Electronic Companion for "Impact of Information Asymmetry and Limited Production Capacity on Business Interruption Insurance"

Appendix EC.1: Optimal Effort with BI Insurance

Given μ_i and δ_j, the firm's expected profit with BI insurance contract (y,s) is

$$\pi^I_{ij1}(K,e,y,s) = -y + \tau p E_{\epsilon_t}[\min(D_{ijt},K)] - ceK + (T-\tau)pE_{\epsilon_t}[\min(D_{ijt},eK)] + (T-\tau-w)spE_{\epsilon_t}\big[\min(D_{ijt},K) - \min(D_{ijt},eK)\big] - C^I_{ij}(K,e,y,s),$$

where

$$C^I_{ij}(K,e,y,s) = \lambda\Big\{ceK + (T-\tau)p\Big(E_{\epsilon_t}[\min(D_{ijt},K)] - E_{\epsilon_t}[\min(D_{ijt},eK)]\Big) - (T-\tau-w)spE_{\epsilon_t}\big[\min(D_{ijt},K) - \min(D_{ijt},eK)\big]\Big\}.$$

Based on this, the (unconstrained) first-order optimality condition with respect to e is

$$\frac{d}{de}\pi^I_{ij1}(K,e,y,s) = (1+\lambda)\big\{-cK + pK[(T-\tau)-(T-\tau-w)s]\big[1-F_{ij}(eK)\big]\big\} = 0.$$

In addition, the second-order derivative of the profit function with respect to effort e is negative. Therefore, the firm's profit is a concave function of effort. The effort e satisfying the (unconstrained) first-order optimality condition is $\frac{1}{K}F^{-1}_{ij}\left(\frac{wp+(T-\tau-w)p(1-s)-c}{wp+(T-\tau-w)p(1-s)}\right)$. However, this value may violate the constraint that $e\le 1$. Thus, the optimal effort subject to this constraint is

$$e^*_{ij}(s) = \begin{cases} \frac{1}{K}F^{-1}_{ij}\left(\frac{wp+(T-\tau-w)p(1-s)-c}{wp+(T-\tau-w)p(1-s)}\right) & \text{if } F^{-1}_{ij}\left(\frac{wp+(T-\tau-w)p(1-s)-c}{wp+(T-\tau-w)p(1-s)}\right) < K,\\ 1 & \text{otherwise,}\end{cases}$$

where

$$F^{-1}_{ij}\left(\frac{wp+(T-\tau-w)p(1-s)-c}{wp+(T-\tau-w)p(1-s)}\right) = \mu_i + \delta_j + \gamma\ln\left(\frac{wp+(T-\tau-w)p(1-s)-c}{c}\right). \quad \text{Q.E.D.}$$

Appendix EC.2: Proofs of Theoretical Results

Proof of Proposition 1(i). The first- and second-order derivative of the firm's optimal effort $e^*_{ij}(s)$ with respect to s are

$$\frac{d}{ds}e^*_{ij}(s) = \begin{cases} \frac{-(T-\tau-w)p\gamma}{K[wp+(T-\tau-w)p(1-s)-c]} < 0 & \text{if } e^*_{ij}(s) = F^{-1}_{ij}\left(\frac{wp+(T-\tau-w)p(1-s)-c}{wp+(T-\tau-w)p(1-s)}\right),\\ 0 & \text{if } e^*_{ij}(s)=1,\end{cases}$$

$$\frac{d^2}{ds^2}e^*_{ij}(s) = \begin{cases} \frac{-(T-\tau-w)^2p^2\gamma}{K^2[wp+(T-\tau-w)p(1-s)-c]^2} < 0 & \text{if } e^*_{ij}(s) = F^{-1}_{ij}\left(\frac{wp+(T-\tau-w)p(1-s)-c}{wp+(T-\tau-w)p(1-s)}\right),\\ 0 & \text{if } e^*_{ij}(s)=1.\end{cases}$$

Because $T-\tau\ge w$, the first- and second-order derivative are both non-positive, and thus $e^*_{ij}(s)$ is non-increasing and concave in s. Q.E.D.

Proof of Proposition 1(ii). We take the first- and second-order derivatives of the firm's expected profit with respect to s. First, if $e^*_{ij}(s)=1$, this implies that the resumed capacity is exactly the initial capacity K. Therefore, no revenue loss can be claimed and the firm receives no compensation. As a result, the coverage percentage s does not influence the expected profit $\pi^I_{ij1}(K,y,s)$ in this case. For $j\in\{H,L\}$, if $e^*_{ij}(s)=1$, then $\frac{d}{ds}\pi^I_{ij1}(K,y,s)=0$.

On the other hand, if $e^*_{ij}(s)\neq 1$, the first-order derivative of $E_{\delta,Z}[\pi^I_i(K,y,s)]$ with respect to s is

$$
\begin{aligned}
&\frac{d}{ds}E_{\delta,Z}[\pi^I_i(K,y,s)]\\
&=\beta\rho\frac{d}{ds}\pi^I_{iH1}(K,y,s)+\beta\rho\frac{d}{ds}\pi^I_{iL1}(K,y,s)\\
&=\beta\rho(1+\lambda)(T-\tau-w)p\left\{\int_{e^*_{iH}(s)K}^{K}\left[x-e^*_{iH}(s)K\right]f_{iH}(x)dx+\int_K^\infty\left[K-e^*_{iH}(s)K\right]f_{iH}(x)dx\right\}\\
&\quad+(1-\beta)\rho(1+\lambda)(T-\tau-w)p\left\{\int_{e^*_{iL}(s)K}^{K}\left[x-e^*_{iL}(s)K\right]f_{iL}(x)dx+\int_K^\infty\left[K-e^*_{iL}(s)K\right]f_{iL}(x)dx\right\}\\
&=\rho(1+\lambda)(T-\tau-w)p\left\{K-\gamma E_\delta\left[\xi(K,\mu_i,\delta)\right]+\gamma\ln\left(\frac{wp+(T-\tau-w)p(1-s)}{wp+(T-\tau-w)p(1-s)-c}\right)\right\}\geq 0.
\end{aligned}
$$

All of the integrals in the preceding expression are non-negative. Therefore, the resulting derivative is non-negative. With regard to the second-order derivative, we note that, if $e^*_{ij}(s)=1$, then $\frac{d^2}{ds^2}\pi^I_{ij1}(K,y,s)=0$. If $e^*_{ij}(s)\neq 1$, then

$$
\begin{aligned}
&\frac{d^2}{ds^2}E_{\delta,Z}[\pi^I_i(K,y,s)]\\
&=\beta\rho\frac{d^2}{ds^2}\pi^I_{iH1}(K,y,s)+(1-\beta)\rho\frac{d^2}{ds^2}\pi^I_{iL1}(K,y,s)\\
&=\beta\rho(1+\lambda)\frac{(T-\tau-w)^2pc\gamma}{[(T-\tau-w)(1-s)+w](wp+(T-\tau-w)p(1-s)-c)}+(1-\beta)\rho(1+\lambda)\frac{(T-\tau-w)^2pc\gamma}{[(T-\tau-w)(1-s)+w](wp+(T-\tau-w)p(1-s)-c)}\\
&=\rho(1+\lambda)\frac{(T-\tau-w)^2pc\gamma}{[(T-\tau-w)(1-s)+w](wp+(T-\tau-w)p(1-s)-c)}\geq 0.
\end{aligned}
$$

Therefore, the second-order derivative is non-negative as well, and the expected profit is non-decreasing and convex in s. Q.E.D.

Proof of Proposition 2. To obtain the desired result, we will take the derivative of the firm's expected profit with respect to s and μ_i. Based on the proof in Proposition 1(ii), we deduce that, if $e^*_{ij}(s)=1$ for any $j\in\{H,L\}$, then $\frac{d^2}{dsd\mu_i}\pi^I_{ij1}(K,y,s)=0$ because $\frac{d}{ds}\pi^I_{ij1}(K,y,s)=0$. If $e^*_{ij}(s)\neq 1$, the derivative with respect to s and μ_i is

$$
\begin{aligned}
&\frac{d^2}{dsd\mu_i}E_{\delta,Z}[\pi^I_i(K,y,s)]\\
&=\beta\rho\frac{d^2}{dsd\mu_i}\pi^I_{iH1}(K,y,s)+(1-\beta)\rho\frac{d^2}{dsd\mu_i}\pi^I_{iL1}(K,y,s)\\
&=-\beta\rho(1+\lambda)(T-\tau-w)p\frac{\exp\left(\frac{\mu_i+\delta_H}{\gamma}\right)}{\exp\left(\frac{K}{\gamma}\right)+\exp\left(\frac{\mu_i+\delta_H}{\gamma}\right)}-(1-\beta)\rho(1+\lambda)(T-\tau-w)p\frac{\exp\left(\frac{\mu_i+\delta_L}{\gamma}\right)}{\exp\left(\frac{K}{\gamma}\right)+\exp\left(\frac{\mu_i+\delta_L}{\gamma}\right)}<0.
\end{aligned}
$$

Thus, the marginal profit per coverage percentage s is non-increasing in μ_i. Q.E.D.

Proofs of Lemma 1 and Proposition 3. To derive the properties of the insurer's profit function $R(s_H, s_L)$, we compute the first- and second-order derivatives and Hessian matrix with respect to s_H and s_L. Given that $K > \mu_H + \delta_H + \gamma \ln\left(\frac{(T-\tau)p-c}{c}\right) > \mu_L + \delta_H + \gamma \ln\left(\frac{(T-\tau)p-c}{c}\right)$, we have

$$\begin{aligned}
&\frac{d}{ds_H} R(s_H, s_L) \qquad \text{(EC.2.1)}\\
&= p\rho(T-\tau-w_H)\\
&\quad \Bigg\{(1-\alpha)(1+\lambda)\gamma E_\delta\left[\xi(K,\mu_L,\delta) - \xi(K,\mu_H,\delta)\right] - \alpha \tfrac{(T-\tau-w_H)c\gamma s_H}{[(T-\tau)(1-s_H)+s_H w_H]([(T-\tau)(1-s_H)+s_H w_H]p-c)}\\
&\qquad + \alpha\lambda\left(K + \gamma\ln\left[\tfrac{[(T-\tau)(1-s_H)+s_H w_H]p}{[(T-\tau)(1-s_H)+s_H w_H]p-c}\right] - \gamma E_\delta\left[\xi(K,\mu_H,\delta)\right]\right)\Bigg\},
\end{aligned}$$

$$\begin{aligned}
&\frac{d}{ds_L} R(s_H, s_L) \qquad \text{(EC.2.2)}\\
&= (1-\alpha)p\rho\Bigg\{\lambda\left(K + \gamma\ln\left[\tfrac{[(T-\tau)(1-s_L)+s_L w_L]p}{[(T-\tau)(1-s_L)+s_L w_L]p-c}\right] - \gamma E_\delta\left[\xi(K,\mu_L,\delta)\right]\right)\\
&\qquad - \tfrac{(T-\tau-w_L)c\gamma s_L}{[(T-\tau)(1-s_L)+s_L w_L]([(T-\tau)(1-s_L)+s_L w_L]p-c)}\Bigg\},
\end{aligned}$$

$$\begin{aligned}
&\frac{d^2}{ds_H^2} R(s_H, s_L) \qquad \text{(EC.2.3)}\\
&= \alpha(T-\tau-w_H)^2 cp\rho\gamma\\
&\quad \cdot \tfrac{\lambda([(T-\tau)(1-s_H)+s_H w_H]p-c)[(T-\tau)(1-s_H)+s_H w_H] + (T-\tau)c - p[(T-\tau)^2(1-s_H^2)+2(T-\tau)s_H^2 w_H - (s_H w_H)^2]}{([(T-\tau)(1-s_H)+s_H w_H]p-c)[(T-\tau)^2(1-s_H^2)+2(T-\tau)s_H^2 w_H-(s_H w_H)^2]},
\end{aligned}$$

$$\begin{aligned}
&\frac{d^2}{ds_L^2} R(s_L, s_L) \qquad \text{(EC.2.4)}\\
&= (1-\alpha)(T-\tau-w_L)^2 cp\rho\gamma\\
&\quad \cdot \tfrac{\lambda([(T-\tau)(1-s_L)+s_L w_L]p-c)[(T-\tau)(1-s_L)+s_L w_L] + (T-\tau)c - p[(T-\tau)^2(1-s_L^2)+2(T-\tau)s_L^2 w_L - (s_L w_L)^2]}{([(T-\tau)(1-s_L)+s_L w_L]p-c)[(T-\tau)^2(1-s_L^2)+2(T-\tau)s_L^2 w_L-(s_L w_L)^2]},
\end{aligned}$$

$$\frac{d^2}{ds_H s_L} R(s_H, s_L) = 0. \qquad \text{(EC.2.5)}$$

Because there is no interaction term of s_H and s_L in the profit function, we can check the optimality separately. First, the denominator in (EC.2.4) is positive, and thus a sufficient condition for the concavity of $R(s_H, s_L)$ in terms of s_L is:

$$\begin{aligned}
&\lambda([(T-\tau)(1-s_L)+s_L w_L]p-c)[(T-\tau)(1-s_L)+s_L w_L]\\
&\qquad + (T-\tau)c - p[(T-\tau)^2(1-s_L^2) + 2(T-\tau)s_L^2 w_L - (s_L w_L)^2] \le 0.
\end{aligned}$$

The left-hand side of the preceding inequality is a quadratic and convex function of s_L. Thus, the preceding sufficient condition is satisfied within the two roots of the quadratic function. We know s_L

has a lower bound 0 and an upper bound $\frac{(T-\tau)p-c}{(T-\tau-w_L)}$, and thus the aforementioned quadratic function achieves its highest value either at $s_L = 0$ or $s_L = \frac{(T-\tau)p-c}{(T-\tau-w_L)}$. Its value is $(\lambda-1)(T-\tau)[(T-\tau)p-c]$ at $s_L = 0$, and $-\frac{[(T-\tau)p-c]c}{p}$ at $s_L = \frac{(T-\tau)p-c}{(T-\tau-w_L)}$. As a result, we can conclude that, if $\lambda \leq 1$, then the profit function is concave in s_L. If $\lambda > 1$, the profit function is convex in s_L when $s_L \leq \hat{s}_L$, and concave when $s_L \geq \hat{s}_L$, where $\hat{s}_L$ is the smaller root of that quadratic function, which is

$$\hat{s}_L = \frac{\lambda[2(T-\tau)p-c] - \sqrt{4(T-\tau)p[(T-\tau)p-c]+(c\lambda)^2}}{2(T-\tau-w_L)p(1+\lambda)}. \tag{EC.2.6}$$

By the condition that $K > \mu_L + \delta_H + \gamma \ln(\frac{(T-\tau)p-c}{c})$ and the arguments in the proof of Proposition 1(ii), we deduce that for any s_L and w_L:

$$K + \gamma \ln\left[\frac{[(T-\tau)(1-s_L)+s_L w_L]p}{[(T-\tau)(1-s_L)+s_L w_L]p-c}\right] - \gamma E_\delta\left[\xi(K,\mu_H,\delta)\right] > 0.$$

Therefore, by letting $s_L = 0$, we can get

$$\frac{d}{ds_L}R(s_H,0) = (1-\alpha)p\rho\left\{\lambda\left(K + \gamma\ln\left[\frac{(T-\tau)p}{(T-\tau)p-c}\right] - \gamma E_\delta\left[\xi(K,\mu_L,\delta)\right]\right)\right\} > 0.$$

This implies that the profit function is increasing at $s_L = 0$. Therefore, if $\lambda > 1$, then the profit function is increasing and convex when $s_L \in [0, \hat{s}_L]$. In general, we conclude that the profit function is quasi-concave in s_L.

Similarly, we can check the optimality of the profit function on s_H. Due to the similar structure of the second-order derivative, the sufficient conditions for concavity are the same: if $\lambda \leq 1$, the profit function is concave in s_H, whereas if $\lambda > 1$, the profit function is convex in s_H when $s_H \leq \hat{s}_H$, and concave when $s_H \geq \hat{s}_H$, where

$$\hat{s}_H = \frac{\lambda[2(T-\tau)p-c] - \sqrt{4(T-\tau)p[(T-\tau)p-c]+(c\lambda)^2}}{2(T-\tau-w_H)p(1+\lambda)}. \tag{EC.2.7}$$

In the first-order derivative of the profit function with respect to s_H, namely $\frac{d}{ds_H}R(s_H,s_L)$, there is an additional term $(1-\alpha)(1+\lambda)\gamma E_\delta\left[\xi(K,\mu_L,\delta) - \xi(K,\mu_H,\delta)\right]$, which is a negative constant. Following the proof argument for quasi-concavity in s_L, we deduce that the profit function is increasing and convex in $0 \leq s_H \leq \hat{s}_H$ if $\frac{d}{ds_H}R(0,s_L)$ is non-negative. As a result, the following condition is sufficient for the quasi-concavity of profit function in s_H:

$$(1-\alpha)(1+\lambda)\gamma E_\delta\left[\xi(K,\mu_L,\delta) - \xi(K,\mu_H,\delta)\right] + \alpha\lambda\left(K + \gamma\ln\left[\frac{(T-\tau)p}{(T-\tau)p-c}\right] - \gamma E_\delta\left[\xi(K,\mu_H,\delta)\right]\right) \geq 0.$$

Q.E.D.

Proof of Proposition 4. We first compare s_H^* and s_L^*. Note that, if $s'_H \le s'_L$, then $s_H^* \le s_L^*$, where s'_H and s'_L are as characterized in (29) and (30). Thus, we focus on the comparison between s'_H and s'_L. Given $w_H = w_L = w$, we deduce from (30) that

$$\lambda\left(K + \gamma \ln\left[\frac{[(T-\tau)(1-s'_L)+s'_L w]p}{[(T-\tau)(1-s'_L)+s'_L w]p - c}\right] - \gamma E_\delta\left[\xi(K,\mu_L,\delta)\right]\right) - \frac{(T-\tau-w)c\gamma s'_L}{\left([(T-\tau)(1-s'_L)+s'_L w]p-c\right)\left[(T-\tau)(1-s'_L)+s'_L w\right]} = 0.$$

Because $\alpha \ge 0$ and $\mu_L < \mu_H$, this implies that

$$\alpha\lambda\left(K + \gamma \ln\left[\frac{[(T-\tau)(1-s'_L)+s'_L w]p}{[(T-\tau)(1-s'_L)+s'_L w]p - c}\right] - \gamma E_\delta\left[\xi(K,\mu_H,\delta)\right]\right) - \alpha\frac{(T-\tau-w)c\gamma s'_L}{\left([(T-\tau)(1-s'_L)+s'_L w]p-c\right)\left[(T-\tau)(1-s'_L)+s'_L w\right]} < 0.$$

Thus,

$$(1-\alpha)(1+\lambda)\gamma E_\delta\left[\xi(K,\mu_L,\delta) - \xi(K,\mu_H,\delta)\right] - \alpha\frac{(T-\tau-w)c\gamma s'_L}{\left([(T-\tau)(1-s'_L)+s'_L w]p-c\right)\left[(T-\tau)(1-s'_L)+s'_L w\right]} + \alpha\lambda\left(K + \gamma \ln\left[\frac{[(T-\tau)(1-s'_L)+s'_L w]p}{[(T-\tau)(1-s'_L)+s'_L w]p - c}\right] - \gamma E_\delta\left[\xi(K,\mu_H,\delta)\right]\right) < 0.$$

Consequently, $\frac{d}{ds_H}R(s'_L, s_L) < 0$; i.e., the first-order derivative of the profit function with respect to s_H is negative at $s_H = s'_L$. Since the profit function is quasi-concave in s_H, this further implies that $s'_H < s'_L$. Therefore $s_H^* \le s_L^*$.

Next, we compare y_H^* and y_L^* via (26)-(27). We note that if $s_H^* = s_L^*$, then we have the premiums $y_H^* = g_H(s_H^*) = g_L(s_H^*, s_L^*) = y_L^*$. Now, consider the case where $s_H^* \le s_L^*$. By the arguments used in the proof of Proposition 1(ii), we have $\frac{d}{ds}E_{\delta,Z}[\pi_L^I(K,s)] \ge 0$. Thus, we can get

$$y_L^* = g_L(s_H^*, s_L^*) = g_H(s_H^*) + E_{\delta,Z}[\pi_L^I(K,s_L)] - E_{\delta,Z}[\pi_L^I(K,s_H)] \ge g_H(s_H^*) = y_H^*.$$

As a result, we have $s_H^* \le s_L^*$ and $y_H^* \le y_L^*$. Q.E.D.

Proof of Proposition 5. The firm optimizes its capacity via a backward solution as well. Thus, once the capacity decision is made, given $K = K_I^*$, the proof would be same to that in Proposition 2, and thus $\frac{d^2}{d\mu_i ds}E_{\delta,Z}[\pi_i^I(K_I^*, y, s)] \le 0$. Q.E.D.

Proof of Proposition 6. First, according to (31) and (32), the firm's profit with and without an option of purchasing BI insurance are as follows:

$$E_{\mu,\delta,Z}[\pi^I(K,y^*,s^*)] = \alpha E_{\delta,Z}[\pi_H^I(K,y_H^*,s_H^*)] + (1-\alpha)E_{\delta,Z}[\pi_L^I(K,y_L^*,s_L^*)] - \eta K,$$
$$E_{\mu,\delta,Z}[\pi^N(K)] = \alpha E_{\delta,Z}[\pi_H^N(K)] + (1-\alpha)E_{\delta,Z}[\pi_L^N(K)] - \eta K.$$

Based on the above expressions, we would like to compare the optimal capacities in these two cases. We let K_I^* and K_N^* denote the optimal capacities with and without the option of purchasing

BI insurance, respectively. We first derive K_N^*. By taking the derivative of the firm's profit with respect to K, we have

$$\frac{d}{dK}E_{\mu,\delta,Z}[\pi^N(K)] = \alpha\frac{d}{dK}E_{\delta,Z}[\pi_H^N(K)] + (1-\alpha)\frac{d}{dK}E_{\delta,Z}[\pi_L^N(K)] - \eta.$$

For optimality, we need to check the second-order derivative of $E_{\delta,Z}[\pi_i^N(K)]$ for $i \in \{H, L\}$, which is

$$\begin{aligned}&\frac{d^2}{dK^2}E_{\delta,Z}[\pi_i^N(K)]\\ &\quad = -\frac{p\{1-[(T-\tau)\lambda+1-\tau]\rho\}\exp\left(\frac{K+\mu_i}{\gamma}\right)\left\{2\exp\left(\frac{K+\mu_i+\delta_H+\delta_L}{\gamma}\right)+E_\delta\left[\exp\left(\frac{2K+\delta}{\gamma}\right)\right]+E_\delta\left[\exp\left(\frac{2\mu_i+\delta_H+\delta_L+\delta}{\gamma}\right)\right]\right\}}{\gamma\prod_{j\in\{H,L\}}\left[\exp\left(\xi(K,\mu_i,\delta)\right)\right]^2}.\end{aligned}$$

Therefore, if $1-[(T-\tau)\lambda+1-\tau]\rho > 0$, then $\frac{d^2}{dK^2}E_{\delta,Z}[\pi_i^N(K)] < 0$ for $i \in \{H, L\}$. In addition, regarding the first-order derivative, we note that

$$\frac{d}{dK}E_{\delta,Z}[\pi_i^N(K)] = \frac{p\{1-[(T-\tau)\lambda+1-\tau]\rho\}\exp\left(\frac{\mu_i}{\gamma}\right)\left\{\exp\left(\frac{\mu_i+\delta_H+\delta_L}{\gamma}\right)+E_\delta\left[\exp\left(\frac{K+\delta}{\gamma}\right)\right]\right\}}{\gamma\prod_{j\in\{H,L\}}\exp\left(\xi(K,\mu_i,\delta)\right)}.$$

Thus, if $1-[(T-\tau)\lambda+1-\tau]\rho > 0$ and $E_{\mu,\delta,Z}[\pi^N(0)] > 0$, there exists an optimal capacity $K_N^* > 0$ that maximizes the profit. If $1-[(T-\tau)\lambda+1-\tau]\rho < 0$, the marginal revenue in capacity K is decreasing convex, which means that it is not profitable for the firm to construct any capacity, and thus the optimal capacity would be 0.

Next, regarding the case where the firm has an option to purchase BI insurance, we assume that the insurer would offer insurance contracts satisfying IC and IR constraints. Therefore, the firm would always purchase the insurance contract corresponding to its early demand forecast. Given μ_i, the first-order derivative with respect to K is

$$\begin{aligned}&\frac{d}{dK}E_{\delta,Z}[\pi_i^I(K,y_i^*,s_i^*)]\\ &= \frac{\partial}{\partial K}E_{\delta,Z}[\pi_i^I(K,y_i^*,s_i^*)] + \frac{\partial}{\partial y_i}E_{\delta,Z}[\pi_i^I(K,y_i^*,s_i^*)]\frac{d}{dK}y_i^* + \frac{\partial}{\partial s_i}E_{\delta,Z}[\pi_i^I(K,y_i^*,s_i^*)]\frac{d}{dK}s_i^*\\ &= \frac{\partial}{\partial K}E_{\delta,Z}[\pi_i^I(K,y_i^*,s_i^*)] - \frac{d}{dK}y_i^* + \frac{\partial}{\partial s_i}E_{\delta,Z}[\pi_i^I(K,y_i^*,s_i^*)]\frac{d}{dK}s_i^*.\end{aligned}$$

With regard to μ_H, we have

$$\frac{d}{dK}E_{\delta,Z}[\pi_H^I(K,y_H^*,s_H^*)] = \frac{\partial}{\partial K}E_{\delta,Z}[\pi_H^I(K,y_H^*,s_H^*)] - \frac{\partial}{\partial K}g_H(s_H^*) = \frac{d}{dK}E_{\delta,Z}[\pi_H^N(K)].$$

Therefore, we find that the high-demand firm's marginal revenue per unit capacity with the option of purchasing insurance is same to that without the option, and thus $\frac{d^2}{dK^2}E_{\delta,Z}[\pi_H^I(K,y_H^*,s_H^*)]<0$ if $1-[(T-\tau)\lambda+1-\tau]\rho>0$. With regard to μ_L, we have

$$\begin{aligned}
&\frac{d}{dK}E_{\delta,Z}[\pi_L^I(K,y_L^*,s_L^*)]\\
&=\frac{\partial}{\partial K}E_{\delta,Z}[\pi_L^I(K,y_L^*,s_L^*)]-\frac{d}{dK}y_L^*+\frac{\partial}{\partial s_L}E_{\delta,Z}[\pi_L^I(K,y_L^*,s_L^*)]\frac{d}{dK}s_L^*\\
&=\frac{\partial}{\partial K}E_{\delta,Z}[\pi_L^I(K,y_H^*,s_H^*)]-\frac{d}{dK}y_H^*+\frac{\partial}{\partial s_H}E_{\delta,Z}[\pi_L^I(K,y_H^*,s_H^*)]\frac{d}{dK}s_H^*.
\end{aligned}$$

Moreover, the second-order derivative is

$$\begin{aligned}
&\frac{d^2}{dK^2}E_{\delta,Z}[\pi_L^I(K,y_L^*,s_L^*)]\\
&=\frac{d}{dK}\left\{\frac{\partial}{\partial K}E_{\delta,Z}[\pi_L^I(K,y_H^*,s_H^*)]-\frac{d}{dK}y_H^*+\frac{\partial}{\partial s_H}E_{\delta,Z}[\pi_L^I(K,y_H^*,s_H^*)]\frac{d}{dK}s_H^*\right\}\\
&=\frac{d}{dK}\frac{\partial}{\partial K}\left\{E_{\delta,Z}[\pi_L^I(K,y_H^*,s_H^*)]-E_{\delta,Z}[\pi_H^I(K,y_H^*,s_H^*)]\right\}+\frac{d^2}{dK^2}E_{\delta,Z}[\pi_H^N(K)]\\
&\qquad+\frac{d}{dK}\left[\frac{\partial}{\partial s_H}\left\{E_{\delta,Z}[\pi_L^I(K,y_H^*,s_H^*)]-E_{\delta,Z}[\pi_H^I(K,y_H^*,s_H^*)]\right\}\frac{d}{dK}s_H^*\right].
\end{aligned}$$

By checking the derivatives, we can get if $1-[(T-\tau)\lambda+1-\tau]\rho>0$,

$$\begin{aligned}
&\frac{d}{dK}\frac{\partial}{\partial K}\left\{E_{\delta,Z}[\pi_L^I(K,y_H^*,s_H^*)]-E_{\delta,Z}[\pi_H^I(K,y_H^*,s_H^*)]\right\}+\frac{d^2}{dK^2}E_{\delta,Z}[\pi_H^N(K)]<0,\\
&\frac{d}{dK}\left[\frac{\partial}{\partial s_H}\left\{E_{\delta,Z}[\pi_L^I(K,y_H^*,s_H^*)]-E_{\delta,Z}[\pi_H^I(K,y_H^*,s_H^*)]\right\}\frac{d}{dK}s_H^*\right]<0,
\end{aligned}$$

and thus we have $\frac{d^2}{dK^2}E_{\mu,\delta,Z}[\pi^I(K,y^*,s^*)]<0$. That is, with the option of purchasing BI insurance, the firm's optimal profit is also concave in K.

Next, we would like to compare the optimal capacity K_I^* and K_N^*. Because we have $\frac{d}{dK}E_{\delta,Z}[\pi_H^I(K,y_H^*,s_H^*)]=\frac{d}{dK}E_{\delta,Z}[\pi_H^N(K)]$, we check whether $\frac{d}{dK}E_{\delta,Z}[\pi_L^I(K,y_L^*,s_L^*)]-\frac{d}{dK}E_{\delta,Z}[\pi_L^N(K)]$ is positive or negative, and we have

$$\begin{aligned}
&\frac{d}{dK}E_{\delta,Z}[\pi_L^I(K,y_L^*,s_L^*)]-\frac{d}{dK}E_{\delta,Z}[\pi_L^N(K)]\\
=&\frac{\partial}{\partial K}E_{\delta,Z}[\pi_L^I(K,y_H^*,s_H^*)]-\frac{d}{dK}E_{\delta,Z}[\pi_L^N(K)]-\left(\frac{\partial}{\partial K}E_{\delta,Z}[\pi_H^I(K,y_H^*,s_H^*)]-\frac{d}{dK}E_{\delta,Z}[\pi_H^N(K)]\right)\\
&+\frac{\partial}{\partial s_H}\left\{E_{\delta,Z}[\pi_L^I(K,y_H^*,s_H^*)]-E_{\delta,Z}[\pi_H^I(K,y_H^*,s_H^*)]\right\}\frac{d}{dK}s_H^*.
\end{aligned}$$

Letting $\mu_H=\mu_L+\varepsilon$, we have the following if $\varepsilon\to 0$:

$$\begin{aligned}
&\frac{\partial}{\partial K}E_{\delta,Z}[\pi_L^I(K,y_H^*,s_H^*)]-\frac{d}{dK}E_{\delta,Z}[\pi_L^N(K)]-\left(\frac{\partial}{\partial K}E_{\delta,Z}[\pi_H^I(K,y_H^*,s_H^*)]-\frac{d}{dK}E_{\delta,Z}[\pi_H^N(K)]\right)\to 0,\\
&\frac{\partial}{\partial s_H}\left\{E_{\delta,Z}[\pi_L^I(K,y_H^*,s_H^*)]-E_{\delta,Z}[\pi_H^I(K,y_H^*,s_H^*)]\right\}\to 0.
\end{aligned}$$

Thus, if $\varepsilon \to 0$, we have

$$\frac{d}{dK}E_{\delta,Z}[\pi_L^I(K,y_L^*,s_L^*)] - \frac{d}{dK}E_{\delta,Z}[\pi_L^N(K)] \to 0.$$

In this case, because both profit functions are concave in K and $\frac{d}{dK}E_{\delta,Z}[\pi_L^I(K,y_L^*,s_L^*)] \to \frac{d}{dK}E_{\delta,Z}[\pi_L^N(K)]$, we deduce that $K_I^* \to K_N^*$ as $\mu_H \to \mu_L$ (i.e., $\varepsilon \to 0$).

Next, we would like to prove that $K_N^* > K_I^*$ for sufficiently small $\varepsilon > 0$. To that end, we first let

$$\begin{aligned}
&\Delta_K(\varepsilon)\\
&= \frac{d}{d\varepsilon}\left\{\frac{d}{dK}E_{\delta,Z}[\pi_L^I(K,y_L^*,s_L^*)] - \frac{d}{dK}E_{\delta,Z}[\pi_L^N(K)]\right\}\\
&= \frac{d}{d\varepsilon}\left\{\frac{\partial}{\partial K}E_{\delta,Z}[\pi_L^I(K,y_H^*,s_H^*)] - \frac{d}{dK}E_{\delta,Z}[\pi_L^N(K)] - \left(\frac{\partial}{\partial K}E_{\delta,Z}[\pi_H^I(K,y_H^*,s_H^*)] - \frac{d}{dK}E_{\delta,Z}[\pi_H^N(K)]\right)\right\}\\
&\quad + \frac{d}{d\varepsilon}\left(\frac{\partial}{\partial s_H}\left\{E_{\delta,Z}[\pi_L^I(K,y_H^*,s_H^*)] - E_{\delta,Z}[\pi_H^I(K,y_H^*,s_H^*)]\right\}\frac{d}{dK}s_H^*\right).
\end{aligned}$$

Regarding the terms on the right-hand side above, we note the following:

$$\begin{aligned}
&\frac{d}{d\varepsilon}\left\{\frac{\partial}{\partial K}E_{\delta,Z}[\pi_L^I(K,y_H^*,s_H^*)] - \frac{d}{dK}E_{\delta,Z}[\pi_L^N(K)] - \left(\frac{\partial}{\partial K}E_{\delta,Z}[\pi_H^I(K,y_H^*,s_H^*)] - \frac{d}{dK}E_{\delta,Z}[\pi_H^N(K)]\right)\right\}\Bigg|_{\varepsilon=0}\\
&= -(T-\tau-w_H)ps_H(1+\lambda)\rho\exp\left(\tfrac{K+\mu_L}{\gamma}\right)\frac{2\exp\left(\frac{K+\mu_L+\delta_H+\delta_L}{\gamma}\right)+E_\delta\left[\exp\left(\frac{2K+\delta}{\gamma}\right)+\exp\left(\frac{2(\mu_L+\delta_H+\delta_L)-\delta}{\gamma}\right)\right]}{\gamma\prod_{j\in\{H,L\}}\exp\left(\xi(K,\mu_i,\delta)\right)^2} < 0.
\end{aligned}$$

$$\begin{aligned}
&\frac{d}{d\varepsilon}\,\frac{\partial}{\partial s_H}\left\{E_{\delta,Z}[\pi_L^I(K,y_H^*,s_H^*)] - E_{\delta,Z}[\pi_H^I(K,y_H^*,s_H^*)]\right\}\frac{d}{dK}s_H^*\Bigg|_{\varepsilon=0}\\
&\qquad = -\frac{\alpha\lambda\exp\left(\frac{\mu_L}{\gamma}\right)\left\{\exp\left(\frac{\mu_L+\delta_H+\delta_L}{\gamma}\right)+E_\delta\left[\exp\left(\frac{K+\delta}{\gamma}\right)\right]\right\}}{\prod_{j\in\{H,L\}}\exp\left(\xi(K,\mu_L,\delta_j)\right)}\frac{d}{dK}s_H^* < 0.
\end{aligned}$$

$$\frac{\partial}{\partial s_H}\left\{E_{\delta,Z}[\pi_L^I(K,y_H^*,s_H^*)] - E_{\delta,Z}[\pi_H^I(K,y_H^*,s_H^*)]\right\}\frac{d}{d\varepsilon}\frac{d}{dK}s_H^*\Bigg|_{\varepsilon=0} = 0.$$

Therefore, we have $\Delta_K(0) < 0$, which implies that given a sufficiently small $\varepsilon > 0$, we have $\frac{d}{dK}E_{\delta,Z}[\pi_L^N(K)] > \frac{d}{dK}E_{\delta,Z}[\pi_L^I(K,y_L^*,s_L^*)]$, and thus $K_N^* > K_I^*$. As a result, there exists $\tilde{\mu} > \mu_L$ such that $K_N^* > K_I^*$ for all $\mu_H \in (\mu_L, \tilde{\mu})$. Q.E.D.

Proof of Proposition 7(i). To show the reverse single-crossing property with delayed recovery, we first extend our model. First, we prove the subcase without an option of the fast recovery. Note that the delay periods overlap the waiting periods. Therefore, the details of the extended model depend on their relative values. First, if $d \le w$, the firm's expected profit with BI insurance would be

$$\begin{aligned}
\tilde{\pi}_{ij1}^I(K,e,y,s) = &-\eta K + \tau p E_{\epsilon_t}[\min(D_{ijt},K)] - y - ceK + (T-\tau-d)pE_{\epsilon_t}[\min(D_{ijt},eK)]\\
&+ (T-\tau-w)spE_{\epsilon_t}\big[\min(D_{ijt},K) - \min(D_{ijt},eK)\big] - \tilde{C}_{ij}^I(K,e,y,s),
\end{aligned} \tag{EC.2.8}$$

where

$$\tilde{C}^I_{ij}(K,e,y,s)=\lambda\Big\{ceK+(T-\tau-d)p\Big(E_{\epsilon_t}[\min(D_{ijt},K)]-E_{\epsilon_t}[\min(D_{ijt},eK)]\Big) \\ -(T-\tau-w)spE_{\epsilon_t}\big[\min(D_{ijt},K)-\min(D_{ijt},eK)\big]\Big\}. \tag{EC.2.9}$$

The only difference between (17) and (EC.2.9) is that with delayed recovery, the firm neither earns profit nor receives insurance compensation for the d periods following the disruption—this is because the insurance has a waiting term $w\geq d$. However, if $d>w$, the firm's expected profit with BI insurance becomes

$$\tilde{\pi}^I_{ij1}(K,e,y,s)=-\eta K+\tau pE_{\epsilon_t}[\min(D_{ijt},K)]-y-ceK+(T-\tau-d)pE_{\epsilon_t}[\min(D_{ijt},eK)] \\ +(d-w)spE_{\epsilon_t}\big[\min(D_{ijt},K)\big] \\ +(T-\tau-d)spE_{\epsilon_t}\big[\min(D_{ijt},K)-\min(D_{ijt},eK)\big]-\tilde{C}^I_{ij}(K,e,y,s), \tag{EC.2.10}$$

where

$$\tilde{C}^I_{ij}(K,e,y,s)=\lambda\Big\{ceK+(T-\tau-d)p\Big(E_{\epsilon_t}[\min(D_{ijt},K)]-E_{\epsilon_t}[\min(D_{ijt},eK)]\Big) \\ -(d-w)spE_{\epsilon_t}\big[\min(D_{ijt},K)\big]-(T-\tau-d)spE_{\epsilon_t}\big[\min(D_{ijt},K)-\min(D_{ijt},eK)\big]\Big\}. \tag{EC.2.11}$$

In this case, once a disruption occurs, the firm neither earns profit nor receives compensation in the first w periods following the disruption. After the waiting periods, the firm receives compensation for all disruption loss in the following $d-w$ periods because the capacity has not been recovered. Following that, the firm starts to earn profit by resumed capacity and receives partial compensation from the insurer.

In this extended model, we re-examine the reverse single-crossing property in these two cases. First, we analyze the case where $d\leq w$, and then the case where $d>w$.

With regard to the case where $d\leq w$, we have the following according to (EC.2.8): given μ_i and δ_j, the firm's optimal effort with BI insurance is

$$\tilde{e}^*_{ij}(s)=\begin{cases}\frac{1}{K}F^{-1}_{ij}\big(\frac{(w-d)p+(T-\tau-w)p(1-s)-c}{(w-d)p+(T-\tau-w)p(1-s)}\big) & \text{if } F^{-1}_{ij}\big(\frac{(w-d)p+(T-\tau-w)p(1-s)-c}{(w-d)p+(T-\tau-w)p(1-s)}\big)<K,\\ 1 & \text{otherwise,}\end{cases} \tag{EC.2.12}$$

where

$$F^{-1}_{ij}\left(\frac{(w-d)p+(T-\tau-w)p(1-s)-c}{(w-d)p+(T-\tau-w)p(1-s)}\right)=\mu_i+\delta_j+\gamma\ln\left(\frac{(w-d)p+(T-\tau-w)p(1-s)-c}{c}\right). \tag{EC.2.13}$$

This is similar to (18). As in (20) and (21), we let $E_{\delta,Z}[\tilde{\pi}_i^I(K,y,s)]$ denote the total expected profit with BI insurance, and note that

$$\begin{aligned}
&\frac{d}{ds}E_{\delta,Z}[\tilde{\pi}_i^I(K,y,s)]\\
&\qquad= \rho(1+\lambda)(T-\tau-w)p\left\{K-\gamma E_\delta\left[\xi(K,\mu_i,\delta)\right]+\gamma\ln\left(\tfrac{(w-d)p+(T-\tau-w)p(1-s)}{(w-d)p+(T-\tau-w)p(1-s)-c}\right)\right\}\geq 0,\\
&\frac{d^2}{ds^2}E_{\delta,Z}[\tilde{\pi}_i^I(K,y,s)] = \rho(1+\lambda)\tfrac{(T-\tau-w)^2pc\gamma}{[(T-\tau-w)(1-s)+w-d]((w-d)p+(T-\tau-w)p(1-s)-c)}\geq 0,\\
&\frac{d^2}{dsd\mu_i}E_{\delta,Z}[\tilde{\pi}_i^I(K,y,s)] = -\rho(1+\lambda)(T-\tau-w)pE_\delta\left[\tfrac{\exp\left(\frac{\mu_i+\delta}{\gamma}\right)}{\exp\left(\frac{K}{\gamma}\right)+\exp\left(\frac{\mu_i+\delta}{\gamma}\right)}\right]<0.
\end{aligned}$$

Thus, the reverse single-crossing property always holds in this case. With regard to the case where $d>w$, the firm's optimal effort with BI insurance in (EC.2.10) is

$$\tilde{e}_{ij}^*(s) = \begin{cases}\frac{1}{K}F_{ij}^{-1}\left(\frac{(T-\tau-d)p(1-s)-c}{(T-\tau-d)p(1-s)}\right)=\frac{1}{K}\left[\mu_i+\delta_j+\gamma\ln\left(\frac{(T-\tau-d)p(1-s)-c}{c}\right)\right] & \text{if } F_{ij}^{-1}\left(\frac{(T-\tau-d)p(1-s)-c}{(T-\tau-d)p(1-s)}\right)<K,\\ 1 & \text{otherwise.}\end{cases} \tag{EC.2.14}$$

Thus, deriving the expression for $E_{\delta,Z}[\tilde{\pi}_i^I(K,y,s)]$ in this case, we have

$$\begin{aligned}
&\frac{d}{ds}E_{\delta,Z}[\tilde{\pi}_i^I(K,y,s)]\\
&\qquad= \rho(1+\lambda)p\left[(T-\tau-w)\left\{K-\gamma E_\delta\left[\xi(K,\mu_i,\delta)\right]+\gamma\ln\left(\tfrac{(T-\tau-d)p(1-s)}{(T-\tau-d)p(1-s)-c}\right)\right\}+(d-w)\mu_i\right]\geq 0,\\
&\frac{d^2}{ds^2}E_{\delta,Z}[\tilde{\pi}_i^I(K,y,s)] = \rho(1+\lambda)\tfrac{(T-\tau-w)pc\gamma}{(1-s)[(T-\tau-d)p(1-s)-c]}\geq 0,\\
&\frac{d^2}{dsd\mu_i}E_{\delta,Z}[\tilde{\pi}_i^I(K,y,s)] = \rho(1+\lambda)pE_\delta\left[\tfrac{d\left[\exp\left(\frac{\mu_i+\delta}{\gamma}\right)+\exp\left(\frac{K}{\gamma}\right)\right]-(T-\tau)\exp\left(\frac{\mu_i+\delta}{\gamma}\right)-w\exp\left(\frac{K}{\gamma}\right)}{\exp\left(\frac{\mu_i+\delta}{\gamma}\right)+\exp\left(\frac{K}{\gamma}\right)}\right].
\end{aligned}$$

To satisfy the reverse single-crossing property, we need to have $\frac{d^2}{dsd\mu_i}E_{\delta,Z}[\tilde{\pi}_i^I(K,y,s)]\leq 0$, and a sufficient condition for this is the following:

$$d\left[\exp\left(\tfrac{\mu_i+\delta_j}{\gamma}\right)+\exp\left(\tfrac{K}{\gamma}\right)\right]-(T-\tau)\exp\left(\tfrac{\mu_i+\delta_j}{\gamma}\right)-w\exp\left(\tfrac{K}{\gamma}\right)\leq 0 \text{ for all } \mu_i \text{ and } \delta_j.$$

Let

$$\bar{d} = \frac{(T-\tau)\exp\left(\frac{\mu_L+\delta_L}{\gamma}\right)+w\exp\left(\frac{K}{\gamma}\right)}{\exp\left(\frac{\mu_L+\delta_L}{\gamma}\right)+\exp\left(\frac{K}{\gamma}\right)}.$$

If $d\leq\bar{d}$, then the reverse single-crossing property holds in this case as well. Q.E.D.

Proof of Proposition 7(ii). Similar to the proof of Proposition 7(i), there are two possible scenarios: $d\leq w$ and $d>w$. We re-examine the reverse single-crossing property in both cases.

With regard to the case where $d \leq w$, we note following: given μ_i and δ_j, the firm's optimal effort with fast recovery, $\bar{e}^*_{ij}(s)$, is given by

$$\bar{e}^*_{ij}(s) = \begin{cases} \frac{1}{K}\left[\mu_i + \delta_j + \gamma \ln\left(\frac{wp+(T-\tau-w)p(1-s)-\kappa c}{\kappa c}\right)\right] & \text{if } \mu_i + \delta_j + \gamma \ln\left(\frac{wp+(T-\tau-w)p(1-s)-\kappa c}{\kappa c}\right) < K, \\ 1 & \text{otherwise.} \end{cases} \tag{EC.2.15}$$

The optimal effort in (EC.2.15) is similar to (18) except that the cost is κc. Comparing (EC.2.15) to the optimal effort with regular recovery, $\tilde{e}^*_{ij}(s)$, in (EC.2.12) and (EC.2.13), we deduce that

$$\bar{e}^*_{ij}(s) > \tilde{e}^*_{ij}(s) \;\Leftrightarrow\; \kappa < \tfrac{(T-\tau-w)(1-s)+w}{(T-\tau-w)(1-s)+w-d}.$$

Because the right-hand side of the preceding inequality is non-decreasing in s, we deduce that if $\kappa < \frac{T-\tau}{T-\tau-d}$, then $\bar{e}^*_{ij}(s) > \tilde{e}^*_{ij}(s)$ for all $s \geq 0$. Now, given μ_i, let $E_{\delta,Z}[\bar{\pi}^I_i(K,y,s)]$ denote the firm's total expected profit with BI insurance and fast recovery, and suppose that $d \leq \bar{d}$. According to Propositions 2 and 7(i), we know that if both high-demand and low-demand firms choose regular recovery, or if they both choose fast recovery, the reverse single-crossing property holds. Thus, it is sufficient to focus on the cases where one type of firm chooses regular recovery and the other chooses fast recovery. We know that the firm would choose fast recovery if $E_{\delta,Z}[\bar{\pi}^I_i(K,y,s)] - E_{\delta,Z}[\tilde{\pi}^I_i(K,y,s)] > 0$. By taking the derivative with respect to μ_i, we obtain the following:

$$\frac{d}{d\mu_i}\Big\{E_{\delta,Z}[\bar{\pi}^I_i(K,y,s)] - E_{\delta,Z}[\tilde{\pi}^I_i(K,y,s)]\Big\} = -(1+\lambda)[(\kappa-1)c - dp].$$

If $\kappa < \frac{T-\tau}{T-\tau-d}$, then $\kappa < \frac{(T-\tau-w)(1-s)+w}{(T-\tau-w)(1-s)+w-d}$, and thus $(\kappa-1)c - dp < 0$. As a result, we deduce that $E_{\delta,Z}[\bar{\pi}^I_i(K,y,s)] - E_{\delta,Z}[\tilde{\pi}^I_i(K,y,s)]$ is increasing in μ_i. That is,

$$E_{\delta,Z}[\bar{\pi}^I_H(K,y,s)] - E_{\delta,Z}[\tilde{\pi}^I_H(K,y,s)] > E_{\delta,Z}[\bar{\pi}^I_L(K,y,s)] - E_{\delta,Z}[\tilde{\pi}^I_L(K,y,s)].$$

Therefore, if $\kappa < \frac{T-\tau}{T-\tau-d}$, it is impossible that the high-demand firm chooses regular recovery while the low-demand firm chooses fast recovery. If the high-demand firm chooses fast recovery and the low-demand firm chooses regular recovery, then we have the following:

$$\begin{aligned} &\frac{d}{ds}\Big(E_{\delta,Z}[\bar{\pi}^I_H(K,y,s)] - E_{\delta,Z}[\tilde{\pi}^I_L(K,y,s)]\Big) \\ &\quad = -p(T-\tau-w)(1+\lambda)\gamma\Big\{E_\delta\left[\xi(K,\mu_H,\delta) - \xi(K,\mu_L,\delta)\right] + \ln\left(\tfrac{(w-d)p+(T-\tau-w)(1-s)p}{(w-d)p+(T-\tau-w)(1-s)p-c}\right) \\ &\qquad\qquad - \ln\left(\tfrac{wp+(T-\tau-w)(1-s)p}{wp+(T-\tau-w)(1-s)p-\kappa c}\right)\Big\} \end{aligned}$$

If $\kappa < \frac{T-\tau}{T-\tau-d}$, we also have $\ln\left(\frac{(w-d)p+(T-\tau-w)(1-s)p}{(w-d)p+(T-\tau-w)(1-s)p-c}\right) - \ln\left(\frac{wp+(T-\tau-w)(1-s)p}{wp+(T-\tau-w)(1-s)p-\kappa c}\right) > 0$. As a result, $\frac{d}{ds}(E_{\delta,Z}[\bar{\pi}^I_H(K,y,s)] - E_{\delta,Z}[\tilde{\pi}^I_L(K,y,s)]) < 0$, and the reverse single-crossing property holds in the case where $d \leq w$.

For the case where $d > w$, we have the following: given μ_i and δ_j, the firm's optimal effort $\bar{e}^*_{ij}(s)$ is as in (EC.2.15). Comparing this to the optimal effort with regular recovery $\tilde{e}^*_{ij}(s)$ in (EC.2.14), we have

$$\bar{e}^*_{ij}(s) > \tilde{e}^*_{ij}(s) \;\Leftrightarrow\; \kappa < \tfrac{wp+(T-\tau-w)p(1-s)}{(T-\tau-d)p(1-s)} = \tfrac{T-\tau}{T-\tau-d} + \tfrac{ws}{(T-\tau-d)(1-s)}.$$

Thus, as in the previous case, we deduce that if $\kappa < \frac{T-\tau}{T-\tau-d}$, then $\bar{e}^*_{ij}(s) > \tilde{e}^*_{ij}(s)$ for all $s \geq 0$. As explained earlier, as long as $d \leq \bar{d}$, we know from Propositions 2 and 7(i) that the reverse single-crossing property holds if both high-demand and low-demand firms choose the same type of recovery (either regular or fast). Therefore, it is sufficient to focus on the cases where one type of firm chooses regular recovery and the other chooses fast recovery. If $\kappa < \frac{T-\tau}{T-\tau-d}$, then

$$\begin{aligned}\frac{d}{d\mu_i}\Big\{E_{\delta,Z}[\bar{\pi}^I_i(K,y,s)] - E_{\delta,Z}[\tilde{\pi}^I_i(K,y,s)]\Big\} &= -(1+\lambda)[(\kappa-1)c - dp(1-s)] \\ &\geq -(1+\lambda)\left[\tfrac{cd}{T-\tau-d} - dp(1-s)\right] \\ &\geq -(1+\lambda)\tfrac{d}{T-\tau-d}\left[c - p(T-\tau-d)(1-s)\right] > 0.\end{aligned}$$

As before, it is impossible that the high-demand firm chooses regular recovery and the low-demand firm chooses fast recovery. If the high-demand firm chooses fast recovery and the low-demand firm chooses regular recovery, we have

$$\frac{d}{ds}\Big(E_{\delta,Z}[\bar{\pi}^I_H(K,y,s)] - E_{\delta,Z}[\tilde{\pi}^I_L(K,y,s)]\Big) = -p(1+\lambda)\Delta_f(d),$$

where

$$\begin{aligned}\Delta_f(d) = d\mu_L + w(\mu_H - \mu_L) + (T-\tau-w)\gamma E_\delta\left[\xi(K,\mu_H,\delta) - \xi(K,\mu_L,\delta)\right] \\ + (T-\tau-d)\ln\left(\tfrac{(T-\tau-d)(1-s)p}{(T-\tau-d)(1-s)p-c}\right) - (T-\tau)\ln\left(\tfrac{(T-\tau)(1-s)p}{(T-\tau)(1-s)p-\kappa c}\right).\end{aligned}$$

Regarding $\Delta_f(d)$, observe that

$$\frac{\partial}{\partial d}\Delta_f(d) = \frac{\mu_L[(T-\tau-d)p(1-s)-c] + \gamma\left\{c - [(T-\tau-d)p(1-s)-c]\ln\left(\frac{(T-\tau-d)(1-s)p}{(T-\tau-d)(1-s)p-c}\right)\right\}}{(T-\tau-d)(1-s)p-c}.$$

Because $\ln\left(\frac{(T-\tau-d)(1-s)p}{(T-\tau-d)(1-s)p-c}\right) < \frac{c}{(T-\tau-d)(1-s)p-c}$, we have $\frac{\partial}{\partial d}\Delta_f(d) > 0$. In addition, we also have

$$\begin{aligned}\Delta_f(w) = w\mu_H + (T-\tau-w)\gamma E_\delta\left[\xi(K,\mu_H,\delta) - \xi(K,\mu_L,\delta)\right] \\ + (T-\tau-w)\ln\left(\tfrac{(T-\tau-w)(1-s)p}{(T-\tau-w)(1-s)p-c}\right) - (T-\tau)\ln\left(\tfrac{(T-\tau)(1-s)p}{(T-\tau)(1-s)p-\kappa c}\right).\end{aligned}$$

Note that if $\kappa < \frac{T-\tau}{T-\tau-d}$, then $\ln\left(\frac{(T-\tau-d)(1-s)p}{(T-\tau-d)(1-s)p-c}\right) > \ln\left(\frac{(T-\tau)(1-s)p}{(T-\tau)(1-s)p-\kappa c}\right)$. In addition, a necessary condition for $E_{\delta,Z}[\bar{\pi}^I_H(K,y,s)] - E_{\delta,Z}[\tilde{\pi}^I_H(K,y,s)] > 0$ (i.e., the high-demand firm chooses fast recovery) is that $\mu_H > \ln\left(\frac{(T-\tau-d)(1-s)p}{(T-\tau-d)(1-s)p-c}\right)$ for all $d > w$. Because the preceding right-hand side is increasing in d, this implies that $\mu_H > \ln\left(\frac{(T-\tau-w)(1-s)p}{(T-\tau-w)(1-s)p-c}\right)$, and thus $\Delta_f(w) > 0$. Because $\frac{\partial}{\partial d}\Delta_f(d) > 0$ and $\Delta_f(w) > 0$, $-p(1+\lambda)\Delta_f(d) < 0$ for all $d > w$. As a result, if $\kappa < \frac{T-\tau}{T-\tau-d}$, then the reverse single-crossing property holds in the case where $d > w$. This completes the proof for both cases. Q.E.D.

Proof of Proposition 8. First, we let $c(e)$ denote the recovery cost as a function of the effort e, where

$$c(e) = \begin{cases} c_1 eK & \text{if } e \le m, \\ c_1 mK + c_2(e-m)K & \text{otherwise.} \end{cases}$$

For purposes of exposition, it is useful to consider the following two cases:

1. The effort cost is linear with unit cost c_1.
2. The effort cost is linear with unit cost c_2, while the total cost is deducted by mK.

Without considering the threshold m, given μ_i, δ_j and a contract (y,s), we know that the profit function is still concave in e and we can get the optimal efforts in these two cases. We let $e_{ij}^{1*}(s)$ and $e_{ij}^{2*}(s)$ denote the optimal efforts in these two cases respectively, which are:

$$e_{ij}^{1*}(s) = \begin{cases} \frac{1}{K}\left[\mu_i + \delta_j + \gamma\ln\left(\frac{wp+(T-\tau-w)p(1-s)-c_1}{c_1}\right)\right] & \text{if } \mu_i + \delta_j + \gamma\ln\left(\frac{wp+(T-\tau-w)p(1-s)-c_1}{c_1}\right) < K, \\ 1 & \text{otherwise,} \end{cases}$$

$$e_{ij}^{2*}(s) = \begin{cases} \frac{1}{K}\left[\mu_i + \delta_j + \gamma\ln\left(\frac{wp+(T-\tau-w)p(1-s)-c_2}{c_2}\right)\right] & \text{if } \mu_i + \delta_j + \gamma\ln\left(\frac{wp+(T-\tau-w)p(1-s)-c_2}{c_2}\right) < K, \\ 1 & \text{otherwise.} \end{cases}$$

It is straightforward to see that $e_{ij}^{1*}(s) \ge e_{ij}^{2*}(s)$ because $c_1 < c_2$ for all s.

With the nonlinear cost function $c(e)$, both $e_{ij}^{1*}(s)$ and $e_{ij}^{2*}(s)$ could be the candidates of the optimal effort, depending on the threshold m and coverage percentage s. Given s, the optimal effort is

$$e_{ij}^{*}(s) = \begin{cases} e_{ij}^{1*}(s) & \text{if} \quad m \ge e_{ij}^{1*}(s) > e_{ij}^{2*}(s), \\ m & \text{if} \quad e_{ij}^{1*}(s) > m \ge e_{ij}^{2*}(s), \\ e_{ij}^{2*}(s) & \text{if} \quad e_{ij}^{1*}(s) > e_{ij}^{2*}(s) \ge m. \end{cases} \tag{EC.2.16}$$

Next, we check whether the reverse single-crossing property holds. Because of the piecewise-linear cost function, the profit function may not be differentiable with respect to s. Therefore, we check $\frac{d}{ds}E_{\delta,Z}[\pi_H^I(K,y,s)] - \frac{d}{ds}E_{\delta,Z}[\pi_L^I(K,y,s)]$ instead. To prove the reverse single-crossing property, it suffices to show that $\frac{d}{ds}\pi_{Hj1}^I(K, e_{Hj}^*(s), y, s) - \frac{d}{ds}\pi_{Lj1}^I(K, e_{Lj}^*(s), y, s) < 0$ for $j \in \{H, L\}$. Because there are several scenarios of $e_{Hj}^*(s)$ and $e_{Lj}^*(s)$ in (EC.2.16), in the following analysis, we check the reverse single-crossing property case by case.

Case 1: $e^*_{Hj}(s) = e^{1*}_{Hj}(s)$. In this case, we know that $m \geq e^{1*}_{Hj}(s) > e^{2*}_{Hj}(s)$, and we deduce that $m \geq e^{1*}_{Lj}(s) > e^{2*}_{Lj}(s)$ as well. Thus, the only possible case for the low-demand firm's optimal effort would be $e^*_{Lj}(s) = e^{1*}_{Lj}(s)$. Hence, we have

$$\frac{d}{ds}\pi^I_{Hj1}(K, e^{1*}_{Hj}(s), y, s) - \frac{d}{ds}\pi^I_{Lj1}(K, e^{1*}_{Lj}(s), y, s) = (T-\tau-w)p\gamma(1+\lambda)\left[\xi(K,\mu_L,\delta_j) - \xi(K,\mu_H,\delta_j)\right] < 0,$$

and thus the reverse single-crossing property holds.

Case 2: $e^*_{Hj}(s) = m$. In this case, we know $e^{1*}_{Hj}(s) > m \geq e^{2*}_{Hj}(s)$, and there are two possible subcases for the low-demand firm's optimal effort:

1. $e^{1*}_{Hj}(s) > m \geq e^{1*}_{Lj}(s)$:

   In this subcase, the low-demand firm's optimal effort is $e^*_{Lj}(s) = e^{1*}_{Lj}(s)$, and we have

$$\begin{aligned}&\frac{d}{ds}\pi^I_{Hj1}(K, m, y, s) - \frac{d}{ds}\pi^I_{Lj1}(K, e^{1*}_{Lj}(s), y, s)\\ &= (T-\tau-w)p(1+\lambda)\Bigg\{-mK + \gamma\Bigg[\xi(mK,\mu_H,\delta_j) + \xi(K,\mu_L,\delta_j)\\ &\qquad\qquad - \xi(K,\mu_H,\delta_j) + \ln\left(\tfrac{wp+(T-\tau-w)p(1-s)}{wp+(T-\tau-w)p(1-s)-c_1}\right)\Bigg]\Bigg\},\end{aligned}$$

   which is not always positive or always negative. Therefore, we cannot claim that the reverse single-crossing property always holds in this subcase.

2. $e^{1*}_{Hj}(s) > e^{1*}_{Lj}(s) > m \geq e^{2*}_{Hj}(s) > e^{2*}_{Lj}(s)$:

   In this subcase, the low-demand firm's optimal effort is $e^*_{Lj}(s) = m$ as well, and we have

$$\begin{aligned}&\frac{d}{ds}\pi^I_{Hj1}(K, m, y, s) - \frac{d}{ds}\pi^I_{Lj1}(K, m, y, s)\\ &= (T-\tau-w)p\gamma(1+\lambda)\left[\xi(K,\mu_L,\delta_j) - \xi(K,\mu_H,\delta_j) - \xi(mK,\mu_L,\delta_j) + \xi(mK,\mu_H,\delta_j)\right] > 0,\end{aligned}$$

   and thus it is the single-crossing property that holds in this subcase.

Case 3: $e^*_{Hj}(s) = e^{2*}_{Hj}(s)$. In this case, we know that $m \geq e^{1*}_{Hj}(s) \geq e^{2*}_{Hj}(s)$, and there are three possible subcases for the low-demand firm's optimal effort:

1. $e^{1*}_{Hj}(s) \geq e^{2*}_{Hj}(s) > m \geq e^{1*}_{Lj}(s) \geq e^{2*}_{Lj}(s)$:

   In this subcase, the low-demand firm's optimal effort is $e^*_{Lj}(s) = e^{1*}_{Lj}(s)$, and we have

$$\begin{aligned}&\frac{d}{ds}\pi^I_{Hj1}(K, e^{2*}_{Hj}(s), y, s) - \frac{d}{ds}\pi^I_{Lj1}(K, e^{1*}_{Lj}(s), y, s)\\ &= (T-\tau-w)p\gamma(1+\lambda)\left[\xi(K,\mu_L,\delta_j) - \xi(K,\mu_H,\delta_j) + \ln\left(\tfrac{wp+(T-\tau-w)p(1-s)-c_1}{wp+(T-\tau-w)p(1-s)-c_2}\right)\right],\end{aligned}$$

   which is not always positive or always negative.

2. $e^{1*}_{Hj}(s) > e^{1*}_{Lj}(s) > m \geq e^{2*}_{Lj}(s)$:

   In this subcase, the low-demand firm's optimal effort is $e^{*}_{Lj}(s) = m$, and we have

$$\begin{aligned}&\frac{d}{ds}\pi^{I}_{Hj1}(K, e^{2*}_{Hj}(s), y, s) - \frac{d}{ds}\pi^{I}_{Lj1}(K, m, y, s)\\&= (T-\tau-w)p(1+\lambda)\Bigg\{mK + \gamma\Bigg[-\xi(mK, \mu_H, \delta_j) + \xi(K, \mu_L, \delta_j)\\&\qquad\qquad - \xi(K, \mu_H, \delta_j) + \ln\left(\tfrac{wp+(T-\tau-w)p(1-s)}{wp+(T-\tau-w)p(1-s)-c_2}\right)\Bigg]\Bigg\},\end{aligned}$$

   which is not always positive or always negative.

3. $e^{1*}_{Hj}(s) > e^{1*}_{Lj}(s) \geq e^{2*}_{Lj}(s) \geq m$:

   In this subcase, the low-demand firm's optimal effort is $e^{*}_{Lj}(s) = e^{2*}_{Lj}(s)$, and we have

$$\frac{d}{ds}\pi^{I}_{Hj1}(K, e^{2*}_{Hj}(s), y, s) - \frac{d}{ds}\pi^{I}_{Lj1}(K, e^{2*}_{Lj}(s), y, s) = (T-\tau-w)p\gamma(1+\lambda)\left[\xi(K, \mu_L, \delta_j) - \xi(K, \mu_H, \delta_j)\right] < 0,$$

   and thus the reverse single-crossing property holds.

To find a sufficient condition that the reverse single-crossing property holds among all the cases, we need to avoid Case 2-2 and find sufficient conditions in Case 2-1, 3-1 and 3-2.

To avoid Case 2-2, we can have $e^{2*}_{Hj}(s) \geq e^{1*}_{Lj}(s)$ for all δ_j and s. By letting $c_2 = ac_1$, we have

$$\begin{aligned}&e^{2*}_{Hj}(s) - e^{1*}_{Lj}(s) \geq 0\\&\Rightarrow \frac{1}{K}\left[\mu_H - \mu_L - \gamma \ln\left(a\tfrac{wp+(T-\tau-w)p(1-s)-c_1}{wp+(T-\tau-w)p(1-s)-ac_1}\right)\right] \geq 0\\&\Rightarrow a \leq \frac{wp+(T-\tau-w)p(1-s)}{[wp+(T-\tau-w)p(1-s)-c_1]\Delta_1 + c_1}, \text{ where } \Delta_1 = \exp\left(\frac{\mu_L-\mu_H}{\gamma}\right).\end{aligned}$$

The right-hand side of the preceding inequality is decreasing in s and increasing in w. Therefore, a general sufficient condition is the following:

$$a \leq \frac{(T-\tau)p(1-\bar{s})}{[(T-\tau)p(1-\bar{s})-c_1]\Delta_1 + c_1}. \tag{EC.2.17}$$

With regard to Case 2-1, we find that

$$\frac{d}{dm}\left\{\frac{d}{ds}\pi^{I}_{Hj1}(K, m, y, s) - \frac{d}{ds}\pi^{I}_{Lj1}(K, e^{1*}_{Lj}(s), y, s)\right\} = -\frac{\exp\left(\frac{\mu_H+\delta_j}{\gamma}\right)K(T-\tau-w)p(1+\lambda)}{\ln[\xi(mK, \mu_H, \delta_j)]} < 0.$$

In addition, under the condition in (EC.2.17), we have $e^{1*}_{Hj}(s) \geq m \geq e^{2*}_{Hj}(s) \geq e^{1*}_{Lj}(s)$, and thus one sufficient condition is as follows:

$$\begin{aligned}&\frac{d}{ds}\pi^{I}_{Hj1}(K, e^{2*}_{Hj}(s), y, s) - \frac{d}{ds}\pi^{I}_{Lj1}(K, e^{1*}_{Lj}(s), y, s) \leq 0\\&\Rightarrow \ln\left[\exp\left(\tfrac{K}{\gamma}\right) + \exp\left(\tfrac{\mu_H+\delta_j}{\gamma}\right)\right] - \ln\left[\exp\left(\tfrac{K}{\gamma}\right) + \exp\left(\tfrac{\mu_L+\delta_j}{\gamma}\right)\right] \leq \ln\left[\tfrac{wp+(T-\tau-w)p(1-s)-c_1}{wp+(T-\tau-w)p(1-s)-ac_1}\right]\\&\Rightarrow a \leq \frac{(T-\tau-w)p(1-s)+wp}{c_1}\left(1 - \frac{\exp\left(\frac{K}{\gamma}\right)+\exp\left(\frac{\mu_L+\delta_j}{\gamma}\right)}{\exp\left(\frac{K}{\gamma}\right)+\exp\left(\frac{\mu_H+\delta_j}{\gamma}\right)}\right) + \frac{\exp\left(\frac{K}{\gamma}\right)+\exp\left(\frac{\mu_L+\delta_j}{\gamma}\right)}{\exp\left(\frac{K}{\gamma}\right)+\exp\left(\frac{\mu_H+\delta_j}{\gamma}\right)}.\end{aligned}$$

As before, we note that the right-hand side of the preceding inequality is decreasing in s and increasing in w. In addition, it is also decreasing in δ_j. Therefore, a general sufficient condition is the following:

$$a \le \frac{(T-\tau)p(1-\bar{s})}{c_1}(1-\Delta_2)+\Delta_2, \text{ where } \Delta_2 = \frac{\exp\left(\frac{K}{\gamma}\right)+\exp\left(\frac{\mu_L+\delta_L}{\gamma}\right)}{\exp\left(\frac{K}{\gamma}\right)+\exp\left(\frac{\mu_H+\delta_L}{\gamma}\right)}. \tag{EC.2.18}$$

We find that the reverse single-crossing property holds in Case 3-1 and Case 3-2 if (EC.2.17) and (EC.2.18) hold. Therefore, our sufficient condition for the reverse single-crossing property is that $a \le \bar{a}$, where

$$\bar{a} = \min\left\{\frac{(T-\tau)p(1-\bar{s})}{[(T-\tau)p(1-\bar{s})-c_1]\Delta_1+c_1}, \frac{(T-\tau)p(1-\bar{s})}{c_1}(1-\Delta_2)+\Delta_2\right\},$$

$$\Delta_1 = \exp\left(\frac{\mu_L-\mu_H}{\gamma}\right), \qquad \Delta_2 = \frac{\exp\left(\frac{K}{\gamma}\right)+\exp\left(\frac{\mu_L+\delta_L}{\gamma}\right)}{\exp\left(\frac{K}{\gamma}\right)+\exp\left(\frac{\mu_H+\delta_L}{\gamma}\right)}. \quad \text{Q.E.D.}$$

Proof of Proposition 9. Given μ_i and δ_j, the firm's expected profit with BI insurance contract (y,s) and the maximum indemnity period v is

$$\begin{aligned}\pi^I_{ij1}(K,e,y,s) = & -y+\tau p E_{\epsilon_t}[\min(D_{ijt},K)] - ceK + (T-\tau)pE_{\epsilon_t}[\min(D_{ijt},eK)] \\ & + (v-w)spE_{\epsilon_t}\big[\min(D_{ijt},K)-\min(D_{ijt},eK)\big] - C^I_{ij}(K,e,y,s),\end{aligned}$$

where

$$\begin{aligned}C^I_{ij}(K,e,y,s) = \lambda\Big\{ & ceK + (T-\tau)p\Big(E_{\epsilon_t}[\min(D_{ijt},K)] - E_{\epsilon_t}[\min(D_{ijt},eK)]\Big) \\ & - (v-w)spE_{\epsilon_t}\big[\min(D_{ijt},K)-\min(D_{ijt},eK)\big]\Big\}.\end{aligned}$$

In this case, the firm's optimal effort with BI insurance is

$$e^*_{ij}(s) = \begin{cases}\frac{1}{K}F^{-1}_{ij}\left(\frac{(T-\tau-v+w)p+(v-w)p(1-s)-c}{(T-\tau-v+w)p+(v-w)p(1-s)}\right) & \text{if } F^{-1}_{ij}\left(\frac{(T-\tau-v+w)p+(v-w)p(1-s)-c}{(T-\tau-v+w)p+(v-w)p(1-s)}\right) < K, \\ 1 & \text{otherwise,}\end{cases}$$

where

$$F^{-1}_{ij}\left(\frac{(T-\tau-v+w)p+(v-w)p(1-s)-c}{(T-\tau-v+w)p+(v-w)p(1-s)}\right) = \mu_i+\delta_j+\gamma\ln\left(\frac{(T-\tau-v+w)p+(v-w)p(1-s)-c}{c}\right).$$

Using this optimal effort with BI insurance, we obtain the firm's profit with insurance $E_{\delta,Z}[\pi^I_i(K,y,s)]$, and by taking the derivatives with respect to s and μ_i, we have

$$\frac{d}{ds}E_{\delta,Z}[\pi^I_i(K,y,s)] = \rho(1+\lambda)(T-\tau-w)p\left\{K-\gamma E_\delta[\xi(K,\mu_i,\delta)]+\gamma\ln\left(\frac{p[T-\tau+s(v-w)]}{p[T-\tau+s(v-w)]-c}\right)\right\} \ge 0,$$

$$\frac{d^2}{ds^2}E_{\delta,Z}[\pi^I_i(K,y,s)] = \rho(1+\lambda)\frac{cp\gamma(v-w)^2}{(p[T-\tau+s(v-w)]-c)[T-\tau+s(v-w)]} \ge 0,$$

$$\frac{d^2}{dsd\mu_i}E_{\delta,Z}[\pi^I_i(K,y,s)] = -\rho(1+\lambda)(v-w)pE_\delta\left[\frac{\exp\left(\frac{\mu_i+\delta}{\gamma}\right)}{\exp\left(\frac{K}{\gamma}\right)+\exp\left(\frac{\mu_i+\delta}{\gamma}\right)}\right] < 0.$$

Based on the above, the reverse single-crossing property holds. Q.E.D.

Appendix EC.3: Effects of Model Parameters on Optimal Contracts

EC.3.1 Conditions for Quasi-concavity of the Insurer's Profit Function

Lemma 1 provides sufficient conditions under which the insurer's profit function $R(s_H, s_L)$ is quasi-concave in s_H and s_L. In this section, we first provide some examples satisfying these conditions, and then we illustrate the scenarios when these conditions are not satisfied.

The sufficient conditions in Lemma 1 are (28), that $K > \mu_L + \delta_H + \gamma \ln\left(\frac{(T-\tau)p-c}{c}\right)$, and that $K > \mu_H + \delta_H + \gamma \ln\left(\frac{(T-\tau)p-c}{c}\right)$. First, because $\mu_H > \mu_L$, it is obvious that $\mu_H + \delta_H + \gamma \ln\left(\frac{(T-\tau)p-c}{c}\right)$ is greater than $\mu_L + \delta_H + \gamma \ln\left(\frac{(T-\tau)p-c}{c}\right)$. In addition, the left-hand side of (28) is increasing in K. All of these conditions require higher capacity K, while which condition is the strictest depends on problem parameters, especially α, μ_H and μ_L. Figure EC.1 illustrates how these conditions depend on α and K. If α is small, condition (28) appears to be the strictest, whereas if α is large, the condition that $K > \mu_H + \delta_H + \gamma \ln\left(\frac{(T-\tau)p-c}{c}\right)$ could become the strictest.

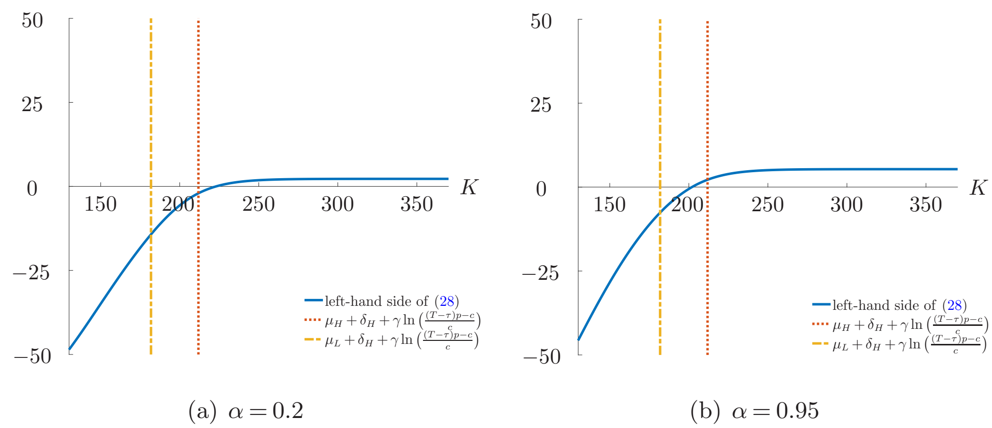


Figure EC.1 Thresholds of the Sufficient Conditions in K

If the capacity K is sufficiently small, the firm would not benefit from an insurance with low coverage percentage s because it would put the full effort on the recovery and receive no compensation. Thus, with a small s, the firm would not purchase insurance and the insurer would make zero profit. As s becomes higher, the firm may purchase the insurance, but this contract may or may not be profitable for the insurer. Figure EC.2 illustrates that the profit function is not necessarily quasi-concave in these cases.

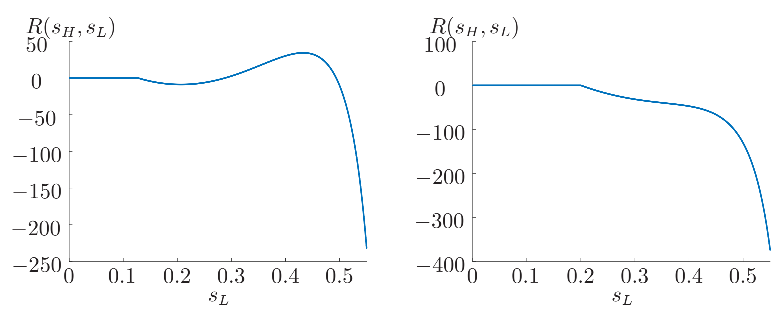


Figure EC.2 Cross Sections of the Profit Function When the Capacity is Sufficiently Small.

We would also like to show that s_H^* may be 0 if (28) is not satisfied. In this case, there are two main categories for the shape of the insurer's profit function $R(s_H, s_L)$ with respect to s_H:

1. $R(s_H, s_L)$ is decreasing and quasi-concave in $s_H \in [0, \bar{s}]$.
2. $R(s_H, s_L)$ is not quasi-concave in $s_H \in [0, \bar{s}]$.

Proof. Recalling the proofs in Lemma 1 and Proposition 3, we let $\hat{s}_H$ denote the threshold of the concavity as in (EC.2.7), and we can check the concavity/convexity of the insurer's profit function $R(s_H, s_L)$ with respect to s_H. Depending on the location of $\hat{s}_H$, all possible cases are as follows: (i) $R(s_H, s_L)$ is concave in $s_H \in [0, \bar{s}]$; (ii) $R(s_H, s_L)$ is convex in $s_H \in [0, \hat{s}_H]$ and concave in $s_H \in [\hat{s}_H, \bar{s}]$; (iii) $R(s_H, s_L)$ is convex in $s_H \in [0, \bar{s}]$. Moreover, when (28) is not satisfied, $R(s_H, s_L)$ is decreasing in s_H at $s_H = 0$. Thus, for case (i), $R(s_H, s_L)$ is decreasing and concave in $s_H \in [0, \bar{s}]$. For cases (ii) and (iii), if there is any point in $s_H \in [0, \bar{s}]$ such that $R(s_H, s_L)$ is increasing in s_H at that point, then $R(s_H, s_L)$ is not quasi-concave. Otherwise, $R(s_H, s_L)$ is quasi-concave as well as decreasing in s_H. To sum up, in the feasible region $[0, \bar{s}]$ where s_H resides, we have the following possibilities: $R(s_H, s_L)$ is either quasi-concave in s_H or not quasi-concave in s_H. If it is quasi-concave in s_H, then it must be decreasing in s_H as well. As a result, if (28) is not satisfied, then there are two main categories regarding the shape of $R(s_H, s_L)$ with respect to s_H, as described above. Q.E.D.

Figure EC.3 illustrates sheds further light on the cases described above. If $R(s_H, s_L)$ is quasi-concave in s_H and it is also decreasing in s_H, in which case the optimal insurance coverage s_H^* is zero (see the top panels in Figure EC.3). If $R(s_H, s_L)$ is not quasi-concave, then the optimal insurance coverage s_H^* could be zero or positive, and moreover, the stationary points of $R(s_H, s_L)$ are not always the local maxima (see the bottom panels in Figure EC.3).

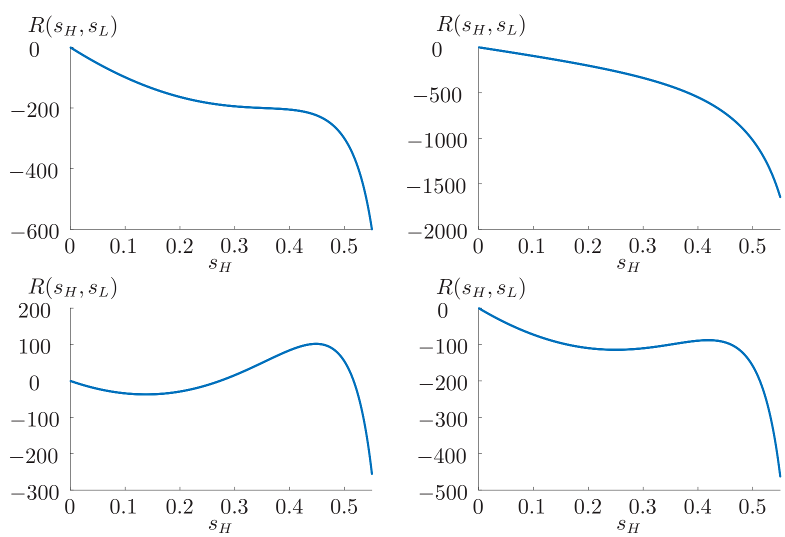


Figure EC.3 Cross Sections of the Profit Function When (28) Does Not Hold

EC.3.2 Endogenous Waiting Periods

In this subsection, we consider the case of endogenous waiting periods and analyze the optimal decisions in this case. In our analysis, we find that if the optimal coverage percentages are stationary points of the insurer's profit function, changing waiting periods does not affect the insurer's optimal profit, and the waiting periods are substitutes to the coverage percentages. That is, if $s_i^* = s_i'$ for $i \in \{H, L\}$, then

$$\frac{d}{dw_i} R(s_H^*, s_L^*) = 0 \text{ and } \frac{d}{dw_i} s_i^* = \frac{s_i^*}{T - \tau - w_i}.$$

To prove the preceding statement, we focus on the optimal contracts when $s_H^* = s_H'$ and $s_L^* = s_L'$, and the insurer's optimal profit is $R(s_H^*, s_L^*)$. First, we would like to show $\frac{d}{dw_H} R(s_H^*, s_L^*) = \frac{d}{dw_L} R(s_H^*, s_L^*) = 0$, and these derivatives are

$$\begin{aligned}
\frac{d}{dw_H} R(s_H^*, s_L^*) &= \frac{\partial}{\partial w_H} R(s_H^*, s_L^*) + \frac{\partial}{\partial s_H} R(s_H^*, s_L^*) \frac{d}{dw_H} s_H^* + \frac{\partial}{\partial s_L} R(s_H^*, s_L^*) \frac{d}{dw_H} s_L^*, \\
\frac{d}{dw_L} R(s_H^*, s_L^*) &= \frac{\partial}{\partial w_L} R(s_H^*, s_L^*) + \frac{\partial}{\partial s_H} R(s_H^*, s_L^*) \frac{d}{dw_L} s_H^* + \frac{\partial}{\partial s_L} R(s_H^*, s_L^*) \frac{d}{dw_L} s_L^*.
\end{aligned}$$

It is easy to check that $\frac{d}{dw_H} s_L^* = \frac{d}{dw_L} s_H^* = 0$. Thus, the above derivatives become

$$\begin{aligned}
\frac{d}{dw_H} R(s_H^*, s_L^*) &= \frac{\partial}{\partial w_H} R(s_H^*, s_L^*) + \frac{\partial}{\partial s_H} R(s_H^*, s_L^*) \frac{d}{dw_H} s_H^*, \\
\frac{d}{dw_L} R(s_H^*, s_L^*) &= \frac{\partial}{\partial w_L} R(s_H^*, s_L^*) + \frac{\partial}{\partial s_L} R(s_H^*, s_L^*) \frac{d}{dw_L} s_L^*.
\end{aligned}$$

Moreover, we also have

$$\frac{\frac{\partial}{\partial w_i} R(s_H^*, s_L^*)}{\frac{\partial}{\partial s_i} R(s_H^*, s_L^*)} = \frac{-s_i^*}{T - \tau - w_i} \text{ for } i \in \{H, L\}.$$

Thus, to conclude that $\frac{d}{dw_H} R(s_H^*, s_L^*) = \frac{d}{dw_L} R(s_H^*, s_L^*) = 0$, it suffices to show that $\frac{d}{dw_i} s_i^* = \frac{s_i^*}{T-\tau-w_i}$ for $i \in \{H, L\}$.

Because there is no closed-form expression of s_H^* and s_L^*, we use the implicit function theorem to compute the derivatives of s_H^* and s_L^* with respect to w_H and w_L. Based on this, we have

$$\frac{d}{dw_i} s_i^* = -\frac{\frac{d^2}{dw_i ds_i} R(s_H^*, s_L^*)}{\frac{d^2}{ds_i^2} R(s_H^*, s_L^*)} \text{ for } i \in \{H, L\}.$$

Note that

$$\begin{aligned}
\frac{d^2R(s_H^*,s_L^*)}{dw_H ds_H} &= cs_H^*\alpha\gamma\Delta_H,\\
\frac{d^2R(s_H^*,s_L^*)}{ds_H^2} &= c(T-\tau-w_H)\alpha\gamma\Delta_H, \text{ where}\\
\Delta_H &= \tfrac{p[(T-\tau)(1-s_H^*)s_H^* w_H]\{(T-\tau)[\lambda(1-s_H^*)-1-s_H^*]+(1+\lambda)s_H^* w_H\}-c\{(T-\tau)[(1-s_H^*)\lambda-1]+\lambda s_H^* w_H\}}{[(T-\tau)(1-s_H^*)+s_H^* w_H]^2\{p[(T-\tau)(1-s_H^*)+s_H^* w_H]-c\}^2}, \text{ and}\\
\frac{d^2R(s_H^*,s_L^*)}{dw_L ds_L} &= cs_L^*\gamma\Delta_L,\\
\frac{d^2R(s_H^*,s_L^*)}{ds_L^2} &= c(T-\tau-w_L)\gamma\Delta_L, \text{ where}\\
\Delta_L &= \tfrac{p[(T-\tau)(1-s_L^*)s_L^* w_L]\{(T-\tau)[\lambda(1-s_L^*)-1-s_L^*]+(1+\lambda)s_L^* w_L\}-c\{(T-\tau)[(1-s_L^*)\lambda-1]+\lambda s_L^* w_L\}}{[(T-\tau)(1-s_L^*)+s_L^* w_L]^2\{p[(T-\tau)(1-s_L^*)+s_L^* w_L]-c\}^2}.
\end{aligned}$$

Thus, we have

$$\begin{aligned}
\frac{d}{dw_i}s_i^* &= \frac{s_i^*}{T-\tau-w_i} \text{ for } i\in\{H,L\}, \text{ and therefore}\\
\frac{d}{dw_H}R(s_H^*,s_L^*) &= \frac{d}{dw_L}R(s_H^*,s_L^*) = 0. \quad \text{Q.E.D.}
\end{aligned}$$

EC.3.3 Comparative Statics of the Optimal Contracts

In this section, we analyze how problem parameters affect the optimal contracts. Given the exogenous waiting periods, we focus on the effects on the optimal premiums and optimal coverage percentages, where the optimal coverage percentages satisfy the first-order conditions. Because $s_i^* = s_i'$ for $i\in\{H,L\}$, we have $\frac{d^2}{ds_H^2}R(s_H^*,s_L^*)\le 0$ and $\frac{d^2}{ds_L^2}R(s_H^*,s_L^*)\le 0$ in the following analysis. Then, similar to the analysis in Section EC.3.2, we rely on the implicit function theorem to get analytical results via (26) and (27). For simplicity, we let $r_H(s_H)$ and $r_L(s_L)$ satisfy

$$\begin{aligned}
r_H(s_H) =& (1-\alpha)(1+\lambda)\gamma E_\delta\left[\xi(K,\mu_L,\delta)-\xi(K,\mu_H,\delta)\right] - \alpha\tfrac{(T-\tau-w_H)c\gamma s_H'}{\left([(T-\tau)(1-s_H')+s_H' w_H]p-c\right)\left[(T-\tau)(1-s_H')+s_H' w_H\right]}\\
&+\alpha\lambda\left(K+\gamma\ln\left[\tfrac{[(T-\tau)(1-s_H')+s_H' w_H]p}{[(T-\tau)(1-s_H')+s_H' w_H]p-c}\right]-\gamma E_\delta\left[\xi(K,\mu_H,\delta)\right]\right), \qquad \text{(EC.3.1)}\\
r_L(s_L) =& \lambda\left(K+\gamma\ln\left[\tfrac{[(T-\tau)(1-s_L')+s_L' w_L]p}{[(T-\tau)(1-s_L')+s_L' w_L]p-c}\right]-\gamma E_\delta\left[\xi(K,\mu_L,\delta)\right]\right)\\
&-\tfrac{(T-\tau-w_L)c\gamma s_L'}{\left([(T-\tau)(1-s_L')+s_L' w_L]p-c\right)\left[(T-\tau)(1-s_L')+s_L' w_L\right]}, \qquad \text{(EC.3.2)}
\end{aligned}$$

so that the first-order conditions imply $r_H(s_H')=r_L(s_L')=0$. Based on these, we can analyze the impacts of the following parameters by taking the derivatives of the expressions in (EC.3.1) and (EC.3.2).

Change in α. First, note that

$$\frac{d}{d\alpha}r_H(s_H) = (1+\lambda)\gamma E_\delta\left[\xi(K,\mu_H,\delta)-\xi(K,\mu_L,\delta)\right] - \frac{(T-\tau-w_H)c\gamma s_H^*}{(p[(T-\tau)(1-s_H^*)+s_Hw_H]-c)[(T-\tau)(1-s_H^*)+s_Hw_H]} + \lambda\left(K+\gamma\ln\left[\frac{p[(T-\tau)(1-s_H^*)+s_Hw_H]}{p[(T-\tau)(1-s_H^*)+s_Hw_H]-c}\right]-\gamma E_\delta\left[\xi(K,\mu_H,\delta)\right]\right).$$

In this derivative, $E_\delta\left[\xi(K,\mu_H,\delta)-\xi(K,\mu_L,\delta)\right]>0$, and by the first-order optimality condition in (29), we have

$$\lambda\left(K+\gamma\ln\left[\frac{p[(T-\tau)(1-s_H^*)+s_Hw_H]}{p[(T-\tau)(1-s_H^*)+s_Hw_H]-c}\right]-\gamma E_\delta\left[\xi(K,\mu_H,\delta)\right]\right) - \frac{(T-\tau-w_H)c\gamma s_H^*}{(p[(T-\tau)(1-s_H^*)+s_Hw_H]-c)[(T-\tau)(1-s_H^*)+s_Hw_H]} > 0.$$

Therefore, $\frac{d}{d\alpha}r_H(s_H)>0$. In addition, because $\frac{d^2}{ds_H^2}R(s_H^*,s_L^*)\le 0$, we have $\frac{d}{ds_H}r_H(s_H)\le 0$, and thus

$$\frac{d}{d\alpha}s_H^* = -\frac{\frac{d}{d\alpha}r_H(s_H)}{\frac{d}{ds_H}r_H(s_H)} > 0.$$

Next, based on (26), the first-order derivative of y_H^* with respect to α is

$$\frac{d}{d\alpha}y_H^* = \frac{\partial}{\partial s_H}E_{\delta,Z}[\pi_H^I(K,y_H^*,s_H^*)]\frac{d}{d\alpha}s_H^* > 0.$$

Hence, both s_H^* and y_H^* are increasing in α. With regard to s_L^*, note that $\frac{d}{d\alpha}r_L(s_L)=0$, and thus

$$\frac{d}{d\alpha}s_L^* = -\frac{\frac{d}{d\alpha}r_L(s_L)}{\frac{d}{ds_L}r_L(s_L)} = 0.$$

With regard to y_L^*, by (27), the first-order derivative of y_L^* with respect to α is

$$\frac{d}{d\alpha}y_L^* = \left(\frac{\partial}{\partial s_H}E_{\delta,Z}[\pi_H^I(K,y_H^*,s_H^*)] - \frac{\partial}{\partial s_H}E_{\delta,Z}[\pi_L^I(K,y_H^*,s_H^*)]\right)\frac{d}{d\alpha}s_H^* + \frac{\partial}{\partial s_L}E_{\delta,Z}[\pi_L^I(K,y_L^*,s_L^*)]\frac{d}{d\alpha}s_L^* < 0.$$

Hence, y_L^* is decreasing in α, but s_L^* does not change in α.

Change in K. The first-order derivative of $r_H(s_H)$ with respect to K is

$$\frac{d}{dK}r_H(s_H) = (1-\alpha)(1+\lambda)E_\delta\left[\frac{\exp\left(\frac{K}{\gamma}\right)}{\exp\left(\frac{K}{\gamma}\right)+\exp\left(\frac{\mu_L+\delta}{\gamma}\right)} - \frac{\exp\left(\frac{K}{\gamma}\right)}{\exp\left(\frac{K}{\gamma}\right)+\exp\left(\frac{\mu_H+\delta}{\gamma}\right)}\right] + \alpha\lambda\left(E_\delta\left[\frac{\exp\left(\frac{\mu_H+\delta}{\gamma}\right)}{\exp\left(\frac{K}{\gamma}\right)+\exp\left(\frac{\mu_H+\delta}{\gamma}\right)}\right]\right) > 0.$$

Therefore, we have

$$\frac{d}{dK}s_H^* = -\frac{\frac{d}{dK}r_H(s_H)}{\frac{d}{ds_H}r_H(s_H)} > 0.$$

The first-order derivative of y_H^* with respect to K is

$$\begin{aligned}\frac{d}{dK}y_H^* =& \frac{\partial}{\partial s_H}E_{\delta,Z}[\pi_H^I(K,y_H^*,s_H^*)]\frac{d}{dK}s_H^* \\ &+ (1+\lambda)\rho(T-\tau-w_H)ps_H^*\frac{\exp\left(\frac{\mu_H}{\gamma}\right)\left\{\exp\left(\frac{\mu_H+\delta_H+\delta_L}{\gamma}\right)+E_\delta\left[\exp\left(\frac{K+\delta}{\gamma}\right)\right]\right\}}{\prod_{j\in\{H,L\}}\exp\left[\xi(K,\mu_H,\delta)\right]}>0.\end{aligned}$$

Thus, both s_H^* and y_H^* is increasing in K. With regard to s_L^*, we have

$$\frac{d}{dK}r_L(s_L) = \lambda\left(E_\delta\left[\frac{\exp\left(\frac{\mu_L+\delta}{\gamma}\right)}{\exp\left(\frac{K}{\gamma}\right)+\exp\left(\frac{\mu_L+\delta}{\gamma}\right)}\right]\right)>0,$$

and thus $\frac{d}{dK}s_L^*>0$. Finally, the first-order derivative of y_L^* with respect to K is

$$\begin{aligned}\frac{d}{dK}y_L^* =& \frac{d}{dK}\left\{E_{\delta,Z}[\pi_H^I(K,s_H)]-E_{\delta,Z}[\pi_H^N(K)]+E_{\delta,Z}[\pi_L^I(K,s_L)]-E_{\delta,Z}[\pi_L^I(K,s_H)]\right\} \\ =& \frac{\partial}{\partial K}\left\{E_{\delta,Z}[\pi_H^I(K,s_H)]-E_{\delta,Z}[\pi_H^N(K)]+E_{\delta,Z}[\pi_L^I(K,s_L)]-E_{\delta,Z}[\pi_L^I(K,s_H)]\right\} \\ &+\frac{\partial}{\partial s_H}\left\{E_{\delta,Z}[\pi_H^I(K,s_H)]-E_{\delta,Z}[\pi_L^I(K,s_H)]\right\}\frac{d}{dK}s_H^* \\ &+\frac{\partial}{\partial s_L}\left\{E_{\delta,Z}[\pi_L^I(K,s_L)]\right\}\frac{d}{dK}s_L^*.\end{aligned}$$

In the previous expression, we have

$$\begin{aligned}&\frac{\partial}{\partial K}\left\{E_{\delta,Z}[\pi_H^I(K,s_H)]-E_{\delta,Z}[\pi_H^N(K)]+E_{\delta,Z}[\pi_L^I(K,s_L)]-E_{\delta,Z}[\pi_L^I(K,s_H)]\right\}>0, \text{ and} \\ &\frac{\partial}{\partial s_L}\left\{E_{\delta,Z}[\pi_L^I(K,s_L)]\right\}\frac{d}{dK}s_L^*>0.\end{aligned}$$

However, because of the reverse single-crossing property in Proposition 2, we know that the second term $\frac{\partial}{\partial s_H}\{E_{\delta,Z}[\pi_H^I(K,s_H)]-E_{\delta,Z}[\pi_L^I(K,s_H)]\}<0$. Thus, y_L^* could either increase or decrease as K increases, but s_L^* is increasing in K.

Change in μ_H. The first-order derivative of $r_H(s_H)$ with respect to μ_H is

$$\frac{d}{d\mu_H}r_H(s_H) = -(1-\alpha+\lambda)\frac{\exp\left(\frac{\mu_H}{\gamma}\right)\left\{\exp\left(\frac{\mu_H+\delta_H+\delta_L}{\gamma}\right)+E_\delta\left[\exp\left(\frac{K+\delta}{\gamma}\right)\right]\right\}}{\prod_{j\in\{H,L\}}\exp\left[\xi(K,\mu_H,\delta)\right]}<0.$$

Therefore, we have

$$\frac{d}{d\mu_H}s_H^* = -\frac{\frac{d}{d\mu_H}r_H(s_H)}{\frac{d}{ds_H}r_H(s_H)}<0.$$

The first-order derivative of y_H^* with respect to μ_H is

$$\frac{d}{d\mu_H}y_H^* = \frac{\partial}{\partial s_H}E_{\delta,Z}[\pi_H^I(K,y_H^*,s_H^*)]\frac{d}{d\mu_H}s_H^*$$
$$-(1+\lambda)\rho(T-\tau-w_H)ps_H^*\frac{\exp\left(\frac{\mu_H}{\gamma}\right)\left\{\exp\left(\frac{\mu_H+\delta_H+\delta_L}{\gamma}\right)+E_\delta\left[\exp\left(\frac{K+\delta}{\gamma}\right)\right]\right\}}{\prod_{j\in\{H,L\}}\exp\left[\xi(K,\mu_H,\delta)\right]}<0.$$

Thus, both s_H^* and y_H^* is decreasing in μ_H. With regard to s_L^*, note that $\frac{d}{d\mu_H}r_L(s_L)=0$, and thus

$$\frac{d}{d\mu_H}s_L^* = -\frac{\frac{d}{d\mu_H}r_L(s_L)}{\frac{d}{ds_L}r_L(s_L)}=0.$$

Thus, s_L^* is irrelevant to μ_H. Next, the first-order derivative of y_L^* with respect to μ_H is

$$\frac{d}{d\mu_H}y_L^* = \frac{d}{d\mu_H}y_H^* + \frac{d}{d\mu_H}\left\{E_{\delta,Z}[\pi_L^I(K,s_L)]-E_{\delta,Z}[\pi_L^I(K,s_H)]\right\}$$
$$=\frac{d}{d\mu_H}y_H^* - \left(K+\gamma\ln\left[\frac{p[(T-\tau)(1-s_H^*)+s_Hw_H]}{p[(T-\tau)(1-s_H^*)+s_Hw_H]-c}\right]-\gamma E_\delta\left[\xi(K,\mu_H,\delta)\right]\right)\frac{d}{d\mu_H}s_H^*.$$

We know that the first term $\frac{d}{d\mu_H}y_H^*$ is negative, while it minuses the remaining negative term. Thus, y_L^* could either increase or decrease as μ_H increases, but s_L^* does not change in μ_H.

Change in μ_L. The first-order derivative of $r_H(s_H)$ with respect to μ_L is

$$\frac{d}{d\mu_L}r_H(s_H)=(1-\alpha)(1+\lambda)\frac{\exp\left(\frac{\mu_L}{\gamma}\right)\left\{\exp\left(\frac{\mu_L+\delta_H+\delta_L}{\gamma}\right)+E_\delta\left[\exp\left(\frac{K+\delta}{\gamma}\right)\right]\right\}}{\prod_{j\in\{H,L\}}\exp\left[\xi(K,\mu_L,\delta)\right]}>0.$$

Therefore, first-order derivative of s_H^* with respect to μ_L is

$$\frac{d}{d\mu_L}s_H^* = -\frac{\frac{d}{d\mu_L}r_H(s_H)}{\frac{d}{ds_H}r_H(s_H)}>0.$$

The first-order derivative of y_H^* with respect to μ_L is

$$\frac{d}{d\mu_L}y_H^* = \frac{\partial}{\partial s_H}E_{\delta,Z}[\pi_H^I(K,y_H^*,s_H^*)]\frac{d}{d\mu_L}s_H^*>0.$$

Thus, both s_H^* and y_H^* are increasing in μ_L. With regard to s_L^*, we have

$$\frac{d}{d\mu_L}r_L(s_L)=-\lambda E_\delta\left[\frac{\exp\left(\frac{\mu_L+\delta}{\gamma}\right)}{\exp\left(\xi(K,\mu_L,\delta)\right)}\right]<0,$$

and thus $\frac{d}{d\mu_L}s_L^*<0$. Finally, the first-order derivative of y_L^* with respect to μ_L is

$$\frac{d}{d\mu_L}y_L^* = \frac{d}{d\mu_L}y_H^* + \frac{d}{d\mu_L}\left\{E_{\delta,Z}[\pi_L^I(K,s_L)]-E_{\delta,Z}[\pi_L^I(K,s_H)]\right\}$$
$$=\frac{\partial}{\partial s_H}\left\{E_{\delta,Z}[\pi_H^I(K,s_H^*)]-E_{\delta,Z}[\pi_L^I(K,s_H^*)]\right\}\frac{d}{d\mu_L}s_H^* + \frac{\partial}{\partial s_L}E_{\delta,Z}[\pi_L^I(K,s_L^*)]\frac{d}{d\mu_L}s_L^*$$
$$=\rho(T-\tau-w_H)p(1+\lambda)\gamma E_\delta\left[\xi(K,\mu_L,\delta)-\xi(K,\mu_H,\delta)\right]\frac{d}{d\mu_L}s_H^* + \frac{d}{ds_L}E_{\delta,Z}[\pi_L^I(K,s_L^*)]\frac{d}{d\mu_L}s_L^*<0.$$

As a result, both s_L^* and y_L^* are decreasing in μ_L.

Change in ρ. The first-order derivatives of $r_H(s_H)$ and $r_L(s_L)$ with respect to ρ are

$$\frac{d}{d\rho} r_H(s_H) = \frac{d}{d\rho} r_L(s_L) = 0.$$

Thus, we have $\frac{d}{d\rho} s_H^* = \frac{d}{d\rho} s_L^* = 0$. With regard to y_H^*, we have

$$\frac{d}{d\rho} y_H^* = \frac{d}{d\rho}\left\{ E_{\delta,Z}[\pi_H^I(K,s_H)] - E_{\delta,Z}[\pi_H^N(K)]\right\}$$

According to (14) and (21), we let $\pi_{ij1}^I(K,s) = y + \pi_{ij1}^I(K,y,s)$, and we have

$$\frac{d}{d\rho} y_H^* = \beta\{\pi_{HH1}^I(K,s_H) - \pi_{HH1}^N(K)\} + (1-\beta)\{\pi_{HL1}^I(K,s_H) - \pi_{HL1}^N(K)\} > 0.$$

Similarly, we can get

$$\frac{d}{d\rho} y_L^* = \frac{d}{d\rho} y_H^* + \beta\{\pi_{LH1}^I(K,s_L) - \pi_{LH1}^I(K,s_H)\} + (1-\beta)\{\pi_{LL1}^I(K,s_L) - \pi_{LL1}^I(K,s_H)\} > 0.$$

As a result, y_H^* and y_L^* are increasing in ρ, but s_H^* and s_L^* do not change in ρ.

Change in λ. The first-order derivative of $r_H(s_H)$ with respect to λ is

$$\begin{aligned}\frac{d}{d\lambda} r_H(s_H) =& (1-\alpha)\gamma E_\delta\left[\xi(K,\mu_L,\delta) - \xi(K,\mu_H,\delta)\right] \\ &+ \alpha\left(K + \gamma \ln\left[\tfrac{[(T-\tau)(1-s'_H)+s'_H w_H]p}{[(T-\tau)(1-s'_H)+s'_H w_H]p-c}\right] - \gamma E_\delta\left[\xi(K,\mu_H,\delta)\right]\right),\end{aligned}$$

Because of the sufficient condition in Lemma 1, we have $\frac{d}{d\lambda} r_H(s_H) > 0$, and thus $\frac{d}{d\lambda} s_H^* > 0$. In addition, because λ is a positive multiplier in $C_{ij}^N(K,e)$ and $C_{ij}^I(K,e,y,s)$, we deduce that

$$\frac{\partial}{\partial\lambda}\left\{ E_{\delta,Z}[\pi_H^I(K,s_H)] - E_{\delta,Z}[\pi_H^N(K)]\right\} > 0,$$

and thus

$$\frac{d}{d\lambda} y_H^* = \frac{\partial}{\partial\lambda}\left\{ E_{\delta,Z}[\pi_H^I(K,s_H)] - E_{\delta,Z}[\pi_H^N(K)]\right\} + \frac{\partial}{\partial s_H}\left\{ E_{\delta,Z}[\pi_H^I(K,s_H)] - E_{\delta,Z}[\pi_H^N(K)]\right\}\frac{d}{d\lambda} s_H^* > 0.$$

Similarly, the first-order derivative of $r_L(s_L)$ with respect to λ is

$$\frac{d}{d\lambda} r_L(s_L) = K + \gamma\ln\left[\tfrac{[(T-\tau)(1-s'_L)+s'_L w_L]p}{[(T-\tau)(1-s'_L)+s'_L w_L]p-c}\right] - \gamma E_\delta\left[\xi(K,\mu_L,\delta)\right] > 0.$$

Therefore, we have $\frac{d}{d\lambda} s_L^* > 0$. Similar to $\frac{d}{d\lambda} y_H^*$, we can get

$$\frac{\partial}{\partial\lambda}\left\{ E_{\delta,Z}[\pi_L^I(K,s_L)] - E_{\delta,Z}[\pi_L^I(K,s_H)]\right\} > 0,$$

and thus

$$\begin{aligned}\frac{d}{d\lambda} y_L^* =& \frac{d}{d\lambda} y_H^* + \frac{\partial}{\partial\lambda}\left\{ E_{\delta,Z}[\pi_L^I(K,s_L)] - E_{\delta,Z}[\pi_L^I(K,s_H)]\right\} \\ &+ \frac{\partial}{\partial s_L}\left\{ E_{\delta,Z}[\pi_L^I(K,s_L)] - E_{\delta,Z}[\pi_L^I(K,s_H)]\right\}\frac{d}{d\lambda} s_L^* > 0.\end{aligned}$$

Consequently, y_H^*, y_L^*, s_H^*, and s_L^* are all increasing in λ. Q.E.D.

EC.3.4 The Effects of the Other Parameters on the Optimal Contracts

In this section, we provide numerical results to show how the optimal contracts are affected by the parameters that are not discussed in Section 6. First, we are interested in the impacts of the parameters β, δ_H and δ_L, which are related to forecast updating. Due to the assumption that $E[\delta] = \beta\delta_H + (1-\beta)\delta_L = 0$, each of these parameters cannot unilaterally change. Thus, we suppose that there is a fixed range of forecast updates, $G = \delta_H - \delta_L$. If any of β, δ_H and δ_L changes, then the others would also change according to the fixed range. Figures EC.4-EC.6 show the impacts of these parameters. In these figures, the premiums and the coverage percentage in the optimal contracts appear to be convex in β, δ_H and δ_L, but neither increasing nor decreasing.

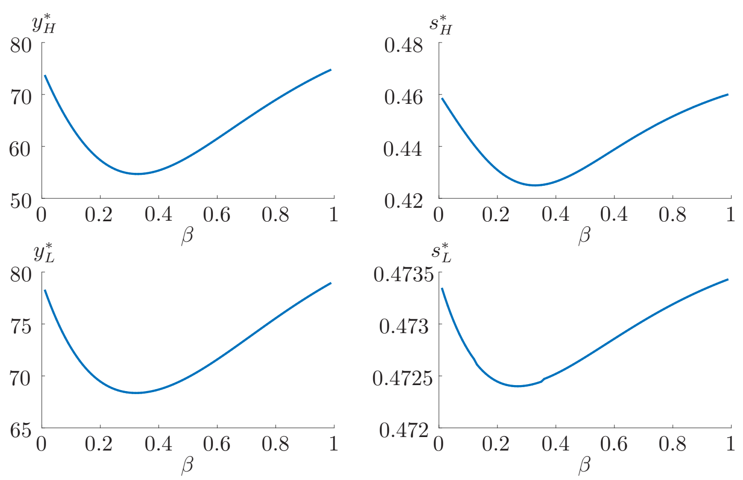


Figure EC.4 The Impact of β on the Optimal Contracts

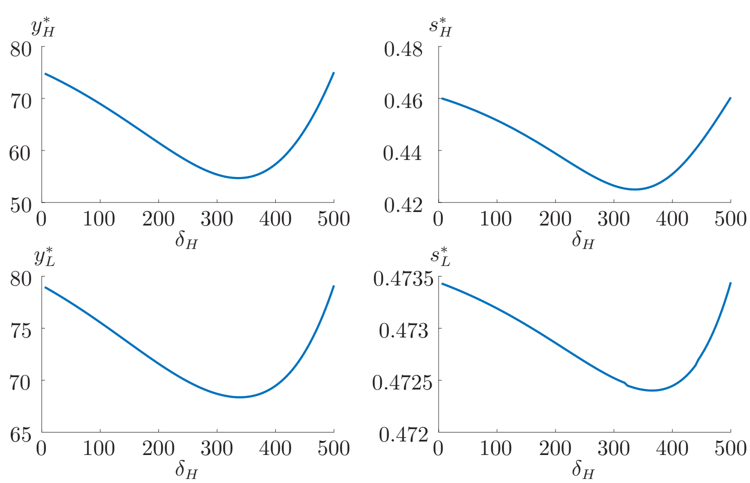


Figure EC.5 The Impact of δ_H on the Optimal Contracts

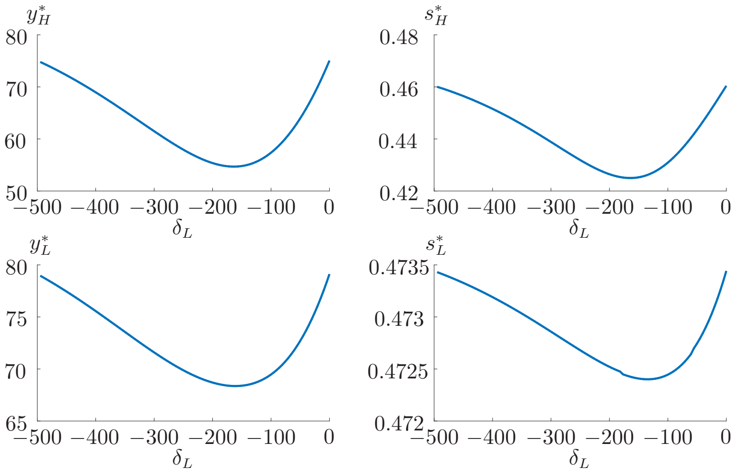
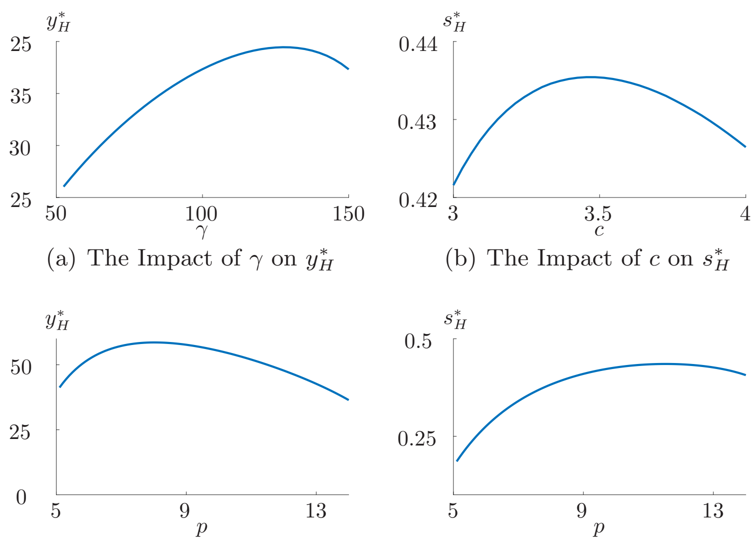


Figure EC.6 The Impact of δ_L on the Optimal Contracts

Similarly, with regard to the remaining parameters γ, p and c, we illustrate that their impacts on the optimal contracts are not necessarily monotone. For example, Figure 7(a) shows that y_H^* may be increasing or decreasing in γ; Figure 7(b) shows that s_H^* does not monotonically change in c; Figure 7(c) and 7(d) shows that the change in p may have different impacts on the optimal contract (y_H^*, s_H^*), depending on the value of p.



(a) The Impact of γ on y_H^* (b) The Impact of c on s_H^*

(c) The Impact of p on y_H^* (d) The Impact of p on s_H^*

Figure EC.7 The Impact of the Parameters on the Optimal Contracts