The Joint Determination of Haircuts and Interest Rates for Collateralized Loans in Shadow Banking

Jinji Hao

Victoria University of Wellington

SUPPLEMENTARY MATERIALS

Links and Contributions to Gottardi et al. (2019) and Simsek (2013)

Both Gottardi et al. (2019) and Simsek (2013) adopt the collateral equilibrium approach and essentially assume the same limited commitment of borrowers as my paper. In this section, I illustrate the links of my paper to these two papers and discuss the differences and contributions of my paper.

First note that Gottardi et al. (2019) differs from my paper in many aspects, leading to results that are not directly comparable. Distinct from my paper and the practice, Gottardi et al. (2019) define the haircut as the dollar amount of the down payment (i.e., the equity component of the asset price) rather than as a percentage of the endogenous collateral asset price and consider loan contracts with state-contingent promises instead of a fixed face value. Gottardi et al. (2019) also consider collateral re-use, non-pecuniary costs of default, and recourse loans. Moreover, in terms of analysis, Gottardi et al. (2019) does not study interest rates, does not distinguish between downside and upside quality of collateral, and does not analyze the effects of depositors' saving needs. In order to see the links more clearly, I pick a single feature of their model—specifically, the quasilinear preference of depositors—and incorporate it into my model. I then repeat all the comparative statics of my paper and show that this single feature alone generates substantial differences in terms of model outcomes and predictions.

The models of Simsek (2013) and my paper are more comparable. Nevertheless, the comparative statics of my paper are completely different from those of Simsek (2013). To draw the comparison, I also repeat all the comparative statics of my paper within the model of Simsek (2013).

S.1. Quasilinear Preferences for Depositors

I consider a variant of my model in the paper that differs in only one way: depositors have quasilinear preferences with their date-0 expected utility given by

$$c_0^d + \mathbb{E}\big[u(c_1^d(\omega))\big].$$

To make the analysis comparable, I consider the same type of equilibrium allocation in this model with quasilinear preferences; that is, banks hold all the risky assets and issue collateralized bonds.[1]

The key difference caused by quasilinear preferences is that the equilibrium bond face value ω^* is determined by

$$u'(e_1^d + K\omega^*) = 1, \tag{S.1}$$

which, in contrast to (29) of my model in the paper, says that the depositors' marginal utility in states $\omega > \omega^*$ at date 1 equals their marginal utility of 1 at date 0. It implies that collateral quality does not affect the equilibrium bond face value ω^*. In particular, it implies that, unlike in my model in the paper, there is no income effect of bond price changes on depositors—bond price changes do not affect depositors' date-1 consumption and thus the equilibrium bond face value ω^*. Depositors consume what is left over after the purchase of bonds at date 0, $c_0^d = e_0^d - Kq$. Equation (S.1) also implies that, unlike in my model in the paper, depositors' initial consumption good endowment does not affect the equilibrium bond face value ω^*. Although Equation (S.1) does imply that the supply of assets affects the equilibrium bond face value ω^* as in my model in the paper, it is for a different reason. The other difference caused by quasilinear preferences is that depositors' marginal rate of substitution is now given by

$$MRS^d(\omega) = u'(e_1^d + K\omega) \text{ for } \omega < \omega^* \text{ and } MRS^d(\omega) = MRS^b(\omega) = 1 \text{ for } \omega \geq \omega^*, \tag{S.2}$$

which is in contrast to (26) of my model in the paper. The bond and the risky asset are still priced according to (27) and (28) in the paper, respectively.

[1] This requires assuming that there exists a solution $\omega^* \in [\underline{\omega}, \bar{\omega}]$ to (S.1).

S.2. Heterogeneous Beliefs

Compared with my model, there are two fundamental differences in Simsek (2013) that result in distinct model predictions. First, in Simsek's model, both depositors and banks are risk-neutral but have heterogeneous beliefs. They maximize their date-0 expected utility

$$c_0^i + \mathbb{E}_i\left[c_1^i(\omega)\right]$$

for $i = d, b$, respectively.[2] Banks are optimistic, while depositors are pessimistic. Their prior beliefs about the state at date 1 are given by the probability distribution F_i over Ω, respectively, which satisfy the hazard-rate order. This feature makes banks the natural buyers of the risky assets.

Second, in Simsek's model, at date 0, banks are cash constrained (e_0^b is small), while depositors are cash rich (e_0^d is large). These two features of the model motivate banks to buy the risky assets by issuing collateralized loans to depositors. The bond is priced by depositors with

$$q = \mathbb{E}_d[\min(\omega, \omega^*)]. \tag{S.3}$$

Note that banks' perceived cost of debt financing is

$$r_b^{per}(\omega^*) \equiv \frac{\mathbb{E}_b[\min(\omega, \omega^*)]}{\mathbb{E}_d[\min(\omega, \omega^*)]} - 1, \tag{S.4}$$

which is greater than 0 due to the disagreement about the probability of default. Therefore, if banks are endowed with all the risky assets as in my model, there would be no trade as banks would not be cash constrained. In Simsek's model, all the risky assets are instead initially endowed to unmodeled agents who sell their assets at date 0. Nevertheless, in my model, modeling the supply of assets in the same way as Simsek (2013) does not change the equilibrium, except for banks' date-0 consumption, which is immaterial. With this interpretation, the two models become comparable.

There are two forces that pin down the equilibrium bond face value ω^* and the risky asset price

[2] In the original model of Simsek (2013), agents only consume at date 1 with the consumption good (a dollar) being storable. But we can equivalently assume that agents consume at both dates and consider the consumption good as perishable in his model due to the risk-neutral nature of the agents and the absence of discounting.

p. First, banks maximize their expected leveraged return on equity

$$R_b^L(\omega^*) \equiv \frac{\mathbb{E}_b[\omega] - \mathbb{E}_b[\min(\omega, \omega^*)]}{p - \mathbb{E}_d[\min(\omega, \omega^*)]}. \tag{S.5}$$

The optimality implies

$$p = \underbrace{\mathbb{E}_d[\min(\omega, \omega^*)]}_{\text{debt component } q} + \underbrace{\frac{1 - F_d(\omega^*)}{1 - F_b(\omega^*)}\mathbb{E}_b[\max(\omega - \omega^*, 0)]}_{\text{equity component } E}, \tag{S.6}$$

which is different from the decomposition in (28) of my model in the paper in two aspects.[3] Depositors' $MRS^d(\omega)$ is endogenous in my paper, while it is constant in Simsek's model due to universal risk-neutrality. Moreover, belief heterogeneity in Simsek's model leads to the extra term $\frac{1-F_d(\omega^*)}{1-F_b(\omega^*)}$ in the decomposition (S.6). As a result, the asset price p is decreasing in the bond face value ω^* in (S.6) but is increasing in ω^* in (28) of my model in the paper. Second, the asset market clears (banks' date-0 budget constraint) with

$$p = \underbrace{\mathbb{E}_d[\min(\omega, \omega^*)]}_{\text{debt component } q} + \underbrace{\frac{e_0^b}{K}}_{\text{equity component } E}. \tag{S.7}$$

Substituting (S.6) into (S.7), we have the bond face value ω^* determined by

$$\frac{1 - F_d(\omega^*)}{1 - F_b(\omega^*)}\mathbb{E}_b[\max(\omega - \omega^*, 0)] = \frac{e_0^b}{K}, \tag{S.8}$$

which is banks' budget constraint in holding the equity component of the risky asset. In contrast, the condition (29) of my model in the paper is depositors' budget constraint in holding the debt component of the risky asset. This implies that banks' date-0 consumption good endowment e_0^b matters in Simsek's model, while depositors' date-0 consumption good endowment e_0^d matters in my model. Note that the equity component on the left-hand side of (S.8) is decreasing in ω^* given the assumed hazard-rate order, while the debt component on the left-hand side of (29) is increasing in ω^*. This implies that the collateral supply K has the opposite effects on the bond face value ω^* in these two models. Finally, (S.8) can also be interpreted as the equality of the two equity components in (S.6) and (S.7). It implies that, unlike in my model, the value of the equity

[3] Equation (S.6) is a rearrangement of equation (11) in Simsek (2013). Equation (S.7) below is equation (16) in Simsek (2013) after the supply of the risky asset is generalized from one unit to K units.

component E of the risky asset in equilibrium, $\frac{e_0^b}{K}$, is not affected by agents' beliefs about collateral quality in Simsek's model.

The discount factor (equivalently the interest rate) and the ratio of equity and debt components of the asset price (equivalently the haircut) are given by

$$\frac{1}{1+R} = \frac{q}{\omega^*} = \mathbb{E}_d\left[\min\left(\frac{\omega}{\omega^*}, 1\right)\right] \quad \text{and} \quad \frac{1}{1-H} - 1 = \frac{E}{q} = \frac{e_0^b/K}{\mathbb{E}_d[\min(\omega, \omega^*)]}, \tag{S.9}$$

respectively.

S.3. Upside Quality of Collateral

Remark 1. *In the model with quasilinear preferences, the upside quality of collateral does not affect the bond face value ω^*, as in my model in the paper, which means that an improvement in the upside quality of collateral has the same effects in this model with quasilinear preferences as in my model in the paper. But the reasons behind this phenomenon differ, as evident from (S.1) and (29). Specifically, the condition (S.1) that characterizes the bond face value ω^* in the model with quasilinear preferences says that the depositors' marginal utility in states $\omega > \omega^*$ at date 1 equals their marginal utility of 1 at date 0. However, the condition (29) that characterizes the bond face value ω^* in my model in the paper is the intersection of depositors' consumption smoothing curve and depositors' bond pricing curve.*

Remark 2. *Simsek (2013) conducts the comparative statics of how disagreements between depositors and banks regarding the upside or downside quality of collateral impact asset prices and haircuts. This involves simultaneously rendering banks more optimistic and depositors more pessimistic. However, I improve the objective quality of collateral in the comparative statics analysis of my model. To make the predictions more comparable, I also conduct comparative statics analysis within his model, where I improve the upside or downside quality of collateral under the subjective beliefs of both depositors and banks.*

In Simsek's model, when banks become more optimistic about the upside quality of collateral over $(\omega^*, \bar{\omega})$, the bond face value ω^* increases; the asset price p increases due to a higher debt *component of the asset price (the equity component does not change), resulting in a lower haircut H; the interest rate R increases due to a higher default risk. However, depositors' belief about the upside quality of collateral over $(\omega^*, \bar{\omega})$ has no effect on the equilibrium.*

In contrast, in my model, when the upside quality of collateral improves, the bond face value ω^* and the interest rate R do not change; the asset price p increases due to a higher equity component of the asset price (the debt component does not change), resulting in a higher haircut H.

S.4. Downside Quality of Collateral

Remark 3. *In the model with quasilinear preferences, the downside quality of collateral does not affect the bond face value ω^*. This is because equation (S.1) that characterizes ω^* does not depend on the downside quality of collateral. This means that when the downside quality of collateral is improved, the bond face value channel in my model in the paper is absent in this model with quasilinear preferences, which would always predict a decrease in the haircut H. The asset price p increases due to a higher debt component (the equity component does not change), and the interest rate falls accordingly.*

Remark 4. *In Simsek's model, banks' beliefs about the downside quality of collateral over $(\underline{\omega}, \omega^*)$ do not impact the equilibrium. However, when depositors become less pessimistic about the downside quality of collateral over $(\underline{\omega}, \omega^*)$, it raises the bond price, despite no effect on the bond face value ω^*. Consequently, it has the same effects as an improvement in the downside quality of collateral in the model with quasilinear preferences for depositors.*

In contrast, as the downside quality of collateral improves in my model, the bond face value ω^* decreases, both reducing the interest rate R; the asset price p increases due to higher both debt and equity components of the asset price; the haircut H decreases if and only if the upside quality is high or the downside quality is low.

Remark 5. *Unlike in my model in the paper, depositors' date-0 consumption good endowment* e_0^d *has no effect on the equilibrium except for depositors' consumption at date 0 in the model with quasilinear preferences. The reason is that the bond face value* ω^* *is determined by (S.1), which does not depends on* e_0^d*. Nevertheless, an increased supply of the risky asset* K *has the same effects on variables of interest in the model with quasilinear preferences as in my model in the paper. The model with quasilinear preferences predicts a decrease in both the bond face value* ω^* *and the asset price* p*, but through a different mechanism. First, the bond face value* ω^* *decreases because* $K\omega^*$ *must be a constant as implied by (S.1). Second, in this model with quasilinear preferences, the asset price* p *decreases because a larger supply of the risky asset* K *reduces depositors' marginal utility at date 1 in states* $\omega < \omega^*$*, while in my model in the paper, it is because a lower* ω^* *increases depositors' marginal utility at date 0. The model with quasilinear preferences also predicts an increase in the haircut and a decrease in the interest rate.*[4]

Remark 6. *As the supply of the risky asset* K *increases, Simsek's (2013) model generates opposite predictions for the variables* ω^**,* H*, and* R *compared to my model. In his model, to finance the purchase of additional risky assets, cash-constrained banks must borrow more by raising the bond face value* ω^**. The interest rate* R *increases due to higher default risk, and the haircut* H *decreases as a result of both reduced equity contributions by banks and larger loans. Nevertheless, Simsek's (2013) model generates the same prediction for the asset price* p *as my model. This is because the tightened date-0 budget constraint of banks depresses the asset price. Technically, this occurs because, unlike in my model, a higher bond face value* ω^* *reduces the risky asset price in his model.*

For the effects of agents' consumption good endowments at date 0, although depositors' consumption good endowment e_0^d *matters in my model, it has no effect on the equilibrium in Simsek's model. However, although a larger banks' consumption good endowment* e_0^b *has no effect on the equilibrium in my model, it relaxes banks' date-0 budget constraint in Simsek's model, having exactly*

[4]A lower ω^* increases the equity component E of the asset price, so a higher haircut H follows. Although a higher K reduces $\frac{q}{\omega^*}$, this effect is dominated (by Assumption 3) by a lower ω^*, which increases $\frac{q}{\omega^*}$. A lower interest rate R follows.

the opposite effects of an increased supply of the risky asset K in his model.

S.6. Depositors' Risk Aversion

Remark 7. *In the model with quasilinear preferences for depositors, the effects of a higher depositors' risk aversion are also the same as those of an improved downside quality of collateral. Under Assumption A.1, (S.1) is reduced to $e_1^d + K\omega^* = 1$, so the bond face value ω^* is unaffected by the risk aversion. A higher risk aversion increases depositors' marginal rate of substitution and thus the bond price q, leading to a higher asset price, a lower interest rate, and a lower haircut.*

Remark 8. *In Simsek's (2013) model, as both types of agent are risk neutral, there is no corresponding comparative statics with respect to risk aversion. Geanakoplos (2003, 2009) notes that heterogeneity in beliefs may be regarded as a reduced-form version of heterogeneity in risk aversion in terms of determining who are the natural buyers of collateral assets because differences in risk aversion mean different risk-adjusted probabilities. Then one may tend to think that an increase in depositors' risk aversion in my model is corresponding to an increase in the belief disagreements in Simsek's model. However, this is not exactly the case. In my model, the equilibrium admits a representation in terms of a risk-neutral measure for the risk-averse depositors.[5] The endogenously determined $MRS^d(\omega)$ of risk-averse depositors is equal to that of the risk-neutral banks in high states but is higher in low states. So the associated risk-adjusted (risk-neutral) belief of risk-averse depositors adjusts probabilities of low states upward and probabilities of high states downward. But importantly, their implicitly associated risk-free discount factor (the mean $MRS^d(\omega)$), which can be represented by a modified time discount factor, is higher than the risk-neutral banks. Therefore, within a risk-neutral representation, the equilibrium features both heterogeneity in beliefs and heterogeneity in time preferences, and these heterogeneities are also endogenous rather than exogenously fixed. This is why the characterizations of equilibria in these two models are different. Not*

[5] To be precise, the equilibrium allocations and prices in the original economy can be supported in a new economy where the originally risk-averse depositors become risk-neural and their belief and the time preference are replaced with their risk-adjusted probabilities and mean pricing kernel, respectively. But note that the equilibrium concept in the original economy is stronger since the equilibrium allocations must be consistent with the pricing kernel, while in the new economy the risk-neutral belief is fixed.

surprisingly, the model predictions are also different. In Simsek's model, when depositors become more pessimistic overall, the bond face value ω^* decreases, the asset price p decreases, and the haircut H increases.

ADDITIONAL REFERENCE

[1] Geanakoplos, John, 2009, The Leverage Cycle, *NBER Macroeconomic Annual* 24, 1-65.