Electronic Companion:

In this e-companion, we present supporting material to the main paper. In§**??**, we present the proof of Proposition 2. In §EC.2, we prove Proposition 3. In §EC.3, we prove Proposition .

In §EC.4, we prove technical lemmas **??**-1. In §EC.10, we present additional empirical support which corresponds to analyzing our first call-center data set, described in §3.

EC.1. Proof of Proposition 2

PROPOSITION EC.1. *In the $M/M/n$ model as either (i) $\rho^n = \frac{\lambda^n}{n\mu} \to \rho < 1$, or (ii) $\rho^n \to 1$ in the QED many-server heavy-traffic regime, as given by (9), we have:*

$$r[W^n(\tau_t^n), W^n(t^n)] \to \rho \quad as \quad n \to \infty. \tag{EC.1}$$

Proof. For the proof of the proposition, we need the following Lemma EC.1 which we prove in the following section.

LEMMA EC.1. *For all $x \geq 0$,*

$$\lim_{n\to\infty} \mathbb{P}(W^n(\tau_t^n) \geq x) = \lim_{n\to\infty} \mathbb{P}(W^n(t^n) \geq x).$$

Moreover, $\lim_{n\to\infty} Var[W^n(\tau_t^n)] = \lim_{n\to\infty} Var[W^n(t^n)]$.

We let γ_t^n denote the time of entry of the LES customer to service, and let $\xi_t^n \equiv t^n - \gamma_t^n$ denote the time between the entry to service of the LES customer and the new arrival time. We note that the following analysis holds irrespective of the initial state of the system; in particular, we do not need to be in steady state. We can write

$$W^n(t^n) = \sum_{i=1}^{N(t^n-\tau_t^n)} X_i = \sum_{i=1}^{N(W^n(\tau_t^n)+\xi_t^n)} X_i = \sum_{i=1}^{N(W^n(\tau_t^n))} X_i + \sum_{i=1}^{N(t^n)-N(\gamma_t^n)-1} X_i = \sum_{i=1}^{N(W^n(\tau_t^n))} X_i + Y^n,$$

where $X_i \sim \text{Exp}(n\mu)$ and $Y^n \equiv \sum_{i=1}^{N(t^n)-N(\gamma_t^n)-1} X_i$ is independent of $W^n(\tau_t^n)$. This is so because γ_t^n is a stopping time for the Poisson arrival process; by the strong Markov property and the memoryless property for the exponential service times, we have independence. Thus, letting $\text{Cov}[X, Y]$ denote the covariance between two random variables X and Y:

$$\begin{aligned}\text{Cov}\left[W^n(\tau_t^n), W^n(t^n)\right] &= \text{Cov}\left[\sum_{i=1}^{N(W^n(\tau_t^n))} X_i, W^n(\tau_t^n)\right] \\ &= \mathbb{E}\left[\left(\sum_{i=1}^{N(W^n(\tau_t^n))} X_i\right) \cdot W^n(\tau_t^n)\right] - \mathbb{E}\left[\left(\sum_{i=1}^{N(W^n(\tau_t^n))} X_i\right)\right] \cdot \mathbb{E}\left[W^n(\tau_t^n)\right]\end{aligned}$$

$$
\begin{aligned}
&= \mathbb{E}\left[\mathbb{E}\left[\left(\sum_{i=1}^{N(W^n(\tau_t^n))} X_i\right)\cdot W^n(\tau_t^n)\right]\Big| W(\tau_t^n)\right] \\
&- \mathbb{E}\left[\mathbb{E}\left[\left(\sum_{i=1}^{N(W^n(\tau_t^n))} X_i\right)\right]\Big| W(\tau_t^n)\right]\cdot \mathbb{E}\left[W^n(\tau_t^n)\right] \\
&= \mathbb{E}\left[\frac{\lambda^n W(\tau_t^n)}{n\mu}\cdot W(\tau_t^n)\right] - \mathbb{E}\left[\frac{\lambda^n W(\tau_t^n)}{n\mu}\right]\mathbb{E}[W(\tau_t^n)] \\
&= \rho^n \mathbb{E}[(W^n(\tau_t^n))^2] - \rho^n(\mathbb{E}[W^n(\tau_t^n)])^2 = \rho^n \mathrm{Var}[W^n(\tau_t^n)],
\end{aligned}
$$

where we used the law of iterated expectations in the previous expressions. Therefore, we deduce that:

$$
r[W^n(\tau_t^n), W^n(t^n)] = \frac{\rho^n \mathrm{Var}[W^n(\tau_t^n)]}{\sqrt{\mathrm{Var}[W^n(\tau_t^n)]\mathrm{Var}[W^n(t^n)]}} \to \rho \text{ as } n \to \infty.
$$

■

EC.2. Proof of Proposition 3

Proposition EC.2. *For the $M/M/n+M$ model in the ED many-server heavy-traffic limiting regime, we have that:*

$$
r[W^n(\tau_t^n), W^n(t^n)] \to \frac{1}{\rho} \quad as \quad n \to \infty. \tag{EC.2}
$$

Proof. For the proof of the proposition, we need the following lemma where $\mathrm{Nor}(\mu, \sigma^2)$ denotes a normal distribution with mean μ and variance σ^2.

Lemma EC.2. *In the ED regime,*

(a) For any $w \geq 0$,

$$
\lim_{n\to\infty} \mathbb{P}(W^n(\tau_t^n) \geq w) = \lim_{n\to\infty} \mathbb{P}(W_S^n \geq w),
$$

where W_S^n is a random variable with the distribution of the virtual waiting time of a customer conditional on the customer being both delayed and served.

(b) As $n \to \infty$,

$$
\sqrt{n}(W_S^n - \bar{w}) \Rightarrow \mathrm{Nor}\left(0, \frac{1}{\theta\mu}\right),
$$

where $\bar{w} \equiv \frac{1}{\theta}\ln(\rho)$.

We are now ready to derive an asymptotic expression for the correlation. To this end, we write:

$$
r[W^n(t^n), W^n(\tau_t^n)] = \frac{\mathrm{Cov}[W^n(t), W^n(\tau_t^n)]}{\sqrt{\mathrm{Var}[W^n(t)]\mathrm{Var}[W^n(\tau_t^n)]}}.
$$

We have,

$$
\begin{aligned}
\mathrm{Cov}[W^n(t^n), W^n(\tau_t^n)] &= \frac{1}{2}\left(\mathrm{Var}[W^n(t)] + \mathrm{Var}[W^n(\tau_t^n)] - \mathrm{Var}[W^n(t) - W^n(\tau_t^n)]\right) \\
&= \frac{1}{2}\left(\mathrm{Var}[W^n(t)] + \mathrm{Var}[W^n(\tau_t^n)]\right) - \frac{1}{2n}\mathrm{Var}[\sqrt{n}(W^n(t) - W^n(\tau_t^n))].
\end{aligned}
$$

By Theorem 4 of Ibrahim and Whitt (2009a), we have that

$$\mathrm{Var}[\sqrt{n}(W^n(t^n) - W^n(\tau_t^n))] \to \frac{2(\rho-1)}{\rho\theta} \text{ as } n \to \infty.$$

Also, we have that that $n\mathrm{Var}[W^n(t^n)] \to \frac{1}{\theta}$, where we use Theorem 6.4 of Talreja and Whitt (2009)[7]. Lemma EC.2 implies that $n\mathrm{Var}[W^n(\tau_t^n)] \to \frac{1}{\theta}$. Thus,

$$\begin{aligned} r[W^n(t^n), W^n(\tau_t^n)] &= \frac{\frac{n}{2}\mathrm{Var}[W^n(t^n)] + \frac{n}{2}\mathrm{Var}[W^n(\tau_t^n)] - \frac{1}{2}\mathrm{Var}[\sqrt{n}(W^n(t^n) - W^n(\tau_t^n))]}{\sqrt{n\mathrm{Var}[W^n(t^n)] \cdot n\mathrm{Var}[W^n(\tau_t^n)]}} \\ &\to 1 - \frac{\frac{\rho-1}{\rho\theta}}{\frac{1}{\theta}}. \end{aligned}$$

That is,

$$r[W^n(t), W^n(\tau_t^n)] \to \frac{1}{\rho} \text{ in the ED regime.}$$

■

EC.3. Proof of Proposition

PROPOSITION EC.3. *For the $M/M/n+M$ model in the many-server heavy-traffic QED regime:*

$$r[W^n(t^n), W^n(\tau_t^n)] \to 1 \quad as \quad n \to \infty. \tag{EC.3}$$

Proof. For the proof, we need the following lemma.

LEMMA EC.3. *In the QED regime,*

(a) For any $w \ge 0$,

$$\lim_{n\to\infty} \mathbb{P}(\sqrt{n}W^n(\tau_t^n) \ge w) = \lim_{n\to\infty} \mathbb{P}(\sqrt{n}W_S^n \ge w),$$

where W_S^n is a random variable with the distribution of a served customer who is delayed.

(b) For any $w \ge 0$,

$$\lim_{n\to\infty} \mathbb{P}(\sqrt{n}W^n(t^n) \ge w) = \lim_{n\to\infty} \mathbb{P}(\sqrt{n}W_S^n \ge w).$$

To prove the proposition, we can write:

$$r(W^n(t^n), W^n(\tau_t^n)) = \frac{\mathrm{Cov}[\sqrt{n}W^n(t^n), \sqrt{n}W^n(\tau_t^n)]}{\sqrt{\mathrm{Var}[\sqrt{n}W^n(t)]\mathrm{Var}[\sqrt{n}W^n(\tau_t^n)]}}.$$

We also have that,

$$\mathrm{Cov}[\sqrt{n}W^n(t^n), \sqrt{n}W^n(\tau_t^n)] = \frac{1}{2}\left(\mathrm{Var}[\sqrt{n}W^n(t^n)] + \mathrm{Var}[\sqrt{n}W^n(\tau_t^n)] - \mathrm{Var}[\sqrt{n}(W^n(t^n) - W^n(\tau_t^n))]\right).$$

[7] Talreja and Whitt (2009) do not condition on $W^n(t) > 0$. However, $\sqrt{n}(W^n(t) - \bar{w})$ and $\sqrt{n}(W^n(t) - \bar{w})$ have the same distribution asymptotically in the ED regime, since $\mathbb{P}(W^n(t) > 0) \to 1$.

We know that $\sqrt{n}(W^n(\tau_t^n) - W^n(t^n)) \Rightarrow 0$ as $n \to \infty$, which follows from Theorem 1 of Ibrahim et al. (2016), so that $\mathrm{Var}[\sqrt{n}(W^n(\tau_t^n) - W^n(t^n))] \to 0$. [8] By Lemma EC.3, together with uniform integrability, we have:

$$\lim_{n\to\infty} \mathrm{Var}[\sqrt{n}W^n(t^n)] = \lim_{n\to\infty} \mathrm{Var}[\sqrt{n}W^n(\tau_t^n)].$$

By Garnett et al. (2002), we have that $\sqrt{n}W^n(t^n)$ converges weakly to a finite random variable so that its variance converges as well to a positive constant. Thus,

$$r[W^n(t^n), W^n(\tau_t^n)] \to 1 \text{ in the QED regime.}$$

■

EC.4. Proofs of Technical Lemmas

EC.4.1. Proof of Lemma EC.1

Proof. First, we prove part (a). The statement holds trivially for $x = 0$. Now, for $x > 0$:

$$\begin{aligned}\mathbb{P}(W^n(\tau_t^n) \geq x) &= \mathbb{P}(W^n(\tau_t^n) \geq x | W^n(\tau_t^n) > 0)\mathbb{P}(W^n(\tau_t^n) > 0)\\ &= \mathbb{P}(W_\infty^n \geq x | W_\infty^n > 0, D^n, E^n)\mathbb{P}(W^n(\tau_t^n) > 0),\end{aligned}$$

where we define the following events:

- D^n: next arrival after entry to service is delayed
- E^n: next arrival is before next entry to service

We also note that:

$$\mathbb{P}(W^n(\tau_t^n) > 0) = 1 - \mathbb{P}(\text{LES customer finds exactly } n-1 \text{ customers in the system upon arrival}).$$

This is so because an LES customer must find at least $n-1$ customers in the system upon arrival. To see why, assume, aiming at a contradiction, that LES encounters $k < n-1$ customers in the system upon arrival. Then, it must be that the next arriving customer is not delayed, i.e., the current customer could not be an LES customer; this is a contradiction. Further,

$$\begin{aligned}\mathbb{P}(W_\infty^n \geq x | W_\infty^n > 0, D^n, E^n) &= \frac{\mathbb{P}(E^n, D^n | W_\infty^n \geq x, W_\infty^n > 0)\mathbb{P}(W_\infty^n \geq x, W_\infty^n > 0)}{\mathbb{P}(W_\infty^n > 0, E^n, D^n)}\\ &= \frac{\mathbb{P}(E^n, D^n | W_\infty^n \geq x)\mathbb{P}(W_\infty^n \geq x | W_\infty^n > 0)}{\mathbb{P}(E^n, D^n | W_\infty^n > 0)}\\ &= \mathbb{P}(W_\infty^n \geq x | W_\infty^n > 0),\end{aligned}$$

[8] We establish the uniform integrability for $\{\sqrt{n}W^n(\tau_t^n), n \geq 1\}$ in the proof of Lemma EC.3.

since $\mathbb{P}(D^n, E^n | W_\infty^n \geq x) = \mathbb{P}(D^n, E^n | W_\infty^n > 0)$. This is so because:

$$\begin{aligned}
\mathbb{P}(D^n, E^n | W_\infty^n \geq x) &= \mathbb{P}(D^n | E^n, W_\infty^n \geq x)\mathbb{P}(E^n | W_\infty^n \geq x) \\
&= \mathbb{P}(E^n | W_\infty^n \geq x) \text{ since } \mathbb{P}(D^n | E^n, W_\infty^n \geq x) = 1 \\
&= \frac{\lambda^n}{\lambda^n + n\mu} \\
&= \mathbb{P}(E^n | W_\infty^n > 0) \\
&= \mathbb{P}(D^n | E^n, W_\infty^n > 0)\mathbb{P}(E^n | W_\infty^n > 0) \text{ since } \mathbb{P}(D^n | E^n, W_\infty^n > 0) = 1 \\
&= \mathbb{P}(D^n, E^n | W_\infty^n > 0).
\end{aligned}$$

Finally, letting π_{n-1} denote the probability that the LES customer encounters $n-1$ customers in the system upon arrival, we must have that $\pi_{n-1}^n \to 0$ as $n \uparrow \infty$. This is so because for every n fixed we have $\sum_{k=0}^{\infty} \pi_k^n = 1$ which implies that $\pi_k^n \to 0$ as $k \to \infty$. Thus, for $k \geq M_\epsilon$, we have: $\pi_k^n < \epsilon$. This implies that for $n \geq M_\epsilon + 1$, we must also have that $\pi_{n-1}^n < \epsilon$. That is, $\pi_{n-1}^n \to 0$ as $n \to \infty$. This implies that $\mathbb{P}(W^n(\tau_t^n) > 0) \to 1$. Thus, we obtain:

$$\lim_{n\to\infty} \mathbb{P}(W^n(\tau_t^n) \geq x) = \lim_{n\to\infty} \mathbb{P}(W^n(t) \geq x) = \mathbb{P}(W_\infty \geq x | W_\infty > 0),$$

as desired, where $[W_\infty | W_\infty > 0]$ is the corresponding limiting steady-state distribution which is proper under both limiting regimes. To show that for every t:

$$\lim_{n\to\infty} \text{Var}[W^n(\tau_t^n)] = \lim_{n\to\infty} \text{Var}[W^n(t)] = \text{Var}[W_\infty \geq x | W_\infty > 0],$$

we need to show that uniform integrability of the sequence $\{W^n(\tau_t^n), n \geq 1\}$ holds. To do so, note that for any $x \geq 0$:

$$\begin{aligned}
\mathbb{P}[W^n(\tau_t^n) \geq x] &= \mathbb{P}[W_\infty^n \geq x | E^n, D^n] \\
&\leq \mathbb{P}[W_\infty^n \geq x | E^n, D^n, W_\infty^n > 0] \\
&= \frac{\mathbb{P}[W_\infty^n \geq x, E^n, D^n | W_\infty^n > 0]}{\mathbb{P}[E^n, D^n | W_\infty^n > 0]} \\
&\leq \frac{\mathbb{P}[W_\infty^n \geq x | W_\infty^n > 0]}{\mathbb{P}[E^n, D^n | W_\infty^n > 0]} \\
&\leq \frac{\mathbb{P}[W_\infty^n \geq x | W_\infty^n > 0]}{\frac{\rho^n}{\rho^n + 1}} \\
&= \mathbb{P}[W_\infty^n \geq x | W_\infty^n > 0] \frac{\rho^n + 1}{\rho^n}.
\end{aligned}$$

Thus, for any $x \geq 0$,

$$x \cdot \mathbb{P}[W^n(\tau_t^n) \geq x] \leq x \cdot \mathbb{P}[W_\infty^n \geq x | W_\infty^n > 0] \frac{\rho^n + 1}{\rho^n}.$$

This implies,

$$\begin{aligned}\mathbb{E}[(W^n(\tau_t^n))^2] = \int_0^\infty 2x\mathbb{P}[W^n(\tau_t^n) \geq x]dx &\leq \frac{\rho^n+1}{\rho^n}\int_0^\infty 2x\mathbb{P}[W_\infty^n \geq x | W_\infty^n > 0]dx \\ &= \frac{\rho^n+1}{\rho^n}\mathbb{E}[(W_\infty^n)^2 | W_\infty^n > 0] \\ &< M,\end{aligned}$$

for $M < \infty$ large enough. Thus, $\sup_{n\geq 1}\{\mathbb{E}[(W^n(\tau_t^n))^2]\} < \infty$, and the sequence $\{W^n(\tau_t^n), n \geq 1\}$ is uniformly integrable: Convergence of moments then follows from the convergence in distribution together with the established uniform integrability. ∎

EC.4.2. Proof of Lemma EC.2

Proof. First, note that $W^n(\tau_t^n) \stackrel{\mathcal{D}}{=} [W^n | S^n, A^n, E^n]$ where

- Event S^n: "customer is served"
- Event A^n: "next arrival after current entry to service is before next entry to service"
- Event E^n: "next arrival is delayed"

Part (a): The lemma holds trivially at $w = 0$. Letting $w > 0$, we have:

$$\begin{aligned}&\mathbb{P}(W^n(\tau^n) \geq w) \\ &= \int_w^\infty f_{W|A,S,E}(x|A^n, S^n, E^n)dx \\ &= \int_w^\infty \frac{f_{W|S,E}(x|S^n, E^n)\mathbb{P}(A^n|W^n = x, S^n, E^n)\mathbb{P}(S^n, E^n)}{\mathbb{P}(A^n, S^n, E^n)}dx \\ &= \frac{\mathbb{P}(S^n, E^n)}{\mathbb{P}(A^n, S^n, E^n)} \times \\ &\quad \int_w^\infty f_{W|S,E}(x|S^n, E^n)(\mathbb{P}(A^n|Q^n = 0, W^n = x, S^n, E^n)\mathbb{P}(Q^n = 0|W^n = x, S^n, E^n) \\ &+ \mathbb{P}(A^n|Q^n > 0, W^n = x, S^n, E^n)\mathbb{P}(Q^n > 0|W^n = x, S^n, E^n))dx.\end{aligned}$$

Since $\mathbb{P}(E^n) \to 1$, we can remove it hereafter from the conditioning event since it does not matter asymptotically. Let Q^n be the number of customers left in queue when LES enters service. Note that $[Q^n | W^n = x, S^n]$ is distributed as the number of customers at time x in an $M/M/\infty$ queue starting out empty a time 0 which is Poisson distributed with rate $\frac{\lambda^n}{\theta}(1 - e^{-\theta x})$. Thus, for $x > 0$,

$$\lim_{n\to\infty} \mathbb{P}(Q^n = 0 | W^n = x, S^n) = 0 \text{ and } \lim_{n\to\infty} \mathbb{P}(Q^n > 0 | W^n = x, S^n) = 1.$$

Also, note that $\mathbb{P}(A^n | Q^n > 0, W^n = x, S^n) = \frac{\lambda^n}{\lambda^n + n\mu} = \frac{\rho}{\rho+1}$. We now show that $\mathbb{P}(A^n | S^n, E^n) \to \frac{\rho}{\rho+1}$ as well. This is obtained by a similar conditioning argument, as follows.

$$\mathbb{P}(A^n | S^n)$$

$$
\begin{aligned}
&= \int_0^\infty \mathbb{P}(A^n|S^n, W^n = x) f_W(x|S^n)dx \\
&= \int_{0^+}^\infty \mathbb{P}(A^n|S^n, W^n = x, Q^n = 0)\mathbb{P}(Q^n = 0|S^n, W^n = x) f_W(x|S^n)dx \\
&+ \int_{0^+}^\infty \mathbb{P}(A^n|S^n, W^n = x, Q^n > 0)\mathbb{P}(Q^n > 0|S^n, W^n = x) f_W(x|S^n)dx \\
&+ \mathbb{P}(W^n = 0|S^n)\mathbb{P}(A^n|S^n, W^n = 0)\mathbb{P}(Q^n = 0|S^n, W^n = 0) \\
&= \frac{\rho}{\rho+1}\int_{0^+}^\infty \mathbb{P}(Q^n > 0|S^n, W^n = x) f_W(x|S^n)dx + \mathbb{P}(W^n = 0|S^n) \\
&\to \frac{\rho}{\rho+1}.
\end{aligned}
$$

Thus,

$$
\lim_{n\to\infty} \mathbb{P}(W^n(\tau_t^n) \geq w) = \lim_{n\to\infty} \mathbb{P}(W^n(t) \geq w|S^n) = \lim_{n\to\infty} \mathbb{P}(W_S^n \geq w).
$$

We note that we can establish the uniform integrability of $\{W^n(\tau_t^n), n \geq 1\}$ by the following argument, building on the above:

$$
\mathbb{P}(W^n(\tau^n) \geq w) = \mathbb{P}(W^n \geq w|A^n, S^n, E^n) \leq \frac{\mathbb{P}(W^n \geq w)}{\mathbb{P}(A^n, S^n, E^n)} \to (\rho+1)\mathbb{P}(W^n \geq w),
$$

which follows because: $\mathbb{P}(A^n, S^n, E^n) = \mathbb{P}(A^n|S^n, E^n)\cdot\mathbb{P}(S^n, E^n) \to \frac{\rho}{\rho+1}\frac{1}{\rho} = \frac{1}{\rho+1}$. Thus, we deduce that $\mathbb{E}[W^n(\tau_t^n)^2] = \int_0^\infty 2x\mathbb{P}(W^n(\tau_t^n) \geq x) \leq (\rho+1)\mathbb{E}[(W^n)^2] < M$ for some large $M < \infty$. Thus, uniform integrability follows.

Part (b) We use three lemmas.

Lemma EC.4. *As $n \to \infty$,*

$$
\sqrt{n}\left(\sum_{i=0}^{\lfloor nt \rfloor} Y_{i,n} - c(t)\right) \Rightarrow \text{Nor}(0, d(t)), \tag{EC.4}
$$

where $c(t) \equiv \frac{1}{\theta}\ln\left(1+\frac{\theta t}{\mu}\right)$ and $d(t) \equiv \frac{t}{\mu(\mu+\theta t)}$ for all $t \geq 0$.

Proof. Let $m_i \equiv \mathbb{E}[Y_{i,n}] = \frac{1}{n\mu+(i+1)\theta}$ and $\sigma_i^2 \equiv \text{Var}(Y_{i,n}) = \left(\frac{1}{n\mu+(i+1)\theta}\right)^2$. Then, for any $t \geq 0$:

$$
\frac{\max_{0\leq j\leq \lfloor nt \rfloor} m_j^2}{\sum_{j=0}^{\lfloor nt \rfloor} m_j^2} \to 0 \text{ as } n \to \infty.
$$

By Lemma EC.6 (which applies the Lindeberg-Feller CLT), we must have that

$$
\frac{\sum_{i=0}^{\lfloor nt \rfloor}(Y_{i,n} - m_i)}{\sqrt{\sum_{i=0}^{\lfloor nt \rfloor}\sigma_i^2}} \Rightarrow \text{Nor}(0,1). \tag{EC.5}
$$

To obtain (EC.4), note that for every $t \geq 0$:

$$
\sum_{i=0}^{\lfloor nt \rfloor} m_i \to c(t) \equiv \frac{1}{\theta}\ln\left(1+\frac{\theta t}{\mu}\right) \text{ and } n\sum_{i=0}^{\lfloor nt \rfloor}\sigma_i^2 \to d(t) \equiv \frac{t}{\mu(\mu+\theta t)}.
$$

Therefore, by the continuous mapping theorem,

$$\frac{\sum_{i=0}^{\lfloor nt\rfloor}(Y_{i,n}-m_i)}{\sqrt{\sum_{i=0}^{\lfloor nt\rfloor}\sigma_i^2}}=\sqrt{n}\frac{\sum_{i=0}^{\lfloor nt\rfloor}(Y_{i,n}-m_i)}{\sqrt{n\sum_{i=0}^{\lfloor nt\rfloor}\sigma_i^2}}\Rightarrow \text{Nor}(0,1) \text{ and } \sqrt{n}\left(\sum_{i=0}^{\lfloor nt\rfloor}Y_{i,n}-c(t)\right)\Rightarrow \text{Nor}(0,d(t)). \tag{EC.6}$$

■

We can use Lemma EC.4 to prove that W_S^n and W^n have asymptotically the same distribution, as follows.

LEMMA EC.5. *As $n\to\infty$,*

$$\sqrt{n}(W_S^n-w)\Rightarrow \text{Nor}(0,\frac{1}{\theta\mu}),$$

where $w\equiv\frac{1}{\theta}\ln(\rho)$.

Proof. We can write:

$$W_S^n=\sum_{i=0}^{Q^n}[Y_{i,n}|Y_{i,n}<T]\stackrel{D}{=}\sum_{i=0}^{Q^n}Y_{i,n},$$

where $T\sim\text{Exp}(\theta)$, since the rank ordering of exponentials and their minimum are independent. Now, using Lemma EC.4 and applying Theorem 6.4 of Talreja and Whitt (2009) yields the convergence. Since we also have that $\sqrt{n}(W^n-w)\Rightarrow\text{Nor}(0,\frac{1}{\theta\mu})$, W_S^n and W^n have the same distribution, asymptotically.

■

Above, we used the following lemma.

LEMMA EC.6. *(Resnick, Chap. 9, problem 19) Let $U_1,U_2,\cdots,U_k$ be a sequence of independent exponential random variables with respective means $m_i, 1\le i\le k$. If*

$$\frac{\max_{1\le i\le k}m_i^2}{\sum_{j=1}^k m_j^2}\to 0 \text{ as } k\to\infty,$$

then

$$\frac{\sum_{j=1}^k(U_j-m_j)}{\sqrt{\sum_{j=1}^k m_j^2}}\Rightarrow \text{Nor}(0,1).$$

■

EC.4.3. Proof of Lemma EC.3

Proof. Note that $W^n(\tau^n)=^{\mathcal{D}}[W^n|S^n,A^n,E^n]$ where

- Event S^n: "customer is served"
- Event A^n: "next arrival after current entry to service is before next entry to service"
- Event E^n: "next arrival is delayed"

Part (a). For $w \geq 0$,

$$
\begin{aligned}
&\mathbb{P}(\sqrt{n}W^n(\tau^n) \geq w) \\
&= \frac{1}{\sqrt{n}} \int_w^\infty f_{W|A,S,E}(x/\sqrt{n}|A^n, S^n, E^n)dx \\
&= \frac{1}{\sqrt{n}} \int_w^\infty \frac{f_{W|S,E}(x/\sqrt{n}|S^n)\mathbb{P}(A^n|W^n = x/\sqrt{n}, S^n, E^n)\mathbb{P}(S^n, E^n)}{\mathbb{P}(A^n, S^n, E^n)} dx \\
&= \frac{1}{\sqrt{n}} \frac{\mathbb{P}(S^n, E^n)}{\mathbb{P}(A^n, S^n, E^n)} \times \\
&[\int_w^\infty f_{W|S,E}(x/\sqrt{n}|S^n, E^n)\mathbb{P}(A^n|Q^n/\sqrt{n} = 0, W^n = x/\sqrt{n}, S^n, E^n)\mathbb{P}(Q^n/\sqrt{n} = 0|W^n = x/\sqrt{n}, S^n, E^n)dx \\
&+ \int_w^\infty f_{W|S,E}(x/\sqrt{n}|S^n, E^n)\mathbb{P}(A^n|Q^n/\sqrt{n} > 0, W^n = x/\sqrt{n}, S^n, E^n)\mathbb{P}(Q^n/\sqrt{n} > 0|W^n = x/\sqrt{n}, S^n, E^n))dx].
\end{aligned}
$$

Note that as $n \to \infty$,

$$
\mathbb{P}[Q^n/\sqrt{n} = 0|W^n = x/\sqrt{n}, S^n, E^n] = \mathbb{P}[Q^n = 0|W^n = x/\sqrt{n}, S^n] = Exp\left(-\frac{\lambda^n}{\theta}\left(1 - e^{-\theta x/\sqrt{n}}\right)\right) \to 1
$$

and $\mathbb{P}(A^n|Q^n/\sqrt{n} = 0, W^n = x/\sqrt{n}, S^n, E^n) = 1$. There remains to show that $\frac{\mathbb{P}(S^n,E^n)}{\mathbb{P}(A^n,S^n,E^n)} \to 1$. To obtain this,

$$
\begin{aligned}
&\mathbb{P}(A^n|S^n, E^n) \\
&= \frac{1}{\sqrt{n}} \int_0^\infty \mathbb{P}(A^n|S^n, W^n = x/\sqrt{n}, E^n) f_W(x/\sqrt{n}|S^n, E^n)dx \\
&= \frac{1}{\sqrt{n}} \int_0^\infty \mathbb{P}(A^n|S^n, W^n = x/\sqrt{n}, Q^n/\sqrt{n} = 0, E^n)\mathbb{P}(Q^n/\sqrt{n} = 0|S^n, W^n = x/\sqrt{n}, E^n) f_W(x/\sqrt{n}|S^n, E^n)dx \\
&+ \frac{1}{\sqrt{n}} \int_0^\infty \mathbb{P}(A^n|S^n, W^n = x/\sqrt{n}, Q^n/\sqrt{n} > 0, E^n)\mathbb{P}(Q^n/\sqrt{n} > 0|S^n, W^n = x/\sqrt{n}, E^n) f_W(x/\sqrt{n}|S^n, E^n)dx \\
&\to 1.
\end{aligned}
$$

Note that we are conditioning on the probability that the *next* customer is delayed, not the current one. However, the difference between the arrival times is asymptotically negligible so that the current customer must have been delayed as well. To establish uniform integrability of the sequence $\{\sqrt{n}W(\tau_t^n), n \geq 1\}$, we note that:

$$
\mathbb{P}(\sqrt{n}W^n(\tau^n) \geq w) \leq \frac{1}{\sqrt{n}} \int_w^\infty \frac{f_{W|S,E}(x/\sqrt{n}|S^n)\mathbb{P}(A^n|W^n = x/\sqrt{n}, S^n, E^n)\mathbb{P}(S^n, E^n)}{\mathbb{P}(A^n, S^n, E^n)} dx \to \mathbb{P}(\sqrt{n}W^n(t^n) \geq w),
$$

so that we can bound the second moment of $\sqrt{n}W^n(\tau^n)$ as in the proof of Lemmas **??** and EC.2.

Part (b). Let $w \geq 0$,

$$
\begin{aligned}
\lim_{n\to\infty} \mathbb{P}(\sqrt{n}W_S^n \geq w | W_S^n > 0) &= \lim_{n\to\infty} \mathbb{P}(\sqrt{n}W^n \geq w | S^n, W^n > 0) \\
&= \lim_{n\to\infty} \frac{\mathbb{P}(\sqrt{n}W^n \geq w, S^n | W^n > 0)}{\mathbb{P}(S^n | W^n > 0)} \\
&= \lim_{n\to\infty} \mathbb{P}(\sqrt{n}W^n \geq w | W^n > 0) \text{ since } \mathbb{P}(S^n) \to 1.
\end{aligned}
$$

■

EC.4.4. Proof of Lemma 1

Proof. We focus on delays for low-priority customers in what follows. We let γ_t^n denote the time of entry of the LES customer (of the low type) to service. For $x>0$:

$$\begin{aligned}
&\lim_{n\to\infty}\mathbb{P}(W^n(\tau_t^n)\geq x)\\
&=\lim_{n\to\infty}\left(\mathbb{P}(W^n(\tau_t^n)\geq x|W^n(\tau_t^n)>0)\mathbb{P}(W^n(\tau_t^n)>0)\right)\\
&=\lim_{n\to\infty}\mathbb{P}(W_\infty^n\geq x|W_\infty^n>0).
\end{aligned}$$

The last step proceeds similarly to our proof for the single-class $M/M/n$ queue, so we omit the relevant details. There remains to show that $\mathbb{P}(W^n(\tau_t^n)>0)\to 1$. For this, we resort to Lemma EC.7 (below) which implies that $\mathbb{P}(t^n-\gamma_t^n<M)\to 1$ for any $M>0$ because convergence in distribution to a constant implies convergence in probability. This implies:

$$\begin{aligned}
&\mathbb{P}(W^n(\tau_t^n)=0)\\
&=\sum_{k=0}^{n-1}\mathbb{P}(\text{LES customers finds } k \text{ customers in service at } \tau_t^n)\\
&\leq\sum_{k=0}^{n-1}\mathbb{P}(\text{at least } n-k-1 \text{ H arrivals in } (\tau_t^n,t^n))\\
&=\left[\sum_{k=0}^{n-1}\mathbb{P}(\text{at least } n-k-1 \text{ H arrivals in } (\tau_t^n,t^n)|t^n-\gamma^n<M)\right]\mathbb{P}(t^n-\gamma_t^n<M)\\
&+\left[\sum_{k=0}^{n-1}\mathbb{P}(\text{at least } n-k-1 \text{ H arrivals in } (\tau_t^n,t^n)|t^n-\gamma^n\geq M)\right]\mathbb{P}(t^n-\gamma_t^n\geq M)\\
&\leq e^{-Cn}+e^{-Kn}\quad\text{for some } C>0 \text{ by Chernoff bound and } K>0 \text{ by proof of Lemma EC.4}\\
&\to 0,
\end{aligned}$$

so that $\mathbb{P}(\text{LES customer was delayed})=\mathbb{P}(W^n(\tau_t^n)>0)\to 1$. ■

LEMMA EC.7. *As $n\to\infty$,*

$$t^n-\gamma_t^n\Rightarrow 0.$$

Proof. Let $\xi_t^n\equiv t^n-\gamma_t^n$ and calculate $\lim_{n\to\infty}\mathbb{P}(\xi_t^n>x)$ for $x\geq 0$. Note that there cannot be H customers in queue at time γ_t^n. We can write:

$$\mathbb{P}(\xi_t^n>x)=\sum_{i=1}^{2}\mathbb{P}(\xi_t^n>x|A_i)\mathbb{P}(A_i),$$

where Q_L denotes the number of L customers in queue and:

- $A_1 \equiv \{Q_L > 0 \text{ at } \gamma_t^n\}$; then

$$\begin{aligned}\mathbb{P}(\xi_t^n > x|A_1) &\leq \mathbb{P}(\text{at least as many H arrivals as service completions in } (0,\xi_t^n), \xi_t^n > x|A_1)\\ &\leq \mathbb{P}(\text{at least as many H arrivals as service completions in } (0,x)|A_1).\end{aligned}$$

By a slight abuse of notation: Conditional on all servers being busy in $(0,\xi_t^n)$, and in particular in $(0,x)$: # service completions $\sim$ Poiss$(n\mu x)$ and # H arrivals $\sim$ Poiss$(\lambda_H^n x)$, where $\lambda_H^n < n\mu$. Indeed, $T^n \equiv$ (# H arrivals - # service completions) has a Skellam $(\lambda_H^n x, n\mu x)$ distribution so that, by a bound on its weight at 0:

$$\mathbb{P}(\text{at least as many H arrivals as service completions in } (0,x)|A_1) = \mathbb{P}(T^n \geq 0) \leq e^{-n(\sqrt{\rho_H x}-\sqrt{x})^2} \to 0.$$

- $A_2 \equiv \{Q_L = 0 \text{ at } \gamma_t^n\}$; then

$$\begin{aligned}\mathbb{P}(\xi_t^n > x|A_2) &= \mathbb{P}(\xi_t^n > x, \text{ no L arrivals in } (0,x)|A_2)\\ &\quad + \mathbb{P}(\xi_t^n > x, \text{ at least one L arrival in } (0,x)|A_2).\end{aligned}$$

For the first part, note that:

$$\mathbb{P}(\xi_t^n > x, \text{ no L arrivals in } (0,x)|A_2) \leq \mathbb{P}(\text{no L arrivals in } (0,x)) \leq e^{-\lambda_L^n x} \to 0.$$

For the second part, note that:

$$\begin{aligned}&\mathbb{P}(\xi_t^n > x, \text{at least one L arrival in } (0,x)|A_2)\\ &= \mathbb{P}(\xi_t^n > x|\text{at least one L arrival in } (0,x), A_2)\\ &\times \mathbb{P}(\text{at least one L arrival in } (0,x)|A_2).\end{aligned}$$

Now, let s^n denote the time of the first L arrival in (γ_t^n, t^n), and define the events:

— $E^n \equiv \{\text{at least one L arrival in } (0,x), A_2\}$

— $F^n \equiv \{\text{at least 1 new H arrival remaining in queue at } s^n\}$

— $\bar{F}^n$ is the complement of F^n

Then, for any s^n:

$$\begin{aligned}\mathbb{P}(\xi_t^n > x|E^n) &= \mathbb{P}(\xi_t^n > x|E^n, F^n)\mathbb{P}(F^n|E^n) + \mathbb{P}(\xi_t^n > x|E^n, \bar{F}^n)\mathbb{P}(\bar{F}^n|E^n)\\ &\leq \mathbb{P}(\xi_t^n > x|E^n, F^n)\mathbb{P}(F^n|E^n)\\ &+ \mathbb{P}(\#\text{SC in } (s^n,t^n) \leq \#\text{H arrivals in } (s^n,t^n)|E^n, \bar{F}^n)\mathbb{P}(\bar{F}^n|E^n)\\ &\to 0,\end{aligned}$$

where SC denotes "service completions". This is so because $\mathbb{P}(F^n|E^n) \to 0$ and #H arrivals in (s^n,t^n) $-$ #service completions in (s^n,t^n) has a Skellam $(\lambda_H^n x, n\mu x)$ distribution. Thus, $\mathbb{P}(\xi_t^n > x|A_2) \to 0$.

Combining the above steps, we must have that:

$$t^n - \gamma_t^n \Rightarrow 0.$$

■

EC.5. Proof of Proposition 6

Proof. Our proof for Proposition 6 makes use of a coupling argument. Before we get to the technical details, we begin by presenting the intuition behind our reasoning. The correlation in (16) can be written as $1/\bar{\rho}$, where we define $\bar{\rho} \equiv \rho_L/(1-\rho_H)$. That is, it is of the same form as the correlation expression in (EC.2), for the overloaded single-class queue with abandonment. This suggests that, from the standpoint of low-priority customers, the system can be approximated by an overloaded single-class $M/M/n+M$ queue where a fraction $1-\rho_H$ of the available capacity is unavailable (consistently busy serving H customers). In concert with that intuition, our proof couples the original system with two bounding systems which, asymptotically, can both be approximated by overloaded single-class queues with traffic intensity $\bar{\rho}$; we then rely on a sandwiching argument to obtain the desired convergence for the correlation in (16).

Lower-bound system. We consider a system with two dedicated server pools, L and H, of respective sizes $n_L = n - \lambda_H$ and $n_H = \lambda_H$. (Here, we ignore integrality for the number of servers, which is justifiable in large systems.) Let $N_H(t)$, $N_L(t)$ be the numbers in service, and $Q_H(t)$, $Q_L(t)$ be the numbers in queue, at time t, for the H and L classes, respectively. There is sharing between the two pools as follows: An arrival of type L may only occupy a server in the H pool if all servers in the L pool are busy and $N_H(t) < n_H$, i.e., there is an idle server in the H pool and no H customers waiting for it; we assume the same condition for H customers to be served by a server in the L pool. Otherwise, L customers are served by the L pool, and H customers by the H pool. The service discipline is work-conserving, i.e., we do not allow a server to idle if there are customers waiting in line, and we use a FCFS discipline within each class. We couple arrivals in this and the original system. We assign service times to servers not to customers, and we randomly create new patience times for all waiting customers at each departure epoch (we can do so because of the exponential assumption on abandonment times); we do so identically in both systems. We initiate both systems empty. The lower-bound and original systems will have identical sample paths until a time epoch t_0 where: There is a departure from service from the L pool at t_0, and there are customers of both types waiting in queue. In the original system, an H customer must be served next since she takes priority over L customers. In the lower-bound system, an L customer must be served instead. We generate the same service time for both customers. We also regenerate the patience times of all customers in queue. Thus, the total number of customers remains identical in both systems, and only the identity of the customers in service changes. We repeat the same argument for similar subsequent epochs. In so doing, we ensure that the number of served L customers, at every point in time, is at least as high in the lower-bound system as in the original system. Therefore, the waiting time of L customers in the lower-bound system will be smaller, in a stochastic ordering sense.

Upper bound system. Once more, we consider a system with two server pools, L and H, of respective sizes $n_L = n - \lambda_H$ and $n_H = \lambda_H$. We now assume that H customers have non-preemptive priority over L customers in the L pool of servers. We also assume that L customers are never allowed in the H pool. That is, if at time t we have $N_L(t) = n_L$, $Q_L(t) > 0$, $N_H(t) < n_H$, and a departure from service occurs in the H pool, then the newly freed server waits for subsequent H customers; i.e., we allow for idling in the H pool. The original and upper bound systems have identical samples paths until a time epoch t_0 where there is an L customer waiting, all servers in the L pool are busy, no H customers are in queue, and at least one server in the H pool is idle (this can either be at a departure epoch from the H pool, or an arrival epoch for an L customer). At this point, we serve the L customer in the original system, and we keep her waiting in the upper bound system. We regenerate patience times for all customers waiting in either system, and all service times for customers in service. Proceeding as such at every subsequent such epoch guarantees that the number of L customers in queue in the original system, at every point in time, is at least as large in the upper-bound system as in the original system. Therefore, the waiting time of L customers in the upper-bound system will be larger, in a stochastic ordering sense.

Analysis in the bounding systems. We index processes in the lower-bound system by I, and in the upper-bound system by II. By the analysis above, the following holds at t^n:

$$W_I^n(t^n) \leq_{st} W^n(t^n) \leq_{st} W_{II}^n(t^n),$$

where $\leq_{st}$ denotes first-order stochastic dominance. Since the LES customer is some served customer, the following must also hold:

$$W_I^n(\tau_t^n) \leq_{st} W^n(\tau_t^n) \leq_{st} W_{II}^n(\tau_t^n).$$

This implies:

$$W_I^n(t^n) \cdot W_I^n(\tau_t^n) \leq_{st} W^n(t^n) \cdot W^n(\tau_t^n) \leq_{st} W_{II}^n(t^n) \cdot W_{II}^n(\tau_t^n),$$

and, taking expectations, we must also have:

$$\mathbb{E}[W_I^n(t^n) \cdot W_I^n(\tau_t^n)] \leq \mathbb{E}[W^n(t^n) \cdot W^n(\tau_t^n)] \leq \mathbb{E}[W_{II}^n(t^n) \cdot W_{II}^n(\tau_t^n)].$$

We now turn to our asymptotic analysis. We let all systems run long enough to reach steady state and let $n \uparrow \infty$. Consider a single-class $M/M/n_{LB} + M$ system, dedicated to L customers, which we denote by "LB". We let $n_{LB} = n_L + n_L^{1/2+\delta}$ servers, for some $\delta > 0$, and identical parameters for the L class as in our original system. Thus, the "LB" system is an overloaded queue with traffic intensity $\bar{\rho} \equiv \rho_L/(1-\rho_H)$. For large n, it is readily seen that $W_{LB}^n(t^n) \leq_{st} W_I^n(t^n)$. Similarly, we consider a single-class $M/M/n_{UB}+M$ system, dedicated to L customers, which we denote by "UB". We let $n_{UB} = n_L - n_L^{1/2+\delta'}$ servers, for some $\delta' > 0$, and identical parameters for the L class as in our original system. The "UB" system is also an overloaded queue with traffic intensity $\bar{\rho} \equiv \rho_L/(1-\rho_H)$. For large n, it is readily seen that $W_{II}^n(t^n) \leq_{st} W_{UB}^n(t^n)$. Using Whitt (2004), the following convergence holds in steady state:

$$W_{LB}^n(\infty) \Rightarrow \frac{1}{\theta}\ln(\bar{\rho}) \quad \text{and} \quad W_{UB}^n(\infty) \Rightarrow \frac{1}{\theta}\ln(\bar{\rho}), \quad \text{as} \quad n \to \infty,$$

where "$\Rightarrow$" denotes convergence in distribution. By a sandwiching argument, noting that the covariance $\text{Cov}[W_I^n(t^n), W_I^n(\tau_t^n)] = \mathbb{E}[W_I^n(t^n) \cdot W_I^n(\tau_t^n)] - \mathbb{E}[W_I^n(t^n)]\mathbb{E}[W_I^n(\tau_t^n)]$, we obtain the desired:

$$r[W^n(\tau_t^n), W^n(t^n)] \to \frac{1}{\bar{\rho}} = \frac{1-\rho_H}{\rho_L} \text{ as } n \to \infty.$$

∎

EC.6. Alternative Error Criterion

In this section, we demonstrate that our correlation-based framework continues to apply when considering an alternative delay criterion which penalizes over and under estimation of delays in a non-symmetric manner. Shah et al. (2019) considers an alternative Newsvendor-type error criterion. We define for a predictor P the error criterion:

$$ERR(P) = a\mathbb{E}[((P-W)^+)^2] + b\mathbb{E}[((W-P)^+)^2].$$

Lemma EC.8. *If $X \equiv P - W$ has a symmetric distribution around 0, then*

$$ERR(P) = \frac{a+b}{2} MSE(P).$$

Proof.

$$\begin{aligned}
ERR(P) &= a\mathbb{E}[((P-W)^+)^2] + b\mathbb{E}[((W-P)^+)^2] \\
&= a\mathbb{E}\left[\frac{1}{4}\left((P-W)+|P-W|\right)^2\right] + b\mathbb{E}\left[\frac{1}{4}\left((W-P)+|W-P|\right)^2\right] \\
&= \frac{a}{4}\left[\mathbb{E}[(P-W)^2] + \mathbb{E}[|P-W|^2] + 2\mathbb{E}[(P-W)|P-W|]\right] \\
&\quad + \frac{b}{4}\left[\mathbb{E}[(W-P)^2] + \mathbb{E}[|W-P|^2] + 2\mathbb{E}[(W-P)|W-P|]\right] \\
&= \frac{a+b}{2} MSE(P) + \frac{a-b}{2}\mathbb{E}[(P-W)|P-W|].
\end{aligned}$$

Recall that $X \equiv (P-W)$ is symmetric around 0. Thus,

$$\mathbb{E}[(P-W)|P-W|] = \mathbb{E}[X|X|] = \mathbb{E}[X^2\mathbf{1}(X \geq 0)] - \mathbb{E}[X^2\mathbf{1}(X \leq 0)] = 0.$$

Thus,

$$ERR(P) = \frac{a+b}{2} MSE(P),$$

as desired. ■

There remains to show that the LES and EA predictors satisfy the symmetry property above. We note that this is true in heavily-loaded large systems. For supporting theory, we refer the reader to earlier papers demonstrating the asymptotic normality of the waiting time conditional on the LES prediction in Ibrahim and Whitt (2009a) and Ibrahim and Whitt (2009b) (which consider both the ED and QED regimes).

EC.6.1. Supporting Numerical Study

In what follows, we present numerical results which substantiate Lemma EC.8 for the LES announcement. In particular, we consider the single-class $M/M/100$ model with $\rho = 0.98$, the $M/M/100+M$ model with $\rho = 1.4$, $\theta = 0.5$ and $\mu = 1$, the two-class $M/M/100$ model with $\rho_L = 0.75$ and $\rho_H = 0.23$, and the two-class $M/M/100+M$ model with $\rho_L = 1.1$, $\rho_H = 0.3$, $\theta_H = \theta_L = 0.5$, and $\mu_L = \mu_H = 1$. The point estimates in Table EC.1 are based on averaging 10 independent simulation runs of length 8 million arrivals each. We consider different values of a and b in Lemma EC.8, calculate the new errors in the Lemma, and relate them to $ASE(LES)$, also calculated in the same simulation runs. The values of a and b are chosen so that $(a+b)/2 = 0.5$, i.e., we expect to see that $ERR(LES)/ASE(LES)$ is roughly equal to 0.5. We note that ASE(LES) varies slightly for different values of a and b because of the different seeds in the simulations. Table EC.1 illustrates that Lemma EC.8 indeed holds for the LES announcement, particularly when a and b are not too asymmetric.

EC.7. An Additional Data Set from a Larger Call Center

The real-life data set that we have analyzed in the previous section is taken from a small call center (number of agents is less than 15). To check the robustness of our results, we now consider an additional data set taken from a larger call center, where calls are handled by a pool of 200 agents. In particular, we use data from the call center of a Dutch company which specializes in delivering business solutions to its clients.

$M/M/100$ with $\rho = 0.98$

a	b	ASE(LES)	ERR(LES)	ERR(LES)/ASE(LES)
0.5	0.5	0.00994	0.00497	0.500
0.75	0.25	0.0104	0.00484	0.464
0.25	0.75	0.0104	0.00545	0.535
0.1	0.9	0.00986	0.00552	0.560
0.9	0.1	0.0103	0.00460	0.445

$M/M/100+M$ with $\rho = 1.4$

a	b	ASE(LES)	ERR(LES)	ERR(LES)/ASE(LES)
0.5	0.5	0.0116	0.00581	0.500
0.75	0.25	0.0116	0.00538	0.463
0.25	0.75	0.0116	0.00625	0.537
0.1	0.9	0.0116	0.00647	0.559
0.9	0.1	0.0116	0.00513	0.440

Two-class $M/M/100$ with $\rho_L = 0.75$ and $\rho_H = 0.23$

a	b	ASE(LES)	ERR(LES)	ERR(LES)/ASE(LES)
0.5	0.5	0.0229	0.0115	0.500
0.75	0.25	0.0228	0.0103	0.454
0.25	0.75	0.0225	0.0123	0.546
0.1	0.9	0.0231	0.0132	0.572
0.9	0.1	0.0229	0.00980	0.427

Two-class $M/M/100+M$ with $\rho_L = 1.1$ and $\rho_H = 0.3$

a	b	ASE(LES)	ERR(LES)	ERR(LES)/ASE(LES)
0.5	0.5	0.0304	0.0152	0.500
0.75	0.25	0.0304	0.0136	0.450
0.25	0.75	0.0303	0.0167	0.551
0.1	0.9	0.0304	0.0177	0.581
0.9	0.1	0.0303	0.0127	0.419

Table EC.1 New error criterion for the LES announcement in Lemma EC.8, for alternative values of a and b.

EC.7.0.1. Description of the Data. There are 11 different queues in the call center, and each queue corresponds to either one or two call types. We focus on one such queue: Queue-30170. We select this queue because it corresponds to a single call type. However, it is important to note that it is served by an agent pool which may be serving other call types at the same time. The total number of agents who serve Queue-30170 is equal to 148. Our data does not contain information about the routing policy for any of the queues. Thus, it reflects a realistic scenario, where the manager of the call center has information about the waiting times of customers, but the routing itself may be done in an ad-hoc manner. The call center is closed on Sundays. The average wait time for delayed customers is close to 90 seconds, and the probability of abandonment is close to 6%.

Our data for Queue-30170 is from May 9, 2012 until September 29, 2012. There are close to 50,000 delayed customers in that set. For each delayed customer, we proceed as before and calculate the LES prediction, the running average wait-time prediction, and the WA prediction which is based on a point estimate of the correlation. We calculate out-of-sample estimates for the correlation and the average waiting time based on a sample of 10,000 delayed customers. We then discard this sample from consideration when calculating the errors corresponding to our alternative predictions. As such, we are left with predictions for a total of 98 consecutive days. For each of the average wait-time and correlation estimates, we include a weekday effect. To compare the accuracies of our alternative predictions, we focused on delays which exceeded 5 seconds.

EC.7.0.2. Accuracy of the WA Prediction. We present in Table EC.2 results which parallel those that we reported in Table 9. Based on Table EC.2, we can make the following observations. First, the LES prediction is increasingly accurate as the size of the system grows. This is to be expected, and is in concert with previous theoretical results establishing the asymptotic accuracy of the LES prediction in large queueing systems, e.g., see Ibrahim et al. (2016). In particular, the LES prediction performs generally better than the average-waiting-time prediction in this case. Second, our new proposed WA prediction has a clear superior performance compared to both the LES and EA predictions. Indeed, WA "wins" on close to 75% of the days in our sample. Third, by restricting attention to days where the LES announcement yields the smallest daily error on average (first column in Table EC.2), we see that it outperforms the EA prediction by a lot in this case: Indeed, the ASE for EA is roughly 13 times the ASE for the LES announcement in that case. In other words, announcing the average can lead to considerable errors. Finally, we have further evidence that WA is a good announcement in practice: *Even when WA is not the most accurate prediction, it remains competitive, i.e., there do not exist days when it is dramatically outperformed by either the LES announcement or EA.*

Queue-30170

	Ratios of (row) ASE to (column) winner ASE			
	EA wins (8%)	WA wins (74%)	LES wins (17%)	Overall
EA	1	2.42	13.1	4.09
WA	1.30	1	2.40	1.25
LES	2.03	1.43	1	1.39

Table EC.2 Comparison of the ASE's of EA, WA, and LES for Queue-30170. In each column, we report estimates of the ratio of the ASE of the prediction in the corresponding row, relative to the ASE of the predictor in the corresponding column.

EC.8. Additional Numerical Results: Time-Varying Arrivals

We now consider time-varying arrival rates. This is practically important because arrival processes to service systems, in real life, typically vary significantly over time. We consider a sinusoidal arrival-rate intensity function to mimic cyclic behavior that is common in arrival processes to service systems:

$$\lambda(u) = \bar{\lambda} + \bar{\lambda}\alpha \sin(\gamma u), \text{ for } 0 \le u < \infty \ , \tag{EC.7}$$

where $\bar{\lambda}$ is the average arrival rate and α is the relative amplitude. Given an appropriate constant staffing level, this arrival-rate function corresponds to alternating periods of underload and overload in the system. As pointed out by Eick et al. (1993), the parameters of the arrival-rate intensity function, $\lambda(u)$ in (EC.7), should be interpreted relative to the mean service time. Then, we speak of γ as the relative frequency. Table EC.3 displays values of the relative frequency as a function of the mean service time, assuming a daily cycle. For interpretation, we also will specify the associated mean service time in minutes, given a daily cycle. Small (large) values of γ correspond to slow (fast) time-variability in the arrival process, relative to the service times.

γ	Cycle length	Mean service time
0.0436	144	10 minutes
0.262	24	1 hour
1.571	4	6 hours

Table EC.3 The relative frequency is the frequency computed with measuring units so that the mean service time is equal to 1.

In Table EC.4, we consider a two-class queueing system with time-varying arrivals, and focus on low-priority customers, as before. We hold the values of ρ_H and ρ_L fixed, and vary γ to increase the frequency in the time-varying arrivals. We let the amplitude be fixed as well: $\alpha = 0.3$. We consider Markovian queues only, to focus on the effect of the time variation in the arrival rates.

Based on Ibrahim and Whitt (2011), we know that the LES prediction can be biased with time-varying arrivals, because delays then vary systematically over time. Thus, the assumptions of Proposition 1, namely that the LES prediction is unbiased and has the same variance as the steady-state delay, fail to hold. Therefore, it is not clear whether the superior performance of the WA predictor, as derived in §4.3, will continue to hold in this case. Indeed, inspecting the point estimates of the correlation in Table EC.4, we find that these estimates vary considerably with γ, even when the traffic intensities in the system are held fixed. Thus, we do not expect simple expressions for the correlations, such as those derived in §5, to continue holding with time-varying arrivals. In comparing ASE(WA) and ASE(WA-run), we find that, while these two predictors remain generally close, WA-run performs slightly better than WA, particularly when γ is large. Interestingly, we find that both WA and WA-run remain superior to both the LES announcement and EA, in almost all cases considered (the only exception is for $\gamma = 1.571$ and no abandonment, in which case EA is superior). This shows that our new WA predictor is robust to time-variation in the arrival rates as well.

$M_t/M/30$ with two classes and sinusoidal arrival rates

γ	ρ_L	ρ_H	LES	EA	WA	WA-run	EXP	Corr	$\mathbb{E}[W\|W>0]$
0	0.2	0.5	0.322	0.279	0.272	0.270	0.343	0.284	0.221
0.0436	0.2	0.5	0.773	0.905	0.746	0.714	0.903	0.637	0.724
0.262	0.2	0.5	0.668	0.646	0.588	0.587	0.671	0.450	0.549
1.571	0.2	0.5	0.384	0.313	0.316	0.310	0.392	0.183	0.271

$M_t/M/100+M$ with two classes and sinusoidal arrival rates

γ	ρ_L	ρ_H	LES	EA	WA	WA-run	EXP	Corr	$\mathbb{E}[W\|W>0]$
0	0.3	0.5	0.107	0.101	0.0946	0.0934	0.132	0.412	0.0857
0.0436	0.3	0.5	0.233	0.273	0.215	0.214	0.408	0.641	0.303
0.262	0.3	0.5	0.224	0.252	0.204	0.204	0.334	0.606	0.274
1.571	0.3	0.5	0.153	0.140	0.133	0.131	0.187	0.391	0.137

Table EC.4 Comparison of the square-root ASE's of the different predictions for low-priority customers and time-varying arrivals. We let $\alpha = 0.3$ and consider alternative values of γ.

EC.9. Additional Numerical Results: Customer Response

In this section, we consider a balking function, $\beta_0(w)$, for which the assumptions of Ibrahim et al. (2016) are violated. In particular, the function $\beta_0(w)$ is both non-monotone and discontinuous at $w = 0.1$, i.e., it violates both the continuity and monotonically increasing assumptions of Ibrahim et al. (2016):

$$\beta_0 = \begin{cases} \frac{w}{1+w} & \text{if } w \le 0.1 \\ exp(-\theta w) & \text{otherwise.} \end{cases} \tag{EC.8}$$

In this case, we investigate the performances of LES and EA in the $M/M/100+M$ model for varying values of ρ, and report corresponding estimates for the correlations. First, we note that the conditions of Corollary 1 are violated, i.e., we do not have that $\beta = \gamma = 1$. Nevertheless, Proposition 1 continues to hold as we illustrate in Table EC.5. There, we report point estimates of β, γ, C_{W_D}, and r as given in the proposition (we add a hat notation to estimates). We also let $A \equiv \left(\hat{\beta}+1+\left(\frac{\hat{\gamma}-1}{\hat{C_{W_D}}}\right)^2-2\cdot\hat{r}[P,W_D]\sqrt{\hat{\beta}}\right)$. According to Proposition 1, we should have that ASE(LES) is roughly equal to $A\times$ ASE(EA). Table EC.5 shows that this is indeed the case. We also note that Corollary 1 does not hold in this case: For example, we see that for $\rho=1.2$ and $\rho=1.4$, we have that the correlation estimate $\hat{r}>0.5$, yet ASE(LES) > ASE(EA).

ρ	$1/\rho$	ASE(LES)	ASE(EA)	$\hat{r}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{C}_{W_D}$	A	ASE(LES)/ASE(EA)
1.2	0.833	0.00175	0.00191	0.566	0.67	0.499	0.713	0.913	0.913
1.4	0.714	0.00255	0.00246	0.533	0.635	0.442	0.661	1.038	1.038
1.6	0.625	0.00356	0.00301	0.493	0.598	0.421	0.636	1.181	1.183
1.8	0.556	0.00478	0.00358	0.445	0.569	0.420	0.617	1.332	1.335
2.0	0.5	0.00623	0.00419	0.390	0.544	0.433	0.607	1.485	1.486

Table EC.5 Point estimates of ASE's and correlations in the heavily-loaded $M/M/100+M$ queue where customers balk with probability $\beta_0(w)$ in (EC.8).

EC.10. Additional Empirical Results

In this section, we describe additional results quantifying the performance of our alternative announcements with the small call-center data set analyzed in §6.2. In §EC.10.1, we consider an alternative EA prediction where we announce a running-average waiting time which we continuously update. That is, we do not consider a static announcement calculated out-of-sample as in the main paper. In §EC.10.2, we present yet another EA-type announcement which accounts for seasonal effects by incorporating a day-of-week effect. In §EC.10.3, we present results for tow additional call types, NE (stock exchange activity) and NW (potential new customers getting information). In §6.3, we consider a data-based predictor which exploits information about the queue-length seen by a delayed customer upon arrival. We present additional tabular results in §EC.10.4.

EC.10.1. Running Average Waiting Time

In this section, we consider a continuously updated EA announcement; thus, we consider our entire data set and do not remove a sample where we compute an out-of-sample estimate for EA as we did in the main paper. We denote that new prediction by EA-C. In Table EC.6, we parallel Table 9 in §6.2: We take a closer look at performance and present data estimates for the ratios of the ASE's of our three predictors. The first sub-table corresponds to IN callers, whereas the second sub-table corresponds to low-priority PS callers. Each column in the table corresponds to days where one of the predictors yields the smallest ASE. For example, for the first column, we restrict attention to those days where the LES announcement yields the smallest ASE: For IN calls, the LES announcement yields the smallest ASE on only 4 days out of 106.

Table EC.6 shows that WA continues to have superior performance over both EA-C and the LES announcement. It is worth noting however that, unlike in Table 9, on the 4 days where the LES announcement yields superior performance over both WA and EA-C, it significantly outperforms those predictors (as can be seen from the first column of the table). Upon closer inspection, we see that, on those four days, customer delays were considerably shorter than usual: The average waiting time on those days is 57 seconds, whereas it is 140 seconds over the entire sample. (We note in passing that three our of these four days fell in the sample that we had initially removed to calculate an out-of-sample estimate of EA.) Thus, since customers have short delays on those days, both the WA and EA-C predictions perform poorly since they fail to capture that these days have unusually short delays.

IN Call Type

	EA-C wins (32.0%)	WA wins (64.2%)	LES wins (3.8%)
EA-C/winner	1	1.22	5.48
WA/winner	1.04	1	3.50
LES/winner	2.12	1.70	1

PS Call Type

	EA-C wins (13.1%)	WA wins (76.6%)	LES wins (10.2%)
EA-C/winner	1	1.49	2.38
WA/winner	1.03	1	1.59
LES/winner	2.02	3.82	1

Table EC.6 Comparison of the ASE's of EA-C, WA, and LES in August-December for IN and PS customers. For EA-C, we use a running average that is continuously updated (not calculated out of sample). In each column, we report estimates of the ratio of the ASE of the prediction in the corresponding row, relative to the ASE of the predictor in the corresponding column.

EC.10.2. Day-Of-Week Effect

In this section, we present results for yet another EA-based prediction. In particular, we account for day-of-week seasonality in calculating the EA announcement, i.e., we make a different EA announcement based on the day of week. We present detailed results in Tables EC.8 and EC.9, which we relegate to §EC.10.4. When inspecting the results of those tables, we noticed that the seasonally-adjusted EA announcement, which we denote by EA-DOW, does not perform better than the EA announcement for IN callers. To explain why that is the case, we plot Figures EC.1

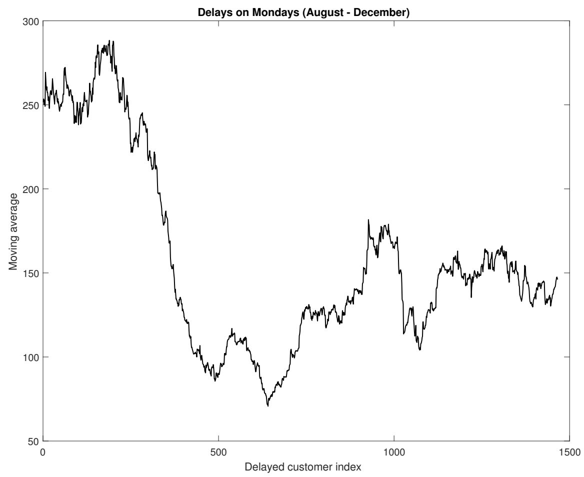

Figure EC.1 **Moving average (window = 100) on Mondays.**

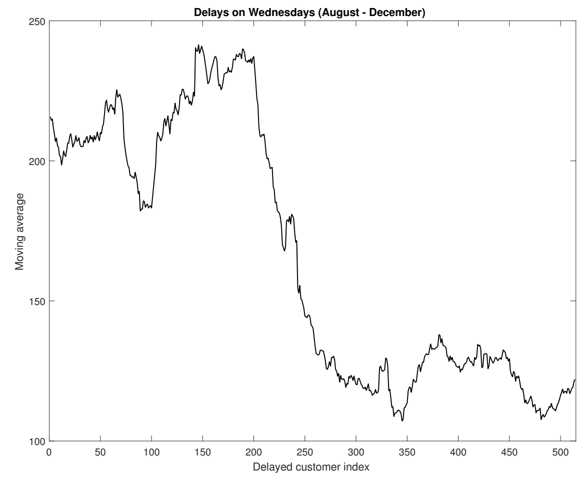

Figure EC.2 **Moving average (window = 100) on Wednesdays.**

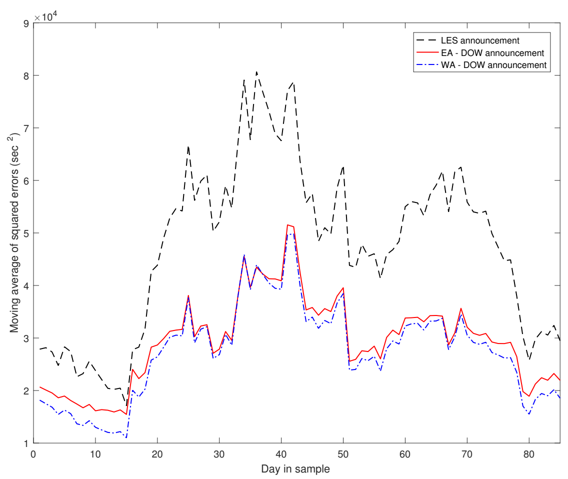

Figure EC.3 **Moving average (window = 10) for squared errors of announcements.**

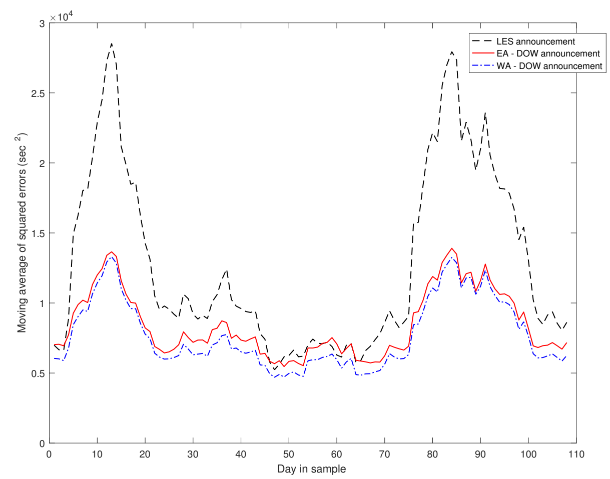

Figure EC.4 **Moving average (window = 10) for squared errors of announcements.**

and EC.2 where the curves correspond to moving averages of delays on Mondays and Wednesdays for IN callers. In calculating the EA-DOW estimates, we take the first 2000 callers as our out of sample, which clearly correspond to longer delays, as the Figures show. Considering the overall EA average, as we did in the main paper, smoothes out those systematic changes in delays that were observed in the data (unfortunately, we do not have an explanation for what happened on those days). Nevertheless, Figures EC.3 and EC.4 show that we continue to observe the superiority of the WA-DOW prediction over both the LES announcement and EA-DOW. In these figures, we plot moving averages of the squared errors with a centered window of length 10.

EC.10.3. Results for Alternative Call Types

We include results for the NE and NW types in Table EC.7; this Table parallels Table EC.6: As before, we observe that WA is superior to the remaining predictions.

NE Call Type

	EA wins (20.0%)	WA wins (71.4%)	LES wins (8.5%)
EA/winner	1	1.35	4.69
WA/winner	1.19	1	2.23
LES/winner	2.63	1.82	1

NW Call Type

	EA wins (16.2%)	WA wins (79.0%)	LES wins (4.8%)
EA/winner	1	1.13	9.42
WA/winner	1.71	1	5.76
LES/winner	13.12	1.93	1

Table EC.7 Comparison of the ASE's of EA, WA, and LES in August-December for IN and PS customers. For EA, we use a running average that is continuously updated (not calculated out of sample). In each column, we report estimates of the ratio of the ASE of the prediction in the corresponding row, relative to the ASE of the predictor in the corresponding column.

EC.10.4. Detailed Tabular Results

Day index	LES	EA	EA-DOW	WA	WA-DOW
1	56989	30413	30333	32542	33205
2	9149	14570	19737	9322	12061
3	21244	17901	18061	13988	14067
4	29101	21381	22558	18790	19180
5	22898	14239	12708	12893	12328
6	29561	14403	17188	13155	14103
7	22349	16054	16171	12790	12836
8	6775	10679	12235	5189	5854
9	56844	21710	21579	24632	22952
10	19173	12449	10458	10074	9370
11	9167	18106	23629	11163	13951
12	15099	12453	12607	8886	8958
13	44859	21545	24403	21784	23097
14	11380	10315	10450	6544	6606
15	5776	13059	14910	6398	7271
16	12757	11324	16045	7203	9429
17	20390	14497	12866	12269	11573
18	9084	11520	16208	6336	8660
19	23289	12899	12995	11003	11028
20	125855	96237	96112	100463	99972
21	14161	2968	6075	170	954
22	49823	23798	24085	24543	24187
23	154173	76856	72893	80711	78190
24	23299	17270	14533	14187	12897

Day index	LES	EA	EA-DOW	WA	WA-DOW
25	57191	25053	27055	24767	25553
26	51238	29925	29971	28429	28443
27	38426	14920	15120	15660	15529
28	4251	11966	17631	4651	7382
29	149194	76286	77503	82586	83961
30	19882	14764	17769	12556	13776
31	51174	26238	26252	26889	26872
32	61803	26728	26551	29635	28926
33	46554	19895	18535	18792	17800
34	41546	20433	22460	19394	20137
35	125947	60480	60558	64087	64127
36	8402	11549	13156	6893	7658
37	162221	104087	99880	112858	108128
38	124690	90693	92573	86387	87901
39	35120	16553	18547	16280	16685
40	149411	56350	56349	61022	61017
41	12871	11591	13278	8155	9016
42	25160	14314	17467	11433	12253
43	3844	12427	18209	4905	7725
44	27726	20422	18928	18772	18367
45	221396	170461	166953	171166	167802
46	25876	9519	9580	9443	9447
47	12869	13052	14942	10040	11103
48	43232	16858	19186	16767	17713
49	52617	23327	23230	24956	25449
50	58500	39435	41608	39134	39482
51	38822	25770	25879	24029	24061
52	13208	10411	11888	5607	6157
53	91173	47784	47534	46936	46799
54	71329	36111	34857	37204	36787
55	30650	23900	27035	20902	21936
56	22084	13134	13257	10645	10688
57	56778	31018	31267	32185	31828
58	20469	14537	17599	12225	13679
59	57450	33307	33630	33045	33830
60	11077	13446	17389	9374	11226
61	84647	66742	66674	66620	66552
62	22474	24852	25807	20780	21098
63	107035	37727	39019	39615	40897
64	137283	65270	66412	70240	71105
65	40456	25570	27502	25493	26026
66	19517	14023	14134	11689	11728
67	32390	20234	22746	18438	18990
68	60359	29902	29259	30160	30430
69	75857	32682	33933	34358	34481
70	37744	16106	16192	16230	16256
71	7240	10308	11995	5973	6824
72	98209	48862	48386	45531	45191
73	116258	84542	85953	83132	84421
74	70747	29064	30358	30342	31129
75	21780	15826	15953	12197	12238
76	16756	9446	10359	7618	7961
77	36619	23854	26279	21799	22955
78	17524	14766	12963	11567	10875

Day index	LES	EA	EA-DOW	WA	WA-DOW
79	50021	29775	31062	29769	29887
80	11892	15891	16067	10273	10354
81	9035	12558	14391	6799	7659
82	29201	17755	21226	15496	17241
83	39211	20788	19419	20408	20037
84	25180	18104	21442	14222	15804
85	62359	39077	39116	40176	40155

Table EC.8: Daily ASE for all predictors from August to December for IN customers.

Day index	LES	EA	EA-DOW	WA	WA-DOW
1	3732	7223	6828	5695	5429
2	5132	6927	7383	5647	5953
3	9001	6523	5800	5980	5512
4	13441	10868	10946	9753	9759
5	3621	8200	4185	6317	3561
6	5042	7602	7173	6157	5861
7	6382	5913	6296	4881	5129
8	24498	13150	13129	12977	12934
9	63404	21849	21687	21949	21944
10	28442	15562	15450	14472	14261
11	21377	10176	9957	10360	10136
12	5992	6743	5693	5646	4889
13	31664	18537	18628	17933	17983
14	38433	17955	17784	17639	17567
15	19829	8632	8631	8782	8704
16	32876	15853	16783	15248	15736
17	18765	8886	8787	8992	8914
18	9141	9418	9734	8011	8220
19	5159	5325	4465	4371	3791
20	15636	7244	5889	7178	6100
21	7249	5808	3993	5168	4075
22	7468	5333	5233	4784	4902
23	7996	8499	8909	7271	7551
24	18617	9949	9783	9902	9767
25	8447	7760	6055	6706	5529
26	6573	6351	6081	5455	5272
27	8931	6503	6740	5986	6146
28	11781	7630	7192	7140	6822
29	2914	7489	5242	5486	3968
30	12035	8007	7787	7162	7003
31	4335	6667	7155	5201	5528
32	24707	14765	14480	13278	13017
33	4710	6525	4798	5181	4040
34	7640	6683	6386	5860	5666
35	4918	7167	7664	5705	6046
36	9188	6969	6165	6308	5769
37	6645	5676	4441	4925	4147
38	23812	16668	16819	14735	14805
39	7898	5965	6265	5370	5569
40	22735	13026	13081	12122	12066

Day index	LES	EA	EA-DOW	WA	WA-DOW
41	11989	6594	5919	6603	6385
42	2917	7441	3260	5509	2656
43	25	13473	7082	9089	4745
44	5926	6052	2834	5231	2937
45	2783	7230	6800	5340	5053
46	8495	7480	7864	6418	6668
47	7898	6613	5856	5861	5343
48	7426	5589	4521	5050	4369
49	3947	7236	6822	5551	5273
50	6398	6983	7424	5733	6020
51	6545	4967	4211	4460	3938
52	6842	6802	5373	5644	4661
53	5347	5296	2895	4397	2892
54	6857	6787	6526	5656	5498
55	6643	6911	7384	5772	6094
56	3697	6882	5785	5259	4519
57	8304	5363	4325	5053	4358
58	15705	14988	17133	13781	15517
59	7893	7045	6807	6165	6007
60	3423	7484	8064	5664	6058
61	6186	8079	6855	6388	5547
62	7918	7612	6151	6378	5360
63	2251	8781	6245	6328	4600
64	826	5497	2062	3808	1499
65	5184	2408	294	803	4
66	12187	9741	9570	9105	8991
67	9423	7267	7694	6552	6843
68	2372	6590	5364	4804	3982
69	9112	7702	6245	6407	5601
70	10558	8891	7514	7960	6994
71	9278	6264	6092	5776	5654
72	12634	6632	6866	6475	6653
73	6538	6881	6114	6100	5588
74	8889	8011	6566	7049	6018
75	13483	7857	7755	7634	7547
76	5408	7724	8201	6249	6570
77	4165	7912	6586	6419	5464
78	6243	5350	4454	4789	4242
79	13374	9041	9044	8510	8482
80	76935	30943	31261	28133	28253
81	9842	6659	6964	5962	6178
82	39061	14801	14921	15364	15396
83	32097	16025	17896	15633	16553
84	20959	11000	11831	11405	12047
85	7063	6785	5173	5688	4668
86	45734	20664	20818	20832	20890
87	18951	11810	11894	10765	10853
88	15405	9545	9240	9437	9244
89	7440	6126	4884	5492	4607
90	19011	10758	10770	10408	10411
91	23340	13578	13475	13254	13201
92	26642	15645	15975	15598	15817
93	10057	5137	4277	5053	4456
94	36332	16951	19512	16447	17722

Day index	LES	EA	EA-DOW	WA	WA-DOW
95	33057	16867	16911	16258	16266
96	15172	9294	9387	8903	8972
97	5512	6802	5679	5452	4695
98	5235	6782	5226	5377	4404
99	7115	7417	5334	6303	4916
100	15914	9077	8981	8491	8411
101	12079	8351	8636	7724	7924
102	4632	6737	3905	5386	3449
103	19149	10097	9970	9891	9793
104	12351	7504	7680	7246	7380
105	4976	5462	4581	4368	3804
106	2060	9829	8217	7142	6031
107	1238	9705	6964	6786	4917
108	11935	6064	5628	5963	5592

Table EC.9: Daily ASE for all predictors from August to December for low priority PS customers.