

On-line Appendix for: A modeling framework for tipping in presence of a social norm

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November 5, 2024

1 APPENDIX

1.1 Proof of Lemma 1

Assuming a customer's tip is non-negative, $v(t; \theta, p)$ is bounded above by $\theta - p$, and since K is positive, it is clear from Equation (2) that if $\theta < p$, then $u(t; \theta, p, m) < 0$, hence, $\theta \notin \mathcal{C}^*$. Assume therefore that $\theta \geq p$, and let $t^*(\theta)$ denote the optimal tip. Noting that $u(t; \theta, p, m)$ is strictly increasing in t over $[0, (\theta - p)/2]$, it must follow that $t^*(\theta) \geq (\theta - p)/2$. Using the definition of L , when $t \geq (\theta - p)/2$ we can rewrite Equation (2) as

$$u(t; \theta, p, m) = (1 - \sigma)(\theta - p) - (1 - 2\sigma)m - L(m - t).$$

A θ -type customer chooses their tip t so as to maximize their utility. By defining $d = m - t$ we get that the maximum utility is obtained at

$$\begin{aligned} t^*(\theta) &= \arg \max_{t \geq (\theta - p)/2} u(t; \theta, p, m) \\ &= m - \arg \min_{d \leq m - (\theta - p)/2} L(d) \\ &= m - \min\{m - (\theta - p)/2, \underline{d}\} \\ &= \max\{(\theta - p)/2, m - \underline{d}\}, \end{aligned}$$

where the second equality follows substitution, the third follows the convexity of L and the definition of \underline{d} , and the last follows basic algebra.

We next show that customer utility is increasing in θ . Consider two customers with types θ_1 and θ_2 such that $\theta_1 < \theta_2$. Let β be such that $t^*(\theta_1) = \beta(\theta_1 - p)$. Then, under rational tipping, it must follow that $\beta \geq 1/2$. Note that the social-pressure cost $K(m - t)$ is decreasing in t for every m . Hence,

$$\begin{aligned} u(t^*(\theta_1); \theta_1, p, m) &= (1 - \sigma)(\theta_1 - p) - (1 - 2\sigma)t^*(\theta_1) - K(m - t^*(\theta_1)) \\ &= (1 - \sigma - \beta(1 - 2\sigma)) \cdot (\theta_1 - p) - K(m - \beta \cdot (\theta_1 - p)) \\ &< (1 - \sigma - \beta(1 - 2\sigma)) \cdot (\theta_2 - p) - K(m - \beta \cdot (\theta_2 - p)) \\ &= u(\beta \cdot (\theta_2 - p); \theta_2, p, m) \\ &\leq u(t^*(\theta_2); \theta_2, p, m). \end{aligned}$$

Hence, by sharing a fraction β of their surplus, the θ_2 -type customer guarantees a higher utility than the θ_1 customer (let alone when tipping optimally). Therefore $u(t^*(\theta); \theta, p, m)$ strictly increases in θ , so that $\theta_1 \in \mathcal{C}^*$ implies $\theta_2 \in \mathcal{C}^*$.

1.2 Proof of Proposition 1

In the proof of Proposition 1, we will make use of the following Lemma 3, which we state and prove below. This lemma proves stronger properties than needed for the proof of Proposition 1 alone, however, we use these results in several other subsequent proofs.

Lemma 3. *Provided the customary tip, m , assume some $\underline{\theta} \in [\theta_L, \theta_H]$ exists such that*

$$u(t^*(\underline{\theta}); \underline{\theta}, p, m) = 0.$$

Then it must follow that $\mathcal{C}^ = [\underline{\theta}, \theta_H]$ and $\underline{\theta} = p + c(m)$, where $c(m)$ is the unique value satisfying*

$$c(m) = \frac{(1 - 2\sigma)m + L(\min\{m - c(m)/2, \underline{d}\})}{1 - \sigma}.$$

Define $\hat{m} = 2(1 - \sigma)\underline{d} + L(\underline{d})$. Then $\hat{m} \geq \underline{d}$, for every m , $m - c(m)/2 \geq \underline{d}$ if and only if $m \geq \hat{m}$, and the following statements hold:

- (i) *For all $m \in [0, \hat{m})$, $c(m)$ is strictly convex increasing, with $c(0) = 0$, whereas for $m \geq \hat{m}$, $c(m) = ((1 - 2\sigma)m + L(\underline{d})) / (1 - \sigma)$;*
- (ii) *For all $\underline{\theta} \in [\theta_L, \theta_H]$, $\hat{\theta}(m, \underline{\theta})$ is strictly increasing in m , and $\hat{\theta}(m, \underline{\theta}) \geq \underline{\theta}$ if and only if $m \geq \hat{m}$, with equality holding if and only if $m = \hat{m}$.*

Proof. From Lemma 1, it follows that $\mathcal{C}^* = [\underline{\theta}, \theta_H]$ for some $\underline{\theta} \in [\theta_L, \theta_H]$ such that $u(t^*(\underline{\theta}); \underline{\theta}, p, m) = 0$. Since the equilibrium utility for consuming customers is strictly monotone, there can be at most one such $\underline{\theta}$. To characterize $\underline{\theta}$, we write $c(m) = \underline{\theta} - p$, and using Lemma 1 and the definition of L , we observe that

$$\begin{aligned} u(t^*(\underline{\theta}); \underline{\theta}, p, m) &= (1 - \sigma)(\theta - p) - (1 - 2\sigma)m - L(m - t^*(\theta)) \\ &= (1 - \sigma)(\underline{\theta} - p) - (1 - 2\sigma)m - L(m - \max\{(\underline{\theta} - p)/2, m - \underline{d}\}) \\ &= (1 - \sigma)(\underline{\theta} - p) - (1 - 2\sigma)m - L(\min\{m - (\underline{\theta} - p)/2, \underline{d}\}) \\ &= (1 - \sigma)c(m) - (1 - 2\sigma)m - L(\min\{m - c(m)/2, \underline{d}\}), \end{aligned}$$

therefore $u(t^*(\underline{\theta}); \underline{\theta}, p, m) = 0$ if and only if

$$c(m) = \frac{(1 - 2\sigma)m + L(\min\{m - c(m)/2, \underline{d}\})}{1 - \sigma}.$$

Recall that L is convex, with \underline{d} being its minimizer, and $L'(0) = -(1 - 2\sigma) < 0$. To show that $\hat{m} \geq \underline{d}$, we note from convexity of L that

$$L(\underline{d}) \geq L(0) + L'(0)(\underline{d} - 0) = -(1 - 2\sigma)\underline{d},$$

thus,

$$\hat{m} = 2(1 - \sigma)\underline{d} + L(\underline{d}) = \underline{d} + ((1 - 2\sigma)\underline{d} + L(\underline{d})) \geq \underline{d}.$$

We now turn to proving item (i) of the lemma. Following the definition of $c(m)$, by taking the derivative with respect to m , we get

$$c'(m) = \frac{1 - 2\sigma + L'(\min\{m - c(m)/2, \underline{d}\}) \frac{d}{dm} \min\{m - c(m)/2, \underline{d}\}}{1 - \sigma}. \quad (10)$$

Since $L'(\underline{d}) = 0$, it can be seen from above that if m is such that $m - c(m)/2 \geq \underline{d}$, then $c'(m) = (1 - 2\sigma)/(1 - \sigma) \in (0, 1)$; otherwise, if $m - c(m)/2 < \underline{d}$, then

$$c'(m) = \frac{1 - 2\sigma + L'(m - c(m)/2)(1 - c'(m)/2)}{1 - \sigma},$$

implying that

$$c'(m) = \frac{1 - 2\sigma + L'(m - c(m)/2)}{1 - \sigma + L'(m - c(m)/2)/2} < 1, \quad (11)$$

where the inequality is because $1 - 2\sigma < 1 - \sigma$ and the fact that $-(1 - 2\sigma) \leq L'(d) < 0$ for all $d \in [0, \underline{d}]$. This shows in addition that both the numerator and the denominator in (11) are positive and hence $c(m)$ is increasing. Moreover, $c'(m) \leq 1$ implies that the function $m - c(m)/2$ is strictly increasing, at a rate of (at least) $1/2$. Noting also that $c(0) = 0 < \underline{d}$, we conclude that there exists some \hat{m} such that $\hat{m} - c(\hat{m})/2 = \underline{d}$. For any $m \geq \hat{m}$, $m - c(m)/2 \geq \underline{d}$, and using the definition of $c(m)$ we get

$$c(m) = \frac{(1 - 2\sigma)m + L(\underline{d})}{1 - \sigma}. \quad (12)$$

Assume otherwise that $m < \hat{m}$, then

$$c(m) = \frac{(1 - 2\sigma)m + L(m - c(m)/2)}{1 - \sigma}.$$

Taking the second derivative of $c(m)$,

$$\begin{aligned} c''(m) &= (1 - c'(m)/2)L''(m - c(m)/2) \frac{1 - \sigma + L'(m - c(m)/2)/2 - (1 - 2\sigma + L'(m - c(m)/2))/2}{(1 - \sigma + L'(m - c(m)/2)/2)^2} \\ &= (1 - c'(m)/2)L''(m - c(m)/2) \frac{1/2}{(1 - \sigma + L'(m - c(m)/2)/2)^2} > 0, \end{aligned}$$

where the inequality follows from the convexity of $L(d)$ and that $c'(m) < 1$. Therefore $c(m)$ is strictly convex over $[0, \hat{m})$.

To prove the second item, fix $\underline{\theta} \in [\theta_L, \theta_H]$, and define

$$\Delta(m) = \hat{\theta}(\underline{\theta}, m) - \underline{\theta} = 2(m - \underline{d}) - c(m),$$

from which we have

$$\Delta'(m) = 2 - c'(m) \geq 1 > 0,$$

so that $\Delta(m)$ is strictly increasing, thus, $\hat{\theta}$ is strictly increasing in m . By substituting $m = 0$ with $c(0) = 0$ we get $\Delta(0) = -2\underline{d} < 0$, implying that $\hat{\theta}(\underline{\theta}, 0) < \underline{\theta}$ and by substituting $m = \hat{m}$, based on (12), we get

$$\begin{aligned}\Delta(\hat{m}) &= 2(\hat{m} - \underline{d}) - \frac{(1 - 2\sigma)\hat{m} + L(\underline{d})}{1 - \sigma} \\ &= 2(2(1 - \sigma)\underline{d} + L(\underline{d}) - \underline{d}) - \frac{(1 - 2\sigma)(2(1 - \sigma)\underline{d} + L(\underline{d})) + L(\underline{d})}{1 - \sigma}, \\ &= 2((1 - 2\sigma)\underline{d} + L(\underline{d})) - \frac{2(1 - \sigma)(1 - 2\sigma)\underline{d} + 2(1 - \sigma)L(\underline{d})}{1 - \sigma} = 0\end{aligned}$$

thus $\hat{\theta}(\underline{\theta}, m) = \underline{\theta}$. The second item then follows from monotonicity of $\Delta(m)$. \square

In light of Lemma 3, the proof of Proposition 1 becomes straightforward: By Lemma 3, the consumption set is given by $[\underline{\theta}, \theta_H]$ where $\underline{\theta} = p - c(m)$. Consider a type $\theta \in [\underline{\theta}, \theta_H]$. If $\theta \leq \hat{\theta}$, then after rearrangement we arrive at

$$m - \underline{d} \geq (\theta - \underline{\theta} + c(m))/2 = (\theta - p)/2,$$

which, by Lemma 1 implies $t^*(\theta) = m - \underline{d}$. Similarly, $\theta > \hat{\theta}$ implies $t^*(\theta) = (\theta - \underline{\theta} + c(m))/2$.

1.3 Proof of Lemma 2

The continuity of θ^{opt} follows from the continuity of $c(m)$ and $\bar{F}(\theta)$ together with the Maximum theorem. To show monotonicity, let m_1 and m_2 satisfy $m_1 < m_2$. For $i = 1, 2$, denote $\underline{\theta}_i = \theta^{\text{opt}}(m_i)$, and for each $\underline{\theta}$ let $\Pi_i(\underline{\theta}) = (\underline{\theta} - c(m_i))\bar{F}(\underline{\theta})$. Let f denote the density of F . Then, for every $\underline{\theta}$,

$$\begin{aligned}\frac{d}{d\underline{\theta}}\Pi_1(\underline{\theta}) &= \frac{d(\underline{\theta} - c(m_1))}{d\underline{\theta}} \cdot \bar{F}(\underline{\theta}) + \frac{d\bar{F}(\underline{\theta})}{d\underline{\theta}} \cdot (\underline{\theta} - c(m_1)) \\ &= \bar{F}(\underline{\theta}) - f(\underline{\theta})(\underline{\theta} - c(m_1)) \\ &< \bar{F}(\underline{\theta}) - f(\underline{\theta})(\underline{\theta} - c(m_2)) = \frac{d}{d\underline{\theta}}\Pi_2(\underline{\theta}),\end{aligned}$$

in which the inequality follows because $c(m)$ is increasing in m (see Lemma 3). Being the maximum point of $\Pi_2(\underline{\theta})$, the optimal cutoff $\underline{\theta}_2$ satisfies, for every $\underline{\theta} \in (\underline{\theta}_2, \theta_H]$,

$$0 \geq \Pi_2(\underline{\theta}) - \Pi_2(\underline{\theta}_2) = \int_{\underline{\theta}_2}^{\underline{\theta}} d\Pi_2 > \int_{\underline{\theta}_2}^{\underline{\theta}} d\Pi_1 = \Pi_1(\underline{\theta}) - \Pi_1(\underline{\theta}_2)$$

meaning that for every $\underline{\theta} \in (\underline{\theta}_2, \theta_H]$, $\Pi_1(\underline{\theta}) < \Pi_1(\underline{\theta}_2)$, which implies that $\underline{\theta}_1 \notin (\underline{\theta}_2, \theta_H]$, thus $\underline{\theta}_1 \leq \underline{\theta}_2$. Furthermore, if $\underline{\theta}_2$ is an interior solution, i.e., $\underline{\theta}_2 \in (\theta_L, \theta_H)$, then it must follow by the first-order condition that

$$0 = \frac{d}{d\underline{\theta}}\Pi_2(\underline{\theta}_2) > \frac{d}{d\underline{\theta}}\Pi_1(\underline{\theta}_2),$$

implying that $\underline{\theta}_1 \neq \underline{\theta}_2$. Similarly, if $\underline{\theta}_1$ is an interior solution, then $\frac{d}{d\underline{\theta}}\Pi_2(\underline{\theta}_1) > 0$, and again, $\underline{\theta}_1 \neq \underline{\theta}_2$. Thus, we conclude that if at least one of $\underline{\theta}_1$ or $\underline{\theta}_2$ is in the interior region (θ_L, θ_H) , then $\underline{\theta}_1 < \underline{\theta}_2$.

1.4 Proof of Proposition 2

In order to prove the proposition, we first introduce the function $\underline{\theta}^{\text{cons.}}(m)$, and then state and prove Lemma 4 below.

Lemma 4.

- (i) For any fixed $\underline{\theta} \in [\theta_L, \theta_H]$, the average tip $ET(m, \underline{\theta})$ as a function of m is strictly increasing, such that for each $m \geq 0$, if $\hat{\theta}(m, \underline{\theta}) < \theta_H$, then $(\partial/\partial m)ET(m, \underline{\theta}) < 1$, otherwise $ET(m, \underline{\theta}) = m - \underline{d}$.
- (ii) For any fixed $m \geq 0$, the average tip $ET(m, \underline{\theta})$ as a function of $\underline{\theta}$ is decreasing, such that for each $\underline{\theta} \in [\theta_L, \theta_H]$, if $\hat{\theta}(m, \underline{\theta}) < \theta_H$, then $(\partial/\partial \underline{\theta})ET(m, \underline{\theta}) < 0$, otherwise $(\partial/\partial \underline{\theta})ET(m, \underline{\theta}) = 0$.
- (iii) There exists a unique value $\bar{m} > 0$ that satisfies $ET(\bar{m}, \theta_L) = \bar{m}$, and for every $m \leq \bar{m}$ there exists a unique $\underline{\theta} \in [\theta_L, \theta_H]$ solving $ET(m, \underline{\theta}) = m$.
- (iv) Define a function $\underline{\theta}^{\text{cons.}} : [0, \bar{m}] \rightarrow [\theta_L, \theta_H]$ as:

$$\underline{\theta}^{\text{cons.}}(m) = \{\underline{\theta} \in [\theta_L, \theta_H] \text{ s.t. } ET(m, \underline{\theta}) = m\}.$$

Then the function $\underline{\theta}^{\text{cons.}}(m)$ is strictly decreasing over $[0, \bar{m}]$, and onto $[\theta_L, \theta_H]$.

Proof. We begin with proving item (i). Fix $\underline{\theta}$, then, for all m , by substituting Equation (4), we can rewrite Equation (6) as

$$ET(m, \underline{\theta}) = \frac{1}{\bar{F}(\underline{\theta})} \int_{\underline{\theta}}^{\theta_H} \max\{(\theta - \underline{\theta} + c(m))/2, m - \underline{d}\} dF(\theta).$$

Note, since $c(m)$ is strictly increasing (Lemma 3), that the integrand above is strictly increasing in m (for all $\theta, \underline{\theta}$), therefore $ET(m, \underline{\theta})$ is increasing in m .

Next, we show that $(\partial/\partial m)ET(m, \underline{\theta}) < 1$. Keep $\underline{\theta}$ fixed, and recall that for any $\theta \leq \hat{\theta}(m, \underline{\theta})$, $(\theta - \underline{\theta} + c(m))/2 \leq m - \underline{d}$ (and vice versa). Note that under the assumption $\hat{\theta}(m, \underline{\theta}) < \theta_H$, $\bar{F}(\hat{\theta}(m, \underline{\theta})) > 0$. Then,

$$\begin{aligned} ET(m, \underline{\theta}) &= \frac{1}{\bar{F}(\underline{\theta})} \left(\int_{\underline{\theta}}^{\hat{\theta}(m, \underline{\theta})} (m - \underline{d}) dF(\theta) + \int_{\hat{\theta}(m, \underline{\theta})}^{\theta_H} (\theta - \underline{\theta} + c(m))/2 dF(\theta) \right) \\ &= \frac{1}{\bar{F}(\underline{\theta})} \left((m - \underline{d})(F(\hat{\theta}(m, \underline{\theta})) - F(\underline{\theta})) + \int_{\hat{\theta}(m, \underline{\theta})}^{\theta_H} (\theta - \underline{\theta} + c(m))/2 dF(\theta) \right). \end{aligned} \quad (13)$$

Taking derivative with respect to m and invoking Leibniz integral rule we get

$$\begin{aligned}
\frac{\partial}{\partial m} ET(m, \underline{\theta}) &= \left(F(\hat{\theta}(m, \underline{\theta})) - F(\underline{\theta}) + (m - \underline{d}) \frac{d}{dm} F(\hat{\theta}(m, \underline{\theta})) \right. \\
&\quad \left. + c'(m) \bar{F}(\hat{\theta}(m, \underline{\theta}))/2 - (\hat{\theta} - \underline{\theta} + c(m))/2 \cdot \frac{d}{dm} F(\hat{\theta}(m, \underline{\theta})) \right) \frac{1}{\bar{F}(\underline{\theta})} \\
&= \left(\bar{F}(\underline{\theta}) - \bar{F}(\hat{\theta}(m, \underline{\theta})) + c'(m) \bar{F}(\hat{\theta}(m, \underline{\theta}))/2 \right) \frac{1}{\bar{F}(\underline{\theta})} \\
&< \left(\bar{F}(\underline{\theta}) - \bar{F}(\hat{\theta}(m, \underline{\theta})) + \bar{F}(\hat{\theta}(m)) \right) \frac{1}{\bar{F}(\underline{\theta})} = 1,
\end{aligned}$$

where the inequality follows from $c'(m) \leq 1$ (see Lemma 3) and $\bar{F}(\hat{\theta}(m, \underline{\theta})) > 0$. If, instead, $\hat{\theta}(m, \underline{\theta}) \geq \theta_H$, then $\bar{F}(\hat{\theta}(m, \underline{\theta})) = 0$, and by Equation (13), $ET(m, \underline{\theta}) = m - \underline{d}$.

We now turn to proving item (ii). Fix m , and let Θ denote a random variable distributed according to F , thus, we can write

$$ET(m, \underline{\theta}) = \mathbb{E}(\max\{(\Theta - \underline{\theta} + c(m))/2, m - \underline{d}\} \mid \Theta \geq \underline{\theta})$$

Note that the expression $\max\{(x + c(m))/2, m - \underline{d}\}$ is increasing in x . Furthermore, because F is IFR, we have (see, e.g., Shaked and Shanthikumar [2007], Theorem 1.A.30), for any two types $\underline{\theta}_1 < \underline{\theta}_2$,

$$[\Theta - \underline{\theta}_1 \mid \Theta > \underline{\theta}_1] \geq_{st} [\Theta - \underline{\theta}_2 \mid \Theta > \underline{\theta}_2],$$

implying that $ET(m, \underline{\theta}_1) \geq ET(m, \underline{\theta}_2)$.

Assume that $\hat{\theta}(m, \underline{\theta}_1) < \theta_H$. For ease of notation, define two random variables D_1 and D_2 on the same probability space, such that for $i = 1, 2$, $D_i =_{st} [\Theta - \underline{\theta}_i \mid \Theta > \underline{\theta}_i]$. Since F is strictly increasing over $[\theta_L, \theta_H]$, we can assume, without loss of generality, that $\Pr(D_1 > D_2) = 1$, thus $(D_1 + c(m))/2 > (D_2 + c(m))/2$ almost surely. Our assumption $\hat{\theta}(m, \underline{\theta}_1) < \theta_H$ implies

$$\Pr((D_1 + c(m))/2 > m - \underline{d}) > 0,$$

therefore,

$$\Pr((D_1 + c(m))/2 > \max\{(D_2 + c(m))/2, m - \underline{d}\}) > 0,$$

implying that

$$\begin{aligned}
ET(m, \underline{\theta}_1) &= \mathbb{E}(\max\{(\Theta - \underline{\theta}_1 + c(m))/2, m - \underline{d}\} \mid \Theta \geq \underline{\theta}_1) \\
&= \mathbb{E}(\max\{(D_1 + c(m))/2, m - \underline{d}\}) \\
&> \mathbb{E}(\max\{(D_2 + c(m))/2, m - \underline{d}\}) \\
&= \mathbb{E}(\max\{(\Theta - \underline{\theta}_2 + c(m))/2, m - \underline{d}\} \mid \Theta \geq \underline{\theta}_2) = ET(m, \underline{\theta}_2),
\end{aligned}$$

thus, ET is strictly decreasing in $\underline{\theta}$. If, on the contrary, $\hat{\theta}(m, \underline{\theta}_1) \geq \theta_H$, then

$$\Pr((D_1 + c(m))/2 > \max\{(D_2 + c(m))/2, m - \underline{d}\}) = 0,$$

implying that $ET(m, \underline{\theta}_1) = m - \underline{d}$.

We move on to the proof of item (iii). First note, by item (i), that whenever $\hat{\theta}(m, \theta_L) < \theta_H$, $m - ET(m, \theta_L)$ is strictly increasing in m , hence, if a solution $\bar{m} - ET(\bar{m}, \theta_L) = 0$ exists such that $\hat{\theta}(\bar{m}, \theta_L) < \theta_H$, it must be unique. Let \hat{m}^{sup} be the unique solution to $\hat{\theta}(\hat{m}^{\text{sup}}, \theta_L) = \theta_H$, thus, by Lemma 3, for every $m < \hat{m}^{\text{sup}}$, $\hat{\theta}(m, \theta_L) < \theta_H$. To show that a solution \bar{m} exists, it suffices to show

$$0 - ET(0, \theta_L) \leq 0 < \hat{m}^{\text{sup}} - ET(\hat{m}^{\text{sup}}, \theta_L).$$

The left segment, $-ET(0, \theta_L) \leq 0$, is directly implied by Equation (6) and the non-negativity of the tip function t . The right segment follows from item (i), observing that, from $\hat{\theta}(\hat{m}^{\text{sup}}, \theta_L) = \theta_H$,

$$\hat{m}^{\text{sup}} - ET(\hat{m}^{\text{sup}}, \theta_L) = \hat{m}^{\text{sup}} - (\hat{m}^{\text{sup}} - \underline{d}) = \underline{d} > 0.$$

Hence, \bar{m} exists uniquely.

Consider now a fixed $m \in [0, \bar{m}]$. This implies that $m - ET(m, \theta_L) \leq \bar{m} - ET(\bar{m}, \theta_L) = 0$, i.e., $m \leq ET(m, \theta_L)$. Recall from Lemma 3 that $c(m) \leq m$, thus,

$$ET(m, \theta_H) = \max\{c(m)/2, m - \underline{d}\} \leq m \leq ET(m, \theta_L).$$

Therefore a solution $\underline{\theta} \in [\theta_L, \theta_H]$ exists such that $ET(m, \underline{\theta}) = m$. Furthermore, if $\underline{\theta}$ is such that $ET(m, \underline{\theta}) = m$, then from item (i) it must hold that $\hat{\theta}(m, \underline{\theta}) \leq \theta_H$. Thus, by item (ii) $ET(m, \underline{\theta})$ is increasing as a function of its second argument around the point $\underline{\theta}$, implying that $\underline{\theta}$ uniquely solve $ET(m, \underline{\theta}) = m$.

To prove the last item, fix an arbitrary $\underline{\theta} \in [\theta_L, \theta_H]$. Because the average tip is non-negative, when $m = 0$,

$$m - ET(m, \underline{\theta}) = -ET(0, \underline{\theta}) \leq 0.$$

From items (ii) and (iii), when $m = \bar{m}$,

$$m - ET(m, \underline{\theta}) = \bar{m} - ET(\bar{m}, \underline{\theta}) \geq \bar{m} - ET(\bar{m}, \theta_L) = 0.$$

By continuity, there must exist some $m_{\underline{\theta}} \in [0, \bar{m}]$ such that $m_{\underline{\theta}} - ET(m_{\underline{\theta}}, \underline{\theta}) = 0$. Furthermore, for this value $m_{\underline{\theta}} \in [0, \bar{m}]$ it must hold that $\hat{\theta}(m_{\underline{\theta}}, \underline{\theta}) < \theta_H$, otherwise we would have $m_{\underline{\theta}} - ET(m_{\underline{\theta}}, \underline{\theta}) = \underline{d} > 0$. Thus, by Lemma 4 item (i), $m - ET(m, \underline{\theta})$ as a function of m must be strictly increasing around the point $m_{\underline{\theta}}$, and hence, there exists no other $m \neq m_{\underline{\theta}}$ solving $m - ET(m, \underline{\theta}) = 0$. In other words, the function $\underline{\theta}^{\text{cons.}}(m)$ is a continuous bijection from $[0, \bar{m}]$ to $[\theta_L, \theta_H]$, and therefore is strictly monotone. Observing that

$$\underline{\theta}^{\text{cons.}}(0) = \theta_H > \theta_L = \underline{\theta}^{\text{cons.}}(\bar{m}),$$

we conclude that $\underline{\theta}^{\text{cons.}}(m)$ is strictly decreasing over $[0, \bar{m}]$. □

We now turn to the proof of Proposition 2. We start by deriving several essential properties of $BR(m)$. The function BR is continuous over the bounded interval $[0, \bar{m}]$ as a composition of such functions. We first show that it is increasing, by showing that $t(\theta; m, \underline{\theta}^{\text{opt.}}(m))$ is increasing in m . Recall that from Lemma 3 we have $\frac{d}{dm}c(m) \leq 1 - 2\sigma$, and from Lemma 2, $\frac{d}{dm}\underline{\theta}^{\text{opt.}} \geq 0$. Thus, using $t^*(\theta)$ in Lemma 1, for all $\theta \in [\underline{\theta}^{\text{opt.}}(m), \theta_H]$

$$\begin{aligned} \frac{d}{dm}t(\theta; m, \underline{\theta}^{\text{opt.}}(m)) &= \frac{d}{dm} \max\{(\theta - \underline{\theta}^{\text{opt.}}(m) + c(m))/2, m - \underline{d}\} \\ &\geq \min \left\{ \left(\frac{d}{dm}c(m) - \frac{d}{dm}\underline{\theta}^{\text{opt.}}(m) \right) / 2, 1 \right\} \\ &= \left(\frac{d}{dm}c(m) - \frac{d}{dm}\underline{\theta}^{\text{opt.}}(m) \right) / 2. \end{aligned}$$

To show that the latter expression is non-negative, recall that f denotes the density of F . Note, by Assumption 1, because $\underline{\theta}^{\text{opt.}}(m)$ satisfies the first-order condition for the profit function, we have

$$0 = \frac{\partial}{\partial \underline{\theta}} \left((\underline{\theta} - c(m)) \bar{F}(\underline{\theta}) \right) \Big|_{\underline{\theta} = \underline{\theta}^{\text{opt.}}(m)} = \bar{F}(\underline{\theta}^{\text{opt.}}(m)) - f(\underline{\theta}^{\text{opt.}}(m))(\underline{\theta}^{\text{opt.}}(m) - c(m)),$$

or equivalently,

$$c(m) = \underline{\theta}^{\text{opt.}}(m) - \frac{\bar{F}(\underline{\theta}^{\text{opt.}}(m))}{f(\underline{\theta}^{\text{opt.}}(m))}, \quad (14)$$

and, by Assumption 2, $\frac{f(\theta)}{F(\theta)}$ is increasing in θ . We thus have that

$$\frac{d}{dm}c(m) = \frac{d}{dm}\underline{\theta}^{\text{opt.}}(m) - \frac{d}{dm} \left(\frac{\bar{F}(\underline{\theta}^{\text{opt.}}(m))}{f(\underline{\theta}^{\text{opt.}}(m))} \right) \geq \frac{d}{dm}\underline{\theta}^{\text{opt.}}(m),$$

implying that $t(\theta; m, \underline{\theta}^{\text{opt.}}(m))$ is indeed increasing in m for all $\theta \geq \underline{\theta}^{\text{opt.}}(m)$.

Now, consider $m_0, m_1 \in [0, \bar{m}]$ such that $m_0 < m_1$, hence, by Lemma 2, $\underline{\theta}^{\text{opt.}}(m_0) \leq \underline{\theta}^{\text{opt.}}(m_1)$. Note, by Lemma 1, that the tip is increasing with type, therefore

$$\frac{\int_{\underline{\theta}^{\text{opt.}}(m_0)}^{\theta_H} t(\theta; m_0, \underline{\theta}^{\text{opt.}}(m_0)) dF(\theta)}{\bar{F}(\underline{\theta}^{\text{opt.}}(m_0))} \leq \frac{\int_{\underline{\theta}^{\text{opt.}}(m_1)}^{\theta_H} t(\theta; m_0, \underline{\theta}^{\text{opt.}}(m_0)) dF(\theta)}{\bar{F}(\underline{\theta}^{\text{opt.}}(m_1))}.$$

Thus,

$$\begin{aligned} BR(m_1) - BR(m_0) &= ET(m_1, \underline{\theta}^{\text{opt.}}(m_1)) - ET(m_0, \underline{\theta}^{\text{opt.}}(m_0)) \\ &= \frac{\int_{\underline{\theta}^{\text{opt.}}(m_1)}^{\theta_H} t(\theta; m_1, \underline{\theta}^{\text{opt.}}(m_1)) dF(\theta)}{\bar{F}(\underline{\theta}^{\text{opt.}}(m_1))} - \frac{\int_{\underline{\theta}^{\text{opt.}}(m_0)}^{\theta_H} t(\theta; m_0, \underline{\theta}^{\text{opt.}}(m_0)) dF(\theta)}{\bar{F}(\underline{\theta}^{\text{opt.}}(m_0))} \\ &\geq \frac{\int_{\underline{\theta}^{\text{opt.}}(m_1)}^{\theta_H} t(\theta; m_1, \underline{\theta}^{\text{opt.}}(m_1)) dF(\theta)}{\bar{F}(\underline{\theta}^{\text{opt.}}(m_1))} - \frac{\int_{\underline{\theta}^{\text{opt.}}(m_1)}^{\theta_H} t(\theta; m_0, \underline{\theta}^{\text{opt.}}(m_0)) dF(\theta)}{\bar{F}(\underline{\theta}^{\text{opt.}}(m_1))} \\ &= \frac{1}{\bar{F}(\underline{\theta}^{\text{opt.}}(m_1))} \int_{\underline{\theta}^{\text{opt.}}(m_1)}^{\theta_H} (t(\theta; m_1, \underline{\theta}^{\text{opt.}}(m_1)) - t(\theta; m_0, \underline{\theta}^{\text{opt.}}(m_0))) dF(\theta) \\ &\geq 0, \end{aligned}$$

where the last inequality is because $t(\theta; m, \underline{\theta}^{\text{opt.}}(m))$ is increasing in m .

We next show that the derivative of BR is bounded. Recall, by Lemma 4, that ET is decreasing in its second argument, and its derivative with respect to the first argument is positive and bounded by 1. We therefore obtain

$$\begin{aligned} 0 \leq \frac{BR(m_1) - BR(m_0)}{m_1 - m_0} &= \frac{ET(m_1, \underline{\theta}^{\text{opt.}}(m_1)) - ET(m_0, \underline{\theta}^{\text{opt.}}(m_0))}{m_1 - m_0} = \\ &\leq \frac{ET(m_1, \underline{\theta}^{\text{opt.}}(m_0)) - ET(m_0, \underline{\theta}^{\text{opt.}}(m_0))}{m_1 - m_0} \leq 1 \end{aligned}$$

implying that BR is increasing at a rate smaller than 1. Assuming a fixed point $m^* = BR(m^*)$ exists, it follows that for every initial value $m_0 \in [0, \bar{m}]$, the sequence of fixed-point iterations $m_k = BR(m_{k-1})$ converges monotonically, from below, to one such fixed point as $k \rightarrow +\infty$. We next show that indeed a fixed point exists uniquely.

To this aim, we recall the definition of $\underline{\theta}^{\text{cons.}}$ in Lemma 4, and we note that a point $m \in [0, \bar{m}]$ is a fixed point of BR if and only if

$$\underline{\theta}^{\text{opt.}}(m) = \underline{\theta}^{\text{cons.}}(m).$$

By Lemma 4 item (iv), $\underline{\theta}^{\text{cons.}}(m)$ is strictly decreasing over $[0, \bar{m}]$ and is onto $[\theta_L, \theta_H]$. Following Lemma 2, $\underline{\theta}^{\text{opt.}}(m)$ is increasing, thus, $\underline{\theta}^{\text{cons.}}(m) - \underline{\theta}^{\text{opt.}}(m)$ is strictly decreasing in m . We conclude therefore that there exists a unique intersection point $m \in [0, \bar{m}]$, satisfying $\underline{\theta}^{\text{cons.}}(m) = \underline{\theta}^{\text{opt.}}(m)$. We thus obtain that $\lim_{k \rightarrow +\infty} m_k = m^*$ such that $\underline{\theta}^{\text{cons.}}(m^*) = \underline{\theta}^{\text{opt.}}(m^*)$, which is further invariant to m_0 . By continuity, we also have that $\underline{\theta}^* = \lim_{k \rightarrow +\infty} \underline{\theta}^{\text{opt.}}(m_k)$ and $p^* = \underline{\theta}^* - c(m^*)$. Finally, Equation (14) implies $\underline{\theta}^{\text{opt.}}(m) - c(m) = \bar{F}(\underline{\theta}^{\text{opt.}}(m))/f(\underline{\theta}^{\text{opt.}}(m))$ which is decreasing in m , therefore, with $m_0 \leq m_1 \leq \dots$ we have $p_0 \geq p_1 \geq \dots$

1.5 Proof of Proposition 3

To show our claim, we show first that BR is pointwise increasing in κ (see Lemma 9 below), the proof of which relies on the following auxiliary results:

Lemma 5. \underline{d} is decreasing in κ , with $\lim_{\kappa \rightarrow +\infty} \underline{d} = 0$ and $\lim_{\kappa \rightarrow 0} \underline{d} = \infty$.

Proof. Recall that ℓ is strictly convex and smooth over $[0, \infty)$, therefore ℓ' is continuous and strictly increasing over $[0, \infty)$, and so is its inverse, ℓ'^{-1} . Also recall that $L(d) = -(1 - 2\sigma)d + \kappa\ell(d)$ is strictly convex and smooth, thus, its minimizer, \underline{d} , satisfies the first-order condition, which after rearrangement can be expressed as

$$\underline{d} = \ell'^{-1} \left(\frac{1 - 2\sigma}{\kappa} \right). \quad (15)$$

By continuity, we therefore have $\lim_{\kappa \rightarrow +\infty} \underline{d} = \ell'^{-1}(0) = 0$. Moreover, as $\kappa \rightarrow 0$, $L(d)$ compactly approaches $-(1 - 2\sigma)d$, which is a strictly decreasing function, thus, for every $M \geq 0$, there exists some sufficiently small $\kappa_M > 0$ such that for all $\kappa \leq \kappa_M$, the minimum of L over the interval $[0, M]$ is obtained at M , and therefore $\underline{d} \geq M$. \square

Lemma 6. $c(m)$ is increasing in κ for every $m \geq 0$, approaches $(1 - 2\sigma)m/(1 - \sigma)$ uniformly as $\kappa \rightarrow +\infty$, and approaches 0 compactly as $\kappa \rightarrow 0$.

Proof. According to Lemma 3, given m , $c(m)$ uniquely solves

$$c = \frac{(1 - 2\sigma)m + L(\min\{m - c/2, \underline{d}\})}{1 - \sigma} \quad (16)$$

By the envelope theorem, $\frac{\partial}{\partial \kappa} L(\underline{d}) = \ell(\underline{d}) > 0$, meaning that $L(\underline{d})$ is increasing in κ , and in addition, for any fixed d (independent of κ), $L(d)$ is also increasing in κ . Thus, for fixed m and c , the expression $L(\min\{m - c/2, \underline{d}\})$ is increasing in κ . Hence, the right-hand side of Equation (16) is increasing in κ . We now consider growth rates with respect to c of the expressions at each side of Equation (16). Clearly, the marginal growth in c of the left-hand side is 1, whereas the on the right hand side, it is bounded by

$$\frac{-L'(\min\{m - c/2, \underline{d}\})}{2(1 - \sigma)} \leq \frac{1}{2} \cdot \frac{1 - 2\sigma}{1 - \sigma} < \frac{1}{2}$$

where the first inequality follows since $-(1 - 2\sigma) \leq L'(d) \leq 0$ for all $d \in [0, \underline{d}]$. It therefore follows that the left-hand side of (16) is increasing in c faster than the right-hand side, thus, it must follow that the solution to (16), namely $c(m)$, is increasing in κ . Furthermore, as $\kappa \rightarrow +\infty$, $\underline{d} \rightarrow 0$, and by continuity, we therefore have $\lim_{\kappa \rightarrow +\infty} L(\underline{d}) = 0$. By Lemma 3 part (i), we have for any $m \geq 0$ that

$$\frac{(1 - 2\sigma)m + L(\underline{d})}{1 - \sigma} \leq c(m) \leq \frac{(1 - 2\sigma)m}{1 - \sigma},$$

hence $c(m)$ approaches $(1 - 2\sigma)m/(1 - \sigma)$ uniformly as $\kappa \rightarrow +\infty$.

To analyze the case $\kappa \rightarrow 0$, recall \underline{d} is the minimum point of L , thus, given m ,

$$\begin{aligned} c(m) &= \frac{(1 - 2\sigma)m + L(\min\{m - c(m)/2, \underline{d}\})}{1 - \sigma} \\ &\leq \frac{(1 - 2\sigma)m + L(m - c(m)/2)}{1 - \sigma} \\ &= \frac{(1 - 2\sigma)c(m)/2 + \kappa\ell(m - c(m)/2)}{1 - \sigma} \\ &\leq \frac{(1 - 2\sigma)c(m)/2 + \kappa\ell(m)}{1 - \sigma} \end{aligned}$$

hence, after rearranging we get $c(m) \leq 2\kappa\ell(m)$. Thus, as $\kappa \rightarrow 0$, $c(m) \rightarrow 0$ for every m . In addition, over any interval $[m_0, m_1]$, ℓ is maximized at m_1 , thus $2\kappa\ell(m_1)$ uniformly bounds $c(m)$ over the interval $[m_0, m_1]$, and the convergence is uniform on compact sets. \square

Lemma 7. For every $m \geq 0$, $\underline{\theta}^{\text{opt.}}(m)$ is increasing in κ .

Proof. By definition of $\underline{\theta}^{\text{opt.}}$, given m , if $\underline{\theta} = \underline{\theta}^{\text{opt.}}(m)$, then following Assumption 1, $\underline{\theta}$ and m satisfy (see Equation (14))

$$c(m) = \underline{\theta} - \frac{\bar{F}(\underline{\theta})}{f(\underline{\theta})}.$$

Note that this equality depends on κ only through the left-hand side, $c(m)$, which, according to Lemma 6, is increasing in κ . The right-hand side, by Assumption 2, is increasing in $\underline{\theta}$, hence, $\underline{\theta}^{\text{opt.}}(m)$ is increasing in κ for every m . \square

Lemma 8. *For every $m \geq 0$ and $\theta \geq \underline{\theta}^{\text{opt.}}(m)$, $t(\theta; m, \underline{\theta}^{\text{opt.}}(m))$ is increasing in κ .*

Proof. From Lemma 1 and Equation (14) we have

$$\begin{aligned} t(\theta; m, \underline{\theta}^{\text{opt.}}(m)) &= \max\{(\theta - \underline{\theta}^{\text{opt.}}(m) + c(m))/2, m - \underline{d}\} \\ &= \max\left\{\left(\theta - \frac{\bar{F}(\underline{\theta}^{\text{opt.}}(m))}{f(\underline{\theta}^{\text{opt.}}(m))}\right)/2, m - \underline{d}\right\}. \end{aligned}$$

From Lemma 5, \underline{d} is decreasing in κ , and from Lemma 7 together with Assumption 2, $\frac{\bar{F}(\underline{\theta}^{\text{opt.}}(m))}{f(\underline{\theta}^{\text{opt.}}(m))}$ is also decreasing in κ for any m . Therefore $t(\theta; m, \underline{\theta}^{\text{opt.}}(m))$ is increasing in κ for any θ and m . \square

Lemma 9. *$BR(m)$ is increasing in κ for every m .*

Proof. Consider m fixed, and for $\kappa_1 < \kappa_2$ and $i = 1, 2$, let $r^{(i)} = BR(m) |_{\kappa=\kappa_i}$ and $\underline{\theta}^{(i)} = \underline{\theta}^{\text{opt.}}(m) |_{\kappa=\kappa_i}$ be the best response and optimal cutoff corresponding with the customary tip m and social-pressure level κ_i . By Lemma 7, $\underline{\theta}^{(1)} \leq \underline{\theta}^{(2)}$. For $i = 1, 2$, denote further, for any $\theta \geq \underline{\theta}^{(i)}$

$$t^{(i)}(\theta) = t(\theta; m, \underline{\theta}^{\text{opt.}}(m)) |_{\kappa=\kappa_i},$$

hence, by Lemma 8 we have $t^{(2)}(\theta) \geq t^{(1)}(\theta)$. Using this notation we can thus write

$$r^{(i)} = \frac{\int_{\underline{\theta}^{(i)}}^{\theta_H} t^{(i)}(\theta) dF(\theta)}{\bar{F}(\underline{\theta}^{(i)})}.$$

Then,

$$\begin{aligned} r^{(2)} - r^{(1)} &= \frac{\int_{\underline{\theta}^{(2)}}^{\theta_H} t^{(2)}(\theta) dF(\theta)}{\bar{F}(\underline{\theta}^{(2)})} - \frac{\int_{\underline{\theta}^{(1)}}^{\theta_H} t^{(1)}(\theta) dF(\theta)}{\bar{F}(\underline{\theta}^{(1)})} \\ &\geq \frac{\int_{\underline{\theta}^{(2)}}^{\theta_H} t^{(2)}(\theta) dF(\theta)}{\bar{F}(\underline{\theta}^{(2)})} - \frac{\int_{\underline{\theta}^{(2)}}^{\theta_H} t^{(1)}(\theta) dF(\theta)}{\bar{F}(\underline{\theta}^{(2)})} \\ &= \frac{1}{\bar{F}(\underline{\theta}^{(2)})} \int_{\underline{\theta}^{(1)}}^{\theta_H} (t^{(2)}(\theta) - t^{(1)}(\theta)) dF(\theta) \\ &\geq 0. \end{aligned}$$

where the first inequality follows due to $t^{(1)}$ being monotone increasing in θ . Thus, $BR(m)$ is increasing in κ for all m . \square

We now turn to the proof of Proposition 3. By Lemma 9, $BR(m)$ is pointwise increasing in κ , therefore its unique fixed point m^* is also increasing in κ . Recalling that $\underline{\theta}^{\text{opt.}}$ is increasing both as a function of m (for every κ , see Lemma 2) and as a function κ (for every m , see Lemma 7),

we therefore obtain that $\underline{\theta}^* = \underline{\theta}^{\text{opt.}}(m^*)$ is increasing in κ . Based on Assumption 1, we have, as in Equation (14), that

$$p^* = \underline{\theta}^* - c(m^*) = \frac{\bar{F}(\underline{\theta}^*)}{f(\underline{\theta}^*)},$$

thus, from Assumption 2 and the monotonicity of $\underline{\theta}^*$, p^* is decreasing in κ . Clearly, θ_H is a uniform upper bound (independent of κ) on the customer utility, thus, under rational tipping, no customer tips more than θ_H , and the average tip amount never exceeds θ_H . As $\underline{\theta}^*$ is confined to $[\theta_L, \theta_H]$ and m^* is confined to $[0, \theta_H]$, we obtain, from monotonicity, that both m^* and $\underline{\theta}^*$ admit finite limits at $\kappa \rightarrow 0$ as well as at $\kappa \rightarrow +\infty$.

Next, we prove the second and third items of Proposition 3, analyzing the case $\kappa \rightarrow 0$ first. From Lemma 5, there exists some sufficiently small $\kappa_0 > 0$ such that for all $\kappa < \kappa_0$, $\underline{d} > \theta_H$. As $m^* \leq \theta_H$, this implies that for all $\kappa < \kappa_0$, $m^* - \underline{d} < 0$, implying by Lemma 3 that $m^* < \hat{m}$.

We now consider the case $\kappa \rightarrow +\infty$. Recall that under Assumption 2, $\frac{f(\theta)}{\bar{F}(\theta)}$ is increasing in θ . We next define a quantity, independent of κ , that will be shown to constitute an upper bound on the limiting cutoff type. Define

$$\theta^{\text{maj}} = \min \left\{ \theta \in [\theta_L, \theta_H) \text{ s.t. } 2\theta - \frac{\bar{F}(\theta)}{f(\theta)} \geq \theta_H \right\} \quad (17)$$

The expression $2\theta - \frac{\bar{F}(\theta)}{f(\theta)}$ is continuous and strictly increasing in θ , and at $\theta = \theta_H$ it is strictly larger than θ_H (and equals $2\theta_H$), thus, θ^{maj} exists (and satisfies $\theta^{\text{maj}} < \theta_H$). Furthermore, for all $\theta \in (\theta^{\text{maj}}, \theta_H]$, because of the strict monotonicity, we have $2\theta - \frac{\bar{F}(\theta)}{f(\theta)} > \theta_H$. We will show that $\lim_{\kappa \rightarrow +\infty} \underline{\theta}^* \leq \theta^{\text{maj}}$.

Denote $\underline{\theta}' = \frac{1}{2}\theta^{\text{maj}} + \frac{1}{2}\lim_{\kappa \rightarrow +\infty} \underline{\theta}^*$, and for the sake of contradiction, assume $\lim_{\kappa \rightarrow +\infty} \underline{\theta}^* > \theta^{\text{maj}}$, so that $\underline{\theta}' > \theta^{\text{maj}}$. Then for any sufficiently large κ with its corresponding equilibrium pair, $\underline{\theta}^*$ and m^* , we have $\underline{\theta}^* > \underline{\theta}' > \theta^{\text{maj}}$, therefore

$$2\underline{\theta}^* - \frac{\bar{F}(\underline{\theta}^*)}{f(\underline{\theta}^*)} > 2\underline{\theta}' - \frac{\bar{F}(\underline{\theta}')}{f(\underline{\theta}')} > \theta_H.$$

Let $\epsilon = 2\underline{\theta}' - \frac{\bar{F}(\underline{\theta}')}{f(\underline{\theta}')} - \theta_H$. From Lemma 5, $\lim_{\kappa \rightarrow +\infty} \underline{d} = 0$ and from Lemma 6, $\lim_{\kappa \rightarrow +\infty} c(m) = (1 - 2\sigma)m/(1 - \sigma)$, uniformly over m , hence,

$$\lim_{\kappa \rightarrow +\infty} 2(\underline{d} - m + c(m)) = -\frac{\sigma}{1 - 2\sigma}m < 0,$$

uniformly for all m . Thus, we can choose κ large enough so that both $\underline{\theta}^* > \underline{\theta}'$ and $2(\underline{d} - m + c(m)) < \epsilon$ for all m . In particular, such a choice of κ implies $2(\underline{d} - m^* + c(m^*)) < \epsilon < 2\underline{\theta}^* - \frac{\bar{F}(\underline{\theta}^*)}{f(\underline{\theta}^*)} - \theta_H$. By Proposition 2, $\underline{\theta}^* = \underline{\theta}^{\text{opt.}}(m^*)$, therefore $\underline{\theta}^*$ and m^* satisfy (see Equation (14)) $c(m^*) = \underline{\theta}^* - \bar{F}(\underline{\theta}^*)/f(\underline{\theta}^*)$. In equilibrium, some customers tip their appreciation tip, i.e., it must follow that

$\hat{\theta}(m^*, \underline{\theta}^*) < \theta_H$ (see Proposition 1). Therefore, from the choice of ϵ ,

$$\begin{aligned}
\theta_H &> \hat{\theta}(m^*, \underline{\theta}^*) \\
&= 2(m^* - \underline{d}) + \underline{\theta}^* - c(m^*) \\
&= 2c(m^*) - 2(\underline{d} - m^* + c(m^*)) + \underline{\theta}^* - c(m^*) \\
&> 2\underline{\theta}^* - 2\frac{\bar{F}(\underline{\theta}^*)}{f(\underline{\theta}^*)} - \epsilon + \frac{\bar{F}(\underline{\theta}^*)}{f(\underline{\theta}^*)} \\
&= 2\underline{\theta}^* - \frac{\bar{F}(\underline{\theta}^*)}{f(\underline{\theta}^*)} - \epsilon \\
&> \theta_H,
\end{aligned}$$

which is a contradiction. Therefore, $\lim_{\kappa \rightarrow +\infty} \underline{\theta}^* \leq \theta^{\text{maj}}$, implying that $\lim_{\kappa \rightarrow +\infty} \bar{F}(\underline{\theta}^*) \geq \bar{F}(\theta^{\text{maj}}) > 0$.

We next show that $\lim_{\kappa \rightarrow +\infty} \hat{\theta}(m^*, \underline{\theta}^*) = \theta_H$. Suppose, for the sake of contradiction, that $\lim_{\kappa \rightarrow +\infty} \hat{\theta}(m^*, \underline{\theta}^*) < \theta_H$, and choose some θ' such that

$$\lim_{\kappa \rightarrow +\infty} \hat{\theta}(m^*, \underline{\theta}^*) < \theta' < \theta_H.$$

Assume κ is sufficiently large, so that the equilibrium pair $(m^*, \underline{\theta}^*)$ satisfies $\hat{\theta}(m^*, \underline{\theta}^*) < \theta'$, and \underline{d} satisfies

$$\underline{d} < \frac{1}{2} \int_{\theta'}^{\theta_H} (\theta - \theta') dF(\theta).$$

Since $(m^*, \underline{\theta}^*)$ is consistent (as in Definition 3), we have (see Equation 13):

$$\begin{aligned}
m^* &= ET(m^*, \underline{\theta}^*) \\
&= m^* - \underline{d} + \frac{1}{\bar{F}(\hat{\theta}(m^*, \underline{\theta}^*))} \int_{\hat{\theta}(m^*, \underline{\theta}^*)}^{\theta_H} ((\theta - \underline{\theta}^* + c(m^*))/2 - (m^* - \underline{d})) dF(\theta) \\
&> m^* - \underline{d} + \int_{\theta'}^{\theta_H} ((\theta - \underline{\theta}^* + c(m^*))/2 - (m^* - \underline{d})) dF(\theta) \\
&= m^* - \underline{d} + \frac{1}{2} \int_{\theta'}^{\theta_H} (\theta - \hat{\theta}(m^*, \underline{\theta}^*)) dF(\theta) \\
&> m^* - \underline{d} + \frac{1}{2} \int_{\theta'}^{\theta_H} (\theta - \theta') dF(\theta) \\
&> m^*
\end{aligned}$$

which is a contradiction. It must follow then that $\lim_{\kappa \rightarrow +\infty} \hat{\theta}(m^*, \underline{\theta}^*) = \theta_H$. The latter, together with the fact that $\lim_{\kappa \rightarrow +\infty} \underline{\theta}^* < \theta_H$, implies that there exists a κ_1 such that if $\kappa > \kappa_1$ then $\hat{\theta}(m^*, \underline{\theta}^*) > \underline{\theta}^*$ and therefore, by Lemma 3, $m^* > \hat{m}$.

Lastly, we assume that ℓ' is concave, and show that \hat{m} is decreasing in κ . Note from the definition of \hat{m} in Lemma 3 that

$$\hat{m} = 2(1 - \sigma)\underline{d} + L(\underline{d}) = \underline{d} + \kappa\ell(\underline{d}).$$

To show that \hat{m} is decreasing in κ , with Lemma 5 it suffices to show that $\kappa\ell(\underline{d})$ is decreasing. Denote $g(d) = \ell'^{-1}(d)$, thus, g is convex increasing with $g(0) = 0$, and therefore $g(d) \leq dg'(d)$ for all d . Following Equation (15), $\underline{d} = g(1/\kappa)$, which further implies

$$\frac{d}{d\kappa}\underline{d} = g'(1/\kappa) \cdot \frac{-1}{\kappa^2}.$$

Note that the fact that ℓ is strictly convex with $\ell(0) = 0$ implies $\ell(d) < d\ell'(d)$ for all $d > 0$. Then, using the convexity of ℓ and g ,

$$\begin{aligned} \frac{d}{d\kappa}(\kappa\ell(\underline{d})) &= \ell(\underline{d}) + \kappa\ell'(\underline{d}) \cdot g'(1/\kappa) \cdot \frac{-1}{\kappa^2} \\ &= \ell(g(1/\kappa)) - \frac{1}{\kappa^2}g'(1/\kappa) \\ &< \frac{1}{\kappa} \left(g(1/\kappa) - \frac{1}{\kappa}g'(1/\kappa) \right) \leq 0. \end{aligned}$$

Thus, \hat{m} is (strictly) decreasing. Because m^* is increasing in κ , there exists some $\hat{\kappa}$ such that $m^* \leq \hat{m}$ if and only if $\kappa \leq \hat{\kappa}$, that is, $\hat{\kappa} = \kappa_0 = \kappa_1$.

1.6 Proof of Proposition 4

Fix $m \geq 0$. We first analyse the limit as $\kappa \rightarrow 0$. Using Assumption 1, we can apply the first-order condition on (5) and solve for $\underline{\theta}^{\text{opt.}}(m)$, obtaining

$$\underline{\theta}^{\text{opt.}}(m) = c(m) + \frac{\bar{F}(\underline{\theta}^{\text{opt.}}(m))}{f(\underline{\theta}^{\text{opt.}}(m))} = c(m) + \theta_H - \underline{\theta}^{\text{opt.}}(m),$$

hence,

$$\underline{\theta}^{\text{opt.}}(m) = \frac{\theta_H + c(m)}{2}. \quad (18)$$

Based on Lemma 6, we get, as $\kappa \rightarrow 0$ that $\underline{\theta}^{\text{opt.}}(m) \rightarrow \frac{\theta_H}{2}$. By continuity, this implies, for every $m \geq 0$

$$\lim_{\kappa \rightarrow 0} \underline{\theta}^{\text{opt.}}(m) - c(m) = \lim_{\kappa \rightarrow 0} \theta^{\text{opt.}}(m) = \frac{\theta_H}{2},$$

and

$$\begin{aligned} \lim_{\kappa \rightarrow 0} BR(m) &= \lim_{\kappa \rightarrow 0} ET(m, \theta_H/2) \\ &= \frac{1}{\bar{F}(\theta_H/2)} \lim_{\kappa \rightarrow 0} \int_{\theta_H/2}^{\theta_H} t(\theta; m, \theta_H/2) dF(\theta) \\ &= \frac{2}{\theta_H} \cdot \frac{1}{2} \int_{\theta_H/2}^{\theta_H} (\theta - \theta_H/2) d\theta = \frac{\theta_H}{8}, \end{aligned}$$

where the first and second transitions follow by definition, and the third follows Equation (4) together with Lemma 5.

To derive the limit as $\kappa \rightarrow +\infty$, note that under uniformly-distributed types, $\underline{\theta}^{\text{opt.}}(m) = \frac{\theta_H + c(m)}{2}$, and recall by Lemma 6 that

$$\lim_{\kappa \rightarrow +\infty} c(m) = \frac{1 - 2\sigma}{1 - \sigma} m,$$

uniformly for all m . Similarly to the case $\kappa \rightarrow 0$, using continuity and the fact that $\lim_{\kappa \rightarrow +\infty} \underline{d} \rightarrow 0$, we get

$$\begin{aligned} \lim_{\kappa \rightarrow 0} BR(m) &= \lim_{\kappa \rightarrow 0} ET(m, (\theta_H + c(m))/2) \\ &= \frac{1}{\bar{F}((\theta_H + c(m))/2)} \lim_{\kappa \rightarrow 0} \int_{(\theta_H + c(m))/2}^{\theta_H} t(\theta; m, (\theta_H + c(m))/2) dF(\theta) \\ &= \frac{2}{\theta_H - c(m)} \int_{(\theta_H + c(m))/2}^{\theta_H} \max\{m, (\theta - (\theta_H - c(m))/2)/2\} d\theta, \end{aligned}$$

hence, for $m \leq \frac{1-\sigma}{3-2\sigma} \theta_H$, we have

$$\begin{aligned} \lim_{\kappa \rightarrow 0} BR(m) &= \frac{2}{\theta_H - \frac{1-2\sigma}{1-\sigma} m} \left(\int_{\frac{\theta_H}{2} + \frac{1-2\sigma}{2(1-\sigma)} m}^{\frac{\theta_H}{2} + \frac{3-2\sigma}{2(1-\sigma)} m} m d\theta + \frac{1}{2} \int_{\frac{\theta_H}{2} + \frac{3-2\sigma}{2(1-\sigma)} m}^{\theta_H} \left(\theta - \frac{\theta_H}{2} + \frac{1-2\sigma}{2(1-\sigma)} m \right) d\theta \right) \\ &= m + \frac{\left(\theta_H - \frac{3-2\sigma}{1-\sigma} m \right)^2}{8 \left(\theta_H - \frac{1-2\sigma}{1-\sigma} m \right)}, \end{aligned}$$

and for $m > \frac{1-\sigma}{3-2\sigma} \theta_H$,

$$\lim_{\kappa \rightarrow 0} BR(m) = \frac{2}{\theta_H - \frac{1-2\sigma}{1-\sigma} m} \int_{\frac{\theta_H}{2} + \frac{1-2\sigma}{2(1-\sigma)} m}^{\theta_H} m d\theta = m.$$

1.7 Illustration of comparative statics with respect to σ

Figure 6 depicts m^*/p^* as a function of κ (which is increasing, see Proposition 3) with $\theta_H = 1$ for $\sigma = 0, 1/10, 2/10, 3/10$ and $4/10$. We observe that m^*/p^* is generally non-monotone in σ . For the displayed values of σ , when κ is large (high social pressure) increased σ decreases the tip-to-price ratio, but over an intermediary range of κ (low-to-moderate social pressure), increased σ weakly increases the tip-to-price ratio. Interestingly, when κ is sufficiently small, m^*/p^* is independent of σ , as discussed in Section 2.3.

1.8 Proof of Proposition 5

Assume that types are uniformly distributed over $[\theta_L, \theta_H]$ and let $K(d) = (\kappa/2)(\{d\}^+)^2$. We further assume $\theta_L < \theta_H/2$, thus, Assumption 1 is satisfied. In Lemma 10 below, we characterize the equilibrium tip-to-price ratio in explicit form. Based on standard arguments we then show its derivative with respect to θ_H is strictly positive.

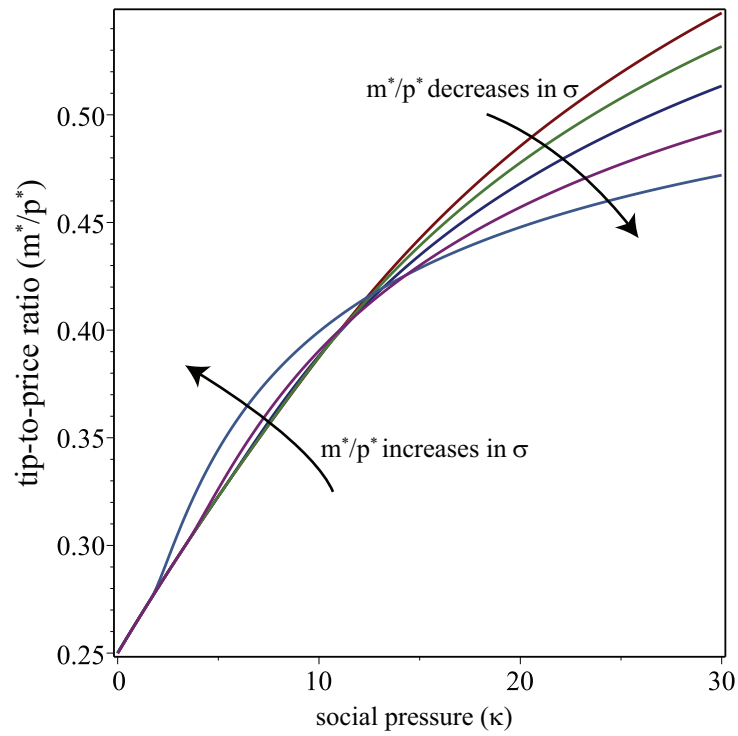


Figure 6: Illustration of the tip-to-price ratio as a function of the social pressure for different values of $\sigma \in \{0/10, 1/10, 2/10, 3/10, 4/10\}$.

Lemma 10. Let $y = \kappa\theta_H$, and define $\hat{y} = 4\sigma^2 - 20\sigma + 9$. If $y \leq \hat{y}$, the equilibrium tip-to-price ratio is

$$\frac{m^*}{p^*} = \frac{\sqrt{\frac{y}{4} + 4} - 1}{4}, \quad (19)$$

whereas if $y \geq \hat{y}$,

$$\frac{m^*}{p^*} = \frac{(1 - \sigma)y + \frac{2\sigma - 1}{3 - 2\sigma} \left\{ (4\sigma + 2\sigma^2 - \frac{13}{2}) - 4(1 - \sigma) \sqrt{\frac{y(3 - 2\sigma) - 2(1 - 2\sigma)^2}{1 - 2\sigma}} \right\}}{y - \frac{(2\sigma - 1)^2}{3 - 2\sigma} \left\{ (2\sigma + 1) - 2\sqrt{\frac{y(3 - 2\sigma) - 2(1 - 2\sigma)^2}{1 - 2\sigma}} \right\}}. \quad (20)$$

Proof. The proof of Lemma 10 is rather technical, and for brevity we omit several lengthy yet straightforward algebraic manipulations. Those can be readily verified using standard computer-algebra tools.

First note that $L(d) = -(1 - 2\sigma)d + (\kappa/2)(d^+)^2$ implies $\underline{d} = (1 - 2\sigma)/\kappa$, and recall from Proposition 1 that $c(m)$ is the unique solution to

$$(1 - \sigma)c(m) = (1 - 2\sigma)m + L(\min\{m - c(m)/2, \underline{d}\}). \quad (21)$$

For convenience, denote $\underline{L} = L(\underline{d}) = -\frac{(1 - 2\sigma)^2}{2\kappa}$, and recall from Lemma 3 the definition of \hat{m} which in the setting of Proposition 5 satisfies

$$\hat{m} = 2(1 - \sigma)\underline{d} + \underline{L} = \frac{1}{2} \frac{(1 - 2\sigma)(3 - 2\sigma)}{\kappa}. \quad (22)$$

According to Lemma 3, if $m > \hat{m}$ then $m - c(m)/2 > \underline{d}$ and therefore

$$c(m) = \frac{(1 - 2\sigma)m + \underline{L}}{1 - \sigma}.$$

Otherwise, if $m \leq \hat{m}$ then $m - c(m)/2 \leq \underline{d}$, and substituting $L(m - c(m)/2)$ in Equation (21) gives

$$\begin{aligned} c(m) &= \frac{(1 - 2\sigma)m - (1 - 2\sigma)(m - c(m)/2) + \frac{\kappa}{2}(m - c(m)/2)^2}{1 - \sigma} \\ &= c(m) + \frac{(1/2)\kappa(m - c(m)/2)^2 - c(m)/2}{1 - \sigma}, \end{aligned}$$

or equivalently,

$$\kappa(c(m)/2 - m)^2 - c(m) = 0$$

Following Proposition 1, this solution must satisfy $c(m) < m$ for all m , hence yielding

$$c(m) = 2 \cdot \frac{\kappa m + 1 - \sqrt{2m\kappa + 1}}{\kappa}. \quad (23)$$

With types uniformly distributed, similarly to Equation (18), we have $\underline{\theta}^{\text{opt.}}(m) = (\theta_H + c(m))/2$ for all m . Note, since the price is given by $\underline{\theta}^{\text{opt.}}(m) - c(m)$, that the latter implies the tip-to-price ratio can be expressed as $m/(\theta_H - \underline{\theta}^{\text{opt.}}(m))$. Equipped with an expression for $c(m)$, we solve for $m^* = ET(\underline{\theta}^{\text{opt.}}(m^*), m^*)$ by separating into two cases: $m^* \leq \hat{m}$ or $m^* > \hat{m}$. We identify each case based on the value of y .

Case I: Assume $m^* \leq \hat{m}$. Then, using Equation (23),

$$\underline{\theta}^* = \frac{1}{2}(\theta_H - c(m^*)) = \frac{1}{2} \left(\theta_H + 2 \frac{m^* \kappa + 1 - \sqrt{2m^* \kappa + 1}}{\kappa} \right). \quad (24)$$

Equation (6) and some basic algebraic manipulations then yield:

$$\begin{aligned} m^* &= ET(\underline{\theta}^*, m^*) \\ &= \frac{\int_{\underline{\theta}^*}^{\theta_H} \frac{1}{2}(\theta - \underline{\theta}^* + 2 \frac{m^* \kappa + 1 - \sqrt{2m^* \kappa + 1}}{\kappa}) d\theta}{\int_{\underline{\theta}^*}^{\theta_H} d\theta} \\ &= \frac{1}{4}(\theta_H - \underline{\theta}^*) + \frac{\kappa m^* + 1 - \sqrt{2m^* \kappa + 1}}{\kappa}, \end{aligned}$$

or

$$2m^* \kappa + 1 = \left(\frac{1}{4} \kappa (\theta_H - \underline{\theta}^*) + 1 \right)^2. \quad (25)$$

Rewriting Equation (24) and substituting (25) gives:

$$\begin{aligned} 0 &= y - 2\underline{\theta}^* \kappa + 2(m^* \kappa + 1 - \sqrt{2m^* \kappa + 1}) \\ &= y - 2\underline{\theta}^* \kappa + 1 + 2m^* \kappa + 1 - 2\sqrt{2m^* \kappa + 1} \\ &= -y + 2(\theta_H - \underline{\theta}^*) \kappa + 1 + \left(\frac{1}{4} \kappa (\theta_H - \underline{\theta}^*) + 1 \right)^2 - 2 \left(\frac{1}{4} \kappa (\theta_H - \underline{\theta}^*) + 1 \right). \end{aligned}$$

The latter is a quadratic equation in $\kappa(\theta_H - \underline{\theta}^*)$ admitting exactly one positive root expressed as $4\sqrt{y+16} - 16$, therefore implying

$$\underline{\theta}^* = \theta_H - 4 \frac{\sqrt{y+16} - 4}{\kappa},$$

and by plugging the root in (25) and isolating m^* we further have

$$m^* = \frac{1}{2} \theta_H + 3 \frac{4 - \sqrt{y+16}}{\kappa}. \quad (26)$$

After substituting $\underline{\theta}^*$ and m^* in the tip-to-price ratio expression ($m^*/(\theta_H - \underline{\theta}^*)$) we get the expression in (19).

Furthermore, from Equations (22) and (26) together, we obtain that $m^* \leq \hat{m}$ if and only if

$$\frac{1}{2} \theta_H + 3 \frac{4 - \sqrt{y+16}}{\kappa} \leq \frac{1}{2} \frac{(1-2\sigma)(3-2\sigma)}{\kappa},$$

or, equivalently, if

$$(y+16) - 16 + 6 \cdot (4 - \sqrt{y+16}) - (1-2\sigma)(3-2\sigma) \leq 0.$$

The latter is a quadratic inequality in $\sqrt{y+16}$ which is satisfied if and only if $\sqrt{y+16} \in [1+2\sigma, 5-2\sigma]$. Because $\sigma < 1/2$ and $y > 0$, we have $1+2\sigma < \sqrt{y+16}$, hence, we conclude that $m^* \leq \hat{m}$ if and only if $\sqrt{y+16} \leq 5-2\sigma$, or, equivalently, if and only if $y \leq \hat{y}$.

Case II: Assume now $m^* > \hat{m}$ (equivalently, $y > \hat{y}$), therefore $c(m^*)$ takes the form in Equation (23). As before, $\underline{\theta}^* = (\theta_H + c(m^*))/2$, and similarly to Case I we find m^* by solving $m^* = ET(\underline{\theta}^*, m^*)$. Recall from Proposition 1 that $\hat{\theta}(\underline{\theta}, m) = \underline{\theta} - c(m) + 2(m - \underline{d})$, which, by Lemma 3, satisfies $\hat{\theta}(\underline{\theta}, m) \geq \underline{\theta}$ whenever $m > \hat{m}$. Then

$$ET(\underline{\theta}^*, m^*) = \frac{\int_{\underline{\theta}^*}^{\hat{\theta}(\underline{\theta}^*, m^*)} (m^* - \underline{d}) d\theta + \int_{\hat{\theta}(\underline{\theta}^*, m^*)}^{\theta_H} \frac{1}{2} (\theta - \underline{\theta}^* + c(m^*)) d\theta}{\int_{\underline{\theta}^*}^{\theta_H} d\theta}$$

After basic integration and algebraic manipulations the equation $m^* = ET(\underline{\theta}^*, m^*)$, which is quadratic in m^* , yields two roots, $\hat{m} + (1 - \sigma) ((\theta_H - \underline{\theta}^*) - 2\sqrt{\underline{d}(\theta_H - \underline{\theta}^*)} \cdot \{-1, 1\})$. Imposing the condition $\{y = \hat{y} \Leftrightarrow m^* = \hat{m}\}$, we are then left with a unique solution:

$$m^* = \hat{m} + (1 - \sigma) ((\theta_H - \underline{\theta}^*) - 2\sqrt{\underline{d}(\theta_H - \underline{\theta}^*)}).$$

To fully characterize m^* it suffices therefore to find a solution for $\underline{\theta}^*$. After plugging m^* in the equation $\underline{\theta}^* = (\theta_H + c(m^*))/2$ (where $c(m^*)$ is given in Equation (23)) we obtain an equation for $\underline{\theta}^*$:

$$y = 2\kappa(\theta_H - \underline{\theta}^*) + \frac{(1 - 2\sigma) \left(\kappa \underline{L} + (1 - \sigma) 2\kappa \underline{d} + (1 - \sigma) \left(\kappa(\theta_H - \underline{\theta}^*) - 2\sqrt{\kappa \underline{d} \kappa (\theta_H - \underline{\theta}^*)} \right) \right) + \kappa \underline{L}}{1 - \sigma}$$

Substituting \underline{d} and \underline{L} and rearranging we get

$$(3 - 2\sigma) \cdot \kappa(\theta_H - \underline{\theta}^*) - 2(1 - 2\sigma)^{3/2} \sqrt{\kappa(\theta_H - \underline{\theta}^*)} + (1 - 2\sigma)^2 - y = 0, \quad (27)$$

Equation (27) is quadratic in $\sqrt{\kappa(\theta_H - \underline{\theta}^*)}$ with a positive quadratic coefficient, a negative linear coefficient. Furthermore, the constant term in the equation (i.e., $(1 - 2\sigma)^2 - y$) is negative, because, by assumption, $y > \hat{y}$, and this implies

$$y > 4\sigma^2 - 20\sigma + 9 = (5 - 2\sigma)^2 - 16 = (1 - 2\sigma)^2 + 8(1 - 2\sigma) > (1 - 2\sigma)^2.$$

It follows that Equation (27) admits only one positive root, that is

$$\sqrt{\kappa(\theta_H - \underline{\theta}^*)} = \frac{2(1 - 2\sigma)^{3/2} + \sqrt{4(1 - 2\sigma)^3 - 4(3 - 2\sigma)((1 - 2\sigma)^2 - y)}}{2(3 - 2\sigma)},$$

from which we can easily isolate $\underline{\theta}^*$, and, as before, substitute it in the tip-to-price ratio expression $m^*/(\theta_H - \underline{\theta}^*)$ to establish, after basic algebra, the expression in Equation (20). \square

We now turn to proving Proposition 5, namely, that the equilibrium tip-to-price ratio increases in θ_H . It can be seen from Lemma 10, that the tip-to-price ratio depends on θ_H and κ only through their product, which we denote by y . Recall from Proposition 3 that the tip-to-price ratio increases in κ . Thus,

$$0 \leq \frac{d}{d\kappa} \left(\frac{m^*}{p^*} \right) = \frac{d}{dy} \left(\frac{m^*}{p^*} \right) \cdot \frac{dy}{d\kappa} = \frac{d}{dy} \left(\frac{m^*}{p^*} \right) \cdot \theta_H.$$

Hence $(d/dy)(m^*/p^*) \geq 0$. It follows that

$$0 \leq \frac{d}{dy} \left(\frac{m^*}{p^*} \right) \cdot \kappa = \frac{d}{dy} \left(\frac{m^*}{p^*} \right) \cdot \frac{dy}{d\theta_H} = \frac{d}{d\theta_H} \left(\frac{m^*}{p^*} \right).$$

and therefore m^*/p^* is increasing in θ_H .

1.9 Appendix for Section 3.2

Under weak social norm ($\kappa \rightarrow 0$), the marginal return to effort is

$$\frac{\partial}{\partial q} ET(m, \underline{\theta}, q) = \frac{1}{2} \frac{\int_{\underline{\theta}}^{\theta_H} (\theta - \underline{\theta}) dF(\theta)}{\int_{\underline{\theta}}^{\theta_H} dF(\theta)} = \frac{\theta_H - \underline{\theta}}{4}.$$

The marginal cost is given by $(d/dq)c^{\text{effort}}(q - q_0) = \gamma(q - q_0)$, and by equating the latter with the marginal return and rearranging we obtain that the optimal quality for the worker is $q_0 + (\theta_H - \underline{\theta})/(4\gamma)$. Under weak social norm, the cost of tipping for the cutoff customer is 0, and the firm sets $\underline{\theta}$ so as to maximize $q\underline{\theta}\bar{F}(\underline{\theta})$, which for uniformly distributed types (and any q) is maximized at $\theta_H/2$. Plugging this into the worker's optimal quality gives $q_0 + \theta_H/(8\gamma)$, namely, the worker's optimal additional effort is $\theta_H/(8\gamma)$.

When the firm bears the effort cost, by taking derivative of the profit function with respect to q we get

$$\frac{\partial}{\partial q} \left(q\underline{\theta} - c^{\text{effort}}(q - q_0) \right) \bar{F}(\underline{\theta}) = (\underline{\theta} - \gamma(q - q_0))\bar{F}(\underline{\theta}),$$

implying, by the first order condition, that the optimal quality q^* and cutoff type $\underline{\theta}^*$ satisfy $q^* = q_0 + \underline{\theta}^*/\gamma$. Similarly,

$$\frac{\partial}{\partial \underline{\theta}} \left(q\underline{\theta} - c^{\text{effort}}(q - q_0) \right) \bar{F}(\underline{\theta}) = \frac{q}{\theta_H - \theta_L} \cdot \left(\theta_H - 2\underline{\theta} + \frac{c^{\text{effort}}(q - q_0)}{q} \right),$$

implying, by the first order condition, that

$$\underline{\theta}^* = \frac{1}{2} \left(\theta_H + \frac{c^{\text{effort}}(q^* - q_0)}{q^*} \right) \geq \frac{\theta_H}{2}.$$