Online Appendix to "Incentive Design for Operations-Marketing Multitasking"

In this online appendix, we provide proofs of the results in the main body, additional technical details, and robustness checks.

OA.1. Proofs of Results Stated in the Paper

Proof of Theorem 1: We need the following two lemmas.

LEMMA OA1. *An optimal contract for* (4) *exists.*

Proof of Lemma OA1. Due to constraint (4d), we may assume w is a function in $L^\infty(\mathcal{X})$, the space of uniformly bounded functions on $\mathcal{X}$. Moreover, those same constraints ensure the feasible region of (4) is bounded in the norm on $L^\infty(\mathcal{X})$. Hence, by Alaoglu's Theorem (see Theorem 5.105 in Aliprantis and Border 2006), the set $\{w : 0 \le w(\vec{x}) \le \overline{w}\}$ is compact in the weak topology $\sigma(L^\infty(\mathcal{X}), L^1(\mathcal{X}))$ (for a definition of this weak topology, see Section 5.14 of Aliprantis and Border (2006)). Because $f(\cdot|\vec{a})$ is in $L^1(\mathcal{X})$ for all $\vec{a} \in \mathcal{A}$, the constraints (4b) and (4c) are continuous in the $\sigma(L^\infty(\mathcal{X}), L^1(\mathcal{X}))$ topology, and so the feasible region is a closed subset of $\{w : 0 \le w(\vec{x}) \le \overline{w}\}$ and thus also compact in the $\sigma(L^\infty(\mathcal{X}), L^1(\mathcal{X}))$ topology. Moreover, the objective function $V(w, a)$ is continuous in the $\sigma(L^\infty(\mathcal{X}), L^1(\mathcal{X}))$ topology, and so, by Weierstrass's Theorem (see Theorem 2.35 in Aliprantis and Border 2006), an optimal contract exists. □

Let W denote the set of feasible contracts to (4). An *extremal contract* of W is a contract that cannot be written as the convex combination of two other feasible contracts. That is, $w \in W$ is an extremal contract if $w^1, w^2 \in W$, and $\lambda_1, \lambda_2 \in (0,1)$ with $\lambda_1 + \lambda_2 = 1$ do not exist such that $w = \lambda_1 w^1 + \lambda_2 w^2$. The next result is a consequence of Barvinok (2002, Proposition III.5.3).

LEMMA OA2. *Every extremal feasible contract to* (4) *is a bang-bang contract.*

Proof of Lemma OA2. This argument can be adapted from Proposition III.5.3 in Barvinok (2002). Our problem (4) is a linear program in $L^\infty(\mathcal{X})$ with finitely many constraints. Proposition III.5.3 in Barvinok (2002) shows that in linear programs in $L^\infty[0,1]$, extremal solutions have a bang-bang structure. This result can be adjusted to the multidimensional setting over the compact set $\mathcal{X}$ using standard arguments. Details are omitted. □

Bauer's Maximum Principle (see Theorem 7.69 in Aliprantis and Border (2006)) states that every lower semicontinuous concave function has an extreme-point minimizer over a compact convex set. The feasible region W is convex because all constraints are linear. The compactness of W and the continuity of the objective of (4) were argued in the proof of Lemma OA1. Hence, by Bauer's

Maximum Principle, an optimal extremal contract exists. Therefore, by Lemma OA2, an optimal bang-bang contract exists. □

Proof of Theorem 2. We use Barvinok (2002) Proposition IV.12.6, which is based on duality and complementary slackness, to characterize the structure of an optimal solution to our linear program over $L^\infty[0,1]$. Adapted to our setting, this method involves setting a Lagrange multiplier μ for the IR constraint (4b) and Lagrange multipliers $\lambda_1, \lambda_2, \ldots, \lambda_m$ for the constraints in (4c). Given a choice of nonnegative dual multipliers μ and λ_i, we define a function

$$p(\vec{x}) \triangleq \max\{0, -f(\vec{x}|\vec{a}^*) + \mu f(\vec{x}|\vec{a}^*) + \sum_{i=1}^{m} \lambda_i R_i(\vec{x}) f(\vec{x}|\vec{a}^*)\} \tag{OA1}$$

and, by Barvinok (2002, Proposition IV.12.6), an optimal bang-bang contract has the form

$$w^*(\vec{x}) = \begin{cases} \overline{w} & \text{if } p(\vec{x}) > 0 \\ 0 & \text{if } p(\vec{x}) = 0 \end{cases}, \tag{OA2}$$

assuming $\{\vec{x} : -f(\vec{x}|\vec{a}^*) + \mu f(\vec{x}|\vec{a}^*) + \sum_{i=1}^{m} \lambda_i R_i(\vec{x}) f(\vec{x}|\vec{a}^*) = 0\}$ has measure zero.

Observe that $p(\vec{x}) > 0$ if and only if

$$-1 + \mu + \sum_{i=1}^{m} \lambda_i R_i(\vec{x}) > 0, \tag{OA3}$$

which can be rewritten as

$$\sum_{i=1}^{m} \omega_i R_i(\vec{x}) > \tfrac{1-\mu}{\sum_{i=1}^{m} \lambda_i},$$

where $\omega_i \triangleq \frac{\lambda_i}{\sum_{i=1}^{m} \lambda_i}$. In the first step, we divide both sides of the argument in the "max" in (OA1) by $f(\vec{x}|\vec{a}^*)$, which is positive for all $\vec{x} \in \mathcal{X}$. In the last step, we divide through by $\sum_{i=1}^{m} \lambda_i$ which we assume is nonzero. This assumption is without loss, because otherwise $\lambda_i = 0$ for all i, and so (OA3) either holds for all x or no x. Thus, if $\sum_{i=1}^{m} \lambda_i = 0$, the extremal contract is either constant at 0 or constant at $\bar{w}$. In either case, the contract is either not feasible (violates IR) or not optimal (pays out the maximum) and thus can be excluded from consideration. Observe that ω_i is nonnegative because λ_i is nonnegative for all i. Using this equivalence and defining

$$t \triangleq \tfrac{1-\mu}{\sum_{i=1}^{m} \lambda_i},$$

we may re-express the optimal bang-bang contract in (OA2) as[16]

$$w^*(\vec{x}) = \begin{cases} \overline{w} & \text{if } \sum_{i=1}^{m} \omega_i R_i(\vec{x}) \geq t. \\ 0 & \text{otherwise.} \end{cases}$$

[16] Note we change the strict inequality in (OA2) to a weak inequality here. Because $p(\vec{x}) = 0$ is assumed to be a measure-zero event, this change can be made without loss.

This completes the proof. □

Proof of Theorem 3. By Theorem 2, an optimal contract to (4) exists that is feasible to (8). Thus, the optimal value of (8) is at least the optimal value of (4). Moreover, because the feasible region of (8) is a restriction of the feasible region of (4) (i.e., it restricts to information-trigger contracts), the value of the former cannot exceed the value of latter. Together, this implies both problems have the same optimal value. An optimal solution (ω^*, t^*) of (8) yields the trigger contract

$$w^*(\vec{x}) = \begin{cases} \overline{w} & \text{if } \sum_{i=1}^m \omega_i^* R_i(\vec{x}) \geq t^* \\ 0 & \text{otherwise,} \end{cases}$$

which is a feasible solution to (4). Moreover, w^* attains the optimal value of (4) because (ω^*, t^*) is optimal to (8) and the values of both problems are equal. Therefore, w^* is optimal to (4). □

Proof of Lemma 1. When the context is clear, we lighten notation to $\Pr(I \leq i, S \leq s)$. First, note

$$\Pr(I \leq i, S \leq s) = \Pr(I \leq i, \min\{Q, I\} \leq s) = \Pr(I \leq i, Q < I, Q \leq s) + \Pr(I \leq s, Q \geq I)$$

because $s \leq i$. We can develop this derivation further by noting that

$$\begin{aligned} \Pr(I \leq i, Q < I, Q \leq s) &= \Pr(Q < I \leq i, Q \leq s) \\ &= \int_0^s \int_q^i f(j|e_o) g(q|e_m) dj dq \\ &= \int_0^s (F(i|e_o) - F(q|e_o)) g(q|e_m) dq \\ &= F(i|e_o) \int_o^s g(q|e_m) dq - \int_0^s F(q|e_o) g(q|e_m) dq \\ &= F(i|e_o) G(s|e_m) - \int_0^s F(q|e_o) g(q|e_m) dq. \end{aligned}$$

It is useful to further analyze this by integration by parts to conclude that

$$\Pr(I \leq i, Q < I, Q \leq s) = F(i|e_o) G(s|e_m) - F(s|e_o) G(s|e_m) + \int_0^s G(q|e_m) f(q|e_o) dq. \tag{OA4}$$

Moreover,

$$\begin{aligned} \Pr(I \leq s, Q \geq I) &= \int_0^s \left(\int_j^{\bar{Q}} g(q|e_m) dq \right) f(j|e_o) dj \\ &= \int_0^s (1 - G(j|e_m)) f(j|e_o) dj \\ &= F(s|e_o) - \int_0^s G(j|e_m) f(j|e_o) dj. \end{aligned} \tag{OA5}$$

From (OA4) and (OA5), we can conclude that

$$\Pr(I \leq i, S \leq s) = \Pr(I \leq i, Q < I, Q \leq s) + \Pr(I \leq s, Q \geq I)$$

$$= F(i|e_o)G(s|e_m) - F(s|e_o)G(s|e_m) + \int_0^s G(q|e_m)f(q|e_o)dq + F(s|e_o) - \int_0^s G(j|e_m)f(j|e_o)dj$$
$$= F(s|e_o) + G(s|e_m)[F(i|e_o) - F(s|e_o)]$$

for $s < i$, and that if $s = i$,

$$\Pr(I \le i, S \le s) = F(i|e_o).$$

To conclude, we have derived the joint cumulative distribution function as

$$\Pr(I \le i, S \le s) = \begin{cases} F(s|e_o) + G(s|e_m)[F(i|e_o) - F(s|e_o)] & \text{if } s < i \\ F(i|e_o) & \text{if } s = i \end{cases}$$

and complete the proof. □

Proof of Proposition 1. This follows from implications of the MLRP assumptions stated in Section 3. To explore the implications of the MLRP assumption, it is useful to represent $R^{\text{NSO}}_{e_o,e_m}(i,s)$ and $R^{\text{SO}}_{e_o,e_m}$ explicitly:

$$R^{\text{NSO}}_{e_o^H,e_m^L}(i,s) = 1 - \frac{g(s|e_m^L)}{g(s|e_m^H)},\ R^{\text{NSO}}_{e_o^L,e_m^H}(i,s) = 1 - \frac{f(i|e_o^L)}{f(i|e_o^H)},\ \text{and } R^{\text{NSO}}_{e_o^L,e_m^L}(i,s) = 1 - \frac{f(i|e_o^L)g(s|e_m^L)}{f(i|e_o^H)g(s|e_m^H)}; \quad \text{(OA6)}$$
$$R^{\text{SO}}_{e_o^H,e_m^L}(i) = 1 - \frac{1-G(i|e_m^L)}{1-G(i|e_m^H)},\ R^{\text{SO}}_{e_o^L,e_m^H}(i) = 1 - \frac{f(i|e_o^L)}{f(i|e_o^H)},\ \text{and } R^{\text{SO}}_{e_o^L,e_m^L}(i) = 1 - \frac{f(i|e_o^L)(1-G(i|e_m^L))}{f(i|e_o^H)(1-G(i|e_m^H))}. \quad \text{(OA7)}$$

The MLRP lends monotonicity to the ratios on the right-hand sides of (9) and (10) that allows us to reveal the structure of the bonus regions B^{NSO} and B^{SO} defined in (12) and (13), respectively. We state these implications in the following lemma.

LEMMA OA3. *The following hold: (i) both $\frac{f(i|e_o^L)}{f(i|e_o^H)}$ and $\frac{1-G(i|e_m^L)}{1-G(i|e_m^L)}$ are nonincreasing and nonconstant in i; and (ii) $\frac{g(s|e_m^L)}{g(s|e_m^H)}$ is nonincreasing and nonconstant in s.*

Finally, note $R^{\text{NSO}}_{e_o^H,e_m^L}(i,s)$ is constant in i and $R^{\text{NSO}}_{e_o^L,e_m^H}(i,s)$ is constant in s.

Returning to the proof of the proposition, let

$$s^*(i) \triangleq \min\{s : \varphi^{\text{NSO}}(i,s) = t\}, \quad \text{(OA8)}$$

where

$$\varphi^{NSO}(i,s) \triangleq \sum_{e_o,e_m} \omega_{e_o,e_m} R^{\text{NSO}}(i,s). \quad \text{(OA9)}$$

These objects are illustrated in Figure OA.1. The critical inventory value i_s is the unique fixed point of s^*. That is, it is defined by

$$s^*(i_s) = i_s. \quad \text{(OA10)}$$

We first note the domain of s^* need not be all $[0, \bar{I}]$, because for a given i, an s might not exist such that $\varphi^{\text{NSO}}(i,s) = t$. However, properties on the $R^{\text{NSO}}_{e_o,e_m}$ imply that once some $\bar{i}$ and s exist such

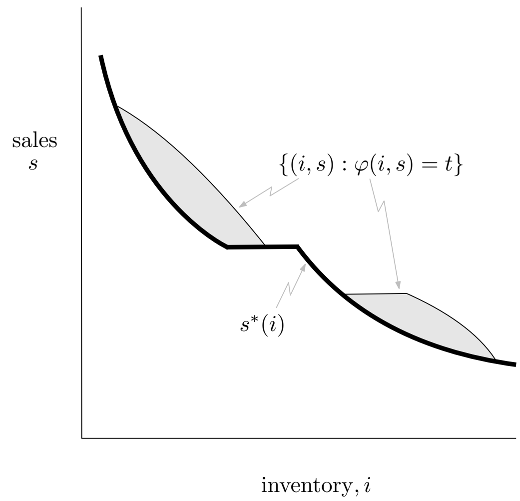


Figure OA.1 **Illustration of the level set** $\{(i,s):\varphi^{\mathsf{NSO}}(i,s)=t\}$ **of the function** φ^{NSO} **(defined in (OA9)) and the function** $s^*(i)$ **(defined in (OA8).**

that $\varphi(\bar{i},s)=t$ is nonempty, the same is true for any i larger than $\bar{i}$. That is, the domain of s^* is an interval of the form $[\bar{i},\bar{I}]$.

Next, we show the mapping $s^*(i)$ is well behaved. Specifically, it is a nonincreasing, continuous, and almost everywhere differentiable function of i on its domain. The reasoning is as follows. As described in the paragraph above (OA8), each of the $R^{\mathrm{NSO}}_{e_o,e_m}(i,s)$ are continuous, nonincreasing, and nonconstant in each of its coordinates. Hence, the same is true of the function $\varphi(i,s)$. Hence the level set $\{(i,s):\varphi(i,s)=t\}$ has the structure illustrated in Figure OA.1. That is, the level set $\{(i,s):\varphi(i,s)=t\}$ is the region between two nonincreasing and continuous functions. Observe that the graph of $s^*(i)$ in (i,s)-space is precisely the lower envelope of $\{(i,s):\varphi(i,s)=t\}$. Thus, we can conclude s^* is a nonincreasing, continuous, and almost everywhere differentiable function of i.

Finally, we prove the existence of uniqueness of i_s. Because $s^*(i)$ is a nonincreasing and continuous function of i and we have assumed B^{NSO} has a positive measure, the arg min in (OA10) is nonempty (by Brouwer's Fixed Point Theorem (Corollary 17.56 in Aliprantis and Border (2006)) and is a singleton (because s^* is nonincreasing and the 45° line is strictly increasing). Hence, a unique choice exists for i_s. □

Proof of Proposition 2. When B^{SO} has positive measure, we set

$$i_m = \min\left\{i:\varphi^{\mathrm{SO}}(i)=t\right\},$$

where

$$\varphi^{\mathrm{SO}}(i) \triangleq \sum_{e_o,e_m}\omega_{e_o,e_m}R^{\mathrm{SO}}_{e_o,e_m}(i).$$

If B^{SO} has zero measure, we simply set $i_m = \bar{I}$.

To see that the set of i such that $\varphi^{\text{SO}}(i) = t$ is nonempty, observe that $\varphi^{\text{SO}}(\bar{I}) \geq t$ because $(\bar{I}, \bar{I})$ must be in every bonus region, and an i exists such that $\varphi^{\text{SO}}(i) < 0$. The latter follows because properties (OA6) and (OA7) imply $\int_{D^{\text{SO}}} R^{\text{SO}}_{e_o,e_m}(i,s) f(i|e_o) g(i|e_m) ds di \leq 0$ for all e_o, e_m, and so for all e_o, e_m, an i exists such that $R^{\text{SO}}_{e_o,e_m}(i) < 0$. Hence, by this continuity of $\varphi^{\text{SO}}(i)$, the set of i such that $\varphi^{\text{SO}}(i) = t$ is nonempty. Again, from the monotonicity properties of φ^{SO}, the bonus region in the stockout case is precisely as defined in Proposition 2. □

Proof of Proposition 3. From the definition of $s^*(i)$, i_s, and i_m, the following holds for every valid choice of t and $(\omega_{e_o^H, e_m^L}, \omega_{e_o^L, e_m^L})$:

$$\begin{aligned}
1 - t &= \omega_{e_o^H, e_m^L} \frac{1 - G(i_m|e_m^L)}{1 - G(i_m|e_m^H)} + \omega_{e_o^L, e_m^H} \frac{f(i_m|e_o^L)}{f(i_m|e_o^H)} + \omega_{e_o^L, e_m^L} \frac{f(i_m|e_o^L)(1 - G(i_m|e_m^L))}{f(i_m|e_o^H)(1 - G(i_m|e_m^H))} \\
&= \omega_{e_o^H, e_m^L} \frac{g(s^*(i_s)|e_m^L)}{g(s^*(i_s)|e_m^H)} + \omega_{e_o^L, e_m^H} \frac{f(i_s|e_o^L)}{f(i_s|e_o^H)}) + \omega_{e_o^L, e_m^L} \frac{f(i_s|e_o^L) g(s^*(i_s)|e_m^L)}{f(i_s|e_o^H) g(s^*(i_s)|e_m^H)} \\
&= \omega_{e_o^H, e_m^L} \frac{g(i_s|e_m^L)}{g(i_s|e_m^H)} + \omega_{e_o^L, e_m^H} \frac{f(i_s|e_o^L)}{f(i_s|e_o^H)}) + \omega_{e_o^L, e_m^L} \frac{f(i_s|e_o^L) g(i_s|e_m^L)}{f(i_s|e_o^H) g(i_s|e_m^H)},
\end{aligned}$$

where the second equality follows because $s^*(i_s) = i_s$. MRLP distributions also have increasing failure rates, so we have $\frac{1 - G(\cdot|e_m^L)}{1 - G(\cdot|e_m^H)} < \frac{g(\cdot|e_m^L)}{g(\cdot|e_m^*)}$, which implies

$$\begin{aligned}
&\omega_{e_o^H, e_m^L} \frac{g(i_s|e_m^L)}{g(i_s|e_m^H)} + \omega_{e_o^L, e_m^H} \frac{f(i_s|e_o^L)}{f(i_s|e_o^H)}) + \omega_{e_o^L, e_m^L} \frac{f(i_s|e_o^L) g(i_s|e_m^L)}{f(i_s|e_o^H) g(i_s|e_m^H)} \\
&\leq \omega_{e_o^H, e_m^L} \frac{g(i_m|e_m^L)}{g(i_m|e_m^H)} + \omega_{e_o^L, e_m^H} \frac{f(i_m|e_o^L)}{f(i_m|e_o^H)}) + \omega_{e_o^L, e_m^L} \frac{f(i_m|e_o^L) g(i_m|e_m^L)}{f(i_m|e_o^H) g(i_m|e_m^H)}. \quad \square
\end{aligned}$$

Because f and g satisfy the MLRP, $\omega_{e_o^H, e_m^L} \frac{g(i|e_m^L)}{g(i|e_m^H)} + \omega_{e_o^L, e_m^H} \frac{f(i|e_o^L)}{f(i|e_o^H)}) + \omega_{e_o^L, e_m^L} \frac{f(i|e_o^L) g(i|e_m^L)}{f(i|e_o^H) g(i|e_m^H)}$ is a decreasing function of i. As result, we can conclude $i_1^* \geq i_2^*$.

Proof of Proposition 5. We first prove (i). For $i < i_m$, notice that $w^*(s', i) = w^*(s'', i) = 0$ for all $(i, s'), (i, s'') \in D$ with $s' \leq s''$. For $i \in [i_m, i_s]$, observe that $w(s, i) = 0$ if $s < i$ and $\bar{w}$ if $s = i$. Thus, $w^*(s', i) \leq w^*(s'', i) = 0$ for all $(i, s'), (i, s'') \in D$ with $s' \leq s''$. Finally, suppose $i > i_s$. By the structure of B^{NSO} and B^{SO} in Propositions 1 and 2, $w^*(s, i) = \bar{w}$ if $s^*(i) \leq s \leq i$, and 0 otherwise. Hence, again, $w^*(s', i) \leq w^*(s'', i) = 0$ for all $(i, s'), (i, s'') \in D$ with $s' \leq s''$. This implies w^* is monotone in s. We now show (ii). First, we show w^* is monotone in i. Suppose $s \geq i_s$. This implies $w(i, s) = \bar{w}$ for all $i \geq s$ (which is needed for $(i, s) \in D$) and so w^* is monotone in i in this region. Suppose otherwise that $s < i_s$. In this case, $w(i, s) = 0$ for $i \leq \min(s^*)^{-1}(s)$ and $\bar{w}$ otherwise, which again yields monotonicity in i. Joint monotonicity now follows from monotonicity in both direction i and s. Turning now to (iii), if $i_m < i_s$, an $\epsilon > 0$ exists such that $w^*(i_m, i_m) = \bar{w}$ but $w^*(i_m + \epsilon, i_m) = 0$, and thus w^* is not monotone in i.

Finally, we show w^* also fails joint monotonicity when $i_m < i_s$. Consider the point $(i^\circ, s^\circ) \triangleq (i_s + \epsilon, s^*(i_1 + \epsilon))$ on the graph of s^*. This means $w^*(i^\circ, s^\circ) = \bar{w}$ because the graph of s^* for $i \geq i_s$

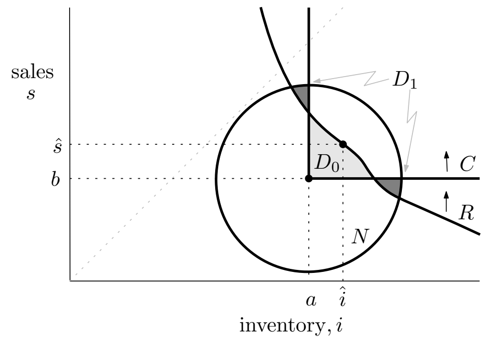


Figure OA.2 Illustration of the objects in the proof of Proposition 6.

lies in the bonus region of w^*. Choose any (i,s) in the open line segment between (i_m,i_m) and (i°,s°) with $i<i_s$. Such a choice is possible because $i_m<i_s$. See Figure 3 for an illustration.

Note (i,s) does not lie in the bonus region. Indeed, $i<i_s$ (by construction) and $s^*(i)<i_s$ (because $s^\circ \le i_s$ and s^* is a nonincreasing function). Hence, $w^*(i,s)=0$. This yields a contradiction of monotonicity. Observe that $(i_m,i_m)<(i,s)<(i^\circ,s^\circ)$ but $w^*(i_m,i_m)=w^*(i^\circ,s^\circ)=\bar{w}$ and $w^*(i,s)=0$.[17] □

Proof of Proposition 6. We prove both by noting that no optimal compensation plan can have a bonus region that has a "corner," defined as follows. Let B denote the bonus region of an optimal contact, that is, where the store manager receives a positive bonus. Let (a,b) be a point in B and let $C\triangleq(a,b)+\mathbb{R}^n_+$ denote the (translated) cone of points that are pointwise no smaller than $(\bar{i},\bar{s})$. We say (a,b) is a corner point of B if (a,b) is an isolated extreme point of B. That is, a nonempty neighborhood N of $(\bar{i},\bar{s})$ exists, such that $B\cap N=C\cap N$. This claim rules out corner compensation plans, but also any compensation plan with a "corner" as defined above.

We first prove the claim in the simplest case, in which on the no-jump constraint for (e_o^L,e_m^L) is tight, the other two no-jump constraints (for (e_o^H,e_m^L) and (e_o^L,e_m^H)) are slack. This case is the easiest to understand and will make clear what needs to be handled in the more challenging settings. Let w^* be an optimal compensation plan with a corner at (a,b) go to a new point $(\hat{i},\hat{s})=(a,b)+\delta(1,1)$, where δ is chosen sufficiently small so that the no-jump constraints for (e_o^H,e_m^L) and for (e_o^L,e_m^H) remain slack. Now, define the set $R=\left\{(i,s):R^{\text{NSO}}_{e_o^L,e_m^L}(i,s)\ge R^{\text{NSO}}_{e_o^L,e_m^L}(\hat{i},\hat{s})\right\}$, which is the $R^{\text{NSO}}_{e_o^L,e_m^L}(\hat{i},\hat{s})$-superlevel sets of $R^{\text{NSO}}_{e_o^L,e_m^L}(i,s)$. Now, δ is also chosen sufficiently small so that both of the sets

$$D_0 \triangleq \{(i,s): s<i\}\cap(C\setminus R)$$

$$D_1 \triangleq \{(i,s): s<i\}\cap(R\cap N\setminus C)$$

[17] Not this proof allows for the possibility that $s^*(i)=s^*(i_s)$ for all $i\ge i_s$, which cannot be ruled out under the MLRP assumptions.

have positive measure (where N is defined when we say (a,b) is a corner). We intersect both sets with $\{(i,s): s<i\}$ so that we only deal with points in the interior of the domain of the optimal compensation plan w^* (which is only defined on $\{(i,s): i \le s\}$). Such a choice for δ is possible because the boundary of the set R expressed by the implicit function theorem by $r(i)$ as a function of i is a *strictly decreasing* function of i. The fact that $r(i)$ is both a function and is strictly decreasing is due to the assumption that f and g satisfy the strict MLRP. See Figure 4 for a visual representation of the sets D_0 and D_1.

Now, consider the perturbation function h defined as follows:

$$h(s,i) = \begin{cases} -\epsilon_0 & \text{if } (i,s) \in D_0 \\ \epsilon_1 & \text{if } (i,s) \in D_1 \\ 0 & \text{otherwise,} \end{cases}$$

where $\epsilon_0, \epsilon_1 > 0$ are chosen so that $\mathbb{E}[h|e_o^H, e_m^H] = 0$. Such a choice is possible because D_0 and D_1 both have positive measure. Now consider the new optimal compensation plan $w' = w^* + h$. We claim w' is also an optimal compensation plan. Indeed,

$$\mathbb{E}[w'|e_o^H, e_m^H] = \mathbb{E}[w^*|e_o^H, e_m^H] + \mathbb{E}[h|e_o^H, e_m^H] = \mathbb{E}[w^*|e_o^H, e_m^H]$$

because $\mathbb{E}[h|e_o^H, e_m^H] = 0$. Thus, if we can show w' is feasible, it is optimal. Because we have assumed the IR constraint is not binding at w^* and ϵ_0, and D_2 can be chosen sufficiently small so that the IR constraint is satisfied at w' (indeed, the condition that $\mathbb{E}[h|e_o^H, e_m^H] = 0$ is only a single linear constraint on ϵ_0 and D_2, and thus a degree of freedom from that requirement allows us to drive ϵ_0 and D_2 arbitrarily small). Now consider the IC constraint. We claim that

$$\begin{aligned} \int_{s \le i} R_{e_o^L, e_m^L}(i,s) w'(i,s) f(i|e_o^L) g(s|e_m^L) \, di ds &> \int_{s \le i} R_{e_o^L, e_m^L}(i,s) w^*(i,s) f(i|e_o^L) g(s|e_m^L) \, di ds \\ &\ge c(e_o^H, e_m^H) - c(e_o^L, e_m^L) \end{aligned} \tag{OA11}$$

holds for all (e_o, e_m). Notice the second inequality holds because w^* is feasible to the IC constraint (here we are taking the form of IC constraints from (4c)). Thus, it remains to show (OA11). This follows because

$$\begin{aligned} &\int_{s<i} R^{\text{NSO}}_{e_o^L, e_m^L}(i,s) h(s,i) f(i|e_o^L) g(s|e_m^L) \, di ds \\ &= -\epsilon_0 \int_{D_0} R^{\text{NSO}}_{e_o^L, e_m^L}(i,s) f(i|e_o^L) g(s|e_m^L) \, di ds + D_2 \int_{D_1} R^{\text{NSO}}_{e_o^L, e_m^L}(i,s) f(i|e_o^L) g(s|e_m^L) \, di ds \\ &> R^{\text{NSO}}_{e_o^L, e_m^L}(a,b) \left[-\epsilon_0 \int_{D_0} f(i|e_o^L) g(s|e_m^L) \, di ds \right] + R^{\text{NSO}}_{e_o^L, e_m^L}(\hat{i}, \hat{s}) \left[D_2 \int_{D_1} f(i|e_o^L) g(s|e_m^L) \, di ds \right] \end{aligned}$$

$$> R(a,b)\left[-\epsilon_0 \int\limits_{D_0} f(i|e_o^L)g(s|e_m^L)dids + D_2 \int\limits_{D_1} f(i|e_o^L)g(s|e_m^L)dids\right] = 0,$$

where the key fact in each step is that $R^{\text{NSO}}_{e_o^L,e_m^L}(i,s)$ is coordinatewise strictly increasing. This implies (OA11). Indeed, observe that it suffices to integrate in the region $s<i$ to conclude the IC constraints bind for w' because $w'(i,s)=w^*(i,s)$ for $i=s$ (due to $h(i,s)=0$ for $i=s$). We thus conclude w' is an optimal compensation plan.

In fact, we have shown something more in (OA11): the IC constraints are slack at optimal compensation plan w'. However, because problem (2) is linear, every optimal compensation plan must be on the boundary of the feasible region. This implies either w' must bind the IR constraint, which violates our assumption of positive rents, which is a contradiction. This completes the proof in the special case of the argument in which only the no-jump constraint for (e_o^L, e_m^L) was initially tight.

We now allow the possibility that at least one of the other two no-jump constraints is tight at (a,b). In the case in which exactly one of the no-jump constraints for (e_o^H, e_m^L) or (e_o^L, e_m^H) is tight, an argument largely analogous to the previous one can be conducted. Here, the region D_1 will consist of only one "piece" (the D_1 in Figure OA.2 has two distinct pieces) because the no-jump constraints for (e_o^H, e_m^L) or (e_o^L, e_m^H) correspond to horizontal and vertical superlevel sets for $R^{\text{NSO}}_{e_o,e_m}(i,s)$ from the discussion following (OA7). The difficulty here is that when both of the no-jump constraints for (e_o^H, e_m^L) or (e_o^L, e_m^H) are tight, "room" remains to construct D_1 as in Figure OA.2. In this setting, we need to construct three regions, D_0, D_1, and D_2, where a tradeoff exists between the value of the perturbation in regions D_1 and D_2 to lead to a strictly increasing covariance as we were able to show in (OA11).

To make this argument, we need the following definitions. We also move a distance δ along $(1,1)$ to the point $(\hat{i},\hat{s})$. The superlevel set R and the region D_0 are defined exactly as before. To define D_1 and D_2, we need the following concepts. At (a,b), $R_{e_o^H,e_m^L}(i,s) \triangleq R_2(s)$ is constant in i at b with value $R_1(b)$ and so we consider the horizontal line through the point (a,b). Where this line intersects R is denoted (i_1,b). The region D_1 is chosen below the horizontal line to the right of (i_1,b), above R, and inside N. Similarly, $R_{e_o^L,e_m^H}(i,s) \triangleq R_2(i)$ is constant in s at a (at the value $R_2(a)$). The region D_2 is chosen to the left of the vertical line above (a,s_2) and above the lower envelope of R. The specific sets D_1 and D_2 and δ are chosen so that an $\bar{\epsilon}>0$ exists such that

$$\mathbb{E}[R_1(s)|D_1] - R_1(b) \le \bar{\epsilon} \text{ and } \mathbb{E}[R_2(i)|D_2] - R_2(a) \le \bar{\epsilon},$$

where (as used above) $\mathbb{E}[\cdot]$ is the expectation with respect to distributions with effort (e_o^H, e_m^H). From the regions D_0, D_1, and D_2, we define the perturbation

$$h(i,s)=\begin{cases}-\epsilon_0 & \text{if } (i,s)\in D_0\\ \epsilon_1 & \text{if } (i,s)\in D_1\\ \epsilon_2 & \text{if } (i,s)\in D_2\\ 0 & \text{otherwise.}\end{cases}$$

Using a logic similar to the simpler case above, it suffices to show that $\mathbb{E}[h(S,I)]=0$ and $\mathbb{E}[R(S,I)h(S,I)]>0$. We now have three degrees of freedom, and so this is possible. For the condition $\mathbb{E}[h(S,I)]=0$, this only requires

$$\epsilon_0\mathbb{P}(D_0)=\epsilon_1\mathbb{P}(D_1)+\epsilon_2\mathbb{P}(D_2),$$

where $\mathbb{P}[\cdot]$ is the probability measure with respect to distributions with effort (e_o^H,e_m^H). As for the covariance condition $\mathbb{E}[R(S,I)h(S,I)]>0$, this analysis is more delicate. We first show how to find ϵ_1 and ϵ_2 such that

$$\begin{aligned}&\frac{\int_{D_0}R_1(s)g(s)f(i)dsdi}{\int_{D_0}g(s)f(i)dsdi}\left(\epsilon_1\int_{D_1}g(s)f(i)dsdi+\epsilon_2\int_{D_2}g(s)f(i)dsdi\right)\\&<\epsilon_1\int_{D_1}R_1(s)g(s)f(i)dsdi+\epsilon_2\int_{D_2}R_1(s)g(s)f(i)dsdi\end{aligned}$$

and

$$\begin{aligned}&\frac{\int_{D_0}R_2(i)g(s)f(i)dsdi}{\int_{D_0}g(s)f(i)dsdi}\left(\epsilon_1\int_{D_1}g(s)f(i)dsdi+\epsilon_2\int_{D_2}g(s)f(i)dsdi\right)\\&<\epsilon_1\int_{D_1}R_2(i)g(s)f(i)dsdi+\epsilon_2\int_{D_2}R_2(i)g(s)f(i)dsdi.\end{aligned}$$

To see that, we note the coefficient of ϵ_1 in the first inequality is

$$\int_{D_1}R_1(s)g(s)f(i)dsdi-\frac{\int_{D_0}R_1(s)g(s)f(i)dsdi}{\int_{D_0}g(s)f(i)dsdi}\int_{D_1}g(s)f(i)dsdi>0,$$

and the coefficient of ϵ_1 in the second inequality is

$$\int_{D_1}R_2(i)g(s)f(i)dsdi-\frac{\int_{D_0}R_2(i)g(s)f(i)dsdi}{\int_{D_0}g(s)f(i)dsdi}\int_{D_1}g(s)f(i)dsdi<0.$$

Therefore, the existence of such ϵ_1 and ϵ_2 depends on

$$\begin{aligned}&\frac{\frac{\int_{D_0}R_2(i)g(s)f(i)dsdi}{\int_{D_0}g(s)f(i)dsdi}\int_{D_2}g(s)f(i)dsdi-\int_{D_2}R_2(i)g(s)f(i)dsdi}{\int_{D_1}R_2(i)g(s)f(i)dsdi-\frac{\int_{D_0}R_2(i)g(s)f(i)dsdi}{\int_{D_0}g(s)f(i)dsdi}\int_{D_1}g(s)f(i)dsdi}\\&>\frac{\epsilon_1}{\epsilon_2}>\frac{\frac{\int_{D_0}R_1(s)g(s)f(i)dsdi}{\int_{D_0}g(s)f(i)dsdi}\int_{D_2}g(s)f(i)dsdi-\int_{D_2}R_1(s)g(s)f(i)dsdi}{\int_{D_1}R_1(s)g(s)f(i)dsdi-\frac{\int_{D_0}R_1(s)g(s)f(i)dsdi}{\int_{D_0}g(s)f(i)dsdi}\int_{D_1}g(s)f(i)dsdi}.\end{aligned}$$

The sufficient condition is

$$\frac{\frac{\int_{D_2} R_2(i)g(s)f(i)dsdi}{\int_{D_2} g(s)f(i)dsdi}-\frac{\int_{D_0} R_2(i)g(s)f(i)dsdi}{\int_{D_0} g(s)f(i)dsdi}}{\frac{\int_{D_0} R_2(i)g(s)f(i)dsdi}{\int_{D_0} g(s)f(i)dsdi}-\frac{\int_{D_1} R_2(i)g(s)f(i)dsdi}{\int_{D_1} g(s)f(i)dsdi}} > \frac{\frac{\int_{D_0} R_1(s)g(s)f(i)dsdi}{\int_{D_0} g(s)f(i)dsdi}-\frac{\int_{D_2} R_1(s)g(s)f(i)dsdi}{\int_{D_2} g(s)f(i)dsdi}}{\frac{\int_{D_1} R_1(s)g(s)f(i)dsdi}{\int_{D_1} g(s)f(i)dsdi}-\frac{\int_{D_0} R_1(s)g(s)f(i)dsdi}{\int_{D_0} g(s)f(i)dsdi}}.$$

We can choose D_0 such that

$$\frac{\int_{D_0} R_2(i)g(s)f(i)dsdi}{\int_{D_0} g(s)f(i)dsdi} - R_2(a) \le \bar{\epsilon} \text{ and } \frac{\int_{D_0} R_1(s)g(s)f(i)dsdi}{\int_{D_0} g(s)f(i)dsdi} - R_1(b) \le \bar{\epsilon}.$$

Choose D_1 and D_2 to make $\frac{\int_{D_2} R_2(i)g(s)f(i)dsdi}{\int_{D_2} g(s)f(i)dsdi}$ as large as possible and $\frac{\int_{D_1} R_2(i)g(s)f(i)dsdi}{\int_{D_1} g(s)f(i)dsdi}$ as close to $R_2(a)$ as possible (indeed the distance is also controlled by $\bar{\epsilon}$), and $\frac{\int_{D_2} R_1(s)g(s)f(i)dsdi}{\int_{D_2} g(s)f(i)dsdi}$ as large as possible. Then, it suffices to have

$$\frac{\frac{\int_{D_2} R_2(i)g(s)f(i)dsdi}{\int_{D_2} g(s)f(i)dsdi}-\frac{\int_{D_0} R_2(i)g(s)f(i)dsdi}{\int_{D_0} g(s)f(i)dsdi}}{\frac{\int_{D_0} R_2(i)g(s)f(i)dsdi}{\int_{D_0} g(s)f(i)dsdi}-R_2(a)+\bar{\epsilon}} > \frac{\frac{\int_{D_0} R_1(s)g(s)f(i)dsdi}{\int_{D_0} g(s)f(i)dsdi}-(R_1(\bar{s})-\bar{\epsilon})}{\frac{\int_{D_1} R_1(s)g(s)f(i)dsdi}{\int_{D_1} g(s)f(i)dsdi}-\frac{\int_{D_0} R_1(s)g(s)f(i)dsdi}{\int_{D_0} g(s)f(i)dsdi}}.$$

Note both $\frac{\int_{D_2} R_2(i)g(s)f(i)dsdi}{\int_{D_2} g(s)f(i)dsdi}$ and $\frac{\int_{D_2} R_1(s)g(s)f(i)dsdi}{\int_{D_1} g(s)f(i)dsdi}$ can be sufficiently large. In the limit case, we can make

$$\frac{\frac{\int_{D_2} R_2(i)g(s)f(i)dsdi}{\int_{D_2} g(s)f(i)dsdi}-R_2(a)}{2\bar{\epsilon}} > \frac{2\bar{\epsilon}}{\frac{\int_{D_1} R_1(s)g(s)f(i)dsdi}{\int_{D_1} g(s)f(i)dsdi}-R_1(b)},$$

which is always possible. □

Proof of Proposition 9: The proof of parts (i) and (ii) are similar to that of Proposition 1. Due to Theorem 2 there exists an optimal information trigger contract involving likelihood ratio functions $R_{e_o,e_m}(i,z)$ as described in (19) and (20).

For part (i), note that $(*)$ equals

$$\frac{\int_i^{\bar{Q}} \frac{1}{q-i}\varphi(\frac{z-i}{q-i})g(q|e_m^L)dq}{\int_i^{\bar{Q}} \frac{1}{q-i}\varphi(\frac{z-i}{q-i})g(q|e_m^H)dq} = \frac{\int_i^{\bar{Q}} \frac{1}{q-i}\varphi(\frac{z-i}{q-i})g(q|e_m^L)dq}{\int_i^{\bar{Q}} \frac{1}{q-i}\varphi(\frac{z-i}{q-i})\frac{g(q|e_m^H)}{g(q|e_m^L)}g(q|e_m^L)dq} \le \frac{1}{\frac{g(i|e_m^H)}{g(i|e_m^L)}},$$

which is bounded, where $\frac{g(q|e_m^H)}{g(q|e_m^L)} \ge \frac{g(i|e_m^H)}{g(i|e_m^L)}$ by MLRP. Now, as $\int_i^{\bar{Q}} \frac{1}{q-i}\varphi(\frac{z-i}{q-i})g(q|e_m^L)dq \to \infty$, it must be that $\int_i^{\bar{Q}} \frac{1}{q-i}\varphi(\frac{z-i}{q-i})\frac{g(q|e_m^H)}{g(q|e_m^L)}g(q|e_m^L)dq \to \infty$ as well. First, by changes of varibles, $\varsigma = q-i$,

$$\frac{\int_0^{\bar{Q}-i} \frac{1}{\varsigma}\varphi(\frac{z-i}{\varsigma})g(\varsigma+i|e_m^L)d\varsigma}{\int_0^{\bar{Q}} \frac{1}{\varsigma}\varphi(\frac{z-i}{\varsigma})g(\varsigma+i|e_m^H)d\varsigma} = \lim_{\epsilon\to 0}\frac{\int_\epsilon^{\bar{Q}-i} \frac{1}{\varsigma}\varphi(\frac{z-i}{\varsigma})g(\varsigma+i|e_m^L)d\varsigma}{\int_\epsilon^{\bar{Q}} \frac{1}{\varsigma}\varphi(\frac{z-i}{\varsigma})g(\varsigma+i|e_m^H)d\varsigma}.$$

Therefore, by L'Hopital's rule, we have

$$\lim_{\epsilon\to 0}\frac{\int_\epsilon^{\bar{Q}-i} \frac{1}{\varsigma}\varphi(\frac{z-i}{\varsigma})g(\varsigma+i|e_m^L)d\varsigma}{\int_\epsilon^{\bar{Q}} \frac{1}{\varsigma}\varphi(\frac{z-i}{\varsigma})g(\varsigma+i|e_m^H)d\varsigma} = \lim_{\epsilon\to 0}\frac{\frac{1}{\epsilon}\varphi(\frac{z-i}{\epsilon})g(\epsilon+i|e_m^L)}{\frac{1}{\epsilon}\varphi(\frac{z-i}{\epsilon})g(\epsilon+i|e_m^H)} = \frac{g(i|e_m^L)}{g(i|e_m^H)},$$

which is independent of z. This establishes the result, because it also implies $R^{\mathrm{LS}}_{e_o,e_m}(i,z) = R^{\mathrm{NLS}}_{e_o,e_m}(i,z)$ at $z=i$, and so B^{NLS} has the structure expressed in the result.

For part (ii), we show $R^{\mathrm{LS}}(i,z|e_o,e_m)$ is nondecreasing in z. This suffices to show an $i_\ell \in (0,\bar{I}]$ exists such that

$$B^{\mathrm{LS}} = \{(i,z) : i \le i_\ell \text{ and } \ell^*(i) \le z \le \bar{Q}\} \cup \{(i,z) : i \ge i_\ell \text{ and } i \le z \le \bar{Q}\},$$

using an argument analogous to that of Proposition 1. The final part of the proof argues $i_\ell = i_s$, establishing the result.

To establish the first part of the argument, we first claim $(*)$ in (20) is nonincreasing in z under Assumption 1. First, we can write

$$\int_i^{\bar{Q}} \gamma'(z|q,i) g(q|e_m) dq = \int_i^{\bar{Q}} \frac{\gamma'(z|q,i)}{\gamma(z|q,i)} \frac{g(q|e_m)}{g(q|e_m^*)} \gamma(z|q,i) g(q|e_m^*) dq.$$

Note $\frac{\gamma'(z|q,i)}{\gamma(z|q,i)}$ is nondecreasing in q by Assumption 1, $\frac{g(q|e_m)}{g(q|e_m^*)}$ is nonincreasing in q. Therefore, the two functions $\frac{\gamma'(z|q,i)}{\gamma(z|q,i)}$ and $\frac{g(q|e_m)}{g(q|e_m^*)}$ are negatively correlated given any z. Therefore,

$$\frac{\int_i^{\bar{Q}} \frac{\gamma'(z|q,i)}{\gamma(z|q,i)} \frac{g(q|e_m)}{g(q|e_m^*)} \gamma(z|q,i) g(q|e_m^*) dq}{\int_i^{\bar{Q}} \gamma(z|q,i) g(q|e_m^*) dq} \le \frac{\int_i^{\bar{Q}} \frac{\gamma'(z|q,i)}{\gamma(z|q,i)} \gamma(z|q,i) g(q|e_m^*) dq}{\int_i^{\bar{Q}} \gamma(z|q,i) g(q|e_m^*) dq} \frac{\int_i^{\bar{Q}} \frac{g(q|e_m)}{g(q|e_m^*)} \gamma(z|q,i) g(q|e_m^*) dq}{\int_i^{\bar{Q}} \gamma(z|q,i) g(q|e_m^*) dq}.$$

By canceling out one of the $\int_i^{\bar{Q}} \gamma(z|q,i) g(q|e_m^*) dq$ in the denominator, we obtain

$$\begin{aligned}\int_i^{\bar{Q}} \frac{\gamma'(z|q,i)}{\gamma(z|q,i)} \frac{g(q|e_m)}{g(q|e_m^*)} \gamma(z|q,i) g(q|e_m^*) dq &\le \int_i^{\bar{Q}} \frac{\gamma'(z|q,i)}{\gamma(z|q,i)} \gamma(z|q,i) g(q|e_m^*) dq \frac{\int_i^{\bar{Q}} \frac{g(q|e_m)}{g(q|e_m^*)} \gamma(z|q,i) g(q|e_m^*) dq}{\int_i^{\bar{Q}} \gamma(z|q,i) g(q|e_m^*) dq} \\ &= \int_i^{\bar{Q}} \gamma'(z|q,i) g(q|e_m^*) dq \frac{\int_i^{\bar{Q}} \gamma(z|q,i) g(q|e_m) dq}{\int_i^{\bar{Q}} \gamma(z|q,i) g(q|e_m^*) dq}.\end{aligned}$$

It remains to show i_s and i_ℓ are equal. This requires that $R^{\mathrm{LS}}_{e_o,e_m}(i,i) = R^{\mathrm{NSL}}_{e_o,e_m}(i,i)$, which occurs if

$$\frac{\int_i^{\bar{q}} \gamma(i|q,i) g(q|e_m) dq}{\int_i^{\bar{q}} \gamma(z|q,i) g(q|e_m^*) dq} = \frac{g(i|e_m)}{g(i|e_m^*)}.$$

But this holds because $\Lambda \in (0,1)$, and so $\gamma(i|q,i)dq$ is the Dirac measure $\delta(i=q)$, thus yielding the result.

Proof of Proposition 10: It suffices to show $\frac{\int_i^{\bar{Q}} \gamma(z|q,i) g(q|e_m) dq}{\int_i^{\bar{Q}} \gamma(z|q,i) g(q|e_m^*) dq}$ is nonincreasing in i. We need

$$\frac{\partial}{\partial i} \left[\frac{\int_i^{\bar{Q}} \gamma(z|q,i) g(q|e_m) dq}{\int_i^{\bar{Q}} \gamma(z|q,i) g(q|e_m^*) dq} \right] \le 0.$$

It is equivalent to show

$$\frac{-h(i|q,i) g(i|e_m) + \int_i^{\bar{Q}} \frac{\partial}{\partial i} \gamma(z|q,i) g(q|e_m) dq}{\int_i^{\bar{Q}} \gamma(z|q,i) g(q|e_m) dq} - \frac{-h(i|q,i) g(i|e_m^*) + \int_i^{\bar{Q}} \frac{\partial}{\partial i} \gamma(z|q,i) g(q|e_m^*) dq}{\int_i^{\bar{Q}} \gamma(z|q,i) g(q|e_m^*) dq} \le 0.$$

The fact that

$$\frac{-h(i|q,i)g(i|e_m)}{\int_i^{\bar{Q}}\gamma(z|q,i)g(q|e_m)dq}+\frac{h(i|q,i)g(i|e_m^*)}{\int_i^{\bar{Q}}\gamma(z|q,i)g(q|e_m^*)dq}<0$$

is due to Assumption 1:

$$\frac{\int_i^{\bar{Q}}\gamma(z|q,i)g(q|e_m^*)dq}{\int_i^{\bar{Q}}\gamma(z|q,i)g(q|e_m)dq}>\frac{g(i|e_m^*)}{g(i|e_m)}.$$

It remains to show

$$\frac{\int_i^{\bar{Q}}\frac{\partial}{\partial i}\gamma(z|q,i)g(q|e_m)dq}{\int_i^{\bar{Q}}\gamma(z|q,i)g(q|e_m)dq}\le\frac{\int_i^{\bar{Q}}\frac{\partial}{\partial i}\gamma(z|q,i)g(q|e_m^*)dq}{\int_i^{\bar{Q}}\gamma(z|q,i)g(q|e_m^*)dq}.$$

By Young's Theorem, Assumption 2 implies $\frac{\frac{\partial}{\partial i}\gamma(z|q,i)}{\gamma(z|q,i)}$ is nondecreasing in q, and so we have

$$\begin{aligned}
&\int_i^{\bar{Q}}\frac{\partial}{\partial i}\gamma(z|q,i)g(q|e_m)dq\\
=&\int_i^{\bar{Q}}\frac{\frac{\partial}{\partial i}\gamma(z|q,i)}{\gamma(z|q,i)}\frac{g(q|e_m)}{g(q|e_m^*)}\gamma(z|q,i)g(q|e_m^*)dq\\
\le&\int_i^{\bar{Q}}\frac{\frac{\partial}{\partial i}\gamma(z|q,i)}{\gamma(z|q,i)}\gamma(z|q,i)g(q|e_m^*)dq\frac{\int_i^{\bar{Q}}\frac{g(q|e_m)}{g(q|e_m^*)}\gamma(z|q,i)g(q|e_m^*)dq}{\int_i^{\bar{Q}}\gamma(z|q,i)g(q|e_m^*)dq}\\
=&\int_i^{\bar{Q}}\frac{\partial}{\partial i}\gamma(z|q,i)g(q|e_m^*)dq\frac{\int_i^{\bar{Q}}\gamma(z|q,i)g(q|e_m)dq}{\int_i^{\bar{Q}}\gamma(z|q,i)g(q|e_m^*)dq},
\end{aligned}$$

which shows the desired inequality.

Proof of Proposition 12. First, note $f(i|e_o)$ may potentially involve $\bar{I}$, so we do some transformation. Let $\tau=i/\bar{I}$ be the ratio. Suppose $\tau\in[0,1]$ has a distribution $\tilde{F}(\tau|e_o)$, which is independent of $\bar{I}$. Then, $F(i|e_o)=\tilde{F}(i/\bar{I}|e_o)$ and $f(i|e_o)=\frac{1}{\bar{I}}\tilde{f}(i/\bar{I}|e_o)$.

We denote $\tilde{s}(\mu,\lambda,\tau)$ as the cut-off solving $\mu+\sum_j\lambda_j(1-\frac{g(s|\hat{e}_m^j)\tilde{f}(\tau|\hat{e}_o^j)}{g(s|e_m^*)\tilde{f}(\tau|e_o^*)})=1$ for $s<\tau\bar{I}$, where j is the index for the j-th IC constraint. And denote $\tau_s(\mu,\lambda)$ as the minimum solution for $\tilde{s}(\mu,\lambda,\tau)=\tau\bar{I}$, $\tau_m(\mu,\lambda)$ as the minimum solution for $\mu+\sum_j\lambda_j(1-\frac{(1-G(\tau\bar{I}|\hat{e}_m^j))\tilde{f}(\tau|\hat{e}_o^j)}{(1-G(\tau\bar{I}s|e_m^*))\tilde{f}(\tau|e_o^*)})=1$.

By strong duality, and Theorem 2, we have

$$\begin{aligned}
-W(\bar{I})&=\min_{\mu,\lambda}\phi(\mu,\lambda,\bar{I})\\
&=\min_{\mu,\lambda}\int_{\tau_s(\mu,\lambda)}^1\int_{\tilde{s}(\mu,\lambda,\tau)}^{\tau\bar{I}}\left(-1+\mu+\sum_j\lambda_j(1-\frac{g(s|\hat{e}_m^j)\tilde{f}(\tau|\hat{e}_o^j)}{g(s|e_m^*)\tilde{f}(\tau|e_o^*)})\right)g(s|e_m^*)\tilde{f}(\tau|e_o^*)dsd\tau\\
&\quad+\int_{\tau_m(\mu,\lambda)}^1\left(-1+\mu+\sum_j\lambda_j(1-\frac{(1-G(\tau\bar{I}|\hat{e}_m^j))\tilde{f}(\tau|\hat{e}_o^j)}{(1-G(\tau\bar{I}s|e_m^*))\tilde{f}(\tau|e_o^*)})\right)(1-G(\tau\bar{I}s|e_m^*))\tilde{f}(\tau|e_o^*)d\tau\\
&\quad-\sum_j\lambda_j[c(e^*)-c(\hat{e}^j)]-\mu[c(e^*)+\underline{U}].
\end{aligned}$$

Note the dual is convex in (μ, λ). Let (μ^*, λ^*) be the solution of the dual minimization. Then, by the envelope theorem, we have

$$\begin{aligned}
-\frac{dW(\bar{I})}{d\bar{I}} &= \frac{\partial}{\partial \bar{I}}\phi(\mu^*, \lambda^*, \bar{I}) \\
&= \int_{\tau_s(\mu^*,\lambda^*)}^{1} \tau \left(-1+\mu^* + \sum_j \lambda_j^* (1 - \frac{g(\tau\bar{I}|\hat{e}_m^j)\tilde{f}(\tau|\hat{e}_o^j)}{g(\tau\bar{I}|e_m^*)\tilde{f}(\tau|e_o^*)})\right) g(\tau\bar{I}|e_m^*)\tilde{f}(\tau|e_o^*)d\tau \\
&\quad - \int_{\tau_m(\mu^*,\lambda^*)}^{1} \tau \left((-1+\mu^*)g(\tau\bar{I}|e_m^*)\tilde{f}(\tau|e_o^*) + \sum_j \lambda_j^* [g(\tau\bar{I}|e_m^*)\tilde{f}(\tau|e_o^*) - g(\tau\bar{I}|\hat{e}_m^j))\tilde{f}(\tau|\hat{e}_o^j)]\right) d\tau \\
&= -\int_{\tau_m(\mu^*,\lambda^*)}^{\tau_s(\mu^*,\lambda^*)} \tau \left(-1+\mu^* + \sum_j \lambda_j^* (1 - \frac{g(\tau\bar{I}|\hat{e}_m^j)\tilde{f}(\tau|\hat{e}_o^j)}{g(\tau\bar{I}|e_m^*)\tilde{f}(\tau|e_o^*)})\right) g(\tau\bar{I}|e_m^*)\tilde{f}(\tau|e_o^*)d\tau \\
&> -\left(-1+\mu^* + \sum_j \lambda_j^* (1 - \frac{g(\tau_s(\mu^*,\lambda^*)\bar{I}|\hat{e}_m^j)\tilde{f}(\tau_s(\mu^*,\lambda^*)|\hat{e}_o^j)}{g(\tau_s(\mu^*,\lambda^*)\bar{I}|e_m^*)\tilde{f}(\tau_s(\mu^*,\lambda^*)|e_o^*)})\right) \int_{\tau_m(\mu^*,\lambda^*)}^{\tau_s(\mu^*,\lambda^*)} \tau g(\tau\bar{I}|e_m^*)\tilde{f}(\tau|e_o^*)d\tau = 0,
\end{aligned}$$

where the inequality is by MLRP and $\tau_s(\mu^*, \lambda^*) \geq \tau_m(\mu^*, \lambda^*)$.

OA.2. Numerical Illustration of a Mast-and-Sail Compensation Plan

We now use a numerical example to illustrate an optimal bonus region of the "mast and sail" structure (as seen in Figure 2(a)) in the case of outcome distributions that satisfy the MLRP. This example shows the delicacy of numerical computation in this setting, which is typical of moral hazard problems. Our extensive numerical simulations use similar logic to that found in this example.

Example OA.1. Consider the following instance of the multitasking store manager. The distribution functions of operating and marketing effort are $F(i|e_o) = i^{e_o}$ and $G(s|e_m) = s^{e_m}$, respectively, where $e_o \in \{e_o^L, e_o^H\}$ and $e_m \in \{e_m^L, e_m^H\}$, where $e_o^L = e_m^L = 1$ and $e_o^H = e_m^H = 2$. The target action is $(e_o^H, e_m^H) = (2, 2)$. The cost function is $c(e_o^H, e_m^H) = 5$, $c(e_o^H, e_m^L) = c(e_o^L, e_m^H) = 4$ and $c(e_o^L, e_m^L) = 2$. The resource constraint for the firm has $\bar{w} = 10$.

For now, we suppose an optimal choice of ω (as guaranteed to exist by Theorem 2) has $\omega_{e_o^L, e_m^L} = 1$ and $\omega_{e_o^H, e_m^L} = \omega_{e_o^L, e_m^H} = 0$. We construct the associated trigger value t below, and also show the resulting compensation plan with these choices of parameters is indeed feasible to (4) and thus optimal.

The condition $R^{\text{NSO}}_{e_o,e_m}(i,s) \geq t$ can be expressed as

$$1 - t \geq \tfrac{f(i|e_o)g(s|e_m)}{f(i|e_o^H)g(s|e_m^H)} = \tfrac{e_o}{e_o^H}\tfrac{e_m}{e_m^H} i^{e_o - e_o^H} s^{e_m - e_m^H}$$

and the condition $R^{\text{SO}}_{e_o,e_m}(i) \geq t$ amounts to

$$1 - t \geq \tfrac{e_o i^{e_o - 1}(1 - i^{e_m})}{e_o^H i^{e_o^H - 1}(1 - i^{e_m^H})}.$$

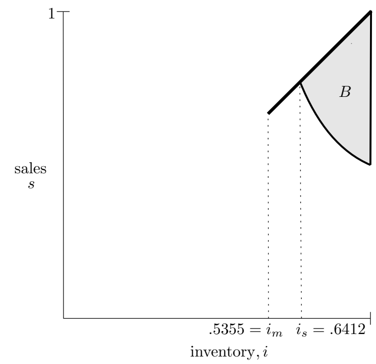


Figure OA.3 The bonus region of the optimal compensation plan for Example OA.1.

These two conditions are equivalent to (suppose $e_o < e_o^H$)

$$i \geq \begin{cases} \frac{1}{4s(1-t)} & \text{if } s < i \\ i_m(t) & \text{if } s = i \end{cases},$$

where $i_m(t) \triangleq \frac{-1+\sqrt{1+2/(1-t)}}{2}$. Therefore, the bonus region B^{NSO} is

$$\{(i,s) : \tfrac{1}{4i(1-t)} \leq s < i \text{ and } i \geq i_s(t)\},$$

where $i_s(t) \triangleq \frac{1}{2\sqrt{1-t}}$.

The next step is to determine t. Because $\omega_{e_o^L, e_m^L} = 1$, the NJ constraint between (e_o^H, e_m^H) and (e_o^L, e_m^L) is tight. We can isolate for t in the resulting equality to determine t. The tight NJ constraint is

$$\int_{R(i,s|e)\geq t} R(i,s) f(i|e_o^*) g(s|e_m^*) ds di = (c(e_o^H, e_m^H) - c(e_o^L, e_m^L)) \cdot \bar{w}^{-1},$$

which can be rewritten as

$$\int_{i_s(t)}^{1} \int_{\frac{1}{4i(1-t)}}^{i} (4si - 1) ds di + \int_{i_m(t)}^{1} (2i(1-i^2) - (1-i)) di = 0.3.$$

Solving for t results in $t^* \simeq 0.3919$.

We check whether the trigger compensation plan w^* with $t^* \simeq 0.3919$, $\omega_{e_o^L, e_m^L} = 1$, and $\omega_{e_o^H, e_m^L} = \omega_{e_o^L, e_m^H} = 0$ is an optimal compensation plan. It suffices to check that the remaining no-jump constraints are feasible. We first check that no profitable deviation to $e = (e_o^H, e_m^L)$ exists. Under the trigger compensation plan, the marginal revenue of deviation to (e_o^H, e_m^L) is

$$\bar{w} \int_{i_s(t)}^{1} \int_{\frac{1}{4i(1-t)}}^{i} (4si - 2i) ds di + \bar{w} \int_{i_m(t)}^{1} (2i(1-i^2) - 2i(1-i)) di.$$

Plugging $t^* \simeq 0.3919$ into the above object yields $1.748 > 1 = c(e_o^H, e_m^H) - c(e_o^H, e_m^L)$. Therefore, the store manager will not deviate to (e_o^H, e_m^L).

Similarly, the store manager's marginal revenue of deviation to (e_o^L, e_m^H) is

$$\bar{w}\int_{i_s(t)}^{1}\int_{\frac{1}{4i(1-t)}}^{i}(4si-2s)dsdi+\bar{w}\int_{i_m(t)}^{1}(2i(1-i^2)-(1-i^2))di.$$

Plugging $t^* \simeq 0.3919$ into the above object yields $1.864 > 1 = c(e_o^H, e_m^H) - c(e_o^L, e_m^H)$. Therefore, the store manager will not deviate to (e_o^L, e_m^H). The bonus region is illustrated in Figure OA.3. Note $i_m(t^*) = 0.5355 < i_s(t^*) = 0.6412$, and so we have a mast-and-sail bonus region as plotted in Figure OA.3. Under this optimal compensation plan, the probability of payout under the optimal mast-and-sail compensation plan is 51.96%.

OA.3. Numerical Illustrations of Corner Compensation Plans

To give a sense of how to compute the optimal corner compensation plan we give two concrete examples. These examples refer to some auxiliary results in the same online appendix that help us analyze corner compensation plans.

Example OA.2 (Example OA.1 continued). Returning to the set up in Example OA.1, the optimal corner compensation plan solves problem (14) and can be simply stated as:

$$\begin{aligned}
\min_{a,b} \quad & (1-b^2)(1-a^2) \\
\text{s.t.} \quad & (1-b^2)(1-a^2)-(1-b)(1-a) \geq 0.3 \\
& (1-b^2)(1-a^2)-(1-b^2)(1-a) \geq 0.2 \\
& (1-b^2)(1-a^2)-(1-b)(1-a^2) \geq 0.2 \\
& b \leq a.
\end{aligned}$$

Note we have formulated the objective to minimize the expected probability of paying out the bonus $\bar{w}$ (following Proposition 7), which is equivalent to the objective function (14a).

Now, the optimality condition for the optimal corner compensation plan

$$\frac{\mathcal{H}^f(a|e_o^*)}{\mathcal{H}^g(b|e_m^*)} = \frac{\mathcal{H}^f(a|e_o)}{\mathcal{H}^g(b|e_m)}$$

from equation (15) implies

$$\frac{\frac{2a}{(1-a^2)}}{\frac{2b}{(1-b^2)}} = \frac{\frac{1}{(1-a)}}{\frac{1}{(1-b)}},$$

which yields $a = b$. Combined with the other case of Proposition 8, we have $a = b$ in optimality. In this case, we can show the optimal choice of ω has $\omega_{e_o^L, e_m^L} = 1$ and $\omega_{e_o^H, e_m^L} = \omega_{e_o^L, e_m^H} = 0$. The optimal corner compensation plan can be obtained through solving $(1-b^2)(1-b^2)-(1-b)^2 = 0.3$, which yields $b^* = 0.5234$. Under the optimal corner compensation plan, the probability of payout is 0.5271. By comparison, the probability of payout under the optimal mast-and-sail compensation plan is 0.5196. So the performance gap is (0.5271-0.5196)/0.5196=1.44%.

EXAMPLE OA.3. Consider an instance with the same setting as in Example OA.1 except that the cost functions are as follows: $c(e_o^H, e_m^H) = 3.4$, $c(e_o^H, e_m^L) = 1.5$, $c(e_o^L, e_m^H) = 1.8$, and $c(e_o^L, e_m^L) = 1$. In this case, we can show the optimal choice of ω has $\omega_{e_o^H, e_m^L} = 1$ and $\omega_{e_o^L, e_m^L} = \omega_{e_o^L, e_m^H} = 0$. Thus, we can solve for b^* by setting $(1-b^2)(1-b^2) - (1-b^2)(1-b) = 0.19$, which yields $b^* = 0.4896$, which corresponds to a probability of payout of 0.5781. By comparison, the optimal mast-and-sail compensation plan has a probability of payout of 0.5148. Thus, using a corner compensation plan leads to an efficiency loss of $(0.5781 - 0.5148)/0.5148 = 12.30\%$.

OA.4. Fully Observed Demand

A natural benchmark is to look at the scenario where demand is fully observed and not censored by inventory; that is, both the firm and store manager can observe Q. This situation is much simpler than the case of censored demand. The analysis in Section 4 still applies and it can be shown that $R_{e_o,e_m}(i,q) = 1 - \frac{f(i|e_o)g(q|e_m)}{f(i|e_o^*)g(q|e_m^*)}$, which is precisely what we analyzed before as $R^{NSO}_{e_o,e_m}(i,s)$ in (9) for $s \le i$. Similar reasoning thus yields the result:

PROPOSITION OA1. *A nonincreasing and continuous function q^f exists such that an optimal contract w^f to the fully observed demand exists with the form*

$$w^f(i,q) = \begin{cases} \overline{w} & \text{if } (i,q) \in B^f \\ 0 & \text{otherwise,} \end{cases}$$

where $B^f = \{(i,q) : q \ge q^f(i)\}$.

Recall, just as in Proposition 1 for s^*, the domain of the function q^f need not be all of $[0, \bar{I}]$, and an $\bar{i}$ may exist such that $B^f \subseteq [\bar{i}, \bar{I}] \times [0, \bar{Q}]$. Thus, the bonus region of the optimal contract in the fully observed demand case is an "expanded sail" that is not truncated by the 45° line.

Next, we use a numerical experiment to demonstrate the efficiency loss due to demand censoring, by comparing the firm's expected additional payment to the store manager — compared to the first-best scenario — for inducing the same effort level. We illustrate our result in Figure OA.4.

OA.5. Approximating Mast-and-Sail Contracts

In this section, we consider approximating mast-and-sail contracts when we allow these contracts to be nonmonotone in nature, in contrast to Section 7.2, where attention was restricted to monotone approximations. The purpose of this exercise is to understand what aspects of the mast-and-sail structure are driving optimality.

We consider the following three nonmonotone approximations, in addition to weighted-sum threshold policies:

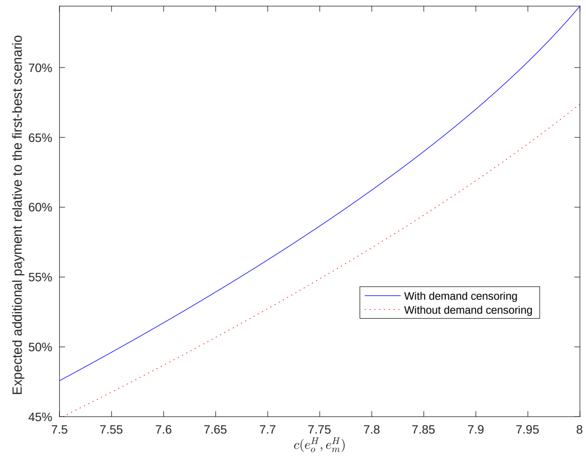
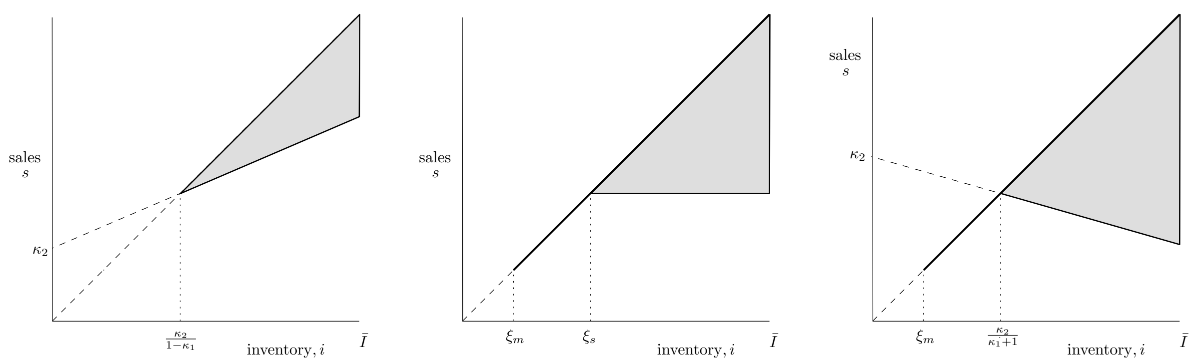


Figure OA.4 Expected additional payment relative to the first-best scenario with and without demand censoring. Parameters: $\bar{w} = 20$, $c(e_o^L, e_m^L) = 1$ and $c(e_o^H, e_m^L) = c(e_o^L, e_m^H) = 5$. We vary $c(e_o^H, e_m^H)$ between 7.5 and 8.



(a) A weighted-difference-threshold compensation plan.

(b) A mast-and-flat-sail compensation plan.

(c) A mast-and-linearized-sail compensation plan.

Figure OA.5 Nonmonotone simplifications of the mast-and-sail contracts.

(a) *Weighted-difference threshold compensation plan*: This compensation plan is similar to a weighted-sum threshold compensation plan, except that the threshold for the bonus payout is a weighted *difference* of the sales quantity and inventory level. Specifically, the agent receives a bonus if the sales quantity s and inventory level i satisfy $s - \kappa_1 \cdot i \geq \kappa_2$, for some $\kappa_1, \kappa_2 \geq 0$.

(b) *Mast-and-flat-sail compensation plan*: This compensation plan has both a "mast" and a "sail" except that the bottom of the sail is flattened. It has two parameters, ξ_m and ξ_s, where $0 < \xi_m \leq \xi_s < 1$, such that the agent receives a bonus under one of the following two conditions: (a) the sales quantity $s \geq \xi_m$ and all inventory is cleared and (b) the sales quantity $s \geq \xi_s$ and not all inventory is cleared.

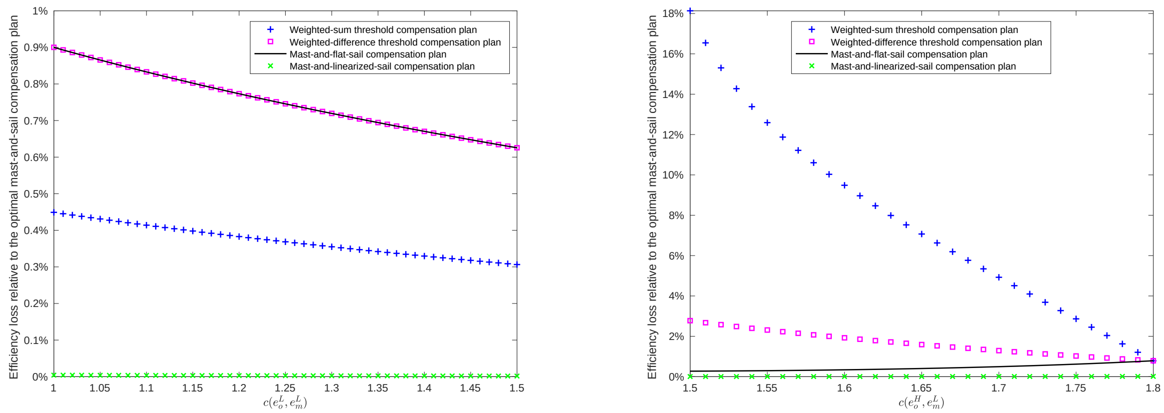
(c) *Mast-and-linearized-sail compensation plan*: This simple contract most closely mimics the structure of the mast-and-sail contract and has three parameters, ξ_m, κ_1 and κ_2, all nonnegative, such that the agent receives a bonus under one of the following two conditions: (a) the sales quantity $s \geq \xi_m$ and all inventory is cleared or (b) the sales quantity s and realized inventory level i satisfy $s + \kappa_1 \cdot i \geq \kappa_2$ and not all inventory is cleared.

These contract types are illustrated visually in Figures OA.5(a) and OA.5(c). One can view the mast-and-flat-sail compensation plan as a "demand-censoring-aware" threshold policy because it can be viewed as a generalized sales-quota-bonus compensation plan with two thresholds, depending on whether inventory is cleared. Similarly, the mast-and-linearized-sail compensation plan is a "demand-censoring-aware" weighted-sum compensation plan. In this case, however, the linear portion is always downward sloping, because it mimics the nonincreasing function s^*.

We evaluate the performance of the above three compensation plans (along with weighted-sum threshold contracts) through extensive numerical experiments, and illustrate two representative scenarios in Figure OA.6. We draw the following observations. First, the performance of the weighted-sum threshold compensation plan is similar to that of the corner (or sales-quota-bonus) compensation plan in that it is near optimal when the marketing and operational activities are sufficiently complementary (see Figure OA.6(a)), yet far from the optimal otherwise (see Figure OA.6(b)). Interestingly, the weighted-difference threshold compensation plan performs reasonably well in the latter case.

Second, the mast-and-flat-sail compensation plan, despite having both a "mast" and a "sail," can be outperformed by a "triangular sail" of the weighted-sum threshold described in Section 7.2 (see Figure OA.6(a)). However, the shape of the triangular sail is important here. Contracts with two "pieces" (a mast and something resembling a sail) can tailor incentives to the "cleared inventory" and "not cleared inventory" cases in isolation from each other. A single "piece" contract (like a weighted threshold compensation plan) needs to effectively "bridge" between these two cases by designing the transition from "cleared inventory" to "not cleared inventory." A perfect "bridge" is not always possible to build, but our experiments show single-piece contracts with strong performance are possible if constructed correctly. The correct construction (whether a weighted-sum or weighted-difference threshold) is the one that most closely mimics the structure of the underlying optimal mast-and-sail contract. For instance, a weighted-difference contract mimics the case in which the bottom tip of the mast is lower than the bottom tip of the sail. Note, however, that weighted-difference compensation plans are also not monotone, again underscoring the fact that non-monotonicity is somehow endemic to the case of pure demand censoring explored here.

This leads to our final observation. The performance of the mast-and-linearized-sail compensation plan is consistently near optimal (or optimal). It treats the "cleared inventory" case and "not cleared inventory" case individually and with relatively little restriction (except that the sail is triangular). Thus, it gains many of the benefits of the mast-and-sail compensation plan but remains simple and easier to compute. The drawback, of course, is that it remains non-monotone and thus susceptible to *ex post* manipulation of inventory just as in the mast-and-sail setting.



(a) Parameters: $\bar{w} = 10$, $c(e_o^H, e_m^L) = c(e_o^L, e_m^H) = 3.0$, and $c(e_o^H, e_m^H) = 3.5$. We vary $c(e_o^L, e_m^L)$ between 1 and 1.5.

(b) Parameters: $\bar{w} = 10$, $c(e_o^L, e_m^L) = 1$, $c(e_o^L, e_m^H) = 1.8$, and $c(e_o^H, e_m^H) = 3.5$. We vary $c(e_o^H, e_m^L)$ between 1.5 and 1.8.

Figure OA.6 Performance of the optimal weighted-sum threshold , weighted-difference threshold, mast-and-flat-sail, and mast-and-linearized-sail compensation plans, relative to the optimal mast-and-sail compensation plan. We assume the same random distributions for I and S as in Figure OA.5.

OA.6. Single-tasking versus Multitasking

A natural question is to compare a multitasking manager to a single-tasking manager responsible for only one of these tasks. In this section, we ask about the optimal incentives for an inventory manager who finds himself in the same situation as the store manager in the main body of the paper, but cannot influence demand through his efforts (i.e., he can only undertake operational effort). Building on our analysis of the general multitasking setting in Section 4 (in particular Theorem 2), one can show the optimal contract for the inventory manager problem is an inventory quota-bonus compensation plan.

Proposition OA2. *There exist nonnegative multipliers ω and a "target" t such that an optimal solution to the single-tasking inventory manager problem of the following form exists:*

$$w^*(i,s) = \begin{cases} \bar{w} & \text{if } R(i,s) \geq t \\ 0 & \text{otherwise} \end{cases},$$

where now $R(i,s) = 1 - \frac{f(i|e^L)}{f(i|e^H)}$, and where the goal is to implement high inventory effort.

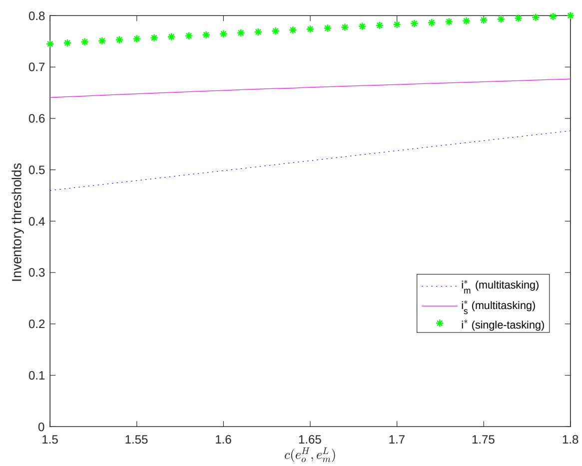


Figure OA.7 **Comparison of inventory thresholds under single-tasking versus multitasking. All the parameters are the same as in Figure 5(b)**

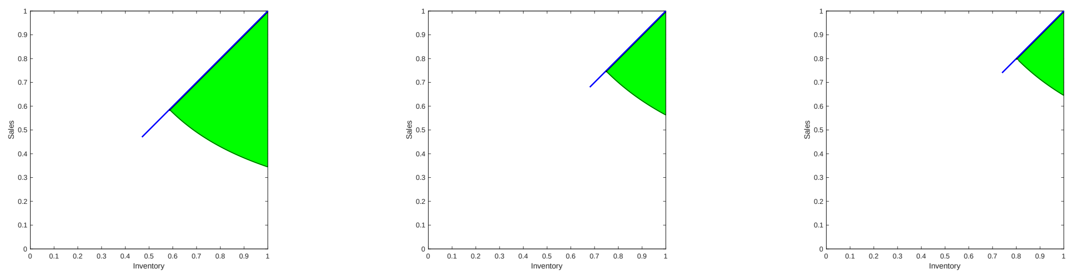
Under the MLRP assumption, it is straightforward to show the condition $R(i,s) \geq t$ translates into an inventory threshold i^* where

$$w^*(i,s) = \begin{cases} \bar{w} & \text{if } i \geq i^*. \\ 0 & \text{if } i < i^*. \end{cases} \tag{OA12}$$

Given the optimality of a quota-bonus structure for the inventory management problem, an interesting question is whether the inventory threshold should be higher for the single-tasking agent than the multitasking agent. We explore this question numerically. Consider the same set of parameters as in Figure 5(b). In the case of single-tasking, we fix the marketing effort e_m as e_m^H and look for an inventory threshold i^* specified in (OA12). We observe from Figure OA.7 that under single-tasking, the firm consistently chooses an inventory threshold i^* that is higher than both inventory thresholds (i.e., i_s^* and i_m^*) derived from the multitasking setting. In particular, this result implies an inventory manager has a more stringent requirement for earning a bonus in terms of inventory than a multitasker who is responsible for inventory (among other things).

OA.7. Effect of Compensation Ceiling on Compensation Plans and Firm Profitability

We now numerically illustrate the effect of the compensation ceiling ($\bar{w}$) on the design of the optimal compensation plan and the firm's profitability. Consider a case in which all the parameters are the same as in Figure 5(a) except that (a) we fix $c(e_o^L, e_m^L) = 1.0$ and (b) we vary the value of $\bar{w}$ from 9 to 21. We compute the optimal compensation plans and plot three scenarios in Figure OA.8, corresponding to the cases of $\bar{w} = 9, 15$, and 21, respectively. In addition, we compute the firm's expected cost of compensating the store manager for each combination of parameters. We do not explicitly report the firm's expected profit, which also depends on its unit revenue; a lower compensation cost suggests a higher expected profit. Figure Figure OA.9 shows the relationship between $\bar{w}$ and the firm's expected cost of compensation.



(a) $\bar{w} = 9$ (b) $\bar{w} = 15$ (c) $\bar{w} = 21$

Figure OA.8 The effect of compensation ceiling on the optimal bonus region. In each panel, the blue line shows the "mast" part of the bonus region, whereas the green area shows the "sail" part.

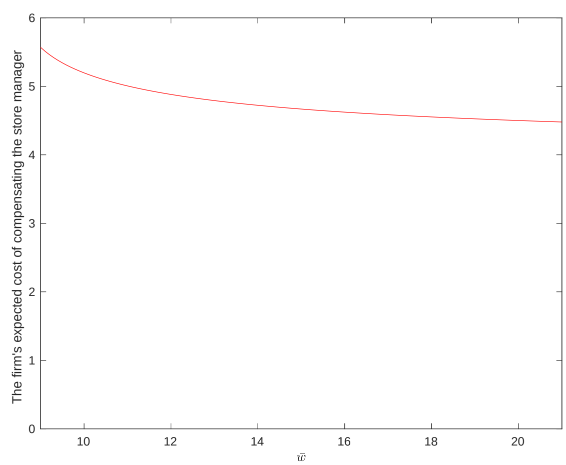


Figure OA.9 The effect of $\bar{w}$ on the firm's expected cost of compensating the store manager.

OA.8. Microfounding the Compensation Ceiling

In this section, we attempt to microfound the compensation ceiling by examining the case in which the agent's probability of receiving the bonus cannot be below a pre-specified threshold (denoted by $\mathcal{P}$). Our analysis is motivated by the observation that in practice, firms aim to ensure each agent, when opting to exert effort, can receive the bonus with a reasonably high likelihood.

In an influential book on salesforce compensation, Zoltners et al. (2006, pp. 53–57) point out two key factors in the effect of contracts to motivate effort in salesforce staff — engagement (measured by an "engagement rate") and excitement (measured by an "excitement index"). The engagement rate "measures what percentage of a sales force receives incentive pay with a plan," whereas the excitement index measures "the rate at which salespeople earn their last incremental dollar" (in our case, the bonus). Clearly, a tension exists between the engagement rate and excitement index. Zoltners et al. (2006) states that too low an engagement rate can hurt the morale of the salesforce. Although we are examining a multitasking setting, we believe the engagement-excitement tradeoff characterized by Zoltners et al. (2006) is relevant to our setting because a higher compensation ceiling ($\bar{w}$) makes the optimal compensation plan more rewarding yet less achievable. Hence, the choice of $\bar{w}$ illustrates this engagement-excitements tradeoff. Zoltners et al. (2006) argue that an "ideal engagement rate" exists and depends on a number of factors. We believe this notion provides practical justification for the notion that $\bar{w}$ could be chosen to match this "ideal engagement rate" that balances the engagement-excitement tradeoff to best motivate the store manager to exert high effort.

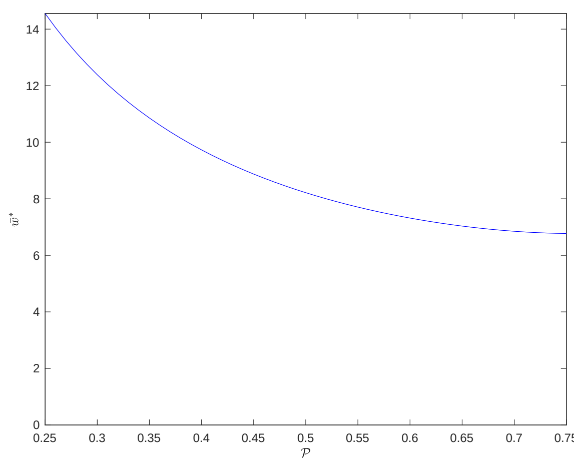


Figure OA.10 **The maximum $\bar{w}$ as a function of the agent's minimum likelihood of receiving the bonus ($\mathcal{P}$), which varies between 25% and 75%. All the parameters are the same as in Figure 5(a) except that we fix $c(e_o^L, e_m^L) = 1.0$.**

Note that for any given $\bar{w}$, we can characterize the optimal compensation plan as in Section 5.2, which gives the optimal solution specified by $t^*(\bar{w})$ such that the bonus region consists of two parts:

$$B^{\text{NSO}}(\bar{w}) \triangleq \left\{ (i,s) \in D^{\text{NSO}} : \sum_{e_o,e_m} \omega_{e_o,e_m} R^{\text{NSO}}_{e_o,e_m}(i,s) \geq t^*(\bar{w}) \right\}$$

and

$$B^{\mathrm{SO}}(\bar{w}) \triangleq \left\{ (i,s) \in D^{\mathrm{SO}} : \sum_{e_o,e_m} \omega_{e_o,e_m} R^{\mathrm{SO}}_{e_o,e_m}(i,s) \geq t^*(\bar{w}) \right\}.$$

Thus, the problem is equivalent to finding the maximum $\bar{w}$ that satisfies

$$\Pr\left(\sum_{e_o,e_m} \omega_{e_o,e_m} R^{\mathrm{NSO}}_{e_o,e_m}(i,s) \geq t^*(\bar{w}) \right) + \Pr\left(\sum_{e_o,e_m} \omega_{e_o,e_m} R^{\mathrm{SO}}_{e_o,e_m}(i,s) \geq t^*(\bar{w}) \right) \geq \mathcal{P}. \tag{OA13}$$

Using (OA13) as an additional constraint and endogenously choosing $\bar{w}$, we conduct a numerical experiment and illustrate in Figure OA.10 a sensitivity analysis showing how $\mathcal{P}$ restricts the range of $\bar{w}$ in the incentive-design problem. Our experiment shows that as such a likelihood increases, as one would expect, the value of $\bar{w}$ decreases. Thus, by incorporating the agent's likelihood of receiving a bonus, this extension can be viewed as a natural way of microfounding $\bar{w}$.

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