Online Appendix for "Dynamic Resource Allocation on Multi-Category Two-Sided Platforms"

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A. Robustness: Theoretical Model without Taking Logarithm

As discussed in Footnote 1, the linear additive functional form of the marketing response functions (e.g., $n_t = \beta_u u_t + \lambda_n n_{t-1} + \theta_n s_{t-1} + \gamma_{nn} \tilde{n}_{t-1} + \gamma_{ns} \tilde{s}_{t-1}$) can be derived by taking log of the dynamic marketing response functions (e.g., $N_t = U_t^{\beta_u} N_{t-1}^{\lambda_n} S_{t-1}^{\theta_n} \tilde{N}_{t-1}^{\gamma_{nn}} \tilde{S}_{t-1}^{\gamma_{ns}}$) in Sridhar et al. (2011). In this section, we use numerical solutions to show that the results of the per-transaction and per-user models continue to hold without taking log of the variables.

Per-Transaction Model The objective function, without taking log of the variables, is $N_t S_t - U_t - V_t = e^{n_t} e^{s_t} - e^{u_t} - e^{v_t}$. The optimal investment problem becomes

$$\max \quad \left(e^{n_1} e^{s_1} - e^{u_1} - e^{v_1}\right) + \left(e^{\tilde{n}_1} e^{\tilde{s}_1} - e^{\tilde{u}_1} - e^{\tilde{v}_1}\right) + \left(e^{n_2} e^{s_2} - e^{u_2} - e^{v_2}\right) + \left(e^{\tilde{n}_2} e^{\tilde{s}_2} - e^{\tilde{u}_2} - e^{\tilde{v}_2}\right)$$

The marketing response functions are Equations 1 and 2 of the paper. The first-order conditions with respect to the focal category investments u_1, v_1, u_2, and v_2 are

$$\frac{\partial \pi}{\partial u_1} = \beta_u e^{\beta_u u_1 + \beta_v v_1} - e^{u_1} + e^{n_2} e^{s_2} \beta_u \left(\lambda_n + \theta_s\right) + e^{\tilde{n}_2} e^{\tilde{s}_2} \beta_u \left(\tilde{\gamma}_{nn} + \tilde{\gamma}_{sn}\right) = 0$$
$$\frac{\partial \pi}{\partial v_1} = \beta_v e^{\beta_u u_1 + \beta_v v_1} - e^{v_1} + e^{n_2} e^{s_2} \beta_v \left(\lambda_s + \theta_n\right) + e^{\tilde{n}_2} e^{\tilde{s}_2} \beta_v \left(\tilde{\gamma}_{ss} + \tilde{\gamma}_{ns}\right) = 0$$
$$\frac{\partial \pi}{\partial u_2} = \beta_u e^{n_2} e^{s_2} - e^{u_2} = 0$$
$$\frac{\partial \pi}{\partial v_2} = \beta_v e^{n_2} e^{s_2} - e^{v_2} = 0$$

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Similarly for $\tilde{u}_1, \tilde{v}_1, \tilde{u}_2, \tilde{v}_2$. As the system is highly non-linear, it is infeasible to obtain closed-form solutions. We numerically solve for the optimal investments, which are high-dimensional functions of the effect parameters. To show how the optimal investment changes with the effects (i.e., the propositions) in a trackable way, we first parametrize the model using values similar to the estimated ones in the empirical model.[1] We then vary the effect parameters one at a time and plot how the optimal investment changes with that particular effect in Figure 1a. For instance, to test Proposition 1 regarding which side to invest, we vary λ_n while keep the rest of the parameters fixed. We then plot how $I = (u_1^* + u_2^*) - (v_1^* + v_2^*)$ changes on the y-axis with respect to λ_n on the x-axis in the first plot of Figure 1a. We find that the results are consistent with the original propositions of the per-transaction model, suggesting that the original propositions continue to hold when we do not take log of the variables. Note that this exercise can also show the robustness of our findings to alternative parametrization of the model.

Per-User Model The objective function, without taking log of the variables, is the following:

$$\max \quad \sum_{t=1,2} \left(\sqrt{e^{n_t}} + \sqrt{e^{s_t}} - e^{u_t} - e^{v_t}\right) + \left(\sqrt{e^{\tilde{n}_t}} + \sqrt{e^{\tilde{s}_t}} - e^{\tilde{u}_t} - e^{\tilde{v}_t}\right)$$

The marketing response functions are Equations 1 and 2 of the paper. The first-order conditions with respect to the focal category investments u_1, v_1, u_2, and v_2 are

$$\frac{\partial \pi}{\partial u_1} = \frac{\beta_u}{2}\sqrt{e^{\beta_u u_1}} - e^{u_1} + \frac{\lambda_n \beta_u}{2}\sqrt{e^{n_2}} + \frac{\theta_s \beta_u}{2}\sqrt{e^{s_2}} + \frac{\tilde{\gamma}_{nn}\beta_u}{2}\sqrt{e^{\tilde{n}_2}} + \frac{\tilde{\gamma}_{sn}\beta_u}{2}\sqrt{e^{\tilde{s}_2}} = 0$$

$$\frac{\partial \pi}{\partial v_1} = \frac{\beta_v}{2}\sqrt{e^{\beta_v v_1}} - e^{v_1} + \frac{\lambda_s \beta_v}{2}\sqrt{e^{s_2}} + \frac{\theta_n \beta_v}{2}\sqrt{e^{n_2}} + \frac{\tilde{\gamma}_{ss}\beta_v}{2}\sqrt{e^{\tilde{s}_2}} + \frac{\tilde{\gamma}_{ns}\beta_v}{2}\sqrt{e^{\tilde{n}_2}} = 0$$

$$\frac{\partial \pi}{\partial u_2} = \frac{\beta_u}{2}\sqrt{e^{n_2}} - e^{u_2} = 0$$

$$\frac{\partial \pi}{\partial v_2} = \frac{\beta_v}{2}\sqrt{e^{s_2}} - e^{v_2} = 0$$

Similar to the per-transaction model, we numerically solve for the optimal investments using the same parametrization and plot how the optimal investment changes with each effect in Figure 1b. We find that the results are consistent with the original propositions of the per-user model. Again, it suggests that the original propositions continue to hold when we do not take log of the variables.

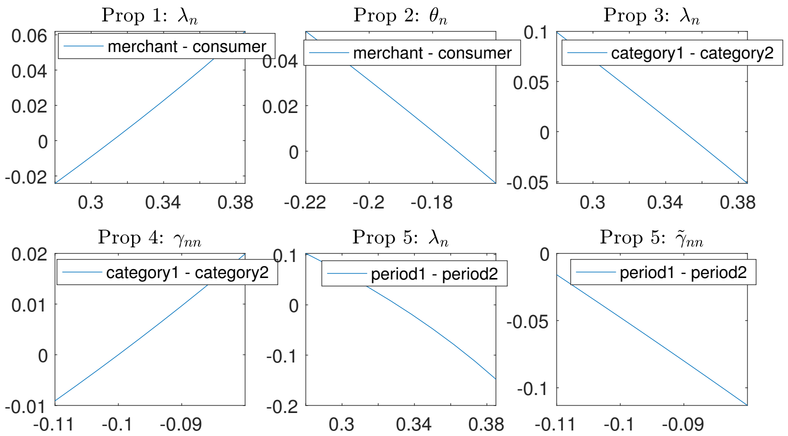
B. Derivation of Model Extensions: Category Asymmetry

Per-Transaction Model Suppose the category-specific fees are $\{a, 1\}$ for categories

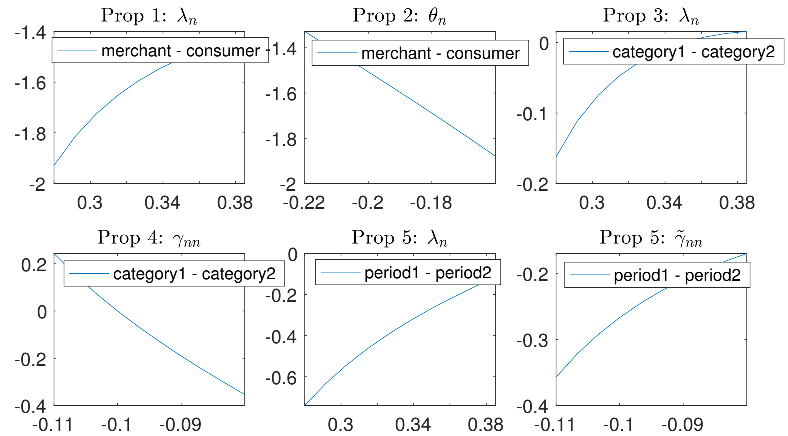
[1] Specifically, we let the focal category parameters take the values of $\lambda_n = \tilde{\lambda}_n = 0.35, \theta_n = \tilde{\theta}_n = -0.2, \lambda_s = \tilde{\lambda}_s = 0.35, \theta_s = \tilde{\theta}_s = -0.2, \gamma_{nn} = \tilde{\gamma}_{nn} = -0.1, \gamma_{ns} = \tilde{\gamma}_{ns} = 0.1, \gamma_{ss} = \tilde{\gamma}_{ss} = -0.1, \gamma_{sn} = \tilde{\gamma}_{sn} = -0.1$. The results are robust if we let the two categories to be asymmetric in the effect sizes and if we adopt alternative parametrization.

Figure 1: Alternative Functional Form: without Taking ln

(a) Per-Transaction Model



(b) Per-User Model



Notes: We plot the optimal investment I on the y-axis and the effect parameter on the x-axis. For Propositions 1 and 2 regarding which side to invest, $I = (u_1^* + u_2^*) - (v_1^* + v_2^*)$. For Propositions 3 and 4 regarding which category to invest, $I = (v_1^* + v_2^* + u_1^* + u_2^*) - (\tilde{v}_1^* + \tilde{v}_2^* + \tilde{u}_1^* + \tilde{u}_2^*)$. For Proposition 5 regarding when to invest, $I = (v_1^* + v_2^* + u_1^* + u_2^*) - (\tilde{v}_1^* + \tilde{v}_2^* + \tilde{u}_1^* + \tilde{u}_2^*)$.

1 and 2. The optimal investment problem is

$$\max \quad (an_1s_1 - u_1 - v_1) + (\tilde{n}_1\tilde{s}_1 - \tilde{u}_1 - \tilde{v}_1) + (an_2s_2 - u_2 - v_2) + (\tilde{n}_2\tilde{s}_2 - \tilde{u}_2 - \tilde{v}_2)$$

The optimal investment solutions are

$$\begin{aligned}
u_1^* &= \frac{1}{a\beta_u\beta_v}\left(1 - \lambda_s - \theta_n\frac{\beta_v}{\beta_u} - \tilde{\gamma}_{ss} - \tilde{\gamma}_{ns}\frac{\beta_v}{\beta_u}\right) \\
v_1^* &= \frac{1}{a\beta_u\beta_v}\left(1 - \lambda_n - \theta_s\frac{\beta_u}{\beta_v} - \tilde{\gamma}_{nn} - \tilde{\gamma}_{sn}\frac{\beta_u}{\beta_v}\right) \\
u_2^* &= \frac{1}{a\beta_u\beta_v} - \left(\lambda_n u_1^* + \theta_n\frac{\beta_v}{\beta_u}v_1^* + \gamma_{nn}\tilde{u}_1^* + \gamma_{ns}\frac{\beta_v}{\beta_u}\tilde{v}_1^*\right) \\
v_2^* &= \frac{1}{a\beta_u\beta_v} - \left(\lambda_s v_1^* + \theta_s\frac{\beta_u}{\beta_v}u_1^* + \gamma_{ss}\tilde{v}_1^* + \gamma_{sn}\frac{\beta_u}{\beta_v}\tilde{u}_1^*\right)
\end{aligned}$$

Similarly for $\tilde{u}_1, \tilde{v}_1, \tilde{u}_2, \tilde{v}_2$. We examine how changes in a affect the optimal investment (e.g., $\frac{\partial u_1^*}{\partial a}$) and whether it amplifies or dampens the propositions (e.g., $\frac{\partial^2\left(u_1^*+u_2^*\right)-\left(v_1^*+v_2^*\right)}{\partial\lambda_n\partial a}$).

First, when a is larger, the investments in the focal category $\{u_1^*, v_1^*, u_2^*, v_2^*\}$ decrease ($\frac{\partial u_1^*}{\partial a} < 0$, $\frac{\partial v_1^*}{\partial a} < 0$, $\frac{\partial u_2^*}{\partial a} < 0$, $\frac{\partial v_2^*}{\partial a} < 0$) while the investments in the other category $\{\tilde{u}_1^*, \tilde{v}_1^*\}$ remain the same for period 1 and $\{\tilde{u}_2^*, \tilde{v}_2^*\}$ increase for period 2, so a higher fee of a category decreases the relative investments to that category. This is consistent with the "compensatory" rule in terms of which category to invest for the per-transaction model.

Second, the propositions still qualitatively hold, while the magnitudes of the effects change with a. In particular, as shown in Table 1a, the column of "original proposition" has the same signs as before, indicating that introducing a produces the same qualitative results. The column of "effect of a" suggests that a higher fee of the focal category a will dampen an effect when the driver of the effect is the focal category (e.g., λ_n, θ_n, $\tilde{\gamma}_{nn}$, $\tilde{\gamma}_{ns}$); a higher fee of the focal category a will have no effect when the driver of the effect is the other category (e.g., γ_{nn}, γ_{ns}).

Per-User Model Suppose the category-specific fees are $\{a, 1\}$ for categories 1 and 2. The optimal investment problem is

$$\begin{aligned}
\max \quad & (a\sqrt{n_1} + a\sqrt{s_1} - u_1 - v_1) + \left(\sqrt{\tilde{n}_1} + \sqrt{\tilde{s}_1} - \tilde{u}_1 - \tilde{v}_1\right) \\
& + (a\sqrt{n_2} + a\sqrt{s_2} - u_2 - v_2) + \left(\sqrt{\tilde{n}_2} + \sqrt{\tilde{s}_2} - \tilde{u}_2 - \tilde{v}_2\right)
\end{aligned}$$

Table 1: Extension: Asymmetric Category-Specific Prices

(a) Per-Transaction Model

		Focal Parameter	Original Proposition	Effect of a	
Which side	Prop 1	λ_n	$\frac{\partial I}{\partial \lambda_n} > 0$	$\frac{\partial^2 I}{\partial \lambda_n \partial a} < 0$	dampen
$I = (u_1^* + u_2^*) - (v_1^* + v_2^*)$	Prop 2	θ_n	$\frac{\partial I}{\partial \theta_n} < 0$	$\frac{\partial^2 I}{\partial \theta_n \partial a} > 0$	dampen
Which category	Prop 3	λ_n	$\frac{\partial I}{\partial \lambda_n} < 0$	$\frac{\partial^2 I}{\partial \lambda_n \partial a} > 0$	dampen
$I = (v_1^* + v_2^* + u_1^* + u_2^*)$		θ_n	$\frac{\partial I}{\partial \theta_n} < 0$	$\frac{\partial^2 I}{\partial \theta_n \partial a} > 0$	dampen
$-(\tilde{v}_1^* + \tilde{v}_2^* + \tilde{u}_1^* + \tilde{u}_2^*)$	Prop 4	γ_{nn}	$\frac{\partial I}{\partial \gamma_{nn}} > 0$	$\frac{\partial^2 I}{\partial \gamma_{nn} \partial a} = 0$	no change
		γ_{ns}	$\frac{\partial I}{\partial \gamma_{ns}} > 0$	$\frac{\partial^2 I}{\partial \gamma_{ns} \partial a} = 0$	no change
When	Prop 5	λ_n	$\frac{\partial I}{\partial \lambda_n} < 0$	$\frac{\partial^2 I}{\partial \lambda_n \partial a} > 0$	dampen
$I = (v_1^* + u_1^*) - (v_2^* + u_2^*)$		θ_n	$\frac{\partial I}{\partial \theta_n} < 0$	$\frac{\partial^2 I}{\partial \theta_n \partial a} > 0$	dampen
		$\tilde{\gamma}_{nn}$	$\frac{\partial I}{\partial \tilde{\gamma}_{nn}} < 0$	$\frac{\partial^2 I}{\partial \tilde{\gamma}_{nn} \partial a} > 0$	dampen
		$\tilde{\gamma}_{ns}$	$\frac{\partial I}{\partial \tilde{\gamma}_{ns}} < 0$	$\frac{\partial^2 I}{\partial \tilde{\gamma}_{ns} \partial a} > 0$	dampen

(b) Per-User Model

		Focal Parameter	Original Proposition	Effect of a	
Which side	Prop 1	λ_n	$\frac{\partial I}{\partial \lambda_n} > 0$	$\frac{\partial^2 I}{\partial \lambda_n \partial a} > 0$	amplify
$I = (u_1^* + u_2^*) - (v_1^* + v_2^*)$	Prop 2	θ_n	$\frac{\partial I}{\partial \theta_n} < 0$	$\frac{\partial^2 I}{\partial \theta_n \partial a} < 0$	amplify
Which category	Prop 3	λ_n	$\frac{\partial I}{\partial \lambda_n} > 0$	$\frac{\partial^2 I}{\partial \lambda_n \partial a} > 0$	amplify
$I = (v_1^* + v_2^* + u_1^* + u_2^*)$		θ_n	$\frac{\partial I}{\partial \theta_n} > 0$	$\frac{\partial^2 I}{\partial \theta_n \partial a} > 0$	amplify
$-(\tilde{v}_1^* + \tilde{v}_2^* + \tilde{u}_1^* + \tilde{u}_2^*)$	Prop 4	γ_{nn}	$\frac{\partial I}{\partial \gamma_{nn}} < 0$	$\frac{\partial^2 I}{\partial \gamma_{nn} \partial a} = 0$	no change
		γ_{ns}	$\frac{\partial I}{\partial \gamma_{ns}} < 0$	$\frac{\partial^2 I}{\partial \gamma_{ns} \partial a} = 0$	no change
When	Prop 5	λ_n	$\frac{\partial I}{\partial \lambda_n} > 0$	$\frac{\partial^2 I}{\partial \lambda_n \partial a} > 0$	amplify
$I = (v_1^* + u_1^*) - (v_2^* + u_2^*)$		θ_n	$\frac{\partial I}{\partial \theta_n} > 0$	$\frac{\partial^2 I}{\partial \theta_n \partial a} > 0$	amplify
		$\tilde{\gamma}_{nn}$	$\frac{\partial I}{\partial \tilde{\gamma}_{nn}} > 0$	$\frac{\partial^2 I}{\partial \tilde{\gamma}_{nn} \partial a} > 0$	amplify
		$\tilde{\gamma}_{ns}$	$\frac{\partial I}{\partial \tilde{\gamma}_{ns}} > 0$	$\frac{\partial^2 I}{\partial \tilde{\gamma}_{ns} \partial a} > 0$	amplify

The optimal solutions are

$$
\begin{aligned}
u_1^* &= \frac{a^2\beta_u}{4\left(1-\lambda_n-\theta_s\frac{\beta_u}{\beta_v}-\tilde{\gamma}_{nn}-\tilde{\gamma}_{sn}\frac{\beta_u}{\beta_v}\right)^2}\\
v_1^* &= \frac{a^2\beta_v}{4\left(1-\lambda_s-\theta_n\frac{\beta_v}{\beta_u}-\tilde{\gamma}_{ss}-\tilde{\gamma}_{ns}\frac{\beta_v}{\beta_u}\right)^2}\\
u_2^* &= \frac{a^2\beta_u}{4}-\left(\lambda_n u_1^*+\theta_n\frac{\beta_v}{\beta_u}v_1^*+\gamma_{nn}\tilde{u}_1^*+\gamma_{ns}\frac{\beta_v}{\beta_u}\tilde{v}_1^*\right)\\
v_2^* &= \frac{a^2\beta_v}{4}-\left(\lambda_s v_1^*+\theta_s\frac{\beta_u}{\beta_v}u_1^*+\gamma_{ss}\tilde{v}_1^*+\gamma_{sn}\frac{\beta_u}{\beta_v}\tilde{u}_1^*\right)
\end{aligned}
$$

Similarly for $\tilde{u}_1, \tilde{v}_1, \tilde{u}_2, \tilde{v}_2$. We examine how changes in a affect the optimal investment (e.g., $\frac{\partial u_1^*}{\partial a}$) and whether it amplifies or dampens the propositions (e.g., $\frac{\partial^2\left(u_1^*+u_2^*\right)-\left(v_1^*+v_2^*\right)}{\partial\lambda_n\partial a}$).

First, when a is larger, the investments in the focal category $\{u_1^*, v_1^*, u_2^*, v_2^*\}$ increase ($\frac{\partial u_1^*}{\partial a}>0$, $\frac{\partial v_1^*}{\partial a}>0$, $\frac{\partial u_2^*}{\partial a}>0$, $\frac{\partial v_2^*}{\partial a}>0$) while the investments in the other category $\{\tilde{u}_1^*, \tilde{v}_1^*\}$ remain the same for period 1 and $\{\tilde{u}_2^*, \tilde{v}_2^*\}$ decrease for period 2, so a higher fee of a category increases the relative investment to that category. This is consistent with the "reinforcing" rule in terms of which category to invest for the per-user model.

Second, the propositions still qualitatively hold, while the magnitudes of the effects change with a. In particular, as shown in Table 1b, the column of "original proposition" has the same signs as before, indicating that introducing a produces the same qualitative results. The column of "effect of a" suggests that a higher fee of the focal category a will amplify an effect when the driver of the effect is the focal category (e.g., λ_n, θ_n, $\tilde{\gamma}_{nn}$, $\tilde{\gamma}_{ns}$); a higher price of the focal category a has no effect when the driver of the effect is the other category (e.g., γ_{nn}, γ_{ns}).

C. Derivation of Model Extensions: Side Asymmetry

We examine side asymmetry for the per-user model as the fee is not charged by side in the per-transaction model. Suppose in the per-user model platforms charge asymmetric fees $\{1, \alpha\}$ on the merchant- and consumer-side users, respectively. The optimal investment problem becomes

$$
\begin{aligned}
\max \quad & \left(\sqrt{n_1}+\alpha\sqrt{s_1}-u_1-v_1\right)+\left(\sqrt{\tilde{n}_1}+\alpha\sqrt{\tilde{s}_1}-\tilde{u}_1-\tilde{v}_1\right)\\
& +\left(\sqrt{n_2}+\alpha\sqrt{s_2}-u_2-v_2\right)+\left(\sqrt{\tilde{n}_2}+\alpha\sqrt{\tilde{s}_2}-\tilde{u}_2-\tilde{v}_2\right)
\end{aligned}
$$

Table 2: Extension: Asymmetric Fees in Per-User Model

		Focal Parameter	Original Proposition	Effect of α	
Which side	Prop 1	λ_n	$\frac{\partial I}{\partial \lambda_n} > 0$	$\frac{\partial^2 I}{\partial \lambda_n \partial \alpha} = 0$	no change
$I = (u_1^* + u_2^*) - (v_1^* + v_2^*)$	Prop 2	θ_n	$\frac{\partial I}{\partial \theta_n} < 0$	$\frac{\partial^2 I}{\partial \theta_n \partial \alpha} < 0$	amplify
Which category	Prop 3	λ_n	$\frac{\partial I}{\partial \lambda_n} > 0$	$\frac{\partial^2 I}{\partial \lambda_n \partial \alpha} = 0$	no change
$I = (v_1^* + v_2^* + u_1^* + u_2^*)$		θ_n	$\frac{\partial I}{\partial \theta_n} > 0$	$\frac{\partial^2 I}{\partial \theta_n \partial \alpha} > 0$	amplify
$-(\tilde{v}_1^* + \tilde{v}_2^* + \tilde{u}_1^* + \tilde{u}_2^*)$	Prop 4	γ_{nn}	$\frac{\partial I}{\partial \gamma_{nn}} < 0$	$\frac{\partial^2 I}{\partial \gamma_{nn} \partial \alpha} = 0$	no change
		γ_{ns}	$\frac{\partial I}{\partial \gamma_{ns}} < 0$	$\frac{\partial^2 I}{\partial \gamma_{ns} \partial \alpha} < 0$	amplify
When	Prop 5	λ_n	$\frac{\partial I}{\partial \lambda_n} > 0$	$\frac{\partial^2 I}{\partial \lambda_n \partial \alpha} = 0$	no change
$I = (v_1^* + u_1^*) - (v_2^* + u_2^*)$		θ_n	$\frac{\partial I}{\partial \theta_n} > 0$	$\frac{\partial^2 I}{\partial \theta_n \partial \alpha} > 0$	amplify
		$\tilde{\gamma}_{nn}$	$\frac{\partial I}{\partial \tilde{\gamma}_{nn}} > 0$	$\frac{\partial^2 I}{\partial \tilde{\gamma}_{nn} \partial \alpha} = 0$	no change
		$\tilde{\gamma}_{ns}$	$\frac{\partial I}{\partial \tilde{\gamma}_{ns}} > 0$	$\frac{\partial^2 I}{\partial \tilde{\gamma}_{ns} \partial \alpha} > 0$	amplify

The optimal solution becomes

$$
\begin{aligned}
u_1^* &= \frac{\beta_u}{4\left(1 - \lambda_n - \theta_s \frac{\beta_u}{\beta_v} - \tilde{\gamma}_{nn} - \tilde{\gamma}_{sn} \frac{\beta_u}{\beta_v}\right)^2} \\
v_1^* &= \frac{\alpha^2 \cdot \beta_v}{4\left(1 - \lambda_s - \theta_n \frac{\beta_v}{\beta_u} - \tilde{\gamma}_{ss} - \tilde{\gamma}_{ns} \frac{\beta_v}{\beta_u}\right)^2} \\
u_2^* &= \frac{\beta_u}{4} - \left(\lambda_n u_1^* + \theta_n \frac{\beta_v}{\beta_u} v_1^* + \gamma_{nn} \tilde{u}_1^* + \gamma_{ns} \frac{\beta_v}{\beta_u} \tilde{v}_1^*\right) \\
v_2^* &= \frac{\alpha^2 \cdot \beta_v}{4} - \left(\lambda_s v_1^* + \theta_s \frac{\beta_u}{\beta_v} u_1^* + \gamma_{ss} \tilde{v}_1^* + \gamma_{sn} \frac{\beta_u}{\beta_v} \tilde{u}_1^*\right)
\end{aligned}
$$

We examine how changes in α affect the optimal investment (e.g., $\frac{\partial u_1^*}{\partial \alpha}$) and whether it amplifies or dampens the propositions (e.g., $\frac{\partial^2 \left(u_1^* + u_2^*\right) - \left(v_1^* + v_2^*\right)}{\partial \lambda_n \partial \alpha}$).

First, when α is larger, the consumer-side investments v_1^*, v_2^* increase ($\frac{\partial v_1^*}{\partial \alpha} > 0$, $\frac{\partial v_2^*}{\partial \alpha} > 0$) while the merchant-side investment u_1^* remains the same and u_2^* decreases ($\frac{\partial u_2^*}{\partial \alpha} < 0$), so a larger fee on the consumer side increases the relative investment to the consumer side. This is consistent with the "reinforcing" rule in terms of which side to invest for the per-user model.

Second, the propositions still qualitatively hold, while the magnitudes of the effects change with α. In particular, as shown in Table 2, the column of "original proposition" has the same signs as before, indicating that introducing α produces the same qualitative results. The column of "effect of α" suggests that a larger fee on the consumer side α will amplify an effect when the driver of the effect is the consumer side (e.g., θ_n, γ_{ns}, $\tilde{\gamma}_{ns}$); a larger fee on the consumer side α will have no effect when the driver of the effect is the merchant side (e.g., λ_n, γ_{nn}, $\tilde{\gamma}_{nn}$).

D. Robustness Check: Vary Assumptions on Observed Investment

The main specification assumes that Groupon's merchant-side investment in a category-market-period is a function of the observed category-market level number of deals and fraction of new merchants in the *current* period. We conduct robustness checks by re-estimating the model under two alternative assumptions: 1) Groupon followed a "reinforcing" rule in practice so that the category-market level investment in a period is positively proportional to the observed category-market level number of deals in the *last* period; 2) Groupon followed a "compensatory" rule in practice so that the category-market level investment in a period is negatively proportional to the observed category-market level number of deals in the *last* period.[2] As shown in Tables 3 and 4, the estimation results are robust to different assumptions on the observed investment.

[2] To operationalize "positively proportional" and "negatively proportional," suppose the number of deals in a particular category-market is n_i and the total number of deals is $N \equiv \sum_i n_i$. "Positively proportional" suggests that the fraction of investment allocated to that category-market is $\frac{n_i}{N}$ so that a larger n_i leads to more investment. "Negatively proportional" suggests that the fraction of investment allocated to that category-market is $\frac{N-n_i}{\sum_i (N-n_i)}$ so that a larger n_i leads to less investment.

Table 3: Robustness Checks: Merchant-side Regression Estimation Results

Merchant-side Regression		Main	Reinforcing	Compensatory
Investment (β_u)		0.479***	0.157***	0.163**
		(0.0713)	(0.0191)	(0.0808)
Own dynamic (λ_{nj})	Beauty	0.288***	0.190***	0.307***
		(0.0683)	(0.0720)	(0.0695)
	Fitness	0.238**	0.250**	0.318***
		(0.101)	(0.105)	(0.102)
	Entertainment	0.374***	0.303***	0.396***
		(0.0547)	(0.0586)	(0.0554)
	Restaurant	0.339***	0.314***	0.404***
		(0.0519)	(0.0538)	(0.0516)
Cross-side (θ_{nj})	Beauty	-0.232***	-0.212***	-0.234***
		(0.0460)	(0.0466)	(0.0461)
	Fitness	-0.0407	0.0578	-0.00698
		(0.0394)	(0.0397)	(0.0408)
	Entertainment	0.141***	0.223***	0.216***
		(0.0445)	(0.0442)	(0.0439)
	Restaurant	-0.0970*	-0.178***	-0.187***
		(0.0533)	(0.0523)	(0.0537)
Cross-category (γ_{nnj})	Beauty	0.0665	0.198**	0.144*
		(0.0762)	(0.0771)	(0.0760)
	Fitness	0.0589	-0.0993	-0.0740
		(0.0740)	(0.0717)	(0.0727)
	Entertainment	-0.147***	-0.0730	-0.103*
		(0.0538)	(0.0546)	(0.0541)
	Restaurant	-0.0731*	-0.0254	-0.0680
		(0.0410)	(0.0420)	(0.0416)
Cross-category (γ_{nsj})	Beauty	0.128***	0.117**	0.137***
		(0.0448)	(0.0455)	(0.0454)
	Fitness	-0.0447	-0.0660	-0.0205
		(0.0435)	(0.0443)	(0.0454)
	Entertainment	-0.148***	-0.253***	-0.252***
		(0.0503)	(0.0490)	(0.0488)
	Restaurant	0.164***	0.217***	0.233***
		(0.0592)	(0.0594)	(0.0609)
Return		0.811***	0.307***	0.314***
		(0.0793)	(0.0243)	(0.0241)
Past experience		0.0694***	0.0656***	0.0664***
		(0.00491)	(0.00490)	(0.00487)
Market fixed effects		YES	YES	YES
Category fixed effects		YES	YES	YES
Market structure		YES	YES	YES
Time		YES	YES	YES
N.		9,663	9,663	9,663
R-squared		0.611	0.577	0.581

Notes : Standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Table 4: Robustness Checks: Consumer-side Regression Estimation Results

Consumer-side Regression		Main	Reinforcing	Compensatory
Investment (β_v)		0.0532***	0.0474***	0.0423**
		(0.0173)	(0.0170)	(0.0173)
Own dynamic (λ_{sj})	Beauty	0.0840	0.0473	0.0599
		(0.0729)	(0.0720)	(0.0728)
	Fitness	0.0744	0.0791	0.0970
		(0.0634)	(0.0628)	(0.0628)
	Entertainment	0.348***	0.363***	0.363***
		(0.0691)	(0.0684)	(0.0694)
	Restaurant	-0.0364	-0.0234	-0.0773
		(0.0828)	(0.0817)	(0.0813)
Cross-side (θ_{sj})	Beauty	-0.466***	-0.524***	-0.500***
		(0.107)	(0.106)	(0.107)
	Fitness	0.105	-0.0274	0.130
		(0.160)	(0.154)	(0.161)
	Entertainment	-0.119	-0.172**	-0.108
		(0.0874)	(0.0851)	(0.0878)
	Restaurant	-0.223***	-0.246***	-0.224***
		(0.0806)	(0.0797)	(0.0811)
Cross-category (γ_{ssj})	Beauty	-0.0135	0.0244	-0.00298
		(0.0715)	(0.0704)	(0.0718)
	Fitness	0.111	0.117*	0.0835
		(0.0713)	(0.0706)	(0.0706)
	Entertainment	-0.188**	-0.198***	-0.208***
		(0.0756)	(0.0748)	(0.0760)
	Restaurant	0.0633	0.0559	0.107
		(0.0942)	(0.0930)	(0.0926)
Cross-category (γ_{snj})	Beauty	0.215*	0.261**	0.263**
		(0.118)	(0.117)	(0.117)
	Fitness	-0.258**	-0.186*	-0.281**
		(0.114)	(0.110)	(0.114)
	Entertainment	-0.164*	-0.133	-0.168*
		(0.0862)	(0.0848)	(0.0864)
	Restaurant	-0.0214	-0.0197	-0.00879
		(0.0648)	(0.0642)	(0.0649)
Discount		0.605***	0.611***	0.614***
		(0.0791)	(0.0779)	(0.0790)
Price		-0.356***	-0.359***	-0.360***
		(0.0160)	(0.0157)	(0.0159)
Duration		-0.105***	-0.136***	-0.144***
		(0.0321)	(0.0316)	(0.0320)
Market fixed effects		YES	YES	YES
Category fixed effects		YES	YES	YES
Market structure		YES	YES	YES
Time		YES	YES	YES
N.		9,663	9,663	9,663
R-squared		0.534	0.532	0.516

Notes : Standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1.