

Appendices to “Impact of Social Interactions on Duopoly Competition with Quality Considerations”

The appendices are divided into two parts. We present the proofs for the main model under the committed pricing scheme (Section 4) in Part A. In Part B, we present the additional findings and proofs for the results under the dynamic pricing scheme (Section 5.1). Table 1 below summarizes the notations used in the paper.

Table 1 Summary of Notations

Symbol	Description
“M”	the scenario of committed pricing with MEE
“VC”	the scenario of VEE-C under both committed pricing and dynamic pricing strategies
“VP”	the scenario of committed pricing with VEE-P
“DM”	the scenario of dynamic pricing with MEE
“DVP”	the scenario of dynamic pricing with VEE-P
“b”	benchmark case without MEE or VEE
r	the strength of MEE
t_c	the strength of VEE-C
t_p	the strength of VEE-P
p_i^s	committed price of firm i under scenario $s \in \{M, VP, VC\}$
p_{in}^s	price of firm i in period $n = 1, 2$, under scenario $s \in \{DM, DVP\}$
d_{in}^s	demand of firm i in period $n = 1, 2$, under scenario $s \in \{M, VP, VC, DM, DVP\}$
d_i^s	total demand of firm i in the two periods under scenario $s \in \{M, VP, VC, DM, DVP\}$
π_{in}^s	profit of firm i in period $n = 1, 2$, under scenario $s \in \{M, VP, VC, DM, DVP\}$
π_i^s	total profit of firm i in two periods under scenario $s \in \{M, VP, VC, DM, DVP\}$
q_i^s	quality level of firm i under scenario $s \in \{M, VP, VC, DM, DVP\}$
“ $\hat{}$ ”	“ $\hat{}$ ” over above symbols to denote the corresponding final equilibrium

A. The Committed Pricing Scheme

A.1. Market Expansion Effect (MEE)

Proof of Lemma 1: Given $t_c = t_p = 0$ and $0 < r \leq 1$ and $0 \leq q_L < q_H \leq 1$, firm i decides its price, p_i , to maximize its total profit:

$$\max_{p_i \geq 0} p_i (d_{i1}^M + d_{i2}^M), \quad (\text{A1})$$

where $i = H, L$. We will show in Proposition 1 that the market is partially covered and each firm has positive demand in the equilibrium. Consumers will purchase from firm H if $u_{Hn} > \max\{0, u_{Ln}\}$

and from firm L if $u_{Ln} > \max\{0, u_{Hn}\}$ in period $n = 1, 2$. Thus, the demand of each firm in each period can be solved as

$$\begin{aligned} d_{H1}^M &= 1 - \frac{p_H - p_L}{q_H - q_L}, d_{L1}^M = \frac{p_H - p_L}{q_H - q_L} - \frac{p_L}{q_L}, \\ d_{H2}^M &= (1 + rd_{H1}^M + rd_{L1}^M) \left(1 - \frac{p_H - p_L}{q_H - q_L} \right), \text{ and } d_{L2}^M = (1 + rd_{H1}^M + rd_{L1}^M) \left(\frac{p_H - p_L}{q_H - q_L} - \frac{p_L}{q_L} \right). \end{aligned} \quad (\text{A2})$$

Given the above demands, one can easily verify that firm i 's total profit $p_i(d_{i1}^M + d_{i2}^M)$ is concave in p_i , i.e., $\frac{\partial^2(p_i(d_{i1}^M + d_{i2}^M))}{\partial p_i^2} < 0$, for $i = H, L$. Thus, solving the two firms' first-order-conditions together, i.e., $\frac{\partial(p_H(d_{H1}^M + d_{H2}^M))}{\partial p_H} = 0$ and $\frac{\partial(p_L(d_{L1}^M + d_{L2}^M))}{\partial p_L} = 0$, leads to the the sub-game equilibrium prices:

$$\begin{aligned} p_H^M(q_H, q_L, r) &= \frac{q_L(8q_H - 10rq_H - q_L(2-r) - X_1) + 12rq_H^2}{8(3q_H - q_L)r} \quad \text{and} \\ p_L^M(q_H, q_L, r) &= \frac{q_L(6rq_H + 8q_H - q_L(3r+2) - X_1)}{4(3q_H - q_L)r}, \end{aligned} \quad (\text{A3})$$

where $X_1 = \sqrt{q_L^2(r-2)^2 - 4q_Hq_L(r^2 + 2r + 8) + 4q_H^2(3r^2 + 12r + 16)}$. Plugging Equations (A3) into Equations (A2), the demand of each firm in each period can be written as:

$$\begin{aligned} d_{H1}^M(q_H, q_L, r) &= \frac{q_L(8q_H - 10rq_H - q_L(2-r) - X_1) + 12rq_H^2}{8(3q_H - q_L)(q_H - q_L)r}, \\ d_{L1}^M(q_H, q_L, r) &= \frac{q_L(4q_H(3-r) - q_L(2-r) - X_1) + 2q_H(X_1 - 8q_H)}{8(3q_H - q_L)(q_H - q_L)r}, \\ d_{H2}^M(q_H, q_L, r) &= \frac{(X_1 - q_L(r+2) + q_H(6r+4))(q_L(q_L(r-2) - X_1 - 10rq_H + 8q_H) + 12rq_H^2)}{32(3q_H - q_L)^2(q_H - q_L)r}, \text{ and} \\ d_{L2}^M(q_H, q_L, r) &= \frac{(X_1 - q_L(r+2) + q_H(6r+4))(q_L(q_L(r-2) - 4rq_H + 12q_H - X_1) + 2q_H(X_1 - 8q_H))}{32(3q_H - q_L)^2(q_H - q_L)r}. \end{aligned} \quad (\text{A4})$$

Moreover, the total demand of firm i is given by $d_i^M(q_H, q_L, r) = d_{i1}^M(q_H, q_L, r) + d_{i2}^M(q_H, q_L, r)$, and firm i 's total profit is $\pi_i^M(q_H, q_L, r) = p_i^M(q_H, q_L, r)d_i^M(q_H, q_L, r)$, for $i = H, L$.

Note that the sub-game equilibrium prices, demands, and profits are all differentiable. For $r \in (0, 1]$ and $0 \leq q_L < q_H \leq 1$, taking derivative with respect to r and after simplification, one can verify that the followings hold

$$\begin{aligned} \frac{\partial p_H^M(q_H, q_L, r)}{\partial r} &< 0, \quad \frac{\partial p_L^M(q_H, q_L, r)}{\partial r} < 0, \\ \frac{\partial d_H^M(q_H, q_L, r)}{\partial r} &> 0, \quad \frac{\partial d_L^M(q_H, q_L, r)}{\partial r} > 0, \\ \frac{\partial \pi_H^M(q_H, q_L, r)}{\partial r} &> 0, \text{ and } \frac{\partial \pi_L^M(q_H, q_L, r)}{\partial r} > 0. \end{aligned}$$

This completes the proof of Lemma 1. \square

Proof of Proposition 1: We complete the proof in three steps: **Step 1.** Solve the potential equilibrium given that the market is partially covered and each firm has positive demand; **Step 2.** Show the existence of the equilibrium derived in Step 1; **Step 3.** Show the uniqueness of the equilibrium. **Step 1.** We first solve the potential equilibrium given that the market is partially covered and each firm has positive demand. We have solved the firms' pricing decisions for given quality levels in

the proof of Lemma 1. Given the firms' pricing decisions in Stage 2, we proceed to solve for firms' quality decisions in the first stage:

$$\begin{aligned} \max_{q_H \in (q_L, 1]} \pi_H^M(q_H, q_L, r) &= p_H^M(q_H, q_L, r) d_H^M(q_H, q_L, r), \text{ and} \\ \max_{q_L \in [0, q_H)} \pi_L^M(q_H, q_L, r) &= p_L^M(q_H, q_L, r) d_L^M(q_H, q_L, r), \end{aligned}$$

where $p_i^M(q_H, q_L, r)$ and $d_i^M(q_H, q_L, r)$ are given by Equations (A3) and (A4), respectively.

First, one can easily verify that $\pi_H^M(q_H, q_L, r)$ increases in q_H for any $q_L \in [0, 1)$ and $r \in (0, 1]$, i.e., $\frac{d\pi_H^M(q_H, q_L, r)}{dq_H} > 0$. Thus, firm H's optimal quality is $\hat{q}_H^M = 1$ for any $q_L \in [0, 1)$ and $r \in (0, 1]$. Given that $q_H = 1$, next we solve firm L's optimal quality q_L . Directly checking firm L's profit function, we find that it is strictly concave in q_L , i.e., $\frac{\partial^2 \pi_L^M(1, q_L, r)}{\partial q_L^2} < 0$, for any $r \in (0, 1]$ and $q_L \in [0, 1)$. Moreover, one can verify that $\frac{\partial \pi_L^M(1, q_L, r)}{\partial q_L} \big|_{q_L=0.1} > 0$ and $\frac{\partial \pi_L^M(1, q_L, r)}{\partial q_L} \big|_{q_L=0.9} < 0$. Therefore, there exists a unique $\hat{q}_L^M(r) \in (0.1, 0.9)$ that can be solved from $\frac{\partial \pi_L^M(1, q_L, r)}{\partial q_L} = 0$ and maximizes $\pi_L^M(1, q_L, r)$.

Hence, given that the market is partially covered and each firm has positive demand, the potential equilibrium outcome is: $q_H = \hat{q}_H^M = 1$, $q_L = \hat{q}_L^M(r)$, $p_H = \hat{p}_H^M(r) = p_H^M(1, \hat{q}_L^M(r), r)$, and $p_L = \hat{p}_L^M(r) = p_L^M(1, \hat{q}_L^M(r), r)$. Let $\sigma^M = (\hat{q}_H^M, \hat{q}_L^M(r), \hat{p}_H^M(r), \hat{p}_L^M(r))$ denote this potential equilibrium, and let $\hat{d}_i^M(r) = d_i^M(1, \hat{q}_L^M(r), r)$ and $\hat{\pi}_i^M(r) = \hat{p}_i^M(r) \hat{d}_i^M(r)$ denote firm i 's total demand and profit at the potential equilibrium σ^M .

Step 2. To show the existence of the equilibrium σ^M , we will show that σ^M is a Sub-game Perfect Equilibrium (SPE). We just need to verify that σ^M satisfies the no-deviation requirements of SPE: (1) Given firms' quality $0 \leq q_L < q_H \leq 1$ and firm i 's price $p_i^M(q_H, q_L, r)$, firm j 's price decision will not deviate from $p_j^M(q_H, q_L, r)$ to other prices; and (2) given both firms' prices $(p_H^M(q_H, q_L, r), p_L^M(q_H, q_L, r))$ in Stage 2 and firm i 's quality $\hat{q}_i^M(r)$, firm j 's quality will not deviate from $\hat{q}_j^M(r)$ to other qualities, i.e., $\hat{\pi}_j^M(r) \geq \max_{q_j \neq \hat{q}_j^M(r)} \pi_j^M(q_j, \hat{q}_i^M(r), r)$, for $i, j \in \{H, L\}$ and $i \neq j$.

For requirement (1), we just need to show that

$$\pi_j^M(q_H, q_L, r) \geq \max_{p_j \neq p_j^M(q_H, q_L, r)} p_j d_j^M(p_j | q_H, q_L, p_i^M(q_H, q_L, r)), \text{ for } i, j \in \{H, L\} \text{ and } i \neq j, \quad (\text{A5})$$

where the general demand functions are given by:

$$\begin{aligned} d_{H1}^M &= (1 - \max\{\frac{p_H - p_L}{q_H - q_L}, \frac{p_H}{q_H}\})^+, & d_{L1}^M &= (\min\{1, \frac{p_H - p_L}{q_H - q_L}\} - \frac{p_L}{q_L})^+, \\ d_{H2}^M &= (1 + r d_{H1}^M + r d_{L1}^M) d_{H1}^M, & \text{and} & & d_{L2}^M &= (1 + r d_{H1}^M + r d_{L1}^M) d_{L1}^M. \end{aligned}$$

Firm i 's total demand of two periods is $d_i^M = d_{i1}^M + d_{i2}^M$. Note that the above demand functions include all situations that market is partially covered or fully covered and both firms have positive demands or one of them has zero demand. By straightforward but tedious algebraic analysis, we can verify that given $r \in (0, 1]$ and firm i 's price $p_i^M(q_H, q_L, r)$, firm j 's optimal price is $p_j^M(q_H, q_L, r)$ and thus will not deviate, i.e., the inequality (A5) holds. Thus, requirement (1) is satisfied.

For requirement (2), we have already shown in Step 1 that firm H's optimal quality is $q_H = 1$ for any $q_L \in [0, 1)$ and that firm L's optimal quality is $q_L = \hat{q}_L^M(r)$ given $q_H = 1$. Thus, requirement (2) holds. Hence, we have shown that σ^M is an SPE.

Step 3. We show the uniqueness of the equilibrium σ^M . Note that in Step 1, we have derived a unique equilibrium σ^M by backward induction under the condition that the market is partially covered and each firm has positive demand. Thus, we just need to show that there does not exist any equilibrium if the above condition does not hold, i.e., the market is fully covered or one of the firms has zero demand. Note that in the case of MEE, $d_{H2} + d_{L2} = 1 + r(d_{H1} + d_{L1})$ if and only if $d_{H1} + d_{L1} = 1$, i.e., the market is fully covered in period 2 if and only if the market is fully covered in period 1; and $d_{i2} = 0$ if and only if $d_{i1} = 0$, i.e., firm i has zero demand in period 2 if and only if it has zero demand in period 1. Thus, to show the uniqueness, we need to verify that: (1) Any strategy that leads to $d_{H1} + d_{L1} = 1$ is not an equilibrium; and (2) any strategy that leads to $d_{H1} = 0$ or $d_{L1} = 0$ is not an equilibrium.

If $d_{H1} + d_{L1} = 1$, it implies that the consumer with $\theta = 0$ makes the purchase. That is, either $p_L = 0$ or $p_H = 0$. It is obvious that any strategy with $p_L = 0$ or $p_H = 0$ cannot be an equilibrium since one of the firm's profit will be zero and always has incentive to deviate to a small enough price to earn a positive profit. Thus, (1) holds. Similarly, if $d_{i1} = 0$, then firm i 's profit is zero. It is obvious that any strategy with $d_{i1} = 0$ cannot be an equilibrium outcome since firm i has incentive to deviate to a lower price to get a positive demand and earn a positive profit. Thus (2) holds.

Hence, we conclude that in the equilibrium, the market is partially covered and each firm has positive demand (i.e., $d_{H1} > 0$, $d_{L1} > 0$, $d_{H2} > 0$, $d_{L2} > 0$, and $d_{H1} + d_{L1} < 1$). Combined with Steps 1 and 2, it shows that σ^M is the unique equilibrium. In what follows, we prove Parts (a)-(d). Note that all the final equilibrium outcomes are differentiable.

Part (a): Since $\hat{q}_L^M(r)$ is the unique solution of $\frac{\partial \pi_L^M(1, q_L, r)}{\partial q_L} = 0$ for $q_L \in [0, 1)$, by the Implicit Function Theorem, we have $\frac{\partial \hat{q}_L^M(r)}{\partial r} = -\frac{\partial^2 \pi_L^M(1, q_L, r)}{\partial q_L \partial r} / \frac{\partial^2 \pi_L^M(1, q_L, r)}{\partial q_L^2} \Big|_{q_L = \hat{q}_L^M(r)}$. We have already shown that $\frac{\partial^2 \pi_L^M(1, q_L, r)}{\partial q_L^2} < 0$ for any $q_L \in [0, 1)$ and $r \in (0, 1]$. One can also verify that $\frac{\partial^2 \pi_L^M(1, q_L, r)}{\partial q_L \partial r} > 0$ for any $q_L \in [0, 0.6)$ and $r \in (0, 1]$. Moreover, we find that $\hat{q}_L^M(r) < 0.6$ for any $r \in (0, 1]$ since $\frac{\partial \pi_L^M(1, q_L, r)}{\partial q_L} < 0$ for any $q_L \in [0.6, 1)$ and $r \in (0, 1]$. Therefore, $\frac{\partial \hat{q}_L^M(r)}{\partial r} > 0$ for $r \in (0, 1]$.

Part (b): Note that $\hat{p}_i^M(r) = p_i^M(1, \hat{q}_L^M(r), r)$, for $i = H, L$. By the chain rule, $\frac{\partial \hat{p}_i^M(r)}{\partial r} = \left(\frac{\partial p_i^M(1, q_L, r)}{\partial r} + \frac{\partial p_i^M(1, q_L, r)}{\partial q_L} \frac{\partial \hat{q}_L^M(r)}{\partial r} \right) \Big|_{q_L = \hat{q}_L^M(r)}$. In the proof of Lemma 1, we have shown that $\frac{\partial p_i^M(1, q_L, r)}{\partial r} < 0$. In Part (a) above, we have shown that $\frac{\partial \hat{q}_L^M(r)}{\partial r} > 0$. Moreover, one can also verify that $\frac{\partial p_i^M(1, q_L, r)}{\partial q_L} < 0$ for any $q_L \in [0, 1)$ and $r \in (0, 1]$, and that $\frac{\partial p_L^M(1, q_L, r)}{\partial q_L} < 0$ for any $q_L \in (0.55, 1)$ and $r \in (0, 1]$. Since $\hat{q}_L^M(r)$ increases in r , we have $\hat{q}_L^M(r) \geq \hat{q}_L^M(0) = \hat{q}^b = 4/7 > 0.55$. Thus $\frac{\partial p_i^M(1, q_L, r)}{\partial q_L} \Big|_{q_L = \hat{q}_L^M(r)} < 0$ holds for any $r \in$

$(0, 1]$. Combining all the above statements, we deduce that $\frac{\partial \hat{p}_i^M(r)}{\partial r} < 0$ for any $r \in (0, 1]$, as desired.

Therefore, $\hat{p}_i^M(r)$ decreases in r and $\hat{p}_i^M(r) < \hat{p}_i^M(0) = \hat{p}_i^b$ for $r \in (0, 1]$ and $i = H, L$.

Part (c): Note that $\hat{d}_i^M(r) = d_i^M(1, \hat{q}_L^M(r), r)$, for $i = H, L$. By the chain rule, $\frac{\partial \hat{d}_i^M(r)}{\partial r} = \left(\frac{\partial d_i^M(1, q_L, r)}{\partial r} + \frac{\partial d_i^M(1, q_L, r)}{\partial q_L} \frac{\partial \hat{q}_L^M(r)}{\partial r} \right) \Big|_{q_L = \hat{q}_L^M(r)}$. In the proof of Lemma 1, we have shown that $\frac{\partial d_i^M(1, q_L, r)}{\partial r} > 0$. In Part (a) above, we have shown that $\frac{\partial \hat{q}_L^M(r)}{\partial r} > 0$. Moreover, one can verify that $\frac{\partial d_i^M(1, q_L, r)}{\partial q_L} > 0$. Thus, $\frac{\partial \hat{d}_i^M(r)}{\partial r} > 0$ for $i = H, L$.

Part (d): Note that $\hat{\pi}_i^M(r) = \pi_i^M(1, \hat{q}_L^M(r), r)$, for $i = H, L$. By the chain rule, $\frac{\partial \hat{\pi}_i^M(r)}{\partial r} = \left(\frac{\partial \pi_i^M(1, q_L, r)}{\partial r} + \frac{\partial \pi_i^M(1, q_L, r)}{\partial q_L} \frac{\partial \hat{q}_L^M(r)}{\partial r} \right) \Big|_{q_L = \hat{q}_L^M(r)}$. One can verify that $\frac{\partial \pi_H^M(1, q_L, r)}{\partial r} + \frac{\partial \pi_H^M(1, q_L, r)}{\partial q_L} \frac{\partial \hat{q}_L^M(r)}{\partial r} > 0$ for any $q_L \in [0, 1]$ and $r \in (0, 1]$. Thus, $\frac{\partial \hat{\pi}_H^M(r)}{\partial r} > 0$. In the proof of Lemma 1, we have shown that $\frac{\partial \pi_L^M(1, q_L, r)}{\partial r} > 0$. In Part (a) above, we have shown that $\frac{\partial \hat{q}_L^M(r)}{\partial r} > 0$. Moreover, $\frac{\partial \pi_L^M(1, q_L, r)}{\partial q_L} \Big|_{q_L = \hat{q}_L^M(r)} = 0$. Thus, $\frac{\partial \hat{\pi}_L^M(r)}{\partial r} > 0$.

Next, we show that $\hat{\pi}_H^M(r) - \hat{\pi}_L^M(r)$ increases in r . By the chain rule: $\frac{\partial (\hat{\pi}_H^M(r) - \hat{\pi}_L^M(r))}{\partial r} = \left(\frac{\partial (\pi_H^M(1, q_L, r) - \pi_L^M(1, q_L, r))}{\partial r} + \frac{\partial (\pi_H^M(1, q_L, r) - \pi_L^M(1, q_L, r))}{\partial q_L} \frac{\partial \hat{q}_L^M(r)}{\partial r} \right) \Big|_{q_L = \hat{q}_L^M(r)}$. First, the definition of $\hat{q}_L^M(r)$ indicates that $\frac{\partial (\pi_H^M(1, q_L, r) - \pi_L^M(1, q_L, r))}{\partial q_L} \Big|_{q_L = \hat{q}_L^M(r)} = 0$; second, one can verify that $\frac{\partial (\pi_H^M(1, q_L, r) - \pi_L^M(1, q_L, r))}{\partial r} + \frac{\partial (\pi_H^M(1, q_L, r) - \pi_L^M(1, q_L, r))}{\partial q_L} \frac{\partial \hat{q}_L^M(r)}{\partial r} > 0$ for any $q_L \in [0, 1]$ and $r \in (0, 1]$. Thus, $\frac{\partial (\hat{\pi}_H^M(r) - \hat{\pi}_L^M(r))}{\partial r} > 0$, which implies $\hat{\pi}_H^M(r) - \hat{\pi}_H^b > \hat{\pi}_L^M(r) - \hat{\pi}_L^b > 0$, for $r \in (0, 1]$.

Finally, we show that $\frac{\hat{\pi}_L^M(r) - \hat{\pi}_H^M(r)}{\hat{\pi}_L^b - \hat{\pi}_H^b}$ increases in r . By the chain rule,

$$\frac{\partial \left(\frac{\hat{\pi}_L^M(r) - \hat{\pi}_H^M(r)}{\hat{\pi}_L^b - \hat{\pi}_H^b} \right)}{\partial r} = \left(\frac{\partial \left(\frac{\pi_L^M(1, q_L, r) - \pi_H^M(1, q_L, r)}{\hat{\pi}_L^b - \hat{\pi}_H^b} \right)}{\partial r} + \frac{\partial \left(\frac{\pi_L^M(1, q_L, r) - \pi_H^M(1, q_L, r)}{\hat{\pi}_L^b - \hat{\pi}_H^b} \right)}{\partial q_L} \frac{\partial \hat{q}_L^M(r)}{\partial r} \right) \Big|_{q_L = \hat{q}_L^M(r)}$$

First, $\frac{\partial \left(\frac{\pi_L^M(1, q_L, r) - \pi_H^M(1, q_L, r)}{\hat{\pi}_L^b - \hat{\pi}_H^b} \right)}{\partial q_L} \Big|_{q_L = \hat{q}_L^M(r)} = 0$; second, one can verify that $\frac{\partial \left(\frac{\pi_L^M(1, q_L, r) - \pi_H^M(1, q_L, r)}{\hat{\pi}_L^b - \hat{\pi}_H^b} \right)}{\partial r} + \frac{\partial \left(\frac{\pi_L^M(1, q_L, r) - \pi_H^M(1, q_L, r)}{\hat{\pi}_L^b - \hat{\pi}_H^b} \right)}{\partial q_L} \frac{\partial \hat{q}_L^M(r)}{\partial r} > 0$

for any $q_L \in (0.5, 0.6)$ and $r \in (0, 1]$; third, $0.5 < \hat{q}_L^M(0) \leq \hat{q}_L^M(r) \leq \hat{q}_L^M(1) < 0.6$ for any $r \in (0, 1]$.

Thus, we have $\frac{\partial \left(\frac{\hat{\pi}_L^M(r) - \hat{\pi}_H^M(r)}{\hat{\pi}_L^b - \hat{\pi}_H^b} \right)}{\partial r} > 0$, which implies that $(\hat{\pi}_L^M(r) - \hat{\pi}_L^b) / \hat{\pi}_L^b > (\hat{\pi}_H^M(r) - \hat{\pi}_H^b) / \hat{\pi}_H^b > 0$

for all $r \in (0, 1]$. This completes the proof. \square

Proof of Corollary 1: Part (a): Let $CS_{in}^M(r)$ denote total consumer surplus of purchasing from firm i in period n in the equilibrium. Thus, $CS_{H1}^M(r)$ and $CS_{L1}^M(r)$ are given by:

$$\begin{aligned} CS_{H1}^M(r) &= \int_{\bar{\theta}_H^M}^1 (\theta - \hat{p}_H^M(r)) d\theta = (1 - 2\hat{p}_H^M(r) + \bar{\theta}_H^M)(1 - \bar{\theta}_H^M)/2 \\ &= (1 - 2\hat{p}_H^M(r) + \frac{\hat{p}_H^M(r) - \hat{p}_L^M(r)}{1 - \hat{q}_L^M(r)})(1 - \frac{\hat{p}_H^M(r) - \hat{p}_L^M(r)}{1 - \hat{q}_L^M(r)})/2 \quad \text{and} \\ CS_{L1}^M(r) &= \int_{\bar{\theta}_L^M}^{\bar{\theta}_H^M} (\theta \hat{q}_L^M(r) - \hat{p}_L^M(r)) d\theta = (\hat{q}_L^M(r)(\bar{\theta}_H^M + \bar{\theta}_L^M) - 2\hat{p}_L^M(r))(\bar{\theta}_H^M - \bar{\theta}_L^M)/2 \\ &= (\hat{q}_L^M(r) \left(\frac{\hat{p}_H^M(r) - \hat{p}_L^M(r)}{1 - \hat{q}_L^M(r)} + \frac{\hat{p}_L^M(r)}{\hat{q}_L^M(r)} \right) - 2\hat{p}_L^M(r)) \left(\frac{\hat{p}_H^M(r) - \hat{p}_L^M(r)}{1 - \hat{q}_L^M(r)} - \frac{\hat{p}_L^M(r)}{\hat{q}_L^M(r)} \right) / 2. \end{aligned}$$

The total consumer surplus in each period is then given by $CS_1^M(r) = CS_{H1}^M(r) + CS_{L1}^M(r)$ and $CS_2^M(r) = (1 + r\hat{d}_{H1}^M(r) + r\hat{d}_{L1}^M(r))CS_1^M(r)$, and total consumer surplus of two periods is $CS^M(r) =$

$CS_1^M(r) + CS_2^M(r)$. We can show that $\frac{dCS_{H1}^M(r)}{dr} > 0$ and $\frac{dCS_{L1}^M(r)}{dr} > 0$. Thus $\frac{dCS_1^M(r)}{dr} > 0$. Moreover one can easily verify that $\frac{d(1+r\hat{a}_{H1}^M(r)+r\hat{a}_{L1}^M(r))}{dr} > 0$. Thus, $\frac{dCS_2^M(r)}{dr} > 0$. Hence, $\frac{dCS^M(r)}{dr} > 0$. Therefore, $CS_1^M(r)$, $CS_2^M(r)$, and $CS^M(r)$ all increase in r . Let CS^b denote the total consumer surplus of two periods in the benchmark case, i.e., $CS^b = CS^M(0)$. Thus, $CS^M(r) > CS^M(0) = CS^b$ for $r \in (0, 1]$.

Part (b): Total social welfare is defined as $SW^M(r) = CS^M(r) + \pi_H^M(r) + \pi_L^M(r)$. We have already shown that $CS^M(r)$ increases in r in (a), and that $\pi_H^M(r)$ and $\pi_L^M(r)$ increase in r in Proposition 1. Hence, $SW^M(r)$ increases in r . \square

A.2. Value Enhancement Effect from Current Consumers (VEE-C)

Proof of Lemma 2: Given $r = t_p = 0$ and $t_c > 0$ and $0 \leq q_L < q_H \leq 1$, firm i decides its price p_i to maximize its total profit:

$$\max_{p_i \geq 0} p_i (d_{i1}^{VC} + d_{i2}^{VC}),$$

where $i = H, L$. In the VEE-C case, each period is identical; thus, each firm's demand is the same in each period (i.e., $d_{H1}^{VC} = d_{H2}^{VC}$ and $d_{L1}^{VC} = d_{L2}^{VC}$). If firm i has zero demand in one period, then it has zero demand in both periods and zero total profit. Thus, it is obvious that in the equilibrium, each firm must have positive demand in each period, i.e., $d_{in} > 0$ for $i = H, L$ and $n = 1, 2$. However, due to the value enhancement effect in each period, it is not obvious that in the equilibrium whether the market is fully covered or not. We will focus on the case where the market is partially covered in the following proof; and we will show in the proof of Proposition 2 that there is no equilibrium in the case where the market is fully covered and thus the market being partially covered is the unique equilibrium outcome when t_c is small enough.

Given that the market is partially covered, let $\bar{\theta}_H^{VC} \in (0, 1)$ denote the consumer who is indifferent in purchasing from firm H and firm L and $\bar{\theta}_L^{VC} \in (0, \bar{\theta}_H^{VC})$ denote the consumer who is indifferent in purchasing from firm L and not purchasing. Then, the two firms' demands in each period are given by $d_{H1}^{VC} = d_{H2}^{VC} = 1 - \bar{\theta}_H^{VC}$ and $d_{L1}^{VC} = d_{L2}^{VC} = \bar{\theta}_H^{VC} - \bar{\theta}_L^{VC}$, respectively; and a consumer's utility of purchasing from firm H is $u_{Hn} = \theta q_H - p_H + t_c(1 - \bar{\theta}_H^{VC})$ and the utility of purchasing from firm L is $u_{Ln} = \theta q_L - p_L + t_c(\bar{\theta}_H^{VC} - \bar{\theta}_L^{VC})$ in period $n = 1, 2$. Thus, $\bar{\theta}_H^{VC}$ and $\bar{\theta}_L^{VC}$ can be solved from:

$$\begin{aligned} \bar{\theta}_H^{VC} q_H - p_H + t_c(1 - \bar{\theta}_H^{VC}) &= \bar{\theta}_H^{VC} q_L - p_L + t_c(\bar{\theta}_H^{VC} - \bar{\theta}_L^{VC}) \quad \text{and} \\ \bar{\theta}_L^{VC} q_L - p_L + t_c(\bar{\theta}_H^{VC} - \bar{\theta}_L^{VC}) &= 0. \end{aligned}$$

Hence, we have:

$$\bar{\theta}_H^{VC} = \frac{p_L q_L + (p_H - t_c)(t_c - q_L)}{q_L^2 + t_c q_L + q_H(t_c - q_L) - t_c^2} \quad \text{and} \quad \bar{\theta}_L^{VC} = \frac{t_c(p_H - t_c) + p_L(q_L - q_H + t_c)}{q_L^2 + t_c q_L + q_H(t_c - q_L) - t_c^2}. \quad (\text{A6})$$

Each firm's demand in each period is then given as below:

$$\begin{aligned} d_{H1}^{VC} = d_{H2}^{VC} = 1 - \bar{\theta}_H^{VC} &= 1 - \frac{p_L q_L + (p_H - t_c)(t_c - q_L)}{q_L^2 + t_c q_L + q_H(t_c - q_L) - t_c^2} \quad \text{and} \\ d_{L1}^{VC} = d_{L2}^{VC} = \bar{\theta}_H^{VC} - \bar{\theta}_L^{VC} &= \frac{p_L(q_H - t_c) + q_L(t_c - p_H)}{q_L^2 + t_c q_L + q_H(t_c - q_L) - t_c^2}. \end{aligned} \quad (A7)$$

Then, firm i 's profit is given by $2p_i d_{i1}^{VC}$. Checking the second-order derivative, we find that firm i 's total profit is concave in p_i , i.e., $\frac{\partial^2 2p_i d_{i1}^{VC}}{\partial p_i^2} < 0$, for $i = H, L$. Solving the two firms' first-order conditions together, i.e., $\frac{\partial(2p_H d_{H1}^{VC})}{\partial p_H} = 0$ and $\frac{\partial(2p_L d_{L1}^{VC})}{\partial p_L} = 0$, we get the following prices:

$$\begin{aligned} p_H^{VC}(q_H, q_L, t_c) &= \frac{2q_H^2(q_L - t_c) - 2q_H(t_c q_L - t_c^2 + q_L^2) + t_c q_L^2}{4q_H(q_L - t_c) - 4t_c q_L + 4t_c^2 - q_L^2} \quad \text{and} \\ p_L^{VC}(q_H, q_L, t_c) &= \frac{q_L(q_H(t_c - q_L) + 2t_c q_L - 2t_c^2 + q_L^2)}{-4q_H(q_L - t_c) + 4t_c q_L - 4t_c^2 + q_L^2}. \end{aligned} \quad (A8)$$

$p_H = p_H^{VC}(q_H, q_L, t_c)$ and $p_L = p_L^{VC}(q_H, q_L, t_c)$ satisfy the conditions of this case $0 < \bar{\theta}_L^{VC} < \bar{\theta}_H^{VC} < 1$ if $0 < t_c < T^{VC}(q_H, q_L)$, where $T^{VC}(q_H, q_L) = (2q_H + q_L - \sqrt{4q_H^2 - 4q_H q_L + 9q_L^2})/4$. Plugging Equations (A8) into Equations (A7), the demand of each firm in each period can be written as:

$$\begin{aligned} d_{H1}^{VC}(q_H, q_L, t_c) = d_{H2}^{VC}(q_H, q_L, t_c) &= \frac{(q_L - t_c)(2q_H^2(q_L - t_c) - 2q_H(t_c q_L - t_c^2 + q_L^2) + t_c q_L^2)}{(-4q_H(q_L - t_c) + 4t_c q_L - 4t_c^2 + q_L^2)(q_H(t_c - q_L) + t_c q_L - t_c^2 + q_L^2)} \quad \text{and} \\ d_{L1}^{VC}(q_H, q_L, t_c) = d_{L2}^{VC}(q_H, q_L, t_c) &= \frac{q_L(q_H^2(q_L - t_c) - q_H(3t_c q_L - 3t_c^2 + q_L^2) + t_c(2t_c q_L - 2t_c^2 + q_L^2))}{(-4q_H(q_L - t_c) + 4t_c q_L - 4t_c^2 + q_L^2)(q_H(t_c - q_L) + t_c q_L - t_c^2 + q_L^2)}. \end{aligned} \quad (A9)$$

Firm i 's total demand of two periods is given by $d_i^{VC}(q_H, q_L, t_c) = d_{i1}^{VC}(q_H, q_L, t_c) + d_{i2}^{VC}(q_H, q_L, t_c)$, and its total profit is $\pi_i^{VC}(q_H, q_L, t_c) = p_i^{VC}(q_H, q_L, t_c) d_i^{VC}(q_H, q_L, t_c)$, for $i \in \{H, L\}$.

Given that the market is partially covered and each firm has positive demand (i.e., $0 < \bar{\theta}_L^{VC} < \bar{\theta}_H^{VC} < 1$), taking derivative with respect to t_c (note that the sub-game equilibrium outcomes are all differentiable), one can verify that, after simplification:

$$\frac{\partial p_L^{VC}(q_H, q_L, t_c)}{\partial t_c} < 0 \quad \text{and} \quad \frac{\partial d_H^{VC}(q_H, q_L, t_c)}{\partial t_c} > 0.$$

$\frac{\partial p_H^{VC}(q_H, q_L, t_c)}{\partial t_c}$, $\frac{\partial d_L^{VC}(q_H, q_L, t_c)}{\partial t_c}$, $\frac{\partial \pi_H^{VC}(q_H, q_L, t_c)}{\partial t_c}$, and $\frac{\partial \pi_L^{VC}(q_H, q_L, t_c)}{\partial t_c}$ could be > 0 , $= 0$ or < 0 . Thus, $p_L^{VC}(q_H, q_L, t_c)$ decreases in t_c , $d_H^{VC}(q_H, q_L, t_c)$ increases in t_c ; and $p_H^{VC}(q_H, q_L, t_c)$, $d_L^{VC}(q_H, q_L, t_c)$, $\pi_H^{VC}(q_H, q_L, t_c)$, and $\pi_L^{VC}(q_H, q_L, t_c)$ may increase or decrease in t_c . This completes the proofs of Parts (a)-(c). \square

Proof of Proposition 2: Similar to the proof of Proposition 1, we complete the proof in three steps:

Step 1. Solve the potential equilibrium given that the market is partially covered and each firm has positive demand in each period; **Step 2.** Show the existence of the equilibrium derived in Step 1 when t_c is small enough; **Step 3.** Show the uniqueness of the equilibrium when t_c is small enough. In the end, we will provide the upper bound for t_c that guarantees t_c is small enough and the equilibrium uniquely exists.

Step 1. We first solve the potential equilibrium given that the market is partially covered and each firm has positive demand in each period. We have solved the firms' pricing decisions for given quality levels in the proof of Lemma 2. Given the firms' pricing decisions in Stage 2, we proceed to solve firms' quality decisions in the first stage:

$$\max_{q_H \in (q_L, 1]} \pi_H^{VC}(q_H, q_L, t_c) = p_H^{VC}(q_H, q_L, t_c) d_H^{VC}(q_H, q_L, t_c) \quad \text{and} \quad \max_{q_L \in [0, q_H)} \pi_L^{VC}(q_H, q_L, t_c) = p_L^{VC}(q_H, q_L, t_c) d_L^{VC}(q_H, q_L, t_c),$$

where $p_i^{VC}(q_H, q_L, t_c)$ and $d_i^{VC}(q_H, q_L, t_c)$ are given by Equations (A8) and (A9), respectively.

First, one can easily verify that given the market is partially covered and each firm has positive demand (i.e., $0 < \bar{\theta}_L^{VC} < \bar{\theta}_H^{VC} < 1$), $\pi_H^{VC}(q_H, q_L, t_c)$ increases in q_H for any $q_L \in [0, 1)$ and $q_H > t_c$ i.e., $\frac{d\pi_H^{VC}(q_H, q_L, t_c)}{dq_H} > 0$. Thus for $q_H > t_c$, the optimal quality is $q_H = 1$. Moreover, it is easy to verify that $\pi_H^{VC}(q_H, q_L, t_c) < \pi_H^{VC}(1, q_L, t_c)$ for any $q_L < q_H \leq t_c$ and $q_L \in [0, 1)$. Thus, firm H's global optimal quality is $\hat{q}_H^{VC} = 1$ for any $q_L \in [0, 1)$. Given that $q_H = 1$, next we solve firm L's optimal quality q_L . Directly checking firm L's profit function, we find that $\pi_L^{VC}(1, q_L, t_c)$ first increases and then decreases in q_L if $0 < t_c < \frac{23-3\sqrt{41}}{40} \approx 0.0948$. Therefore, there exists a unique $\hat{q}_L^{VC}(t_c)$ that can be solved from $\frac{\partial \pi_L^{VC}(1, q_L, t_c)}{\partial q_L} = 0$ when t_c is small enough.

Hence, when t_c is small enough, given that the market is partially covered and each firm has positive demand in each period, the potential equilibrium outcome is: $q_H = \hat{q}_H^{VC} = 1$, $q_L = \hat{q}_L^{VC}(t_c)$, $p_H = \hat{p}_H^{VC}(t_c) = p_H^{VC}(1, \hat{q}_L^{VC}(t_c), t_c)$, and $p_L = \hat{p}_L^{VC}(t_c) = p_L^{VC}(1, \hat{q}_L^{VC}(t_c), t_c)$. Let $\sigma^{VC} = (\hat{q}_H^{VC}, \hat{q}_L^{VC}(t_c), \hat{p}_H^{VC}(t_c), \hat{p}_L^{VC}(t_c))$ denote this potential equilibrium, and let $\hat{d}_i^{VC}(t_c) = d_i^{VC}(1, \hat{q}_L^{VC}(t_c), t_c)$ and $\hat{\pi}_i^{VC}(t_c) = \hat{p}_i^{VC}(t_c) \hat{d}_i^{VC}(t_c)$ be firm i 's total demand and profit at the potential equilibrium σ^{VC} .

Step 2. To show the existence of the equilibrium σ^{VC} , we will show that σ^{VC} is a Sub-game Perfect Equilibrium (SPE) when t_c is small enough. We just need to verify that σ^{VC} satisfies the no-deviation requirements of SPE: (1) Given firms' quality $0 \leq q_L < q_H \leq 1$ and firm i 's price $p_i^{VC}(q_H, q_L, t_c)$, firm j 's price decision will not deviate from $p_j^{VC}(q_H, q_L, t_c)$ to any other price. (2) Given both firms' pricing strategies $(p_H^{VC}(q_H, q_L, t_c), p_L^{VC}(q_H, q_L, t_c))$ in Stage 2 and firm i 's quality $\hat{q}_i^{VC}(r)$, firm j 's quality decision will not deviate from $\hat{q}_j^{VC}(t_c)$ to any other quality, i.e., $\hat{\pi}_j^{VC}(t_c) \geq \max_{q_j \neq \hat{q}_j^{VC}(t_c)} \pi_j^{VC}(q_j, \hat{q}_i^{VC}(t_c), t_c)$, for $i, j \in \{H, L\}$ and $i \neq j$.

To show (1), we just need to show

$$\pi_j^{VC}(q_H, q_L, t_c) \geq \max_{p_j \neq p_j^{VC}(q_H, q_L, t_c)} p_j d_j^{VC}(p_j | q_H, q_L, p_i^{VC}(q_H, q_L, t_c)), \text{ for } i, j \in \{H, L\} \text{ and } i \neq j, \quad (\text{A10})$$

where the general demand functions in period $n = 1, 2$ are given by:

$$(d_{Hn}^{VC}(q_H, q_L, p_H, p_L), d_{Ln}^{VC}(q_H, q_L, p_H, p_L)) = \begin{cases} \left(1 - \frac{p_L q_L + (p_H - t_c)(t_c - q_L)}{q_L^2 + t_c q_L + q_H(t_c - q_L) - t_c^2}, \frac{p_L(q_H - t_c) + q_L(t_c - p_H)}{q_L^2 + t_c q_L + q_H(t_c - q_L) - t_c^2}\right), & \text{if } (p_H, p_L) \in C_1 \\ \left(1 - \frac{p_H - p_L - t_c}{q_H - q_L - 2t_c}, \frac{p_H - p_L - t_c}{q_H - q_L - 2t_c}\right), & \text{if } (p_H, p_L) \in C_2 \\ \left(1 - \frac{p_H - t_c}{q_H - t_c}, 0\right), & \text{if } (p_H, p_L) \in C_3 \\ (1, 0), & \text{if } (p_H, p_L) \in C_4 \\ \left(0, 1 - \frac{p_L - t_c}{q_L - t_c}\right), & \text{if } (p_H, p_L) \in C_5 \\ (0, 1), & \text{if } (p_H, p_L) \in C_6 \end{cases}$$

Note that the above demand functions include all the situations that market is partially or fully covered and each firm has positive or zero demand. In the above equation, C_i ($i = 1, 2, \dots, 6$) is a set of conditions that defines a region for the price pair: $C_1 = \{0 < \frac{t_c(p_H - t_c) + p_L(q_L - q_H + t_c)}{q_L^2 + t_c q_L + q_H(t_c - q_L) - t_c^2} < \frac{p_L q_L + (p_H - t_c)(t_c - q_L)}{q_L^2 + t_c q_L + q_H(t_c - q_L) - t_c^2} < 1\}$, in which the market is partially covered and each firm has positive demand; $C_2 = \{0 < \frac{p_H - p_L - t_c}{q_H - q_L - 2t_c} < 1, t_c \frac{p_H - p_L - t_c}{q_H - q_L - 2t_c} \geq p_L\}$, in which the market is fully covered and each firm has positive demand; $C_3 = \{0 < \frac{p_H - t_c}{q_H - t_c} < 1, \frac{p_H - t_c}{q_H - t_c} q_L - p_L < 0\}$, in which the market is partially covered and firm L has no demand; $C_4 = \{t_c \geq p_H\}$, in which the market is fully covered and firm L has no demand; $C_5 = \{0 < \frac{p_L - t_c}{q_L - t_c} < 1, q_H - p_H \leq q_L - p_L + t_c(1 - \frac{p_L - t_c}{q_L - t_c})\}$, in which the market is partially covered and firm H has no demand; and $C_6 = \{t_c \geq p_L, q_H - p_H \leq q_L - p_L + t_c\}$, in which the market is fully covered and firm H has no demand. Firm i 's total demand of two periods is given by $d_i^{VC} = d_{i1}^{VC} + d_{i2}^{VC}$, for $i \in \{H, L\}$. Through straightforward yet tedious algebraic analysis, we can verify that firm L will not deviate from $p_L^{VC}(q_H, q_L, t_c)$ when t_c is small enough (i.e., $0 < t_c < T^{VC}(q_H, q_L)$ and $t_c < \frac{23 - 3\sqrt{41}}{40} \approx 0.0948$); and firm H will not deviate from $p_H^{VC}(q_H, q_L, t_c)$ when $t_c < T_1^{VC}(q_H, q_L)$, where $T_1^{VC}(q_H, q_L)$ is the smallest root of the polynomial $-16t_c^6(5q_H + 2q_L) + 4t_c^5(42q_H q_L + 32q_H^2 - 5q_L^2) - 8t_c^4(35q_H^2 q_L - 3q_H q_L^2 + 10q_H^3 - 6q_L^3) + t_c^3(184q_H^3 q_L + 61q_H^2 q_L^2 - 162q_H q_L^3 + 16q_H^4 + 9q_L^4) + t_c^2 q_L(-97q_H^3 q_L + 153q_H^2 q_L^2 + q_H q_L^3 - 40q_H^4 - 17q_L^4) + t_c q_L^2(q_H - q_L)^2(33q_H q_L + 32q_H^2 - 5q_L^2) + 16t_c^7 + 8q_H q_L^6 - 24q_H^2 q_L^5 + 24q_H^3 q_L^4 - 8q_H^4 q_L^3 = 0$. Thus, inequality (A10) holds and requirement (1) is satisfied when t_c is small enough.

For requirement (2), we have shown in Step 1 that when $t_c < 0.0948$, firm H's optimal quality is $q_H = 1$ for any $q_L \in [0, 1)$ and firm L's optimal quality is $q_L = \hat{q}_L^{VC}(t_c)$ given $q_H = 1$. Thus, both firms will not deviate and requirement (2) holds. Hence, we have shown that σ^{VC} is an SPE when t_c is small enough.

Step 3. We will show the uniqueness of the equilibrium σ^{VC} when t_c is small enough. Note that in Step 1 we have derived the unique equilibrium σ^{VC} by backward induction under the condition that the market is partially covered and each firm has positive demand in each period. Thus, we just need to show that there does not exist any equilibrium if the above condition does not hold, i.e., the market is fully covered or one of the firms has zero demand. Since the two periods are identical, we just need to verify that: (1) Any strategy that leads to $d_{H1} = 0$ or $d_{L1} = 0$ is not an equilibrium. (2) Any strategy that leads to $d_{H1} + d_{L1} = 1$ is not an equilibrium.

If $d_{i1} = 0$, then $d_{i2} = 0$ and thus firm i 's profit is zero. It is obvious that any strategy with $d_{i1} = 0$ cannot be an equilibrium outcome since firm i has incentive to deviate to a lower price to get a positive demand and earn a positive profit. Thus (1) holds.

If $d_{H1} + d_{L1} = 1$, then the market is fully covered. We will show that there does not exist any SPE in this case by contradiction. Assume there exists an SPE in this case, then we can solve it by backward induction. Let $\bar{\theta}_{H,F}^{VC} \in (0, 1)$ denote the consumer who is indifferent in purchasing from firm H and firm L. Since the market is fully covered by the two firms, then each firm's demand in each period is given by $d_{H1}^{VC} = d_{H2}^{VC} = 1 - \bar{\theta}_{H,F}^{VC}$ and $d_{L1}^{VC} = d_{L2}^{VC} = \bar{\theta}_{H,F}^{VC}$. Thus, $\bar{\theta}_{H,F}^{VC}$ can be solved from $\bar{\theta}_{H,F}^{VC} q_H - p_H + t_c(1 - \bar{\theta}_{H,F}^{VC}) = \bar{\theta}_{H,F}^{VC} q_L - p_L + t_c \bar{\theta}_{H,F}^{VC}$. Thus, we get $\bar{\theta}_{H,F}^{VC} = \frac{p_H - p_L - t_c}{q_H - q_L - 2t_c}$. Note that in this case, consumer with $\theta = 0$ will purchase from firm L and has non-negative utility, i.e., $u_{L1}|_{\theta=0} = -p_L + t_c \bar{\theta}_{H,F}^{VC} \geq 0$. Each firm's demand in each period is then given as below:

$$d_{H1}^{VC} = d_{H2}^{VC} = 1 - \bar{\theta}_{H,F}^{VC} = 1 - \frac{p_H - p_L - t_c}{q_H - q_L - 2t_c} \text{ and } d_{L1}^{VC} = d_{L2}^{VC} = \bar{\theta}_{H,F}^{VC} = \frac{p_H - p_L - t_c}{q_H - q_L - 2t_c}.$$

In Stage 2, given quality levels $0 \leq q_L < q_H \leq 1$, firm i decides price p_i to maximize its total profit $2p_i d_{i1}^{VC}$. Checking the derivative, we find that firm i 's total profit is concave in p_i if $q_H - q_L - 2t_c > 0$ and increases in p_i if $q_H - q_L - 2t_c < 0$, i.e., $\frac{\partial^2(2p_i d_{i1}^{VC})}{\partial p_i^2} < 0$ if $q_H - q_L - 2t_c > 0$ and $\frac{\partial(2p_i d_{i1}^{VC})}{\partial p_i} > 0$ if $q_H - q_L - 2t_c < 0$, for $i = H, L$. When $q_H - q_L - 2t_c > 0$, solving the two firms' first-order conditions together (i.e., $\frac{\partial(2p_H d_{H1}^{VC})}{\partial p_H} = 0$ and $\frac{\partial(2p_L d_{L1}^{VC})}{\partial p_L} = 0$), we can get the following prices:

$$p_{H,F}^{VC}(q_H, q_L, t_c) = \frac{2q_H - 2q_L - 3t_c}{3} \quad \text{and} \quad p_{L,F}^{VC}(q_H, q_L, t_c) = \frac{q_H - q_L - 3t_c}{3}.$$

However, the above prices violate the conditions of $0 < \bar{\theta}_H^{VC} = \frac{p_H - p_L - t_c}{q_H - q_L - 2t_c} < 1$ and $u_{L1}|_{\theta=0} = -p_L + t_c \bar{\theta}_H^{VC} \geq 0$. When $q_H - q_L - 2t_c < 0$, firm i 's profit increases in p_i and thus its optimal price is on the boundary in this case. We find that for any p_H , there does not exist optimal p_L for firm L that satisfies $0 < \bar{\theta}_H^{VC} = \frac{p_H - p_L - t_c}{q_H - q_L - 2t_c} < 1$ and $u_{L1}|_{\theta=0} = -p_L + t_c \bar{\theta}_H^{VC} \geq 0$. Thus, no pricing equilibrium exists in the case of fully covered market. That is, any strategy that leads to $d_{H1} + d_{L1} = 1$ cannot be an equilibrium. Therefore, (2) holds.

Hence, we conclude that in the equilibrium, the market is partially covered and each firm has positive demand in each period (i.e., $d_{Hi} > 0$, $d_{Li} > 0$, and $d_{Hi} + d_{Li} < 1$, for $i = 1, 2$).

Combining Steps 1-3, we show that σ^{VC} is the unique SPE when t_c is small enough. Moreover, in the main body of the paper and the appendix, we define $\bar{t}_c = 0.07$ and assume $0 < t_c \leq \bar{t}_c$ to make sure t_c is small enough which guarantees the existence and uniqueness of the equilibrium σ^{VC} .

Finally, we proceed to prove parts (a)-(d) below. Note that all the final equilibrium outcomes are differentiable.

Part (a): Since $\hat{q}_L^{VC}(t_c)$ is the unique solution to $\frac{\partial \pi_L^{VC}(1, q_L, t_c)}{\partial q_L} = 0$, by the Implicit Function Theorem, we have: $\frac{\partial \hat{q}_L^{VC}(t_c)}{\partial t_c} = -\frac{\partial^2 \pi_L^{VC}(1, q_L, t_c)}{\partial q_L \partial t_c} / \frac{\partial^2 \pi_L^{VC}(1, q_L, t_c)}{\partial q_L^2} \Big|_{q_L = \hat{q}_L^{VC}(t_c)}$. One can verify that $\frac{\partial^2 \pi_L^{VC}(1, q_L, t_c)}{\partial q_L^2} < 0$ and $\frac{\partial^2 \pi_L^{VC}(1, q_L, t_c)}{\partial q_L \partial t_c} < 0$ for any $q_L \in [0.26, 0.6]$ and $0 < t_c \leq \bar{t}_c$. Moreover, we find that $0.26 < \hat{q}_L^{VC}(t_c) < 0.6$

holds for $0 < t_c \leq \bar{t}_c$, since $\frac{\partial \pi_L^{VC}(1, q_L, t_c)}{\partial q_L} < 0$ for any $q_L \in [0.6, 1)$ and $\frac{\partial \pi_L^{VC}(1, q_L, t_c)}{\partial q_L} > 0$ for any $q_L \in [0, 0.26]$. Therefore, $\frac{\partial \hat{q}_L^{VC}(t_c)}{\partial t_c} < 0$.

Part (b): Note that $\hat{p}_i^{VC}(t_c) = p_i^{VC}(1, \hat{q}_L^{VC}(t_c), t_c)$, for $i = H, L$. By the chain rule, $\frac{\partial \hat{p}_i^{VC}(t_c)}{\partial t_c} = \left(\frac{\partial p_i^{VC}(1, q_L, t_c)}{\partial t_c} + \frac{\partial p_i^{VC}(1, q_L, t_c)}{\partial q_L} \frac{\partial \hat{q}_L^{VC}(t_c)}{\partial t_c} \right) \Big|_{q_L = \hat{q}_L^{VC}(t_c)}$. One can verify that $\frac{\partial p_H^{VC}(1, q_L, t_c)}{\partial t_c} + \frac{\partial p_H^{VC}(1, q_L, t_c)}{\partial q_L} \frac{\partial \hat{q}_L^{VC}(t_c)}{\partial t_c} > 0$ and $\frac{\partial p_L^{VC}(1, q_L, t_c)}{\partial t_c} + \frac{\partial p_L^{VC}(1, q_L, t_c)}{\partial q_L} \frac{\partial \hat{q}_L^{VC}(t_c)}{\partial t_c} < 0$ for any $q_L \in [0.26, 0.6]$ and $0 < t_c \leq \bar{t}_c$. As already shown in Part (a) above, $\hat{q}_L^{VC}(t_c) \in (0.26, 0.6)$. Therefore, $\frac{\partial \hat{p}_H^{VC}(t_c)}{\partial t_c} > 0$ and $\frac{\partial \hat{p}_L^{VC}(t_c)}{\partial t_c} < 0$. Hence, $\hat{p}_H^{VC}(t_c) > \hat{p}_H^{VC}(0) = \hat{p}_H^b$ and $\hat{p}_L^{VC}(t_c) < \hat{p}_L^{VC}(0) = \hat{p}_L^b$ for $0 < t_c \leq \bar{t}_c$.

Part (c): Note that $\hat{d}_i^{VC}(t_c) = d_i^{VP}(1, \hat{q}_L^{VC}(t_c), t_c)$, for $i = H, L$. By the chain rule, $\frac{\partial \hat{d}_i^{VC}(t_c)}{\partial t_c} = \left(\frac{\partial d_i^{VP}(1, q_L, t_c)}{\partial t_c} + \frac{\partial d_i^{VP}(1, q_L, t_c)}{\partial q_L} \frac{\partial \hat{q}_L^{VC}(t_c)}{\partial t_c} \right) \Big|_{q_L = \hat{q}_L^{VC}(t_c)}$. One can verify that $\frac{\partial d_H^{VP}(1, q_L, t_c)}{\partial t_c} + \frac{\partial d_H^{VP}(1, q_L, t_c)}{\partial q_L} \frac{\partial \hat{q}_L^{VC}(t_c)}{\partial t_c} > 0$ for any for any $q_L \in [0.26, 0.6]$ and $0 < t_c \leq \bar{t}_c$. Since $\hat{q}_L^{VC}(t_c) \in (0.26, 0.6)$, we have $\frac{\partial \hat{d}_H^{VC}(t_c)}{\partial t_c} > 0$, i.e., $\hat{d}_H^{VC}(t_c)$ increases in t_c . Moreover, one can verify that $\frac{\partial d_L^{VP}(1, q_L, t_c)}{\partial t_c} + \frac{\partial d_L^{VP}(1, q_L, t_c)}{\partial q_L} \frac{\partial \hat{q}_L^{VC}(t_c)}{\partial t_c}$ is negative when t_c is close to 0 and is positive when t_c is close to \bar{t}_c . Thus, $\hat{d}_L^{VC}(t_c)$ may decrease or increase in t_c .

Part (d): Note that $\hat{\pi}_i^{VC}(t_c) = \pi_i^{VC}(1, \hat{q}_L^{VC}(t_c), t_c)$. By the chain rule, $\frac{\partial \hat{\pi}_i^{VC}(t_c)}{\partial t_c} = \left(\frac{\partial \pi_i^{VC}(1, q_L, t_c)}{\partial t_c} + \frac{\partial \pi_i^{VC}(1, q_L, t_c)}{\partial q_L} \frac{\partial \hat{q}_L^{VC}(t_c)}{\partial t_c} \right) \Big|_{q_L = \hat{q}_L^{VC}(t_c)}$. One can verify that $\frac{\partial \pi_H^{VC}(1, q_L, t_c)}{\partial t_c} + \frac{\partial \pi_H^{VC}(1, q_L, t_c)}{\partial q_L} \frac{\partial \hat{q}_L^{VC}(t_c)}{\partial t_c} > 0$ and $\frac{\partial \pi_L^{VC}(1, q_L, t_c)}{\partial t_c} + \frac{\partial \pi_L^{VC}(1, q_L, t_c)}{\partial q_L} \frac{\partial \hat{q}_L^{VC}(t_c)}{\partial t_c} < 0$ for $q_L \in (0.26, 0.6)$ and $0 < t_c \leq \bar{t}_c$. Since $\hat{q}_L^{VC}(t_c) \in (0.26, 0.6)$, we have $\frac{\partial \hat{\pi}_H^{VC}(t_c)}{\partial t_c} > 0$ and $\frac{\partial \hat{\pi}_L^{VC}(t_c)}{\partial t_c} < 0$, i.e., $\hat{\pi}_H^{VC}(t_c)$ increases in t_c and $\hat{\pi}_L^{VC}(t_c)$ decreases in t_c . Therefore, $\hat{\pi}_H^{VC}(t_c) - \hat{\pi}_H^b > 0 > \hat{\pi}_L^{VC}(t_c) - \hat{\pi}_L^b$. \square

Proof of Corollary 2: Part (a): Let $CM_{in}^{VC}(t_c)$ denote consumer monetary surplus of purchasing from firm i in period n in the equilibrium, which is given by:

$$CM_{H1}^{VC}(t_c) = CM_{H2}^{VC}(t_c) = \int_{\bar{\theta}_H^{VC}}^1 (\theta - \hat{p}_H^{VC}(t_c)) d\theta = (1 - 2\hat{p}_H^{VC}(t_c) + \bar{\theta}_H^{VC})(1 - \bar{\theta}_H^{VC})/2 \quad \text{and}$$

$$CM_{L1}^{VC}(t_c) = CM_{L2}^{VC}(t_c) = \int_{\bar{\theta}_L^{VC}}^{\bar{\theta}_H^{VC}} (\theta \hat{q}_L^{VC}(t_c) - \hat{p}_L^{VC}(t_c)) d\theta = (\hat{q}_L^{VC}(t_c)(\bar{\theta}_H^{VC} + \bar{\theta}_L^{VC}) - 2\hat{p}_L^{VC}(t_c))(\bar{\theta}_H^{VC} - \bar{\theta}_L^{VC})/2,$$

where $\bar{\theta}_H^{VC} = \frac{\hat{p}_L^{VC}(t_c)\hat{q}_L^{VC}(t_c) + (\hat{p}_H^{VC}(t_c) - t_c)(t_c - \hat{q}_L^{VC}(t_c))}{\hat{q}_L^{VC}(t_c)^2 + t_c\hat{q}_L^{VC}(t_c) + (t_c - \hat{q}_L^{VC}(t_c)) - t_c^2}$ and $\bar{\theta}_L^{VC} = \frac{t_c(\hat{p}_H^{VC}(t_c) - t_c) + \hat{p}_L^{VC}(t_c)(\hat{q}_L^{VC}(t_c) - 1 + t_c)}{\hat{q}_L^{VC}(t_c)^2 + t_c\hat{q}_L^{VC}(t_c) + (t_c - \hat{q}_L^{VC}(t_c)) - t_c^2}$. $\bar{\theta}_H^{VC}$ is the consumer who is indifferent in purchasing from firm H and firm L; $\bar{\theta}_L^{VC}$ is the consumer who is indifferent in purchasing from firm L and not purchasing. Consumer monetary surplus in period n is then given by $CM_n^{VC}(t_c) = CM_{Hn}^{VC}(t_c) + CM_{Ln}^{VC}(t_c)$, and total consumer monetary surplus of two periods is given by $CM^{VC}(t_c) = CM_1^{VC}(t_c) + CM_2^{VC}(t_c)$.

We can show that $\frac{dCM_{Hn}^{VC}(t_c)}{dt_c} < 0$ and $\frac{dCM_{Ln}^{VC}(t_c)}{dt_c} < 0$ for $0 < t_c \leq \bar{t}_c$ and $n = 1, 2$. Thus $\frac{dCM_1^{VC}(t_c)}{dt_c} < 0$, $\frac{dCM_2^{VC}(t_c)}{dt_c} < 0$, and $\frac{dCM^{VC}(t_c)}{dt_c} < 0$. Therefore, $CM_1^{VC}(t_c)$, $CM_2^{VC}(t_c)$, and $CM^{VC}(t_c)$ all decrease in t_c .

Let CM^b denote the total consumer monetary surplus of two periods in the benchmark case, i.e., $CM^b = CM^{VC}(0)$. Thus, $CM^{VC}(t_c) < CM^{VC}(0) = CM^b$ for $0 < t_c \leq \bar{t}_c$.

Part (b): Let $CS_{in}^{VC}(t_c)$ denote consumer total surplus of purchasing from firm i in period n in the equilibrium, which is given by:

$$\begin{aligned} CS_{Hn}^{VC}(t_c) &= \int_{\bar{\theta}_H^{VC}}^1 (\theta - \hat{p}_H^{VC}(t_c) + t_c \hat{d}_{Hn}^{VC}(t_c)) d\theta \\ &= \frac{1}{2} (1 - 2\hat{p}_H^{VC}(t_c) + 2t_c \hat{d}_{Hn}^{VC}(t_c) + \bar{\theta}_H^{VC})(1 - \bar{\theta}_H^{VC}) \quad \text{and} \\ CS_{Ln}^{VC}(t_c) &= \int_{\bar{\theta}_L^{VC}}^{\bar{\theta}_H^{VC}} (\theta \hat{q}_L^{VC}(t_c) - \hat{p}_L^{VC}(t_c) + t_c \hat{d}_{Ln}^{VC}(t_c)) d\theta \\ &= \frac{1}{2} (\hat{q}_L^{VC}(t_c) (\bar{\theta}_H^{VC} + \bar{\theta}_L^{VC}) - 2\hat{p}_L^{VC}(t_c) + 2t_c \hat{d}_{Ln}^{VC}(t_c)) (\bar{\theta}_H^{VC} - \bar{\theta}_L^{VC}), \text{ for } n = 1, 2. \end{aligned}$$

Consumer total surplus in period n is given by $CS_n^{VC}(t_c) = CS_{Hn}^{VC}(t_c) + CS_{Ln}^{VC}(t_c)$, and consumer total surplus of two periods is given by $CS^{VC}(t_c) = CS_1^{VC}(t_c) + CS_2^{VC}(t_c)$.

Given $0 < t_c \leq \bar{t}_c$, we can show that $\frac{dCS_{Hn}^{VC}(t_c)}{dt_c} < 0$ if $t_c < 0.0128246$ and $\frac{dCS_{Hn}^{VC}(t_c)}{dt_c} \geq 0$ otherwise; and $\frac{dCS_{Ln}^{VC}(t_c)}{dt_c} < 0$, for $n = 1, 2$. Moreover, we can also show that $\frac{dCS^{VC}(t_c)}{dt_c} < 0$ if $t_c < 0.0603381$ and $\frac{dCS^{VC}(t_c)}{dt_c} \geq 0$ otherwise. Therefore, $CS^{VC}(t_c)$ first decreases and then increases in t_c .

Part (c): The monetary term of social welfare is defined as $SM^{VC}(t_c) = \pi_H^{VC}(t_c) + \pi_L^{VC}(t_c) + CM^{VC}(t_c)$. We have already shown that $CM^{VC}(t_c)$ decreases in t_c in part (a), and that $\pi_H^{VC}(t_c)$ increases in t_c and $\pi_L^{VC}(t_c)$ decreases in t_c in Proposition 2(d). We can verify that $\frac{dSM^{VC}(t_c)}{dt_c} < 0$, indicating $SM^{VC}(t_c)$ decreases in t_c . Thus, $SM^{VC}(t_c) < SM^{VC}(0) = SM^b$.

Part (d): The social welfare is defined as $SW^{VC}(t_c) = \pi_H^{VC}(t_c) + \pi_L^{VC}(t_c) + CS^{VC}(t_c)$. Given $0 < t_c \leq \bar{t}_c$, we can verify that $\frac{dSW^{VC}(t_c)}{dt_c} > 0$, indicating $SW^{VC}(t_c)$ increases in t_c . Thus, $SW^{VC}(t_c) > SW^{VC}(0) = SW^b$. \square

A.3. Value Enhancement Effect from Previous Consumers (VEE-P)

Proof of Lemma 3: Given $r = t_c = 0$ and $t_p > 0$ and $0 \leq q_L < q_H \leq 1$, firm i decides its price p_i to maximize its total profit:

$$\max_{p_i \geq 0} p_i d_{i1}^{VP} + p_i d_{i2}^{VP}, \text{ where } i = H, L.$$

It is obvious that in the equilibrium $p_H > 0$ and $p_L > 0$, which indicates that the market is partially covered by the two firms in period 1 since a consumer with $\theta = 0$ always receives negative utility of purchasing from either firm. Moreover, if firm i has zero demand in period 1, then there is no value enhancement effect for firm i in period 2 and its demand in period 2 is also zero, which leads to zero total profit. Thus, in the equilibrium, each firm must have positive demand in period 1, i.e., $0 < \frac{p_L}{q_L} < \frac{p_H - p_L}{q_H - q_L} < 1$. Thus, the demand of each firm in period 1 can be solved as below:

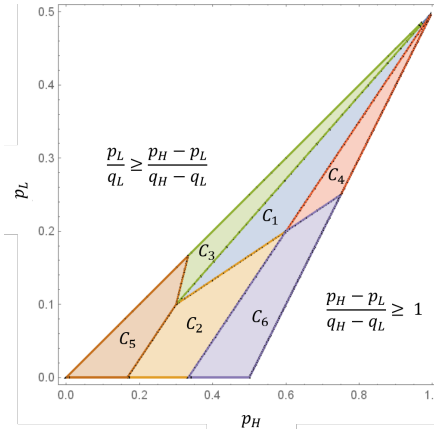
$$d_{H1}^{VP} = 1 - \frac{p_H - p_L}{q_H - q_L} \quad \text{and} \quad d_{L1}^{VP} = \frac{p_H - p_L}{q_H - q_L} - \frac{p_L}{q_L}. \quad (\text{A11})$$

However, due to the value enhancement effect, in period 2, it is not obvious that in the equilibrium whether the market is fully covered or not and whether each firm has zero demand or not. The general demand of each firm in period 2 can be solved as below:

$$(d_{H2}^{VP}, d_{L2}^{VP}) = \begin{cases} \left(1 - \frac{(p_H - t_p d_{H1}^{VP}) - (p_L - t_p d_{L1}^{VP})}{q_H - q_L}, \frac{(p_H - t_p d_{H1}^{VP}) - (p_L - t_p d_{L1}^{VP})}{q_H - q_L} - \frac{p_L - t_p d_{L1}^{VP}}{q_L}\right), & \text{if } (p_H, p_L) \in C_1; \\ \left(1 - \frac{(p_H - t_p d_{H1}^{VP}) - (p_L - t_p d_{L1}^{VP})}{q_H - q_L}, \frac{(p_H - t_p d_{H1}^{VP}) - (p_L - t_p d_{L1}^{VP})}{q_H - q_L}\right), & \text{if } (p_H, p_L) \in C_2; \\ (1 - (p_H - t_p d_{H1}^{VP}), 0), & \text{if } (p_H, p_L) \in C_3; \\ \left(0, 1 - \frac{p_L - t_p d_{L1}^{VP}}{q_L}\right), & \text{if } (p_H, p_L) \in C_4; \\ (1, 0), & \text{if } (p_H, p_L) \in C_5; \\ (0, 1), & \text{if } (p_H, p_L) \in C_6. \end{cases} \quad (\text{A12})$$

In the above, C_i ($i = 1, 2, \dots, 6$) is a set of conditions that defines a region for the price pair: $C_1 = \{0 < \frac{(p_L - t_p d_{L1}^{VP})}{q_L} < \frac{(p_H - t_p d_{H1}^{VP}) - (p_L - t_p d_{L1}^{VP})}{q_H - q_L} < 1\}$; $C_2 = \{\frac{(p_L - t_p d_{L1}^{VP})}{q_L} \leq 0 < \frac{(p_H - t_p d_{H1}^{VP}) - (p_L - t_p d_{L1}^{VP})}{q_H - q_L} < 1\}$; $C_3 = \{0 \leq p_H - t_p d_{H1}^{VP} \leq \frac{(p_L - t_p d_{L1}^{VP})}{q_L}\}$; $C_4 = \{0 \leq p_L - t_p d_{L1}^{VP} \leq \min\{q_L, q_L - q_H + (p_H - t_p d_{H1}^{VP})\}\}$; $C_5 = \{p_H - t_p d_{H1}^{VP} \leq \min\{0, p_L - t_p d_{L1}^{VP}\}\}$; and $C_6 = \{p_L - t_p d_{L1}^{VP} \leq \min\{0, q_L - q_H + (p_H - t_p d_{H1}^{VP})\}\}$. See Figure A1 for an illustration. We will say ‘‘under Condition i ’’ if $(p_H, p_L) \in C_i$. Under Condition 1, the market is not fully covered and each firm has positive demand in period 2; under Condition 2, the market is fully covered in period 2 and each firm has positive demand in period 2; under other conditions, one of the firms has zero demand in period 2.

Figure A1 Six Cases for Demand in Period 2 under VEE-P



Note: in the figure, $q_H = 1$ and $q_L = 0.5$, and C_i presents Condition i for $i = 1, 2, \dots, 6$.

Next, we solve the firms’ pricing decisions for given quality in the equilibrium in two steps. First, we find all the possible sub-game equilibrium candidates in each region of C_i . Second, we show when and which candidate could be an equilibrium outcome. We will show that when t_p is small enough, the unique equilibrium only exists in the region C_1 where the market is partially covered and each firm has positive demand in each period.

Under Condition 1: $0 < \frac{(p_L - t_p d_{L1}^{VP})}{q_L} < \frac{(p_H - t_p d_{H1}^{VP}) - (p_L - t_p d_{L1}^{VP})}{q_H - q_L} < 1$. Each firm's profit can be written as:

$$\begin{aligned} \pi_H^{VP}(q_H, q_L, t_p, p_H, p_L) &= p_H \left(1 - \frac{p_H - p_L}{q_H - q_L} \right) + p_H \left(1 - \frac{(p_H - t_p(1 - \frac{p_H - p_L}{q_H - q_L})) - (p_L - t_p(\frac{p_H - p_L}{q_H - q_L} - \frac{p_L}{q_L}))}{q_H - q_L} \right), \text{ and} \\ \pi_L^{VP}(q_H, q_L, t_p, p_L, p_H) &= \\ p_L \left(\frac{p_H - p_L}{q_H - q_L} - \frac{p_L}{q_L} \right) &+ p_L \left(\frac{(p_H - t_p(1 - \frac{p_H - p_L}{q_H - q_L})) - (p_L - t_p(\frac{p_H - p_L}{q_H - q_L} - \frac{p_L}{q_L}))}{q_H - q_L} - \frac{p_L - t_p(\frac{p_H - p_L}{q_H - q_L} - \frac{p_L}{q_L})}{q_L} \right). \end{aligned}$$

One can verify that firm i 's total profit is concave in p_i , i.e., $\frac{\partial^2 \pi_i^{VP}}{\partial p_i^2} < 0$, for $i = H, L$. Suppose that there exists an equilibrium outcome $(p_H^{VP}(q_H, q_L, t_p), p_L^{VP}(q_H, q_L, t_p))$ under Condition 1, which satisfies $\frac{\partial \pi_H^{VP}}{\partial p_H} \Big|_{p_H=p_H^{VP}(q_H, q_L, t_p), p_L=p_L^{VP}(q_H, q_L, t_p)} = 0$ and $\frac{\partial \pi_L^{VP}}{\partial p_L} \Big|_{p_H=p_H^{VP}(q_H, q_L, t_p), p_L=p_L^{VP}(q_H, q_L, t_p)} = 0$. Thus, $(p_H^{VP}(q_H, q_L, t_p), p_L^{VP}(q_H, q_L, t_p))$ can be solved as below:

$$\begin{aligned} p_H^{VP}(q_H, q_L, t_p) &= \frac{(q_H - q_L) \left(4q_H^3 (2q_L + t_p) + 2q_H^2 (t_p^2 - 8q_L^2) \right) + q_H q_L \left(-2q_L t_p + 8q_L^2 - t_p^2 \right) + q_L^2 t_p (t_p - 2q_L)}{8q_H^3 (2q_L + t_p) + q_H^2 (4q_L t_p - 36q_L^2 + 7t_p^2) + 2q_H q_L (-4q_L t_p + 12q_L^2 - t_p^2) + q_L^2 (-4q_L t_p - 4q_L^2 + 7t_p^2)}, \\ p_L^{VP}(q_H, q_L, t_p) &= \frac{q_L (q_H - q_L) \left(2q_H^2 (2q_L + t_p) + q_H (-2q_L t_p - 8q_L^2 + t_p^2) \right) - 3q_L t_p^2 + 4q_L^3}{8q_H^3 (2q_L + t_p) + q_H^2 (4q_L t_p - 36q_L^2 + 7t_p^2) + 2q_H q_L (-4q_L t_p + 12q_L^2 - t_p^2) + q_L^2 (-4q_L t_p - 4q_L^2 + 7t_p^2)}. \end{aligned} \tag{A13}$$

One can verify that $p_H = p_H^{VP}(q_H, q_L, t_p)$ and $p_L = p_L^{VP}(q_H, q_L, t_p)$ satisfy Condition 1 iff $0 \leq t_p < T^{VP}(q_H, q_L)$, where $T^{VP}(q_H, q_L)$ is the third (or largest) root of the polynomial $(q_L + q_H)^2 t_p^3 + (-5q_L^3 + 2q_H q_L^2 + q_H^2 q_L + 2q_H^3) t_p^2 + (2q_H q_L^3 - 4q_H^2 q_L^2 + 2q_H^3 q_L) t_p - 4q_H^3 q_L^2 + 4q_L^5 - 12q_H q_L^4 + 12q_H^2 q_L^3 = 0$. Thus, $(p_H^{VP}(q_H, q_L, t_p), p_L^{VP}(q_H, q_L, t_p))$ is a possible equilibrium candidate when $0 \leq t_p < T^{VP}(q_H, q_L)$.

Under Conditions 2-6: Similar as the analysis under Condition 1, we can solve the equilibrium candidates under each condition and verify whether this equilibrium satisfies the condition requirements. Through tedious but straightforward algebraic analysis, we can show that when $0 \leq t_p < T^{VP}(q_H, q_L)$, there does not exist any equilibrium under Conditions 2-6. Detailed analysis is available from authors upon request.

Therefore, for any given $0 \leq q_L < q_H \leq 1$ and $0 \leq t_p < T^{VP}(q_H, q_L)$, the firms' prices are $(p_H^{VP}(q_H, q_L, t_p), p_L^{VP}(q_H, q_L, t_p))$, under which the market is partially covered and each firm has positive demand in each period. In the proof of Proposition 3, we will further show that when t_p is small enough, the market is partially covered and each firm has positive demand in each period in the SPE. Plugging Equations (A13) into Equation (A12), the demand of each firm in each period can be written as:

$$\begin{aligned}
d_{H1}^{VP}(q_H, q_L, t_p) &= \frac{4q_H^3(2q_L + t_p) + q_H^2(6q_L t_p - 16q_L^2 + 5t_p^2) + 8q_H q_L^2(q_L - t_p) + q_L^2 t_p(3t_p - 2q_L)}{8q_H^3(2q_L + t_p) + q_H^2(4q_L t_p - 36q_L^2 + 7t_p^2) + 2q_H q_L(-4q_L t_p + 12q_L^2 - t_p^2) + q_L^2(-4q_L t_p - 4q_L^2 + 7t_p^2)}, \\
d_{L1}^{VP}(q_H, q_L, t_p) &= \frac{2q_H^3(2q_L + t_p) + q_H^2(2q_L t_p - 8q_L^2 + t_p^2) + 2q_H q_L(-q_L t_p + 2q_L^2 + t_p^2) + q_L^2 t_p(t_p - 2q_L)}{8q_H^3(2q_L + t_p) + q_H^2(4q_L t_p - 36q_L^2 + 7t_p^2) + 2q_H q_L(-4q_L t_p + 12q_L^2 - t_p^2) + q_L^2(-4q_L t_p - 4q_L^2 + 7t_p^2)}, \\
d_{H2}^{VP}(q_H, q_L, t_p) &= \frac{X_3 - q_H q_L(-10q_L^2 t_p + 3q_L t_p^2 + 8q_L^3 + 2t_p^3) + q_L^2 t_p(-3q_L t_p + 2q_L^2 + 2t_p^2)}{(q_H - q_L) X_2}, \\
d_{L2}^{VP}(q_H, q_L, t_p) &= \frac{X_4 + q_H q_L^2(-8q_L^2 t_p + 5q_L t_p^2 - 4q_L^3 + t_p^3) + q_L^3 t_p(q_L t_p + 2q_L^2 - 3t_p^2)}{q_L(q_H - q_L) X_2},
\end{aligned} \tag{A14}$$

where $X_2 = (8q_H^3(2q_L + t_p) - q_H^2(36q_L^2 - 7t_p^2 - 4q_L t_p) + 2q_H q_L(12q_L^2 - t_p^2 - 4q_L t_p) - q_L^2(4q_L t_p + 4q_L^2 - 7t_p^2))$, $X_3 = 4q_H^4(2q_L + t_p) + q_H^3(6q_L t_p - 24q_L^2 + 7t_p^2) + q_H^2(-22q_L^2 t_p - q_L t_p^2 + 24q_L^3 + 4t_p^3)$, and $X_4 = 2q_H^4(3q_L t_p + 2q_L^2 + t_p^2) + q_H^3(-16q_L^2 t_p - q_L t_p^2 - 12q_L^3 + t_p^3) + q_H^2 q_L(16q_L^2 t_p - 7q_L t_p^2 + 12q_L^3 - 3t_p^3)$.

Firm i 's total demand of two periods is given by $d_i^{VP}(q_H, q_L, t_p) = d_{i1}^{VP}(q_H, q_L, t_p) + d_{i2}^{VP}(q_H, q_L, t_p)$, and its total profit is $\pi_i^{VP}(q_H, q_L, t_p) = p_i^{VP}(q_H, q_L, t_p) d_i^{VP}(q_H, q_L, t_p)$, for $i \in \{H, L\}$.

For $0 \leq q_L < q_H \leq 1$ and $0 < t_p < T^{VP}(q_H, q_L)$, taking derivative with respect to t_p (note that the sub-game equilibrium outcomes are all differentiable), one can verify that, after simplification:

$$\frac{\partial p_H^{VP}(q_H, q_L, t_p)}{\partial t_p} < 0, \quad \frac{\partial p_L^{VP}(q_H, q_L, t_p)}{\partial t_p} < 0, \quad \text{and} \quad \frac{\partial d_H^{VP}(q_H, q_L, t_p)}{\partial t_p} > 0.$$

However, $\frac{\partial d_L^{VP}(q_H, q_L, t_p)}{\partial t_p}$, $\frac{\partial \pi_H^{VP}(q_H, q_L, t_p)}{\partial t_p}$, and $\frac{\partial \pi_L^{VP}(q_H, q_L, t_p)}{\partial t_p}$ could be > 0 , $= 0$ or < 0 . Thus, $p_H^{VP}(q_H, q_L, t_p)$ and $p_L^{VP}(q_H, q_L, t_p)$ decrease in t_p ; $d_H^{VP}(q_H, q_L, t_p)$ increases in t_p ; and $d_L^{VP}(q_H, q_L, t_p)$, $\pi_H^{VP}(q_H, q_L, t_p)$, and $\pi_L^{VP}(q_H, q_L, t_p)$ may increase or decrease in t_p . This completes the proofs of Parts (a)-(c). \square

Proof of Proposition 3: Similar to the proof of Proposition 1, we complete the proof in three steps:

Step 1. Solve the potential equilibrium given that the market is partially covered and each firm has positive demand in each period; **Step 2.** Show the existence of the equilibrium derived in Step 1 when t_p is small enough; **Step 3.** Show the uniqueness of the equilibrium when t_p is small enough. In the end, we will provide the upper bound for t_p guarantees t_p is small enough and the equilibrium uniquely exists.

Step 1. We first solve the potential equilibrium given that the market is partially covered and each firm has positive demand in each period. We have solved the firms' pricing decisions for given quality levels in the proof of Lemma 3. Given the firms' pricing decisions in Stage 2, we proceed to solve firms' quality decisions in the first stage:

$$\begin{aligned}
\max_{q_H \in (q_L, 1]} \pi_H^{VP}(q_H, q_L, t_p) &= p_H^{VP}(q_H, q_L, t_p) d_H^{VP}(q_H, q_L, t_p), \text{ and} \\
\max_{q_L \in [0, q_H]} \pi_L^{VP}(q_H, q_L, t_p) &= p_L^{VP}(q_H, q_L, t_p) d_L^{VP}(q_H, q_L, t_p),
\end{aligned}$$

where $p_i^{VP}(q_H, q_L, t_p)$ and $d_i^{VP}(q_H, q_L, t_p)$ are given by Equations (A13) and (A14), respectively.

First, one can easily verify that $\pi_H^{VP}(q_H, q_L, t_p)$ increases in q_H for any $q_L \in [0, 1)$ and $t_p > 0$, i.e., $\frac{d\pi_H^{VP}(q_H, q_L, t_p)}{dq_H} > 0$. Thus, firm H's optimal quality is $\hat{q}_H^{VP} = 1$ for any $q_L \in [0, 1)$ and $t_p > 0$. Given that $q_H = 1$, next we solve firm L's optimal quality q_L . Directly checking firm L's profit function, we find that $\pi_L^{VP}(1, q_L, t_p)$ first increases and then decreases in q_L for $0 < t_p < T^{VP}(1, q_L)$. Therefore, there exists a unique $\hat{q}_L^{VP}(t_p)$ that can be solved from $\frac{\partial \pi_L^{VP}(1, q_L, t_p)}{\partial q_L} = 0$.

Therefore, given that the market is partially covered and each firm has positive demand in each period, the potential equilibrium outcome is: $q_H = \hat{q}_H^{VP} = 1$, $q_L = \hat{q}_L^{VP}(t_p)$, $p_H = \hat{p}_H^{VP}(t_p) = p_H^{VP}(1, \hat{q}_L^{VP}(t_p), t_p)$, and $p_L = \hat{p}_L^{VP}(t_p) = p_L^{VP}(1, \hat{q}_L^{VP}(t_p), t_p)$. Let $\sigma^{VP} = (\hat{q}_H^{VP}, \hat{q}_L^{VP}(t_p), \hat{p}_H^{VP}(t_p), \hat{p}_L^{VP}(t_p))$ denote this potential equilibrium, and let $\hat{d}_i^{VP}(t_p) = d_i^{VP}(1, \hat{q}_L^{VP}(t_p), t_p)$ and $\hat{\pi}_i^{VP}(t_p) = \hat{p}_i^{VP}(t_p) \hat{d}_i^{VP}(t_p)$ denote firm i 's total demand and profit at the potential equilibrium σ^{VP} .

Step 2. To show the existence of the equilibrium σ^{VP} , we will show that σ^{VP} is a Sub-game Perfect Equilibrium (SPE) when t_p is small enough. We just need to verify that σ^{VP} satisfies the no-deviation requirements of SPE: (1) Given firms' quality $0 \leq q_L < q_H \leq 1$ and firm i 's price $p_i^{VP}(q_H, q_L, t_p)$, firm j 's price decision will not deviate from $p_j^{VP}(q_H, q_L, t_p)$ to any other price. (2) Given both firms' pricing strategies $(p_H^{VP}(q_H, q_L, t_p)$ and $p_L^{VP}(q_H, q_L, t_p))$ in Stage 2 and firm i 's quality $\hat{q}_i^{VP}(t_p)$, firm j 's quality decision will not deviate from $\hat{q}_j^{VP}(t_p)$ to any other quality, i.e., $\hat{\pi}_j^{VP}(t_p) \geq \max_{q_j \neq \hat{q}_j^{VP}(t_p)} \pi_j^{VP}(q_j, \hat{q}_i^{VP}(t_p), t_p)$, for $i, j \in \{H, L\}$ and $i \neq j$.

To show (1), we just need to show that

$$\pi_j^{VP}(q_H, q_L, t_p) \geq \max_{p_j \neq p_j^{VP}(q_H, q_L, t_p)} p_j d_j^{VP}(p_j | q_H, q_L, p_i^{VP}(q_H, q_L, t_p)), \text{ for } i, j \in \{H, L\} \text{ and } i \neq j, \quad (\text{A15})$$

where the general demand functions are given by:

$$\begin{aligned} d_{H1}^{VP} &= (1 - \max\{\frac{p_H - p_L}{q_H - q_L}, \frac{p_H}{q_H}\})^+, & d_{L1}^{VP} &= (\min\{1, \frac{p_H - p_L}{q_H - q_L}\} - \frac{p_L}{q_L})^+, \\ d_{H2}^{VP} &= (1 - \max\{\frac{(p_H - t_p d_{H1}^{VP}) - (p_L - t_p d_{L1}^{VP})}{q_H - q_L}, \frac{p_H - t_p d_{H1}^{VP}}{q_H}\})^+, \text{ and} \\ d_{L2}^{VP} &= (\min\{1, \frac{(p_H - t_p d_{H1}^{VP}) - (p_L - t_p d_{L1}^{VP})}{q_H - q_L}\} - \frac{p_L - t_p d_{L1}^{VP}}{q_L})^+. \end{aligned}$$

Firm i 's total demand of two periods is given by $d_i^{VP} = d_{i1}^{VP} + d_{i2}^{VP}$, for $i = H, L$. Note that the above demand functions include all the situations that market is partially covered or fully covered and both firms have positive demand or zero demand in some period. Through straightforward yet tedious algebraic analysis, we can verify that given $0 < t_p < T^{VP}(q_H, q_L)$ and firm i 's price $p_i^{VP}(q_H, q_L, t_p)$, firm j 's optimal price is $p_j^{VP}(q_H, q_L, t_p)$ and thus will not deviate. That is, the inequality (A15) holds. Thus, requirement (1) holds.

For requirement (2), we have already shown in Step 1 that firm H's optimal quality is $q_H = 1$ for any $q_L \in [0, 1)$ and that firm L's optimal quality is $q_L = \hat{q}_L^{VP}(t_p)$ given $q_H = 1$. Thus, requirement (2) holds. Hence, we have shown that σ^{VP} is an SPE.

Step 3. We will show the uniqueness of the equilibrium σ^{VP} when t_p is small enough. Note that in Step 1 we have derived the unique equilibrium σ^{VP} by backward induction under the condition that the market is partially covered and each firm has positive demand in each period. Thus, we just need to show that there does not exist any equilibrium if the above condition does not hold, i.e., the market is fully covered or one of the firms has zero demand in period 1 or 2. That is, to show the uniqueness, we need to verify that: (1) Any strategy that leads to $d_{H1} + d_{L1} = 1$ is not an equilibrium; (2) any strategy that leads to $d_{H1} = 0$ or $d_{L1} = 0$ is not an equilibrium; (3) any strategy that leads to $d_{H2} + d_{L2} = 1$ is not an equilibrium; and (4) any strategy that leads to $d_{H2} = 0$ or $d_{L2} = 0$ is not an equilibrium.

If $d_{H1} + d_{L1} = 1$, then that means the consumer with $\theta = 0$ makes the purchase in period 1. That is, either $p_L = 0$ or $p_H = 0$. It is obvious that any strategy with $p_L = 0$ or $p_H = 0$ cannot be an equilibrium since one of the firm's profit will be zero and always has incentive to deviate to a small enough price to earn a positive profit. Thus, (1) holds.

If $d_{i1} = 0$, then $d_{i2} = 0$ since there is no value enhancement effect in period 2 and $u_{i2} = u_{i1}$, for $i = H, L$. Thus, firm i 's profit is zero. It is obvious that any strategy with $d_{i1} = 0$ cannot be an equilibrium outcome since firm i has incentive to deviate to a lower price to get a positive demand in period 1 and earn a positive profit. Thus (2) holds.

If $d_{H2} + d_{L2} = 1$ or $d_{H2} = 0$ or $d_{L2} = 0$, then the equilibrium is in the region $C_2 \cup C_3 \cup C_4 \cup C_5 \cup C_6$. In the proof of Lemma 3, we have shown that there does not exist any price equilibrium of Stage 2 in region C_i for given quality $0 \leq q_L < q_H \leq 1$ when t_p is small enough. Thus, (3) and (4) hold.

Hence, we conclude that in the equilibrium, the market is partially covered and each firm has positive demand in each period (i.e., $d_{Hi} > 0$, $d_{Li} > 0$, and $d_{Hi} + d_{Li} < 1$, for $i = 1, 2$).

Combining Steps 1-3, we show that σ^{VP} is the unique equilibrium when t_p is small enough. In the main body of the paper and the appendix, we define that $\bar{t}_p = 0.1$ and assume $0 < t_p \leq \bar{t}_p$ to make sure t_p is small enough to guarantee the existence and uniqueness of the equilibrium σ^{VP} .

Finally, we proceed to prove Parts (a)-(d) below. Note that all the final equilibrium outcomes are differentiable.

Part (a): Since $\hat{q}_L^{VP}(t_p)$ is the unique solution of $\frac{\partial \pi_L^{VP}(1, q_L, t_p)}{\partial q_L} = 0$ for $0 < t < T^{VP}(1, q_L)$, by the Implicit Function Theorem, we have: $\frac{\partial \hat{q}_L^{VP}(t_p)}{\partial t_p} = -\frac{\partial^2 \pi_L^{VP}(1, q_L, t_p)}{\partial q_L \partial t_p} / \frac{\partial^2 \pi_L^{VP}(1, q_L, t_p)}{\partial q_L^2} \Big|_{q_L = \hat{q}_L^{VP}(t_p)}$. One can verify

that $\frac{\partial^2 \pi_L^{VP}(1, q_L, t_p)}{\partial q_L^2} < 0$ and $\frac{\partial^2 \pi_L^{VP}(1, q_L, t_p)}{\partial q_L \partial t_p} < 0$ for any $q_L \in [0, 0.65]$ and $0 < t_p \leq \bar{t}_p$. Moreover, we find that $\hat{q}_L^{VP}(t_p) < 0.65$ for $0 < t_p \leq \bar{t}_p$, since $\frac{\partial \pi_L^{VP}(1, q_L, t_p)}{\partial q_L} < 0$ for any $q_L \in [0.65, 1]$. Therefore, $\frac{\partial \hat{q}_L^{VP}(t_p)}{\partial t_p} < 0$.

Part (b): Note that $\hat{p}_i^{VP}(t_p) = p_i^{VP}(1, \hat{q}_L^{VP}(t_p), t_p)$, for $i = H, L$. By the Chain Rule, $\frac{\partial \hat{p}_i^{VP}(t_p)}{\partial t_p} = \left(\frac{\partial p_i^{VP}(1, q_L, t_p)}{\partial t_p} + \frac{\partial p_i^{VP}(1, q_L, t_p)}{\partial q_L} \frac{\partial \hat{q}_L^{VP}(t_p)}{\partial t_p} \right) \Big|_{q_L = \hat{q}_L^{VP}(t_p)}$. One can verify that $\frac{\partial p_H^{VP}(1, q_L, t_p)}{\partial t_p} + \frac{\partial p_H^{VP}(1, q_L, t_p)}{\partial q_L} \frac{\partial \hat{q}_L^{VP}(t_p)}{\partial t_p} > 0$ and $\frac{\partial p_L^{VP}(1, q_L, t_p)}{\partial t_p} + \frac{\partial p_L^{VP}(1, q_L, t_p)}{\partial q_L} \frac{\partial \hat{q}_L^{VP}(t_p)}{\partial t_p} < 0$ for any $q_L \in [0, 0.65]$ and $0 < t_p \leq \bar{t}_p$. As already shown in Part (a) above, $\hat{q}_L^{VP}(t_p) < 0.65$. Therefore, $\frac{\partial \hat{p}_H^{VP}(t_p)}{\partial t_p} > 0$ and $\frac{\partial \hat{p}_L^{VP}(t_p)}{\partial t_p} < 0$. Hence, $\hat{p}_H^{VP}(t_p) > \hat{p}_H^{VP}(0) = \hat{p}_H^b$ and $\hat{p}_L^{VP}(t_p) < \hat{p}_L^{VP}(0) = \hat{p}_L^b$ for $0 < t_p \leq \bar{t}_p$.

Part (c): Note that $\hat{d}_i^{VP}(t_p) = d_i^{VP}(1, \hat{q}_L^{VP}(t_p), t_p)$, for $i = H, L$. By the chain rule, $\frac{\partial \hat{d}_i^{VP}(t_p)}{\partial t_p} = \left(\frac{\partial d_i^{VP}(1, q_L, t_p)}{\partial t_p} + \frac{\partial d_i^{VP}(1, q_L, t_p)}{\partial q_L} \frac{\partial \hat{q}_L^{VP}(t_p)}{\partial t_p} \right) \Big|_{q_L = \hat{q}_L^{VP}(t_p)}$. One can verify that $\frac{\partial d_H^{VP}(1, q_L, t_p)}{\partial t_p} + \frac{\partial d_H^{VP}(1, q_L, t_p)}{\partial q_L} \frac{\partial \hat{q}_L^{VP}(t_p)}{\partial t_p} > 0$ for any $q_L \in [0, 0.65]$ and $0 < t_p \leq \bar{t}_p$. Since $\hat{q}_L^{VP}(t_p) < 0.65$, we have $\frac{\partial \hat{d}_H^{VP}(t_p)}{\partial t_p} > 0$, i.e., $\hat{d}_H^{VP}(t_p)$ increases in t_p . Moreover, one can verify that $\frac{\partial d_L^{VP}(1, q_L, t_p)}{\partial t_p} + \frac{\partial d_L^{VP}(1, q_L, t_p)}{\partial q_L} \frac{\partial \hat{q}_L^{VP}(t_p)}{\partial t_p}$ is negative when t_p is close to 0 and is positive when t_p is close to \bar{t}_p . Thus, $\hat{d}_L^{VP}(t_p)$ may decrease or increase in t_p .

Part (d): Note that $\hat{\pi}_i^{VP}(t_p) = \pi_i^{VP}(1, \hat{q}_L^{VP}(t_p), t_p)$. By the chain rule, $\frac{\partial \hat{\pi}_i^{VP}(t_p)}{\partial t_p} = \left(\frac{\partial \pi_i^{VP}(1, q_L, t_p)}{\partial t_p} + \frac{\partial \pi_i^{VP}(1, q_L, t_p)}{\partial q_L} \frac{\partial \hat{q}_L^{VP}(t_p)}{\partial t_p} \right) \Big|_{q_L = \hat{q}_L^{VP}(t_p)}$. One can verify that $\frac{\partial \pi_H^{VP}(1, q_L, t_p)}{\partial t_p} + \frac{\partial \pi_H^{VP}(1, q_L, t_p)}{\partial q_L} \frac{\partial \hat{q}_L^{VP}(t_p)}{\partial t_p} > 0$ and $\frac{\partial \pi_L^{VP}(1, q_L, t_p)}{\partial t_p} + \frac{\partial \pi_L^{VP}(1, q_L, t_p)}{\partial q_L} \frac{\partial \hat{q}_L^{VP}(t_p)}{\partial t_p} < 0$ for $q_L \in [0, 0.65]$ and $0 < t_p \leq \bar{t}_p$. Since $\hat{q}_L^{VP}(t_p) < 0.65$, we have $\frac{\partial \hat{\pi}_H^{VP}(t_p)}{\partial t_p} > 0$ and $\frac{\partial \hat{\pi}_L^{VP}(t_p)}{\partial t_p} < 0$, i.e., $\hat{\pi}_H^{VP}(t_p)$ increases in t_p and $\hat{\pi}_L^{VP}(t_p)$ decreases in t_p . Therefore, $\hat{\pi}_H^{VP}(t_p) - \hat{\pi}_H^b > 0 > \hat{\pi}_L^{VP}(t_p) - \hat{\pi}_L^b$. \square

Proof of Corollary 3: Part (a): Let $CM_{in}^{VP}(t_p)$ denote consumer monetary surplus of purchasing from firm i in period n in the equilibrium, which is given by:

$$\begin{aligned} CM_{H1}^{VP}(t_p) &= \int_{\bar{\theta}_{H1}^{VP}}^1 (\theta - \hat{p}_H^{VP}(t_p)) d\theta \\ &= (1 - 2\hat{p}_H^{VP}(t_p) + \bar{\theta}_{H1}^{VP})(1 - \bar{\theta}_{H1}^{VP})/2 \\ &= (1 - 2\hat{p}_H^{VP}(t_p) + \frac{\hat{p}_H^{VP}(t_p) - \hat{p}_L^{VP}(t_p)}{1 - \hat{q}_L^{VP}(t_p)})(1 - \frac{\hat{p}_H^{VP}(t_p) - \hat{p}_L^{VP}(t_p)}{1 - \hat{q}_L^{VP}(t_p)})/2, \\ CM_{L1}^{VP}(t_p) &= \int_{\bar{\theta}_{L1}^{VP}}^{\bar{\theta}_{H1}^{VP}} (\theta \hat{q}_L^{VP}(t_p) - \hat{p}_L^{VP}(t_p)) d\theta \\ &= (\hat{q}_L^{VP}(t_p)(\bar{\theta}_{H1}^{VP} + \bar{\theta}_{L1}^{VP}) - 2\hat{p}_L^{VP}(t_p))(\bar{\theta}_{H1}^{VP} - \bar{\theta}_{L1}^{VP})/2 \\ &= (\hat{q}_L^{VP}(t_p)(\frac{\hat{p}_H^M(t_p) - \hat{p}_L^M(t_p)}{1 - \hat{q}_L^{VP}(t_p)} + \frac{\hat{p}_L^{VP}(t_p)}{\hat{q}_L^{VP}(t_p)}) - 2\hat{p}_L^{VP}(t_p))(\frac{\hat{p}_H^{VP}(t_p) - \hat{p}_L^{VP}(r)}{1 - \hat{q}_L^{VP}(t_p)} - \frac{\hat{p}_L^{VP}(t_p)}{\hat{q}_L^{VP}(t_p)})/2, \\ CM_{H2}^{VP}(t_p) &= \int_{\bar{\theta}_{H2}^{VP}}^1 (\theta - \hat{p}_H^{VP}(t_p)) d\theta \\ &= (1 - 2\hat{p}_H^{VP}(t_p) + \bar{\theta}_{H2}^{VP})(1 - \bar{\theta}_{H2}^{VP})/2, \quad \text{and} \\ CM_{L2}^{VP}(t_p) &= \int_{\bar{\theta}_{L2}^{VP}}^{\bar{\theta}_{H2}^{VP}} (\theta \hat{q}_L^{VP}(t_p) - \hat{p}_L^{VP}(t_p)) d\theta \\ &= (\hat{q}_L^{VP}(t_p)(\bar{\theta}_{H2}^{VP} + \bar{\theta}_{L2}^{VP}) - 2\hat{p}_L^{VP}(t_p))(\bar{\theta}_{H2}^{VP} - \bar{\theta}_{L2}^{VP})/2, \end{aligned}$$

where $\bar{\theta}_{H2}^{VP} = \frac{(\hat{p}_H^{VP}(t_p) - t_p \hat{d}_{H1}^{VP}(t_p)) - (\hat{p}_L^{VP}(t_p) - t_p \hat{d}_{L1}^{VP}(t_p))}{1 - \hat{q}_L^{VP}(t_p)}$ and $\bar{\theta}_{L2}^{VP} = \frac{\hat{p}_L^{VP}(t_p) - t_p \hat{d}_{L1}^{VP}(t_p)}{\hat{q}_L^{VP}(t_p)}$. The consumer monetary surplus in period n is given by $CM_n^{VP}(t_p) = CM_{Hn}^{VP}(t_p) + CM_{Ln}^{VP}(t_p)$ for $n = 1, 2$, and total consumer

monetary surplus of two periods is given by $CM^{VP}(t_p) = CM_1^{VP}(t_p) + CM_2^{VP}(t_p)$.

We can show that $\frac{dCM_{H1}^{VP}(t_p)}{dt_p} < 0$, $\frac{dCM_{L1}^{VP}(t_p)}{dt_p} < 0$, $\frac{dCM_{H2}^{VP}(t_p)}{dt_p} < 0$ and $\frac{dCM_{L2}^{VP}(t_p)}{dt_p} < 0$ for $0 < t_p \leq \bar{t}_p$. Thus $\frac{dCM_1^{VP}(t_p)}{dt_p} < 0$, $\frac{dCM_2^{VP}(t_p)}{dt_p} < 0$, and $\frac{dCM^{VP}(t_p)}{dt_p} < 0$. Therefore, $CM_1^{VP}(t_p)$, $CM_2^{VP}(t_p)$, and $CM^{VP}(t_p)$ all decrease in t_p . Let CM^b denote the total consumer monetary surplus of two periods in the benchmark case, i.e., $CM^b = CM^{VP}(0)$. Thus, $CM^{VP}(t_p) < CM^{VP}(0) = CM^b$ for $0 < t_p \leq \bar{t}_p$.

Part (b): Let $CS_{in}^{VP}(t_p)$ denote consumer total surplus of purchasing from firm i in period n in the equilibrium, which is given by:

$$\begin{aligned} CS_{H1}^{VP}(t_p) &= CM_{H1}^{VP}(t_p), \quad CS_{L1}^{VP}(t_p) = CM_{L1}^{VP}(t_p), \\ CS_{H2}^{VP}(t_p) &= \int_{\bar{\theta}_{H2}^{VP}}^1 (\theta - \hat{p}_H^{VP}(t_p) + t_p \hat{d}_{H1}^{VP}(t_p)) d\theta \\ &= \frac{1}{2} (1 - 2\hat{p}_H^{VP}(t_p) + 2t_p \hat{d}_{H1}^{VP}(t_p) + \bar{\theta}_{H2}^{VP} (1 - \bar{\theta}_{H2}^{VP})) \quad \text{and} \\ CS_{L2}^{VP}(t_p) &= \int_{\bar{\theta}_{L2}^{VP}}^{\bar{\theta}_{H2}^{VP}} (\theta \hat{q}_L^{VP}(t_p) - \hat{p}_L^{VP}(t_p) + t_p \hat{d}_{L1}^{VP}(t_p)) d\theta \\ &= \frac{1}{2} (\hat{q}_L^{VP}(t_p) (\bar{\theta}_{H2}^{VP} + \bar{\theta}_{L2}^{VP}) - 2\hat{p}_L^{VP}(t_p) + 2t_p \hat{d}_{L1}^{VP}(t_p)) (\bar{\theta}_{H2}^{VP} - \bar{\theta}_{L2}^{VP}), \text{ for } n = 1, 2. \end{aligned}$$

Consumer total surplus in period n is given by $CS_n^{VP}(t_p) = CS_{Hn}^{VP}(t_p) + CS_{Ln}^{VP}(t_p)$, and consumer total surplus of two periods is given by $CS^{VP}(t_p) = CS_1^{VP}(t_p) + CS_2^{VP}(t_p)$.

Given $0 < t_p \leq \bar{t}_p$, we can show that $\frac{dCS^{VP}(t_p)}{dt_p} < 0$ if $t_p < 0.0382251$ and $\frac{dCS^{VP}(t_p)}{dt_p} \geq 0$ otherwise. Thus, $CS^{VP}(t_p)$ first decreases and then increases in t_p .

Part (c): The monetary term of social welfare is defined as $SM^{VP}(t_p) = \pi_H^{VP}(t_p) + \pi_L^{VP}(t_p) + CM^{VP}(t_p)$. We have shown that $CM^{VP}(t_p)$ decreases in t_p in (a), and that $\pi_H^{VP}(t_p)$ increases in t_p and $\pi_L^{VP}(t_p)$ decreases in t_p in Proposition 3(d). We can verify that $\frac{dSM^{VP}(t_p)}{dt_p}$ could be ≥ 0 or ≤ 0 . Thus $SM^{VP}(t_p)$ may increase or decrease in t_p .

Part (d): The social welfare is defined as $SW^{VP}(t_p) = \pi_H^{VP}(t_p) + \pi_L^{VP}(t_p) + CS^{VP}(t_p)$. Given $0 < t_p \leq \bar{t}_p$, we can verify that $\frac{dSW^{VP}(t_p)}{dt_p} > 0$, indicating $SW^{VP}(t_p)$ increases in t_p . Thus, $SW^{VP}(t_p) > SW^{VP}(0) = SW^b$. \square

B. The Dynamic Pricing Scheme

In this section, we analyze the model with dynamic pricing scheme, and show the equilibrium results parallel to those in our main model. Since each period is independent and identical in the case of VEE-C, the model and results are exactly the same under different pricing schemes. Hence, in the sequel, we only focus on MEE in Section B.1 and VEE-P in Section B.2. Note that the following results are proved along with Propositions 4 and 5.

B.1. Market Expansion Effect (MEE)

We show the results and proofs of the MEE case under dynamic pricing in this section. Similar to the analysis under committed pricing scheme, we solve the equilibrium of MEE by backward induction. First, for given firms' quality levels $0 \leq q_L < q_H \leq 1$, we solve the two firms' pricing decisions and summarize the results in Lemma B1, which is directly comparable to Lemma 1.

LEMMA B1. *Suppose that firms adopt dynamic pricing strategy. In the MEE case (i.e., $t_c = t_p = 0$ and $0 < r \leq 1$), for any given $0 \leq q_L < q_H \leq 1$, the firms' sub-game equilibrium prices in period n are uniquely given by $(p_{Hn}^{DM}(q_H, q_L, r), p_{Ln}^{DM}(q_H, q_L, r))$, for $n = 1, 2$.*

Next, we proceed to solve the quality decisions in Stage 1, and summarize the final equilibrium and the corresponding result in Proposition B1, a counterpart of Proposition 1.

PROPOSITION B1. *Suppose that firms adopt dynamic pricing strategy. In the MEE case (i.e., $t_c = t_p = 0$ and $0 < r \leq 1$), there exists a unique equilibrium, in which firm H 's quality is $\hat{q}_H^{DM} = 1$ and firm L 's quality $0 < \hat{q}_L^{DM}(r) < 1$. In the equilibrium, the market is not fully covered and each firm has positive demand. Moreover, the following statements hold.*

- (a) $\hat{q}_L^{DM}(r)$ increases in r .
- (b) $\hat{p}_{in}^{DM}(r)$ decreases in r and $\hat{p}_{i1}^{DM}(r) < \hat{p}_{i2}^{DM}(r) < \hat{p}_H^b$, for $i = H, L$ and $n = 1, 2$.
- (c) $\hat{d}_H^{DM}(r)$ and $\hat{d}_L^{DM}(r)$ increase in r .
- (d) $\hat{\pi}_H^{DM}(r)$ and $\hat{\pi}_L^{DM}(r)$ increase in r . Moreover, $\hat{\pi}_H^{DM}(r) - \hat{\pi}_H^b > \hat{\pi}_L^{DM}(r) - \hat{\pi}_L^b > 0$ while $0 < (\hat{\pi}_H^{DM}(r) - \hat{\pi}_H^b) / \hat{\pi}_H^b < (\hat{\pi}_L^{DM}(r) - \hat{\pi}_L^b) / \hat{\pi}_L^b$.

Proof of Lemma B1 and Proposition B1: We remark that the proof of Lemma B1 and Proposition B1 is very similar to that of Lemma 1 and Proposition 1, and therefore we suppress some steps for succinct exhibition. We solve the equilibrium of the MEE case under dynamic pricing by backward induction in three steps: **Step 1.** Given the quality levels and demands in period 1, we solve the price decisions in period 2 of Stage 2. **Step 2.** Given the quality levels, we solve the price decisions in period 1 of Stage 2. **Step 3.** We solve the quality decisions in Stage 1.

Step 1: Price Decisions in Period 2. When $t_c = t_p = 0$ and $r \in (0, 1]$, given that $0 \leq q_L < q_H \leq 1$ and each firm's demand in period 1, d_{H1}^{DM} and d_{L1}^{DM} , firm i decides p_{i2} to maximize its profit of period 2:

$$\max_{p_{i2} \geq 0} \pi_{i2}^{DM} = p_{i2} d_{i2}^{DM}, \quad \text{for } i = H, L.$$

In the equilibrium, $p_{H2} > 0$ and $p_{L2} > 0$ hold, and thus the market will not be fully covered by the two firms in period 2. Moreover, in the equilibrium, each firm's demand is positive in period 2, i.e., $0 < \frac{p_{L2}}{q_L} < \frac{p_{H2} - p_{L2}}{q_H - q_L} < 1$. Therefore, two firms' demands in period 2 are given by

$$d_{H2}^{DM} = (1 + r d_{H1}^{DM} + r d_{L1}^{DM}) \left(1 - \frac{p_{H2} - p_{L2}}{q_H - q_L} \right) \quad \text{and} \quad d_{L2}^{DM} = (1 + r d_{H1}^{DM} + r d_{L1}^{DM}) \left(\frac{p_{H2} - p_{L2}}{q_H - q_L} - \frac{p_{L2}}{q_L} \right).$$

One can verify that π_{i2}^{DM} is concave in p_{i2} , i.e., $\frac{\partial^2 \pi_{i2}^{DM}}{\partial p_{i2}^2} < 0$. Solving the two firms' first-order conditions together leads to the prices in period 2 as below:

$$p_{H2}^{DM}(q_H, q_L) = \frac{2q_H(q_H - q_L)}{4q_H - q_L} \quad \text{and} \quad p_{L2}^{DM}(q_H, q_L) = \frac{q_L(q_H - q_L)}{4q_H - q_L}. \quad (B1)$$

The corresponding demand and profit of each firm in period 2 are given as below:

$$\begin{aligned} d_{H2}^{DM}(q_H, q_L, r, d_{H1}^{DM}, d_{L1}^{DM}) &= (1 + rd_{H1}^{DM} + rd_{L1}^{DM}) \frac{2q_H}{4q_H - q_L}, \\ d_{L2}^{DM}(q_H, q_L, r, d_{H1}^{DM}, d_{L1}^{DM}) &= (1 + rd_{H1}^{DM} + rd_{L1}^{DM}) \frac{q_H}{4q_H - q_L}, \\ \pi_{H2}^{DM}(q_H, q_L, r, d_{H1}^{DM}, d_{L1}^{DM}) &= (1 + rd_{H1}^{DM} + rd_{L1}^{DM}) \frac{4q_H^2(q_H - q_L)}{(4q_H - q_L)^2}, \text{ and} \\ \pi_{L2}^{DM}(q_H, q_L, r, d_{H1}^{DM}, d_{L1}^{DM}) &= (1 + rd_{H1}^{DM} + rd_{L1}^{DM}) \frac{q_H q_L (q_H - q_L)}{(4q_H - q_L)^2}. \end{aligned} \quad (B2)$$

Step 2: Price Decisions in Period 1. Given the firms' prices in period 2, firm i decides price in period 1, i.e., p_{i1} , to maximize its total profit of two periods:

$$\max_{p_{i1} \geq 0} \pi_i^{DM} = p_{i1} d_{i1}^{DM} + \pi_{i2}^{DM}(q_H, q_L, r, d_{i1}^{DM}, d_{j1}^{DM}), \quad (B3)$$

where $i, j = H, L$ and $i \neq j$, and the demands in period 1 are given by

$$(d_{H1}^{DM}, d_{L1}^{DM}) = \begin{cases} \left(1 - \frac{p_{H1} - p_{L1}}{q_H - q_L}, \frac{p_{H1} - p_{L1}}{q_H - q_L} - \frac{p_{L1}}{q_L}\right), & \text{if } (p_{H1}, p_{L1}) \in \bar{C}_1; \\ \left(1 - \frac{p_{H1} - p_{L1}}{q_H - q_L}, \frac{p_{H1} - p_{L1}}{q_H - q_L}\right), & \text{if } (p_{H1}, p_{L1}) \in \bar{C}_2; \\ \left(1 - \frac{p_{H1}}{q_H}, 0\right), & \text{if } (p_{H1}, p_{L1}) \in \bar{C}_3; \\ \left(0, 1 - \frac{p_{L1}}{q_L}\right), & \text{if } (p_{H1}, p_{L1}) \in \bar{C}_4; \\ (1, 0), & \text{if } (p_{H1}, p_{L1}) \in \bar{C}_5; \\ (0, 1), & \text{if } (p_{H1}, p_{L1}) \in \bar{C}_6. \end{cases}$$

In the above, \bar{C}_i ($i = 1, 2, \dots, 6$) is a set of conditions that defines a region for the price pair in period

1. More specifically, $\bar{C}_1 = \{0 < \frac{p_{L1}}{q_L} < \frac{p_{H1} - p_{L1}}{q_H - q_L} < 1\}$; $\bar{C}_2 = \{\frac{p_{L1}}{q_L} \leq 0 < \frac{p_{H1} - p_{L1}}{q_H - q_L} < 1\}$; $\bar{C}_3 = \{0 \leq \frac{p_{H1}}{q_H} \leq \frac{p_{L1}}{q_L}\}$; $\bar{C}_4 = \{0 \leq p_{L1} \leq q - 1 + p_{H1}\}$; $\bar{C}_5 = \{p_{H1} \leq \min\{0, p_{L1}\}\}$; and $\bar{C}_6 = \{p_{L1} \leq \min\{0, q_L - q_H + p_{H1}\}\}$. We refer to $(p_{H1}, p_{L1}) \in \bar{C}_i$ simply as "under Condition i ". Under Condition 1, the market is not fully covered and each firm has positive demand in period 1; under Condition 2, the market is fully covered and each firm has positive demand in period 1; under other conditions, one of the firms has zero demand in period 1.

We can apply similar approach as in the proof of Lemma 1 to show that the market is partially covered and each firm's demand is positive in period 1, i.e., $0 < \frac{p_{L1}}{q} < \frac{p_{H1} - p_{L1}}{1 - q} < 1$. Therefore, we just focus on Condition 1. Under Condition 1, one can easily verify that firm i 's total profit is concave in p_{i1} ; and thus the firms' first-period prices can be solved from the first-order conditions and are given by:

$$\begin{aligned} p_{H1}^{DM}(q_H, q_L, r) &= \frac{q_H(q_H - q_L)(32q_H^2 + (r+2)q_L^2 - (r+16)q_H q_L)}{(4q_H - q_L)^3} \text{ and} \\ p_{L1}^{DM}(q_H, q_L, r) &= \frac{q_L(q_H - q_L)(2(r-4)q_H q_L - 2(r-8)q_H^2 + q_L^2)}{(4q_H - q_L)^3}. \end{aligned} \quad (B4)$$

Therefore, the corresponding demand of each firm in period 1 is

$$\begin{aligned} d_{H1}^{DM}(q_H, q_L, r) &= \frac{q_H(32q_H^2 + (r+2)q_L^2 - (r+16)q_H q_L)}{(4q_H - q_L)^3} \text{ and} \\ d_{L1}^{DM}(q_H, q_L, r) &= \frac{q_H(2(r+8)q_H^2 + (r+1)q_L^2 - (3r+8)q_H q_L)}{(4q_H - q_L)^3}. \end{aligned} \quad (B5)$$

Plugging Equations (B5) into Equations (B2) yields the second-period demands as functions of (q_H, q_L, r) :

$$\begin{aligned} d_{H2}^{DM}(q_H, q_L, r) &= \left(1 + \frac{rq_H(-4(r+6)q_Hq_L + 2(r+24)q_H^2 + (2r+3)q_L^2)}{(4q_H - q_L)^3}\right) \frac{2q_H}{4q_H - q_L} \text{ and} \\ d_{L2}^{DM}(q_H, q_L, r) &= \left(1 + \frac{rq_H(-4(r+6)q_Hq_L + 2(r+24)q_H^2 + (2r+3)q_L^2)}{(4q_H - q_L)^3}\right) \frac{q_H}{4q_H - q_L}. \end{aligned} \quad (\text{B6})$$

Moreover, firm i 's total demand of two periods $d_i^{DM}(q_H, q_L, r) = d_{i1}^{DM}(q_H, q_L, r) + d_{i2}^{DM}(q_H, q_L, r)$ and its total profit $\pi_i^{DM}(q_H, q_L, r) = p_{i1}^{DM}(q_H, q_L, r)d_{i1}^{DM}(q_H, q_L, r) + p_{i2}^{DM}(q_H, q_L, r)d_{i2}^{DM}(q_H, q_L, r)$.

Step 3. Quality Decisions. Given the firms' dynamic pricing decisions in Stage 2, we proceed to solve firms' quality decisions in Stage 1:

$$\max_{q_H \in (q_L, 1]} \pi_H^{DM}(q_H, q_L, r) \quad \text{and} \quad \max_{q_L \in [0, q_H)} \pi_L^{DM}(q_H, q_L, r).$$

First, one can easily verify that $\pi_H^{DM}(q_H, q_L, r)$ increases in q_H for any $q_L \in [0, 1]$ and $r \in (0, 1]$, i.e., $\frac{d\pi_H^{DM}(q_H, q_L, r)}{dq_H} > 0$. Thus, firm H's optimal quality is $\hat{q}_H^{DM} = 1$ for any $q_L \in [0, 1]$. Given that $q_H = 1$, next we solve firm L's optimal quality q_L . Directly checking firm L's profit function, we find that $\pi_L^{DM}(1, q_L, r)$ is concave in q_L , i.e., $\frac{d^2\pi_L^{DM}(1, q_L, r)}{dq_L^2} < 0$, for any $q_L \in [0, 1]$ and $r \in (0, 1]$. Therefore, firm L's optimal quality $\hat{q}_L^{DM}(r)$ is solved from $\frac{d\pi_L^{DM}(1, q_L, r)}{dq_L} = 0$.

Thus, the equilibrium outcome is: $q_H = \hat{q}_H^{DM} = 1$, $q_L = \hat{q}_L^{DM}(r)$, $p_{i1} = \hat{p}_{i1}^{DM}(r) = p_{i1}^{DM}(1, \hat{q}_L^{DM}(r), r)$, and $p_{i2} = \hat{p}_{i2}^{DM}(r) = p_{i2}^{DM}(1, \hat{q}_L^{DM}(r), r)$, for $i = H, L$. Let $\hat{d}_{in}^{DM}(r) = d_{in}^{DM}(1, \hat{q}_L^{DM}(r), r)$ and $\hat{\pi}_{in}^{DM}(r) = \hat{p}_{in}^{DM}(r)\hat{d}_{in}^{DM}(r)$ denote firm i 's demand and profit in period n for $i = H, L$ and $n = 1, 2$. Firm i 's total demand and total profit are given by $\hat{d}_i^{DM}(r) = \hat{d}_{i1}^{DM}(r) + \hat{d}_{i2}^{DM}(r)$ and $\hat{\pi}_i^{DM}(r) = \hat{\pi}_{i1}^{DM}(r) + \hat{\pi}_{i2}^{DM}(r)$. Note that we can apply similar approach as in the proof of Proposition 1 to show the existence and uniqueness of the above equilibrium in which the market is partially covered and each firm's demand is positive in each period. The detailed proof is omitted and is available from authors upon request. The proof of Proposition B1(a)-(d) are provided in the proof of Proposition 4. \square

Proof of Proposition 4: Note that all the final equilibrium outcomes are differentiable.

Part (a): Since $\hat{q}_L^{DM}(r)$ is the unique solution of $\frac{\partial \pi_L^{DM}(1, q_L, r)}{\partial q_L} = 0$, by the Implicit Function Theorem, we have: $\frac{\partial \hat{q}_L^{DM}(r)}{\partial r} = -\frac{\partial^2 \pi_L^{DM}(1, q_L, r)}{\partial q_L \partial r} / \frac{\partial^2 \pi_L^{DM}(1, q_L, r)}{\partial q_L^2} \Big|_{q=\hat{q}_L^{DM}(r)}$. We have already shown that $\frac{\partial^2 \pi_L^{DM}(1, q_L, r)}{\partial q_L^2} < 0$. Besides, we can verify that $\frac{\partial^2 \pi_L^{DM}(1, q_L, r)}{\partial q_L \partial r} > 0$ for any $q_L \in (0, 0.6)$. Moreover, since $\frac{\partial \pi_L^{DM}(1, q_L, r)}{\partial q_L} < 0$ for any $q_L \in [0.6, 1)$, we deduce that $\hat{q}_L^{DM}(r) < 0.6$. Therefore, $\frac{\partial \hat{q}_L^{DM}(r)}{\partial r} > 0$, i.e. $\hat{q}_L^{DM}(r)$ increases in r and $\hat{q}_L^{DM}(r) > \hat{q}^b$ for $r \in (0, 1]$.

Next, we show that $\hat{q}_L^M(r) > \hat{q}_L^{DM}(r)$ for any $r \in (0, 1]$. One can verify that $\frac{\partial \pi_L^M(1, q_L, r)}{\partial q_L} > \frac{\partial \pi_L^{DM}(1, q_L, r)}{\partial q_L}$ for any $q_L \in [0.4, 1]$; moreover, $\frac{\partial \pi_L^M(1, q_L, r)}{\partial q_L} > 0$ and $\frac{\partial \pi_L^{DM}(1, q_L, r)}{\partial q_L} > 0$ for any $q_L \in (0, 0.4]$. Since $\hat{q}_L^M(r)$ is the unique solution to $\frac{\partial \pi_L^M(1, q_L, r)}{\partial q_L} = 0$ and $\hat{q}_L^{DM}(r)$ to $\frac{\partial \pi_L^{DM}(1, q_L, r)}{\partial q_L} = 0$ (concavity of the profit functions), we have $\hat{q}_L^M(r) > \hat{q}_L^{DM}(r) > \hat{q}_L^b$.

Part (b): Note that $\hat{p}_i^{DM}(r) = p_i^{DM}(1, \hat{q}_L^{DM}(r), r)$, for $i = H, L$. By the chain rule, $\frac{\partial \hat{p}_i^{DM}(r)}{\partial r} = \left(\frac{\partial p_i^{DM}(1, q_L, r)}{\partial r} + \frac{\partial p_i^{DM}(1, q_L, r)}{\partial q_L} \frac{\partial \hat{q}_L^{DM}(r)}{\partial r} \right) \Big|_{q_L = \hat{q}_L^{DM}(r)}$. One can verify that $\frac{\partial p_i^{DM}(1, q_L, r)}{\partial r} < 0$. We already have $\frac{\partial \hat{q}_L^{DM}(r)}{\partial r} > 0$. Moreover, one can verify that $\frac{\partial p_{H1}^{DM}(1, q_L, r)}{\partial q_L} < 0$ for any $q_L \in (0, 1]$, and that $\frac{\partial p_{L1}^{DM}(1, q_L, r)}{\partial q_L} < 0$ for any $q_L \in (0.56, 1)$. Since $\hat{q}_L^{DM}(r)$ increases in r , we have $\hat{q}_L^{DM}(r) \geq \hat{q}_L^{DM}(0) = \hat{q}^b = 4/7 > 0.56$. Thus, $\frac{\partial p_{L1}^{DM}(1, q_L, r)}{\partial q_L} \Big|_{q_L = \hat{q}_L^{DM}(r)} < 0$. Based on all these facts, we show that $\frac{\partial \hat{p}_i^{DM}(r)}{\partial r} < 0$. Therefore, $\hat{p}_i^{DM}(r)$ decreases in r and $\hat{p}_i^{DM}(r) < \hat{p}_i^{DM}(0) = \hat{p}_i^b$ for $r \in (0, 1]$ and $i = H, L$.

Similarly, note that $\hat{p}_{i2}^{DM}(r) = p_{i2}^{DM}(1, \hat{q}_L^{DM}(r))$, for $i = H, L$. By the chain rule, $\frac{\partial \hat{p}_{i2}^{DM}(r)}{\partial r} = \left(\frac{\partial p_{i2}^{DM}(1, q_L)}{\partial q_L} \frac{\partial \hat{q}_L^{DM}(r)}{\partial r} \right) \Big|_{q_L = \hat{q}_L^{DM}(r)}$. We already have $\frac{\partial \hat{q}_L^{DM}(r)}{\partial r} > 0$. Moreover, we verify that $\frac{\partial p_{H2}^{DM}(1, q_L)}{\partial q_L} < 0$ for any $q_L \in (0, 1]$, and that $\frac{\partial p_{L2}^{DM}(1, q_L)}{\partial q_L} < 0$ for any $q_L \in (0.56, 1)$. Thus, $\frac{\partial \hat{p}_{i2}^{DM}(r)}{\partial r} < 0$. Therefore, $\hat{p}_{i2}^{DM}(r)$ decreases in r and $\hat{p}_{i2}^{DM}(r) < \hat{p}_{i2}^{DM}(0) = \hat{p}_i^b$ for $r \in (0, 1]$.

Next, we will show that $\hat{p}_i^{DM}(r) < \hat{p}_i^M(r) < \hat{p}_{i2}^{DM}(r)$ for any $r \in (0, 1]$. Due to concavity, $\hat{q}_L^{DM}(r)$ is the unique solution of $\frac{\partial \pi_L^{DM}(1, q_L, r)}{\partial q_L} = 0$ for $q_L \in [0, 1]$; that is $\hat{q}_L^{DM}(r)$ is the first (or smallest) root of the polynomial $q_L^5(11r + 14) + q_L^4(8r^2 - 116r - 232) + q_L^3(28r^2 + 492r + 1472) + q_L^2(-96r^2 - 1232r - 4352) + q_L(76r^2 + 1856r + 5632) - 16r^2 - 768r - 2048 = 0$. Similarly, $\hat{q}_L^M(r)$ can be solved as the first (or smallest) root of the polynomial $2q_L^5(r^3 - 9r^2 + 20r - 28) + q_L^4(-9r^3 + 72r^2 - 76r + 480) + 2q_L^3(12r^3 - 87r^2 - 232r - 884) + q_L^2(-11r^3 + 396r^2 + 1596r + 2992) - 4q_L(15r^3 + 168r^2 + 520r + 640) + 36r^3 + 288r^2 + 768r + 768 = 0$. Substituting the equilibrium quality to $p_i^{DM}(1, \hat{q}_L^{DM}(r), r)$ (Equation (B4)), $p_{i2}^{DM}(1, \hat{q}_L^{DM}(r))$ (Equation (B1)), and $p_i^M(1, \hat{q}_L^M(r), r)$ (Equation (A3)), and after some straightforward simplifications, we can verify that $\hat{p}_i^{DM}(r) < \hat{p}_i^M(r) < \hat{p}_{i2}^{DM}(r) < \hat{p}_i^b$, for $i = H, L$.

Part (c): Note that $\hat{d}_i^{DM}(r) = d_i^{DM}(1, \hat{q}_L^{DM}(r), r)$, for $i = H, L$. By the chain rule, $\frac{\partial \hat{d}_i^{DM}(r)}{\partial r} = \left(\frac{\partial d_i^{DM}(1, q_L, r)}{\partial r} + \frac{\partial d_i^{DM}(1, q_L, r)}{\partial q_L} \frac{\partial \hat{q}_L^{DM}(r)}{\partial r} \right) \Big|_{q_L = \hat{q}_L^{DM}(r)}$. We already have $\frac{\partial \hat{q}_L^{DM}(r)}{\partial r} > 0$, and can verify that $\frac{\partial d_i^{DM}(1, q_L, r)}{\partial r} > 0$ and $\frac{\partial d_i^{DM}(1, q_L, r)}{\partial q_L} > 0$. Thus, $\frac{\partial \hat{d}_i^{DM}(r)}{\partial r} > 0$, i.e., $\hat{d}_i^{DM}(r)$ increases in r for $i = H, L$.

Similar to Part (c) above, substituting $\hat{q}_L^{DM}(r)$ into $d_i^{DM}(1, \hat{q}_L^{DM}(r), r)$ and $\hat{q}_L^M(r)$ into $d_i^M(1, \hat{q}_L^M(r), r)$, we can verify that $\hat{d}_i^{DM}(r) > \hat{d}_i^M(r) > \hat{d}_i^b$ for $r \in (0, 1]$ and $i = H, L$.

Part (d): Note that $\hat{\pi}_i^{DM}(r) = \pi_i^{DM}(1, \hat{q}_L^{DM}(r), r)$, for $i = H, L$. By the chain rule, $\frac{\partial \hat{\pi}_i^{DM}(r)}{\partial r} = \left(\frac{\partial \pi_i^{DM}(1, q_L, r)}{\partial r} + \frac{\partial \pi_i^{DM}(1, q_L, r)}{\partial q_L} \frac{\partial \hat{q}_L^{DM}(r)}{\partial r} \right) \Big|_{q_L = \hat{q}_L^{DM}(r)}$. One can verify that $\frac{\partial \pi_H^{DM}(1, q_L, r)}{\partial r} + \frac{\partial \pi_H^{DM}(1, q_L, r)}{\partial q_L} \frac{\partial \hat{q}_L^{DM}(r)}{\partial r} > 0$ for any $q_L \in (0, 1]$. Thus, $\frac{\partial \hat{\pi}_H^{DM}(r)}{\partial r} > 0$. One can also verify that $\frac{\partial \pi_L^{DM}(1, q_L, r)}{\partial r} > 0$. Since $\frac{\partial \pi_L^{DM}(1, q_L, r)}{\partial q_L} \Big|_{q_L = \hat{q}_L^{DM}(r)} = 0$, we conclude that $\frac{\partial \hat{\pi}_L^{DM}(r)}{\partial r} > 0$. Therefore, both $\hat{\pi}_H^{DM}(r)$ and $\hat{\pi}_L^{DM}(r)$ increase in r .

Substituting $\hat{q}_L^{DM}(r)$ into $\pi_i^{DM}(1, \hat{q}_L^{DM}(r), r)$ and $\hat{q}_L^M(r)$ into $\pi_i^M(1, \hat{q}_L^M(r), r)$, we can verify that $\hat{\pi}_i^{DM}(r) > \hat{\pi}_i^M(r) > \hat{\pi}_i^b$ for any $r \in (0, 1]$ and $i = H, L$.

This completes the proof of Proposition 4 and Proposition B1 parts(a)-(d). \square

B.2. Value Enhancement Effect from Previous Consumers (VEE-P)

We show the results and proofs of the VEE-P case under dynamic pricing in this section. Similar to the analysis under committed pricing scheme, we solve the equilibrium of VEE-P by backward induction. First, for given firms' quality levels, we solve the pricing decisions and summarize the results in Lemma B2, which is comparable to Lemma 3. Then we solve the quality decisions in Stage 1 and show results of final equilibrium in Proposition B2, which is comparable to Proposition 3.

LEMMA B2. *Suppose that firms adopt dynamic pricing strategy. In the VEE-P case, for any given $0 \leq q_L < q_H \leq 1$, the firms' equilibrium prices in period n are uniquely given by $(p_{Hn}^{DVP}(q_H, q_L, t_p), p_{Ln}^{DVP}(q_H, q_L, t_p))$, for $n = 1, 2$.*

PROPOSITION B2. *Suppose that firms adopt dynamic pricing strategy. In the VEE-P case, there exists a unique equilibrium, in which firm H 's quality is $\hat{q}_H^{DVP} = 1$ and firm L 's quality $0 < \hat{q}_L^{DVP}(t_p) < 1$. In the equilibrium, the market is not fully covered and each firm has positive demand. Moreover, the following statements hold.*

- (a) $\hat{q}_L^{DVP}(t_p)$ decreases in t_p .
- (b) $\hat{p}_{H1}^{DVP}(t_p)$ first decreases in t_p and then increases in t_p ; $\hat{p}_{L1}^{DVP}(t_p)$ decreases in t_p ; $\hat{p}_{H1}^{DVP}(t_p) < \hat{p}_H^b$ and $\hat{p}_{L1}^{DVP}(t_p) < \hat{p}_L^b$; $\hat{p}_{H2}^{DVP}(t_p)$ and $\hat{p}_{L2}^{DVP}(t_p)$ increase in t_p ; $\hat{p}_{H2}^{DVP}(t_p) > \hat{p}_H^b$ and $\hat{p}_{L2}^{DVP}(t_p) > \hat{p}_L^b$.
- (c) $\hat{d}_H^{DVP}(t_p)$ increases in t_p ; $\hat{d}_L^{DVP}(t_p)$ first decreases and then increases in t_p .
- (d) $\hat{\pi}_H^{DVP}(t_p)$ increases in t_p and $\hat{\pi}_L^{DVP}(t_p)$ decreases in t_p ; $\hat{\pi}_H^{DVP}(t_p) - \hat{\pi}_H^b > 0 > \hat{\pi}_L^{DVP}(t_p) - \hat{\pi}_L^b$.

Proof of Lemma B2 and Proposition B2: We remark that the proof of Lemma B2 and Proposition B2 is very similar to that of Lemma 3 and Proposition 3, and therefore we suppress some steps for succinct exhibition. We solve the equilibrium of the VEE-P case under dynamic pricing by backward induction in three steps: **Step 1.** Given the quality levels and the demands in period 1, we solve the price decisions in period 2 of Stage 2. **Step 2.** Given the quality levels, we solve the price decisions in period 1 of Stage 2. **Step 3.** We solve the firms' quality decisions in Stage 1.

Step 1: Price Decisions in Period 2. When $r = t_c = 0$ and $t_p > 0$ and is small enough, given that $0 \leq q_L < q_H \leq 1$ and each firm's demand in period 1, d_{H1}^{DVP} and d_{L1}^{DVP} , firm i decides price in period 2 to maximize its second-period profit: $\max_{p_{i2} \geq 0} \pi_{i2}^{DVP} = p_{i2} d_{i2}^{DVP}$, for $i = H, L$.

Similar to the analysis under committed pricing, we only focus on the case that the market is partially covered, which is true when t_p is sufficiently small. Thus, the demands of period 2 are:

$$d_{H2}^{DVP} = 1 - \frac{(p_{H2} - t_p d_{H1}^{DVP}) - (p_{L2} - t_p d_{L1}^{DVP})}{q_H - q_L} \quad \text{and} \quad d_{L2}^{DVP} = \frac{(p_{H2} - t_p d_{H1}^{DVP}) - (p_{L2} - t_p d_{L1}^{DVP})}{q_H - q_L} - \frac{p_{L2} - t_p d_{L1}^{DVP}}{q_L}.$$

One can easily verify that π_{i2}^{DVP} is concave in p_{i2} , i.e., $\frac{\partial^2 \pi_{i2}^{DVP}}{\partial p_{i2}^2} < 0$. Solving the two firms' first-order-conditions together leads to the prices in period 2 as below:

$$\begin{aligned} p_{H2}^{DVP}(q_H, q_L, t_p, d_{H1}^{DVP}, d_{L1}^{DVP}) &= \frac{2q_H^2 + (2t_p q_H - t_p q_L)d_{H1}^{DVP} - t_p q_H d_{L1}^{DVP} - 2q_H q_L}{4q_H - q_L} \text{ and} \\ p_{L2}^{DVP}(q_H, q_L, t_p, d_{H1}^{DVP}, d_{L1}^{DVP}) &= \frac{q_H q_L + (2t_p q_H - t_p q_L)d_{L1}^{DVP} - t_p q_L d_{H1}^{DVP} - q_L^2}{4q_H - q_L}. \end{aligned} \quad (B7)$$

The corresponding demand and profit of each firm in period 2 are as below:

$$\begin{aligned} d_{H2}^{DVP}(q_H, q_L, t_p, d_{H1}^{DVP}, d_{L1}^{DVP}) &= \frac{2q_H^2 + (2t_p q_H - t_p q_L)d_{H1}^{DVP} - t_p q_H d_{L1}^{DVP} - 2q_H q_L}{4q_H^2 - 5q_H q_L + q_L^2}, \\ d_{L2}^{DVP}(q_H, q_L, t_p, d_{H1}^{DVP}, d_{L1}^{DVP}) &= \frac{(2t_p q_H^2 - t_p q_H q_L)d_{L1}^{DVP} - t_p q_H q_L d_{H1}^{DVP} + q_H^2 q_L - q_H q_L^2}{q_L(4q_H^2 - 5q_H q_L + q_L^2)}, \\ \pi_{H2}^{DVP}(q_H, q_L, t_p, d_{H1}^{DVP}, d_{L1}^{DVP}) &= \frac{((t_p q_L - 2t_p q_H)d_{H1}^{DVP} + t_p q_H d_{L1}^{DVP} + 2q_H q_L - 2q_H^2)^2}{(4q_H - q_L)^2(q_H - q_L)}, \text{ and} \\ \pi_{L2}^{DVP}(q_H, q_L, t_p, d_{H1}^{DVP}, d_{L1}^{DVP}) &= \frac{q_H((2t_p q_H - t_p q_L)d_{L1}^{DVP} - t_p q_L d_{H1}^{DVP} + q_H q_L - q_L^2)^2}{(4q_H - q_L)^2(q_H - q_L)q_L}. \end{aligned} \quad (B8)$$

Step 2: Price Decisions in Period 1. Given the firms' prices in period 2, firm i decides price in period 1, i.e., p_{i1} , to maximize its total profit of two periods:

$$\max_{p_{i1} \geq 0} \pi_i^{DVP} = p_{i1} d_{i1}^{DVP} + \pi_{i2}^{DVP}(q_H, q_L, t_p, d_{H1}^{DVP}, d_{L1}^{DVP}), \text{ for } i, j = H, L \text{ and } i \neq j.$$

Again, following the same logic as in the proof of Proposition 3, we can show that the market is partially covered and each firm's demand is positive in each period in the equilibrium for sufficiently small t_p . Thus, each firm's demand in period 1 can be written as: $d_{H1}^{DVP} = 1 - \frac{p_{H1} - p_{L1}}{q_H - q_L}$ and $d_{L1}^{DVP} = \frac{p_{H1} - p_{L1}}{q_H - q_L} - \frac{p_L}{q_L}$. For sufficiently small t_p (i.e., $0 < t_p < T^{DVP}(q_H, q_L) = \frac{4q_L q_H^2 - 5q_H q_L^2 + q_L^3}{2q_H^2 - q_H q_L + q_L^2}$), π_i^{DVP} is concave in p_{i1} , and thus firms' prices in period 1 can be solved from the first-order-conditions, i.e., $\frac{d\pi_i^{DVP}}{dp_{H1}} = 0$ and $\frac{d\pi_i^{DVP}}{dp_{L1}} = 0$. Hence, the prices in period 1 are given as below:

$$p_{H1}^{DVP}(q_H, q_L, t_p) = \frac{2q_H(X_5 + X_6)}{X_7} \quad \text{and} \quad p_{L1}^{DVP}(q_H, q_L, t_p) = \frac{q_L X_5}{X_7}, \quad (B9)$$

where $X_5 = -q_H q_L^3 \left(-24q_L^2 t_p^2 + 4q_L t_p^3 + 15q_L^4 + 4t_p^4 \right) + q_H^2 q_L^2 \left(8q_L^3 t_p - 98q_L^2 t_p^2 + 12q_L t_p^3 + 87q_L^4 + 16t_p^4 \right) - q_H^3 q_L \left(40q_L^3 t_p - 168q_L^2 t_p^2 + 4q_L t_p^3 + 245q_L^4 + 20t_p^4 \right) + 32q_H^6 \left(-q_L t_p + 2q_L^2 - t_p^2 \right) + 8q_H^5 \left(5q_L^2 t_p + 11q_L t_p^2 - 30q_L^3 + 3t_p^3 \right) + 4q_H^4 \left(6q_L^3 t_p - 37q_L^2 t_p^2 + q_L t_p^3 + 87q_L^4 + 6t_p^4 \right) - 2q_L^6 t_p^2 + q_L^8$, $X_6 = t_p \left(-q_H q_L + 2q_H^2 + q_L^2 \right) \left(q_H q_L \left(-3q_L^2 t_p + 10q_L t_p^2 - 10q_L^3 + 2t_p^3 \right) + 8q_H^4 \left(2q_L + t_p \right) - 2q_H^3 \left(11q_L t_p + 20q_L^2 + 3t_p^2 \right) + q_H^2 \left(17q_L^2 t_p - 10q_L t_p^2 + 33q_L^3 - 6t_p^3 \right) - 2q_L^3 t_p^2 + q_L^5 \right)$, and $X_7 = 64q_H^6 \left(4q_L^2 - t_p^2 \right) - 16q_H^5 \left(48q_L^3 - 7q_L t_p^2 \right) + 24q_H^4 \left(-10q_L^2 t_p^2 + 36q_L^4 + t_p^4 \right) + q_H^3 \left(238q_L^3 t_p^2 - 20q_L t_p^4 - 464q_L^5 \right) + q_H^2 \left(-118q_L^4 t_p^2 + 16q_L^2 t_p^4 + 129q_L^6 \right) - 2q_H \left(-13q_L^5 t_p^2 + 2q_L^3 t_p^4 + 9q_L^7 \right) - 2q_L^6 t_p^2 + q_L^8$.

Given the firms' prices in period 1, the corresponding demands in period 1 are as below:

$$\begin{aligned} d_{H1}^{DVP}(q_H, q_L, t_p) &= 1 - \frac{p_{H1}^{DVP}(q_H, q_L, t_p) - p_{L1}^{DVP}(q_H, q_L, t_p)}{q_H - q_L}, \text{ and} \\ d_{L1}^{DVP}(q_H, q_L, t_p) &= \frac{p_{H1}^{DVP}(q_H, q_L, t_p) - p_{L1}^{DVP}(q_H, q_L, t_p)}{q_H - q_L} - \frac{p_{L1}^{DVP}(q_H, q_L, t_p)}{q_L}. \end{aligned} \quad (B10)$$

The corresponding price, demand, and profit of each firm in period 2 are obtained by substituting Equations (B10) into Equations (B7) and (B8):

$$p_{H2}^{DVP}(q_H, q_L, t_p) = p_{H2}^{DVP}(q_H, q_L, t_p), d_{H1}^{DVP}(q_H, q_L, t_p), d_{L1}^{DVP}(q_H, q_L, t_p)), \quad (B11a)$$

$$p_{L2}^{DVP}(q_H, q_L, t_p) = p_{L2}^{DVP}(q_H, q_L, t_p), d_{L1}^{DVP}(q_H, q_L, t_p), d_{L1}^{DVP}(q_H, q_L, t_p)), \quad (B11b)$$

$$d_{H2}^{DVP}(q_H, q_L, t_p) = d_{H2}^{DVP}(q_H, q_L, t_p), d_{H1}^{DVP}(q_H, q_L, t_p), d_{L1}^{DVP}(q_H, q_L, t_p)), \quad (B11c)$$

$$d_{L2}^{DVP}(q_H, q_L, t_p) = d_{L2}^{DVP}(q_H, q_L, t_p), d_{L1}^{DVP}(q_H, q_L, t_p), d_{L1}^{DVP}(q_H, q_L, t_p)), \quad (B11d)$$

$$\pi_{H2}^{DVP}(q_H, q_L, t_p) = \pi_{H2}^{DVP}(q_H, q_L, t_p), d_{H1}^{DVP}(q_H, q_L, t_p), d_{L1}^{DVP}(q_H, q_L, t_p)), \text{ and} \quad (B11e)$$

$$\pi_{L2}^{DVP}(q_H, q_L, t_p) = \pi_{L2}^{DVP}(q_H, q_L, t_p), d_{L1}^{DVP}(q_H, q_L, t_p), d_{L1}^{DVP}(q_H, q_L, t_p)), \quad (B11f)$$

Moreover, firm i 's total demand is $d_i^{DVP}(q_H, q_L, t_p) = d_{i1}^{DVP}(q_H, q_L, t_p) + d_{i2}^{DVP}(q_H, q_L, t_p)$ and its total profit is $\pi_i^{DVP}(q_H, q_L, t_p) = p_{i1}^{DVP}(q_H, q_L, t_p)d_{i1}^{DVP}(q_H, q_L, t_p) + p_{i2}^{DVP}(q_H, q_L, t_p)d_{i2}^{DVP}(q_H, q_L, t_p)$.

Step 3: Quality Decisions. Given the firms' pricing decision in Stage 2, we proceed to solve firms' quality decisions in Stage 1:

$$\max_{q_H \in (q_L, 1]} \pi_H^{DVP}(q_H, q_L, t_p) \quad \text{and} \quad \max_{q_L \in [0, q_H)} \pi_L^{DVP}(q_H, q_L, t_p).$$

First, one can easily verify that $\pi_H^{DVP}(q_H, q_L, t_p)$ increases in q_H given that the market is partially covered and each firm has positive demand for sufficiently small t_p . Thus, firm H's optimal quality is $\hat{q}_H^{DVP} = 1$. Given that $q_H = 1$, next we solve firm L's optimal quality q_L . Taking derivative with respect to q_L , one can verify that $\pi_L^{DVP}(1, q_L, t_p)$ is concave in q_L given that the market is partially covered and each firm has positive demand for sufficiently small t_p . Therefore, firm L's optimal quality $\hat{q}_L^{DVP}(t_p)$ is solved from $\frac{d\pi_L^{DVP}(1, q_L, t_p)}{dq_L} = 0$.

Thus, the equilibrium outcome is: $q_H = \hat{q}_H^{DVP} = 1$, $q_L = \hat{q}_L^{DVP}(t_p)$, and $p_{in} = \hat{p}_{in}^{DVP}(t_p) = p_{ni}^{DM}(1, \hat{q}_L^{DVP}(t_p), t_p)$, for $i = H, L$ and $n = 1, 2$. Let $\hat{d}_{in}^{DVP}(t_p) = d_{in}^{DVP}(1, \hat{q}_L^{DVP}(t_p), t_p)$ and $\hat{\pi}_{in}^{DVP}(t_p) = \hat{p}_{in}^{DVP}(t_p)\hat{d}_{in}^{DVP}(t_p)$ denote firm i 's demand and profit in period n . Firm i 's total demand and total profit are given by $\hat{d}_i^{DVP}(t_p) = \hat{d}_{i1}^{DVP}(t_p) + \hat{d}_{i2}^{DVP}(t_p)$ and $\hat{\pi}_i^{DVP}(t_p) = \hat{\pi}_{i1}^{DVP}(t_p) + \hat{\pi}_{i2}^{DVP}(t_p)$.

Note that we can apply similar approach as in the proof of Proposition 3 to show the existence and uniqueness of the above equilibrium in which the market is partially covered and each firm's demand is positive in each period. The detailed proof is omitted and available from authors upon request. The proof of Proposition B2 parts(a)-(d) are provided in proof of Proposition 5. \square

Proof of Proposition 5: Note that all the final equilibrium outcomes are differentiable.

Part (a): $\hat{q}_L^{DVP}(t_p)$ is the unique solution of $\frac{\partial \pi_L^{DVP}(1, q_L, t_p)}{\partial q_L} = 0$. By the Implicit Function Theorem, we have $\frac{\partial \hat{q}_L^{DVP}(t_p)}{\partial t_p} = -\frac{\partial^2 \pi_L^{DVP}(1, q_L, t_p)}{\partial q_L \partial t_p} / \frac{\partial^2 \pi_L^{DVP}(1, q_L, t_p)}{\partial q_L^2} |_{q_L = \hat{q}_L^{DVP}(t_p)}$. We have already shown $\frac{\partial^2 \pi_L^{DVP}(1, q_L, t_p)}{\partial q_L^2} < 0$, and can verify that $\frac{\partial^2 \pi_L^{DVP}(1, q_L, t_p)}{\partial q_L \partial t_p} < 0$ for any $q_L \in (0.2, 0.77)$ and $t_p \in (0, \bar{t}_p]$. Moreover, we find that $\hat{q}_L^{DVP}(t_p) \in (0.2, 0.77)$ for any $t_p \in (0, \bar{t}_p]$. Therefore, $\frac{\partial \hat{q}_L^{DVP}(t_p)}{\partial t_p} < 0$, i.e. $\hat{q}_L^{DVP}(t_p)$ decreases in t_p and $\hat{q}_L^{DVP}(t_p) < \hat{q}_L^b$. Next, we compare $\hat{q}_L^{DVP}(t_p)$ with $\hat{q}_L^{VP}(t_p)$. Directly solving $\frac{\partial \pi_L^{DVP}(1, q_L, t_p)}{\partial q_L} = 0$ and $\frac{\partial \pi_L^{VP}(1, q_L, t_p)}{\partial q_L} = 0$, we can check that $\hat{q}_L^b > \hat{q}_L^{DVP}(t_p) > \hat{q}_L^{VP}(t_p)$ when $t_p < \tilde{t}$ and $\hat{q}_L^{DVP}(t_p) < \hat{q}_L^{VP}(t_p) < \hat{q}_L^b$ when $\tilde{t} < t_p < \bar{t}_p$, where $\tilde{t} \approx 0.0829548$.

Part (b): Note that $\hat{p}_{in}^{DVP}(t_p) = p_{in}^{DVP}(1, \hat{q}_L^{DVP}(t_p), t_p)$, for $i = H, L$ and $n = 1, 2$. By the chain rule, $\frac{\partial \hat{p}_{in}^{DVP}(t_p)}{\partial t_p} = \left(\frac{\partial p_{in}^{DVP}(1, q_L, t_p)}{\partial t_p} + \frac{\partial p_{in}^{DVP}(1, q_L, t_p)}{\partial q_L} \frac{\partial \hat{q}_L^{DVP}(t_p)}{\partial t_p} \right) \Big|_{q_L = \hat{q}_L^{DVP}(t_p)}$. Similar to the previous analysis, one can verify that: 1) $\hat{p}_{H1}^{DVP}(t_p)$ first decreases and then increases in t_p ; 2) $\hat{p}_{L1}^{DVP}(t_p)$ decreases in t_p ; and 3) $\hat{p}_{H2}^{DVP}(t_p)$ and $\hat{p}_{L2}^{DVP}(t_p)$ increase in t_p . Comparing with the benchmark prices, we can further show that $\hat{p}_{H1}^{DVP}(t_p) < \hat{p}_H^b$ and $\hat{p}_{L2}^{DVP}(t_p) > \hat{p}_L^b$, for $i = H, L$.

Next, substitute $\hat{q}_L^{DVP}(t_p)$ into $p_{i1}^{DVP}(1, \hat{q}_L^{DVP}(t_p), t_p)$ (Equations (B9)) and $p_{i2}^{DVP}(1, \hat{q}_L^{DVP}(t_p), t_p)$ (Equations (B11a) and (B11b)); and substitute $\hat{q}_L^{VP}(t_p)$ into $p_i^{VP}(1, \hat{q}_L^{VP}(t_p), t_p)$ (from Equations (A13)). After some tedious but straightforward simplifications, one can verify that $\hat{p}_{H1}^{DVP}(t_p) < \hat{p}_H^b < \hat{p}_H^{VP}(t_p) < \hat{p}_{H2}^{DVP}(t_p)$ and $\hat{p}_{L1}^{DVP}(t_p) < \hat{p}_L^{VP}(t_p) < \hat{p}_L^b < \hat{p}_{L2}^{DVP}(t_p)$.

Part (c): Note that $\hat{d}_i^{DVP}(t_p) = d_i^{DVP}(1, \hat{q}_L^{DVP}(t_p), t_p)$, for $i = H, L$. By the chain rule, $\frac{\partial \hat{d}_i^{DVP}(t_p)}{\partial t_p} = \left(\frac{\partial d_i^{DVP}(1, q_L, t_p)}{\partial t_p} + \frac{\partial d_i^{DVP}(1, q_L, t_p)}{\partial q_L} \frac{\partial \hat{q}_L^{DVP}(t_p)}{\partial t_p} \right) \Big|_{q_L = \hat{q}_L^{DVP}(t_p)}$. Taking similar approach, we can check that: 1) $\hat{d}_H^{DVP}(t_p)$ increases in t_p ; and 2) $\hat{d}_L^{DVP}(t_p)$ first decreases and then increases in t_p .

Next, substitute $\hat{q}_L^{DVP}(t_p)$ into $d_i^{DVP}(1, \hat{q}_L^{DVP}(t_p), t_p)$ and $\hat{q}_L^{VP}(t_p)$ into $d_i^{VP}(1, \hat{q}_L^{VP}(t_p), t_p)$. After some straightforward yet tedious simplifications, one can verify that $\hat{d}_H^{DVP}(t_p) < \hat{d}_H^{VP}(t_p)$ and $\hat{d}_L^{DVP}(t_p) > \hat{d}_L^{VP}(t_p)$.

Part (d): Note that $\hat{\pi}_i^{DVP}(t_p) = \pi_i^{DVP}(1, \hat{q}_L^{DVP}(t_p), t_p)$, for $i = H, L$. By the chain rule, $\frac{\partial \hat{\pi}_i^{DVP}(t_p)}{\partial t_p} = \left(\frac{\partial \pi_i^{DVP}(1, q_L, t_p)}{\partial t_p} + \frac{\partial \pi_i^{DVP}(1, q_L, t_p)}{\partial q_L} \frac{\partial \hat{q}_L^{DVP}(t_p)}{\partial t_p} \right) \Big|_{q_L = \hat{q}_L^{DVP}(t_p)}$. Again, via similar approach, we can verify that $\hat{\pi}_H^{DVP}(t_p)$ increases in t_p and $\hat{\pi}_L^{DVP}(t_p)$ decreases in t_p .

Substitute $\hat{q}_L^{DVP}(t_p)$ into $\pi_i^{DVP}(1, \hat{q}_L^{DVP}(t_p), t_p)$ and $\hat{q}_L^{VP}(t_p)$ into $\pi_i^{VP}(1, \hat{q}_L^{VP}(t_p), t_p)$. After tedious but straightforward simplifications, one can verify that: 1) $\hat{\pi}_L^b > \hat{\pi}_L^{DVP}(t_p) > \hat{\pi}_L^{VP}(t_p)$ for any $t_p \in (0, \bar{t}_p]$; and 2) $\hat{\pi}_H^{VP}(t_p) > \hat{\pi}_H^{DVP}(t_p) > \hat{\pi}_H^b$ when $t_p < \hat{t}$ and $\hat{\pi}_H^{DVP}(t_p) > \hat{\pi}_H^{VP}(t_p) > \hat{\pi}_H^b$ when $\hat{t} < t_p \leq \bar{t}_p$, where $\hat{t} \approx 0.058082$.

This completes the proof of Proposition 5 and Proposition B2 parts(a)-(d). \square