Online Appendix
"Informing the Public about a Pandemic"

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A Proofs of Results

Before providing proofs, we first provide some definitions used in the statements of the results in the paper and the proofs. First, let $\gamma \triangleq \sigma 2\sqrt{3}$ (which also corresponds to $\gamma = \sup \mathcal{C} - \inf \mathcal{C}$).

The closed-form expression of the equilibrium threshold $c^\star(\mu)$ is

$$c^\star(\mu) = \frac{\mu\kappa(\theta + \sigma\sqrt{3})}{\sigma 2\sqrt{3} + \mu\kappa}\,.$$

The thresholds over μ are given by

$$\mu_\lambda^{\mathsf{DP}} \triangleq \rho_\lambda^{\mathsf{DP}}(\omega_h - \omega_\ell) + \omega_\ell \tag{A.1}$$

$$\mu_\lambda^{\mathsf{EX}} \triangleq \rho_\lambda^{\mathsf{EX}}(\omega_h - \omega_\ell) + \omega_\ell \tag{A.2}$$

where

$$\rho_\lambda^{\mathsf{DP}} \triangleq \frac{\gamma^2(5\lambda - 4) - 2\kappa^2\omega_\ell\omega_h(2\lambda - 1) - \gamma\kappa(2(1-\lambda)\omega_h + \lambda\omega_\ell)}{\kappa(\omega_h - \omega_\ell)(2\kappa\omega_h(2\lambda - 1) + \gamma\lambda)} \tag{A.3}$$

$$\rho_\lambda^{\mathsf{EX}} \triangleq \frac{\Big(2\kappa\omega_\ell(1 - 2\lambda) - \gamma(4 - 5\lambda)\Big)(\kappa\omega_\ell + \gamma)}{\kappa(\omega_h - \omega_\ell)(2\kappa\omega_\ell(2\lambda - 1) + \gamma\lambda)}\,. \tag{A.4}$$

Then, the optimal downplaying and exaggerating probabilities (we use subscripts to highlight the differences) are in order respectively:

$$\pi^\star_{\mathsf{DP}}(m_h \,|\, \omega_h) = \frac{\omega_h - \omega_\ell}{\mu^\circ - \omega_\ell}\left(\frac{\mu^\circ - \mu_\lambda^{\mathsf{DP}}}{\omega_h - \mu_\lambda^{\mathsf{DP}}}\right) \text{ and } \pi^\star_{\mathsf{DP}}(m_\ell \,|\, \omega_\ell) = 1\,, \tag{A.5}$$

$$\pi^{\star}_{\mathsf{EX}}(m_h\,|\,\omega_h)=1 \text{ and } \pi^{\star}_{\mathsf{EX}}(m_\ell\,|\,\omega_\ell)=\frac{(\mu^{\mathsf{EX}}_{\lambda}-\mu^{\circ})(\omega_h-\omega_\ell)}{(\omega_h-\mu^{\circ})(\mu^{\mathsf{EX}}_{\lambda}-\omega_\ell)}\,. \tag{A.6}$$

Furthermore, we define for a given γ (and hence σ),

$$\lambda_1(\kappa)\triangleq\max\left(-\epsilon, G_\ell^{-1}\left(\frac{\kappa}{\gamma}\right)\right) \tag{A.7}$$

$$\lambda_2(\kappa)\triangleq\max\left(-\epsilon, G_2^{-1}\left(\frac{\kappa}{\gamma}\right)\right) \tag{A.8}$$

$$\lambda_3(\kappa)\triangleq\min\left(1+\epsilon, G_1^{-1}\left(\frac{\kappa}{\gamma}\right)\right) \tag{A.9}$$

$$\lambda_4(\kappa)\triangleq\min\left(1+\epsilon, G_h^{-1}\left(\frac{\kappa}{\gamma}\right)\right) \tag{A.10}$$

for $\epsilon>0$ and $Y^{-1}(\cdot)$ represents the inverse of a function Y. Note here that we project functions λ_i, $i=1,\ldots,4$ in order to extend their domains to positive real numbers because $\kappa/\gamma\geq 0$. Since $\lambda\in[0,1]$, the extended parts ($1+\epsilon$ and $-\epsilon$) do not affect our analysis.

$$G_h(\lambda)\triangleq\frac{1-\frac{5\lambda}{4}}{\omega_h\left(\frac{1}{2}-\lambda\right)} \text{ and } G_\ell(\lambda)\triangleq\frac{1-\frac{5\lambda}{4}}{\omega_\ell\left(\frac{1}{2}-\lambda\right)} \text{ for } \lambda\in(-\infty,1/2)\cup(1/2,\infty)\,, \tag{A.11}$$

$$G_1(\lambda)\triangleq\sqrt{\frac{5\lambda-4}{2(2\lambda-1)\omega_h\omega_\ell}+\left(\frac{2(1-\lambda)\omega_h+\lambda\omega_\ell}{4(2\lambda-1)\omega_h\omega_\ell}\right)^2}-\frac{2(1-\lambda)\omega_h+\lambda\omega_\ell}{4(2\lambda-1)\omega_h\omega_\ell} \text{ for } \lambda\in(4/5,\infty)\,, \tag{A.12}$$

$$G_2(\lambda)\triangleq\sqrt{\frac{4-5\lambda}{2(1-2\lambda)\omega_h\omega_\ell}+\left(\frac{\lambda\omega_h+2(1-\lambda)\omega_\ell}{4(1-2\lambda)\omega_h\omega_\ell}\right)^2}+\frac{\lambda\omega_h+2(1-\lambda)\omega_\ell}{4(1-2\lambda)\omega_h\omega_\ell} \text{ for } \lambda\in(-\infty,1/2)\,. \tag{A.13}$$

We also define

$$\begin{aligned}\overline{\kappa}_0&\triangleq\gamma G_\ell(0)\\ \underline{\kappa}_0&\triangleq\gamma G_2(0)\\ \overline{\kappa}_1&\triangleq\gamma G_1(1)\\ \underline{\kappa}_1&\triangleq\gamma G_h(1)\,.\end{aligned} \tag{A.14}$$

In the proofs of results, we denote the first and second derivatives of an arbitrary function $Y(x)$ with respect to x by $Y'(x)$ and, respectively $Y''(x)$.

Proof of Proposition 1. Fix μ. It is straightforward to see from (1) and (2), the equilibrium action $a^{\star}(c,\mu)$ in (7) of an individual with cost c is the unique solution to

$$\min_{\alpha\in\{0,1\}}\alpha c+(1-\alpha)\mu P_{a^{\star},\mu}\kappa \tag{A.15}$$

for $c^\star(\mu) = \mu P_{a^\star,\mu}\kappa$. We next show that $P_{a^\star,\mu}$ is uniquely pinned down by $a^\star(c,\mu)$ for $c^\star(\mu) = \mu P_{a^\star,\mu}\kappa$ and (2). Let $F(\cdot)$ denote the c.d.f. of the uniform distribution with support $\mathcal{C}$.

$$P_{a^\star,\mu} = 1 - F\left(\mu P_{a^\star,\mu}\kappa\right). \tag{A.16}$$

Let $h(x) \triangleq x - 1 + F(\mu x\kappa)$ for all $x \in [0,1]$. The function h is continuous and strictly increasing. Furthermore, $h(0) = -1 < 0$ since $\theta - \gamma/2 \geq 0$ and $h(1) = F(\mu\kappa) \geq 0$. Thus, we conclude that $P_{a^\star,\mu}$ is the unique point such that $P_{a^\star,\mu} = h^{-1}(0)$. Finally, solving the unique $P_{a^\star,\mu}$ of $P_{a^\star,\mu} = h^{-1}(0)$ for a uniform distribution over $[\theta - \gamma/2, \theta + \gamma/2]$ leads to $P_{a^\star,\mu} = (\theta + \sigma\sqrt{3})/(2\sigma\sqrt{3} + \mu\kappa)$. Using the fact that $c^\star(\mu) = \mu P_{a^\star,\mu}\kappa$, we obtain $c^\star(\mu) = \dfrac{\mu\kappa(\theta + \sigma\sqrt{3})}{\sigma 2\sqrt{3} + \mu\kappa}$. It is straightforward to show the monotonicity and concavity properties by taking first and second derivatives of $P_{a^\star,\mu}$ and $c^\star(\mu)$ with respect to μ. Q.E.D.

Proof of Theorem 1. Before proving Theorem 1, we first provide four lemmas and some new notation. We delegate the proofs of those lemmas to the end. For simplicity, we define $\tilde{K}_i(\mu) \triangleq K_i(c^\star(\mu))$ for $i \in \{e,h\}$ and so $\tilde{K}_\lambda(\mu) \triangleq K_\lambda(c^\star(\mu))$ for $\lambda \in [0,1]$.

Lemma A.1. *Functions G_h, G_ℓ, G_1 and G_2 defined respectively in* (A.11)-(A.13) *are strictly increasing. Furthermore, $G_1(\lambda) > G_h(\lambda)$, $G_\ell(\lambda) > G_1(\lambda)$ for $\lambda \in (4/5,1]$, and $G_\ell(\lambda) > G_2(\lambda)$, $G_2(\lambda) > G_h(\lambda)$ for $\lambda \in [0,1/2)$.*

Lemma A.2. *Consider λ_i, for $i = 1,\ldots,4$ defined in* (A.7)-(A.10), *then $\lambda_1 \leq \lambda_2 \leq 1/2 \leq 4/5 \leq \lambda_3 \leq \lambda_4$. Further, thresholds $\lambda_1, \lambda_2, \lambda_3$ and λ_4 are all nondecreasing in κ with $\lambda_1(\overline{\kappa}_0) = \lambda_2(\underline{\kappa}_0) = 0$ and $\lambda_3(\overline{\kappa}_1) = \lambda_4(\underline{\kappa}_1) = 1$ for positive cutoffs $\overline{\kappa}_0 > \underline{\kappa}_0 > \overline{\kappa}_1 > \underline{\kappa}_1$ defined in* (A.14).

Lemma A.3. *We have the following.*

A. *The function $\tilde{K}_\lambda(\mu)$ is concave if either one of the following conditions holds.*
   1. $\lambda \in [1/2, 4/5]$.
   2. $\lambda \in (4/5,1] \wedge \kappa/\gamma \geq G_\ell(\lambda)$.
   3. $\lambda \in [0,1/2) \wedge G_h(\lambda) \geq \kappa/\gamma$.

B. *The function $\tilde{K}_\lambda(\mu)$ is convex if either one of the following conditions holds.*
   1. $\lambda \in [0,1/2) \wedge \kappa/\gamma \geq G_\ell(\lambda)$,
   2. $\lambda \in (4/5,1] \wedge G_h(\lambda) \geq \kappa/\gamma$.

C. *If $\lambda \in (4/5,1] \wedge G_\ell(\lambda) > \kappa/\gamma \geq G_1(\lambda)$, then $\tilde{K}_\lambda(\mu)$ is first convex and then concave with inflection point $\mu_{in} \in (\omega_\ell, \omega_h)$ and it satisfies $\tilde{K}'_\lambda(\omega_\ell)(\omega_h - \omega_\ell) \geq \tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\omega_\ell)$.*

D. If $\lambda \in [0,1/2) \wedge G_2(\lambda) \geq \kappa/\gamma > G_h(\lambda)$*, then* $\tilde{K}_\lambda(\mu)$ *is first concave and then convex with inflection point* $\tilde{\mu}_{in} \in (\omega_\ell, \omega_h)$ *and it satisfies* $\tilde{K}_\lambda(\omega_h) - \tilde{K}'_\lambda(\omega_h)(\omega_h - \omega_\ell) \geq \tilde{K}_\lambda(\omega_\ell)$*.*

E. If $\lambda \in (4/5,1] \wedge G_1(\lambda) > \kappa/\gamma > G_h(\lambda)$*, then* $\tilde{K}_\lambda(\mu)$ *is first convex and then concave with inflection point* $\mu_{in} \in (\omega_\ell, \omega_h)$ *such that* $\mu_\lambda^{DP} \leq \mu_{in}$*. Moreover,* $\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\mu_\lambda^{DP}) = \tilde{K}'(\mu_\lambda^{DP})(\omega_h - \mu_\lambda^{DP})$*.*

F. If $\lambda \in [0,1/2) \wedge G_\ell(\lambda) > \kappa/\gamma > G_2(\lambda)$*, then* $\tilde{K}_\lambda(\mu)$ *is first concave and then convex with inflection point* $\tilde{\mu}_{in} \in (\omega_\ell, \omega_h)$ *such that* $\mu_\lambda^{EX} \geq \tilde{\mu}_{in}$*. Moreover,* $\tilde{K}_\lambda(\mu_\lambda^{EX}) - \tilde{K}_\lambda(\omega_\ell) = \tilde{K}'_\lambda(\mu_\lambda^{EX})(\mu_\lambda^{EX} - \omega_\ell)$*.*

We further define the following conditions.

$$\mathrm{FD}_1 : \lambda \in [1/2, 4/5] \tag{A.17}$$

$$\mathrm{FD}_2 : \lambda \in (4/5, 1] \wedge \kappa/\gamma \geq G_1(\lambda) \tag{A.18}$$

$$\mathrm{FD}_3 : \lambda \in [0, 1/2) \wedge G_2(\lambda) \geq \kappa/\gamma \tag{A.19}$$

$$\mathrm{ND}_1 : \lambda \in [0, 1/2) \wedge \kappa/\gamma \geq G_\ell(\lambda) \tag{A.20}$$

$$\mathrm{ND}_2 : \lambda \in (4/5, 1] \wedge G_h(\lambda) \geq \kappa/\gamma \tag{A.21}$$

$$\mathrm{DP} : \lambda \in (4/5, 1] \wedge G_1(\lambda) > \kappa/\gamma > G_h(\lambda) \tag{A.22}$$

$$\mathrm{EX} : \lambda \in [0, 1/2) \wedge G_\ell(\lambda) > \kappa/\gamma > G_2(\lambda) \tag{A.23}$$

LEMMA A.4. *The lower convex envelope* $k_\lambda(\mu)$ *of* $\tilde{K}_\lambda(\mu)$ *is*

$$k_\lambda(\mu) \triangleq \begin{cases} \tilde{K}_\lambda(\omega_\ell) + \frac{\mu - \omega_\ell}{\omega_h - \omega_\ell}[\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\omega_\ell)] & \text{if } \mathrm{FD}_1 \vee \mathrm{FD}_2 \vee \mathrm{FD}_3\,, \\ \tilde{K}_\lambda(\mu) & \text{if } \mathrm{ND}_1 \vee \mathrm{ND}_2\,, \\ \tilde{K}_\lambda(\mu) 1_{\{\mu \leq \mu_\lambda^{DP}\}} + \left[\tilde{K}_\lambda(\mu_\lambda^{DP}) + (\mu - \mu_\lambda^{DP}) \frac{\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\mu_\lambda^{DP})}{(\omega_h - \mu_\lambda^{DP})}\right] 1_{\{\mu > \mu_\lambda^{DP}\}} & \text{if } \mathrm{DP}\,, \\ \tilde{K}_\lambda(\mu) 1_{\{\mu \geq \mu_\lambda^{EX}\}} + \left[\tilde{K}_\lambda(\omega_\ell) + (\mu - \omega_\ell) \frac{\tilde{K}_\lambda(\mu_\lambda^{EX}) - \tilde{K}_\lambda(\omega_\ell)}{(\mu_\lambda^{EX} - \omega_\ell)}\right] 1_{\{\mu < \mu_\lambda^{EX}\}} & \text{if } \mathrm{EX}\,. \end{cases} \tag{A.24}$$

where $1_{\{\cdot\}}$ *represents the indicator function.*

Combining these lemmas, we are now ready to prove the theorem.

Following Corollary 2 of Kamenica and Gentzkow (2011), we know that the optimal cost corresponding to any prior mean μ is given by $k_\lambda(\mu)$ because $k_\lambda(\mu)$ is the lower convex envelope of $\tilde{K}_\lambda(\mu)$ (see Lemma A.4). We first provide a set of conditions, and characterize the optimal information disclosure policy for each bullet point.

- If FD_1 or FD_2 or FD_3, full disclosure is optimal. This item follows because we have $k_\lambda(\lambda) = \tilde{K}_\lambda(\omega_\ell) + (\mu - \omega_\ell)[\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\omega_\ell)]/(\omega_h - \omega_\ell)$ so the optimal value can be achieved by the perfect information policy which induces posterior means ω_h and ω_ℓ. Thus, the binary message space $\mathcal{M}_{\mathsf{B}} = \{m_\ell, m_h\}$ with $\pi^\star(m_i \,|\, \omega_i) = 1$ for $i \in \{\ell, h\}$ achieves the optimal cost and hence constitute the optimal policy.

  Next, we show that if $\lambda_2 \le \lambda \le \lambda_3$, then FD_1 or FD_2 or FD_3 and hence full disclosure is optimal.

  — When $4/5 < \lambda \le \lambda_3$, ($4/5 \le \lambda_3$, see Lemma A.2), we have $\lambda \le 1$ and $\lambda \le G_1^{-1}(\kappa/\gamma)$ from $\lambda \le \lambda_3$. Thus, it follows that $\lambda \in (4/5, 1]$ and $\kappa/\gamma \ge G_1(\lambda)$ and hence FD_2.

  — When $1/2 \le \lambda \le 4/5$, we directly obtain FD_1.

  — When $\lambda_2 \le \lambda < 1/2$, ($\lambda_2 \le 1/2$, see Lemma A.2), we have $\lambda \ge 0$ and $\lambda \ge G_2^{-1}(\kappa/\gamma)$ from $\lambda \ge \lambda_2$. Thus, it follows that $\lambda \in [0, 1/2)$ and $G_2(\lambda) \ge \kappa/\gamma$ and hence FD_3.

- If ND_1 or ND_2, no disclosure is optimal. The lower convex envelope $k_\lambda(\mu)$ is equal to the function $\tilde{K}_\lambda(\mu)$. Thus, no disclosure policy is the uniquely optimal. Note that $\lambda \le \lambda_1$ implies ND_1, and $\lambda \ge \lambda_4$ implies ND_2 from the definitions of λ_1 and λ_4. Hence, we prove the second bullet point of the theorem.

- If DP and $\mu^\circ > \mu_\lambda^{\mathsf{DP}}$, it is optimal to downplay the risk. The optimal cost can be achieved by no disclosure policy if $\mu^\circ \le \mu_\lambda^{\mathsf{DP}}$ because $k_\lambda(\mu) = \tilde{K}_\lambda(\mu)$ in that case. Otherwise, the optimal cost can be achieved by inducing posterior means ω_h with probability $(\mu^\circ - \mu_\lambda^{\mathsf{DP}})/(\omega_h - \mu_\lambda^{\mathsf{DP}})$ and $\mu_\lambda^{\mathsf{DP}}$ with the remaining probability. Specifically, the following probability distribution provided in (A.5) with a binary message space $\mathcal{M}_{\mathsf{B}}$ achieves this posterior distribution. We next show that $\lambda_3 < \lambda < \lambda_4$ implies DP. Since, $\lambda_3 \ge 4/5$, we obtain that $\lambda \in (4/5, 1]$. Definitions of λ_3 and λ_4 (see (A.7)-(A.10)) imply $G_1(\lambda) > \kappa/\gamma > G_h(\lambda)$.

- If EX and $\mu^\circ < \mu_\lambda^{\mathsf{EX}}$, it is optimal to exaggerate the risk. The optimal cost can be achieved by no disclosure policy if $\mu^\circ \ge \mu_\lambda^{\mathsf{EX}}$. Otherwise, the optimal cost can be achieved by inducing posterior means ω_ℓ with probability $(\mu_\lambda^{\mathsf{EX}} - \mu^\circ)/(\mu_\lambda^{\mathsf{EX}} - \omega_\ell)$ and $\mu_\lambda^{\mathsf{EX}}$ with the remaining probability. Specifically, the following probability distribution provided in (A.6) with a binary message space $\mathcal{M}_{\mathsf{B}}$ achieves this posterior distribution. We conclude the proof by showing that $\lambda_1 < \lambda < \lambda_2$ implies EX. Since $\lambda_2 \le 1/2$, we obtain that $\lambda \in [0, 1/2)$. Definitions of λ_3 and λ_4 (see (A.7)-(A.10)) $G_2(\lambda) < \kappa/\gamma < G_\ell(\lambda)$.

Proof of Lemma A.1. To prove this result, we first provide the derivatives of G_ℓ and G_h with respect to λ.

$$G_h'(\lambda) = \frac{3}{2\omega_h(1-2\lambda)^2} \text{ and } G_\ell'(\lambda) = \frac{3}{2\omega_\ell(1-2\lambda)^2} \tag{A.25}$$

Since these terms above are positive, the claim follows. Now consider, G_2. The terms $\frac{4-5\lambda}{2(1-2\lambda)\omega_h\omega_\ell}$ and $\frac{\lambda\omega_h+2(1-\lambda)\omega_\ell}{4(1-2\lambda)\omega_h\omega_\ell}$ are increasing functions of λ. Hence, G_2 is also increasing. In order to prove, G_1 is increasing we define the following notation for simplicity. Let $A_1(\lambda) \triangleq \frac{5\lambda-4}{2(2\lambda-1)\omega_h\omega_\ell}$, $A_2(\lambda) \triangleq \frac{2(1-\lambda)\omega_h+\lambda\omega_\ell}{4(2\lambda-1)\omega_h\omega_\ell}$, i.e., $G_1(\lambda) = \sqrt{A_1(\lambda) + (A_2(\lambda))^2} - A_2(\lambda)$. Taking derivative of A_1 and A_2, it can be shown that A_1 is increasing and A_2 is decreasing. Furthermore, it follows that

$$G_1'(\lambda) = \frac{A_1'(\lambda)}{2\sqrt{A_1(\lambda) + (A_2(\lambda)^2)}} - A_2'(\lambda)\left[1 - \frac{1}{\sqrt{\frac{A_1(\lambda)}{(A_2(\lambda))^2}+1}}\right]. \tag{A.26}$$

This expression reveals that $G_1'(\lambda) > 0$ because $A_1'(\lambda)$ is positive, and $A_2'(\lambda)$ is negative.

Next, we prove $G_1(\lambda) > G_h(\lambda)$ and $G_\ell(\lambda) \geq G_1(\lambda)$ for $\lambda \in (4/5, 1]$. Note that we can write $G_h(\lambda) = A_1(\lambda)\omega_\ell$ and $G_\ell(\lambda) = A_1(\lambda)\omega_h$. First, we consider $G_1(\lambda) \geq G_h(\lambda)$, we need to show that

$$\sqrt{A_1(\lambda) + [A_2(\lambda)]^2} > A_1(\lambda)\omega_\ell + A_2(\lambda)$$

Using simple algebraic operations, we equivalently represent the inequality above as

$$1 > \frac{(1-\lambda)\omega_h + (3\lambda-2)\omega_\ell}{\omega_h(2\lambda-1)} \iff \omega_h > \omega_\ell\,.$$

For the last inequality we use the fact that $\lambda \in (4/5, 1]$ so $(3\lambda - 2) > 0$. Now, we consider $G_\ell(\lambda) > G_1(\lambda)$, we need to show that

$$A_1(\lambda)\omega_h + A_2(\lambda) > \sqrt{A_1(\lambda) + [A_2(\lambda)]^2}$$

Algebraic operations yield that the above inequality can be written as follows:

$$\frac{(3\omega_h + \omega_\ell)\lambda - 2\omega_h}{4\lambda\omega_\ell - 2\omega_\ell} > 1 \iff \omega_h > \omega_\ell$$

Here, we use the fact that $\lambda \in (4/5, 1]$ so $(3\lambda - 2) > 0$.

We next prove $G_\ell(\lambda) > G_2(\lambda)$, $G_2(\lambda) > G_h(\lambda)$ for $\lambda \in [0, 1/2)$. For simplicity we define $B_1(\lambda) = G_\ell(\lambda)/\omega_h$ and $B_2(\lambda) \triangleq \frac{\lambda\omega_h+2(1-\lambda)\omega_\ell}{4(1-2\lambda)\omega_h\omega_\ell}$. Using this notation, we first need to show that

$$\omega_h B_1(\lambda) > \sqrt{B_1(\lambda) + [B_2(\lambda)]^2} + B_2(\lambda)$$

Simplifying this inequality we obtain that

$$\frac{((3\omega_h - \omega_\ell)\lambda - 2\omega_h + \omega_\ell)}{(2\lambda - 1)\omega_\ell} > 1 \iff \omega_h > \omega_\ell\,.$$

To obtain the last inequality, we use the fact that $\lambda \in [0, 1/2)$ to show $2 - 3\lambda > 0$. Thus, $G_\ell(\lambda) > G_2(\lambda)$ follows.

Finally, $G_2(\lambda) > G_h(\lambda)$ which can equivalently represented as follows:

$$\sqrt{B_1(\lambda) + [B_2(\lambda)]^2} + B_2(\lambda) > \omega_\ell B_1(\lambda)$$

Using straightforward algebraic operations, we get the equivalent inequality.

$$1 > \frac{\omega_\ell(2 - 3\lambda) - \lambda\omega_h}{2\omega_h(1 - 2\lambda)} \iff \omega_h > \omega_\ell$$

As in the previous steps, we the fact that $\lambda \in [0, 1/2)$. Q.E.D.

Proof of Lemma A.2 Since we know G_h, G_ℓ, G_1 and G_2 (see (A.11)-(A.13)) are strictly increasing in their domains from Lemma A.1, their inverse are also strictly increasing (see, Binmore 1982, p. 111). Since κ/γ is strictly increasing in κ, thresholds λ_i are all nondecreasing in κ.

Because $G_1(4/5) = G_h(4/5) = 0$, it follows that $\lambda_3 \geq 4/5$ and $\lambda_4 \geq 4/5$. Furthermore, $G_1(\lambda) > G_h(\lambda)$ for $\lambda \in (4/5, 1]$ (see Lemma A.1). Therefore, $\lambda_4 \geq \lambda_3 \geq 4/5$. Note that $\lim_{\lambda \to 1/2} G_\ell(\lambda) = \lim_{\lambda \to 1/2} G_2(\lambda) = \infty$, and G_ℓ and G_2 are increasing so we get $1/2 \geq \lambda_2$ and $1/2 \geq \lambda_1$. Since, $G_\ell(\lambda) > G_2(\lambda)$ for $\lambda \in [0, 1/2)$ it follows that $1/2 \geq \lambda_2 \geq \lambda_1$.

Using the definitions of G_h, G_ℓ, G_1 and G_2, we compute $\overline{\kappa}_0 = \gamma G_\ell(0) = 2\gamma/\omega_\ell$, $\underline{\kappa}_0 = \gamma G_2(0) = \gamma(\sqrt{2/(\omega_h\omega_\ell) + 1/\omega_h} - (1/\omega_h)^2)$, $\overline{\kappa}_1 = \gamma G_1(1) = \gamma\left(\sqrt{1/(2\omega_h\omega_\ell) + (1/(4\omega_h))^2} - 1/(4\omega_h)\right)$ and $\underline{\kappa}_1 = \gamma G_h(1) = \gamma/(2\omega_h)$. Then, we obtain $\overline{\kappa}_0 > \underline{\kappa}_0 > \overline{\kappa}_1 > \underline{\kappa}_1$.

Finally, the definitions of $\overline{\kappa}_i, \underline{\kappa}_i$ for $i \in \{0, 1\}$ imply that $\lambda_1(\overline{\kappa}_0) = \lambda_2(\underline{\kappa}_0) = 0$ and $\lambda_3(\overline{\kappa}_1) = \lambda_4(\underline{\kappa}_1) = 1$. Q.E.D.

Proof of Lemma A.3. Using the derived expression of $c^\star(\mu)$ from Proposition 1 and the definition of $\tilde{K}_\lambda(\mu)$, we obtain that

$$\tilde{K}_\lambda(\mu) = \frac{(3\lambda - 2)}{2\gamma}\left(\frac{\mu\kappa(\theta + \gamma/2)}{\gamma + \mu\kappa}\right)^2 + (1 - \lambda)\left(\frac{\mu\kappa(\theta + \gamma/2)}{\gamma + \mu\kappa}\right)\frac{\theta + \gamma/2}{\gamma} - \frac{\lambda(\theta - \gamma/2)^2}{2\gamma}\,. \tag{A.27}$$

The second derivative of $\tilde{K}_\lambda$ with respect to μ is given by

$$\tilde{K}''_\lambda(\mu) = -\frac{4(\theta + \gamma/2)^2\kappa^2\gamma\left[\frac{\kappa}{\gamma}\mu\left(\lambda - \frac{1}{2}\right) + \left(1 - \frac{5\lambda}{4}\right)\right]}{(\kappa\mu + \gamma)^4}\,. \tag{A.28}$$

Part A. Note that $\tilde{K}''_\lambda(\mu)$ in (A.28) is negative when $\lambda\in[1/2,4/5]$. Thus, the first item follows. Recalling the definition of G_h and G_ℓ from (A.11), one can show that the second and third items imply negative $\tilde{K}''_\lambda(\mu)$ because $\frac{\kappa}{\gamma}\mu\left(\lambda-\frac{1}{2}\right)+\left(1-\frac{5\lambda}{4}\right)$ is positive under those conditions.

Part B. To prove this part, we again use (A.28). Simple algebra yields that $\frac{\kappa}{\gamma}\mu\left(\lambda-\frac{1}{2}\right)+\left(1-\frac{5\lambda}{4}\right)$ is negative and hence $\tilde{K}''_\lambda(\mu)$ is positive when the first and second conditions hold.

Part C. First note that $G_1(\lambda)>G_h(\lambda)$ (see Lemma A.1). Thus, we know the parameter regime is such that neither the second item of Part A nor the second item of Part B holds. This implies there exists an inflection point μ_{in} such that $\tilde{K}_\lambda(\mu)$ is first concave and then convex because $\tilde{K}''_\lambda(\mu)$ is first negative and then positive. We next derive $\tilde{K}_\lambda(\omega_h)-\tilde{K}_\lambda(\omega_\ell)-\tilde{K}'_\lambda(\omega_\ell)(\omega_h-\omega_\ell)$ and show that it is nonpositive when $\lambda\in(4/5,1]\wedge G_\ell(\lambda)>\kappa/\gamma\geq G_1(\lambda)$.

$$-\frac{2[(\omega_h-\omega_\ell)(\theta+\gamma/2)\kappa\gamma]^2(\lambda-\frac{1}{2})\left[\omega_\ell\omega_h\left(\frac{\kappa}{\gamma}\right)^2+\frac{(2(1-\lambda)\omega_h+\lambda\omega_\ell)}{2(2\lambda-1)}\frac{\kappa}{\gamma}-\frac{(5\lambda-4)}{2(2\lambda-1)}\right]}{(\kappa\omega_h+\gamma)^2(\kappa\omega_\ell+\gamma)^3} \tag{A.29}$$

In order to prove $\tilde{K}_\lambda(\omega_h)-\tilde{K}_\lambda(\omega_\ell)-\tilde{K}'_\lambda(\omega_\ell)(\omega_h-\omega_\ell)\leq 0$, it is sufficient to show the following inequality holds.

$$\left(\frac{\kappa}{\gamma}\right)^2+\frac{(2(1-\lambda)\omega_h+\lambda\omega_\ell)}{2(2\lambda-1)\omega_h\omega_\ell}\frac{\kappa}{\gamma}-\frac{(5\lambda-4)}{2(2\lambda-1)\omega_h\omega_\ell}\geq 0 \tag{A.30}$$

We can rewrite the left-hand side of the above inequality as

$$\left(\frac{\kappa}{\gamma}\right)^2+\frac{(2(1-\lambda)\omega_h+\lambda\omega_\ell)}{2(2\lambda-1)\omega_h\omega_\ell}\frac{\kappa}{\gamma}-\frac{(5\lambda-4)}{2(2\lambda-1)\omega_h\omega_\ell} \tag{A.31}$$

$$=\left(\frac{\kappa}{\gamma}+\frac{(2(1-\lambda)\omega_h+\lambda\omega_\ell)}{4(2\lambda-1)\omega_h\omega_\ell}\right)^2-\left(\frac{(2(1-\lambda)\omega_h+\lambda\omega_\ell)}{4(2\lambda-1)\omega_h\omega_\ell}\right)^2-\frac{(5\lambda-4)}{2(2\lambda-1)\omega_h\omega_\ell}. \tag{A.32}$$

This expression above is nonnegative whenever $\kappa/\gamma\geq G_1(\lambda)$ and we conclude the proof of this part.

Part D. Note that $G_\ell(\lambda)>G_2(\lambda)$ (see Lemma A.1) so the parameter regime in this part is such that neither the third item of Part A nor the first item of Part B holds. Thus, there exists an inflection point $\tilde{\mu}_{in}\in(\omega_\ell,\omega_h)$ such that $\tilde{K}_\lambda(\mu)$ is first convex and then concave because $\tilde{K}''_\lambda(\mu)$ is first positive and then negative. Next, we show that $\tilde{K}_\lambda(\omega_h)-\tilde{K}'_\lambda(\omega_h)(\omega_h-\omega_\ell)-\tilde{K}_\lambda(\omega_\ell)$ is nonnegative when $\lambda\in[0,1/2)\wedge G_2(\lambda)\geq\kappa/\gamma>G_h(\lambda)$. The expression is given by

$$-\frac{2[(\omega_h-\omega_\ell)(\theta+\gamma/2)\kappa\gamma]^2\left(\frac{1}{2}-\lambda\right)\left[\omega_\ell\omega_h\left(\frac{\kappa}{\gamma}\right)^2+\frac{(2(1-\lambda)\omega_\ell+\omega_h\lambda)}{2(1-2\lambda)}\frac{\kappa}{\gamma}-\frac{(5\lambda-4)}{2(1-2\lambda)}\right]}{(\kappa\omega_h+\gamma)^3(\kappa\omega_\ell+\gamma)^2}.$$

To show $\tilde{K}_\lambda(\omega_h) - \tilde{K}'_\lambda(\omega_h)(\omega_h - \omega_\ell) - \tilde{K}_\lambda(\omega_\ell) \geq 0$, we can directly focus on the following inequality.

$$\left(\frac{\kappa}{\gamma}\right)^2 + \frac{(2(1-\lambda)\omega_\ell + \omega_h\lambda)}{2(1-2\lambda)\omega_h\omega_\ell}\frac{\kappa}{\gamma} - \frac{(5\lambda-4)}{2(1-2\lambda)\omega_h\omega_\ell} \leq 0 \tag{A.33}$$

Rewriting this expression, we obtain the following.

$$\left(\frac{\kappa}{\gamma}\right)^2 + \frac{(2(1-\lambda)\omega_\ell + \omega_h\lambda)}{2(1-2\lambda)\omega_h\omega_\ell}\frac{\kappa}{\gamma} - \frac{(5\lambda-4)}{2(1-2\lambda)\omega_h\omega_\ell} \tag{A.34}$$

$$= \left(\frac{\kappa}{\gamma} + \frac{(2(1-\lambda)\omega_\ell + \omega_h\lambda)}{4(1-2\lambda)\omega_h\omega_\ell}\right)^2 - \left(\frac{(2(1-\lambda)\omega_\ell + \omega_h\lambda)}{4(1-2\lambda)\omega_h\omega_\ell}\right)^2 - \frac{(5\lambda-4)}{2(1-2\lambda)\omega_h\omega_\ell} \leq 0 \tag{A.35}$$

The last inequality holds because $\kappa/\gamma \leq G_2(\lambda)$.

Part E. Since $G_\ell(\lambda) > G_1(\lambda)$ (see Lemma A.1), there exists an inflection point μ_{in} as in Part C. Differently from Part C, this time $G_1(\lambda) > \kappa/\gamma$ so $\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\omega_\ell) - \tilde{K}'_\lambda(\omega_\ell)(\omega_h - \omega_\ell) > 0$. Moreover, we know $\tilde{K}_\lambda(\mu) + \tilde{K}'_\lambda(\mu)(\omega_h - \mu)$ is an increasing function of $\mu \in (\omega_\ell, \mu_{in})$ due to convexity and $\tilde{K}_\lambda(\mu_{in}) + \tilde{K}'_\lambda(\mu_{in})(\omega_h - \mu_{in}) > \tilde{K}_\lambda(\omega_h)$ due to concavity for $\mu \in (\mu_{in}, \omega_h)$. Thus, $\tilde{K}_\lambda(\mu) + \tilde{K}'_\lambda(\mu)(\omega_h - \mu)$ crosses $\tilde{K}_\lambda(\omega_h)$ for some $\mu = \mu_\lambda^{\mathsf{DP}}$ such that $\mu_\lambda^{\mathsf{DP}} \leq \mu_{in}$.

Part F. Note that $G_2(\lambda) > G_h(\lambda)$ (see Lemma A.1) therefore there exists an inflection point $\tilde{\mu}_{in}$ as in Part D. Unlike Part D, $\kappa/\gamma > G_2(\lambda)$ thus $\tilde{K}_\lambda(\omega_h) - \tilde{K}'_\lambda(\omega_h)(\omega_h - \omega_\ell) - \tilde{K}_\lambda(\omega_\ell) < 0$. Due to concavity in $(\omega_\ell, \tilde{\mu}_{in})$, we know $\tilde{K}_\lambda(\omega_\ell) < \tilde{K}_\lambda(\tilde{\mu}_{in}) - \tilde{K}'_\lambda(\tilde{\mu}_{in})(\tilde{\mu}_{in} - \omega_\ell)$. Besides, $\tilde{K}_\lambda(\mu) - \tilde{K}'_\lambda(\mu)(\mu - \omega_\ell)$ is a decreasing function of $\mu \in (\tilde{\mu}_{in}, \omega_h]$ due to convexity after the inflection point $\tilde{\mu}_{in}$. Thus, there must be a point $\mu_\lambda^{\mathsf{EX}}$ in $(\tilde{\mu}_{in}, \omega_h)$ such that $\tilde{K}_\lambda(\mu_\lambda^{\mathsf{EX}}) - \tilde{K}'_\lambda(\mu_\lambda^{\mathsf{EX}})(\mu_\lambda^{\mathsf{EX}} - \omega_\ell) = \tilde{K}_\lambda(\omega_\ell)$. Q.E.D.

Proof of Lemma A.4. In order to check if the proposed function k_λ is the lower convex envelope of $\tilde{K}_\lambda$ or not, we use the verification approach provided in Oberman (2007). In particular, we show that the function k_λ satisfies (Ob) of Oberman (2007) equation for $\tilde{K}_\lambda$. This equation is

$$\max\{k_\lambda(\mu) - \tilde{K}_\lambda(\mu), -k''_\lambda(\mu)\} = 0\,. \tag{Ob}$$

- $\mathrm{FD}_1 \vee \mathrm{FD}_2 \vee \mathrm{FD}_3$: We need to show $\tilde{K}_\lambda(\mu) \geq v(\mu)$ because $k''_\lambda(\mu) = 0$. Since $k_\lambda(\mu)$ is in fact the line connecting $\tilde{K}_\lambda(\omega_\ell)$ and $\tilde{K}_\lambda(\omega_h)$ in this case, the result immediately follows when $\tilde{K}_\lambda$ is concave. Note that Part A of Lemma A.3 shows that $\tilde{K}_\lambda$ is concave when FD_1. For FD_2 and FD_3, we need to complement that the same part of the lemma by considering $\lambda \in (4/5, 1] \wedge G_\ell(\lambda) > \kappa/\gamma \geq G_1(\lambda)$ and $\lambda \in [0, 1/2) \wedge G_2(\lambda) \geq \kappa/\gamma > G_h(\lambda)$, respectively.

Assume that $\lambda \in (4/5,1] \wedge G_\ell(\lambda) > \kappa/\gamma \geq G_1(\lambda)$. For $\mu \leq \mu_{in}$, Taylor's theorem implies that $\tilde{K}_\lambda(\omega_\ell) + \tilde{K}'_\lambda(\omega_\ell)(\mu - \omega_\ell) < \tilde{K}_\lambda(\mu)$ because $\tilde{K}_\lambda$ is convex in $[\omega_\ell, \mu_{in}]$. Using Part C of Lemma A.3 to replace $\tilde{K}'_\lambda(\omega_\ell)$ with its lower bound, we obtain that $\tilde{K}_\lambda(\mu) > \tilde{K}_\lambda(\omega_\ell) + \frac{\mu-\omega_\ell}{\omega_h-\omega_\ell}[\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\omega_\ell)]$. When $\mu \geq \mu_{in}$, we know $\tilde{K}_\lambda(\mu) > \tilde{K}_\lambda(\mu_{in})x + \tilde{K}_\lambda(\omega_h)(1-x)$ for any $x \in (0,1)$ due to concavity. Furthermore, we know that $\tilde{K}_\lambda(\mu_{in}) \geq \tilde{K}_\lambda(\omega_\ell) + \frac{\mu_{in}-\omega_\ell}{\omega_h-\omega_\ell}[\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\omega_\ell)]$ from the previous analysis. Thus, we obtain that

$$\tilde{K}_\lambda(\mu) > \left(1 - \frac{\mu_{in}-\omega_\ell}{\omega_h-\omega_\ell}\right) x\tilde{K}_\lambda(\omega_\ell) + \left(x\frac{\mu_{in}-\omega_\ell}{\omega_h-\omega_\ell} + 1 - x\right)\tilde{K}_\lambda(\omega_h)$$

Here, we can set $\left(1 - \frac{\mu_{in}-\omega_\ell}{\omega_h-\omega_\ell}\right)x = \left(1 - \frac{\mu-\omega_\ell}{\omega_h-\omega_\ell}\right)$ because $0 \leq \frac{1-\frac{\mu-\omega_\ell}{\omega_h-\omega_\ell}}{1-\frac{\mu_{in}-\omega_\ell}{\omega_h-\omega_\ell}} \leq 1$ since $\mu \geq \mu_{in}$. Thus, we obtain that $\tilde{K}_\lambda(\mu) > \tilde{K}_\lambda(\omega_\ell) + \frac{\mu-\omega_\ell}{\omega_h-\omega_\ell}[\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\omega_\ell)]$ for $\mu \geq \mu_{in}$, too.

Next, assume that $\lambda \in [0,1/2) \wedge G_2(\lambda) \geq \kappa/\gamma > G_h(\lambda)$. Taylor's theorem implies that $\tilde{K}_\lambda(\mu) > \tilde{K}_\lambda(\omega_h) + \tilde{K}'_\lambda(\omega_h)(\mu - \omega_h)$ for $\mu > \tilde{\mu}_{in}$ due to convexity. Since $\mu - \omega_h$ is negative, we replace $\tilde{K}'_\lambda(\omega_h)$ with its upper bound obtained in Part D of Lemma A.3. Thus, it follows that $\tilde{K}_\lambda(\mu) > \tilde{K}_\lambda(\omega_\ell) + \frac{\mu-\omega_\ell}{\omega_h-\omega_\ell}[\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\omega_\ell)]$ for $\mu > \tilde{\mu}_{in}$. When $\mu \leq \tilde{\mu}_{in}$, we know that $\tilde{K}_\lambda(\mu) \geq \tilde{K}_\lambda(\tilde{\mu}_{in})x + (1-x)\tilde{K}_\lambda(\omega_\ell)$ for any $x \in [0,1]$ because $\tilde{K}_\lambda$ is concave in $[\omega_\ell, \tilde{\mu}_{in}]$. Moreover, we know that $\tilde{K}_\lambda(\tilde{\mu}_{in}) \geq \tilde{K}_\lambda(\omega_\ell) + \frac{\tilde{\mu}-\omega_\ell}{\omega_h-\omega_\ell}[\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\omega_\ell)]$ from the previous analysis. Combining these observations, we obtain

$$\tilde{K}_\lambda(\mu) > \tilde{K}_\lambda(\omega_\ell)x + \frac{\tilde{\mu}_{in}-\omega_\ell}{\omega_h-\omega_\ell}[\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\omega_\ell)]x + (1-x)\tilde{K}_\lambda(\omega_\ell) \tag{A.36}$$

$$= \tilde{K}_\lambda(\omega_\ell) + x\frac{\tilde{\mu}_{in}-\omega_\ell}{\omega_h-\omega_\ell}[\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\omega_\ell)] \tag{A.37}$$

Here, we can set $x = \left(\frac{\mu-\omega_\ell}{\omega_h-\omega_\ell}\right) / \left(\frac{\tilde{\mu}_{in}-\omega_\ell}{\omega_h-\omega_\ell}\right)$ because $\mu \leq \tilde{\mu}_{in}$, and obtain that $\tilde{K}_\lambda(\mu) > \tilde{K}_\lambda(\omega_\ell) + \frac{\mu-\omega_\ell}{\omega_h-\omega_\ell}[\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\omega_\ell)]$. This completes the proof of this item of the lemma.

- $\mathrm{ND}_1 \vee \mathrm{ND}_2$: Part B of Lemma A.3 shows that $\tilde{K}_\lambda$ is convex, thus its lower convex envelope is equal to $\tilde{K}_\lambda(\mu)$ itself (see Boyd and Vandenberghe 2004, pp. 94).
- DP: We consider two cases separately. First, assume that $\mu \leq \mu_\lambda^{\mathsf{DP}}$, then $k_\lambda(\mu) = \tilde{K}_\lambda(\mu)$ and $\tilde{K}_\lambda(\mu)$ is convex because $\mu_\lambda^{\mathsf{DP}} \leq \mu_{in}$ (see Part E of Lemma A.3). Next, assume that $\mu \geq \mu_\lambda^{\mathsf{DP}}$, then $k''_\lambda(\mu) = 0$ by its definition. Thus, we need to show that $k_\lambda(\mu) - \tilde{K}_\lambda(\mu) \leq 0$ for $\mu \geq \mu_\lambda^{\mathsf{DP}}$ to prove that $k_\lambda(\mu)$ satisfies (Ob). Taylor's theorem implies that $\tilde{K}_\lambda(\mu) > \tilde{K}_\lambda(\mu_\lambda^{\mathsf{DP}}) + (\mu - \mu_\lambda^{\mathsf{DP}})\tilde{K}'_\lambda(\mu_\lambda^{\mathsf{DP}})$ for $\mu_{in} \geq \mu$ due to convexity. From Part E of Lemma A.3, we also know $k_\lambda(\mu) = \tilde{K}_\lambda(\mu_\lambda^{\mathsf{DP}}) + (\mu - \mu_\lambda^{\mathsf{DP}})\tilde{K}'_\lambda(\mu_\lambda^{\mathsf{DP}})$. Thus, $k_\lambda(\mu) - \tilde{K}_\lambda(\mu) \leq 0$ for $\mu_{in} \geq \mu \geq \mu_\lambda^{\mathsf{DP}}$.

Next, we consider $\mu \geq \mu_{in}$. When we look at the boundary $\mu = \mu_{in}$, the former observation implies $\tilde{K}_\lambda(\mu_{in}) \geq k_\lambda(\mu_{in})$. Because $\tilde{K}_\lambda$ is concave when $\mu \geq \mu_{in}$, it follows that $\tilde{K}_\lambda(\mu) \geq \tilde{K}_\lambda(\mu_{in}) + (\mu - \mu_{in})[\tilde{K}_\lambda(\omega_h) - \tilde{K}_\lambda(\mu_{in})]/(\omega_h - \mu_{in})$ which constitutes the line connecting $\tilde{K}_\lambda(\mu_{in})$ to $\tilde{K}_\lambda(\omega_h)$. Because $k_\lambda(\mu)$ is in fact the line connecting $k_\lambda(\mu_{in})$ to $k_\lambda(\omega_h) = \tilde{K}_\lambda(\omega_h)$ by definition, it follows that $\tilde{K}_\lambda(\mu) \geq k_\lambda(\mu)$ for $\mu \geq \mu_{in}$, too.

Combining these, we conclude that $k_\lambda(\mu)$ satisfies (Ob).

- EX: We consider two cases separately. If $\mu \geq \mu_\lambda^{\mathsf{EX}}$, then $k_\lambda(\mu) = \tilde{K}_\lambda(\mu)$ and $\tilde{K}_\lambda(\mu)$ is convex from Part F of Lemma A.3 because $\mu_\lambda^{\mathsf{EX}} \geq \tilde{\mu}_{in}$. We next consider $\mu_\lambda^{\mathsf{EX}} \geq \mu$. By its definition, $k_\lambda''(\mu) = 0$ when $\mu_\lambda^{\mathsf{EX}} \geq \mu$. Thus, we need to show that $k_\lambda(\mu) - \tilde{K}_\lambda(\mu) \leq 0$ for $\mu_\lambda^{\mathsf{EX}} \geq \mu$ to prove that $k_\lambda(\mu)$ satisfies (Ob).

  When $\mu_\lambda^{\mathsf{EX}} \geq \mu \geq \tilde{\mu}_{in}$, Taylor's theorem implies that $\tilde{K}_\lambda(\mu) \geq \tilde{K}_\lambda(\mu_\lambda^{\mathsf{EX}}) + (\mu - \mu_\lambda^{\mathsf{EX}})\tilde{K}_\lambda'(\mu_\lambda^{\mathsf{EX}})$ because $\tilde{K}_\lambda(\mu)$ is convex for $\mu \geq \tilde{\mu}_{in}$. Furthermore, we know $k_\lambda(\mu) = \tilde{K}_\lambda(\mu_\lambda^{\mathsf{EX}}) + (\mu - \mu_\lambda^{\mathsf{EX}})\tilde{K}_\lambda'(\mu_\lambda^{\mathsf{EX}})$ because $\tilde{K}_\lambda(\mu_\lambda^{\mathsf{EX}}) - \tilde{K}_\lambda(\omega_\ell) = \tilde{K}_\lambda'(\mu_\lambda^{\mathsf{EX}})(\mu_\lambda^{\mathsf{EX}} - \omega_\ell)$ (see Part F of Lemma A.3). Thus, $k_\lambda(\mu) \leq \tilde{K}_\lambda(\mu)$ for $\mu_\lambda^{\mathsf{EX}} \geq \mu \geq \tilde{\mu}_{in}$.

  Next, we consider $\tilde{\mu}_{in} \geq \mu$. By definition $k_\lambda(\omega_\ell) = \tilde{K}_\lambda(\omega_\ell)$, and $k_\lambda(\mu)$ is the line connecting $k_\lambda(\omega_\ell)$ and $k_\lambda(\tilde{\mu}_{in})$ for $\tilde{\mu}_{in} \geq \mu$. We also know $\tilde{K}_\lambda(\mu)$ is above the line connecting $\tilde{K}_\lambda(\omega_\ell)$ and $\tilde{K}_\lambda(\tilde{\mu}_{in})$ because $\tilde{K}_\lambda$ is concave for $\tilde{\mu}_{in} \geq \mu$. Hence, we need to show $k_\lambda(\tilde{\mu}_{in}) \leq \tilde{K}_\lambda(\tilde{\mu}_{in})$ to complete the proof. The previous observation also implies that $k_\lambda(\tilde{\mu}_{in}) \leq \tilde{K}_\lambda(\tilde{\mu}_{in})$ so the claim follows. Q.E.D.

Proof of Theorem 2. We first prove the monotonicity of downplaying and exaggerating probabilities in γ. Because $\gamma = 2\sigma\sqrt{3}$, we also obtain the same monotonicity behavior in σ.

Note that the optimal downplaying probability, see (A.5), is increasing in $\mu_\lambda^{\mathsf{DP}}$ when $\lambda_1 < \lambda < \lambda_2$, and exaggerating probability, see (A.6), is decreasing $\mu_\lambda^{\mathsf{EX}}$ when $\lambda_3 < \lambda < \lambda_4$. Thus, we focus on the derivatives of $\mu_\lambda^{\mathsf{DP}}$ and $\mu_\lambda^{\mathsf{EX}}$ with respect to γ.

The derivative of $\mu_\lambda^{\mathsf{DP}}$ with respect to γ is given by

$$\frac{\gamma^2(5\lambda-4)\left[-8(1-\lambda)\frac{\left(\lambda-\frac{1}{2}\right)}{(5\lambda-4)}\frac{\kappa^2\omega_h^2}{\gamma^2}+8\left(\lambda-\frac{1}{2}\right)\frac{\kappa\omega_h}{\gamma}+\lambda\right]}{16\left(\kappa\omega_h\left(\lambda-\frac{1}{2}\right)+\frac{\gamma\lambda}{4}\right)^2\kappa}. \tag{A.38}$$

Since λ is inside $(4/5, 1]$ when downplaying, the derivative is positive if and only if the term inside square brackets is positive. Thus, we focus on that term.

$$-8(1-\lambda)\frac{\left(\lambda-\frac{1}{2}\right)}{(5\lambda-4)}\frac{\kappa^2\omega_h^2}{\gamma^2}+8\left(\lambda-\frac{1}{2}\right)\frac{\kappa\omega_h}{\gamma}+\lambda = 8\left(\lambda-\frac{1}{2}\right)\frac{\kappa\omega_h}{\gamma}\left(1-\frac{(1-\lambda)}{(5\lambda-4)}\frac{\kappa\omega_h}{\gamma}\right)+\lambda \tag{A.39}$$

Next, we show that $\left(1-\frac{(1-\lambda)}{(5\lambda-4)}\frac{\kappa\omega_h}{\gamma}\right)$ is positive because the remaining terms are positive. To do so, we use the upper bound $G_1(\lambda)$ on κ/γ. In particular, we prove that $\frac{5\lambda-4}{1-\lambda}\geq\omega_h G_1(\lambda)$. First note that

$$\begin{aligned}&\frac{5\lambda-4}{1-\lambda}-\omega_h G_1(\lambda)\\&=\frac{5\lambda-4}{1-\lambda}-\sqrt{\frac{(5\lambda-4)\omega_h}{2\,(2\lambda-1)\,\omega_\ell}+\left(\frac{2\,(1-\lambda)\,\omega_h+\lambda\omega_\ell}{4\,(2\lambda-1)\,\omega_\ell}\right)^2}+\frac{2\,(1-\lambda)\,\omega_h+\lambda\omega_\ell}{4\,(2\lambda-1)\,\omega_\ell}.\end{aligned}\tag{A.40}$$

In the following, we show that the sum of positive terms above is larger than the negative term.

$$\left[\frac{5\lambda-4}{1-\lambda}+\frac{2\,(1-\lambda)\,\omega_h+\lambda\omega_\ell}{4\,(2\lambda-1)\,\omega_\ell}\right]^2-\frac{(5\lambda-4)\omega_h}{2\,(2\lambda-1)\,\omega_\ell}-\left(\frac{2\,(1-\lambda)\,\omega_h+\lambda\omega_\ell}{4\,(2\lambda-1)\,\omega_\ell}\right)^2\tag{A.41}$$

$$=\frac{(5\lambda-4)\omega_h}{2\,(2\lambda-1)\,\omega_\ell}\left\{\frac{2(2\lambda-1)\omega_\ell}{(1-\lambda)\omega_h}\left[\frac{5\lambda-4}{1-\lambda}+\frac{2\,(1-\lambda)\,\omega_h+\lambda\omega_\ell}{2\,(2\lambda-1)\,\omega_\ell}\right]-1\right\}\tag{A.42}$$

$$=\frac{(5\lambda-4)\omega_h}{2\,(2\lambda-1)\,\omega_\ell}\left\{\frac{2\,(2\lambda-1)\,\omega_\ell(5\lambda-4)}{\omega_h(1-\lambda)^2}+\frac{\lambda\omega_\ell}{\omega_h(1-\lambda)}+1\right\}>0\tag{A.43}$$

The last inequality follows because all terms are positive. Combining these, we conclude that $\mu_\lambda^{\mathsf{DP}}$ is increasing in γ.

The derivative of $\mu_\lambda^{\mathsf{EX}}$ with respect to γ is given by

$$\frac{\gamma^2(4-5\lambda)\left[8(1-\lambda)\frac{\kappa^2\omega_\ell^2}{\gamma^2}\frac{\left(\frac{1}{2}-\lambda\right)}{(4-5\lambda)}+8\frac{\kappa\omega_\ell}{\gamma}\left(\frac{1}{2}-\lambda\right)-\lambda\right]}{16\left(\kappa\omega_\ell\left(\lambda-\frac{1}{2}\right)+\frac{\gamma\lambda}{4}\right)^2\kappa}.\tag{A.44}$$

Recall that λ is inside $[0,1/2)$ when exaggerating, so the derivative is positive if and only if the term inside square brackets is positive. Hence, we focus on that term.

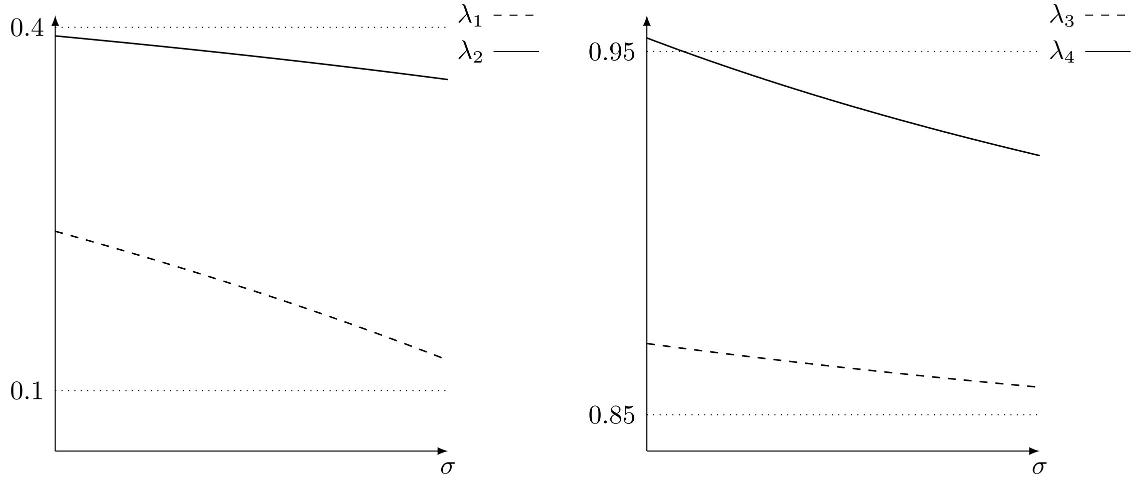
$$8(1-\lambda)\frac{\kappa^2\omega_\ell^2}{\gamma^2}\frac{\left(\frac{1}{2}-\lambda\right)}{(4-5\lambda)}+8\frac{\kappa\omega_\ell}{\gamma}\left(\frac{1}{2}-\lambda\right)-\lambda=8\frac{\kappa\omega_\ell}{\gamma}\left(\frac{1}{2}-\lambda\right)\left(\frac{\kappa\omega_\ell}{\gamma}\frac{(1-\lambda)}{4-5\lambda}+1\right)-\lambda\tag{A.45}$$

Here, note that $\left(\frac{\kappa\omega_\ell}{\gamma}\frac{(1-\lambda)}{4-5\lambda}+1\right)$ is larger than 1, thus we next show that $8\frac{\kappa\omega_\ell}{\gamma}(1/2-\lambda)\geq\lambda$ to complete the proof. To do so, we use the lower bound on κ/γ.

$$\frac{\kappa}{\gamma}8\omega_\ell(1/2-\lambda)\geq G_2(\lambda)8\omega_\ell(1/2-\lambda)=\lambda+\frac{2(1-\lambda)\omega_\ell}{\omega_h}\geq\lambda.\tag{A.46}$$

Thus, we conclude that $\mu_\lambda^{\mathsf{EX}}$ is increasing in γ.

The remaining part of the theorem follows because functions G_2 and G_ℓ on $[0,1/2)$ and G_1, G_h on $(4/5,1]$ are increasing functions and κ/γ is a decreasing function of σ. Figure A1 provides an illustration for this result. Q.E.D.



(a) λ_1 and λ_2 and $\kappa = 150$

(b) λ_3 and λ_4 and $\kappa = 9$

Figure A1 **Threshold functions λ_i, $i = 1, \ldots, 4$ as a function of σ for $\omega_\ell = 0.3$, $\omega_h = 0.8$ and $\theta = 10$.**

B Belief Update using Bayes' Rule

The prior belief of the population is ρ°, and suppose that the sender commits to an information disclosure policy $\Gamma = (\pi, \mathcal{M})$. After receiving message m, receivers update their beliefs from ρ° to ρ_m according Bayes' rule as follows.

$$\rho_m \triangleq \mathbb{P}(\omega = \omega_h \,|\, m) = \frac{\pi(m \,|\, \omega_h)\rho^\circ}{\pi(m \,|\, \omega_h)\rho^\circ + \pi(m \,|\, \omega_\ell)(1-\rho^\circ)}\,. \tag{B.1}$$

Accordingly, the posterior mean corresponding to the posterior belief is

$$\mu_m \triangleq \rho_m \omega_h + (1-\rho_m)\omega_\ell\,. \tag{B.2}$$

C Technical Intuition of Our Results

In this section of the appendix, we provide further intuition regarding Theorem 1. To do so, we introduce the notation $c^\star(\mu\kappa)$ such that $c^\star(\mu\kappa) = c^\star(\mu)$ in Proposition 1 with some abuse of notation.

As discussed in Section 3, the shape of $K_\lambda(c^\star(\mu\kappa))$ is jointly determined by that of $K_i(\cdot)$ $(i \in \{e, h\})$ and that of the threshold $c^\star(\mu\kappa)$, which we recall is an increasing concave function of μ due to negative externalities among the receivers (as implied by Proposition 1). Therefore, we next explore the shape of $K_i(\cdot)$ $(i \in \{e, h\})$.

PROPOSITION C.1. *For any $x \in \mathcal{C}$, the function $K_e(x)$ is increasing convex, whereas the function $K_h(x)$ is concave and unimodal with the peak point at $x = (\theta + \sigma\sqrt{3})/2$.*

Proof of Proposition C.1. We first derive the expressions for $K_i(x)$ for $i \in \{e,h\}$ for uniform distribution with mean θ and support $\mathcal{C}$. Note that we assume $x \in \mathcal{C}$.

$$K_e(x) = \mathbb{E}_c[1_{\{c \leq x\}} c] = \int_{\theta-\gamma/2}^{x} \frac{x}{\gamma} \mathrm{d}x = \frac{1}{2\gamma}\left[(x)^2 - \left(\theta - \frac{\gamma}{2}\right)^2\right] \tag{C.1}$$

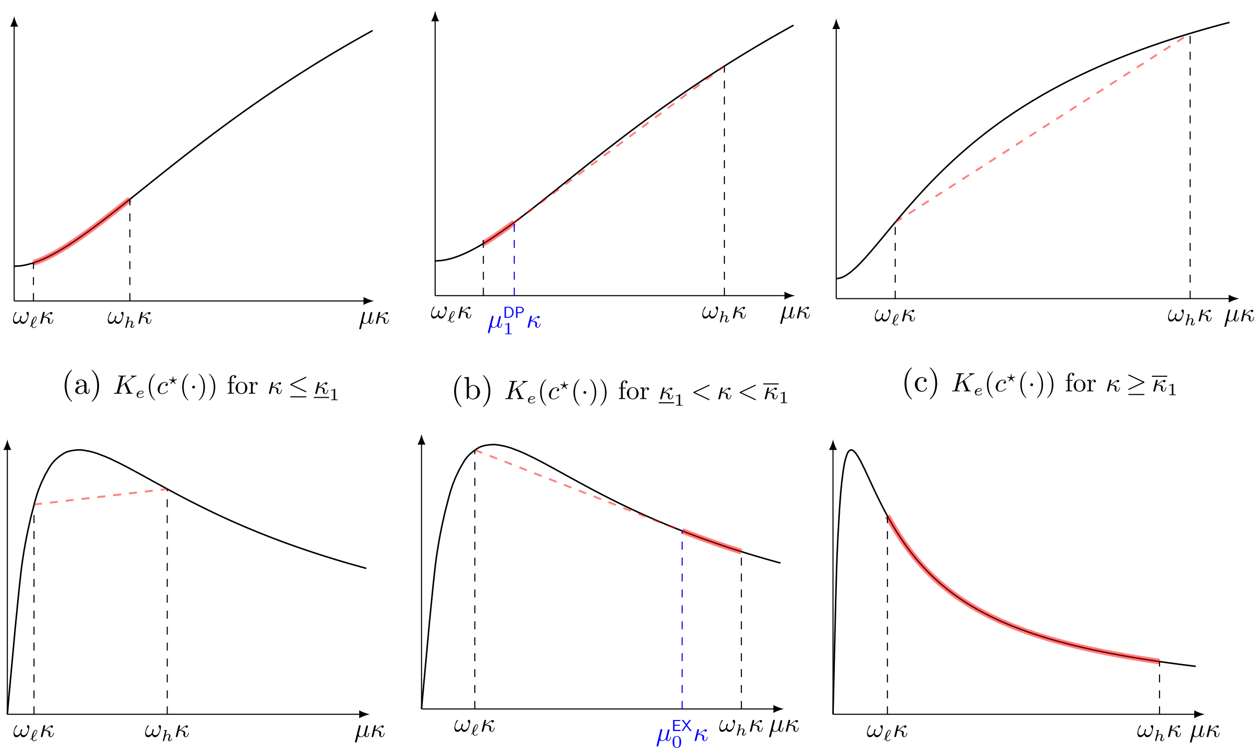
$$K_h(x) = x(1-F(x)) = x\left(1 - \frac{x-\theta+\gamma/2}{\gamma}\right) \tag{C.2}$$

Using these, we get $K_e'(x) = x/\gamma$ and $K_e''(x) = 1/\gamma$ and prove the first bullet point. Furthermore, it follows that $K_h'(x) = [(\theta+\gamma/2) - 2x]/\gamma$ and $K_h''(x) = -2/\gamma$. Because the second derivative is negative, it follows that $K_h(x)$ is concave. Since $K_h'(x) \geq 0$ when $x \leq (\theta+\gamma/2)/2$, and $K_h'(x) \leq 0$ when $x \geq (\theta+\gamma/2)/2$, the result follows. Q.E.D.

Thus, as equilibrium threshold $c^\star$ increases, more individuals incurring increasingly higher economic cost remain in confinement. Hence, the overall economic cost increases at an increasing rate (i.e., $K_e(\cdot)$ is increasing convex). In contrast, total healthcare cost $K_h(\cdot)$ first increases as $c^\star$ increases, because the expected healthcare cost per individual is given by $\mu\kappa P_{a^\star,\mu} = c^\star$. However, as threshold $c^\star$ keeps increasing, fewer individuals choose to engage in social interaction, eventually lowering the total healthcare cost (i.e., $K_h(\cdot)$ then decreases).

As the optimal policy is obtained through the lower convex envelop of $K_\lambda\big(c^\star(\mu\kappa)\big)$ as a function of μ, the nature of the optimal information policy characterized above is fundamentally driven by the second-order behavior of function $K_\lambda\big(c^\star(\cdot)\big)$. Specifically, as illustrated by Figure C1, $K_e\big(c^\star_\gamma(\cdot)\big)$ is first convex and then concave, whereas $K_h\big(c^\star_\gamma(\cdot)\big)$ is the opposite, i.e, first concave and then convex. The domain of these functions are $[\omega_\ell\kappa, \omega_h\kappa]$ because $\mu \in [\omega_\ell, \omega_h]$. In effect, κ acts to control the active domain of function $K_\lambda\big(c^\star(\cdot)\big)$, over which the lower convex envelope will be constructed.

Take the economy-biased government ($\lambda = 1$) as an example. When the healthcare cost is low ($\kappa \leq \underline{\kappa}_1$), function $K_e\big(c^\star(\cdot)\big)$ is entirely convex over $[\omega_\ell\kappa, \omega_h\kappa]$ and hence its lower convex envelope is itself (see Figure C1a), suggesting that the optimal martingale split of μ° is simply not to split and hence no disclosure is optimal. When the healthcare cost moves into the intermediate range ($\underline{\kappa}_1 < \kappa < \overline{\kappa}_1$), the inflection point of function $K_\lambda\big(c^\star(\cdot)\big)$ falls within $[\omega_\ell\kappa, \omega_h\kappa]$ and hence its lower convex envelope consists of itself over $[\omega_\ell\kappa, \mu_1^{\mathsf{DP}}\kappa]$ and the straight line connecting $\big(\mu_1^{\mathsf{DP}}\kappa, K_e\big(c^\star(\mu_1^{\mathsf{DP}}\kappa)\big)\big)$ and $\big(\omega_h\kappa, K_e\big(c^\star(\omega_h\kappa)\big)\big)$ (see Figure C1b), where the straight line and function $K_e\big(c^\star(\cdot)\big)$ are tangent with each other at $\big(\mu_1^{\mathsf{DP}}\kappa, K_e\big(c^\star(\mu_1^{\mathsf{DP}}\kappa)\big)\big)$ (see (A.1) for the closed-form expression of μ_1^{DP}). Thus, as in the previous case, no disclosure is optimal if $\mu^\circ \le \mu_1^{\mathsf{DP}}$; otherwise, the optimal martingale split of μ° would induce posterior beliefs at μ_1^{DP} and ω_h, which can be implemented by downplaying (see (A.5) in for exact expression of $\pi^\star(m_\ell\,|\,\omega_h)$). When the healthcare cost is sufficiently large ($\kappa \ge \overline{\kappa}_1$), function $K_e\big(c^\star(\cdot)\big)$ is entirely concave over $[\omega_\ell\kappa, \omega_h\kappa]$ and hence its lower convex envelope is the straight line connecting $\big(\omega_\ell\kappa, K_e\big(c_\gamma^\star(\omega_\ell\kappa)\big)\big)$ and $\big(\omega_h\kappa, K_e\big(c_\gamma^\star(\omega_h\kappa)\big)\big)$ (see Figure C1c), suggesting that the optimal martingale split of μ° would induce posterior beliefs at ω_ℓ and ω_h and hence full disclosure is optimal.



(a) $K_e(c^\star(\cdot))$ for $\kappa \le \underline{\kappa}_1$ (b) $K_e(c^\star(\cdot))$ for $\underline{\kappa}_1 < \kappa < \overline{\kappa}_1$ (c) $K_e(c^\star(\cdot))$ for $\kappa \ge \overline{\kappa}_1$

(d) $K_h(c^\star(\cdot))$ for $\kappa \le \underline{\kappa}_0$ (e) $K_h(c^\star(\cdot))$ for $\underline{\kappa}_0 < \kappa < \overline{\kappa}_0$ (f) $K_h(c^\star(\cdot))$ for $\kappa \ge \overline{\kappa}_0$

Figure C1 Convexification of $K_\lambda\big(c^\star(\mu\kappa)\big)$ as a function of μ for $\lambda \in \{1,0\}$, $\omega_\ell = 0.15$, $\omega_h = 0.9$, $\gamma = 20$ and $\theta = 10$. Solid red lines represent that $K_\lambda\big(c^\star(\mu\kappa)\big)$ coincides with its lower convex envelope, whereas dashed red lines represent $K_\lambda\big(c^\star(\mu\kappa)\big)$ is strictly above its lower convex envelope.

Similarly, the healthcare-biased government's optimal policy can be constructed from the lower convex envelope of $K_h\big(c^\star(\cdot)\big)$ (see Figures C1d-C1f), albeit in the direction opposite to that of the economy-biased policy. This is because $K_h\big(c^\star(\cdot)\big)$ demonstrates the second-order behavior in contrast to that of $K_e\big(c^\star(\cdot)\big)$. Therefore, when the government needs to make a trade off between the economic and healthcare costs by minimizing $K_\lambda(c^\star(\mu\kappa))$ for some $\lambda \in [0,1]$, the opposing behaviors of $K_e\big(c^\star(\cdot)\big)$ and $K_h\big(c^\star(\cdot)\big)$ may offset each other and subsequently damp down the information distortion in the optimal policy.

D Numerical Study

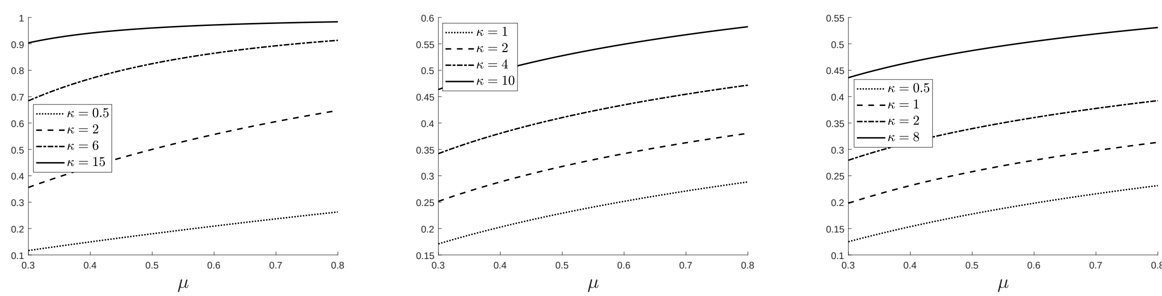
In this section of the appendix, we provide a numerical study where we consider different probability distributions for the economic cost c. For any given value of λ and κ we first numerically characterize the objective function of the sender $K_\lambda(c^\star(\mu))$ using the equilibrium threshold $c^\star(\mu)$. Next, we analyze the lower convex envelope of $K_\lambda(c^\star(\mu))$ to characterize the optimal information policy. Moreover, we numerically illustrate the effect of economic inequality on downplaying and exaggerating probabilities.

D.1 Optimal Policy

In order to derive the sender's objective function $K_\lambda(c^\star(\mu))$, we first need to compute the equilibrium threshold as a function of μ using the unique solution of the following equation.

$$\frac{c^\star(\mu)}{\mu\kappa} = 1 - F\left(c^\star(\mu)\right) \tag{D.1}$$

In Proposition 1, we solve $c^\star(\mu)$ in closed form for uniform distribution. Due to nega-

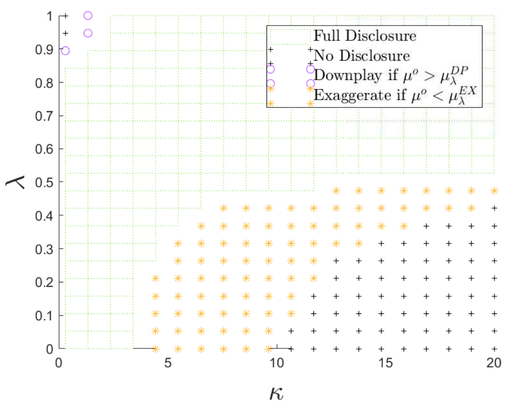


(a) $a = 0.5$ and $b = 0.5$ (b) $a = 1$ and $b = 3$ (c) $a = 2$ and $b = 5$

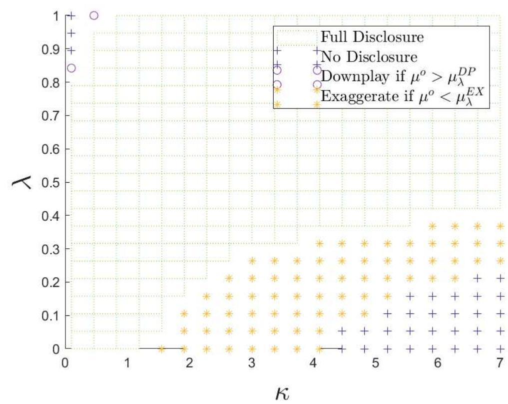
Figure D1 Equilibrium threshold $c^\star(\mu)$ for $\omega_\ell = 0.3$ and $\omega_h = 0.8$ and beta distributions with different a and b

tive externalities (captured by $P_{a^\star,\mu}$), the equilibrium threshold increases albeit with a diminishing rate for uniform distribution. Although the same properties (monotonicity and concavity) hold for distributions with an increasing density function, it is not straightforward to see whether negative externalities affect the equilibrium threshold in the same way for different distributions. To analyze this, we consider three beta distributions whose density functions are decreasing (with parameters $a = 1$ and $b = 3$), unimodal with single peak point (with parameters $a = 2$ and $b = 5$), and nonmonotone with single lowest point (with parameters $a = 0.5$ and $b = 0.5$). Figure D1 illustrates that the same properties of $c^\star(\mu)$, and hence the effect of negative externalities on the equilibrium, are indeed extended to those cases.

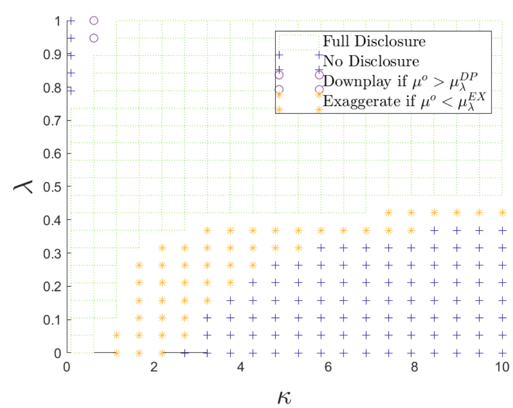
Next, we numerically analyze the optimal information policy in the (κ, λ)-space.



(a) $a = 0.5$ and $b = 0.5$



(b) $a = 1$ and $b = 3$

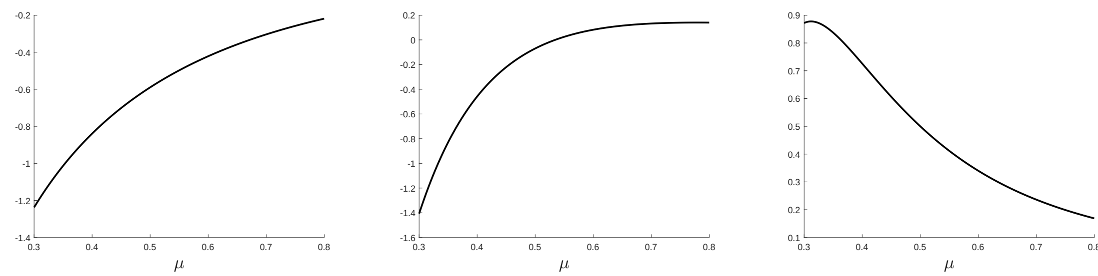


(c) $a = 2$ and $b = 5$

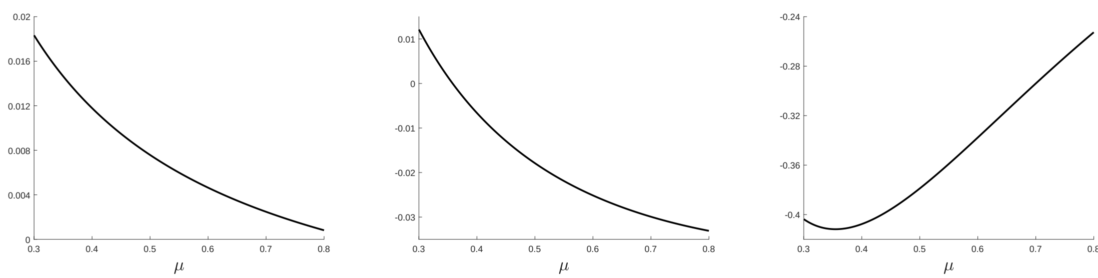
Figure D2 Optimal Policy in (κ, λ)-space for $\omega_\ell = 0.3$, $\omega_h = 0.8$ and beta distributions with parameters a and b

For the same three beta distributions, Figure D2 plots the government's optimal policy in the (κ, λ)-space, which demonstrates qualitatively the same patterns as those in Figure 1 of the paper. Hence, the main insights of Theorem 1 are also present for different beta distributions. More specifically, when the government weighs the economy and population health sufficiently equally, i.e., when λ is sufficiently close to $1/2$, the government fully discloses its information about the severity of the epidemic. However, a more polarized government (i.e., λ is sufficiently far from $1/2$), may misrepresent its information. As in the case of uniform distribution, this happens when health costs also take on extreme values. When λ is small and κ is large, the government exaggerates the epidemic's severity if the population's perception of the risk is low ($\mu^\circ < \mu_\lambda^{\mathsf{EX}}$). On the other hand, when λ is large and κ small, the government downplays the epidemic's severity if the population's perception of the risk is high ($\mu^\circ > \mu_\lambda^{\mathsf{DP}}$). If both λ and κ take on very extreme values, the government prefers not to disclose any information.

The technical intuition behind the results in Figure D2 is driven by the second order behavior of $K_\lambda(c^\star(\mu))$ as discussed in Appendix C (see Figure C1). Therefore, to conclude this section, we numerically compute and illustrate the second derivative of objective function $K_\lambda(c^\star(\mu))$.



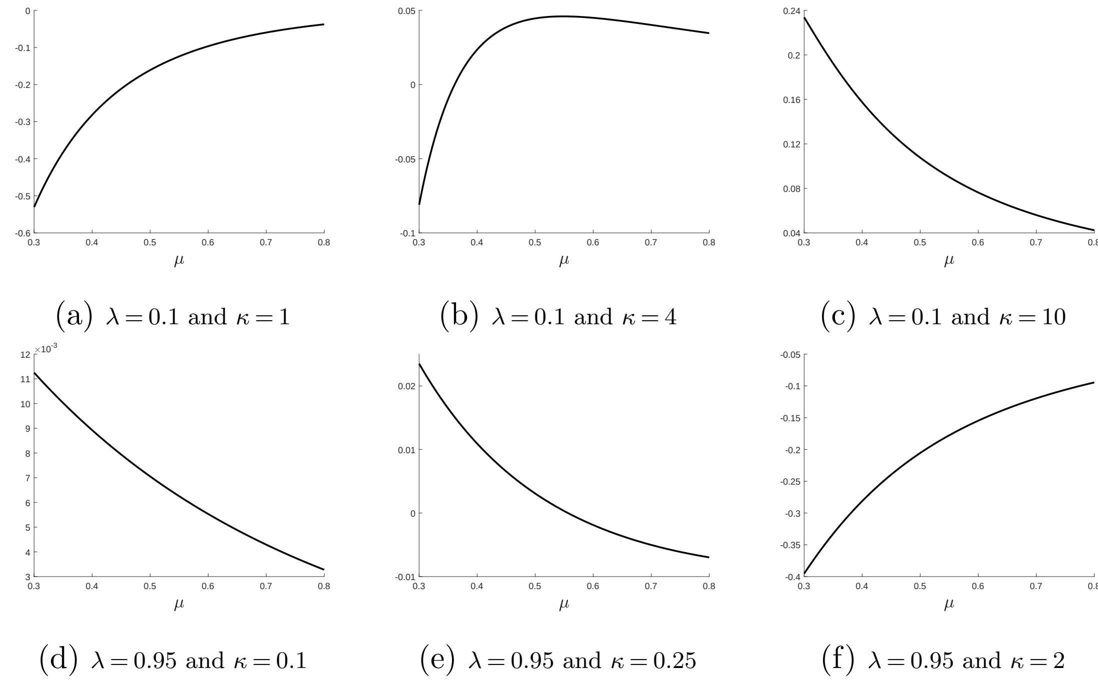
(a) $\lambda = 0.1$ and $\kappa = 2$ (b) $\lambda = 0.1$ and $\kappa = 6$ (c) $\lambda = 0.1$ and $\kappa = 15$



(d) $\lambda = 0.95$ and $\kappa = 0.5$ (e) $\lambda = 0.95$ and $\kappa = 1.2$ (f) $\lambda = 0.95$ and $\kappa = 4$

Figure D3 Second derivative of $K_\lambda(c^\star(\mu))$ for $\omega_\ell = 0.3$ and $\omega_h = 0.8$ and the beta distribution with $a = b = 0.5$

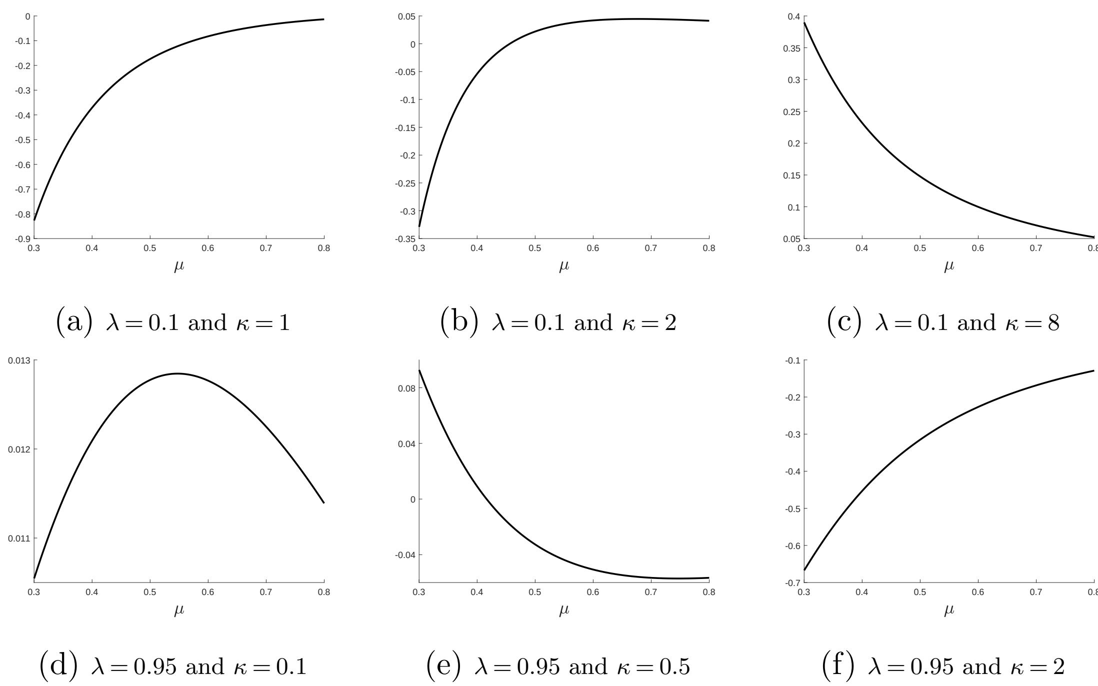
In Figures D3-D5, we provide examples which correspond to no disclosure (positive second derivative as in Figures D3c, D3d,D4c, D4d, D5c, D5d), full disclosure (negative second derivative as in Figures D3a, D3f, D4a, D4f, D5a, D5f), downplaying (first positive then negative second derivative as in Figures D3e, D4e, D5e), and exaggerating (first negative then positive second derivative as in Figures D3b, D4b, D5b) information policies.



(a) $\lambda = 0.1$ and $\kappa = 1$ (b) $\lambda = 0.1$ and $\kappa = 4$ (c) $\lambda = 0.1$ and $\kappa = 10$

(d) $\lambda = 0.95$ and $\kappa = 0.1$ (e) $\lambda = 0.95$ and $\kappa = 0.25$ (f) $\lambda = 0.95$ and $\kappa = 2$

Figure D4 **Second derivative of $K_\lambda(c^\star(\mu))$ for $\omega_\ell = 0.3$ and $\omega_h = 0.8$ and the beta distribution with $a = 1$ and $b = 3$**



(a) $\lambda = 0.1$ and $\kappa = 1$ (b) $\lambda = 0.1$ and $\kappa = 2$ (c) $\lambda = 0.1$ and $\kappa = 8$

(d) $\lambda = 0.95$ and $\kappa = 0.1$ (e) $\lambda = 0.95$ and $\kappa = 0.5$ (f) $\lambda = 0.95$ and $\kappa = 2$

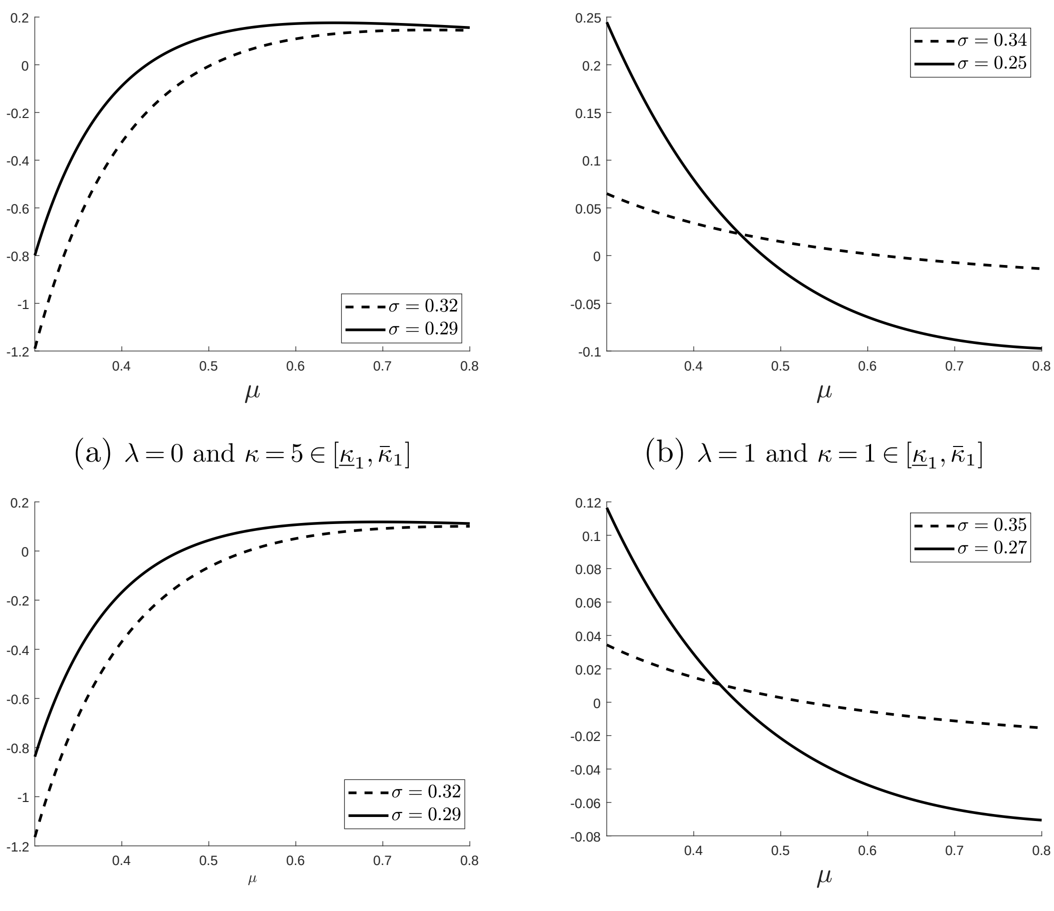
Figure D5 **Second derivative of $K_\lambda(c^\star(\mu))$ for $\omega_\ell = 0.3$ and $\omega_h = 0.8$ and the beta distribution with $a = 2$ and $b = 5$**

D.2 Effect of Economic Inequality

We next analyze with our numerical study how probabilities of downplaying and exaggerating change when the standard deviation of the economic cost distribution increases and its mean remains the same, see Figure D7.

Probability distributions in the beta distribution family are characterized by two parameters a and b. If these parameters are equal to each other, i.e., $a = b = x$ for some x, the mean of the distribution equals to $a/(a+b) = x/(2x) = 1/2$, and its standard deviation equals to $\sqrt{1/(8x+4)}$. Therefore, we parameterize the beta distributions by taking $a = b = x$ and varying x below, so that we only vary the standard deviation of a beta distribution but keep its mean fixed at $1/2$.

Moreover, we identify ranges of σ values for each (κ, λ) pair such that optimal policy is either downplaying or exaggerating. Figure D6 illustrates the second derivative of $K_\lambda(c^\star(\mu))$



(a) $\lambda = 0$ and $\kappa = 5 \in [\underline{\kappa}_1, \bar{\kappa}_1]$

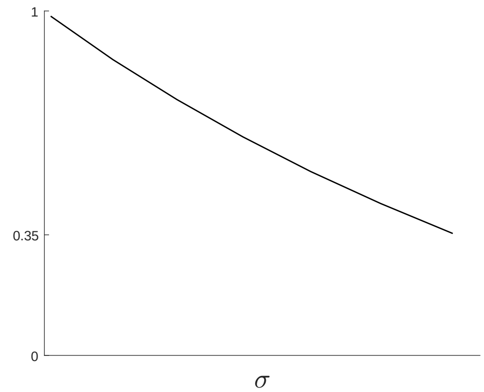
(b) $\lambda = 1$ and $\kappa = 1 \in [\underline{\kappa}_1, \bar{\kappa}_1]$

(c) $\lambda = 0.1$ and $\kappa = 5 \in [\underline{\kappa}_0, \bar{\kappa}_0]$

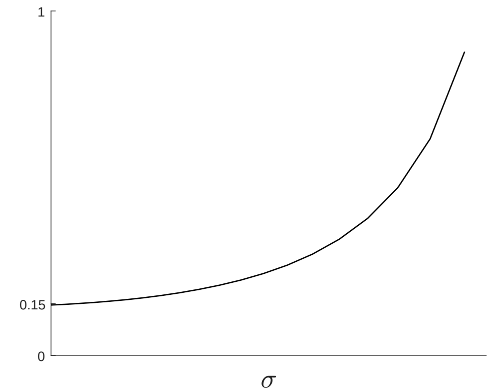
(d) $\lambda = 0.97$ and $\kappa = 1 \in [\underline{\kappa}_1, \bar{\kappa}_1]$

Figure D6 **Second derivative of $K_\lambda(c^\star(\mu))$ for $\omega_\ell = 0.3$, $\omega_h = 0.8$, and the beta distributions with mean $1/2$.**

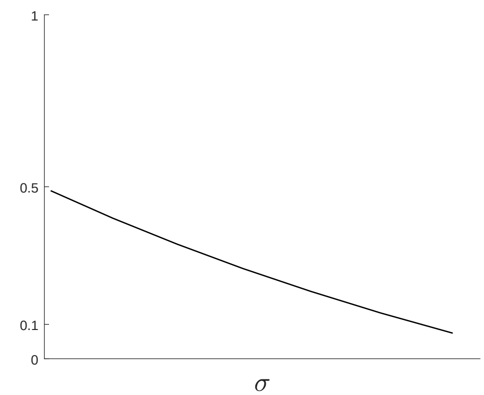
for different (κ, λ) pairs and σ values. Take for example Figure D6a. In that figure, the objective function $K_\lambda(c^\star(\mu))$ is first concave and then convex for $\sigma \in [0.29, 0.32]$. Therefore, we could use $[0.29, 0.32]$ as the range of σ when we analyze how $\pi^\star_{\mathsf{EX}}(m_h \mid \omega_\ell)$ changes in σ for $\lambda = 0$ and $\kappa = 5$.



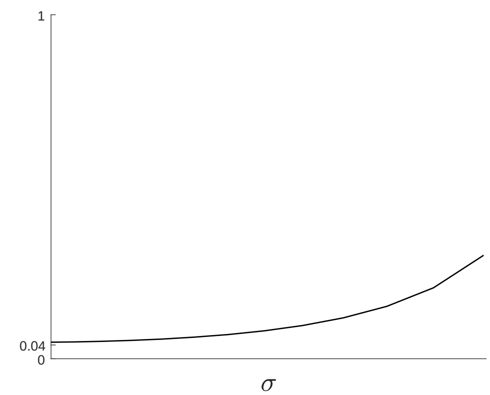
(a) $\pi^{\star}_{\mathsf{EX}}(m_h \,|\, \omega_\ell)$ for $\lambda = 0$ and $\kappa = 5 \in [\underline{\kappa}_0, \bar{\kappa}_0]$



(b) $\pi^{\star}_{\mathsf{DP}}(m_\ell \,|\, \omega_h)$ for $\lambda = 1$ and $\kappa = 1 \in [\underline{\kappa}_1, \bar{\kappa}_1]$



(c) $\pi^{\star}_{\mathsf{EX}}(m_h \,|\, \omega_\ell)$ for $\lambda = 0.1$ and $\kappa = 5 \in [\underline{\kappa}_0, \bar{\kappa}_0]$



(d) $\pi^{\star}_{\mathsf{DP}}(m_\ell \,|\, \omega_h)$ for $\lambda = 0.97$ and $\kappa = 1 \in [\underline{\kappa}_1, \bar{\kappa}_1]$

Figure D7 **Optimal distortion probabilities as a function of σ for $\omega_\ell = 0.3$, $\omega_h = 0.8$, and the beta distributions with mean $1/2$.**

Finally, Figure D7 exemplifies that the main insights in Theorem 2 (as in Figure 2 of our paper) can be extended to distributions other than uniform.

E Extensive Literature Review

In this section of the appendix, we provide an extensive review of the studies which are omitted in the main body of our paper, and elaborate on the differences between them and ours.

Kamenica and Gentzkow (2011) were among the first to establish the Bayesian persuasion framework and developed a general solution approach based on the notion of concavification. In a generic model setting, Kamenica and Gentzkow (2011) consider a single receiver and simplify her response in a *reduced form*, i.e., as an *exogenous* function of the posterior belief (induced by the sender's message and information policy). Subsequently, the sender's payoff is rendered as a function of the posterior belief, and the optimal policy is then identified through the concavification of that sender's reduced-form payoff function.[1]

Much of following research applies this general framework to various contexts by specifically modeling the micro-foundation for the (possibly multiple) receivers' responses and strategic interactions in order to derive the receivers' response functions (rather than assuming them in a reduced form). As such, the specific features of these contexts translate, through the receiver's response functions, to specific curvatures of the sender's payoff function, which in turn determines different types of optimal policies.

The main feature that our model captures is the *externality* that engaging in social interactions creates due to the nature of an epidemic disease. Accordingly, our analysis derives specific properties of the population's equilibrium response in a nonatomic game (see Proposition 1) and subsequently the curvature of the government's payoff function (see Appendix C), based on which our main results (Theorems 1 and 2) hinge.

More precisely, our results stem from the joint consideration of several components: sender's bias (captured by λ), public information policy for multiple receivers (all receivers get the same information), a heterogeneous population (captured by c), and negative externalities (captured by P). Without any of those components, our results will disappear.

Some representative papers applying the information design framework in other (mostly operations) contexts include the following.

- As one of the pioneers in this area, Rayo and Segal (2010) study how a platform/seller should recommend its ads/products based on its proprietary information about the profitability and value-to-consumer of its offerings. In that model, there is a single receiver, while we analyze a multiple receiver setting with heterogeneous economic costs and negative externalities.
- Lingenbrink and Iyer (2019) study how a designer can reveal informative signals on the queue length (in the long-run equilibrium) to influence customers' joining decisions. Their setting is de facto a single receiver one because, each receiver is short-lived and gets a different signal upon arrival depending on the queue length whereas a continuum of receivers are simultaneously present in our setup and get the same information.

[1] Of course, the concavification-based approach is not the unique solution method for Bayesian persuasion. Depending on the nature of problems, some other equivalent methods are available, such as using Rothschild and Stiglitz's (1970) mean-preserving spread (Gentzkow and Kamenica 2016), formulation as linear programs (Lingenbrink and Iyer 2019, Küçükgül et al. 2019).

- Candogan and Drakopoulos (2020) add a Bayesian persuasion game onto a network game (with quadratic payoffs à la Ballester et al. 2006) to study a social network platform's information policy to influence receivers' beliefs about signal quality and subsequently their engagement decisions. The externalities studied in that paper are *positive*. Specifically, receivers derive higher utilities from a certain action if their connections also do the same. However, we analyze a setting with *negative* externalities (specifically driven by the nature of an epidemic setting) where taking a certain action becomes less favorable for a receiver if more receivers do the same.
- Drakopoulos et al. (2020) and Lingenbrink and Iyer (2018) demonstrate that obfuscating inventory and demand information can create stockout risk which motivates buyers to make early purchases. The nature of the strategic interaction among receivers in those two papers is characteristically different from the strategic externalities introduced by the homogeneous mixing nature of an epidemic in our setting. More specifically, Drakopoulos et al. (2020) focus on private information policies under which each receiver gets a different signal/information. On the other hand, we focus on public information policies which represents better how governments inform their populations because all receivers get the same information. This is a fundamentally different setup. Moreover, public information policies are degenerate for Drakopoulos et al. (2020) while the optimal public information policy takes different forms in our setup based on the model parameters (λ and κ). Despite studying public information policies, Lingenbrink and Iyer (2018) focus on a homogeneous population and they numerically show in their appendix that their results are not extended to heterogeneous population. We however study a heterogeneous population consisting of receivers with different economic cost c's. Moreover, we characterize the effect of this inequality on the optimal information policy.

The settings studied by the above examples are essentially *static*; recent development in this area has extended the information design framework to *dynamic* settings, where methodological innovation may be needed. Examples include:

- In a two-arm bandit setting, Papanastasiou et al. (2018) (also see Bimpikis and Papanastasiou 2019, for a review of other works in similar model settings) study how to strategically disclose previous agents' outcomes to induce higher level of exploration (as opposed to exploitation) among self-interested agents (who have the right to choose between the arms).

- In a two-period setting, Alizamir et al. (2020) study a much related topic in the context of WHO's declaration of a Public Health Emergency of International Concern. However, their focus is on the WHO's reputation concern when trading off elicitation of today's response with tomorrow's reputation (so as to elicit proper responses tomorrow) due to the possibility of false alarms or missed alerts.
- In a revenue management setting, Küçükgül et al. (2019) design the information disclosure of previous customers' purchase decisions in order to influence upcoming customers when each customer has a piece of private information about the underlying product. In other words, the sender uses information instrument to engineer agents' social learning.

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