Platform Competition with Multihoming on Both Sides: Subsidize or Not?

— ONLINE APPENDIX —

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In this appendix, we show that assuming $A_x = B_x$ and $A_y = B_y$ simplifies exposition of the benchmark cases without any loss to the our results.

1 General Model Set-up and Benchmarks

We consider a setting with two types of potential participants (sides), X and Y, which are spatially differentiated and uniformly distributed; specifically $x \sim U[0,1]$ for side X and $y \sim U[0,Y]$ for side Y. We allow Y to be smaller, greater or equal to 1. There is two-sided Hoteling competition between the two platforms, A and B that are located at the ends of these segments, with A at 0 in both sides and B respectively at 1 and Y. The platforms charge participation fees p_i, $i = A, B$ on side X and r_i on side Y, and incur zero marginal cost in serving additional users.

A user located at x on side X (respectively y on Y) receives utility from joining platform $i = A, B$:

$$\begin{aligned} u(x;A) =& A_x + \alpha y_A - p_A - zx\,, \\ u(x;B) =& B_x + \alpha(Y - y_B) - p_B - z(1-x)\,, \\ u(y;A) =& A_y + \beta x_A - r_A - qy\,, \\ u(y;B) =& B_y + \beta(1 - x_B) - r_B - q(Y-y)\,, \end{aligned} \tag{1}$$

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where for platform A a mass of y_A agents participate on side Y and a mass x_A agents participate on side X, while for platform B a mass of $1 - x_B$ agents participate on side X and a mass of $Y - y_B$ agents participate on side Y; α and β is the "network effect" of the other side on side X and Y respectively; A_x, B_x and A_y, B_y are the stand-alone values users on side X and Y obtain from joining the respective platform; and z and q are the respective "transportation cost," i.e., the loss of utility due to preference mis-match or set-up costs. We assume that $qz > \alpha\beta$, i.e., that network effects are weaker than transportation costs, which is a typical assumption in models of competition with network effects on Hoteling line, as these models focus on the effects of differentiation.

1.1 Single-homing benchmark

We begin by analyzing as a benchmark the case with full coverage and single-homing, as is typical in most of the literature on platform competition. Imposing single-homing, under full coverage, the platforms share the market according to $x_A = x_B = \tilde{x}$ s.t. $u(\tilde{x}; A) = u(\tilde{x}; B)$ and $y_A = y_B = \tilde{y}$ s.t., $u(\tilde{y}; A) = u(\tilde{y}; B)$; i.e.,

$$\tilde{x} = \frac{z + A_x - B_x + 2\alpha\tilde{y} - \alpha Y - p_A + p_B}{2z},$$

$$\tilde{y} = \frac{qY + A_y - B_y + 2\beta\tilde{x} - \beta - r_A + r_B}{2q}.$$

First, solving for $\tilde{x}$ and $\tilde{y}$ for given prices yields

$$\tilde{x} = \frac{1}{2} + \frac{q(A_x - B_x) + \alpha(A_y - B_y) - q(p_A - p_B) - \alpha(r_A - r_B)}{2(zq - \alpha\beta)},$$

$$\tilde{y} = \frac{Y}{2} + \frac{\beta(A_x - B_x) + z(A_y - B_y) - \beta(p_A - p_B) - z(r_A - r_B)}{2(zq - \alpha\beta)}.$$

Platform A is choosing p_A and r_A to maximize its profit, $\Pi_A = p_A\tilde{x} + r_A\tilde{y}$. The FOCs wrt p_A and r_A yield

$$zq - \alpha\beta + q(A_x - B_x) + \alpha(A_y - B_y) + qp_B + \alpha r_B - 2qp_A - r_A(\alpha + \beta) = 0,$$

$$Y(zq - \alpha\beta) + \beta(A_x - B_x) + z(A_y - B_y) + \beta p_B + zr_B - (\alpha + \beta)p_A - 2zr_A = 0.$$

Platform B is choosing p_B and r_B to maximize its profit, $\Pi_B = p_A(1 - \tilde{x}) + r_A(Y - \tilde{y})$. The

FOCs wrt p_B and r_B yield

$$zq - \alpha\beta - q(A_x - B_x) - \alpha(A_y - B_y) + qp_A + \alpha r_A - 2qp_B - r_B(\alpha + \beta) = 0\,,$$

$$Y(zq - \alpha\beta) - \beta(A_x - B_x) - z(A_y - B_y) + \beta p_A + zr_A - (\alpha + \beta)p_B - 2zr_B = 0\,.$$

Using these FOC to solve for the equilibrium, we obtain

$$\tilde{x}^S = \frac{1}{2} + \frac{3q(A_x - B_x) + (\alpha + 2\beta)(A_y - B_y)}{18(zq - \alpha\beta) - 4(\alpha - \beta)^2}\,,$$

$$\tilde{y}^S = \frac{Y}{2} + \frac{(2\alpha + \beta)(A_x - B_x) + 3z(A_y - B_y)}{18(zq - \alpha\beta) - 4(\alpha - \beta)^2}\,,$$

$$p_A^S = z - Y\beta + \frac{(A_x - B_x)(3qz - \beta(2\alpha + \beta)) + (A_y - B_y)z(\alpha - \beta)}{9(zq - \alpha\beta) - 2(\alpha - \beta)^2}\,,$$

$$r_A^S = qY - \alpha + \frac{(A_y - B_y)(3qz - \alpha(\alpha + 2\beta) - q(A_x - Bx)(\alpha - \beta)}{9(zq - \alpha\beta) - 2(\alpha - \beta)^2}\,,$$

$$p_B^S = z - Y\beta - \frac{(A_x - B_x)(3qz - \beta(2\alpha + \beta)) + (A_y - B_y)z(\alpha - \beta)}{9(zq - \alpha\beta) - 2(\alpha - \beta)^2}\,,$$

$$r_B^S = qY - \alpha - \frac{(A_y - B_y)(3qz - \alpha(\alpha + 2\beta) - q(A_x - B_x)(\alpha - \beta)}{9(zq - \alpha\beta) - 2(\alpha - \beta)^2}\,.$$

For this equilibrium to exist, we need to have the utilities of marginal users, $\tilde{x}^S$ and $\tilde{y}^S$ to be positive, and profits of the platforms Π_A^S and Π_B^S to be positive as well.
We get $u(\tilde{x}^*, A) = u(\tilde{x}^*, B) = \frac{1}{2}(A_x + B_x + Y(\alpha + 2\beta) - 3z)$ and $u(\tilde{y}^*, A) = u(\tilde{y}^*, B) = \frac{1}{2}(A_y + B_y + 2\alpha + \beta - 3qY)$. They are positive when

$$A_x + B_x > 3z - Y(\alpha + 2\beta) \qquad \text{and} \qquad A_y + B_y > 3qY - (2\alpha + \beta)\,.$$

Note that the requirement that stand alone values are positive for the equilibrium is a general property of the Hoteling model, with or without network effects.

1.2 Multihoming on one side

We now examine the second benchmark, with single homing imposed on side X, multihoming allowed on side Y, and full coverage of both sides. As before, $\tilde{x}$ is characterized by $u(\tilde{x}, A) = u(\tilde{x}, B)$. A user on side Y who multihomes, i.e., joins both platforms, obtains utility $u(y;A\&B) = A_x + B_x + \beta - r_A - r_B - qY = u(y;A) + u(y;B)$. It is preferable for user y to join both platforms when $u(y;A) > 0$ and $u(y;B) > 0$. Therefore, all users $y < y_A$ join platform A, where y_A is characterized by $u(y_A;A) = 0$. All users $y > y_B$ join platform B, where y_B is characterized by $u(y_B;B) = 0$. Users $y \in (y_B, y_A)$ multihome.

Solving for $\tilde{x}$, y_A and y_B for given prices yields

$$\tilde{x} = \frac{1}{2} + \frac{q(A_x - B_x) + \alpha(A_y - B_y)}{2\,(zq - \alpha\beta)} - \frac{q(p_{AM} - p_{BM}) + \alpha(r_{AM} - r_{BM})}{2\,(zq - \alpha\beta)},$$

$$y_A = \frac{(A_x - B_x)q\beta + (A_y - B_y)\alpha\beta + (zq - \alpha\beta)(2A_y + \beta) - \beta q(p_{AM} - p_{BM}) - (2qz - \alpha\beta)r_{AM} + \alpha\beta r_{BM}}{2q(zq - \alpha\beta)},$$

$$y_B = Y - \frac{(B_x - A_x)q\beta + (B_y - A_y)\alpha\beta + (zq - \alpha\beta)(2B_y + \beta) + \beta q(p_{AM} - p_{BM}) + \alpha\beta r_{AM} - (2qz - \alpha\beta)r_{BM}}{2q(zq - \alpha\beta)}.$$

Note that formula for $\tilde{x}$ is the same as in the case of single-homing on both sides.

In equilibrium, we obtain

$$\tilde{x}^M = \frac{1}{2} + \frac{2q(A_x - B_x) + (\alpha + \beta)(A_y - B_y)}{2(6(qz - \alpha\beta) - (\alpha - \beta)^2)},$$

$$y_A^M = \frac{2A_y + \alpha + \beta}{4q} + \frac{(\alpha - \beta)(2q(A_x - B_x) + (\alpha + \beta)(A_y - B_y)}{4q(6(qz - \alpha\beta) - (\alpha - \beta)^2},$$

$$y_B^M = Y - \frac{2B_y + \alpha + \beta}{4q} - \frac{(\alpha - \beta)(2q(A_x - B_x) + (\alpha + \beta)(A_y - B_y)}{4q(6(qz - \alpha\beta) - (\alpha - \beta)^2},$$

$$p_A^M = \frac{4(qz - \alpha\beta) + \beta(\alpha - \beta) - \beta(A_y + B_y)}{4q} + \frac{q(4(qz - \alpha\beta) + \beta(\alpha - \beta))(A_x - B_x) + (\alpha(qz - \alpha\beta) + qz(\alpha - \beta))(A_y - B_y)}{2q(6(qz - \alpha\beta) - (\alpha - \beta)^2)},$$

$$p_B^M = \frac{4(qz - \alpha\beta) + \beta(\alpha - \beta) - \beta(A_y + B_y)}{4q} - \frac{q(4(qz - \alpha\beta) + \beta(\alpha - \beta))(A_x - B_x) + (\alpha(qz - \alpha\beta) + qz(\alpha - \beta))(A_y - B_y)}{2q(6(qz - \alpha\beta) - (\alpha - \beta)^2)},$$

$$r_A^M = \frac{2A_y - \alpha + \beta}{4} - \frac{(\alpha - \beta)(2q(A_x - B_x) + (\alpha + \beta)(A_y - B_y)}{4(6(qz - \alpha\beta) - (\alpha - \beta)^2)},$$

$$r_B^M = \frac{2B_y - \alpha + \beta}{4} + \frac{(\alpha - \beta)(2q(A_x - B_x) + (\alpha + \beta)(A_y - B_y)}{4(6(qz - \alpha\beta) - (\alpha - \beta)^2)}.$$

Such equilibrium exists if the platforms make positive profits and the marginal users obtain positive utility. Marginal users obtain positive utility when $A_y + B_y + \alpha + \beta > 2qY$ and $(\alpha + \beta)(A_y + B_y) + 2q(A_x + B_x) > 6(qz - \alpha\beta) - (\alpha - \beta)^2$.

2 Allowing for multihoming on both sides

We now allow for multihoming on both sides of a platform, i.e., $x_A > x_B$ and $y_A > y_B$. In such a case, multihoming agents on side X and Y obtain respective utility

$$u(x; A\&B) = A_x + B_x + \alpha Y - p_A - p_B - z\,,$$
$$u(y; A\&B) = A_y + B_y + \beta - r_A - r_B - q\,.$$

An agent multihomes when multihoming yields higher utility than joining only platform A, only platform B, or not joining either of the platforms.

Utility of an agent joining A without having joined the other platform is given by $u(x; A)$ as in (1). Therefore, the agent indifferent between joining platform A only and not joining any platform at all, $\bar{x}_A$, is characterized by $u(\bar{x}_A; A) = 0$, i.e.,

$$\bar{x}_A = \frac{A_x + \alpha\, y_A - p_A}{z}\,. \tag{2}$$

I.e., $\bar{x}_A$ would be the market captured by platform A if it was the only platform.

Similarly, given only the choice of platform B or no platform, all users $x > \bar{x}_B$ would prefer to join B, while $x < \bar{x}_B$ would not join, where

$$\bar{x}_B = 1 - \frac{B_x + \alpha\,(Y - y_B) - p_B}{z}\,.$$

However, user x's utility from joining A in addition to B is given by

$$u(x; A|B) = u(x; A\&B) - u(x; B)\,. \tag{3}$$

If there is multihoming on side Y, this incremental utility $u(x; A|B)$ is smaller than $u(x; A)$. Thus, platform A's market coverage on side X is $\hat{x}_A$, characterized by $u(\hat{x}_A; A|B) = 0$ which is equivalent to $u(\hat{x}_A; A\&B) = u(\hat{x}_A; B)$. I.e.,

$$\hat{x}_A = \frac{A_x + \alpha\, y_B - p_A}{z}\,. \tag{4}$$

Since $y_A > y_B$, then $\hat{x}_A < \bar{x}_A$.

Partial multihoming on both sides occurs in equilibrium when $0 < \hat{x}_B < \hat{x}_A < 1$ and

$0 < \hat{y}_B < \hat{y}_A < 1$, where

$$\begin{aligned}
\hat{x}_A &= \frac{A_x + \alpha \hat{y}_B - p_A}{z}\,, \\
\hat{x}_B &= 1 - \frac{B_x + \alpha(Y - \hat{y}_A) - p_B}{z}\,, \\
\hat{y}_A &= \frac{A_y + \beta \hat{x}_B - r_A}{q}\,, \\
\hat{y}_B &= Y - \frac{B_y + \beta(1 - \hat{x}_A) - r_B}{q}\,.
\end{aligned} \tag{5}$$

After solving platforms' profit-maximizing problems, the pure strategy equilibrium with partial multihoming on both sides is characterized by

$$\begin{aligned}
\hat{x}_A^{MM} &= \frac{q[(2qz - \alpha\beta)(A_x + \alpha Y) - \alpha z(B_y + \beta)]}{(qz - \alpha\beta)(4qz - \alpha\beta)}\,, \\
\hat{y}_A^{MM} &= \frac{z[(2qz - \alpha\beta)(A_y + \beta) - \beta q(B_x + Y\alpha)]}{(qz - \alpha\beta)(4qz - \alpha\beta)}\,, \\
\hat{x}_B^{MM} &= \frac{(2qz - \alpha\beta)^2 - (2qz - \alpha\beta)q(B_x + Y\alpha) + \alpha qzA_y}{(qz - \alpha\beta)(4qz - \alpha\beta)}\,, \\
\hat{y}_B^{MM} &= \frac{Y(2qz - \alpha\beta)^2 - (2qz - \alpha\beta)z(B_y + \beta) + \beta qzA_x}{(qz - \alpha\beta)(4qz - \alpha\beta)}\,.
\end{aligned} \tag{6}$$

And

$$\begin{aligned}
p_A^{MM} &= \frac{(2qz - \alpha\beta)(A_x + Y\alpha) - \alpha z(B_y + \beta)}{4qz - \alpha\beta}\,, \\
r_A^{MM} &= \frac{(2qz - \alpha\beta)(A_y + \beta) - \beta q(B_x + Y\alpha)}{4qz - \alpha\beta}\,, \\
p_B^{MM} &= \frac{(2qz - \alpha\beta)(B_x + Y\alpha) - \alpha z(A_y + \beta)}{4qz - \alpha\beta}\,, \\
r_B^{MM} &= \frac{(2qz - \alpha\beta)(B_y + \beta) - \beta q(A_x + Y\alpha)}{4qz - \alpha\beta}\,.
\end{aligned}$$

Proposition 1 *For range of parameters described by conditions (i)-(vi) below, there exists a pure strategy equilibrium with partial multihoming on both sides.*

(i) $zq(4zq + A_y\alpha) > q(Y\alpha + B_x)(2qz - \alpha\beta) + \alpha\beta(4qz - \alpha\beta)$

(ii) $q(A_x + Y\alpha)(2qz - \alpha\beta) < (qz - \alpha\beta)(4qz - \alpha\beta) + \alpha qz(B_y + \beta)$

(iii) $zq(4zq + A_y\alpha + B_y\alpha + \alpha\beta) < q(2qz - \alpha\beta)(A_x + B_x + 2Y\alpha) + \alpha\beta(4qz - \alpha\beta)$

(iv) $Y(2qz - \alpha\beta)^2 + qz\beta A_x > z(B_y + \beta)(2qz - \alpha\beta)$

(v) $z(A_y + \beta)(2qz - \alpha\beta) < (qz - \alpha\beta)(4qz - \alpha\beta) + \beta zq(B_x + Y\alpha)$

(vi) $Y(2qz - \alpha\beta)^2 + \beta zq(A_x + B_x + Y\alpha) < z(2qz - \alpha\beta)(A_y + B_y + 2\beta)$

Proof. Suppose parameters satisfy conditions (i)-(vi). It is straightforward to provide examples illustrating that the region of parameters satisfying these conditions is non-empty. Then $\hat{x}_A^{MM}$, $\hat{x}_B^{MM}$, $\hat{y}_A^{MM}$ and $\hat{y}_B^{MM}$ calculated according to (6) are such that $0 < \hat{x}_B^{MM} < \hat{x}_A^{MM} < 1$ and $0 < \hat{y}_B^{MM} < \hat{y}_A^{MM} < 1$. Conditions $\hat{x}_B^{MM} < \hat{x}_A^{MM}$ and $\hat{y}_B^{MM} < \hat{y}_A^{MM}$ indicate multihoming on both sides. And since all four thresholds are strictly between 0 and 1, the multihoming is partial. Moreover, multihoming brings positive utility on each side, i.e.,

$$\alpha z(A_y + B_y) + 2qz(A_x + B_x) > \alpha\beta(z - \alpha Y) + 4z(qz - \alpha\beta)\,,$$

$$\beta q(A_x + B_x) + 2qz(A_y + B_y) > \alpha\beta(qY - \beta) + 4qY(qz - \alpha\beta)\,.$$

The final condition for existence of the equilibrium, platforms' positive profits, are assured by the result that the prices are non-negative. ■

Proposition 2 *In an equilibrium with partial multihoming on both sides, there are no subsidies, i.e.,* p_A^{MM}*,* r_A^{MM}*,* p_B^{MM} *and* r_B^{MM} *are strictly positive.*

Proof. Consider parameters for which conditions (i)-(vi) above are satisfied. Since the conditions imply $0 < \hat{x}_B^{MM} < \hat{x}_A^{MM} < 1$ and $0 < \hat{y}_B^{MM} < \hat{y}_A^{MM} < 1$, they must also imply $\hat{x}_A^{MM} > 0$, $\hat{x}_B^{MM} < 1$, $\hat{y}_A^{MM} > 0$ and $\hat{y}_B^{MM} < 1$. Direct algebraic manipulations reveal that $\hat{x}_A^{MM} > 0 \iff p_A^{MM} > 0$, $\hat{y}_A^{MM} > 0 \iff r_A^{MM}$, $\hat{x}_B^{MM} < 1 \iff p_B^{MM} > 0$, and $\hat{y}_B^{MM} < 1 \iff r_B^{MM} > 0$. ■

With these result, we achieve the goal of this appendix: showing that the assumption $A_x = B_x$ and $A_y = B_y$ simplifies exposition of the benchmark cases without any loss to the our results.