ONLINE APPENDIX

A1. Proof of Lemma 4

Lemma 4. *Define* $\underline{v}^*(\phi\theta) = Max\{\phi\theta\left(\pi_B^L(n) + w + X(2,n)\right) - w, -1\}$. *Then, there exists a symmetric equilibrium in the early market where the bidding strategy of a buyer with idiosyncratic valuation v is given by:*

i) *Bid its true valuation* $(w + v)$ *if* $v \geq \underline{v}^*(\phi\theta)$;

ii) *Do not bid if* $v < \underline{v}^*(\phi\theta)$.

Proof: In the construction of the equilibrium, we assume that buyers with a valuation $v \geq R(\phi\theta)$ follow a symmetric bidding strategy *b(v)* where $\frac{\partial b(v)}{\partial v} > 0$. Also, assume for now that $\phi\theta\left(\pi_B^L(n) + w + X(2,n)\right) - w \geq -1$. A buyer of type $\underline{v}^*(\phi\theta)$ bids $b\left(\underline{v}^*(\phi\theta)\right)$ and wins only if all other buyers have valuations below $\underline{v}^*(\phi\theta)$. Hence, whenever a buyer of type $\underline{v}^*(\phi\theta)$ wins, it pays the reservation price $R(\phi\theta)$. For a buyer of type $\underline{v}^*(\phi\theta)$ the alternative to bidding $b\left(\underline{v}^*(\phi\theta)\right)$ is not bidding. If all other bidders have valuations below $\underline{v}^*(\phi\theta)$, the expected profit from not bidding is $\phi\theta\pi_B^L(n)$, which is the expected profit of the buyer when the startup is sold in the late market. In equilibrium, the marginal type $\underline{v}^*(\phi\theta)$ has to be indifferent between bidding and not bidding:

$$F\left(\underline{v}^*(\phi\theta), m-1\right)\left(\underline{v}^*(\phi\theta) + w - R(\phi\theta)\right) = F(\underline{v}^*(\phi\theta), m-1)\phi\theta\pi_B^L(n) \Leftrightarrow$$

$$\underline{v}^*(\phi\theta) = \phi\theta\pi_B^L(n) + R(\phi\theta) - w = \phi\theta\left(\pi_B^L(n) + w + X(2,n)\right) - w.$$

Consider now types $v > \underline{v}^*(\phi\theta)$. Two situations can occur: there are either competing buyers or not. If there are competing buyers, it is optimal to bid the true valuation. The startup is sold in the early market because there is at least one other buyer that bids above $R(\phi\theta)$, and it follows from standard second-price auction logic that bidding the true valuation is optimal. In the other case, without competing buyers, it is also optimal to bid the true valuation. The startup is then bought at the price $R(\phi\theta)$ which (i) results in higher profits than buying in the late market since $v > \underline{v}^*(\phi\theta)$, and (ii) $R(\phi\theta)$ is the lowest price at which the startup can be acquired.

If $\phi\theta\left(\pi_B^L(n) + w + X(2,n)\right) - w < -1$, a similar argument establishes that $\underline{v}^*(\phi\theta) = -1$ and it is optimal for a buyer to bid its valuation for all v. □

A2. Proofs of Lemma 5 and Proposition 1

We first characterize the profit from pursuing flexibility and compare it to the alternatives for $c = 0$, that is $\phi = 1$. The results for this case are developed in Lemmata A2.2-A2.4. Afterwards, we prove the results in the main text where investing in seeking early offers and in developing the invention simultaneously come at a cost, $0 < c \leq R$, that is $\phi < 1$. We start by proving a condition (A2.1) that will be helpful in subsequent steps of the proofs.

Lemma A2.1. *Consider the uniform distribution and the following inequality:*

$$\frac{G(\underline{v}^*(\theta),m)}{g(\underline{v}^*(\theta),m)} \geq \theta\pi_B^L(n)\left(1+\frac{\pi_B^L(n)}{w+X(2,n)}\right) \tag{A2.1}$$

Suppose first that n is finite. Then, there exists some $\theta_0 \in \left(\frac{w-1}{w+X(2,n)+\pi_B^L(n)},1\right)$ such that (A2.1) holds if and only if $\theta > \theta_0$. If instead $n \to \infty$, inequality (A2.1) holds for all values of θ.

Proof: For the uniform distribution, it holds that (i) $G(-1,m)/g(-1,m) = 0$, and that (ii) $G(v,m)/g(v,m)$ is increasing and strictly convex in *v*. Consider first *n* finite for which $\pi_B^L(n) > 0$. For $\theta \leq \left(\frac{w-1}{w+X(2,n)+\pi_B^L(n)}\right)$, we have that $\underline{v}^*(\theta) = -1$. As $\frac{G(-1,m)}{g(-1,m)} = 0$ and the right hand side of inequality (A2.1) is strictly positive, it follows that (A2.1) does not hold for $\theta \leq \left(\frac{w-1}{w+X(2,n)+\pi_B^L(n)}\right)$. Consider now $\theta = 1$. The right hand side of inequality (A2.1) is decreasing in *w*. Furthermore, calculations show that inequality (A2.1) holds for $w = 1$ and $\theta = 1$ for all permissible values of *n*. Since $w > 1$ by assumption, we have that inequality (A2.1) holds for $\theta = 1$. Since the left hand side of inequality (A2.1) is increasing and strictly convex in θ, whereas the right hand side of (A2.1) is increasing and linear in θ, these results imply together with continuity that there exists some $\theta_0 \in \left(\frac{w-1}{w+X(2,n)+\pi_B^L(n)},1\right)$ such that (A2.1) holds if and only if for $\theta > \theta_0$. Finally, for $n \to \infty$, $\pi_B^L(n) \to 0$ and inequality (A2.1) holds for all values of θ. □

A2.1. The case of c = 0

Lemma A2.2. *Suppose that the startup invests both in seeking early offers and in developing the invention into an innovation. For $c = 0$, the startup's profit from flexibility is continuous in θ, and the profit function consists of two parts:*

(i) *For* $\theta \le \frac{w-1}{w+X(2,n)+\pi_B^L(n)}$, *the startup's profit from flexibility is independent of* θ *and is equal to* $\Pi^F = w + X(2,m)$.

(ii) *For* $\theta > \frac{w-1}{w+X(2,n)+\pi_B^L(n)}$, *the startup's profit from flexibility first declines and then increases with* θ *for n finite. The startup's profit from flexibility increases with* θ *for* $n \to \infty$.

Proof: We have that $\underline{v}^*(\theta) = -1$ for $\theta \le \frac{w-1}{w+X(2,n)+\pi_B^L(n)}$. Hence, the startup is always sold in the early market, and the startup's expected profit is $w + X(2,m)$. This proves part (i) of Lemma A2.2. Consider now $\theta > \frac{w-1}{w+X(2,n)+\pi_B^L(n)}$ where the startup may continue to the late market. Notice that $\Pi^F \to w + X(2,m)$ for $\theta \to \left(\frac{w-1}{w+X(2,n)+\pi_B^L(n)}\right)_+$ since $\underline{v}^*(\theta) \to -1$ for $\theta \to \left(\frac{w-1}{w+X(2,n)+\pi_B^L(n)}\right)_+$. Therefore, Π^F is continuous, including at $\theta = \frac{w-1}{w+X(2,n)+\pi_B^L(n)}$. Differentiating Π^F with respect to θ and simplifying yields:

$$\partial\Pi^F/_{\partial\theta} = G\big(\underline{v}^*(\theta), m\big)\big(w + X(2,n)\big) - \theta g\big(\underline{v}^*(\theta), m\big)\pi_B^L(n)\left(w + X(2,n) + \pi_B^L(n)\right).$$

Consider first *n* finite. It follows from Lemma A2.1 that $\partial\Pi^F/_{\partial\theta} \ge 0$ if and only if $\theta \ge \theta_0$. Finally, for *n* $\to \infty$, Lemma A2.1 implies that $\partial\Pi^F/_{\partial\theta} > 0$ for $\theta > \frac{w-1}{w+X(2,n)+\pi_B^L(n)}$. Together, these two arguments prove part (ii) of Lemma A2.2. □

Lemma A2.3. *For* $c = 0$, *the following holds:*

(i) $\partial\Pi^F/_{\partial\theta} < \partial\Pi^L/_{\partial\theta}$ *for all* θ, *except for* $\theta = \frac{w-1}{w+X(2,n)+\pi_B^L(n)}$ *where* Π^F *is not differentiable.*

(ii) $\Pi^F > \Pi^L$ *for all* θ,

Proof: Consider first $\theta \le \frac{w-1}{w+X(2,n)+\pi_B^L(n)}$. In this range of θ, the startup's expected profit is $w + X(2,m)$, and $\partial\Pi^F/_{\partial\theta} = 0$. Consider now $\theta > \frac{w-1}{w+X(2,n)+\pi_B^L(n)}$ where the startup may continue to the late market. Differentiating Π^F with respect to θ and simplifying the expression yields:

$$\partial\Pi^F/_{\partial\theta} = G\big(\underline{v}^*(\theta), m\big)\big(w + X(2,n)\big) - \theta g\big(\underline{v}^*(\theta), m\big)\pi_B^L(n)\left(w + X(2,n) + \pi_B^L(n)\right). \quad \text{(A2.2)}$$

Π^F is continuous, but not differentiable at $\theta = \frac{w-1}{w+X(2,n)+\pi_B^L(n)}$ where $\partial\Pi^F/_{\partial\theta} = 0$ for $\theta \to \left(\frac{w-1}{w+X(2,n)+\pi_B^L(n)}\right)_-$ and $\partial\Pi^F/_{\partial\theta}$ is given by equation (A2.2) for $\theta \to \left(\frac{w-1}{w+X(2,n)+\pi_B^L(n)}\right)_+$. Since $\partial\Pi^L/_{\partial\theta} = w + X(2,n)$, part (i) of Lemma A2.3 follows immediately from comparing $\partial\Pi^F/_{\partial\theta}$ and $\partial\Pi^L/_{\partial\theta}$. For

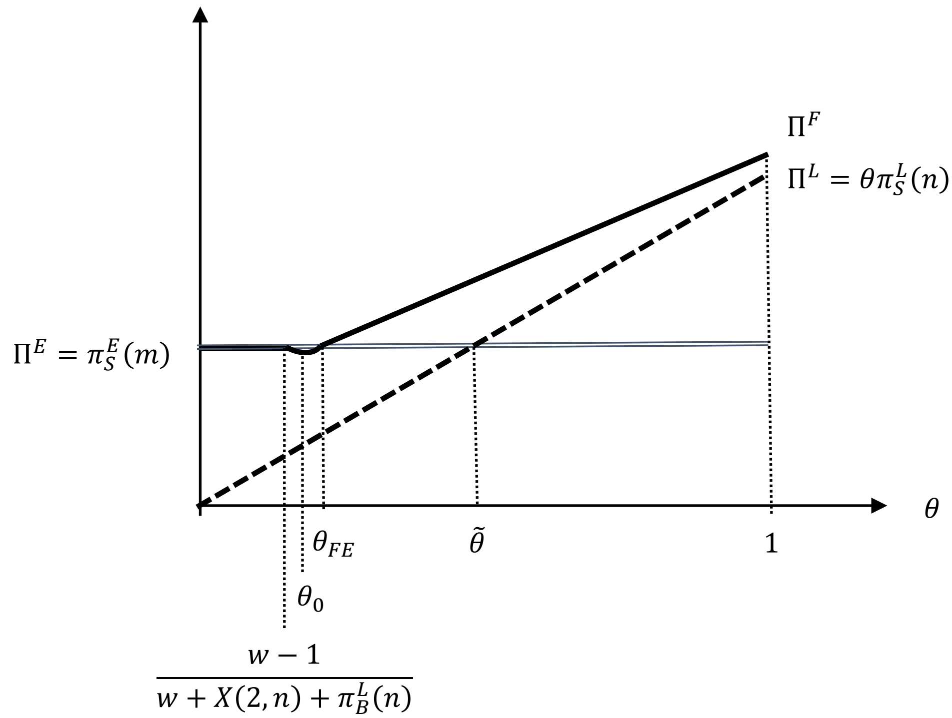
$\theta = 1$, $\Pi^F - \Pi^L$ simplifies to $\int_{\underline{v}^*(1)}^{1}\big(x - X(2,n)\big)g(x,m)dx > 0$. Since $\Pi^F > \Pi^L$ for $\theta = 1$ and $\partial\Pi^F/\partial\theta < \partial\Pi^L/\partial\theta$ everywhere except at $\theta = \frac{w-1}{w+X(2,n)+\pi_B^L(n)}$, part (ii) follows from continuity of Π^L and Π^F. □

Lemma A2.4. *For* $c = 0$, *there exists a value* $\theta_{FE} \in \left(\frac{w-1}{w+X(2,n)+\pi_B^L(n)}, \tilde{\theta}\right)$ *such that* $\Pi^F > \Pi^E$ *if and only if* $\theta > \theta_{FE}$.

Proof: We assume that there is an infinitesimal opportunity cost of time (left out of the equations for simplicity) such that the startup invests in seeking early offers and developing the invention if and only if Π^F is strictly greater than Π^E. Using part (i) of Lemma A2.2, this implies immediately that the startup commits to entering the early market for $\theta \leq \frac{w-1}{w+X(2,n)+\pi_B^L(n)}$. Consider now $\theta > \frac{w-1}{w+X(2,n)+\pi_B^L(n)}$. For *n* finite, it follows from Lemma A2.2 part (ii) that Π^F first decreases in θ for $\theta \leq \theta_0$, and afterward increases in θ for $\theta > \theta_0$. For $\theta = \theta_0$, it follows immediately that $\Pi^F < \Pi^E$. For $\theta = 1$, on the other hand, it follows from Lemma A2.2 and equation (3) in the paper that $\Pi^F > \Pi^L > \Pi^E$. Since Π^F is strictly increasing in θ for $\theta \in (\theta_0, 1]$, there exists a unique threshold of θ denoted θ_{FE} such that $\Pi^F > \Pi^E$ if and only if $\theta > \theta_{FE}$. Furthermore, it follows from $\partial\Pi^F/\partial\theta < \partial\Pi^L/\partial\theta$ and $\Pi^F > \Pi^L$ for $\theta = 1$ that $\theta_{FE} < \tilde{\theta}$. For $n \rightarrow \infty$, Lemma A2.1 implies that the startup's profit from flexibility is strictly greater than $w + X(2,m)$ if and only if $\theta > \frac{w-1}{w+X(2,n)+\pi_B^L(n)} \equiv \theta_{FE}$ where again $\theta_{FE} < \tilde{\theta}$. Taken together, these results prove Lemma A2.4. □

The results for the case of $c = 0$ are illustrated in Figure A1.

Figure A1: *Profit from pursuing the three strategies as function of θ for $c = 0$.*



A2.2. The case of $0 < c \leq R$

Lemma 5. *Let c be small enough and n finite. Suppose that the startup allocates resources to both trying the early market and developing the invention into an innovation. Then, if* $\theta \leq \frac{w-1}{\phi[w+X(2,n)+\pi_B^L(n)]}$, *the startup's profit from flexibility is independent of* θ *and is equal to* $\Pi^E = w + X(2,m)$. *If* $\theta > \frac{w-1}{\phi[w+X(2,n)+\pi_B^L(n)]}$, *the startup's profit from flexibility first declines and then increases with* θ.

<u>Proof</u>: In the general case, Π^F is a function of $\phi\theta$ rather than θ. Then, it follows immediately from Lemma A2.2 that $\Pi^F = w + X(2,m)$ if $\phi\theta \leq \frac{w-1}{w+X(2,n)+\pi_B^L(n)}$, which proves the first part of the lemma. Consider now $\phi\theta > \frac{w-1}{w+X(2,n)+\pi_B^L(n)}$. From the proof of Lemma A2.4 it follows that Π^F first decreases and then increases in θ if and only if $\phi > \theta_0$. If this condition holds, $\phi\theta > \theta_0$ for θ sufficiently close to 1, which ensures that Π^F is increasing in θ for $\theta > \frac{\theta_0}{\phi}$. Denote the unique value of c for which $\phi = \theta_0$ by c_0. Then, the proposition holds for $c < c_0$. □

Proposition 1. *There exist* $\underline{c}$ *and* $\overline{c}$, *0 <* $\underline{c} < \overline{c} < R$, *such that the startup's exit strategies are as follows:*

(i) *If* $c \leq \underline{c}$, *the startup always tries the early market. There exists a threshold* $\theta_L \in (0, \tilde{\theta})$ *such that it also invests in developing the invention, thereby choosing flexibility, if and only if* $\theta > \theta_L$.

(ii) *If* $\underline{c} < c < \overline{c}$, *there exists a threshold* $\theta_H \in (\tilde{\theta}, 1)$ *such that a startup with* $\theta \geq \theta_H$ *commits to the late market. A startup with* $\theta \leq \theta_H$ *always tries the early market; it does also invest in developing the invention if* $\theta_H > \theta > \theta_L$, *thereby choosing flexibility.*

(iii) *If* $c \geq \overline{c}$, *a startup with low* $\theta\,(\theta \leq \tilde{\theta})$ *tries the early market, but does not invest in developing the invention, thereby committing to the early market. A startup with high* $\theta\,(\theta > \tilde{\theta})$ *commits to the late market.*

<u>Proof</u>: It follows from the proof of Lemma A2.4 that $\Pi^F > \Pi^E$ for $\theta > \frac{\theta_{FE}}{\phi} \equiv \theta_L$. Furthermore, it can be shown as in the proof of Lemma A2.3 that $\partial\Pi^F/\partial\theta < \partial\Pi^L/\partial\theta$. Hence, the flexible strategy is chosen for some interval of θ greater than θ_L if and only if $\Pi^F > \Pi^L$ for $\theta = \theta_L \Leftrightarrow w + X(2,m) > \theta_{FE}/\phi\,(w + X(2,n)) \Leftrightarrow \phi > \theta_{FE}\frac{w+X(2,n)}{w+X(2,m)} = \theta_{FE}/\tilde{\theta}$. Denote the unique value of c for which $\phi = \theta_{FE}/\tilde{\theta}$ by $\overline{c}$. If $c \geq \overline{c}$, the startup never chooses flexibility, and the optimal choice of strategy detailed in part (iii) of the proposition follows from the analysis in the main text. Consider now $c < \overline{c}$. Here, commitment to early exit is optimal for $\theta \leq \theta_L$, and flexibility is optimal for some range of θ greater than θ_L. Define $\underline{c}$ implicitly by $\Pi^F = \Pi^L$ for $\theta = 1$. Notice that $\Pi^F = \Pi^L$ for $\theta = \tilde{\theta}$ and $\phi = \phi(R - \overline{c})$ as well as for $\theta = 1$ and $\phi = \phi(R - \underline{c})$. Since $0 < \partial\Pi^F/\partial\theta < \partial\Pi^L/\partial\theta$ in the relevant range of θ, this implies that $\phi(R - \overline{c}) < \phi(R - \underline{c}) \Leftrightarrow \underline{c} < \overline{c}$. Then, if $\underline{c} < c < \overline{c}$, we have that $\Pi^F > \Pi^L$ for $\theta = \theta_L$ and $\Pi^L > \Pi^F$ for $\theta = 1$. It follows from continuity of the profit functions and from $0 < \partial\Pi^F/\partial\theta < \partial\Pi^L/\partial\theta$ that there exists some $\theta_H \in (\theta_L, 1)$ such that $\Pi^F = \Pi^L$. Flexibility and commitment to late exit are profit maximizing for $\theta_L < \theta \leq \theta_H$ and for $\theta_H < \theta \leq 1$, respectively. Taken together, these results prove part (ii) of the proposition. Finally, part (i) follows from the observation that $\Pi^F > \Pi^L$ for $\theta > \theta_L$ for $c \leq \underline{c}$. □

A3. Proof of Proposition 2

Proposition 2 (comparative statics). *Let* $\underline{c} < c < \overline{c}$ *and m < n. Then, the following holds:*

i) $\frac{\partial \theta_L}{\partial w} > 0.$

ii) $\frac{\partial \theta_L}{\partial n} < 0$ *and* $\frac{\partial \theta_H}{\partial n} < 0.$

iii) $\frac{\partial \theta_L}{\partial m} = 0$ *for* $n \to \infty$, *and* $\frac{\partial \theta_H}{\partial m} > 0.$

iv) $\frac{\partial \theta_L}{\partial c} > 0$ *and* $\frac{\partial \theta_H}{\partial c} < 0.$

Proof: Let $\Delta^{FL} = \Pi^F - \Pi^L$ and $\Delta^{FE} = \Pi^F - \Pi^E$. Let γ be any exogenous parameter of the model. Then, $\text{sign}\left\{\frac{\partial \theta_L}{\partial \gamma}\right\} = \text{sign}\left\{\frac{-\frac{\partial \Delta^{FE}}{\partial \gamma}}{\frac{\partial \Delta^{FE}}{\partial \theta}}\right\}$ and $\text{sign}\left\{\frac{\partial \theta_H}{\partial \gamma}\right\} = \text{sign}\left\{\frac{-\frac{\partial \Delta^{FL}}{\partial \gamma}}{\frac{\partial \Delta^{FL}}{\partial \theta}}\right\}$. It follows from the proof of Lemma 5 Proposition 1 that $\frac{\partial \Delta^{FE}}{\partial \theta} > 0$ and that $\frac{\partial \Delta^{FL}}{\partial \theta} < 0$ for any $\theta > \frac{\theta_0}{\phi}$ where $\theta_L, \theta_H \in \left(\frac{\theta_0}{\phi}, 1\right)$. Thus,

$$\text{sign}\left\{\frac{\partial \theta_L}{\partial \gamma}\right\} = -\text{sign}\left\{\frac{\partial \Delta^{FE}}{\partial \gamma}\right\} \text{ and } \text{sign}\left\{\frac{\partial \theta_H}{\partial \gamma}\right\} = \text{sign}\left\{\frac{\partial \Delta^{FL}}{\partial \gamma}\right\}. \tag{A3.1}$$

Consider the comparative statics with respect to *w*. Some simple calculations show that $\frac{\partial \Delta^{FE}}{\partial w} = (1 - \phi\theta)\left[\phi\theta g\left(\underline{v}^*(\phi\theta), m\right)\pi_B^L(n) - G\left(\underline{v}^*(\phi\theta), m\right)\right]$. Using inequality (A2.1) of Lemma A2.1 and the fact that $\theta > \frac{\theta_0}{\phi}$, it follows that $\frac{\partial \Delta^{FE}}{\partial w} < 0$ in the range of θ considered. In turn, using (A3.1), the sign of $\frac{\partial \Delta^{FE}}{\partial w}$ implies that $\frac{\partial \theta_L}{\partial w} > 0$.

Consider instead the comparative statics with respect to *n*. Notice, $\partial \underline{v}^*(\phi\theta) / \partial n = \partial\left(\phi\theta\left(X(2,n) + \pi_B^L(n)\right)\right) / \partial n > 0$. Hence,

$$\frac{\partial \Delta^{FE}}{\partial n} = -\phi\theta\pi_B^L(n) g\left(\underline{v}^*(\phi\theta), m\right) \partial \underline{v}^*(\phi\theta) / \partial n + \phi\theta G\left(\underline{v}^*(\phi\theta), m\right) \partial X(2,n) / \partial n > 0 \Leftrightarrow$$

$$G\left(\underline{v}^*(\phi\theta), m\right) / g\left(\underline{v}^*(\phi\theta), m\right) > \phi\theta\pi_B^L(n)\left(1 + \frac{\partial \pi_B^L(n)}{\partial n} \bigg/ \frac{\partial X(2,n)}{\partial n}\right).$$

Since $\frac{\partial \pi_B^L(n)}{\partial n} < 0 < \frac{\partial X(2,n)}{\partial n}$, it follows from Lemma A2.1 and $\theta > \frac{\theta_0}{\phi}$ that $\frac{\partial \Delta^{FE}}{\partial n} > 0$. Similarly, $\frac{\partial \Delta^{FL}}{\partial n} = -\theta\left(\phi g\left(\underline{v}^*(\phi\theta), m\right)\pi_B^L(n) + \left(1 - \phi G\left(\underline{v}^*(\phi\theta), m\right)\right)\frac{\partial X(2,n)}{\partial n}\right) < 0$. In turn, using (A3.1), the signs of $\frac{\partial \Delta^{FE}}{\partial n}$ and $\frac{\partial \Delta^{FL}}{\partial n}$ imply that $\frac{\partial \theta_L}{\partial n} < 0$ and $\frac{\partial \theta_H}{\partial n} < 0$.

Consider instead the comparative statics with respect to *m* and assume first that $n \to \infty$. It follows from Lemma A2.2 that $\Delta^{FE} > 0$ for $\theta > \frac{w-1}{\phi(w+1)}$. Hence, using the notation of Proposition 1, $\theta_L = \frac{w-1}{\phi(w+1)}$, which is independent of *m*. Turning to Δ^{FL}, consider any $n > m$. We have:

$$\frac{\partial \Delta^{FL}}{\partial m} = \int_{\underline{v}^*(\theta)}^{1}\left(w + x - \theta\left(w + X(2,n)\right)\right)\partial g(x,m)/\partial m\, dx.$$

Notice that $w + x - \theta\left(w + X(2,n)\right) > 0$ for $x \geq \underline{v}^*(\theta)$. Furthermore, $\underline{v}^*(\theta) \geq X(2,m)$ for all $\theta \geq \tilde{\theta}$, which can be shown to imply that $\partial g(x,m)/\partial m \geq 0$ for $x \in \left[\underline{v}^*(\theta), 1\right]$. Using (A3.1) and Proposition 1 showing that $\theta_H \in (\tilde{\theta}, 1)$, it follows that $\frac{\partial \theta_H}{\partial m} > 0$.

Finally, consider the comparative statics with respect to *c*. First, we derive the comparative statics with respect to ϕ. We have that $\frac{\partial \Delta^{FE}}{\partial \phi} = \frac{\partial \Delta^{FL}}{\partial \phi} = \theta \frac{\partial \Pi^F}{\partial(\phi\theta)} > 0$ as θ_L and θ_H are greater than $\frac{\theta_0}{\phi}$. Finally, because $\phi'(c) < 0$, the comparative statics with respect to *c* follow immediately. □

A4. Buyers' valuations are constant over time

In this appendix, we consider a variant of the model where buyers' valuations are the same in the early and late market. We consider the case of flexibility as this assumption only affects results if the startup can collect offers in both the early and the late market. It is also assumed that $c = 0 \Leftrightarrow \phi = 1$, but the analysis can be extended to $c > 0$ in the same way as it is done in Section 4 of the main text. For simplicity, we model the early and the late market as an ascending auction. In the model considered in the main text, this type of auction is equivalent to a closed-bid second-price auction. We choose an ascending auction because the seller does not learn the exact value of the highest valuation in the early market and cannot make the decision on whether to continue or not to the late market contingent on this piece of information.[1] This simplifies the Bayesian updating and makes the model easier to solve.

[1] Alternatively, we could assume that there is an auctioneer who reveals the outcome of the auction in the early market but not the individual bids.

Buyers in the early market only bid for valuations greater than or equal to some $\underline{v}^*(\theta)$. For most parts of the proofs, we leave out the dependence on θ in order to make the notation and the expressions more compact. The first part of the analysis consists of (i) characterizing the threshold $\underline{v}^*$ for which an early buyer is exactly indifferent between bidding the true valuation in the early market and not bidding and waiting for the late market, and (ii) showing that the startup in the early market accepts offers greater than or equal to $\underline{v}^*$. Having characterized the equilibrium outcome of the early market, we derive the threshold $\underline{\hat{v}}(\theta)$ below which it is efficient to wait for the late market. Again, we leave out from the notation the dependence on θ when convenient. In the last part of the proof, we show that $\underline{v}^* \leq \underline{\hat{v}}$ (with strict inequality for some values of θ), implying that the startup tends to be sold too early.

A4.1. Equilibrium in the early market

A buyer of type $\underline{v}^*$ has to be indifferent between bidding or not. Bidding in the early market yields an expected profit of $H(\underline{v}^*)^{m-1}(w + \underline{v}^* - R)$ where *R* is the reserve price. That is, a buyer of type $\underline{v}^*$ earns profit of $w + \underline{v}^* - R$ if and only if all other buyers in the early market have valuations below $\underline{v}^*$. If the buyer does not bid, there is a probability $H(\underline{v}^*)^{m-1}$ that all other buyers have valuations below $\underline{v}^*$ and that the startup is sold in the late market. If the startup continues to the late market, the focal buyer of type $\underline{v}^*$ is faced with two sets of competing buyers: *n* – *m* buyers that participate only in the late market ("late buyers") and *m - 1* buyers that have participated in the early market already ("early buyers"). The valuation of a late buyer is distributed on *[w - 1, w + 1]* according to *H(.)*. Conditional on no sale in the early market, the updated belief of the focal buyer is that the valuation of an early buyer is distributed on $[w-1, w + \underline{v}^*)$ according to $H(.)/H(\underline{v}^*)$. The cumulative distribution function of the highest valuation among the other buyers in the late market is thus:

$$F_B^L(x,(m-1)\cap(n-m)) = \begin{cases} H(x)^{n-1}/H(\underline{v}^*)^{m-1} & \text{for } x \leq \underline{v}^* \\ H(x)^{n-m} & \text{for } \underline{v}^* < x \leq 1 \end{cases}.$$

The corresponding continuous density function is:

$$f_B^L(x,(m-1)\cap(n-m)) = \begin{cases} (n-1)h(x)H(x)^{n-2}/H(\underline{v}^*)^{m-1} & \text{for } x \leq \underline{v}^* \\ (n-m)h(x)H(x)^{n-m-1} & \text{for } \underline{v}^* < x \leq 1 \end{cases}.$$

All buyers bid their true valuations in the late market. Therefore, the expected profit of the buyer from not bidding is:

$$\theta H(\underline{v}^*)^{m-1}\int_{-1}^{\underline{v}^*} f_B^L(x,(m-1)\cap(n-m))(\underline{v}^* - x)dx.$$

Simplifying expressions, a buyer of type $\underline{v}^*$ is indifferent between bidding and not bidding if and only if:

$$w + \underline{v}^* - R = \theta \int_{-1}^{\underline{v}^*} f_B^L(x, (m-1) \cap (n-m))(\underline{v}^* - x)dx. \qquad \text{(A4.1)}$$

Following the analysis in the main text, we impose that the reserve price has to be credible. In particular, if there is only one bidder, the startup's profit from accepting the reserve price has to be equal to the expected profit from not selling in the early market and hoping for better offers in the late market. Suppose that there is only one bidder that starts bidding at *R*. In this case, the startup knows that (i) there is one early buyer with a valuation greater than or equal to $\underline{v}^*$, and that (ii) there are *(m – 1)* early buyers with valuations below $\underline{v}^*$. Since valuations are constant across time, the startup uses this information to update its belief about the expected profit from selling in the late market. In particular, the updated belief is that (i) there is one early buyer in the late market with a valuation distributed on $[w + \underline{v}^*, w+1]$ according to the cumulative distribution function $\left(H(.) - H(\underline{v}^*)\right) / \left(1 - H(\underline{v}^*)\right)$, and that (ii) there are *(m – 1)* early buyers with valuations distributed on $[w-1, w + \underline{v}^*)$ according to the cumulative distribution function $H(.)/H(\underline{v}^*)$. Since the valuation of each of the *n – m* late buyers is distributed on $[w-1, w+1]$ according to $H(.)$, the cumulative distribution function of the highest valuation among the bidders in the late market is:

$$F_S^L(x, m \cap (n-m)) = \begin{cases} 0 & \text{for } x \leq \underline{v}^* \\ \dfrac{H(x)^{n-m} \left(H(x) - H(\underline{v}^*)\right)}{1 - H(\underline{v}^*)} & \text{for } \underline{v}^* < x \leq 1 \end{cases}.$$

The cumulative distribution function of the second-highest valuation among all bidders in the late market is:

$$G_S^L(x, m \cap (n-m)) =$$

$$\begin{cases} \dfrac{H(x)^{n-1}}{H(\underline{v}^*)^{m-1}} & \text{for } x \leq \underline{v}^* \\ \dfrac{H(x)^{n-m}\left(H(x) - H(\underline{v}^*)\right)}{1 - H(\underline{v}^*)} + \dfrac{(n-m)H(x)^{n-m-1}(1-H(x))\left(H(x) - H(\underline{v}^*)\right)}{1 - H(\underline{v}^*)} + \dfrac{H(x)^{n-m}(1-H(x))}{1-H(\underline{v}^*)} & \text{for } \underline{v}^* < x \leq 1 \end{cases}.$$

This can be simplified to:

$$G_S^L(x, m \cap (n-m)) = \begin{cases} \dfrac{H(x)^{n-1}}{H(\underline{v}^*)^{m-1}} & \text{for } x \leq \underline{v}^* \\ H(x)^{n-m} + (n-m)\left(1 - H(x)\right)H(x)^{n-m-1}\left(\dfrac{H(x) - H(\underline{v}^*)}{1 - H(\underline{v}^*)}\right) & \text{for } \underline{v}^* < x \leq 1 \end{cases}.$$

The reserve price is credible if it is equal to the expected profit in the late market:

$$R = \theta\left(w + \int_{-1}^{1} g_S^L(x, m \cap (n-m))x dx\right). \qquad \text{(A4.2)}$$

Plugging *R* into (A4.1) and simplifying, we obtain:

$$w + \underline{v}^* = \theta\left(w + \underline{v}^* H(\underline{v}^*)^{n-m} + \int_{\underline{v}^*}^{1} g_S^L(x, m \cap (n-m))x dx\right).$$

Integrating by parts, this simplifies to:

$$w + \underline{v}^* = \theta\left(w + 1 - \int_{\underline{v}^*}^{1} G_S^L(x, m \cap (n-m))dx\right). \qquad \text{(A4.3)}$$

Lemma A4.1. *There exists some value* $\bar{\bar{\theta}} \in (0,1)$ *such that* $\underline{v}^*(\theta) = -1$ *for* $\theta \leq \bar{\bar{\theta}}$, *and* $\underline{v}^*(\theta)$ *is increasing in* θ *for* $\theta \in (\bar{\bar{\theta}}, 1]$ *with* $\underline{v}^*(1) = 1$.

Proof: The derivative of the LHS of (A4.3) with respect to $\underline{v}^*$ is equal to 1. The derivative of the RHS of (A4.3) with respect to $\underline{v}^*$ is equal to:

$$\theta\left(G_S^L(\underline{v}^*, m \cap (n-m)) - \int_{\underline{v}^*}^{1} \frac{\partial G_S^L(x, m \cap (n-m))}{\partial \underline{v}^*} dx\right).$$

Since $\frac{\partial G_S^L(x, m \cap (n-m))}{\partial \underline{v}^*} \leq 0$, the RHS of (A4.3) is weakly increasing in $\underline{v}^*$ The second-order derivative of the RHS of (A4.3) with respect to $\underline{v}^*$is equal to:

$$\theta \frac{\partial G_S^L(\underline{v}^*, m \cap (n-m))}{\partial \underline{v}^*} + \theta \frac{\partial G_S^L(x, m \cap (n-m))}{\partial \underline{v}^*}\bigg|_{x=\underline{v}^*} - \theta \int_{\underline{v}^*}^{1} \frac{\partial^2 G_S^L(x, m \cap (n-m))}{\partial {\underline{v}^*}^2} dx,$$

which simplifies to $-\theta \int_{\underline{v}^*}^{1} \frac{\partial^2 G_S^L(x, m \cap (n-m))}{\partial {\underline{v}^*}^2} dx$. Calculations show:

$$\frac{\partial^2 G_S^L(x, m \cap (n-m))}{\partial {\underline{v}^*}^2} = -(n-m)(1-H(x))^2 H(x)^{n-m-1}\left(\frac{h'(\underline{v}^*)\left(1-H(\underline{v}^*)\right) + 2h(\underline{v}^*)}{\left(1-H(\underline{v}^*)\right)^3}\right).$$

Using the fact that *h(v) = 1/2* for all *v*, this simplifies to:

$$\frac{\partial^2 G_S^L(x, m \cap (n-m))}{\partial {\underline{v}^*}^2} = -\frac{(n-m)(1-H(x))^2 H(x)^{n-m-1}}{\left(1-H(\underline{v}^*)\right)^3} < 0.$$

We conclude that the RHS of (A4.3) is convex in $\underline{v}^*$. Finally, the derivative of the RHS of (A4.3) with respect to $\underline{v}^*$ is equal to θ for $\underline{v}^* = 1$. Hence, using the fact that the RHS of (A4.3) is convex in $\underline{v}^*$, it follows that the derivative of the RHS of (A4.3) with respect to $\underline{v}^*$ is positive and no greater than θ for all $\underline{v}^* \in [-1,1]$. Taken together, these arguments imply that the LHS of (A4.3) is increasing in $\underline{v}^*$ at higher rate than the RHS of (A4.3). Consider first $\theta < 1$ where the LHS of (A4.3) > RHS of (A4.3) for $\underline{v}^* = 1$. There is a unique $\underline{v}^* \in (0,1)$ that solves (A4.3) if and only if the following condition holds: LHS of (A4.3) < RHS of (A4.3) for $\underline{v}^* = -1$. Simplifying expressions, we have that LHS of (A4.3) = RHS of (A4.3) for $\underline{v}^* = -1$ if and only if the following condition holds:

$$\frac{w-1}{\left(w+1-\int_{-1}^{1} G_{Late}^{Seller}(x, m \cap (n-m))dv\right)} = \theta. \qquad \text{(A4.4)}$$

Integrating by parts, this can be rewritten as:

$$\frac{w-1}{\left(w+\int_{-1}^{1} x g_S^L(x, m\cap(n-m))dv\right)} = \theta. \tag{A4.5}$$

Since the LHS of (A4.5) is strictly between 0 and 1, there exists some $\bar{\bar{\theta}} \in (0,1)$ that solves (A4.5). For $\theta > \bar{\bar{\theta}}$, LHS of (A4.3) < RHS of (A4.3) for $\underline{v}^* = -1$, and there exists some $\underline{v}^* \in (0,1)$ that solves (A4.3). For $\theta \le \bar{\bar{\theta}}$, this is not the case, and $\underline{v}^* = -1$. Notice also that for $\theta = 1$ the solution to equation (A4.3) is $\underline{v}^* = 1$. Finally, total differentiating equation (A4.3) and using the fact that the RHS increases in $\underline{v}^*$ at a lower rate than the LHS of equation (A4.3), it follows immediately that $\underline{v}^*$ is strictly increasing in θ for $\theta > \bar{\bar{\theta}}$. □

Lemma A4.1 implies that a startup that does not face execution risk always sells late. The intuition is that the early offers remain, and there are additional and potentially better offers to be had from waiting for the late market.

So far, we have analyzed the case in which there is only one active buyer in the early market (i.e. only one buyer with valuation greater than or equal to $\underline{v}^*$) and thus the startup always accepts the reserve price. For the above to constitute an equilibrium, we also need to ensure that the startup accepts the highest offer when there is more than one active buyer in the early market and the highest offer is above *R*.

Lemma A4.2. *The startup accepts the best offer above* $\underline{v}^*$ *in the early market.*

Proof: The reserve price derived in the proof of Lemma A1.1 ensures that the startup accepts the offer when there is only one bidder in the early market. Consider now situations where there are two or more bidders. Suppose that the second highest valuation in the early market $x(2,m) = k$, $k \ge \underline{v}^*$. The startup observes $x(2,m) = k$ and updates its belief about the expected profit in the late market. The startup's belief is that the valuation of the buyer with the highest valuation in the early market is distributed on $(k, 1]$ according to $\left(H(.) - H(k)\right)/\left(1 - H(k)\right)$, and the valuations of the late buyers are distributed on $[-1, 1]$ according to $H(.)$. The distribution of the second highest bid is thus:

$G_S^L(x, m \cap (n-m) | x(2,m) = k)$=

$$\begin{cases} 0 & for\ x \in [0, k] \\ H(x)^{n-m} + \frac{(n-m)H(x)^{n-m-1}(1-H(x))(H(x)-H(k))}{1-H(k)} & \text{for } k < x \le 1 \end{cases}.$$

Using this, we can write down the equation ensuring that the startup prefers to trade in the early market:

$$w + k \ge \theta\left(w + 1 - \int_k^1 G_S^L(x, m \cap (n-m) | x(2,m) = k)\, dx\right). \tag{A4.6}$$

Notice that $G_S^L(x, m \cap (n-m) | x(2,m) = k) = G_S^L\big(x, m \cap (n-m)\big)$ for $k = \underline{v}^*$. Therefore, since $\underline{v}^*$ solves equation (A4.3), equation (A4.6) holds with equality for $k = \underline{v}^*$. Furthermore, it follows directly from the proof of Lemma A4.1 that equation (A4.6) holds with strict inequality for $k > \underline{v}^*$. □

A4.2. Efficiency

Trade takes place in the early market if the highest valuation is greater than or equal to $\underline{v}^*$. Let us derive the threshold $\underline{\hat{v}}$ for which trade should take place early from the point of view of efficiency. The value created in the early market is equal to the value created in the late market if and only if:

$$w + \underline{\hat{v}} = \theta\left(w + F(\underline{\hat{v}}, n-m)\underline{\hat{v}} + \int_{\underline{\hat{v}}}^{1} f(x, n-m)x dx\right).$$

Integrating by parts, this simplifies to:

$$w + \underline{\hat{v}} = \theta\left(w + 1 - \int_{\underline{\hat{v}}}^{1} F(x, n-m)dx\right). \tag{A4.7}$$

Lemma A4.3. *When trying the early market is costless, the startup is more likely to be sold early compared to the efficient timing.*

<u>Proof</u>: Suppose first that $\theta < 1$. Then, we have that the LHS of (A4.7) > RHS of (A4.7) for $\underline{\hat{v}} = 1$ and that the RHS increases in $\underline{\hat{v}}$ at a lower rate than the LHS of equation (A4.7). Arguing as in the proof of Lemma A4.1, we find that there is a unique $\underline{\hat{v}}$ that solves (A4.7) if and only if LHS of (A4.7) < RHS of (A4.7) for $\underline{\hat{v}} = -1$. This condition simplifies to:

$$\theta > \frac{w-1}{w+X(1,n-m)}. \tag{A4.8}$$

Otherwise, for $\theta \leq \frac{w-1}{w+X(1,n-m)}$, we have that $\underline{\hat{v}} = -1$. Suppose instead that $\theta = 1$. Then, it follows from equation (A4.7) that $\underline{\hat{v}} = 1$; that is, all startups that face no execution risk should sell late from the point of view of efficiency. Finally, arguing as in the proof of Lemma A4.1, it can be shown that $\underline{\hat{v}}$ is increasing in θ for $\theta > \frac{w-1}{w+X(1,n-m)}$. This characterizes $\underline{\hat{v}}$ as a function of θ. As the next step, we compare the equilibrium level of activity in the early market to the efficient one. Here, in order to make the dependence on θ explicit, we use the notation in main text, $\underline{\hat{v}}(\theta)$ and $\underline{v}^*(\theta)$. We have $\int_{\underline{v}}^{1} G_S^L(x, m \cap (n-m))dx > \int_{\underline{v}}^{1} F(x, n-m)dx$ for $1 > \underline{v}$. Using equations (A4.3) and (A4.7), this implies that $\underline{\hat{v}}(\theta) \geq$

$\underline{v}^*(\theta)$. Furthermore, if $1 > \underline{\hat{v}}(\theta) > -1$, then $\underline{v}^*(\theta) < \underline{\hat{v}}(\theta)$. Using equation (A4.8), we have that $\underline{\hat{v}}(\theta) > \underline{v}^*(\theta)$ for all $\theta \in \left(\frac{w-1}{w+X(1,n-m)}, 1\right)$. Therefore, it follows:

$$\int_0^1 \left(1 - F\left(\underline{\hat{v}}(\theta), m\right)\right) d\theta < \int_0^1 \left(1 - F\left(\underline{v}^*(\theta), m\right)\right) d\theta \qquad \text{(A4.9)}$$

where the LHS of inequality (A4.9) is the value maximizing number of the early deals and the RHS of inequality (A4.9) is the equilibrium number of early deals. Proof follows. □

The intuition behind Lemma A4.3 is very similar to the one detailed in main text: the decision of the highest valuation buyer and the startup to trade in the early market does not take into account the expected profit of other buyers, resulting in too many early deals in equilibrium. Notice, unlike the model with uncorrelated valuations, there is no externality on the other early buyers with lower valuations: these buyers would not acquire the startup in the late market in any case. Still, the externality on the buyers that are active only in the late market remains.

A5. Equilibrium model with m endogenous

We analyze here the case in which buyers must invest in absorptive capacity to make deals in the early market. Assume that buyers need to pay a fixed amount, *T* to accumulate absorptive capacity, which is thus their entry cost in the early market. Buyers make this decision before the startup decides its exit timing. Buyers enter the early market as long as their expected profits cover *T*. Thus, a free entry condition determines the endogenous number of early buyers, m^*. Notice that, because we have assumed unlimited acquisition capacity, all buyers are active in the late market. We consider the two polar cases considered in the main text, commitment ($c = R$) and flexibility ($c = 0$). Without loss of insight, we focus on $b = 0$ in the formal proofs in order to reduce the number of case distinctions.

A5.1. Commitment Case

We first show the conditions for the existence of a stable early market equilibrium with a positive number of buyers $n > m^* \geq 2$ investing in absorptive capacity. Then, we analyze the efficiency properties of such an equilibrium.

The incentive to invest in absorptive capacity will depend on the probability of a startup to participate in the early market, $Q(\tilde{\theta})$. Ignoring integer constraints, the number of incumbents *m* investing in absorptive capacity and participating in the early market is given by:

$$Q(\tilde{\theta})\pi_B^E(m) = T. \qquad \text{(A5.1)}$$

Assuming θ is uniformly distributed on *[b, 1]*, we can rewrite equation (3) from the main text and equation (A5.1) as follows:

$$\tilde{\theta}_{Seller}(m) = \frac{w+\frac{m-3}{m+1}}{w+\frac{n-3}{n+1}}, \qquad \text{(A5.2)}$$

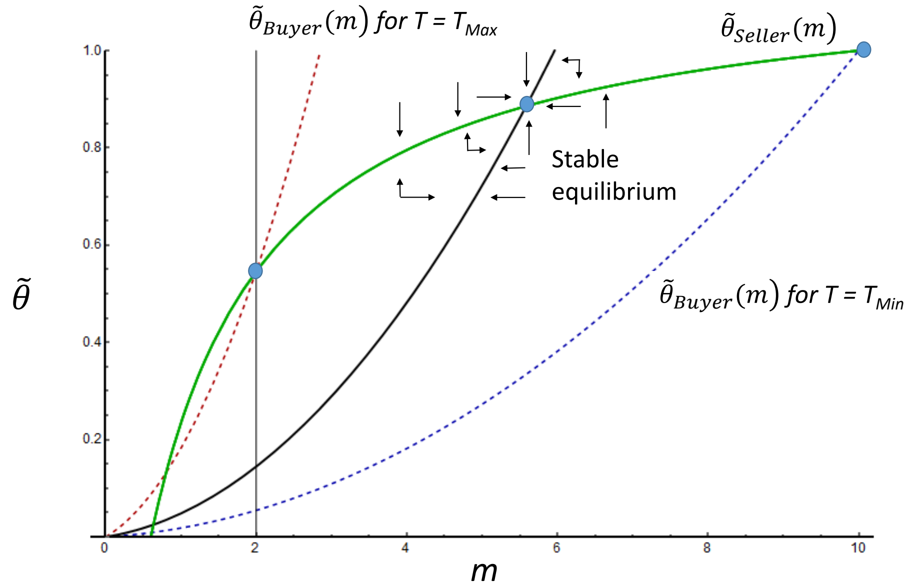
$$m_{Buyer}(\tilde{\theta}) = \frac{1}{2}\left(\sqrt{1+\frac{8(\tilde{\theta}-b)}{T(1-b)}}-1\right). \qquad \text{(A5.3)}$$

In order to plot the functions, it is useful to invert (A5.3), which yields:

$$\tilde{\theta}_{Buyer}(m) = \ b + \frac{1}{2}(1-b)Tm(1+m). \qquad \text{(A5.4)}$$

The functional forms in equations (A5.2) and (A5.4) immediately imply that $\tilde{\theta}_{Buyer}(m)$ is increasing and convex in *m* and that $\tilde{\theta}_{Seller}(m)$ is increasing and concave in *m*. An equilibrium is characterized by m^*such that $\tilde{\theta}_{Seller}(m^*) = \tilde{\theta}_{Buyer}(m^*) = \tilde{\theta}$. For the early and the late markets to be both active, the condition $n > m^* \geq 2$ has to hold. Indeed, no startup type enters the early market if there is no competition ($m^* < 2$), and all startup types enter the early market to avoid execution risk if all buyers are present there ($m^* = n$). Finally, stability requires that the $\tilde{\theta}_{Buyer}(m)$-curve is steeper than the $\tilde{\theta}_{Seller}(m)$-curve at the intersection. Figure A5.1 provides a graphical representation of the equilibrium in θ and *m*.

Figure A5.1. *Illustration of the candidate equilibria* (*n = 10, w = 3/2, T = 1/22*)



The following proposition identifies the conditions (i.e. acceptable values of our exogenous parameters) for the existence of a stable early market equilibrium.

Proposition A5.1. *Define* $T_{min} \equiv \frac{2}{n(n+1)}$ *and* $T_{max} \equiv \frac{1}{3} - \frac{4(n-2)}{9(n-3+w(n+1))}$. *Then, there exists a stable equilibrium where* $n > m^* \geq 2$ *if and only if* $T_{min} < T \leq T_{max}$.

<u>Sketch of the proof</u>: We start by determining the lower bound on T, T_{Min}. This is the value of T for which $m^* = n$, and all startup types commit to the early market. This threshold is defined by the following equation:

$$\tilde{\theta}_{Seller}(m = n) = \tilde{\theta}_{Buyer}(m = n) = 1 \Leftrightarrow T_{Min} \equiv \frac{2}{n(n+1)}.$$

Figure A5.1 illustrates how $\tilde{\theta}_{Seller}(m = n) = \tilde{\theta}_{Buyer}(m = n) = 1$ for $T = T_{Min}$. If $T < T_{Min}$, $m^* = n$ and the late market does not exist. We now turn to the upper bound on T, T_{Max}. This is given by the unique value of T for which $\tilde{\theta}_{Seller}(m = 2) = \tilde{\theta}_{Buyer}(m = 2)$. For $T_{Min} < T < T_{Max}$, we have that $\tilde{\theta}_{Buyer}(2) < \tilde{\theta}_{Seller}(2)$ and $\tilde{\theta}_{Buyer}(n) > \tilde{\theta}_{Seller}(n)$. Since $\tilde{\theta}_{Buyer}(m)$ and $\tilde{\theta}_{Seller}(m)$ are increasing and convex and increasing and concave in m, respectively, there exists a unique value m^* for which $\tilde{\theta}_{Buyer}(m^*) = \tilde{\theta}_{Seller}(m^*)$. Furthermore, $\partial\tilde{\theta}_{Buyer}(m)/\partial m > \partial\tilde{\theta}_{Seller}(m)/\partial m$ for $m = m^*$, which implies that the equilibrium is stable. Finally, it can be verified that the stability condition also holds for $T = T_{Max}$ and $m^* = 2$. □

The interpretation of the condition $T > T_{Min}$ in Proposition A5.1 is that T should be sufficiently large to avoid that all buyers (and afterwards all startup types) enter into the early market. On the other hand, if $T > T_{Max}$, entry barriers into the early market are prohibitively high, and the early market collapses. In addition to the stable equilibrium analyzed above, it should be noted that an equilibrium with no early market always exists due to the possibility of coordination failure among buyers. Indeed, if a buyer expects other buyers not to invest in absorptive capacity, it has no incentive to invest itself because two or more competing buyers are needed for the startup to invest in seeking early offers.

Assuming that buyers coordinate on the stable equilibrium whenever it exists, we study now whether too few or too many incumbents invest in absorptive capacity. We have shown in the main text that for a given number of early buyers, a startup has too high a probability of choosing the late market from the point of view of efficiency. That is, $\hat{\theta} > \tilde{\theta}$. Foreseeing this outcome, we show next that too few buyers enter the early market.

Proposition A5.2. *The equilibrium number of early buyers maximizes value creation conditional on the probability of the startup entering the early market. Since the probability of early market entry is too low from the point of view of efficiency, the number of buyers that invest in absorptive capacity is also lower than the number that maximizes value creation.*

Proof: A buyer's profit from entry into the early market is:

$$\boldsymbol{Q(\tilde{\theta})\int_{-1}^{1}\left(\int_{-1}^{v} f(x,m-1)(v-x)dx\right)h(v)dv - T}.$$

The additional value created by an additional buyer is:

$$\boldsymbol{Q(\hat{\theta})\int_{-1}^{1}\left(\int_{-1}^{v} f(x,m-1)(v-x)dx\right)h(v)dv - T}.$$

Hence, if $\boldsymbol{Q(\tilde{\theta}) = Q(\hat{\theta})}$, the equilibrium number of buyers in the early market maximizes value creation. This proves part (i) of the proposition. Since $\boldsymbol{Q(\tilde{\theta}) < Q(\hat{\theta})}$, there are too few early buyers from the point of view of value creation. □

Define the *efficiency gap* as the ratio between $\boldsymbol{\hat{\theta}}$ and $\boldsymbol{\tilde{\theta}}$, that is:

$$\text{Efficiency gap} = \frac{\hat{\theta}}{\tilde{\theta}} = \frac{\frac{w+X(1,m)}{w+X(1,n)}}{\frac{w+X(2,m)}{w+X(2,n)}}. \tag{A5.5}$$

Lemma A5.1. *The efficiency gap declines with m.*

Proof: We have that $\boldsymbol{\partial\left(\frac{\hat{\theta}}{\tilde{\theta}}\right)\Big/\partial m} = \frac{\frac{\partial X(1,m)}{\partial m}\tilde{\theta} - \frac{\partial X(2,m)}{\partial m}\hat{\theta}}{(\tilde{\theta})^2} < \boldsymbol{0}$ because $\boldsymbol{\hat{\theta} > \tilde{\theta}}$ and $\boldsymbol{\frac{\partial X(2,m)}{\partial m} > \frac{\partial X(1,m)}{\partial m}}$. □

Proposition A5.3. *The efficiency gap increases with b for $m^* < n$.*

Proof: Total differentiating the equilibrium condition $\tilde{\theta}_{Buyer}(m^*) = \tilde{\theta}_{Seller}(m^*)$, we obtain:

$$\frac{dm}{db} = -\frac{\left.\partial\tilde{\theta}_{Buyer}(m)\Big/\partial b\right|_{m=m^*}}{\left.\partial\tilde{\theta}_{Buyer}(m)\Big/\partial m\right|_{m=m^*} - \left.\partial\tilde{\theta}_{Seller}(m)\Big/\partial m\right|_{m=m^*}}.$$

Using equation (A5.4), we have that $\left.\partial\tilde{\theta}_{Buyer}(m)\Big/\partial b\right|_{m=m^*} = 1 - \frac{1}{2}Tm^*(1+m^*) = 1 - \tilde{\theta} > 0$ for $b =$ *0* and $m^* < n$. Stability implies that $\left.\partial\tilde{\theta}_{Buyer}(m)\Big/\partial m\right|_{m=m^*} - \left.\partial\tilde{\theta}_{Seller}(m)\Big/\partial m\right|_{m=m^*} > 0$. Hence, $\frac{dm}{db} < 0$. As an increase in *b* decreases entry into the early market, it follows from Lemma A5.1 that the efficiency gap increases. □

A5.2. Flexibility Case

In the *flexibility* case, computing the equilibrium number of buyers is logically similar, but mathematically more complex. However, the analysis is simplified by the fact that all startup types seek early offers as long as there are two or more buyers, and that there is a positive probability that the startup continues to the late market even if $m = n$. Hence, there exist active early and late markets if T is sufficiently low to ensure that entry into the early market is profitable for two or more buyers.

A buyer's expected profit from its participation in the early market when bidding for a start-up of type θ is denoted $\Delta\pi_B^E(\theta, m)$ and is given by:

$$\Delta\pi_B^E(\theta, m) =$$

$$\int_{\underline{v}^*(\theta)}^{1}\left(\underbrace{F\left(\underline{v}^*(\theta), m-1\right)\left(w + v - R^*(\theta) - \theta\pi_B^L(n)\right)}_{\substack{\text{The highest valuation among the}\\ \text{competing buyers is less than or equal to } \underline{v}^*(\theta)}} + \underbrace{\int_{\underline{v}^*(\theta)}^{v} f(x, m-1)(v - x)\,dx}_{\substack{\text{The highest valuation among the}\\ \text{competing buyers is above } \underline{v}^*(\theta)}}\right) h(v)dv. \qquad \text{(A5.6)}$$

A buyer acquires the startup and earns a strictly positive profit from its participation in the early market if (i) it has the highest valuation among the early buyers, and (ii) its valuation exceeds $\underline{v}^*(\theta)$.[2] Integrating by parts, $\Delta\pi_B^E(\theta, m)$ can be written as:

$$\Delta\pi_B^E(\theta, m) = \int_{\underline{v}^*(\theta)}^{1} \left(\int_{\underline{v}^*(\theta)}^{v} F(x, m-1)\, dx\right) h(v)dv. \tag{A5.7}$$

It follows immediately that $\Delta\pi_B^E(\theta, m)$ is decreasing in m as $F(x, m-1)$ is decreasing in m. Integrating over startup types, an equilibrium with an active early market exists if and only if:

$$\int_0^1 \Delta\pi_B^E(\theta, 2)\, q(\theta)d\theta - T \geq 0. \tag{A5.8}$$

If condition (A5.9) holds, the equilibrium number of buyers in the early market m^* is given either by the zero profit condition

$$\int_0^1 \Delta\pi_B^E(\theta, m^*)\, q(\theta)d\theta - T = 0, \tag{A5.9}$$

or by $m^* = n$ if $\int_0^1 \Delta\pi_B^E(\theta, n)\, q(\theta)d\theta - T \geq 0$. Notice, since $\Delta\pi_B^E(\theta, m)$ is decreasing in m, if there exists a solution to (A5.9), this solution is unique. We will in the following assume that T takes on a value such that (A5.9) holds for some $m^* \in [2, n]$.

Proposition A5.4. *In the flexible case, startups tend to sell inefficiently early, and the number of buyers that invest in absorptive capacity is greater than the number that maximizes efficiency.*

Proof. For simplicity, we divide the proof in two claims. We show first that buyers over-invest in absorptive capacity. We then show that this behavior results in too many early deals from the point of view of value creation.

Claim 1: Buyers overinvest in absorptive capacity.

Suppose that a startup of type θ is sold in the early market for some $x(1, m) \geq \underline{\hat{v}}(\theta) \geq -1$. There is a reserve price $\hat{R}(\theta)$ in the auction representing the expected value creation in the late market. Then, applying the same steps as in the derivation of $\Delta\pi_B^E(\theta, m)$, the contribution of the marginal early buyer to value creation can be written as:

$$\Delta V^E(\theta, m) \equiv \int_{\underline{\hat{v}}(\theta)}^{1} \left(F\left(\underline{\hat{v}}(\theta), m-1\right)\left(w + \underline{\hat{v}}(\theta) - \hat{R}(\theta)\right) + \int_{\underline{\hat{v}}(\theta)}^{v} F(x, m-1)\, dx\right) h(v)dv, \tag{A5.10}$$

[2] If there is only one early buyer with a valuation above $\underline{v}^*(\theta)$, its profit from participation in the early market is the profit earned in the early market, $w + v - R^*(\theta)$, minus the expected profit that it would have earned had it not been in the early market, $\theta\pi_B^L(n)$.

where $\underline{\hat{v}}(\theta)$ is the threshold value that maximizes value creation. The value created is maximized for $\partial \Delta V^E(\theta,m) / \partial \underline{\hat{v}}(\theta) = 0 \Leftrightarrow \underline{\hat{v}}(\theta) = \hat{R}(\theta) - w$ if $-1 < \hat{R}(\theta) - w$. Otherwise, $\underline{\hat{v}}(\theta) = -1$. As argued in the main text, the expected value creation in the late market is $\hat{R}(\theta) = \theta\big(w + X(1,n)\big)$. Hence,

$$\underline{\hat{v}}(\theta) \equiv Max\big\{\theta\big(w + X(1,n)\big) - w, -1\big\}.$$

Plugging $\underline{\hat{v}}(\theta)$ into $\Delta V^E(\theta,m)$, we obtain:

$$\Delta V^E(\theta,m) = \int_{\underline{\hat{v}}(\theta)}^{1} \left(\int_{\underline{\hat{v}}(\theta)}^{v} F(x, m-1)\, dx\right) h(v)\, dv. \tag{A5.11}$$

Finally, arguing as above, we can show that $\Delta V^E(\theta,m)$ is decreasing in m. Hence, the value maximizing number of early buyers is determined by the following equation:

$$\int_0^1 \Delta V^E(\theta,m)\, q(\theta) d\theta - T = 0. \tag{A5.12}$$

Comparing equations (A5.11) and (A5.8), and using $\underline{v}^*(\theta) \leq \underline{\hat{v}}(\theta)$, we obtain $\Delta V^E(\theta,m) \leq \Delta\pi_B^E(\theta,m)$. Furthermore, $\Delta V^E(\theta,m) < \Delta\pi_B^E(\theta,m)$ for $\theta > \frac{w-1}{w+X(1,n)}$ as $\underline{v}^*(\theta) < \underline{\hat{v}}(\theta)$. Using equations (A5.12) and (A5.9), this implies that $m^* > \hat{m}$.

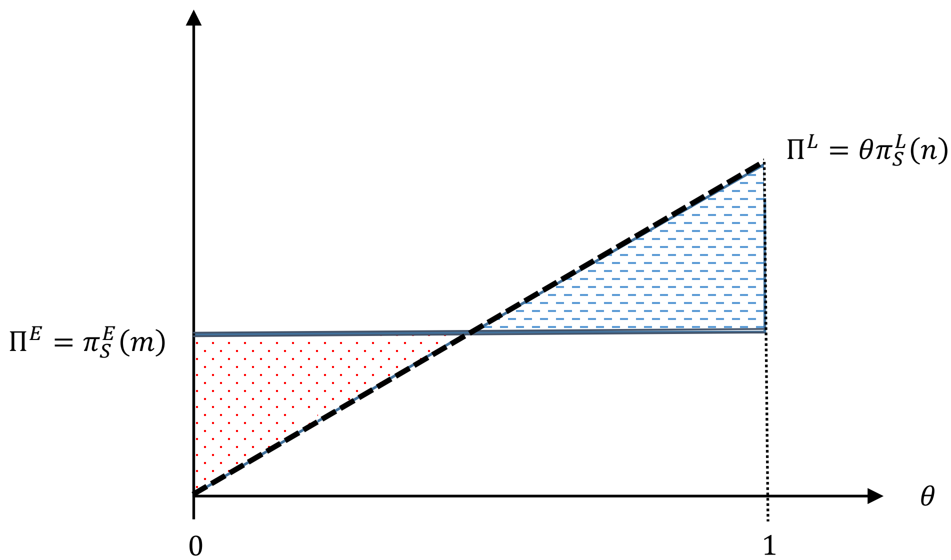
Claim 2: Too many early deals.

The expected number of deals in the early market is given by $\int_0^1 \Big(1 - F\big(\underline{v}^*(\theta), m^*\big)\Big) q(\theta) d\theta$ which is greater than the number of the value maximizing number of deals $\int_0^1 \Big(1 - F\big(\underline{\hat{v}}(\theta), \hat{m}\big)\Big) q(\theta) d\theta$ since $\underline{v}^*(\theta) \leq \underline{\hat{v}}(\theta)$ and $m^* > \hat{m}$. □

A6. Change of timing

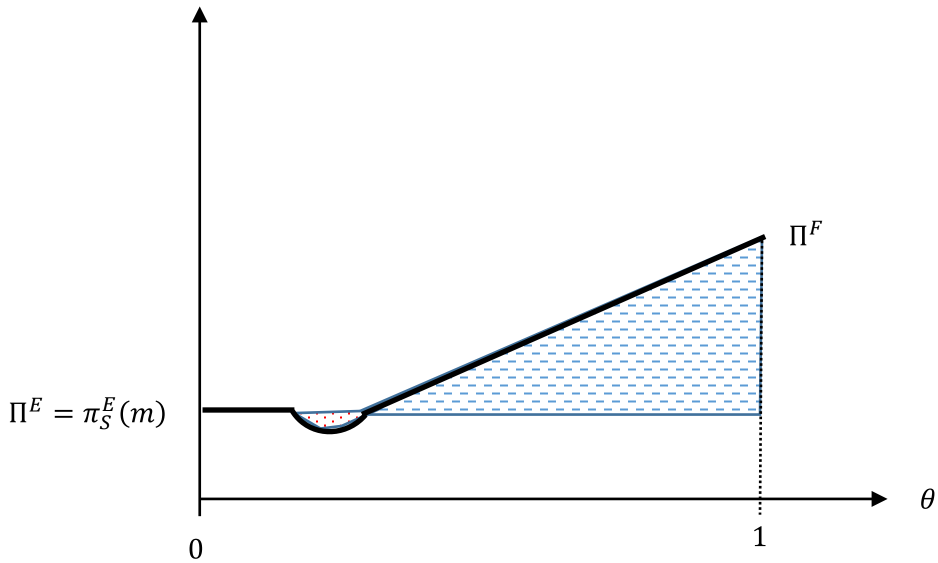
In order to understand what happens if decisions about the allocation of resources between trying the early market and developing the invention are made before the type θ is drawn, it is useful to make pairwise comparisons of the three exit strategies.

a. Commitment to an early vs. a late exit



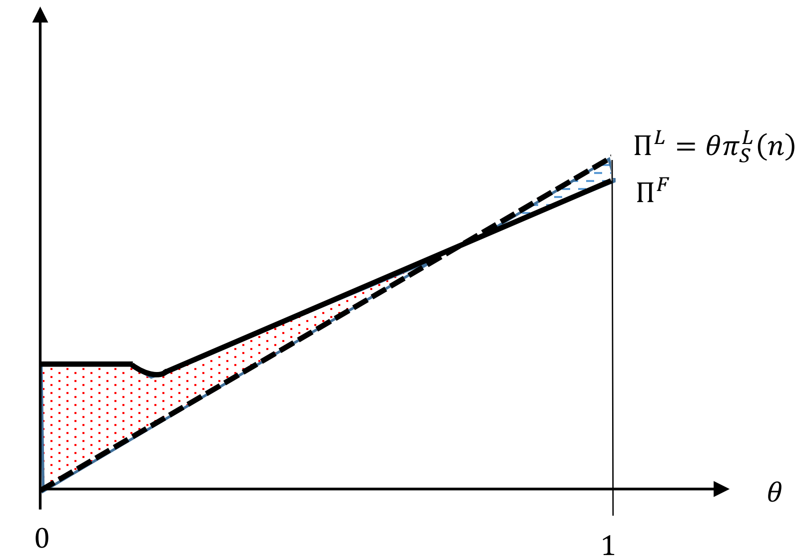
If the startup chooses to commit to an early exit, it earns higher (lower) profits when the realized θ is low (high) compared to commitment to a late exit. The profit gain (loss) when θ is low (high) is represented by the dotted red (dashed blue) triangle. In this case, it is easy to compare the two exit strategies explicitly. Indeed, commitment to an early exit is preferred to commitment to a late exit if and only if $\pi_S^E(m) > E(\theta)\pi_S^L(n)$ where $E(\theta)$ is the expected value of θ.

b. Commitment to an early exit vs. flexibility



As noted in the manuscript, flexibility results in lower profits than commitment to an early exit for a range of θ, because early buyers prefer not to bid and wait for the late market for some values of *v*. Indeed, the dotted red area represents the additional profit that commitment to an early exit results in for these values of θ compared to flexibility. For high values of θ flexibility is more profitable, and the additional profit compared to commitment to an early exit is represented by the dashed blue area. For the uniform distribution of θ, flexibility dominates commitment to an early exit if the blue area is greater than the red area. In the above illustration, it is obvious that flexibility is preferred to commitment to an early exit, and this is an intuitive finding: Flexibility gives the startup the possibility to adjust the exit strategy to the realized value of θ. However, due to the functional form of the profit function in the case of flexibility, it is not possible to derive closed-form expressions for the red and the blue areas, not even for the uniform distribution. Numerical simulations for chosen parameter values support the logic described above.

c. Commitment to a late exit vs. flexibility



For low values of θ flexibility is more profitable than commitment to a late exit, and the additional profit is represented by the dotted red area. For high values of θ commitment to a late exit is more profitable, and the additional profit is represented by the dashed blue area. For the uniform distribution of θ, flexibility dominates commitment to an early exit if the red area is greater than the blue area. In the above illustration, flexibility is preferred to commitment to a late exit. It is not possible to compare the two exit strategies analytically, although one can provide a numerical simulation under different parameter values.

d. Summary of the three pairwise comparisons

If the startup chooses the allocation of resources between trying the early market and developing the invention before knowing its execution capability, flexibility has the advantage that the startup is better able to adjust its actions to the realized execution capability. While this is intuitive, it is not possible to show this result analytically as it involves integrating over the highly non-linear profit function in the case of flexibility.