Online Supplement I: Analysis of the Subgame Between the Firm and Consumers Given the Availability and Price of Offsets

For $x \geq 0$ and $\alpha \in [0,1]$, denote $e_{x,\alpha} = \max(\min(\tilde{e}_{x,\alpha}, \bar{e}), 0)$, where $\tilde{e}_{x,\alpha}$ is the solution to the equation $[x + c'(\tilde{e}_{x,\alpha})]\alpha m + f'(\tilde{e}_{x,\alpha}) = 0$.

I1 With Carbon Offsets

Lemma I1.1. *If $g > \xi$, then $t^*(g) = 0$.*

By Lemma I1.1, if $g > \xi$, then the firm does not buy any offsets, and thus the subgame will be the same as the game without carbon offsets (see Section I2).

If $g \leq \xi$, then the firm has the following six options to choose from:

1. Sell to high-value eco-unconscious consumers alone (i.e., $d_c = 0$, $d_u = (1-\alpha)\beta_u m$) at $p = v_h$. Then, the firm's profit maximization problem is
$$\begin{aligned} &\max\,(v_h - c(e) - gt)(1-\alpha)\beta_u m - f(e) \\ &s.t. \quad e \in [0, \bar{e}] \\ &\qquad\;\; t \in [0, e + e_o] \end{aligned}$$
The optimal solution is as follows: $e^{(1)} = \bar{e}$, $t^{(1)} = 0$ and the corresponding $\pi^{(1)} = (v_h - c(\bar{e}))(1-\alpha)\beta_u m$.

2. Sell to high-value eco-conscious consumers and high-value eco-unconscious consumers (i.e., $d_c = \alpha\beta_c m$, $d_u = (1-\alpha)\beta_u m$) at $p = v_h - \xi(e + e_o - t)$. Then, the firm's profit maximization problem is
$$\begin{aligned} &\max[v_h - \xi(e + e_o - t) - c(e) - gt]((1-\alpha)\beta_u + \alpha\beta_c)m - f(e) \\ &s.t. \quad v_h - \xi(e + e_o - t) \geq v_l \\ &\qquad\;\; e \in [0, \bar{e}] \\ &\qquad\;\; t \in [0, e + e_o] \end{aligned}$$
The optimal solution is as follows: $e^{(2)} = e_{g,(1-\alpha)\beta_u+\alpha\beta_c}$, $t^{(2)} = e_{g,(1-\alpha)\beta_u+\alpha\beta_c} + e_o$ and the corresponding $\pi^{(2)} = (v_h - c(e_{g,(1-\alpha)\beta_u+\alpha\beta_c}) - ge_{g,(1-\alpha)\beta_u+\alpha\beta_c} - ge_o)((1-\alpha)\beta_u + \alpha\beta_c)m - f(e_{g,(1-\alpha)\beta_u+\alpha\beta_c})$.

3. Sell to all eco-unconscious consumers (i.e., $d_c = 0$, $d_u = (1-\alpha)m$) at $p = v_l$. Then, the firm's profit maximization problem is
$$\begin{aligned} &\max\,(v_l - c(e) - gt)(1-\alpha)m - f(e) \\ &s.t. \quad v_h - \xi(e + e_o - t) \leq v_l \\ &\qquad\;\; e \in [0, \bar{e}] \\ &\qquad\;\; t \in [0, e + e_o] \end{aligned}$$
The optimal solution is as follows:
   - If $v_h - \xi e_o - \xi\bar{e} \leq v_l$, then $e^{(3)} = \bar{e}$, $t^{(3)} = 0$ and the corresponding $\pi^{(3)} = (v_l - c(\bar{e}))(1-\alpha)m$;
   - If $v_h - \xi e_o - \xi\bar{e} > v_l$, then there is no feasible solution.

4. Sell to high-type eco-conscious consumers and all eco-unconscious consumers (i.e., $d_c = \alpha\beta_c m$,

$d_u = (1-\alpha)m$) at $p = v_l$. Then the firm's profit maximization problem is

$$\begin{aligned} &\max(v_l - c(e) - gt)(1-\alpha+\alpha\beta_c)m - f(e) \\ &s.t. \quad v_h - \xi(e+e_o-t) \geq v_l \\ &\qquad\ \ e \in [0, \overline{e}] \\ &\qquad\ \ t \in [0, e+e_o] \end{aligned}$$

- If $v_h - \xi\bar{e} - \xi e_o \geq v_l$, then the optimal solution is $e^{(4)} = \bar{e}$, $t^{(4)} = 0$, and the corresponding $\pi = (v_l - c(\bar{e}))(1-\alpha+\alpha\beta_c)m$.
- If $v_h - \xi\bar{e} - \xi e_o < v_l$, then the first constraint must be binding (i.e., $t = \frac{v_l+\xi e_o - v_h + \xi e}{\xi}$). Thus, the profit maximization problem above is equivalent to

$$\begin{aligned} &\max(v_l - c(e) - \tfrac{g}{\xi}(v_l + \xi e_o - v_h + \xi e))(1-\alpha+\alpha\beta_c)m - f(e) \\ &s.t. \quad e \in [0, \overline{e}] \\ &\qquad\ \ \tfrac{v_l+\xi e_o - v_h + \xi e}{\xi} \in [0, e+e_o] \end{aligned}$$

  - If $v_h - \xi e_{g,1-\alpha+\alpha\beta_c} - \xi e_o < v_l$, then the optimal solution is $e^{(4)} = e_{g,1-\alpha+\alpha\beta_c}$, $t^{(4)} = \frac{v_l+\xi e_o - v_h + \xi e}{\xi}$, and the corresponding $\pi^{(4)} = (v_l - c(e_{g,1-\alpha+\alpha\beta_c}) - \frac{v_l - v_h}{\xi} g - g e_{g,1-\alpha+\alpha\beta_c} - g e_o)(1-\alpha+\alpha\beta_c)m - f(e_{g,1-\alpha+\alpha\beta_c})$;
  - If $v_h - \xi e_{g,1-\alpha+\alpha\beta_c} - \xi e_o \geq v_l$, then we must have $t = 0$. Thus, the optimal solution is $e^{(4)} = \frac{v_h - v_l - \xi e_o}{\xi}$, $t^{(4)} = 0$, and the corresponding $\pi^{(4)} = (v_l - c(\frac{v_h - v_l - \xi e_o}{\xi}))(1-\alpha+\alpha\beta_c)m - f(\frac{v_h - v_l - \xi e_o}{\xi})$.

5. Sell to high-type eco-conscious consumers and all eco-unconscious consumers (i.e., $d_c = \alpha\beta_c m$, $d_u = (1-\alpha)m$) at $p = v_h - \xi(e+e_o-t)$. Then the firm's profit problem is

$$\begin{aligned} &\max(v_h - \xi(e+e_o-t) - c(e) - gt)(1-\alpha+\alpha\beta_c)m - f(e) \\ &s.t. \quad v_h - \xi(e+e_o-t) \leq v_l \\ &\qquad\ \ e \in [0, \overline{e}] \\ &\qquad\ \ t \in [0, e+e_o] \end{aligned}$$

   Since $g \leq \xi$, the first constraint must be binding (otherwise we have $t = e + e_o$ resulting in $v_h \leq v_l$ which contradicts $v_h > v_l$). Thus, this optimization problem is dominated by Option 4 above.

6. Sell to all consumers (i.e., $d_c = \alpha m$, $d_u = (1-\alpha)m$) at $p = v_l - \xi(e+e_o-t)$. Then the firm's profit maximization problem is

$$\begin{aligned} &\max(v_l - \xi(e+e_o-t) - c(e) - gt)m - f(e) \\ &s.t. \quad e \in [0, \overline{e}] \\ &\qquad\ \ t \in [0, e+e_o] \end{aligned}$$

   The optimal solution is as follows: $e^{(6)} = e_{g,1}$, $t^{(6)} = e_{g,1} + e_o$ and the corresponding $\pi^{(6)} = (v_l - c(e_{g,1}) - g e_{g,1} - g e_o)m - f(e_{g,1})$.

Then, given g, the equilibrium of the subgame is one of the six options above in which the firm obtains the highest profit π.

I2 Without Carbon Offsets

In this section, we consider the case when there are no carbon offsets. Then, the firm has only two decision variables, i.e., the product price p and the controllable emissions level e. The profit function is as follows.

$$\pi(p,e) = (p - c(e)) \cdot d(p,e) - f(e)$$

where the demand function $d(p,e)$ is given by (2) with $t = 0$.

If $v_h - \xi e_o \leq v_l$, then the firm has four options to choose from:

(a1) Sell to high-value eco-unconscious consumers alone (i.e., $d_c = 0$, $d_u = (1-\alpha)\beta_u m$) at $p = v_h$. Then, the firm's profit maximization problem is

$$\max\,(v_h - c\,(e))\,(1-\alpha)\,\beta_u m - f\,(e)$$
$$s.t. \quad e \in [0, \overline{e}]$$

The optimal solution is as follows: $e^{(a1)} = \bar{e}$ and the corresponding $\pi^{(a1)} = (v_h - c(\bar{e}))(1-\alpha)\beta_u m$.

(a2) Sell to all eco-unconscious consumers (i.e., $d_c = 0$, $d_u = (1-\alpha)m$) at $p = v_l$. Then, the firm's profit maximization problem is

$$\max\,(v_l - c\,(e))\,(1-\alpha)\,m - f\,(e)$$
$$s.t. \quad e \in [0, \overline{e}]$$

The optimal solution is as follows: $e^{(a2)} = \bar{e}$ and the corresponding $\pi^{(a2)} = (v_l - c(\bar{e}))(1-\alpha)m$.

(a3) Sell to high-type eco-conscious consumers and all eco-unconscious consumers (i.e., $d_c = \alpha\beta_c m$, $d_u = (1-\alpha)m$) at $p = v_h - \xi(e + e_o)$. Then the firm's profit problem is

$$\max(v_h - \xi(e + e_o) - c\,(e))(1 - \alpha + \alpha\beta_c)m - f\,(e)$$
$$s.t. \quad e \in [0, \overline{e}]$$

The optimal solution is as follows: $e^{(a3)} = e_{\xi,1-\alpha+\alpha\beta_c}$ and the corresponding $\pi^{(a3)} = (v_h - \xi e_o - \xi e_{\xi,1-\alpha+\alpha\beta_c} - c(e_{\xi,1-\alpha+\alpha\beta_c}))(1-\alpha+\alpha\beta_c)m - f(e_{\xi,1-\alpha+\alpha\beta_c})$.

(a4) Sell to all consumers (i.e., $d_c = \alpha m$, $d_u = (1-\alpha)m$) at $p = v_l - \xi(e + e_o)$. Then the firm's profit maximization problem is

$$\max(v_l - \xi(e + e_o - t) - c\,(e))m - f\,(e)$$
$$s.t. \quad e \in [0, \overline{e}]$$

The optimal solution is as follows: $e^{(a4)} = e_{\xi,1}$ and the corresponding $\pi^{(a4)} = (v_l - \xi e_{\xi,1} - \xi e_o - c(e_{\xi,1}))m - f(e_{\xi,1})$.

If $v_h - \xi e_o > v_l$, then the firm has six options to choose from:

(b1) Sell to high-value eco-unconscious consumers alone (i.e., $d_c = 0$, $d_u = (1-\alpha)\beta_u m$) at $p = v_h$. Then, the firm's profit maximization problem is

$$\max\,(v_h - c\,(e))\,(1-\alpha)\,\beta_u m - f\,(e)$$
$$s.t. \quad e \in [0, \overline{e}]$$

The optimal solution is as follows: $e^{(b1)} = \bar{e}$ and the corresponding $\pi^{(b1)} = (v_h - c(\bar{e}))(1-\alpha)\beta_u m$.

(b2) Sell to high-value eco-conscious consumers and high-value eco-unconscious consumers (i.e., $d_c = \alpha\beta_c m$, $d_u = (1-\alpha)\beta_u m$) at $p = v_h - \xi(e + e_o)$. Then, the firm's profit maximization problem is

$$\max[v_h - \xi(e + e_o) - c\,(e)]((1-\alpha)\beta_u + \alpha\beta_c)m - f\,(e)$$
$$s.t. \quad v_h - \xi(e + e_o) \geq v_l$$
$$e \in [0, \overline{e}]$$

- If $v_h - \xi e_{\xi,(1-\alpha)\beta_u+\alpha\beta_c} - \xi e_o \geq v_l$, then optimal solution is as follows: $e^{(b2)} = e_{\xi,(1-\alpha)\beta_u+\alpha\beta_c}$ and the corresponding $\pi^{(b2)} = (v_h - \xi e_{\xi,(1-\alpha)\beta_u+\alpha\beta_c} - \xi e_o - c(e_{\xi,(1-\alpha)\beta_u+\alpha\beta_c}))((1-\alpha)\beta_u + \alpha\beta_c)m - f(e_{\xi,(1-\alpha)\beta_u+\alpha\beta_c})$;
- If $v_h - \xi e_{\xi,(1-\alpha)\beta_u+\alpha\beta_c} - \xi e_o < v_l$, then the first constraint must be binding. Then, the profit maximization problem is dominated by that in Option (b5) below.

(b3) Sell to all eco-unconscious consumers (i.e., $d_c = 0$, $d_u = (1-\alpha)m$) at $p = v_l$. Then, the firm's profit maximization problem is

$$\begin{aligned} &\max\, (v_l - c(e))(1-\alpha)m - f(e) \\ &s.t. \quad v_h - \xi(e + e_o) \leq v_l \\ &\qquad\quad e \in [0, \bar{e}] \end{aligned}$$

The optimal solution is as follows:

- If $v_h - \xi e_o - \xi \bar{e} \leq v_l$, then $e^{(b3)} = \bar{e}$ and the corresponding $\pi^{(b3)} = (v_l - c(\bar{e}))(1-\alpha)m$;
- If $v_h - \xi e_o - \xi \bar{e} > v_l$, then there is no feasible solution.

(b4) Sell to high-type eco-conscious consumers and all eco-unconscious consumers (i.e., $d_c = \alpha\beta_c m$, $d_u = (1-\alpha)m$) at $p = v_l$. Then the firm's profit maximization problem is

$$\begin{aligned} &\max (v_l - c(e))(1-\alpha+\alpha\beta_c)m - f(e) \\ &s.t. \quad v_h - \xi(e + e_o) \geq v_l \\ &\qquad\quad e \in [0, \bar{e}] \end{aligned}$$

The optimal solution is as follows: $e^{(b4)} = \min(\bar{e}, \frac{v_h - \xi e_o - v_l}{\xi})$ and the corresponding $\pi^{(b4)} = (v_l - c(\min(\bar{e}, \frac{v_h - \xi e_o - v_l}{\xi})))m - f(\min(\bar{e}, \frac{v_h - \xi e_o - v_l}{\xi}))$.

(b5) Sell to high-type eco-conscious consumers and all eco-unconscious consumers (i.e., $d_c = \alpha\beta_c m$, $d_u = (1-\alpha)m$) at $p = v_h - \xi(e + e_o)$. Then the firm's profit problem is

$$\begin{aligned} &\max (v_h - \xi(e + e_o) - c(e))(1-\alpha+\alpha\beta_c)m - f(e) \\ &s.t. \quad v_h - \xi(e + e_o) \leq v_l \\ &\qquad\quad e \in [0, \bar{e}] \end{aligned}$$

The optimal solution is as follows:

- If $v_h - \xi e_o - \xi e_{\xi,1-\alpha+\alpha\beta_c} \leq v_l$, then $e^{(b5)} = e_{\xi,1-\alpha+\alpha\beta_c}$ and the corresponding $\pi^{(b5)} = (v_h - \xi e_o - \xi e_{\xi,1-\alpha+\alpha\beta_c} - c(e_{\xi,1-\alpha+\alpha\beta_c}))(1-\alpha+\alpha\beta_c)m - f(e_{\xi,1-\alpha+\alpha\beta_c})$;
- If $v_h - \xi e_o - \xi e_{\xi,1-\alpha+\alpha\beta_c} > v_l$, then the first constraint must be binding. Then the optimization problem is dominated by Option (b4) above.

(b6) Sell to all consumers (i.e., $d_c = \alpha m$, $d_u = (1-\alpha)m$) at $p = v_l - \xi(e + e_o)$. Then the firm's profit maximization problem is

$$\begin{aligned} &\max (v_l - \xi(e + e_o - t) - c(e))m - f(e) \\ &s.t. \quad e \in [0, \bar{e}] \end{aligned}$$

The optimal solution is as follows: $e^{(b6)} = e_{\xi,1}$ and the corresponding $\pi^{(b6)} = (v_l - \xi e_{\xi,1} - \xi e_o - c(e_{\xi,1}))m - f(e_{\xi,1})$.

The firm will compare the profit with each option and choose the best one in equilibrium, i.e., (p^b, e^b).

Online Supplement II: Generalization of Propositions 1, 2 and 4

For the results (i) (results (ii)) of Propositions 1, 2 and 4, we have assumed $\beta_u = 1$ ($\beta_c = 1$). In this appendix, we show in Propositions II1(i), II2(i) and II3(i) (Propositions II1(ii), II2(ii) and II3(ii)) below that the results in Propositions 1(i), 2(i) and 4(i) (Propositions 1(ii), 2(ii) and 4(ii)) can still hold when $\beta_u < 1$ ($\beta_c < 1$).

Proposition II1. *Suppose the firm chooses to purchase offsets ($t^*(g) > 0$).*

(i) There exists a threshold $\mathfrak{B}_u < 1$ such that if $\beta_u > \mathfrak{B}_u$, then if $\beta_c \in (\mathfrak{B}_1, \mathfrak{B}_2)$ for some $\mathfrak{B}_1, \mathfrak{B}_2 \geq 0$, then there exist $g_1 < g_2$ such that $e^*(g_1) \leq e^*(g_2)$, where the emissions inequality is strict if $\bar{e}$ is large enough.

(ii) If there exist $\tilde{g}_1 < \tilde{g}_2$ such that $e^*(\tilde{g}_1) < e^*(\tilde{g}_2)$ when $\beta_c = 1$, then there exists a threshold $\mathfrak{B}_c < 1$ such that $e^*(\tilde{g}_1) < e^*(\tilde{g}_2)$ if $\beta_c > \mathfrak{B}_c$.

Proposition II2.

(i) There exists a threshold $\mathfrak{B}_u < 1$ such that if $\beta_u > \mathfrak{B}_u$, then $\theta^*(g_1) \geq \theta^*(g_2)$ for any $g_1 < g_2$.

(ii) If there exist $\tilde{g}_1 < \tilde{g}_2$ such that $\theta^*(\tilde{g}_1) < \theta^*(\tilde{g}_2)$ when $\beta_c = 1$, then there exists a threshold $\mathfrak{B}_c < 1$ such that $\theta^*(\tilde{g}_1) < \theta^*(\tilde{g}_2)$ if $\beta_c > \mathfrak{B}_c$.

Proposition II3.

(i) There exists a threshold $\mathfrak{B}_u < 1$ such that if $\beta_u > \mathfrak{B}_u$, then $g^* = c_g$.

(ii) If there exist $c_g \in (0, G)$ such that $g^* > c_g$ when $\beta_c = 1$, then there exists a threshold $\mathfrak{B}_c < 1$ such that $g^* > c_g$ if $\beta_c > \mathfrak{B}_c$.

Online Supplement III: Details of Footnote 4

III1 Analysis of the Special Case where $\beta_u = 0$ and $\beta_c > 0$

First, note that Proposition 3 has been proved for the general case where $\beta_u, \beta_c \in [0,1]$ and thus it holds under this special case. Thus, we only need to check the other three results (i.e., Propositions 1, 2, and 4) under this special case.

Note that the condition $\beta_u = 0$ and $\beta_c > 0$ implies a positive correlation between the two types of consumer preferences (similar as in Case P). As we can see from the following propositions, the results are similar to those in Case P (i.e., Propositions 1(ii), 2(ii) and 4(ii)).

Proposition III1. *Suppose the firm chooses to purchase offsets ($t^*(g) > 0$). There exist $g_1 < g_2$ such that the firm reduces its controllable emissions if the offset price decreases from g_2 to g_1 (i.e., $e^*(g_1) \leq e^*(g_2)$), if the share of high-value eco-conscious consumers is moderate (i.e., $\beta_c \in (\mathcal{B}_1, \mathcal{B}_2)$ for some $\mathcal{B}_1, \mathcal{B}_2 \geq 0$) and $v_h - v_l < \xi e_o$, where the inequality in $e^*(g_1) \leq e^*(g_2)$ is strict if $\bar{e}$ is large enough. Moreover, under these instances, there are more eco-unconscious consumers buying the product while the demand from the eco-conscious segment remains unchanged as the offset price decreases (i.e., $d_u^*(g_1) > d_u^*(g_2)$ and $d_c^*(g_1) = d_c^*(g_2)$).*

Proposition III1 shows that the insight in Proposition 1(ii), i.e., a lower offset price may cause the firm to reduce its controllable emissions per unit of product, is still valid.

Proposition III2. *There exist $g_1 < g_2$ such that the firm reduces the portion of its emissions to offset if the offset price decreases from g_2 to g_1 (i.e., $\theta^*(g_1) < \theta^*(g_2)$) under the conditions in Proposition III1.*

Proposition III2 shows that the insight in Proposition 2(ii) continues to hold. Specifically, when consumers' preferences for the product function and for its environmental attributes are positively correlated, then a lower offset may prompt the firm to expand into the eco-unconscious market segment and offset less of its emissions.

Proposition III3. *There exists $c_g \in (0, G)$ such that the NGO sets offset price higher than its cost (i.e., $g^* > c_g$), under the conditions in Proposition III1.*

Proposition III3 shows that the insight in Proposition 4(ii) continues to hold.

III2 Different High-Type Valuations Between Eco-Conscious and Eco-Unconscious Consumers

In this section, we consider an extension of the base model. Specifically, suppose high-type eco-conscious consumers have a higher valuation for the product's function (denoted as v_{hc}) than high-type eco-unconscious consumers (whose valuation for the product's function is still denoted as v_h). As for the low types, their valuation continues to be v_l for both eco-conscious and eco-unconscious consumers. We specify the details for each one of the four types of consumers in Table III1.

Table III1: Consumer Types, Their Utilities and Respective Segment Sizes (Section III2)

	Eco-Conscious	Eco-Unconscious
High Valuation for Product Function	$u_{c,h} = v_{hc} - \xi\mathcal{E} - p$ size $= m\alpha\beta_c$	$u_{u,h} = v_h - p$ size $= m(1-\alpha)\beta_u$
Low Valuation for Product Function	$u_{c,l} = v_l - \xi\mathcal{E} - p$ size $= m\alpha(1-\beta_c)$	$u_{u,l} = v_l - p$ size $= m(1-\alpha)(1-\beta_u)$

Lemma III1. *The firm purchases offsets if and only if their price g is low enough. (Formally, there exists a threshold G such that $t^*(g) > 0$ if and only if $g \le G$.)*

We resort to a numerical approach to solve this extended model. As in Section 6, we assume $c(e) = \frac{1}{2}k_c(\bar{e} - e)^2 + \bar{c}$ and $f(e) = \frac{1}{2}k_f(\bar{e} - e)^2$, and the same distributions for all of the randomly generated parameter values. In addition, we have $v_{hc} = (1+\chi)v_h$, where $\chi \sim U(0, 0.5)$.

We randomly generate 2,000,000 scenarios based on the distributions above. For each scenario, given any feasible value of the offset price $g \ge c_g$, we can find out the firm's best responses $(p^*(g), e^*(g), t^*(g))$ by solving the following profit optimization problem with the new utility functions:

$$\begin{aligned} &\max_{p,e,t}\ [p - c(e) - gt] \cdot d(p,e,t) - f(e) \\ &s.t.\ \ e \in [0, \bar{e}], t \in [0, e + e_o] \end{aligned} \tag{III1}$$

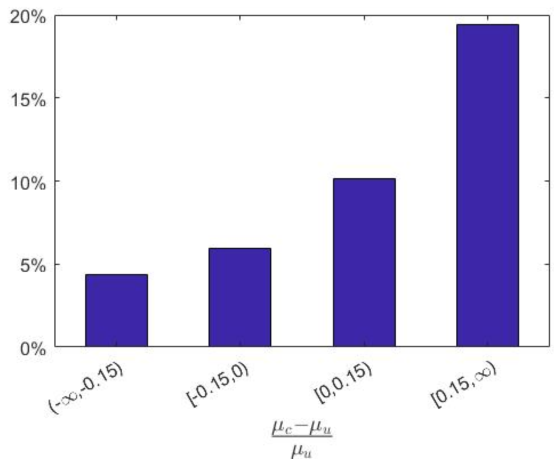
where the demand function $d(p,e,t)$ is given by (2) with the new valuation variable. We thereafter conduct an exhaustive search and evaluate the firm's optimal decisions $(p^*(g), e^*(g), t^*(g))$ over the following grid of g values $\{g_i = c_g + 0.001(i-1)\}_{i=1}^{+\infty} \triangleq \mathbb{G}$. Note that we do not need to go through the entire grid (which has infinite number of points); according to Lemma III1, if $t^*(g_I) = 0$, then we can stop the search as the firm will not purchase any offset for any larger value of g and thus the offset price does not have any impact on the subgame, implying that the firm's best response functions for any index $i > I$ should be the same as that when $i = I$. Since the main focus of this paper is the role of carbon offsets, we remove the scenario where $t^*(g_1) = 0$, which implies that the firm does not have any incentive to purchase offsets for any feasible value of $g \ge c_g$. As a result, we end up having 1,337,820 randomly generated scenarios.

As before, we consider the following four ranges for the relative average valuation between the two groups of customers $\frac{\mu_c - \mu_u}{\mu_u}$, where $\mu_c = \beta_c v_{hc} + (1-\beta_c)v_l$ and $\mu_u = \beta_u v_h + (1-\beta_u)v_l$: $(-\infty, -0.15)$, $[-0.15, 0)$, $[0, 0.15)$, $[0.15, \infty)$. (Note that μ_c depends on v_{hc} rather than v_h in this extended model.) Then, for each one of the four ranges of $\frac{\mu_c - \mu_u}{\mu_u}$, we calculate the fraction of instances where the corresponding results hold.

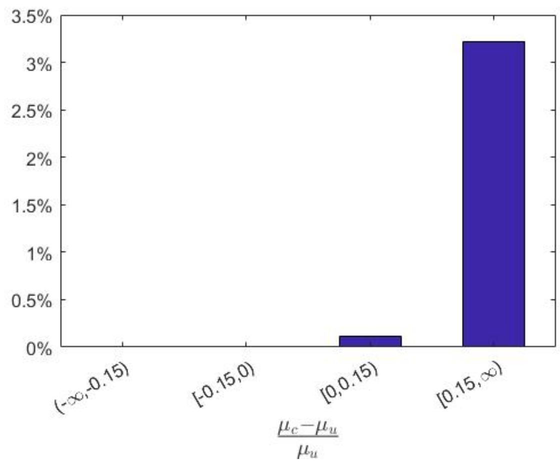
First, we study the impacts of offset price g on firm's decisions and verify the results in Propositions 1 and 2. Figure III1a shows the likelihood that the main result in Proposition 1 continues to be valid in this extended model, specifically that the firm reduces its controllable emissions as offset prices decrease. Consistent with the numerical results for our main model, this figure indicates that such instances are not uncommon with this generalized model. Figure III1b shows the likelihood that the result in

Figure III1: Proportion of instances (in model extension with different high-type valuations between eco-conscious and eco-unconscious consumers) where...

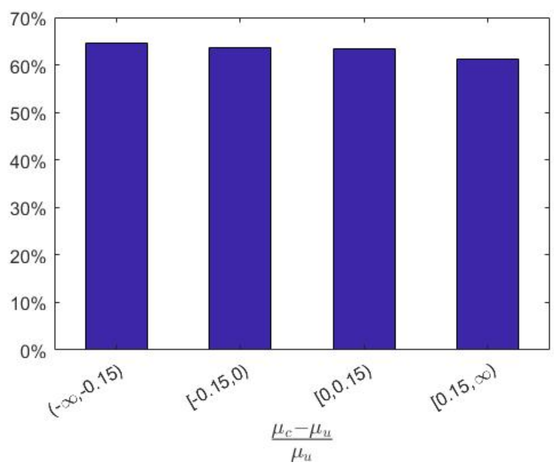
(a) There exists a pair of offset prices when the firm offsets at both prices and strictly reduces its controllable emissions at a lower offset price



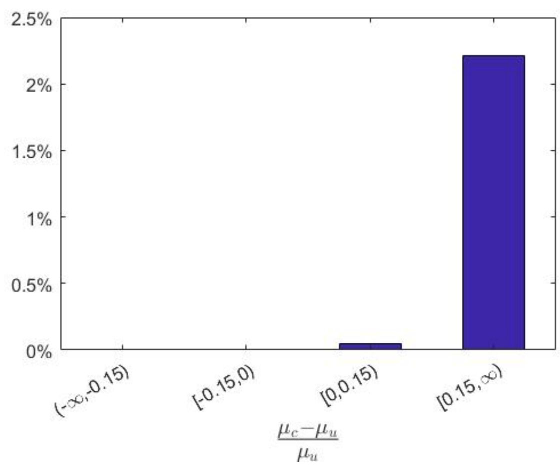
(b) There exists a pair of offset prices when the firm strictly offsets a lower share of its emissions at a lower offset price



(c) The firm strictly reduces its emissions and purchases offsets as offsets become available



(d) The NGO sets offset price strictly higher than its cost



Propositions 2 holds, that is, when, in response to an offset price decrease, the firm offsets a lower share of its emissions. Consistent with both our analytical finding in the main model, this figure shows that the result is more likely to occur when the two types of consumer preferences are positively correlated (i.e., $\frac{\mu_c-\mu_u}{\mu_u} > 0$).

Next, we study the equilibrium of the entire game and verify the results in Propositions 3 and 4. For each one of the 1,337,820 scenarios above, we are able to find the optimal offset price for the NGO ($g^* = \min[\arg\min_{g\geq c_g} \mathcal{E}^*(g)]$) through the exhaustive grid search described earlier. In addition, as for the case when carbon offsets are not available, we can calculate the firm's optimal decisions (p^b, e^b) by solving the profit optimization problem (i.e., (III1) with an additional constraint that $t = 0$). We again consider the same four ranges for the ratio $\frac{\mu_c-\mu_u}{\mu_u}$ as before. Then, for each one of the four ranges of $\frac{\mu_c-\mu_u}{\mu_u}$, we calculate the fraction of instances where the corresponding results hold.

Figure III1c shows the likelihood that the main insight in Proposition 3, when the firm strictly reduces its controllable emissions and purchases offsets as offsets become available, holds in this extended model. The results suggest the robustness of our insight that such instances are relatively common. Figure III1d shows the likelihood that it is optimal for the NGO to set the offset price strictly higher than its cost (i.e., $g^* > c_g$). Consistent with our analytical finding, this figure indicates that the result is more likely to happen when the two types of consumer preferences are positively correlated (i.e., $\frac{\mu_c - \mu_u}{\mu_u} > 0$).

Online Supplement IV: Derivation of Data in Table 4

Organic products are believed to be healthier/safer than conventional alternatives, because of benefits such as having fewer synthetic pesticides and fertilizers and being free of hormones and antibiotics.[18] In Gil et al. (2000), the authors conducted a survey in two Spanish regions: Navarra and Madrid. Based on participants' answers to various lifestyle questions in the questionnaire, the authors first used the K-means cluster analysis technique to identify market segments in relation to organic food products. Specifically, three segments in Navarra and four segments in Madrid were identified. Results from the cluster analysis and market segments characterization are given in Tables 6 and 7 in Gil et al. (2000). What is unique about this survey is that people were asked to express their attitude towards environmental issues via multiple questions, by which the authors were able to generate a factor called *Environmental concerns*, which reflects if consumers are conscious about environmental issues. This will help us estimate the difference in product valuations between eco-conscious and eco-unconscious consumers.

In Gil et al. (2000), the authors consider a wide range of organic food products, including vegetables, potatoes, cereals, fruits, eggs, chicken, and red meat. Participants were in the survey asked whether or not they are willing to pay a certain premium to buy an organic food product instead of a conventional one. Then, consumer valuations (in terms of the percentage over the price of the conventional product) are estimated using the contingent valuation approach[19]. We denote the average of such percentage premium as δ_i for consumer segment i. Tables IV1 and IV2 below summarize the results for Navarra and Madrid, respectively. All the data in these two tables are extracted from Tables 8 and 9 in Gil et al. (2000).

Now, we are ready to estimate the ratio $\frac{\mu_c - \mu_u}{\mu_u}$ using the available data in Gil et al. (2000). If we denote the price of conventional product as x_j for product $j \in \{$vegetables, potatoes, cereals, fruits, eggs, chicken, red meat$\}$, then the average valuation for the organic product will be $\mu_i = (1 + \delta_i/100)x_j$ for each segment i.

Let us first look at Navarra. As we mentioned earlier, there is a key variable in Gil et al. (2000), *Environmental concerns*, which measures consumer's eco-consciousness. From the last line in Table 6 in Gil et al. (2000), we know that both Segments 1 and 2 are conscious about environmental issues while Segment 3 is not. So, we combine Segments 1 and 2 as the eco-conscious segment and treat Segment 3 as the eco-unconscious segment. To calculate μ_c, we take the weighted average of μ_1 and μ_2 based on their relative population size, i.e., $\mu_c = \mu_1 w_1 + \mu_2 w_2$, where the weights $w_1 = \frac{0.25}{0.25+0.52}$ and $w_2 = \frac{0.52}{0.25+0.52}$ are derived based on the data in Table 6 in Gil et al. (2000). As for μ_u, it equals to μ_3. Therefore, we can obtain the ratio $\frac{\mu_c - \mu_u}{\mu_u} = \frac{\mu_1 w_1 + \mu_2 w_2 - \mu_3}{\mu_3} = \frac{[(1+\delta_1/100)w_1 + (1+\delta_2/100)w_2 - (1+\delta_3/100)]}{1+\delta_3/100}$. (Note that x_j is canceled out in the calculation of the ratio.) Specifically,

[18] See for example: https://time.com/4871915/health-benefits-organic-food/ and https://ota.com/organic-101/health-benefits-organic.

[19] The contingent valuation method was first proposed by Michael Hanemann in the seminal paper: Hanemann, W. M. (1984). Welfare evaluations in contingent valuation experiments with discrete responses. *American Journal of Agricultural Economics, 66*(3), 332-341.
This method has been widely used in literature to analyze survey data.

Table IV1: Average Valuations for Organic Products in Navarra (% premium over the conventional product price)

	Segment 1 (δ_1)	Segment 2 (δ_2)	Segment 3 (δ_3)
Vegetables	21.43	23.77	2.71
Potatoes	17.25	14.89	7.46
Cereals	16.00	17.46	8.33
Fruits	23.20	22.60	3.00
Eggs	20.13	17.61	9.33
Chicken	23.55	22.70	2.33
Red meat	18.46	21.54	2.67

- Vegetables:
$\frac{\mu_c-\mu_u}{\mu_u} = \frac{[(1+21.43/100)\frac{0.25}{0.25+0.52}+(1+23.77/100)\frac{0.52}{0.25+0.52}-(1+2.71/100)]}{1+2.71/100} \approx 0.20.$
- Potatoes:
$\frac{\mu_c-\mu_u}{\mu_u} = \frac{[(1+17.25/100)\frac{0.25}{0.25+0.52}+(1+14.89/100)\frac{0.52}{0.25+0.52}-(1+7.46/100)]}{1+7.46/100} \approx 0.08.$
- Cereals:
$\frac{\mu_c-\mu_u}{\mu_u} = \frac{[(1+16.00/100)\frac{0.25}{0.25+0.52}+(1+17.46/100)\frac{0.52}{0.25+0.52}-(1+8.33/100)]}{1+8.33/100} \approx 0.08.$
- Fruits:
$\frac{\mu_c-\mu_u}{\mu_u} = \frac{[(1+23.20/100)\frac{0.25}{0.25+0.52}+(1+22.60/100)\frac{0.52}{0.25+0.52}-(1+3.00/100)]}{1+3.00/100} \approx 0.19.$
- Eggs:
$\frac{\mu_c-\mu_u}{\mu_u} = \frac{[(1+20.13/100)\frac{0.25}{0.25+0.52}+(1+17.61/100)\frac{0.52}{0.25+0.52}-(1+9.33/100)]}{1+9.33/100} \approx 0.08.$
- Chicken:
$\frac{\mu_c-\mu_u}{\mu_u} = \frac{[(1+23.55/100)\frac{0.25}{0.25+0.52}+(1+22.70/100)\frac{0.52}{0.25+0.52}-(1+2.33/100)]}{1+2.33/100} \approx 0.20.$
- Red meat:
$\frac{\mu_c-\mu_u}{\mu_u} = \frac{[(1+18.46/100)\frac{0.25}{0.25+0.52}+(1+21.54/100)\frac{0.52}{0.25+0.52}-(1+2.67/100)]}{1+2.67/100} \approx 0.17.$

Table IV2: Average Valuations for Organic Products in Madrid (% premium over the conventional product price)

	Segment 1 (δ_1)	Segment 2 (δ_2)	Segment 3 (δ_3)	Segment 4 (δ_4)
Vegetables	11.00	15.38	15.69	21.09
Potatoes	5.61	4.33	3.33	13.63
Cereals	5.13	7.91	4.00	11.33
Fruits	9.23	16.38	15.79	25.29
Eggs	1.83	10.00	21.00	13.71
Chicken	6.33	2.40	8.00	13.38
Red meat	7.43	4.50	14.00	19.00

Next, let us look at the other region, Madrid. Note that there are four segments identified in this region. Similar to before, from the last line in Table 7 in Gil et al. (2000), we know that both Segments 3 and 4 are conscious about environmental issues and the other two are not.[20] So, we combine Segments

[20]While consumers in Segment 3 are concerned about environment, they do not have an active interest in reducing the effect of environmental degradation by recycling products and using recycled products, which is measured by the variable

3 and 4 as the eco-conscious segment, and we further combine Segments 1 and 2 as the eco-unconscious segment. To calculate μ_c, we take the weighted average of μ_3 and μ_4 based on their relative population size, i.e., $\mu_c = \mu_3 w_3 + \mu_4 w_4$, where the weights $w_3 = \frac{0.22}{0.22+0.35}$ and $w_4 = \frac{0.35}{0.22+0.35}$ are derived based on the data in Table 7 in Gil et al. (2000). As for μ_u, we take the weighted average of μ_1 and μ_2 based on their relative population size, i.e., $\mu_u = \mu_1 w_1 + \mu_2 w_2$, where the weights $w_1 = \frac{0.23}{0.23+0.20}$ and $w_2 = \frac{0.20}{0.23+0.20}$ are derived based on the data in Table 7 in Gil et al. (2000). Therefore, we can obtain the ratio $\frac{\mu_c-\mu_u}{\mu_u} = \frac{\mu_3 w_3+\mu_4 w_4-\mu_1 w_1-\mu_2 w_2}{\mu_1 w_1+\mu_2 w_2} = \frac{[(1+\delta_3/100)w_3+(1+\delta_4/100)w_4-(1+\delta_1/100)w_1-(1+\delta_2/100)w_2]}{(1+\delta_1/100)w_1+(1+\delta_2/100)w_2}$. (Note that x_j is canceled out in the calculation of the ratio.) Specifically,

- Vegetables:
$$\frac{\mu_c-\mu_u}{\mu_u} = \frac{[(1+15.69/100)\frac{0.22}{0.22+0.35}+(1+21.09/100)\frac{0.35}{0.22+0.35}-(1+11.00/100)\frac{0.23}{0.23+0.20}-(1+15.38/100)\frac{0.20}{0.23+0.20}]}{(1+11.00/100)\frac{0.23}{0.23+0.20}+(1+15.38/100)\frac{0.20}{0.23+0.20}} \approx 0.05.$$
- Potatoes:
$$\frac{\mu_c-\mu_u}{\mu_u} = \frac{[(1+3.33/100)\frac{0.22}{0.22+0.35}+(1+13.63/100)\frac{0.35}{0.22+0.35}-(1+5.61/100)\frac{0.23}{0.23+0.20}-(1+4.33/100)\frac{0.20}{0.23+0.20}]}{(1+5.61/100)\frac{0.23}{0.23+0.20}+(1+4.33/100)\frac{0.20}{0.23+0.20}} \approx 0.04.$$
- Cereals:
$$\frac{\mu_c-\mu_u}{\mu_u} = \frac{[(1+4.00/100)\frac{0.22}{0.22+0.35}+(1+11.33/100)\frac{0.35}{0.22+0.35}-(1+5.13/100)\frac{0.23}{0.23+0.20}-(1+7.91/100)\frac{0.20}{0.23+0.20}]}{(1+5.13/100)\frac{0.23}{0.23+0.20}+(1+7.91/100)\frac{0.20}{0.23+0.20}} \approx 0.02.$$
- Fruits:
$$\frac{\mu_c-\mu_u}{\mu_u} = \frac{[(1+15.79/100)\frac{0.22}{0.22+0.35}+(1+25.29/100)\frac{0.35}{0.22+0.35}-(1+9.23/100)\frac{0.23}{0.23+0.20}-(1+16.38/100)\frac{0.20}{0.23+0.20}]}{(1+9.23/100)\frac{0.23}{0.23+0.20}+(1+16.38/100)\frac{0.20}{0.23+0.20}} \approx 0.08.$$
- Eggs:
$$\frac{\mu_c-\mu_u}{\mu_u} = \frac{[(1+21.00/100)\frac{0.22}{0.22+0.35}+(1+13.71/100)\frac{0.35}{0.22+0.35}-(1+1.83/100)\frac{0.23}{0.23+0.20}-(1+10.00/100)\frac{0.20}{0.23+0.20}]}{(1+1.83/100)\frac{0.23}{0.23+0.20}+(1+10.00/100)\frac{0.20}{0.23+0.20}} \approx 0.10.$$
- Chicken:
$$\frac{\mu_c-\mu_u}{\mu_u} = \frac{[(1+8.00/100)\frac{0.22}{0.22+0.35}+(1+13.38/100)\frac{0.35}{0.22+0.35}-(1+6.33/100)\frac{0.23}{0.23+0.20}-(1+2.40/100)\frac{0.20}{0.23+0.20}]}{(1+6.33/100)\frac{0.23}{0.23+0.20}+(1+2.40/100)\frac{0.20}{0.23+0.20}} \approx 0.07.$$
- Red meat:
$$\frac{\mu_c-\mu_u}{\mu_u} = \frac{[(1+14.00/100)\frac{0.22}{0.22+0.35}+(1+19.00/100)\frac{0.35}{0.22+0.35}-(1+7.43/100)\frac{0.23}{0.23+0.20}-(1+4.50/100)\frac{0.20}{0.23+0.20}]}{(1+7.43/100)\frac{0.23}{0.23+0.20}+(1+4.50/100)\frac{0.20}{0.23+0.20}} \approx 0.10.$$

Environmental conservation, according to the result in Table 7 in Gil et al. (2000). If we count this segment as eco-unconscious, we can obtain similar results regarding the value of $\frac{\mu_c-\mu_u}{\mu_u}$: Vegetables (0.06), Potatoes (0.09), Cereals (0.05), Fruits (0.10), Eggs (0.03), Chicken (0.07), and Red meat (0.09). Note that all of them continue to be less than or equal to 0.10.

Online Supplement V: Details of Model Extensions

V1 Utility Dependent on Total Carbon Footprint

Here, we provide an analysis of the model extension where the utility of eco-conscious consumers depends on the total carbon footprint of the firm. Same as in the main model, we denote $(p^*(g), e^*(g), t^*(g))$ as the firm's best response functions given the offset price and g^* as the optimal offset price in equilibrium. In addition, we continue to use superscript $\cdot^b$ to denote the equilibrium outcome for the case without the availability of carbon offsets. For $x \geq 0$ and $\alpha \in [0, 1]$, with a slight abuse of notation, denote $e_{x,\alpha} = \max(\min(\tilde{e}_{x,\alpha}, \bar{e}), 0)$, where $\tilde{e}_{x,\alpha}$ is the solution to the equation $[x\alpha m + c'(\tilde{e}_{x,\alpha})]\alpha m + f'(\tilde{e}_{x,\alpha}) = 0$. Note the definition of $\tilde{e}_{x,\alpha}$ is slightly different from that in the main model.

Given that demand is deterministic, it is not optimal for the firm to overproduce (i.e., $q > d(p, e, q, t)$). Thus, the total sales must equal to the total production quantity in this extended model, i.e., $\min(q, d(p, e, q, t)) = q$. (Note that the firm may choose to underproduce (i.e., $q < d(p, e, q, t)$) as a lower q leads to lower total emissions and higher consumer utility.)

Lemma V1.1. *The firm purchases offsets (i.e., $t^*(g) > 0$) if and only if their price g is low enough (i.e., $g \leq G$).*

Lemma V1.2. *If $m < \frac{v_l - c(0)}{3\xi(e_o + \bar{e})}$, then $q = d(p, e, q, t)$ in equilibrium with and without the availability of carbon offsets.*

In practice, firms typically prioritize product sales over carbon footprint reduction. To rule out the scenario where the firm strategically creates supply shortage just for reducing carbon footprint (i.e., $q < d(p, e, q, t)$), we assume $m < \frac{v_l - c(0)}{3\xi(e_o + \bar{e})}$ in the following analysis. We will later numerically verify the robustness of our results in the general case in Section V1.1.

Proposition V1.1. *Suppose the firm chooses to purchase offsets (i.e., $t^*(g) > 0$).*

(i) For Case N, there exist $g_1 < g_2$ such that the firm reduces its controllable emissions if the offset price decreases from g_2 to g_1 (i.e., $e^*(g_1) \leq e^*(g_2)$) if the share of high-value eco-conscious consumers is moderate (i.e., $\beta_c \in (B_1, B_2)$ for some $B_1, B_2 \geq 0$) and $\xi > Z$ for some $Z \geq 0$, where the inequality in $e^*(g_1) \leq e^*(g_2)$ is strict if $\bar{e}$ is large enough. Moreover, under these instances, there are more eco-conscious consumers buying the product while the demand from the eco-unconscious segment remains unchanged as the offset price decreases (i.e., $d_u^*(\tilde{g}_1) = d_u^*(\tilde{g}_2)$ and $d_c^*(\tilde{g}_1) > d_c^*(\tilde{g}_2)$).

(ii) For Case P, there exist $\tilde{g}_1 < \tilde{g}_2$ such that the firm reduces its controllable emissions if the offset price decreases from $\tilde{g}_2$ to $\tilde{g}_1$ (i.e., $e^*(\tilde{g}_1) \leq e^*(\tilde{g}_2)$), if the share of high-value eco-unconscious consumers is moderate (i.e., $\beta_u \in (\tilde{B}_1, \tilde{B}_2)$ for some $\tilde{B}_1, \tilde{B}_2 \geq 0$) and $v_l > v_h - \xi e_o m$ and $\xi > \tilde{Z}$ for some $\tilde{Z} \geq 0$, where the inequality in $e^*(\tilde{g}_1) \leq e^*(\tilde{g}_2)$ is strict if $\bar{e}$ is large enough.. Moreover, under these instances, there are more eco-unconscious consumers buying the product while the demand from the eco-conscious segment remains unchanged as the offset price decreases (i.e., $d_u^*(\tilde{g}_1) > d_u^*(\tilde{g}_2)$ and $d_c^*(\tilde{g}_1) = d_c^*(\tilde{g}_2)$).

Proposition V1.1 shows that the insight in Proposition 1, i.e., a lower offset price may cause the firm to reduce its controllable emissions per unit of product, is still valid.

Proposition V1.2.

(i) For Case N, given a lower offset price, the firm increases the portion of its emissions to offset (i.e., $\theta^*(g_1) \geq \theta^*(g_2)$ for any $g_1 < g_2$).

(ii) For Case P, there exist $\tilde{g}_1 < \tilde{g}_2$ such that the firm reduces the portion of its emissions to offset if the offset price decreases from $\tilde{g}_2$ to $\tilde{g}_1$ (i.e., $\theta^*(\tilde{g}_1) < \theta^*(\tilde{g}_2)$) under the conditions in Proposition V1.1(ii).

Proposition V1.2 shows that Proposition 2 continues to hold. Specifically, a lower offset price leads to an increase in the share of emissions to offset in Case N but may prompt the firm to offset less of its emissions in Case P.

Proposition V1.3. *Suppose $c_g \leq G$.*

(i) For Case N, compared to the scenarios when offsets are not available, the firm purchases offsets and reduces its controllable emissions (i.e., $t^*(g^*) > 0$ and $e^*(g^*) \leq e^b$) if the size of the eco-conscious segment is not large (i.e., $\alpha \in (0, A)$ for some $A > 0$), where the inequality in $e^*(g^*) \leq e^b$ is strict if $\bar{e}$ is large enough.

(ii) For Case P, compared to the scenarios when offsets are not available, the firm purchases offsets and reduces its controllable emissions (i.e., $t^*(g^*) > 0$ and $e^*(g^*) \leq e^b$) if the size of the eco-conscious segment is not large (i.e., $\alpha \in (0, \tilde{A})$ for some $\tilde{A} > 0$) and $v_l < v_h - \xi(\bar{e} + e_o)$, where the inequality in $e^*(g^*) \leq e^b$ is strict if $\bar{e}$ is large enough.

Proposition V1.3 shows that the insight in Proposition 3, i.e., the presence of carbon offsets may increase the firm's internal emissions reduction effort (i.e., $e^*(g^*) < e^b$), is still valid.

Proposition V1.4.

(i) For Case N, the NGO should set offset price equal to its cost, i.e., $g^* = c_g$.

(ii) For Case P, there exists $c_g > 0$ such that the NGO sets offset price higher than its cost (i.e., $g^* > c_g$), under the conditions in Proposition V1.1(ii).

Proposition V1.4 shows that Proposition 4 continues to hold.

V1.1 Numerical Study

As in Section 6, we assume $c(e) = \frac{1}{2}k_c(\bar{e} - e)^2 + \bar{c}$ and $f(e) = \frac{1}{2}k_f(\bar{e} - e)^2$, and the same distributions for the randomly generated parameter values $\bar{e}$, e_o, ξ, α, β_c, β_u, v_l, v_h, k_c and $\bar{c}$. For m, we use $m = \eta_m \frac{v_l - c(\bar{e})}{3\xi(e_o + \bar{e})}$ where $\eta_m \sim U(0, 20)$ (and thus our numerical setting is not restricted to the assumption in Lemma V1.2). For k_f, we use $k_f = \eta_f k_c m$ where $\eta_f \sim U(0, 10000)$.

We randomly generate 2,000,000 scenarios based on the distributions above. For each scenario, given any feasible value of the offset price $g \geq c_g$, we can find the firm's best responses $(p^*(g), e^*(g), q^*(g), t^*(g))$

by solving the following profit optimization problem with the new utility functions:

$$\begin{aligned} &\max_{p,e,q,t}\ p\cdot\min(q,d(p,e,q,t))-(c(e)+gt)q-f(e) \\ &s.t.\ \ e\in[0,\bar{e}],t\in[0,e+e_o] \end{aligned} \tag{V1}$$

where the demand function $d(p,e,q,t)$ is given by (2) with the new utility functions. We thereafter conduct an exhaustive search and evaluate the firm's optimal decisions $(p^*(g),e^*(g),q^*(g),t^*(g))$ over the following grid of g values $\{g_i=c_g+0.001(i-1)\}_{i=1}^{+\infty}\triangleq\mathbb{G}$. Note that we do not need to go through the entire grid (which has infinite number of points); according to Lemma V1.1, if $t^*(g_I)=0$, then we can stop the search as the firm will not purchase any offset for any larger value of g and thus the offset price does not have any impact on the subgame, implying that the firm's best response functions for any index $i>I$ should be the same as that when $i=I$. Since the main focus of this paper is the role of carbon offsets, we remove the scenario where $t^*(g_1)=0$, which implies that the firm does not have any incentive to purchase offsets for any feasible value of $g\geq c_g$. As a result, we end up having 1,290,737 randomly generated scenarios.

As before, we consider the following four ranges for the relative average valuation between the two groups of customers $\frac{\mu_c-\mu_u}{\mu_u}$, where $\mu_c=\beta_c v_h+(1-\beta_c)v_l$ and $\mu_u=\beta_u v_h+(1-\beta_u)v_l$: $(-\infty,-0.15)$, $[-0.15,0)$, $[0,0.15)$, $[0.15,\infty)$. Then, for each one of the four ranges of $\frac{\mu_c-\mu_u}{\mu_u}$, we calculate the fraction of instances where the corresponding results hold.

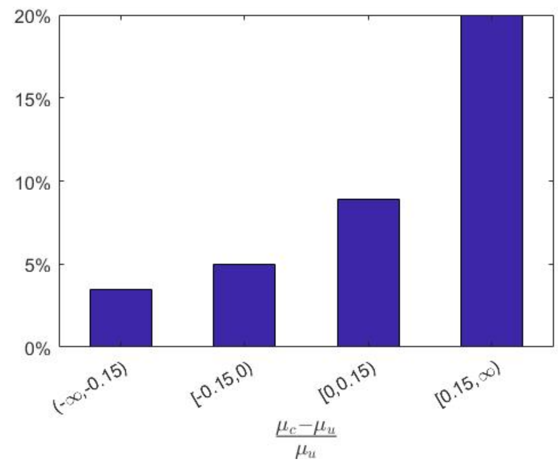
First, we study the impacts of offset price g on firm's decisions and verify the results in Propositions 1 and 2. Figure V1a shows the likelihood that the main result in Proposition 1 continues to be valid in this extended model, specifically that the firm reduces its controllable emissions as offset prices decrease. Consistent with the numerical results for our main model, this figure indicates that such instances are not uncommon with this generalized model. Figure V1b shows the likelihood that the result in Propositions 2 holds, that is, when, in response to an offset price decrease, the firm offsets a lower share of its emissions. Consistent with both our analytical finding in the main model, this figure shows that the result is more likely to occur when the two types of consumer preferences are positively correlated (i.e., $\frac{\mu_c-\mu_u}{\mu_u}>0$).

Next, we study the equilibrium of the entire game and verify the results in Propositions 3 and 4. For each one of the 1,290,737 scenarios above, we are able to find the optimal offset price for the NGO ($g^*=\min[\arg\min_{g\geq c_g}\mathcal{E}^*(g)]$) through the exhaustive grid search described earlier. In addition, as for the case when carbon offsets are not available, we can calculate the firm's optimal decisions (p^b,e^b) by solving the profit optimization problem (i.e., (V1) with an additional constraint that $t=0$). We again consider the same four ranges for the ratio $\frac{\mu_c-\mu_u}{\mu_u}$ as before. Then, for each one of the four ranges of $\frac{\mu_c-\mu_u}{\mu_u}$, we calculate the fraction of instances where the corresponding results hold.

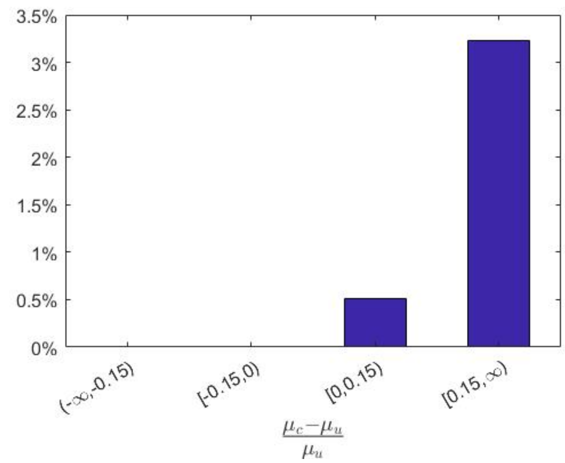
Figure V1c shows the likelihood that the main insight in Proposition 3, when the firm strictly reduces its controllable emissions and purchases offsets as offsets become available, holds in this extended model. The results suggest the robustness of our insight that such instances are relatively common. Figure V1d shows the likelihood that it is optimal for the NGO to set the offset price strictly higher than its cost (i.e., $g^*>c_g$). Consistent with our analytical finding, this figure indicates that the result is more likely

Figure V1: Proportion of instances (in model when consumer utility depends on total carbon footprint) where...

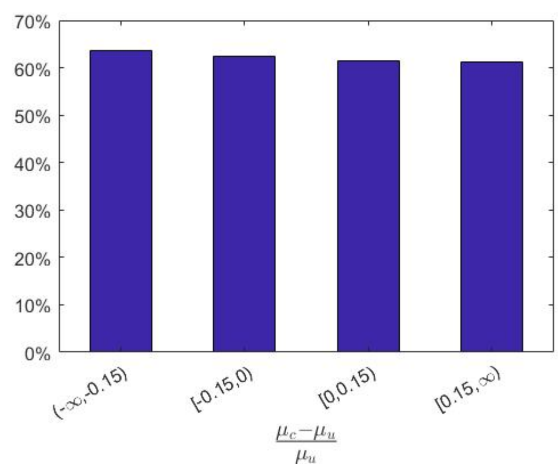
(a) There exists a pair of offset prices when the firm offsets at both prices and strictly reduces its controllable emissions at a lower offset price



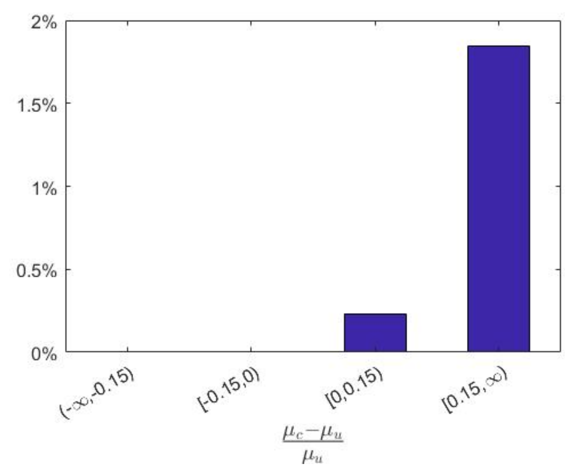
(b) There exists a pair of offset prices when the firm strictly offsets a lower share of its emissions at a lower offset price



(c) The firm strictly reduces its emissions and purchases offsets as offsets become available



(d) The NGO sets offset price strictly higher than its cost



to happen when the two types of consumer preferences are positively correlated (i.e., $\frac{\mu_c - \mu_u}{\mu_u} > 0$).

V2 Uncertain Market Size

We here provide an analysis of the model extension where there is demand uncertainty. Same as in the main model, we denote $(p^*(g), e^*(g), t^*(g))$ as the firm's best response functions given the offset price and g^* as the optimal offset price in equilibrium. In addition, we continue to use superscript $\cdot^b$ to denote the equilibrium outcome for the case without the availability of carbon offsets. For $x \geq 0$ and $\alpha \in [0, 1]$, similar to the main model, denote $e_{x,\alpha} = \max(\min(\tilde{e}_{x,\alpha}, \bar{e}), 0)$, where $\tilde{e}_{x,\alpha}$ is the solution to the equation $[x + c'(\tilde{e}_{x,\alpha})]\alpha m + f'(\tilde{e}_{x,\alpha}) = 0$. Also, for ease of exposition, denote $\mu = \phi + (1 - \phi)\lambda$.

In the main model, the firm simply needs to decide the market segment it wants to cover and the production quantity equals the size of the corresponding market segment. When market size is uncertain, the firm also needs to make nontrivial production quantity decision q. Given the Bernoulli distribution where M could be m or λm, the firm can choose to have either $q = \alpha\beta_c m I_{u_{c,h}\geq 0} + \alpha(1-\beta_c) m I_{u_{c,l}\geq 0} + (1-\alpha)\beta_u m I_{u_{u,h}\geq 0} + (1-\alpha)(1-\beta_u) m I_{u_{u,l}\geq 0}$ or $q = \alpha\beta_c \lambda m I_{u_{c,h}\geq 0} + \alpha(1-\beta_c)\lambda m I_{u_{c,l}\geq 0} + (1-\alpha)\beta_u \lambda m I_{u_{u,h}\geq 0} + (1-\alpha)(1-\beta_u)\lambda m I_{u_{u,l}\geq 0}$, given the market segment to be covered (i.e., $I_{u_{c,h}\geq 0}$, $I_{u_{c,l}\geq 0}$, $I_{u_{u,h}\geq 0}$ and $I_{u_{u,l}\geq 0}$). In the former (latter) case, the firm has the risk of overstocking (understocking) inventory, i.e., $q \geq (\leq) D(p,e,t)$, where $D(p,e,t)$ is given by (2) with the random total market size M.

Lemma V2.1. *The firm purchases offsets (i.e., $t^*(g) > 0$) if and only if their price g is low enough (i.e., $g \leq G$).*

Lemma V2.2. *If $\phi > \frac{c(0)+\xi\mu\bar{e}+\xi\mu e_o}{(v_l-\xi\bar{e}-\xi e_o)^+}$, then the firm always chooses to overstock (i.e., $q = \alpha\beta_c m I_{u_{c,h}\geq 0} + \alpha(1-\beta_c) m I_{u_{c,l}\geq 0} + (1-\alpha)\beta_u m I_{u_{u,h}\geq 0} + (1-\alpha)(1-\beta_u) m I_{u_{u,l}\geq 0}$) in equilibrium with and without the availability of carbon offsets.*

To focus on the case where the production quantity is greater than the sales volume (which cannot be captured in the main model), we assume $\phi > \frac{c(0)+\xi\mu\bar{e}+\xi\mu e_o}{(v_l-\xi\bar{e}-\xi e_o)^+}$ in the following analysis. We will later numerically verify the robustness of our results in the general case in Section V2.1.

Consider another model where the market size is deterministic. We use the hat notation to denote the parameters of this newly constructed model, the details of which are given as follows:

- Consumer utility function: $\hat{u} = \hat{V} - \hat{\Xi}\mathcal{E} - p$, where $\hat{V} = \mu V$ and $\hat{\Xi} = \mu\Xi$;
- Demand parameters: $\hat{\alpha} = \alpha$, $\hat{\beta}_u = \beta_u$, $\hat{\beta}_c = \beta_c$ and the market size $\hat{M} = m$ with probability 1;
- Firm's cost functions: $\hat{c}(e) = c(e)$, $\hat{f}(e) = f(e)$.

Note that this new model is the same as the main model in the paper, simply with V and Ξ being scaled by μ. We will next show that the extended model in Section 7.2 and this new model have the same result regarding the firm' emission decisions (e,t) and the NGO's offset pricing decision (g), and thus all of the results in the paper should continue to hold with the model extension. The result is summarized in Proposition V2.1. Here, we use the hat notation to denote the equilibrium outcomes (i.e., $\hat{p}^*(g), \hat{e}^*(g), \hat{t}^*(g), \hat{g}^*, \hat{p}^b, \hat{e}^b$) in the new model above.

Proposition V2.1. *Suppose $\phi > \frac{c(0)+\xi\mu\bar{e}+\xi\mu e_o}{(v_l-\xi\bar{e}-\xi e_o)^+}$. When offsets are available, then $g^* = \hat{g}^*$, $p^*(g) = \hat{p}^*(g)/\mu$, $e^*(g) = \hat{e}^*(g)$ and $t^*(g) = \hat{t}^*(g)$. When offsets are not available, then $p^b = \hat{p}^b/\mu$, $e^b = \hat{e}^b$.*

V2.1 Numerical Study

As in Section 6, we assume $c(e) = \frac{1}{2}k_c(\bar{e}-e)^2 + \bar{c}$ and $f(e) = \frac{1}{2}k_f(\bar{e}-e)^2$, and the same distributions for all of the randomly generated parameter values. In addition, for λ and ϕ, we use $\lambda \sim U(0,1)$ and $\phi \sim U(0,1)$ (and thus our numerical setting is not restricted to the assumption in Lemma V2.2).

We randomly generate 2,000,000 scenarios based on the distributions above. For each scenario, given any feasible value of the offset price $g \geq c_g$, we can find the firm's best responses $(p^*(g), e^*(g), q^*(g), t^*(g))$

by solving the following profit optimization problem with the new utility functions:

$$\begin{aligned} \max_{p,e,q,t} \; & p \cdot \mathbb{E}_M \min(q, D(p,e,t)) - (c(e) + gt)q - f(e) \\ s.t. \;\; & e \in [0, \bar{e}], t \in [0, e + e_o] \end{aligned} \tag{V2}$$

where the demand function $D(p, e, t)$ is given by (2) with the random total market size M. We thereafter conduct an exhaustive search and evaluate the firm's optimal decisions $(p^*(g), e^*(g), q^*(g), t^*(g))$ over the following grid of g values $\{g_i = c_g + 0.001(i-1)\}_{i=1}^{+\infty} \triangleq \mathbb{G}$. Note that we do not need to go through the entire grid (which has infinite number of points); according to Lemma V2.1, if $t^*(g_I) = 0$, then we can stop the search as the firm will not purchase any offset for any larger value of g and thus the offset price does not have any impact on the subgame, implying that the firm's best response functions for any index $i > I$ should be the same as that when $i = I$. Since the main focus of this paper is the role of carbon offsets, we remove the scenario where $t^*(g_1) = 0$, which implies that the firm does not have any incentive to purchase offsets for any feasible value of $g \geq c_g$. As a result, we end up having 1,154,211 randomly generated scenarios.

As before, we consider the following four ranges for the relative average valuation between the two groups of customers $\frac{\mu_c - \mu_u}{\mu_u}$, where $\mu_c = \beta_c v_h + (1-\beta_c)v_l$ and $\mu_u = \beta_u v_h + (1-\beta_u)v_l$: $(-\infty, -0.15)$, $[-0.15, 0)$, $[0, 0.15)$, $[0.15, \infty)$. Then, for each one of the four ranges of $\frac{\mu_c - \mu_u}{\mu_u}$, we calculate the fraction of instances where the corresponding results hold.

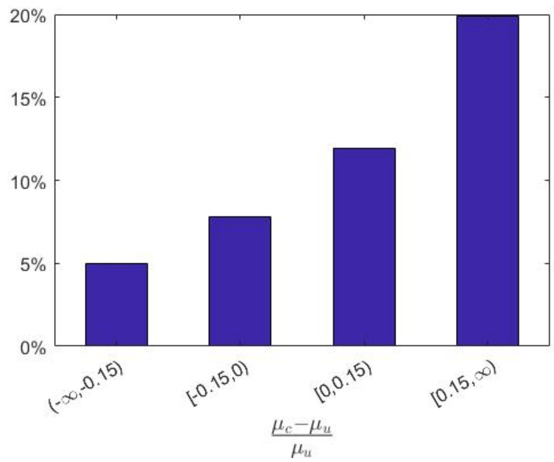
First, we study the impacts of offset price g on firm's decisions and verify the results in Propositions 1 and 2. Figure V2a shows the likelihood that the main result in Proposition 1 continues to be valid in this extended model, specifically that the firm reduces its controllable emissions as offset prices decrease. Consistent with the numerical results for our main model, this figure indicates that such instances are not uncommon with this generalized model. Figure V2b shows the likelihood that the result in Propositions 2 holds, that is, when, in response to an offset price decrease, the firm offsets a lower share of its emissions. Consistent with both our analytical finding in the main model, this figure shows that the result is more likely to occur when the two types of consumer preferences are positively correlated (i.e., $\frac{\mu_c - \mu_u}{\mu_u} > 0$).

Next, we study the equilibrium of the entire game and verify the results in Propositions 3 and 4. For each one of the 1,154,211 scenarios above, we are able to find the optimal offset price for the NGO ($g^* = \min[\arg\min_{g \geq c_g} \mathcal{E}^*(g)]$) through the exhaustive grid search described earlier. In addition, as for the case when carbon offsets are not available, we can calculate the firm's optimal decisions (p^b, e^b) by solving the profit optimization problem (i.e., (V2) with an additional constraint that $t = 0$). We again consider the same four ranges for the ratio $\frac{\mu_c - \mu_u}{\mu_u}$ as before. Then, for each one of the four ranges of $\frac{\mu_c - \mu_u}{\mu_u}$, we calculate the fraction of instances where the corresponding results hold.

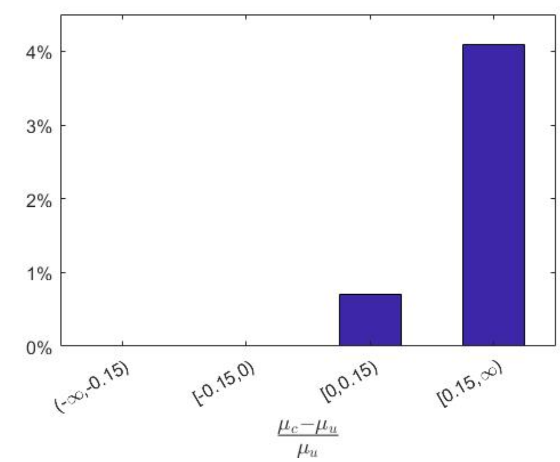
Figure V2c shows the likelihood that the main insight in Proposition 3, when the firm strictly reduces its controllable emissions and purchases offsets as offsets become available, holds in this extended model. The results suggest the robustness of our insight that such instances are relatively common. Figure V2d shows the likelihood that it is optimal for the NGO to set the offset price strictly higher than its cost (i.e., $g^* > c_g$). Consistent with our analytical finding, this figure indicates that the result is more likely

Figure V2: Proportion of instances (in model extension with uncertain market size) where...

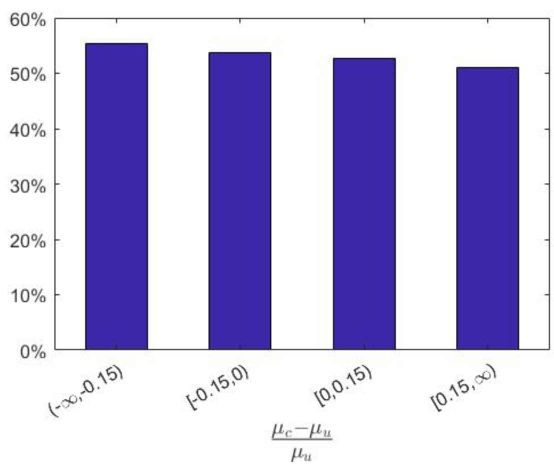
(a) There exists a pair of offset prices when the firm offsets at both prices and strictly reduces its controllable emissions at a lower offset price



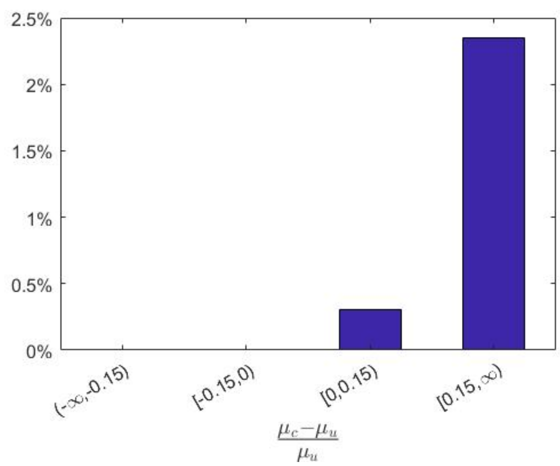
(b) There exists a pair of offset prices when the firm strictly offsets a lower share of its emissions at a lower offset price



(c) The firm strictly reduces its emissions and purchases offsets as offsets become available



(d) The NGO sets offset price strictly higher than its cost



to happen when the two types of consumer preferences are positively correlated (i.e., $\frac{\mu_c-\mu_u}{\mu_u} > 0$).

V3 Valuation Dependent on Controllable Emission Level

We here provide an analysis of the model extension where the firm's controllable emission decision e can have a negative impact on the product's base quality and thus on consumer valuation. Same as in the main model, we denote $(p^*(g), e^*(g), t^*(g))$ as the firm's best response functions given the offset price and g^* as the optimal offset price in equilibrium. In addition, we continue to use superscript $\cdot^b$ to denote the equilibrium outcome for the case without the availability of carbon offsets.

Consider another model where the firm's decision e has no impact on consumer's base valuation. We use the hat notation to denote the parameters of this newly constructed model, the details of which are given as follows:

- Consumer utility function: $\hat{u} = V - \Xi\mathcal{E} - p$;
- Demand parameters: $\hat{\alpha} = \alpha$, $\hat{\beta}_u = \beta_u$, $\hat{\beta}_c = \beta_c$ and $\hat{m} = m$;
- Firm's cost functions: $\hat{c}(e) = c(e) + w(e)$, $\hat{f}(e) = f(e)$.

Note that the variable cost function $\hat{c}(e)$ is convexly decreasing because both $c(e)$ and $w(e)$ are convexly decreasing. Thus, this new model is exactly the same as the main model in the paper. We will next show that the extended model in Section 7.3 and this new model have the same result regarding the firm' emission decisions (e, t) and the NGO's offset pricing decision (g), and thus all of the results in the paper should continue to hold with the model extension. The result is summarized in Proposition V3.1. Here, we use the hat notation to denote the equilibrium outcomes (i.e., $\hat{p}^*(g), \hat{e}^*(g), \hat{t}^*(g), \hat{g}^*, \hat{p}^b, \hat{e}^b$) in the new model above.

Proposition V3.1. *When offsets are available, then* $g^* = \hat{g}^*$, $p^*(g) = \hat{p}^*(g) - w(e^*(g))$, $e^*(g) = \hat{e}^*(g)$ *and* $t^*(g) = \hat{t}^*(g)$. *When offsets are not available, then* $p^b = \hat{p}^b - w(e^b)$, $e^b = \hat{e}^b$.

V4 Utility Dependent on Emissions and Offsetting Details

In the main model, we have focused on the case where the firm discloses only its effective unit emissions $\mathcal{E}$, or product carbon footprint, to consumers. Some firms, such as some of the examples previously discussed (Apple, IKEA, Patagonia, Dell) report a more detailed breakdown of their emissions (including details on the sources of emissions for each of the scopes 1, 2 and 3), so consumers could in principle have their own (rough) estimates of e and e_o. As discussed before, although scopes 1 and 2 emissions are controllable, only a portion of scope 3 emissions are controllable. In addition, although it is not common for firms to communicate the exact amount of carbon offsets they purchase in their websites and sustainability reports, consumers could in principle obtain such data from the CDP (formerly, Carbon Disclosure Project). In this section, we consider a model extension, where consumers have full information on the specific amounts of controllable and uncontrollable emissions (i.e., e and e_o) as well as the amount of carbon offsets when making their purchasing decisions. In addition, different types of emissions may have different marginal effects on consumers' utility, and consumers may value carbon offsets lower than emissions reduction in the firm's operations. More specifically, while the firm may be viewed as not fully responsible for its uncontrollable supply chain emissions, consumers may perceive offsetting controllable emissions as taking the easy way out. We therefore revise eco-conscious consumers' utility functions to incorporate these effects as follows:

$$u_{c,h} = v_h - \xi(e - \psi\tau) - \psi_o\xi(e_o - \tau_o) - p;$$
$$u_{c,l} = v_l - \xi(e - \psi\tau) - \psi_o\xi(e_o - \tau_o) - p,$$

where $\tau \in [0, e]$ and $\tau_o \in [0, e_o]$ are the amount of offsets the firm purchases for its controllable and uncontrollable emissions, respectively. In addition, $\psi \in [0, 1]$ and $\psi_o \in [0, 1]$ are two discount factors to reflect (i) consumers' potential negative reaction to offsets for a firm's controllable emissions and (ii) the fact that they may assign a smaller degree of responsibility to the firm for its uncontrollable emissions. As for eco-unconscious consumers, their utility functions remain the same as before, i.e., $u_{u,h} = v_h - p$ and $u_{u,l} = v_l - p$. Note that the main model in the paper can be regarded as a special case of this general model with $\psi = \psi_o = 1$.

Lemma V4.1. *The firm purchases offsets if and only if their price g is low enough. (Formally, there exists a threshold G such that $\tau^*(g) + \tau_o^*(g) > 0$ if and only if $g \leq G$.)*

Due to its complexity, and multiple degrees of freedom, we resort to a numerical approach to solve the game with the new utility functions. As in Section 6, we assume $c(e) = \frac{1}{2}k_c(\bar{e} - e)^2 + \bar{c}$ and $f(e) = \frac{1}{2}k_f(\bar{e} - e)^2$, and the same distributions for all of the randomly generated parameter values. In addition, we have $\psi \sim U(0.8, 1)$ and $\psi_o \sim U(0.8, 1)$.

We randomly generate 2,000,000 scenarios based on the distributions above. For each scenario, given any feasible value of the offset price $g \geq c_g$, we can find the firm's best responses $(p^*(g), e^*(g), \tau^*(g), \tau_o^*(g))$ by solving the following profit optimization problem with the new utility functions:

$$\begin{aligned} \max_{p,e,\tau,\tau_o} \ & [p - c(e) - g\tau - g\tau_o] \cdot d(p, e, \tau, \tau_o) - f(e) \\ s.t. \ \ & e \in [0, \bar{e}], \tau \in [0, e], \tau_o \in [0, e_o] \end{aligned} \tag{V3}$$

where the demand function $d(p, e, \tau, \tau_o)$ is given by (2) with the new utility functions. We thereafter conduct an exhaustive search and evaluate the firm's optimal decisions $(p^*(g), e^*(g), \tau^*(g), \tau_o^*(g))$ over the following grid of g values $\{g_i = c_g + 0.001(i-1)\}_{i=1}^{+\infty} \triangleq \mathbb{G}$. Note that we do not need to go through the entire grid (which has infinite number of points); according to Lemma V4.1, if $\tau^*(g_I) + \tau_o^*(g_I) = 0$, then we can stop the search as the firm will not purchase any offset for any larger value of g and thus the offset price does not have any impact on the subgame, implying that the firm's best response functions for any index $i > I$ should be the same as that when $i = I$. Since the main focus of this paper is the role of carbon offsets, we remove the scenario where $\tau^*(g_1) + \tau_o^*(g_1) = 0$, which implies that the firm does not have any incentive to purchase offsets for any feasible value of $g \geq c_g$. As a result, we end up having 824,665 randomly generated scenarios.[21]

As before, we consider the following four ranges for the relative average valuation between the two groups of customers $\frac{\mu_c - \mu_u}{\mu_u}$, where $\mu_c = \beta_c v_h + (1 - \beta_c)v_l$ and $\mu_u = \beta_u v_h + (1 - \beta_u)v_l$: $(-\infty, -0.15)$, $[-0.15, 0)$, $[0, 0.15)$, $[0.15, \infty)$. Then, for each one of the four ranges of $\frac{\mu_c - \mu_u}{\mu_u}$, we calculate the fraction of instances where the corresponding results hold.

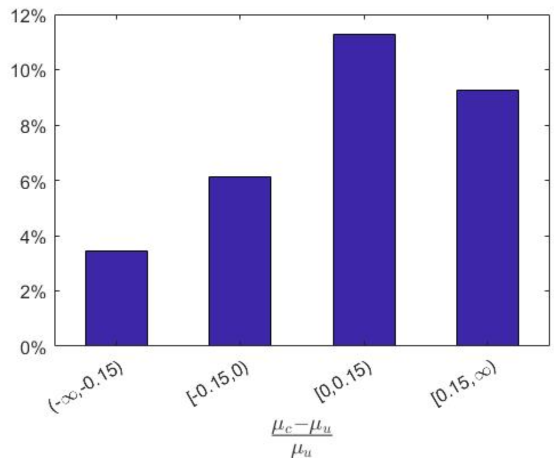
First, we study the impacts of offset price g on firm's decisions and verify the results in Propositions 1 and 2. Figure V3a shows the likelihood that the main result in Proposition 1 continues to be valid in this extended model, specifically that the firm reduces its controllable emissions as offset prices decrease. Consistent with the numerical results for our main model, this figure indicates that such instances are not uncommon with this generalized model. Figure V3b shows the likelihood that the result in Propositions 2 holds, that is, when, in response to an offset price decrease, the firm offsets a lower share of its emissions. Consistent with both our analytical finding in the main model, this figure shows that the result is more likely to occur when the two types of consumer preferences are positively correlated (i.e., $\frac{\mu_c - \mu_u}{\mu_u} > 0$).

Next, we study the equilibrium of the entire game and verify the results in Propositions 3 and 4.

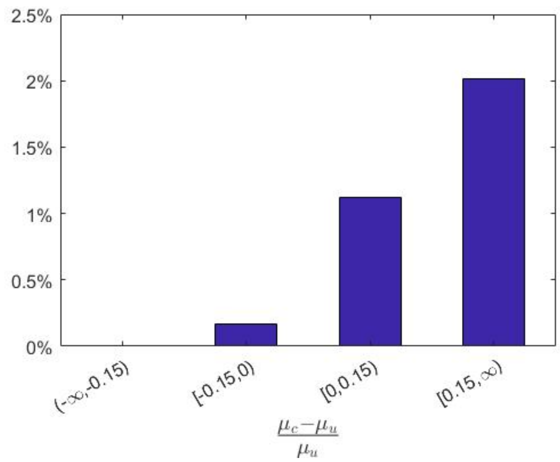
[21] When consumers have negative reaction to offsets (as in the extended model), the firm will have less incentive to purchase offsets (and it is thus more likely to have $\tau^*(g) + \tau_o^*(g) = 0$ for all feasible value of $g \geq c_g$) compared to the main model in the paper.

Figure V3: Proportion of instances in general utility model where...

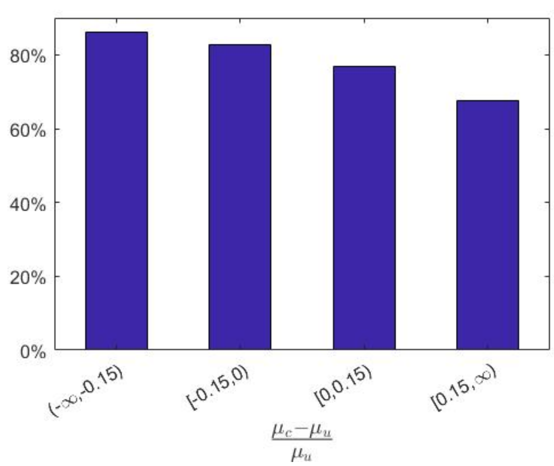
(a) There exists a pair of offset prices when the firm offsets at both prices and strictly reduces its controllable emissions at a lower offset price



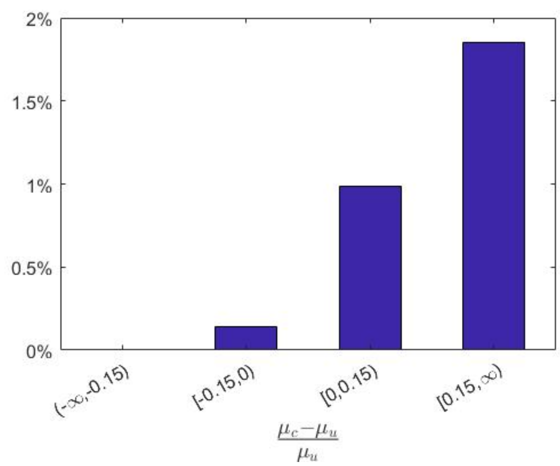
(b) There exists a pair of offset prices when the firm strictly offsets a lower share of its emissions at a lower offset price



(c) The firm strictly reduces its emissions and purchases offsets as offsets become available



(d) The NGO sets offset price strictly higher than its cost



For each one of the 824,665 scenarios above, we are able to find the optimal offset price for the NGO ($g^* = \min[\arg\min_{g \geq c_g} \mathcal{E}^*(g)]$) through the exhaustive grid search described earlier. In addition, as for the case when carbon offsets are not available, we can calculate the firm's optimal decisions (p^b, e^b) by solving the profit optimization problem (i.e., (V3) with an additional constraint that $\tau = \tau_o = 0$). We again consider the same four ranges for the ratio $\frac{\mu_c-\mu_u}{\mu_u}$ as before. Then, for each one of the four ranges of $\frac{\mu_c-\mu_u}{\mu_u}$, we calculate the fraction of instances where the corresponding results hold.

Figure V3c shows the likelihood that the main insight in Proposition 3, when the firm strictly reduces its controllable emissions and purchases offsets as offsets become available, holds in this extended model. The results suggest the robustness of our insight that such instances are relatively common. Figure V3d shows the likelihood that it is optimal for the NGO to set the offset price strictly higher than its cost (i.e., $g^* > c_g$). Consistent with our analytical finding, this figure indicates that the result is more likely to happen when the two types of consumer preferences are positively correlated (i.e., $\frac{\mu_c - \mu_u}{\mu_u} > 0$).

Online Supplement VI: Proofs

Lemma VI1. *$e_{x,\alpha}$ is weakly decreasing in both x and α.*

Proof of Lemma VI1: Note $e_{x,\alpha} = \max(\min(\tilde{e}_{x,\alpha}, \bar{e}), 0)$, where $\tilde{e}_{x,\alpha}$ is the solution to the equation $[x + c'(\tilde{e}_{x,\alpha})]\alpha m + f'(\tilde{e}_{x,\alpha}) = 0$. Then, we have $\frac{\partial \tilde{e}_{x,\alpha}}{\partial \alpha} = \frac{-xm - c'(\tilde{e}_{x,\alpha})m}{\alpha m c''(\tilde{e}_{x.\alpha}) + f''(\tilde{e}_{x,\alpha})} = \frac{f'(\tilde{e}_{x,\alpha})/\alpha}{\alpha m c''(\tilde{e}_{x.\alpha}) + f''(\tilde{e}_{x,\alpha})} < 0$ and $\frac{\partial \tilde{e}_{x,\alpha}}{\partial x} = \frac{-\alpha m}{\alpha m c''(\tilde{e}_{x.\alpha}) + f''(\tilde{e}_{x,\alpha})} < 0$. Thus, we can conclude the result. □

Proof of Lemma 1: Given the offset price g, the firm's profit function is $\pi(p, e, t|g) = (p - c(e) - gt)d(p, e, t) - f(e)$. Suppose there exist $g_1 < g_2$ such that $t^*(g_1) = 0 < t^*(g_2)$. Then, $\pi(p^*(g_2), e^*(g_2), t^*(g_2)|g_2) > \pi(p^*(g_1), e^*(g_1), t^*(g_1)|g_2) = \pi(p^*(g_1), e^*(g_1), t^*(g_1)|g_1) > \pi(p^*(g_2), e^*(g_2), t^*(g_2)|g_1)$, where the equality is because $t^*(g_1) = 0$. However, this is contradictory to the fact that $\pi(p^*(g_2), e^*(g_2), t^*(g_2)|g_2) - \pi(p^*(g_2), e^*(g_2), t^*(g_2)|g_1) = (g_1 - g_2)t^*(g_2)d(p^*(g_2), e^*(g_2), t^*(g_2)) < 0$. Thus, we can conclude the result. □

Proof of Proposition 1: Let us first look at Case N. Denote $\pi(p, e, t|g)$ as the profit given firm's decisions (p, e, t) and offset price g. If $t > 0$, according to the analysis of the subgame in Online Appendix I1 the equilibrium in the subgame must be one of the following:

1. $p = v_h$, $e = e_{g,1-\alpha+\alpha\beta_c}$, $t = e_{g,1-\alpha+\alpha\beta_c} + e_o$, $d_c = \alpha\beta_c m$, $d_u = (1-\alpha)m$;
2. $p = v_l$, $e = e_{g,1}$, $t = e_{g,1} + e_o$, $d_c = \alpha m$, $d_u = (1-\alpha)m$.

Define function $h(g) \triangleq \pi(v_l, e_{g,1}, e_{g,1} + e_o|g) - \pi(v_h, e_{g,1-\alpha+\alpha\beta_c}, e_{g,1-\alpha+\alpha\beta_c} + e_o|g) = (v_l - c(e_{g,1}) - ge_{g,1} - ge_o)m - f(e_{g,1}) - [(v_h - c(e_{g,1-\alpha+\alpha\beta_c}) - ge_{g,1-\alpha+\alpha\beta_c} - ge_o)(1 - \alpha + \alpha\beta_c)m - f(e_{g,1-\alpha+\alpha\beta_c})]$. Note that $\lim_{g \to +\infty} h'(g) = -e_o\alpha(1 - \beta_c)m < 0$. Then, if $h(0) > 0$ (i.e., $\beta_c < B_2 \triangleq \frac{\frac{v_l - c(\bar{e})}{v_h - c(\bar{e})} - 1 + \alpha}{\alpha}$), then there exists $X = \min\{g|h(g) = 0\}$ and $X > 0$. Then, if the following condition hold:

(1) $\pi(v_l, e_{X,1}, e_{X,1} + e_o|X) > \pi(v_h, \bar{e}, 0|X)$, i.e., $(v_l - c(e_{X,1}) - Xe_{X,1} - Xe_o)m - f(e_{X,1}) > (v_h - c(\bar{e}))(1 - \alpha)m$,

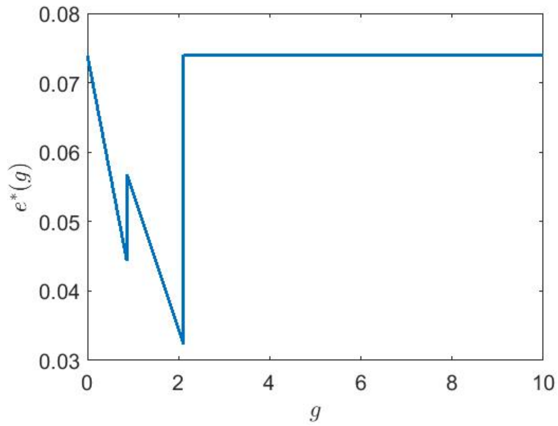
then we must have $X < G$ and for a small positive number $\epsilon > 0$,

- if $g \in (0, X)$, then $p^* = v_l$, $e^* = e_{g,1}$ and $t^* = e_{g,1} + e_o$;
- if $g \in (X, X + \epsilon)$, then $p^* = v_h$, $e^* = e_{g,1-\alpha+\alpha\beta_c}$ and $t^* = e_{g,1-\alpha+\alpha\beta_c} + e_o$.

Therefore, there exist g_1, g_2 and $\tilde{\epsilon} \in (0, \epsilon)$ such that $X - \tilde{\epsilon} < g_1 < X < g_2 < X + \tilde{\epsilon}$ and $e^*(g_1) = e_{g_1,1} \le e_{g_2,1-\alpha+\alpha\beta_c} = e^*(g_2)$, where the inequality is due to Lemma VI1 and is strict if $\bar{e}$ is large enough; also, in this case, we have $d_c^*(g_1) = \alpha m > \alpha\beta_c m = d_c^*(g_2)$ and $d_u^*(g_1) = (1-\alpha)m = d_u^*(g_2)$. As for condition (1), note that $\frac{\partial h}{\partial \beta_c} < 0$ and $\frac{\partial h}{\partial g}|_{g=X} < 0$, and thus we have $\frac{\partial X}{\partial \beta_c} < 0$. Since $\frac{\partial (v_l - c(e_{X,1}) - Xe_{X,1} - Xe_o)m - f(e_{X,1}))}{\partial X} < 0$ (due to Envelope Theorem), condition (1) is equivalent to β_c being large (i.e., $\beta_c > B_1$).

Figure VI1 depicts a numerical example for Proposition 1(i).

Figure VI1: A Numerical Example for Proposition 1(i)



Note: In this example, $m = 80$, $c(e) = \frac{1}{2}k_c(\bar{e} - e)^2 + \bar{c}$, $f(e) = \frac{1}{2}k_f(\bar{e} - e)^2$, $\bar{c} = 1.6$, $k_c = 3$, $k_f = 2091$, $\bar{e} = 0.074$, $e_o = 0.154$, $\xi = 15$, $\alpha = 0.5$, $\beta_c = 0.1$, $\beta_u = 1$, $v_l = 4.32$, $v_h = 6.4$.

Next, let us look at Case P. Denote $\pi(p, e, t|g)$ as the profit given firm's decisions (p, e, t) and offset price g. If $v_h < v_l + \xi e_o$, then $v_l > v_h - \xi(\bar{e} + e_o)$ and $\max(e_{g,1}, \frac{v_h - v_l - \xi e_o}{\xi}) = e_{g,1}$, and thus if $g \le G$ (i.e., $t > 0$), according to the analysis of the subgame in Online Appendix I1, the equilibrium in the subgame must be one of the following:

1. $p = v_h$, $e = e_{g,\alpha+(1-\alpha)\beta_u}$, $t = e_{g,\alpha+(1-\alpha)\beta_u} + e_o$, $d_c = \alpha m$, $d_u = (1 - \alpha)\beta_u m$;
2. $p = v_l$, $e = e_{g,1}$, $t = e_{g,1} + e_o - \frac{v_h - v_l}{\xi}$, $d_c = \alpha m$, $d_u = (1 - \alpha)m$.

Define function $h(g) \triangleq \pi(v_l, e_{g,1}, \frac{v_l - v_h + \xi e_{g,1} + \xi e_o}{\xi}|g) - \pi(v_h, e_{g,\alpha+(1-\alpha)\beta_u}, e_{g,\alpha+(1-\alpha)\beta_u} + e_o|g) = (v_l - c(e_{g,1}) - g\frac{v_l - v_h}{\xi} - ge_{g,1} - ge_o)m - f(e_{g,1}) - [(v_h - c(e_{g,\alpha+(1-\alpha)\beta_u}) - ge_{g,\alpha+(1-\alpha)\beta_u} - ge_o)(\alpha + (1 - \alpha)\beta_u)m - f(e_{g,\alpha+(1-\alpha)\beta_u})]$. Note that $h(0) = (v_l - c(\bar{e}))m - (v_h - c(\bar{e}))(\alpha + (1 - \alpha)\beta_u)m$ and $\lim_{g\to+\infty} h'(g) = \frac{v_h - v_l}{\xi}m - e_o(1-\alpha)(1-\beta_u)m$. If $h(0) > 0$ and $\lim_{g\to+\infty} h'(g) < 0$ (i.e., $\beta_u < \tilde{B}_2 \triangleq \min(\frac{(v_l - c(\bar{e})) - (v_h - c(\bar{e}))\alpha}{(v_h - c(\bar{e}))(1-\alpha)}, \frac{\xi e_o(1-\alpha) - (v_h - v_l)}{\xi e_o(1-\alpha)})$), then there exists $X = \min\{g|h(g) = 0\}$ and $X > 0$. Then, if the following condition hold:

(1) $\pi(v_h, e_{X,\alpha+(1-\alpha)\beta_u}, e_{X,\alpha+(1-\alpha)\beta_u} + e_o|X) > \max(\pi(v_h, \bar{e}, 0|X), \pi(v_l, \bar{e}, 0|X))$, i.e., $(v_h - c(e_{X,\alpha+(1-\alpha)\beta_u}) - Xe_{X,\alpha+(1-\alpha)\beta_u} - Xe_o)(\alpha + (1-\alpha)\beta_u)m - f(e_{X,\alpha+(1-\alpha)\beta_u}) > \max((v_h - c(\bar{e}))(1-\alpha)\beta_u m, (v_l - c(\bar{e}))(1-\alpha)m)$;

then we must have $X < G$ and for a small positive number $\epsilon > 0$,

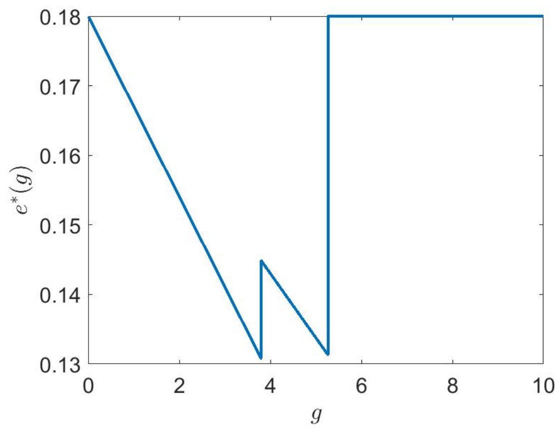
- if $g \in (0, X)$, then $p^* = v_l$, $e^* = e_{g,1}$, $t^* = e_{g,1} + e_o - \frac{v_h - v_l}{\xi}$,
- if $g \in (X, X + \epsilon)$, then $p^* = v_h$, $e^* = e_{g,\alpha+(1-\alpha)\beta_u}$, $t^* = e_{g,\alpha+(1-\alpha)\beta_u} + e_o$.

Therefore, there exist g_1, g_2 and $\tilde{\epsilon} \in (0, \epsilon)$ such that $X - \tilde{\epsilon} < g_1 < X < g_2 < X + \tilde{\epsilon}$ and $e^*(g_1) = e_{g_1,1} \le e_{g_2,\alpha+(1-\alpha)\beta_u} = e^*(g_2)$, where the inequality is due to Lemma VI1 and is strict if $\bar{e}$ is large enough; also, in this case, we have $d_c^*(g_1) = \alpha m = d_c^*(g_2)$ and $d_u^*(g_1) = (1 - \alpha)m > (1 - \alpha)\beta_u = d_u^*(g_2)$. As for condition (1), since $\beta_u < \frac{(v_l - c(\bar{e})) - (v_h - c(\bar{e}))\alpha}{(v_h - c(\bar{e}))(1-\alpha)} < \frac{v_l - c(\bar{e})}{v_h - c(\bar{e})}$, condition (1) is equivalent to

$(v_h - c(e_{X,\alpha+(1-\alpha)\beta_u}) - Xe_{X,\alpha+(1-\alpha)\beta_u} - Xe_o)(\alpha + (1-\alpha)\beta_u)m - f(e_{X,\alpha+(1-\alpha)\beta_u}) > (v_l - c(\bar{e}))(1 - \alpha)m$. Note that $\frac{\partial h}{\partial \beta_u} < 0$ and $\frac{\partial h}{\partial g}|_{g=X} < 0$, and thus we have $\frac{\partial X}{\partial \beta_u} < 0$. By Envelope Themore, we have $\frac{\partial((v_h - c(e_{X,\alpha+(1-\alpha)\beta_u}) - Xe_{X,\alpha+(1-\alpha)\beta_u} - Xe_o)(\alpha+(1-\alpha)\beta_u)m - f(e_{X,\alpha+(1-\alpha)\beta_u}))}{\partial X} < 0$, and thus condition (1) is equivalent to β_u being large (i.e., $\beta_u > \tilde{B}_1$).

Figure VI2 depicts a numerical example for Proposition 1(ii).

Figure VI2: A Numerical Example for Proposition 1(ii)



Note: In this example, $m = 70$, $c(e) = \frac{1}{2}k_c(\bar{e} - e)^2 + \bar{c}$, $f(e) = \frac{1}{2}k_f(\bar{e} - e)^2$, $\bar{c} = 2$, $k_c = 40$, $k_f = 2600$, $\bar{e} = 0.18$, $e_o = 0.16$, $\xi = 45$, $\alpha = 0.3$, $\beta_c = 1$, $\beta_u = 0.35$, $v_l = 6$, $v_h = 8.8$.

□

Proof of Footnote 9: If $f(e) = 0$ for any $e \in [0, \bar{e}]$, then for any g and α, we have $e_{g,\alpha} = e_g$, which is independent of α. From the subgame analysis in Online Appendix I1, when $t^* > 0$, we must have $e^* = e_g$, which is (weakly) decreasing in g (by Lemma VI1). □

Proof of Proposition 2: Let us first look at Case N. According to the analysis of the subgame in Online Appendix I1, if $t > 0$, the equilibrium in the subgame must be one of the following:

1. $p = v_h$, $e = e_{g,1-\alpha+\alpha\beta_c}$, $t = e_{g,1-\alpha+\alpha\beta_c} + e_o$, $d_c = \alpha\beta_c m$, $d_u = (1-\alpha)m$;
2. $p = v_l$, $e = e_{g,1}$, $t = e_{g,1} + e_o$, $d_c = \alpha m$, $d_u = (1-\alpha)m$.

Thus, if $t > 0$, we have $\theta = 1$ in equilibrium. Given Lemma 1, we can conclude that

- if $g > G$, then $\theta^* = 0$;
- if $g \leq G$, then $\theta^* = 1$.

Thus, we can conclude Proposition 2(i).

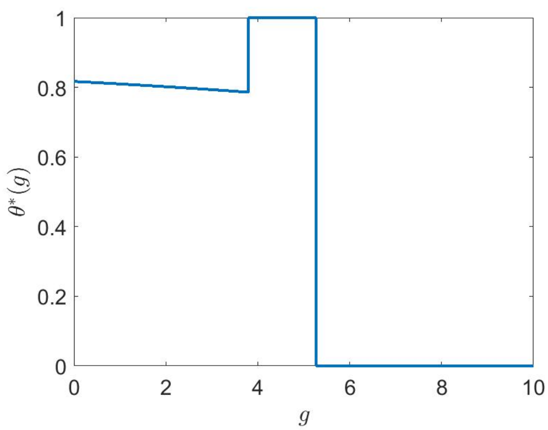
Next, let us look at Case P. From the proof of Proposition 1(ii), if $\beta_u \in (\tilde{B}_1, \tilde{B}_2)$ and $v_h - v_l < \xi e_o$, then

- if $g_1 \in (0, X)$,[22] then $\theta^*(g_1) = 1 - \frac{v_h - v_l}{\xi(e_{g,1}+e_o)}$;
- if $g_2 \in (X, X+\epsilon)$, then $\theta^*(g_2) = 1$.

Thus, $\theta^*(g_1) < \theta^*(g_2)$.

Below, we provide a numerical example for Propositions 2(ii) in Figure VI3.

Figure VI3: A Numerical Example for Propositions 2(ii)



Note: In this example, the parameter values are the same as in Figure VI2.

□

Proof of Lemma 2: If $c_g \geq G$, then for any $g \geq c_g$, according to Lemma 1, $t^*(g) = 0$. Thus, the firm will not choose to purchase any offset from the NGO, given any feasible choice offset price $g \geq c_g$.

Next, let us consider the case when $c_g \leq G$. First, from the Lemma I1.1 and Lemma 1, we have $G \leq \xi$.

- If $G < \xi$, then from the subgame analysis in Online Appendix I1,
  - if $g \leq G$, then $t^*(g) > 0$ and thus $\mathcal{E}^*(g) \in \{0, \frac{v_h - v_l}{\xi}\}$
  - if $g \in (G, \xi]$, then $t^*(g) = 0$ and thus $\mathcal{E}^*(g) \in \{\frac{v_h - v_l}{\xi}, \bar{e} + e_o\}$

  Note that the equilibrium in the subgame (including $\mathcal{E}^*$) is independent of g when $g > G$, because $t^*(g) = 0$. Thus, for $g > \xi$, we also have $\mathcal{E}^*(g) \in \{\frac{v_h - v_l}{\xi}, \bar{e} + e_o\}$. Note that $\mathcal{E}^*(g) \leq \bar{e} + e_o$ for any g. Thus, for any $g_1 \in [c_g, G]$ and $g_2 > G$, we have $\mathcal{E}^*(g_1) \leq \mathcal{E}^*(g_2)$. Thus, $g^* \in [c_g, G]$. Then, by Lemma 1, we have $t^*(g^*) > 0$.
- If $G = \xi$, then from the subgame analysis in Online Appendix I1, then $t^*(G) > 0$ and thus $\mathcal{E}^*(G) \in \{0, \frac{v_h - v_l}{\xi}\}$
  - If $\mathcal{E}^*(G) = 0$, then $\mathcal{E}^*(G) \leq \mathcal{E}^*(g_2)$ for any $g_2 > G$. Thus, $g^* \in [c_g, G]$. Then, by Lemma 1, we have $t^*(g^*) > 0$.

[22] Note that we have assumed $g > 0$ throughout the paper. If $g = 0$, the firm's best response functions may not be unique. Specifically, if $g = 0 < X$, we can find that $p^* = v_l$, $e^* = \bar{e}$ and $t^* \in [\frac{v_l - v_h + \xi(\bar{e}+e_o)}{\xi}, \bar{e} + e_o]$. In this case, the firm does not need to offset all of its emissions (despite zero offset price) to attract all consumers to purchase the product at price $p = v_l$.

– If $\mathcal{E}^*(G) = \frac{v_h - v_l}{\xi}$, then from the subgame analysis in Online Appendix I1, given that $t^*(G) > 0$, we must have $e_{G,1-\alpha+\alpha\beta_c} + e_o - \frac{v_h - v_l}{G} > 0$ (i.e., $e_{\xi,1-\alpha+\alpha\beta_c} + e_o - \frac{v_h - v_l}{\xi} > 0$) and $(\pi^{(4)} - \pi^{(6)})|_{g=G=\xi} > 0$. Since $t^*(G) > 0$, we have $(\pi^{(4)} - \pi^{(6)})|_{g=G=\xi} = \pi^{(a3)} - \pi^{(a4)} = \pi^{(b5)} - \pi^{(b6)}$, and thus $\pi^{(a3)} - \pi^{(a4)} > 0$ and $\pi^{(b5)} - \pi^{(b6)} > 0$. Also, since $e_{\xi,1-\alpha+\alpha\beta_c} + e_o - \frac{v_h - v_l}{\xi} > 0$ and $e_{\xi,1-\alpha+\alpha\beta_c} \leq e_{\xi,(1-\alpha)\beta_u+\alpha\beta_c}$, we have $v_h - \xi e_{\xi,(1-\alpha)\beta_u+\alpha\beta_c} - \xi e_o < v_l$. Thus, when $g > G$, the outcome in the subgame must be from option (a1), (a2), (a3), (b1), (b3), (b4), (b5). Note that $\mathcal{E}^{(a1)} = \mathcal{E}^{(a2)} = \mathcal{E}^{(b1)} = \mathcal{E}^{(b3)} = \bar{e} + e_o \geq \mathcal{E}^*(G)$, $\mathcal{E}^{(a3)} = e_{\xi,1-\alpha+\alpha\beta_c} + e_o > \frac{v_h - v_l}{\xi} = \mathcal{E}^*(G)$, $\mathcal{E}^{(b4)} = \frac{v_h - v_l}{\xi} = \mathcal{E}^*(G)$, and $\mathcal{E}^{(b5)} = e_{\xi,1-\alpha+\alpha\beta_c} + e_o > \frac{v_h - v_l}{\xi} = \mathcal{E}^*(G)$. Therefore, $\mathcal{E}^*(G) \leq \mathcal{E}^*(g_2)$ for any $g_2 > G$. Thus, $g^* \in [c_g, G]$. Then, by Lemma 1, we have $t^*(g^*) > 0$.

□

Proof of Proposition 3: Let's first look at the case when carbon offsets are not available. The analysis of the game is given in Appendix I2.

Lemma VI2. *When offsets are not available, there exists a threshold $A > 0$ such that if $\alpha \in (0, A)$, then $e^b = \bar{e}$.*

Proof of Lemma VI2: If $v_h - \xi e_o \leq v_l$, then we have $v_h - \xi e_o - \xi e_{\xi,1-\alpha+\alpha\beta_c} < v_l$.

- If $(v_h - c(\bar{e}))\beta_u > (v_l - c(\bar{e}))$, then we just need to compare cases (a1), (a3) and (a4), where $e^{(a1)} = \bar{e}$, $e^{(a3)} = e_{\xi,1-\alpha+\alpha\beta_c}$ and $e^{(a4)} = e_{\xi,1}$. By Envelope Theorem, we have $\frac{\partial(\pi^{(a1)} - \pi^{(a3)})}{\partial\alpha} = -(v_h - c(\bar{e}))\beta_u m + (v_h - \xi e_o - \xi e_{\xi,1-\alpha+\alpha\beta_c} - c(e_\xi, 1-\alpha+\alpha\beta_c))(1-\beta_c)m < -(v_l - c(\bar{e}))m + (v_l - c(e_\xi, 1-\alpha+\alpha\beta_c))(1-\beta_c)m < 0$ and $\frac{\partial(\pi^{(a1)} - \pi^{(a4)})}{\partial\alpha} = -(v_h - c(\bar{e}))\beta_u m < 0$. Also, note that $\lim_{\alpha\to 0}(\pi^{(a1)} - \pi^{(a3)}) = (v_h - c(\bar{e}))\beta_u m - [(v_h - \xi e_o - \xi e_{\xi,1} - c(e_{\xi,1}))m - f(e_{\xi,1})] > (v_l - c(\bar{e}))m - (v_l - c(e_{\xi,1}))m + f(e_{\xi,1}) > 0$ and $\lim_{\alpha\to 0}(\pi^{(a1)} - \pi^{(a4)}) = (v_h - c(\bar{e}))\beta_u m - [(v_l - \xi e_o - \xi e_{\xi,1} - c(e_{\xi,1}))m - f(e_{\xi,1})] > (v_l - c(\bar{e}))m - (v_l - \xi e_o - \xi e_{\xi,1} - c(e_{\xi,1}))m + f(e_{\xi,1}) > 0$. Thus, there exists $A > 0$ such that if $\alpha \in (0, A)$, then $\pi^{(a1)} > \max\{\pi^{(a3)}, \pi^{(a4)}\}$ and thus $e^b = e^{(a1)} = \bar{e}$.
- If $(v_h - c(\bar{e}))\beta_u \leq (v_l - c(\bar{e}))$, then we just need to compare cases (a2), (a3) and (a4), where $e^{(a2)} = \bar{e}$, $e^{(a3)} = e_{\xi,1-\alpha+\alpha\beta_c}$ and $e^{(a4)} = e_{\xi,1}$. By Envelope Theorem, we have $\frac{\partial(\pi^{(a2)} - \pi^{(a3)})}{\partial\alpha} = -(v_l - c(\bar{e}))m + (v_h - \xi e_o - \xi e_{\xi,1-\alpha+\alpha\beta_c} - c(e_\xi, 1-\alpha+\alpha\beta_c))(1-\beta_c)m < -(v_l - c(\bar{e}))m + (v_l - c(e_\xi, 1-\alpha+\alpha\beta_c))(1-\beta_c)m < 0$ and $\frac{\partial(\pi^{(a2)} - \pi^{(a4)})}{\partial\alpha} = -(v_l - c(\bar{e}))m < 0$. Also, note that $\lim_{\alpha\to 0}(\pi^{(a2)} - \pi^{(a3)}) = (v_l - c(\bar{e}))m - [(v_h - \xi e_o - \xi e_{\xi,1} - c(e_{\xi,1}))m - f(e_{\xi,1})] > (v_l - c(\bar{e}))m - (v_l - c(e_{\xi,1}))m + f(e_{\xi,1}) > 0$ and $\lim_{\alpha\to 0}(\pi^{(a2)} - \pi^{(a4)}) = (v_l - c(\bar{e}))m - [(v_l - \xi e_o - \xi e_{\xi,1} - c(e_{\xi,1}))m - f(e_{\xi,1})] > (v_l - c(\bar{e}))m - (v_l - \xi e_o - \xi e_{\xi,1} - c(e_{\xi,1}))m + f(e_{\xi,1}) > 0$. Thus, there exists $A > 0$ such that if $\alpha \in (0, A)$, then $\pi^{(a2)} > \max\{\pi^{(a3)}, \pi^{(a4)}\}$ and thus $e^b = e^{(a2)} = \bar{e}$.

If $v_h - \xi\bar{e} - \xi e_o \leq v_l < v_h - \xi e_o$:

- If $(v_h - c(\bar{e}))\beta_u > (v_l - c(\bar{e}))$, then we just need to compare cases (b1), (b2), (b4), (b5) and (b6), where $e^{(b1)} = \bar{e}$, $e^{(b2)} = e_{\xi,(1-\alpha)\beta_u+\alpha\beta_c}$, $e^{(b4)} = \frac{v - \xi e_o - v_l}{\xi}$, $e^{(a5)} = e_{\xi,1-\alpha+\alpha\beta_c}$ and $e^{(a6)} = e_{\xi,1}$. By Envelope Theorem, we have the following results:

* If $v_h - \xi e_{\xi,(1-\alpha)\beta_u+\alpha\beta_c} - \xi e_o \geq v_l$, then $\frac{\partial(\pi^{(b1)}-\pi^{(b2)})}{\partial\alpha} = -(v_h - c(\bar{e}))\beta_u m + (v_h - \xi e_o - \xi e_{\xi,(1-\alpha)\beta_u+\alpha\beta_c} - c(e_{\xi,(1-\alpha)\beta_u+\alpha\beta_c}))(\beta_u - \beta_c)m < 0$.
* $\frac{\partial(\pi^{(b1)}-\pi^{(b4)})}{\partial\alpha} = -(v_h - c(\bar{e}))\beta_u m + (v_l - c(\frac{v_h-v_l-\xi e_o}{\xi}))(1-\beta_c)m < -(v_l - c(\bar{e}))m + (v_l - c(\frac{v_h-v_l-\xi e_o}{\xi}))(1-\beta_c)m < 0$
* If $v_h - \xi e_{\xi,1-\alpha+\alpha\beta_c} - \xi e_o \geq v_l$, then $\frac{\partial(\pi^{(b1)}-\pi^{(b5)})}{\partial\alpha} = -(v_h - c(\bar{e}))\beta_u m + (v_h - \xi e_o - \xi e_{\xi,1-\alpha+\alpha\beta_c} - c(e_{\xi,1-\alpha+\alpha\beta_c}))(1-\beta_c)m < -(v_l - c(\bar{e}))m + (v_l - c(e_{\xi,1-\alpha+\alpha\beta_c}))(1-\beta_c)m < 0$.
* $\frac{\partial(\pi^{(b1)}-\pi^{(b6)})}{\partial\alpha} = -(v_h - c(\bar{e}))\beta_u m < 0$

Also, note that:

* If $v_h - \xi e_{\xi,(1-\alpha)\beta_u+\alpha\beta_c} - \xi e_o \geq v_l$, then $\lim_{\alpha\to 0}(\pi^{(b1)} - \pi^{(b2)}) = (v_h - c(\bar{e}))\beta_u m - [(v_h - \xi e_o - \xi e_{\xi,\beta_u} - c(e_{\xi,\beta_u}))\beta_u m - f(e_{\xi,\beta_u})] > 0$
* $\lim_{\alpha\to 0}(\pi^{(b1)} - \pi^{(b4)}) = (v_h - c(\bar{e}))\beta_u m - [(v_l - c(\frac{v_h-v_l-\xi e_o}{\xi}))m - f(\frac{v_h-v_l-\xi e_o}{\xi})] > (v_l - c(\bar{e}))m - [(v_l - c(\frac{v_h-v_l-\xi e_o}{\xi}))m - f(\frac{v_h-v_l-\xi e_o}{\xi})] > 0$
* If $v_h - \xi e_{\xi,1-\alpha+\alpha\beta_c} - \xi e_o \geq v_l$, then $\lim_{\alpha\to 0}(\pi^{(b1)} - \pi^{(b5)}) = (v_h - c(\bar{e}))\beta_u m - [(v_h - \xi e_o - \xi e_{\xi,1} - c(e_{\xi,1}))m - f(e_{\xi,1})] > (v_l - c(\bar{e}))m - [(v_l - c(e_{\xi,1}))m - f(e_{\xi,1})] > 0$
* $\lim_{\alpha\to 0}(\pi^{(b1)} - \pi^{(b6)}) = (v_h - c(\bar{e}))\beta_u m - [(v_l - \xi e_o - \xi e_{\xi,1} - c(e_{\xi,1}))m - f(e_{\xi,1})] > (v_l - c(\bar{e}))m - (v_l - \xi e_o - \xi e_{\xi,1} - c(e_{\xi,1}))m + f(e_{\xi,1}) > 0$

Thus, there exists $A > 0$ such that if $\alpha \in (0, A)$, then $\pi^{(b1)} > \max\{\pi^{(b2)}, \pi^{(b4)}, \pi^{(b5)}, \pi^{(b6)}\}$ and thus $e^b = e^{(b1)} = \bar{e}$.

- If $(v_h - c(\bar{e}))\beta_u \leq (v_l - c(\bar{e}))$, then we just need to compare cases (b2), (b3), (b4), (b5) and (b6), where $e^{(b2)} = e_{\xi,(1-\alpha)\beta_u+\alpha\beta_c}$, $e^{(b3)} = \bar{e}$, $e^{(b4)} = \frac{v_-\xi e_o - v_l}{\xi}$, $e^{(a5)} = e_{\xi,1-\alpha+\alpha\beta_c}$ and $e^{(a6)} = e_{\xi,1}$. By Envelope Theorem, we have the following results:

* If $v_h - \xi e_{\xi,(1-\alpha)\beta_u+\alpha\beta_c} - \xi e_o \geq v_l$, then $\frac{\partial(\pi^{(b3)}-\pi^{(b2)})}{\partial\alpha} = -(v_l - c(\bar{e}))m + (v_h - \xi e_o - \xi e_{\xi,(1-\alpha)\beta_u+\alpha\beta_c} - c(e_{\xi,(1-\alpha)\beta_u+\alpha\beta_c}))(\beta_u - \beta_c)m < -(v_h - c(\bar{e}))\beta_u m + (v_h - \xi e_o - \xi e_{\xi,(1-\alpha)\beta_u+\alpha\beta_c} - c(e_{\xi,(1-\alpha)\beta_u+\alpha\beta_c}))(\beta_u - \beta_c)m < 0$.
* $\frac{\partial(\pi^{(b3)}-\pi^{(b4)})}{\partial\alpha} = -(v_l - c(\bar{e}))m + (v_l - c(\frac{v_h-v_l-\xi e_o}{\xi}))(1-\beta_c)m < 0$
* If $v_h - \xi e_{\xi,1-\alpha+\alpha\beta_c} - \xi e_o \geq v_l$, then $\frac{\partial(\pi^{(b3)}-\pi^{(b5)})}{\partial\alpha} = -(v_l - c(\bar{e}))m + (v_h - \xi e_o - \xi e_{\xi,1-\alpha+\alpha\beta_c} - c(e_{\xi,1-\alpha+\alpha\beta_c}))(1-\beta_c)m < -(v_l - c(\bar{e}))m + (v_l - c(e_{\xi,1-\alpha+\alpha\beta_c}))(1-\beta_c)m < 0$.
* $\frac{\partial(\pi^{(b3)}-\pi^{(b6)})}{\partial\alpha} = -(v_l - c(\bar{e}))m < 0$

Also, note that:

* If $v_h - \xi e_{\xi,(1-\alpha)\beta_u+\alpha\beta_c} - \xi e_o \geq v_l$, then $\lim_{\alpha\to 0}(\pi^{(b3)} - \pi^{(b2)}) = (v_l - c(\bar{e}))m - [(v_h - \xi e_o - \xi e_{\xi,\beta_u} - c(e_{\xi,\beta_u}))\beta_u m - f(e_{\xi,\beta_u})] > (v_h - c(\bar{e}))\beta_u m - [(v_h - \xi e_o - \xi e_{\xi,\beta_u} - c(e_{\xi,\beta_u}))\beta_u m - f(e_{\xi,\beta_u})] > 0$
* $\lim_{\alpha\to 0}(\pi^{(b3)} - \pi^{(b4)}) = (v_l - c(\bar{e}))m - [(v_l - c(\frac{v_h-v_l-\xi e_o}{\xi}))m - f(\frac{v_h-v_l-\xi e_o}{\xi})] > 0$
* If $v_h - \xi e_{\xi,1-\alpha+\alpha\beta_c} - \xi e_o \geq v_l$, then $\lim_{\alpha\to 0}(\pi^{(b3)} - \pi^{(b5)}) = (v_l - c(\bar{e}))m - [(v_h - \xi e_o - \xi e_{\xi,1} - c(e_{\xi,1}))m - f(e_{\xi,1})] > (v_l - c(\bar{e}))m - [(v_l - c(e_{\xi,1}))m - f(e_{\xi,1})] > 0$
* $\lim_{\alpha\to 0}(\pi^{(b3)} - \pi^{(b6)}) = (v_l - c(\bar{e}))m - [(v_l - \xi e_o - \xi e_{\xi,1} - c(e_{\xi,1}))m - f(e_{\xi,1})] > 0$

Thus, there exists $A > 0$ such that if $\alpha \in (0, A)$, then $\pi^{(b3)} > \max\{\pi^{(b2)}, \pi^{(b4)}, \pi^{(b5)}, \pi^{(b6)}\}$ and thus $e^b = e^{(b3)} = \bar{e}$.

If $v_l < v_h - \xi\bar{e} - \xi e_o$, then we have $v_l < v_h - \xi e_{\xi,(1-\alpha)\beta_u+\alpha\beta_c} - \xi e_o$ and $v_l < v_h - \xi e_{\xi,1-\alpha+\alpha\beta_c} - \xi e_o$.

Thus, we just need to compare cases (b1), (b2), (b4), and (b6), where $e^{(b1)} = \bar{e}$, $e^{(b2)} = e_{\xi,(1-\alpha)\beta_u+\alpha\beta_c}$, $e^{(b4)} = \bar{e}$, and $e^{(a6)} = e_{\xi,1}$. By Envelope Theorem, we have $\frac{\partial(\pi^{(b1)}-\pi^{(b2)})}{\partial\alpha} = -(v_h - c(\bar{e}))\beta_u m + (v_h - \xi e_o - \xi e_{\xi,(1-\alpha)\beta_u+\alpha\beta_c} - c(e_{\xi,(1-\alpha)\beta_u+\alpha\beta_c}))(\beta_u - \beta_c)m < 0$ and $\frac{\partial(\pi^{(b4)}-\pi^{(b5)})}{\partial\alpha} = -(v_l - c(\bar{e}))(1-\beta_c)m < 0$. Also, note that $\lim_{\alpha\to 0}(\pi^{(b1)} - \pi^{(b2)}) = (v_h - c(\bar{e}))\beta_u m - [(v_h - \xi e_o - \xi e_{\xi,\beta_u} - c(e_{\xi,\beta_u}))\beta_u m - f(e_{\xi,\beta_u})] > 0$ and $\lim_{\alpha\to 0}(\pi^{(b4)} - \pi^{(b5)}) = (v_l - c(\bar{e}))m - [(v_l - \xi e_o - \xi e_{\xi,1} - c(e_{\xi,1}))m - f(e_{\xi,1})] > 0$. Thus, there exists $A > 0$ such that if $\alpha \in (0, A)$, then $\max(\pi^{(b1)}, \pi^{(b4)} > \max\{\pi^{(b2)}, \pi^{(b5)}\}$ and thus e^b is either $e^{(b1)}$ or $e^{(b4)}$ (i.e., $e^b = \bar{e}$). □

With offsets, if $c_g \le G$, by Lemma 2, we have $t^*(g^*) > 0$. According to the analysis of the subgame in Online Appendix I1, if $t > 0$, we have $e \in \{e_{g,(1-\alpha)\beta_u+\alpha\beta_c}, e_{g,1-\alpha+\alpha\beta_c}, e_{g,1}\} \le \bar{e}$ and the inequality is strict if $\bar{e}$ is large enough. Thus, we must have $e^*(g^*) \in \{e_{g^*,(1-\alpha)\beta_u+\alpha\beta_c}, e_{g^*,1-\alpha+\alpha\beta_c}, e_{g^*,1}\} \le \bar{e}$ and the inequality is strict if $\bar{e}$ is large enough.

Therefore, if $c_g \le G$ and $\alpha \in (0, A)$, we have $e^*(g^*) \in \{e_{g^*,(1-\alpha)\beta_u+\alpha\beta_c}, e_{g^*,1-\alpha+\alpha\beta_c}, e_{g^*,1}\} \le \bar{e} = e^b$, where the inequality is strict if $\bar{e}$ is large enough. □

Proof of Proposition 4: Let us first look at Case N. From the proof of Proposition 1(i), we have the following result:

- if $g > G$, then $\theta^*(g) = 0$;
- if $g \le G$, then $\theta^*(g) = 1$ and thus $\mathcal{E}^*(g) = 0$.

Thus, given that $c_g \le G$, it is easy to find that $g^* = c_g$, under which we have $\theta^*(g^*) = 1$ and thus $\mathcal{E}^*(g^*) = 0$.

Next, let us look at Case P. If $v_h - v_l < \xi e_o$ and $\beta_u \in (\tilde{B}_1, \tilde{B}_2)$, according to the proof of Proposition 1(ii), there exist $g_1, g_2 \in (0, G)$, such that $g_1 < g_2 < G$ and $\theta^*(g_1) = 1 - \frac{v_h - v_l}{\xi(e_{g_1,1}+e_o)} < 1 = \theta^*(g_2)$, and thus $\mathcal{E}^*(g_1) > 0 = \mathcal{E}^*(g_2)$. Therefore, if $c_g = g_1$, then we can conclude the optimal offset price $g^* = \arg\min_{g\ge c_g} \mathcal{E}^*(g) > c_g$. □

Proof of Footnote 16: First note that it is certainly not profitable for the firm to produce more than the targeted demand $d(p, e, t)$. Second, if the firm produces less than the demand $d(p, e, t)$, which is deterministic and independent of the firm's production quantity, the firm can be better off by increasing its production quantity and sells more of its products in the market. Therefore, in equilibrium, we must have production quantity equal to the demand $d(p, e, t)$. □

Proof of Proposition II1: Let us first prove Proposition II1(i). According to the subgame analysis in Online Appendix I1, the firm will choose the best among Options 1,2,3,4,6. If $\beta_u > \frac{v_l - c(\bar{e})}{v_h - c(\bar{e})}$, then $\pi^{(1)} > \pi^{(3)}$ for any $g \in (0, \xi)$ and $\beta_c \in [0, 1]$. Also, note that $\lim_{\beta_u\to 1}(\pi^{(2)} - \pi^{(4)}) > 0$, and thus there exists $b_u < 1$ such that $\pi^{(2)} - \pi^{(4)} > 0$ for any $g \in (0, \xi)$ and $\beta_c \in [0, 1]$. Thus, if $\beta_u > \mathfrak{B}_u \triangleq \max(\frac{v_l - c(\bar{e})}{v_h - c(\bar{e})}, b_u)$, then we just need to compare Option 1,2,6:

- Option 1: $p = v_h$, $e = \bar{e}$, $t = 0$, $d_c = 0$, $d_u = (1-\alpha)\beta_u m$;
- Option 2: $p = v_h$, $e = e_{g,(1-\alpha)\beta_u+\alpha\beta_c}$, $t = e_{g,(1-\alpha)\beta_u+\alpha\beta_c} + e_o$, $d_c = \alpha\beta_c m$, $d_u = (1-\alpha)\beta_u m$;
- Option 6: $p = v_l$, $e = e_{g,1}$, $t = e_{g,1} + e_o$, $d_c = \alpha m$, $d_u = (1-\alpha)m$.

Define function $h(g) \triangleq \pi(v_l, e_{g,1}, e_{g,1} + e_o|g) - \pi(v_h, e_{g,(1-\alpha)\beta_u+\alpha\beta_c}, e_{g,(1-\alpha)\beta_u+\alpha\beta_c} + e_o|g) = (v_l - c(e_{g,1}) - ge_{g,1} - ge_o)m - f(e_{g,1}) - [(v_h - c(e_{g,(1-\alpha)\beta_u+\alpha\beta_c}) - ge_{g,(1-\alpha)\beta_u+\alpha\beta_c} - ge_o)((1-\alpha)\beta_u + \alpha\beta_c)m - f(e_{g,(1-\alpha)\beta_u+\alpha\beta_c})]$. Note that $\lim_{g\to+\infty} h'(g) = -e_o(1-(1-\alpha)\beta_u - \alpha\beta_c)m < 0$. Then, if $h(0) > 0$ (i.e., $\beta_c < \mathfrak{B}_2 \triangleq \frac{\frac{v_l - c(\bar{e})}{v_h - c(\bar{e})} - (1-\alpha)\beta_u}{\alpha}$), then there exists $X = \min\{g|h(g) = 0\}$ and $X > 0$. Then, if the following condition hold:

(1) $\pi(v_l, e_{X,1}, e_{X,1}+e_o|X) > \pi(v_h, \bar{e}, 0|X)$, i.e., $(v_l - c(e_{X,1}) - Xe_{X,1} - Xe_o)m - f(e_{X,1}) > (v_h - c(\bar{e}))(1-\alpha)\beta_u m$,

then for a small positive number $\epsilon > 0$, we must have $X < G$ and

- if $g \in (0, X)$, then $p^* = v_l$, $e^* = e_{g,1}$ and $t^* = e_{g,1} + e_o$;
- if $g \in (X, X+\epsilon)$, then $p^* = v_h$, $e^* = e_{g,(1-\alpha)\beta_u+\alpha\beta_c}$ and $t^* = e_{g,(1-\alpha)\beta_u+\alpha\beta_c} + e_o$.

Therefore, there exist g_1, g_2 and $\tilde{\epsilon} \in (0, \epsilon)$ such that $X - \tilde{\epsilon} < g_1 < X < g_2 < X + \tilde{\epsilon}$ and $e^*(g_1) = e_{g_1,1} \leq e_{g_2,(1-\alpha)\beta_u+\alpha\beta_c} = e^*(g_2)$, where the inequality is due to Lemma VI1 and is strict if $\bar{e}$ is large enough; also, in this case, we have $d_c^*(g_1) > d_c^*(g_2)$ and $d_u^*(g_1) = d_u^*(g_2)$. As for condition (1), note that $\frac{\partial h}{\partial \beta_c} < 0$ and $\frac{\partial h}{\partial g}|_{g=X} < 0$, and thus we have $\frac{\partial X}{\partial \beta_c} < 0$. Since $\frac{\partial(v_l - c(e_{X,1}) - Xe_{X,1} - Xe_o)m - f(e_{X,1}))}{\partial X} < 0$ (due to Envelope Theorem), condition (1) is equivalent to β_c being large (i.e., $\beta_c > \mathfrak{B}_1$).

Next, let us prove Proposition II1(ii). When $\beta_c = 1$, then if there exist $\tilde{g}_1 < \tilde{g}_2$ such that $e^*(\tilde{g}_1) < e^*(\tilde{g}_2)$, then according to the proof of Proposition 1(ii), we have the following results:

- If $g = \tilde{g}_1$, then the firm chooses Option 4 with $p^{(4)} = v_l$, $e^{(4)} = e_{\tilde{g}_1,1}$, $t^{(4)} = e_{\tilde{g}_1,1} + e_o - \frac{v_h - v_l}{\xi}$, and $\pi^{(4)}(\tilde{g}_1) > \max(\pi^{(1)}(\tilde{g}_1), \pi^{(2)}(\tilde{g}_1), \pi^{(3)}(\tilde{g}_1), \pi^{(6)}(\tilde{g}_1))$.
- If $g = \tilde{g}_2$, then the firm chooses Option 2 with $p^{(2)} = v_h$, $e^{(2)} = e_{\tilde{g}_2,(1-\alpha)\beta_u+\alpha}$, $t^{(2)} = e_{\tilde{g}_2,(1-\alpha)\beta_u+\alpha} + e_o$, and $\pi^{(2)}(\tilde{g}_2) > \max(\pi^{(1)}(\tilde{g}_2), \pi^{(3)}(\tilde{g}_2), \pi^{(4)}(\tilde{g}_2), \pi^{(6)}(\tilde{g}_2))$.

Moreover, $e_{\tilde{g}_1,1} < e_{\tilde{g}_2,(1-\alpha)\beta_u+\alpha}$.

Since $\pi^{(i)}$ $(i = 1,2,3,4,6)$ is continuous in β_c, we should continue to have $\pi^{(2)}(\tilde{g}_1) > \max(\pi^{(1)}(\tilde{g}_1), \pi^{(3)}(\tilde{g}_1), \pi^{(4)}(\tilde{g}_1), \pi^{(6)}(\tilde{g}_1))$ and $\pi^{(4)}(\tilde{g}_2) > \max(\pi^{(1)}(\tilde{g}_2), \pi^{(2)}(\tilde{g}_2), \pi^{(3)}(\tilde{g}_2), \pi^{(6)}(\tilde{g}_2))$ when β_c is smaller than one but large enough (i.e., $\beta_c > b_c$ for some $b_c < 1$). As a result,

- If $g = \tilde{g}_1$, then the firm chooses Option 4 with $p^{(4)} = v_l$, $e^{(4)} = e_{\tilde{g}_1,1-\alpha+\alpha\beta_c}$, $t^{(4)} = e_{\tilde{g}_1,1-\alpha+\alpha\beta_c} + e_o - \frac{v_h - v_l}{\xi}$, and $\pi^{(4)}(\tilde{g}_1) > \max(\pi^{(1)}(\tilde{g}_1), \pi^{(2)}(\tilde{g}_1), \pi^{(3)}(\tilde{g}_1), \pi^{(6)}(\tilde{g}_1))$.
- If $g = \tilde{g}_2$, then the firm chooses Option 2 with $p^{(2)} = v_h$, $e^{(2)} = e_{\tilde{g}_2,(1-\alpha)\beta_u+\alpha\beta_c}$, $t^{(2)} = e_{\tilde{g}_2,(1-\alpha)\beta_u+\alpha\beta_c} + e_o$, and $\pi^{(2)}(\tilde{g}_2) > \max(\pi^{(1)}(\tilde{g}_2), \pi^{(3)}(\tilde{g}_2), \pi^{(4)}(\tilde{g}_2), \pi^{(6)}(\tilde{g}_2))$.

Thus, $e^*(\tilde{g}_1) = e^{(4)} = e_{\tilde{g}_1,1-\alpha+\alpha\beta_c}$ and $e^*(\tilde{g}_2) = e^{(2)} = e_{\tilde{g}_2,(1-\alpha)\beta_u+\alpha\beta_c}$. Since $e_{\tilde{g}_1,1} < e_{\tilde{g}_2,(1-\alpha)\beta_u+\alpha}$ and both $e_{\tilde{g}_1,1-\alpha+\alpha\beta_c}$ and $e_{\tilde{g}_2,(1-\alpha)\beta_u+\alpha\beta_c}$ are continuous in β_c. Thus, there exists $\hat{b}_c < 1$ such that

$e_{\tilde{g}_1,1-\alpha+\alpha\beta_c} < e_{\tilde{g}_2,(1-\alpha)\beta_u+\alpha\beta_c}$ if $\beta_c > \hat{b}_c$. Define $\mathfrak{B}_c = \max(b_c, \hat{b}_c)$ and we can conclude that $e^*(\tilde{g}_1) < e^*(\tilde{g}_2)$ if $\beta_c > \mathfrak{B}_c$. □

Proof of Proposition II2: Let us first prove Proposition II2(i). According to the proof of Proposition II1(i), if $\beta_u > \mathfrak{B}_u$ (where $\mathfrak{B}_u$ is defined in Proposition II1(i)), then we just need to compare Option 1,2,6:

- Option 1: $p = v_h$, $e = \bar{e}$, $t = 0$, $d_c = 0$, $d_u = (1-\alpha)\beta_u m$;
- Option 2: $p = v_h$, $e = e_{g,(1-\alpha)\beta_u+\alpha\beta_c}$, $t = e_{g,(1-\alpha)\beta_u+\alpha\beta_c} + e_o$, $d_c = \alpha\beta_c m$, $d_u = (1-\alpha)\beta_u m$;
- Option 6: $p = v_l$, $e = e_{g,1}$, $t = e_{g,1} + e_o$, $d_c = \alpha m$, $d_u = (1-\alpha)m$.

Thus, if $t > 0$, we have $\theta = 1$ in equilibrium. Given Lemma 1, we can conclude that

- if $g > G$, then $\theta^* = 0$;
- if $g \le G$, then $\theta^* = 1$.

Thus, we can conclude Proposition II2(i).

Next, let us prove Proposition II2(ii). When $\beta_c = 1$, then if there exist $\tilde{g}_1 < \tilde{g}_2$ such that $\theta^*(\tilde{g}_1) < \theta^*(\tilde{g}_2)$, then according to the proof of Proposition 2(ii), we have the following results:

- If $g = \tilde{g}_1$, then the firm chooses Option 4 with $p^{(4)} = v_l$, $e^{(4)} = e_{\tilde{g}_1,1}$, $t^{(4)} = e_{\tilde{g}_1,1} + e_o - \frac{v_h - v_l}{\xi}$, and $\pi^{(4)}(\tilde{g}_1) > \max(\pi^{(1)}(\tilde{g}_1), \pi^{(2)}(\tilde{g}_1), \pi^{(3)}(\tilde{g}_1), \pi^{(6)}(\tilde{g}_1))$.
- If $g = \tilde{g}_2$, then the firm chooses Option 2 with $p^{(2)} = v_h$, $e^{(2)} = e_{\tilde{g}_2,(1-\alpha)\beta_u+\alpha}$, $t^{(2)} = e_{\tilde{g}_2,(1-\alpha)\beta_u+\alpha} + e_o$, and $\pi^{(2)}(\tilde{g}_2) > \max(\pi^{(1)}(\tilde{g}_2), \pi^{(3)}(\tilde{g}_2), \pi^{(4)}(\tilde{g}_2), \pi^{(6)}(\tilde{g}_2))$.

Since $\pi^{(i)}$ $(i = 1, 2, 3, 4, 6)$ is continuous in β_c, we should continue to have $\pi^{(2)}(\tilde{g}_1) > \max(\pi^{(1)}(\tilde{g}_1), \pi^{(3)}(\tilde{g}_1), \pi^{(4)}(\tilde{g}_1), \pi^{(6)}(\tilde{g}_1))$ and $\pi^{(4)}(\tilde{g}_2) > \max(\pi^{(1)}(\tilde{g}_2), \pi^{(2)}(\tilde{g}_2), \pi^{(3)}(\tilde{g}_2), \pi^{(6)}(\tilde{g}_2))$ when β_c is smaller than one but large enough (i.e., $\beta_c > \mathfrak{B}_c$ for some $\mathfrak{B}_c < 1$). As a result,

- If $g = \tilde{g}_1$, then the firm chooses Option 4 with $p^{(4)} = v_l$, $e^{(4)} = e_{\tilde{g}_1,1-\alpha+\alpha\beta_c}$, $t^{(4)} = e_{\tilde{g}_1,1-\alpha+\alpha\beta_c} + e_o - \frac{v_h - v_l}{\xi}$, and $\pi^{(4)}(\tilde{g}_1) > \max(\pi^{(1)}(\tilde{g}_1), \pi^{(2)}(\tilde{g}_1), \pi^{(3)}(\tilde{g}_1), \pi^{(6)}(\tilde{g}_1))$.
- If $g = \tilde{g}_2$, then the firm chooses Option 2 with $p^{(2)} = v_h$, $e^{(2)} = e_{\tilde{g}_2,(1-\alpha)\beta_u+\alpha\beta_c}$, $t^{(2)} = e_{\tilde{g}_2,(1-\alpha)\beta_u+\alpha\beta_c} + e_o$, and $\pi^{(2)}(\tilde{g}_2) > \max(\pi^{(1)}(\tilde{g}_2), \pi^{(3)}(\tilde{g}_2), \pi^{(4)}(\tilde{g}_2), \pi^{(6)}(\tilde{g}_2))$.

Thus, $\theta^*(\tilde{g}_1) = \frac{t^{(4)}}{e^{(4)}+e_o} = 1 - \frac{v_h - v_l}{\xi(e_{\tilde{g}_1,1-\alpha+\alpha\beta_c}+e_o)}$ and $\theta^*(\tilde{g}_2) = \frac{t^{(2)}}{e^{(2)}+e_o} = 1$. Therefore, $\theta^*(\tilde{g}_1) < \theta^*(\tilde{g}_2)$. □

Proof of Proposition II3: Let us first prove Proposition II3(i). From the proof of Proposition II1(i), if $\beta_u > \mathfrak{B}_u$ (where $\mathfrak{B}_u$ is defined in Proposition II1(i)), we have the following result:

- if $g > G$, then $\theta^*(g) = 0$;
- if $g \le G$, then $\theta^*(g) = 1$ and thus $\mathcal{E}^*(g) = 0$.

Thus, given that $c_g \le G$, it is easy to find that $g^* = c_g$, under which we have $\theta^*(g^*) = 1$ and thus $\mathcal{E}^*(g^*) = 0$.

Next, let us prove Proposition II3(ii). According to the proof of Proposition 4(ii), when $\beta_c = 1$, if there exists $c_g \in (0, G)$ such that $g^* > c_g$, then $\theta^*(c_g) < 1 = \theta^*(g^*)$. Then, by Proposition II2(ii) and its proof, if $\beta_c > \mathfrak{B}_c$ (where $\mathfrak{B}_c$ is defined in Proposition II2(ii)), then $\theta^*(c_g) < 1 = \theta^*(g^*)$, and thus $\mathcal{E}^*(c_g) > 0 = \mathcal{E}^*(g^*)$. Thus, we can conclude the result. □

Proof of Lemma I1.1: The firm's profit function is $\pi(p, e, t) = (p - c(e) - gt)d(p, e, t) - f(e)$. Suppose $g > \xi$ and the optimal solution is (p^*, e^*, t^*) where $t^* > 0$. Consider another option $t^\triangle = 0$, $e^\triangle = e^*$ and $p^\triangle = p^* - \xi t^*$. Then, based on consumer utility functions, we can find that $d(p^*, e^*, t^*) \leq d(p^\triangle, e^\triangle, t^\triangle)$. Therefore, we have

$$\begin{aligned}\pi(p^*, e^*, t^*) &= (p^* - c(e^*) - gt^*)d(p^*, e^*, t^*) - f(e^*)\\ &< (p^* - c(e^*) - \xi t^*)d(p^*, e^*, t^*) - f(e^*)\\ &\leq (p^* - c(e^*) - \xi t^*)d(p^\triangle, e^\triangle, t^\triangle) - f(e^*)\\ &= (p^\triangle - c(e^\triangle) - gt^\triangle)d(p^\triangle, e^\triangle, t^\triangle) - f(e^\triangle)\\ &= \pi(p^\triangle, e^\triangle, t^\triangle)\end{aligned}$$

where the first inequality is due to $\xi < g$ and $t^* > 0$. This is contradictory to the fact that (p^*, e^*, t^*) is the optimal solution. Thus, if $g > \xi$, we must have $t^* = 0$. □

Proof of Proposition III1: Denote $\pi(p, e, t|g)$ as the profit given firm's decisions (p, e, t) and offset price g. Given g, if $v_h < v_l + \xi e_o$, then $v_l > v_h - \xi(\bar{e} + e_o)$ and $\max(e_{g,1-\alpha+\alpha\beta_c}, \frac{v_h - v_l - \xi e_o}{\xi}) = e_{g,1-\alpha+\alpha\beta_c}$ for any $g \in (0, \xi)$, and thus according to the analysis of the subgame in Online Appendix I1, the equilibrium in the subgame must be one of the following:

1. $p = v_h$, $e = e_{g,\alpha\beta_c}$, $t = e_{g,\alpha\beta_c} + e_o$, $d_c = \alpha\beta_c m$, $d_u = 0$;
2. $p = v_l$, $e = \bar{e}$, $t = 0$, $d_c = 0$, $d_u = (1 - \alpha)m$;
3. $p = v_l$, $e = e_{g,1-\alpha+\alpha\beta_c}$, $t = e_{g,1-\alpha+\alpha\beta_c} + e_o - \frac{v_h - v_l}{\xi}$, $d_c = \alpha\beta_c m$, $d_u = (1 - \alpha)m$;
4. $p = v_l$, $e = e_{g,1}$, $t = e_{g,1} + e_o$, $d_c = \alpha m$, $d_u = (1 - \alpha)m$.

Define function $h(g) \triangleq \pi(v_l, e_{g,1-\alpha+\alpha\beta_c}, \frac{v_l - v_h + \xi e_{g,1-\alpha+\alpha\beta_c} + \xi e_o}{\xi}|g) - \pi(v_h, e_{g,\alpha\beta_c}, e_{g,\alpha\beta_c} + e_o|g) = (v_l - c(e_{g,1-\alpha+\alpha\beta_c}) - g\frac{v_l - v_h}{\xi} - ge_{g,1-\alpha+\alpha\beta_c} - ge_o)(1 - \alpha + \alpha\beta_c)m - f(e_{g,1-\alpha+\alpha\beta_c}) - [(v_h - c(e_{g,\alpha\beta_c}) - ge_{g,\alpha\beta_c} - ge_o)\alpha\beta_c m - f(e_{g,\alpha\beta_c})]$. Note that $h(0) = (v_l - c(\bar{e}))(1 - \alpha + \alpha\beta_c)m - (v_h - c(\bar{e}))\alpha\beta_c m$, which is positive if and only if $\beta_c < \check{B}_2 \triangleq \frac{(v_l - c(\bar{e}))(1-\alpha)}{(v_h - v_l)\alpha}$. Also, note that $\lim_{g\to+\infty} h'(g) = \frac{v_h - v_l}{\xi}(1 - \alpha + \alpha\beta_c)m - e_o(1 - \alpha)m$, which is negative if and only if $\beta_c < \check{\check{B}}_2 \triangleq \frac{(-v_h + v_l + \xi e_o)(1-\alpha)}{(v_h - v_l)\alpha}$. If $h(0) > 0$ and $\lim_{g\to+\infty} h'(g) < 0$ (i.e., $\beta_c < \mathcal{B}_2 \triangleq \min(\check{B}_2, \check{\check{B}}_2)$), then there exists $X = \min\{g|h(g) = 0\}$ and $X > 0$. Then, if the following conditions hold:

(1) $\pi(v_l, e_{X,1-\alpha+\alpha\beta_c}, e_{X,1-\alpha+\alpha\beta_c} + e_o|X) > \pi(v_l, \bar{e}, 0|X)$, i.e., $(v_l - c(e_{X,1-\alpha+\alpha\beta_c}) - X\frac{v_l - v_h}{\xi} - Xe_{X,1-\alpha+\alpha\beta_c} - Xe_o)(1 - \alpha + \alpha\beta_c)m - f(e_{X,1-\alpha+\alpha\beta_c}) > (v_l - c(\bar{e}))(1 - \alpha)m$;

(2) $\pi(v_l, e_{X,1-\alpha+\alpha\beta_c}, e_{X,1-\alpha+\alpha\beta_c} + e_o|X) > \pi(v_l, e_{g,1}, e_{g,1} + e_o|X)$, i.e., $(v_l - c(e_{X,1-\alpha+\alpha\beta_c}) - X\frac{v_l - v_h}{\xi} - Xe_{X,1-\alpha+\alpha\beta_c} - Xe_o)(1 - \alpha + \alpha\beta_c)m - f(e_{X,1-\alpha+\alpha\beta_c}) > (v_l - c(e_{X,1}) - Xe_{X,1} - Xe_o)m - f(e_{X,1})$,

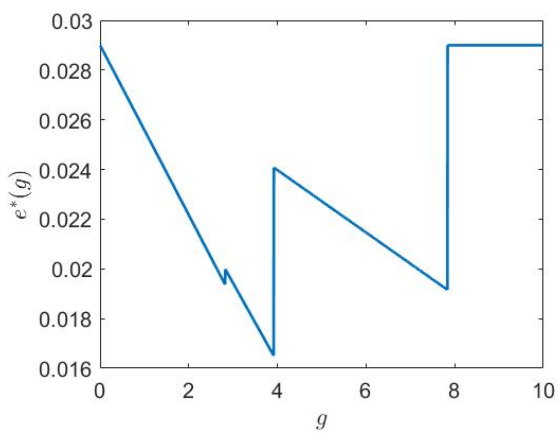
then for a small positive number $\epsilon > 0$, we must have

- if $g \in (X-\epsilon, X)$, then $p^* = v_l$, $e^* = e_{g,1-\alpha+\alpha\beta_c}$, $t^* = e_{g,1-\alpha+\alpha\beta_c} + e_o - \frac{v_h - v_l}{\xi}$,
- if $g \in (X, X+\epsilon)$, then $p^* = v_h$, $e^* = e_{g,\alpha\beta_c}$, $t^* = e_{g,\alpha\beta_c} + e_o$.

Therefore, there exist g_1, g_2 and $\tilde{\epsilon} \in (0, \epsilon)$ such that $X - \tilde{\epsilon} < g_1 < X < g_2 < X + \tilde{\epsilon}$ and $e^*(g_1) = e_{g_1,1-\alpha+\alpha\beta_c} \leq e_{g_2,\alpha\beta_c} = e^*(g_2)$, where the inequality is due to Lemma VI1 and is strict if $\bar{e}$ is large enough; also, in this case, we have $d_c^*(g_1) = \alpha\beta_c m = d_c^*(g_2) < 1$ and $d_u^*(g_1) = (1-\alpha)m > 0 = d_u^*(g_2)$. As for condition (1), note that $\pi(v_l, e_{0,1-\alpha+\alpha\beta_c}, e_{0,1-\alpha+\alpha\beta_c} + e_o|0) - \pi(v_l, \bar{e}, 0|0) = (v_l - c(\bar{e}))\alpha\beta_c m > 0$ and $\lim_{\beta_c \to \check{B}_2} X = 0$ and $\lim_{\beta_c \to \check{B}_2} \frac{\partial h}{\partial \beta_c} = -(v_h - v_l) < 0$ and thus there exists $\check{B}_1 < \check{B}_2$ such that $(\pi(v_l, e_{X,1-\alpha+\alpha\beta_c}, e_{X,1-\alpha+\alpha\beta_c} + e_o|X) - \pi(v_l, \bar{e}, 0|X)) > 0$ (i.e., condition (1) holds) if $\beta_c \in (\check{B}_1, \check{B}_2)$. As for condition (2), note that if $\beta_c = 1$, then $(v_l - c(e_{X,1-\alpha+\alpha\beta_c}) - X\frac{v_l - v_h}{\xi} - Xe_{X,1-\alpha+\alpha\beta_c} - Xe_o)(1-\alpha+\alpha\beta_c)m - f(e_{X,1-\alpha+\alpha\beta_c}) > (v_l - c(e_{X,1}) - Xe_{X,1} - Xe_o)m - f(e_{X,1})$ for any $0 < X < \xi$. Thus, by the continuity of the profit functions, we can conclude that $\exists \check{\check{B}}_1 < 1$ such that condition (2) holds if $\beta_c > \check{\check{B}}_1$. Finally, we define $\mathcal{B}_1 = \max(\check{B}_1, \check{\check{B}}_1)$ and we can conclude the result.

Figure VI4 depicts a numerical example for Proposition III1.

Figure VI4: A Numerical Example for Proposition III1



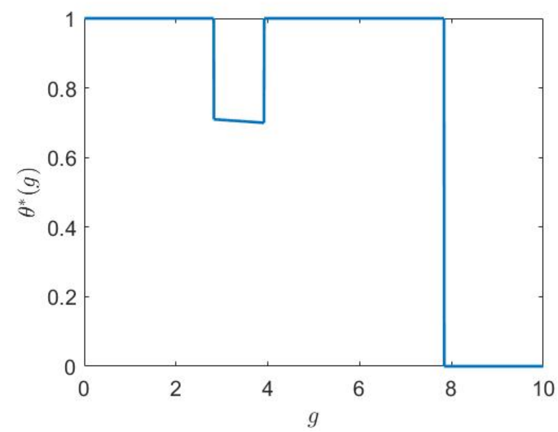
Note: In this example, $m = 19$, $c(e) = \frac{1}{2}k_c(\bar{e} - e)^2 + \bar{c}$, $f(e) = \frac{1}{2}k_f(\bar{e} - e)^2$, $\bar{c} = 1$, $k_c = 56$, $k_f = 4256$, $\bar{e} = 0.029$, $e_o = 0.086$, $\xi = 49$, $\alpha = 0.4$, $\beta_c = 0.8$, $\beta_u = 0$, $v_l = 2.3$, $v_h = 4.3$.

$\square$

Proof of Proposition III2: From the proof of Proposition III1, if $\beta_c \in (\mathcal{B}_1, \mathcal{B}_2)$ and $v_h - v_l < \xi e_o$, then there exist g_1, g_2 such that $X - \epsilon < g_1 < X < g_2 < X + \epsilon$ and $\theta^*(g_1) = 1 - \frac{v_h - v_l}{\xi(e_{g,1-\alpha+\alpha\beta_c} + e_o)} < 1 = \theta^*(g_2)$.

Below, we provide a numerical example for Propositions III2 in Figure VI5.

Figure VI5: A Numerical Example for Propositions III2



Note: In this example, the parameter values are the same as in Figure VI4.

□

Proof of Proposition III3: Under the conditions in Proposition III1, according to the proof of Proposition III1, there exist $g_1, g_2 \in (0, G)$, such that $g_1 < g_2 < G$ and $\theta^*(g_1) = 1 - \frac{v_h - v_l}{\xi(e_{g_1,1-\alpha+\alpha\beta_c} + e_o)} < 1 = \theta^*(g_2)$, and thus $\mathcal{E}^*(g_1) > 0 = \mathcal{E}^*(g_2)$. Therefore, if $c_g = g_1$, then we can conclude the optimal offset price $g^* = \arg\min_{g \geq c_g} \mathcal{E}^*(g) > c_g$. □

Proof of Lemma III1: Given g, the firm's profit function is $\pi(p, e, t|g) = (p - c(e) - gt)d(p, e, t) - f(e)$. Suppose there exist $g_1 < g_2$ such that $t^*(g_1) = 0 < t^*(g_2)$. Then, $\pi(p^*(g_2), e^*(g_2), t^*(g_2)|g_2) > \pi(p^*(g_1), e^*(g_1), t^*(g_1)|g_2) = \pi(p^*(g_1), e^*(g_1), t^*(g_1)|g_1) > \pi(p^*(g_2), e^*(g_2), t^*(g_2)|g_1)$, where the equality is because $t^*(g_1) = 0$. However, this is contradictory to the fact that $\pi(p^*(g_2), e^*(g_2), t^*(g_2)|g_2) - \pi(p^*(g_2), e^*(g_2), t^*(g_2)|g_1) = (g_1 - g_2)t^*(g_2)d(p^*(g_2), e^*(g_2), t^*(g_2)) < 0$. Thus, we can conclude the result. □

Lemma VI3. *If* $e_{x,\alpha} = \max(\min(\tilde{e}_{x,\alpha}, \bar{e}), 0)$*, where* $\tilde{e}_{x,\alpha}$ *is the solution to the equation* $[x\alpha m + c'(\tilde{e}_{x,\alpha})]\alpha m + f'(\tilde{e}_{x,\alpha}) = 0$*, then* $e_{x,\alpha}$ *is weakly decreasing in both* x *and* α*.*

Proof of Lemma VI3: Note we have $\frac{\partial \tilde{e}_{x,\alpha}}{\partial \alpha} = \frac{-2x\alpha m^2 - c'(\tilde{e}_{x,\alpha})m}{\alpha m c''(\tilde{e}_{x.\alpha}) + f''(\tilde{e}_{x,\alpha})} = \frac{f'(\tilde{e}_{x,\alpha})/\alpha - x\alpha m^2}{\alpha m c''(\tilde{e}_{x.\alpha}) + f''(\tilde{e}_{x,\alpha})} < 0$ and $\frac{\partial \tilde{e}_{x,\alpha}}{\partial x} = \frac{-\alpha^2 m^2}{\alpha m c''(\tilde{e}_{x.\alpha}) + f''(\tilde{e}_{x,\alpha})} < 0$. Thus, we can conclude the result. □

Proof of Lemma V1.1: Given g, the firm's profit function is $\pi(p, e, t, q|g) = (p - c(e) - gt)\min(q, d(p, e, t, q)) - f(e)$. Suppose there exist $g_1 < g_2$ such that $t^*(g_1) = 0 < t^*(g_2)$. Then, $\pi(p^*(g_2), e^*(g_2), t^*(g_2), q^*(g_2)|g_2) > \pi(p^*(g_1), e^*(g_1), t^*(g_1), q^*(g_1)|g_2) = \pi(p^*(g_1), e^*(g_1), t^*(g_1), q^*(g_1)|g_1) > \pi(p^*(g_2), e^*(g_2), t^*(g_2), q^*(g_2)|g_1)$, where the equality is because $t^*(g_1) = 0$. However, this is contradictory to the fact that

$$\begin{aligned}
&\pi(p^*(g_2), e^*(g_2), t^*(g_2), q^*(g_2)|g_2) - \pi(p^*(g_2), e^*(g_2), t^*(g_2), q^*(g_2)|g_1) \\
&= (g_1 - g_2)t^*(g_2)\min(q^*(g_2), d(p^*(g_2), e^*(g_2), t^*(g_2), q^*(g_2))) \\
&< 0
\end{aligned}$$

Thus, we can conclude the result. □

Lemma VI4. *If $g > \xi m$, then it is not optimal for the firm to buy any offset, i.e., $t^*(g) = 0$.*

Proof of Lemma VI4: The firm's profit function is $\pi(p, e, t, q) = (p - c(e) - gt)\min(q, d(p, e, t, q)) - f(e)$. Suppose $g > \xi m$ and the optimal solution is (p^*, e^*, t^*, q^*) where $t^* > 0$. Consider another option $t^\triangle = 0$, $e^\triangle = e^*$, $p^\triangle = p^* - \xi t^* q^*$ and $q^\triangle = q^*$. Then, based on consumer utility functions, we can find that $d(p^*, e^*, t^*, q^*) \le d(p^\triangle, e^\triangle, t^\triangle, q^\triangle)$. Therefore, we have

$$\begin{aligned}
\pi(p^*, e^*, t^*, q^*) &= (p^* - c(e^*) - gt^*)\min(q^*, d(p^*, e^*, t^*, q^*)) - f(e^*) \\
&< (p^* - c(e^*) - \xi q^* t^*)\min(q^*, d(p^*, e^*, t^*, q^*)) - f(e^*) \\
&\le (p^* - c(e^*) - \xi q^* t^*)\min(q^*, d(p^\triangle, e^\triangle, t^\triangle, q^\triangle)) - f(e^*) \\
&= (p^\triangle - c(e^\triangle) - g^\triangle t^\triangle)\min(q^\triangle, d(p^\triangle, e^\triangle, t^\triangle, q^\triangle)) - f(e^\triangle) \\
&= \pi(p^\triangle, e^\triangle, t^\triangle, q^\triangle)
\end{aligned}$$

where the first inequality is due to $q \le m$ and $\xi m < g$ and $t^* > 0$. This is contradictory to the fact that (p^*, e^*, t^*, q^*) is the optimal solution. Thus, we can conclude the result. □

Proof of Lemma V1.2: Since demand is deterministic, we must have $q \le d(p, e, t, q)$. Thus, the firm's demand function is $\pi(p, e, t, q) = (p - c(e) - gt)q - f(e)$. Given consumer utility functions, the optimal price must be one of the following four values: v_h, v_l, $v_h - \xi(e + e_o - t)q$ and $v_l - \xi(e + e_o - t)q$.

(1) If $p = v_h$, then $\pi^{(1)}(e, t, q) = (v_h - c(e) - gt)q - f(e)$;
(2) If $p = v_l$, then $\pi^{(2)}(e, t, q) = (v_l - c(e) - gt)q - f(e)$;
(3) If $p = v_h - \xi(e + e_o - t)q$, then $\pi^{(3)}(e, t, q) = (v_h - \xi(e + e_o - t)q - c(e) - gt)q - f(e)$;
(4) If $p = v_l - \xi(e + e_o - t)q$, then $\pi^{(4)}(e, t, q) = (v_l - \xi(e + e_o - t)q - c(e) - gt)q - f(e)$.

Note that $\pi^{(i)}(e, t, q)$ are all concave in q. Given that $q \le d(p, e, t, q) \le m$ and Lemma VI4 we can conclude that if the following two conditions hold:

(a) $m < \frac{v_l - c(e) - gt}{2\xi(e + e_o - t)}$ for any $g \in (0, \xi m]$ and $e \in [0, \bar{e}]$, $t \in [0, \bar{e} + e_o]$;
(b) $m < \frac{v_l - c(e)}{2\xi(e + e_o)}$ for any $g > \xi m$ and $e \in [0, \bar{e}]$,

then $\frac{\partial \pi^{(i)}}{\partial q}|_{q=m} > 0$ and thus q must equal to its upperbound $d(p, e, t, q)$ (since $d(p, e, t, q) \le m$). Note that a sufficient condition for both conditions (a) and (b) is $m < \frac{v_l - c(0)}{3\xi(\bar{e} + e_o)}$. □

Before we prove Propositions V1.1-V1.4, let's first look at firm's profit optimization problem given the availability of offset and its price g:

For Case N:

- If carbon offsets are available: If $g > \xi m$, then Lemma VI4 implies that the firm does not purchase any offsets, and thus the subgame is the same as the game without offsets (see below). If $g \leq \xi m$, then the firm has three options to choose from:
  - Sell to all of the consumers at $p = v_l - \xi(e + e_o - t)m$. Then the firm's profit maximization problem is
$$\begin{aligned} &\max(v_l - \xi(e + e_o - t)m - c(e) - gt)m - f(e) \\ &s.t. \quad e \in [0, \bar{e}] \\ &\qquad\quad t \in [0, e + e_o] \end{aligned}$$
    The optimal solution is as follows: $e = e_{g/m,1}$, $t = e_{g/m,1} + e_o$ and the corresponding $\pi = (v_l - c(e_{g/m,1}) - ge_{g/m,1} - ge_o)m - f(e_{g/m,1})$.
  - Sell to both types of high-value consumers at $p = v_h - \xi(e + e_o - t)(1 - \alpha + \alpha\beta_c)m$. Then the firm's profit maximization problem is
$$\begin{aligned} &\max(v_h - \xi(e + e_o - t)(1 - \alpha + \alpha\beta_c)m - c(e) - gt)(1 - \alpha + \alpha\beta_c)m - f(e) \\ &s.t. \quad e \in [0, \bar{e}] \\ &\qquad\quad t \in [0, e + e_o] \end{aligned}$$
    The optimal solution is as follows:
    * if $g > \xi(1 - \alpha + \alpha\beta_c)m$, then $e = e_{\xi,1-\alpha+\alpha\beta_c}$ and $t = 0$, and the corresponding $\pi = (v_h - \xi(e_{\xi,1-\alpha+\alpha\beta_c} + e_o) - c(e_{\xi,1-\alpha+\alpha\beta_c})(1 - \alpha + \alpha\beta_c)m - f(e_{\xi,1-\alpha+\alpha\beta_c})$;
    * if $g \leq \xi(1-\alpha+\alpha\beta_c)m$, then $e = e_{g/((1-\alpha+\alpha\beta_c)m),1-\alpha+\alpha\beta_c}$, $t = e_{g/((1-\alpha+\alpha\beta_c)m),1-\alpha+\alpha\beta_c} + e_o$ and the corresponding $\pi = (v_h - c(e_{g/((1-\alpha+\alpha\beta_c)m),1-\alpha+\alpha\beta_c}) - ge_{g/((1-\alpha+\alpha\beta_c)m),1-\alpha+\alpha\beta_c} - ge_o)(1 - \alpha + \alpha\beta_c)m - f(e_{g/((1-\alpha+\alpha\beta_c)m),1-\alpha+\alpha\beta_c})$.
  - Sell to high-value eco-unconscious consumers at $p = v_h$. Then the firm's profit maximization problem is
$$\begin{aligned} &\max\,(v_h - c(e))\,(1 - \alpha)\,m - f(e) \\ &s.t. \quad e \in [0, \bar{e}] \end{aligned}$$
    The optimal solution is as follows: $e = \bar{e}$, $t = 0$ and the corresponding $\pi = (v_h - c(\bar{e}))(1-\alpha)m$.
- If carbon offsets are not available, then the firm has three options to choose from
  (n1) Sell to all of the consumers at $p = v_l - \xi e_o m - \xi em$. Then the firm's profit maximization problem is
$$\begin{aligned} &\max(v_l - \xi e_o m - \xi em - c(e))m - f(e) \\ &s.t. \quad e \in [0, \bar{e}] \end{aligned}$$
  The optimal solution is as follows: $e^{(n1)} = e_{\xi,1}$ and the corresponding $\pi^{(n1)} = (v_l - \xi e_o m - \xi e_{\xi,1} m - c(e_{\xi,1}))m - f(e_{\xi,1})$.
  (n2) Sell to both types of high-value consumers at $p = v_h - \xi(e + e_o)(1 - \alpha + \alpha\beta_c)m$. Then the firm's profit maximization problem is
$$\begin{aligned} &\max(v_h - \xi(e + e_o)(1 - \alpha + \alpha\beta_c)m - c(e))(1 - \alpha + \alpha\beta_c)m - f(e) \\ &s.t. \quad e \in [0, \bar{e}] \end{aligned}$$
  The optimal solution is as follows: $e^{(n2)} = e_{\xi,1-\alpha+\alpha\beta_c}$ and the corresponding $\pi^{(n2)} = (v_h - \xi(e_{\xi,1-\alpha+\alpha\beta_c} + e_o)(1 - \alpha + \alpha\beta_c)m - c(e_{\xi,1-\alpha+\alpha\beta_c}))(1 - \alpha + \alpha\beta_c)m - f(e_{\xi,1-\alpha+\alpha\beta_c})$.
  (n3) Sell to high-value eco-unconscious consumers at $p = v_h$. Then the firm's profit maximization

problem is

$$\max\left(v_h - c\left(e\right)\right)\left(1-\alpha\right)m - f\left(e\right)$$
$$s.t. \quad e \in [0,\bar{e}]$$

The optimal solution is as follows: $e^{(n3)} = \bar{e}$ and the corresponding $\pi^{(n3)} = (v_h - c(\bar{e}))(1-\alpha)m$.

For Case P:

- If carbon offsets are available: If $g > \xi m$, then Lemma VI4 implies that the firm does not purchase any offsets, and thus the subgame is the same as the game without offsets (see below). If $g \le \xi m$, then the firm has the following five options to choose from.
  (1) Sell to all of the consumers at $p = v_l$. Then the firm's profit maximization problem is

$$\begin{aligned}&\max(v_l - c\left(e\right) - gt)m - f\left(e\right)\\ &s.t. \quad v_h - \xi(e+e_o-t)m \ge v_l\\ &\qquad e \in [0,\bar{e}]\\ &\qquad t \in [0, e+e_o]\end{aligned}$$

  * If $v_h - \xi\bar{e}m - \xi e_o m \ge v_l$, then the optimal solution is $e = \bar{e}$, $t = 0$, and the corresponding $\pi = (v_l - c(\bar{e}))m$.
  * If $v_h - \xi\bar{e}m - \xi e_o m < v_l$, then the first constraint must be binding (i.e., $t = \frac{v_l + \xi e_o m - v_h + \xi e}{\xi m}$). Thus, the profit maximization problem above is equivalent to

$$\begin{aligned}&\max(v_l - c\left(e\right) - \tfrac{g}{\xi m}\left(v_l + \xi e_o m - v_h + \xi e m\right))m - f\left(e\right)\\ &s.t. \quad e \in [0,\bar{e}]\\ &\qquad \tfrac{v_l + \xi e_o m - v_h + \xi e m}{\xi m} \in [0, e+e_o]\end{aligned}$$

    Therefore, if $v_h - \xi e_{g/m,1} m - \xi e_o m < v_l$, then the optimal solution is $e = e_{g/m,1}$, $t = \frac{v_l + \xi e_o m - v_h + \xi e_{g/m,1} m}{\xi m}$, and the corresponding $\pi = (v_l - c(e_{g/m,1}) - \frac{v_l - v_h}{\xi m} g - g e_{g/m,1} - g e_o)m - f(e_{g/m,1})$; if $v_h - \xi e_{g/m,1} m - \xi e_o m \ge v_l$, then we must have $t = 0$. Thus, the optimal solution is $e = \frac{v_h - v_l - \xi e_o m}{\xi m}$, $t = 0$, and the corresponding $\pi = (v_l - c(\frac{v_h - v_l - \xi e_o m}{\xi m}))m - f(\frac{v_h - v_l - \xi e_o m}{\xi m})$.
  (2) Sell to all of the consumers at $p = v_h - \xi(e + e_o - t)m$. Then the firm's profit problem is

$$\begin{aligned}&\max(v_h - \xi(e+e_o-t)m - c\left(e\right) - gt)m - f\left(e\right)\\ &s.t. \quad v_h - \xi(e+e_o-t)m \le v_l\\ &\qquad e \in [0,\bar{e}]\\ &\qquad t \in [0, e+e_o]\end{aligned}$$

  Since $g \le \xi m$, the first constraint must be binding (otherwise we have $t = (e+e_o)m$ resulting in $v_h \le v_l$ which contradicts $v_h > v_l$). Thus, this optimization problem is dominated by case (1).
  (3) Sell to both types of eco-unconscious consumers at $p = v_l$. Then the firm's profit maximization problem is

$$\begin{aligned}&\max\left(v_l - c\left(e\right) - gt\right)\left(1-\alpha\right)m - f\left(e\right)\\ &s.t. \quad v_h - \xi(e+e_o)(1-\alpha)m \le v_l\\ &\qquad e \in [0,\bar{e}]\\ &\qquad t \in [0, e+e_o]\end{aligned}$$

The optimal solution is as follows:

- ∗ If $v_h - \xi(e+e_o)(1-\alpha)m \leq v_l$, then $e = \bar{e}$, $t = 0$ and the corresponding $\pi = (v_l - c(\bar{e}))(1-\alpha)m$;
- ∗ If $v_h - \xi(e+e_o)(1-\alpha)m > v_l$, then there is no feasible solution.

(4) Sell to both types of high-value consumers at $p = v_h - \xi(e+e_o-t)(\alpha+(1-\alpha)\beta_u)m$. Then the firm's profit maximization problem is

$$\begin{aligned} &\max\left(v_h - \xi(e+e_o-t)(\alpha+(1-\alpha)\beta_u)m - c\left(e\right) - gt\right)\left(\alpha+(1-\alpha)\beta_u\right)m - f\left(e\right) \\ &s.t. \quad v_h - \xi(e+e_o-t)(\alpha+(1-\alpha)\beta_u)m \geq v_l \\ &\qquad\ e \in [0,\bar{e}] \\ &\qquad\ t \in [0, e+e_o] \end{aligned}$$

The optimal solution is as follows:

- ∗ if $g > \xi(\alpha+(1-\alpha)\beta_u)m$, then $e = \min(e_{\xi,\alpha+(1-\alpha)\beta_u}, \frac{v_h-v_l-\xi e_o(\alpha+(1-\alpha)\beta_u)m}{\xi(\alpha+(1-\alpha)\beta_u)m})$, $t = 0$ and the corresponding profit

$$\begin{aligned} \pi = &(v_h - \xi(\min(e_{\xi,\alpha+(1-\alpha)\beta_u}, \tfrac{v_h-v_l-\xi e_o(\alpha+(1-\alpha)\beta_u)m}{\xi(\alpha+(1-\alpha)\beta_u)m}) + e_o)(\alpha+(1-\alpha)\beta_u)m \\ &-c(\min(e_{\xi,\alpha+(1-\alpha)\beta_u}, \tfrac{v_h-v_l-\xi e_o(\alpha+(1-\alpha)\beta_u)m}{\xi(\alpha+(1-\alpha)\beta_u)m}))(\alpha+(1-\alpha)\beta_u)m \\ &-f(\min(e_{\xi,\alpha+(1-\alpha)\beta_u}, \tfrac{v_h-v_l-\xi e_o(\alpha+(1-\alpha)\beta_u)m}{\xi(\alpha+(1-\alpha)\beta_u)m})); \end{aligned}$$

- ∗ if $g > \xi(\alpha+(1-\alpha)\beta_u)m$, then $e = e_{g/((\alpha+(1-\alpha)\beta_u)m),\alpha+(1-\alpha)\beta_u}$, $t = e_{g/((\alpha+(1-\alpha)\beta_u)m),\alpha+(1-\alpha)\beta_u} + e_o$, and the corresponding $\pi = (v_h - c(e_{g/((\alpha+(1-\alpha)\beta_u)m),\alpha+(1-\alpha)\beta_u}) - g(e_{g/((\alpha+(1-\alpha)\beta_u)m),\alpha+(1-\alpha)\beta_u} + e_o))(\alpha+(1-\alpha)\beta_u)m - f(e_{g/((\alpha+(1-\alpha)\beta_u)m),\alpha+(1-\alpha)\beta_u})$.

(5) Sell to high-value eco-unconscious consumers at $p = v_h$. Then the firm's profit maximization problem is

$$\begin{aligned} &\max\left(v_h - c\left(e\right)\right)\left(1-\alpha\right)\beta_u m - f\left(e\right) \\ &s.t. \quad e \in [0,\bar{e}] \end{aligned}$$

The optimal solution is as follows: $e = \bar{e}$ and the corresponding $\pi = (v_h - c(\bar{e}))(1-\alpha)\beta_u m$.

- If carbon offsets are not available:
  - if $v_h - \xi e_o m < v_l$, then the firm has three options to choose from:

(p1) Sell to all of the consumers at $p = v_h - \xi e_o m - \xi e m$. Then the firm's profit maximization problem is

$$\begin{aligned} &\max(v_h - \xi e_o m - \xi e m - c\left(e\right))m - f\left(e\right) \\ &s.t. \quad e \in [0,\bar{e}] \end{aligned}$$

The optimal solution is as follows: $e^{(p1)} = e_{\xi,1}$ and the corresponding $\pi^{(p1)} = (v_h - \xi e_o m - \xi e_{\xi,1} m - c(e_{\xi,1}))m - f(e_{\xi,1})$.

(p2) Sell to both types of eco-unconscious consumers at $p = v_l$. Then the firm's profit maximization problem is

$$\begin{aligned} &\max\left(v_l - c\left(e\right)\right)\left(1-\alpha\right)m - f\left(e\right) \\ &s.t. \quad e \in [0,\bar{e}] \end{aligned}$$

The optimal solution is as follows: $e^{(p2)} = \bar{e}$ and the corresponding $\pi^{(p2)} = (v_l - c(\bar{e}))(1-\alpha)m$.

(p3) Sell to high-value eco-unconscious consumers at $p = v_h$. Then the firm's profit maximization problem is

$$\max\left(v_h - c\left(e\right)\right)\left(1-\alpha\right)\beta_u m - f\left(e\right)$$
$$s.t. \quad e \in [0, \bar{e}]$$

The optimal solution is as follows: $e^{(p3)} = \bar{e}$ and the corresponding $\pi^{(p3)} = (v_h - c(\bar{e}))(1-\alpha)\beta_u m$.

- $v_h - \xi e_o m \geq v_l$, then the firm has five options to choose from:

(p1) Sell to all of the consumers at $p = v_l$. Then the firm's profit maximization problem is

$$\max\left(v_l - c\left(e\right)\right)m - f\left(e\right)$$
$$s.t. \quad v_h - \xi e_o m - \xi e m \geq v_l$$
$$e \in [0, \bar{e}]$$

The optimal solution is as follows: $e^{(p1)} = \min(\bar{e}, \frac{v_h - \xi e_o m - v_l}{\xi m})$ and the corresponding $\pi^{(p1)} = (v_l - c(\min(\bar{e}, \frac{v_h - \xi e_o - v_l}{\xi})))m - f(\min(\bar{e}, \frac{v_h - \xi e_o - v_l}{\xi}))$.

(p2) Sell to all of the consumers at $p = v_h - \xi e_o m - \xi e m$. Then the firm's profit maximization problem is

$$\max(v_h - \xi e_o m - \xi e m - c\left(e\right))m - f\left(e\right)$$
$$s.t. \quad v_h - \xi e_o m - \xi e m \leq v_l$$
$$e \in [0, \bar{e}]$$

The optimal solution is as follows:

· If $v_h - \xi e_o m - \xi e_{\xi,1} m \leq v_l$, then $e^{(p2)} = e_{\xi,1}$ and the corresponding $\pi^{(p2)} = (v_h - \xi e_o m - \xi e_{\xi,1} m - c(e_{\xi,1}))m - f(e_{\xi,1})$;

· If $v_h - \xi e_o m - \xi e_{\xi,1} m > v_l$, then the first constraint must be binding. Then the optimization problem is dominated by case (p1).

(p3) Sell to both types of high-value consumers at $p = v_h - \xi(e + e_o)(\alpha + (1-\alpha)\beta_u)m$. Then the firm's profit maximization problem is

$$\max(v_h - \xi(e + e_o)(\alpha + (1-\alpha)\beta_u)m - c\left(e\right))(\alpha + (1-\alpha)\beta_u)m - f\left(e\right)$$
$$s.t. \quad v_h - \xi(e + e_o)(\alpha + (1-\alpha)\beta_u)m \geq v_l$$
$$e \in [0, \bar{e}]$$

The optimal solution is as follows:

· If $v_h - \xi(e_{\xi,\alpha+(1-\alpha)\beta_u} + e_o)(\alpha + (1-\alpha)\beta_u)m \geq v_l$, then $e^{(p3)} = e_{\xi,\alpha+(1-\alpha)\beta_u}$ and the corresponding $\pi^{(p3)} = (v_h - \xi(e_{\xi,\alpha+(1-\alpha)\beta_u} + e_o)(\alpha + (1-\alpha)\beta_u)m - c(e_{\xi,\alpha+(1-\alpha)\beta_u}))(\alpha + (1-\alpha)\beta_u)m - f(e_{\xi,\alpha+(1-\alpha)\beta_u})$;

· If $v_h - \xi(e_{\xi,\alpha+(1-\alpha)\beta_u} + e_o)(\alpha + (1-\alpha)\beta_u)m < v_l$, then the first constraint must be binding. Then the optimization problem is dominated by case (p2).

(p4) Sell to both types of eco-unconscious consumers at $p = v_l$. Then the firm's profit maximization problem is

$$\max\left(v_l - c\left(e\right)\right)\left(1-\alpha\right)m - f\left(e\right)$$
$$s.t. \quad v_h - \xi(e + e_o)(1-\alpha)m \leq v_l$$
$$e \in [0, \bar{e}]$$

The optimal solution is as follows:

· If $v_h - \xi(\bar{e} + e_o)(1-\alpha)m \leq v_l$, then $e^{(p4)} = \bar{e}$ and the corresponding $\pi^{(p4)} = (v_l -$

$c(\bar{e}))(1-\alpha)m$;

· If $v_h - \xi(\bar{e}+e_o)(1-\alpha)m > v_l$, then there is no feasible solution.

(p5) Sell to high-value eco-unconscious consumers at $p = v_h$. Then the firm's profit maximization problem is

$$\max\left(v_h - c\left(e\right)\right)\left(1-\alpha\right)\beta_u m - f\left(e\right)$$
$$s.t.\quad e \in [0,\bar{e}]$$

The optimal solution is as follows: $e^{(p5)} = \bar{e}$ and the corresponding $\pi^{(p5)} = (v_h - c(\bar{e}))(1-\alpha)\beta_u m$.

Proof of Proposition V1.1: Let us first look at Case N. Denote $\pi(p,e,t|g)$ as the profit given firm's decisions (p,e,t) and offset price g. If $t>0$ and $\xi > \frac{g}{(1-\alpha+\alpha\beta_c)m}$, according to the analysis of the subgame above, the equilibrium in the subgame must be one of the following:

1. $p = v_h$, $e = e_{g/((1-\alpha+\alpha\beta_c)m),1-\alpha+\alpha\beta_c}$, $t = e_{g/((1-\alpha+\alpha\beta_c)/m),1-\alpha+\alpha\beta_c} + e_o$, $d_c = \alpha\beta_c m$, $d_u = (1-\alpha)m$;
2. $p = v_l$, $e = e_{g/m,1}$, $t = e_{g/m,1} + e_o$, $d_c = \alpha m$, $d_u = (1-\alpha)m$.

Define function $h(g) \triangleq \pi(v_l, e_{g/m,1}, e_{g/m,1}+e_o|g) - \pi(v_h, e_{g/((1-\alpha+\alpha\beta_c)m),1-\alpha+\alpha\beta_c}, e_{g/((1-\alpha+\alpha\beta_c)m),1-\alpha+\alpha\beta_c} + e_o|g) = (v_l - c(e_{g/m,1}) - g e_{g/m,1} - g e_o)m - f(e_{g/m,1}) - [(v_h - c(e_{g/((1-\alpha+\alpha\beta_c)m),1-\alpha+\alpha\beta_c}) - g e_{g/((1-\alpha+\alpha\beta_c)m),1-\alpha+\alpha\beta_c} - g e_o)(1-\alpha+\alpha\beta_c)m - f(e_{g/((1-\alpha+\alpha\beta_c)m),1-\alpha+\alpha\beta_c})]$. Note that $\lim_{g\to+\infty} h'(g) = -e_o\alpha(1-\beta_c)m < 0$. Then, if $h(0) > 0$ (i.e., $\beta_c < B_2 \triangleq \frac{\frac{v_l - c(\bar{e})}{v_h - c(\bar{e})} - 1 + \alpha}{\alpha}$), then there exists $X = \min\{g|h(g)=0\}$ and $X > 0$. Then, if the following condition hold:

(1) $\pi(v_l, e_{X/m,1}, e_{X/m,1}+e_o|X) > \pi(v_h, \bar{e}, 0|X)$, i.e., $(v_l - c(e_{X/m,1}) - X e_{X/m,1} - X e_o)m - f(e_{X/m,1}) > (v_h - c(\bar{e}))(1-\alpha)m$,

then we must have $X < G$ and for a small positive number $\epsilon > 0$,

- if $g \in (0, X)$, then $p^* = v_l$, $e^* = e_{g/m,1}$ and $t^* = e_{g/m,1} + e_o$;
- if $g \in (X, X+\epsilon)$, then $p^* = v_h$, $e^* = e_{g/((1-\alpha+\alpha\beta_c)m),1-\alpha+\alpha\beta_c}$ and $t^* = e_{g/((1-\alpha+\alpha\beta_c)m),1-\alpha+\alpha\beta_c} + e_o$.

Therefore, there exist g_1, g_2 and $\tilde{\epsilon} \in (0,\epsilon)$ such that $X - \tilde{\epsilon} < g_1 < X < g_2 < X + \tilde{\epsilon}$ and $e^*(g_1) = e_{g_1/m,1} \leq e_{g_2/((1-\alpha+\alpha\beta_c)m),1-\alpha+\alpha\beta_c} = e^*(g_2)$, where the inequality is due to Lemma VI3 and is strict if $\bar{e}$ is large enough; also, in this case, we have $d_c^*(g_1) > d_c^*(g_2)$ and $d_u^*(g_1) = d_u^*(g_2)$. As for condition (1), note that $\frac{\partial h}{\partial \beta_c} < 0$ and $\frac{\partial h}{\partial g}|_{g=X} < 0$, and thus we have $\frac{\partial X}{\partial \beta_c} < 0$. Since $\frac{\partial(v_l - c(e_{X/m,1}) - X e_{X/m,1} - X e_o)m - f(e_{X/m,1}))}{\partial X} < 0$ (due to Envelope Theorem), condition (1) is equivalent to β_c being large (i.e., $\beta_c > B_1$).

Next, let us look at Case P. Denote $\pi(p,e,t|g)$ as the profit given firm's decisions (p,e,t) and offset price g. If $v_h < v_l + \xi e_o m$, then $v_l > v_h - \xi(\bar{e}+e_o)m$ and $\max(e_{g/m,1}, \frac{v_h - v_l - \xi e_o m}{\xi m}) = e_{g/m,1}$, and thus if $g \leq G$ (i.e., $t > 0$) and $\xi > \frac{g}{(1-\alpha+\alpha\beta_c)m}$, according to the analysis of the subgame above, the equilibrium in the subgame must be one of the following:

1. $p = v_h$, $e = e_{g/((\alpha+(1-\alpha)\beta)m),\alpha+(1-\alpha)\beta_u}$, $t = e_{g/((\alpha+(1-\alpha)\beta)m),\alpha+(1-\alpha)\beta_u} + e_o$, $d_c = \alpha m$, $d_u = (1-\alpha)\beta_u m$;

2. $p = v_l$, $e = e_{g/m,1}$, $t = e_{g/m,1} + e_o - \frac{v_h - v_l}{\xi m}$, $d_c = \alpha m$, $d_u = (1-\alpha)m$.

Define function $h(g) \triangleq \pi(v_l, e_{g/m,1}, \frac{v_l - v_h + \xi e_{g/m,1} m + \xi e_o m}{\xi m}|g) - \pi(v_h, e_{g/(\tilde{\alpha} m),\tilde{\alpha}}, e_{g/(\tilde{\alpha} m),\tilde{\alpha}} + e_o|g) = (v_l - c(e_{g/m,1}) - g\frac{v_l - v_h}{\xi m} - g e_{g/m,1} - g e_o)m - f(e_{g/m,1}) - [(v_h - c(e_{g/(\tilde{\alpha} m),\tilde{\alpha}}) - g e_{g/(\tilde{\alpha} m),\tilde{\alpha}} - g e_o)\tilde{\alpha} m - f(e_{g/(\tilde{\alpha} m),\tilde{\alpha}})]$, where $\tilde{\alpha} = \alpha + (1-\alpha)\beta_u$. Note that $h(0) = (v_l - c(\bar{e}))m - (v_h - c(\bar{e}))(\alpha + (1-\alpha)\beta_u)m$ and $\lim_{g \to +\infty} h'(g) = \frac{v_h - v_l}{\xi m} m - e_o(1-\alpha)(1-\beta_u)m$. If $h(0) > 0$ and $\lim_{g \to +\infty} h'(g) < 0$ (i.e., $\beta_u < \tilde{B}_2 \triangleq \min(\frac{(v_l - c(\bar{e})) - (v_h - c(\bar{e}))\alpha}{(v_h - c(\bar{e}))(1-\alpha)}, \frac{\xi m e_o(1-\alpha) - (v_h - v_l)}{\xi m e_o(1-\alpha)})$), then there exists $X = \min\{g | h(g) = 0\}$ and $X > 0$. Then, if the following condition hold:

(1) $\pi(v_h, e_{X/(\tilde{\alpha} m),\tilde{\alpha}}, e_{X/(\tilde{\alpha} m),\tilde{\alpha}} + e_o|X) > \max(\pi(v_h, \bar{e}, 0|X), \pi(v_l, \bar{e}, 0|X))$, i.e., $(v_h - c(e_{X/(\tilde{\alpha} m),\tilde{\alpha}}) - X e_{X/(\tilde{\alpha} m),\tilde{\alpha}} - X e_o)\tilde{\alpha} m - f(e_{g/(\tilde{\alpha} m),\tilde{\alpha}}) > \max((v_h - c(\bar{e}))(1-\alpha)\beta_u m, (v_l - c(\bar{e}))(1-\alpha)m)$;

then we must have $X < G$ and for a small positive number $\epsilon > 0$,

- if $g \in (0, X)$, then $p^* = v_l$, $e^* = e_{g/m,1}$, $t^* = e_{g/m,1} + e_o - \frac{v_h - v_l}{\xi m}$,
- if $g \in (X, X+\epsilon)$, then $p^* = v_h$, $e^* = e_{g/((\alpha+(1-\alpha)\beta_u)m),\alpha+(1-\alpha)\beta_u}$, $t^* = e_{g/((\alpha+(1-\alpha)\beta_u)m),\alpha+(1-\alpha)\beta_u} + e_o$.

Therefore, there exist g_1, g_2 and $\tilde{\epsilon} \in (0, \epsilon)$ such that $X - \tilde{\epsilon} < g_1 < X < g_2 < X + \tilde{\epsilon}$ and $e^*(g_1) = e_{g_1/m,1} \leq e_{g_2/((\alpha+(1-\alpha)\beta_u)m),\alpha+(1-\alpha)\beta_u} = e^*(g_2)$, where the inequality is due to Lemma VI1 and is strict if $\bar{e}$ is large enough; also, in this case, we have $d_c^*(g_1) = d_c^*(g_2)$ and $d_u^*(g_1) > d_u^*(g_2)$. As for condition (1), since $\beta_u < \frac{(v_l - c(\bar{e})) - (v_h - c(\bar{e}))\alpha}{(v_h - c(\bar{e}))(1-\alpha)} < \frac{v_l - c(\bar{e})}{v_h - c(\bar{e})}$, condition (1) is equivalent to $(v_h - c(e_{X/(\tilde{\alpha} m),\tilde{\alpha}}) - X e_{X/(\tilde{\alpha} m),\tilde{\alpha}} - X e_o)\tilde{\alpha} m - f(e_{g/(\tilde{\alpha} m),\tilde{\alpha}}) > (v_l - c(\bar{e}))(1-\alpha)m$. Note that $\frac{\partial h}{\partial \beta_u} < 0$ and $\frac{\partial h}{\partial g}|_{g=X} < 0$, and thus we have $\frac{\partial X}{\partial \beta_u} < 0$. Since $\frac{\partial((v_h - c(e_{X/(\tilde{\alpha} m),\tilde{\alpha}}) - X e_{X/(\tilde{\alpha} m),\tilde{\alpha}} - X e_o)\tilde{\alpha} m - f(e_{g/(\tilde{\alpha} m),\tilde{\alpha}}))}{\partial X} < 0$ (due to Envelope Theorem), condition (1) is equivalent to β_u being large (i.e., $\beta_u > \tilde{B}_1$). □

Proof of Proposition V1.2: Let us first look at Case N. According to the analysis of the subgame above, if $t > 0$ and $\xi > \frac{g}{(1-\alpha+\alpha\beta_c)m}$, then the equilibrium in the subgame must be one of the following:

1. $p = v_h$, $e = e_{g/((1-\alpha+\alpha\beta_c)m),1-\alpha+\alpha\beta_c}$, $t = e_{g/((1-\alpha+\alpha\beta_c)m),1-\alpha+\alpha\beta_c} + e_o$, $d_c = \alpha\beta_c m$, $d_u = (1-\alpha)m$;
2. $p = v_l$, $e = e_{g/m,1}$, $t = e_{g/m,1} + e_o$, $d_c = \alpha m$, $d_u = (1-\alpha)m$.

Thus, if $t > 0$, we have $\theta = 1$ in equilibrium. Given Lemma V1.1, we can conclude that

- if $g > G$, then $\theta^* = 0$;
- if $g \leq G$, then $\theta^* = 1$.

Thus, we can conclude Proposition V1.2(i).

Next, let us look at Case P. From the proof of Proposition V1.1(ii), under the conditions in Proposition V1.1(ii), there exist g_1, g_2 such that $0 < g_1 < X < g_2 < X + \epsilon$ and $\theta^*(g_1) = 1 - \frac{v_h - v_l}{\xi m(e_{g/m,1} + e_o)} < 1 = \theta^*(g_2)$. □

Proof of Proposition V1.3: Let us first look at Case N. When offsets are not available, from the game analysis above, we have $\lim_{\alpha\to 0}(\pi^{(n3)} - \pi^{(n1)}) = (v_h - c(\bar{e}))m - (v_l - \xi e_o m - \xi e_{\xi,1} m - c(e_{\xi,1}))m + f(e_{\xi,1}) > 0$ and $\lim_{\alpha\to 0}(\pi^{(n3)} - \pi^{(n2)}) = (v_h - c(\bar{e}))m - (v_h - \xi(e_{\xi,1} + e_o)m - c(e_{\xi,1}))m + f(e_{\xi,1}) > 0$. Thus, there exists a threshold $A > 0$ such that if $\alpha \in (0, A)$, then $\pi^{(n3)} > \max\{\pi^{(n1)}, \pi^{(n2)}\}$ and thus $e^b = e^{(n3)} = \bar{e}$. When offsets are available, from the subgame analysis above, if $t^*(g) > 0$, then we must have $\mathcal{E}^*(g) = 0$. By Lemma V1.1, if $c_g \le G$, then NGO will choose to set $g^* = c_g$ and have $t^*(g^*) > 0$ and $\mathcal{E}^*(g^*) = 0$. Note that if $t > 0$, we have $e \in \{e_{g/((1-\alpha+\alpha\beta_c)m),1-\alpha+\alpha\beta_c}, e_{g/m,1}\} \le \bar{e}$ and the inequality is strict if $\bar{e}$ is large enough. Therefore, if $c_g \le G$ and $\alpha \in (0, A)$, we have $e^*(g^*) \in \{e_{g/((1-\alpha+\alpha\beta_c)m),1-\alpha+\alpha\beta_c}, e_{g/m,1}\} \le \bar{e} = e^b$, where the inequality is strict if $\bar{e}$ is large enough.

Next, let us look at Case P. When offsets are not available, if $v_l < v_h - \xi(\bar{e} + e_o)$, then from the game analysis above, the potential equilibrium must be from option (p1), (p3) and (p5). Note that $\lim_{\alpha\to 0}(\pi^{(p5)} - \pi^{(p3)}) = (v_h - c(\bar{e}))\beta_u m - (v_h - \xi(e_{\xi,\beta_u} + e_o)\beta_u m - c(e_{\xi,\beta_u}))\beta_u m + f(e_{\xi,\beta_u}) > 0$. Thus, there exists a threshold $\tilde{A} > 0$ such that if $\alpha \in (0, \tilde{A})$, then $\max\{\pi^{(p1)}, \pi^{(p5)}\} > \pi^{(p3)}$ and thus $e^b = \bar{e}$. When offsets are available, if $v_l < v_h - \xi(\bar{e} + e_o)$, then from the game analysis above, if $t^*(g) > 0$, then we must have $\mathcal{E}^*(g) = 0$. By Lemma V1.1, if $c_g \le G$, then NGO will choose to set $g^* = c_g$ and have $t^*(g^*) > 0$ and $\mathcal{E}^*(g^*) = 0$. Note that if $t > 0$, we have $e = e_{g/((\alpha+(1-\alpha)\beta_u)m),\alpha+(1-\alpha)\beta_u} \le \bar{e}$ and the inequality is strict if $\bar{e}$ is large enough. Therefore, if $c_g \le G$ and $\alpha \in (0, A)$ and $v_l < v_h - \xi(\bar{e} + e_o)$, we have $e^*(g^*) e_{g/((\alpha+(1-\alpha)\beta_u)m),\alpha+(1-\alpha)\beta_u} \le \bar{e} = e^b$, where the inequality is strict if $\bar{e}$ is large enough. □

Proof of Proposition V1.4: Let us first look at Case N. From the proof of Proposition V1.2(i), we have the following result:

- if $g > G$, then $\theta^*(g) = 0$;
- if $g \le G$, then $\theta^*(g) = 1$ and thus $\mathcal{E}^*(g) = 0$.

Thus, given that $c_g \le G$, it is easy to find that $g^* = c_g$, under which we have $\theta^*(g^*) = 1$ and thus $\mathcal{E}^*(g^*) = 0$.

Next, let us look at Case P. Under the conditions in Proposition V1.1(ii), according to the proof of Proposition V1.1(ii), there exist $g_1, g_2 \in (0, G)$, such that $g_1 < g_2 < G$ and $\theta^*(g_1) = 1 - \frac{v_h - v_l}{\xi m(e_{g_1/m,1} + e_o)} < 1 = \theta^*(g_2)$, and thus $\mathcal{E}^*(g_1) > 0 = \mathcal{E}^*(g_2)$. Therefore, if $c_g = g_1$, then we can conclude the optimal offset price $g^* = \arg\min_{g \ge c_g} \mathcal{E}^*(g) > c_g$. □

Proof of Lemma V2.1: Given the offset price g, the firm's profit function is $\pi(p, e, t, q|g) = p\mathbb{E}\min(q, D(p, e, t)) - (c(e) + gt)q - f(e)$. Suppose there exist $g_1 < g_2$ such that $t^*(g_1) = 0 < t^*(g_2)$. Then, we have $\pi(p^*(g_2), e^*(g_2), t^*(g_2), q^*(q_2)|g_2) > \pi(p^*(g_1), e^*(g_1), t^*(g_1), q^*(g_1)|g_2) = \pi(p^*(g_1), e^*(g_1), t^*(g_1), q^*(g_1)|g_1) > \pi(p^*(g_2), e^*(g_2), t^*(g_2), q^*(g_2)|g_1)$, where the equality is because $t^*(g_1) = 0$. However, this is contradictory to the fact that

$$
\begin{aligned}
&\pi(p^*(g_2), e^*(g_2), t^*(g_2), q^*(g_2)|g_2) - \pi(p^*(g_2), e^*(g_2), t^*(g_2), q^*(g_2)|g_1) \\
&= (g_1 - g_2)t^*(g_2)q^*(g_2) \\
&< 0
\end{aligned}
$$

Thus, we can conclude the result. □

Proof of Lemma V2.2: Denote $y = \alpha\beta_c m I_{u_{c,h}\geq 0} + \alpha(1-\beta_c)m I_{u_{c,l}\geq 0} + (1-\alpha)\beta_u m I_{u_{u,h}\geq 0} + (1-\alpha)(1-\beta_u)m I_{u_{u,l}\geq 0}$.

Let's first look at the case where the firm cannot purchase carbon offsets. Then, the firm solves the following optimization problem

$$
\begin{aligned}
\max\ & p\mathbb{E}\min(q, D(p,e)) - c(e)q - f(e) \\
s.t.\ & e \in [0, \bar{e}] \\
& q \in [0, m]
\end{aligned}
$$

Since total market size follows the Bernoulli distribution and the demand D is independent of q, we can conclude that q must be either y or λy in the optimal solution.

- If $q = y$, then the profit is $\pi^{\triangle}(p,e) = p\mu y - c(e)y - f(e)$;
- If $q = \lambda y$, then the profit is $\pi^{\triangle\triangle}(p,e) = p\lambda y - c(e)\lambda y - f(e)$;

Note that $\frac{\pi^{\triangle} - \pi^{\triangle\triangle}}{(1-\lambda)y} = p\phi - c(e)$. We can check that one of the utility functions (i.e., $u_{c,h}$, $u_{c,l}$, $u_{u,h}$ or $u_{u,l}$) must be 0 in equilibrium, i.e., p^* must equal to one of the following: v_h, v_l, $v_h - \xi(e+e_o)$ or $v_l - \xi(e+e_o)$. Thus, if $\phi > \frac{c(0)}{(v_l - \xi\bar{e} - \xi e_o)^+}$, then we must have $p\phi - c(e) > 0$ (i.e., $\pi^{\triangle} > \pi^{\triangle\triangle}$) for any $e \in [0, \bar{e}]$. Thus, in equilibrium, we must have $q = y$.

When the firm can choose to purchase carbon offsets, the firm solves the following optimization problem

$$
\begin{aligned}
\max\ & p\mathbb{E}\min(q, D(p,e,t)) - (c(e) + gt)q - f(e) \\
s.t.\ & e \in [0, \bar{e}] \\
& t \in [0, e + e_o] \\
& q \in [0, m]
\end{aligned}
$$

Since total market size follows the Bernoulli distribution and the demand D is independent of q, we can conclude that q must be either y or λy in the optimal solution.

- If $q = y$, then the profit is $\pi^{\diamond}(p,e,t) = p\mu y - c(e)y - gty - f(e)$;
- If $q = \lambda y$, then the profit is $\pi^{\diamond\diamond}(p,e,t) = p\lambda y - c(e)\lambda y - gt\lambda y - f(e)$;

Lemma VI5. *If $g > \xi\mu$, then $t^*(g) = 0$.*

Proof of Lemma VI5: We can check that one of the utility functions (i.e., $u_{c,h}$, $u_{c,l}$, $u_{u,h}$ or $u_{u,l}$) must be 0 in equilibrium, i.e., p^* must equal to one of the following: v_h, v_l, $v_h - \xi(e + e_o - t)$ or $v_l - \xi(e + e_o - t)$. Given $p \in \{v_h, v_l, v_h - \xi(e+e_o-t), v_l - \xi(e+e_o-t)\}$, if $g > \xi\mu$, we have $\frac{\partial \pi^{\diamond}(p,e,t)}{\partial t} < 0$ and $\frac{\partial \pi^{\diamond\diamond}(p,e,t)}{\partial t} < 0$ for any e and t. Thus, we must have $t^*(g) = 0$. □

Note that $\frac{\pi^{\diamond}-\pi^{\diamond\diamond}}{(1-\lambda)y} = p\phi - c(e) - gt$. By Lemma VI5, since $p^* \in \{v_h, v_l, v_h - \xi(e+e_o-t), v_l - \xi(e+e_o-t)\}$, if $\phi > \frac{c(0)+g\bar{e}+ge_o}{(v_l-\xi\bar{e}-\xi e_o)^+}$ for any $g \in (0, \xi\mu]$ ($\Leftrightarrow \phi > \frac{c(0)+\xi\mu\bar{e}+\xi\mu e_o}{(v_l-\xi\bar{e}-\xi e_o)^+}$), then we must have $p\phi - c(e) - gt > 0$ (i.e., $\pi^{\diamond} > \pi^{\diamond\diamond}$) for any $e \in [0, \bar{e}]$ and $t \in [0, e+e_o]$. Thus, in equilibrium, we must have $q = y$. □

Proof of Proposition V2.1: Let us first look at the case where offsets are available. Define $p^{\dagger}(g) = \mu p^*(g)$, $e^{\dagger}(g) = e^*(g)$, $t^{\dagger}(g) = t^*(g)$. If $\phi > \frac{c(0)+\xi\mu\bar{e}+\xi\mu e_o}{(v_l-\xi\bar{e}-\xi e_o)^+}$, by Lemma V2.2, $q = \alpha\beta_c m I_{u_{c,h}\geq 0} + \alpha(1-\beta_c) m I_{u_{c,l}\geq 0} + (1-\alpha)\beta_u m I_{u_{u,h}\geq 0} + (1-\alpha)(1-\beta_u) m I_{u_{u,l}\geq 0}$.

Suppose $(\hat{p}^*(g), \hat{e}^*(g), \hat{t}^*(g)) \neq (\mu p^*(g), e^*(g), t^*(g))$ (i.e., $(\hat{p}^*(g), \hat{e}^*(g), \hat{t}^*(g)) \neq (p^{\dagger}(g), e^{\dagger}(g), t^{\dagger}(g))$). Thus, for any g,

$$\begin{aligned}
&\pi^*(p^*(g), e^*(g), t^*(g)) \\
&= [p^*(g)\mu - c(e^*(g)) - gt^*(g)] \Pr(V - \Xi(e^*(g) + e_o - t^*(g)) - p^*(g) \geq 0) m - f(e^*(g)) \\
&= [p^{\dagger}(g) - \hat{c}(e^{\dagger}(g)) - gt^{\dagger}(g)] \Pr(\mu V - \mu\Xi(e^{\dagger}(g) + e_o - t^{\dagger}(g)) - p^{\dagger}(g) \geq 0) m - \hat{f}(e^{\dagger}(g)) \\
&< [\hat{p}^*(g) - \hat{c}(\hat{e}^*(g)) - g\hat{t}^*(g)] \Pr(\mu V - \mu\Xi(\hat{e}^*(g) + e_o - \hat{t}^*(g)) - \hat{p}^*(g) \geq 0) m - \hat{f}(\hat{e}^*(g)) \\
&= [p^{\triangle}(g)\mu - c(\hat{e}^*(g)) - g\hat{t}^*(g)] \Pr(V - \Xi(\hat{e}^*(g) + e_o - \hat{t}^*(g)) - p^{\triangle}(g) \geq 0) m - f(\hat{e}^*(g)) \\
&= \pi^*(p^{\triangle}(g), \hat{e}^*(g), \hat{t}^*(g))
\end{aligned}$$

where $p^{\triangle}(g) = \hat{p}^*(g)/\mu$ and the inequality is due to the fact that $(\hat{p}^*(g), \hat{e}^*(g), \hat{t}^*(g))$ is the firm's optimal solution in the newly defined model and $(\hat{p}^*(g), \hat{e}^*(g), \hat{t}^*(g)) \neq (p^{\dagger}(g), e^{\dagger}(g), t^{\dagger}(g))$. However, the result above contradicts the fact that $(p^*(g), e^*(g), t^*(g))$ is the firm's optimal solution in the original model. Thus, must have $(\hat{p}^*(g), \hat{e}^*(g), \hat{t}^*(g)) = (\mu p^*(g), e^*(g), t^*(g))$ for any g. As a result, $\mathcal{E}^*(g) = \hat{\mathcal{E}}^*(g)$ for any g, and therefore $g^* = \hat{g}^*$.

Next, let us look at the case where offsets are not available. Define $p^{\dagger} = \mu p^b$, $e^{\dagger} = e^b$. If $\phi > \frac{c(0)+\xi\mu\bar{e}+\xi\mu e_o}{(v_l-\xi\bar{e}-\xi e_o)^+}$, by Lemma V2.2, $q = \alpha\beta_c m I_{u_{c,h}\geq 0} + \alpha(1-\beta_c) m I_{u_{c,l}\geq 0} + (1-\alpha)\beta_u m I_{u_{u,h}\geq 0} + (1-\alpha)(1-\beta_u) m I_{u_{u,l}\geq 0}$.

Suppose $(\hat{p}^b, \hat{e}^b) \neq (\mu p^b, e^b)$ (i.e., $(\hat{p}^b, \hat{e}^b) \neq (p^{\dagger}, e^{\dagger})$). Thus, for any g,

$$\begin{aligned}
&\pi^b(p^b, e^b) \\
&= [p^b\mu - c(e^b)] \Pr(V - \Xi(e^b + e_o) - p^b \geq 0) m - f(e^b) \\
&= [p^{\dagger} - \hat{c}(e^{\dagger})] \Pr(\mu V - \mu\Xi(e^{\dagger} + e_o) - p^{\dagger} \geq 0) m - \hat{f}(e^{\dagger}) \\
&< [\hat{p}^b - \hat{c}(\hat{e}^b)] \Pr(\mu V - \mu\Xi(\hat{e}^b + e_o) - \hat{p}^b \geq 0) m - \hat{f}(\hat{e}^b) \\
&= [p^{\triangle}\mu - c(\hat{e}^b)] \Pr(V - \Xi(\hat{e}^b + e_o) - p^{\triangle} \geq 0) m - f(\hat{e}^b) \\
&= \pi^b(p^{\triangle}, \hat{e}^b)
\end{aligned}$$

where $p^{\triangle} = \hat{p}^b/\mu$ and the inequality is due to the fact that $(\hat{p}^b, \hat{e}^b)$ is the firm's optimal solution in the newly defined model and $(\hat{p}^b, \hat{e}^b) \neq (p^{\dagger}, e^{\dagger})$. However, the result above contradicts the fact that (p^b, e^b) is the firm's optimal solution in the original model. Thus, must have $(\hat{p}^b, \hat{e}^b) = (\mu p^b, e^b)$. □

Proof of Proposition V3.1: Let us first look at the case where offsets are available. Define $p^{\dagger}(g) = p^*(g) + w(e^*(g))$, $e^{\dagger}(g) = e^*(g)$, $t^{\dagger}(g) = t^*(g)$. Suppose $(\hat{p}^*(g), \hat{e}^*(g), \hat{t}^*(g)) \neq (p^*(g) + w(e^*(g)), e^*(g), t^*(g))$ (i.e.,

$(\hat{p}^*(g), \hat{e}^*(g), \hat{t}^*(g)) \neq (p^\dagger(g), e^\dagger(g), t^\dagger(g)))$. Thus, for any g,

$$
\begin{aligned}
&\pi^*(p^*(g), e^*(g), t^*(g)) \\
&= [p^*(g) - c(e^*(g)) - gt^*(g)] \Pr(V - w(e^*(g)) - \Xi(e^*(g) + e_o - t^*(g)) - p^*(g) \geq 0)m - f(e^*(g)) \\
&= [p^\dagger(g) - \hat{c}(e^\dagger(g)) - gt^\dagger(g)] \Pr(V - \Xi(e^\dagger(g) + e_o - t^\dagger(g)) - p^\dagger(g) \geq 0)m - \hat{f}(e^\dagger(g)) \\
&< [\hat{p}^*(g) - \hat{c}(\hat{e}^*(g)) - g\hat{t}^*(g)] \Pr(V - \Xi(\hat{e}^*(g) + e_o - \hat{t}^*(g)) - \hat{p}^*(g) \geq 0)m - \hat{f}(\hat{e}^*(g)) \\
&= [p^\triangle(g) - c(\hat{e}^*(g)) - g\hat{t}^*(g)] \Pr(V - w(\hat{e}^*(g)) - \Xi(\hat{e}^*(g) + e_o - \hat{t}^*(g)) - p^\triangle(g) \geq 0)m - f(\hat{e}^*(g)) \\
&= \pi^*(p^\triangle(g), \hat{e}^*(g), \hat{t}^*(g))
\end{aligned}
$$

where $p^\triangle(g) = \hat{p}^*(g) - w(\hat{e}^*(g))$ and the inequality is due to the fact that $(\hat{p}^*(g), \hat{e}^*(g), \hat{t}^*(g))$ is the firm's optimal solution in the newly defined model and $(\hat{p}^*(g), \hat{e}^*(g), \hat{t}^*(g)) \neq (p^\dagger(g), e^\dagger(g), t^\dagger(g))$. However, the result above contradicts the fact that $(p^*(g), e^*(g), t^*(g))$ is the firm's optimal solution in the original model. Thus, must have $(\hat{p}^*(g), \hat{e}^*(g), \hat{t}^*(g)) = (p^*(g) + w(e^*(g)), e^*(g), t^*(g))$ for any g. As a result, $\mathcal{E}^*(g) = \hat{\mathcal{E}}^*(g)$ for any g, and therefore $g^* = \hat{g}^*$.

Next, let us look at the case where offsets are not available. Define $p^\dagger = p^b + w(e^b)$, $e^\dagger = e^b$. Suppose $(\hat{p}^b, \hat{e}^b) \neq (p^b + w(e^b), e^b)$ (i.e., $(\hat{p}^b, \hat{e}^b) \neq (p^\dagger, e^\dagger)$). Thus, for any g,

$$
\begin{aligned}
&\pi^b(p^b, e^b) \\
&= [p^b - c(e^b)] \Pr(V - w(e^b) - \Xi(e^b + e_o) - p^b \geq 0)m - f(e^b) \\
&= [p^\dagger - \hat{c}(e^\dagger)] \Pr(V - \Xi(e^\dagger + e_o) - p^\dagger \geq 0)m - \hat{f}(e^\dagger) \\
&< [\hat{p}^b - \hat{c}(\hat{e}^b)] \Pr(V - \Xi(\hat{e}^b + e_o) - \hat{p}^b \geq 0)m - \hat{f}(\hat{e}^b) \\
&= [p^\triangle - c(\hat{e}^b)] \Pr(V - w(\hat{e}^b) - \Xi(\hat{e}^b + e_o) - p^\triangle \geq 0)m - f(\hat{e}^b) \\
&= \pi^b(p^\triangle, \hat{e}^b)
\end{aligned}
$$

where $p^\triangle = \hat{p}^b - w(\hat{e}^b)$ and the inequality is due to the fact that $(\hat{p}^b, \hat{e}^b)$ is the firm's optimal solution in the newly defined model and $(\hat{p}^b, \hat{e}^b) \neq (p^\dagger, e^\dagger)$. However, the result above contradicts the fact that (p^b, e^b) is the firm's optimal solution in the original model. Thus, must have $(\hat{p}^b, \hat{e}^b) = (p^b + w(e^b), e^b)$. $\square$

Proof of Lemma V4.1: Given g, the firm's profit function is $\pi(p, e, \tau, \tau_o|g) = (p - c(e) - g(\tau + \tau_o))d(p, e, \tau, \tau_o) - f(e)$. Suppose there exist $g_1 < g_2$ such that $\tau^*(g_1) + \tau_o^*(g_1) = 0 < \tau^*(g_2) + \tau_o^*(g_2)$. Then,

$$
\begin{aligned}
&\pi(p^*(g_2), e^*(g_2), \tau^*(g_2), \tau_o^*(g_2)|g_2) \\
&> \pi(p^*(g_1), e^*(g_1), \tau^*(g_1), \tau_o^*(g_1)|g_2) \\
&= \pi(p^*(g_1), e^*(g_1), \tau^*(g_1), \tau_o^*(g_1)|g_1) \\
&> \pi(p^*(g_2), e^*(g_2), \tau^*(g_2), \tau_o^*(g_2)|g_1),
\end{aligned}
$$

where the equality is because $\tau^*(g_1) + \tau_o^*(g_1) = 0$. However, this is contradictory to the fact that

$$
\begin{aligned}
&\pi(p^*(g_2), e^*(g_2), \tau^*(g_2), \tau_o^*(g_2)|g_2) - \pi(p^*(g_2), e^*(g_2), \tau^*(g_2), \tau_o^*(g_2)|g_1) \\
&= (g_1 - g_2)(\tau^*(g_2) + \tau_o^*(g_2))d(p^*(g_2), e^*(g_2), \tau^*(g_2), \tau_o^*(g_2)) < 0
\end{aligned}
$$

Thus, we can conclude the result. $\square$