

# Online Appendix

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# 1 Asset buyers' posterior beliefs

We derive the functional form of  $\omega_M^B(\hat{\theta})$ , where  $M \in \{1, 2\}$ , i.e., expression (6) in the manuscript. Asset buyers evaluate the posterior probability of  $s = B$  as  $\omega_M^B(\hat{\theta})$  given their beliefs of creditors' equilibrium strategy and the observation of  $M$  bank runs. By Bayes' rule

$$\begin{aligned}\omega_M^B(\hat{\theta}) &\equiv \text{Prob}(s = B | \theta < \hat{\theta}, M) = \frac{\text{Prob}(s = B, \theta < \hat{\theta}, M)}{\text{Prob}(\theta < \hat{\theta}, M)} \\ &= \frac{\text{Prob}(s = B) \cdot \text{Prob}(\theta < \hat{\theta}, M | s = B)}{\text{Prob}(s = B) \cdot \text{Prob}(\theta < \hat{\theta}, M | s = B) + \text{Prob}(s = G) \cdot \text{Prob}(\theta < \hat{\theta}, M | s = G)}.\end{aligned}$$

We can further express  $\omega_M^B(\hat{\theta})$  as follows:

$$\begin{aligned}\omega_M^B(\hat{\theta}) &= \frac{(1 - \alpha) \left(\frac{\hat{\theta} - \underline{\theta}_B}{\bar{\theta} - \underline{\theta}_B}\right)^M \left(\frac{\bar{\theta} - \hat{\theta}}{\bar{\theta} - \underline{\theta}_B}\right)^{2-M}}{(1 - \alpha) \left(\frac{\hat{\theta} - \underline{\theta}_B}{\bar{\theta} - \underline{\theta}_B}\right)^M \left(\frac{\bar{\theta} - \hat{\theta}}{\bar{\theta} - \underline{\theta}_B}\right)^{2-M} + \alpha \left(\frac{\hat{\theta} - \underline{\theta}_G}{\bar{\theta} - \underline{\theta}_G}\right)^M \left(\frac{\bar{\theta} - \hat{\theta}}{\bar{\theta} - \underline{\theta}_G}\right)^{2-M}} \\ &= \frac{(1 - \alpha) \left(\frac{\hat{\theta} - \underline{\theta}_B}{\bar{\theta} - \underline{\theta}_B}\right)^M}{(1 - \alpha) \left(\frac{\hat{\theta} - \underline{\theta}_B}{\bar{\theta} - \underline{\theta}_B}\right)^M + \alpha \left(\frac{\hat{\theta} - \underline{\theta}_G}{\bar{\theta} - \underline{\theta}_G}\right)^M \left(\frac{\bar{\theta} - \hat{\theta}}{\bar{\theta} - \underline{\theta}_G}\right)^{2-M}} = \frac{(\hat{\theta} - \underline{\theta}_B)^M}{(\hat{\theta} - \underline{\theta}_B)^M + \kappa (\hat{\theta} - \underline{\theta}_G)^M}.\end{aligned}$$

For simplicity, we define  $\kappa \equiv \frac{\alpha}{1 - \alpha} \left(\frac{\bar{\theta} - \underline{\theta}_B}{\bar{\theta} - \underline{\theta}_G}\right)^2$  as an exogenous parameter, which does not depend on the endogenous threshold  $\hat{\theta}$ .

## 2 Complementary results for the global game analyses

This section derives two parametric restrictions on the precision of signals to ensure creditors' posterior beliefs are well defined, proves the monotonicity of  $L^i(x_j^i, y^{-i})$  and  $L^{-i}(x_j^i, y^{-i})$  (in Appendix A.2) and calculates the representative creditor's payoff difference function (in Appendix B2).

### 2.1 Restrictions on the precision of signals

Recall that the prior distribution of  $\theta^i$  is  $\tilde{\theta}^i \sim U(\underline{\theta}_s, \bar{\theta})$  for each state  $s$  and a creditor  $j$  of a bank  $i$  observes a private signal  $x_j^i = \theta^i + \epsilon_j^i$  with  $\epsilon_j^i$  drawn from  $U(-\epsilon, \epsilon)$ . For  $x_j^i$  such that  $x_j^i - \epsilon > \underline{\theta}_G$  and  $x_j^i + \epsilon < \bar{\theta}$ , we have that for every value of  $\theta^i$  that the signal assigns a positive density, the prior distribution also assigns a positive density to it. The uniform prior of  $\theta^i$  becomes uninformative when the creditor  $j$  Bayesian updates the posterior. Note that both  $\theta^i$  and  $x_j^i$  have the same prior density  $f(\theta^i) = f(x_j^i) = \frac{1}{\bar{\theta} - \underline{\theta}_s}$  in a given state  $s$ , and the density of  $x_j^i$

conditional on  $\theta^i$  is  $f(x_j^i|\theta^i) = \frac{1}{2\epsilon}$ . By Bayes's rule, we have:

$$f(\theta^i|x_j^i) = \frac{f(\theta^i) \cdot f(x_j^i|\theta^i)}{f(x_j^i)} = \frac{\frac{1}{\bar{\theta}-\underline{\theta}_s} \cdot \frac{1}{2\epsilon}}{\frac{1}{\bar{\theta}-\underline{\theta}_s}} = \frac{1}{2\epsilon}. \quad (1)$$

The posterior belief of  $\theta^i$  conditional on  $x_j^i$  is simply  $\tilde{\theta}^i|x_j^i \sim U(x_j^i - \epsilon, x_j^i + \epsilon)$  for such a signal.

Recall that we need to analyze the creditors' posterior beliefs explicitly only if they do not have dominant strategies, i.e., when  $x_j^i \in [x^L, x^U] \equiv [\theta^L - \epsilon, \theta^U + \epsilon]$ . To ensure that the prior of  $\theta^i$  is uninformative and the posterior belief of a creditor  $j$  when  $x_j^i \in [x^L, x^U]$  is given by (1), we let  $x^L - \epsilon > \underline{\theta}_G$  and  $x^U + \epsilon < \bar{\theta}$ . The condition then becomes  $2\epsilon < \min\{\theta^L - \underline{\theta}_G, \bar{\theta} - \theta^U\}$ .

Similarly, for a signal  $y^i$  such that  $y^i - \eta > \underline{\theta}_G$  and  $y^i + \eta < \bar{\theta}$ , we also have  $\tilde{\theta}^i|y^i \sim U(y^i - \eta, y^i + \eta)$ . From an ex-ante perspective,  $\tilde{x}_j^i$  and  $\tilde{y}^i$  are correlated and the conditional density  $f(y^i|x_j^i)$  is given by expression (A.24) in Appendix A.2 of the manuscript. When  $x_j^i \in [x^L, x^U]$ , the lowest and highest possible  $y^i$  are  $x^L - \eta - \epsilon$  and  $x^U + \eta + \epsilon$ , respectively. Therefore, we let  $x^L - \eta - \epsilon - \eta > \underline{\theta}_G$  and  $x^U + \eta + \epsilon + \eta < \bar{\theta}$ , that is  $2\eta < \min\{\theta^L - \underline{\theta}_G - 2\epsilon, \bar{\theta} - \theta^U - 2\epsilon\}$ , to ensure the prior of  $\theta^i$  is uninformative and the conditional density  $f(y^i|x_j^i)$  is non-zero and given by expression (A.24) for any  $x_j^i \in [x^L, x^U]$ .

In sum, when creditors' signals are not too noisy, i.e.,  $\epsilon$  and  $\eta$  are small relative to the support of banks' fundamentals, the creditors' posterior beliefs are well-defined.

## 2.2 The monotonicity of $L^i(x_j^i, y^{-i})$ and $L^{-i}(x_j^i, y^{-i})$

We prove  $\frac{\partial L^i(x_j^i, y^{-i})}{\partial x_j^i} \leq 0$ ,  $\frac{\partial L^i(x_j^i, y^{-i})}{\partial y^{-i}} \leq 0$ ,  $\frac{\partial L^{-i}(x_j^i, y^{-i})}{\partial x_j^i} \leq 0$  and  $\frac{\partial L^{-i}(x_j^i, y^{-i})}{\partial y^{-i}} \leq 0$ .

Recall the functional form of  $L^i(x_j^i, y^{-i})$  in Appendix A.2 of the manuscript. We have  $\frac{\partial L^i(x_j^i, y^{-i})}{\partial x_j^i} = -\frac{x^*(y^{-i}) - x_j^i + 2\epsilon}{4\epsilon^2} < 0$  when  $x_j^i \in (x^*(y^{-i}), x^*(y^{-i}) + 2\epsilon]$ ,  $\frac{\partial L^i(x_j^i, y^{-i})}{\partial x_j^i} = -\frac{x_j^i - x^*(y^{-i}) + 2\epsilon}{4\epsilon^2} < 0$  when  $x_j^i \in [x^*(y^{-i}) - 2\epsilon, x^*(y^{-i})]$ , and  $\frac{\partial L^i(x_j^i, y^{-i})}{\partial x_j^i} = 0$  otherwise. So we have  $\frac{\partial L^i(x_j^i, y^{-i})}{\partial x_j^i} \leq 0$ ,  $\forall x_j^i$  and  $y^{-i}$ . Similarly, we have  $\frac{\partial L^i(x_j^i, y^{-i})}{\partial x^*(y^{-i})} = \frac{x^*(y^{-i}) - x_j^i + 2\epsilon}{4\epsilon^2} > 0$  when  $x_j^i \in (x^*(y^{-i}), x^*(y^{-i}) + 2\epsilon]$ ,  $\frac{\partial L^i(x_j^i, y^{-i})}{\partial x^*(y^{-i})} = \frac{x_j^i - x^*(y^{-i}) + 2\epsilon}{4\epsilon^2} > 0$  when  $x_j^i \in [x^*(y^{-i}) - 2\epsilon, x^*(y^{-i})]$ , and  $\frac{\partial L^i(x_j^i, y^{-i})}{\partial x^*(y^{-i})} = 0$  otherwise. Provided that  $x^*(y^{-i})$  is monotonically decreasing in  $y^{-i}$ , we have  $\frac{\partial L^i(x_j^i, y^{-i})}{\partial y^{-i}} \leq 0$ ,  $\forall x_j^i$  and  $y^{-i}$ .

Upon observing  $x_j^i$ , the conditional distribution function  $F(y^i|x_j^i)$  of the signal  $y^i$  is:

$$F_{y^i}(y^i|x_j^i) = Pr(\tilde{y}^i \leq y^i|x_j^i) = \begin{cases} 0 & y^i \leq x_j^i - \epsilon - \eta \\ \frac{[(y^i+\eta)-(x_j^i-\epsilon)]^2}{8\eta\epsilon} & x_j^i - \epsilon - \eta < y^i \leq x_j^i + \epsilon - \eta \\ \frac{y^i+\eta-x_j^i}{2\eta} & x_j^i + \epsilon - \eta < y^i \leq x_j^i - \epsilon + \eta \\ 1 - \frac{[(y^i-\eta)-(x_j^i+\epsilon)]^2}{8\eta\epsilon} & x_j^i - \epsilon + \eta < y^i \leq x_j^i + \epsilon + \eta \\ 1 & y^i > x_j^i + \epsilon + \eta. \end{cases}$$

One can also verify that  $F(y^i|x')$  first-order stochastically dominates  $F(y^i|x)$  for any  $x' > x$ , i.e., the conditional distribution of  $y^i$  “shifts left” when  $x_j^i$  becomes smaller. Provided that  $x^*(y^i)$  is non-increasing in  $y^i$ , the representative creditor  $j$  believes that it is more likely for the bank  $-i$ 's creditors to take a higher threshold signal  $x^*(y^i)$  when observing a lower  $x_j^i$ . Combined with the fact that  $L^{-i}(\theta^{-i}, x^*(y^i))$  also monotonically increases in  $x^*(y^i)$ , we have  $\frac{\partial L^{-i}(x_j^i, y^{-i})}{\partial x_j^i} \leq 0$ . Lastly from expression (A.25) in the manuscript, one can calculate the following:

$$\begin{aligned} & \frac{\partial}{\partial y^{-i}} \int_{y^{-i}-\eta}^{y^{-i}+\eta} L^{-i}(\theta^{-i}, x^*(y^i)) d\theta^{-i} \\ &= \frac{\partial}{\partial y^{-i}} \int_{y^{-i}-\eta}^{y^{-i}+\eta} \max \left\{ \min \left\{ \frac{x^*(y^i) - \theta^{-i} + \epsilon}{2\epsilon}, 1 \right\}, 0 \right\} d\theta^{-i} \\ &= \max \left\{ \min \left\{ \frac{x^*(y^i) - y^{-i} - \eta + \epsilon}{2\epsilon}, 1 \right\}, 0 \right\} - \max \left\{ \min \left\{ \frac{x^*(y^i) - y^{-i} + \eta + \epsilon}{2\epsilon}, 1 \right\}, 0 \right\} \leq 0. \end{aligned}$$

We have  $\frac{\partial L^{-i}(x_j^i, y^{-i})}{\partial y^{-i}} \leq 0$  as the conditional density  $f(y^i|x_j^i)$  is everywhere non-negative.

### 2.3 The representative creditor's payoff difference function

We derive explicitly the representative creditor's payoff difference  $E[DW(L^i, \theta^i, P^*)|x_j^i, y^{-i}]$  – given that all other creditors take the equilibrium threshold strategy and that the creditor observes signals  $(x_j^i, y^{-i})$  following the standard global game approach (e.g., [Liu and Mello \(2011\)](#) and [Angeletos and Lian \(2016\)](#)). This corresponds to expression (B.26) of Appendix B.2 in the manuscript.

Observing a signal  $y^{-i}$  about  $\theta^{-i}$ , the representative creditor  $j$  knows that all other creditors in the bank  $i$  withdraw if and only if their private signals are below  $x^*(y^{-i})$ . Suppose now the creditor  $j$  also anticipates the bank  $i$  to sell assets at a price  $P^*$  if runs happen. Given all other creditors' strategy, the bank  $i$  fails if and only if its cash flow  $\theta^i$  falls below a critical  $\theta^*(y^{-i})$ , or equivalently the fraction  $L^i$  of creditors who withdraw early in the bank exceeds a critical value  $L^c(\theta^*(y^{-i}), P^*)$ . Depending on the realized  $x_j^i$ , the functional form of the creditor  $j$ 's expected payoff difference can be characterized by the following 5 cases.

*Case (1):* When  $x_j^i > x^*(y^{-i}) + 2\epsilon$ , the representative creditor  $j$  perceives  $L(\theta^i, x^*(y^{-i})) = 0$  with probability 1. Consequently, we have the creditor  $j$ 's expected payoff difference as:

$$E[DW(L^i, \theta^i, P^*) | x_j^i, y^{-i}] = (1 - q) \frac{D_1}{q}.$$

*Case (2):* When  $x^*(y^{-i}) < x_j^i \leq x^*(y^{-i}) + 2\epsilon$ , the representative creditor believes  $L^i$  has a mixed distribution: with a positive probability mass  $\frac{x_j^i - x^*(y^{-i})}{2\epsilon}$  at  $L^i = 0$  and being uniformly distributed on  $(0, 1 - \frac{x_j^i - x^*(y^{-i})}{2\epsilon}]$  with density 1. Notice that  $1 - \frac{x_j^i - x^*(y^{-i})}{2\epsilon}$  decreases in  $x_j^i$ , with  $1 - \frac{x_j^i - x^*(y^{-i})}{2\epsilon} = 1$  when  $x_j^i = x^*(y^{-i})$  and  $1 - \frac{x_j^i - x^*(y^{-i})}{2\epsilon} = 0$  when  $x_j^i = x^*(y^{-i}) + 2\epsilon$ . Since  $L^c(\theta^*(y^{-i}), P^*) \in (0, 1)$ , there exists a  $\bar{x} \in (x^*(y^{-i}), x^*(y^{-i}) + 2\epsilon]$  such that  $L^c(\theta^*(y^{-i}), P^*) = 1 - \frac{\bar{x} - x^*(y^{-i})}{2\epsilon}$ . When  $x_j^i \in (\bar{x}, x^*(y^{-i}) + 2\epsilon]$ , we have  $1 - \frac{x_j^i - x^*(y^{-i})}{2\epsilon} < L^c(\theta^*(y^{-i}), P^*)$  and the expected payoff difference as

$$E[DW(L^i, \theta^i, P^*) | x_j^i, y^{-i}] = \frac{x_j^i - x^*(y^{-i})}{2\epsilon} (1 - q) \frac{D_1}{q} + \int_0^{1 - \frac{x_j^i - x^*(y^{-i})}{2\epsilon}} (1 - q) \frac{D_1}{q} dL^i = (1 - q) \frac{D_1}{q}.$$

When  $x_j^i \in (x^*(y^{-i}), \bar{x}]$ , we have  $1 - \frac{x_j^i - x^*(y^{-i})}{2\epsilon} > L^c(\theta^*(y^{-i}), P^*)$  and the payoff difference as

$$\begin{aligned} E[DW(L^i, \theta^i, P^*) | x_j^i, y^{-i}] &= \frac{x_j^i - x^*(y^{-i})}{2\epsilon} (1 - q) \frac{D_1}{q} + \int_0^{L^c(\theta^*(y^{-i}), P^*)} (1 - q) \frac{D_1}{q} dL^i + \int_{L^c(\theta^*(y^{-i}), P^*)}^{1 - \frac{x_j^i - x^*(y^{-i})}{2\epsilon}} (-D_1) dL^i \\ &= \frac{D_1}{q} \cdot [L^c(\theta^*(y^{-i}), P^*) - q] + \frac{D_1}{q} \frac{x_j^i - x^*(y^{-i})}{2\epsilon}. \end{aligned}$$

*Case (3):* When  $x_j^i = x^*(y^{-i})$ , the representative creditor  $j$  believes  $L^i \sim U(0, 1)$ . We have

$$\begin{aligned} E[DW(L^i, \theta^i, P^*) | x_j^i, y^{-i}] &= \int_0^{L^c(\theta^*(y^{-i}), P^*)} (1 - q) \frac{D_1}{q} dL^i + \int_{L^c(\theta^*(y^{-i}), P^*)}^1 (-D_1) dL^i \\ &= \frac{D_1}{q} \cdot [L^c(\theta^*(y^{-i}), P^*) - q]. \end{aligned}$$

*Case (4):* When  $x^*(y^{-i}) - 2\epsilon \leq x_j^i < x^*(y^{-i})$ , the representative creditor  $j$  perceives that  $L^i$  has a mixed distribution: with a positive probability mass  $\frac{x^*(y^{-i}) - x_j^i}{2\epsilon}$  at  $L^i = 1$  and being uniformly distributed on  $[\frac{x^*(y^{-i}) - x_j^i}{2\epsilon}, 1)$  with density 1. So there exists a  $\underline{x} \in [x^*(y^{-i}) - 2\epsilon, x^*(y^{-i})]$  such that  $\frac{x^*(y^{-i}) - \underline{x}}{2\epsilon} = L^c(\theta^*(y^{-i}), P^*)$ . When  $x_j^i \in [\underline{x}, x^*(y^{-i})]$ , we have  $\frac{x^*(y^{-i}) - x_j^i}{2\epsilon} < L^c(\theta^*(y^{-i}), P^*)$ . The expected payoff difference can be written as

$$\begin{aligned} E[DW(L^i, \theta^i, P^*) | x_j^i, y^{-i}] &= \int_{\frac{x^*(y^{-i}) - x_j^i}{2\epsilon}}^{L^c(\theta^*(y^{-i}), P^*)} (1 - q) \frac{D_1}{q} dL + \int_{L^c(\theta^*(y^{-i}), P^*)}^1 (-D_1) dL + \frac{x^*(y^{-i}) - x_j^i}{2\epsilon} (-D_1) \\ &= \frac{D_1}{q} \cdot [L^c(\theta^*(y^{-i}), P^*) - q] - \frac{D_1}{q} \frac{x^*(y^{-i}) - x_j^i}{2\epsilon}. \end{aligned}$$

When  $x_j^i \in [x^*(y^{-i}) - 2\epsilon, \underline{x}]$ , we have  $\frac{x^*(y^{-i}) - x_j^i}{2\epsilon} > L^c(\theta^*(y^{-i}), P^*)$  and

$$E[DW(L^i, \theta^i, P^*) | x_j^i, y^{-i}] = \int_{\frac{x^*(y^{-i}) - x_j^i}{2\epsilon}}^1 (-D_1) dL + \frac{x^*(y^{-i}) - x_j^i}{2\epsilon} (-D_1) = -D_1.$$

*Case (5):* When  $x_j^i < x^*(y^{-i}) - 2\epsilon$ , the creditor  $j$  perceives  $L^i(\theta^i, x^*(y^{-i})) = 1$  with probability

1. Therefore, we have

$$E[DW(L^i, \theta^i, P^*) | x_j^i, y^{-i}] = -D_1, \text{ for } x_j^i < x^*(y^{-i}) - 2\epsilon.$$

To summarize the 5 cases, we obtain expression (B.26) of the manuscript.

### 3 Robustness checks of the proposed policy

In Section 4 of the manuscript, we consider various extensions to the proposed committed liquidity support. We now collect the omitted proofs in this section.

#### 3.1 Proof of Proposition 4: banks' voluntary participation in the arrangement

*Proof.* The timing of the game is summarized in Figure 1.

Figure 1: Timing of the game with the regulator's offer and banks' decision

t = 0	t = 1	t = 2
<ol style="list-style-type: none"> <li>1. Banks are established, with their portfolios and liability structures as given.</li> <li>2. The regulator offers a contract to buy bank assets at <math>P_A</math> in case any bank run happens.</li> <li>3. The banks' managements simultaneously decide whether to accept the offer.</li> </ol>	<ol style="list-style-type: none"> <li>1. State <math>s</math> and <math>(\theta^1, \theta^2)</math> are realized sequentially.</li> <li>2. Having observed <math>P_A</math> and banks' participation in the regulator's program, creditors receive noisy signals and simultaneously decide on whether to run.</li> <li>3. When experiencing runs, a bank that participates in the regulator's program sells assets to the regulator at <math>P_A</math>; and a bank that refused to join the arrangement sells assets to the secondary-market asset buyers.</li> <li>4. When called to move, the secondary-market asset buyers bid for and acquire assets on sale.</li> </ol>	<ol style="list-style-type: none"> <li>1. Bank assets pay off.</li> <li>2. Remaining obligations are settled.</li> </ol>

We denote  $a_i \in \{A, NA\}$ ,  $i \in \{1, 2\}$ , as a bank  $i$ 's manager's action, where 'A' and 'NA' stand for 'accept the regulator's offer' and 'refuse the regulator's offer' respectively. At the start of  $t = 1$ , the regulator's offer  $P_A \in (P_2^*, P_1^*)$  and two bank managers' strategy profile  $(a_i, a_{-i})$  define a subgame  $g_{a_i, a_{-i}}(P_A)$ . We establish a condition under which both banks accepting the contract for the regulator's liquidity support is a Nash equilibrium of the  $t = 0$  stage game. For

completeness, we first work with the non-limiting case where  $\epsilon$  and  $\eta$  are away from zero, then take the limit after the equilibrium is solved.

**Step 1:** We derive the manager  $i$ 's equilibrium payoffs in subgame  $g_{A,A}(P_A)$  and  $g_{NA,A}(P_A)$ . In subgame  $g_{A,A}(P_A)$ , the analyses are similar to that in Section 3.3. The equilibrium threshold signal, and the equilibrium critical cash flows that trigger runs and failures are  $x^*(P_A) = \theta^*(P_A) + (2q - 1)\epsilon$ ,  $\hat{\theta}(P_A) \equiv x^*(P_A) + \epsilon$  and  $\theta^*(P_A) = \frac{D_2 - D_1}{1 - qD_1/P_A}$ , respectively.

In subgame  $g_{NA,A}(P_A)$ , only the bank  $-i$ 's manager signs the contract for the regulator's liquidity support. Knowing with certainty that the bank's asset price will be  $P_A$ , creditors in the bank  $-i$  no longer need to form rational beliefs about the number of runs (i.e., their signal  $y^i$  becomes redundant). In the non-limiting case, the equilibrium threshold signal, the critical cash flows that trigger failures and runs for the bank  $-i$  will be  $x^*(P_A)$ ,  $\theta^*(P_A)$  and  $\hat{\theta}(P_A)$ .

The bank  $i$ 's manager does not sign the contract with the regulator. Upon runs happen, the bank still raises liquidity from the secondary market where the asset buyers bid competitively to purchase the bank's assets on sale. The analyses of the bank run game and the asset bidding game resemble that in Section 3.2.2, the bank  $i$ 's critical cash flows triggers failures and runs  $\theta^*(y^{-i})$  and  $\hat{\theta}(y^{-i})$ , the creditors' threshold signal  $x^*(y^{-i})$ , and the equilibrium asset price  $P_M^*$  jointly solve the following system of equations.<sup>1</sup>

$$\begin{cases} P_M^* = \omega_M^B(\hat{\theta}'_M, P_A) \cdot \frac{\theta_B + \hat{\theta}'_M}{2} + \omega_M^G(\hat{\theta}'_M, P_A) \cdot \frac{\theta_G + \hat{\theta}'_M}{2} \\ x_M^* = \theta^*(y^{-i}) + (2q - 1)\epsilon = x^*(y^{-i}) \\ \theta_M^* = \frac{D_2 - D_1}{1 - qD_1/P_M^*} = \theta^*(y^{-i}) \\ \hat{\theta}'_M = x^*(y^{-i}) + \epsilon \equiv \hat{\theta}(y^{-i}) \end{cases} \quad \text{with } M = \begin{cases} 1 & \text{if } y^{-i} > y^U \\ 2 & \text{if } y^{-i} < y^L. \end{cases} \quad (2)$$

In particular, the asset buyers' posteriors beliefs about  $s$  are characterized by (3).

$$\omega_M^B(\hat{\theta}'_M, P_A) = \frac{\left(\frac{\hat{\theta}'_M - \theta_B}{\theta - \theta_B}\right) \cdot \omega_{M-1}^B(P_A)}{\left(\frac{\hat{\theta}'_M - \theta_B}{\theta - \theta_B}\right) \cdot \omega_{M-1}^B(P_A) + \left(\frac{\hat{\theta}'_M - \theta_G}{\theta - \theta_G}\right) \cdot \omega_{M-1}^G(P_A)} = 1 - \omega_M^G(\hat{\theta}'_M, P_A), \quad (3)$$

where

$$\begin{aligned} \omega_1^B(P_A) &= Prob(s = B | \theta^{-i} < \hat{\theta}(P_A)) = \frac{\left(\frac{\hat{\theta}(P_A) - \theta_B}{\theta - \theta_B}\right) \cdot (1 - \alpha)}{\left(\frac{\hat{\theta}(P_A) - \theta_B}{\theta - \theta_B}\right) \cdot (1 - \alpha) + \left(\frac{\hat{\theta}(P_A) - \theta_G}{\theta - \theta_G}\right) \cdot \alpha} = 1 - \omega_1^G(P_A), \\ \omega_0^B(P_A) &= Prob(s = B | \theta^{-i} > \hat{\theta}(P_A)) = \frac{\left(\frac{\bar{\theta} - \hat{\theta}(P_A)}{\theta - \theta_B}\right) \cdot (1 - \alpha)}{\left(\frac{\bar{\theta} - \hat{\theta}(P_A)}{\theta - \theta_B}\right) \cdot (1 - \alpha) + \left(\frac{\bar{\theta} - \hat{\theta}(P_A)}{\theta - \theta_G}\right) \cdot \alpha} = 1 - \omega_0^G(P_A). \end{aligned}$$

<sup>1</sup>When the regulator intervenes, we use the "prime" on the combination  $(\theta_M^*, x_M^*, P_M^*)$  to differentiate it from the equilibrium combination  $(\theta_M^*, x_M^*, P_M^*)$  in the laissez-faire economy.

Conditional on whether a run happens to the bank  $-i$ , the buyers' beliefs about  $s$  become  $\omega_1^s(P_A)$  or  $\omega_0^s(P_A)$ . When being called upon to purchase the bank  $i$ 's assets, the buyers update beliefs once more. Their posteriors about  $s$  become  $\omega_2^s(\hat{\theta}_2, P_A)$  when runs happen to both banks or  $\omega_1^s(\hat{\theta}_1, P_A)$  when a run only happens to the bank  $i$  as expressed in (3). We have the following Result 1.

**Result 1.** *When  $P_A \in (P_2^*, P_1^*)$ , the combination  $(\theta_M^*, x_M^*, P_M^*)$  is the unique solution to the system of equations (2) for each  $M \in \{1, 2\}$ . It holds that  $P_2^* < P_2^*$ ,  $\theta_2^* > \theta_2^*$  and  $x_2^* > x_2^*$ , while  $P_1^* = P_1^*$ ,  $\theta_1^* = \theta_1^*$  and  $x_1^* = x_1^*$ . A bank who signs the contract for the regulator's liquidity support exerts only a negative externality on the other bank who does not sign the contract.*

*Proof.* Following exactly the proof of Lemma 3 in the manuscript, we can establish the existence and uniqueness of the solution  $(P_M^*, x_M^*, \theta_M^*)$ , for  $M \in \{1, 2\}$ .<sup>2</sup>

We then show  $P_1^* = P_1^*$ ,  $x_1^* = x_1^*$ , and  $\theta_1^* = \theta_1^*$ . Indeed,  $\forall \hat{\theta} \in [\theta^L, \theta^U]$ , we have

$$\omega_0^B(P_A) = \frac{(1 - \alpha) \cdot (\bar{\theta} - \underline{\theta}_G)}{(1 - \alpha) \cdot (\bar{\theta} - \underline{\theta}_G) + \alpha \cdot (\bar{\theta} - \underline{\theta}_B)},$$

and asset buyers' posterior  $\omega_1^B(\hat{\theta}, P_A)$  as

$$\omega_1^B(\hat{\theta}, P_A) = \frac{\left(\frac{\hat{\theta} - \underline{\theta}_B}{\bar{\theta} - \underline{\theta}_B}\right) \cdot \omega_0^B(P_A)}{\left(\frac{\hat{\theta} - \underline{\theta}_B}{\bar{\theta} - \underline{\theta}_B}\right) \cdot \omega_0^B(P_A) + \left(\frac{\hat{\theta} - \underline{\theta}_B}{\bar{\theta} - \underline{\theta}_G}\right) \cdot \omega_0^G(P_A)} = \frac{(\hat{\theta} - \underline{\theta}_B)}{(\hat{\theta} - \underline{\theta}_B) + \kappa(\hat{\theta} - \underline{\theta}_G)} = \omega_1^B(\hat{\theta}).$$

This means  $\omega_1^B(\hat{\theta}, P_A) = \omega_1^B(\hat{\theta})$ , the expression of which is the private asset buyers' posterior when observing  $M = 1$  in the laissez-faire market. So the system of equations when  $M = 1$  is the same as the system of equations (18) in the laissez-faire market. The latter system has a unique combination of solution  $(\theta_1^*, x_1^*, P_1^*)$ . Then, we have  $(\theta_1^*, x_1^*, P_1^*) = (\theta_1^*, x_1^*, P_1^*)$ .

One can similar show  $\forall \hat{\theta}$ ,  $\frac{\omega_2^B(\hat{\theta}, P_A)}{\omega_2^G(\hat{\theta}, P_A)} > \frac{\omega_2^B(\hat{\theta})}{\omega_2^G(\hat{\theta})}$ , where  $\omega_2^B(\hat{\theta})$  is the private asset buyers' posterior when observing  $M = 2$  in the laissez-faire market. Follow Step 2 of the proof of Lemma 3, we have  $\theta_2^* > \theta_2^*$ ,  $x_2^* > x_2^*$  and  $P_2^* < P_2^*$ .  $\square$

In Result 2, we characterize the bank  $-i$ 's creditors' equilibrium threshold strategy and the asset buyers' equilibrium bids for the bank's assets on sale.

**Result 2.** *When the bank  $i$  does not accept the regulator's offer  $P_A \in (P_2^*, P_1^*)$ , its bank run game has a unique equilibrium. A representative creditor of the bank withdraws early if and only if his private signal falls below*

<sup>2</sup>The details are omitted to avoid replication and can be provided upon any request.

$$x^*(y^{-i}, P_A) = \begin{cases} x_2^* & y^{-i} < x^*(P_A) + \eta + \epsilon \\ x_1^* & y^{-i} \geq x^*(P_A) + \eta + \epsilon. \end{cases} \quad (4)$$

The asset buyers offer price  $P_2^*$  to purchase the bank  $i$ 's assets when observing two bank runs and offer price  $P_1^*$  when observing only one run happening to the bank  $i$ .

*Proof.* We only sketch the proof here as it resembles that of Proposition 2 of the manuscript.

When observing  $y^{-i} < y^L$ , the representative creditor expects  $L^{-i}(x_j^i, y^{-i}) = 1$ , independent of his private signal  $x_j^i$ . When observing  $y^{-i} > y^U$ , the creditor holds a belief that  $L^{-i}(x_j^i, y^{-i}) = 0$ , independent of  $x_j^i$ . Similar to Section 3.2.2, when observing  $y^{-i} > y^U$ , the representative creditor anticipates the asset price to be  $P_1^*$ . Similarly, upon observing  $y^{-i} < y^L$ , the representative creditor anticipates that the bank  $i$  will have to sell assets for a price  $P_2^*$ . The asset price, creditors' threshold signal in bank  $i$ , and the bank's critical cash flows solve the system of equations (2).

We again have that the creditors' equilibrium threshold signal  $x^*(y^{-i})$  must be a step function with only two possible values  $x_1^*$  and  $x_2^*$ . We again establish a point of discontinuity  $\hat{y}^{-i} \in [y^L, y^U]$  such that  $x^*(y^{-i}) = x_2^*$  for  $y^{-i} < \hat{y}^{-i}$ , and  $x^*(y^{-i}) = x_1^*$  for  $y^{-i} \geq \hat{y}^{-i}$ . As  $y^{-i}$  rises above the cutoff value  $\hat{y}^{-i}$ , the creditor's expected asset price changes from  $P_2^*$  to  $P_1^*$ .

One can show that the unique point of discontinuity  $\hat{y}^{-i}$  equals  $x^*(P_A) + \eta + \epsilon$ . To see it, we prove that when  $y^{-i} \in [y^L, x_1^* - \eta + \epsilon]$ , creditor  $j$  rationally expects  $L^{-i}(x_j^i, y^{-i}) > 0$  independent of  $x_j^i$  and withdraws with the critical signal  $x^*(y^{-i}) = x_2^*$ . When  $y^{-i} \in [x^*(P_A) + \eta + \epsilon, y^U]$ , creditor  $j$  rationally expects  $L^{-i}(x_j^i, y^{-i}) = 0$  independent of  $x_j^i$  and withdraws with the critical signal  $x^*(y^{-i}) = x_1^*$ . Second, we show that given all other creditors take the equilibrium strategy (4), the representative creditor expects  $L^{-i}(x_j^i, y^{-i}) > 0$  when he observes an outside signal  $y^{-i} \in (x_1^* - \eta + \epsilon, x^*(P_A) + \eta + \epsilon)$  and a private signal  $x_j^i \leq x_2^* + 2\epsilon$ . This means when  $y^{-i} \in (x_1^* - \eta + \epsilon, x^*(P_A) + \eta + \epsilon)$ , the creditor optimally withdraws from the bank  $i$  if and only if  $x_j^i < x_2^*$ , i.e.,  $x^*(y^{-i}) = x_2^*$ . The strategy (4) is indeed an equilibrium. Lastly, the uniqueness can be established by contradiction following exactly **Step 4** of the Proof of Proposition 2 in the manuscript.  $\square$

In the limiting case where  $\eta$  and  $\epsilon$  approach zero, we have  $x_M^* = \hat{\theta}_M = \theta_M^* = \frac{D_2 - D_1}{1 - qD_1/P_M^*}$  and  $x^*(P_A) = \hat{\theta}(P_A) = \theta^*(P_A) = \frac{D_2 - D_1}{1 - qD_1/P_A}$ .

**Step 2:** We analyze the bank  $i$ 's manager's optimal choice regarding whether to accept the regulator's offer  $P_A$ .

We move back to  $t = 0$  to consider the optimal response of the bank  $i$ 's manager (thereafter the manager  $i$ ) to the bank  $-i$ 's manager's action  $a_{-i} = A$ . The manager  $i$ 's objective is  $[1 - Pr(\text{failure of the bank } i)] \times b$ , where

$$Pr(\text{failure of the bank } i | a_{-i} = A) = \begin{cases} Pr(A, A) & \text{if } a_i = A, \\ Pr(NA, A) & \text{if } a_i = NA. \end{cases}$$

If  $Pr(A, A) < Pr(NA, A)$ , the manager  $i$ 's best response is  $a_i^* = A$ , the strategy profile  $(a_i^*, a_{-i}^*) = (A, A)$  becomes a Nash equilibrium of the stage game.

We derive explicitly  $Pr(A, A)$  and  $Pr(NA, A)$  in the limiting case. It can be seen easily

$$Pr(A, A) = \sum_{s=G,B} Pr(s) \cdot \left( \frac{\theta^*(P_A) - \underline{\theta}_s}{\bar{\theta} - \underline{\theta}_s} \right).$$

Consider the manager  $i$ 's choice  $a_i = NA$ . The bank  $i$  fails if  $\theta^i < \theta_2^*$  conditional on  $\theta^{-i} < \theta^*(P_A)$ , while it fails if  $\theta^i < \theta_1^*$  conditional on  $\theta^{-i} \geq \theta^*(P_A)$ . The bank  $i$ 's probability of failure is

$$Pr(NA, A) = \sum_{s=G,B} Pr(s) \cdot \left[ \left( \frac{\theta_1^* - \underline{\theta}_s}{\bar{\theta} - \underline{\theta}_s} \right) \cdot \left( \frac{\bar{\theta} - \theta^*(P_A)}{\bar{\theta} - \underline{\theta}_s} \right) + \left( \frac{\theta_2^* - \underline{\theta}_s}{\bar{\theta} - \underline{\theta}_s} \right) \cdot \left( \frac{\theta^*(P_A) - \underline{\theta}_s}{\bar{\theta} - \underline{\theta}_s} \right) \right].$$

Define an auxiliary function  $F(P_A) = Pr(A, A)(P_A) - Pr(NA, A)(P_A)$ . We have

$$F(P_A) = \sum_{s=G,B} Pr(s) \cdot \left[ \left( \frac{\theta^*(P_A) - \underline{\theta}_s}{\bar{\theta} - \underline{\theta}_s} \right) - \left( \frac{\theta_1^* - \underline{\theta}_s}{\bar{\theta} - \underline{\theta}_s} \right) \cdot \left( \frac{\bar{\theta} - \theta^*(P_A)}{\bar{\theta} - \underline{\theta}_s} \right) - \left( \frac{\theta_2^* - \underline{\theta}_s}{\bar{\theta} - \underline{\theta}_s} \right) \cdot \left( \frac{\theta^*(P_A) - \underline{\theta}_s}{\bar{\theta} - \underline{\theta}_s} \right) \right].$$

We then establish a  $P_A^C \in (P_2^*, P_1^*)$  such that  $F(P_A^C) = 0$ . When the regulator's price support  $P_A = P_A^C$ , the manager  $i$  is indifferent between actions  $A$  and  $NA$ , given  $a_{-i} = A$ . We have

$$\begin{aligned} \lim_{P_A \rightarrow P_2^*} F(P_A) &= \sum_{s=G,B} \frac{Pr(s)}{(\bar{\theta} - \underline{\theta}_s)^2} \cdot (\theta_2^* - \theta_1^*) \cdot (\bar{\theta} - \theta_2^*) > 0, \\ \lim_{P_A \rightarrow P_1^*} F(P_A) &= \sum_{s=G,B} \frac{Pr(s)}{(\bar{\theta} - \underline{\theta}_s)^2} \cdot (\theta_1^* - \underline{\theta}_s) \cdot (\theta_1^* - \lim_{P_A \rightarrow P_1^*} \theta_2^*) < 0. \end{aligned}$$

Note that  $\theta_2^* > \theta_2^*$  for  $P_A < P_2^*$ , so  $\lim_{P_A \rightarrow P_1^*} \theta_2^* > \theta_2^* > \theta_1^*$  must be true. Moreover, we have  $\lim_{P_A \rightarrow P_2^*} \theta_2^* = \theta_2^*$  because the secondary market asset buyers' posteriors about  $s$   $\omega_2^s(\hat{\theta}, P_A) \rightarrow \omega_2^s(\hat{\theta}, \theta_2^*)$  when  $P_A \rightarrow P_2^*$ . So the equilibrium critical cash flow  $\theta_2^*$  converges to  $\theta_2^*$  when  $P_A \rightarrow P_2^*$ . By the continuity of  $F(P_A)$ , there must exist a  $P_A^C \in (P_2^*, P_1^*)$  such that  $F(P_A^C) = 0$ .

On the other hand, it can be checked that

$$\frac{dF(P_A)}{dP_A} = \sum_{s=G,B} \frac{Pr(s)}{(\bar{\theta} - \underline{\theta}_s)^2} \cdot \left[ \frac{d\theta^*(P_A)}{dP_A} \cdot ((\bar{\theta} - \theta_2^*(P_A)) + (\theta_1^* - \underline{\theta}_s)) - \frac{d\theta_2^*}{dP_A} \cdot (\theta^*(P_A) - \underline{\theta}_s) \right] < 0.$$

We have  $\frac{d\theta_2^*}{dP_A} > 0$  because  $\omega_2^B(\theta, P_A)$  increases in  $P_A$ . Conditional on a run happening to the bank  $-i$ , the asset buyers' posterior beliefs about  $s$  deteriorate as the regulator's price  $P_A$  becomes higher. As a consequence,  $P_A^C$  is also unique.

From the above analyses,  $F(P_A) < 0$  when  $P_A \in (P_A^C, P_1^*)$ , and  $F(P_A) \geq 0$  when  $P_A \in (P_2^*, P_A^C]$ . To minimize the bank  $i$ 's probability of failure, the manager  $i$  optimally chooses  $a_i^* = A$  when  $P_A \in (P_A^C, P_1^*)$  and  $a_i^* = NA$  when  $P_A \in (P_2^*, P_A^C]$ .

Lastly, under a sufficient condition  $V_A(P_A^C) > 0$ , the regulator's optimal price support  $P_A^*$  can induce both banks' managers to choose the action  $A$ . To see this, recall that  $\frac{dV_A(P_A)}{dP_A} < 0$  and the optimal  $P_A^*$  satisfying the regulator's break-even condition  $V_A(P_A^*) = 0$ . Therefore, we have  $P_A^* > P_A^C$ , in which case  $F(P_A^*) < 0$  and  $Pr(A, A) < Pr(NA, A)$ . Both banks' managers accepting the liquidity support offered by the regulator is a Nash equilibrium of the stage game.  $\square$

### 3.2 The bank run equilibrium with the unilateral commitment

We provide a sketch of the proof for the result: When  $P_U \in [P_2^*, P_1^*)$ , a wholesale creditor of the bank  $i$  withdraw if and only if his private signal falls below a critical signal  $x^*(y^{-i}, P_U)$ .

$$x^*(y^{-i}, P_U) = \begin{cases} x^*(P_U) & y^{-i} < x^*(P_U) + \eta + \epsilon \\ x_1^* & y^{-i} \geq x^*(P_U) + \eta + \epsilon, \end{cases} \quad (5)$$

When observing  $y^{-i} < y^L$ , the representative creditor expects  $L^{-i}(x_j^i, y^{-i}) = 1$ , independent of his private signal  $x_j^i$ . The creditor anticipates that the bank  $i$  will liquidate its assets at a price  $P_U$  when experiencing runs. Since the secondary market asset buyers will offer a price  $P_2^* < P_U$ , the bank  $i$  optimally sell assets to the regulator. Consequently, the representative creditor optimally withdraw if and only if  $x_j^i$  is below  $x^*(y^{-i}) = x^*(P_U) = \theta^*(P_U) + (2q - 1)\epsilon$  and the bank  $i$  fails if and only if  $\theta^i$  is below  $\theta^*(y^{-i}) = \theta^*(P_U) = \frac{D_2 - D_1}{1 - qD_1/P_U}$ .

When observing  $y^{-i} > y^U$ , the creditor holds a belief that  $L^{-i}(x_j^i, y^{-i}) = 0$ , independent of  $x_j^i$ . He rationally expects the bank  $i$ 's liquidation price to be  $P_1^*$  upon runs happen to the bank. Since the regulator's pre-committed price  $P_U$  is lower, the bank  $i$  optimally sells assets to the secondary market buyers. Consequently, the representative creditor optimally withdraw if and only if  $x_j^i < x_1^*$  and the bank  $i$  fails if and only if  $\theta^i < \theta_1^*$ . Note that  $x_1^* < x^*(P_U)$  and  $\theta_1^* < \theta^*(P_U)$ .

Again, the creditors' equilibrium threshold signal  $x^*(y^{-i})$  must be a step function with only two possible values  $x_1^*$  and  $x^*(P_U)$ . We establish a point of discontinuity  $\hat{y}^{-i} \in [y^L, y^U]$  such that  $x^*(y^{-i}) = x^*(P_U)$  for  $y^{-i} < \hat{y}^{-i}$  and  $x^*(y^{-i}) = x_1^*$  for  $y^{-i} \geq \hat{y}^{-i}$ . As  $y^{-i}$  rises above the cutoff value  $\hat{y}^{-i}$ , the creditor's expected asset price changes from  $P_U$  to  $P_1^*$ .

One can show that the unique point of discontinuity  $\hat{y}^{-i}$  equals  $x^*(P_U) + \eta + \epsilon$ . To see it, we prove that when  $y^{-i} \in [y^L, x_1^* - \eta + \epsilon]$ , creditor  $j$  rationally expects  $L^{-i}(x_j^i, y^{-i}) > 0$  independent

of  $x_j^i$  and withdraws with the critical signal  $x^*(y^{-i}) = x^*(P_U)$ . When  $y^{-i} \in [x^*(P_U) + \eta + \epsilon, y^U]$ , creditor  $j$  rationally expects  $L^{-i}(x_j^i, y^{-i}) = 0$  independent of  $x_j^i$  and withdraws with the critical signal  $x^*(y^{-i}) = x_1^*$ . Second, we show that given all other creditors take the equilibrium strategy (5), the representative creditor expects  $L^{-i}(x_j^i, y^{-i}) > 0$  when he observes a semi-public signal  $y^{-i} \in (x_1^* - \eta + \epsilon, x^*(P_U) + \eta + \epsilon)$  and a private signal  $x_j^i \leq x^*(P_U) + 2\epsilon$ . This means when  $y^{-i} \in (x_1^* - \eta + \epsilon, x^*(P_U) + \eta + \epsilon)$ , the creditor optimally withdraws from the bank  $i$  if and only if  $x_j^i < x^*(P_U)$ , i.e.,  $x^*(y^{-i}) = x^*(P_U)$ . The strategy (5) is indeed an equilibrium. Again, the uniqueness can be established by contradiction following **Step 4** of the Proof of Proposition 2.

### 3.3 Proof of Proposition 5: the regulator's unilateral commitment

*Proof.* The proof consists of two steps. First, we analyze the optimal design of ex-ante liquidity support where the regulator unilaterally commits at  $t = 0$  to purchasing banks' assets on sale for a price  $P_U$ . We then compare its stability effects with that of our proposed policy.

**Step 1:** We first characterize the regulator's program. The regulator has the same objective under both arrangements, so  $\text{SYS}(P_U)$  is continuous and decreasing in  $P_U$ . We then consider the continuity and the monotonicity of the regulator's expected payoff function  $V_U(P_U)$  when  $P_U \geq P_2^*$ . As explained in the manuscript, when  $P_U \geq P_1^*$  we have:

$$V_U(P_U) = V_A(P_U) = \sum_{M=1}^2 \left( \sum_{s=G,B} \text{Pr}(s) \cdot \text{Pr}(\theta < \theta^*(P_U)|s)^M \cdot \text{Pr}(\theta > \theta^*(P_U)|s)^{2-M} \right) C_2^M \cdot M \cdot \frac{D_1}{P_U} \cdot \Pi_M(P_U). \quad (6)$$

Additionally, when  $P_U \in [P_2^*, P_1^*)$ ,  $V_U(P_U)$  can be re-arranged into:

$$V_U(P_U) = \left( \sum_{s=G,B} \text{Pr}(s) \cdot \text{Pr}(\theta < \theta^*(P_U)|s)^2 \cdot 2 \cdot \frac{D_1}{P_U} \right) \cdot \Pi_2(P_U). \quad (7)$$

When  $P_U = P_1^*$ , expressions (6) and (7) are equal because  $\Pi_1(P_1^*) = 0$ . Consequently,  $V_U(P_U)$  is continuous in  $P_U$ . Note that  $V_U(P)$  has the same form as  $V_A(P)$  for any price  $P \geq P_1^*$ , hence  $V_U(P_U)$  decreases in  $P_U$  when  $P_U \geq P_1^*$ . When  $P_U \in [P_2^*, P_1^*)$ , one can also verify that  $V_U(P_U)$  is decreasing in  $P_U$  provided that  $\alpha$ , the probability of  $s = G$ , is sufficiently large. We assume this is the case, therefore,  $V_U(P_U)$  decreases in  $P_U$  when  $P_U \geq P_2^*$ .

We solve the supporting price that minimizes the systemic risk under the arrangement with the regulator's unilateral commitment. Consider first  $\bar{V} = 0$ , so the regulator must break even. Then, it is not optimal for her to pre-commit to a price  $P_U \geq P_1^*$  since her expected payoff, i.e., given by (6), will be strictly negative. Indeed, we have:  $\Pi_1(P_U) \leq 0$  and  $\Pi_2(P_U) < 0$  for  $P_U \geq P_1^*$ . Instead, by pre-committing to a price  $P_U \in [P_2^*, P_1^*)$ , the regulator's expected payoff

$V_U(P_U)$ , i.e., given by (7), is smaller than or equal to 0. We have:  $\Pi_2(P_U) = 0$  when  $P_U = P_2^*$  and  $\Pi_2(P_U) < 0$  when  $P_U > P_2^*$ . The only price that allows the regulator to break even in expectation is  $P_U^* = P_2^*$ . Then, consider  $\bar{V} > 0$ . Both  $\text{SYS}(P_U)$  and  $V_U(P_U)$  decrease in  $P_U$ . To minimize the systemic risk, the regulator optimally chooses the unique supporting price  $P_U^{**}$  to exhaust her loss-bearing capacity, i.e.,  $V_U(P_U^{**}) = -\bar{V}$ . Like the case of mutual commitments, we focus on  $0 < \bar{V} < -V_U(qD_2)$ , so  $P_2^* < P_U^{**} < qD_2$  and  $\theta^*(P_U^{**}) > D_2$ .

**Step 2:** We compare the stability effects of the two arrangements. When the regulator cannot make any expected loss, i.e.,  $\bar{V} = 0$ , the ex-ante liquidity support with the mutual commitments strictly dominates that with the regulator's unilateral commitment. The regulator can offer a price  $P_A^* \in (P_2^*, P_1^*)$  under the former arrangement, while she can only offer  $P_2^*$  under the latter arrangement. Therefore, we have  $\text{SYS}(P_A^*) < \text{SYS}(P_U^*) = \text{SYS}(\theta_2^*)$ .

The former arrangement also outperforms the latter when the regulator has a moderate loss-bearing capacity  $0 < \bar{V} < -V_A(P_1^*) = -V_U(P_1^*)$ . The feasible choices of  $P_A$  and  $P_U$  are bounded below  $P_1^*$ . The regulator can offer a  $P_A^{**} \in (P_A^*, P_1^*)$  under the mutual commitments, while she can only offer a lower  $P_U^{**} \in (P_2^*, P_A^{**})$  with her unilateral commitment. To show  $P_U^{**} < P_A^{**}$ , observe that for any price  $P \in [P_2^*, P_1^*)$ , the regulator's expected payoffs under the two arrangements are linked as follows:

$$V_A(P) = V_U(P) + \left( \sum_{s=G,B} Pr(s) \cdot Pr(\theta < \theta^*(P)|s) \cdot Pr(\theta > \theta^*(P)|s) \cdot 2 \cdot \frac{D_1}{P} \right) \cdot \Pi_1(P). \quad (8)$$

Evaluating expression (8) at  $P_A^{**}$ , we obtain

$$-\bar{V} = V_A(P_A^{**}) = V_U(P_A^{**}) + \left( \sum_{s=G,B} Pr(s) \cdot Pr(\theta < \theta^*(P_A^{**})|s) \cdot Pr(\theta > \theta^*(P_A^{**})|s) \cdot 2 \cdot \frac{D_1}{P_A^{**}} \right) \cdot \Pi_1(P_A^{**}). \quad (9)$$

Note that  $\theta^*(P_A^{**}) = \frac{D_2 - D_1}{1 - qD_1/P_A^{**}}$  and  $\Pi_1(P_A^{**}) > 0$ . Therefore, we have  $V_U(P_A^{**}) < -\bar{V}$  from (9). By the monotonicity of  $V_U(P_U)$ , we must have  $P_U^{**} < P_A^{**}$ .

When the regulator can make losses  $\bar{V} \geq -V_A(P_1^*)$ , both  $P_A$  and  $P_U$  can be set beyond  $P_1^*$ . She would receive the same expected payoff under both arrangements when committing to a price  $P \geq P_1^*$ . Therefore, the two arrangements are equally effective at the optimum.  $\square$

### 3.4 Proof of Proposition 6: ex-ante asset purchase vs. ex-ante collateralized lending

*Proof.* The proof consists of two steps. We first analyze the optimal design of ex-ante liquidity support where an information-constrained regulator commits ex-ante to lending to banks that

require her liquidity assistance. We then compare its stability effects with that of our proposed policy, i.e., the regulator commits to purchasing banks' assets at a price  $P_A^*$ . We establish a sufficient condition for our proposed asset purchasing policy to dominate the lending policy.

In the case of lending, the regulator announces at  $t = 0$  an interest rate  $r > \frac{1-q}{q}$  and a collateral requirement up to a  $\beta \in (0, 1)$  fraction of a bank's assets as the terms of lending to any bank that may suffer from runs at  $t = 1$ .<sup>3</sup> If surviving from runs at  $t = 2$ , a bank repays the regulator  $(1 + r)$  on the amount of liquidity borrowed at  $t = 1$ . Instead, if the bank fails, the regulator can seize up to  $\beta$  fraction of its assets before liquidation happens at  $t = 2$ .  $\beta$  can be seen as the maximum amount of bank assets that can be pledged as collateral.<sup>4</sup> To conduct a fair comparison, we assume a bank can dispose of the same maximum amount of assets either for asset sale or for use as collateral, i.e.,  $\beta = \frac{D_1}{P_A^*}$ .

**Step 1:** We analyze the optimal design of intervention when the regulator commits *ex-ante* to a lending rate  $r$  before any run happens. Note that the regulator chooses  $r$  to minimize  $\text{SYS}(r)$  subject to an *ex-ante* break-even constraint  $V_L(r) \geq 0$ , where  $\text{SYS}(r)$  denotes the systemic risk under the intervention and  $V_L(r)$  denotes the regulator's expected payoff.<sup>5</sup>

Observing a  $r > \frac{1-q}{q}$ , creditors' bank run game can be analyzed following the classic global-games approach. The equilibrium critical cash flows that trigger failures and runs are  $\theta_L^*(r) = (D_2 - D_1) + qD_1(1 + r)$  and  $\hat{\theta}_L(r) = x_L^*(r) + \epsilon$ , and the equilibrium critical signal is  $x_L^*(r) = \theta_L^*(r) + (2q - 1)\epsilon$ .<sup>6</sup> We have  $\text{SYS}(r) = \alpha \cdot \left( \frac{\theta_L^*(r) - \underline{\theta}_G}{\bar{\theta} - \underline{\theta}_G} \right)^2 + (1 - \alpha) \cdot \left( \frac{\theta_L^*(r) - \underline{\theta}_B}{\bar{\theta} - \underline{\theta}_B} \right)^2$ , with  $\frac{d\text{SYS}(r)}{dr} > 0$ .

The regulator's payoff  $V_L(r)$  can be expressed as follows:

$$V_L(r) = 2 \sum_{s=G,B} Pr(s) \cdot \left( \frac{\hat{\theta}_L(r) - \underline{\theta}_s}{\bar{\theta} - \underline{\theta}_s} \right) \cdot \left[ \int_{\underline{\theta}_s}^{\hat{\theta}_L(r) - 2\epsilon} (\beta\theta - D_1) d\theta + \int_{\hat{\theta}_L(r) - 2\epsilon}^{\theta_L^*(r)} \left( \beta\theta - \frac{x_L^*(r) - \theta + \epsilon}{2\epsilon} \cdot D_1 \right) d\theta + \int_{\theta_L^*(r)}^{\hat{\theta}_L(r)} \frac{x_L^*(r) - \theta + \epsilon}{2\epsilon} \cdot [D_1(1 + r) - D_1] d\theta \right] \cdot \frac{1}{\hat{\theta}_L(r) - \underline{\theta}_s}. \quad (10)$$

The term in the square bracket is her payoff from lending to a bank at an interest rate  $r$  in a given state  $s$ . Recall  $L(\theta, x_L^*(r)) = \max \left\{ \min \left\{ \frac{x_L^*(r) - \theta + \epsilon}{2\epsilon}, 1 \right\}, 0 \right\}$ , so  $\hat{\theta}_L(r) - 2\epsilon$  and  $\hat{\theta}_L(r)$  are two cutoffs of  $\theta$  such that  $L(\hat{\theta}_L(r) - 2\epsilon, x_L^*(r)) = 1$  and  $L(\hat{\theta}_L(r), x_L^*(r)) = 0$ . In addition,  $\theta_L^*(r)$  is the equilibrium

<sup>3</sup>When possessing information on banks' solvency, the regulator can, again, costlessly limit the systemic risk to  $\text{SYS}(D_2)$  by directly lending only to solvent banks at an interest rate  $r \leq \frac{1-q}{q}$ . There is also no need for her to demand collateral since the lending will be risk-free. Once informationally constrained about banks' solvency, the regulator has to increase  $r$  above  $\frac{1-q}{q}$  to compensate her losses from lending to insolvent banks.

<sup>4</sup>Note that the regulator will optimally demand the maximum amount  $\beta$  to reduce the lending rate  $r$ . Intuitively, the equilibrium critical cash flow that triggers bank failures increases in  $r$ , but does not directly depend on  $\beta$ .

<sup>5</sup>From the analysis in Section 3.3, the regulator's constraint of saving no insolvent bank never binds when she has to break even. So we omit this constraint for simplicity.

<sup>6</sup>The solution of the global game when creditors observe an interest rate  $r$  can be provided upon any request.

critical cash flow that triggers failures, i.e.,  $L(\theta_L^*(r), x_L^*(r)) = q$ . When  $\theta \in [\underline{\theta}_s, \hat{\theta}_L(r) - 2\epsilon]$ , the total withdrawal is  $L = 1$  and the bank needs to borrow  $D_1$  from the regulator. When  $\theta \in (\hat{\theta}_L(r) - 2\epsilon, \theta_L^*(r))$ , the bank needs to borrow  $LD_1 = D_1 \cdot \frac{x_L^*(r) - \theta + \epsilon}{2\epsilon}$ . In those cases, the bank fails at  $t = 2$  even with the regulator's liquidity injection since  $\theta < \theta_L^*(r)$ . Thus, the regulator will seize the collateral  $\beta\theta$  at  $t = 2$ . When  $\theta \in [\theta_L^*(r), \hat{\theta}_L(r))$ , the bank survives from runs with the regulator's liquidity assistance. It then repays  $D_1(1 + r) \cdot \frac{x_L^*(r) - \theta + \epsilon}{2\epsilon}$  to the regulator at  $t = 2$ . We presume that  $\frac{dV_L(r)}{dr} > 0$ .

For both  $\text{SYS}(r)$  and  $V_L(r)$  increase in  $r$ , the regulator optimally chooses  $r^*$  such that  $V_L(r^*) = 0$  to minimize the systemic risk. One can derive the equilibrium as follows: there exists a unique  $r^* > \frac{1-q}{q}$  such that the regulator breaks even ex-ante; a wholesale creditor  $j$  of a bank  $i$  withdraws if and only if his private signal falls below  $x_L^*$ . The unique  $(r^*, x_L^*, \theta_L^*)$  jointly solve the system  $V_L(r^*) = 0$ ,  $x_L^* = x_L^*(r^*)$  and  $\theta_L^* = \theta_L^*(r^*)$ . The equilibrium systemic risk under ex-ante lending is  $\text{SYS}(\theta_L^*) = \alpha \cdot \left(\frac{\theta_L^* - \theta_G}{\theta - \theta_G}\right)^2 + (1 - \alpha) \cdot \left(\frac{\theta_L^* - \theta_B}{\theta - \theta_B}\right)^2$ .

The optimal design of the ex-ante asset purchase policy in the non-limiting case resembles the analysis of the limiting case in Section 3.3 of the manuscript. Observing a  $P_A \in [\underline{P}, qD_2)$ , the equilibrium critical cash flows that trigger failures and runs are  $\theta^*(P_A) = \frac{D_2 - D_1}{1 - qD_1/P_A}$  and  $\hat{\theta}(r) = x^*(P_A) + \epsilon$ , and the equilibrium critical signal is  $x^*(P_A) = \theta^*(P_A) + (2q - 1)\epsilon$ . The regulator's objective is  $\text{SYS}(P_A) = \alpha \cdot \left(\frac{\theta^*(P_A) - \theta_G}{\theta - \theta_G}\right)^2 + (1 - \alpha) \cdot \left(\frac{\theta^*(P_A) - \theta_B}{\theta - \theta_B}\right)^2$ . Note that the regulator's expected payoff when pre-committing a price  $P_A$  in the non-limiting case is as follows:

$$V_A(P_A) = 2 \sum_{s=G,B} Pr(s) \left( \frac{\hat{\theta}(P_A) - \theta_s}{\theta - \theta_s} \right) \cdot \left[ \int_{\theta_s}^{\hat{\theta}(P_A) - 2\epsilon} (\theta - P_A) \cdot \frac{D_1}{P_A} d\theta + \int_{\hat{\theta}(P_A) - 2\epsilon}^{\hat{\theta}(P_A)} (\theta - P_A) \cdot \left( \frac{x^*(P_A) - \theta + \epsilon D_1}{2\epsilon} \right) \frac{D_1}{P_A} d\theta \right] \cdot \frac{1}{\hat{\theta}(P_A) - \theta_s}. \quad (11)$$

Similar to the lending case, the term in the square bracket is the regulator's payoff from purchasing a bank's assets at a price  $P_A$  in a given state  $s$ .  $\hat{\theta}(P_A) - 2\epsilon$  and  $\hat{\theta}(P_A)$  are two threshold cash flows solving  $L(\hat{\theta}(P_A) - 2\epsilon, x^*(P_A)) = 1$  and  $L(\hat{\theta}(P_A), x^*(P_A)) = \frac{x^*(P_A) - \hat{\theta}(P_A) + \epsilon}{2\epsilon} = 0$ . When  $\theta \in [\underline{\theta}_s, \hat{\theta}(P_A) - 2\epsilon]$ , the total withdrawal is  $L = 1$ , so the bank needs to sell  $\frac{D_1}{P_A}$  fraction of assets to meet its liquidity demand  $D_1$ . When  $\theta \in (\hat{\theta}(P_A) - 2\epsilon, \hat{\theta}(P_A))$ , the total withdrawal becomes  $L \in (0, 1)$ , therefore it only needs to sell  $L \cdot \frac{D_1}{P_A}$  fraction of assets. In the expression,  $\theta - P_A$  denotes the regulator's unit payoff from purchasing assets. One can verify that  $\frac{dV_A(P_A)}{dP_A} < 0$ .<sup>7</sup>

<sup>7</sup>Note that when  $\epsilon \rightarrow 0$ , we have  $\hat{\theta}(P_A) \rightarrow \theta^*(P_A)$ ,  $x^*(P_A) \rightarrow \theta^*(P_A)$  and  $V_A(P_A) \rightarrow \sum_{s=G,B} Pr(s) \cdot Pr(\theta < \theta^*(P_A)|s) \cdot \left(2 \cdot \frac{D_1}{P_A} \cdot \pi(P_A|s)\right)$ , which is, after some manipulation, expression (22) in the manuscript.

For both  $\text{SYS}(P_A)$  and  $V_A(P_A)$  decrease in  $P_A$ , the regulator optimally chooses  $P_A^*$  such that  $V_A(P_A^*) = 0$  to minimize the systemic risk. One can again derive the equilibrium as follows: there exists a unique  $P_A^* \in [\underline{P}, qD_2)$  such that the regulator breaks even ex-ante; a wholesale creditor  $j$  of a bank  $i$  withdraws if and only if  $x_j^i$  falls below  $x_A^*$ . The unique  $(P_A^*, x_A^*, \theta_A^*)$  jointly solve the system  $V_A(P_A^*) = 0$ ,  $x_A^* = x^*(P_A^*)$  and  $\theta_A^* = \theta^*(P_A^*)$ . The equilibrium systemic risk under ex-ante asset purchase is  $\text{SYS}(\theta_A^*) = \alpha \cdot \left(\frac{\theta_A^* - \theta_G}{\theta - \theta_G}\right)^2 + (1 - \alpha) \cdot \left(\frac{\theta_A^* - \theta_B}{\theta - \theta_B}\right)^2$ .

**Step 2:** We compare the stability effects of two policies and derive a sufficient condition under which the ex-ante asset purchase dominates the ex-ante collateralized lending.

To proceed, we focus on a lending rate  $\hat{r}$  such that  $\theta_L^*(\hat{r}) = \theta_A^*$ , i.e., the regulator achieves the equilibrium critical cash flow  $\theta_A^*$  when she pre-commits to lending at  $\hat{r}$ . We have:

$$1 + \hat{r} = \frac{D_2 - D_1}{P_A^* - qD_1} \Leftrightarrow \theta_L^*(\hat{r}) = (D_2 - D_1) + qD_1(1 + \hat{r}) = \frac{D_2 - D_1}{1 - qD_1/P_A^*} = \theta_A^*.$$

Note that  $\hat{r}$  is unique. The equilibrium critical signal associated with the critical cash flow is  $x_L^*(\hat{r}) = \theta_L^*(\hat{r}) + (2q - 1)\epsilon = x_A^*$ . We then calculate the regulator's expected payoff  $V_L(\hat{r})$ .

To start with, we insert the optimal allocation  $(P_A^*, x_A^*, \theta_A^*)$  into (11) and obtain:

$$0 = 2 \sum_{s=G,B} Pr(s) \left( \frac{\theta_A^* - \theta_s}{\theta - \theta_s} \right) \cdot \left[ \int_{\theta_s}^{\theta_A^* + 2q\epsilon - 2\epsilon} (\theta - P_A^*) \cdot \frac{D_1}{P_A^*} d\theta + \int_{\theta_A^* + 2q\epsilon - 2\epsilon}^{\theta_A^* + 2q\epsilon} (\theta - P_A^*) \cdot \left( \frac{x_A^* - \theta + \epsilon}{2\epsilon} \cdot \frac{D_1}{P_A^*} \right) d\theta \right] \cdot \frac{1}{\theta_A^* + 2q\epsilon - \theta_s} = V_A(P_A^*).$$

The term in the square bracket can be further expressed as:<sup>8</sup>

$$\underbrace{\frac{D_1}{P_A^*} \int_{\theta_s}^{\theta_A^* + 2q\epsilon - 2\epsilon} \theta d\theta + \frac{D_1}{P_A^*} \int_{\theta_A^* + 2q\epsilon - 2\epsilon}^{\theta_A^* + 2q\epsilon} \theta \cdot \frac{x_A^* - \theta + \epsilon}{2\epsilon} d\theta}_{(*)} - \underbrace{\left( \int_{\theta_s}^{\theta_A^* + 2q\epsilon - 2\epsilon} D_1 d\theta + \int_{\theta_A^* + 2q\epsilon - 2\epsilon}^{\theta_A^* + 2q\epsilon} D_1 \cdot \frac{x_A^* - \theta + \epsilon}{2\epsilon} d\theta \right)}_{(**)} \quad (12)$$

We then insert  $(\hat{r}, x_L^*(\hat{r}), \theta_L^*(\hat{r})) = (\hat{r}, x_A^*, \theta_A^*)$  into (10) and express the regulator's payoff from lending at the interest rate  $\hat{r}$  as:

$$V_L(\hat{r}) = 2 \sum_{s=G,B} Pr(s) \left( \frac{\theta_A^* - \theta_s}{\theta - \theta_s} \right) \cdot \left[ \int_{\theta_s}^{\theta_A^* + 2q\epsilon - 2\epsilon} (\beta\theta - D_1) d\theta + \int_{\theta_A^* + 2q\epsilon - 2\epsilon}^{\theta_A^*} \left( \beta\theta - \frac{x_A^* - \theta + \epsilon}{2\epsilon} \cdot D_1 \right) d\theta + \int_{\theta_A^*}^{\theta_A^* + 2q\epsilon} \frac{x_A^* - \theta + \epsilon}{2\epsilon} \cdot [D_1 \cdot (1 + \hat{r}) - D_1] d\theta \right] \cdot \frac{1}{\theta_A^* + 2q\epsilon - \theta_s}.$$

Again, the term in the square bracket of  $V_L(\hat{r})$  is:

$$\underbrace{\beta \int_{\theta_s}^{\theta_A^*} \theta d\theta + \frac{D_1}{P_A^*} \int_{\theta_A^*}^{\theta_A^* + 2q\epsilon} \theta_A^* \cdot \frac{x_A^* - \theta + \epsilon}{2\epsilon} d\theta}_{(***)} - \underbrace{\left( \int_{\theta_s}^{\theta_A^* + 2q\epsilon - 2\epsilon} D_1 d\theta + \int_{\theta_A^* + 2q\epsilon - 2\epsilon}^{\theta_A^* + 2q\epsilon} D_1 \cdot \frac{x_A^* - \theta + \epsilon}{2\epsilon} d\theta \right)}_{(**)} \quad (13)$$

<sup>8</sup>For simplicity, we drop the density  $\frac{1}{\theta_A^* + 2q\epsilon - \theta_s}$  as it will not affect our comparison.

In this expression, we have:  $D_1(1 + \hat{r}) = D_1 \cdot \frac{D_2 - D_1}{P_A^* - qD_1} = \frac{D_1}{P_A^*} \cdot \frac{D_2 - D_1}{1 - qD_1/P_A^*} = \frac{D_1}{P_A^*} \cdot \theta_A^*$ .

As  $V_A(P_A^*) = 0$ , we can further express  $V_L(\hat{r})$  as  $V_L(\hat{r}) - V_A(P_A^*)$  and obtain:

$$V_L(\hat{r}) = 2 \sum_{s=G,B} Pr(s) \left( \frac{\theta_A^* - \theta_s}{\theta - \theta_s} \right) \cdot \underbrace{\left\{ \left( \beta - \frac{D_1}{P_A^*} \right) \cdot \int_{\theta_s}^{\theta_A^*} \theta d\theta \right.}_{\textcircled{1}} \\ \left. + \frac{D_1}{P_A^*} \left[ \int_{\theta_A^* + 2q\epsilon - 2\epsilon}^{\theta_A^*} \theta \cdot \left( 1 - \frac{x_A^* - \theta + \epsilon}{2\epsilon} \right) d\theta + \int_{\theta_A^*}^{\theta_A^* + 2q\epsilon} (\theta_A^* - \theta) \cdot \frac{x_A^* - \theta + \epsilon}{2\epsilon} d\theta \right] \right\} \cdot \frac{1}{\theta_A^* + 2q\epsilon - \theta_s}. \quad (14)$$

②

from (12) and (13). With the same equilibrium critical cash flow  $\theta_A^*$ , the regulator injects the same amount of liquidity to banks, no matter it purchases assets at  $P_A^*$  or lends to banks at  $\hat{r}$  (i.e., (\*\*\*) in (12) and (13)). The quality of assets being acquired (i.e., (\*) of (12) and (\*\*\*) of (13)) determines her expected payoffs under the two forms of intervention.

Note that  $\textcircled{1} + \textcircled{2} = \textcircled{***} - \textcircled{*}$  and we have:  $sgn(V_L(\hat{r})) = sgn(\textcircled{1} + \textcircled{2})$ . First, observe that  $\textcircled{1} = 0$  because  $\beta = \frac{D_1}{P_A^*}$ . Second, with the equilibrium condition  $x_A^* = \theta_A^* + (2q - 1)\epsilon$ , we can calculate explicitly  $\textcircled{2}$  as follow:

$$\textcircled{2} = \left[ (\theta_A^* - 2\epsilon) \cdot (1 - q)^2 + \epsilon \cdot (1 - 2q) + \frac{\epsilon}{3} \right] \cdot \epsilon \cdot \frac{D_1}{P_A^*}.$$

When  $q \rightarrow 1$ , we have:  $\textcircled{2} \rightarrow -\frac{2}{3}\epsilon^2 \cdot \frac{D_1}{P_A^*} < 0$ ; when  $q \rightarrow \frac{1}{2}$ , we have:  $\textcircled{2} \rightarrow \left( \frac{\theta_A^* - 2\epsilon}{4} + \frac{\epsilon}{3} \right) \epsilon \cdot \frac{D_1}{P_A^*} > 0$  provided with  $\epsilon$  to be small. By continuity,  $\textcircled{2} < 0$  when  $q$  is sufficiently close to 1. To simplify the discussion, we further assume that  $\textcircled{2}$  decreases in  $q$ .<sup>9</sup> Therefore, there exists a unique cutoff  $q^c \in \left( \frac{1}{2}, 1 \right)$  such that  $\textcircled{2} < 0$  when  $q^c < q < 1$ . As  $sgn(V_L(\hat{r})) = sgn(\textcircled{1} + \textcircled{2}) = sgn(\textcircled{2})$ , we have: there exists a unique cutoff  $q^c \in \left( \frac{1}{2}, 1 \right)$  such that  $V_L(\hat{r}) < 0$  when  $q^c < q < 1$ .<sup>10</sup>

Recall that in **Step 1**, the optimal interest rate  $r^*$  that minimizes the systemic risk is such that  $V_L(r^*) = 0$ . When  $q^c < q < 1$ , we have  $V_L(r^*) = 0 > V_L(\hat{r})$ . Therefore,  $r^* > \hat{r}$  because  $\frac{dV_L(r)}{dr} > 0$ . This, in turn, means  $\theta_L^* = (D_2 - D_1) + qD_1(1 + r^*) > (D_2 - D_1) + qD_1(1 + \hat{r}) = \theta_L^*(\hat{r}) = \theta_A^*$ .

<sup>9</sup>Actually, one can check that  $\frac{d}{dq} \left[ (\theta_A^* - 2\epsilon) \cdot (1 - q)^2 + \epsilon \cdot (1 - 2q) + \frac{\epsilon}{3} \right] = -2(1 - q)(\theta_A^* - 2\epsilon) - 2\epsilon < 0$ .

<sup>10</sup>To better understand the result, consider again term  $\textcircled{2}$ . Note that the expression  $\frac{D_1}{P_A^*} \int_{\theta_A^* + 2q\epsilon - 2\epsilon}^{\theta_A^*} \theta \cdot \left( 1 - \frac{x_A^* - \theta + \epsilon}{2\epsilon} \right) d\theta$  is positive and reflects the benefits of the committed lending to the regulator: a bank with a cash flow  $\theta \in [\theta_A^* + 2q\epsilon - 2\epsilon, \theta_A^*]$  still fails despite the interventions. The regulator would seize a  $\beta\theta = \frac{D_1}{P_A^*}\theta$  amount of the bank's assets at  $t = 2$  due to the collateralization of her lending. Instead, she would only acquire a  $\frac{LD_1}{P_A^*}\theta = \frac{x_A^* - \theta + \epsilon}{2\epsilon} \frac{D_1}{P_A^*}\theta$  amount of the bank's assets at  $t = 1$  when committing to purchasing the bank's assets. On the other hand, the expression  $\frac{D_1}{P_A^*} \int_{\theta_A^*}^{\theta_A^* + 2q\epsilon} (\theta_A^* - \theta) \cdot \frac{x_A^* - \theta + \epsilon}{2\epsilon} d\theta$  is negative and reflects the benefits of the committed asset purchase: a bank with a cash flow  $\theta \in [\theta_A^*, \theta_A^* + 2q\epsilon]$  survives runs because of the interventions. The regulator's return on the borrowed liquidity is bounded by the pre-specified interest rate payment  $LD_1(1 + \hat{r}) = \frac{x_A^* - \theta + \epsilon}{2\epsilon} \frac{D_1}{P_A^*}\theta_A^*$ . By contrast, she can access the higher upside gain  $\frac{LD_1}{P_A^*}\theta = \frac{x_A^* - \theta + \epsilon}{2\epsilon} \frac{D_1}{P_A^*}\theta$  when purchasing the bank's assets. When  $q$  increases to 1, the length of interval  $[\theta_A^* + 2q\epsilon - 2\epsilon, \theta_A^*]$  diminishes to 0 while the length of interval  $[\theta_A^*, \theta_A^* + 2q\epsilon]$  expands to  $2\epsilon$ . Therefore, the regulator's payoff difference  $V_L(\hat{r}) - V_A(P_A^*) = V_L(\hat{r}) < 0$  when  $q$  is sufficiently large.

To conclude, when  $q^c < q < 1$ , the regulator achieves a lower systemic risk when committing ex-ante to purchasing banks' assets at  $P_A^*$ , i.e.,  $\text{SYS}(\theta_A^*) < \text{SYS}(\theta_L^*)$ .

Lastly, consider the limiting case where  $\epsilon \rightarrow 0$ . It can be seen easily that term ② in expression (14) approaches zero. Therefore,  $\hat{r} = r^*$  and the two interventions achieve the same allocation  $\theta_A^* = \theta_L^*$ .  $\square$

### 3.5 Ex-ante interventions v.s. Ex-post interventions

#### 3.5.1 Ex-ante v.s. ex-post asset purchase

We compare our proposed policy where an information-constrained regulator commits ex-ante to purchasing banks' assets with ex-post intervention where she purchases assets upon runs happen, in the non-limiting case where  $\epsilon$  is away from 0.

Consider an arrangement where the regulator offers a supporting price  $P_{A,M}$  to purchase banks' assets on sale after observing  $M$  bank runs at  $t = 1$ . The intervention is *ex-post*, in the sense that each  $P_{A,M}$  is set conditional on the observation of  $M \in \{1, 2\}$  actual bank runs. The regulator is also assumed to make no loss in each contingency of  $M$  runs.

Upon observing  $M$  runs, the regulator knows that creditors withdraw from a bank if and only if their private signals are below  $x_{A,M}^*$  and that a bank run happens if and only if the bank's cash flow is below  $\hat{\theta}_{A,M}$ . The regulator's ex-post payoff  $V_{A,M}(P_{A,M})$  from purchasing banks' assets when observing  $M$  bank runs can be expressed as:

$$V_{A,M}(P_{A,M}) = M \sum_{s=G,B} \omega_M^s(\hat{\theta}_{A,M}) \cdot \left[ \int_{\underline{\theta}_s}^{\hat{\theta}_{A,M}-2\epsilon} (\theta - P_{A,M}) \cdot \frac{D_1}{P_{A,M}} d\theta + \int_{\hat{\theta}_{A,M}-2\epsilon}^{\hat{\theta}_{A,M}} (\theta - P_{A,M}) \cdot \left( \frac{x_{A,M}^* - \theta + \epsilon}{2\epsilon} \cdot \frac{D_1}{P_{A,M}} \right) d\theta \right] \cdot \frac{1}{\hat{\theta}_{A,M} - \underline{\theta}_s}.$$

Here  $\omega_M^s(\hat{\theta}_{A,M})$  (functional form as expression (6) of the manuscript) denotes the regulator's posterior beliefs about  $s$  upon observing  $M$  runs. The term in the square bracket represents her payoff from purchasing a bank's assets at a price  $P_{A,M}$  in a given state  $s$ . We also have:  $\frac{dV_{A,M}(P_{A,M})}{dP_{A,M}} < 0, \forall M \in \{1, 2\}$ . The regulator faces an ex-post break-even constraint  $V_{A,M}(P_{A,M}) \geq 0$  in each contingency of  $M$  runs.

Anticipating higher price supports, banks' creditors have less incentive to withdraw, which reduces the critical cash flows that trigger failures and runs. Therefore, the best price support

$P_{A,M}^*$  that the regulator can offer satisfies  $V_{A,M}(P_{A,M}^*) = 0, \forall M \in \{1, 2\}$ .<sup>11</sup> One can derive the unique equilibrium of the game as follows: a wholesale creditor  $j$  of a bank  $i$  withdraws if and only if  $x_j^i$  falls below  $x^*(y^{-i})$ , with  $x^*(y^{-i}) = x_{A,2}^*$  when  $y^{-i} < x_{A,2}^* + \eta + \epsilon$  and  $x^*(y^{-i}) = x_{A,1}^*$  when  $y^{-i} \geq x_{A,2}^* + \eta + \epsilon$ ; the regulator can break even ex-post at each contingency  $M \in \{1, 2\}$  with the unique price  $P_{A,M}^*$ . The unique  $(P_{A,M}^*, x_{A,M}^*, \theta_{A,M}^*)$  jointly solves the system  $V_{A,M}(P_{A,M}^*) = 0, x_{A,M}^* = \theta_{A,M}^* + (2q - 1)\epsilon$  and  $\theta_{A,M}^* = \frac{D_2 - D_1}{1 - qD_1/P_{A,M}^*}$  for each  $M \in \{1, 2\}$ .<sup>12</sup> The equilibrium systemic risk with the ex-post asset purchase becomes:  $\text{SYS}(\theta_{A,2}^*) = \alpha \cdot \left( \frac{\theta_{A,2}^* - \theta_G}{\theta - \theta_G} \right)^2 + (1 - \alpha) \cdot \left( \frac{\theta_{A,2}^* - \theta_B}{\theta - \theta_B} \right)^2$ .

We then turn to *ex-ante* liquidity intervention where the regulator commits before any run happens to purchasing banks' assets at  $P_A^*$ . Recall that the regulator optimally chooses  $P_A^*$  such that  $V_A(P_A^*) = 0$ . The equilibrium systemic risk with the ex-ante asset purchase is  $\text{SYS}(\theta_A^*) = \alpha \cdot \left( \frac{\theta_A^* - \theta_G}{\theta - \theta_G} \right)^2 + (1 - \alpha) \cdot \left( \frac{\theta_A^* - \theta_B}{\theta - \theta_B} \right)^2$ .

The following Result 3 compares the ex-ante asset purchase with the ex-post asset purchase in reducing systemic risks.

**Result 3.** *When the regulator has no information on banks' solvency and injects liquidity in the form of purchasing banks' assets, committing ex-ante at a price  $P_A^*$  achieves a lower risk of systemic bank failures than offering supporting price  $P_{A,M}^*$  ex-post based on the number of runs  $M$ . In particular, we have  $\text{SYS}(\theta_A^*) < \text{SYS}(\theta_{A,2}^*)$ .*

*Proof.* We can express the regulator's ex-post payoff for a given price  $P \in [\underline{P}, qD_2]$  as:

$$V_{A,M}(P) = M \cdot \sum_{s=G,B} \omega_M^s(\hat{\theta}(P)) \cdot \left[ \int_{\underline{\theta}_s}^{\hat{\theta}(P) - 2\epsilon} (\theta - P) \cdot \frac{D_1}{P} d\theta + \int_{\hat{\theta}(P) - 2\epsilon}^{\hat{\theta}(P)} (\theta - P) \cdot \left( \frac{x^*(P) - \theta + \epsilon}{2\epsilon} \cdot \frac{D_1}{P} \right) d\theta \right] \cdot \frac{1}{\hat{\theta}(P) - \underline{\theta}_s},$$

where  $\theta^*(P) = \frac{D_2 - D_1}{1 - qD_1/P}$ ,  $x^*(P) = \theta^*(P) + (2q - 1)\epsilon$  and  $\hat{\theta}(P) = \theta^*(P) + 2q\epsilon$ . We suppose  $\frac{dV_{A,M}(P)}{dP} < 0$ . The equilibrium ex-post supporting price satisfies  $V_{A,M}(P_{A,M}^*) = 0, \forall M \in \{1, 2\}$ .

We can re-arrange the regulator's ex-ante payoff  $V_A(P)$  for a given price  $P$  as follows:

$$V_A(P) = \sum_{M=1}^2 \left( \sum_{s=G,B} Pr(s) \cdot Pr(\theta < \hat{\theta}(P)|s)^M \cdot Pr(\theta > \hat{\theta}(P)|s)^{2-M} \right) \cdot C_2^M \cdot V_{A,M}(P).$$

<sup>11</sup>When  $\epsilon \rightarrow 0$ , we have  $x_{A,M}^* \rightarrow \theta_{A,M}^*$ ,  $\hat{\theta}_{A,M} \rightarrow \theta_{A,M}^*$  and  $V_{A,M}(P_{A,M}) \rightarrow M \cdot \sum_{s=G,B} \omega_M^s(\theta_{A,M}^*) \cdot \pi(P_{A,M}|s) \cdot \frac{D_1}{P_{A,M}} = M \cdot \Pi_M(P_{A,M}) \cdot \frac{D_1}{P_{A,M}}$ . It can be seen that the equilibrium market price  $P_M^*$  also solves  $V_{A,M}(P_M^*) = 0$ . Provided with the monotonicity of  $V_{A,M}(\cdot)$ , we obtain the unique solution  $P_{A,M}^* = P_M^*$ . Therefore, the efficiency of the ex-post asset purchase policy is bounded by the market allocation in the limiting case.

<sup>12</sup>Recall that  $\hat{\theta}_{A,M} = x_{A,M}^* + \epsilon$  and one can, again, show  $P_{A,1}^* > P_{A,2}^*$ ,  $x_{A,1}^* < x_{A,2}^*$  and  $\theta_{A,1}^* < \theta_{A,2}^*$  following the Proof of Lemma 3 in the manuscript.

When intervening ex-ante at a price  $P_A^*$ , the regulator does not require  $V_{A,M}(P_A^*) = 0$  but instead  $V_A(P_A^*) = 0$ . In fact, the regulator expects to make losses in the contingency of  $M = 2$  and to profit in the contingency of  $M = 1$ . Note that  $V_{A,1}(P_{A,2}^*) > 0 = V_{A,1}(P_{A,1}^*)$  and  $V_{A,2}(P_{A,1}^*) < 0 = V_{A,2}(P_{A,2}^*)$  by the equilibrium condition  $V_{A,M}(P_{A,M}^*) = 0$ , the monotonicity of  $V_{A,M}(P)$  and  $P_{A,1}^* > P_{A,2}^*$ . Consequently,  $V_A(P) > 0$  for  $P = P_{A,2}^*$  and  $V_A(P) < 0$  for  $P = P_{A,1}^*$ . Since  $V_A(P)$  is also monotonic and continuous in  $P$ , the unique ex-ante break-even price  $P_A^* \in (P_{A,2}^*, P_{A,1}^*)$ . Since  $P_A^* > P_{A,2}^*$ , we have  $\theta_A^* = \theta^*(P_A^*) < \theta^*(P_{A,2}^*) = \theta_{A,2}^*$  and  $\text{SYS}(\theta_A^*) < \text{SYS}(\theta_{A,2}^*)$ .  $\square$

### 3.5.2 Ex-ante v.s. ex-post collateralized lending

We compare ex-ante intervention where the regulator commits to lending to banks at a pre-specified interest rate with ex-post intervention when she sets interest rates upon runs.

Lending ex-post, the regulator sets an interest rate  $r_M$  to banks that demand liquidity assistance at  $t = 1$ . The arrangement is *ex-post*, in the sense that each  $r_M$  is offered after the observation of  $M \in \{1, 2\}$  actual bank runs. We also assume the same  $\beta$  to be the maximum fraction of a bank's assets that can be used as eligible collateral in this comparison. The regulator must make no loss in each contingency  $M$  as before.

Upon observing  $M$  runs, the regulator knows that creditors withdraw from a bank if and only if their private signals are below  $x_{L,M}^*$  and that a bank run happens if the bank's cash flow is below  $\hat{\theta}_{L,M}$ . Her ex-post payoff  $V_{L,M}(r_M)$  from lending to banks after observing  $M$  bank runs can be expressed as:

$$V_{L,M}(r_M) = M \sum_{s=G,B} \omega_M^s(\hat{\theta}_{L,M}) \cdot \left[ \int_{\underline{\theta}_s}^{\hat{\theta}_{L,M}-2\epsilon} (\beta\theta - D_1) d\theta + \int_{\hat{\theta}_{L,M}-2\epsilon}^{\theta_{L,M}^*} \left( \beta\theta - \frac{x_{L,M}^* - \theta + \epsilon}{2\epsilon} \cdot D_1 \right) d\theta \right. \\ \left. + \int_{\theta_{L,M}^*}^{\hat{\theta}_{L,M}} \frac{x_{L,M}^* - \theta + \epsilon}{2\epsilon} \cdot [D_1(1 + r_M) - D_1] d\theta \right] \cdot \frac{1}{\hat{\theta}_{L,M} - \underline{\theta}_s}.$$

One can also check that  $\frac{dV_{L,M}(r_M)}{dr_M} > 0$ . The regulator's ex-post break-even constraint then writes as  $V_{L,M}(r_M) \geq 0, \forall M \in \{1, 2\}$ .

Like the ex-post asset purchase case, the best interest rate that the regulator would offer is  $r_M^*$  satisfying  $V_{L,M}(r_M^*) = 0, \forall M \in \{1, 2\}$ . The unique equilibrium of the game is: a wholesale creditor  $j$  of a bank  $i$  withdraws if and only if  $x_j^i$  falls below  $x^*(y^{-i})$ , with  $x^*(y^{-i}) = x_{L,2}^*$  when  $y^{-i} < x_{L,2}^* + \eta + \epsilon$  and  $x^*(y^{-i}) = x_{L,1}^*$  when  $y^{-i} \geq x_{L,2}^* + \eta + \epsilon$ ; the regulator can break even ex-post at each contingency  $M \in \{1, 2\}$  with the unique interest rate  $r_M^*$ . The unique combination  $(r_M^*, x_{L,M}^*, \theta_{L,M}^*)$  jointly solve the system  $V_{L,M}(r_M^*) = 0, x_{L,M}^* = \theta_{L,M}^* + (2q - 1)\epsilon$  and  $\theta_{L,M}^* =$

$(D_2 - D_1) + qD_1(1 + r_M^*)$  for each  $M \in \{1, 2\}$ . The equilibrium systemic risk with the ex-post lending is:  $\text{SYS}(\theta_{L,2}^*) = \alpha \cdot \left(\frac{\theta_{L,2}^* - \underline{\theta}_G}{\theta - \underline{\theta}_G}\right)^2 + (1 - \alpha) \cdot \left(\frac{\theta_{L,2}^* - \underline{\theta}_B}{\theta - \underline{\theta}_B}\right)^2$ .

We then turn to liquidity intervention where the regulator commits *ex-ante* before any run happens to lend to banks a pre-specified interest rate. To minimize the systemic crisis, the regulator optimally chooses  $r^*$  such that  $V_L(r^*) = 0$ . The equilibrium systemic risk with the ex-ante lending is  $\text{SYS}(\theta_L^*) = \alpha \cdot \left(\frac{\theta_L^* - \underline{\theta}_G}{\theta - \underline{\theta}_G}\right)^2 + (1 - \alpha) \cdot \left(\frac{\theta_L^* - \underline{\theta}_B}{\theta - \underline{\theta}_B}\right)^2$ .

The following Result 4 compares the ex-ante lending and the ex-post lending policies.

**Result 4.** *When the regulator has no information on banks' solvency and injects liquidity in the form of collateralized lending, committing ex-ante at an interest rate  $r^*$  achieves a lower risk of systemic bank failures than offering interest rates  $r_M^*$  ex-post based on the number of runs  $M$ . In particular, we have  $\text{SYS}(\theta_L^*) < \text{SYS}(\theta_{L,2}^*)$ .*

*Proof.* Again, we can express the regulator's ex-post payoff for a given interest rate  $r$  as:

$$V_{L,M}(r) = M \cdot \sum_{s=G,B} \omega_M^s(\hat{\theta}_L(r)) \cdot \left[ \int_{\underline{\theta}_s}^{\hat{\theta}_L(r)-2\epsilon} (\beta\theta - D_1) d\theta + \int_{\hat{\theta}_L(r)-2\epsilon}^{\theta_L^*(r)} \left( \beta\theta - \frac{x_L^*(r) - \theta + \epsilon}{2\epsilon} \cdot D_1 \right) d\theta \right. \\ \left. + \int_{\theta_L^*(r)}^{\hat{\theta}_L(r)} \frac{x_L^*(r) - \theta + \epsilon}{2\epsilon} \cdot [D_1(1+r) - D_1] d\theta \right] \cdot \frac{1}{\hat{\theta}_L(r) - \underline{\theta}_s}.$$

We suppose  $\frac{dV_{L,M}(r)}{dr} > 0$ . Then, the equilibrium interest rate satisfies  $V_{L,M}(r_M^*) = 0, \forall M \in \{1, 2\}$ . The regulator's payoff from committing ex-ante to lending to banks at an interest rate  $r$  can be re-arranged as:

$$V_L(r) = \sum_{M=1}^2 \left( \sum_{s=G,B} Pr(s) \cdot Pr(\theta < \hat{\theta}_L(r)|s)^M \cdot Pr(\theta > \hat{\theta}_L(r)|s)^{2-M} \right) \cdot C_2^M \cdot V_{L,M}(r).$$

Again, there exists a unique ex-ante interest rate  $r^* \in (r_1^*, r_2^*)$ . Since  $r^* > r_2^*$ , we obtain  $\theta_L^* = \theta_L^*(r^*) < \theta_L^*(r_2^*) = \theta_{L,2}^*$  and  $\text{SYS}(\theta_L^*) < \text{SYS}(\theta_{L,2}^*)$ .  $\square$

To complete the horse-racing between the popular forms of liquidity interventions in the non-limiting case, we present the following Result 5.

**Result 5.** *When  $q > \hat{q} \in (\frac{1}{2}, 1)$  and  $\epsilon > 0$ , committing ex-ante to purchasing banks' assets on sale at a price  $P_A^*$  achieves a strictly lower risk of systemic bank failures than committing ex-ante to lending to banks at an interest rate  $r^*$ , which, in turn, outperforms ex-post lending at an interest rate  $r_M^*$  upon  $M$  runs happen. In particular, we have  $\text{SYS}(\theta_A^*) < \text{SYS}(\theta_L^*) < \text{SYS}(\theta_{L,2}^*)$ .*

*Proof.* Immediately follows from Proposition 6 in the manuscript and Result 4.  $\square$

## 4 Robustness checks of the information structure

### 4.1 Alternative information structure: $y$ being observable to all creditors

We consider an alternative information structure of the model and solve the laissez-faire market equilibrium. Now, a representative creditor  $j \in [0, 1]$  of a bank  $i \in \{1, 2\}$  is assumed to still observe a private signal  $x_j^i = \theta^i + \epsilon_j^i$  about his own bank's fundamental. In addition, all creditors of the bank  $i$  also observe semi-public signals  $y^i$  and  $y^{-i}$  about their own bank's and the other bank's fundamentals, with  $y^i = \theta^i + \eta^i$ ,  $y^{-i} = \theta^{-i} + \eta^{-i}$ .<sup>13</sup> We assume independent noises, with both  $\tilde{\eta}^i, \tilde{\eta}^{-i} \sim U(-\eta, \eta)$  and  $\tilde{\epsilon}_j^i \sim U(-\epsilon, \epsilon)$ . Buyers in the secondary asset market can observe the number of runs  $M \in \{1, 2\}$  when called upon to move but still observe neither the aggregate state  $s$  nor any signal about the banks' fundamentals.

We further make the following assumption:

$$\epsilon \text{ diminishes to } 0 \text{ for a fixed } \eta. \quad (15)$$

The creditor  $j$ 's private signal  $x_j^i$  about  $\theta^i$  is much more precise than the semi-public signal  $y^i$  received by all creditors.

Before solving the equilibrium, we first establish a preliminary result regarding the creditor  $j$ 's posterior of  $\theta^i$  when receiving  $x_j^i \in [x^L, x^U]$  and  $y^i$ .

**Result 6.** *When  $\epsilon$  diminishes to 0 for a fixed  $\eta$ , we have:  $\tilde{\theta}^i|_{(x_j^i, y^i)} \rightarrow \tilde{\theta}^i|_{x_j^i} \sim U(x_j^i - \epsilon, x_j^i + \epsilon)$ .*

*Proof.* We derive the representative creditor's posterior beliefs about  $\tilde{\theta}^i$  upon observing both the private signal  $x_j^i$  and the semi-public signal  $y^i$ .<sup>14</sup> By Bayes' law, we have:

$$f(\theta^i|x_j^i, y^i) = f(\theta^i|x_j^i) \cdot \frac{f(y^i|\theta^i)}{f(y^i|x_j^i)} \quad (16)$$

as  $\tilde{x}_j^i|_{\theta^i}$  and  $\tilde{y}^i|_{\theta^i}$  are independent. Recall that, for any  $x_j^i \in [x^L, x^U]$ ,  $\tilde{\theta}^i|_{x_j^i} \sim U(x_j^i - \epsilon, x_j^i + \epsilon)$  and  $\tilde{y}^i|_{x_j^i}$  has the following density function

$$f(y^i|x_j^i) = \begin{cases} \frac{y^i - (x_j^i - \eta - \epsilon)}{4\eta\epsilon} & x_j^i - \eta - \epsilon < y^i \leq x_j^i - \eta + \epsilon \\ \frac{1}{2\eta} & x_j^i - \eta + \epsilon < y^i \leq x_j^i + \eta - \epsilon \\ \frac{(x_j^i + \eta + \epsilon) - y^i}{4\eta\epsilon} & x_j^i + \eta - \epsilon < y^i \leq x_j^i + \eta + \epsilon \\ 0 & \text{otherwise.} \end{cases} \quad (17)$$

<sup>13</sup>Note that the signal  $y^i$  is now observable to the two groups of creditors, whereas it is observable only to the bank  $-i$ 's creditors in the manuscript. The creditor  $j$ 's information set now becomes  $((x_j^i, y^i), y^{-i})$  with  $(x_j^i, y^i)$  to be the signals about his own bank's fundamentals and  $y^{-i}$  to be the signal about the other bank's fundamentals.

<sup>14</sup>Note that  $y^{-i} = \theta^{-i} + \eta^{-i}$  provides no information about  $\theta^i$ .

Furthermore, when  $\epsilon$  diminishes to 0 for a fixed  $\eta$ , one can compute:

$$\begin{aligned} \text{Prob}(\tilde{y}^i \leq x_j^i - \eta + \epsilon | x_j^i) &= \int_{x_j^i - \eta - \epsilon}^{x_j^i - \eta + \epsilon} \frac{y^i - (x_j^i - \eta - \epsilon)}{4\eta\epsilon} \cdot dy^i = \frac{\epsilon}{2\eta} \rightarrow 0, \\ \text{Prob}(\tilde{y}^i \geq x_j^i + \eta - \epsilon | x_j^i) &= \int_{x_j^i + \eta - \epsilon}^{x_j^i + \eta + \epsilon} \frac{(x_j^i + \eta + \epsilon) - y^i}{4\eta\epsilon} \cdot dy^i = \frac{\epsilon}{2\eta} \rightarrow 0. \end{aligned}$$

So, we obtain:

$$f(y^i | x_j^i) \rightarrow f(y^i | \theta^i) = \begin{cases} \frac{1}{2\eta} & \theta^i - \eta < y^i \leq \theta^i + \eta \\ 0 & \text{otherwise.} \end{cases}$$

Therefore, we have  $\frac{f(y^i | \theta^i)}{f(y^i | x_j^i)} \rightarrow 1$  and  $f(\theta^i | x_j^i, y^i) \rightarrow f(\theta^i | x_j^i) = \frac{1}{2\epsilon}$  when  $x_j^i \in [x^L, x^U]$ .  $\square$

#### 4.1.1 Players' strategies

In the  $t = 1$ 's bank run game, we again focus on the symmetric threshold strategy: after receiving the information  $((x_j^i, y^i), y^{-i})$ , a representative creditor  $j$  from a bank  $i$  withdraws from his bank if and only if  $x_j^i$  is below a threshold  $x(y^{-i})$ :

$$((x_j^i, y^i), y^{-i}) \mapsto \begin{cases} \text{withdraw} & x_j^i < x(y^{-i}) \\ \text{wait} & x_j^i \geq x(y^{-i}) \end{cases} \quad (18)$$

We presume that the creditor  $i$ 's withdrawal decision and the critical signal *do not* directly depend on  $y^i$  for two reasons. First, given a critical signal,  $x_j^i$  provides sufficient information about the bank  $i$ 's cash flow and the bank's other creditors' private signals as shown in Result 6. Second,  $y^{-i}$  alone provides information about the bank  $-i$ 's cash flow and its creditors' private signals. However,  $y^i$  may affect the functional form of  $x(\cdot)$  as the creditor *no longer* needs to infer the critical signal  $x(y^i)$  adopted by the bank  $-i$ 's creditors based on  $x_j^i$ . In particular, the creditor's expectations on the total withdrawal in the bank  $-i$  becomes:

$$L^{-i}(y^i, y^{-i}) = E \left[ L^{-i}(\theta^{-i}, x^*(y^i)) | y^i, y^{-i} \right] = \int_{y^{-i} - \eta}^{y^{-i} + \eta} L^{-i}(\theta^{-i}, x^*(y^i)) \cdot \frac{1}{2\eta} d\theta^{-i}.$$

Different from expression (A.25) of  $L^{-i}(x_j^i, y^{-i})$  in the manuscript, the creditor  $j$ 's expectations about the aggregate withdrawals in the bank  $-i$  now depends directly on the semi-public signals  $y^i$  and  $y^{-i}$ .<sup>15</sup>

The secondary market asset buyers' strategy is still a price schedule  $(P_1, P_2)$  with

$$M \mapsto P_M, M \in \{1, 2\}. \quad (19)$$

<sup>15</sup>As will be shown, the observability of  $y^i$  complicates the analysis of the representative creditor  $j$ 's best response but brings no qualitative change to our main results.

We again solve the PBE of the model, starting with the asset buyers' competitive bidding. The competitive asset price can be expressed explicitly like before:

$$P_M^* = \omega_M^B(\hat{\theta}) \cdot \frac{\theta_B + \hat{\theta}}{2} + \omega_G^B(\hat{\theta}) \cdot \frac{\theta_G + \hat{\theta}}{2} = \frac{E_s(\theta_s | M) + \hat{\theta}}{2}, \quad (20)$$

where  $\hat{\theta}$  is, again, the threshold of cash flow that triggers runs. The equilibrium asset price  $P_M^*$  must belong to interval  $[\underline{P}, qD_2)$ , where  $\underline{P} = \frac{\theta_B + D_2}{2}$ .

When information is complete, for a given price  $P$ , a bank  $i$  with a fundamental  $\theta^i$  fails at  $t = 2$  if and only if the aggregate withdrawals  $L^i$  in the bank exceeds a critical value  $L^c(\theta^i, P) \equiv \frac{P \cdot (\theta^i - D_2)}{D_1 \cdot (\theta^i - P/q)}$ . The creditor  $j$ 's payoff difference between 'wait' and 'withdraw' is again  $DW(L^i, \theta^i, P) = (1 - q) \frac{D_1}{q}$  if  $L^i \in [0, L^c(\theta^i, P)]$  and  $DW(L^i, \theta^i, P) = -D_1$  if  $L^i \in (L^c(\theta^i, P), 1]$ .

When information is incomplete, the creditor has to form beliefs about  $\theta^i$ ,  $L^i$ , and  $P$ , based on his information  $((x_j^i, y^i), y^{-i})$  and the other players' equilibrium strategies. To solve the incomplete information game, we again start with the lower and upper dominance regions of fundamentals  $[\underline{\theta}, D_2)$  and  $(\frac{F}{1-D_1/P}, \bar{\theta}]$ . It is dominant for the creditor  $j$  to withdraw early (wait) when observing  $x_j^i < x^L = D_2 - \epsilon$  ( $x_j^i > x^U = \frac{F}{1-D_1/P} + \epsilon$ ), independent of his beliefs about  $L^i$  and for any asset price  $P \in [\underline{P}, qD_2)$ . We then analyze the creditor  $j$ 's best response to other players' equilibrium strategies for  $x_j^i \in [x^L, x^U]$  to derive the equilibrium.

#### 4.1.2 Equilibrium without the aggregate uncertainty

We analyze the case where  $\underline{\theta}_G = \underline{\theta}_B = \underline{\theta}$ . The asset buyers' single equilibrium break-even price is again  $P^* = \frac{\theta + \hat{\theta}}{2}$ . For the bank run game, recall that the model now features no cross-bank coordination. The critical signal in a bank  $i$  is a single value  $x^*$ . We have:

**Result 7.** *When  $\epsilon$  diminishes to 0 for a fixed  $\eta$ , the game without the aggregate uncertainty has a unique symmetric equilibrium. A wholesale creditor of a bank  $i$  withdraws if and only if his private signal falls below  $x^*$ , a unique threshold independent of the semi-public signals  $y^i$  and  $y^{-i}$ . The asset buyers offer a price  $P^*$  to buy banks' assets, independent of the number of runs.  $P^*$ ,  $x^*$ , and  $\theta^*$  are the same as the ones established in Proposition 1 of the manuscript.*

*Proof.* As argued, the asset price and critical signal are both single values, i.e.,  $P^*$  and  $x^*$ , which do not depend on  $y^{-i}$ . The only thing that needs to be verified is that the creditor  $j$ 's expectations about  $L^i$  based on his private signal  $x_j^i \in [x^L, x^U]$  and the semi-public signal  $y^i$  about  $\theta^i$  has the same functional form as before. This is indeed true by Result 6. The rest of the proof then follows exactly the proof of Proposition 1 in the manuscript.  $\square$

### 4.1.3 Equilibrium with the aggregate uncertainty

With aggregate uncertainty, the creditor  $j$  needs to form rational beliefs about the aggregate withdrawals in both banks to forecast the secondary market asset price. When  $y^{-i} < y^L = x^L - \eta - \epsilon$  and  $y^{-i} > y^U = x^U + \eta + \epsilon$ , the creditor  $j$  expects  $L^{-i}(y^i, y^{-i}) = 1$  and  $L^{-i}(y^i, y^{-i}) = 0$  and rationally anticipates the asset price to be  $P_2^*$  and  $P_1^*$ , respectively, if his own bank is forced into liquidation. As  $\tilde{\theta}^i|_{(x_j^i, y^i)} \rightarrow \tilde{\theta}^i|_{x_j^i}$ ,  $y^i$  does not affect creditor  $j$ 's expected payoff difference functions in these two cases. The asset price, the threshold signal, and the critical cash flows have the same functional form as the ones in the manuscript and solve the system:

$$\begin{cases} P_M^* = \omega_M^B(\hat{\theta}_M) \cdot \frac{\theta_B + \hat{\theta}_M}{2} + \omega_M^G(\hat{\theta}_M) \cdot \frac{\theta_G + \hat{\theta}_M}{2} \\ x_M^* = \theta^*(y^{-i}) + (2q - 1)\epsilon = x^*(y^{-i}) \\ \theta_M^* = \frac{D_2 - D_1}{1 - qD_1/P_M^*} = \theta^*(y^{-i}) \\ \hat{\theta}_M \equiv x^*(y^{-i}) + \epsilon = \hat{\theta}(y^{-i}) \end{cases} \quad \text{with} \quad M = \begin{cases} 1 & \text{if } y^{-i} > y^U \\ 2 & \text{if } y^{-i} < y^L. \end{cases} \quad (21)$$

The system has a unique solution  $(P_M^*, x_M^*, \theta_M^*)$  for each  $M \in \{1, 2\}$ , and  $P_1^* > P_2^*$ ,  $x_1^* < x_2^*$ , and  $\theta_1^* < \theta_2^*$ . The threshold signal  $x^*(y^{-i}) = x_2^*$  when  $y^{-i} < y^L$  and  $x^*(y^{-i}) = x_1^*$  when  $y^{-i} > y^U$ . We then analyze the bank run game for  $y^{-i} \in [y^L, y^U]$  and determine the functional form of  $x^*(y^{-i})$ . We prove the following Result 8.

**Result 8.** *When  $\epsilon$  diminishes to 0 for a fixed  $\eta$ , the game with aggregate uncertainty has a unique symmetric equilibrium: a wholesale creditor of a bank  $i$  withdraws if and only if his private signal falls below  $x^*(y^{-i})$ , with*

$$x^*(y^{-i}) = \begin{cases} x_2^* & y^{-i} < x_2^* + \eta + \epsilon \\ x_1^* & y^{-i} \geq x_2^* + \eta + \epsilon. \end{cases} \rightarrow \begin{cases} \theta_2^* & y^{-i} < \theta_2^* + \eta \\ \theta_1^* & y^{-i} \geq \theta_2^* + \eta. \end{cases} \quad (22)$$

The asset buyers offer price  $P_M^*$  when observing  $M$  bank runs,  $M = 1, 2$ .

*Proof. Existence.* Given that all other creditors follow strategy (22), we show that the creditor  $j$ 's best response is to follow the same strategy. We have already proved that  $x^*(y^{-i}) = x_1^*$  when  $y^{-i} < y^L$  and  $x^*(y^{-i}) = x_2^*$  when  $y^{-i} > y^U$ . To fully characterize the equilibrium, we now analyze the case when  $y^{-i} \in [y^L, y^U]$ . Like before, there must exist a switching point  $\hat{y} \in [y^L, y^U]$  of  $y^{-i}$  such that  $x^*(y^{-i}) = x_2^*$  when  $y^{-i} < \hat{y}$  and  $x^*(y^{-i}) = x_1^*$  when  $y^{-i} \geq \hat{y}$ . We show in three steps that  $\hat{y}$  can still be  $x_2^* + \eta + \epsilon$ .

**Step 1:** We prove that upon observing a  $y^{-i} \in [x_2^* + \eta + \epsilon, y^U]$ , the creditor  $j$  rationally expects  $L^{-i}(y^i, y^{-i}) = 0$  and optimally withdraws if and only if  $x_j^i < x_1^*$ ,  $\forall y^i$ .

When observing a  $y^{-i} \geq x_2^* + \eta + \epsilon$ , the creditor knows that all private signals received by the bank  $-i$ 's creditors are above  $x_2^*$ . So he expects all creditors in the bank  $-i$  to wait in their bank. The creditor  $j$  rationally expects  $L^{-i}(y^i, y^{-i}) = 0, \forall y^i$ , and the asset price to be  $P_1^*$  if the bank  $i$  is forced into liquidation. Since  $\tilde{\theta}^i|_{(x_j^i, y^i)} \rightarrow \tilde{\theta}^i|_{x_j^i}$ , the creditor's payoff difference is not a function of  $y^i$  given the asset price. The representative creditor optimally withdraws if and only if  $x_j^i < x_1^*$ . So  $x^*(y^{-i}) = x_1^*$  for  $y^{-i} \in [x_2^* + \eta + \epsilon, y^U]$  and  $\forall y^i$ .

**Step 2:** We prove that upon observing a  $y^{-i} \in [y^L, x_1^* + \eta + \epsilon)$ , the creditor  $j$  rationally expects  $L^{-i}(y^i, y^{-i}) > 0$  and optimally withdraws if and only if  $x_j^i < x_2^*, \forall y^i$ .

When observing a  $y^{-i} \in [y^L, x_1^* + \eta + \epsilon)$ , the creditor  $j$  believes that the lowest possible private signals received by the bank  $-i$ 's creditors are below  $x_1^*$ . So he rationally expects  $L^{-i}(y^i, y^{-i}) > 0, \forall y^i$ ,<sup>16</sup> and the asset price to be  $P_2^*$  if a run also occurs in the bank  $i$ . By the same token, the representative creditor optimally withdraws if and only if  $x_j^i < x_2^*$ . So  $x^*(y^{-i}) = x_2^*$  for  $y^{-i} \in [y^L, x_1^* + \eta + \epsilon)$  and  $\forall y^i$ .

**Step 3:** We prove that upon observing a  $y^{-i} \in [x_1^* + \eta + \epsilon, x_2^* + \eta + \epsilon)$ , the creditor  $j$  expects  $L^{-i}(y^i, y^{-i}) > 0$  for  $y^i < x_2^* + \eta + \epsilon$  and  $L^{-i}(y^i, y^{-i}) = 0$  for  $y^i \geq x_2^* + \eta + \epsilon$ . However, he still optimally withdraws if and only if  $x_j^i < x_2^*, \forall y^i$ .

When observing a  $y^{-i} \in [x_1^* + \eta + \epsilon, x_2^* + \eta + \epsilon)$ , the creditor  $j$  believes that the lowest possible private signal received by the bank  $-i$ 's creditors is below  $x_2^*$  but above  $x_1^*$ . Therefore, he anticipates  $L^{-i}(y^i, y^{-i}) > 0$  when  $y^i < x_2^* + \eta + \epsilon$ , and  $L^{-i}(y^i, y^{-i}) = 0$  when  $y^i \geq x_2^* + \eta + \epsilon$ . As his belief about  $L^{-i}$  switches from  $L^{-i} > 0$  to  $L^{-i} = 0$  when  $y^i$  switches from  $y^i < x_2^* + \eta + \epsilon$  to  $y^i \geq x_2^* + \eta + \epsilon$ , the creditor  $j$ 's expectation about the would-be asset price for his own bank also switches. The creditor's expected payoff difference differs from the one studied in the manuscript when banks' creditors hold *no* common knowledge on  $y^i$  and  $y^{-i}$ .

Recall that, under strategy (22), all other creditors in the bank  $i$  take  $x^*(y^{-i}) = x_2^*$  as a critical signal as  $y^{-i} \in [x_1^* + \eta + \epsilon, x_2^* + \eta + \epsilon)$ . We analyze the creditor  $j$ 's best response in detail.

*Case (I):* When  $x^L < x_j^i < x_2^* - 2\epsilon$ , the creditor  $j$  expects  $\theta^i|_{(x_j^i, y^i)} \rightarrow \theta^i|_{x_j^i} < x_2^* - \epsilon$ , and  $x_k^i|_{(x_j^i, y^i)} \rightarrow x_k^i|_{x_j^i} < x_2^*, \forall k \in [0, 1]/\{i\}$ . Therefore, he rationally expects  $L^i(x_j^i, y^{-i}) = 1$ . Additionally, the creditor knows that the bank  $-i$ 's creditors also follow  $x^*(y^i) = x_2^*$  because  $\max y^i|_{x_j^i < x_2^* - 2\epsilon} < (x_2^* - 2\epsilon) + \eta + \epsilon = x_2^* + \eta - \epsilon$  from (17). Together with  $y^{-i} < x_2^* + \eta + \epsilon$ , the

<sup>16</sup>Even if the creditors from the bank  $-i$  follow the critical signal  $x^*(y^i) = x_1^*$ , the creditor  $j$  expects  $L^{-i}(y^i, y^{-i}) = E[L^{-i}(\theta^{-i}, x_1^*)|y^{-i}] \geq \int_{y^{-i}-\eta}^{x_1^*+\epsilon} \frac{x_1^*-\theta^{-i}+\epsilon}{2\epsilon} \cdot \frac{1}{2\eta} \cdot d\theta^{-i} = \frac{(x_1^*+\eta+\epsilon-y^{-i})^2}{8\eta\epsilon} > 0$  when observing a  $y^{-i} < x_1^* + \eta + \epsilon$ . Note that  $E[L^{-i}(\theta^{-i}, x_1^*)|y^{-i}]$  decreases in  $y^{-i}$  and  $L^{-i}(\theta^{-i}, x_1^*)$  is  $\frac{x_1^*-\theta^{-i}+\epsilon}{2\epsilon}$  if  $y^{-i} - \eta \leq \theta^{-i} < x_1^* + \epsilon$  and 0 if  $x_1^* + \epsilon \leq \theta^{-i} \leq y^{-i} + \eta$ , conditional on  $y^{-i}$  slightly lower than  $x_1^* + \eta + \epsilon$  (i.e.,  $y^{-i} \in (x_1^* + \eta - \epsilon, x_1^* + \eta + \epsilon)$ ).

creditors  $j$  rationally expects

$$L^{-i}(y^i, y^{-i}) = E[L^{-i}(\theta^{-i}, x_2^*)|y^{-i}] \geq \int_{y^{-i}-\eta}^{x_2^*+\epsilon} \frac{x_2^* - \theta^{-i} + \epsilon}{2\epsilon} \cdot \frac{1}{2\eta} \cdot d\theta^{-i} = \frac{(x_2^* + \eta + \epsilon - y^{-i})^2}{8\eta\epsilon} > 0$$

and the asset price to be  $P_2^*$ .<sup>17</sup> Consequently, his expected payoff difference is

$$E[DW(L^i(x_j^i, y^{-i}), \theta_2^*, P_2^*)] = -D_1 < 0.$$

$\theta_2^*$  is the critical cash flow associated with  $x_2^*$  and  $P_2^*$ . The creditor  $j$  optimally withdraws.

*Case (2):* When  $x_2^* - 2\epsilon \leq x_j^i < x_2^*$ , the creditor  $j$  perceives  $L^i(x_j^i, y^{-i}) = \frac{1}{2} + \frac{x_2^* - x_j^i}{2\epsilon} - \frac{(x_2^* - x_j^i)^2}{8\epsilon^2} > 0$  from Appendix A.2 in the manuscript. On the other hand, the creditor  $j$  expects

$$L^{-i}(y^i, y^{-i}) \geq \frac{(x_2^* + \eta + \epsilon - y^{-i})^2}{8\eta\epsilon} > 0$$

because the bank  $-i$ 's creditors still follow  $x^*(y^i) = x_2^*$  as the critical signal. Indeed, we have:  $\max y^i|_{x_j^i < x_2^*} < x_2^* + \eta + \epsilon$ . Again, the creditor rationally expects the asset price to be  $P_2^*$ . When  $x_j^i \in (x_2^* - 2\epsilon, \underline{x}]$ , we have:

$$E[DW(L^i(x_j^i, y^{-i}), \theta_2^*, P_2^*)] = \int_{\frac{x_2^* - x_j^i}{2\epsilon}}^1 (-D_1) dL^i + \frac{x_2^* - x_j^i}{2\epsilon} \cdot (-D_1) = -D_1 < 0.$$

Instead, when  $x_j^i \in (\underline{x}, x_2^*)$ , we have:

$$\begin{aligned} E[DW(L^i(x_j^i, y^{-i}), \theta_2^*, P_2^*)] &= \int_{\frac{x_2^* - x_j^i}{2\epsilon}}^{L^c(\theta_2^*, P_2^*)} (1 - q) \frac{D_1}{q} dL^i + \int_{L^c(\theta_2^*, P_2^*)}^1 (-D_1) dL^i + \frac{x_2^* - x_j^i}{2\epsilon} \cdot (-D_1) \\ &= \frac{D_1}{q} \cdot [L^c(\theta_2^*, P_2^*) - q] - \frac{D_1}{q} \cdot \frac{x_2^* - x_j^i}{2\epsilon} = -\frac{D_1}{q} \cdot \frac{x_2^* - x_j^i}{2\epsilon} < 0. \end{aligned}$$

Given the asset price to be  $P_2^*$  and the critical cash flow to be  $\theta_2^*$ , the bank  $i$  survives if and only if  $L^i < L^c(\theta_2^*, P_2^*)$ . Recall that  $\underline{x}$  is defined as the value of  $x_j^i$  such that  $\frac{x_2^* - \underline{x}}{2\epsilon} = L^c(\theta_2^*, P_2^*)$ . Moreover,  $L^c(\theta_2^*, P_2^*) = q$  must hold by the equilibrium condition. The creditor  $j$  still optimally withdraws in the bank  $i$ .

*Case (3):* When  $x_j^i = x_2^*$ , the representative creditor believes  $\tilde{L}^i \sim U(0, 1)$ , so  $L^i(x_j^i, y^{-i}) = \frac{1}{2}$  by the Laplacian property. Again, he expects

$$L^{-i}(y^i, y^{-i}) \geq \frac{(x_2^* + \eta + \epsilon - y^{-i})^2}{8\eta\epsilon} > 0$$

because  $y^i < x_2^* + \eta + \epsilon$  with probability 1 (i.e.,  $\max y^i|_{x_j^i = x_2^*} = x_2^* + \eta + \epsilon$ ). The creditor  $j$  rationally expects the asset price to be  $P_2^*$ . We have:

<sup>17</sup>Again, recall that  $E[L^{-i}(\theta^{-i}, x_2^*)|y^{-i}]$  decreases in  $y^{-i}$  and the functional form of  $L^{-i}(\theta^{-i}, x_2^*)$  is  $\frac{x_2^* - \theta^{-i} + \epsilon}{2\epsilon}$  if  $y^{-i} - \eta \leq \theta^{-i} < x_2^* + \epsilon$  and 0 if  $x_2^* + \epsilon \leq \theta^{-i} \leq y^{-i} + \eta$ , conditional on  $y^{-i}$  slightly lower than  $x_2^* + \eta + \epsilon$  (i.e.,  $y^{-i} \in (x_2^* + \eta - \epsilon, x_2^* + \eta + \epsilon)$ ).

$$E[DW(L^i(x_j^i, y^{-i}), \theta_2^*, P_2^*)] = \int_0^{L^c(\theta_2^*, P_2^*)} (1-q) \frac{D_1}{q} dL^i + \int_{L^c(\theta_2^*, P_2^*)}^1 (-D_1) dL^i = \frac{D_1}{q} [L^c(\theta_2^*, P_2^*) - q] = 0.$$

The creditor is indifferent between waiting and withdrawing when observing  $x_j^i = x_2^*$ .

*Case (4):* When  $x_2^* < x_j^i \leq x_2^* + 2\epsilon$ , the observability of  $y^i$  brings in intricacies. The creditor  $j$  expects  $L^i(x_j^i, y^{-i}) = \frac{(x_2^* - x_j^i + 2\epsilon)^2}{8\epsilon^2} > 0$ . Additionally,  $y^i$  can be either higher or lower than  $x_2^* + \eta + \epsilon$ ,  $\forall x_2^* < x_j^i \leq x_2^* + 2\epsilon$  (i.e.,  $\max y^i |_{x_j^i > x_2^*} > x_2^* + \eta + \epsilon$  and  $\min y^i |_{x_j^i \leq x_2^* + 2\epsilon} \leq (x_2^* + 2\epsilon) - \eta - \epsilon = x_2^* - \eta + \epsilon < x_2^* + \eta + \epsilon$ ). We consider these two cases depending on the value of  $y^i$ .

*Case (4.1):* When  $y^i < x_2^* + \eta + \epsilon$ , the creditors in the bank  $-i$ , again, take  $x^*(y^i) = x_2^*$  as the critical signal. So the creditor  $j$  still expects

$$L^{-i}(y^i, y^{-i}) \geq \frac{(x_2^* + \eta + \epsilon - y^{-i})^2}{8\eta\epsilon} > 0.$$

and the asset price to be  $P_2^*$ . When  $x_j^i \in (\bar{x}, x_2^* + 2\epsilon]$ , we have

$$E[DW(L^i(x_j^i, y^{-i}), \theta_2^*, P_2^*)] = \frac{x_j^i - x_2^*}{2\epsilon} \cdot (1-q) \frac{D_1}{q} + \int_0^{1 - \frac{x_j^i - x_2^*}{2\epsilon}} (1-q) \frac{D_1}{q} dL^i = (1-q) \frac{D_1}{q} > 0.$$

And when  $x_j^i \in (x_2^*, \bar{x}]$ , we have:

$$\begin{aligned} E[DW(L^i(x_j^i, y^{-i}), \theta_2^*, P_2^*)] &= \frac{x_j^i - x_2^*}{2\epsilon} \cdot (1-q) \frac{D_1}{q} + \int_0^{L^c(\theta_2^*, P_2^*)} (1-q) \frac{D_1}{q} dL^i + \int_{L^c(\theta_2^*, P_2^*)}^{1 - \frac{x_j^i - x_2^*}{2\epsilon}} (-D_1) dL^i \\ &= \frac{D_1}{q} \cdot [L^c(\theta_2^*, P_2^*) - q] + \frac{D_1}{q} \cdot \frac{x_j^i - x_2^*}{2\epsilon} = \frac{D_1}{q} \cdot \frac{x_j^i - x_2^*}{2\epsilon} > 0. \end{aligned}$$

$\bar{x}$  is defined as the value of  $x_j^i$  such that  $1 - \frac{\bar{x} - x_2^*}{2\epsilon} = L^c(\theta_2^*, P_2^*)$ . The creditor  $j$  now optimally waits in the bank  $i$  when expecting  $P_2^*$  to be the asset price as in the manuscript.

*Case (4.2):* When  $y^i > x_2^* + \eta + \epsilon$ , the creditors in the bank  $-i$  take  $x^*(y^i) = x_1^*$  as the critical signal. Now the lowest private signal is above  $x_1^*$  as  $y^{-i} > x_1^* + \eta + \epsilon$ . The creditor  $j$  expects

$$L^{-i}(y^i, y^{-i}) = E[L^{-i}(\theta^{-i}, x_1^*) | y^{-i}] = 0$$

and the asset price to be  $P_1^*$ . As  $x_j^i \rightarrow \theta_2^*$ , the creditor knows that the critical aggregate withdrawals above which the bank  $i$  fails becomes  $L^c(\theta_2^*, P_1^*)$ . Note that  $L^c(\theta_2^*, P_1^*) > L^c(\theta_2^*, P_2^*)$  as  $P_1^* > P_2^*$ . We then assume, w.l.o.g.,  $L^c(\theta_2^*, P_1^*) > 1$ , so that the bank  $i$  with a cash flow  $\theta_2^*$  will survive when the asset price is  $P_1^*$ . As  $1 - \frac{x_j^i - x_2^*}{2\epsilon} < 1 < L^c(\theta_2^*, P_1^*)$ , the creditor  $j$ 's expected payoff difference becomes:

$$E[DW(L^i(x_j^i, y^{-i}), \theta_2^*, P_1^*)] = \frac{x_j^i - x_2^*}{2\epsilon} \cdot (1-q) \frac{D_1}{q} + \int_0^{1 - \frac{x_j^i - x_2^*}{2\epsilon}} (1-q) \frac{D_1}{q} dL^i = (1-q) \frac{D_1}{q} > 0.$$

The creditor  $j$  still optimally waits in the bank  $i$ .<sup>18</sup>

*Case (5):* When  $x^U > x_j^i > x_2^* + 2\epsilon$ , the creditor believes  $L^i(x_j^i, y^{-i}) = 0$  because, again,  $\theta^i|_{(x_j^i, y^i)} \rightarrow \theta^i|_{x_j^i} > x_2^* + \epsilon$  and  $x_k^i|_{(x_j^i, y^i)} \rightarrow x_k^i|_{x_j^i} > x_2^*$ ,  $\forall k \in [0, 1]/\{i\}$ . The (solvent) bank  $i$  always survives when  $L^i = 0$ . His expected payoff difference is

$$E[DW] = (1 - q) \frac{D_1}{q} > 0.$$

The representative creditor  $j$  optimally waits in the bank  $i$ .

To summarize, when  $y^{-i} \in [x_1^* + \eta + \epsilon, x_2^* + \eta + \epsilon)$ , the representative creditor  $j$  optimally withdraws if and only if  $x_j^i < x_2^*$  given that all other creditors follow the equilibrium strategy (22). Combined the results from **Step 1** to **Step 3**, we proved the existence.

**Step 4:** We establish the uniqueness by showing that any other threshold strategy

$$x^*(y^{-i}) = \begin{cases} x_2^* & y^{-i} < \hat{y} \\ x_1^* & y^{-i} \geq \hat{y}. \end{cases} \quad (23)$$

with a  $\hat{y} \neq x_2^* + \eta + \epsilon$  cannot be an equilibrium.

Again,  $\hat{y}$  must belong to  $[x_1^* + \eta + \epsilon, x_2^* + \eta + \epsilon)$ . Suppose that all other creditors follow strategy (23), we show that it is *not* the representative creditor  $j$ 's best response to follow the same strategy.

Suppose that the creditor  $j$  observes a  $y^{-i} \in [\hat{y}, x_2^* + \eta + \epsilon)$ . He is certain that all other creditors in the bank  $i$  take  $x_1^*$  as the critical signal. Suppose that the creditor also observes a private signal  $x_j^i = x_1^*$ . From (17),  $y^i$  must belong to interval  $[x_1^* - \eta - \epsilon, x_1^* + \eta + \epsilon)$ . The creditor  $j$  then knows that the creditors in the bank  $-i$  take  $x_2^*$  as the critical signal (i.e., Step 2). Consequently, he rationally expects both  $L^i > 0$  and  $L^{-i} > 0$  and the asset price to be  $P_2^*$ .

Consider the creditor's payoff difference when observing  $x_j^i = x_1^* \rightarrow \theta_1^*$ . He knows  $L^i(x_j^i, y^{-i}) = \frac{1}{2}$  and the bank  $i$ 's cash flow is  $\theta_1^*$ . However, the critical aggregate withdrawals above which the bank  $i$  fails becomes  $L^c(\theta_1^*, P_2^*)$ . Note that  $0 < L^c(\theta_1^*, P_2^*) < L^c(\theta_1^*, P_1^*) = q$  as  $P_1^* > P_2^*$ . We have:

$$E[DW(L^i(x_j^i, y^{-i}), \theta_1^*, P_2^*)] = \int_0^{L^c(\theta_1^*, P_2^*)} (1 - q) \frac{D_1}{q} dL^i + \int_{L^c(\theta_1^*, P_2^*)}^1 (-D_1) dL^i = \frac{D_1}{q} \cdot [L^c(\theta_1^*, P_2^*) - q] < 0.$$

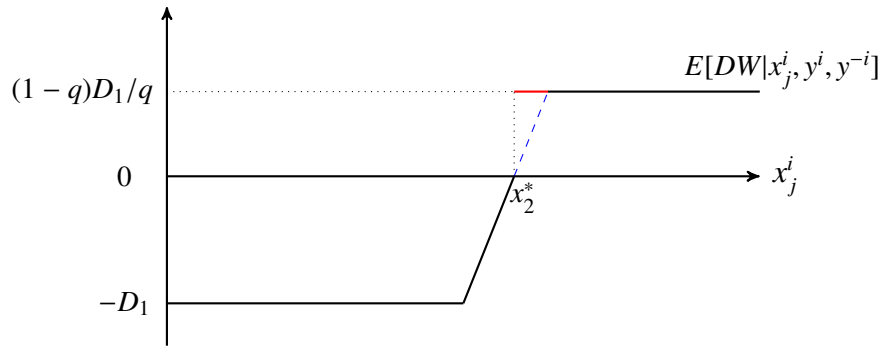
The creditor  $j$  will 'withdraw' instead of being indifferent between 'withdraw' and 'wait' when observing  $x_j^i = x_1^*$ , contradicting the definition of the threshold equilibrium. As  $\hat{y}$  is arbitrary in  $[x_1^* + \eta + \epsilon, x_2^* + \eta + \epsilon)$ , the only switching point is  $x_2^* + \eta + \epsilon$ . This establishes the uniqueness.

<sup>18</sup>When  $L^c(\theta_2^*, P_2^*) < L^c(\theta_2^*, P_1^*) < 1$ , the same qualitative result can be established. We can show that  $E[DW(L^i(x_j^i, y^{-i}), \theta_2^*, P_1^*)] > E[DW(L^i(x_j^i, y^{-i}), \theta_2^*, P_2^*)] > 0, \forall x_j^i \in (x_2^*, x_2^* + 2\epsilon]$ .

Lastly, we take the limit to obtain  $\lim_{\epsilon \rightarrow 0} x_2^* = \theta_2^*$ ,  $\lim_{\epsilon \rightarrow 0} x_1^* = \theta_1^*$ , and the switching point  $y^{-i}$  of the critical signal  $\lim_{\epsilon \rightarrow 0} x_2^* + \eta + \epsilon = \theta_2^* + \eta$ .  $\square$

In Figure 2, We plot the representative creditor's expected payoff difference as a function of  $x_j^i$  when  $y^{-i} \in [x_1^* + \eta + \epsilon, x_2^* + \eta + \epsilon)$ . Comparison to Figure 1 in the manuscript shows that the representative creditor still optimally responds to 'wait' in the bank  $i$  after observing  $x_j^i > x_2^*$  because his expected payoff difference between 'wait' and 'withdraw' is always larger than 0 in this situation.

Figure 2: Payoff differences when  $y^{-i} \in [x_1^* + \eta + \epsilon, x_2^* + \eta + \epsilon)$



The creditor  $j$  now observes both  $x_j^i$  and  $y^i$ . For any  $x_j^i \in (x_2^*, x_2^* + 2\epsilon]$ ,  $y^i$  can be either higher or lower than  $x_2^* + \eta + \epsilon$ . Observing a  $y^i \geq x_2^* + \eta + \epsilon$ , the creditor expects the asset price to be  $P_1^*$  and his expected payoff difference to be in the "thick red" line. Observing a  $y^i < x_2^* + \eta + \epsilon$ , the creditor anticipates the asset price to be  $P_2^*$  and his expected payoff difference to be in the "dash blue" line. When  $x_j^i$  increases, the creditor's payoff difference may even decrease because  $y^i$  may switch from a value higher than  $x_2^* + \eta + \epsilon$  to a value lower than  $x_2^* + \eta + \epsilon$ . However, the creditor optimally 'waits' in the bank  $i$  for  $x_j^i > x_2^*$  as his expected payoff difference between 'wait' and 'withdraw' is always larger than 0.

## 4.2 The equilibrium when $\eta \leq \epsilon$

We solve the equilibrium with the information structure of the manuscript but with  $\eta < \epsilon$ .

Since the secondary market asset buyers only observe the number of runs  $M$ , their equilibrium price schedule  $(P_1^*, P_2^*)$  will have the same expression as before, with  $P_M^*$  given by (7).

We then consider a bank  $i$ 's representative creditor  $j$ 's rational expectations on the total withdrawals  $L^i$  and  $L^{-i}$ —given that all other creditors take the equilibrium threshold strategy and that the creditor observes signals  $(x_j^i, y^{-i})$ . Knowing that all other creditors withdraw if and only if their private signals are below  $x^*(y^{-i})$ , the creditor's expectation  $L^i(x_j^i, y^{-i})$  is the same as expression (12) of the manuscript.  $L^{-i}(x_j^i, y^{-i})$  also has the same functional form as (A.25).

However, when  $\eta < \epsilon$ , the density function  $f(y^i|x_j^i)$  is different from the original (A.24) of the manuscript. In particular, when  $\eta < \epsilon$ , we have:

$$f(y^i|x_j^i) = \begin{cases} \frac{y^i - (x_j^i - \eta - \epsilon)}{4\eta\epsilon} & x_j^i - \eta - \epsilon < y^i \leq x_j^i + \eta - \epsilon \\ \frac{1}{2\epsilon} & x_j^i + \eta - \epsilon < y^i \leq x_j^i - \eta + \epsilon \\ \frac{(x_j^i + \eta + \epsilon) - y^i}{4\eta\epsilon} & x_j^i - \eta + \epsilon < y^i \leq x_j^i + \eta + \epsilon \\ 0 & \text{otherwise.} \end{cases} \quad (24)$$

One can also verify that the monotonicity of  $L^i(x_j^i, y^{-i})$  and  $L^{-i}(x_j^i, y^{-i})$  is the same as before.

Follow the same approaches in Section 3.2.2, the representative creditor  $j$  rationally expects  $L^{-i}(x_j^i, y^{-i}) = 0$  when  $y^{-i} > y^U$  and  $L^{-i}(x_j^i, y^{-i}) = 1$  when  $y^{-i} < y^L$ . Consequently, when observing  $y^{-i} > y^U$ , a creditor in a bank  $i$  withdraws if and only if his private signal falls below  $x_1^*$  and expects his bank to sell assets for a price  $P_1^*$  in runs. Similarly, when observing  $y^{-i} < y^L$ , the creditor withdraws if and only if his private signal falls below  $x_2^*$  and expects his bank to sell assets for a price  $P_2^*$  in runs. The combination  $(P_M^*, x_M^*, \theta_M^*)$  still solves the system of equations (18) of the manuscript. We again prove that there exists a unique perfect Bayesian equilibrium of our dynamic game: a representative creditor of a bank  $i$  withdraws if and only if his private signal falls below  $x^*(y^{-i})$ , with

$$x^*(y^{-i}) = \begin{cases} x_2^* & y^{-i} < x_2^* + \eta + \epsilon \\ x_1^* & y^{-i} \geq x_2^* + \eta + \epsilon. \end{cases} \quad (25)$$

The asset buyers offer  $P_M^*$  when observing  $M$  bank runs,  $M = 1, 2$ . The proof consists 4 steps.

**Step 1:** We prove that upon observing  $y^{-i} \in [y^L, x_1^* - \eta + \epsilon]$  and  $y^{-i} \in [x_2^* + \eta + \epsilon, y^U]$ , the representative creditor expects  $L^{-i}(x_j^i, y^{-i}) > 0$  and  $L^{-i}(x_j^i, y^{-i}) = 0$  respectively, both independent of his private signal  $x_j^i$ .

We can derive the representative creditor  $j$ 's expectation of  $L^{-i}(x_j^i, y^{-i})$  when he observes  $y^{-i} = x_1^* - \eta + \epsilon$  and has a posterior  $\theta^{-i}|_{y^{-i}} \sim U(x_1^* - 2\eta + \epsilon, x_1^* + \epsilon)$ . The creditors in the bank  $-i$  can follow either  $x_2^*$  or  $x_1^*$  as the critical signal depending on their outside signal  $y^i$ . If creditors in the bank  $-i$  withdraw according to the critical signal  $x_1^*$ , we can compute:

$$E[L^{-i}(\theta^{-i}, x_1^*)|x_1^* - \eta + \epsilon] = \int_{x_1^* - 2\eta + \epsilon}^{x_1^* + \epsilon} \frac{x_1^* - \theta^{-i} + \epsilon}{2\epsilon} \cdot \frac{1}{2\eta} d\theta^{-i} = \frac{\eta}{2\epsilon} > 0.$$

Since  $\eta < \epsilon$ , we have  $x_1^* - \epsilon < x_1^* - 2\eta + \epsilon$ . Therefore,  $L^{-i}(\theta^{-i}, x_1^*) = \frac{x_1^* - \theta^{-i} + \epsilon}{2\epsilon}$  when  $x_1^* - 2\eta + \epsilon < \theta^{-i} < x_1^* + \epsilon$ . Instead, if they withdraw according to  $x_2^*$ , one can verify that  $L^{-i}(\theta^{-i}, x_2^*) = 1$  and

$$E[L^{-i}(\theta^{-i}, x_2^*)|x_1^* - \eta + \epsilon] = \int_{x_1^* - 2\eta + \epsilon}^{x_1^* + \epsilon} \frac{1}{2\eta} \cdot d\theta^{-i} = 1.$$

Again, the representative creditor expects  $L^{-i}(x_j^i, y^{-i}) > 0$ , because  $f(y^i|x_j^i)$  is everywhere non-negative and strictly positive when  $y^i \in [x_j^i - \eta - \epsilon, x_j^i + \eta + \epsilon]$ . By the monotonicity of  $L^{-i}(x_j^i, y^{-i})$  with respect to  $y^{-i}$ , we establish that  $L^{-i}(x_j^i, y^{-i}) > 0$  when  $y^{-i} \in [y^L, x_1^* - \eta + \epsilon]$ .

Consider the representative creditor  $j$ 's expectation of  $L^{-i}(x_j^i, y^{-i})$  when observing  $y^{-i} > x_2^* + \eta + \epsilon$ . He knows with certainty that the lowest possible private signal received by the bank  $-i$ 's creditors is still higher than  $x_2^*$ . The representative creditor then expects  $L^{-i}(x_j^i, y^{-i}) = 0$ . We can obtain  $x^*(y^{-i}) = x_2^*$  when  $y^{-i} \in [y^L, x_1^* - \eta + \epsilon]$  and  $x^*(y^{-i}) = x_1^*$  when  $y^{-i} \in (x_2^* + \eta + \epsilon, y^U]$ .

**Step 2:** Given that all other creditors follow the strategy (25), we prove that the representative creditor  $j$  of a bank  $i$  expects  $L^{-i}(x_j^i, y^{-i}) > 0$  when observing  $y^{-i} \in (x_1^* - \eta + \epsilon, x_2^* + \eta + \epsilon)$  and  $x_j^i \leq x_2^* + 2\epsilon$ . Again, we prove this result by first establishing that it holds for  $x_j^i = x_2^* + 2\epsilon$ . Then by the monotonicity of  $L^{-i}(x_j^i, y^{-i})$ , we have  $L^{-i}(x_j^i, y^{-i}) > 0, \forall x_j^i \leq x_2^* + 2\epsilon$ .

Upon observing  $x_j^i = x_2^* + 2\epsilon$ , the representative creditor forms beliefs about the outside signal  $y^i$  received by the other bank's creditors. He can calculate, by (24), that  $y^i$  has a positive conditional density  $f(y^i|x_2^* + 2\epsilon)$  on the interval  $[x_2^* - \eta + \epsilon, x_2^* + \eta + 3\epsilon]$ .

$$f(y^i|x_2^* + 2\epsilon) = \begin{cases} \frac{y^i - (x_2^* - \eta + \epsilon)}{4\eta\epsilon} & x_2^* - \eta + \epsilon < y^i \leq x_2^* + \eta + \epsilon \\ \frac{1}{2\epsilon} & x_2^* + \eta + \epsilon < y^i \leq x_2^* - \eta + 3\epsilon \\ \frac{(x_2^* + \eta + 3\epsilon) - y^i}{4\eta\epsilon} & x_2^* - \eta + 3\epsilon < y^i \leq x_2^* + \eta + 3\epsilon \\ 0 & \text{otherwise.} \end{cases}$$

Under strategy (25), the bank  $-i$ 's creditors will follow the critical signal  $x^*(y^i) = x_2^*$  when observing  $y^i < x_2^* + \eta + \epsilon$  and  $x^*(y^i) = x_1^*$  when observing  $y^i \geq x_2^* + \eta + \epsilon$ . So the creditor  $j$  expects that the creditors in the bank  $-i$  will follow the critical signal  $x_2^*$  with a probability

$$\text{Prob}(x^*(y^i) = x_2^* | x_j^i = x_2^* + 2\epsilon) = \int_{x_2^* - \eta + \epsilon}^{x_2^* + \eta + \epsilon} \frac{y^i - (x_2^* - \eta + \epsilon)}{4\eta\epsilon} dy^i = \frac{\eta}{2\epsilon} > 0.$$

Conditional on that creditors from the bank  $-i$  actually follow the critical signal  $x_2^*$ , the representative creditor can calculate the expected aggregate withdrawals in the bank  $-i$  as  $E[L^{-i}(\theta^{-i}, x_2^*)|y^{-i}] = \int_{y^{-i} - \eta}^{y^{-i} + \eta} L^{-i}(\theta^{-i}, x_2^*) \cdot \frac{1}{2\eta} \cdot d\theta^{-i}$  since  $\theta^{-i}|_{y^{-i}} \sim U(y^{-i} - \eta, y^{-i} + \eta)$ . When the representative creditor observes an outside signal  $y^{-i} < x_2^* + \eta + \epsilon$ , he knows that the expected aggregate withdrawal has a lower bound

$$E[L^{-i}(\theta^{-i}, x_2^*)|y^{-i}] \geq \int_{y^{-i} - \eta}^{x_2^* + \epsilon} \frac{x_2^* - \theta^{-i} + \epsilon}{2\epsilon} \cdot \frac{1}{2\eta} \cdot d\theta^{-i} = \frac{(x_2^* + \eta + \epsilon - y^{-i})^2}{8\eta\epsilon} > 0. \quad (26)$$

The first inequality in (26) is true because  $E[L^{-i}(\theta^{-i}, x_2^*)|y^{-i}]$  decreases in  $y^{-i}$  and the functional form of  $L^{-i}(\theta^{-i}, x_2^*)$  is  $\frac{x_2^* - \theta^{-i} + \epsilon}{2\epsilon}$  if  $y^{-i} - \eta \leq \theta^{-i} < x_2^* + \epsilon$  and 0 if  $x_2^* + \epsilon \leq \theta^{-i} \leq y^{-i} + \eta$ , conditional on  $y^{-i}$  slightly lower than  $x_2^* + \eta + \epsilon$  (i.e.,  $y^{-i} \in (x_2^* + \eta - \epsilon, x_2^* + \eta + \epsilon)$ ).

When the fundamentals  $(\theta^1, \theta^2)$  are realized,  $\theta^{-i}|_{y^{-i}}$  and  $y^i|_{x_j^i}$  are independent. A representative creditor, who observes  $x_j^i = x_2^* + 2\epsilon$  and  $y^{-i} \in (x_1^* - \eta + \epsilon, x_2^* + \eta + \epsilon)$ , will calculate

$$\begin{aligned} L^{-i}(x_2^* + 2\epsilon, y^{-i}) &= \int_{x_2^* - \eta + \epsilon}^{x_2^* + \eta + \epsilon} E \left[ L^{-i}(\theta^{-i}, x_2^*) | y^{-i} \right] \cdot f(y^i | x_2^* + 2\epsilon) \cdot dy^i + \int_{x_2^* + \eta + \epsilon}^{x_2^* + \eta + 3\epsilon} E \left[ L^{-i}(\theta^{-i}, x_1^*) | y^{-i} \right] \cdot f(y^i | x_2^* + 2\epsilon) \cdot dy^i \\ &\geq \int_{x_2^* - \eta + \epsilon}^{x_2^* + \eta + \epsilon} \frac{(x_2^* + \eta + \epsilon - y^{-i})^2}{8\eta\epsilon} \cdot f(y^i | x_2^* + 2\epsilon) \cdot dy^i \\ &= \frac{(x_2^* + \eta + \epsilon - y^{-i})^2}{8\eta\epsilon} \cdot \text{Prob}(x^*(y^i) = x_2^* | x_j^i = x_2^* + 2\epsilon) \\ &= \frac{(x_2^* + \eta + \epsilon - y^{-i})^2}{8\eta\epsilon} \cdot \frac{\eta}{2\epsilon} = \frac{(x_2^* + \eta + \epsilon - y^{-i})^2}{16\epsilon^2} > 0. \end{aligned}$$

The inequality in the second line follows (26),  $E \left[ L^{-i}(\theta^{-i}, x_1^*) | y^{-i} \right]$  being non-negative and the density of  $y^i$  being non-negative everywhere.

**Step 3:** We establish the existence of the equilibrium strategy (25) by analyzing the representative creditor's best response.

We have already proved that  $x^*(y^{-i}) = x_2^*$  for  $y^{-i} \leq x_1^* - \eta + \epsilon$  and  $x^*(y^{-i}) = x_1^*$  for  $y^{-i} \geq x_2^* + \eta + \epsilon$ . Moreover, for  $y^{-i} \in (x_1^* - \eta + \epsilon, x_2^* + \eta + \epsilon)$ , given that all other creditors follow the strategy (25), the representative creditor expects  $L^i(x_j^i, y^{-i}) > 0$  when  $x_j^i \leq x_2^* + 2\epsilon$ , i.e., a positive mass of withdrawals in his own bank. Therefore, for  $y^{-i} \in (x_1^* - \eta + \epsilon, x_2^* + \eta + \epsilon)$  and  $x_j^i \leq x_2^* + 2\epsilon$ , the creditor expects the number of runs to be  $M = 2$  and the asset price to be  $P_2^*$ . Instead, when observing  $x_j^i > x_2^* + 2\epsilon$ , he expects  $L^i(x_j^i, y^{-i}) = 0$ , i.e., no run in his own bank. We can follow the same procedure in the proof of Proposition 1 and of Lemma 3, to show that the representative creditor optimally withdraws if and only if  $x_j^i < \hat{x}(y^{-i})$ , where  $\hat{x}(y^{-i}) = x_2^* - 2\epsilon[L^c(\theta^*(y^{-i}), P_2^*) - q]$  for  $\forall y^{-i} \in [y^L, x_2^* + \eta + \epsilon)$ . A symmetric equilibrium, if exists, features  $L^c(\theta^*(y^{-i}), P_2^*) = q$ . The bank's critical cash flow, the bank  $i$ 's creditors' threshold signal, and the asset price, jointly solve the system of equations (18) of the manuscript for  $M = 2$ , with the unique solution being  $(\theta_2^*, x_2^*, P_2^*)$  by Lemma 3. To summarize, when all other creditors in the bank  $i$  and  $-i$  take the strategy (25), the representative creditor's best response is to follow the same strategy.

**Step 4:** We prove the uniqueness by contradiction. Suppose that all other creditors follow a threshold strategy with a discontinuity point  $\hat{y} < x_2^* + \eta + \epsilon$ , so that  $x^*(y^{-i}) = x_1^*$  when  $y^{-i} \in (\hat{y}, x_2^* + \eta + \epsilon)$ .

Suppose that the representative creditor  $j$  observes a private signal  $x_j^i = x_1^*$ . Following the same procedure in Step 2, the probability that creditors in the bank  $-i$  following the critical signal  $x_2^*$  is positive:  $\text{Prob}(x^*(y^i) = x_2^* | x_j^i = x_1^*) = \int_{x_1^* - \eta - \epsilon}^{x_1^* + \eta - \epsilon} \frac{y^i - (x_1^* - \eta - \epsilon)}{4\eta\epsilon} \cdot dy^i + \int_{x_1^* + \eta - \epsilon}^{x_1^* - \eta + \epsilon} \frac{1}{2\epsilon} dy^i = \frac{2\epsilon - \eta}{2\epsilon} > 0$ . Conditional on that creditors from the bank  $-i$  follow the threshold signal  $x_2^*$  and that the

representative creditor observes  $y^{-i} \in (\hat{y}, x_2^* + \eta + \epsilon)$ , the aggregate withdrawals again satisfy the inequality (26). The representative creditor  $j$  rationally expects  $L^{-i}(x_1^*, y^{-i}) > 0$ .

$$L^{-i}(x_1^*, y^{-i}) \geq \frac{(x_2^* + \eta + \epsilon - y^{-i})^2}{8\eta\epsilon} \cdot \frac{2\epsilon - \eta}{2\epsilon} > 0.$$

By monotonicity, the representative creditor expects  $M = 2$  and the secondary market asset price to be  $P_2^*$  when observing  $x_j^i \leq x_1^*$  and  $y^{-i} \in (\hat{y}, x_2^* + \eta + \epsilon)$ . Consequently, the representative creditor expects his own bank (i.e., the bank  $i$ ) to sell its assets for the price  $P_2^*$  in case runs happen. The expected price  $P_2^*$ , however, contradicts the threshold signal  $x_1^*$  as dictated by the alternative threshold strategy. Therefore, we establish the uniqueness by contradiction.

Lastly, one can follow, again, the same steps to analyze the case  $\eta = \epsilon$ , and prove that there exists a unique perfect Bayesian equilibrium of our dynamic game: a representative creditor of a bank  $i$  withdraws if and only if his private signal falls below  $x^*(y^{-i})$ , with

$$x^*(y^{-i}) = \begin{cases} x_2^* & y^{-i} < x_2^* + 2\epsilon \\ x_1^* & y^{-i} \geq x_2^* + 2\epsilon. \end{cases} \quad (27)$$

The asset buyers offer price  $P_M^*$  when observing  $M$  bank runs,  $M = 1, 2$ .

## References

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