

Appendix to “Social Influence Undermines the Wisdom of the Crowd in Sequential Decision-Making”, Frey & van de Rijt, Management Science

**Appendix A: Additional information on analyses**

This appendix contains two tables and one figure, presenting results mentioned in Sections 3.2 and 4.2.

Table A1. Number of correct majorities by session and experimental condition. Considered are the 25 questions with  $d < \frac{1}{2}$ . There was one tie (March 14), while in all other sessions the independent condition produced more correct majorities than the social influence condition.

<i>Session</i>	<i>Number of correct majorities</i>	
	<i>Influence condition</i>	<i>Independent condition</i>
March 2, 2017	21	24
March 6, 2017	17	24
March 7, 2017	19	21
March 8, 2017	21	22
March 9, 2017 A	22	23
March 9, 2017 B	19	23
March 13, 2017	20	21
March 14, 2017	21	21

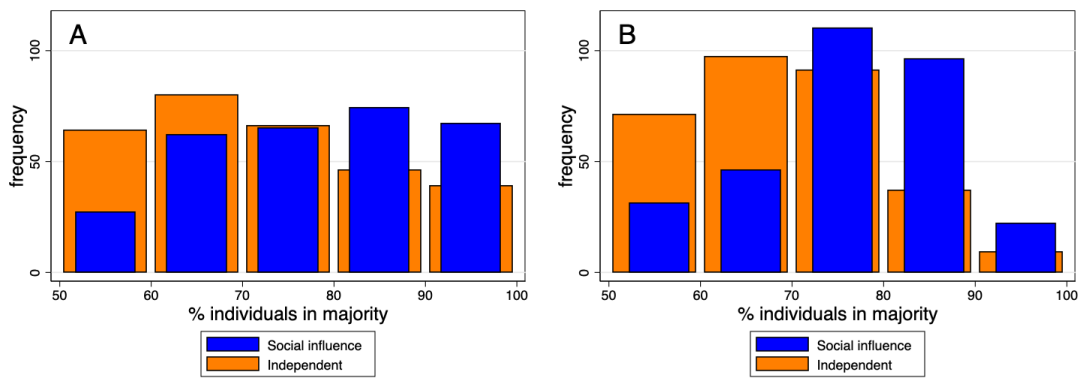


Figure A1. Distribution of majority size by experimental condition in the online experiment; A: small groups (15), B: large groups (100)

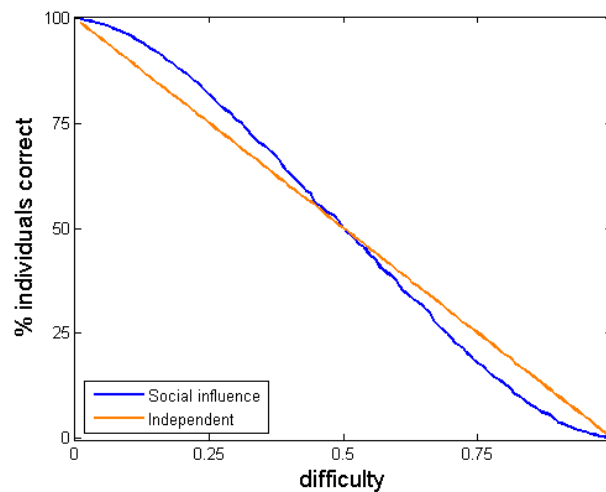
Table A2. The strength of social influence in the laboratory experiment (model (1)), in 15-person groups in the online experiment (model (2)), among the first 50 subjects of the 100-person groups (model (3)), and among the second 50 subjects of the 100-person groups (model (4)). The dependence of the strength of social influence in 100-person groups on the number of previous votes (model (5)) and on the incentive scheme (model (6)). Results from multilevel logistic regressions of the odds of a subject choosing the correct answer on the proportion of previous correct choices (Proportion\_correct), including dummy variables for question fixed effects (not reported) and random effects at the subject level for the intercept and the susceptibility to social influence. (Data from the social influence condition only)

	(1)	(2)	(3)	(4)	(5)	(6)
<i>Fixed-effects parameters</i>						
Proportion_correct	3.548*** (.298)	2.359*** (.227)	2.361*** (.152)	3.745*** (.223)	1.980*** (.186)	2.447*** (.217)
Num_previous_votes					-.016*** (.003)	
Proportion_correct * Num_previous_votes					.025*** (.004)	
Individual_reward						.593** (.202)
Proportion_correct * Individual_reward						-.783** (.300)
Intercept	-.643 (.362)	-1.028*** (.247)	-.572*** (.149)	-1.471*** (.187)	-.317* (.144)	-1.068*** (.198)
<i>Random-effects parameters</i>						
var(Proportion_correct)	3.286 (1.111)	5.805 (1.343)	6.626 (.962)	10.769 (1.577)	7.732 (.855)	5.793 (.985)
var(Intercept)	.716 (.352)	1.859 (.553)	1.798 (.388)	3.187 (.607)	2.120 (.353)	2.048 (.432)
cov(Proportion_correct, intercept)	-1.498 (.591)	-3.209 (.835)	-3.382 (.597)	-5.858 (.981)	-4.015 (.541)	-3.302 (.632)
<i>N</i> choices	2,024	3,672	13,471	14,003	27,474	7,637
<i>N</i> subjects	94	349	1,225	1,243	2,468	699

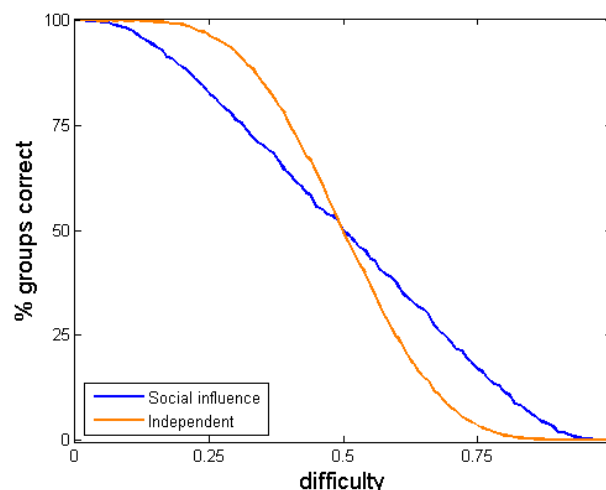
\*:  $p < .05$ , \*\*:  $p < .01$ , \*\*\*:  $p < .001$  (provided only for fixed-effects parameters)

## Appendix B: Analysis for questions with difficulty $d > 0.5$

Our theoretical analysis assumed that independent answers are more likely right than wrong ( $d < \frac{1}{2}$ ), and the empirical analysis was accordingly restricted to questions with observed difficulty  $d < \frac{1}{2}$ . The model predictions reverse for  $d > \frac{1}{2}$ . If  $d > \frac{1}{2}$ , the majority choice in a large group is almost certainly wrong – the wisdom of crowds becomes the “dumbness of crowds” (cf. Grofman, Owen & Feld 1983, p. 264). For  $d > \frac{1}{2}$ , our theoretical model predicts that social influence *decreases* the probability that an individual chooses correctly (H1b) but *increases* the probability that the majority choice is correct (H2b). Figures B1 and B2 show the model predictions for  $d$  ranging from 0 to 1, where for  $d < \frac{1}{2}$  Figures B1 and B2 replicate Figures 1 and 2 presented in the main text.



**Figure B1.** Predicted percentages of correct choices as a function of difficulty, with social influence ( $s_i \sim N(3,1)$ ) and without ( $s_i = 0 \ \forall i$ ). Shown are averages of 10,000 runs for groups of size 12, for each difficulty level and for both scenarios.



**Figure B2.** Predicted percentages of correct majorities as a function of difficulty, with social influence ( $s_i \sim N(3,1)$ ) and without social influence ( $s_i = 0 \ \forall i$ ). Shown are averages of 10,000 runs for groups of size 12, for each difficulty level and for both scenarios.

We here report results for the questions with observed difficulty  $d > \frac{1}{2}$ . It is important to note, however, that the experiments were designed with the intention to maximize statistical power for the detection of the effects of social influence when  $d < \frac{1}{2}$ . On average, participants of the online experiments were presented merely 2.9 questions with observed difficulty  $d > \frac{1}{2}$  while in the laboratory experiment each participant answered 5 questions with  $d > \frac{1}{2}$ .

Table B1 shows that hypothesis 1b is not supported: In none of the conditions was the percentage of correct answers per participant significantly larger in the independent condition than in the social influence condition. In the laboratory experiment, the percentage of correct answers per participant was even significantly higher in the social influence condition than in the independent condition, contrary to H1b.

The data lend some support for the expectation that social influence increases the chance of a correct majority answer if  $d > \frac{1}{2}$  (H2b). The average percentage of correct majority answers per group was among 100-person groups significantly higher in the influence condition than in the independent condition, and the lab experiment shows the same pattern at a somewhat lower significance level (Table B2). The data thus indicate that social influence can increase the chance of correct group answers to questions that independent individuals tend to answer incorrectly. The literature on group problem solving knows a similar result from studies on tasks that group can solve correctly if just one group member finds the correct solution and can explain the solution to the rest of the group (Laughlin 1999; Maciejovsky & Budescu 2007). The mechanism that drives the result in our experiments may be somewhat related as a few early votes for the correct answer may induce subsequent group members to focus on aspects that lead them to the correct answer.

**Table B1.** Average percentages of correct answers by individual participants in the independent condition and the social influence condition. *p*-values from Mann-Whitney U tests.

	Laboratory		Online	
	(Small groups, mixed reward)	Small groups, mixed reward	Large groups, mixed reward	Small groups, individual reward
Independent	40.7	42.4	38.3	36.9
Social influence	56.8	40.7	37.1	37.2
<i>p</i> -value	.000	.640	.510	.907

**Table B2.** Average percentages of correct majority answers in the independent condition and the social influence condition. *p*-values from a Wilcoxon signed-rank test (laboratory) and Mann-Whitney U tests (online).

	Laboratory		Online	
	(Small groups, mixed reward)	Small groups, mixed reward	Large groups, mixed reward	Small groups, individual reward
Independent	35.0	37.7	9.0	30.0
Social influence	57.5	37.1	30.0	26.7
<i>p</i> -value	.075	.809	.002	.537

## Appendix C: The questions used in the experiments

In the laboratory experiment, there were six questions of each of the categories **visual**, **art**, **equations**, **history**, and **geometry** (questions v1, v2, ..., v6, a1, a2, ..., g6). In the online experiment, there were likewise thirty questions in total, ten visual questions, ten art questions, and ten geometry questions. The sets of ten questions used online normally included the six questions from the laboratory experiment, but some questions were changed, so that we have for example two questions v4: v4-lab and v4-web. Below we list all questions by category. The three attention check questions used online are shown last.

The cyclic order of questions in the laboratory experiment was as follows: v1, a1, e1, h1, g1, v2, a2, e2, ..., v6, a6, e6, h6, g6.

For the online experiment, five decks of fifteen questions were drawn. For small groups, large groups, as well as small groups with purely individual incentives, each deck was used for five groups in the social influence condition as well as in the independent condition.

Deck 1: a1, v1, g1, a2, v2, g2, a3, v3, g3, a4, v4, g4, a5, v5, g5

Deck 2: a3, v3, g3, a4, v4, g4, a5, v5, g5, a6, v6, g6, a7, v7, g7

Deck 3: a5, v5, g5, a6, v6, g6, a7, v7, g7, a8, v8, g8, a9, v9, g9

Deck 4: a7, v7, g7, a8, v8, g8, a9, v9, g9, a10, v10, g10, a1, v1, g1

Deck 5: a9, v9, g9, a10, v10, g10, a1, v1, g1, a2, v2, g2, a3, v3, g3

## List of questions

The answering categories are in parentheses, with the correct answer underlined. Shapes and images were shown to subjects in larger size.

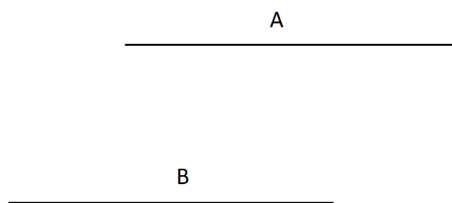
Shown in brackets is the difficulty  $d$  – the proportion of correct answers in the independent condition – separately for the laboratory experiment (“lab”), small groups and large groups in the online experiment (“web-15” and “web-100”, respectively), and small groups with purely individual incentives (“web-15-indiv”).

### VISUAL QUESTIONS

v1. Which line is longer?

(A, B)

[ $d$ : lab: .200, web-15: .267, web-100: .245, web-15-indiv: .283]



v2. Which shape has the larger area?

(Circle, Rectangle)

[ $d$ : lab: .337, web-15: .353, web-100: .431, web-15-indiv: .366]



v3. Which shape has the larger area?

(Triangle, Circle)

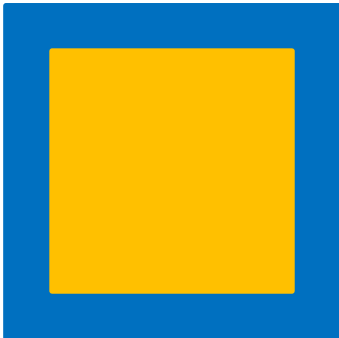
[ $d$ : lab: .323, web-15: .536, web-100: .539, web-15-indiv: .551]



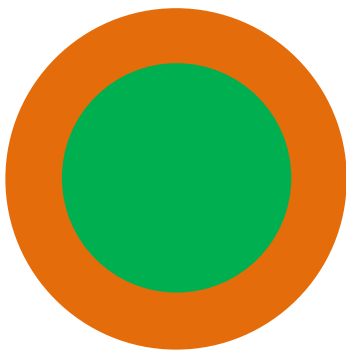
v4-lab. Which area is larger?  
(Blue edge, Yellow center)  
[d: lab: .296]



v4-web. Which area is larger?  
(Blue edge, Yellow center)  
[d: web-15: .213, web-100: .253, web-15-indiv: .276]  
(compared to v4-lab, the yellow center is slightly smaller)



v5-lab. Which area is larger?  
(Orange ring, Green center)  
[d: lab: .156]

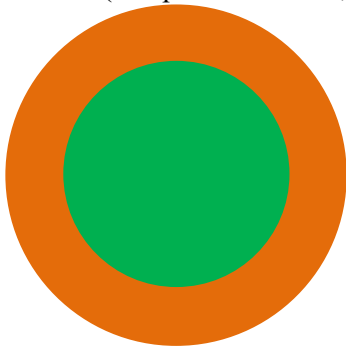


v5-web. Which area is larger?

(Orange ring, Green center)

[d: web-15: .348, web-100: .455, web-15-indiv: .383]

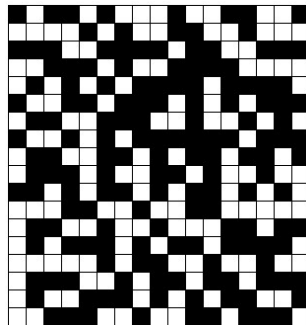
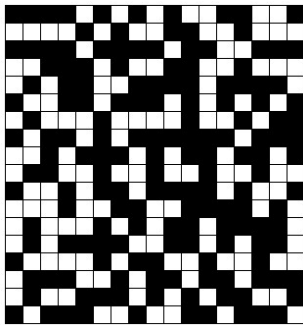
(compared to v5-lab, the green area is slightly smaller)



v6. Which square has more black cells?

(Left, Right)

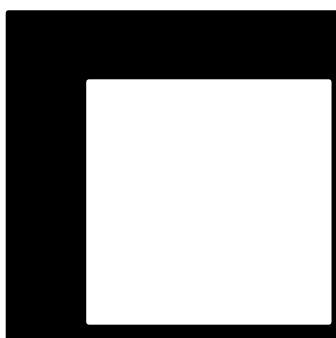
[d: lab: .340, web-15: .366, web-100: .441, web-15-indiv: .375]



v7. Which area is larger?

(Black edge, White center)

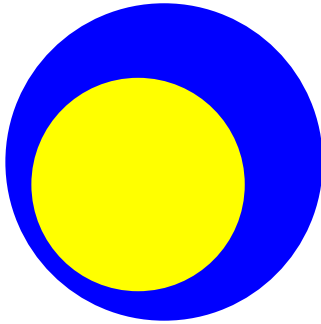
[d: web-15: .341, web-100: .258, web-15-indiv: .262]



v8. Which area is larger?

(Blue edge, Yellow center)

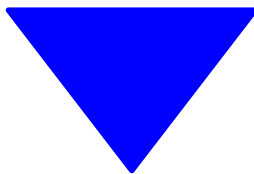
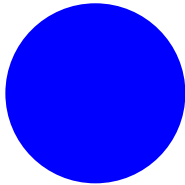
[d: web-15: .213, web-100: .336, web-15-indiv: .247]



v9. Which shape has the larger area?

(Triangle, Circle)

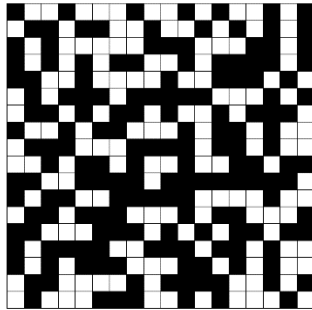
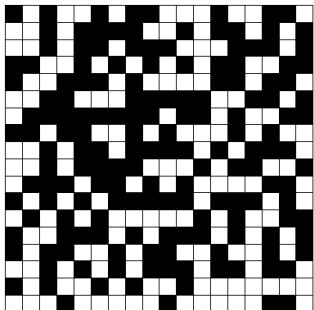
[d: web-15: .444, web-100: .452, web-15-indiv: .465]



v10. Which square has more black cells?

(Left, Right)

[d: web-15: .517, web-100: .454, web-15-indiv: .444]



## ART QUESTIONS

The pictures shown to subjects are omitted due to copyright restrictions. Information on which specific pictures were used is available from the authors on request.

a1-lab. Were these pieces of furniture designed by the same designer?

Picture 1: LC3 Divano, by Le Corbusier

Picture 2: Eames Lounge Chair and Ottoman, by Charles and Ray Eames

(Same designer, Different designers)

[d: lab: .198]

a1-web. Are these art works from the same artist?

Picture 1: Michael Jackson, by Andy Warhol

Picture 2: Mickey Mouse, by Andy Warhol

(Same artist, Different artists)

[d: web-15: .350, web-100: .334, web-15-indiv: .399]

a2-lab. Which building is an example of baroque architecture?

Left picture: Einsiedeln Abbey

Right picture: Cologne Cathedral

(Left, Right)

[d: lab: .402]

a2-web. Which building is an example of baroque architecture?

Left picture: Einsiedeln Abbey

Right picture: Grote of Sint-Laurenskerk

(Left, Right)

[d: web-15: .371, web-100: .396, web-15-indiv: .496]

a3. Which painting is a Vincent van Gogh?

Left picture: Evening, by Edvard Munch

Right picture: Walk Along the Banks of the Seine Near Asnières, by Vincent Van Gogh

(Left, Right)

[d: lab: .198, web-15: .353, web-100: .325, web-15-indiv: .371]

a4-lab. From which period is this lamp?

Picture: Hala Pinocchio table lamp, by H. Busquet

(Before 1990, After 1990)

[d: lab: .495]

a4-web. From which period is this lamp?

Picture: Hala Pinocchio table lamp, by H. Busquet

(Before 1920, After 1920)

[d: web-15: .144, web-100: .153, web-15-indiv: .182]

a5. What is the style of this painting?

Picture: Bain à la Grenouillère, by Claude Monet

(Impressionism, Expressionism)

[d: lab: .204, web-15: .255, web-100: .248, web-15-indiv: .176]

a6-lab. Who created this statue?

Picture: David statue, by Leonardo da Vinci

(Leonardo da Vinci, Michelangelo)

[d: lab: .134]

a6-web. Where is this statue located?

Picture: David statue, by Leonardo da Vinci

(Marseille, Florence)

[d: web-15: .071, web-100: .081, web-15-indiv: .047]

a7. Which building is older?

Left picture: City hall, Leuven, Belgium

Right picture: Federal Palace of Switzerland

(Left, Right)

[d: web-15: .237, web-100: .234, web-15-indiv: .248]

a8. Who painted this picture?

Picture: Guernica, by Pablo Picasso

(Picasso, Dali)

[d: web-15: .396, web-100: .345, web-15-indiv: .338]

a9. In which country was this picture taken?

Picture: Brandenburg Gate

(Germany, France)

[d: web-15: .544, web-100: .562, web-15-indiv: .594]

a10. Who painted this picture?

Picture: Gardanne, by Paul Cézanne

(Paul Cézanne, Paul Klee)

[d: web-15: .164, web-100: .191, web-15-indiv: .237]

## EQUATIONS

e1. What is  $x$  approximately?  $x = [ (1600/8) + 300 ] / 2.5$

( $x = 125$ ,  $x = 200$ )

[d: lab: .333]

e2. What is  $x$ ?  $25x = 950 + 5000 / (100)$

( $x = 40$ ,  $x = 50$ )

[d: lab: .293]

e3. What is  $x$ ?  $-(20 + x) = -(72 / 2)$

( $x = 12$ ,  $x = 16$ )

[d: lab: .110]

e4. What is  $x$ ?  $-x + 38 / 2 = 20$

( $x = -1$ ,  $x = -2$ )

[d: lab: .489]

e5. What is  $x$  approximately?  $628x = \pi$

( $x \cong .005$ ,  $x \cong .05$ )

[d: lab: .438]

e6. What is  $x$  approximately?  $1 / x = 1 + x$

( $x \cong 0.62$ ,  $x \cong 0.82$ )

[d: lab: .322]

## HISTORY QUESTIONS

h1. When was the sovereignty over Hong Kong transferred from the United Kingdom to China?

(1990, 1997)

[d: lab: .515]

h2. When did Germany invade Denmark?

(1939, 1940)

[d: lab: .388]

h3. To what island was Napoleon banned in 1814?

(Elba, Saint Helena)

[d: lab: .571]

h4. Whose wife was Helen of Troy?

(Agamemnon, Menelaus)

[d: lab: .633]

h5. When did the French Revolution start?

(1776, 1789)

[d: lab: .423]

h6. Which country was a founding member of the European Economic Community?

(Luxemburg, Spain)

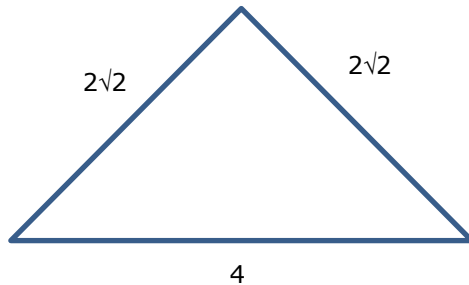
[d: lab: .134]

## GEOMETRY QUESTIONS

g1. What is the bottom-left angle of this triangle?

(30 degrees, 45 degrees)

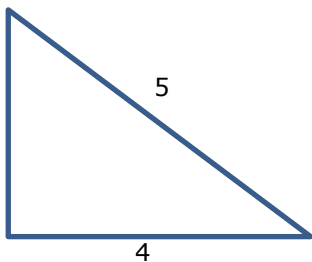
[d: lab: .275, web-15: .328, web-100: .348, web-15-indiv: .350]



g2-lab. How long is the left side of this triangle?

(2.8, 3.0)

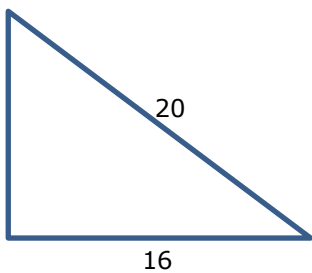
[d: lab: .118]



g2-web. How long is the left side of this triangle?

(12, 13)

[d: web-15: .241, web-100: .240, web-15-indiv: .223]



g3-lab. The sides of the square have length 3. What is the area of the circle?

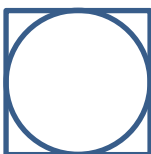
(1.50pi, 2.25pi)

[d: lab: .596]

g3-web. The sides of the square have length 3. What is the area of the circle?

(9pi, 2.25pi)

[d: web-15: .561, web-100: .636, web-15-indiv: .605]



g4. The area of the largest square is 16. What is the area of the smallest square?

(2, 3)

[d: lab: .175, web-15: .231, web-100: .270, web-15-indiv: .278]



g5-lab. The right side of the blue shape below has length 1. What is the size of the blue shape?

(.625, .675)

[d: lab: .457]

g5-web. The right side of the blue shape below has length 1. What is the size of the blue shape?

(.625, .75)

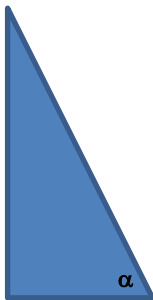
[d: web-15: .590, web-100: .590, web-15-indiv: .594]



g6-lab. What is the tangent of  $\alpha$ ?

(2,  $\frac{1}{2}$ )

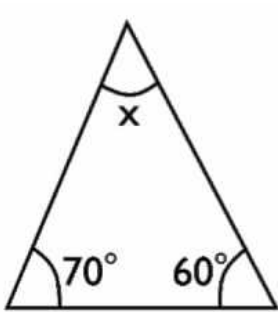
[d: lab: .670]



g6-web. g6. How many degrees is angle  $x$ ?

(40°, 50°)

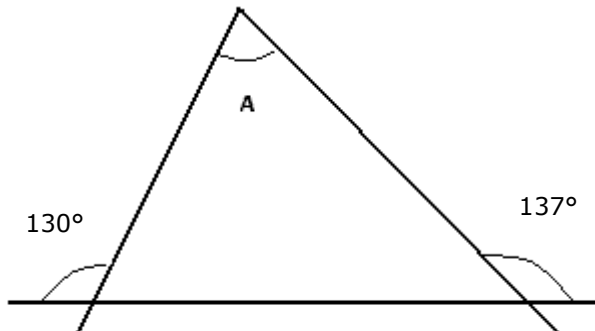
[d: web-15: .275, web-100: .265, web-15-indiv: .224]



g7. What is the size of angle A?

(87, 80)

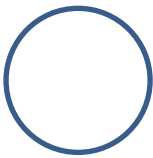
[d: web-15: .388, web-100: .406, web-15-indiv: .314]



g8. The circumference of the circle is  $\pi$ . What is the radius of the circle?

( $\frac{1}{2}$ ,  $\frac{3}{2}$ )

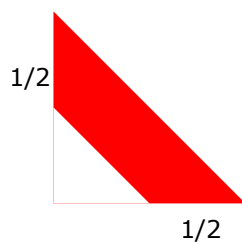
[d: web-15: .286, web-100: .318, web-15-indiv: .395]



g9. What is the area of the red shape?

( $\frac{5}{16}$ ,  $\frac{3}{8}$ )

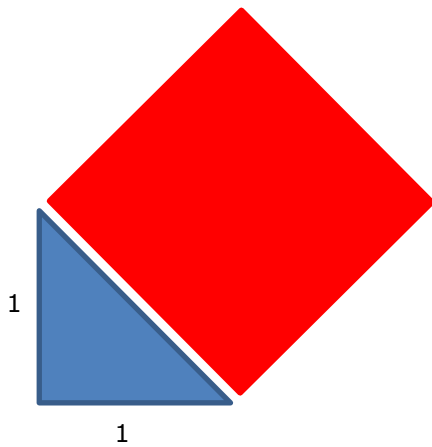
[d: web-15: .418, web-100: .327, web-15-indiv: .343]



g10. What is the size of the red square?

(2, 4)

[d: web-15: .814, web-100: .849, web-15-indiv: .836]



#### ATTENTION CHECK QUESTIONS

1. Which shape has the larger area?

(Circle, Rectangle)



2. Which of these two people is a former President of the U.S.?

Left picture: Barack Obama

Right picture: Karin Keller-Sutter

(Left, Right)

3. What do you get if you melt ice?

(Water, Asphalt)

## Appendix D: Analysis of the model

We analyze the model with zero variance in influence:  $s_i = s = \mu$ .

### Equilibrium

In equilibrium, change in  $p$  is zero,  $c(p) = \left(1 + \frac{d}{1-d} e^{s(1-2p)}\right)^{-1} - p = 0$ , which yields:  $s = \frac{\ln \frac{dp}{(1-d)(1-p)}}{2p-1}$

### Stability

Stability of equilibria is given by negative values of the derivative of  $c(p)$ :  $\frac{\partial c}{\partial p} = \frac{2s \frac{1-d}{d} e^{s(2p-1)}}{\left(\frac{1-d}{d} e^{s(2p-1)} + 1\right)^2} - 1$

### Stable equilibria

At low levels of  $s$ , there exists a unique equilibrium that has a majority choosing correctly, while at high levels of  $s$ , there are two stable equilibria, one with a correct, one with a wrong majority:

Stable equilibria $p^*$	$s = 2$		$s = 2.5$		$s = 3$		$s = 4$	
	majority correct	majority wrong	majority correct	majority wrong	majority correct	majority wrong	majority correct	majority wrong
$d = .1$								
$d = .2$							1.00	
$d = .3$					.98		.99	<b>.07</b>
$d = .4$			.93		.98	<b>.19</b>	.99	<b>.04</b>
$d = .49$	.69		.87	<b>.16</b>	.93	<b>.08</b>	.98	<b>.03</b>