Appendices for

Negotiations in Competing Supply Chains: The Kalai-Smorodinsky Bargaining Solution

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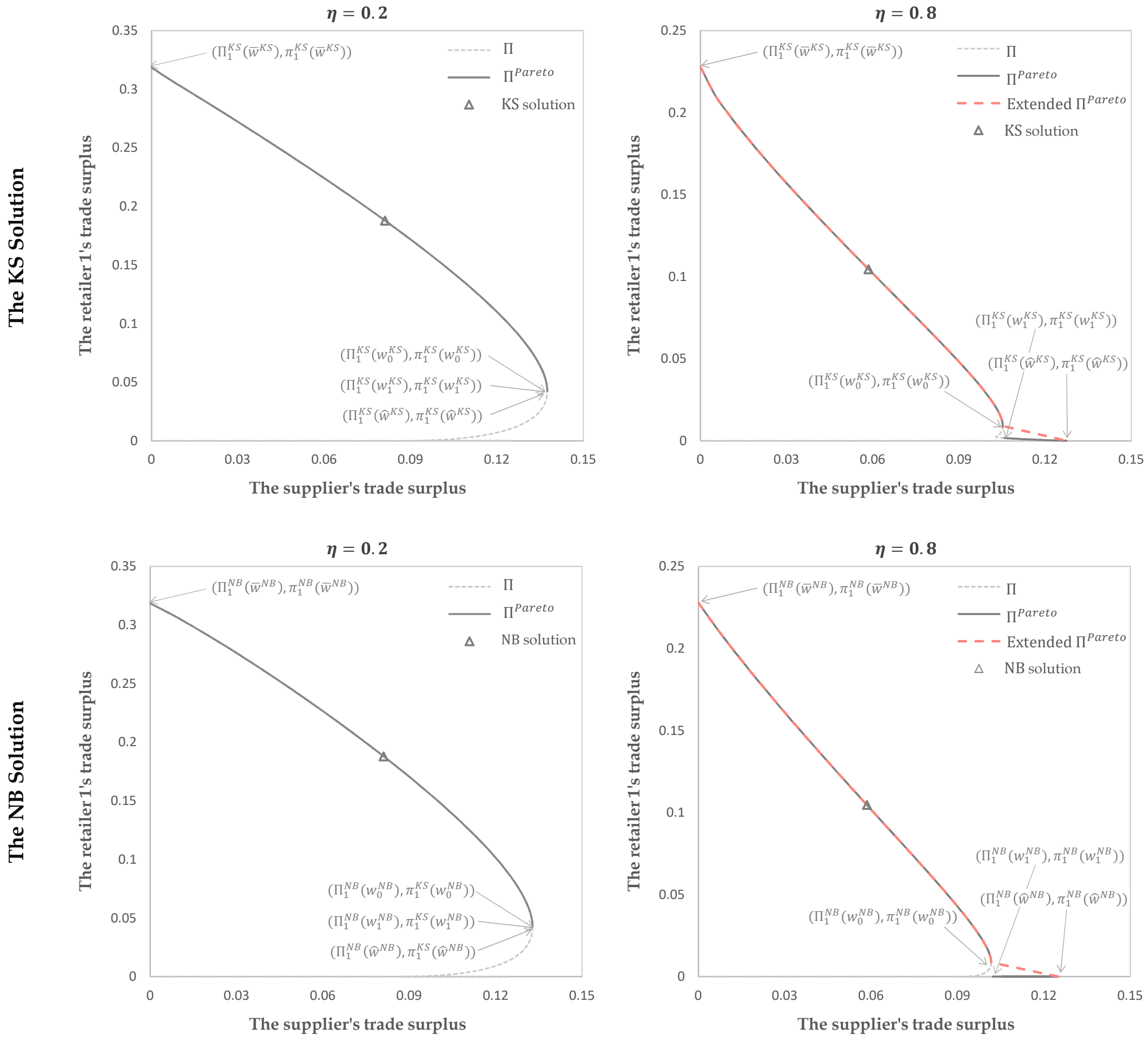
Remark 1 (The Feasible Profit Allocation Set)**.** In general, the feasible set of profit allocation is a curve, which is not a convex set under the wholesale-price contract. To facilitate our analysis with the nonconvex feasible set, the common approach is to allow randomized contracts. That is, the decision becomes a choice of probability distribution over all possible wholesale prices. This randomization convexifies the profit allocation set. When no contingency terms are imposed on contracting (see Feng and Lu 2013b), the convexified region is determined by the original profit allocation curve and the two axis. The Pareto set of the convexified set, where the negotiation outcome lies, is a subset of the original profit allocation curve. Thus, it is sufficient to consider only non-randomized contracts in this case.

When contingency terms are imposed in contract execution, however, the profit allocation set can reveal complex structure. To see that, we consider the first trade in the sequential negotiation over contingent contracts, and refer to an example depicted in Figure 11. The profit allocation set is $\mathbf{\Pi} = \{(\Pi_1^i(w_1), \pi_1^i(w_1)) : \bar{w}^i \leq w_1 \leq a\}$, where $\bar{w}^i = \min\{w_1 : \Pi_1^i(w_1) \geq 0\}$, $i \in \{NB, KS\}$, and the Pareto set is $\mathbf{\Pi}^{Pareto} = \{(\Pi_1^i(w_1), \pi_1^i(w_1)) : \bar{w}^i \leq w_1 \leq w_0^i \text{ and } w_1^i \leq w_1 \leq \hat{w}^i\}$, where $\hat{w}^i = \arg\max\{\Pi_1^i(w_1)\}$, $w_0^i = \min\{w_1 : \mathrm{d}\Pi_1^i(w_1)/\mathrm{d}w_1 \leq 0\}$ and $w_1^i = \mathbb{I}_{\{\hat{w}^i = w_0^i\}} w_0^i + \mathbb{I}_{\{\hat{w}^i > w_0^i\}} \min\{w_1 : w_1 > w_0^i \text{ and } \Pi_1^i(w_1) \geq \Pi_1^i(w_0^i)\}$. When the level of competition is low (i.e., when η is small), indeed $\mathbf{\Pi}^{Pareto} \subset \mathbf{\Pi}$ (or $w_0^i = w_1^i = \hat{w}^i$). For a large η, however, this is not true. In this case, the convexified region for the entire set of $\{\mathbf{\Pi}\}$ would make part of the original Pareto set non-Pareto, inducing discontinuity of the bargaining solution. As we would like to focus on non-randomized strategies, we take an alternative approach. Specifically, we extend the Pareto set $\mathbf{\Pi}^{Pareto}$ by including the segment connecting $(\Pi_1^i(\hat{w}^i), \pi_1^i(\hat{w}^i))$ and $(\Pi_1^i(w_0^i), \pi_1^i(w_0^i))$.



Note. $a = 1$, $b = 1$, $c = 0$.

Figure 11: The profit allocation set of the first trade in the sequential negotiation with contingencies.

A Derivation of the Bargaining Solutions

In this section, we provide the expressions of the firms' profit functions and the derivation of the bargaining solutions. We shall note that under the linear demands, the firms' profits can be expressed as a product of $(a-c)^2/b$ and some function of η. Thus, it is without loss of generality to take $a = 1$, $b = 1$ and $c = 0$ in our analysis. For general values of a, b and c, the corresponding firms' profits would be $(a-c)^2/b\Pi$ and $(a-c)^2/b\pi$, and the corresponding wholesale price would be $c + (a-c)w$, where Π, π and w are computed for $a = 1$, $b = 1$ and $c = 0$.

A.1 One-to-Two Channel

By Lemma 1, we can derive the trade profits for the supplier and the retailers, respectively, as

$$\Pi_i(w_i,w_j) = \sum_{i=1}^{2} w_i q_i^*(\boldsymbol{w}) = \begin{cases} w_j \frac{1-w_j}{2} & 1-w_i \leq \frac{\eta}{2}(1-w_j), \\ \sum_{i=1}^{2} w_i \frac{2(1-w_i)-\eta(1-w_j)}{4-\eta^2} & \frac{\eta}{2}(1-w_j) < 1-w_i < \frac{2}{\eta}(1-w_j), \\ w_i \frac{1-w_i}{2} & 1-w_i \geq \frac{2}{\eta}(1-w_j). \end{cases} \quad (17)$$

$$\pi_i(w_i,w_j) = (p_i^*(\boldsymbol{w})-w_i)q_i^*(\boldsymbol{w}) = \begin{cases} 0 & 1-w_i \leq \frac{\eta}{2}(1-w_j), \\ \frac{(2(1-w_i)-\eta(1-w_j))^2}{(4-\eta^2)^2} & \frac{\eta}{2}(1-w_j) < 1-w_i < \frac{2}{\eta}(1-w_j), \\ \frac{(1-w_i)^2}{4} & 1-w_i \geq \frac{2}{\eta}(1-w_j). \end{cases} \quad (18)$$

The supplier's disagreement point under simultaneous negotiation without contingency is

$$D_i(w_j) = w_j \frac{1-w_j}{2}. \quad (19)$$

Lemma A.1. *In the one-to-two channel without contingency, the maximum profits of the supplier and the retailers are, respectively,*

$$\overline{\Pi}_i(w_j) = \begin{cases} \Pi_i(\frac{2-\eta+2\eta w_j}{4}, w_j) = \frac{2-\eta}{8(2+\eta)} + \frac{1}{2}w_j(1-w_j) & w_j < \frac{1}{2}+\frac{1}{2}\sqrt{\frac{2-\eta}{2+\eta}}, \\ \Pi_i(\frac{1}{2}, w_j) = \frac{1}{8} & w_j > \frac{1}{2}+\frac{1}{2}\sqrt{\frac{2-\eta}{2+\eta}}, \end{cases} \quad (20)$$

$$\overline{\pi}_i(w_j) = \begin{cases} \pi_i(\frac{\eta w_j}{2}, w_j) = \frac{1}{(2+\eta)^2} & w_j < \frac{2}{2+\eta}, \\ \pi_i(1-w_j, w_j) = \frac{w_j^2}{4} & w_j > \frac{2}{2+\eta}. \end{cases} \quad (21)$$

Moreover, the profit allocation $(\Pi_i(w_i,w_j), \pi_i(w_i,w_j))$ is Pareto-dominated for w_i above the maximizer that attains $\overline{\Pi}_i(w_j)$.

Proof. We first note that any feasible (w_i, w_j) should lead to nonnegative trade surpluses (i.e., $\Pi_i(w_i,w_j) - D_i(w_j) \geq 0$ and $\pi_i(w_i,w_j) - d_i(w_j) \geq 0$). Thus, we have $w_i \geq \eta w_j/2$ in the second pieces and $w_i \geq 1-w_j$ in the third pieces in (17) and (18), respectively.

To derive (20), we note that $\Pi_i(w_i,w_j)$ is constant in w_i in the first piece in (17). The second piece is maximized at $w_i^m = (2-\eta+2\eta w_j)/4$ and leads to a maximum value of $\Pi^m = (2-\eta)/(8(2+\eta)) + (1/2)w_j(1-w_j)$. The third piece is maximized at $w_i^r = 1/2$ and leads to a maximum value of $\Pi^r = 1/8$. It is easy to check that $(2-\eta)/4 < w_i^m < (2+\eta)/4$ and $(2-\eta)/4 < (1-w_i^m) < (2+\eta)/4$.

For w_i^r be the maximizer, we must have two cases: *Case (i):* $1-w_i^r > (2/\eta)(1-w_j)$ (or $w_j > w^a \equiv 1-\eta/4$), $1-w_i^m < (2/\eta)(1-w_j)$ (or $w_j < w^b \equiv 1-\eta/(2(2+\eta))$) and $\Pi^m < \Pi^r$ (or $w_j < w^c \equiv (1/2) - \sqrt{(2-\eta)/(2+\eta)}/2$ or $w_j > w^d \equiv (1/2) + \sqrt{(2-\eta)/(2+\eta)}/2$). It is easy to show that $w^a < w^d < w^b$ and the above conditions lead to $w^d < w_j < w^b$. *Case (ii):*

$1 - w_i^r > (2/\eta)(1 - w_j)$ (or $w_j > w^a$) and $1 - w_i^m > (2/\eta)(1 - w_j)$ (or $w_j > w^b$). These give the relation $w_j > w^b$.

For w_i^m to be the maximizer, we can have two cases: *Case (i):* $1 - w_i^r > (2/\eta)(1 - w_j)$ (or $w_j > w^a$), $1 - w_i^m < (2/\eta)(1 - w_j)$ (or $w_j < w^b$) and $\Pi^m > \Pi^r$ (or $w^c < w_j < w^d$). This leads to $w^a < w_j < w^d$. *Case (ii):* $1 - w_i^r < (2/\eta)(1 - w_j)$ (or $w_j < w^a$) and $1 - w_i^m < (2/\eta)(1 - w_j)$ (or $w_j < w^b$). These give the relation $w_j < w^a$.

Combining the above cases, we obtain the expression of (20).

Now we note that the second piece of $\pi_i(w_i, w_j)$ in (18) is decreasing in w_i and is maximized at $w_i = \eta w_j/2 \equiv w_i^p$. The third piece is decreasing in w_i and is maximized at $w_i = 1 - w_j \equiv w_i^q$. For w_i^p to be the maximizer, we must have $1 - w_i^p < (2/\eta)(1 - w_j)$ (or $w_j < 2/(2 + \eta) \equiv w^e$). For w_i^q to be the maximizer, we must have $1 - w_i^q > (2/\eta)(1 - w_j)$ (or $w_j > w^e$). This leads to the expression of (21). □

Lemma A.2. *In the one-to-two channel without contingency, for a given w_j in bargaining unit j, the negotiated wholesale prices in unit i under the NB and KS solutions are*

$$w_i^{NB}(w_j) = \begin{cases} \frac{(2-\eta)(1-\theta)+2\eta w_j}{4} & w_j < \bar{w}_j^a, \\ \frac{3-\theta-\sqrt{(1+\theta)^2-16\theta w_j(1-w_j)}}{4} & w_j > \bar{w}_j^a, \end{cases} \tag{22}$$

$$w_i^{KS}(w_j) = \begin{cases} \frac{\eta w_j}{2} + \frac{2-\eta}{10} & w_j < \frac{2}{2+\eta}, \\ \frac{\eta w_j}{2} + \frac{2-\eta}{2+2(2+\eta)^2 w_j^2} & \frac{2}{2+\eta} < w_j < \bar{w}_j^b, \\ \frac{1}{2} - \frac{-2+\eta+4\sqrt{w_j^3(2+\eta)(4w_j-4(2+\eta)w_j^2(1-w_j)-2+\eta)}}{2(2-\eta+4(2+\eta)w_j^2)} & w_j^b < w_j < \frac{1}{2} + \frac{1}{2}\sqrt{\frac{2-\eta}{2+\eta}}, \\ \frac{1}{2} - \frac{-(2w_j-1)^2+4\sqrt{w_j^3(2w_j-1)^3}}{2(1-4w_j+8w_j^2)} & w_j > \frac{1}{2} + \frac{1}{2}\sqrt{\frac{2-\eta}{2+\eta}}, \end{cases} \tag{23}$$

where $\bar{w}_j^a$ is some value within $[(4 - \eta(1 + \theta))/(4 - \eta^2\theta), (4 + \eta(1 - \theta))/(2(2 + \eta))]$ and $\bar{w}_j^b$ is some value within $[2/(2 + \eta), (4 - \eta)/4]$.

Proof. Applying (17), (18), (19) and $d_i(w_j) = 0$, the Nash product for trade i is

$$\Omega_i(w_i, w_j) = \begin{cases} 0 & 1 - w_i \leq \frac{\eta(1-w_j)}{2}, \\ \left(\frac{(2w_i-\eta w_j)(2(1-w_i)-\eta(1-w_j))}{2(4-\eta^2)}\right)^{1-\theta}\left(\frac{(2(1-w_i)-\eta(1-w_j))^2}{(4-\eta^2)^2}\right)^{\theta} & \frac{\eta(1-w_j)}{2} < 1 - w_i < \frac{2(1-w_j)}{\eta}, \\ \left(\frac{w_i(1-w_i)-w_j(1-w_j)}{2}\right)^{1-\theta}\left(\frac{(1-w_i)^2}{4}\right)^{\theta} & 1 - w_i \geq \frac{2(1-w_j)}{\eta}. \end{cases}$$

Setting $\partial \ln(\Omega)/\partial w_i = 0$ in the second case gives

$$(1 - \theta)\frac{2}{2w_i - \eta w_j} + (1 + \theta)\frac{-2}{2(1 - w_i) - \eta(1 - w_j)} = 0.$$

This gives $w_i^m = ((1-\theta)(2-\eta) + 2\eta w_j)/4$. We shall note that $w_i^m \geq \eta w_j/2$ so that the supplier's surplus is nonnegative under this solution. Setting $\partial \ln(\Omega)/\partial w_i = 0$ in the third case gives

$$(1-\theta)\frac{1-2w_i}{w_i - w_i^2 - w_j + w_j^2} + 2\theta\frac{-1}{1-w_i} = 0.$$

This gives $w_i^r = (3 - \theta - \sqrt{(1+\theta)^2 - 16\theta w_j(1-w_j)})/4 \leq 1/2$. Note that the above expression has two roots within $[0,1]$ and the maximizer of Ω_i should be the smaller root.

For w_i^m be the best response, we can have two cases: *Case (i):* $1 - w_i^m < (2/\eta)(1-w_j)$ (or $w_j < w^a \equiv (4+\eta(1-\theta))/(2(2+\eta)))$ and $1 - w_i^r < (2/\eta)(1-w_j)$ (or $w_j < w^b \equiv (4-\eta(1+\theta))/(4-\eta^2\theta))$. It is easy to show that $w^a > w^b$ and the above conditions lead to $w_j < w^b$. *Case (ii):* $1 - w_i^m < (2/\eta)(1-w_j)$ (or $w_j < w^a$), $1 - w_i^r > (2/\eta)(1-w_j)$ (or $w_j > w^b$) and $\Omega_i(w_i^m, w_j) > \Omega_i(w_i^r, w_j)$. Note that $\Omega_i(w_i^m, w_j) = ((2-\eta)(1-\theta^2))^{1-\theta}(1+\theta)^{2\theta}/(2^{3-\theta}(2+\eta)^{1+\theta})$ and $\Omega_i(w_i^r, w_j) = ((1-\theta)(1+\theta - 8w_j(1-w_j) + \sqrt{(1+\theta)^2 - 16\theta w_j(1-w_j)}))^{1-\theta}(1+\theta+\sqrt{(1+\theta)^2 - 16\theta w_j(1-w_j)})^{2\theta}/2^{4+2\theta}$. It is clear that $\Omega_i(w_i^m, w_j)$ is constant in w_j and $\Omega_i(w_i^r, w_j)$ is increasing in w_j for $w_j > w^b > 1/2$. We also note that $\Omega_i(w_i^r, w^b) < \Omega_i(w_i^m, w_j) < \Omega_i(w_i^r, w^a)$. We deduce that there exists a $\bar{w}_j^a$ such that $\Omega_i(w_i^m, w_j) > [<]\Omega_i(w_i^r, w_j)$ for $w_j < [>]\bar{w}_j^a$. These give the relation $w^b < w_j < \bar{w}_j^a$.

For w_i^r be the best response, we must have two cases: *Case (i):* $1 - w_i^m > (2/\eta)(1-w_j)$ (or $w_j > w^a$) and $1 - w_i^r > (2/\eta)(1-w_j)$ (or $w_j > w^b$). This leads to $w_j > w^a$. *Case (ii):* $1 - w_i^m < (2/\eta)(1-w_j)$ (or $w_j < w^a$), $1 - w_i^r > (2/\eta)(1-w_j)$ (or $w_j > w^b$) and $\Omega_i(w_i^m, w_j) < \Omega_i(w_i^r, w_j)$ (or $w_j > \bar{w}_j^a$). This leads to $\bar{w}_j^a < w_j < w^a$.

Combining the above cases, we obtain the expression of (22).

To derive (23), we substitute the expressions of (17), (18), (20), (21), (19) and $d_i(w_j) = 0$ into (2) and obtain

$$\begin{cases} \frac{(2(1-w_i)-\eta(1-w_j))^2/(4-\eta^2)^2}{(2w_i-\eta w_j)(2(1-w_i)-\eta(1-w_j))/(2(4-\eta^2))} = \frac{1/(2+\eta)^2}{(2-\eta)/(8(2+\eta))} & w_j < \frac{2}{2+\eta}, \\ \frac{(2(1-w_i)-\eta(1-w_j))^2/(4-\eta^2)^2}{(2w_i-\eta w_j)(2(1-w_i)-\eta(1-w_j))/(2(4-\eta^2))} = \frac{w_j^2/4}{(2-\eta)/(8(2+\eta))} & \frac{2}{2+\eta} < w_j < \frac{1}{2} + \frac{1}{2}\sqrt{\frac{2-\eta}{2+\eta}} \textit{ and } \frac{\eta}{2} < \frac{1-w_i}{1-w_j} < \frac{2}{\eta}, \\ \frac{(1-w_i)^2/4}{(w_i(1-w_i)-w_j(1-w_j))/2} = \frac{w_j^2/4}{(2-\eta)/(8(2+\eta))} & \frac{2}{2+\eta} < w_j < \frac{1}{2} + \frac{1}{2}\sqrt{\frac{2-\eta}{2+\eta}} \textit{ and } \frac{1-w_i}{1-w_j} \geq \frac{2}{\eta}, \\ \frac{(1-w_i)^2/4}{(w_i(1-w_i)-w_j(1-w_j))/2} = \frac{w_j^2/4}{1/8-w_j(1-w_j)/2} & w_j > \frac{1}{2} + \frac{1}{2}\sqrt{\frac{2-\eta}{2+\eta}}. \end{cases}$$

We omit the expressions for w_i above the maximizer that attains $\overline{\Pi}_i(w_j)$ as the best response should lead to a Pareto profit allocation. Let $w^c \equiv 2/(2+\eta)$ and $w^d \equiv (1/2) + \sqrt{(2-\eta)/(2+\eta)}/2$.

The first piece gives $w_i^p = (2 - \eta + 5\eta w_j)/10$.

The second piece gives $w_i^q = \eta w_j/2 + (2-\eta)/(2 + 2(2+\eta)^2 w_j^2)$. For w_i^q be the best response, we must have $1 - w_i^q < (2/\eta)(1-w_j)$, i.e.,

$$(4\eta - \eta^3)(1 + (2+\eta)^2 w_j^2)(1 - w_j + 2(2+\eta)w_j^2 - (2+\eta)^2 w_j^3) > 0.$$

Note that the first and second terms are positive and the third term is a cubic function in w_j with the coefficient of w_j^3 being negative. Also, the third term equals $\eta/(2+\eta) > 0$ at $w_j = w^c$, equals $-\eta(48 - 32\eta - 12\eta^2 + 8\eta^3 - \eta^4)/64 < 0$ at $w_j = (4-\eta)/4 \equiv w^e$, and its first-order condition gives $-1 + 4(2+\eta)w_j - 3(2+\eta)^2 w_j^2$, which is negative for w_j within $[w^c, w^e]$. Hence, there exists only one root within $[w^c, w^e]$. Let $\bar{w}_j^b$ denote this root. These give the relation $w^c < w_j < \bar{w}_j^b$.

The third piece gives $w_i^s = 1/2 - (-2+\eta+4\sqrt{w_j^3(2+\eta)(4w_j - 4(2+\eta)w_j^2(1-w_j) - 2 + \eta)})/(2(2-\eta+4(2+\eta)w_j^2))$. Note that the third piece has two roots within $[0,1]$ and the best response should be the smaller root. For w_i^s be the best response, we must have $1 - w_i^s > (2/\eta)(1-w_j)$, i.e.,

$$(1-w_j)(2-\eta)(2-\eta+4(2+\eta)w_j^2)(1-w_j+2(2+\eta)w_j^2 - (2+\eta)^2 w_j^3) < 0.$$

The above inequality implies $w_j > \bar{w}_j^b$. Thus, these yield the relation $\bar{w}_j^b < w_j < w^d$. We shall note that for a $w_j \in [w^e, w^d]$, the profit allocation $(\Pi_i(w_i, w_j), \pi_i(w_i, w_j))$ is Pareto-dominated for some $w_i \in (1/2, (2-\eta+2\eta w_j)/4)$. Because $w_i^s < 1/2$ for any w_j within $[w^e, w^d]$, w_i^s always leads to a Pareto profit allocation.

The fourth piece gives $w_i^t = 1/2 - (-(2w_j-1)^2 + 4\sqrt{w_j^3(2w_j-1)^3})/(2(1-4w_j+8w_j^2)) < 1/2$. Note that the fourth piece has two roots within $[0,1]$ and the best response should be the smaller root. Combining the above cases, we obtain the expression of (23). □

Lemma A.3. *In the one-to-two channel under sequential negotiation, suppose the supplier first negotiates with retailer j.*

i) Without contingency and the negotiated price in unit i is given by (22), the supplier and retailer j's profits are

$$\Pi_j(w_j) \equiv \Pi_j(w_j, w_i^{NB}(w_j)) = \begin{cases} \frac{w_j(1-w_j)}{2} + \frac{(2-\eta)(1-\theta^2)}{8(2+\eta)} & w_j < \bar{w}_j^a, \\ \frac{8\theta w_j(1-w_j) + (1-\theta)(1+\theta+\sqrt{(1+\theta)^2 - 16\theta w_j(1-w_j)})}{16} & w_j > \bar{w}_j^a, \end{cases} \tag{24}$$

$$\pi_j(w_j) \equiv \pi_j(w_j, w_i^{NB}(w_j)) = \begin{cases} \frac{(4+\eta(1-\theta)-2(2+\eta)w_j)^2}{16(2+\eta)^2} & w_j < \bar{w}_j^a, \\ 0 & w_j > \bar{w}_j^a, \end{cases} \tag{25}$$

and the supplier's disagreement point is

$$D_j = \frac{1-\theta^2}{8}. \tag{26}$$

ii) With contingency and the negotiated price in unit i is given by (22), the supplier's profit is

$$\Pi_j(w_j) = \begin{cases} \frac{w_j(1-w_j)}{2} + \frac{(2-\eta)(1-\theta^2)}{8(2+\eta)} & w_j < \bar{w}_j^a, \\ 0 & w_j > \bar{w}_j^a, \end{cases} \tag{27}$$

and retailer j's profit is same as that without contingency, and the supplier's disagreement point is 0.

iii) *Without contingency and the negotiated price in unit i is given by (23), the supplier and retailer j's profits are*

$$\begin{aligned}\Pi_j(w_j) &\equiv \Pi_j(w_j, w_i^{KS}(w_j)) \\ &= \begin{cases} \frac{w_j(1-w_j)}{2} + \frac{2(2-\eta)}{25(2+\eta)} & w_j < \frac{2}{2+\eta}, \\ \frac{w_j(1+(3-\eta^2)w_j+(2+\eta)^2w_j^2(1-w_j)(2+(2+\eta)^2w_j^2))}{2(1+(2+\eta)^2w_j^2)^2} & \frac{2}{2+\eta} < w_j < \bar{w}_j^b, \\ \frac{1}{8} - \frac{(-2+\eta+4\sqrt{w_j^3(2+\eta)(4w_j-4(2+\eta)w_j^2(1-w_j)-2+\eta)})^2}{8(2-\eta+4(2+\eta)w_j^2)^2} & \bar{w}_j^b < w_j < \frac{1}{2}+\frac{1}{2}\sqrt{\frac{2-\eta}{2+\eta}}, \\ \frac{1}{8} - \frac{(-(2w_j-1)^2+4\sqrt{w_j^3(2w_j-1)^3})^2}{8(1-4w_j+8w_j^2)^2} & w_j > \frac{1}{2}+\frac{1}{2}\sqrt{\frac{2-\eta}{2+\eta}}, \end{cases}\end{aligned} \tag{28}$$

$$\pi_j(w_j) \equiv \pi_j(w_j, w_i^{KS}(w_j)) = \begin{cases} \frac{(10+\eta-5(2+\eta)w_j)^2}{100(2+\eta)^2} & w_j < \frac{2}{2+\eta}, \\ \frac{(1-w_j+2(2+\eta)w_j^2-(2+\eta)^2w_j^3)^2}{4(1+(2+\eta)^2w_j^2)^2} & \frac{2}{2+\eta} < w_j < \bar{w}_j^b, \\ 0 & w_j > \bar{w}_j^b, \end{cases} \tag{29}$$

their maximum profits are

$$\overline{\Pi}_j = \Pi_j\big(\frac{1}{2}\big) = \frac{82+9\eta}{200(2+\eta)}, \tag{30}$$

$$\overline{\pi}_j = \pi_j\big(\frac{1}{2} - \frac{1}{10}\sqrt{\frac{50-7\eta}{2+\eta}}\big) = \frac{\big(10-3\eta+\sqrt{(50-7\eta)(2+\eta)}\big)^2}{400(2+\eta)^2}, \tag{31}$$

and the supplier's disagreement point is

$$D_j = \frac{2}{25}. \tag{32}$$

iv) *With contingency and the negotiated price in unit i is given by (23), the supplier's profit is*

$$\Pi_j(w_j) = \begin{cases} \frac{w_j(1-w_j)}{2} + \frac{2(2-\eta)}{25(2+\eta)} & w_j < \frac{2}{2+\eta}, \\ \frac{w_j(1+(3-\eta^2)w_j+(2+\eta)^2w_j^2(1-w_j)(2+(2+\eta)^2w_j^2))}{2(1+(2+\eta)^2w_j^2)^2} & \frac{2}{2+\eta} < w_j < \bar{w}_j^b, \\ 0 & w_j > \bar{w}_j^b, \end{cases} \tag{33}$$

and retailer j's profit is same as that of without contingency. Moreover, the supplier's maximum profit is same as that of without contingency and retailer j's maximum profit is

$$\overline{\pi}_j = \pi_j\big(\frac{1}{2} - \frac{1}{10}\sqrt{\frac{82+9\eta}{2+\eta}}\big) = \frac{(10-3\eta+\sqrt{(82+9\eta)(2+\eta)})^2}{400(2+\eta)^2}, \tag{34}$$

and the supplier's disagreement point is 0.

Moreover, the profit allocation $(\Pi_j(w_j), \pi_j(w_j))$ *is Pareto-dominated for* w_j *above the maximizer that attains* $\overline{\Pi}_j$*.*

Proof. To see part (i), we substitute the expression of (22) into (17) and (18) to derive (24) and (25), respectively. We substitute the negotiated price $w^{NB} = (1-\theta)/2$ in the one-to-one channel (see Proposition 1) into (19) to derive (26). This concludes part (i).

To see part (ii), we note that when the supplier negotiates with retailer j with contingency, her profit becomes zero if retailer j does not order positive quantity (i.e., $w_j > \bar{w}_j^a$). Modifying the second piece in (24) leads to that of (27). We conclude part (ii).

To see part (iii), we substitute the expression of (23) into (17) and (18) to derive (28) and (29), respectively. We substitute the negotiated price $w^{KS} = 1/5$ in the one-to-one channel (see Proposition 1) into (19) to derive (32).

We now derive the supplier and retailer j's maximum profits. We shall note that any feasible w_j should lead to nonnegative trade surpluses (i.e., $\Pi_j(w_j) - D_j \geq 0$ and $\pi_j(w_j) - d_j \geq 0$). Thus, we have $w_j \geq 1/2 - (1/10)\sqrt{(50-7\eta)/(2+\eta)} \equiv w^a$ in the first pieces in (28) and (29), respectively.

To derive (30), we note that the first piece of $\Pi_j(w_j)$ in (28) is maximized at $w_j^m = 1/2$ and leads to a maximum value of $\Pi^m = 1/8 + (2/25)(2-\eta)/(2+\eta)$. The second piece is decreasing in w_j and is thus maximized at $w_j^r = 2/(2+\eta)$. This gives a maximum value of $\Pi^r = \eta/(2+\eta)^2 + (2/25)(2-\eta)/(2+\eta)$. It is easy to check that the maximum values of the third and fourth pieces are both smaller than 1/8. Because $\Pi^m > \max\{1/8, \Pi^r\}$, we can obtain the expression of (30).

To derive (31), we note that the first piece of $\pi_j(w_j)$ in (29) is decreasing in w_j and is maximized at $w_j^p = w^a$. This leads to a maximum value of $\pi^p = (10 - 3\eta + \sqrt{(50-7\eta)(2+\eta)})^2/(400(2+\eta)^2)$. The second piece is convex in w_j and thus the maximum value is $\pi^q = \max\{\pi_j(2/(2+\eta)), \pi_j(\bar{w}_j^b)\}$. It is easy to check that $\pi^p > \pi^q$. We conclude part (iii).

To see part (iv), we note that when the supplier negotiates with retailer j with contingency, her profit becomes zero if retailer j does not order positive quantity (i.e., $w_j > \bar{w}_j^b$), which leads to the expression of (33). We note that any feasible w_j should lead to nonnegative trade surpluses and thus we have $w_j \geq 1/2 - (1/10)\sqrt{(82+9\eta)/(2+\eta)} \equiv w^c$. It is then easy to see that (29) is maximized at w^c, which gives the expression of (34). We conclude part (iv). □

A.2 Two-to-One Channel

By Lemma 1, we can derive the trade profits for the suppliers and the retailer, respectively, as

$$\Pi_i(w_i,w_j) = w_i q_i^*(\boldsymbol{w}) = \begin{cases} 0 & 1-w_i \leq \eta(1-w_j), \\ w_i \frac{(1-w_i)-\eta(1-w_j)}{2(1-\eta^2)} & \eta(1-w_j) < 1-w_i < \frac{1}{\eta}(1-w_j), \\ w_i \frac{1-w_i}{2} & 1-w_i \geq \frac{1}{\eta}(1-w_j). \end{cases} \tag{35}$$

$$\pi_i(w_i,w_j) = \sum_{i=1}^{2}(p_i^*(\boldsymbol{w})-w_i)q_i^*(\boldsymbol{w}) = \begin{cases} \frac{(1-w_j)^2}{4} & 1-w_i \leq \eta(1-w_j), \\ \frac{\sum_{i=1}^{2}(1-w_i)((1-w_i)-\eta(1-w_j))}{4(1-\eta^2)} & \eta(1-w_j) < 1-w_i < \frac{1}{\eta}(1-w_j), \\ \frac{(1-w_i)^2}{4} & 1-w_i \geq \frac{1}{\eta}(1-w_j). \end{cases} \tag{36}$$

The retailer's disagreement point under simultaneous negotiation without contingency is

$$d_i(w_j) = \frac{(1-w_j)^2}{4}. \tag{37}$$

Lemma A.4. *In the two-to-one channel without contingency, the maximum profits of the suppliers and the retailer are, respectively,*

$$\overline{\Pi}_i(w_j) = \begin{cases} \Pi_i(\frac{1-\eta+\eta w_j}{2}, w_j) = \frac{(1-\eta(1-w_j))^2}{8(1-\eta^2)} & w_j < \frac{(2+\eta)(1-\eta)}{2-\eta^2}, \\ \Pi_i(\frac{w_j-1+\eta}{\eta}, w_j) = \frac{(1-w_j)(w_j-1+\eta)}{2\eta^2} & \frac{(2+\eta)(1-\eta)}{2-\eta^2} < w_j < \frac{2-\eta}{2}, \\ \Pi_i(\frac{1}{2}, w_j) = \frac{1}{8} & w_j > \frac{2-\eta}{2}, \end{cases} \tag{38}$$

$$\overline{\pi}_i(w_j) = \begin{cases} \pi_i(0,w_j) = \frac{(1-w_j)^2-2\eta(1-w_j)+1}{4(1-\eta^2)} & w_j < 1-\eta, \\ \pi_i(0,w_j) = \frac{1}{4} & w_j > 1-\eta. \end{cases} \tag{39}$$

Moreover, the profit allocation $(\Pi_i(w_i,w_j), \pi_i(w_i,w_j))$ *is Pareto-dominated for* w_i *above the maximizer that attains* $\overline{\Pi}_i(w_j)$*.*

Proof. We first note that any feasible (w_i, w_j) should lead to nonnegative trade surpluses (i.e., $\Pi_i(w_i,w_j) - D_i(w_j) \geq 0$ and $\pi_i(w_i,w_j) - d_i(w_j) \geq 0$). Thus, we have $w_i \geq 0$ in the second and third pieces in (35) and (36), respectively.

To derive (38), we note that $\Pi_i(w_i,w_j)$ is constant in w_i in the first piece in (35). The second piece is maximized at $w_i^m = (1-\eta+\eta w_j)/2$ and leads to a maximum value of $\Pi^m = (1-\eta(1-w_j))^2/(8(1-\eta^2))$. The third piece is maximized at $w_i^r = 1/2$ and leads to a maximum value of $\Pi^r = 1/8$.

For w_i^r be the maximizer, we must have two cases: *Case (i):* $1-w_i^r > (1/\eta)(1-w_j)$ (or $w_j > w^a \equiv 1-\eta/2$), $1-w_i^m < (1/\eta)(1-w_j)$ (or $w_j < w^b \equiv (2+\eta)(1-\eta)/(2-\eta^2)$) and $\Pi^m < \Pi^r$ (or $w_j > w^c \equiv 1-(1/\eta)(1+\sqrt{1-\eta^2})$ and $w_j < w^d \equiv 1-(1/\eta)(1-\sqrt{1-\eta^2})$). It is easy to show that $w^b < w^d < w^a$ and $w^c < 0$ and thus no feasible w_i satisfies the above conditions. *Case (ii):* $1 - w_i^r > (1/\eta)(1 - w_j)$ (or $w_j > w^a$) and $1 - w_i^m > (1/\eta)(1 - w_j)$ (or $w_j > w^b$). These give the relation $w_j > w^a$.

For w_i^m to be the maximizer, we can have two cases: *Case (i):* $1 - w_i^r > (1/\eta)(1 - w_j)$ (or $w_j > w^a$), $1 - w_i^m < (1/\eta)(1 - w_j)$ (or $w_j < w^b$) and $\Pi^m > \Pi^r$ (or $w_j < w^c$ or $w_j > w^d$). It is easy to check that no feasible w_i satisfies the above conditions. *Case (ii):* $1 - w_i^r < (1/\eta)(1 - w_j)$ (or $w_j < w^a$) and $1 - w_i^m < (1/\eta)(1 - w_j)$ (or $w_j < w^b$). These give the relation $w_j < w^b$.

For $w^a < w_j < w^b$, neither w_i^r nor w_i^m is attainable and thus the maximizer should be $w_i^o = 1 - (1/\eta)(1 - w_j)$, which leads to a maximum value of $\Pi^o = (1 - w_j)(w_j - 1 + \eta)/(2\eta^2)$.

Combining the above cases, we obtain the expression of (38).

Now we note that the second and third pieces of $\pi_i(w_i, w_j)$ in (36) are both decreasing in w_i. Thus, $\pi_i(w_i, w_j)$ is maximized at $w_i = 0$ and leads to the expression of (39). □

Lemma A.5. *In the two-to-one channel without contingency, for a given w_j in bargaining unit j, the negotiated wholesale prices in unit i under the NB and KS solutions are*

$$w_i^{NB}(w_j) = \begin{cases} \frac{(1-\theta)(1-\eta+\eta w_j)}{2} & w_j < 1 - \frac{\eta(1+\theta)}{2-\eta^2(1-\theta)}, \\ \frac{\eta-1+w_j}{\eta} & 1 - \frac{\eta(1+\theta)}{2-\eta^2(1-\theta)} < w_j < 1 - \frac{\eta(1+\theta)-\eta^3(1-\theta)}{2(1-\eta^2(1-\theta))}, \\ w_i^{\alpha}(w_j) & w_j > 1 - \frac{\eta(1+\theta)-\eta^3(1-\theta)}{2(1-\eta^2(1-\theta))}, \end{cases} \tag{40}$$

$$w_i^{KS}(w_j) = \begin{cases} \frac{1-\eta+\eta w_j}{5} & w_j < 1 - \eta, \\ \frac{(1-\eta+\eta w_j)^3}{(1-\eta)^2+2(4+\eta-5\eta^2)w_j-(4-5\eta^2)w_j^2} & 1 - \eta < w_j < \bar{w}_j^c, \\ \frac{(1-\eta)^2+2(2+\eta-3\eta^2)w_j-(2-3\eta^2)w_j^2-\sqrt{\Delta_1}}{(1-\eta)^2+2(4+\eta-5\eta^2)w_j-(4-5\eta^2)w_j^2} & \bar{w}_j^c < w_j < \frac{(2+\eta)(1-\eta)}{2-\eta^2}, \\ 1 + \frac{\eta^2(2w_j-w_j^2)+\sqrt{\Delta_2}}{2(1-\eta-(2-\eta+2\eta^2)w_j+(1+\eta^2)w_j^2)} & \frac{(2+\eta)(1-\eta)}{2-\eta^2} < w_j < \frac{2-\eta}{2}, \\ \frac{1+4w_j-2w_j^2-\sqrt{1+6w_j-3w_j^2}}{1+8w_j-4w_j^2} & w_j > \frac{2-\eta}{2}, \end{cases} \tag{41}$$

where $w_i^{\alpha}(w_j)$ is the unique root of (42) within the range $[0, 1/2]$, $\Delta_1 = (1-\eta)^4+2(1-\eta)^3(3+7\eta)w_j-(1-\eta)^2(3-38\eta-67\eta^2)w_j^2-4\eta(17-14\eta-28\eta^2+25\eta^3)w_j^3+\eta(40-54\eta-60\eta^2+75\eta^3)w_j^4-2\eta(4-12\eta-6\eta^2+15\eta^3)w_j^5-\eta^2(4-5\eta^2)w_j^6$, $\Delta_2 = 4(1-\eta)^2-8(3-5\eta+3\eta^2-\eta^3)w_j+4(15-20\eta+15\eta^2-7\eta^3+\eta^4)w_j^2-4(20-20\eta+20\eta^2-9\eta^3+\eta^4)w_j^3+(60-40\eta+60\eta^2-20\eta^3+\eta^4)w_j^4-4(6-2\eta+6\eta^2-\eta^3)w_j^5+4(1+\eta^2)w_j^6$ *and $\bar{w}_j^c$ is some value within* $[1 - \eta, (2 + \eta)(1 - \eta)/(2 - \eta^2)]$.

Proof. Applying (35), (36), (37) and $D_i(w_j) = 0$, the Nash product for trade i is

$$\Omega_i(w_i, w_j) = \begin{cases} 0 & 1 - w_i \leq \eta(1 - w_j), \\ (\frac{w_i(1-w_i-\eta(1-w_j))}{2(1-\eta^2)})^{1-\theta}(\frac{(1-w_i-\eta(1-w_j))^2}{4(1-\eta^2)})^{\theta} & \eta(1 - w_j) < 1 - w_i < \frac{1-w_j}{\eta}, \\ (\frac{w_i(1-w_i)}{2})^{1-\theta}(\frac{(1-w_i)^2-(1-w_j)^2}{4})^{\theta} & 1 - w_i \geq \frac{1-w_j}{\eta}. \end{cases}$$

Setting $\partial \ln(\Omega)/\partial w_i = 0$ in the second case gives

$$(1-\theta)\frac{1}{w_i} + (1+\theta)\frac{-1}{(1-w_i) - \eta(1-w_j)} = 0.$$

This gives $w_i^m = (1-\theta)(1-\eta+\eta w_j)/2$. Setting $\partial \ln(\Omega)/\partial w_i = 0$ in the third case gives

$$(1-\theta)\frac{1-2w_i}{w_i(1-w_i)} + \theta\frac{-2(1-w_i)}{(1-w_i)^2 - (1-w_j)^2} = 0.$$

Rearranging the terms, we obtain

$$\Phi(w_i, w_j) \equiv -2w_i^3 + (5-\theta)w_i^2 - 2\big(1 + (1-\theta)(2w_j - w_j^2)\big)w_i + (1-\theta)(2w_j - w_j^2) = 0. \tag{42}$$

We note that $\Phi(w_i, w_j)$ is a cubic function in w_i with the coefficient of w_i^3 being negative. Since $\Phi(0, w_j) = (1-\theta)(2w_j - w_j^2) > 0$ and $\Phi(1/2, w_j) = -\theta/4 < 0$, there exists at least one root in $[0, 1/2]$ that is a local maximizer of $\Omega_i(w_i, w_j)$. Moreover,

$$\frac{\partial}{\partial w_i}\Phi(w_i, w_j) = -6\Big(w_i - \frac{5-\theta}{6}\Big)^2 + 2(1-\theta)(1-w_j)^2 + \frac{(1+\theta)^2}{6}.$$

The above expression is symmetric with respect to the vertical line $w_i = (5-\theta)/6 > 1/2$, and $\partial\Phi(w_i, w_j)/\partial w_i\big|_{w_i=0} = 2(1-\theta)(1-w_j)^2 - 2(2-\theta) < 0$. Thus, the sign of $\partial\Phi(w_i, w_j)/\partial w_i$ changes at most once and the change is from negative to positive within $w_i \in [0, 1/2]$. This suggests that Φ has a unique root within $[0, 1/2]$ that maximizes the Nash product. Let $w_i^\alpha(w_j)$ denote this root.

For w_i^m be the best response, we can have two cases: *Case (i):* $1 - w_i^m < (1/\eta)(1-w_j)$ (or $w_j < w^a \equiv 1 - \eta(1+\theta)/(2 - \eta^2(1-\theta)))$ and $1 - w_i^\alpha(w_j) < (1/\eta)(1-w_j)$ (or $\Phi(1-(1/\eta)(1-w_j), w_j) > 0$). It is easy to show that $\Phi(1-(1/\eta)(1-w_j), w_j) > 0$ gives $w_j < w^b \equiv 1 - (\eta(1+\theta) - \eta^3(1-\theta))/(2(1-\eta^2(1-\theta)))$. These give the relation $w_j < w^a$. *Case (ii):* $1 - w_i^m < (1/\eta)(1-w_j)$ (or $w_j < w^a$), $1 - w_i^\alpha(w_j) > (1/\eta)(1-w_j)$ (or $w_j > w^b$) and $\Omega_i(w_i^m, w_j) > \Omega_i(w_i^\alpha(w_j), w_j)$. Because $w^a < w^b$, no feasible w_i satisfies the first two conditions.

For $w_i^\alpha(w_j)$ be the best response, we must have two cases: *Case (i):* $1 - w_i^m > (1/\eta)(1-w_j)$ (or $w_j > w^a$) and $1 - w_i^\alpha(w_j) > (1/\eta)(1-w_j)$ (or $w_j > w^b$). This leads to $w_j > w^b$. *Case (ii):* $1 - w_i^m < (1/\eta)(1-w_j)$ (or $w_j < w^a$), $1 - w_i^\alpha(w_j) > (1/\eta)(1-w_j)$ (or $w_j > w^b$) and $\Omega_i(w_i^m, w_j) < \Omega_i(w_i^\alpha(w_j), w_j)$. It is clear that no feasible w_i satisfies the conditions.

For $w^a < w_j < w^b$, neither $w_i^{\alpha}(w_j)$ nor w_i^m is attainable and thus the maximizer should be $w_i^o = 1 - (1/\eta)(1 - w_j)$.

Combining the above cases, we obtain the expression of (40).

To derive (41), we substitute the expressions of (35), (36), (38), (39), (37) and $D_i(w_j) = 0$ into (2) and obtain

$$\begin{cases} \frac{(1-w_i-\eta(1-w_j))^2/(4(1-\eta^2))}{w_i(1-w_i-\eta(1-w_j))/(2(1-\eta^2))} = \frac{(1-\eta(1-w_j))^2/(4(1-\eta^2))}{(1-\eta(1-w_j))^2/(8(1-\eta^2))} & w_j < 1-\eta, \\ \frac{(1-w_i-\eta(1-w_j))^2/(4(1-\eta^2))}{w_i(1-w_i-\eta(1-w_j))/(2(1-\eta^2))} = \frac{1/4-(1-w_j)^2/4}{(1-\eta(1-w_j))^2/(8(1-\eta^2))} & 1-\eta < w_j < \frac{(2+\eta)(1-\eta)}{2-\eta^2} \ \textit{and}\ \eta < \frac{1-w_i}{1-w_j} < \frac{1}{\eta}, \\ \frac{(1-w_i)^2/4-(1-w_j)^2/4}{w_i(1-w_i)/2} = \frac{1/4-(1-w_j)^2/4}{(1-\eta(1-w_j))^2/(8(1-\eta^2))} & 1-\eta < w_j < \frac{(2+\eta)(1-\eta)}{2-\eta^2} \ \textit{and}\ \frac{1-w_i}{1-w_j} > \frac{1}{\eta}, \\ \frac{(1-w_i)^2/4-(1-w_j)^2/4}{w_i(1-w_i)/2} = \frac{1/4-(1-w_j)^2/4}{(1-w_j)(w_j-1+\eta)/(2\eta^2)} & \frac{(2+\eta)(1-\eta)}{2-\eta^2} < w_j < \frac{2-\eta}{2}, \\ \frac{(1-w_i)^2/4-(1-w_j)^2/4}{w_i(1-w_i)/2} = \frac{1/4-(1-w_j)^2/4}{1/8} & w_j > \frac{2-\eta}{2}. \end{cases}$$

We omit the expressions for w_i above the maximizer that attains $\overline{\Pi}_i(w_j)$ as the best response should lead to a Pareto profit allocation. Let $w^c \equiv 1 - \eta$ and $w^d \equiv (2+\eta)(1-\eta)/(2-\eta^2)$.

The first piece gives $w_i^p = (1 - \eta + \eta w_j)/5$.

The second piece gives $w_i^q = (1-\eta+\eta w_j)^3/\big((1-\eta)^2 + 2(4+\eta-5\eta^2)w_j - (4-5\eta^2)w_j^2\big)$. For w_i^q be the best response, we must have $1 - w_i^q < (1/\eta)(1 - w_j)$, i.e.,

$$\Phi(w_j) = (1-\eta)^2 - (3\eta^2 + 4\eta - 7)w_j + (3\eta^2 + 2\eta - 12)w_j^2 + (4 - \eta^2)w_j^3 > 0.$$

We note that $\Phi(w_j)$ is a cubic function in w_j with the coefficient of w_j^3 being positive. Also, $\Phi(w^c) = \eta(1-\eta^2)^2 > 0$, $\Phi(w^d) = -4\eta(3-\eta^2)(1-\eta^2)^2/(2-\eta^2)^3 < 0$ and $\mathrm{d}\Phi(w_j)/\mathrm{d}w_j = 3(4 - \eta^2)(1-w_j)^2 - 4\eta(1-w_j) - 5 < 0$ for w_j within $[w^c, w^d]$. Hence, there exists only one root within $[w^c, w^d]$. Let $\bar{w}_j^c$ denote this root. These give the relation $w^c < w_j < \bar{w}_j^c$.

The third piece gives $w_i^s = \big((1-\eta)^2 + 2(2+\eta-3\eta^2)w_j - (2-3\eta^2)w_j^2 - \sqrt{\Delta_1}\big)/\big((1-\eta)^2 + 2(4+\eta-5\eta^2)w_j - (4-5\eta^2)w_j^2\big)$, where $\Delta_1 = (1-\eta)^4 + 2(1-\eta)^3(3+7\eta)w_j - (1-\eta)^2(3-38\eta-67\eta^2)w_j^2 - 4\eta(17-14\eta-28\eta^2+25\eta^3)w_j^3 + \eta(40-54\eta-60\eta^2+75\eta^3)w_j^4 - 2\eta(4-12\eta-6\eta^2+15\eta^3)w_j^5 - \eta^2(4-5\eta^2)w_j^6$. Note that the third piece has two roots and the best response should be the smaller root. For w_i^s be the best response, we must have $1 - w_i^s > (1/\eta)(1 - w_j)$. This implies $\Phi(w_j) < 0$ (or $w_j > \bar{w}_j^c$) and thus $\bar{w}_j^c < w_j < w^d$.

The fourth piece gives $w_i^t = 1 + \big(\eta^2(2w_j - w_j^2) + \sqrt{\Delta_2}\big)/\big(2(1-\eta-(2-\eta+2\eta^2)w_j + (1+\eta^2)w_j^2)\big)$, where $\Delta_2 = 4(1-\eta)^2 - 8(3-5\eta+3\eta^2-\eta^3)w_j + 4(15-20\eta+15\eta^2-7\eta^3+\eta^4)w_j^2 - 4(20-20\eta+20\eta^2-9\eta^3+\eta^4)w_j^3 + (60-40\eta+60\eta^2-20\eta^3+\eta^4)w_j^4 - 4(6-2\eta+6\eta^2-\eta^3)w_j^5 + 4(1+\eta^2)w_j^6$. Note that the fourth piece has two roots and the best response should be the smaller root.

The fifth piece gives $w_i^u = (1 + 4w_j - 2w_j^2 - \sqrt{1 + 6w_j - 3w_j^2})/(1 + 8w_j - 4w_j^2)$. Note that the fifth piece has two roots and the best response should be the smaller root.

Combining the above cases, we obtain the expression of (41). $\square$

Lemma A.6. *In the two-to-one channel under sequential negotiation, suppose the retailer first negotiates with supplier j.*

i) Without contingency and the negotiated price in unit i is given by (40), the supplier j and retailer's profits are

$$\Pi_j(w_j) \equiv \Pi_j(w_j, w_i^{NB}(w_j)) = \begin{cases} \frac{w_j((2-\eta^2(1-\theta))(1-w_j)-\eta(1+\theta))}{4(1-\eta^2)} & w_j < 1 - \frac{\eta(1+\theta)}{2-\eta^2(1-\theta)}, \\ 0 & w_j > 1 - \frac{\eta(1+\theta)}{2-\eta^2(1-\theta)}, \end{cases} \tag{43}$$

$$\begin{aligned} \pi_j(w_j) &\equiv \pi_j(w_j, w_i^{NB}(w_j)) \\ &= \begin{cases} \frac{(1-w_j)^2}{4} + \frac{(1+\theta)^2(1-\eta+\eta w_j)^2}{16(1-\eta^2)} & w_j < 1 - \frac{\eta(1+\theta)}{2-\eta^2(1-\theta)}, \\ \frac{(1-w_j)^2}{4\eta^2} & 1 - \frac{\eta(1+\theta)}{2-\eta^2(1-\theta)} < w_j < 1 - \frac{\eta(1+\theta)-\eta^3(1-\theta)}{2(1-\eta^2(1-\theta))}, \\ \frac{(1-w_i^{\alpha}(w_j))^2}{4} & w_j > 1 - \frac{\eta(1+\theta)-\eta^3(1-\theta)}{2(1-\eta^2(1-\theta))}, \end{cases} \end{aligned} \tag{44}$$

and the retailer's disagreement point is

$$d_j = \frac{(1+\theta)^2}{16}. \tag{45}$$

ii) With contingency and the negotiated price in unit i is given by (40), the supplier j's profit is same as that without contingency and retailer's profit is

$$\pi_j(w_j) = \begin{cases} \frac{(1-w_j)^2}{4} + \frac{(1+\theta)^2(1-\eta+\eta w_j)^2}{16(1-\eta^2)} & w_j < 1 - \frac{\eta(1+\theta)}{2-\eta^2(1-\theta)}, \\ 0 & w_j > 1 - \frac{\eta(1+\theta)}{2-\eta^2(1-\theta)}, \end{cases} \tag{46}$$

and the retailer's disagreement point is 0.

iii) Without contingency and the negotiated price in unit i is given by (41), the supplier j and retailer's profits are

$$\begin{aligned} \Pi_j(w_j) &\equiv \Pi_j(w_j, w_i^{KS}(w_j)) \\ &= \begin{cases} \frac{w_j((5-\eta^2)(1-w_j)-4\eta)}{10(1-\eta^2)} & w_j < 1 - \eta, \\ \frac{w_j(1-\eta-w_j)}{2(1-\eta^2)} + \frac{\eta w_j(1-\eta+\eta w_j)^3}{2(1-\eta^2)\left[(1-\eta)^2+2(4+\eta-5\eta^2)w_j-(4-5\eta^2)w_j^2\right]} & 1 - \eta < w_j < \bar{w}_j^c, \\ 0 & w_j > \bar{w}_j^c, \end{cases} \end{aligned} \tag{47}$$

$$\pi_j(w_j) \equiv \pi_j(w_j, w_i^{KS}(w_j))$$

$$= \begin{cases} \frac{(1-w_j)^2}{4} + \frac{4(1-\eta+\eta w_j)^2}{25(1-\eta^2)} & w_j < 1-\eta, \\ \frac{(1-w_j)^2}{4} + \frac{4(1-\eta^2)w_j^2(2-w_j)^2(1-\eta+\eta w_j)^2}{((1-\eta)^2+2(4+\eta-5\eta^2)w_j-(4-5\eta^2)w_j^2)^2} & 1-\eta < w_j < \bar{w}_j^c, \\ \frac{(2(1-\eta^2)(2w_j-w_j^2)+\sqrt{\Delta_1})^2}{4((1-\eta)^2+2(4+\eta-5\eta^2)w_j-(4-5\eta^2)w_j^2)^2} & \bar{w}_j^c < w_j < \frac{(2+\eta)(1-\eta)}{2-\eta^2}, \\ \frac{(\eta^2(2w_j-w_j^2)+\sqrt{\Delta_2})^2}{16(1-\eta-(2-\eta+2\eta^2)w_j+(1+\eta^2)w_j^2)^2} & \frac{(2+\eta)(1-\eta)}{2-\eta^2} < w_j < \frac{2-\eta}{2}, \\ \frac{(2(2w_j-w_j^2)+\sqrt{1+6w_j-3w_j^2})^2}{4(1+8w_j-4w_j^2)^2} & w_j > \frac{2-\eta}{2}, \end{cases} \quad (48)$$

their maximum profits are

$$\overline{\Pi}_j = \Pi_j\big(\frac{(1-\eta)(5+\eta)}{2(5-\eta^2)}\big) = \frac{(1-\eta)(5+\eta)^2}{40(1+\eta)(5-\eta^2)}, \quad (49)$$

$$\overline{\pi}_j = \pi_j(0) = \frac{41+9\eta}{100(1+\eta)}, \quad (50)$$

and the retailer's disagreement point is

$$d_j = \frac{4}{25}. \quad (51)$$

iv) With contingency and the negotiated price in unit i is given by (41), the supplier j's profit is same as that without contingency and retailer's profit is

$$\pi_j(w_j) = \begin{cases} \frac{(1-w_j)^2}{4} + \frac{4(1-\eta+\eta w_j)^2}{25(1-\eta^2)} & w_j < 1-\eta, \\ \frac{(1-w_j)^2}{4} + \frac{4(1-\eta^2)w_j^2(2-w_j)^2(1-\eta+\eta w_j)^2}{((1-\eta)^2+2(4+\eta-5\eta^2)w_j-(4-5\eta^2)w_j^2)^2} & 1-\eta < w_j < \bar{w}_j^c, \\ 0 & w_j > \bar{w}_j^c, \end{cases} \quad (52)$$

their maximum profits are respectively same as those without contingency and the retailer's disagreement point is 0.

Moreover, the profit allocation $(\Pi_j(w_j), \pi_j(w_j))$ *is Pareto-dominated for* w_j *above the maximizer that attains* $\overline{\Pi}_j$*.*

Proof. To see part (i), we substitute the expression of (40) into (35) and (36) to derive (43) and (44), respectively. We substitute the negotiated price $w^{NB} = (1-\theta)/2$ in the one-to-one channel (see Proposition 1) into (37) to derive (45). This concludes part (i).

To see part (ii), we note that when the retailer negotiates with supplier j with contingency, his profit becomes zero if he does not order positive quantity (i.e., $w_j > 1 - \eta(1+\theta)/(2-\eta^2(1-\theta))$). Modifying the expression of (44) leads to that of (46). We conclude part (ii).

To see part (iii), we substitute the expression of (41) into (35) and (36) to derive (47) and (48), respectively. We substitute the negotiated price $w^{KS} = 1/5$ in the one-to-one channel (see Proposition 1) into (37) to derive (51).

Now we consider the supplier j and retailer's maximum profits. We shall note that any feasible w_j should lead to nonnegative trade surpluses (i.e., $\Pi_j(w_j) - D_j \geq 0$ and $\pi_j(w_j) - d_j \geq 0$). Thus, we have $w_j \geq 0$ in the first pieces in (47) and (48), respectively.

To derive (49), we note that the first piece in (47) is maximized at $w_j^m = (1-\eta)(5+\eta)/(2(5-\eta^2))$ and leads to a maximum value of $\Pi^m = (1-\eta)(5+\eta)^2/(40(1+\eta)(5-\eta^2))$. The second piece is decreasing in w_j and thus is maximized at $w_j^r = 1-\eta$. This leads to a maximum value of $\Pi^r = \eta(1-\eta)/10$. It is easy to check that $\Pi^m > \Pi^r$, which leads to the expression of (49).

To derive (50), we note that the first piece in (48) is decreasing in w_j and is maximized at $w_j^p = 0$. This leads to a maximum value of $\pi^p = (41+9\eta)/(100(1+\eta)) \geq 1/4$. It is easy to check that the second piece is convex in w_j for $w_j \in (1-\eta, (2+\eta)(1-\eta)/(2-\eta^2))$ and is maximized at $w_j^q = 1-\eta$. This leads to a maximum value of $\pi^q = \pi_j(1-\eta) = (9\eta^2+16)/100 < \pi^p$. Note that the third, forth and fifth pieces are degenerated to the one-to-one channel and thus the maximum value in turn must be smaller than 1/4. This concludes part (iii).

To see part (iv), we note that when the retailer negotiates with supplier j with contingency, his profit becomes zero if he does not order positive quantity (i.e., $w_j > \bar{w}_j^c$), which leads to the expression of (52). Also note that any feasible w_j should lead to nonnegative trade surpluses and thus we have $w_j \geq 0$. This implies that the maximum profits are same as those without contingency. We conclude part (iv). $\square$

B Proofs of Formal Results

Proof of Proposition 1. Substituting the expressions of $\Pi(w)$, $\pi(w)$, $\overline{\Pi}$ and $\overline{\pi}$ into (2) yields

$$\frac{(1-w)^2/4}{w(1-w)/2} = \frac{1/4}{1/8}.$$

Solving for w leads to $w^{KS} = 1/5$.

To compare with the NB solution, we differentiate the Nash product in (1) with respect to w to obtain

$$\frac{\mathrm{d}\Omega}{\mathrm{d}w} = \frac{(1-w)^\theta}{2^{\theta+1}w^\theta}\big((1-\theta)(1-w) - (1+\theta)w\big) = 0.$$

Solving for w leads to $w^{NB} = (1-\theta)/2$. Setting $w^{KS} = w^{NB}$ gives $\theta = 0.6$. $\square$

Proof of Lemma 2. To see part (i), we first give the expressions of boundaries of the feasible region of (w_i, w_j) (see the top panels of Figure 4), i.e., $\hat{w}_i^1(w_j) = 1 - (\eta/2)(1-w_j)$, $\hat{w}_i^2(w_j) =$

$1-(2/\eta)(1-w_j)$, $\hat{w}_i^3(w_j)=(\eta/2)w_j$ and $\hat{w}_i^4(w_j)=(2\eta w_j+2-\eta)/4$ (see Lemmas A.1 and A.2 for detailed derivation). It is easy to check that the second piece of $\Pi_i(w_i,w_j)$ in (17) is increasing [decreasing] in w_j for $w_j<[>](2\eta w_i+2-\eta)/4\equiv\bar{w}_j(w_i)$ and the second piece of $\pi_i(w_i,w_j)$ in (18) is increasing in w_j. We now substitute the expression of (22) into (17) and (18) to derive the supplier's and retailer i's negotiated profits under the NB solution, respectively, as

$$\Pi_i(w_i^{NB}(w_j),w_j)=\frac{w_j(1-w_j)}{2}+\frac{(2-\eta^2)(1-\theta^2)}{8(2+\eta)}\text{ and }\pi_i(w_i^{NB}(w_j),w_j)=\frac{(1+\theta)^2}{4(2+\eta)^2}$$

for $w_j<\bar{w}_j^a$ (i.e., both products have positive outputs). It is easy to see that $\Pi_i(w_i^{NB}(w_j),w_j)$ is increasing [decreasing] in w_j for $w_j<[>]1/2$ and $\pi_i(w_i^{NB}(w_j),w_j)$ is constant in w_j.

We now substitute the expressions of first two pieces of (23) into (17) and (18) to derive the supplier's and retailer i's negotiated profits under the KS solution, respectively, as

$$\Pi_i(w_i^{KS}(w_j),w_j)=\begin{cases}\frac{w_j(1-w_j)}{2}+\frac{2(2-\eta)}{25(2+\eta)} & w_j<\frac{2}{2+\eta},\\ \frac{w_j(1+(3-\eta^2)w_j+(2+\eta)^2w_j^2(1-w_j)(2+(2+\eta)^2w_j^2))}{2(1+(2+\eta)^2w_j^2)^2} & \frac{2}{2+\eta}<w_j<\bar{w}_j^b,\end{cases}$$

$$\pi_i(w_i^{KS}(w_j),w_j)=\begin{cases}\frac{16}{25(2+\eta)^2} & w_j<\frac{2}{2+\eta},\\ \frac{(2+\eta)^2w_j^4}{(1+(2+\eta)^2w_J^2)^2} & \frac{2}{2+\eta}<w_j<\bar{w}_j^b.\end{cases}$$

In the case that $\bar{w}_j>w_j^b$, the common supplier ends up trading with only retailer j and thus we omit these cases. It is easy to check that the first piece of $\Pi_i(w_i^{KS}(w_j),w_j)$ is increasing [decreasing] in w_j for $w_j<[>]1/2$, and the second piece is decreasing in w_j. Further, we note that the first piece of $\pi_i(w_i^{KS}(w_j),w_j)$ is constant in w_j and the second piece is increasing in w_j. This conclude part (i).

To see part (ii), we first give the expressions of boundaries of the feasible region of (w_i,w_j) (see the bottom panels of Figure 4), i.e., $\breve{w}_i^1(w_j)=1-\eta(1-w_j)$, $\breve{w}_i^2(w_j)=1-(1/\eta)(1-w_j)$ and $\breve{w}_i^3(w_j)=(1-\eta+\eta w_j)/2$ (see Lemmas A.4 and A.5 for detailed derivation). It is easy to see that the second piece of $\Pi_i(w_i,w_j)$ in (35) is increasing in w_j and the second piece of $\pi_i(w_i,w_j)$ in (36) is decreasing in w_j. We now substitute the expression of (40) into (35) and (36) to derive the supplier i's and retailer's negotiated profits under the NB solution, respectively, as

$$\Pi_i(w_i^{NB}(w_j),w_j)=\frac{(1-\theta^2)(1-\eta+\eta w_j)^2}{8(1-\eta^2)}\text{ and }\pi_i(w_i^{NB}(w_j),w_j)=\frac{(1+\theta)^2(1-\eta+\eta w_j)^2}{16(1-\eta^2)}+\frac{(1-w_j)^2}{4}$$

for $w_j<1-\eta(1+\theta)/(2-\eta^2(1-\theta))$. It is easy to see that $\Pi_i(w_i^{NB}(w_j),w_j)$ is increasing in w_j and $\pi_i(w_i^{NB}(w_j),w_j)$ is decreasing in w_j.

We substitute the expressions of first two pieces of (41) into (35) and (36) to derive the supplier i's and retailer's negotiated profits under the KS solution, respectively, as

$$\Pi_i(w_i^{KS}(w_j), w_j) \;=\; \begin{cases} \frac{2(1-\eta+\eta w_j)^2}{25(1-\eta^2)} & w_j < 1-\eta, \\ \frac{2(2-w_j)w_j(1-\eta+\eta w_j)^4}{((1-\eta)^2+2(4+\eta-5\eta^2)w_j-(4-5\eta^2)w_j^2)^2} & 1-\eta < w_j < \bar{w}_j^c, \end{cases}$$

$$\pi_i(w_i^{KS}(w_j), w_j) \;=\; \begin{cases} \frac{(1-w_j)^2}{4} + \frac{4(1-\eta+\eta w_j)^2}{25(1-\eta^2)} & w_j < 1-\eta, \\ \frac{(1-w_j)^2}{4} + \frac{4(1-\eta^2)w_j^2(2-w_j)^2(1-\eta+\eta w_j)^2}{((1-\eta)^2+2(4+\eta-5\eta^2)w_j-(4-5\eta^2)w_j^2)^2} & 1-\eta < w_j < \bar{w}_j^c. \end{cases}$$

In the case that $w_j > \bar{w}_j^c$, the common retailer ends up trading with only supplier j and thus we omit these cases. It is easy to check that $\Pi_i(w_i^{KS}(w_j), w_j)$ is increasing in w_j and $\pi_i(w_i^{KS}(w_j), w_j)$ is decreasing in w_j for $w_j < \bar{w}_j^c$. Thus, we conclude part (ii). □

Proof of Proposition 2. We first consider the one-to-two channel. By (22), (23) and symmetry, we can derive negotiated prices under the NB and KS solutions as $\hat{w}_{sim}^{NB} = (1-\theta)/2$ and $\hat{w}_{sim}^{KS} = 1/5$, respectively. Setting $\hat{w}_{sim}^{NB} = \hat{w}_{sim}^{KS}$ gives $\hat{\theta}_{sim}^{KS} = 0.6$.

Now we consider the two-to-one channel. By (40), (41) and symmetry, we can derive negotiated prices under the NB and KS solutions as $\check{w}_{sim}^{NB} = (1-\theta)(1-\eta)/(2-(1-\theta)\eta)$ and $\check{w}_{sim}^{KS} = (1-\eta)/(5-\eta)$, respectively. Setting $\check{w}_{sim}^{NB} = \check{w}_{sim}^{KS}$ gives $\check{\theta}_{sim}^{KS} = 0.6$. □

Proof of Proposition 3. To see part (i), we apply (24), (25), (26) and $d_j = 0$ to (1) and obtain the Nash product for unit j as

$$\Omega_j(w_j) = \begin{cases} (\frac{w_j(1-w_j)}{2} + \frac{(2-\eta)(1-\theta_i^2)}{8(2+\eta)} - \frac{1-\theta_i^2}{8})^{1-\theta_j}(\frac{(4+\eta(1-\theta_i)-2(2+\eta)w_j)^2}{16(2+\eta)^2})^{\theta_j} & w_j < \bar{w}_j^a, \\ 0 & w_j > \bar{w}_j^a. \end{cases}$$

Setting $\mathrm{d}\ln\Omega_j(w_j)/\mathrm{d}w_j = 0$ in the first case gives

$$(1-\theta_j)\frac{1-2w_j}{2(2+\eta)w_j(1-w_j)-\eta(1-\theta_i^2)} + 2\theta_j\frac{-1}{4+\eta(1-\theta_i)-2(2+\eta)w_j} = 0.$$

This gives

$$\hat{w}_j^{NB} = \frac{6-2\theta_j+\eta(2-\theta_i+\theta_i\theta_j) - \sqrt{(2-\eta\theta_i)^2(1+\theta_j)^2+4\eta\theta_j((4+\eta)\theta_i^2+4\theta_i-\eta)}}{4(\eta+2)}. \tag{53}$$

Note that the above expression has two roots within $[0,1]$ and the maximizer of Ω_j should be the smaller root.

We then substitute the expressions of (28), (29), (30), (31), (32) and $d_j = 0$ into (2) and obtain

$$\frac{(10+\eta-5(2+\eta)w_j)^2/(100(2+\eta)^2)}{w_j(1-w_j)/2+2(2-\eta)/(25(2+\eta))-2/25} = \frac{(10-3\eta+\sqrt{(50-7\eta)(2+\eta)})^2/(400(2+\eta)^2)}{(82+9\eta)/(200(2+\eta))-2/25}$$

for $w_j < 2/(2+\eta)$. We omit the expressions for w_j above the maximizer that attains $\overline{\Pi}_j$ (i.e., $w_j > 1/2$) as the equilibrium prices should lead to a Pareto profit allocation. This gives

$$\hat{w}_j^{KS} = \frac{2(10-\eta)(2+\eta)(50+\eta) + (5(2+\eta)(10-3\eta) - \sqrt{2\Delta_3})\sqrt{(50-7\eta)(2+\eta)}}{5(2+\eta)(300+12\eta-5\eta^2+(20-6\eta)\sqrt{(50-7\eta)(2+\eta)})}, \tag{54}$$

where $\Delta_3 = (2+\eta)(50-7\eta)(100-36\eta+5\eta^2) + (10-3\eta)(100+12\eta-3\eta^2)\sqrt{(50-7\eta)(2+\eta)}$. Note that the above expression has two roots within $[0,1]$ and the price should be the smaller root. It is easy to check that $\mathrm{d}\hat{w}_j^{KS}/\mathrm{d}\eta > 0$ and thus $\hat{w}_j^{KS} \in [0.2, 0.27)$. It follows that $\mathrm{d}\hat{w}_i^{KS}/\mathrm{d}\eta = (\eta/2)\mathrm{d}\hat{w}_j^{KS}/\mathrm{d}\eta + (5\hat{w}_j^{KS} - 1)/10 > 0$.

Finally, we derive the equivalent bargaining power. Recall that the supplier negotiates with retailer j first. We note that $\hat{\theta}_i^{KS}$ should lead to the same expression of the first pieces in (22) and (23). This yields that $\hat{\theta}_i^{KS} = 0.6$. We then set $\hat{w}_j^{NB}\big|_{\theta_i=0.6} = \hat{w}_j^{KS}$ and obtain

$$\hat{\theta}_j^{KS}(\eta) = \frac{5(1-2\hat{w}_j^{KS})(10+\eta-5(2+\eta)\hat{w}_j^{KS})}{50(1-\hat{w}_j^{KS}) - \eta(11-15\hat{w}_j^{KS})}.$$

It is clear that $\mathrm{d}\hat{\theta}_j^{KS}(\eta)/\mathrm{d}\eta < 0$ for $\eta \in (-2, 6)$ and thus $\hat{\theta}_j^{KS}(\eta) \in (0.5, 0.6]$. We conclude part (i).

To see part (ii), we apply (43), (44), (45) and $D_j = 0$ to (1) and obtain the Nash product for unit j as

$$\Omega_j(w_j) = \begin{cases} (\frac{w_j((2-\eta^2(1-\theta_i))(1-w_j)-\eta(1+\theta_i))}{4(1-\eta^2)})^{1-\theta_j}(\frac{(1-w_j)^2}{4} + \frac{(1+\theta_i)^2(1-\eta+\eta w_j)^2}{16(1-\eta^2)} - \frac{(1+\theta_i)^2}{16})^{\theta_j} & w_j < w^a, \\ 0 & w_j > w^a, \end{cases}$$

where $w^a = 1 - \eta(1+\theta_i)/(2-\eta^2(1-\theta_i))$. Setting $\mathrm{d}\ln\Omega_j(w_j)/\mathrm{d}w_j = 0$ in the first case gives

$$\frac{(1-\theta_j)((2-\eta^2(1-\theta_i))(1-2w_j)-\eta(1+\theta_i))}{w_j((2-\eta^2(1-\theta_i))(1-w_j)-\eta(1+\theta_i))} + \frac{\theta_j(-8(1-\eta^2)w_j+2\eta(1+\theta_i)^2(1-\eta+\eta w_j))}{(1+\theta_i)^2(1-\eta+\eta w_j)^2+(1-\eta^2)(4(1-w_j)^2-(1+\theta_i)^2)} = 0.$$

Rearranging the terms gives

$$\Phi(w_j) = \Gamma_0 + \Gamma_1 w_j + \Gamma_2 w_j^2 + \Gamma_3 w_j^3, \tag{55}$$

where $\Gamma_0 = 2(1-\eta)^2(1-\theta_j)(2+\eta(1-\theta_i))(2+\eta(1-2\theta_i-\theta_i^2))$, $\Gamma_1 = 2(1-\eta)\big((1-\theta_i)(5-6\theta_i-3\theta_i^2 - 2(1-2\theta_i-\theta_i^2)\theta_j)\eta^3 + (1-\theta_i)(11+4\theta_i+\theta_i^2-4\theta_j)\eta^2 - 2(3-8\theta_i-3\theta_i^2-2(1-2\theta_i-\theta_i^2)\theta_j)\eta - 8(2-\theta_j)\big)$, $\Gamma_2 = (1-\eta)\big(8(5-\theta_j) + 4(1-\theta_i)(7+2\theta_i-(\theta_i+2)\theta_j)\eta - 2(1-\theta_i)(11+\theta_i-(1-\theta_i)\theta_j)\eta^2 - (1-\theta_i)^2(3+\theta_i)(5-\theta_j)\eta^3\big)$ and $\Gamma_3 = -16 + 4(1-\theta_i)(5+\theta_i)\eta^2 - 2(1-\theta_i)^2(3+\theta_i)\eta^4$.

We note that $\Phi(w_j)$ is a cubic function with the coefficient of w_j^3 being negative (i.e., $\Gamma_3 < 0$). Also, note that $\Phi(w_j) = \Gamma_0 \geq 0$ and

$$\Phi(w_j^o) = -\frac{(1-\eta)^3(2+\eta-\theta_i\eta)^2\big(8+4(2+\theta_i)(1-\theta_i)\eta - 2(1-\theta_i)^2\eta^2 - (1-\theta_i)^2(3+\theta_i)\eta^3\big)\theta_j}{4(2-(1-\theta_i)\eta^2)^2} < 0$$

for $\eta < 1$ and $\theta_j < 1$, where $w_j^o = (1/2)w^a$. In the case that $\eta = 1$ or $\theta_j = 1$, it is easy to check that $\breve{w}_j^{NB} = 0$. This implies that there exists at least one root within the range $[0, w_j^o]$. Note that $\mathrm{d}\Phi(w_j)/\mathrm{d}w_j$ is a quadratic function with the coefficient of w_j^2 being negative (i.e., $3\Gamma_3 < 0$) and is maximized at $w_j^q = -(1/3)\Gamma_2/\Gamma_3 > 0$. Also, note that $\mathrm{d}\Phi(w_j)/\mathrm{d}w_j\big|_{w_j=0} = \Gamma_1 < 0$ and $w_j^q > w_j^o$, which implies that the sign of $\mathrm{d}\Phi(w_j)/\mathrm{d}w_j$ changes at most once within the range $[0, w_j^o]$ and the change is from negative to positive. This suggests that there exists a unique root that is the maximizer of $\Phi(w_j)$ within the range $[0, w_j^o]$. Let $\breve{w}_j^{NB}$ denote this root.

We then substitute the expressions of (47), (48), (49), (50), (51) and $D_j = 0$ into (2) and obtain

$$\frac{(1-w_j)^2/4 + 4(1-\eta+\eta w_j)^2/(25(1-\eta^2)) - 4/25}{w_j((5-\eta^2)(1-w_j) - 4\eta)/(10(1-\eta^2))} = \frac{(41+9\eta)/(100(1+\eta)) - 4/25}{(1-\eta)(5+\eta)^2/(40(1+\eta)(5-\eta^2))}$$

for $w_j < 1 - \eta$. We omit the expressions for w_j above the maximizer that attains $\overline{\Pi}_j$ (i.e., $w_j > (1-\eta)(5-\eta)/(2(5-\eta^2))$) as the equilibrium prices should lead to a Pareto profit allocation. This gives

$$\breve{w}_j^{KS} = \frac{(1-\eta)(5+\eta)\big(375 - 125\eta - 111\eta^2 + 5\eta^3 - 2\sqrt{\Delta_4}\big)}{3125 - 1075\eta - 1450\eta^2 + 390\eta^3 + 181\eta^4 - 19\eta^5},$$

where $\Delta_4 = (1-\eta)(5+\eta)(3125+250\eta-1120\eta^2-266\eta^3+27\eta^4)$. Note that the above expression has two roots within $[0,1]$ and the price should be the smaller root. It is easy to check that $\mathrm{d}\breve{w}_j^{KS}/\mathrm{d}\eta < 0$ and thus $\breve{w}_j^{KS} \in [0, 0.2]$. It follows that $\mathrm{d}\breve{w}_i^{KS}/\mathrm{d}\eta = (\eta/5)\mathrm{d}\breve{w}_j^{KS}/\mathrm{d}\eta - (1-\breve{w}_j^{KS})/5 < 0$.

Finally, we derive the equivalent bargaining power. Recall that the supplier j negotiates with retailer first. We note that $\breve{\theta}_i^{KS}$ should lead to the same expression of the first pieces in (40) and (41). This implies that $\breve{\theta}_i^{KS} = 0.6$. We then substitute $\theta_i = 0.6$ into (55) and obtain

$$\begin{aligned}&(1-\eta)^2(5+\eta)(25-7\eta)(1-\theta_j) + 2(1-\eta)\big((2+7\theta_j)\eta^3 + (86-25\theta_j)\eta^2 + (90-35\theta_j)\eta - 125(2-\theta_j)\big)w_j\\ &\quad -(1-\eta)\big(9(5-\theta_j)\eta^3 + 5(29-\theta_j)\eta^2 - (205-65\theta_j)\eta - 125(5-\theta_j)\big)w_j^2 - 2(125-70\eta^2+9\eta^4)w_j^3 = 0.\end{aligned}$$

Substituting $\breve{w}_j^{KS}$ into the above expression yields

$$\breve{\theta}_j^{KS}(\eta) = \frac{\big((1-\eta)(5+\eta)-2(5-\eta^2)\breve{w}_j^{KS}\big)\big((1-\eta)(25-7\eta)-2(1-\eta)(25+9\eta)\breve{w}_j^{KS}+(25-9\eta^2)(\breve{w}_j^{KS})^2\big)}{(1-\eta)\big((1-\eta)(5+\eta)(25-7\eta)-2(25-7\eta)(5-\eta^2)\breve{w}_j^{KS}+(125+65\eta-5\eta^2-9\eta^3)(\breve{w}_j^{KS})^2\big)}.$$

It is easy to check that $\mathrm{d}\breve{\theta}_j^{KS}(\eta)/\mathrm{d}\eta > 0$ for $\eta \in (-0.26, 1)$ and thus $\breve{\theta}_j^{KS}(\eta) \in [0.6, 1]$. We conclude part (ii). We shall note that parameters with subscript j (i) correspond to those with subscript *seq*1 (*seq*2). □

Proof of Lemma 3. To see part (i), we substitute the expression of (60) into (56) and (57) to derive the supplier's and retailer i's negotiated profits under the NB solution, respectively, as

$$\begin{aligned}\Pi_i(w_i^{NB}(w_j), w_j) &= (1-\theta)\left(\frac{(1-w_j)w_j}{2} + \frac{(1-\theta)(2-\eta)+\sqrt{(2-\eta)^2(1+\theta)^2+16\theta(4-\eta^2)(1-w_j)w_j}}{16(2+\eta)}\right),\\ \pi_i(w_i^{NB}(w_j), w_j) &= \frac{\left((1+\theta)(2-\eta)+\sqrt{(2-\eta)^2(1+\theta)^2+16\theta(4-\eta^2)(1-w_j)w_j}\right)^2}{16(4-\eta^2)^2},\end{aligned}$$

for $w_j < (2-\eta)(4+\eta(1-\theta))/(8-2\eta^2(1-\theta))$. In the case that $w_j > (2-\eta)(4+\eta(1-\theta))/(8-2\eta^2(1-\theta))$, retailer j earns only ϵ profit and equilibrium prices never arise. Hence we omit this case. It is easy to check that $\Pi_i(w_i^{NB}(w_j), w_j)$ and $\pi_i(w_i^{NB}(w_j), w_j)$ exhibit the same monotone property as $w_j(1-w_j)$ with respect to w_j. Thus, the supplier's and retailer i's negotiated profits are increasing [decreasing] in w_j for $w_j <$ [>]1/2.

We now substitute the expression of (62) into (56) and (57) to derive the supplier's retailer i's negotiated profits under the KS solution, respectively, as

$$\Pi_i(w_i^{KS}(w_j), w_j) = \Pi_i(w_i^m, w_j) \quad \text{and} \quad \pi_i(w_i^{KS}(w_j), w_j) = \pi_i(w_i^m, w_j)$$

for $w_j < (2-\eta)/2$, where $w_i^m = (2-\eta+2\eta w_j)/4 + (\Delta_6 - 2\sqrt{2\Delta_5\Delta_6})/(4(6-3\eta+8(\eta+2)(1-w_j)w_j + 2\sqrt{\Delta_6}))$ with Δ_5 and Δ_6 defined in the proof of Proposition 4. In this case that $w_j > (2-\eta)/2$, equilibrium prices never arise and thus we omit this case. It is easy to check that $\mathrm{d}\Pi_i(w_i^m, w_j)/\mathrm{d}w_j >$ [<]0 and $\mathrm{d}\pi_i(w_i^m, w_j)/\mathrm{d}w_j >$ [<]0 for $w_j <$ [>]1/2. Thus, $\Pi_i(w_i^m, w_j)$ and $\pi_i(w_i^m, w_j)$ are increasing [decreasing] in w_j for $w_j <$ [>]1/2. This concludes part (i).

To see part (ii), we focus on the firms' negotiated profits under the KS solution. Substituting the expression of (69) into (64) and (65) respectively, we obtain

$$\Pi_i(w_i^{KS}(w_j), w_j) = \Pi_i(w_i^s, w_j) \quad \text{and} \quad \pi_i(w_i^{KS}(w_j), w_j) = \pi_i(w_i^s, w_j)$$

for $w_j < 1-\eta$, where $w_i^s = (1/2)(1-\eta+\eta w_j)[1 + ((1-\eta+\eta w_j)^2 - 2(1-\eta+\eta w_j)\sqrt{\Delta_8})/((4+\eta^2)(1-w_j)^2 - 10\eta(1-w_j)+5)]$ with Δ_8 defined in the proof of Proposition 4. In the case that $w_j > 1-\eta$, equilibrium prices never arise and thus we omit this case. It is easy to check that $\mathrm{d}\Pi_i(w_i^s, w_j)/\mathrm{d}w_j > 0$ gives $\tilde{\eta} < \eta < 1$, where $\tilde{\eta}$ is the smaller root of

$$\begin{aligned}\Xi(\eta) &= 4(1-w_j)^6\eta^7 - (1-w_j)^5(4w_j^2 - 8w_j + 125)\eta^6 + 4(1-w_j)^4(35w_j^2 - 70w_j + 288)\eta^5\\ &\quad -(1-w_j)^3(28w_j^4 - 112w_j^3 + 1411w_j^2 - 2598w_j + 3081)\eta^4\\ &\quad +4(1-w_j)^2(104w_j^4 - 416w_j^3 + 1053w_j^2 - 1274w_j + 887)\eta^3\\ &\quad +(48w_j^7 - 336w_j^6 + 1324w_j^5 - 3260w_j^4 + 5830w_j^3 - 7138w_j^2 + 5419w_j - 1887)\eta^2\\ &\quad +8(9w_j^4 - 36w_j^3 + 87w_j^2 - 102w_j + 52)\eta - 27(1-w_j)\end{aligned}$$

for $\eta \in [0,1]$ and $w_j \in [0,1]$. Because $\Xi(0) \le 0$ and $\Xi(0.15) > 0$ for any $w_j \in [0,1]$, we must have $\bar{\eta} < 0.15$. We can also show that $\mathrm{d}\pi_i(w_i^s, w_j)/\mathrm{d}w_j < 0$ for $w_j < 1-\eta$. This concludes part (ii).

$\square$

Proof of Proposition 4. To see part (i), we can modify (17) and (18) to derive the trade profits for the supplier and the retailers, respectively, as

$$\Pi_i(w_i, w_j) = \begin{cases} \sum_{i=1}^{2} w_i \frac{2(1-w_i)-\eta(1-w_j)}{4-\eta^2} & \frac{\eta}{2}(1-w_j) < 1-w_i < \frac{2}{\eta}(1-w_j), \\ 0 & 1-w_i \le \frac{\eta}{2}(1-w_j) \text{ or } 1-w_i \ge \frac{2}{\eta}(1-w_j). \end{cases} \tag{56}$$

$$\pi_i(w_i, w_j) = \begin{cases} \frac{(2(1-w_i)-\eta(1-w_j))^2}{(4-\eta^2)^2} & \frac{\eta}{2}(1-w_j) < 1-w_i < \frac{2}{\eta}(1-w_j), \\ 0 & 1-w_i \le \frac{\eta}{2}(1-w_j) \text{ or } 1-w_i \ge \frac{2}{\eta}(1-w_j). \end{cases} \tag{57}$$

The supplier's and retailers' disagreement points under simultaneous negotiation with contingency are 0. Their maximum profits are, respectively,

$$\overline{\Pi}_i(w_j) = \begin{cases} \Pi_i(\frac{2-\eta+2\eta w_j}{4}, w_j) = \frac{2-\eta}{8(2+\eta)} + \frac{1}{2}w_j(1-w_j) & w_j < 1-\frac{\eta}{2(2+\eta)}, \\ \Pi_i(\frac{2w_j-2+\eta}{\eta}+\epsilon, w_j) = \frac{(1-w_j)(2w_j-2+\eta)}{\eta^2} - \epsilon & w_j > 1-\frac{\eta}{2(2+\eta)}, \end{cases} \tag{58}$$

$$\overline{\pi}_i(w_j) = \begin{cases} \pi_i(w^a, w_j) = \frac{(2-\eta+\sqrt{(2-\eta)^2+4(4-\eta^2)(1-w_j)w_j})^2}{4(4-\eta^2)^2} & w_j < \frac{2-\eta}{2}, \\ \pi_i(\frac{2w_j-2+\eta}{\eta}+\epsilon, w_j) = \frac{(1-w_j)^2}{\eta^2} - \epsilon & w_j > \frac{2-\eta}{2}, \end{cases} \tag{59}$$

where $w^a = (1/4)((2-\eta+2\eta w_j) - \sqrt{(2-\eta)^2+4(4-\eta^2)(1-w_j)w_j})$, and ϵ is a sufficiently small positive number. Note that any feasible (w_i, w_j) should lead to nonnegative trade surpluses (i.e., $\Pi_i(w_i, w_j) \ge 0$ and $\pi_i(w_i, w_j) \ge 0$). Thus, we have $w_i \ge w^a$ and $1/2 - \sqrt{1/(2+\eta)} \le w_j \le 1$ in the first pieces in (56) and (57), respectively.

To derive (58), we note that the first piece of $\Pi_i(w_i, w_j)$ in (56) is maximized at $w_i^m = (2-\eta+2\eta w_j)/4$ and leads to a maximum value of $\Pi^m = (2-\eta)/(8(2+\eta)) + (1/2)w_j(1-w_j)$. For w_i^m be the maximizer, we must have $1-w_i^m < (2/\eta)(1-w_j)$ (or $w_j < w^b \equiv 1-\eta/(2(2+\eta))$). In the case that $w_j > w^b$, it is easy to check that the maximizer is $w_i^r = 1-(2/\eta)(1-w_j)+\epsilon$ and leads to a maximum value of $\Pi^r = (1-w_j)(2w_j-2+\eta)/\eta^2 - \epsilon$.

To derive (59), we note that the first piece of $\pi_i(w_i, w_j)$ in (57) is maximized at $w_i^q = w^a$ and leads to a maximum value of $\pi^q = (2-\eta+\sqrt{(2-\eta)^2+4(4-\eta^2)(1-w_j)w_j})^2/(4(4-\eta^2)^2)$. For w^a be the maximizer, we must have $1-w^a < (2/\eta)(1-w_j)$ (or $w_j < (2-\eta)/2$). In the case that $w_j > (2-\eta)/2$, it is easy to check that the maximizer is $w_i^r = 1-(2/\eta)(1-w_j)+\epsilon$. This leads to the second piece of (59).

Now we derive the equilibrium prices under the NB and KS solutions. Applying (56), (57) and

$D_i(w_j) = d_i(w_j) = 0$, the Nash product for trade i is

$$\Omega_i(w_i, w_j) = \begin{cases} (\sum_{i=1}^2 w_i \frac{2(1-w_i)-\eta(1-w_j)}{4-\eta^2})^{1-\theta}(\frac{(2(1-w_i)-\eta(1-w_j))^2}{(4-\eta^2)^2})^{\theta} & \frac{\eta}{2} < \frac{1-w_i}{1-w_j} < \frac{2}{\eta}, \\ 0 & \frac{1-w_i}{1-w_j} \leq \frac{\eta}{2} \text{ or } \frac{1-w_i}{1-w_j} \geq \frac{2}{\eta}. \end{cases}$$

Setting $\partial \ln \Omega_i(w_i, w_j)/\partial w_i = 0$ in the first case gives

$$(1-\theta)\frac{2(1-2w_i)-\eta(1-2w_j)}{\sum_{i=1}^2 w_i(2(1-w_i)-\eta(1-w_j))} + 2\theta\frac{-2}{2(1-w_i)-\eta(1-w_j)} = 0.$$

This gives $w_i^s = (1/8)\big((3-\theta)(2-\eta) + 4\eta w_j - \sqrt{(2-\eta)^2(1+\theta)^2 + 16\theta(4-\eta^2)(1-w_j)w_j}\big)$. For w_i^s be the best response, we must have $1 - w_i^s < (2/\eta)(1-w_j)$ (or $w_j < w^c \equiv (2-\eta)(4+\eta(1-\theta))/(8-2\eta^2(1-\theta))$). In the case that $w_j > w^c$, it is easy to check that the best response should be $1-(2/\eta)(1-w_j)+\epsilon$. This gives

$$w_i^{NB}(w_j) = \begin{cases} \frac{(3-\theta)(2-\eta)+4\eta w_j - \sqrt{(2-\eta)^2(1+\theta)^2+16\theta(4-\eta^2)(1-w_j)w_j}}{8} & w_j < \frac{(2-\eta)(4+\eta(1-\theta))}{8-2\eta^2(1-\theta)}, \\ \frac{2w_j-2+\eta}{\eta} + \epsilon & w_j > \frac{(2-\eta)(4+\eta(1-\theta))}{8-2\eta^2(1-\theta)}. \end{cases} \tag{60}$$

By symmetry, we have

$$\hat{w}_{sim}^{NB} = \frac{(2-\eta)(1-\theta)}{2(2-\eta+2\theta+\eta\theta)}. \tag{61}$$

We now substitute (56), (57), (58), (59) and $D_i(w_j) = d_i(w_j) = 0$ into (2) and obtain

$$\frac{(2(1-w_i)-\eta(1-w_j))^2/(4-\eta^2)^2}{\sum_{i=1}^2 w_i(2(1-w_i)-\eta(1-w_j))/(4-\eta^2)} = \begin{cases} \frac{(2-\eta+\sqrt{(2-\eta)^2+4(4-\eta^2)(1-w_j)w_j})^2/4(4-\eta^2)^2}{(2-\eta)/(8(2+\eta))+(1/2)w_j(1-w_j)} & w_j < \frac{2-\eta}{2}, \\ \frac{(1-w_j)^2/\eta^2-\epsilon}{(2-\eta)/(8(2+\eta))+(1/2)w_j(1-w_j)} & \frac{2-\eta}{2} < w_j < \frac{4+\eta}{2(2+\eta)}, \\ \frac{(1-w_j)^2/\eta^2-\epsilon}{(1-w_j)(2w_j-2+\eta)/\eta^2-\epsilon} & w_j > \frac{4+\eta}{2(2+\eta)}. \end{cases}$$

The first piece gives $w_i^m = \big(2(2-\eta)^2 + 3(2-\eta)(4+3\eta)w_j - 2(2+\eta)(6-7\eta)w_j^2 - 8\eta(2+\eta)w_j^3 + (2-\eta+2\eta w_j - \sqrt{2\Delta_5})\sqrt{\Delta_6}\big)/(2(6-3\eta+8(\eta+2)(1-w_j)w_j+2\sqrt{\Delta_6}))$, where $\Delta_5 = (2-\eta+(2+\eta)(1-w_j)w_j)(2-\eta+4(2+\eta)(1-w_j)w_j) + (2-\eta+3(2+\eta)(1-w_j)w_j)\sqrt{\Delta_6}$ and $\Delta_6 = (2-\eta)^2 + 4(4-\eta^2)(1-w_j)w_j$. Note that the first piece has two roots and the best response should be the smaller root.

The second piece gives $w_i^q = \eta w_j/2 + \big((2-\eta)^2(8+8\eta+3\eta^2) - 8(4-\eta^2)(2-\eta^2)w_j + 2(4-3\eta^2)(4-\eta^2)w_j^2 - 2\sqrt{\Delta_7}\big)/(2(2-\eta)(16+16\eta+5\eta^2) - 8(2+\eta)(8-3\eta^2)w_j + 16(2+\eta)(2-\eta^2)w_j^2) + \epsilon$, where $\Delta_7 = (1-w_j)^3(2-\eta)^2(2+\eta)^3(2-\eta+4(2+\eta)(1-w_j)w_j)(2+(2-\eta^2)(1-2w_j))$. Note that the second piece has two roots and the best response should be the smaller root.

The third piece gives $w_i^p = (2w_j - 2 + \eta)/\eta + \epsilon$.

Combining the above cases leads to

$$w_i^{KS}(w_j) = \begin{cases} \frac{2-\eta+2\eta w_j}{4} + \frac{\Delta_6 - 2\sqrt{2\Delta_5\Delta_6}}{4(6-3\eta+8(\eta+2)(1-w_j)w_j+2\sqrt{\Delta_6})} & w_j < \frac{2-\eta}{2}, \\ \frac{2-\eta+2\eta w_j}{4} - \frac{(4-\eta^2)\eta^2(1-2w_j)^2-4(2-\eta)\eta^2+4\sqrt{\Delta_7}}{32(2+\eta)(2-\eta^2)(1-w_j)^2+16(2+\eta)\eta^2(1-w_j)+4(2-\eta)\eta^2} + \epsilon & \frac{2-\eta}{2} < w_j < \frac{\eta+4}{2(\eta+2)}, \\ \frac{2w_j-2+\eta}{\eta} + \epsilon & w_j > \frac{\eta+4}{2(\eta+2)}. \end{cases} \tag{62}$$

By symmetry, the first piece gives

$$\Phi(w) = 16(6-\eta)^2(2+\eta)^2w^5 - 16(6-\eta)(10-3\eta)(2+\eta)^2w^4 + 8(4-\eta^2)(28+32\eta-7\eta^2)w^3 + 32(3-\eta)(2-\eta)^2(2+\eta)w^2 + (2-\eta)^3(2-9\eta)w - (2-\eta)^4. \tag{63}$$

Note that $\Phi(0) = -(2-\eta)^4 < 0$ and $\Phi(1/5) = (4/3125)(2+\eta)(7688 - 6732\eta + 1350\eta^2 - 81\eta^3) > 0$. This implies that there exists at least one root within $w \in [0, 1/5]$. Also, note that

$$\frac{1}{16(2+\eta)}\frac{\partial^2\Phi(w)}{\partial w^2} = 20(2+\eta)(6-\eta)^2w^3 - 12(2+\eta)(6-\eta)(10-3\eta)w^2 + 3(2-\eta)(28+32\eta-7\eta^2)w + 4(3-\eta)(2-\eta)^2 > 0$$

for $w \in [0, 1/5]$. Thus, there exists a unique root within $w \in [0, 1/5]$. Let $\hat{w}_{sim}^{KS}(\eta)$ denote this root. Note that $\hat{w}_{sim}^{KS}(0) \approx 0.1299 < 0.13$, $\hat{w}_{sim}^{KS}(1/2) \approx 0.1055$, $\hat{w}_{sim}^{KS}(1) \approx 0.0767 > 0.07$. Also,

$$\frac{\partial\Phi(w)}{\partial\eta} = 32(1-3w-20w^2+32w^3-64w^4+48w^5) - 16(1-2w)^2(3-3w-16w^2+4w^3)\eta + 24(1-2w)^3(1-w+2w^2)\eta^2 - 4(1-2w)^4(1-w)\eta^3.$$

Note that $\partial\Phi(w)/\partial\eta$ is a cubic function in η with the coefficient of η^3 being negative. It is easy to check that $\partial\Phi(w)/\partial\eta\big|_{\eta=0} > 0$, $\partial\Phi(w)/\partial\eta\big|_{\eta=1/2} > 0$, $\partial^2\Phi(w)/\partial\eta^2 < 0$, and $\partial\Phi(w)/\partial\eta\big|_{\eta=1} > [<]0$ for $w < [>]0.107$. We deduce that $\Phi(w)$ is increasing in $\eta \in [0, 1/2]$ for $w \in [0.07, 0.13]$ and is increasing in $\eta \in [1/2, 1]$ for $w \in [0.07, 0.107]$. Thus, we conclude that $\hat{w}_{sim}^{KS}$ is decreasing in η.

By symmetry, the second piece gives $w = 1/2 \pm 2\sqrt{16+32\eta+18\eta^2+\eta^3-\eta^4}/(16+16\eta+2\eta^2-\eta^3)$, which are not within the range $[(2-\eta)/2, (\eta+4)/(2(2+\eta))]$. It is easy to check that the third piece gives no feasible solution.

Now we derive the equivalent bargaining power. Substituting (61) into (63) yields

$$\Phi\Big(\frac{(2-\eta)(1-\theta)}{2(2-\eta+2\theta+\eta\theta)}\Big) = \frac{128(2-\eta)^4}{(2(1+\theta)-\eta(1-\theta))^5}\Xi(\theta,\eta) = 0,$$

where $\Xi(\theta,\eta) = (1-\eta)(2-\eta) + (6+3\eta-5\eta^2)\theta - 2(2+\eta)(2-5\eta)\theta^2 - 2(2+\eta)(6+5\eta)\theta^3 + 5(2+\eta)^2\theta^4 - (2+\eta)^2\theta^5$. Note that $\Xi(1/2,\eta) = (1/32)(6-\eta)^2 > 0$ and $\Xi(3/5,\eta) = -(2/3125)(286+111\eta-16\eta^2) < 0$. Because $\hat{w}_{sim}^{NB}$ is (strictly) decreasing in θ, there must exist a unique root within $\theta \in [1/2, 3/5]$. Let $\hat{\theta}_{sim}^{KS}(\eta)$ denote this root. We note that $\partial\Xi(\theta,\eta)/\partial\eta = 2\eta(1-\theta)^5 - 3 + 3\theta + 16\theta^2 - 32\theta^3 + 20\theta^4 - 4\theta^5 < 0$ for $\theta \in [0, 3/5]$. This implies that $\Xi(\theta,\eta)$ is decreasing in η, which is equivalent to $\hat{\theta}_{sim}^{KS}(\eta)$ being decreasing in η. Finally, note that $\hat{\theta}_{sim}^{KS}(0) < 0.59$ and $\hat{\theta}_{sim}^{KS}(1) > 0.57$. This concludes part (i).

To see part (ii), we can modify (35) and (36) to derive the trade profits for the suppliers and the retailer, respectively as

$$\Pi_i(w_i, w_j) = \begin{cases} w_i \frac{(1-w_i)-\eta(1-w_j)}{2(1-\eta^2)} & \eta(1-w_j) < 1-w_i < \frac{1}{\eta}(1-w_j), \\ 0 & 1-w_i \le \eta(1-w_j) \text{ or } 1-w_i \ge \frac{1}{\eta}(1-w_j). \end{cases} \tag{64}$$

$$\pi_i(w_i, w_j) = \begin{cases} \sum_{i=1}^{2} \frac{(1-w_i)((1-w_i)-\eta(1-w_j))}{4(1-\eta^2)} & \eta(1-w_j) < 1-w_i < \frac{1}{\eta}(1-w_j), \\ 0 & 1-w_i \le \eta(1-w_j) \text{ or } 1-w_i \ge \frac{1}{\eta}(1-w_j). \end{cases} \tag{65}$$

The suppliers' and retailer's disagreement point under simultaneous negotiation with contingency are 0. We can easily modify (38) and (39) to derive their maximum profits as

$$\overline{\Pi}_i(w_j) = \begin{cases} \Pi_i(\frac{1-\eta+\eta w_j}{2}, w_j) = \frac{(1-\eta(1-w_j))^2}{8(1-\eta^2)} & w_j < \frac{(2+\eta)(1-\eta)}{2-\eta^2}, \\ \Pi_i(\frac{w_j-1+\eta}{\eta}+\epsilon, w_j) = \frac{(1-w_j)(w_j-1+\eta)}{2\eta^2} - \epsilon & w_j > \frac{(2+\eta)(1-\eta)}{2-\eta^2}, \end{cases} \tag{66}$$

$$\overline{\pi}_i(w_j) = \begin{cases} \pi_i(0, w_j) = \frac{(1-w_j)^2-2\eta(1-w_j)+1}{4(1-\eta^2)} & w_j < 1-\eta, \\ \pi_i(\frac{w_j-1+\eta}{\eta}+\epsilon, w_j) = \frac{(1-w_j)^2}{4\eta^2} - \epsilon & w_j > 1-\eta. \end{cases} \tag{67}$$

Now we are ready to the equilibrium prices under the NB and KS solutions. Applying (64), (65) and $D_i(w_j) = d_i(w_j) = 0$, the Nash product for trade i is

$$\Omega_i(w_i, w_j) = \begin{cases} (w_i \frac{(1-w_i)-\eta(1-w_j)}{2(1-\eta^2)})^{1-\theta} (\sum_{i=1}^{2} \frac{(1-w_i)((1-w_i)-\eta(1-w_j))}{4(1-\eta^2)})^{\theta} & \eta < \frac{1-w_i}{1-w_j} < \frac{1}{\eta}, \\ 0 & \frac{1-w_i}{1-w_j} \le \eta \text{ or } \frac{1-w_i}{1-w_j} \ge \frac{1}{\eta}. \end{cases}$$

Setting $\partial \ln \Omega_i(w_i, w_j)/\partial w_i = 0$ in the first case gives

$$(1-\theta)\frac{1-2w_i-\eta(1-w_j)}{w_i(1-w_i-\eta(1-w_j))} + \theta \frac{-2(1-w_i-\eta(1-w_j))}{\sum_{i=1}^{2}(1-w_i)((1-w_i)-\eta(1-w_j))} = 0.$$

By symmetry, we have

$$\breve{w}_{sim}^{NB} = \frac{(1-\eta)(1-\theta)}{2-\eta-\theta}. \tag{68}$$

We now substitute (64), (65), (66), (67) and $D_i(w_j) = d_i(w_j) = 0$ into (2) and obtain

$$\frac{\sum_{i=1}^{2}(1-w_i)((1-w_i)-\eta(1-w_j))/(4(1-\eta^2))}{w_i((1-w_i)-\eta(1-w_j))/(2(1-\eta^2))} = \begin{cases} \frac{((1-w_j)^2-2\eta(1-w_j)+1)/(4(1-\eta^2))}{(1-\eta(1-w_j))^2/(8(1-\eta^2))} & w_j < 1-\eta, \\ \frac{(1-w_j)^2/(4\eta^2)-\epsilon}{(1-\eta(1-w_j))^2/(8(1-\eta^2))} & 1-\eta < w_j < \frac{(2+\eta)(1-\eta)}{2-\eta^2}, \\ \frac{(1-w_j)^2/(4\eta^2)-\epsilon}{(1-w_j)(w_j-1+\eta)/(2\eta^2)-\epsilon} & w_j > \frac{(2+\eta)(1-\eta)}{2-\eta^2}. \end{cases}$$

Following the lines of the proof of the one-to-two channel, we have

$$w_i^{KS}(w_j) = \begin{cases} \frac{1-\eta+\eta w_j}{2}\left[1 + \frac{(1-\eta+\eta w_j)^2-2(1-\eta+\eta w_j)\sqrt{\Delta_8}}{(4+\eta^2)(1-w_j)^2-10\eta(1-w_j)+5}\right] & w_j < 1-\eta, \\ \frac{1-\eta+\eta w_j}{2}\left[1 + \frac{\eta^2(1-\eta+\eta w_j)^2-2(1-w_j)\sqrt{\Delta_9}}{(2-\eta^2)^2(1-w_j)^2-2\eta^3(1-w_j)+\eta^2}\right] + \epsilon & 1-\eta < w_j < \frac{(2+\eta)(1-\eta)}{2-\eta^2}, \\ \frac{w_j-1+\eta}{\eta} + \epsilon & w_j > \frac{(2+\eta)(1-\eta)}{2-\eta^2}, \end{cases} \tag{69}$$

where $\Delta_8 = (3+\eta^2)(1-w_j)^2 - 8\eta(1-w_j) + 4$ and $\Delta_9 = (1-\eta^2)((4(1-\eta^2)^2 - \eta^6)(1-w_j)^2 + 2\eta^5(1-w_j) - \eta^4)$.

By symmetry, the first piece gives

$$\Gamma(w) = (2+\eta^2)w^3 - (4-6\eta+3\eta^2)w^2 + (1-\eta)(5-3\eta)w - (1-\eta)^2. \tag{70}$$

Note that $\Gamma(w)$ is a cubic function in w with the coefficient of w^3 being positive. For $\eta = 1$, it is clear that $\Gamma(0) = 0$. For $\eta < 1$, we have $\Gamma(0) = -(1-\eta)^2 < 0$, $\Gamma((2\eta - 3 + \sqrt{9-8\eta})/(2\eta)) = (1/\eta^3)(-27 + 27\eta + 5\eta^2 - 7\eta^3 + (9 - 5\eta - 3\eta^2 + \eta^3)\sqrt{9-8\eta}) > 0$ and $\partial\Gamma(w)/\partial w > 0$. Thus, there exists a unique root within $w \in [0, (2\eta - 3 + \sqrt{9-8\eta})/(2\eta)]$. Let $\check{w}^{KS}_{sim}$ denote this root. Because $\partial\Gamma(w)/\partial\eta = 2(1-w)(1-\eta-(3-2\eta)w - \eta w^2) > 0$ for $w \in [0, (2\eta - 3 + \sqrt{9-8\eta})/(2\eta)]$, $\check{w}^{KS}_{sim}$ is decreasing in η.

By symmetry, the second piece gives $w = (1 - \eta^2 - \eta^3 + \eta^4 \pm \sqrt{1 - 4\eta^2 + 2\eta^3 + 3\eta^4 - 2\eta^5})/(2 - 2\eta^2 + \eta^4)$. The smaller root is not within the feasible range and the bigger root leads to a Pareto-dominated profit allocation. It is easy to check that the third piece gives no feasible solution.

Finally, we derive the equivalent bargaining power. Substituting (68) into (70) yields

$$\Gamma\Big(\frac{(1-\eta)(1-\theta)}{2-\eta-\theta}\Big) = \frac{(1-\theta)^3}{(2-\eta-\theta)^3}\Big[6 - 2(7+2\eta)\theta + 9(1+\eta)\theta^2 - (2+\eta)(1+\eta)\theta^3\Big] = 0.$$

Setting the second term being zero gives $\eta(\theta) = (-4 + 9\theta - 3\theta^2 - (1-\theta)\sqrt{16 - 16\theta + \theta^2})/(2\theta^2)$. Note that $\mathrm{d}\eta(\theta)/\mathrm{d}\theta > 0$ for $\theta \in (0,1)$ and thus $\check{\theta}^{KS}_{sim}(\eta)$ is increasing in η. Note that $\check{\theta}^{KS}_{sim}(0) > 0.68$ and $\check{\theta}^{KS}_{sim}(1) = 1$. We conclude part (ii). □

Proof of Proposition 5. To see part (i), we first consider the firms' trade profits under the KS solution. By (17), (18), Lemma A.3 and Proposition 3, we have

$$\hat{\Pi}^{KS}_{sim} = \frac{8}{25(2+\eta)} \le \frac{(1-\hat{w}^{KS}_{seq1})\hat{w}^{KS}_{seq1}}{2} + \frac{2(2-\eta)}{25(2+\eta)} = \hat{\Pi}^{KS}_{seq},$$
$$\hat{\pi}^{KS}_{sim} = \hat{\pi}^{KS}_{seq2} = \frac{16}{25(2+\eta)^2} \ge \frac{(10+\eta-5(2+\eta)\hat{w}^{KS}_{seq1})^2}{100(2+\eta)^2} = \hat{\pi}^{KS}_{seq1}.$$

The above inequalities follow because $\hat{w}^{KS}_{seq1} \in [0.2, 0.27)$ from the proof of Proposition 3.

Now we consider the firms' trade profits under the NB solution. We have

$$\hat{\Pi}^{NB}_{sim} = \frac{(1-\theta^2)}{2(2+\eta)} \le \frac{(1-\hat{w}^{NB}_{seq1})\hat{w}^{NB}_{seq1}}{2} + \frac{(2-\eta)(1-\theta^2)}{8(2+\eta)} = \hat{\Pi}^{NB}_{seq},$$
$$\hat{\pi}^{NB}_{sim} = \hat{\pi}^{NB}_{seq2} = \frac{(1+\theta)^2}{4(2+\eta)^2} \ge \frac{(4+\eta(1-\theta)-2(2+\eta)\hat{w}^{NB}_{seq1})^2}{16(2+\eta)^2} = \hat{\pi}^{NB}_{seq1},$$

where $\hat{w}^{NB}_{seq1} = \big(6 - 2\theta + \eta(2 - \theta + \theta^2) - \sqrt{(2 - \eta\theta)^2(1 + \theta)^2 + 4\eta\theta((4 + \eta)\theta^2 + 4\theta - \eta)}\big)/(4(\eta + 2))$. Because $\partial\hat{w}^{NB}_{seq1}/\partial\theta \le 0$ and $\partial\hat{w}^{NB}_{seq1}/\partial\eta \ge 0$, we have $\hat{w}^{NB}_{seq1} \ge \hat{w}^{NB}_{seq1}\big|_{\eta=0} = (1 - \theta)/2$. The above inequalities then follow. We conclude part (i).

To see part (ii), we first consider the firms' trade profits under the KS solution. By (35), (36), Lemma A.6 and Proposition 3, we have

$$\begin{aligned}
\check{\Pi}^{KS}_{sim} &= \frac{2(1-\eta)}{(1+\eta)(5-\eta)^2},\\
\check{\pi}^{KS}_{sim} &= \frac{8}{(1+\eta)(5-\eta)^2},\\
\check{\Pi}^{KS}_{seq1} &= \frac{\check{w}^{KS}_{seq1}((5-\eta^2)(1-\check{w}^{KS}_{seq1}) - 4\eta)}{10(1-\eta^2)},\\
\check{\Pi}^{KS}_{seq2} &= \frac{2(1-\eta+\eta\check{w}^{KS}_{seq1})^2}{25(1-\eta^2)},\\
\check{\pi}^{KS}_{seq} &= \frac{(25-9\eta^2)(1-\check{w}^{KS}_{seq1})^2 - 32\eta(1-\check{w}^{KS}_{seq1}) + 16}{100(1-\eta^2)}.
\end{aligned}$$

Comparing $\check{\Pi}^{KS}_{sim}$ and $\check{\Pi}^{KS}_{seq1}$, we obtain

$$\check{\Pi}^{KS}_{sim} - \check{\Pi}^{KS}_{seq1} = \frac{1}{10(1-\eta^2)(5-\eta)^2}\Xi_1(\eta),$$

where $\Xi_1(\eta) = 20(1-\eta)^2 - (5-\eta)^2\check{w}^{KS}_{seq1}((5-\eta^2)(1-\check{w}^{KS}_{seq1}) - 4\eta)$. Note that $\Xi_1(\eta)$ is a quadratic function with the coefficient of $(\check{w}^{KS}_{seq1})^2$ being positive. $\Xi_1(\eta) = 0$ has two roots, $w^{(1)} = (1-\eta)/(5-\eta)$ and $w^{(2)} = 20(1-\eta)/((5-\eta)(5-\eta^2))$. We have $w^{(1)} \le \check{w}^{KS}_{seq1} \le (1-\eta)(5+\eta)/(2(5-\eta^2)) \le w^{(2)}$ and thus $\Xi_1(\eta) \le 0$.

Comparing $\check{\Pi}^{KS}_{sim}$ and $\check{\Pi}^{KS}_{seq2}$, we obtain

$$\check{\Pi}^{KS}_{sim} - \check{\Pi}^{KS}_{seq2} = \frac{2\eta}{25(1-\eta^2)(5-\eta)^2}\Xi_2(\eta),$$

where $\Xi_2(\eta) = (10-\eta)(1-\eta)^2 - (5-\eta)^2\check{w}^{KS}_{seq1}(2(1-\eta) + \eta\check{w}^{KS}_{seq1})$. Note that $\Xi_2(\eta)$ is a quadratic function with the coefficient of $(\check{w}^{KS}_{seq1})^2$ being negative. $\Xi_2(\eta) = 0$ has two roots, $w^{(1)} = (1-\eta)/(5-\eta)$ and $w^{(3)} = -(1-\eta)(10-\eta)/(\eta(5-\eta))$. We have $w^{(3)} \le 0 \le w^{(1)} \le \check{w}^{KS}_{seq1}$ and thus $\Xi_2(\eta) \le 0$.

Comparing $\check{\Pi}^{KS}_{seq1}$ and $\check{\Pi}^{KS}_{seq2}$, we obtain

$$\check{\Pi}^{KS}_{seq1} - \check{\Pi}^{KS}_{seq2} = \frac{1}{50(1-\eta^2)}\Xi_3(\eta),$$

where $\Xi_3(\eta) = -4(1-\eta)^2 + (25-\eta)(3-\eta)\check{w}^{KS}_{seq1} - (25-\eta^2)(\check{w}^{KS}_{seq1})^2$. Note that $\Xi_3(\eta)$ is a quadratic function with the coefficient of $(\check{w}^{KS}_{seq1})^2$ being negative. $\Xi_3(\eta) = 0$ has two roots, $w^{(1)} = (1-\eta)/(5-\eta)$ and $w^{(4)} = 4(1-\eta)/(5+\eta)$. We have $w^{(1)} \le \check{w}^{KS}_{seq1}$ and $\check{w}^{KS}_{seq1} < [>] w^{(4)}$ for $\eta < [>] \bar{\eta}^{KS} \approx 0.994576$ and thus $\Xi_3(\eta) \ge [\le] 0$ for $\eta \le [\ge] \bar{\eta}^{KS}$.

Comparing $\breve{\pi}^{KS}_{sim}$ and $\breve{\pi}^{KS}_{seq}$, we obtain

$$\breve{\pi}^{KS}_{sim} - \breve{\pi}^{KS}_{seq} = \frac{1}{100(1-\eta^2)(5-\eta)^2}\Xi_4(\eta),$$

where $\Xi_4(\eta) = -(1-\eta)^2(225+40\eta-9\eta^2)+2(5-\eta)^2(25-16\eta-9\eta^2)\breve{w}^{KS}_{seq1}-(5-\eta)^2(25-9\eta^2)(\breve{w}^{KS}_{seq1})^2$. Note that $\Xi_4(\eta)$ is a quadratic function with the coefficient of $(\breve{w}^{KS}_{seq1})^2$ being negative. $\Xi_4(\eta)=0$ has two roots, $w^{(1)}=(1-\eta)/(5-\eta)$ and $w^{(5)}=(225-185\eta-49\eta^2+9\eta^3)/(125-25\eta-45\eta^2+9\eta^3)$. We have $w^{(1)} \le \breve{w}^{KS}_{seq1} \le w^{(5)}$ and thus $\Xi_4(\eta) \ge 0$.

We now consider the firms' trade profits under the NB solution. We can derive

$$\begin{aligned}
\breve{\Pi}^{NB}_{sim} &= \frac{(1-\eta)(1-\theta^2)}{2(1+\eta)(2-\eta(1-\theta))^2},\\
\breve{\pi}^{NB}_{sim} &= \frac{(1+\theta)^2}{2(1+\eta)(2-\eta(1-\theta))^2},\\
\breve{\Pi}^{NB}_{seq1} &= \frac{\breve{w}^{NB}_{seq1}((2-\eta^2(1-\theta))(1-\breve{w}^{NB}_{seq1})-\eta(1+\theta))}{4(1-\eta^2)},\\
\breve{\Pi}^{NB}_{seq2} &= \frac{(1-\theta^2)(1-\eta+\eta\breve{w}^{NB}_{seq1})^2}{8(1-\eta^2)},\\
\breve{\pi}^{NB}_{seq} &= \frac{(1-\breve{w}^{NB}_{seq1})^2}{4} + \frac{(1+\theta)^2(1-\eta+\eta\breve{w}^{NB}_{seq1})^2}{16(1-\eta^2)},
\end{aligned}$$

where $\breve{w}^{NB}_{seq1}$ is the unique root of (55) with $\theta_i=\theta_j=\theta$. Let $w^l \equiv (1-\theta)(1-\eta)/(2-\eta(1-\theta))$ and $w^u \equiv (1-\eta)(2+\eta(1-\theta))/(2(2-\eta^2(1-\theta)))$. Because $\Psi(w^l)\ge 0$ and $\Psi(w^u)\le 0$, we have $w^l \le \breve{w}^{NB}_{seq1} \le w^u$.

Comparing $\breve{\Pi}^{NB}_{sim}$ and $\breve{\Pi}^{NB}_{seq1}$, we obtain

$$\breve{\Pi}^{NB}_{sim} - \breve{\Pi}^{NB}_{seq1} = \frac{1}{4(1-\eta^2)(2-\eta(1-\theta))^2}\Xi_1(\eta,\theta),$$

where $\Xi_1(\eta,\theta) = 2(1-\eta)^2(1-\theta^2)-(2-\eta(1-\theta))^2\breve{w}^{NB}_{seq1}((2-\eta^2(1-\theta))(1-\breve{w}^{NB}_{seq1})-\eta(1+\theta))$. Note that $\Xi_1(\eta,\theta)$ is a quadratic function with the coefficient of $(\breve{w}^{NB}_{seq1})^2$ being positive. $\Xi_1(\eta,\theta)=0$ has two roots, $w^{(a)}=(1-\theta)(1-\eta)/(2-\eta(1-\theta))$ and $w^{(b)}=2(1-\eta)(1+\theta)/(4-2\eta(1+\eta)(1-\theta)+\eta^3(1-\theta)^2)$. We have $w^{(a)}=w^l \le \breve{w}^{NB}_{seq1} \le w^u \le w^{(b)}$ and thus $\Xi_1(\eta,\theta)\le 0$.

Comparing $\breve{\Pi}^{NB}_{sim}$ and $\breve{\Pi}^{NB}_{seq2}$, we obtain

$$\breve{\Pi}^{NB}_{sim} - \breve{\Pi}^{NB}_{seq2} = \frac{(1-\theta^2)}{8(1-\eta^2)(2-\eta(1-\theta))^2}\Xi_2(\eta,\theta),$$

where $\Xi_2(\eta,\theta) = 4(1-\eta)^2-(2-\eta(1-\theta))^2(1-\eta+\eta\breve{w}^{NB}_{seq1})^2$. Note that $\Xi_2(\eta,\theta)$ is a quadratic function with the coefficient of $(\breve{w}^{NB}_{seq1})^2$ being negative. $\Xi_2(\eta,\theta)=0$ has two roots, $w^{(a)}=(1-\theta)(1-\eta)/(2-$

$\eta(1-\theta))$ and $w^{(c)} = -(1-\eta)(4-\eta(1-\theta))/(\eta(2-\eta(1-\theta)))$. We have $w^{(c)} \leq 0 \leq w^{(a)} \leq \breve{w}^{NB}_{seq1}$ and thus $\Xi_2(\eta,\theta) \leq 0$.

Comparing $\breve{\Pi}^{NB}_{seq1}$ and $\breve{\Pi}^{NB}_{seq2}$, we obtain

$$\breve{\Pi}^{NB}_{seq1} - \breve{\Pi}^{NB}_{seq2} = \frac{1}{8(1-\eta^2)}\Xi_3(\eta,\theta),$$

where $\Xi_3(\eta,\theta) = 2\breve{w}^{NB}_{seq1}((2-\eta^2(1-\theta))(1-\breve{w}^{NB}_{seq1}) - \eta(1+\theta)) - (1-\theta^2)(1-\eta+\eta\breve{w}^{NB}_{seq1})^2$. Note that $\Xi_3(\eta,\theta)$ is a quadratic function with the coefficient of $(\breve{w}^{NB}_{seq1})^2$ being negative. $\Xi_3(\eta,\theta) = 0$ has two roots, $w^{(a)} = (1-\theta)(1-\eta)/(2-\eta(1-\theta))$ and $w^{(d)} = (1+\theta)(1-\eta)/(2+\eta(1-\theta))$. It is easy to check that $w^{(a)} \geq [\leq] w^{(d)}$ for $\eta \geq [\leq] 2\theta/(1-\theta)$. We have two cases depending on the value of η.

Case 1: $\eta \geq 2\theta/(1-\theta)$. We have $w^{(d)} \leq w^{(a)} \leq \breve{w}^{NB}_{seq1}$ and thus $\Xi_3(\eta,\theta) \leq 0$.

Case 2: $\eta < 2\theta/(1-\theta)$. By (55) with $\theta_i = \theta_j = \theta$, we have

$$\frac{(2+\eta(1-\theta))^3}{(1-\eta)^2(1-\theta)^2}\Psi(w^{(d)}) = \Gamma_0 + \Gamma_1\eta + \Gamma_2\eta^2 + \Gamma_3\eta^3 + \Gamma_4\eta^4 + \Gamma_5\eta^5 \equiv G(\eta,\theta),$$

where $\Gamma_0 = -32\theta$, $\Gamma_1 = 8(2-19\theta+\theta^3)$, $\Gamma_2 = 4(19-68\theta-2\theta^2+4\theta^3-\theta^4)$, $\Gamma_3 = 4(1-\theta)(35-13\theta-7\theta^2+\theta^3)$, $\Gamma_4 = 113-29\theta+50\theta^2+30\theta^3-3\theta^4-\theta^5$ and $\Gamma_5 = (3+\theta)(11+6\theta^2-\theta^4)$. We note that $G(0,\theta) = \Gamma_0 \leq 0$, $\partial G(\eta,\theta)/\partial\eta\big|_{\eta=0} = \Gamma_1 \geq [\leq] 0$ for $\theta \leq [\geq] 0.105$, $\partial^2 G(\eta,\theta)/\partial\eta^2\big|_{\eta=0} = 2\Gamma_2 \geq [\leq] 0$ for $\theta \leq [\geq] 0.278$, and $\partial^3 G(\eta,\theta)/\partial\eta^3 = 6\Gamma_3 + 24\Gamma_4\eta + 60\Gamma_5\eta^2 \geq 0$. This implies that $\partial G(\eta,\theta)/\partial\eta$ is convex in $\eta \in [0,1]$. Moreover, the sign of $\partial G(\eta,\theta)/\partial\eta$ (i) is always positive or (ii) changes at most once and the change is from negative to positive. This further implies that $G(\eta,\theta) = 0$ has at most one root within the range $\eta \in [0,1]$. Let $\bar{\eta}^{NB}$ denote this root. In the case that no root exists within the range, we set $\bar{\eta}^{NB} = 1$. Similarly, we can show that $\partial G(\eta,\theta)/\partial\theta < 0$ and thus $\bar{\eta}^{NB}$ is increasing in θ. Thus, we deduce that $\Xi_3(\eta,\theta) \geq [\leq] 0$ for $\eta \leq [\geq] \bar{\eta}^{NB}$.

Comparing $\breve{\pi}^{NB}_{sim}$ and $\breve{\pi}^{NB}_{seq}$, we obtain

$$\breve{\pi}^{NB}_{sim} - \breve{\pi}^{NB}_{seq} = \frac{1}{16(1-\eta^2)(2-\eta(1-\theta))^2}\Xi_4(\eta,\theta),$$

where $\Xi_4(\eta,\theta) = 8(1+\theta)^2(1-\eta) - (2-\eta(1-\theta))^2(4(1-\eta)^2(1-\breve{w}^{NB}_{seq1})^2 + (1+\theta)^2(1-\eta+\eta\breve{w}^{NB}_{seq1})^2)$. Note that $\Xi_4(\eta,\theta)$ is a quadratic function with the coefficient of $(\breve{w}^{NB}_{seq1})^2$ being negative. $\Xi_4(\eta,\theta) = 0$ has two roots, $w^{(a)} = (1-\theta)(1-\eta)/(2-\eta(1-\theta))$ and $w^{(e)} = (1-\eta)((4-\eta^2(1-\theta)^2)(3+\theta) + 4\eta(1-\theta^2))/(8-4\eta(1-\theta)-2\eta^2(3+\theta)(1-\theta)+\eta^3(1-\theta)^2(3+\theta))$. We have $w^{(a)} \leq \breve{w}^{NB}_{seq1} \leq w^u \leq w^{(e)}$ and thus $\Xi_4(\eta,\theta) \geq 0$. This concludes part (ii). □

Proof of Proposition 6. We first consider the profit comparison under the NB solution. From the proof of Lemma 3, we can show that $\hat{\Pi}^{NB}_{sim} \geq [\leq] \hat{\Pi}^{NB}_{seq}$ and $\hat{\pi}^{NB}_{seq1} \geq \hat{\pi}^{NB}_{sim} \geq \hat{\pi}^{NB}_{seq2}$ $[\hat{\pi}^{NB}_{seq1} \leq \hat{\pi}^{NB}_{sim} \leq \hat{\pi}^{NB}_{seq2}]$

for $\hat{w}_{sim}^{NB} \geq [\leq] \hat{w}_{seq1}^{NB}$. Thus, it suffices to compare the difference between $\hat{w}_{sim}^{NB}$ and $\hat{w}_{seq1}^{NB}$. The Nash product for the first trade in sequential negotiation is

$$\Omega_1(w_1) = \Pi_1(w_1, \hat{w}_2^{NB}(w_1))^{1-\theta} \pi_1(w_1, \hat{w}_2^{NB}(w_1))^{\theta},$$

where $\Pi_1(\cdot,\cdot)$ and $\pi_1(\cdot,\cdot)$ are respectively first piece in (56) and (57), and $\hat{w}_2^{NB}(\cdot)$ is given by (60).

Setting $\mathrm{d}\ln\Omega_1(w_1)/\mathrm{d}w_1 = 0$ gives

$$\begin{aligned}&\frac{\partial}{\partial w_1}\Big[\ln \Pi_1(w_1,w_2)^{(1-\theta)}\pi_1(w_1,w_2)^{\theta}\Big]\Big|_{w_2=\hat{w}_2^{NB}(w_1)} \\ &\qquad + \frac{\partial}{\partial w_2}\Big[\ln \Pi_1(w_1,w_2)^{(1-\theta)}\pi_1(w_1,w_2)^{\theta}\Big]\frac{\mathrm{d}\hat{w}_2^{NB}(w_1)}{\mathrm{d}w_1}\Big|_{w_2=\hat{w}_2^{NB}(w_1)} \equiv \Gamma_1 + \Gamma_2 = 0.\end{aligned}$$

We note that $\Gamma_1 \geq [\leq] 0$ for $w_1 \leq [\geq] \hat{w}_{sim}^{NB}$. Also note that the first term of Γ_2 is above zero and the second term of Γ_2 gives

$$\frac{\mathrm{d}\hat{w}_2^{NB}(w_1)}{\mathrm{d}w_1}\Big|_{w_1=\hat{w}_{sim}^{NB}} = \frac{\eta}{2} + \frac{4(2+\eta)\theta^2}{\eta(1-\theta)^2 + 2(\theta^2 - 4\theta - 1)} \geq 0$$

for $(1+5\theta - \sqrt{(1+\theta)(1+9\theta)})/(1-\theta) < \eta < 1$ and $0 < \theta < (5+2\sqrt{13})/27$. Thus, we deduce that $\bar{\eta}^{NB} = \big((1+5\theta - \sqrt{(1+\theta)(1+9\theta)})/(1-\theta)\big) \wedge 1$ and $\bar{\eta}^{NB}$ is increasing in θ. This yields that $\hat{w}_{sim}^{NB} \leq [\geq] \hat{w}_{seq1}^{NB}$ for $\eta \geq [\leq] \bar{\eta}^{NB}$.

Now we consider the profit comparison under the KS solution. We focus on the case when no extension of Pareto profit allocation set is needed (see the detailed discussion in Remark 1), i.e., $\eta \in [0, 0.39]$. Now we compare $\hat{w}_{sim}^{KS}$ and $\hat{w}_{seq1}^{KS}$. Let $\Pi_1(w_1) \equiv \Pi_1(w_1, \hat{w}_2^{KS}(w_1))$ and $\pi_1(w_1) \equiv \pi_1(w_1, \hat{w}_2^{KS}(w_1))$, where $\hat{w}_2^{KS}(\cdot)$ is given by (62). By Proposition 4, the supplier's and retailer 1's maximum profits are $\overline{\Pi}_1 = \Pi_1(1/2)$ and $\overline{\pi}_1 = \pi_1(1/2 - \sqrt{1/(2+\eta)})$, respectively. By (2), $\hat{w}_{seq1}^{KS}$ and $\hat{w}_{sim}^{KS}$ should satisfy

$$\frac{\pi_1(w_1)}{\Pi_1(w_1)} = \frac{\overline{\pi}_1}{\overline{\Pi}_1} \quad \text{and} \quad \frac{\pi_1(w_1)}{\Pi_1(w_1)} = \frac{\overline{\pi}_1(\hat{w}_2^{KS}(w_1))}{\overline{\Pi}_1(\hat{w}_2^{KS}(w_1))},$$

where $\overline{\Pi}_1(\cdot)$ and $\overline{\pi}_1(\cdot)$ are respectively first pieces in (58) and (59). We first note that $\pi_1(w_1)/\Pi_1(w_1)$ is decreasing in w_1. We also note that $\overline{\pi}_1(\hat{w}_2^{KS}(w_1))/\overline{\Pi}_1(\hat{w}_2^{KS}(w_1))$ is increasing in w_1 for $w_1 \leq 0.13$ (i.e., the maximum possible value of $\hat{w}_{sim}^{KS}$), and $\overline{\pi}_1/\overline{\Pi}_1 > \overline{\pi}_1(\hat{w}_2^{KS}(0.13))/\overline{\Pi}_1(\hat{w}_2^{KS}(0.13))$. Thus, we conclude that $\hat{w}_{seq1}^{KS} < \hat{w}_{sim}^{KS} < \hat{w}_{seq2}^{KS}$. □

Proof of Corollary 1. To see part (i), we can compute the supplier's and retailer's profits as

$$\Pi(w) = wq^*(w) = w\big(\frac{1-w}{k+1}\big)^{1/k} \quad \text{and} \quad \pi(w) = q^*(w)(p^*(w) - w) = k\big(\frac{1-w}{k+1}\big)^{1/k+1},$$

and their maximum profits are

$$\overline{\Pi} = \Pi\big(\frac{k}{k+1}\big) = k\big(\frac{1}{k+1}\big)^{2/k+1} \text{ and } \overline{\pi} = \pi(0) = k\big(\frac{1}{k+1}\big)^{1/k+1}.$$

Substituting the above expressions into (1) and (2) gives $w^{NB} = k(1-\theta)/(k+1)$ and $w^{KS} = k/((k+1)^{1/k+1}+k)$, respectively. Setting $w^{NB} = w^{KS}$ gives $\theta^{KS}(k) = 1-(k+1)/((k+1)^{1/k+1}+k)$. This concludes part (i).

To see part (ii), we can compute the supplier's and retailer's profits as

$$\Pi(w) = wq^*(w) = w\exp\big(\frac{1-k-w}{k}\big) \text{ and } \pi(w) = q^*(w)(p^*(w)-w) = k\exp\big(\frac{1-k-w}{k}\big),$$

and their maximum profits are

$$\overline{\Pi} = \Pi(k) = k\exp\big(\frac{1-2k}{k}\big) \text{ and } \overline{\pi} = \pi(0) = k\exp\big(\frac{1-k}{k}\big).$$

Substituting the above expressions into (1) and (2) gives $w^{NB} = k(1-\theta)$ and $w^{KS} = k/e$, respectively. Setting $w^{NB} = w^{KS}$ gives $\theta^{KS}(k) = 1-1/e$. This concludes part (ii).

To see part (iii), we can compute the supplier's and retailer's profits as

$$\Pi(w) = (w-c)q^*(w) = (w-c)\big(\frac{1-k}{w}\big)^{1/k}, \text{ and } \pi(w) = q^*(w)(p^*(w)-w) = k\big(\frac{1-k}{w}\big)^{1/k-1},$$

and their maximum profits are

$$\overline{\Pi} = \Pi\big(\frac{c}{1-k}\big) = \frac{kc}{1-k}\big(\frac{(1-k)^2}{c}\big)^{1/k} \text{ and } \overline{\pi} = \pi(c) = k\big(\frac{1-k}{c}\big)^{1/k-1}.$$

Substituting the above expressions into (1) and (2) gives $w^{NB} = c(1-k\theta)/(1-k)$ and $w^{KS} = c/(1-k(1-k)^{1/k-1})$. Setting $w^{NB} = w^{KS}$ gives $\theta^{KS}(k) = (1-(1-k)^{1/k-1})/(1-k(1-k)^{1/k-1})$. This concludes part (iii). ☐

Proof of Proposition 7. There are eight cases to analyze, depending on the industry structures, contingency terms and negotiation sequence. We present the detailed analysis for one case and omit the others as they follow in the similar way. Specifically, we focus on the simultaneous bargaining without contingency in the one-to-two channel. We consider the negotiation in unit i for a given (v_j, F_j) from unit j. The supplier's and retailer i's profits are

$$\Pi_i(v_i, v_j, F_i, F_j) = R_i(v_i, v_j, F_i) + R_j(v_j, v_i, F_j) \text{ and } \pi_i(v_i, v_j, F_i) = r_i(v_i, v_j, F_i).$$

The retailer i has zero disagreement point (i.e., $d_i(v_j, F_j) = 0$) and the supplier's disagreement point is $D_i(v_j, F_j) = R_j(v_j, a-\eta(a-v_j)/2, F_j)$. Their maximum profits are

$$\begin{aligned} \overline{\Pi}(v_j, F_j) &= \max\{\Pi_i(v_i, v_j, F_i, F_j) : \pi_i(v_i, v_j, F_i) \geq d_i(v_j, F_j)\}, \\ \overline{\pi}(v_j, F_j) &= \max\{\pi_i(v_i, v_j, F_i) : \Pi_i(v_i, v_j, F_i, F_j) \geq D_i(v_j, F_j)\}. \end{aligned}$$

We note that though trade parties' profits and disagreement points may depend on F_j, their total trade surplus $\Pi_i(v_i, v_j, F_i, F_j) + \pi_i(v_i, v_j, F_i) - D_i(v_j, F_j) - d_i(v_j, F_j)$ is independent of F_j. This implies that total trade surplus can be allocated between the supplier and the retailer i by varying F_i. Thus, we have $\overline{\Pi}(v_j, F_j) - D_i(v_j, F_j) = \overline{\pi}(v_j, F_j) - d_i(v_j, F_j)$. Consequently, F_i should split the total surplus evenly between the trade parties and unit payment v_i should maximize the total trade surplus. Therefore, the KS solution coincides with the symmetric NB solution. □

Proof of Lemma 4. We first consider the one-to-two channel. Setting $\widetilde{\widetilde{q}}_i^*(\boldsymbol{u}, \boldsymbol{\ell}) = q_i^*(\boldsymbol{v})$ gives $v_i(\boldsymbol{u}, \boldsymbol{\ell}) = u_i/\ell_i$, where $\widetilde{\widetilde{q}}_i^*(\cdot)$ and $q_i^*(\cdot)$ are derived in Lemma C.1-(i) and Lemma 1-(i), respectively. Thus, the fixed payments resulting the same trade profits are

$$\begin{aligned}
F_i(\boldsymbol{u}, \boldsymbol{\ell}) &= q_i^*(\boldsymbol{v}(\boldsymbol{u}, \boldsymbol{\ell}))(p_i^*(\boldsymbol{v}(\boldsymbol{u}, \boldsymbol{\ell})) - v_i(\boldsymbol{u}, \boldsymbol{\ell})) - \big(\ell_i \widetilde{\widetilde{p}}_i^*(\boldsymbol{u}, \boldsymbol{\ell}) \widetilde{\widetilde{q}}_i^*(\boldsymbol{u}, \boldsymbol{\ell}) - u_i \widetilde{\widetilde{q}}_i^*(\boldsymbol{u}, \boldsymbol{\ell})\big) \\
&= q_i^*(\boldsymbol{v}(\boldsymbol{u}, \boldsymbol{\ell}))(p_i^*(\boldsymbol{v}(\boldsymbol{u}, \boldsymbol{\ell})) - v_i(\boldsymbol{u}, \boldsymbol{\ell})) - \big(\ell_i p_i^*(\boldsymbol{v}(\boldsymbol{u}, \boldsymbol{\ell})) q_i^*(\boldsymbol{v}(\boldsymbol{u}, \boldsymbol{\ell})) - u_i q_i^*(\boldsymbol{v}(\boldsymbol{u}, \boldsymbol{\ell}))\big) \\
&= q_i^*(\boldsymbol{v}(\boldsymbol{u}, \boldsymbol{\ell}))\big((1 - \ell_i) p_i^*(\boldsymbol{v}(\boldsymbol{u}, \boldsymbol{\ell})) + u_i - v_i(\boldsymbol{u}, \boldsymbol{\ell})\big).
\end{aligned}$$

Now we turn to the two-to-one channel. Similarly, we can set $\widetilde{\widetilde{q}}_i^*(\boldsymbol{u}, \boldsymbol{\ell}) = q_i^*(\boldsymbol{v})$ and obtain $v_i(\boldsymbol{u}, \boldsymbol{\ell}) = \frac{2u_i((2-\eta^2)\ell_j - \eta^2 \ell_i) + \eta(\ell_j - \ell_i)(((2-\eta)\ell_j - \eta \ell_i)a - 2u_j)}{4\ell_i \ell_j - (\ell_i + \ell_j)\eta^2}$, where $\widetilde{\widetilde{q}}_i^*(\cdot)$ and $q_i^*(\cdot)$ are derived in Lemma C.1-(ii) and Lemma 1-(ii), respectively. Correspondingly, the fixed payments are $F_i(\boldsymbol{u}, \boldsymbol{\ell}) = q_i^*(\boldsymbol{v}(\boldsymbol{u}, \boldsymbol{\ell}))((1 - \ell_i) p_i^*(\boldsymbol{v}(\boldsymbol{u}, \boldsymbol{\ell})) + u_i - v_i(\boldsymbol{u}, \boldsymbol{\ell}))$. This concludes the proof. □

Proof of Lemma 5. We first consider the one-to-two channel. Setting $\tilde{q}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma}) = q_i^*(\boldsymbol{v})$ gives $v_i(\boldsymbol{\omega}, \boldsymbol{\gamma}) = \max\{\omega_i - \gamma_i b \tilde{q}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma}), c\}$, where $\tilde{q}_i^*(\cdot)$ is derived in Lemma C.2-(i). Therefore, the fixed payments resulting the same trade profits are

$$\begin{aligned}
F_i(\boldsymbol{\omega}, \boldsymbol{\gamma}) &= q_i^*(\boldsymbol{v}(\boldsymbol{\omega}, \boldsymbol{\gamma}))(p_i^*(\boldsymbol{v}(\boldsymbol{\omega}, \boldsymbol{\gamma})) - v_i(\boldsymbol{\omega}, \boldsymbol{\gamma})) - \big(\tilde{q}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma}) \tilde{p}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma}) - T_i(\tilde{q}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma}))\big) \\
&= T_i(\tilde{q}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma})) - \tilde{q}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma}) v_i(\boldsymbol{\omega}, \boldsymbol{\gamma}).
\end{aligned}$$

Now we turn to the two-to-one channel. We can follow the similar logic and show that $v_i(\boldsymbol{\omega}, \boldsymbol{\gamma}) = \max\{\omega_i - \gamma_i b \tilde{q}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma}), c\}$ and $F_i(\boldsymbol{\omega}, \boldsymbol{\gamma}) = T_i(\tilde{q}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma})) - \tilde{q}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma}) v_i(\boldsymbol{\omega}, \boldsymbol{\gamma})$, where $\tilde{q}_i^*(\cdot)$ is derived in Lemma C.2-(ii). This concludes the proof. □

C Market Equilibrium under Coordinating Contracts

In this subsection, we provide the expressions of retail quantities and prices when the negotiations are conducted over revenue-sharing and quality discount contracts. We shall note that the proofs are analogous to that of Lemma 1.

Lemma C.1 (Retail Quantities and Prices: Revenue Sharing Contracts)**.** *Given the negotiated contract parameters* $(\boldsymbol{u}, \boldsymbol{\ell}) = (u_i, u_j, \ell_i, \ell_j)$, $i, j \in \{1, 2\}$ *and* $i \neq j$.

- *i)* **One-to-two channel:** *the competing retailers set the following equilibrium quantities and prices: a) if* $\frac{\ell_i a - u_i}{\ell_j a - u_j} \leq \frac{\eta \ell_i}{2\ell_j}$, $\tilde{\tilde{q}}_i^*(\boldsymbol{u}, \boldsymbol{\ell}) = 0$ *and* $\tilde{\tilde{p}}_i^*(\boldsymbol{u}, \boldsymbol{\ell}) = \frac{2\ell_i a - \eta(\ell_i a - u_i)}{2\ell_i}$; *b) if* $\frac{\eta \ell_i}{2\ell_j} < \frac{\ell_i a - u_i}{\ell_j a - u_j} < \frac{2\ell_i}{\eta \ell_j}$, $\tilde{\tilde{q}}_i^*(\boldsymbol{u}, \boldsymbol{\ell}) = \frac{2\ell_j(\ell_i a - u_i) - \eta \ell_i(\ell_j a - u_j)}{\ell_i \ell_j b(4 - \eta^2)}$ *and* $\tilde{\tilde{p}}_i^*(\boldsymbol{u}, \boldsymbol{\ell}) = \frac{u_i}{\ell_i} + b\tilde{\tilde{q}}_i^*(\boldsymbol{u}, \boldsymbol{\ell})$; *c) if* $\frac{\ell_i a - u_i}{\ell_j a - u_j} \geq \frac{2\ell_i}{\eta \ell_j}$, $\tilde{\tilde{q}}_i^*(\boldsymbol{u}, \boldsymbol{\ell}) = \frac{\ell_i a - u_i}{2\ell_i b}$ *and* $\tilde{\tilde{p}}_i^*(\boldsymbol{u}, \boldsymbol{\ell}) = \frac{\ell_i a + u_i}{2}$.

- *ii)* **Two-to-one channel:** *the common retailer sets the following equilibrium quantities and prices: a) if* $\frac{\ell_i a - u_i}{\ell_j a - u_j} \leq \frac{\eta(\ell_i + \ell_j)}{2\ell_j}$, $\tilde{\tilde{q}}_i^*(\boldsymbol{u}, \boldsymbol{\ell}) = 0$ *and* $\tilde{\tilde{p}}_i^*(\boldsymbol{u}, \boldsymbol{\ell}) = \frac{2\ell_i a - \eta(\ell_i a - u_i)}{2\ell_i}$; *b) if* $\frac{\eta(\ell_i + \ell_j)}{2\ell_j} < \frac{\ell_i a - u_i}{\ell_j a - u_j} < \frac{2\ell_i}{\eta(\ell_i + \ell_j)}$, $\tilde{\tilde{q}}_i^*(\boldsymbol{u}, \boldsymbol{\ell}) = \frac{2\ell_j(\ell_i a - u_i) - \eta(\ell_i + \ell_j)(\ell_j a - u_j)}{b(4\ell_i \ell_j - (\ell_i + \ell_j)^2 \eta^2)}$ *and* $\tilde{\tilde{p}}_i^*(\boldsymbol{u}, \boldsymbol{\ell}) = \frac{\ell_j(2 - \eta^2)(\ell_i a + u_i) - \eta^2(\ell_j^2 a + \ell_i u_i)}{4\ell_i \ell_j - (\ell_i + \ell_j)^2 \eta^2} + \frac{\eta(\ell_j - \ell_i)(\ell_j a - u_j)}{4\ell_i \ell_j - (\ell_i + \ell_j)^2 \eta^2}$; *c) if* $\frac{\ell_i a - u_i}{\ell_j a - u_j} \geq \frac{2\ell_i}{\eta(\ell_i + \ell_j)}$, $\tilde{\tilde{q}}_i^*(\boldsymbol{u}, \boldsymbol{\ell}) = \frac{\ell_i a - u_i}{2\ell_i b}$ *and* $\tilde{\tilde{p}}_i^*(\boldsymbol{u}, \boldsymbol{\ell}) = \frac{\ell_i a + u_i}{2}$.

Lemma C.2 (Retail Quantities and Prices: Quantity Discount Contracts)**.** *Given the negotiated contract parameters* $(\boldsymbol{\omega}, \boldsymbol{\gamma}) = (\omega_i, \omega_j, \gamma_i, \gamma_j)$, $i, j \in \{1, 2\}$ *and* $i \neq j$.

- *i)* **One-to-two channel:** *the competing retailers set the following equilibrium quantities and prices: a) if* $\frac{a - \omega_i}{a - \omega_j} \leq \frac{\eta}{2 - \gamma_j}$ *and* $\frac{\omega_i - c}{a - c} > \frac{2 - \eta}{2}$, $\tilde{q}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma}) = 0$ *and* $\tilde{p}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma}) = a - \eta \min\left\{\frac{a - w_j}{2 - \gamma_j}, \frac{a - c}{2}\right\}$; *b) if* $\frac{\eta}{2 - \gamma_j} \leq \frac{a - \omega_i}{a - \omega_j} \leq \frac{2 - \gamma_i}{\eta}$, $\frac{\omega_i - c}{a - c} > \frac{\gamma_i}{2 + \eta}$ *and* $\frac{\gamma_j \eta}{4 - 2\gamma_i - \eta^2} \leq \frac{(2 + \eta)(\omega_j - c) - \gamma_j(a - c)}{(2 + \eta)(\omega_i - c) - \gamma_i(a - c)} \leq \frac{4 - 2\gamma_j - \eta^2}{\gamma_i \eta}$, $\tilde{q}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma}) = \frac{(2 - \gamma_j)(a - \omega_i) - \eta(a - \omega_j)}{b((2 - \gamma_i)(2 - \gamma_j) - \eta^2)}$ *and* $\tilde{p}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma}) = \omega_i + (1 - \gamma_i)b\tilde{q}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma})$; *c) if* $\frac{\gamma_i}{2 + \eta} \leq \frac{\omega_i - c}{a - c} \leq \frac{2 - \eta}{2}$ *and* $\frac{(2 + \eta)(\omega_j - c) - \gamma_j(a - c)}{(2 + \eta)(\omega_i - c) - \gamma_i(a - c)} < \frac{\gamma_j \eta}{4 - 2\gamma_i - \eta^2}$, $\tilde{q}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma}) = \frac{2(a - \omega_i) - \eta(a - c)}{b(4 - 2\gamma_i - \eta^2)}$ *and* $\tilde{p}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma}) = w_i + (1 - \gamma_i)b\tilde{q}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma})$; *d) if* $\frac{\gamma_j}{2 + \eta} \leq \frac{\omega_j - c}{a - c} \leq \frac{2 - \eta}{2}$ *and* $\frac{(2 + \eta)(\omega_i - c) - \gamma_i(a - c)}{(2 + \eta)(\omega_j - c) - \gamma_j(a - c)} < \frac{\gamma_i \eta}{4 - 2\gamma_j - \eta^2}$, $\tilde{q}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma}) = \frac{(2 - \gamma_j)(a - c) - \eta(a - w_j)}{b(4 - 2\gamma_j - \eta^2)}$ *and* $\tilde{p}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma}) = c + b\tilde{q}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma})$; *e) if* $\frac{\omega_i - c}{a - c} < \frac{\gamma_i}{2 + \eta}$ *and* $\frac{\omega_j - c}{a - c} < \frac{\gamma_j}{2 + \eta}$, $\tilde{q}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma}) = \frac{a - c}{b(2 + \eta)}$ *and* $\tilde{p}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma}) = c + b\tilde{q}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma})$; *f) if* $\frac{a - \omega_i}{a - \omega_j} \geq \frac{2 - \gamma_i}{\eta}$ *and* $\frac{\omega_j - c}{a - c} > \frac{2 - \eta}{2}$, $\tilde{q}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma}) = \min\left\{\frac{a - w_i}{(2 - \gamma_i)b}, \frac{a - c}{2b}\right\}$ *and* $\tilde{p}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma}) = a - \min\left\{\frac{a - w_i}{(2 - \gamma_i)}, \frac{a - c}{2}\right\}$.

- *ii)* **Two-to-one channel:** *the common retailer sets the following equilibrium quantities and prices: a) if* $\frac{a - \omega_i}{a - \omega_j} \leq \frac{2\eta}{2 - \gamma_j}$ *and* $\frac{\omega_i - c}{a - c} > 1 - \eta$, $\tilde{q}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma}) = 0$ *and* $\tilde{p}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma}) = a - \eta \min\left\{\frac{a - w_j}{2 - \gamma_j}, \frac{a - c}{2}\right\}$; *b) if* $\frac{2\eta}{2 - \gamma_j} \leq \frac{a - \omega_i}{a - \omega_j} \leq \frac{2 - \gamma_i}{2\eta}$, $\frac{\omega_i - c}{a - c} > \frac{\gamma_i}{2(1 + \eta)}$ *and* $\frac{\gamma_j \eta}{2 - \gamma_i - 2\eta^2} \leq \frac{2(1 + \eta)(\omega_j - c) - \gamma_j(a - c)}{2(1 + \eta)(\omega_i - c) - \gamma_i(a - c)} \leq \frac{2 - \gamma_j - 2\eta^2}{\gamma_i \eta}$, $\tilde{q}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma}) = \frac{(2 - \gamma_j)(a - \omega_i) - 2\eta(a - \omega_j)}{b((2 - \gamma_i)(2 - \gamma_j) - 4\eta^2)}$ *and* $\tilde{p}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma}) = \frac{a + \omega_i - \gamma_i b\tilde{q}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma})}{2}$; *c) if* $\frac{\gamma_i}{2(1 + \eta)} \leq \frac{\omega_i - c}{a - c} \leq 1 - \eta$ *and* $\frac{2(1 + \eta)(\omega_j - c) - \gamma_j(a - c)}{2(1 + \eta)(\omega_i - c) - \gamma_i(a - c)} < \frac{\gamma_j \eta}{2 - \gamma_i - 2\eta^2}$, $\tilde{q}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma}) = \frac{(a - \omega_i) - \eta(a - c)}{b(2 - \gamma_i - 2\eta^2)}$ *and* $\tilde{p}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma}) = \frac{a + \omega_i - \gamma_i b\tilde{q}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma})}{2}$; *d) if* $\frac{\gamma_j}{2(1 + \eta)} \leq \frac{\omega_j - c}{a - c} \leq 1 - \eta$ *and* $\frac{2(1 + \eta)(\omega_i - c) - \gamma_i(a - c)}{2(1 + \eta)(\omega_j - c) - \gamma_j(a - c)} < \frac{\gamma_i \eta}{2 - \gamma_j - 2\eta^2}$, $\tilde{q}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma}) = \frac{(2 - \gamma_j)(a - c) - 2\eta(a - w_j)}{2b(2 - \gamma_j - 2\eta^2)}$ *and* $\tilde{p}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma}) = \frac{a + c}{2}$; *e) if* $\frac{\omega_i - c}{a - c} < \frac{\gamma_i}{2(1 + \eta)}$ *and* $\frac{\omega_j - c}{a - c} < \frac{\gamma_j}{2(1 + \eta)}$, $\tilde{q}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma}) = \frac{a - c}{2b(1 + \eta)}$ *and* $\tilde{p}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma}) = \frac{a + c}{2}$; *f) if* $\frac{a - \omega_i}{a - \omega_j} \geq \frac{2 - \gamma_i}{2\eta}$ *and* $\frac{\omega_j - c}{a - c} > 1 - \eta$, $\tilde{q}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma}) = \min\left\{\frac{a - w_i}{(2 - \gamma_i)b}, \frac{a - c}{2b}\right\}$ *and* $\tilde{p}_i^*(\boldsymbol{\omega}, \boldsymbol{\gamma}) = a - \min\left\{\frac{a - w_i}{(2 - \gamma_i)}, \frac{a - c}{2}\right\}$.