Online Appendix A. Deriving variances using Z-transforms

This Appendix gives an overview of how Z-transforms, a basic analysis tool in control theory, were used to calculate the inventory and order variance. For the unfamiliar reader, we first derive the familiar variances for the full off-shoring setting, before tackling dual sourcing systems.

8.1. Deriving the full off-shoring order and inventory variances

We start with the $Z-$transform of the inventory balance equation,

$$i_t = i_{t-1} - d_t + q^g_{t-L}, \tag{55}$$

for the single sourcing setting:

$$I_s[z] = z^{-1} I_s[z] - D[z] + z^{-L} Q_g[z] \Rightarrow I_s[z] = \left(z^{-L} Q_g[z] - D[z]\right) \frac{z}{z-1}. \tag{56}$$

This defines the inventory transfer function, $I_s[z]$, as a function of the order and demand transforms $Q_g[z]$ and $D[z]$. This inventory relationship can be graphically represented in a control block diagram[20] by blocks labeled b, c, d in Fig. 10 (the letters refer to the short-hand notation in the top-left corner of the blocks in Fig. 10). In our single off-shore supplier setting, the OUT replenishment policy (75) is the optimal linear policy. The order transfer function is found by first transforming the inventory position (the sum of current inventory and outstanding receipts) to yield

$$IP[z] = I_s[z] + \sum_{i=1}^{L-1} z^{-i} Q_g[z] \Rightarrow IP[z] = I_s[z] + \frac{1 - z^{1-L}}{z-1} Q_g[z], \tag{57}$$

represented by block e in Fig. 10. Next, we convert the demand and forecasting difference equations ($d_t = \mu + \epsilon_t$ and $\hat{d}^\star_{t+n,t} = \mu$ for iid; and (22)-(30) for AR(1)), into transfer functions. The transform of the demand ($D[z]$) and the forecast of demand over $L+1$ periods ($A[z]$) is shown in Table 7 for all three demand processes (iid, AR(1), and IMA(0,1,1)). These transforms correspond to block d and a in Fig. 10, completing the block diagram.

Remark. In the time domain, under a random demand, the long-run expected inventory $\mathbb{E}[i_t] = i^\star$. As the variance is the expected squared deviation from the mean, the specific value of $i^\star$ has no impact on var$[i_t]$, which we aim to calculate. Likewise, the long-run expected demand, $\mathbb{E}[d_t]$ and the long-run expected replenishment quantity $\mathbb{E}[q^g_t]$ have no influence on the var$[d_t]$ and var$[q^g_t]$ respectively. □

The $Z-$transform of the order-up-to replenishment policy (75) is

$$Q_g[z] = A[z] D[z] - IP[z]. \tag{58}$$

[20] Block diagrams are a standard control theory technique, see Nise [2004].

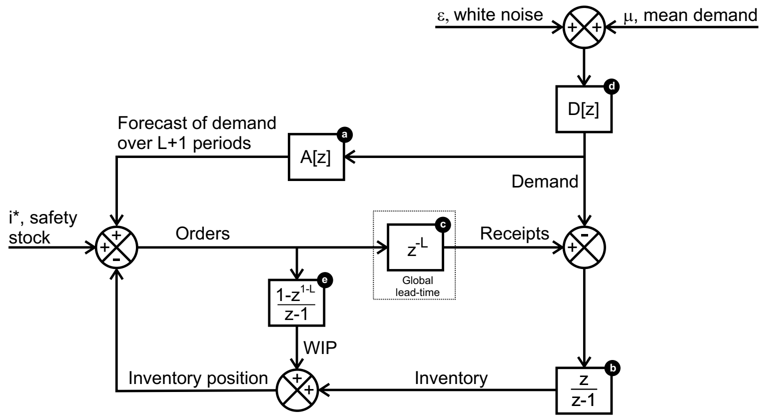

Figure 10 Block diagram of the full off-shoring setting.

Transfer function	iid demand	AR(1) demand	IMA(0,1,1) demand
A[z]	0	$\frac{\rho(\rho^L-1)}{\rho-1}$	$\frac{z\beta L}{z-1+\beta}$
D[z]	1	$\frac{z}{z-\rho}$	$1+\frac{\beta}{z-1}$

Table 7 Full off-shoring transfer functions.

Substituting (56) and (57) into (58) results in

$$Q_g[z] = A[z]D[z] - IP[z] \tag{59}$$

$$= A[z]D[z] - I_s[z] - \frac{1-z^{1-L}}{z-1}Q_g[z] \tag{60}$$

$$= A[z]D[z] - (z^{-L}Q_g[z] - D[z])\frac{z}{z-1} - \frac{1-z^{1-L}}{z-1}Q_g[z]. \tag{61}$$

Collecting together all the $Q_g[z]$ and simplifying gives

$$Q_g[z]\left(1 + z^{-L}\frac{z}{z-1} + \frac{1-z^{1-L}}{z-1}\right) = A[z]D[z] + D[z]\frac{z}{z-1} \tag{62}$$

$$Q_g[z] = \frac{A[z]D[z] + D[z]\frac{z}{z-1}}{1 + z^{-L}\frac{z}{z-1} + \frac{1-z^{1-L}}{z-1}} \tag{63}$$

$$Q_g[z] = D[z]\left(1 + A[z] - \frac{A[z]}{z}\right). \tag{64}$$

After substituting in the relevant expressions for $A[z]$ and $D[z]$ as given in Table 7, simple algebra leads to the transfer functions for the global orders $Q_g[z]$. Knowing $Q_g[z]$, the inventory transfer function $I_s[z]$ can be found from (56).

We are now ready to determine the variances of the orders and inventory levels from the relevant transfer function using Tsypkin's Relation (Lemma 3). Putting $A[z] = 0$ and $D[z] = 1$ into (64) provides the transfer function of the global orders under iid demand:

$$Q_g[z] = 1. \tag{65}$$

Taking the inverse z-transform of (65) gives the order impulse response function

$$q_t^g = \mathscr{Z}^{-1}[1] = \delta[t], \tag{66}$$

which equals the impulse function (Kronecker delta). Using Tyspkin's relation,

$$\sigma_{q,s}^2 = \sigma^2 \sum_{t=0}^{\infty} \delta[t]^2 = \sigma^2. \tag{67}$$

Note, the OUT policy behaves as a *pass-on-orders* system and as expected $\sigma_{q,s}^2 = \sigma^2$.

Using (65) inside of (56) provides the inventory transfer function,

$$I_s[z] = \frac{z(z^{-L}-1)}{z-1} \tag{68}$$

which has a time domain response given by it inverse z-transform,

$$i_t^s = \mathscr{Z}^{-1}\left[\frac{z(z^{-L}-1)}{z-1}\right] = -\mathscr{Z}^{-1}\left[\sum_{t=0}^{L-1} z^{-t}\right] = h[t-L] - h[t], \tag{69}$$

where $h[\cdot]$ is the unit step function; that is, $h[t<0]=0$, otherwise $h[t \geq 0]=1$. Applying Tyspkin's relation from Lemma 3 provides

$$\sigma_{i,s}^2|_{\mathrm{IID}} = \sigma^2 \sum_{t=0}^{\infty} (h[t-L] - h[t])^2 = \sigma^2 L. \tag{70}$$

The variance of inventory in the presence of AR(1) and IMA(0,1,1) demand is obtained in exactly the same manner. As these expressions can be quite lengthy, we used Mathematica (Wolfram Research) to help with the algebra involved. For clarity, we also provide a sketch of the derivation of the remaining variance expressions in what remains of Online Appendix A.

8.1.1. Deriving the full off-shoring inventory variance under AR(1) demand Departing from (64), we substitute in the transfer function of demand, $D[z] = z/(z-\rho)$, and the transfer function of the mechanism that converts the demand into the forecast of demand over the lead-time, $A[z] = \rho(\rho^L - 1)/(\rho - 1)$; simplifying yields the order transfer function,

$$Q_{g,s}[z] = \frac{z}{z-\rho}\left(1 + \frac{\rho(\rho^L-1)}{\rho-1} - \frac{\rho(\rho^L-1)}{z(\rho-1)}\right) = \frac{\rho - z + (z-1)\rho^{1+L}}{(z-\rho)(\rho-1)}. \tag{71}$$

Using (71) inside (56) and simplifying provides the inventory transfer function,

$$I_s[z] = \frac{\rho^{L+1}}{\rho-1}\left(\frac{z^{1-L}}{z-\rho}\right) + \frac{1}{1-\rho}\left(\frac{z^{1-L}}{z-1} + \frac{(\rho-1)z^2}{(z-1)(z-\rho)}\right). \tag{72}$$

Taking the inverse $Z-$transform of reveals the inventory impulse response,

$$i_t^s = \frac{\rho^{L+1}}{\rho-1}\left(\rho^{t-L} h[t-L]\right) + \frac{1}{1-\rho}\left(h[t-L] + \rho^{t+1} - 1\right) = \begin{cases} \frac{\rho^{t+1}-1}{1-\rho} & \text{if } t < L, \\ 0 & \text{if } t \geq L. \end{cases} \tag{73}$$

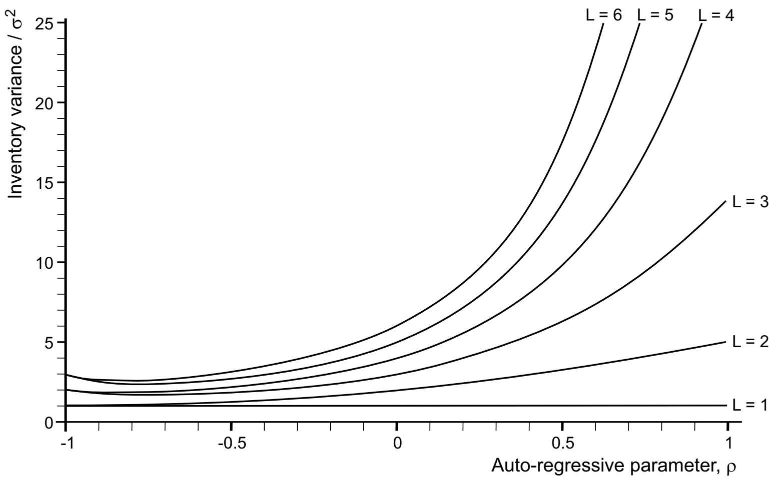

Figure 11 The effect of auto-correlated demand on the inventory in a fully off-shored supply chain. (Recall that $\sigma_{i,s}^2/\sigma^2 = L$ for iid demand).

Tyspkin's sum of squares then provides the inventory variance,

$$\sigma_{i,s}^2|_{\mathrm{AR}} = \frac{\sigma^2}{(\rho-1)^2}\left(\frac{\rho(\rho(\rho^L-1)-2)(\rho^L-1)}{\rho^2-1}+L\right) \tag{74}$$

which matches the inventory variance given by Disney and Lambrecht [2008]. With unit lead time $L=1$, $\sigma_{i,s}^2=\sigma^2$. As with iid demand, the inventory variance is increasing in lead time L, but superlinearly if $\rho>0$ and sublinearly if $\rho<0$. It also is convex in ρ, see Figure 11.

8.1.2. Deriving the single sourcing inventory variance under IMA(0,1,1) demand For IMA(0,1,1) demand and its forecast introduced in §6.2, single-sourcing under a linearized OUT replenishment policy becomes

$$q_t^g = \underbrace{\left(i^\star + \sum_{i=1}^{L} \hat{d}_{t+i,t}^\star\right)}_{\text{Target inventory position}} - \underbrace{\left(i_t + \sum_{i=1}^{L-1} q_{t-i}^g\right)}_{\text{Inventory position}} \tag{75}$$

$$= i^\star + L\hat{d}_{t+n,t}^\star - i_t - \sum_{i=1}^{L-1} q_{t-i}^g. \tag{76}$$

Remark 1. At time $t=0$, the long-run variance of the demand and forecast is infinite. As the forecast is added directly into the replenishment order (see (76)), the long-run variance of the orders, q_t^g, is also infinite. We assume the supplier can provide items at a unit purchase price, p, and the supplier either has sufficient in-house capacity, or can subcontract excess demand to another supplier with the same lead time, quality, and cost characteristics. □

Remark 2. The demand variance under IMA(0,1,1) demand is given by $\sigma_d^2 = \sigma^2(1+\sum_{n=1}^{\infty}\beta^2)$; the variance of the inventory maintained by the OUT policy is $\sigma_{q,g}^2 = \sigma^2((1+L\beta)^2+\sum_{n=1}^{\infty}\beta^2)$. The difference $\sigma_{q,g}^2 - \sigma_d^2 = \sigma^2 L\beta(2+L\beta)$ reveals bullwhip is always generated as $\beta>0$ and $L\geq 1$. □

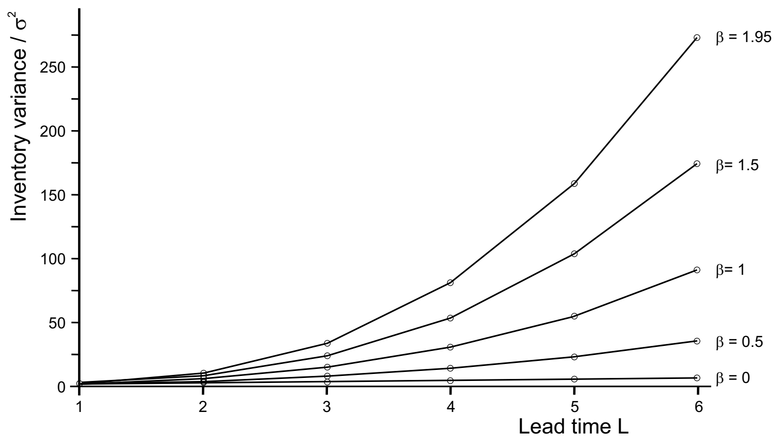

Figure 12 The variance of the inventory levels when outsourcing all production to an off-shore supplier under IMA(0,1,1) demand.

Graves [1999] shows that the variance of the inventory levels under single sourcing is given by (33). With unit lead time $L=1$, $\sigma_{i,s}^2=\sigma^2$. As noted by Graves [1999] and shown in Figure 12, the inventory variance, which was linear in the lead-time L for iid demand, is now convex increasing in L and in β. This means when β is sufficiently large, the inventory costs will also be convex in L. When $\beta=0$, the iid expression is recovered. When $\beta=1$, the IMA(0,1,1) demand process degenerates into the AR(1) process with $\rho=1$ and $\sigma_{i,s}=\sigma^2(L^2+L(2L^2-3L+1))/6$.

To verify the inventory variance expression of Graves [1999] with our z-transform approach, we depart from (64), this time substituting in $D[z]=1+\beta/(z-1)$, for the demand and, $A[z]=z\beta L/(z-1+\beta)$, for the lead-time demand forecast generating mechanism. Simplifying yields the transfer function of the orders,

$$Q_{g,s}[z]=1+\beta\left(L+\frac{1}{z-1}\right). \tag{77}$$

Placing (77) inside (56) and simplifying yields the IMA(0,1,1) inventory transfer function,

$$I_s[z]=\frac{z}{z-1}\left(z^{-L}\left(1+\beta L+\frac{\beta}{z-1}\right)-\frac{\beta}{z-1}-1\right). \tag{78}$$

Taking the inverse $Z-$transform gives the inventory transfer function in the time domain;

$$i_t^s=(1+\beta L+\beta(t-L))h[n-L]-t\beta-1=\begin{cases}-1-t\beta & \text{if } t<L,\\ 0 & \text{if } t\geq L.\end{cases} \tag{79}$$

Finally, summing the squared impulse response provides the inventory variance,

$$\sigma_{i,s}^2|_{\mathrm{IMA}}=\sigma^2\sum_{t=0}^{L-1}(-1-t\beta)^2=\sigma^2L\big(1+\beta(L-1)+\beta^2(L-1)(2L-1)/6\big), \tag{80}$$

confirming Graves [1999] and used in (33).

Transfer function	iid demand	AR(1) demand	IMA(0,1,1) demand
$A[z]$	0	0	$\frac{\beta}{1-(1-\beta)z^{-1}}$
$B[z]$	0	ρ	$\frac{\beta}{1-(1-\beta)z^{-1}}$
$C[z]$	0	0	$\frac{\beta z^{1-L}}{1-(1-\beta)z^{-1}}$
$D[z]$	1	$\frac{z}{z-\rho}$	$\frac{\beta+z-1}{z-1}$

Table 8 Dual sourcing transfer functions.

8.2. Obtaining the dual sourcing order and inventory variances

We now consider the variance of the inventory levels and the replenishment orders in the dual sourcing setting. A new block diagram is required, see Fig. 13. This is based on the dual sourcing inventory balance equation (1), the demand processes and their forecasts, (22)-(30), the replenishment rules, (9) and (32), and the off-shore orders, (8) and (31). Specific demand processes require substitutions within the block diagram as detailed in Table 8. The OUT policy can be accessed by setting $\alpha = 0$, POUT requires $-1 < \alpha \geq 1$ for stability.

Equation (63) hints at another way to obtain the transfer function for the local orders directly from the block diagram. First sum all the *feed-forward routes*, from the white noise input to the output, the local orders; $(bd - cd + dfh - adefh)$. Next, divide by $1 -$ *sum of the feedback loops*, from the local orders back to the local orders leading to

$$\begin{aligned} Q[z] =& (bd - cd + dfh - adefh)/(1 + fgh) \\ =& \frac{D[z]z^{-L}\left(A[z](\alpha-1)z + z^L(B[z](z-1) + C[z](z-1) - \alpha z + z)\right)}{z-\alpha}. \end{aligned} \tag{81}$$

After substituting and simplifying, we arrive at transfer functions for the local replenishment orders, $Q[z]$; Table 8 provides the required information. The levels $\{i^\star, \mu\gamma, (1-\gamma)\mu\}$ have no consequence and can be ignored when determining the variance. The transfer function for the inventory levels in dual sourcing mode, $I[z]$, can be found from

$$I_d[z] = \frac{z}{z-1}\big(Q[z]z^{-1} + D[z](A[z]z^{-L} - 1)\big). \tag{82}$$

8.2.1. Dual sourcing variances under iid demand

Placing $A[z] = B[z] = C[z] = 0$ and $D[z] = 1$, see Table 8, into (81) and simplifying yields the transfer function of the orders:

$$Q_d[z] = \frac{z(1-\alpha)}{z-\alpha}. \tag{83}$$

Remark. The poles and zeros (the roots of the denominator and numerator w.r.t. z respectively) of a system's transfer function, see (84) or (84), are required to lie within the unit circle in the complex plane for stability. Thus, stability[21] is achieved when $-1 < \alpha \leq 1$. □

[21] Poles and zeros are allowed to lie on the unit circle at $z = 1$; hence the inequality $\alpha \leq 1$.

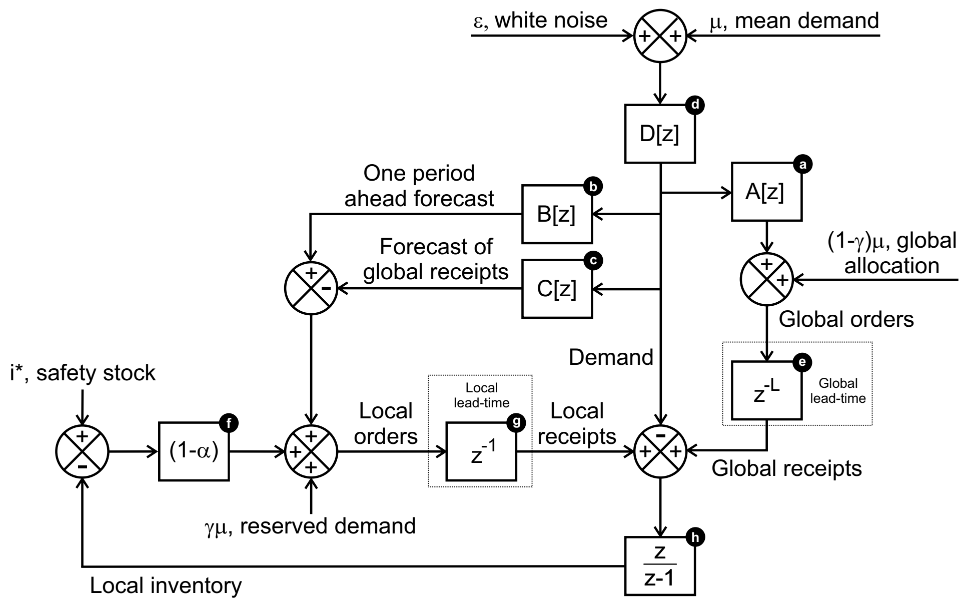

Figure 13 Generic block diagram of the dual sourcing for both the TBS and DYN settings.

Taking the inverse $Z-$transform of the transfer function of the orders, (83) results in

$$q_t^d = \mathscr{Z}^{-1}\left[\frac{z(1-\alpha)}{z-\alpha}\right] = (1-\alpha)\mathscr{Z}^{-1}\left[\frac{1}{1-\alpha z^{-1}}\right] = (1-\alpha)\mathscr{Z}^{-1}\left[\sum_{t=0}^{\infty}\alpha^t z^{-t}\right] = (1-\alpha)\alpha^t. \tag{84}$$

Applying Tsypkin's relation to (84), we obtain the variance expression,

$$\sigma_q^2|_{\text{IID}} = \sigma^2\sum_{t=0}^{\infty}((1-\alpha)\alpha^t)^2 = \sigma^2\left(\frac{1-\alpha}{1+\alpha}\right). \tag{85}$$

Putting (83) into (82) and simplifying gives the inventory transfer function,

$$I_d[z] = \frac{z}{\alpha - z}. \tag{86}$$

The variance of the inventory levels requires us to take the inverse z-transform of the inventory level transfer function. For the iid case,

$$i_t^d = \mathscr{Z}^{-1}\left[\frac{z}{\alpha - z}\right] = (1-\alpha)\mathscr{Z}^{-1}\left[\sum_{t=0}^{\infty}\alpha^t z^{-t}\right] = -\alpha^t, \tag{87}$$

to which we apply Tsypkin's relation to obtain the following expression for the inventory variance,

$$\sigma_i^2|_{\text{IID}} = \sigma^2\sum_{t=0}^{\infty}(-\alpha^t)^2 = \sigma^2\left(\frac{1}{1-\alpha^2}\right). \tag{88}$$

8.2.2. Dual sourcing variances under AR(1) demand The local order transfer function in the dual sourcing setting is found by substituting $A[z]$, $B[z]$, $C[z]$, and $D[z]$ from Table 8 into (81) and simplifying,

$$Q[z] = z\left(\frac{z-1}{\alpha - z} + \frac{z}{z-\rho}\right). \tag{89}$$

Taking the inverse $Z-$transform provides the impulse response of the local orders in response in the dual sourcing scenario under AR(1) demand,

$$q_t = \mathscr{Z}^{-1}[Q[z]] = \alpha^t - \alpha^{t+1} + \rho^{t+1}, \tag{90}$$

which can be used inside Tsypkin's relation, (54), to provide the variance of the local orders under AR(1) demand shown in (26).

The transfer function of inventory levels is found by placing (89) into (82) and simplifying,

$$I[z] = \frac{z}{\alpha - z}. \tag{91}$$

Taking the inverse $Z-$transform of (91) provides the impulse response the inventory levels under AR(1) demand in the time domain,

$$i_t = -\alpha^t. \tag{92}$$

By Tsypkin's relation the sum square of (92) provides the variance of the FGI in the dual sourcing setting given by (27).

8.2.3. Dual sourcing variances under IMA(0,1,1) demand The transfer function of the local orders in the dual sourcing setting is obtained by substituting the relevant $A[z]$, $B[z]$, $C[z]$, and $D[z]$ from Table 8 into (81) and simplifying:

$$Q_d[z] = \frac{z\left(\alpha(1-\beta) + \beta z^{1-L}(\alpha - z) + z(\beta - \alpha + 1) - 1\right)}{(z-1)(z-\alpha)}. \tag{93}$$

The time-domain impulse response of the local orders is given by the inverse $Z-$transform of (93),

$$q_t = \mathscr{Z}^{-1}[Q_d[z]] = \frac{(\beta - \alpha\beta)h[t+1-L] + (\alpha - \beta - 1)\left(1 - \alpha^{t+1}\right) - \left(\alpha(\beta - 1) + 1\right)\left(\alpha^t - 1\right)h[t-1]}{\alpha - 1}. \tag{94}$$

Summing the square of (94) over $t = 0$ to ∞ gives the variance expression in (34).

The inventory variance is obtained by first using (93) in (82) to find the inventory transfer function,

$$I[z] = \frac{z}{\alpha - z}. \tag{95}$$

Taking the inverse $Z-$transform reveals the inventory impulse response,

$$i_t = \mathscr{Z}^{-1}[I[z]] = -\alpha^t. \tag{96}$$

Squaring (96) inside the Tsypkin sum provides the inventory variance expression in (34).

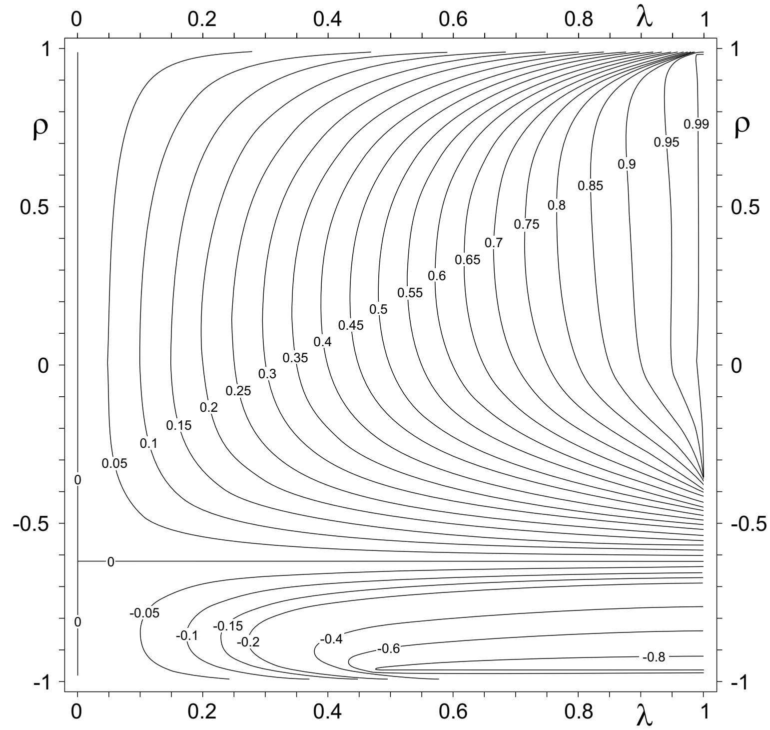

Figure 14 The contour lines of the optimal smoothing $\alpha^\star$ as a function of capacity intensity λ (horizontal axis) and serial correlation ρ (vertical axis) for the TBS-POUT policy under AR(1) demand. Higher capacity intensity requires more production smoothing. Around the natural neighborhood of $\rho > 0$, higher serial correlation leads to less smoothing.

Online Appendix B. Analytic optimization under AR(1) and IMA(0,1,1) demand

This Appendix presents the analytic optimization of α under AR(1) and IMA(0,1,1).

8.3. Correlated AR(1) demand

$f_{\mathrm{AR}}[\alpha^\star] = \lambda$ is easily solved numerically for $\alpha^\star \in [-1, 1]$ given $\lambda \in [0, 1]$ and $\rho \in [-1, 1]$, as shown in Fig. 14. Unfortunately, the analytic solution of $f_{\mathrm{AR}}[\alpha^\star] = \lambda$ requires the roots of a 7-th order polynomial and we know that no general analytic solutions exist for polynomials above 4-th order. For the practicing manager/analyst Online Appendix C provides the VBA code for a Microsoft Excel Add-in that can numerically determine $\alpha^\star$ for a given cost function under AR(1) demand. The algorithm is based on the Regula-Falsi method. We also provide an approximate analytic solution that is a bound:

Lemma 4 *For TBS-POUT with AR(1) demand, the optimal smoothing $\alpha^\star = g_{AR}[\rho]\lambda + o[\lambda]$ where*

$$g_{AR}[\rho] = 1/f'_{AR}[0] = \frac{1+\rho-\rho^2}{\sqrt{\frac{2\rho^3-2\rho-1}{\rho^2-1}}}. \tag{97}$$

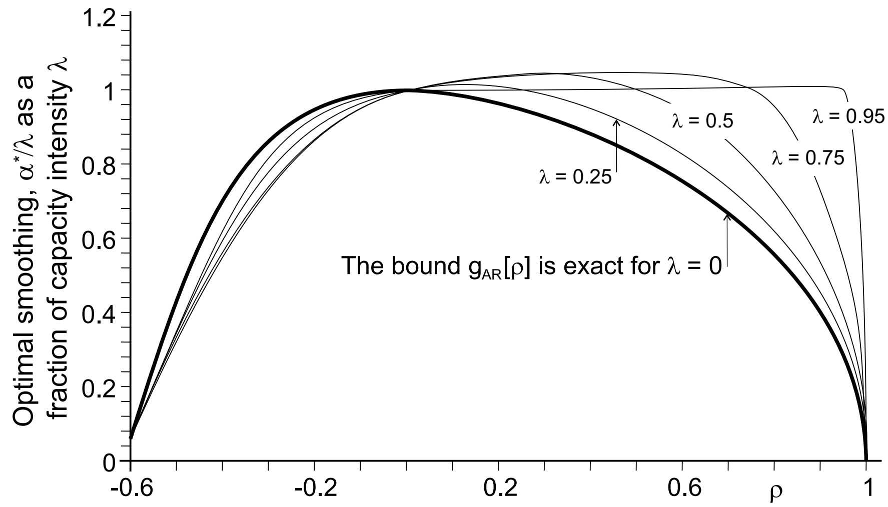

Figure 15 The graph shows $\alpha^\star/\lambda$ for five capacity intensities $\lambda \in \{0, 0.25, 0.5, 0.75, 0.95\}$ and the linear approximation $g_{\mathsf{AR}}[\rho]$ which is exact for $\lambda = 0$ and a bound otherwise. For practical (positive) correlation values, AR(1) demand series with larger serial correlation require less local order smoothing.

The linear approximation $g_{AR}[\rho]\lambda$ is a lower (upper) bound for $\alpha^\star$ when serial correlation ρ is positive (negative) and is asymptotically correct for (i) $\lambda \to 0$ and (ii) $\rho \to 0$. $\square$

Lemma 4 shows that, in the natural neighborhood of $\rho > 0$, higher serial correlation leads to less smoothing, as shown in Fig. 15. This is sensible as more negative serial correlation typically results in higher demand fluctuations for which inventory benefits from more smoothing. Lemma 4 also shows that $\alpha^\star$ is positive if $\rho > \overline{\rho} = -0.6180$ (the negative zero of $g_{\mathrm{AR}}[\rho]$ which interestingly is the reciprocal of the golden ratio). Positive smoothing is the natural regime to consider as negative smoothing implies that deviations from the target inventory are over-corrected in each replenishment decision. As most real demand patterns are positively auto-correlated, a positive α is likely in practice.

8.4. IMA(0,1,1) demand

$f_{\mathrm{IMA}}[\alpha^\star] = \lambda$ is easily solved numerically for $\alpha^\star \in [-1, 1]$ given $\lambda \in [0, 1]$ and $\beta \in [0, 2]$, as shown in Fig. 16 for different lead times L. Online Appendix D provides the VBA code to determine $\alpha^\star$ numerically with the Regula-Falsi method in a Microsoft Excel Add-in. If $L = 2$, optimal smoothing roughly equals the capacity intensity λ, almost irrespective of β. Yet as lead-times increase, the optimal smoothing decreases as the non-stationarity β increases, except for high capacity intensity for which smoothing is extremely high (and insensitive to β). Unfortunately, there is no general analytic solution of $f_{\mathrm{IMA}}[\alpha^\star] = \lambda$ but we can provide an approximate analytic solution that is a bound:

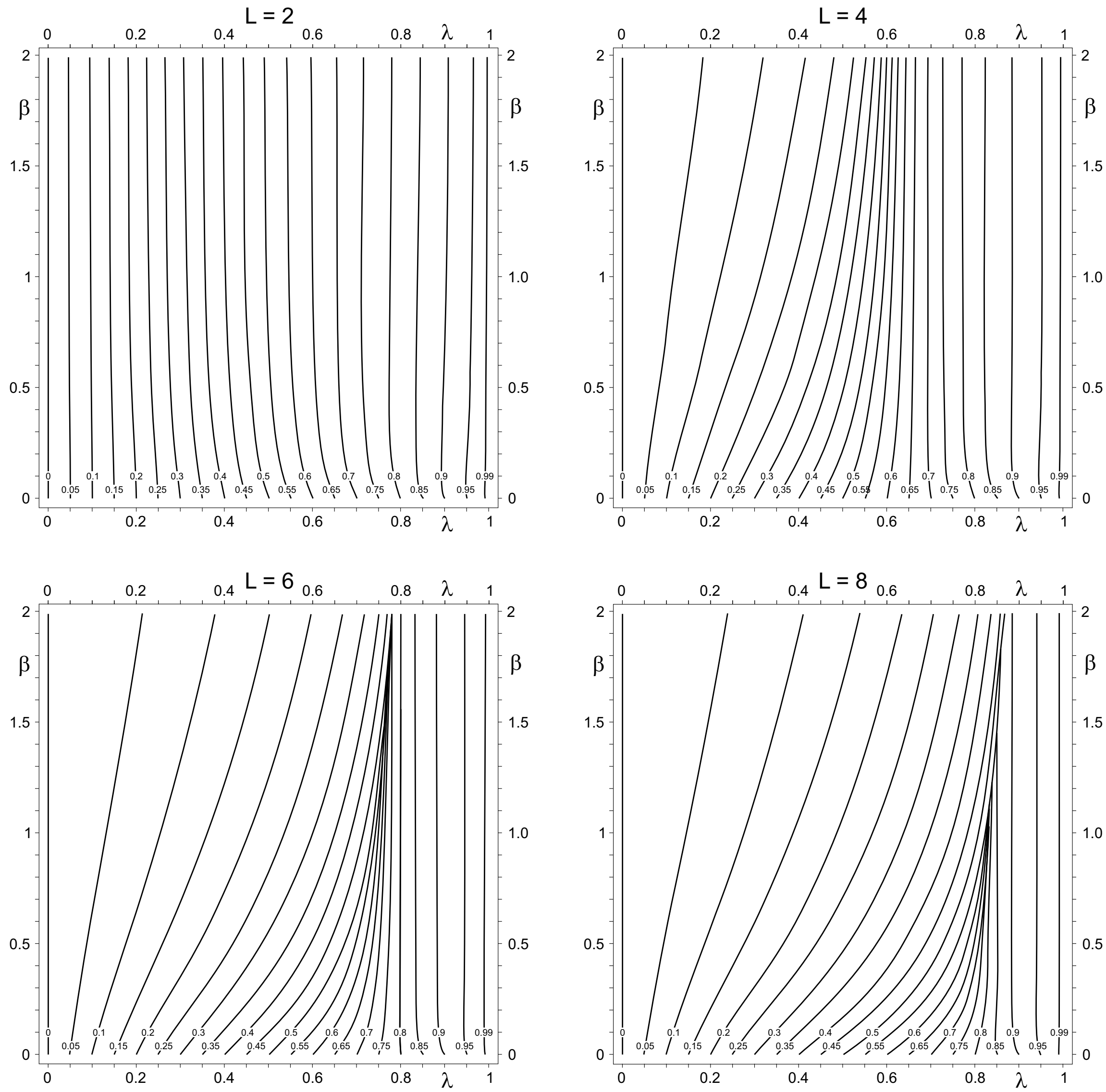

Figure 16 The contour lines of the optimal smoothing $\alpha^\star$ under IMA(0,1,1) demand: as lead-times increase, the optimal smoothing decreases as β increases, except for high capacity intensity for which smoothing is extremely high (and insensitive to β).

Lemma 5 *For DYN-POUT with IMA(0,1,1) demand, the optimal smoothing $\alpha^\star = g_{IMA}[\beta, L]\lambda + o[\lambda]$ where*

$$g_{IMA}[\beta, L] = 1/f'_{IMA}[0] = \begin{cases} 1 & \text{if } L = 2 \\ 1/\sqrt{(L-1)\beta^2 + 2\beta + 1} & \text{if } L > 2 \end{cases}. \tag{98}$$

The linear approximation $g_{IMA}[\beta, L]\lambda$ is a lower bound for $\alpha^\star$ and is asymptotically correct for (i) $\lambda \to 0$ and (ii) $\beta \to 0$. $\square$

Lemma 5 shows that for small capacity intensities λ, optimal smoothing decreases in β, as shown in Fig. 17. Indeed, $\alpha^\star$ is roughly proportional to $1/\beta$ and to $1/\sqrt{L}$, implying that optimal smoothing decreases in both the non-stationarity β and the off-shore lead time L.

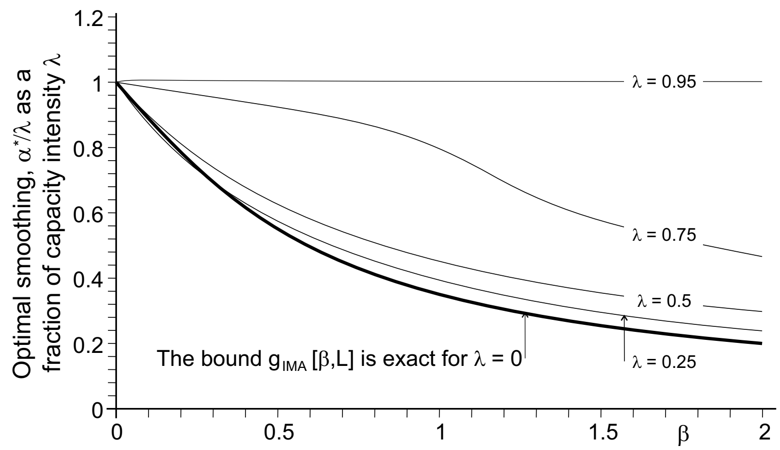

Figure 17 The graph shows $\alpha^\star/\lambda$ for $L=6$ and for five capacity intensities $\lambda \in \{0, 0.25, 0.5, 0.75, 0.95\}$ and the linear approximation $g_{\mathsf{IMA}}[\beta, L]$ which is exact for $\lambda = 0$ and a lower bound otherwise. IMA(0,1,1) demand series with higher non-stationarity require less local order smoothing.

Online Appendix C. Finding $\alpha^\star$ numerically under AR(1) and IMA(0,1,1) demand with the Regula-Falsi method

A user defined function can be added to Microsoft Excel for numerically finding $\alpha^\star$ using the following VBA code. The function is based on the Regula-Falsi method and is guaranteed to find a solution. For AR(1) demand the $\alpha^\star$ for the TBS-POUT policy can be accessed with "=DSAlphaStar(0,ρ,λ)"; for IMA(0,1,1) demand, the $\alpha^\star$ for the DYN-POUT policy can be accessed by "=DSAlphaStar(L,β,λ)". Due to an asymptote in the AR(1) case, the method is not reliable when $\rho < 0.61803$, so we a have included an escape that outputs a warning when this issue has the potential to happen.

```
Option Explicit
Function Fp(p As Double, L As Integer, br As Double, lambda As Double)
Dim p1, p2 As Double

If L > 0 Then
    p1 = ((-1 + 2 / (1 + p) + br * (2 - 2 * p ^ (L - 1) + (L - 1) * br)) ^ 0.5)
    p2 = ((((p - 1) ^ 2) * (1 / (1 - p ^ 2)) ^ 0.5 * (p ^ 2 + (L - 1) * (p ^ L) * ((1 + p) ^ 2) * br)) / (p ^ 3))
Else
    p1 = (-(-1 + p + (-2 + p + p ^ 2) * br - 2 * p * br ^ 2 + 2 * br ^ 3) / ((1 + p) * (-1 + p * br) * (-1 + br * br))) ^ 0.5
    p2 = (((1 / (1 - p ^ 2)) ^ 0.5) * (-1 + p) ^ 2 * (1 + br * (1 + p ^ 2 - br - 2 * p * br))) / (p * (-1 + p * br) ^ 2)
End If
Fp = (p1 / (p2 + p1)) - lambda
End Function

Function DSAlphaStar(L As Integer, br As Double, lambda As Double)
Dim A, B, C, Fa, Fb, Fc, p1, p2 As Double
Dim grandloop As Integer
Dim p As Double

If br < -0.61803398875 Then
    DSAlphaStar = "When rho<-0.61803 this method is not reliable"
Else:
    A = -0.999999
    B = 0.999999
    C = 0
    For grandloop = 1 To 1000
       p = A
       Fa = Fp(p, L, br, lambda)
        p = B
       Fb = Fp(p, L, br, lambda)
        C = (A * Fb - B * Fa) / (Fb - Fa)
        p = C
        Fc = Fp(p, L, br, lambda)
        If Fc * Fa > 0 Then
```

```
            A = C
        Else: B = C
        End If
        If Abs(Fc) < 0.00000000001 Then
           grandloop = 1000
    End If
Next grandloop
DSAlphaStar = C
End If
End Function
```