

A Appendix

A.1 Proofs of Propositions

Proof of Lemma 1

Proof. $p_{1C}^{TS*} = (6t_C(6t_C - 3\alpha_S - 3\alpha_T + \gamma_S - \gamma_T) + (\alpha_S + \alpha_T)(\alpha_S + \alpha_T - \gamma_S)(\alpha_S + \alpha_T - 2(\gamma_S + \gamma_T)) - t_C(\alpha_S^2 + 3\alpha_T^2 - 4\alpha_T(\gamma_S + \gamma_T) + \alpha_S(4\alpha_T - 6(\gamma_S + \gamma_T)) + 2(\gamma_S + \gamma_T)^2)) / (36t_C - (2(\alpha_S + \alpha_T) - \gamma_S - \gamma_T)(\alpha_S + \alpha_T - 2(\gamma_S + \gamma_T)))$,

$$p_{2C}^{TS*} = (6t_C(6t_C - 3\alpha_S - 3\alpha_T - \gamma_S + \gamma_T) + (\alpha_S + \alpha_T)(\alpha_S + \alpha_T - \gamma_T)(\alpha_S + \alpha_T - 2(\gamma_S + \gamma_T)) - t_C(3\alpha_S^2 + \alpha_T^2 + 4\alpha_S(\alpha_T - \gamma_S - \gamma_T) - 6\alpha_T(\gamma_S + \gamma_T) + 2(\gamma_S + \gamma_T)^2)) / (36t_C - (2(\alpha_S + \alpha_T) - \gamma_S - \gamma_T)(\alpha_S + \alpha_T - 2(\gamma_S + \gamma_T)))$$
,
$$p_{1A}^{TS*} = (36t_C - (\alpha_S - \gamma_S - \gamma_T)(2(\alpha_S + \alpha_T) - \gamma_S - \gamma_T)(\gamma_S + \gamma_T) - (2\alpha_S^2 + 4\alpha_S\alpha_T + 2\alpha_T^2 - 6\alpha_S\gamma_S - 6\alpha_T\gamma_S + \gamma_S^2 - 4\gamma_T(\alpha_S + \alpha_T - \gamma_S)) - (3\gamma_T^2 - 6t_C(\alpha_S - \alpha_T - 3\gamma_S - 3\gamma_T))) / (36t_C - (2(\alpha_S + \alpha_T) - \gamma_S - \gamma_T)(\alpha_S + \alpha_T - 2(\gamma_S + \gamma_T)))$$
,
$$p_{2A}^{TS*} = (36t_C - (\alpha_T - \gamma_S - \gamma_T)(2(\alpha_S + \alpha_T) - \gamma_S - \gamma_T)(\gamma_S + \gamma_T) - (2\alpha_S^2 + 4\alpha_S\alpha_T + 2\alpha_T^2 - 4\alpha_S\gamma_S - 4\alpha_T\gamma_S + 3\gamma_S^2 - 2\gamma_T(3\alpha_S + 3\alpha_T - 2\gamma_S)) - (\gamma_T^2 - 6t_C(\alpha_S - \alpha_T + 3\gamma_S + 3\gamma_T))) / (36t_C - (2(\alpha_S + \alpha_T) - \gamma_S - \gamma_T)(\alpha_S + \alpha_T - 2(\gamma_S + \gamma_T)))$$
.

Below we show that when $t_C > t_C = \max\{t_{C1}, t_{C2}, t_{C3}, t_{C4}\}$ (which is a sufficient condition), $\frac{\partial p_{1C}^{TS*}}{\partial \alpha_S} < 0$, $\frac{\partial p_{2C}^{TS*}}{\partial \alpha_S} < 0$, $\frac{\partial p_{1A}^{TS*}}{\partial \alpha_S} < 0$ and $\frac{\partial p_{2A}^{TS*}}{\partial \alpha_S} > 0$.

First, $\frac{\partial p_{1C}^{TS*}}{\partial \alpha_S} = \{(36t_C - (2(\alpha_S + \alpha_T) - \gamma_S - \gamma_T)(\alpha_S + \alpha_T - 2(\gamma_S + \gamma_T)))(\alpha_S + \alpha_T)(\alpha_S + \alpha_T - \gamma_S) - 2t_C(9 + \alpha_S + 2\alpha_T - 3\gamma_S - 3\gamma_T) + ((\alpha_S + \alpha_T) - 2(\gamma_S + \gamma_T)(\alpha_S + \alpha_T - \gamma_S)(\alpha_S + \alpha_T - 2\gamma_S - 2\gamma - T)) - ((5(\gamma_S + \gamma_T) - 4(\alpha_S + \alpha_T))(36t_C^2 + (\alpha_S + \alpha_T)(\alpha_S + \alpha_T - S)(\alpha_S + \alpha_T - 2\gamma_S - 2\gamma_T) - t_C(\alpha_S^2 + 3\alpha_T^2 + 4\alpha_S\alpha_T + 6(3\alpha_T + \gamma_S - \gamma_T) + 6\alpha_S(3 - \gamma_S - \gamma_T) - 4\alpha_T(\gamma_S + \gamma_T) + 2(\gamma_S + \gamma_T)^2))\} / (36t_C - (2(\alpha_S + \alpha_T) - \gamma_S - \gamma_T)(\alpha_S + \alpha_T - 2(\gamma_S + \gamma_T)))^2$.

The sign of the above expression is the sign of its numerator, and it will be negative when $t_C > t_{C1}$ since the numerator is a quadratic, concave function of t_C , where t_{C1} is the larger root and is given below:

$$t_{C1} = \{[(\alpha_S^2(4\alpha_T - 7\gamma_S - 7\gamma_T - 72) + 2\alpha_S(4\alpha_T^2 - 3\alpha_T(\gamma_S + \gamma_T)24) + 2(\gamma_S + \gamma_T)^2 + 96\gamma_S + 84\gamma_T) + \alpha_T^2(\gamma_S + \gamma_T - 72) + 4\alpha_T^3 - 2\alpha_T(2(5\gamma_S - 42)\gamma_T + \gamma_S(5\gamma_S - 96) + 5\gamma_T^2) + 2(\gamma_S + \gamma_T)((2\gamma_S - 33)\gamma_T + (\gamma_S - 39)\gamma_S + \gamma_T^2)]^2 + 144(2\alpha_S + \gamma_S + \gamma_T - 18)(2\alpha_S^2 + 4\alpha_S\alpha_T - 2\alpha_S(\gamma_S + \gamma_T) + 2\alpha_T^2 - 2\alpha_T(\gamma_S + \gamma_T) + \gamma_S(\gamma_S + \gamma_T))(\alpha_S + \alpha_T - 2(\gamma_S + \gamma_T))^2]^{1/2} + [2\alpha_T(2\alpha_S^2 - 3\alpha_S(24 - \gamma_S - \gamma_T) - 5(\gamma_S + \gamma_T)^2 - 12(8\gamma_S + 7\gamma_T)) - 7\alpha_S^2\gamma_S - 7\alpha_S^2\gamma_T - 72\alpha_S^2 + \alpha_T^2(8\alpha_S + \gamma_S + \gamma_T - 72) + 4\alpha_S\gamma_S^2 + 8\alpha_S\gamma_S\gamma_T + 192\alpha_S\gamma_S + 4\alpha_S\gamma_T^2 + 168\alpha_S\gamma_T + 4\alpha_T^3 + 6\gamma_S^2\gamma_T + 2\gamma_S^3 - 78\gamma_S^2 + 6\gamma_S\gamma_T^2 - 144\gamma_S\gamma_T + 2\gamma_T^3 - 66\gamma_T^2]\} / (72(18 - 2\alpha_S - \gamma_S - \gamma_T))$$
.

Second, $\frac{\partial p_{2C}^{TS*}}{\partial \alpha_S} = \{(36t_C - (2(\alpha_S + \alpha_T) - \gamma_S - \gamma_T)(\alpha_S + \alpha_T - 2(\gamma_S + \gamma_T)))(\alpha_S + \alpha_T)(3\alpha_S + \alpha_T - 4\gamma_S) - 2t_C(9 + 3\alpha_S + 2\alpha_T - 2\gamma_S - 2\gamma_T) - 2(3(\alpha_S + \alpha_T) - \gamma_S) - 2\gamma_T^2\} + \{(4(\alpha_S + \alpha_T) - 5(\gamma_S + \gamma_T))(36t_C^2 + (\alpha_S + \alpha_T)(\alpha_S + \alpha_T - T)(\alpha_S + \alpha_T - 2\gamma_S - 2\gamma_T) - t_C(3\alpha_S^2 + \alpha_T^2 + 2\alpha_S(9 + 2\alpha_T - 2\gamma_S - 2\gamma_T) + 6(\gamma_S - \gamma_T) + 6\alpha_T(3 - \gamma_S - \gamma_T) + 2(\gamma_S + \gamma_T)^2))\} / \{36t_C - (2(\alpha_S + \alpha_T) - \gamma_S - \gamma_T)(\alpha_S + \alpha_T - 2(\gamma_S + \gamma_T))\}^2$.

The sign of the above expression is the sign of its numerator, and similar to the proof above, it will be negative when $t_C > t_{C2}$, where t_{C2} is given by

$$t_{C2} = \{[(\alpha_S^2(4\alpha_T - 7\gamma_S - 7\gamma_T + 72) + 2\alpha_S(4\alpha_T^2 - 3\alpha_T(\gamma_S + \gamma_T) - 24) + 2(\gamma_S + \gamma_T)^2 - 12(7\gamma_S + 8\gamma_T)) + \alpha_T^2\gamma_S + \gamma_T + 72) + 4\alpha_T^3 - 2\alpha_T(2(5\gamma_S + 48)\gamma_T + \gamma_S(5\gamma_S + 84) + 5\gamma_T^2) + 2(\gamma_S + \gamma_T)((2\gamma_S + 39)\gamma_T + \gamma_S(\gamma_S +$$

$$33) + \gamma_T^2))^2 - 144(2\alpha_S + \gamma_S + \gamma_T + 18)(\gamma_T(\gamma_S - 2(\alpha_S + \alpha_T)) + 2(\alpha_S + \alpha_T)(\alpha_S + \alpha_T - \gamma_S) + \gamma_T^2)(\alpha_S + \alpha_T - 2(\gamma_S + \gamma_T))^2]^{1/2} + [3\alpha_S(\gamma_S + \gamma_T - 24) - 2\alpha_T(-2\alpha_S^2 + 5(\gamma_S + \gamma_T)^2 + 84\gamma_S + 96\gamma_T) - 7\alpha_S^2\gamma_S - 7\alpha_S^2\gamma_T + 72\alpha_S^2 + \alpha_T^2(8\alpha_S + \gamma_S + \gamma_T + 72) + 4\alpha_S\gamma_S^2 + 8\alpha_S\gamma_S\gamma_T - 168\alpha_S\gamma_S + 4\alpha_S\gamma_T^2 - 192\alpha_S\gamma_T + 4\alpha_T^3 + 6\gamma_S^2\gamma_T + 2\gamma_S^3 + 66\gamma_S^2 + 6\gamma_S\gamma_T^2 + 144\gamma_S\gamma_T + 2\gamma_T^3 + 78\gamma_T^2] / (72(18 + 2\alpha_S + \gamma_S + \gamma_T)).$$

$$\text{Third, } \frac{\partial p_{1A}^{TS*}}{\partial \alpha_S} = \{[36t_C - (2(\alpha_S + \alpha_T) - \gamma_S - \gamma_T)(\alpha_S + \alpha_T - 2(\gamma_S + \gamma_T))][6\gamma_S - 6t_C + 4\gamma_T + 3(\gamma_S + \gamma_T)^2 - 2\alpha_T(2 + \gamma_S + \gamma_T) + 4\alpha_S(1 + \gamma_S + \gamma_T)] + (4(\alpha_S + \alpha_T) - 5(\gamma_S + \gamma_T))[-2\alpha_S^2(\gamma_S + \gamma_T + 1) + \alpha_S(-2\alpha_T(\gamma_S + \gamma_T + 2) + 3(\gamma_S + \gamma_T)^2 + 6\gamma_S + 4\gamma_T) - 2\alpha_T^2 + 2\alpha_T\gamma_S^2 + \gamma_T^2(2\alpha_T - 3(\gamma_S + 1)) + \gamma_T(4\alpha_T(\gamma_S + 1) - \gamma_S(3\gamma_S + 4)) + 6\alpha_T\gamma_S - \gamma_S^3 - \gamma_S^2 - \gamma_T^3 + 6t_C(-\alpha_S + \alpha_T + 3(\gamma_S + \gamma_T + 2))]\} / \{36t_C - (2(\alpha_S + \alpha_T) - \gamma_S - \gamma_T)(\alpha_S + \alpha_T - 2(\gamma_S + \gamma_T))\}^2.$$

The sign of the above expression is the sign of its numerator, and it will be negative when $t_C > t_{C3}$, where t_{C3} is given by

$$t_{C3} = \{[4\alpha_S^2(-2\alpha_T^2 + 10\alpha_T(\gamma_S + \gamma_T) + 7(\gamma_S + \gamma_T)^2 + 6(\gamma_S - \gamma_T)) - 16\alpha_S^3(\alpha_T - 3(\gamma_S + \gamma_T)) + 4\alpha_S^4 + 8\alpha_S(-4\alpha_T^2(\gamma_S + \gamma_T) + 6\alpha_T^3 - \alpha_T(2(\gamma_S + 9)\gamma_T + (\gamma_S - 18)\gamma_S + \gamma_T^2) - 3(\gamma_S + \gamma_T)(2(\gamma_S - 1)\gamma_T + \gamma_S(\gamma_S + 2) + \gamma_T^2)) - 8\alpha_T^2((8\gamma_S + 15)\gamma_T + \gamma_S(4\gamma_S - 15) + 4\gamma_T^2) - 24\alpha_T^3(\gamma_S + \gamma_T) + 36\alpha_T^4 + 4\alpha_T(\gamma_S + \gamma_T)(2(5\gamma_S + 3)\gamma_T + \gamma_S(5\gamma_S - 6) + 5\gamma_T^2) + \gamma_S^2((\gamma_S - 108)\gamma_S + 36) + 4(\gamma_S + 27)\gamma_T^3 + 6(\gamma_S(\gamma_S + 18) + 6)\gamma_T^2 + 4\gamma_S((\gamma_S - 27)\gamma_S - 18)\gamma_T + \gamma_T^4]^{1/2} - 2\alpha_S^2 + 4\alpha_S(\alpha_T - 3(\gamma_S + \gamma_T)) + 6\alpha_T^2 - 10\alpha_T(\gamma_S + \gamma_T) + 5(\gamma_S + \gamma_T)^2 + 6\gamma_S - 6\gamma_T\} / 72.$$

$$\text{Finally, } \frac{\partial p_{2A}^{TS*}}{\partial \alpha_S} = \{(36t_C - (2(\alpha_S + \alpha_T) - \gamma_S - \gamma_T)(\alpha_S + \alpha_T - 2(\gamma_S + \gamma_T)))2(3t_C - 2\alpha_S + 2\gamma_S + 3\gamma_T + (\gamma_S + \gamma_T)^2 - \alpha_T(2 + \gamma_S + \gamma_T)) - [(5(\gamma_S + \gamma_T) - 4(\alpha_S + \alpha_T))(36t_C - (\alpha_T - \gamma_S - \gamma_T)(2(\alpha_S + \alpha_T) - \gamma_S - \gamma_T)(\gamma_S + \gamma_T) - (2\alpha_S^2 + 4\alpha_S\alpha_T + 2\alpha_T^2 - 4\alpha_S\gamma_S - 4\alpha_T\gamma_S + 3\gamma_S^2 - 2\gamma_T(3\alpha_S + 3\alpha_T - 2\gamma_S)) - (\gamma_T^2 - 6t_C(\alpha_S - \alpha_T + 3\gamma_S + 3\gamma_T)))]\} / \{36t_C - (2(\alpha_S + \alpha_T) - \gamma_S - \gamma_T)(\alpha_S + \alpha_T - 2(\gamma_S + \gamma_T))\}^2.$$

The sign of the above expression is the sign of its numerator, and it will be positive when $t_C > t_{C4}$, where t_{C4} is given by

$$t_{C4} = \{[(4\alpha_S^2(-2\alpha_T^2 + 10\alpha_T(\gamma_S + \gamma_T) + 7(\gamma_S + \gamma_T)^2 + 6(\gamma_S - \gamma_T)) - 16\alpha_S^3(\alpha_T - 3(\gamma_S + \gamma_T)) + 4\alpha_S^4 + 8\alpha_S(-4\alpha_T^2\gamma_S + \gamma_T) + 6\alpha_T^3 - \alpha_T(2(\gamma_S + 9)\gamma_T + (\gamma_S - 18)\gamma_S + \gamma_T^2) - 3(\gamma_S + \gamma_T)(2(\gamma_S - 1)\gamma_T + \gamma_S(\gamma_S + 2) + \gamma_T^2)) - 8\alpha_T^2((8\gamma_S + 15)\gamma_T + \gamma_S(4\gamma_S - 15) + 4\gamma_T^2) - 24\alpha_T^3(\gamma_S + \gamma_T) + 36\alpha_T^4 + 4\alpha_T(\gamma_S + \gamma_T)(2(5\gamma_S + 3)\gamma_T + \gamma_S(5\gamma_S - 6) + 5\gamma_T^2) + \gamma_S^2((\gamma_S - 108)\gamma_S + 36) + 4(\gamma_S + 27)\gamma_T^3 + 6(\gamma_S(\gamma_S + 18) + 6)\gamma_T^2 + 4\gamma_S((\gamma_S - 27)\gamma_S - 18)\gamma_T + \gamma_T^4]^{1/2} - 2\alpha_S^2 + 4\alpha_S(\alpha_T - 3(\gamma_S + \gamma_T)) + 6\alpha_T^2 - 10\alpha_T(\gamma_S + \gamma_T) + 5(\gamma_S + \gamma_T)^2 + 6\gamma_S - 6\gamma_T\} / 72.$$

□

Proof of Proposition 1

$$\text{Proof. } \Pi_1^{TT} - \Pi_1^{ST} = \frac{\gamma_T - \alpha_T}{2} + \frac{(2(\alpha_T + \alpha_S - \gamma_T - \alpha_S)(\alpha_T - \gamma_T - \gamma_S)(\alpha_S + \alpha_T - \gamma_S - \gamma_T) + t_C(\alpha_S - \alpha_T)(2\alpha_S + \gamma_S + \gamma_T))}{2((2(\alpha_S + \alpha_T) - \gamma_S - \gamma_T)(\alpha_S + \alpha_T - 2(\gamma_S - \gamma_T)) - 36t_C)} + \frac{((\alpha_S + \alpha_T + 2\gamma_S)(\gamma_S - \gamma_T) - 6t_C(\alpha_S + 5\alpha_T - \gamma_S - 5\gamma_T))}{2((2(\alpha_S + \alpha_T) - \gamma_S - \gamma_T)(\alpha_S + \alpha_T - 2(\gamma_S - \gamma_T)) - 36t_C)} > 0 \text{ if and only if } \alpha^* > \alpha_S > \alpha_1 \text{ where}$$

$$\alpha^* = \frac{6t_C + 2t_C\alpha_T - 2\gamma_S^2 + \gamma_T - t_C\gamma_T - \alpha_T\gamma_T + \gamma_T^2 - \gamma_S(1 + t_C - \alpha_T + \gamma_T)}{4(t_C - \gamma_S)} + \frac{\sqrt{((\alpha_T - 2\gamma_S - 1)\gamma_S + t_C(6 + 2\alpha_T - \gamma_S - \gamma_T) + \gamma_T - (\alpha_T + \gamma_S)\gamma_T + \gamma_T^2)^2 + 8(t_C - \gamma_S)(2\gamma_S\gamma_T - \gamma_S(\alpha_T + \alpha_T^2 + 2\gamma_S) + \alpha_T(1 - \alpha_T + \gamma_S)\gamma_T + \alpha_T\gamma_T^2 - t_C(6(\gamma_S - \gamma_T) - \alpha_T(6 - \gamma_S - \gamma_T)))}}{4(t_C - \gamma_S)},$$

$$\alpha_1 = \frac{6t_C + 2t_C\alpha_T - 2\gamma_S^2 + \gamma_T - t_C\gamma_T - \alpha_T\gamma_T + \gamma_T^2 - \gamma_S(1 + t_C - \alpha_T + \gamma_T)}{4(t_C - \gamma_S)} - \frac{\sqrt{((\alpha_T - 2\gamma_S - 1)\gamma_S + t_C(6 + 2\alpha_T - \gamma_S - \gamma_T) + \gamma_T - (\alpha_T + \gamma_S)\gamma_T + \gamma_T^2)^2 + 8(t_C - \gamma_S)(2\gamma_S\gamma_T - \gamma_S(\alpha_T + \alpha_T^2 + 2\gamma_S) + \alpha_T(1 - \alpha_T + \gamma_S)\gamma_T + \alpha_T\gamma_T^2 - t_C(6(\gamma_S - \gamma_T) - \alpha_T(6 - \gamma_S - \gamma_T)))}}{4(t_C - \gamma_S)}.$$

Similarly, $\Pi_1^{SS} - \Pi_1^{TS} = \frac{\gamma_S - \alpha_S + t_C}{2} - \frac{(2(\alpha_S - \gamma_S - \gamma_T)((\alpha_S + \alpha_T - \gamma_S)(\alpha_S + \alpha_T - \gamma_S - \gamma_T) + t_C(\gamma_S + \gamma_T - \alpha_S - 3\alpha_T))}{2(-2(\alpha_S + \alpha_T) - \gamma_S - \gamma_T)(\alpha_S + \alpha_T - 2(\gamma_S - \gamma_T)) + 36t_C}$
 $-\frac{(36t_C^2 - t_C(5\alpha_S + \alpha_T - 5\gamma_S - \gamma_T) - (\gamma_S - \gamma_T)(\alpha_S + \alpha_T + 2\gamma_T))}{2(-2(\alpha_S + \alpha_T) - \gamma_S - \gamma_T)(\alpha_S + \alpha_T - 2(\gamma_S - \gamma_T)) + 36t_C} > 0$ if and only if $\alpha_2 > \alpha_S > \alpha^{**}$ where

$$\alpha_2 = \frac{\gamma_S - \alpha_T \gamma_S + \gamma_S^2 - \gamma_T + \alpha_T \gamma_T + \gamma_S \gamma_T - t_C(6 - 2\alpha_T - \gamma_S - \gamma_T)}{2(\gamma_S + \gamma_T)} + \frac{\sqrt{(\gamma_S(1 - \alpha_T + \gamma_S) - (1 - \alpha_T - \gamma_S)\gamma_T - t_C(6 - 2\alpha_T - \gamma_S - \gamma_T))^2 + 4(\gamma_S + \gamma_T)(\alpha_T \gamma_S(1 + \gamma_S) + (2\alpha_T^2 + 2\gamma_S - \alpha_T(1 + \gamma_S))\gamma_T - 2(1 + \alpha_T)\gamma_T^2 + t_C(6(\gamma_S - \gamma_T) - \alpha_T(6 - 2\alpha_T - \gamma_S - \gamma_T)))}}{4(\gamma_S + \gamma_T)},$$

$$\alpha^{**} = \frac{\gamma_S - \alpha_T \gamma_S + \gamma_S^2 - \gamma_T + \alpha_T \gamma_T + \gamma_S \gamma_T - t_C(6 - 2\alpha_T - \gamma_S - \gamma_T)}{2(\gamma_S + \gamma_T)} - \frac{\sqrt{(\gamma_S(1 - \alpha_T + \gamma_S) - (1 - \alpha_T - \gamma_S)\gamma_T - t_C(6 - 2\alpha_T - \gamma_S - \gamma_T))^2 + 4(\gamma_S + \gamma_T)(\alpha_T \gamma_S(1 + \gamma_S) + (2\alpha_T^2 + 2\gamma_S - \alpha_T(1 + \gamma_S))\gamma_T - 2(1 + \alpha_T)\gamma_T^2 + t_C(6(\gamma_S - \gamma_T) - \alpha_T(6 - 2\alpha_T - \gamma_S - \gamma_T)))}}{4(\gamma_S + \gamma_T)}.$$

It can also be easily shown that the signs (which are equal to the numerators) of both profit differences are quadratic and concave functions of α_S . The upper bound on α_S in Equation (7) in Section 3.4, $\bar{\alpha}_S$, is given by $\bar{\alpha}_S = \min\{[6t_C + 2t_C\alpha_T - 2\gamma_S^2 + \gamma_T - t_C\gamma_T - \alpha_T\gamma_T + \gamma_T^2 - \gamma_S(1 + t_C - \alpha_T + \gamma_T) + \{((\alpha_T - 2\gamma_S - 1)\gamma_S + t_C(6 + 2\alpha_T - \gamma_S - \gamma_T) + \gamma_T - (\alpha_T + \gamma_S)\gamma_T + \gamma_T^2)^2 + 8(t_C - \gamma_S)(2\gamma_S\gamma_T - \gamma_S(\alpha_T + \alpha_T^2 + 2\gamma_S) + \alpha_T(1 - \alpha_T + \gamma_S)\gamma_T + \alpha_T\gamma_T^2 - t_C(6(\gamma_S - \gamma_T) - \alpha_T(6 - \gamma_S - \gamma_T)))\}^{1/2}]/(4(t_C - \gamma_S)), (\gamma_T - \gamma_S(\gamma_S + \gamma_T + 1) - t_C(\gamma_S + \gamma_T - 6))/(\gamma_T + 2t_C - \gamma_S), (18 - \gamma_S - \gamma_T)/2\}$.

From Equation (7), it can be derived that $\alpha^* > \alpha_S > 0 > \alpha^{**}$. Thus, for the equilibrium analysis, we only need to investigate the following two regions in terms of α_S : (i) $\alpha_1 > \alpha_2 > 0$ and (ii) $\alpha_2 > \alpha_1 > 0$.

Given the above profit differences, the equilibrium conditions in the first region ($\alpha_1 > \alpha_2 > 0$) are as follows:

When $\alpha_S > \alpha_1$, TT is the unique equilibrium. When $\alpha_1 > \alpha_S > \alpha_2$, TS/ST is the unique equilibrium. When $\alpha_2 > \alpha_S > 0$, SS is the unique equilibrium. This completes the proof of Proposition 1.

Given the above profit differences, the equilibrium conditions in the second region ($\alpha_2 > \alpha_1 > 0$) are as follows:

When $\alpha_S > \alpha_2$, TT is the unique equilibrium. When $\alpha_2 > \alpha_S > \alpha_1$, TT and SS are both equilibria. When $\alpha_1 > \alpha_S > 0$, SS is the unique equilibrium. This completes the proof of Corollary 1.

Also, $\Pi_1^{SS} - \Pi_1^{TS} > 0$ if $\gamma_S > \gamma_A$ or $\gamma_S < \gamma_B$ and $\Pi_1^{TT} - \Pi_1^{ST} > 0$ if $\gamma_C > \gamma_S > \gamma_D$ where,

$$\gamma_A = \frac{\alpha_S^2 + \alpha_T \gamma_T - \alpha_T - 2\gamma_T - \alpha_S(-\alpha_T + \gamma_T + t_C + 1) + \alpha_T t_C - 6t_C}{2(\alpha_S + \alpha_T)} + \frac{\sqrt{4(\alpha_S + \alpha_T)(\gamma_T(\alpha_S + \alpha_T)(\alpha_S - 2\alpha_T + 1) + 2(\alpha_T + 1)\gamma_T^2 + \gamma_T t_C(-\alpha_S + \alpha_T + 6) - 2(\alpha_T - 3)t_C(\alpha_S - \alpha_T)) + (\gamma_T(\alpha_S - \alpha_T + 2) - (\alpha_S - 1)(\alpha_S + \alpha_T) + t_C(\alpha_S - \alpha_T + 6))^2}}{2(\alpha_S + \alpha_T)},$$

$$\gamma_B = \frac{\alpha_S^2 + \alpha_T \gamma_T - \alpha_T - 2\gamma_T - \alpha_S(-\alpha_T + \gamma_T + t_C + 1) + \alpha_T t_C - 6t_C}{2(\alpha_S + \alpha_T)} - \frac{\sqrt{4(\alpha_S + \alpha_T)(\gamma_T(\alpha_S + \alpha_T)(\alpha_S - 2\alpha_T + 1) + 2(\alpha_T + 1)\gamma_T^2 + \gamma_T t_C(-\alpha_S + \alpha_T + 6) - 2(\alpha_T - 3)t_C(\alpha_S - \alpha_T)) + (\gamma_T(\alpha_S - \alpha_T + 2) - (\alpha_S - 1)(\alpha_S + \alpha_T) + t_C(\alpha_S - \alpha_T + 6))^2}}{2(\alpha_S + \alpha_T)},$$

$$\gamma_C = \frac{2\alpha_S^2 - \alpha_T^2 + \alpha_T \gamma_T - \alpha_T + 2\gamma_T - \alpha_S(-\alpha_T + \gamma_T + t_C + 1) + \alpha_T t_C - 6t_C}{4(1 + \alpha_S)} + \frac{\sqrt{(-2\alpha_S^2 + \gamma_T(\alpha_S - \alpha_T - 2) - \alpha_S \alpha_T + \alpha_S + \alpha_T^2 + \alpha_T + t_C(\alpha_S - \alpha_T + 6))^2 + 8(\alpha_S + 1)(\gamma_T(\alpha_S + \alpha_T)(-\alpha_T + \gamma_T + 1) + \gamma_T t_C(-\alpha_S + \alpha_T + 6) - 2(\alpha_S - 3)t_C(\alpha_S - \alpha_T))}}{4(1 + \alpha_S)},$$

$$\gamma_D = \frac{2\alpha_S^2 - \alpha_T^2 + \alpha_T \gamma_T - \alpha_T + 2\gamma_T - \alpha_S(-\alpha_T + \gamma_T + t_C + 1) + \alpha_T t_C - 6t_C}{4(1 + \alpha_S)} - \frac{\sqrt{(-2\alpha_S^2 + \gamma_T(\alpha_S - \alpha_T - 2) - \alpha_S \alpha_T + \alpha_S + \alpha_T^2 + \alpha_T + t_C(\alpha_S - \alpha_T + 6))^2 + 8(\alpha_S + 1)(\gamma_T(\alpha_S + \alpha_T)(-\alpha_T + \gamma_T + 1) + \gamma_T t_C(-\alpha_S + \alpha_T + 6) - 2(\alpha_S - 3)t_C(\alpha_S - \alpha_T))}}{4(1 + \alpha_S)}.$$

Finally, we characterize the equilibrium outcomes in terms of γ_S . It can be easily shown that the sign of $(\Pi_1^{SS} - \Pi_1^{TS})$ is a quadratic and convex function of γ_S , whereas the sign of $(\Pi_1^{TT} - \Pi_1^{ST})$ is a quadratic

and concave function of γ_S . Similar to the assumption on the upper bound on α_S , we impose a lower bound on $\gamma_S, \underline{\gamma}_S$, which is given by

$$\underline{\gamma}_S = \max\{(\alpha_S^2 + \alpha_T \gamma_T - \alpha_T - 2\gamma_T - \alpha_S(-\alpha_T + \gamma_T + t_C + 1) + \alpha_T t_C - 6t_C - (4(\alpha_S + \alpha_T)(\gamma_T(\alpha_S + \alpha_T)(\alpha_S - 2\alpha_T + 1) + 2(\alpha_T + 1)\gamma_T^2 + \gamma_T t_C(-\alpha_S + \alpha_T + 6) - 2(\alpha_T - 3)t_C(\alpha_S - \alpha_T)) + (\gamma_T(\alpha_S - \alpha_T + 2) - (\alpha_S - 1)(\alpha_S + \alpha_T) + t_C(\alpha_S - \alpha_T + 6))^2)/(2(\alpha_S + \alpha_T)), (2\alpha_S^2 - \alpha_T^2 + \alpha_T \gamma_T - \alpha_T + 2\gamma_T - \alpha_S(-\alpha_T + \gamma_T + t_C + 1) + \alpha_T t_C - 6t_C - ((-2\alpha_S^2 + \gamma_T(\alpha_S - \alpha_T - 2) - \alpha_S \alpha_T + \alpha_S + \alpha_T^2 + \alpha_T + t_C(\alpha_S - \alpha_T + 6))^2 + 8(\alpha_S + 1)(\gamma_T(\alpha_S + \alpha_T)(-\alpha_T + \gamma_T + 1) + \gamma_T t_C(-\alpha_S + \alpha_T + 6) - 2(\alpha_S - 3)t_C(\alpha_S - \alpha_T)))^{1/2}/(4(1 + \alpha_S))\}.$$

Given the assumption $\gamma_S > \underline{\gamma}_S$, we only need to investigate the equilibrium outcomes in the following two regions in terms of γ_S : (i) $\gamma_A > \gamma_C > 0$ and (ii) $\gamma_C > \gamma_A > 0$.

Given the above profit differences, the equilibrium conditions in the first region ($\gamma_A > \gamma_C > 0$) are as follows:

When $\gamma_S > \gamma_A$, SS is the unique equilibrium. When $\gamma_A > \gamma_S > \gamma_C$, TS/ST is the unique equilibrium. When $\gamma_C > \gamma_S > 0$, TT is the unique equilibrium.

Given the above profit differences, the equilibrium conditions in the second region ($\gamma_C > \gamma_A > 0$) are as follows:

When $\gamma_S > \gamma_C$, SS is the unique equilibrium. When $\gamma_C > \gamma_S > \gamma_A$, TT and SS are both equilibria. When $\gamma_A > \gamma_S > 0$, TT is the unique equilibrium. This completes the proof of the discussion following Corollary 1. □

Proof of Proposition 2

Proof. $\Pi_i^{TT*} = \frac{t_C + 1 - \alpha_T + \gamma_T}{2}$ and $\Pi_i^{SS*} = \frac{t_C + 1 - \alpha_S + \gamma_S}{2}$. Thus, $\Pi_i^{SS*} < \Pi_i^{TT*}$ iff $(\gamma_S - \gamma_T) < (\alpha_S - \alpha_T)$. Recall that when $\alpha_1 > \alpha_2 > \alpha_S > 0$ or $\alpha_2 > \alpha_1 > \alpha_S > 0$, SS is unique equilibrium. As a result, in this parameter range, SS is a Prisoner's Dilemma outcome when $(\gamma_S - \gamma_T) < (\alpha_S - \alpha_T)$. □

Proof of Proposition 3

Proof. Consumer surplus when both platforms offer traditional ads is $CS(TT) = 2 \int_0^{1/2} (u_0 - t_C x - p_{1C}^{TT} - \gamma_T/2) dx = u_0 + \alpha_T - 5t_C/4 - \gamma_T/2$. By contrast, consumer surplus when both platforms offer sponsored content ads is $CS(SS) = 2 \int_0^{1/2} (u_0 - t_C x - p_{1C}^{SS} - \gamma_S/2) dx = u_0 + \alpha_S - 5t_C/4 - \gamma_S/2$. Therefore, the difference in consumer surplus across the two advertising formats is $CS(SS) - CS(TT) = \alpha_S - \alpha_T - \gamma_S/2 + \gamma_T/2$. This is positive when $\gamma_S \leq \gamma_T + 2(\alpha_S - \alpha_T) = \gamma_1$.

Advertiser surplus when both platforms offer traditional ads is $AS(TT) = 2 \int_0^{1/2} (\alpha_T/2 - x - p_{1A}^{TT}) dx = \alpha_T/2 - 5/4 - \gamma_T$. By contrast, advertiser surplus when both platforms offer sponsored

content ads is $AS(SS) = 2 \int_0^{1/2} (\alpha_S/2 - x - p_{1A}^{SS}) dx = \alpha_S/2 - 5/4 - \gamma_S$. Therefore, the difference in advertisers' surplus across the two advertising formats is $AS(SS) - AS(TT) = \frac{\alpha_S - \alpha_T - 2\gamma_S}{2} + \gamma_T$. This is positive when $\gamma_S \leq \gamma_T + \frac{(\alpha_S - \alpha_T)}{2} = \gamma_2$.

Because $\alpha_S > \alpha_T$, we find that $\gamma_T + 2(\alpha_S - \alpha_T) > \gamma_T + \frac{(\alpha_S - \alpha_T)}{2}$, so that compared to advertiser surplus, it is more likely for consumer surplus to improve when platforms shift from traditional ads to sponsored content ads. □

Proof of Lemma 2

Proof. When advertisers are multi-homing, consumers pay $p_{iC}^{SS*,M} = \frac{4t_C - \alpha_S(\alpha_S - 3\gamma_S)}{4}$, and when advertisers are single-homing, consumers pay $p_{iC}^{SS*} = t_C - \alpha_S$. First, note that $p_{iC}^{SS*,M}$ is always higher than p_{iC}^{SS*} (otherwise it would violate the condition that multi-homing advertisers' demand must be less than 1). Second, we also know that consumers experience more advertising under multi-homing as compared to that under single-homing. As a result, given that consumers' utility function is $U_{iC}^S = u_0 - t_C|x_{iC}^S - l_i| - p_{iC}^S - \gamma_S x_{iA}^{eS}$, and both p_{iC}^S and x_{iA}^{eS} are higher under multi-homing, total consumer surplus is lower under multi-homing compared to that under single-homing.

Advertisers' surplus when some of the advertisers are multi-homing is $AS(SS)^M = 2 \int_0^{(\alpha_S - \gamma_S)/4} (\alpha_S/2 - x - p_{1A}^{SS,M}) dx = \frac{(\alpha_S - \gamma_S)^2}{16}$. Advertisers' surplus when all the advertisers are single-homing is $AS(SS)^S = 2 \int_0^{1/2} (\alpha_S/2 - x - p_{1A}^{SS}) dx = \alpha_S/2 - 5/4 - \gamma_S$. Note that $(AS(SS)^M - AS(SS)^S)$ is a quadratic and convex function of γ_S . Thus, $AS(SS)^M > AS(SS)^S$ when $\gamma_S > \alpha_S - 8 + 2\sqrt{11 - 2\alpha_S}$ or $\gamma_S < \alpha_S - 8 - 2\sqrt{11 - 2\alpha_S}$. When $\alpha_S > 5.5$, both of the above cutoff values of γ_S are complex numbers, implying that $AS(SS)^M > AS(SS)^S$ always holds. On the other hand, when $\alpha_S < 5.5$, both of the above cutoff values of γ_S are negative numbers again implying that $AS(SS)^M > AS(SS)^S$. Thus, advertisers are better off under multi-homing.

Platforms' profit when some of the advertisers are multi-homing are $\Pi_i^{SS*,M} = \frac{8t_C - \alpha_S^2 + 6\alpha_S\gamma_S - \gamma_S^2}{16}$. When all the advertisers are single-homing, platforms' profits are $\Pi_i^{SS*,S} = \frac{1+t_C - \alpha_S + \gamma_S}{2}$. $\Pi_i^{SS*,M} > \Pi_i^{SS*,S}$ when $4 + 3\gamma_S - 2\sqrt{2}(1 + \gamma_S) < \alpha_S < 4 + 3\gamma_S + 2\sqrt{2}(1 + \gamma_S)$. Given that each platform's demand from advertisers in equilibrium is between $\frac{1}{2}$ and 1 in the case of advertiser multi-homing, we obtain the following condition: $2 < \alpha_S - \gamma_S < 4$. Based on this condition, it can be easily shown that the inequalities $4 + 3\gamma_S - 2\sqrt{2}(1 + \gamma_S) < \alpha_S < 4 + 3\gamma_S + 2\sqrt{2}(1 + \gamma_S)$ always hold. Thus, platforms are better off under multi-homing.

□

Proof of Proposition 4

Proof. In the case of multi-homing, consumers are better off in SS compared to that in TT when $\gamma_S \leq \gamma_3 = 2\alpha_S - \sqrt{3\alpha_S^2 + \alpha_T^2 + \gamma_T^2 - 4\alpha_T\gamma_T}$. By contrast, advertisers are better off in SS compared to that in TT when $\gamma_S \leq \gamma_4 = \gamma_T + (\alpha_S - \alpha_T)$. Recall that γ_1 and γ_2 are the corresponding thresholds in the case of single-homing.

$\gamma_3 < \gamma_1 = \gamma_T + 2(\alpha_S - \alpha_T) \Leftrightarrow (3\alpha_S^2 + \alpha_T^2 + \gamma_T^2 - 4\alpha_T\gamma_T) > (2\alpha_T - \gamma_T)^2$, which is always true. Similarly, it can be easily shown that $\gamma_4 > \gamma_2 = \gamma_T + \frac{(\alpha_S - \alpha_T)}{2}$ is also always true.

□

Proof of Lemma 3

Proof. We first examine the impact of γ_S and α_S on the price for consumers. $\frac{\partial p_{iC}^{SS*}}{\partial \gamma_S} = \frac{6\alpha_S}{5} > 0$, $\frac{\partial p_{iC}^{SS*}}{\partial \alpha_S} = \frac{-2(1+6\alpha_S-9\gamma_S)}{15} < 0$ when $\frac{7+12\gamma_S}{9} < \alpha_S < \frac{4(1+\gamma_S)}{3}$ if $\gamma_S < \frac{17}{3}$.

Next, we examine the impact of γ_S and α_S on the price for advertisers. $\frac{\partial p_{iA}^{SS*}}{\partial \gamma_S} = \frac{2}{5} > 0$, $\frac{\partial p_{iA}^{SS*}}{\partial \alpha_S} = \frac{1}{5} > 0$.

□

Proof of Proposition 5

Proof. In the SS equilibrium, platform i 's profit is $\Pi_i^{SS*} = \frac{1-21\alpha_S^2-24\gamma_S^2+75t_C-4\alpha_S+2\gamma_S+96\alpha_S\gamma_S}{150}$, and thus we have $\frac{\partial \Pi_i^{SS*}}{\partial \alpha_S} = -\frac{2+21\alpha_S-48\gamma_S}{75}$.

It is easy to see that $\frac{\partial \Pi_i^{SS*}}{\partial \alpha_S} \Big|_{\alpha_S = \frac{7+12\gamma_S}{9}} = \frac{12\gamma_S-11}{45}$ and $\frac{\partial \Pi_i^{SS*}}{\partial \alpha_S} \Big|_{\alpha_S = \frac{4(1+\gamma_S)}{3}} = \frac{4\gamma_S-6}{15}$. Therefore, when $\gamma_S < \frac{11}{12}$, $\frac{\partial \Pi_i^{SS*}}{\partial \alpha_S} < 0$ holds; when $\gamma_S > \frac{3}{2}$, $\frac{\partial \Pi_i^{SS*}}{\partial \alpha_S} > 0$ holds; otherwise, there exists $\frac{7+12\gamma_S}{9} < \alpha_S^* < \frac{4(1+\gamma_S)}{3}$ such that $\frac{\partial \Pi_i^{SS*}}{\partial \alpha_S} > 0$ if $\alpha_S < \alpha_S^*$ and $\frac{\partial \Pi_i^{SS*}}{\partial \alpha_S} \leq 0$ if $\alpha_S \geq \alpha_S^*$.

A.2 Impact of Content from Sponsored Content Ads

Define the threshold $u^* = 6t_C - \alpha_T^2 + 3\alpha_T\gamma_T - \alpha_T - 2\gamma_T^2 - \gamma_T$ and assume that $u^* > |u_S|$.

$$\begin{aligned} \Pi_1^{ST} - \Pi_1^{TT} &= \frac{u_S(u_S+6t_C-\alpha_T^2+3\alpha_T\gamma_T-\alpha_T-2\gamma_T^2-\gamma_T)}{2(9t_C-2\alpha_T^2+5\alpha_T\gamma_T-2\gamma_T^2)} = \frac{u_S(u_S+u^*)}{2(9t_C-2\alpha_T^2+5\alpha_T\gamma_T-2\gamma_T^2)}, \text{ and} \\ \Pi_1^{SS} - \Pi_1^{TS} &= \frac{u_S(-u_S+6t_C-\alpha_T^2+3\alpha_T\gamma_T-\alpha_T-2\gamma_T^2-\gamma_T)}{2(9t_C-2\alpha_T^2+5\alpha_T\gamma_T-2\gamma_T^2)} = \frac{u_S(u^*-u_S)}{2(9t_C-2\alpha_T^2+5\alpha_T\gamma_T-2\gamma_T^2)}. \end{aligned}$$

$\Pi_1^{ST} - \Pi_1^{TT} > 0$ when $u_S > 0$ or when $u_S < 0$ and $|u_S| > u^*$ (which violates the assumption that $u^* > |u_S|$). By contrast, $\Pi_1^{SS} - \Pi_1^{TS} > 0$ when $0 < u_S < u^*$. As a result, SS is the unique equilibrium when $0 < u_S < u^*$. On the other hand, TT is the unique equilibrium when

$\Pi_1^{ST} - \Pi_1^{TT} < 0$ and $\Pi_1^{SS} - \Pi_1^{TS} < 0$, which translates to the following conditions: $u_S < 0$ and $|u_S| < u^*$.

A.3 Total Impact of Sponsored Content Ads

$\Pi_1^{ST} - \Pi_1^{TT} = \frac{\alpha_T - \gamma_T}{2} - \frac{(2(\alpha_T + \alpha_S - \gamma_T - u_S)(\alpha_T - \gamma_T - \gamma_S)(\alpha_S + \alpha_T - \gamma_S - \gamma_T) + t_C(\alpha_S - \alpha_T)(2\alpha_S + \gamma_S + \gamma_T))}{2((2(\alpha_S + \alpha_T) - \gamma_S - \gamma_T)(\alpha_S + \alpha_T - 2(\gamma_S - \gamma_T)) - 36t_C)}$
 $- \frac{((2u_S - \alpha_S - \alpha_T - 2\gamma_S)(2u_S - \gamma_S + \gamma_T) + 6t_C(4u_S - \alpha_S - 5\alpha_T + \gamma_S + 5\gamma_T))}{2((2(\alpha_S + \alpha_T) - \gamma_S - \gamma_T)(\alpha_S + \alpha_T - 2(\gamma_S - \gamma_T)) - 36t_C)} > 0$ when $\alpha_S > \alpha_{S1}$ or $\alpha_S < \alpha_{S2}$, where

$$\alpha_{S1} = \{ (6t_C + 2t_C\alpha_T - 2\gamma_S^2 + \gamma_T - t_C\gamma_T - \alpha_T\gamma_T + \gamma_T^2 - \gamma_S(1 + t_C - \alpha_T + \gamma_T) + 2u_S(\alpha_T - \gamma_S - \gamma_T + 1)) + [((\alpha_T - 2\gamma_S - 1)\gamma_S + t_C(6 + 2\alpha_T - \gamma_S - \gamma_T) + 2u_S(\alpha_T - \gamma_S - \gamma_T + 1) + \gamma_T - (\alpha_T + \gamma_S)\gamma_T + \gamma_T^2)^2 + 8(t_C - \gamma_S)(2\gamma_S\gamma_T - 4u_S^2\gamma_S(\alpha_T + \alpha_T^2 + 2\gamma_S) + \alpha_T(1 - \alpha_T + \gamma_S)\gamma_T + \alpha_T\gamma_T^2 + 2u_S(\alpha_T^2 + \alpha_T(-2\gamma_S - 2\gamma_T + 1) + (\gamma_S + \gamma_T)^2 + 3\gamma_S - \gamma_T) - t_C(6(\gamma_S - \gamma_T) - \alpha_T(6 - \gamma_S - \gamma_T) - 24u_S))]^{1/2} \} / (4(t_C - \gamma_S)),$$

$$\alpha_{S2} = \{ (6t_C + 2t_C\alpha_T - 2\gamma_S^2 + \gamma_T - t_C\gamma_T - \alpha_T\gamma_T + \gamma_T^2 - \gamma_S(1 + t_C - \alpha_T + \gamma_T) + 2u_S(\alpha_T - \gamma_S - \gamma_T + 1)) - [((\alpha_T - 2\gamma_S - 1)\gamma_S + t_C(6 + 2\alpha_T - \gamma_S - \gamma_T) + 2u_S(\alpha_T - \gamma_S - \gamma_T + 1) + \gamma_T - (\alpha_T + \gamma_S)\gamma_T + \gamma_T^2)^2 + 8(t_C - \gamma_S)(2\gamma_S\gamma_T - 4u_S^2\gamma_S(\alpha_T + \alpha_T^2 + 2\gamma_S) + \alpha_T(1 - \alpha_T + \gamma_S)\gamma_T + \alpha_T\gamma_T^2 + 2u_S(\alpha_T^2 + \alpha_T(-2\gamma_S - 2\gamma_T + 1) + (\gamma_S + \gamma_T)^2 + 3\gamma_S - \gamma_T) - t_C(6(\gamma_S - \gamma_T) - \alpha_T(6 - \gamma_S - \gamma_T) - 24u_S))]^{1/2} \} / (4(t_C - \gamma_S)).$$

Similarly, $\Pi_1^{SS} - \Pi_1^{TS} = \frac{\gamma_S - \alpha_S + t_C}{2} - \frac{(2(\alpha_S - \gamma_S - \gamma_T)((u_S + \alpha_S + \alpha_T - \gamma_S)(\alpha_S + \alpha_T - \gamma_S - \gamma_T) + t_C(\gamma_S + \gamma_T - \alpha_S - 3\alpha_T))}{2(-2(\alpha_S + \alpha_T) - \gamma_S - \gamma_T)(\alpha_S + \alpha_T - 2(\gamma_S - \gamma_T)) + 36t_C}$
 $- \frac{(36t_C^2 - t_C(4u_S + 5\alpha_S + \alpha_T - 5\gamma_S - \gamma_T) + (2u_S - \gamma_S + \gamma_T)(2u_S + \alpha_S + \alpha_T + 2\gamma_T))}{2(-2(\alpha_S + \alpha_T) - \gamma_S - \gamma_T)(\alpha_S + \alpha_T - 2(\gamma_S - \gamma_T)) + 36t_C} > 0$ if $\alpha_S3 > \alpha_S > \alpha_{S4}$, where

$$\alpha_{S3} = \{ (2u_S(2\gamma_S + 2\gamma_T - 1 - \alpha_T)\gamma_S - \alpha_T\gamma_S + \gamma_S^2 - \gamma_T + \alpha_T\gamma_T + \gamma_S\gamma_T - t_C(6 - 2\alpha_T - \gamma_S - \gamma_T)) + [\{ (\gamma_S(1 - \alpha_T + \gamma_S) + 2u_S(1 + \alpha_T - 2\gamma_S - 2\gamma_T) - (1 - \alpha_T - \gamma_S)\gamma_T - t_C(6 - 2\alpha_T - \gamma_S - \gamma_T))^2 + 4(2u_S + \gamma_S + \gamma_T)(4u_S^2 + \alpha_T\gamma_S(1 + \gamma_S) + (2\alpha_T^2 + 2\gamma_S - \alpha_T(1 + \gamma_S))\gamma_T - 2(1 + \alpha_T)\gamma_T^2 - 2u_S(\alpha_T(\gamma_S + \gamma_T - 1) + (\gamma_S + \gamma_T)^2 - \gamma_S + 3\gamma_T) + t_C(24u_S + 6(\gamma_S - \gamma_T) - \alpha_T(6 - 2\alpha_T - \gamma_S - \gamma_T))) \}^{1/2}] \} / (2(2u_S + \gamma_S + \gamma_T)),$$

$$\alpha_{S4} = \{ (2u_S(2\gamma_S + 2\gamma_T - 1 - \alpha_T)\gamma_S - \alpha_T\gamma_S + \gamma_S^2 - \gamma_T + \alpha_T\gamma_T + \gamma_S\gamma_T - t_C(6 - 2\alpha_T - \gamma_S - \gamma_T)) - [\{ (\gamma_S(1 - \alpha_T + \gamma_S) + 2u_S(1 + \alpha_T - 2\gamma_S - 2\gamma_T) - (1 - \alpha_T - \gamma_S)\gamma_T - t_C(6 - 2\alpha_T - \gamma_S - \gamma_T))^2 + 4(2u_S + \gamma_S + \gamma_T)(4u_S^2 + \alpha_T\gamma_S(1 + \gamma_S) + (2\alpha_T^2 + 2\gamma_S - \alpha_T(1 + \gamma_S))\gamma_T - 2(1 + \alpha_T)\gamma_T^2 - 2u_S(\alpha_T(\gamma_S + \gamma_T - 1) + (\gamma_S + \gamma_T)^2 - \gamma_S + 3\gamma_T) + t_C(24u_S + 6(\gamma_S - \gamma_T) - \alpha_T(6 - 2\alpha_T - \gamma_S - \gamma_T))) \}^{1/2}] \} / (2(2u_S + \gamma_S + \gamma_T)).$$

□

A.4 Multi-homing Analysis

When advertisers are multi-homing, the equilibrium profit differences are as follows,

$$\Pi_1^{TT} - \Pi_1^{ST} = \frac{(\alpha_S^2 + \alpha_T^2 + \gamma_S^2 + \gamma_T^2 - 12t_C - 4\alpha_S\gamma_S - 4\alpha_T\gamma_T)^2(8t_C + 6\alpha_T\gamma_T - \alpha_T^2 - \gamma_T^2) - 4((\alpha_T - \gamma_T)^2 - 2\alpha_S\gamma_S - 6t_C)^2(8t_C - (\alpha_S - \gamma_S)^2 + 4\alpha_T\gamma_T)}{16(\alpha_S^2 + \alpha_T^2 + \gamma_S^2 + \gamma_T^2 - 12t_C - 4\alpha_S\gamma_S - 4\alpha_T\gamma_T)^2}.$$

The above expression would be positive under the following sufficient conditions: $(8t_C + 6\alpha_T\gamma_T - \alpha_T^2 - \gamma_T^2) > 0$ and $(8t_C - (\alpha_S - \gamma_S)^2 + 4\alpha_T\gamma_T) < 0$. These two inequalities are satisfied when $\alpha_S > \alpha_{Sa} = \gamma_S + 2\sqrt{\alpha_T\gamma_T + 2t_C}$ and $\alpha_T < \alpha_{Ta} = 3\gamma_T + 2\sqrt{2}\sqrt{\gamma_T^2 + t_C}$.

Similarly, $\Pi_1^{TS} - \Pi_1^{SS} = \frac{4((\alpha_S - \gamma_S)^2 - 2\alpha_T\gamma_T - 6t_C)^2(8t_C - (\alpha_T - \gamma_T)^2 + 4\alpha_S\gamma_S) - (8t_C + 6\alpha_S\gamma_S - \alpha_S^2 - \gamma_S^2)(\alpha_S^2 + \alpha_T^2 + \gamma_S^2 + \gamma_T^2 - 12t_C - 4\alpha_S\gamma_S - 4\alpha_T\gamma_T)^2}{16(\alpha_S^2 + \alpha_T^2 + \gamma_S^2 + \gamma_T^2 - 12t_C - 4\alpha_S\gamma_S - 4\alpha_T\gamma_T)^2}$.

The above expression would be positive under the following sufficient conditions: $(8t_C - (\alpha_T - \gamma_T)^2 + 4\alpha_S\gamma_S) > 0$ and $(8t_C + 6\alpha_S\gamma_S - \alpha_S^2 - \gamma_S^2) < 0$. These two inequalities are satisfied when $\alpha_S > \alpha_{Sb} = 3\gamma_S + 2\sqrt{2}\sqrt{\gamma_S^2 + t_C}$ and $\alpha_T < \alpha_{Tb} = \gamma_T + 2\sqrt{\alpha_S\gamma_S + 2t_C}$.

Therefore, when $\alpha_S > \max\{\alpha_{Sa}, \alpha_{Sb}\}$ and $\alpha_T < \min\{\alpha_{Ta}, \alpha_{Tb}\}$, TT is the unique equilibrium. When $\alpha_S < \min\{\alpha_{Sa}, \alpha_{Sb}\}$ and $\alpha_T > \max\{\alpha_{Ta}, \alpha_{Tb}\}$, SS is the unique equilibrium. When $\alpha_{Sb} < \alpha_S < \alpha_{Sa}$ and $\alpha_{Ta} < \alpha_T < \alpha_{Tb}$, asymmetric equilibrium TS/ST occurs. Finally, when $\alpha_{Sa} < \alpha_S < \alpha_{Sb}$ and $\alpha_{Tb} < \alpha_T < \alpha_{Ta}$, multiple equilibria arise where both SS and TT are equilibria.

Prices and profits under each subgame are as follows,

$$p_{iC}^{TT^*,M} = \frac{4t_C - \alpha_T(\alpha_T - 3\gamma_T)}{4}, \quad (1)$$

$$p_{iA}^{TT^*,M} = \frac{(\alpha_T + \gamma_T)}{4}, \quad (2)$$

$$\Pi_i^{TT^*,M} = \frac{8t_C - \alpha_T^2 + 6\alpha_T\gamma_T - \gamma_T^2}{16}, \quad (3)$$

$$p_{iC}^{SS^*,M} = \frac{4t_C - \alpha_S(\alpha_S - 3\gamma_S)}{4}, \quad (4)$$

$$p_{iA}^{SS^*,M} = \frac{(\alpha_S + \gamma_S)}{4}, \quad (5)$$

$$\Pi_i^{SS^*,M} = \frac{8t_C - \alpha_S^2 + 6\alpha_S\gamma_S - \gamma_S^2}{16}, \quad (6)$$

$$p_{1C}^{TS^*,M} = \frac{(6t_C - (\alpha_S - \gamma_S)^2 + 2\alpha_T\gamma_T)(4t_C + 2\alpha_S\gamma_S - \alpha_T(\alpha_T - \gamma_T))}{24t_C - 2(\alpha_S^2 - 4\alpha_S\gamma_S + \alpha_T^2 - 4\alpha_T\gamma_T + \gamma_S^2 + \gamma_T^2)}, \quad (7)$$

$$p_{2C}^{TS^*,M} = \frac{(6t_C - (\alpha_T - \gamma_T)^2 + 2\alpha_S\gamma_S)(4t_C + 2\alpha_T\gamma_T - \alpha_S(\alpha_S - \gamma_S))}{24t_C - 2(\alpha_S^2 - 4\alpha_S\gamma_S + \alpha_T^2 - 4\alpha_T\gamma_T + \gamma_S^2 + \gamma_T^2)}, \quad (8)$$

$$p_{1A}^{TS^*,M} = \frac{(6t_C - (\alpha_S - \gamma_S)^2 + 2\alpha_T\gamma_T)(\alpha_T + \gamma_T)}{24t_C - 2(\alpha_S^2 - 4\alpha_S\gamma_S + \alpha_T^2 - 4\alpha_T\gamma_T + \gamma_S^2 + \gamma_T^2)}, \quad (9)$$

$$p_{2A}^{TS^*,M} = \frac{(6t_C - (\alpha_T - \gamma_T)^2 + 2\alpha_S\gamma_S)(\alpha_S + \gamma_S)}{24t_C - 2(\alpha_S^2 - 4\alpha_S\gamma_S + \alpha_T^2 - 4\alpha_T\gamma_T + \gamma_S^2 + \gamma_T^2)}, \quad (10)$$

$$\Pi_1^{TS^*,M} = \frac{((\alpha_S - \gamma_S)^2 - 2\alpha_T\gamma_T - 6t_C)^2(8t_C - (\alpha_T - \gamma_T)^2 + 4\alpha_S\gamma_S)}{4(\alpha_S^2 - 4\alpha_S\gamma_S + \alpha_T^2 - 4\alpha_T\gamma_T + \gamma_S^2 + \gamma_T^2 - 12t_C)}, \quad (11)$$

$$\Pi_2^{TS^*,M} = \frac{((\alpha_T - \gamma_T)^2 - 2\alpha_S\gamma_S - 6t_C)^2(8t_C - (\alpha_S - \gamma_S)^2 + 4\alpha_T\gamma_T)}{4(\alpha_S^2 - 4\alpha_S\gamma_S + \alpha_T^2 - 4\alpha_T\gamma_T + \gamma_S^2 + \gamma_T^2 - 12t_C)}. \quad (12)$$

Due to the analytical complexity, the analysis above uses a set of sufficient conditions to characterize the equilibrium outcomes. We can also fully characterize the equilibrium (with sufficient

and necessary conditions) using the parameter t_C . Note that the signs of $(\Pi_1^{TT} - \Pi_1^{ST})$ and $(\Pi_1^{TS} - \Pi_1^{SS})$ are both concave functions of t_C . Denote the two roots of t_{Ca} and t_{Cb} (where $t_{Ca} > t_{Cb}$) such that $\Pi_1^{TT} - \Pi_1^{ST} = 0$ when $t_C = t_{Ca}$ or $t_C = t_{Cb}$. Similarly, denote the two roots of t_{Cc} and t_{Cd} (where $t_{Cc} > t_{Cd}$) such that $\Pi_1^{TS} - \Pi_1^{SS} = 0$ when $t_C = t_{Cc}$ or $t_C = t_{Cd}$. As a result, the necessary and sufficient conditions for each equilibrium outcome is given as follows. When $t_C > \max\{t_{Ca}, t_{Cc}\}$, or $t_C < \min\{t_{Cb}, t_{Cd}\}$, SS is the unique equilibrium. When $\max\{t_{Cb}, t_{Cd}\} < t_C < \min\{t_{Ca}, t_{Cc}\}$, TT is the unique equilibrium. When $\max\{t_{Ca}, t_{Cd}\} < t_C < t_{Cc}$, or $t_{Cd} < t_C < \min\{t_{Cc}, t_{Cb}\}$, asymmetric equilibrium TS/ST arises. Finally, when $\max\{t_{Cb}, t_{Cc}\} < t_C < t_{Ca}$, or $t_{Cb} < t_C < \min\{t_{Ca}, t_{Cd}\}$, both TT and SS are equilibria.

A.5 Incompletely Covered Advertising Market Analysis

In this section, we analyze the extension in which the consumer market is fully covered but the advertising market is partially covered. First, we present the optimal strategies for two platforms when they choose the same advertising strategies.

When both platforms offer sponsored content ads, platform 1's demand from consumers and advertisers are given by (those of platform 2's are symmetrically defined):

$$\begin{aligned}
x_{1C}^{SS} &= \frac{p_{2C}^{SS} - p_{1C}^{SS} + t_C - \gamma_S x_{1A}^{eSS} + \gamma_S x_{2A}^{eSS}}{2t_C}, \\
y_{1A}^{SS} &= \frac{3(p_{1A}^{SS} - p_{2A}^{SS}) + 1 + 3\alpha_S(x_{1C}^{eSS} - x_{2C}^{eSS})}{6}, \text{ advertisers located between } (1/3, 2/3), \\
z_{1A}^{SS} &= \alpha_S x_{1C}^{eSS} - p_{1A}^{SS}, \text{ advertisers located between } (0, 1/3), \\
x_{1A}^{SS} &= y_{1A}^{SS} + z_{1A}^{SS} = \frac{3p_{2A}^{SS} - 9p_{1A}^{SS} + 1 + 9\alpha_S x_{1C}^{eSS} - 3\alpha_S x_{2C}^{eSS}}{6}.
\end{aligned}$$

With the assumption of rational expectations, $x_{iC}^{SS} = x_{iC}^{eSS}$ and $x_{iA}^{SS} = x_{iA}^{eSS}$, we obtain the following prices and demand when both platforms offer sponsored content ads after solving the

platforms' optimization problems:

$$\begin{aligned}
p_{iC}^{SS*} &= \frac{(15t_C - 2\alpha_S) - 6\alpha_S(\alpha_S - 3\gamma_S)}{15}, \\
p_{iA}^{SS*} &= \frac{1 + 3\alpha_S + 6\gamma_S}{15}, \\
x_{iC}^{SS*} &= \frac{1}{2}, \\
x_{iA}^{SS*} &= \frac{1 + 3\alpha_S - 4\gamma_S}{10}.
\end{aligned}$$

The platforms' profits are $\Pi_i^{SS*} = \frac{1-21\alpha_S^2-24\gamma_S^2+(75t_C-4\alpha_S+2\gamma_S)+96\alpha_S\gamma_S}{150}$.

When both platforms offer traditional ads, we obtain the following prices and demand after solving the platforms' optimization problems:

$$\begin{aligned}
p_{iC}^{TT*} &= \frac{(15t_C - 2\alpha_T) - 6\alpha_T(\alpha_T - 3\gamma_T)}{15}, \\
p_{iA}^{TT*} &= \frac{1 + 3\alpha_T + 6\gamma_T}{15}, \\
x_{iC}^{TT*} &= \frac{1}{2}, \\
x_{iA}^{TT*} &= \frac{1 + 3\alpha_T - 4\gamma_T}{10}.
\end{aligned}$$

The platforms' profits are $\Pi_i^{TT*} = \frac{1-21\alpha_T^2-24\gamma_T^2+(75t_C-4\alpha_T+2\gamma_T)+96\alpha_T\gamma_T}{150}$.

When two platforms offer different types of ads (e.g., platform 1 offers traditional ads and platform 2 offers sponsored content ads), we obtain the following prices after solving the platforms' optimization problem:

$$\begin{aligned}
p_{1C}^{TS*} &= [(14t_C(90t_C - 5\alpha_S - 7\alpha_T + 3\gamma_S - 3\gamma_T) - (-6\alpha_S^3 - 26\alpha_S^2\alpha_T - 26\alpha_S\alpha_T^2 - 6\alpha_T^3 + 42\alpha_S^2\gamma_S + 60\alpha_S\alpha_T\gamma_S + 26\alpha_T^2\gamma_S - 36\alpha_S\gamma_S^2 - 24\alpha_T\gamma_S^2 + 8\alpha_S^2\gamma_T + 36\alpha_S\alpha_T\gamma_T + 36\alpha_T^2\gamma_T + 20\alpha_S\gamma_S\gamma_T - 24\alpha_T\gamma_S\gamma_T + 7\alpha_T - 3\gamma_S + 3\gamma_T) + 3t_C(99\alpha_S^2 + 76\alpha_S\alpha_T + 153\alpha_T^2 - 498\alpha_S\gamma_S - 76\alpha_T\gamma_S + 102\gamma_S^2 - 102\alpha_S\gamma_T - 348\alpha_T\gamma_T + 52\gamma_S\gamma_T + 6\gamma_T^2)) + 12(3\alpha_S^3(\alpha_T - 6\gamma_S - \gamma_T) + \alpha_S^2(10\alpha_T^2 - 15\alpha_T\gamma_S + 36\gamma_S^2 - 11\alpha_T\gamma_T + 15\gamma_S\gamma_T + \gamma_T^2) - \alpha_T(3\alpha_T^2(\gamma_S + 6\gamma_T) + \gamma_S(3\gamma_S^2 + 10\gamma_S\gamma_T + 3\gamma_T^2) - 2\alpha_T(5\gamma_S^2 + 6\gamma_S\gamma_T + 9\gamma_T^2)) + \alpha_S(3\alpha_T^3 - 2\gamma_S(3\gamma_S + \gamma_T)^2 - 4\alpha_T^2(5\gamma_S + 3\gamma_T) + \alpha_T(15\gamma_S^2 + 56\gamma_S\gamma_T + 9\gamma_T^2)))]/G,
\end{aligned}$$

$$\begin{aligned}
p_{1A}^{TS*} &= (84t_C - (12\alpha_S^2 + 16\alpha_S\alpha_T + 4\alpha_T^2 - 42\alpha_S\gamma_S - 22\alpha_T\gamma_S + 9\gamma_S^2 + 18t_C(3\alpha_S - 17\alpha_T - 9\gamma_S - 19\gamma_T) - 2\alpha_S\gamma_T - 38\alpha_T\gamma_T + 2\gamma_S\gamma_T + 21\gamma_T^2) - 3((3\gamma_S + \gamma_T)^2(\gamma_S + 3\gamma_T) + 6\alpha_S^2(4\alpha_T + 3\gamma_S + 5\gamma_T) - 8\alpha_T^2(\gamma_S + 6\gamma_T) + 2\alpha_T(9\gamma_S^2 - 14\gamma_S\gamma_T - 27\gamma_T^2) + \alpha_S(8\alpha_T^2 - 42\alpha_T\gamma_S - 27\gamma_S^2 + 10\alpha_T\gamma_T - 66\gamma_S\gamma_T - 19\gamma_T^2)))/G,
\end{aligned}$$

$$\begin{aligned}
p_{2C}^{TS*} &= [14t_C(90t_C - 7\alpha_S - 5\alpha_T - 3\gamma_S + 3\gamma_T) - (-6\alpha_S^3 - 26\alpha_S^2\alpha_T - 26\alpha_S\alpha_T^2 - 6\alpha_T^3 + 36\alpha_S^2\gamma_S + 36\alpha_S\alpha_T\gamma_S + 8\alpha_T^2\gamma_S + 26\alpha_S^2\gamma_T + 60\alpha_S\alpha_T\gamma_T + 42\alpha_T^2\gamma_T - 24\alpha_S\gamma_S\gamma_T + 20\alpha_T\gamma_S\gamma_T - 24\alpha_S\gamma_T^2 - 36\alpha_T\gamma_T^2 + 3t_C(153\alpha_S^2 + 99\alpha_T^2 + 6\gamma_S^2 + 4\alpha_S(19\alpha_T - 87\gamma_S - 19\gamma_T) + 52\gamma_S\gamma_T + 102\gamma_T^2 - 6\alpha_T(17\gamma_S + 83\gamma_T)) + 12(3\alpha_S^3(\alpha_T - 6\gamma_S -
\end{aligned}$$

$$\begin{aligned}
& \gamma_T) + 2\alpha_S^2(5\alpha_T^2 + 9\gamma_S^2 + 6\gamma_S\gamma_T + 5\gamma_T^2 - 2\alpha_T(3\gamma_S + 5\gamma_T)) + \alpha_S(3\alpha_T^3 - \alpha_T^2(11\gamma_S + 15\gamma_T) - \gamma_T(3\gamma_S^2 + 10\gamma_S\gamma_T + \\
& 3\gamma_T^2) + \alpha_T(9\gamma_S^2 + 56\gamma_S\gamma_T + 15\gamma_T^2)) + \alpha_T(-2\gamma_T(\gamma_S + 3\gamma_T)^2 - 3\alpha_T^2(\gamma_S + 6\gamma_T) + \alpha_T(\gamma_S^2 + 15\gamma_S\gamma_T + 36\gamma_T^2)))/G, \\
& p_{2A}^{TS*} = (84t_C + (-4\alpha_S^2 - 16\alpha_S\alpha_T - 12\alpha_T^2 + 38\alpha_S\gamma_S + 2\alpha_T\gamma_S - 21\gamma_S^2 + 22\alpha_S\gamma_T + 42\alpha_T\gamma_T - 2\gamma_S\gamma_T - \\
& 9\gamma_T^2 + 18t_C(17\alpha_S - 3\alpha_T + 19\gamma_S + 9\gamma_T)) - 3(8\alpha_S^2(\alpha_T - 6\gamma_S - \gamma_T) + (3\gamma_S + \gamma_T)(\gamma_S + 3\gamma_T)^2 + 6\alpha_T^2(5\gamma_S + \\
& 3\gamma_T) + 2\alpha_S(12\alpha_T^2 + 5\alpha_T\gamma_S - 27\gamma_S^2 - 21\alpha_T\gamma_T - 14\gamma_S\gamma_T + 9\gamma_T^2) - \alpha_T(19\gamma_S^2 + 66\gamma_S\gamma_T + 27\gamma_T^2)))/G, \\
& \text{where } G = 3(420t_C - 54\alpha_S^2 - 54\alpha_T^2 + 53\alpha_T\gamma_S - 54\gamma_S^2 + 207\alpha_T\gamma_T - 52\gamma_S\gamma_T - 54\gamma_T^2 + \alpha_S(-52\alpha_T + \\
& 207\gamma_S + 53\gamma_T)).
\end{aligned}$$

We can obtain platforms' profits by plugging in these optimal prices. Clearly, SS is the equilibrium when $\Pi_i^{SS*} > \Pi_i^{TS*}$ and $\Pi_i^{ST*} > \Pi_i^{TT*}$. Similarly, TT is the equilibrium when $\Pi_i^{TT*} > \Pi_i^{ST*}$ and $\Pi_i^{TS*} > \Pi_i^{SS*}$.

B Additional Analyses

B.1 Analysis of Monopoly

When the monopoly platform offers traditional ads, we assume that markets for both consumers and advertisers are incompletely covered. In this case, the platform's demand from consumers and advertisers is given by (superscript e denotes the expected value),

$$x_C^T = \frac{u_0 - p_C^T - \gamma_T x_A^{eT}}{t_C} \in (0, 1), \quad (13)$$

$$x_A^T = \frac{\alpha_T x_C^{eT} - p_A^T}{t_A} \in (0, 1). \quad (14)$$

As in the main model, we assume $t_A = 1$. Given consumers' disutility from seeing the ads, their demand decreases in the externality parameter γ_T and their expected number of ads on the platform, x_A^{eT} . By contrast, given advertisers' utility from reaching consumers, their demand increases in the externality parameter α_T and their expected number of consumers on the platform, x_C^{eT} . The platform's profit is then given by $\Pi^T = p_C^T x_C^T + p_A^T x_A^T$. We assume that both consumers and advertisers have rational expectations: $x_C^T = x_C^{eT}$ and $x_A^T = x_A^{eT}$. Solving the platform's optimization problem, we obtain the following prices, demand and profits when the platform offers traditional ads.

$$p_C^{T*} = \frac{2t_C u_0 - u_0 \alpha_T (\alpha_T - \gamma_T)}{4t_C - (\alpha_T - \gamma_T)^2}, \quad (15)$$

$$p_A^{T*} = \frac{u_0 (\alpha_T + \gamma_T)}{4t_C - (\alpha_T - \gamma_T)^2}, \quad (16)$$

$$x_C^{T*} = \frac{2u_0}{4t_C - (\alpha_T - \gamma_T)^2}, \quad (17)$$

$$x_A^{T*} = \frac{u_0 (\alpha_T - \gamma_T)}{4t_C - (\alpha_T - \gamma_T)^2}, \quad (18)$$

$$\Pi^{T*} = \frac{u_0^2}{4t_C - (\alpha_T - \gamma_T)^2}. \quad (19)$$

Next, we analyze the optimal strategies for the monopoly platform when it offers sponsored content ads. Similarly, we assume that markets for consumers and advertisers are incompletely covered. In this case, consumers' utility is given by $u_0 + u_S - t_C x_C^S - p_C^S - \gamma_S x_A^{eS}$, and advertisers' utility is given by $\alpha_S x_C^{eS} - x_A^S - p_A^S$. Therefore, the platform's demand from consumers and advertisers are given by,

$$x_C^S = \frac{u_0 + u_S - p_C^S - \gamma_S x_A^{eS}}{t_C} \in (0, 1), \quad (20)$$

$$x_A^S = \alpha_S x_C^{eS} - p_A^S \in (0, 1). \quad (21)$$

The platform's profit is given by $\Pi^S = p_C^S x_C^S + p_A^S x_A^S$. Given our assumptions on rational expectations, we again have $x_C^S = x_C^{eS}$, and $x_A^S = x_A^{eS}$. Solving the platform's optimization problem, we obtain the following prices, demand and profits when the platform offers sponsored content ads.

$$p_C^{S*} = \frac{2t_C(u_0 + u_S) - (u_0 + u_S)\alpha_S(\alpha_S - \gamma_S)}{4t_C - (\alpha_S - \gamma_S)^2}, \quad (22)$$

$$p_A^{S*} = \frac{(u_0 + u_S)(\alpha_S + \gamma_S)}{4t_C - (\alpha_S - \gamma_S)^2}, \quad (23)$$

$$x_C^{S*} = \frac{2(u_0 + u_S)}{4t_C - (\alpha_S - \gamma_S)^2}, \quad (24)$$

$$x_A^{S*} = \frac{(u_0 + u_S)(\alpha_S - \gamma_S)}{4t_C - (\alpha_S - \gamma_S)^2}, \quad (25)$$

$$\Pi^{S*} = \frac{(u_0 + u_S)^2}{4t_C - (\alpha_S - \gamma_S)^2}. \quad (26)$$

Comparing the platform's profits with the two different ad formats, we see that if the externality parameters in both cases are identical (i.e., $\alpha_S = \alpha_T, \gamma_S = \gamma_T$), then for any $u_S > 0$, offering sponsored content ads is the strictly dominant equilibrium strategy. Similarly, for any $u_S < 0$, offering traditional ads is the strictly dominant equilibrium strategy.

When the externality parameters are different and $u_S = 0$, then offering sponsored content ads would be a strictly dominant strategy as long as $(\gamma_S - \gamma_T) < (\alpha_S - \alpha_T)$.

B.2 Free Content for Consumers

We already know that consumers' equilibrium price from sponsored content advertising is $p_{iC}^{SS*} = t_C - \alpha_S$. When $\alpha_S > t_C$, the media content is free for the consumers. We assume that α_T is also greater than t_C , so that regardless of the ad format, the media content is always free for the consumers. As a result, platforms' revenues now only depend on the revenues from the advertisers' side: $\Pi_i = p_{iA}^{vw} x_{iA}^{vw}$.

When both platforms offer sponsored content ads, we obtain the following prices, demands and profit

functions after solving the platforms' optimization problems.

$$p_{iC}^{SS*} = 0, \quad (27)$$

$$p_{iA}^{SS*} = 1 + \frac{\alpha_S \gamma_S}{t_C}, \quad (28)$$

$$x_{iC}^{SS*} = \frac{1}{2} = x_{iA}^{SS*}, \quad (29)$$

$$\Pi_i^{SS*} = \frac{1}{2} \left(1 + \frac{\alpha_S \gamma_S}{t_C} \right). \quad (30)$$

The prices, demand and profit functions from the case of TT are symmetric and given as follows,

$$p_{iC}^{TT*} = 0, p_{iA}^{TT*} = 1 + \frac{\alpha_T \gamma_T}{t_C}, x_{iC}^{TT*} = x_{iA}^{TT*} = \frac{1}{2}, \text{ and } \Pi_i^{TT*} = \frac{1}{2} \left(1 + \frac{\alpha_T \gamma_T}{t_C} \right).$$

The prices, demand and profit functions from the case of TS/ST are as follows,

$$p_{1C}^{TS*} = 0, \quad p_{1A}^{TS*} = \frac{t_C(6 - \alpha_S + \alpha_T) + (\alpha_S + \alpha_T)(2\gamma_S + \gamma_T)}{6t_C}, \quad (31)$$

$$p_{2C}^{TS*} = 0, \quad p_{2A}^{TS*} = \frac{t_C(6 + \alpha_S - \alpha_T) + (\alpha_S + \alpha_T)(\gamma_S + 2\gamma_T)}{6t_C}, \quad (32)$$

$$x_{1C}^{TS*} = \frac{12t_C^2 + (\alpha_S + \alpha_T)(\gamma_S - \gamma_T)(\gamma_S + \gamma_T) + 2t_C((3 + 2\alpha_S + \alpha_T)\gamma_S - (3 - 2\alpha_S - \alpha_T)\gamma_T)}{6t_C(4t_C + (\alpha_S + \alpha_T)(\gamma_S + \gamma_T))}, \quad (33)$$

$$x_{2C}^{TS*} = \frac{12t_C^2 - (\alpha_S + \alpha_T)(\gamma_S - \gamma_T)(\gamma_S + \gamma_T) + 2t_C((3 + \alpha_S + 2\alpha_T)\gamma_T - (3 - \alpha_S - 2\alpha_T)\gamma_S)}{6t_C(4t_C + (\alpha_S + \alpha_T)(\gamma_S + \gamma_T))}, \quad (34)$$

$$x_{1A}^{TS*} = \frac{t_C(6 - \alpha_S + \alpha_T) + (\alpha_S + \alpha_T)(2\gamma_S + \gamma_T)}{3(4t_C + (\alpha_S + \alpha_T)(\gamma_S + \gamma_T))}, \quad (35)$$

$$x_{2A}^{TS*} = \frac{t_C(6 + \alpha_S - \alpha_T) + (\alpha_S + \alpha_T)(\gamma_S + 2\gamma_T)}{3(4t_C + (\alpha_S + \alpha_T)(\gamma_S + \gamma_T))}, \quad (36)$$

$$\Pi_1^{TS*} = \frac{(t_C(6 - \alpha_S + \alpha_T) + (\alpha_S + \alpha_T)(2\gamma_S + \gamma_T))^2}{18t_C(4t_C + (\alpha_S + \alpha_T)(\gamma_S + \gamma_T))}, \quad (37)$$

$$\Pi_2^{TS*} = \frac{(t_C(6 + \alpha_S - \alpha_T) + (\alpha_S + \alpha_T)(\gamma_S + 2\gamma_T))^2}{18t_C(4t_C + (\alpha_S + \alpha_T)(\gamma_S + \gamma_T))}. \quad (38)$$

We see that the biggest change of results comes in terms of the effect of an increase in α_S on platforms' profits with sponsored content advertising. Now, as α_S increases, so do the profits (from sponsored content ads) of the platforms. Since the consumers do not pay any price, an increase in α_S does not reduce consumers' price. As a result, the only effect of an increase in α_S is an increase in advertisers' price which in turn increases platforms' profits (recall that when the content is not free for the consumers, α_S does not affect advertisers' price). Furthermore, as long as $\alpha_S \gamma_S > \alpha_T \gamma_T$ (which always holds under the assumption $\alpha_S > \alpha_T$ and $\gamma_S > \gamma_T$), advertisers pay a higher price and platforms earn greater profits under sponsored content ads compared to that under traditional ads. The equilibrium conditions can be easily obtained by comparing the profit differences across different subgames.

B.3 The Case with No Ads

In the main text, we assumed that each platform displays either the traditional ads or the sponsored content ads. In this extension, we analyze the situation where the platforms do not offer either type of ads. In particular, the two competing platforms trade off between choosing the sponsored content ads and the no ads strategy (denoted by N).

When neither platform offers any ad, they compete only in the consumer market. Their profits are given by $\Pi_i^{NN} = p_{iC}^{NN} x_{iC}^{NN}$. The optimal prices and profits are given as follows:

$$p_{1C}^{NN*} = p_{2C}^{NN*} = t_C, \quad \Pi_1^{NN*} = \Pi_2^{NN*} = \frac{t_C}{2}.$$

When both platforms offer the sponsored content ads, they compete in both sides of the market. Their profit functions are given by $\Pi_i^{SS} = p_{iC}^{SS} x_{iC}^{SS} + p_{iA}^{SS} x_{iA}^{SS}$. The optimal prices and profits are given as follows:

$$p_{1C}^{SS*} = p_{2C}^{SS*} = t_C - \alpha_S, \quad p_{1A}^{SS*} = p_{1A}^{SS*} = 1 + \gamma_S, \quad \Pi_1^{SS*} = \Pi_2^{SS*} = \frac{1 + t_C - \alpha_S + \gamma_S}{2}.$$

When one platform (e.g., platform 1) offers the sponsored content ads and the other platform (e.g., platform 2) offers no ads, they compete in the consumer market. At the same time, the platform offering the sponsored content ads acts as a monopoly in the advertising market. The two platforms' profits are given by $\Pi_1^{SN} = p_{1C}^{SN} x_{1C}^{SN} + p_{1A}^{SN} x_{1A}^{SN}$ and $\Pi_2^{SN} = p_{2C}^{SN} x_{2C}^{SN}$. The optimal prices and profits are given below:

$$\begin{aligned} p_{1C}^{SN*} &= \frac{(3t_C + \alpha_S \gamma_S)(4t_C + \alpha_S \gamma_S - \alpha_S^2)}{12t_C + 4\alpha_S \gamma_S - \alpha_S^2 - \gamma_S^2}, \\ p_{2C}^{SN*} &= \frac{(6t_C - (\alpha_S - \gamma_S)^2)(2t_C + \alpha_S \gamma_S)}{12t_C + 4\alpha_S \gamma_S - \alpha_S^2 - \gamma_S^2}, \\ p_{1A}^{SN*} &= \frac{(\alpha_S + \gamma_S)(3t_C + \alpha_S \gamma_S)}{12t_C + 4\alpha_S \gamma_S - \alpha_S^2 - \gamma_S^2}, \\ \Pi_1^{SN*} &= \frac{(8t_C - (\alpha_S - \gamma_S)^2)(3t_C + \alpha_S \gamma_S)^2}{(12t_C + 4\alpha_S \gamma_S - \alpha_S^2 - \gamma_S^2)^2}, \\ \Pi_2^{SN*} &= \frac{(2t_C + \alpha_S \gamma_S)(6t_C - (\alpha_S - \gamma_S)^2)^2}{(12t_C + 4\alpha_S \gamma_S - \alpha_S^2 - \gamma_S^2)^2}. \end{aligned}$$

Clearly, when $\Pi_1^{SS*} \geq \Pi_1^{NS*}$ and $\Pi_1^{SN*} \geq \Pi_1^{NN*}$, both platforms prefer to offer the sponsored content ads, and SS is the dominant outcome. Similarly, we can analyze the competing platforms' trade-off between offering the traditional ads and no ads. When the following conditions are satisfied, NN (neither platform offering any ad) is never going to be the equilibrium:

$$\begin{aligned} (1 + t_C - \alpha_v + \gamma_v)(12t_C + 4\alpha_v \gamma_v - \alpha_v^2 - \gamma_v^2)^2 - 2(2t_C + \alpha_v \gamma_v)(6t_C - (\alpha_v - \gamma_v)^2)^2 &> 0, \\ 2(8t_C - (\alpha_v - \gamma_v)^2)(3t_C + \alpha_v \gamma_v)^2 - t_C(12t_C + 4\alpha_v \gamma_v - \alpha_v^2)^2 &> 0, \quad v \in \{S, T\}. \end{aligned}$$

In other words, NN is dominated by SS or TT when these conditions are satisfied.