

Appendix A Proofs of Results

Proof of Theorem 1: To prove Theorem 1, we first prove the following statement: Let $f(x)$ and $g(x)$ be two unimodal continuous and differentiable functions on a closed interval $[a, b]$, and x_f^* and x_g^* be the x values that maximize $f(x)$ and $g(x)$, respectively. If $f'(x) \geq g'(x)$ for all x in the domain, then $x_f^* \geq x_g^*$.

Because $f'(x) \geq g'(x)$, we have $f'(x_g^*) \geq g'(x_g^*)$. It means that the value of $f(x)$ can increase by choosing the value of x above x_g^* . Thus, there exists a better feasible solution for optimizing $f(x)$, and that feasible solution is higher than x_g^* . Since $f(x)$ is a unimodal function, we have $x_f^* \geq x_g^*$.

We now proceed to proving the main result in Theorem 1. The derivative of the two profit functions are:

$$\begin{aligned}\pi'_{\text{ET}}(s) &= \frac{\alpha_p \gamma_r}{(1 - \gamma_p s)^2} + \frac{\alpha_r \gamma_r}{(1 - \gamma_r s)^2} + \frac{\beta_p \gamma_p}{(1 - \gamma_p s)^2} - \frac{\beta_p \gamma_r}{(1 - \gamma_p s)^2} - \tau'(s), \quad s \in [0, \frac{1}{2\gamma_r}]; \\ \pi'_{\text{EO}}(s) &= \frac{\alpha_p \gamma_r}{(1 - \gamma_r s)^2} + \frac{\alpha_r \gamma_r}{(1 - \gamma_r s)^2} + \frac{\beta_p \gamma_p}{(1 - \gamma_p s)^2} - \frac{\beta_p \gamma_r}{(1 - \gamma_r s)^2} - \tau'(s), \quad s \in [0, \frac{1}{2\gamma_r}].\end{aligned}$$

It is easy to verify that for $\tau(s) = \frac{ks}{1 - \gamma_r s}$, $\pi_{\text{ET}}(s)$ and $\pi_{\text{EO}}(s)$ are both unimodal functions. Using above equations, $\forall s \in [0, \frac{1}{2\gamma_r}]$ we have

$$\begin{aligned}\pi'_{\text{ET}}(s) - \pi'_{\text{EO}}(s) &= \left(\frac{\alpha_p \gamma_r}{(1 - \gamma_p s)^2} - \frac{\beta_p \gamma_r}{(1 - \gamma_p s)^2} \right) - \left(\frac{\alpha_p \gamma_r}{(1 - \gamma_r s)^2} - \frac{\beta_p \gamma_r}{(1 - \gamma_r s)^2} \right), \\ &= \gamma_r (\beta_p - \alpha_p) \left(\frac{1}{(1 - \gamma_r s)^2} - \frac{1}{(1 - \gamma_p s)^2} \right) \geq 0.\end{aligned}$$

Using the statement proven in the beginning of this proof, we have $s^{\text{ET}} \geq s^{\text{EO}}$. ■

Proof of Proposition 2: For any given $s \in [0, \frac{1}{2\gamma_r}]$, we have

$$\begin{aligned}\pi_{\text{ET}}(s) - \pi_{\text{EO}}(s) &= \left(\frac{\alpha_p \gamma_r s}{1 - \gamma_p s} - \frac{\beta_p \gamma_r s}{1 - \gamma_p s} \right) - \left(\frac{\alpha_p \gamma_r s}{1 - \gamma_r s} - \frac{\beta_p \gamma_r s}{1 - \gamma_r s} \right) \\ &= \gamma_r (\beta_p - \alpha_p) \left(\frac{s}{1 - \gamma_r s} - \frac{s}{1 - \gamma_p s} \right) \geq 0.\end{aligned}$$

Therefore,

$$\pi_{\text{ET}}(s^{\text{EO}}) \geq \pi_{\text{EO}}(s^{\text{EO}}).$$

Since s^{ET} is the optimal amount of learning effort under ET,

$$\pi_{\text{ET}}(s^{\text{ET}}) \geq \pi_{\text{ET}}(s^{\text{EO}}).$$

By transitivity,

$$\pi_{\text{ET}}(s^{\text{ET}}) \geq \pi_{\text{EO}}(s^{\text{EO}}),$$

i.e., $\pi_{\text{ET}}^* \geq \pi_{\text{EO}}^*$. ■

Proof of Proposition 3:

$$\begin{aligned}\phi_r^{ET} &= \frac{1 - c_r^{ET}}{1 - \gamma_r s^{ET}} = \frac{\gamma_r s^{ET}}{1 - \gamma_r s^{ET}}, \\ \phi_r^{EO} &= \frac{1 - c_r^{EO}}{1 - \gamma_r s^{EO}} = \frac{\gamma_r s^{EO}}{1 - \gamma_r s^{EO}}.\end{aligned}$$

Theorem 1 shows

$$s^{ET} \geq s^{EO},$$

therefore,

$$\frac{\gamma_r s^{ET}}{1 - \gamma_r s^{ET}} \geq \frac{\gamma_r s^{EO}}{1 - \gamma_r s^{EO}},$$

i.e., $\phi_r^{ET} \geq \phi_r^{EO}$. ■

Proof of Theorem 2: First Order Condition gives us:

$$\pi'_{ET}(s^{ET}) = \frac{\alpha_p \gamma_r}{(1 - \gamma_p s^{ET})^2} + \frac{\alpha_r \gamma_r}{(1 - \gamma_r s^{ET})^2} + \frac{\beta_p \gamma_p}{(1 - \gamma_p s^{ET})^2} - \frac{\beta_p \gamma_r}{(1 - \gamma_p s^{ET})^2} - \frac{k}{(1 - \gamma_r s^{ET})^2} = 0; \quad (47)$$

$$\pi'_{EO}(s^{EO}) = \frac{\alpha_p \gamma_r}{(1 - \gamma_p s^{EO})^2} + \frac{\alpha_r \gamma_r}{(1 - \gamma_r s^{EO})^2} + \frac{\beta_p \gamma_p}{(1 - \gamma_p s^{EO})^2} - \frac{\beta_p \gamma_r}{(1 - \gamma_p s^{EO})^2} - \frac{k}{(1 - \gamma_r s^{EO})^2} = 0. \quad (48)$$

Multiply (47) by $(1 - \gamma_r s^{ET})^2$ and arrange the equation:

$$\left(\frac{1 - \gamma_r s^{ET}}{1 - \gamma_p s^{ET}}\right)^2 \cdot (\alpha_p \gamma_r + \beta_p \gamma_p - \beta_p \gamma_r) = k - \alpha_r \gamma_r \quad (49)$$

Note that

$$\frac{1 - \gamma_r s^{ET}}{1 - \gamma_p s^{ET}} = 1 - (\gamma_r - \gamma_p) \frac{s^{ET}}{1 - \gamma_p s^{ET}}, \quad (50)$$

Substituting (50) into (49), with some algebra we have

$$\phi_p^{ET} \equiv \frac{\gamma_r s^{ET}}{1 - \gamma_p s^{ET}} = \frac{\gamma_r}{\gamma_r - \gamma_p} \left[1 - \left(\frac{k - \alpha_r \gamma_r}{\alpha_p \gamma_r + \beta_p \gamma_p - \beta_p \gamma_r} \right)^{\frac{1}{2}} \right] \quad (51)$$

Similarly, from (48) we derive the expression for the coverage rate under equal opportunity:

$$\phi_p^{EO} \equiv \frac{\gamma_r s^{EO}}{1 - \gamma_p s^{EO}} = \frac{\gamma_r}{\gamma_r - \gamma_p} \left[\left(\frac{\beta_p \gamma_p}{k - \alpha_r \gamma_r + \beta_p \gamma_r - \alpha_p \gamma_r} \right)^{\frac{1}{2}} - 1 \right] \quad (52)$$

Therefore, we have:

$$\phi_p^{ET} - \phi_p^{EO} = \frac{\gamma_r}{\gamma_r - \gamma_p} \left[2 - \left(\frac{k - \alpha_r \gamma_r}{\alpha_p \gamma_r + \beta_p \gamma_p - \beta_p \gamma_r} \right)^{\frac{1}{2}} - \left(\frac{\beta_p \gamma_p}{k - \alpha_r \gamma_r + \beta_p \gamma_r - \alpha_p \gamma_r} \right)^{\frac{1}{2}} \right] \quad (53)$$

$$= \frac{1}{\gamma_r - \gamma_p} \left[2 - \sigma - \left(\frac{\beta_p \gamma_p}{\sigma^2 \beta_p \gamma_p + (1 - \sigma^2)(\beta_p \gamma_r - \alpha_p \gamma_r)} \right)^{\frac{1}{2}} \right] \quad (54)$$

When $\frac{\alpha_p}{\beta_p} \leq 1 - \frac{1-(2-\sigma)^2\sigma^2}{(1-\sigma^2)(2-\sigma)^2} \cdot \frac{\gamma_p}{\gamma_r}$, we have

$$(\beta_p - \alpha_p)\gamma_r \geq \frac{1 - (2 - \sigma)^2\sigma^2}{(1 - \sigma^2)(2 - \sigma)^2} \cdot \beta_p\gamma_p, \quad (55)$$

$$[1 - (2 - \sigma)^2\sigma^2]\beta_p\gamma_p \leq (1 - \sigma^2)(2 - \sigma)^2(\beta_p\gamma_r - \alpha_p\gamma_r), \quad (56)$$

$$\beta_p\gamma_p \leq (2 - \sigma)^2\sigma^2\beta_p\gamma_p + (2 - \sigma)^2(1 - \sigma^2)(\beta_p\gamma_r - \alpha_p\gamma_r), \quad (57)$$

$$\frac{\beta_p\gamma_p}{\sigma^2\beta_p\gamma_p + (1 - \sigma^2)(\beta_p\gamma_r - \alpha_p\gamma_r)} \leq (2 - \sigma)^2, \quad (58)$$

therefore,

$$\left(\frac{\beta_p\gamma_p}{\sigma^2\beta_p\gamma_p + (1 - \sigma^2)(\beta_p\gamma_r - \alpha_p\gamma_r)}\right)^{\frac{1}{2}} \leq 2 - \sigma, \quad (59)$$

which implies that $\phi_p^{\text{ET}} > \phi_p^{\text{EO}}$, since $\gamma_r - \gamma_p > 0$. ■

Proof of Theorem 3: Let NS_p^{ET} and NF_p^{ET} be the number of successful acceptance and the number of failed acceptance in the protected group under equal treatment, respectively. Then,

$$\begin{aligned} NS_p^{\text{ET}} &= \frac{1 - c^{\text{ET}}}{1 - \gamma_p s^{\text{ET}}}(1 - d_p) = \frac{\gamma_r s^{\text{ET}}}{1 - \gamma_p s^{\text{ET}}}(1 - d_p); \\ NF_p^{\text{ET}} &= \frac{1 - \gamma_p s^{\text{ET}} - c^{\text{ET}}}{1 - \gamma_p s^{\text{ET}}}d_p = \frac{(\gamma_r - \gamma_p)s^{\text{ET}}}{1 - \gamma_p s^{\text{ET}}}d_p. \end{aligned}$$

Therefore, we have

$$\delta_p^{\text{ET}} = \frac{NS_p^{\text{ET}}}{NS_p^{\text{ET}} + NF_p^{\text{ET}}} = \frac{(1 - d_p)\gamma_r}{\gamma_r - \gamma_p d_p}.$$

Similarly, under equal opportunity, we have

$$\begin{aligned} NS_p^{\text{EO}} &= \frac{1 - c_p^{\text{EO}}}{1 - \gamma_p s^{\text{EO}}}(1 - d_p) = \frac{\gamma_r s^{\text{EO}}}{1 - \gamma_r s^{\text{EO}}}(1 - d_p) \\ NF_p^{\text{EO}} &= \frac{1 - \gamma_p s^{\text{EO}} - c_p^{\text{EO}}}{1 - \gamma_p s^{\text{EO}}}d_p = \left(\frac{\gamma_r s^{\text{EO}}}{1 - \gamma_r s^{\text{EO}}} - \frac{\gamma_p s^{\text{EO}}}{1 - \gamma_p s^{\text{EO}}}\right) \cdot d_p \end{aligned}$$

Thus,

$$\delta_p^{\text{EO}} = \frac{NS_p^{\text{EO}}}{NS_p^{\text{ET}} + NF_p^{\text{ET}}} = \frac{(1 - d_p)\gamma_r}{\gamma_r - \frac{1 - \gamma_r s^{\text{EO}}}{1 - \gamma_p s^{\text{EO}}} \cdot \gamma_p d_p} < \frac{(1 - d_p)\gamma_r}{\gamma_r - \gamma_p d_p} = \delta_p^{\text{ET}}. \quad \text{■}$$

Appendix B Proofs of the Results in the Two Other Cases

B.1 Medium Expected Loss

In this section, we show that our results hold for the case of medium expected loss, i.e.,

$$\alpha_p + \alpha_r < \beta_p \leq \alpha_p + \frac{1 - \gamma_p s}{1 - \gamma_r s} \alpha_r.$$

From the analysis in section 3.3, we know that in this case

$$\begin{aligned} c_p^{\text{ET}} = c_r^{\text{ET}} &= 1 - \gamma_r s, & \pi_{\text{ET}}(s) &= \frac{\alpha_p \gamma_r s}{1 - \gamma_p s} + \frac{\alpha_r \gamma_r s}{1 - \gamma_r s} + \frac{\beta_p \gamma_p s}{1 - \gamma_p s} - \frac{\beta_p \gamma_r s}{1 - \gamma_p s} - \tau(s) \\ c_p^{\text{EO}} = 1 - \gamma_p s, c_r^{\text{EO}} &= 1 - \frac{1 - \gamma_r s}{1 - \gamma_p s} \gamma_p s, & \pi_{\text{EO}}(s) &= \frac{\alpha_p \gamma_p s}{1 - \gamma_p s} + \frac{\alpha_r \gamma_p s}{1 - \gamma_p s} - \tau(s); \end{aligned}$$

Learning

The derivative of the two profit functions are:

$$\begin{aligned} \pi'_{\text{ET}}(s) &= \frac{\alpha_p \gamma_r}{(1 - \gamma_p s)^2} + \frac{\alpha_r \gamma_r}{(1 - \gamma_r s)^2} + \frac{\beta_p \gamma_p}{(1 - \gamma_p s)^2} - \frac{\beta_p \gamma_r}{(1 - \gamma_p s)^2} - \tau'(s) \\ \pi'_{\text{EO}}(s) &= \frac{\alpha_p \gamma_p}{(1 - \gamma_p s)^2} + \frac{\alpha_r \gamma_p}{(1 - \gamma_p s)^2} - \tau'(s); \end{aligned}$$

Therefore,

$$\begin{aligned} \pi'_{\text{ET}}(s) - \pi'_{\text{EO}}(s) &= \frac{\alpha_p \gamma_r}{(1 - \gamma_p s)^2} + \frac{\alpha_r \gamma_r}{(1 - \gamma_r s)^2} + \frac{\beta_p \gamma_p}{(1 - \gamma_p s)^2} - \frac{\beta_p \gamma_r}{(1 - \gamma_p s)^2} - \frac{\alpha_p \gamma_p}{(1 - \gamma_p s)^2} - \frac{\alpha_r \gamma_p}{(1 - \gamma_p s)^2} \\ &= \frac{(\beta_p - \alpha_p - \alpha_r) \gamma_p}{(1 - \gamma_p s)^2} + \left[\frac{\alpha_r}{(1 - \gamma_r s)^2} - \frac{\beta_p}{(1 - \gamma_p s)^2} \right] \gamma_r \end{aligned}$$

As

$$\beta_p \leq \alpha_p + \frac{1 - \gamma_p s}{1 - \gamma_r s} \alpha_r,$$

we have

$$\beta_p < \frac{1 - \gamma_p s}{1 - \gamma_r s} \alpha_r < \left(\frac{1 - \gamma_p s}{1 - \gamma_r s} \right)^2 \alpha_r,$$

thus,

$$\frac{\beta_p}{(1 - \gamma_p s)^2} < \frac{\alpha_r}{(1 - \gamma_r s)^2}.$$

Also

$$\beta_p > \alpha_p + \alpha_r.$$

Therefore,

$$\pi'_{\text{ET}}(s) - \pi'_{\text{EO}}(s) > 0.$$

By the statement shown in the proof of Theorem 1, we have $s^{\text{ET}} \geq s^{\text{EO}}$. ■

Impact on the firm

For any given $s \in [0, \frac{1}{2\gamma_r}]$, we have

$$\pi_{\text{ET}}(s) - \pi_{\text{EO}}(s) = \frac{(\beta_p - \alpha_p - \alpha_r) \gamma_p s}{1 - \gamma_p s} + \left[\frac{\alpha_r}{1 - \gamma_r s} - \frac{\beta_p}{1 - \gamma_p s} \right] \gamma_r s \geq 0$$

Therefore,

$$\pi_{\text{ET}}(s^{\text{EO}}) \geq \pi_{\text{EO}}(s^{\text{EO}}).$$

Since s^{ET} is the optimal amount of learning effort under ET,

$$\pi_{\text{ET}}(s^{\text{ET}}) \geq \pi_{\text{ET}}(s^{\text{EO}}).$$

By transitivity,

$$\pi_{\text{ET}}(s^{\text{ET}}) \geq \pi_{\text{EO}}(s^{\text{EO}}),$$

i.e., $\pi_{\text{ET}}^* \geq \pi_{\text{EO}}^*$. ■

Impact on the Regular Group

$$\begin{aligned} \phi_r^{\text{ET}} &= \frac{1 - c_r^{\text{ET}}}{1 - \gamma_r s^{\text{ET}}} = \frac{\gamma_r s^{\text{ET}}}{1 - \gamma_r s^{\text{ET}}}, \\ \phi_r^{\text{EO}} &= \frac{1 - c_r^{\text{EO}}}{1 - \gamma_r s^{\text{EO}}} = \frac{\gamma_p s^{\text{EO}}}{1 - \gamma_r s^{\text{EO}}}. \end{aligned}$$

Since

$$s^{\text{ET}} \geq s^{\text{EO}}, \gamma_r \geq \gamma_p$$

therefore,

$$\frac{\gamma_r s^{\text{ET}}}{1 - \gamma_r s^{\text{ET}}} \geq \frac{\gamma_r s^{\text{EO}}}{1 - \gamma_r s^{\text{EO}}} \geq \frac{\gamma_p s^{\text{EO}}}{1 - \gamma_r s^{\text{EO}}},$$

i.e., $\phi_r^{\text{ET}} \geq \phi_r^{\text{EO}}$.

Since $c_p^{\text{ET}}, c_p^{\text{EO}} \geq 1 - \gamma_r s$, we have $\delta_r^{\text{ET}} = \delta_r^{\text{EO}} = 1$. ■

Impact on the Protected Group

$$\begin{aligned} \phi_p^{\text{ET}} &= \frac{1 - c_p^{\text{ET}}}{1 - \gamma_p s^{\text{ET}}} = \frac{\gamma_r s^{\text{ET}}}{1 - \gamma_p s^{\text{ET}}}, \\ \phi_p^{\text{EO}} &= \frac{1 - c_p^{\text{EO}}}{1 - \gamma_p s^{\text{EO}}} = \frac{\gamma_p s^{\text{EO}}}{1 - \gamma_p s^{\text{EO}}}. \end{aligned}$$

Since

$$s^{\text{ET}} \geq s^{\text{EO}}, \gamma_r \geq \gamma_p,$$

we have $\phi_p^{\text{ET}} \geq \phi_p^{\text{EO}}$. ■

B.2 Large Expected Loss

In this section, we show that our results hold for the case of large expected loss, i.e.,

$$\beta_p > \alpha_p + \frac{1 - \gamma_p s}{1 - \gamma_r s} \alpha_r.$$

From the analysis in section 3.3, we know that in this case

$$\begin{aligned} c_p^{\text{ET}} = c_r^{\text{ET}} &= 1 - \gamma_p s, & \pi_{\text{ET}}(s) &= \frac{\alpha_p \gamma_p s}{1 - \gamma_p s} + \frac{\alpha_r \gamma_p s}{1 - \gamma_r s} - \tau(s); \\ c_p^{\text{EO}} = 1 - \gamma_p s, c_r^{\text{EO}} &= 1 - \frac{1 - \gamma_r s}{1 - \gamma_p s} \gamma_p s, & \pi_{\text{EO}}(s) &= \frac{\alpha_p \gamma_p s}{1 - \gamma_p s} + \frac{\alpha_r \gamma_p s}{1 - \gamma_p s} - \tau(s). \end{aligned}$$

Learning

The derivative of the two profit functions are:

$$\begin{aligned} \pi'_{\text{ET}}(s) &= \frac{\alpha_p \gamma_p}{(1 - \gamma_p s)^2} + \frac{\alpha_r \gamma_p}{(1 - \gamma_r s)^2} - \tau'(s); \\ \pi'_{\text{EO}}(s) &= \frac{\alpha_p \gamma_p}{(1 - \gamma_p s)^2} + \frac{\alpha_r \gamma_p}{(1 - \gamma_p s)^2} - \tau'(s). \end{aligned}$$

Therefore,

$$\pi'_{\text{ET}}(s) - \pi'_{\text{EO}}(s) = \frac{\alpha_r \gamma_p}{(1 - \gamma_r s)^2} - \frac{\alpha_r \gamma_p}{(1 - \gamma_p s)^2} \geq 0.$$

By the statement shown in the proof of Theorem 1, we have $s^{\text{ET}} \geq s^{\text{EO}}$. ■

Impact on the decision maker

For any given $s \in [0, \frac{1}{2\gamma_r}]$, we have

$$\pi_{\text{ET}}(s) - \pi_{\text{EO}}(s) = \frac{\alpha_r \gamma_p s}{1 - \gamma_r s} - \frac{\alpha_r \gamma_p s}{1 - \gamma_p s} \geq 0$$

Therefore,

$$\pi_{\text{ET}}(s^{\text{EO}}) \geq \pi_{\text{EO}}(s^{\text{EO}}).$$

Since s^{ET} is the optimal amount of learning effort under ET,

$$\pi_{\text{ET}}(s^{\text{ET}}) \geq \pi_{\text{ET}}(s^{\text{EO}}).$$

By transitivity,

$$\pi_{\text{ET}}(s^{\text{ET}}) \geq \pi_{\text{EO}}(s^{\text{EO}}),$$

i.e., $\pi_{\text{ET}}^* \geq \pi_{\text{EO}}^*$. ■

Impact on the Regular Group

$$\begin{aligned}\phi_r^{ET} &= \frac{1 - c_r^{ET}}{1 - \gamma_r s^{ET}} = \frac{\gamma_p s^{ET}}{1 - \gamma_r s^{ET}}, \\ \phi_r^{EO} &= \frac{1 - c_r^{EO}}{1 - \gamma_r s^{EO}} = \frac{\gamma_p s^{EO}}{1 - \gamma_p s^{EO}}.\end{aligned}$$

Since

$$s^{ET} \geq s^{EO}, \gamma_r \geq \gamma_p$$

therefore,

$$\frac{\gamma_p s^{ET}}{1 - \gamma_r s^{ET}} \geq \frac{\gamma_p s^{ET}}{1 - \gamma_p s^{ET}} \geq \frac{\gamma_p s^{EO}}{1 - \gamma_p s^{EO}},$$

i.e, $\phi_r^{ET} \geq \phi_r^{EO}$.

As $c_p^{ET}, c_p^{EO} \geq 1 - \gamma_r s$, we have $\delta_r^{ET} = \delta_r^{EO} = 1$. ■

Impact on the Protected Group

$$\begin{aligned}\phi_p^{ET} &= \frac{1 - c_p^{ET}}{1 - \gamma_p s^{ET}} = \frac{\gamma_p s^{ET}}{1 - \gamma_p s^{ET}}, \\ \phi_p^{EO} &= \frac{1 - c_p^{EO}}{1 - \gamma_p s^{EO}} = \frac{\gamma_p s^{EO}}{1 - \gamma_p s^{EO}}.\end{aligned}$$

Since

$$s^{ET} \geq s^{EO}$$

we have $\phi_p^{ET} \geq \phi_p^{EO}$. ■