item 4)

In this section, we prove that monopoly AV leads to a strictly higher price than perfectly competitive AVs in every scenario. Note that the decisions under monopoly and perfect competition are different both at the stage of choosing the AV capacity as well as the AV fleet size (the number of AVs to dispatch) in each scenario. Therefore, we analyze and conduct a comparison for both steps, the results of which are summarized in the following lemmas:

LEMMA 6 **(Monopoly vs. Competition: Capacity)**. *The equilibrium AV capacity under perfect competition is always strictly higher than the optimal AV capacity selected by a monopoly supplier. That is, compared to a perfectly competitive setting, a monopoly AV supplier will limit capacity to optimize profit.*

LEMMA 7 **(Monopoly vs. Competition: Dispatch and Pricing)**. *Assume that AV capacities are set as described in Lemma 6. Then in all scenarios, a monopoly supplier consistently dispatches fewer AVs than in a perfectly competitive setting. That is to say, a monopoly supplier not only purchases fewer AVs but also deploys fewer of them in every scenario. Moreover, in each scenario, the price is strictly higher under a monopoly than under perfect competition.*

Combining the two lemmas together, one can immediately draw the conclusion that the prices are always lower under perfect competition than under monopoly in every scenario. Next, we prove each lemma.

F.5.1. Proof of Lemma 6 In this section, we prove that the equilibrium AV capacity under perfect competition is strictly lower than the optimal AV capacity under monopoly. Our proof for Lemma 6 takes the following steps:

Step 1. Characterize the optimal AV capacity N^* that maximizes a monopoly supplier's net aggregate profit, which is given by

$$N^* = \arg\max_N \sum_1^I P(m_i)\left\{\max_{n_{AV,i}\le N} \pi_i(n_{AV,i})n_{AV,i}\right\} - c_f N$$

which involves two sub-steps:

Step 1a. (Proof) Given an arbitrary AV capacity N, derive the optimal AV fleet size in every scenario i. In other words, solve

$$\max_{n_{AV,i}\le N} \pi_i(n_{AV,i})n_{AV,i}, \text{ for all } i$$

Step 1b. (Proof) Solve for the optimal AV capacity N^*.

Step 2. (Proof) Derive the equilibrium AV fleet size N' under perfect competition, where N' satisfies the perfect competition equilibrium condition Eq. (6):

$$\sum_1^I P(m_i)\max\{\pi_i(N'), 0\} = c_f \tag{129}$$

Step 3. (Proof) Prove that Eq. (129) can only hold under $N' > N^*$.

Proof of Step 1a. Consider an arbitrary fleet size N. Consider any scenario i. Then a monopoly supplier solves the following constrained optimization problem (a sub-problem of Eq. (120)):

$$\max_{n_{AV}\le N} \pi_i(n_{AV})n_{AV} \tag{130}$$

We omit the subscript i from $n_{AV,i}$ for brevity since we are focusing on one scenario in this part. At this point, we have known a number of characteristics of $\pi_i(n_{AV})$. When n_{AV} is low, AVs and HVs coexist in the market, and the relationship between the equilibrium HV fleet size n_{HV} and the AV fleet size n_{AV} is characterized by Eq. (118). In this case, the variable profit per AV in this scenario is given by

$$\pi_i(n_{AV}) = w(n_{HV}) - c_v = w\left(\frac{(\bar{u}V - w_0) - \frac{\bar{u}^2 V}{m_i t_2}n_{AV}}{\alpha + \frac{\bar{u}^2 V}{m_i t_2}}\right) - c_v$$

which is a linear function in n_{AV}. The simplicity of the linear form allows us to solve an unconstrained problem, $\max_{n_{AV}} \pi_i(n_{AV})n_{AV}$, by setting its first-order derivative to zero; in fact, one can check the unconstrained problem is strictly concave. We denote this optimal size as $n^*_{AV,i}$ and give it expression below:

$$n^*_{AV,i} = \frac{1}{2\alpha C}(w_0 C + \alpha\bar{u}V \cdot m_i), \text{ where } C \text{ is a constant and } C = \frac{\bar{u}^2 V}{t_2} \tag{131}$$

Clearly, $n^*_{AV,i}$ is strictly increasing in m_i. Going back to the constrained optimization problem Eq. (130), concavity implies that the optimal solution satisfies:

$$n_{AV,i} = \min(N, n^*_{AV,i}) \tag{132}$$

In other words, in relatively low demand scenario where the AV capacity N is above the unconstrained maximizer $n^*_{AV,i}$, a monopoly supplier does not deploy all of its capacity; instead, it only deploys $n^*_{AV,i}$ AVs and leaves the rest $(N - n^*_{AV,i})$ idle, to maintain a high price of the market; as the demand gets denser, the price is high enough even if the supplier deploys all of its capacity.

Next, let K be such that $N \in (n^*_{AV,K}, n^*_{AV,K+1})$. In the case where $N > n^*_{AV,I}$, let $K = I$. Then, we can rewrite the aggregate profit function Eq. (120) by replacing the AV fleet size in each scenario with the optimal decision Eq. (132):

$$\max_N \left(\sum_1^K P(m_i)\pi_i(n^*_{AV,i})n^*_{AV,i} \right) + \left(\sum_{K+1}^I P(m_i)\pi_i(N)N \right) - c_f N \tag{133}$$

That is, from scenario 1 to K, the AV capacity N is sufficient to cover the optimal fleet size; from scenario $K+1$ to I, the supplier has to use up all its capacity N. Note that only the second and third terms from left to right in Eq. (133) depend on N; moreover, as we have shown above, $\pi_i(N)$ is linear and strictly decreasing in N. Thus, we can take the first-order derivative of the expression in Eq. (133) over N, which gives:

$$\left(\sum_{K+1}^I P(m_i) \left(\frac{\partial \pi_i}{\partial N} N + \pi_i(N) \right) \right) - c_f \tag{134}$$

providing that $n^*_{AV,K} < N < n^*_{AV,K+1}$ holds. The second-order derivative can be computed as well; the expression is given by

$$\sum_{K+1}^I P(m_i) \left(\frac{\partial^2 \pi_i}{\partial N^2} N + 2\frac{\partial \pi_i}{\partial N} \right)$$

Since π_i is linear in N, the first term from left to right in the parenthesis equals zero. Thus, the second-order derivative is equal to:

$$2\sum_{K+1}^I P(m_i)\frac{\partial \pi_i}{\partial N} < 0$$

The value of $\frac{\partial \pi_i}{\partial N}$ can be found near Eq. (123).

Therefore, we have proved that the aggregate net profit Eq. (120) is piece-wise strictly concave and continuous in the AV capacity N. Now we would like to focus on the endpoints ($n^*_{AV,1}$, $n^*_{AV,2}$, ..., $n^*_{AV,I}$) and show that Eq. (120) is indeed smooth near these points:

Consider an arbitrary scenario $i = K$ with $n^*_{AV,K}$. First, compute the derivatives in the neighborhood $(n^*_{AV,K} - \epsilon, n^*_{AV,K})$ on the left of $n^*_{AV,K}$. The aggregate net profit Eq. (120) is given by

$$\sum_{1}^{K-1} P(m_i)\pi_i(n^*_{AV,i})n^*_{AV,i} + \sum_{K}^{I} P(m_i)\pi_i(N)N - c_f N$$

since the capacity N in this neighborhood is lower than $n^*_{AV,K}$. Taking the first-order derivative gives:

$$\sum_{K}^{I} P(m_i)(\frac{\partial \pi_i}{\partial N}N + \pi_i(N)) - c_f, \text{ for } N \in (n^*_{AV,K} - \epsilon, n^*_{AV,K}) \tag{135}$$

Now compute the derivatives in the neighborhood $(n^*_{AV,K}, n^*_{AV,K} + \epsilon)$ in a similar fashion; the first-order derivative is given by

$$\sum_{K+1}^{I} P(m_i)(\frac{\partial \pi_i}{\partial N}N + \pi_i(N)) - c_f, \text{ for } N \in (n^*_{AV,K}, n^*_{AV,K} + \epsilon) \tag{136}$$

One can check that as ϵ approaches zero and N approaches $n^*_{AV,K}$, Eq. (135) and Eq. (136) are equal. To see why, let $N = n^*_{AV,K}$ and deduct Eq. (135) by Eq. (136):

$$\left(\sum_{K}^{I} P(m_i)(\frac{\partial \pi_i}{\partial N}N + \pi_i(N))\Big|_{N=n^*_{AV,K}} - c_f\right) - \left(\sum_{K+1}^{I} P(m_i)(\frac{\partial \pi_i}{\partial N}N + \pi_i(N))\Big|_{n^*_{AV,K}} - c_f\right)$$
$$= P(m_K)(\frac{\partial \pi_K}{\partial N}N + \pi_K(N))\Big|_{N=n^*_{AV,K}}$$

But by the definition of $n^*_{AV,K}$ (see the discussion above Eq. (131)), it is the maximizer of the unconstrained problem in scenario K; that is,

$$\frac{\partial \pi_K}{\partial N}N + \pi_K(N))\Big|_{N=n^*_{AV,K}} = 0$$

Therefore, Eq. (135) and Eq. (136) are equal, proving the smoothness of the aggregate net profit function. Therefore, the first-order derivative of Eq. (120) is continuous and strictly decreasing, implying that Eq. (120) is strictly concave in the AV capacity N.

Proof of Step 1b. Hence, the optimal capacity N^* for the monopoly supplier must satisfy that the first-order derivative (Eq. (134)) equals zero, which is equivalent to

$$\sum_{K+1}^{I} P(m_i)(\frac{\partial \pi_i}{\partial N}|_{N=N^*} N^* + \pi_i(N^*)) = c_f \tag{137}$$

Proof of Step 2. Now consider the perfect competitive equilibrium. By the equilibrium definition Eq. (124), if we denote the equilibrium AV capacity as N', then N' must satisfy the following:

$$\sum_{1}^{I} P(m_i)\max\{\pi_i(N'),0\} = c_f \tag{138}$$

There must exist a scenario K', such that

$$\pi_i(N') \leq 0 \text{ for all } i \leq K', \pi_{i+1}(N') > 0 \text{ for all } i > K' \tag{139}$$

and such a K' is unique. In other words, scenario K' is the scenario with the highest potential demand mass such that the variable profit per AV remains at or below zero; any scenario with $i > K'$ will have a strictly positive variable profit. All else being equal, as the potential demand mass further increases, the market is more "profitable", and the variable profit goes above zero. We formally state the monotonicity of π_i in i in Lemma 8 and provide proof right after the lemma.

Thus, we can rewrite the equilibrium condition Eq. (138) as

$$\sum_{i=K'+1}^{I} P(m_i)\pi_i(N') = c_f \tag{140}$$

Combining Eq. (140) with Eq. (137), we have

$$\sum_{i=K'+1}^{I} P(m_i)\pi_i(N') = \sum_{K+1}^{I} P(m_i)(\frac{\partial \pi_i}{\partial N}\mid_{N=N^*} N^* + \pi_i(N^*)) \tag{141}$$

with N' and N^* being the perfectly competitive equilibrium capacity and the monopoly supplier's optimal capacity, respectively.

Proof of Step 3. Next, we show that Eq. (141) cannot hold if $N' = N^*$ or $N' < N^*$. We start with $N' = N^*$:

If $N' = N^*$, then it must be true that $K' \leq K$. That is, at an AV capacity $N = N' = N^*$, in any scenario i, if the variable profit per AV when all N AVs are dispatched, i.e. $\pi_i(N)$, is below zero, then it must be true that a monopoly supplier only dispatches part of its AV capacity. This is intuitive because dispatching all AVs will lead to a negative profit for the monopoly supplier. Then we can rewrite Eq. (141) to be the following:

$$\sum_{i=K'+1}^{K} P(m_i)\pi_i(N) + \sum_{i=K+1}^{I} P(m_i)\pi_i(N) = \sum_{K+1}^{I} P(m_i)(\frac{\partial \pi_i}{\partial N}\mid_N N + \pi_i(N)) \tag{142}$$

where $N = N' = N^*$. By eliminating the term $\sum_{i=K+1}^{I} P(m_i)\pi_i(N)$ from both sides, we have that

$$\sum_{i=K'+1}^{K} P(m_i)\pi_i(N) = \sum_{K+1}^{I} P(m_i)(\frac{\partial \pi_i}{\partial N}\mid_N N)$$

But this cannot be possible: on the left-hand side, by the definition of K', the variable profit per AV, $\pi_i(N)$, should be strictly positive for $i = K'+1, ..., I$ (Eq. (139)), making the left-hand side strictly positive if $K > K'+1$, or equal to zero if $K = K'$; on the right-hand side, we have proved in Eq. (123) that $\frac{\partial \pi_i}{\partial N}|_N N < 0$ for all i. Thus, unless $K+1 > I$ and $K = K'$ hold at the same time, the two sides cannot be equal. However, $K+1 = K'+1 > I$ cannot be true, otherwise, the variable profit per AV, $\pi_i(N)$, will be strictly negative for all $i = 1, 2, ..., I$ by the definition of K' (Eq. (139)), implying that AVs are not profitable, which contradicts Item 2 in Proposition 10. Therefore, to conclude, it is not possible to have $N' = N^*$. Next, we show that it is also not possible to have $N' < N^*$, either. That is, the perfectly competitive AV capacity cannot be lower than the monopoly supplier's optimal AV capacity. The proof largely remains the same as the $N' = N^*$ case and we only need to change a few details.

If $N' < N^*$, it is still true that $K' \leq K$. That is, at an AV capacity N', in any scenario i, if the variable profit per AV when all N' AVs are dispatched, i.e. $\pi_i(N')$, is below zero (the case for $i \leq K'$ defined in Eq. (139)), then it must be true that a monopoly supplier does not dispatch all of its AV capacity N^* (which is greater than N') in this scenario. Then we have equality similar to Eq. (142):

$$\sum_{i=K'+1}^{K} P(m_i)\pi_i(N') + \sum_{i=K+1}^{I} P(m_i)\pi_i(N') = \sum_{K+1}^{I} P(m_i)(\frac{\partial \pi_i}{\partial N}|_{N=N^*} N^* + \pi_i(N^*))$$

where $N' < N^*$. Rearranging the terms on both sides gives

$$\sum_{i=K+1}^{I} P(m_i)\pi_i(N') - \sum_{i=K+1}^{I} P(m_i)\pi_i(N^*) = -\sum_{i=K'+1}^{K} P(m_i)\pi_i(N') + \sum_{K+1}^{I} P(m_i)(\frac{\partial \pi_i}{\partial N}|_{N=N^*} N^*) \tag{143}$$

Now similar to the case for $N' = N^*$, it is easy to verify that

$$\pi_i(N') > 0 \text{ for } i \geq K'+1 \tag{144}$$

and

$$\frac{\partial \pi_i}{\partial N}|_{N=N^*} < 0 \text{ for all } i = 1, ..., I \tag{145}$$

Combining *Eq.* (144) and *Eq.* (145), we know that the right-hand side of Eq. (143) is strictly negative.[12] Eq. (143) thus leads to the following inequality:

$$\sum_{i=K+1}^{I} P(m_i)(\pi_i(N') - \pi_i(N^*)) < 0$$

[12] Similar to the analysis for $N' = N^*$, the right-hand side, $-\sum_{i=K'+1}^{K} P(m_i)\pi_i(N') + \sum_{K+1}^{I} P(m_i)(\frac{\partial \pi_i}{\partial N}|_{N=N^*} N^* + \pi_i(N^*))$, cannot be zero because this will require $K' = K > I$, in which case AVs cannot make any positive profit in any scenario.

Since $\pi_i(N)$ is strictly decreasing in N for all i, it implies that

$$\pi_i(N') < \pi_i(N^*) \Leftrightarrow N' > N^*$$

which is a contradiction to the assumption that $N' < N^*$. We have thus proved that the perfectly competitive equilibrium AV capacity is strictly higher than the monopoly supplier's optimal AV capacity. That is,

$$N' > N^*$$

F.5.2. Proof of Lemma 7 Next, we show that the prices under perfect competition are strictly lower than that under monopoly, providing the equilibrium capacity N' and optimal monopoly capacity N^* in the proof of Lemma 6. By Lemma 6, we have shown that $N' > N^*$. That is, the equilibrium AV capacity under perfect competition is strictly higher than the optimal capacity under monopoly.

Now knowing that $N' > N^*$, it is straightforward to show that the prices are always lower under perfect competition. Suppose not. Then there exists a scenario i, in which the price under perfect competition is higher than or equal to the price under monopoly. Denote the price as $p^*_{comp,i}$ (which stands for perfect competition) and $p^*_{mono,i}$ (which stands for monopoly), respectively. Denote the AV fleet size as $n^*_{AV,mono,i}$ and $n^*_{AV,comp,i}$ for perfect competition and monopoly, respectively. There are four cases:

1. $n^*_{AV,mono,i} = N^*$, $n^*_{AV,comp,i} = N'$. In other words, under both settings, the AV fleet size is at its capacity; the monopoly supplier chooses to use up all of its N^* AVs, and the perfectly competitive AVs can all make a positive profit after all N' of them join. Since $N' > N^*$, more AVs participate under perfect competition than under monopoly ($n^*_{AV,comp,i} > n^*_{AV,mono,i}$), implying a lower price under perfect competition. That the price is strictly decreasing in the AV fleet size in a given scenario is a result implied by the monotonicity of the variable profit per AV $\pi_i(n_{AV})$; this term by definition is determined by the product of price and utilization (see Eq. (121) for its expression when HVs participate in scenario i; when HVs do not participate, $\pi_i(n_{AV})$ is just the product of the price and utilization minus the constant cost term, $p_i(d_{AV})\bar{u} - c_v$, where $d_{AV} = n_{AV}\bar{u}/t_2$). Since the utilization $\bar{u}$ is constant, the price is also strictly decreasing in n_{AV}. Therefore, $p^*_{mono,i} > p^*_{comp,i}$.
2. $n^*_{AV,mono,i} < N^*$, $n^*_{AV,comp,i} < N'$. Under both settings, the AV fleet size is strictly below its capacity; that is, the monopoly supplier chooses the optimal fleet size $n^*_{AV,mono,i}$, and the perfectly competitive AVs already make a zero variable profit per AV at a fleet size $n^*_{AV,comp,i}$ before all of its capacity N' are deployed (i.e. $\pi_i(N') < 0$). In this case, $p^*_{comp,i} = 0$. Clearly, a monopoly supplier will never choose a fleet size that leads to the price being zero. Thus, $p^*_{mono,i} > p^*_{comp,i} = 0$. Note that in this case, it also must be true that $n^*_{AV,mono,i} < n^*_{AV,comp,i}$, because the price is strictly decreasing in the AV fleet size in a given scenario.

3. $n^*_{AV,mono,i} < N^*$, $n^*_{AV,comp,i} = N'$. Under perfect competitive AVs, the AV fleet size is at its capacity N', but under monopoly AV, the AV fleet size is strictly below its capacity N^*. Again, since $N' > N^*$, this implies there are more AVs participating under perfect competition, which leads to a lower price.
4. $n^*_{AV,mono,i} = N^*$, $n^*_{AV,comp,i} < N'$. Under perfect competitive AVs, the AV fleet size is strictly below its capacity N', but under monopoly AV, the AV fleet size is at its capacity N^*. Again, this implies that the price under perfect competition is down to zero (which is why not all AVs participate.), while the monopoly supplier will never make a decision that induces zero price. Therefore, $p^*_{mono,i} > p^*_{comp,i} = 0$. Again, the monotonicity of price in the AV fleet size implies that $n^*_{AV,mono,i} < n^*_{AV,comp,i}$.

Therefore, we have proved that the price is strictly lower under perfectly competitive AVs than a monopoly AV supplier.

Our last part is to provide formal proof for $\pi_i(n_{AV})$ being increasing in i for any given n_{AV}. We formally state it as a lemma:

LEMMA 8. *Consider the variable profit per AV in scenario i, denoted as $\pi_i(n_{AV})$, as a function of the AV fleet size n_{AV}:*

$$\pi_i(n_{AV}) = \begin{cases} p_i(d_{AV} + d_{HV,i})\bar{u} - c_v = w(n_{HV,i}) - c_v & \text{if } d_{HV,i} > 0 \\ p_i(d_{AV})\bar{u} - c_v & \text{if } d_{HV,i} \le 0 \end{cases} \tag{146}$$

where d_{AV}, $d_{HV,i}$ and $n_{HV,i}$ are given by Eq. (117) *and Eq.* (118).[13] *Then, fixing n_{AV}, $\pi_i(n_{AV})$ is strictly increasing in i. In other words, $\pi_S(n_{AV}) < \pi_{S+1}(n_{AV})$ for $m_S < m_{S+1}$.*

Proof. Just as a reminder,

$$n_{HV,i} = \frac{(\bar{u}V - w_0) - \frac{\bar{u}^2 V}{m_i t_2} n_{AV}}{\alpha + \frac{\bar{u}^2 V}{m_i t_2}}$$

Moreover, by Eq. (117),

$$d_{HV,i} = n_{HV,i} t_2/\bar{u}, \; d_{AV} = n_{AV} t_2/\bar{u}$$

Thus,

$$d_{HV,i} = \frac{(\bar{u}V - w_0) - \frac{\bar{u}^2 V}{m_i t_2} n_{AV}}{\alpha + \frac{\bar{u}^2 V}{m_i t_2}} t_2/\bar{u}$$

which is strictly increasing in m_i. That is,

$$d_{HV,S} < d_{HV,S+1}, \text{ for } m_S < m_{S+1}$$

[13] It should be noted that we include the subscript i for those terms affected by m_i when n_{AV} is fixed. This is because we are comparing two scenarios here. When conducting analysis within a single scenario, it is not necessary to highlight i, which is why the i subscripts were not included in earlier expressions.

and as a result:

$$n_{HV,S} < n_{HV,S+1}, \text{ for } m_S < m_{S+1}$$

In other words, keeping the AV fleet size fixed, more HVs participate in a higher demand scenario. There are three cases:

1. $d_{HV,S} > 0$, $d_{HV,S+1} > 0$. That is, in both scenarios, HVs participate in the ride-hailing market. Then by Eq. (146).

$$\pi_S(n_{AV}) - \pi_{S+1}(n_{AV}) = w(n_{HV,S}) - w(n_{HV,S+1}) < 0$$

This is because with the fixed AV fleet size, as the potential demand gets higher, the price increases, inducing more HVs to participate and serve the residual demand; in equilibrium, the reservation earnings of the marginal HV are higher; since AVs and HVs face the same price and utilization, this also makes AVs more profitable on a per-AV basis.

2. $d_{HV,S} \le 0$, $d_{HV,S+1} > 0$. That is, HVs participate in the higher scenario of the two but do not participate in the lower scenario. Then by Eq. (146),

$$\pi_S(n_{AV}) - \pi_{S+1}(n_{AV}) = p_S(d_{AV})\bar{u} - w(n_{HV,S+1})$$

The fact that HVs do not participate in scenario S implies that the expected earnings per HV cannot cover the lowest HV reservation earnings for HV participation; that is,

$$p_S(d_{AV})\bar{u} \le w(0) = w_0$$

On the flip side, that HVs participate in scenario $S+1$ implies the opposite; that is,

$$w(n_{HV,S+1}) > w(0) = w_0$$

Therefore, we conclude that

$$\pi_S(n_{AV}) - \pi_{S+1}(n_{AV}) < 0$$

3. $d_{HV,S} \le 0$, $d_{HV,S+1} \le 0$. That is, HVs do not participate in either scenario. Then again by Eq. (146),

$$\pi_S(n_{AV}) - \pi_{S+1}(n_{AV}) = p_S(d_{AV})\bar{u} - p_{S+1}(d_{AV})\bar{u}$$

Note that the price is given by

$$p_i(d_{AV}) = V(1 - d_{AV}/m_i)$$

as defined by Eq. (2). Thus, at the same d_{AV}, the price is higher in the scenario with a larger potential demand mass. That is,

$$p_S(d_{AV}) < p_{S+1}(d_{AV})$$

Therefore, we have

$$\pi_S(n_{AV}) - \pi_{S+1}(n_{AV}) = p_S(d_{AV})\bar{u} - p_{S+1}(d_{AV})\bar{u} < 0$$

In summary, we have proved that across all scenarios, the variable profit per AV, $\pi_i(n_{AV})$, is strictly increasing in n_{AV}. □

G. Extensions

Next, we analyze some extensions to the main analysis. Appendix G.1 discusses the implications of AVs on driver welfare. Appendix G.2 discusses the role that demand variability plays in the competitive advantages of AVs and HVs.

G.1. Driver welfare

In this section, we investigate how AVs impact driver welfare and how the impact may differ under common and independent platform markets. We find that in an independent platform market, consistent with the conventional wisdom, AVs always reduce driver surplus (Proposition 15); however, in a common platform market, surprisingly, AVs may improve driver surplus, providing that the HV supply is sufficiently inelastic (i.e., when the cost parameter α is large). We start with the formal definition of driver surplus and then discuss each dispatch platform design, respectively.

G.1.1. Definition Driver surplus is defined as the monetary gain that HVs get by providing service above the lowest earnings that they are willing to provide service at (i.e., the reservation earnings). Given the HV supply curve $w(n_{HV})$ and an equilibrium HV fleet size n^*_{HV}, the driver surplus is given by

$$\int_0^{n^*_{HV}} \left(w(n^*_{HV}) - w(n_{HV})\right)\, dn_{HV} \tag{147}$$

Driver surplus (147) depends on the shape of the HV supply curve, $w(n_{HV})$, as well as the level of HV supply in equilibrium, n^*_{HV}. When the cost parameter $\alpha = 0$, which indicates a perfectly elastic HV supply, the reservation earnings are constant at w_0 and exactly equal to the expected earnings $w(n^*_{HV})$ from providing service; therefore, driver surplus is zero regardless of the equilibrium HV supply. When the cost parameter $\alpha > 0$, which indicates a finitely elastic HV supply, the reservation earnings $w(n_{HV})$ are strictly below the expected earnings $w(n^*_{HV})$ for all but the marginal HV that provide service in equilibrium; therefore, driver surplus is strictly above zero.

Furthermore, when $\alpha > 0$, driver surplus (147) is strictly increasing in the equilibrium HV fleet size n^*_{HV}. A larger HV fleet in equilibrium indicates that more HVs with a positive surplus participate in the market; moreover, it also requires higher expected earnings to attract HVs with higher reservation earnings, which increases the surplus of all participating HVs. Therefore, when comparing the driver surplus in two settings, given the same supply curve $w(n_{HV})$, it is equivalent to comparing the equilibrium HV fleet size n^*_{HV}.

G.1.2. Independent platform market Proposition 15 formally states how driver surplus changes after AVs are introduced to an independent platform market:

PROPOSITION 15 **(Driver surplus)**. *Consider a market with a finitely elastic supply curve ($\alpha > 0$). Compared with a pure-HV market, the driver surplus* (147) *is strictly lower in an independent platform market in any scenario where AVs participate.*

Proof. In this proof, we show that HVs are strictly worse off in any demand scenario when AVs participate in an independent platform market compared with a pure-HV market. By the definition of driver surplus Eq. (147), in a given scenario, driver surplus is strictly increasing in the number of HVs participating in the market. Therefore, it is sufficient to compare the equilibrium HV fleet size for each demand scenario under the independent platform market ($n^*_{HV,IPM,i}$) and the pure-HV market ($n^*_{HV,i}$).

For any given scenario i, there are two possibilities:

1. HVs do not participate in the independent platform market. In other words, $n^*_{HV,IPM,i} = 0$. Since HVs always participate in any scenario in a pure-HV market (Assumption 2), it must be true that $n^*_{HV,i} > n^*_{HV,IPM,i}$. Therefore, HVs are worse off after introducing AVs.
2. HVs participate in the independent platform market. In other words, $n^*_{HV,IPM,i} > 0$. The rest of the proof focuses on this case, and we show that $n^*_{HV,i} > n^*_{HV,IPM,i,}$ still holds here. By the equilibrium condition Eq. (21) in Definition 2 for a market with both AVs and HVs, under the independent platform market, given an arbitrary $d_{AV} > 0$ which represents the demand level served by AVs, the HV fleet size $n^*_{HV,IPM,i}$ satisfies:

$$R_i(d^*_{HV,IPM,i}; d_{AV}) = w(n^*_{HV,IPM,i})n^*_{HV,IPM,i} \tag{148}$$

where

$$R_i(d^*_{HV,IPM,i}; d_{AV}) = p_i(d^*_{HV,IPM,i} + d_{AV}) \cdot d^*_{HV,IPM,i} \cdot t_2 \tag{149}$$

and

$$n^*_{HV,IPM,i} = \underline{n}(d^*_{HV,IPM,i})$$

Furthermore, by the equilibrium condition Eq. (9), in a pure-HV market, the HV fleet size $n^*_{HV,i}$ satisfies

$$r_i(d^*_{HV,i}) = w(n^*_{HV,i})n^*_{HV,i} \tag{150}$$

where

$$r_i(d^*_{HV,i}) = p_i(d^*_{HV,i}) \cdot d^*_{HV,i} \cdot t_2$$

and

$$n^*_{HV,i} = \underline{n}(d^*_{HV,i})$$

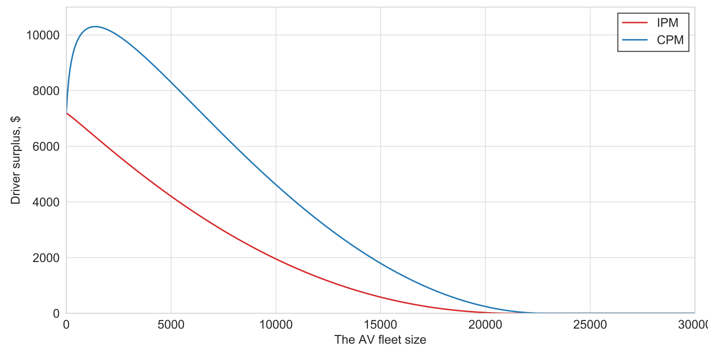
Since $d_{AV} > 0$, it must be true that the left-hand side of Eq. (149) satisfies

$$r_i(d^*_{HV,IPM,i}; d_{AV}) < p_i(d^*_{HV,IPM,i}) \cdot d^*_{HV,IPM,i} \cdot t_2$$

Thus, by Eq. (148), it holds that

$$w(n^*_{HV,IPM,i})n^*_{HV,IPM,i} < p_i(d^*_{HV,IPM,i}) \cdot d^*_{HV,IPM,i} \cdot t_2 \tag{151}$$

Figure 8 Driver surplus in the change of the AV fleet size in a single demand scenario



Note. In both CPM and IPM, the potential demand is $m_i = 50,000$ requests/hour.

Comparing Eq. (151) with the pure-HV equilibrium condition Eq. (150), we know that $n_{HV} = n^*_{HV,IPM,i}$ cannot be an equilibrium point under a pure-HV market; the total HV revenue at $n_{HV} = n^*_{HV,IPM,i}$ is strictly higher than the total HV cost, which will induce more HVs to participate, until the HV fleet size reaches $n^*_{HV,i}$ and the equilibrium condition Eq. (150) is satisfied. Therefore, we must have

$$n^*_{HV,i} > n^*_{HV,IPM,i}$$

This concludes our proof for the surplus comparison. □

The proof leverages the idea in Appendix G.1.1 that comparing driver surplus is equivalent to comparing the equilibrium HV fleet size. It shows that the equilibrium HV fleet size under an independent platform market with any AV participation is strictly lower than that under a pure-HV market. The high-level idea is that the equilibrium HV fleet size depends on the expected earnings for HVs, which are driven by two main factors, the market price, and the HV utilization. Keeping the HV fleet size fixed, if AVs are introduced into an independent platform market, then the HV utilization remains the same; however, the price will go down due to the increase in supply, which lowers the expected earnings of HVs. Thus, when AVs provide service in the market, compared with a pure-HV market, some HVs (those with higher reservation earnings) will exit the market; this will reduce the density of the HV dispatch platform and lower the HV utilization rate, which further reduces the expected earnings of HVs. As a result of both the price and efficiency reduction, the equilibrium HV fleet size is a strictly decreasing function of the AV fleet size, which is numerically illustrated by the red curve in Fig. 8.

G.1.3. Common platform market In contrast, the implication of AVs on driver surplus is more nuanced under a common platform market. For example, in Fig. 8, driver surplus under a common platform market (the blue curve) is not monotone in the AV fleet size, unlike that under an independent platform market.

Fig. 9 provides a detailed view of how driver surplus in Fig. 8 is derived and why it is not monotone in the AV fleet size under a common platform market. Fig. 9(a) and (b) consider two particular demand scenarios, with the potential demand mass $m_i = 500$ and $m_i = 50,000$, respectively. In each demand scenario, two types of curves are plotted: the colored curves are the expected earnings per HV as a function of the HV fleet size, with the colors representing different exogenously given AV fleet sizes; the black dashed line is the HV supply curve, $w(n_{HV})$. The intersection of a colored curve and the black curve is then the equilibrium HV fleet size under the corresponding AV fleet size.

Fig. 9(a) shows a scenario where HVs cannot break even in a pure-HV market but can break even in a common platform market with AV participation. The red curve, which is the expected earnings per HV in a pure-HV market, does not intersect with the HV supply curve, implying that the market is not thick enough to support the cost of HVs. However, with more AVs, the HV earnings curve moves upwards (red curve $\rightarrow$ purple curve $\rightarrow$ blue curve). Not only the supply curve intersects with the earnings curve when the AV fleet size $n_{AV} = 5$ and $n_{AV} = 10$, but the equilibrium HV fleet size is higher as the AV fleet size increases from 5 to 10. Since driver surplus is increasing in the equilibrium HV fleet size as indicated by Eq. (147), the example illustrates how driver surplus can be increasing in the AV fleet size under a common platform market.

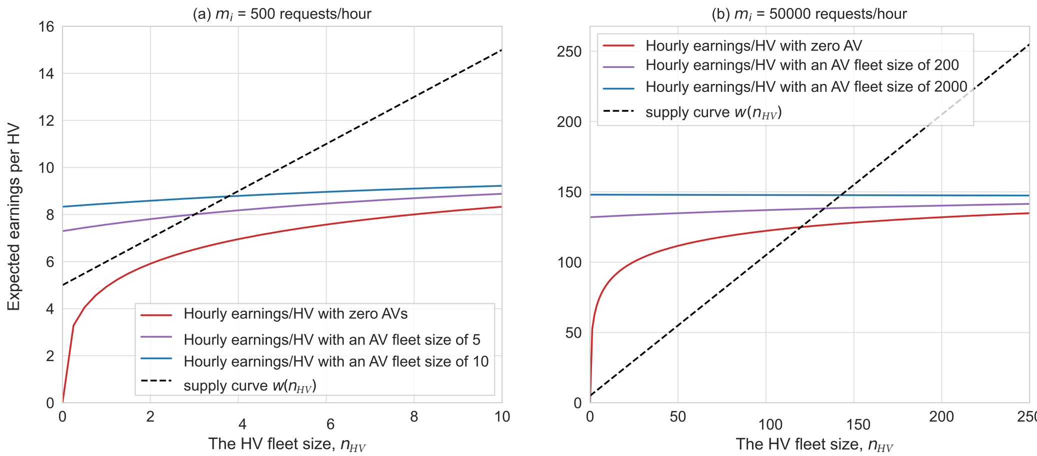
Fig. 9(b) conducts a similar analysis under a significantly higher potential demand mass. Unlike Fig. 9(a), in this case, HVs can break even in a pure-HV market (the dashed line intersects with the red curve). However, the result is consistent: as the AV fleet size increases, the HV earnings curve moves upwards, leading to a higher equilibrium HV fleet size. This example confirms that the introduction of AVs may benefit HVs, regardless of whether HVs can break even on their own.

Even though the examples in Fig. 9 consider exogenously given AV fleet sizes, which may not necessarily be the optimal decisions of the AV supplier(s), this presents a clear contrast with the strong result in Proposition 15, where any AV participation strictly reduces driver surplus under an independent platform market. This surplus comparison again highlights the important role that the dispatch platform market plays in determining the welfare implication of AVs.

G.2. Demand variability

As we have discussed in the main analysis, there is a trade-off between AVs and HVs, arising from their distinct cost structures. AVs are capital-intensive assets that require upfront investment, while HVs are flexible and incur costs only when being utilized. In this section, we discuss how demand variability impacts the equilibrium outcome. We consider the setting with perfect supply

Figure 9 Hourly earnings per HV at different AV fleet sizes in a single demand scenario, CPM



elasticity ($\alpha = 0$) and positive density elasticity ($r > 0$). Our conclusion is that, without demand variability, AVs fully replace HVs by serving the market alone in all demand scenarios and market configurations; in other words, HVs have no competitive advantage in the absence of demand variability. However, a high demand variability is not sufficient to guarantee a high HV participation; when the cost of AVs is sufficiently low, the AV supplier may choose a large AV capacity to hedge the demand uncertainty, which may indeed reduce the level of HV participation.

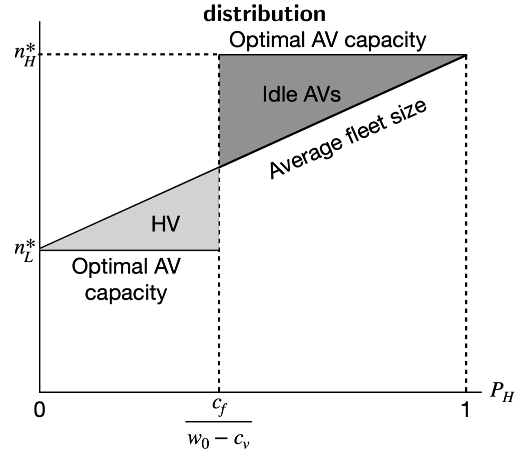
We start by considering a common platform market and a monopoly AV supplier. For simplicity, consider a two-point distribution with two demand scenarios, high (H) and low (L). That is, $i \in \{H, L\}$ and $m_L < m_H$. The distribution of the potential demand mass M is given by

$$M = \begin{cases} m_L, & \text{with probability } P_L \\ m_H, & \text{with probability } P_H = 1 - P_L \end{cases}$$

Recall that under this market configuration, as stated by Proposition 8, the monopoly supplier's decision is a newsvendor problem and thus has a closed-form solution. We then derive the optimal AV capacity N^* and the optimal AV fleet sizes in the two demand scenarios, as a function of the probability mass of the high demand scenario P_H, which we illustrate in Fig. 10.

Fig. 10 plots two functions: "Average fleet size" is defined as the sum of the HV and AV equilibrium fleet size, averaged across the two scenarios. "Optimal AV capacity", N^*, is a step function of P_H, with $N^* = n^*_{HV,L}$ when P_H is below the critical fractile $(c_f/(w_0 - c_v))$, and $N^* = n^*_{HV,H}$ when P_H is at or above the critical fractile. When P_H is below the critical fractile, AVs serve the market alone in the low scenario and coexist with HVs in the high scenario (except when $P_H = 0$). Furthermore, the difference between the average fleet size and the AV capacity equals the average HV fleet size (the light gray area in Fig. 10), which we use as a measure for HV participation. When P_H is at or above the critical fractile, AVs serve the market alone in all scenarios, and there

Figure 10 The optimal AV capacity and average total fleet sizes, in the change of the potential demand distribution



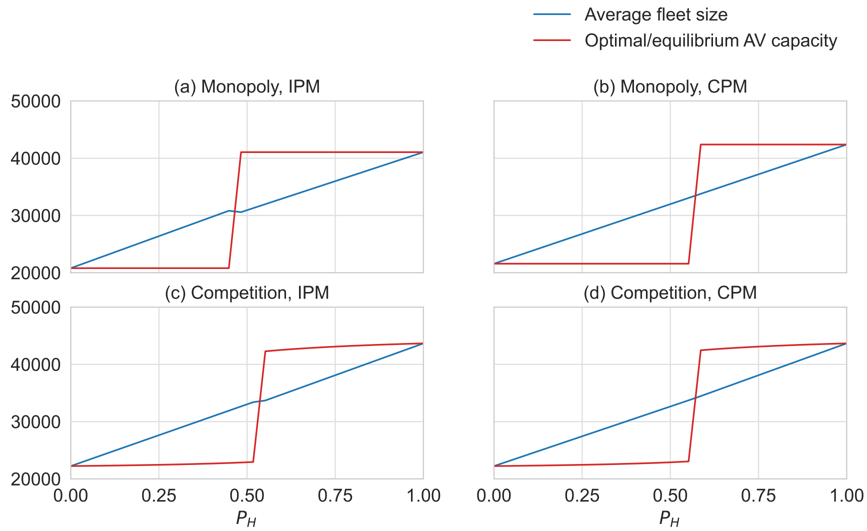
Note. When $P_H < c_f/(w_0 - c_v)$, the optimal AV capacity $N^* = n^*_{HV,L}$, which is the equilibrium HV fleet size for the low demand scenario; AVs are always 100% utilized, and HVs participate in the high demand scenario at a fleet size of $(n^*_{HV,H} - n^*_{HV,L})$. When $P_H \geq c_f/(w_0 - c_v)$, $N^* = n^*_{HV,H}$, AVs serve the market alone; in the low demand scenario, there is a number of $(n^*_{HV,H} - n^*_{HV,L})$ idle AVs.

is no HV participation; the difference between the AV capacity and the average fleet size equals the average number of idle AVs.

Although no demand variability ($P_H = 0$ or $P_H = 1$) results in no HV participation, increasing variability doesn't guarantee increased HV participation. Using the variance of M as a metric, the variability is maximized when $P_H = 0.5$. Consider the case of increasing variability as P_H goes from 0 to 0.5. Then when AVs are relatively more expensive ($c_f > 0.5(w_0 - c_v)$), the average HV fleet size is increasing in variability. The intuition is that, when the cost for AVs is relatively high, the monopoly supplier would rather lose the opportunity to make more profit in the high-demand scenario, rather than let AVs sit idle in the low-demand scenario. Thus, with more demand variability, HVs serve a more important role in fulfilling the residual demand in the high demand scenario. In contrast, when AVs are relatively less expensive ($c_f \leq 0.5(w_0 - c_v)$), the average HV fleet size can indeed decrease in variability (for example, the average HV fleet size jumps down to zero as P_H exceeds the critical fractile). This is because, given the low AV costs, the supplier cares more about losing demand than having AVs idle. Thus, they can afford to hedge demand variability with a high AV capacity, which eliminates HV participation.

Fig. 11 is a numerical study of the analysis in Fig. 10 for all four market configurations. The shapes of the average fleet size and the AV capacity exhibit similar patterns across all configurations, yielding consistent insights as Fig. 10: when P_H is at 0 or 1, implying no demand variability, AVs serve the market alone and there is no HV participation; as P_H gradually increases from 0 to 0.5 (or decrease from 1 to 0.5), HVs start to participate, but the HV fleet size is not always monotone in P_H, implying that higher demand variability does not necessarily lead to more HV participation.

Figure 11 Numerical examples of the supply composition and optimal AV capacity, in the change of the potential demand distribution



Note. "Average fleet size": average number of vehicles (including both AVs and HVs) operating in the two scenarios; "Optimal/equilibrium AV capacity": the optimal AV capacity for a monopoly supplier, or the equilibrium AV capacity with perfectly competitive AVs. Parameters: In all four examples, $c_f/(w_0 - c_v) = 0.57$.

H. Numerical Analysis Supplementary

In this section, we provide additional details on the numerical analysis in Section 5. We start with the definition of rider surplus (see Appendix H.1). We then provide the rationales behind our selection of parameters in Appendix H.3. A specific focus is given to the parameter V, elucidating its significance as the riders' maximum valuation of a trip in Appendix H.4. Concluding this section, we present additional numerical analysis, namely the welfare and AV revenue for settings with finite supply elasticity and density effect, in Appendix H.5

H.1. Rider surplus definition

In scenario i, rider surplus depends on the demand curve $d_i(p)$. Given a market-clearing price p_i, the rider's surplus is given by

$$\int_{p_i}^{V} d_i(p)pdp$$

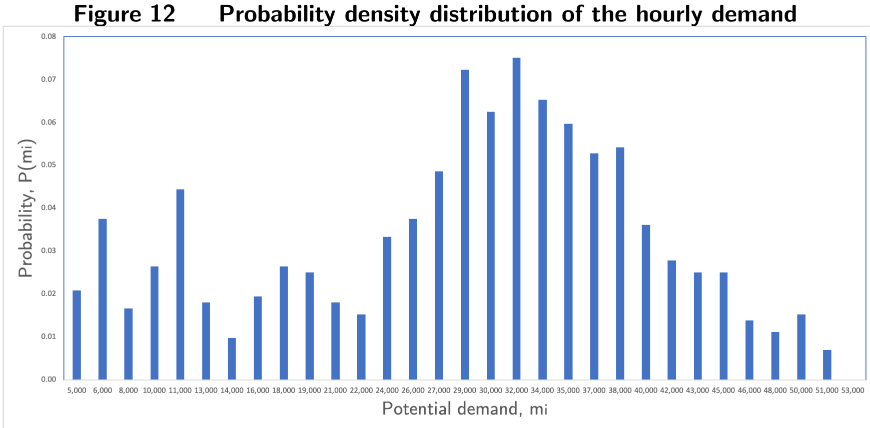
where V is the price that demand will fall to 0. Throughout the paper, we assume a linear demand curve (Eq. (1)); hence, the rider surplus can be further simplified as

$$\int_{p_i}^{V} d_i(p)pdp = \frac{1}{2}d_i(p_i)(V - p_i) = \frac{m_i}{2V}(V - p_i)^2$$

Aggregating all demand scenarios, the total rider surplus is then given by

$$\sum_{i=1}^{I} \mathbb{P}(m_i)\frac{m_i}{2V}(V - p_i)^2$$

Figure 12 Probability density distribution of the hourly demand



H.2. Demand distribution

The demand distribution is obtained from the FHV trip record by NYC Taxi & Limousine Commission (TLC) from June 1, 2019 to July 1, 2019, shown in Fig. 12. The number of trips is counted for each hour during this period and aggregated into 30 demand scenarios, with the lowest scenario being $4,720$ requests/hour and the highest scenario being $51,166$ requests/hour.

H.3. Choice of parameters

Table 7 shows how parameters are chosen in Table 2.

Table 7 Data and rationale behind calculations

	Data source and rationales behind calculations
a	$\approx 3(\text{area})^{1/2}$ (Kolesar 1975), area=300 sq ml (NYC, Population (2022))
t_2	25 min
r	Kolesar (1975)
w_0	GOBankingRates (2022)
c_v	Fuel and maintenance cost per hour
c_f	Straight-line depreciation of \$180k total cost over 3 years.
V	Average taxi trip price in the U.S. (Appendix H.4)

H.4. Parameter V: riders' maximum valuation of a trip

The choice of parameter V in Table 2, Section 5.3 is supported by the taxi price from major U.S. cities. According to our calculation, the average price among the cities is around \$101.85/hour. We round the number to \$100/hour and use it as a proxy for the valuation of ride-hailing service for an average rider, which corresponds to $V/2$ in our model because we assume a uniform distribution in $[0, V]$ for riders' trip valuation. Thus, in Table 2, we set $V = \$200$/ride-hour.

Table 8 provides the relevant data for calculating the taxi price. For each city, Column "Price" represents the average price per hour charged by taxis, which is computed using data from Column

Table 8 Average taxi price in major U.S. cities

City	Price ($/hour)	Initial charge ($)	Per mile charge ($)	Avg. trip distance (miles)	Avg. travel speed (miles/hour)
NYC	86.67	2.5	2.5	3.0	26.0
San Francisco	84.66	3.5	2.75	5.5	25.0
Boston	88.16	2.6	2.8	4.4	26.0
Chicago	82.39	3.25	2.25	5.5	29.0
Washing-ton DC	59.64	3.25	2.16	5.9	22.0
Los Angeles	118.76	2.85	2.7	6.7	38.0
San Diego	166.06	2.8	3	7.2	49.0
Dallas	100.59	2.25	1.8	8.9	49.0
Phoenix	129.77	5	2.3	7.7	44.0
Average	101.85				

Note. Data source: Initial charge and per mile change – Taxi fares (2015), `https://www.taxifarefinder.com/rates.php`; avg. trip distance – Rideguru(2018), `https://ride.guru/lounge/p/what-is-the-average-trip-distance-for-an-uber-or-lyft-ride`; avg. travel speed – Geotab, `https://www.geotab.com/`.

"Initial Charge" to Column "Average travel speed". The detailed calculation of the taxi price in Table 8 is the following:

Avg. price per trip = Initial charge + per mile charge × avg. trip distance

Avg. duration per trip = avg. trip distance/avg. travel speed

Price = Avg. price per trip/Avg. duration per trip

Thus, we obtain an estimation of the average taxi price per hour in Table 8, Column "Price". By taking an average over all cities, we obtain the number $101.85/hour during the trip.

H.5. Additional numerical analysis

In this section, we provide additional numerical analysis for the finite supply elasticity and density effect setting. We consider two different values for the HV cost parameter α.

One may observe that, in both examples, the average HV employment is positive across all market configurations, indicating that HVs serve the market together with AVs under finite supply elasticity and the density effect.

Below, we provide details for each setting:

H.5.1. Parameter values: $\alpha = 2e^{-4}$, $r = 0.4$. See the surplus, welfare, and AV revenue analysis in Table 9 and Table 10.

H.5.2. Parameter values: $\alpha = 8e^{-4}$, $r = 0.4$. See the surplus, welfare, and AV revenue analysis in Table 11 and Table 12.

Table 9 Surplus and HV employment impact ($\alpha = 2e^{-4}$, $r = 0.4$)

	Surplus Rider	AV	Driver	Total	Driver Avg. #
Pure-HV	$2,233,965	$0	$20,097	$2,254,063	13,231
Monopoly, CPM	$2,325,078	$47,805	$1,551	$2,374,434	2,715
Competition, CPM	$2,450,100	$0	$698	$2,450,798	1,500
Monopoly, IPM	$2,136,637	$113,804	$231	$2,250,671	553
Competition, IPM	$2,443,394	$0	$574	$2,443,968	1,194

Table 10 Equilibrium AV capacity, average demand fulfilled by AVs and HVs, and revenue for AVs ($\alpha = 2e^{-4}$, $r = 0.4$)

	AV capacity	Total demand	AV revenue	Average price
Monopoly, CPM	11,822	25,689/hr	$159,381/hr	$18.97/hr
Competition, CPM	13,922	26,355/hr	$130,568/hr	$11.85/hr
Monopoly, IPM	16,488	24,621/hr	$262,502/hr	$27.76/hr
Competition, IPM	14,698	26,309/hr	$137,176/hr	$11.91/hr

Table 11 Surplus and HV employment impact ($\alpha = 8e^{-4}$, $r = 0.4$)

	Surplus Rider	AV	Driver	Total	Driver Avg. #
Pure-HV	$1,975,380	$0	$71,024	$2,046,403	12,487
Monopoly, CPM	$2,220,070	$68,422	$10,651	$2,299,142	3,991
Competition, CPM	$2,442,938	$0	$1,893	$2,444,831	1,157
Monopoly, IPM	$2,114,790	$121,475	$785	$2,237,050	511
Competition, IPM	$2,437,195	$0	$1,530	$2,438,725	897

Table 12 Equilibrium AV capacity, average demand fulfilled by AVs and HVs, and revenue for AVs ($\alpha = 8e^{-4}$, $r = 0.4$)

	AV capacity	Total demand	AV revenue	Average price
Monopoly, CPM	9,645	25,100/hr	$160,412/hr	$22/hr
Competition, CPM	14,494	26,309/hr	$135,451/hr	$12/hr
Monopoly, IPM	16,458	24,498/hr	$269,904/hr	$28/hr
Competition, IPM	15,197	26,266/hr	$141,340/hr	$12/hr