Appendix B (Online Appendix).

"Demographic Diversity and Collusion in Teams" by Jonathan Glover and Eunhee Kim

This online appendix consists of i) the proofs for two extensions, team turnover and continuous effort, ii) the analysis for ex ante collusion, and iii) the analysis for strong substitutability.

Proof of Proposition 3

We first construct the cooperation and collusion constraints to show that bilateral and unilateral replacement are equivalent in terms of the constraints imposed. We then show that the optimal assignment qualitatively remains the same as in Proposition 2.

With bilateral termination, agent i enjoys $f(2)w_k^i - c$ as long as neither agent deviates, and they continue their play with probability $\delta_i\delta_j$, thus the payoff is $\left(f(2)w_k^i - c\right)\sum_{n=0}^{\infty}\delta_i^n\delta_j^n = \frac{f(2)w_k^i - c}{1-\delta_i\delta_j}$. Upon deviation in period t, agent i enjoys $f(1)w_k^i$ and obtains $f(0)w_k^i$ or $f(1)w_k^i - c$ in the following period $t+1$ discounted by $\delta_i\delta_j$, by $\left(\delta_i\delta_j\right)^2$ in period $t+2$, and so on. Thus, the (Cooperation) constraint is written as:

$$\frac{f(2)w_k^i - c}{1-\delta_i\delta_j} \geq f(1)w^i + \frac{\delta_i\delta_j}{1-\delta_i\delta_j}f(0)w_k^i \Rightarrow f(2)w_k^i - c \geq \left(1-\delta_i\delta_j\right)f(1)w_k^i + \delta_i\delta_j f(0)w_k^i,$$

$$\frac{f(2)w_k^i - c}{1-\delta_i\delta_j} \geq f(1)w^i + \frac{\delta_i\delta_j}{1-\delta_i\delta_j}\left(f(1)w_k^i - c\right)$$

$$\Rightarrow f(2)w_k^i - c \geq \left(1-\delta_i\delta_j\right)f(1)w_k^i + \delta_i\delta_j\left(f(1)w_k^i - c\right).$$

With unilateral termination, if a new agent follows the previously agreed play, then analysis is the same as for our main model presented in the paper. Suppose instead that the new agent resets the play. As long as the two agents play as agreed, agent i's payoff from playing joint work is given as $\frac{f(2)w_k^i - c}{1-\delta_i}$. Upon deviation, agent i is penalized by the teammate agent as long as he survives. This periodic payoff is discounted by $\delta_i\delta_j$; thus, he enjoys a discounted payoff of $f(0)w_k^i\sum_{n=1}^{\infty}\delta_i^n\delta_j^n = \frac{\delta_i\delta_j f(0)w_k^i}{1-\delta_i\delta_j}$ or $(f(1)w_k^i - c)\sum_{n=1}^{\infty}\delta_i^n\delta_j^n = \frac{\delta_i\delta_j\left(f(1)w_k^i - c\right)}{1-\delta_i\delta_j}$. If a teammate exits, a new agent joins and resets the play. Upon deviation in period t, the reset play occurs with probability $\delta_i(1-\delta_j)$ in period $t+1$, $\ldots\delta_i^2\left\{\delta_j\left(1-\delta_j\right) + \left(1-\delta_j\right)\delta_j + \left(1-\delta_j\right)^2\right\} = \delta_i^2 - \delta_i^2\delta_j^2$ in period $t+2$, … and $\delta_i^n - \delta_i^n\delta_j^n$ in period $t+n$, thus enjoying the payoff of

$$\left(f(2)w_k^i - c\right)\sum_{n=1}^{\infty}\left(\delta_i^n - \delta_i^n\delta_j^n\right) = \left(f(2)w_k^i - c\right)\frac{\delta_i(1-\delta_j)}{(1-\delta_i)(1-\delta_i\delta_j)}.$$

Then, the (Cooperation) constraint under unilateral termination is:

$$\frac{f(2)w_k^i - c}{1-\delta_i} \geq f(1)w^i + \frac{\delta_i\delta_j f(0)w_k^i}{1-\delta_i\delta_j} + \left(f(2)w_k^i - c\right)\frac{\delta_i(1-\delta_j)}{(1-\delta_i)(1-\delta_i\delta_j)}$$

$$\Rightarrow f(2)w_k^i - c \geq \left(1-\delta_i\delta_j\right)f(1)w_k^i + \delta_i\delta_j(f(1)w_k^i - c).$$

The same logic is extended to the case where the deviating agent plays work as a punishment. Thus, we have the identical (Cooperation) constraints in both bilateral and unilateral termination cases.

With bilateral termination, the (Pareto) constraint becomes:

$$\frac{f(2)w_k^i - c}{1-\delta_i\delta_j} \geq f(1)w_k^i - c + \frac{\delta_i\delta_j}{1-\delta_i\delta_j}U_i\left(\beta_k; w_k^i\right)$$

$$\Rightarrow f(2)w_k^i - c \geq \left(1-\delta_i\delta_j\right)\left(f(1)w_k^i - c\right) + \delta_i\delta_j U_i\left(\beta_k; w_k^i\right),$$

where $U_i\left(\beta_k; w_k^i\right) = f(1)w_k^i - \beta_k c$ (or, $f(1)w_k^i - (1-\beta_k)c$). We will shortly derive β_k. Thus, the functional form of the optimal contract remains the same except for $\delta_i\delta_j$ instead of δ_i.

With unilateral termination, if the new agent continues the collusive play, then the model is again equivalent to our main model. If the replacement agent resets the play by playing joint work,

$$\frac{f(2)w_k^i - c}{1-\delta_i} \geq f(1)w_k^i - c + \frac{\delta_i\delta_j U_i\left(\beta_k; w_k^i\right)}{1-\delta_i\delta_j} + \left(f(2)w_k^i - c\right)\frac{\delta_i(1-\delta_j)}{(1-\delta_i)(1-\delta_i\delta_j)}$$

$$\Rightarrow f(2)w_k^i - c \geq \left(1-\delta_i\delta_j\right)\left(f(1)w_k^i - c\right) + \delta_i\delta_j U_i\left(\beta_k; w_k^i\right),$$

which is the same constraint as under bilateral termination.

We now derive a feasible set of β_k. Consider diverse assignment. With team horizon $\delta_h\delta_l$, agent h does not regret if $\beta_d \leq 1 - \frac{f(2)-f(1)}{c\delta_h\delta_l}w_d^h$. That is, agent h is willing to work up to $1 - \frac{f(2)-f(1)}{c\delta_h\delta_l}w_d^h$. Similarly, agent l is willing to collude if $\beta_d \geq \frac{f(2)-f(1)}{c\delta_h\delta_l}w_d^l$. Thus, for any contract $\boldsymbol{w_d}$ that ensures cooperation, the set of feasible β_d is:

$$B(\boldsymbol{w_d}) = \left[\frac{f(2)-f(1)}{c\delta_h\delta_l}w_d^l, 1 - \frac{f(2)-f(1)}{c\delta_h\delta_l}w_d^h\right].$$

This set is non-empty if $\frac{f(2)-f(1)}{c\delta_h\delta_l}w_d^l < 1 - \frac{f(2)-f(1)}{c\delta_h\delta_l}w_d^h$, or $\frac{w_d^l}{\delta_h\delta_l} + \frac{w_d^h}{\delta_h\delta_l} < \frac{c}{f(2)-f(1)}$. The cooperative wage is $w_d^h = \frac{c}{f(2)-f(1)} \cdot \frac{1}{1+\delta_h\delta_l(x-1)} = w_d^l$, where $x = \frac{f(2)-f(0)}{f(2)-f(1)}$ is defined in Assumption 2. The set $B(\boldsymbol{w_d})$ is non-empty if:

$$2 < \delta_h\delta_l + (\delta_h\delta_l)^2(x-1) \Rightarrow \delta_h\delta_l > \frac{\sqrt{8x-7}-1}{2(x-1)} = \delta^*.$$

The above threshold, δ^*, is the same for homogenous assignments. As in the main setting, it can be shown that any regret-free collusion β_d is self-enforcing for any $w_d^i \geq w_d^{i*}$.

From the (Pareto) constraint for the agent who works with $(1-\beta_d)$,

$$\big(f(2)-f(1)\big)\big(w_d^h + w_d^l\big) \geq \delta_h\delta_l c.$$

With team horizon $\delta_i\delta_j$, even in diverse teams, the principal randomly selects one agent to receive the cooperative wage. Since $w_d^i \geq \frac{c}{f(2)-f(1)} \cdot \frac{1}{1+\delta_h\delta_l(x-1)}$ for cooperation,

$$\frac{1}{\delta_h\delta_l} \cdot \frac{1}{1+\delta_h\delta_l(x-1)} \leq \beta_d \leq 1 - \frac{1}{\delta_h\delta_l} \cdot \frac{1}{1+\delta_h\delta_l(x-1)}.$$

The same logic is extended to homogeneous assignment.

To summarize, under both bilateral and unilateral terminations, the (Cooperation) and (Pareto) constraints qualitatively remain the same. Compared to Section 5, the only change is the adjustment by $\delta_i\delta_j$ instead of δ_i. Therefore, the optimal contracts we identified remain qualitatively the same except for the adjustment by $\delta_i\delta_j$.

When cooperation incentives are in place, we have $W_d > W_s$ because:

$$\frac{4c(1-\delta_h\delta_l)}{f(2)-f(1)} > \frac{2c\left(1-\delta_h^2\right)}{f(2)-f(1)} + \frac{2c\left(1-\delta_l^2\right)}{f(2)-f(1)} \Rightarrow 0 < (\delta_h - \delta_l)^2,$$

if the stage game equilibrium is (*work, shirk*) and (*shirk, work*), or

$$\frac{4c}{f(2)-f(1)} \cdot \frac{1}{1+\delta_h\delta_l(x-1)} > \frac{2c}{f(2)-f(1)} \cdot \frac{1}{1+\delta_h^2(x-1)} + \frac{2c}{f(2)-f(1)} \cdot \frac{1}{1+\delta_l^2(x-1)} \Rightarrow 2 > \delta_h\delta_l(x-1),$$

if the stage game is (*shirk, shirk*). The last inequality is because $x-1 < 2$ and $\delta_h\delta_l \leq 1$.

On the other hand, when the (Pareto) constraints bind across all teams, we have $W_d < W_s$:

$$\frac{2c\delta_h\delta_l}{f(2)-f(1)} < \frac{c\left(\delta_h^2+\delta_l^2\right)}{f(2)-f(1)} \Rightarrow 0 < (\delta_h - \delta_l)^2.$$

When the (Pareto) constraints bind under diverse assignment and the team of two agent hs but not under the team of two agent ls, we have $W_d < W_s$:

$$\frac{4c\delta_h\delta_l}{f(2)-f(1)} < \frac{2c\delta_h^2}{f(2)-f(1)} + \frac{2c}{f(2)-f(1)} \cdot \frac{1}{1+\delta_l^2(x-1)} \Rightarrow 2\delta_h\delta_l < \delta_h^2 + \frac{1}{1+\delta_l^2(x-1)}.$$

The last inequality is because the (Pareto) constraint does not bind under the team of two agent ls, thus $\frac{1}{1+\delta_l^2(x-1)} > \delta_l^2$, and replacing $\frac{1}{1+\delta_l^2(x-1)}$ with δ_l^2 in the above inequality leads to $0 < (\delta_h - \delta_l)^2$. Lastly, when the (Pareto) constraint binds only under the team of two agent hs, we have $W_d < W_s$ if:

$$\frac{4c}{f(2)-f(1)} \cdot \frac{1}{1+\delta_h\delta_l(x-1)} < \frac{2c\delta_h^2}{f(2)-f(1)} + \frac{2c}{f(2)-f(1)} \cdot \frac{1}{1+\delta_l^2(x-1)}. \qquad (A1)$$

Observe that when the (Pareto) constraint begins to bind for the team of two agent hs (i.e., at the threshold $\delta_h^2 = \delta^*$), we already showed that $W_d > W_s$. In the limit, if δ_h converges to $\bar{\delta}$ (which is arbitrarily close to 1), we have:

$$W_d < W_s \Rightarrow \delta_l < 2 + \delta_l^2(x-1)$$

which is always satisfied. Since the left-hand side of inequality (A1) is decreasing in δ_h, whereas the right-hand side is increasing in δ_h, there exists a unique $\delta^T \in [\delta^*, \bar{\delta}]$ such that $W_d < W_s$ for $\delta_h > \delta^T$ and $W_d > W_s$ for $\delta_h < \delta^T$. ■

Proof of Proposition 4

Given wage w_k^i and teammate's effort e_j, agent i chooses effort e_i to maximize his payoff:

$$\frac{2}{5}\left(e_i + e_j + 2\right)^{\frac{1}{2}} w_k^i - e_i.$$

It is immediate to see that the incentive wage (in a one-shot game) eliciting effort 1 from each agent is $w_k^N = 10$.

Homogeneous Assignment. From each agent's first-order condition,

$$e_i = \frac{1}{25} {w_s^i}^2 - 2 - e_j, e_j = \frac{1}{25} {w_s^j}^2 - 2 - e_i.$$

Note that if $w_s^i = w_s^j = w_s$ and assuming symmetric effort for homogenous assignment, then $e_i = e_j = \frac{w_s^2}{50} - 1$. As long as $\frac{w_s^2}{50} - 1 \geq 0$, the stage game equilibrium is $\left(e_s^i, e_s^j\right) = \left(\frac{w_s^2}{50} - 1, \frac{w_s^2}{50} - 1\right)$. If $\frac{w_s^2}{50} - 1 < 0$, then (0,0) is the stage game equilibrium. We will shortly see that for $\delta_l \geq 0.5$, (0,0) is the stage game equilibrium. Given the teammate agent's effort 1, the best one-shot deviation effort of the agent i is $e_i^*(1) = \frac{1}{25} {w_s^i}^2 - 3$, and $e_i^*(1) \in [0,1]$ provided that $0 \leq \frac{1}{25} {w_s^i}^2 - 3 \leq 1$. If $\frac{1}{25} {w_s^i}^2 - 3 < 0$, then his best one-shot deviation effort is 0. We again shortly confirm that for $\delta_l \geq 0.5$, 0 is the best one-shot deviation effort. Then, the cooperative wage is found by solving the following equation as an equality:

$$\frac{2}{5}(2+2)^{\frac{1}{2}}w_s^i - 1 \geq (1-\delta_i)\left(\frac{2}{5}\left(3+e^*(1)\right)^{\frac{1}{2}}w_s^i - e_i^*(1)\right) + \delta_i\left(\frac{2}{5}\left(2+e_s^i+e_s^j\right)^{\frac{1}{2}}w_s^i - e_s\right),$$

which yields $w_s^{i*} = \frac{5}{2(2-\sqrt{3}+(\sqrt{3}-\sqrt{2})\delta_i)}$. It is immediate to see that $\frac{\left(w_s^{i*}\right)^2}{50} - 1 < 0$ and $\frac{1}{25}{w_s^{i*}}^2 - 3 < 0$ for $0.5 \leq \delta_l < \delta_h$, confirming $e_s^i = e_i^*(1) = 0$.

Let c denote the agents' collusive strategy combination. To maximize the total payoffs, c can be found from the first order condition of their total stage game payoffs:

$$\frac{2}{5} \times (c+c+2)^{\frac{1}{2}}\left(w_s^i + w_s^j\right) - c - c \Rightarrow c = \frac{\left(w_s^i+w_s^j\right)^2}{50} - 1.$$

For $0 \leq c < 1$, $0 \leq \frac{\left(w_s^i+w_s^j\right)^2}{50} - 1 < 1$ is required. To see if collusion c is Pareto-dominant, observe that by playing collusion c, the total payoffs are:

$$\frac{2}{5} \times (c+c+2)^{\frac{1}{2}}\left(w_s^i + w_s^j\right) - c - c\Big|_{c=\frac{\left(w_s^i+w_s^j\right)^2}{50}-1} = \frac{\left(w_s^i+w_s^j\right)^2}{25} + 2,$$

whereas by joint working, they get:

$$\frac{2}{5} \times (2+2)^{\frac{1}{2}}\left(w_s^i + w_s^j\right) - 2 = \frac{4}{5}\left(w_s^i + w_s^j\right) - 2.$$

Collusion c is Pareto dominant for the agents if:

$$\frac{\left(w_s^i+w_s^j\right)^2}{25} > \frac{4}{5}\left(w_s^i + w_s^j\right) - 4 \Rightarrow 10 > w_s^i + w_s^j.$$

To see if this collusion is also self-enforcing, note that given the teammate agent's effort c, the best one-shot deviation effort of the agent i is:

$$e_i^*(c) = \frac{1}{25}{w_s^i}^2 - 2 - c.$$

Each agent's payoff under collusion c can be expressed as $c+2$. Then, (c,c) is self-enforcing if:

$$(1-\delta_i)\left(\frac{2}{5}\left(c+e_i^*(c)+2\right)^{\frac{1}{2}}w_s^i - e_i^*(c)\right) + \delta\left(\frac{2}{5}\left(e_s^i+e_s^j+2\right)^{\frac{1}{2}}w_s^i - e_s\right) \leq c+2. \qquad (SE)$$

Since collusion arises for a high discount factor (as we will confirm shortly), the stage game equilibrium is $\left(e_s^i, e_s^j\right) = (0,0)$ and $e_i^*(c) = 0$. Then, the above inequality is always satisfied, thus ensuring that (c,c) is self-enforcing under homogenous assignment. To combat collusion, the principal ensures that $w_s^{i**} + w_s^{j**} = 10$.

Diverse Assignment. Under diverse assignment, the two agents have different discount factors. Using each agent's first order condition:

$$e_h = \frac{1}{25}{w_d^h}^2 - 2 - e_l, e_l = \frac{1}{25}{w_d^l}^2 - 2 - e_h.$$

To ensure that there is a stage game equilibrium from each agent's first order condition, consider $e_h + e_l + 2 = \frac{1}{25}{w_d^h}^2 < \frac{1}{25}{w_d^l}^2$, $e_l = 0$ and $e_h = \frac{1}{25}{w_d^h}^2 - 2$. Given agent l's choice, e_h is agent h's best response; given agent h's choice, zero is agent l's best response. When the teammate agent's effort is 1, the best one-shot deviation effort of agent i is $e_i^*(1) = \frac{1}{25}{w_d^i}^2 - 3$, and $e_i^*(1) \in [0,1]$ is required for feasibility. If w_d^i is small enough that $\frac{1}{25}{w_d^i}^2 < 3$, then his best one-shot deviation effort is 0. Then, the cooperative wage is found as follows:

$$\frac{2}{5}(2+2)^{\frac{1}{2}}w_d^h - 1 \geq (1-\delta_h)\left(\frac{2}{5}\left(3+e_h^*(1)\right)^{\frac{1}{2}}w_d^h - e_h^*(1)\right) + \delta_h\left(\frac{2}{5}(e_h + e_l + 2)^{\frac{1}{2}}w_d^h - \left(\frac{1}{25}{w_d^h}^2 - 2\right)\right) \text{ for agent } h,$$

$$\frac{2}{5}(2+2)^{\frac{1}{2}}w_d^l - 1 \geq (1-\delta_l)\left(\frac{2}{5}\left(3+e_l^*(1)\right)^{\frac{1}{2}}w_d^l - e_l^*(1)\right) + \delta_l\left(\frac{2}{5}(e_h + e_l + 2)^{\frac{1}{2}}w_d^l\right) \text{ for agent } l.$$

We can shortly confirm that for $\delta_l \geq 0.5$, we have $e_h = 0 = e_h^*(1) = e_l^*(1)$. Thus, the optimal cooperative wages are:

$$w_d^h = \frac{5}{2(2-\sqrt{3}+(\sqrt{3}-\sqrt{2})\delta_h)}, w_d^l = \frac{5}{2(2-\sqrt{3}+(\sqrt{3}-\sqrt{2})\delta_l)} \text{ for } e_h = 0 = e_h^*(1) = e_l^*(1).$$

It is straightforward to see that $\frac{1}{25}{w_d^h}^2 - 2 < 0$ and $\frac{1}{25}{w_d^l}^2 < 3$ given $0.5 \leq \delta_l < \delta_h$.

In diverse assignment, the agents' willingness to collude differs. Extending the same logic as under homogeneous assignment, consider $\frac{\left(w_d^h + w_d^l\right)^2}{25} - 2 = c_h + c_l$. The argument that collusion c Pareto dominates joint working is extended to (c_h, c_l). To see if the above collusion is Pareto-dominant relative to joint working for both agents:

$$\frac{2}{5} \times (c_h + c_l + 2)^{\frac{1}{2}}\left(w_d^h + w_d^l\right) - c_h - c_l\Big|_{c_h + c_l = \frac{\left(w_d^h + w_d^l\right)^2}{25} - 2} = \frac{\left(w_d^h + w_d^l\right)^2}{25} + 2 \geq \frac{4}{5}\left(w_d^h + w_d^l\right) - 2,$$

which is true for $w_d^h + w_d^l < 10$. Note that there are continuously many combinations of c_h, c_l such that $\frac{\left(w_d^h + w_d^l\right)^2}{25} - 2 = c_h + c_l$. For instance, consider the following collusion:

$$c_h = \frac{1}{25}w_d^h\left(w_d^h + w_d^l\right) - 1, c_l = \frac{1}{25}w_d^l\left(w_d^h + w_d^l\right) - 1,$$

which generates a per period payoff of $c_h + 2$ for agent h and $c_l + 2$ for agent l, and both agents strictly prefer collusion if $50 > w_d^h(20 - w_d^h - w_d^l)$ for agent h and $50 > w_d^l(20 - w_d^h - w_d^l)$ for agent l.

To see if this collusion is also self-enforcing, the best one-shot deviation of each agent is:

$$e_h^*(c_l) = \frac{1}{25} {w_d^h}^2 - 2 - c_l, e_l^*(c_h) = \frac{1}{25} {w_d^l}^2 - 2 - c_h.$$

Since collusion arises for sufficiently high discount factors (as we will confirm shortly), we have $e_h^*(c_l) = e_l^*(c_h) = e_h = e_l = 0$. Then, (c_h, c_l) is self-enforcing if:

$$(1 - \delta_h)\left(\frac{2}{5}\left(2 + c_l + e_h^*(c_l)\right)^{\frac{1}{2}} w_d^h - e_h^*(c_l)\right) + \delta_h \left(\frac{2}{5}(2 + e_h + e_l)^{\frac{1}{2}} w_d^h - e_h\right) \leq c_h + 2. \quad (SE_h)$$

$$(1 - \delta_l)\left(\frac{2}{5}\left(2 + c_h + e_l^*(c_h)\right)^{\frac{1}{2}} w_d^l - e_l^*(c_h)\right) + \delta_l \left(\frac{2}{5}(2 + e_h + e_l)^{\frac{1}{2}} w_d^l - e_l\right) \leq c_l + 2. \quad (SE_l)$$

It can be shown that when $\delta_h > \delta_l \geq 0.5$, with the cooperative wage w_d^{i*}, the above inequalities are always satisfied, thus ensuring that (c_h, c_l) is self-enforcing under diverse assignment.

To derive the discount factor thresholds, note that:

$$2w_s^{i*} < 10 \Rightarrow \delta_i > \frac{2\sqrt{3}-3}{2(\sqrt{3}-\sqrt{2})} \equiv \delta_c^* \approx 0.73 \text{ for homogeneous assignment,}$$

$$\text{given } \delta_h > \delta_c^*, w_d^{l*} + w_d^{h*} < 10 \Rightarrow \delta_l > \underline{\delta_c}(\delta_h) \text{ for diverse assignment,}$$

where $\underline{\delta_c}(\delta_h) = \frac{2(12-7\sqrt{3})+\left(7(\sqrt{3}-\sqrt{2})-4(3-\sqrt{6})\right)\delta_h}{4(3-\sqrt{6})-7(\sqrt{3}-\sqrt{2})-4(5-2\sqrt{6})\delta_h}$. $\underline{\delta_c}(\delta_h)$ is a decreasing function of δ_h and $\underline{\delta_c}(\delta_h) = \delta_c^*$ at $\delta_h = \delta_c^*$.

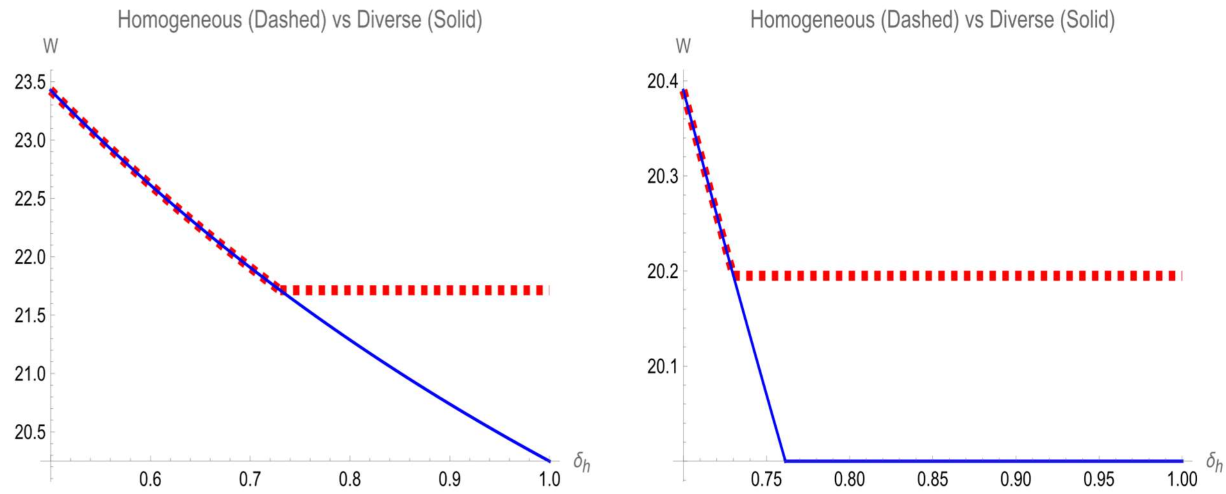
If $\delta_c^* \leq \delta_l < \delta_h$ (i.e., collusion arises across all teams), we have $w_k^{i**} + w_k^{j**} = 10$, thus diverse and homogeneous assignments perform equally. However, if $\underline{\delta_c}(\delta_h) \leq \delta_l < \delta_c^* < \delta_h$ (i.e., collusion arises in the team of two agent hs and diverse teams, but not in the team of two agent ls), diverse assignment is optimal:

$$W_s > W_d \Rightarrow 2w_s^{l*} + 10 > 20 \Rightarrow 10 < 2w_d^{l*},$$

which is true since the (Pareto) constraint does not bind in the team of two agent ls. If $\delta_l < \underline{\delta_c}(\delta_h) < \delta_c^* < \delta_h$ (i.e., collusion arises only in the team of two agent hs, diverse assignment is optimal:

$$W_s > W_d \Rightarrow 2w_s^{l*} + 10 > 2w_d^{l*} + 2w_d^{h*} \Rightarrow 10 > 2w_d^{h*},$$

which is true since the (Pareto) constraint binds in the team of two agent hs. In sum, whenever $\delta_l < \delta_c^* < \delta_h$, diverse assignment is strictly optimal. ■



(a) $\delta_l = 0.5$. Collusion arises only under the team of two agent hs.

(b) $\delta_l = 0.7$. Collusion arises in the team of two agent hs and the two teams under diverse assignment.

Figure 3. Total Wage Comparison with Continuous Effort

Ex Ante Collusion. In this extension, we consider an alternative notion of collusion: instead of requiring collusion to be Pareto-optimal at any point in the game (including after the agents' correlated randomization device assigns them to play particular actions), we now require only that collusion be Pareto-optimal as of the beginning of the game (ex ante). We maintain the requirement that collusion be self-enforcing, i.e., a subgame perfect equilibrium. To show how our results are changed (or maintained) under ex ante collusion, we consider both contracting and team design problems as in Section 5.

To break collusion under the ex-ante notion of Pareto optimality, the contract must satisfy the following (Pareto′) constraints:

$$f(2)w_k^i - c \geq \beta_k\big(f(1)w_k^i - c\big) + (1-\beta_k)f(1)w_k^i \text{ or}$$
$$f(2)w_k^j - c \geq \beta_k f(1)w_k^j + (1-\beta_k)\big(f(1)w_k^j - c\big) \text{ for any } \beta_k \in B(\boldsymbol{w_k}), \qquad \text{(Pareto}'\text{)}$$

where we normalize both sides of the constraints by multiplying by $(1-\delta_i)$. Again, we use weak rather than strict inequalities for expositional convenience. The left-hand side of the (Pareto′) constraint is the payoff from working, and the right-hand side represents the payoff under the proposed collusion. Observe that neither side of (Pareto′) constraints depends on the agents' discount factors. That is, under ex ante collusion, neither of the agents discounts the bribe at the point it is being considered (at the start of the game), because they do not yet know which action they will be assigned to play in the first period of the game. However, as in our main analysis, agent h has a stronger incentive for collusion and, thus, is willing to offer a greater bribe, and whether or not the (Pareto′) constraints bind depends on team assignment. When the (Pareto′) constraints bind across all assignments, the total collusion-proof wages are the same across all teams. Hence, if the collusion problem arises under all assignments (or, collusion does not arise under any assignment), then the principal is indifferent between diverse and homogenous assignment. In the remaining cases, the principal strictly prefers diverse assignment.

Proposition 5. (Ex-ante Collusion) *The minimum necessary collusion-proof* wages *satisfy* $w_k^{i**} + w_k^{j**} = \frac{c}{f(2)-f(1)}$. *Under homogeneous assignment, the (Pareto') constraint binds for* $\delta_i > 1/(x-1)$*, whereas under diverse assignment, the (Pareto') constraint binds for* $\delta_h \delta_l > 1/(x-1)^2$.

i) *For* $\delta_l < \delta_h \leq \delta^m$ *or* $\delta_h > \delta_l \geq 1/(x-1)$*, we have* $W_s = W_d$.

ii) *For* $\delta^m \leq \delta_l \leq 1/\delta_h(x-1)^2$ *and* $1/(x-1) < \delta_h$*, we have* $W_s > W_d$.

iii) *For* $1/\delta_h(x-1)^2 < \delta_l < 1/(x-1) < \delta_h$*, we have* $W_s > W_d$.

Under ex ante collusion, the focus is on the total wages paid to a team—that total must be large enough in order to deter collusion. The advantage of diverse assignment can be twofold. First, by pairing with agent l, it eliminates agent h's incentive for collusion. Second, it lowers the amount of extra wages that have to be paid to the team as a whole in order to ensure they will not collude but cooperate. This is because cross-subsidization (of incentives) exists within a diverse team: by combatting collusion, the saved wage from agent h can be transferred to agent l to ensure cooperative incentives. However, there is no such cross-subsidization within the team of two agent ls as they have identical implicit incentives, thus requiring more expensive cooperative wages compared to the collusion-proof wages paid to a diverse team. There is no longer a difference between the cost of the bribe to the briber and the benefit of the bribe to bribee to exploit, as the principal does under regret-free collusion.

Proof of Proposition 5

For ex-ante collusion, we have:

$$B(\boldsymbol{w_k}) = \left[\frac{f(2)-f(1)}{c} w_k^j, 1 - \frac{f(2)-f(1)}{c} w_k^i\right].$$

$B(\boldsymbol{w_k})$ is non-empty if $w_k^i + w_k^j \leq \frac{c}{f(2)-f(1)}$. As in regret-free collusion, it can be shown that any $\beta_k \in B(\boldsymbol{w_k})$ is also self-enforcing for both agents. Thus, we derive the optimal collusion-proof contract using the (Pareto′) constraint. Recall the ex-ante Pareto constraints:

$$f(2)w_k^i - c \geq U_i(\beta_k; w_k^i) \text{ for any } \beta_k \in B(\boldsymbol{w_k}), \quad \text{(Pareto}')$$

Consider a homogeneous team first. Recall that agent $i1$ works with probability β_s and agent $i2$ works with probability $1 - \beta_s$. As in regret-free collusion, the targeted agent's payoff obtains the maximum at $\beta_s = \frac{f(2)-f(1)}{c} w_s^{i2}$ and/or $\beta_s = 1 - \frac{f(2)-f(1)}{c} w_s^{i1}$. If $\beta_s = \frac{f(2)-f(1)}{c} w_s^{i2}$ is the binding collusion, we target agent $i1$.

$$f(2)w_s^{i1} - c \geq f(1)w_s^{i1} - \big(f(2) - f(1)\big)w_s^{i2} \Leftrightarrow \big(f(2) - f(1)\big)\big(w_s^{i1} + w_s^{i2}\big) \geq c.$$

Thus, the minimum necessary collusion-proof wages must satisfy $w_s^{i1} + w_s^{i2} = \frac{c}{f(2)-f(1)}$. If $\beta_s = 1 - \frac{f(2)-f(1)}{c} w_s^{i1}$, we target agent $i2$ and obtain the same result. Since ensuring $\big(f(2) - f(1)\big)\big(w_s^{i1} + w_s^{i2}\big) \geq c$ prevents collusion, any combination of two wages that ensure cooperative incentives while satisfying the inequality are collusion-proof, and the optimal correlated randomization is characterized as $\frac{f(2)-f(1)}{c} w_s^{i2} \leq \beta_s \leq 1 - \frac{f(2)-f(1)}{c} w_s^{i1}$.

Because both w_s^{i1} and w_s^{i2} must ensure the cooperation incentive, $w_s^{in} \geq w_s^{i*}, n = 1,2$ is required. The (Pareto′) constraint binds for $\delta_i > \frac{1}{x-1}$, because

$$\big(f(2) - f(1)\big)\big(w_s^{i*} + w_s^{i*}\big) < c \Leftrightarrow \delta_i > \frac{1}{x-1}.$$

That is, given that the cooperative wages are offered, agents find collusion Pareto-dominant and self-enforcing if $\delta_i > \frac{1}{x-1}$.

Now, consider diverse teams. Because our argument in homogeneous teams does not depend on the agents' discount factors, we make the same conclusion as above. Whether agent h is targeted or not, the minimum necessary collusion-proof wages satisfy $w_d^h + w_d^l = \frac{c}{f(2)-f(1)}$ where $w_d^i \geq w_d^{i*}, i = h, l$. The (Pareto′) constraint binds for $\delta_h \delta_l > \frac{1}{(x-1)^2}$. Because

$$\big(f(2) - f(1)\big)(w_s^{h*} + w_s^{l*}) < c \Leftrightarrow \delta_h \delta_l > \frac{1}{(x-1)^2}.$$

Whenever $\delta_h > \frac{1}{x-1}$, we have $\frac{1}{\delta_h (x-1)^2} < \frac{1}{x-1}$. Thus, for $\delta_l \leq \frac{1}{\delta_h (x-1)^2}$ and $\frac{1}{x-1} < \delta_h$, the (Pareto′)

constraint does not bind since $\delta_h\delta_l \leq \frac{1}{(x-1)^2}$. For $\frac{1}{\delta_h(x-1)^2} < \delta_l < \frac{1}{x-1} < \delta_h$ or $\frac{1}{x-1} < \delta_l < \delta_h$, the (Pareto′) constraint binds in diverse teams.

Let W_k denote the sum of wages under team $k \in \{s, d\}$. Since the cooperative wages do not depend on team assignment, if collusion does not arise under any assignment, we have $W_s = W_d$. If collusion arises across all teams, $\delta_h > \delta_l \geq 1/(x-1)$, we have $W_s = W_d$. However, if $\delta_l \leq 1/\delta_h(x-1)^2$ and $1/(x-1) < \delta_h$, collusion arises under the team of agent hs (homogeneous team) but does not arise under the team of agent ls and diverse teams. In this case:

$$W_s = \frac{c}{f(2)-f(1)}\left(1 + \frac{2}{1+\delta_l\,(x-1)}\right)$$
$$> W_d = \frac{c}{f(2)-f(1)}\left(\frac{2}{1+\delta_h\,(x-1)} + \frac{2}{1+\delta_l\,(x-1)}\right) \Leftrightarrow \delta_h > \frac{1}{x-1},$$

which is true since the (Pareto′) constraint binds under the team of agent hs for $1/(x-1) < \delta_h$. For $1/\delta_h(x-1)^2 < \delta_l < 1/(x-1) < \delta_h$, collusion arises under the team of agent hs and diverse teams, but does not arise under the team of agent ls. However, as in our main analysis, we have the following result.

$$W_s = \frac{c}{f(2)-f(1)}\left(1 + \frac{2}{1+\delta_l\,(x-1)}\right) > W_d = \frac{2c}{f(2)-f(1)} \Leftrightarrow \delta_l < \frac{1}{x-1}. \blacksquare$$

Strong Substitutability, $\boldsymbol{\frac{f(2)-f(0)}{f(2)-f(1)} \geq \frac{3+\sqrt{5}}{2}}$**.** In this extension, we relax Assumption 2 to examine the case $x \geq \frac{3+\sqrt{5}}{2}$. Recall that whenever collusion arises, for diverse assignment, the optimal contracts are asymmetric, whereas for homogeneous assignment, both symmetric and asymmetric contracts perform equally. For the ease of presentation, we specify the optimal asymmetric contracts for both assignments whenever there is no confusion. Our results are summarized in the following proposition.

Proposition 6. (Strong Substitutability) *The optimal contract is characterized as follows.*

i) For homogenous assignment,

a. If $\frac{3+\sqrt{5}}{2} < x \leq 4$*, the optimal contract is given as* S_1 *as long as* $\frac{2}{x-1} \geq \delta_i$*. If* $\frac{2}{x-1} < \delta_i$*, then paying* $w_s^{i**\prime} = \frac{c}{f(1)-f(0)}$ *to agent* $i1$ *and paying* $w_s^{i*\prime} = \frac{c\delta_i}{f(2)-f(1)} - \frac{c}{f(1)-f(0)}$ *to agent* $i2$ *is optimal in which case agent* $i1$*'s working with probability* $p^* \in (0,1)$ *(and shirking with probability* $1 - p^*$*) and agent* $i2$*'s playing shirking are used as a stage game equilibrium.*

b. *If* $x > 4$ *and* $\delta_i < \frac{2}{3} < \delta^m$*, then no collusion arises and* $w_s^{i*} = \frac{c(1-\delta_i)}{f(2)-f(1)}$ *is optimal. If* $\delta^m > \delta_i > \frac{2}{3}$*, the principal pays* w_s^{i1**}, w_s^{i2**} *such that* $w_s^{i1**} + w_s^{i2**} = \frac{c\delta_i}{f(2)-f(1)}$ *and* $\frac{c(1-\delta_i)}{f(2)-f(1)} \leq w_s^{in**} \leq \frac{c}{f(2)-f(1)}$ *for* $n = 1,2$*. If* $\delta^m < \delta_i$*, then paying* $w_s^{i**\prime} = \frac{c}{f(1)-f(0)}$ *to agent* $i1$ *with* $p^* \in (0,1)$ *and paying* $w_s^{i*\prime} = \frac{c\delta_i}{f(2)-f(1)} - \frac{c}{f(1)-f(0)}$ *to agent* $i2$ *is optimal in which case agent* $i1$*'s working with probability* $p^* \in (0,1)$ *(and shirking with probability* $1 - p^*$*) and agent* $i2$*'s playing shirking are used as a stage game equilibrium.*

ii) *For diverse assignment,*

a. *If* $\frac{3+\sqrt{5}}{2} < x \leq 2 + \sqrt{2}$*, then the optimal contract is given as* S_2 *provided that* $\delta_h > \underline{\delta}^1(x)$*. If* $\delta_h \leq \underline{\delta}^1(x)$*, then the optimal contract identified in Lemma 3 remains optimal.*

b. *If* $2 + \sqrt{2} < x$ *and* $\delta_h \leq \underline{\delta}^2(x)$*, then* S_2 *remains to be optimal provided that* $x \leq \frac{\delta_h+\delta_l}{\delta_h\delta_l} + 1$*; if* $x > \frac{\delta_h+\delta_l}{\delta_h\delta_l} + 1$*, then* $w_d^{l**} = \frac{c}{f(1)-f(0)}, w_d^{h*\prime} = \frac{c}{f(2)-f(1)}\left(\delta_h - \frac{\delta_h}{\delta_l} \cdot \frac{1}{x-1}\right)$ *accompanied by agent* l*'s working with probability* $q^* \in (0,1)$ *(and shirking with* $1 - q^*$*) and agent* h*'s shirking as a stage game equilibrium is optimal.*

c. *If* $2 + \sqrt{2} < x$ *and* $\delta_h > \underline{\delta}^2(x)$*, the optimal contract is* $w_d^{h*} = \frac{c(1-\delta_h)}{f(2)-f(1)}$*,* $w_d^{l**} = \frac{c\delta_l}{f(2)-f(1)}\left(1 - \frac{1-\delta_h}{\delta_h}\right)$ *for* $\delta_l > \frac{\delta_h}{3\delta_h - 1}$ *and* $\delta^m > \delta_h$*. If* $\delta_l \leq \frac{\delta_h}{3\delta_h-1}$*, the optimal contract is* $w_d^{i*} = \frac{c(1-\delta_i)}{f(2)-f(1)}$ *for* $\delta_i < \delta^m$*. If* $\delta^m < \delta_l < \delta_h$*, then* S_2 *remains to be optimal provided that* $x \leq \frac{\delta_h+\delta_l}{\delta_h\delta_l} + 1$*; if* $x > \frac{\delta_h+\delta_l}{\delta_h\delta_l} + 1$*,* $w_d^{l**} = \frac{c}{f(1)-f(0)}, w_d^{h*\prime} = \frac{c}{f(2)-f(1)}\left(\delta_h - \frac{\delta_h}{\delta_l}\frac{1}{x-1}\right)$ *accompanied by agent* l*'s working with probability* $q^* \in (0,1)$ *(and shirking with* $1 - q^*$*) and agent* h*'s shirking as a stage game equilibrium is optimal.*

iii) *When cooperation incentives only exist, both assignments perform equally. When collusion is a pressing concern, diverse assignment dominates whenever the (Pareto) constraint binds under the team of two agent* h*s.*

The contracts S_1 *and* S_2*, and the expressions for* p^* *and* q^* *are presented in the proof.*

Proof of Proposition 6.

Homogenous Assignment: First, consider a homogenous team. Since the agents' discount factors are the same within each team, without loss of generality, we consider a team of two agent hs. We first derive a condition in which the collusion-proof wage upsets the (*shirk, shirk*) equilibrium that is used for sustaining the cooperation equilibrium (*work, work*). Consider asymmetric contracting in which the bribee agent who receives $w_s^{h**} = \frac{c}{f(2)-f(1)}\left(\frac{\delta_h(1+\delta_h(x-1))-1}{1+\delta_h(x-1)}\right)$. First, the bribee prefers working while the briber is shirking if:

$$f(1)w_s^h - c > f(0)w_s^h \Rightarrow w_s^h > \frac{c}{f(1)-f(0)} \qquad (A2)$$

is required. Plug w_s^{h**} into the above inequality (A2), we have $\delta_h > \frac{\sqrt{x}}{x-1} \equiv \delta^S$.

For $x > \frac{3+\sqrt{5}}{2}$, the threshold $\delta^S < 1$, thus $\delta_h > \delta^S$ is well-defined. For $\delta^S \geq \delta^*$:

$$\frac{\sqrt{x}}{x-1} \geq \frac{\sqrt{8x-7}-1}{2(x-1)} \Rightarrow x \leq 4,$$

is required. Thus, given $\frac{3+\sqrt{5}}{2} < x \leq 4$, if $\delta_h > \delta^S$, then the collusion-proof wage upsets the (*shirk, shirk*) stage game equilibrium. Thus, if w_s^{h**} is paid to the bribee and w_s^{h*} is paid to the briber, then the stage game equilibrium is (*work, shirk*). Since (*shirk, shirk*) is no longer the stage game equilibrium and the bribee agent prefers working with the wage w^{h**} instead of shirking, the briber agent expects to enjoy $f(1)w_s^h$ after his free-riding. Thus, if he receives w_s^{h*}, while the briber is receiving w_s^{h**}, the briber deviates by shirking.

To ensure that the collusion-proof wage does not upset (*shirk, shirk*) as a stage game equilibrium, it is immediate from (A2) that the bribee's wage must be bounded above by $w_s^{h**'} = \frac{c}{f(1)-f(0)}$. Since the total collusion-proof wages are $\frac{c}{f(2)-f(1)}\delta_h$, the briber's wage must be at least $w_s^{h*'} = \frac{c}{f(2)-f(1)}\delta_h - w_s^{h**'} = \frac{c}{f(2)-f(1)}\left(\delta_h - \frac{1}{x-1}\right)$ in case the bribee receives $w_s^{h**\prime}$. For $\delta_h > \delta^S$, we have $w_s^{h*'} > 0$. With $w_s^{h*'}$, the briber agent still prefers shirking to working,

$$f(0)w^{h*'} > f(1)w^{h*'} - c \Rightarrow \frac{2}{x-1} > \delta_h,$$

which is always satisfied if $x \leq 3$. If instead $4 \geq x > 3$ and $\frac{2}{x-1} < \delta_h$, then, the briber's wage $w_s^{h*\prime}$ now upsets the (*shirk, shirk*) stage game equilibrium. i.e., The total collusion-proof wages are too high that no wages ensure (*shirk, shirk*) to be a stage game equilibrium if:

$$\frac{c\delta_h}{f(2)-f(1)} > \frac{2c}{f(1)-f(0)} \Rightarrow \frac{2}{x-1} < \delta_h.$$

That is, if $4 \geq x > 3$ and $\frac{2}{x-1} < \delta_h$, there is no pure strategy stage game equilibrium that is used as a punishment upon freeriding. In this case, there is a mixed strategy stage game equilibrium that can be used as a punishment. Consider paying $w_s^{h**'} = \frac{c}{f(1)-f(0)}$ to agent $h1$, and $\frac{c\delta_h}{f(2)-f(1)} - \frac{c}{f(1)-f(0)}$ to the other agent $h2$. In this case, agent $h2$ strictly prefers working when the other agent $h1$ shirks with certainty. To find a punishment for the agents upon freeriding, consider agent $h1$ plays work with probability p and shirk with $1-p$. Agent $h1$'s mixed strategy makes agent $h2$ want to shirk if:

$$\left(pf(1) + (1-p)f(0)\right)\left(\frac{c\delta_h}{f(2)-f(1)} - \frac{c}{f(1)-f(0)}\right) \geq \left(pf(2) + (1-p)f(1)\right)\left(\frac{c\delta_h}{f(2)-f(1)} - \frac{c}{f(1)-f(0)}\right) - c$$

$$\Rightarrow 1 - p\delta^m \leq \frac{1}{\delta_h(x-1)-1} \Rightarrow p \geq \frac{1}{\delta^m} \cdot \frac{\delta_h(x-1)-2}{\delta_h(x-1)-1} \equiv p^*,$$

where we use $\delta^m = \frac{2f(1)-f(2)-f(0)}{f(1)-f(0)} = \frac{x-2}{x-1}$. Observe that $p^* > 0$ since $\frac{2}{x-1} < \delta_h$ and $p^* < 1$ since:

$$\frac{1}{\delta^m} \cdot \frac{\delta_h(x-1)-2}{\delta_h(x-1)-1} < 1 \Rightarrow \delta_h - \frac{1}{x-1} < 1,$$

which is always satisfied. Thus, given agent $h1$ randomizes his play with p^*, shirking is self-enforcing for agent $h2$. Since agent $h1$ receives $\frac{c}{f(1)-f(0)}$, he is indifferent between shirking and working, thus randomizing with p^* is also self-enforcing for him. Now, we check the wages, $\frac{c}{f(1)-f(0)}$ and $\frac{c\delta_h}{f(2)-f(1)} - \frac{c}{f(1)-f(0)}$, and p^* ensure cooperation incentives. For agent $h2$,

$$f(2)\left(\frac{c\delta_h}{f(2)-f(1)} - \frac{c}{f(1)-f(0)}\right) - c \geq (1-\delta_h)f(1)\left(\frac{c\delta_h}{f(2)-f(1)} - \frac{c}{f(1)-f(0)}\right) + \delta_h\left(p^*f(1)\left(\frac{c\delta_h}{f(2)-f(1)} - \frac{c}{f(1)-f(0)}\right) + (1-p^*)f(0)\left(\frac{c\delta_h}{f(2)-f(1)} - \frac{c}{f(1)-f(0)}\right)\right)$$

$$\Rightarrow (2-\delta_h)\delta_h(x-1)^2 \geq x(x-2).$$

Note that $(2-\delta_h)\delta_h$ is strictly increasing for $\delta_h < 1$. Thus, if the above inequality is satisfied for $\delta_h = \frac{2}{x-1}$, then it is also satisfied for any $\delta_h > \frac{2}{x-1}$:

$$(2-\delta_h)\delta_h(x-1)^2\big|_{\delta_h=\frac{2}{x-1}} = 4(x-2) \geq x(x-2),$$

which is always satisfied for $4 \geq x > 3$. Therefore, the wage $\frac{c\delta_h}{f(2)-f(1)} - \frac{c}{f(1)-f(0)}$ with p^* ensures agent $h2$'s cooperation incentive. Meanwhile, for agent $h1$:

$$f(2)\frac{c}{f(1)-f(0)} - c \geq (1-\delta_h)f(1)\frac{c}{f(1)-f(0)} + \delta_h\left(p^*\left(f(1)\frac{c}{f(1)-f(0)} - c\right) + (1 - p^*)f(0)\frac{c}{f(1)-f(0)}\right)$$

$$\Rightarrow \left(f(2)-f(1)\right)\frac{1}{f(1)-f(0)} \geq 1-\delta_h \Rightarrow \delta_h \geq \frac{x-2}{x-1}, \quad (A3)$$

which is always satisfied for $\frac{2}{x-1} < \delta_h$ and $4 \geq x > 3$.

Provided that $\frac{2}{x-1} \geq \delta_h > \delta^S$, the wage $w_s^{h*\prime}$ is greater than w_s^{h*} for a high discount factor:

$$\frac{c}{f(2)-f(1)}\left(\delta_h - \frac{1}{x-1}\right) > \frac{c}{f(2)-f(1)}\frac{1}{1+\delta_h(x-1)} \Rightarrow \delta_h > \frac{\sqrt{x}}{x-1} = \delta^S.$$

Thus, for $\delta_h > \delta^S$, the principal offers $w_s^{h**\prime} = \frac{c}{f(1)-f(0)}$ and $w_s^{h*\prime} = \frac{c}{f(2)-f(1)}\left(\delta_h - \frac{1}{x-1}\right)$.

In sum, for $x \leq 4$ and $\frac{2}{x-1} \geq \delta_h$,

$$S_1 = \begin{cases} w_s^{h*} = \frac{c}{f(2)-f(1)}(1-\delta_h) \text{ if } \delta_h < \delta^m \\ w_s^{h*} = \frac{c}{f(2)-f(1)} \cdot \frac{1}{1+\delta_h(x-1)} \text{ if } \delta^m \leq \delta_h < \delta^* \\ w_s^{h*} = \frac{c}{f(2)-f(1)} \cdot \frac{1}{1+\delta_h(x-1)}, w_s^{h**} = \frac{c}{f(2)-f(1)}\left(\delta_h - \frac{1}{1+\delta_h(x-1)}\right) \text{ if } \delta^* < \delta_h < \delta^S \\ w_s^{h*} = \frac{c}{f(2)-f(1)}\left(\delta_h - \frac{1}{x-1}\right), w_s^{h**} = \frac{c}{f(1)-f(0)} \text{ if } \delta_h \geq \delta^S. \end{cases}$$

Note that under S_1, the total collusion-proof wages are $\frac{c}{f(2)-f(1)}$, thus any wage combinations ensuring the total wages to be $\frac{c}{f(2)-f(1)}$ are collusion-proof as long as $w_s^{h*} \leq w_s^{h**} \leq \frac{c}{f(1)-f(0)}$. For $x \leq 4$ and $\frac{2}{x-1} < \delta_h$, the wages $w_s^{h**} = \frac{c}{f(1)-f(0)}$ to agent $h1$ and $w_h^* = \frac{c\delta_h}{f(2)-f(1)} - \frac{c}{f(1)-f(0)}$ to agent $h2$ (which ensures a mixed strategy stage game equilibrium, agent $h1$'s $p^* = \frac{1}{\delta^m} \cdot \frac{\delta_h(x-1)-2}{\delta_h(x-1)-1}$ and agent $h2$'s shirking) is optimal. For $\delta_h \geq \delta^S$, strong substitutability makes the briber agent better off because his cooperation constraint is not binding.

If instead $\delta^S < \delta^*$ (which arises for $x > 4$), we claim that the collusion arises when $\delta_h < \delta^m = \frac{x-2}{x-1}$, i.e., when the cooperative wage is $w_s^{h*} = \frac{c(1-\delta_h)}{f(2)-f(1)}$. $B(\boldsymbol{w_s})$ is not empty if $\delta_h > \frac{2}{3}$ and $\frac{2}{3}$ is less than δ^m for $x > 4$. In this case, the agents use (*work, shirk*) or (*shirk, work*) stage game equilibrium to enforce collusion. Moreover, we can show that as in the main analysis, any collusion in $B(\boldsymbol{w_s})$ is self-enforcing. Thus, if $\frac{2}{3} < \delta_h < \delta^m$, the principal pays w_s^{h1**}, w_s^{h2**} such that $w_s^{h1**} + w_s^{h2**} = \frac{c\delta_h}{f(2)-f(1)}$ and $\frac{c(1-\delta_h)}{f(2)-f(1)} \leq w_s^{hn**} \leq \frac{c}{f(2)-f(1)}$ for $n = 1,2$ to combat collusion. Any wage combination does not upset the (*work, shirk*) or (*shirk, work*) stage game equilibrium. If $\delta_h > \delta^m$, the stage game equilibrium is now (*shirk, shirk*), while collusion is a still concern. Observe that $\frac{c\delta_h}{f(2)-f(1)} > \frac{2c}{f(1)-f(0)} \Rightarrow \frac{2}{x-1} < \delta_h$. Since $\delta^m = \frac{x-2}{x-1} > \frac{2}{x-1}$ for $x > 4$, whenever $\delta_h > \delta^m$, we have $\frac{2}{x-1} < \delta_h$. Thus, as shown before, there does not exist a wage combination to combat collusion while ensuring (*shirk, shirk*) as a stage game equilibrium. In this case, the principal pays the first agent $\frac{c}{f(1)-f(0)}$ and the second agent $\frac{c\delta_h}{f(2)-f(1)} - \frac{c}{f(1)-f(0)}$, and the first agent's playing p^* and the second agent's playing shirking ensures cooperation. This is because the second agent's cooperation incentive is not affected by $x > 4$ and the first agent's incentive captured in (A3) is always satisfied since $\delta_h > \delta^m = \frac{x-2}{x-1}$.

To summarize the optimal contract for homogenous assignment:

a) If $\frac{3+\sqrt{5}}{2} < x \leq 4$, the optimal contract is given as S_1 for $\frac{2}{x-1} \geq \delta_h$. If $\frac{2}{x-1} < \delta_h$, then paying $w_s^{h**\prime} = \frac{c}{f(1)-f(0)}$ to agent $h1$ and paying $w_s^{h*\prime} = \frac{c\delta_h}{f(2)-f(1)} - \frac{c}{f(1)-f(0)}$ agent $h2$ is optimal in which case the first agent's playing $p^* \in (0,1)$ and the second agent's playing shirking is used as a stage game equilibrium.

b) If $x > 4$ and $\delta_h < \frac{2}{3} < \delta^m$, then no collusion arises and $w_s^{h*} = \frac{c(1-\delta_h)}{f(2)-f(1)}$ is optimal. If $\delta^m > \delta_h > \frac{2}{3}$, the principal pays w_s^{h1**}, w_s^{h2**} such that $w_s^{h1**} + w_s^{h2**} = \frac{c\delta_h}{f(2)-f(1)}$ and $\frac{c(1-\delta_h)}{f(2)-f(1)} \leq w_s^{hn**} \leq \frac{c}{f(2)-f(1)}$ for $n = 1,2$. If $\delta^m < \delta_h$, then since $\delta^m = \frac{x-2}{x-1} > \frac{2}{x-1}$ and $\delta^m < \delta_h$, we have $\frac{c\delta_h}{f(2)-f(1)} > \frac{2c}{f(1)-f(0)}$, thus paying $w_s^{h**\prime} = \frac{c}{f(1)-f(0)}$ to agent $h1$ with $p^* \in (0,1)$ and paying $w_s^{h*\prime} = \frac{c\delta_h}{f(2)-f(1)} - \frac{c}{f(1)-f(0)}$ to agent $h2$ is optimal in which case the first agent's playing $p^* \in (0,1)$ and the second agent's playing shirking is used as a stage game equilibrium.

Diverse Assignment: The similar argument can be extended to a diverse team. If $w_d^{l**} = \frac{c\delta_l}{f(2)-f(1)}\left(1 - \frac{1}{\delta_h(1+\delta_h(x-1))}\right)$ upsets the stage game equilibrium (*shirk, shirk*), then the principal offers $w_d^{l**\prime} = \frac{c}{f(1)-f(0)}$ and $w_d^{h*\prime} = \frac{c}{f(2)-f(1)}\left(\delta_h - \frac{\delta_h}{\delta_l} \cdot \frac{1}{x-1}\right)$, where $w^{h*\prime}$ is derived from the binding (Pareto) constraint for agent l (i.e., the inequality (3)). We derive the discount factor threshold above which this case arises: $\frac{c\delta_l}{f(2)-f(1)}\left(1 - \frac{1}{\delta_h(1+\delta_h(x-1))}\right) > \frac{c}{f(1)-f(0)}$

$$\Rightarrow \delta_l > \frac{1}{x-1} \cdot \frac{\delta_h(1+\delta_h(x-1))}{\delta_h(1+\delta_h(x-1))-1} \equiv \delta^S(\delta_h).$$

In this case, the wage $w_s^{h*'}$ is greater than w_s^{h*}, thus ensuring agent h's cooperation incentive:

$$\frac{c}{f(2)-f(1)}\left(\delta_h - \frac{\delta_h}{\delta_l} \cdot \frac{1}{x-1}\right) > \frac{c}{f(2)-f(1)} \cdot \frac{1}{1+\delta_h(x-1)} \Rightarrow \delta_l > \delta^S(\delta_h).$$

Note that given $x > \frac{3+\sqrt{5}}{2}$, the threshold $\delta^S(\delta_h) < 1$ if δ_h is sufficiently large:

$$\frac{1}{x-1} \cdot \frac{\delta_h(1+\delta_h(x-1))}{\delta_h(1+\delta_h(x-1))-1} < 1 \Rightarrow \delta_h > \frac{\sqrt{\left(\frac{1}{x-1}\right)^2 + \frac{4}{x-2}} - \frac{1}{x-1}}{2} \equiv \underline{\delta}^1(x).$$

It can be shown that at $x = \frac{3+\sqrt{5}}{2}$, we have $\underline{\delta}^1(x) = 1$, and $\underline{\delta}^1(x)$ decreases in x. i.e., we have $\underline{\delta}^1(x) < 1$ for $x > \frac{3+\sqrt{5}}{2}$. Thus, $\delta^S(\delta_h) < 1$ as long as $\delta_h > \underline{\delta}^1(x)$, which is feasible for $x > \frac{3+\sqrt{5}}{2}$. Observe that if either $x < \frac{3+\sqrt{5}}{2}$ (as in our main analysis) or $x \geq \frac{3+\sqrt{5}}{2}$ and $\delta_h \leq \underline{\delta}^1(x)$, then we have $\delta^S(\delta_h) \geq 1$. In these cases, $w_d^{l**} = \frac{c\delta_l}{f(2)-f(1)}\left(1 - \frac{1}{\delta_h(1+\delta_h(x-1))}\right)$ never upsets the (*shirk, shirk*) stage game equilibrium, thus the optimal contract in Lemma 3 remains to be optimal.

Provided that $\delta^S(\delta_h) < 1$ (i.e., when $x > \frac{3+\sqrt{5}}{2}$ and $\delta_h > \underline{\delta}^1(x)$), we check if $\delta^S(\delta_h)$ is greater than $\underline{\delta}(\delta_h) = \frac{\sqrt{1+4(x-1)H}-1}{2(x-1)}$, where $H = \frac{\delta_h^2(x-1)+\delta_h}{\delta_h^2(x-1)+\delta_h-1} > 1$:

$$\frac{1}{x-1} \cdot \frac{\delta_h(1+\delta_h(x-1))}{\delta_h(1+\delta_h(x-1))-1} \geq \frac{\sqrt{1+4(x-1)H}-1}{2(x-1)} \Rightarrow x-2 \geq (x-3)(\delta_h^2(x-1)+\delta_h).$$

If $x \leq 3$, the above inequality is always satisfied for any δ_h, confirming $\delta^S(\delta_h) \geq \underline{\delta}(\delta_h)$. However, if $x > 3$, the above inequality is satisfied for $\delta_h \leq \frac{\sqrt{(x-3)^2+4(x-3)(x-1)(x-2)}-(x-3)}{2(x-3)(x-1)} \equiv \underline{\delta}^2(x)$. Note that $\underline{\delta}^2(x) < 1$ if $x > 2+\sqrt{2}$. If $x \leq 2+\sqrt{2}$, then $\underline{\delta}^2(x) > 1$. Since $\delta_h \leq \bar{\delta} < 1$, the above inequality is always satisfied for $x \leq 2+\sqrt{2}$. Together, if $x \leq 2+\sqrt{2}$, we have $\delta^S(\delta_h) \geq \underline{\delta}(\delta_h)$. Or, if $x > 2+\sqrt{2}$, we have $\delta^S(\delta_h) \geq \underline{\delta}(\delta_h)$ only if $\delta_h \leq \underline{\delta}^2(x)$. Otherwise, if $\delta_h > \underline{\delta}^2(x)$, we have $\delta^S(\delta_h) < \underline{\delta}(\delta_h)$. Therefore, if $x \leq 2+\sqrt{2}$ or $x > 2+\sqrt{2}$ and $\delta_h \leq \underline{\delta}^2(x)$, the bribee's collusion-proof wage, w_d^{l**}, upsets the (*shirk, shirk*) stage game equilibrium for $\delta_l > \delta^S(\delta_h)$. In this case, the principal pays w_d^{l**} and w_d^{h*} for $\underline{\delta}(\delta_h) < \delta_l \leq \delta^S(\delta_h)$ and $w_d^{l**'} = \frac{c}{f(1)-f(0)}$ and $w_d^{h*'} = \frac{c}{f(2)-f(1)}\left(\delta_h - \frac{\delta_h}{\delta_l} \cdot \frac{1}{x-1}\right)$ for $\delta_l > \delta^S(\delta_h)$. To ensure that $w_d^{h*'}$ does not upset the (*shirk, shirk*) stage game equilibrium:

$$w_d^{h*\prime} \leq \frac{c}{f(1)-f(0)} \Rightarrow x \leq \frac{\delta_h+\delta_l}{\delta_h\delta_l} + 1.$$

Thus, the condition $x \leq \frac{\delta_h+\delta_l}{\delta_h\delta_l} + 1$ is required for $w_d^{l**\prime} = \frac{c}{f(1)-f(0)}$ and $w_d^{h*\prime} = \frac{c}{f(2)-f(1)}\left(\delta_h - \frac{\delta_h}{\delta_l} \cdot \frac{1}{x-1}\right)$ to be optimal for $\delta_l > \delta^S(\delta_h)$.

If $x > \frac{\delta_h+\delta_l}{\delta_h\delta_l} + 1$, briber agent h's wage, $w_d^{h*\prime} = \frac{c}{f(2)-f(1)}\left(\delta_h - \frac{\delta_h}{\delta_l} \cdot \frac{1}{x-1}\right)$, upsets the (*shirk, shirk*) stage game equilibrium. As in homogeneous assignment, we derive a mixed strategy stage game equilibrium. Consider paying $w_d^{l**\prime} = \frac{c}{f(1)-f(0)}$ and $w_d^{h*\prime} = \frac{c}{f(2)-f(1)}\left(\delta_h - \frac{\delta_h}{\delta_l} \cdot \frac{1}{x-1}\right)$ and agent l's randomization with q. Agent l's mixed strategy makes agent h want to shirk if:

$$\big(qf(1) + (1-q)f(0)\big)\frac{c}{f(2)-f(1)}\left(\delta_h - \frac{\delta_h}{\delta_l} \cdot \frac{1}{x-1}\right)$$

$$\geq \big(qf(2) + (1-q)f(1)\big)\frac{c}{f(2)-f(1)}\left(\delta_h - \frac{\delta_h}{\delta_l} \cdot \frac{1}{x-1}\right) - c$$

$$\Rightarrow q \geq \frac{\delta_h\big((x-1)\delta_l - 1\big) - \delta_l}{\delta^m\delta_h\big((x-1)\delta_l - 1\big)} \equiv q^*,$$

where we use $\delta^m = \frac{x-2}{x-1}$. Observe that when $\delta_h = \delta_l$, we have $q^* = p^*$. Observe also that $q^* > 0$ since $x > \frac{\delta_h+\delta_l}{\delta_h\delta_l} + 1$ and $q^* < 1$ can be written as $\delta_l\left(1 - \frac{1}{\delta_h}\right) < \frac{1}{x-1}$, which is always satisfied because the left-hand side is nonpositive for $\delta_h \leq 1$, whereas the right-hand side is strictly positive. Thus, given agent l's randomization with q^*, shirking is self-enforcing for agent h. Agent l is indifferent between working and shirking given wage $\frac{c}{f(1)-f(0)}$, thus randomizing q^* is also self-enforcing. To see the randomization q^* also ensures cooperation, first consider agent h:

$$f(2)\frac{c}{f(2)-f(1)}\left(\delta_h - \frac{\delta_h}{\delta_l} \cdot \frac{1}{x-1}\right) - c \geq (1-\delta_h)f(1)\frac{c}{f(2)-f(1)}\left(\delta_h - \frac{\delta_h}{\delta_l} \cdot \frac{1}{x-1}\right)$$

$$+\delta_h\left(q^*f(1)\frac{c}{f(2)-f(1)}\left(\delta_h - \frac{\delta_h}{\delta_l} \cdot \frac{1}{x-1}\right) + (1-q^*)f(0)\frac{c}{f(2)-f(1)}\left(\delta_h - \frac{\delta_h}{\delta_l} \cdot \frac{1}{x-1}\right)\right),$$

$$\Rightarrow \frac{\delta_h}{\delta_l}\big((2-\delta_h)\delta_l(x-2)(x-1) + (x-1)(\delta_h + \delta_l - \delta_h\delta_l) - (x-2)\big) \geq (x-2)(x-1).$$

Notice that the left-hand side is increasing in $\delta_h \leq 1$, and we have $\delta_h \geq \delta_l$. Thus, if the above inequality is satisfied for $\delta_h = \delta_l$ given $x > \frac{\delta_h+\delta_l}{\delta_h\delta_l} + 1$ (or, $\delta_l > \frac{2}{x-1}$), then it is satisfied for any $\delta_h \geq \delta_l$:

$$\frac{\delta_h}{\delta_l}\big((2-\delta_h)\delta_l(x-2)(x-1) + (x-1)(\delta_h + \delta_l - \delta_h\delta_l) - (x-2)\big)\big|_{\delta_h=\delta_l} \geq (x-2)(x-1)$$

$$\Rightarrow (2-\delta_l)\delta_l(x-1)^2 \geq (x-2)x,$$

which is always satisfied for $\delta_l > \frac{2}{x-1}$. Therefore, the wage $\frac{c}{f(2)-f(1)}\left(\delta_h - \frac{\delta_h}{\delta_l}\cdot\frac{1}{x-1}\right)$ with q^*ensures agent h's cooperation incentive. Meanwhile, for agent l:

$$f(2)\frac{c}{f(1)-f(0)} - c \geq (1-\delta_l)f(1)\frac{c}{f(1)-f(0)}$$

$$+\delta_l\left(q^*\left(f(1)\frac{c}{f(1)-f(0)} - c\right) + (1-q^*)f(0)\frac{c}{f(1)-f(0)}\right),$$

$$\Rightarrow \left(\frac{1}{x-1} + \delta_l\right) \geq 1.$$

Recall that $\delta_l \geq \delta^S(\delta_h) = \frac{1}{x-1}\cdot\frac{\delta_h(1+\delta_h(x-1))}{\delta_h(1+\delta_h(x-1))-1}$. Plug this into the left-hand side, we have:

$$\frac{1}{x-1}\left(1 + \frac{\delta_h(1+\delta_h(x-1))}{\delta_h(1+\delta_h(x-1))-1}\right) \geq 1 \Rightarrow \frac{1}{\delta_h(1+\delta_h(x-1))-1} \geq x-3. \quad (A4)$$

If $x \leq 3$, then inequality (A4) is always satisfied because the left-hand side is positive, whereas the right-hand side is nonpositive. For $x > 3$, we know that $\delta^S(\delta_h) > \underline{\delta}(\delta_h)$, and this can be written as $x-2 \geq (x-3)(\delta_h^2(x-1)+\delta_h)$ as we derived before. This inequality, $x-2 \geq (x-3)(\delta_h^2(x-1)+\delta_h)$, can be expressed as:

$$\Rightarrow \frac{x-2}{x-3} - 1 \geq \delta_h^2(x-1) + \delta_h - 1 \Rightarrow \frac{1}{\delta_h^2(x-1)+\delta_h-1} \geq x-3.$$

Therefore, (A4) is always satisfied, which confirms that the wage $\frac{c}{f(1)-f(0)}$ with q^* ensures agent l's cooperation incentive.

If $x > 2+\sqrt{2}$ and $\delta_h > \underline{\delta}^2(x)$, then $\delta^S(\delta_h) < \underline{\delta}(\delta_h)$. In this case, as in homogeneous teams, collusion arises for $\delta_i < \delta^m$, i.e., under the wage $w_d^{i*} = \frac{c(1-\delta_i)}{f(2)-f(1)}$. In this case, $B(\boldsymbol{w_d})$ is not empty if $\frac{1-\delta_h}{\delta_h} + \frac{1-\delta_l}{\delta_l} < 1$, which can be written as $\delta_l > \frac{\delta_h}{3\delta_h-1}$. Again, using the bribee l's (Pareto) constraint given in (3), the optimal contract is $w_d^{h*} = \frac{c(1-\delta_h)}{f(2)-f(1)}$, $w_d^{l**\prime} = \frac{c\delta_l}{f(2)-f(1)}\left(1 - \frac{1-\delta_h}{\delta_h}\right)$, and the (Pareto) constraint binds if $\delta_l > \frac{\delta_h}{3\delta_h-1}$. Since $\delta_l < \delta_h$, we have $\frac{\delta_h}{3\delta_h-1} < \delta_h$, which implies that $\delta_h > \frac{2}{3}$. To see whether $w_d^{l**\prime} = \frac{c\delta_l}{f(2)-f(1)}\left(1 - \frac{1-\delta_h}{\delta_h}\right)$ can ever upset the (*work, shirk*) stage game equilibrium:

$$\frac{c\delta_l}{f(2)-f(1)}\left(1 - \frac{1-\delta_h}{\delta_h}\right) < \frac{c}{f(2)-f(1)},$$

which is always satisfied. Therefore, given $x > 2+\sqrt{2}$ and $\delta_h > \underline{\delta}^2(x)$, the optimal contract is

$w_d^{h*} = \frac{c(1-\delta_h)}{f(2)-f(1)}$, $w_d^{l**\prime} = \frac{c\delta_l}{f(2)-f(1)}\left(1-\frac{1-\delta_h}{\delta_h}\right)$ if $\delta_l > \frac{\delta_h}{3\delta_h-1}$ and $\delta^m > \delta_h > \frac{2}{3}$.

If $\delta_l \leq \frac{\delta_h}{3\delta_h-1}$, the (Pareto) constraint doesn't bind, and the optimal contract is $w_d^{i*} = \frac{c(1-\delta_i)}{f(2)-f(1)}$. If $\delta_h > \delta_l > \delta^m$, the stage game equilibrium is (*shirk, shirk*), while collusion is a still concern. As in homogenous assignment, the optimal contract is $w_d^{l**} = \frac{c}{f(1)-f(0)}$, $w_d^{h*\prime} = \frac{c}{f(2)-f(1)}\left(\delta_h - \frac{\delta_h}{\delta_l}\cdot\frac{1}{x-1}\right)$ provided that $x \leq \frac{\delta_h+\delta_l}{\delta_h\delta_l}+1$. If $x > \frac{\delta_h+\delta_l}{\delta_h\delta_l}+1$, then paying $w_d^{l**} = \frac{c}{f(1)-f(0)}$, $w_d^{h*\prime} = \frac{c}{f(2)-f(1)}\left(\delta_h - \frac{\delta_h}{\delta_l}\cdot\frac{1}{x-1}\right)$ and agent l's randomization q^* and agent h's shirking are used as a stage game equilibrium.

To summarize the optimal contract for diverse assignment:

a) If $\frac{3+\sqrt{5}}{2} < x \leq 2+\sqrt{2}$, then the optimal contract is given as S_2 provided that $\delta_h > \underline{\delta}^1(x)$. If $\delta_h \leq \underline{\delta}^1(x)$, then the optimal contract identified in Lemma 3 remains optimal.

b) If $2+\sqrt{2} < x$ and $\delta_h \leq \underline{\delta}^2(x)$, then S_2 remains to be optimal provided that $x \leq \frac{\delta_h+\delta_l}{\delta_h\delta_l}+1$; if $x > \frac{\delta_h+\delta_l}{\delta_h\delta_l}+1$, then $w_d^{l**} = \frac{c}{f(1)-f(0)}$, $w_d^{h*\prime} = \frac{c}{f(2)-f(1)}\left(\delta_h - \frac{\delta_h}{\delta_l}\cdot\frac{1}{x-1}\right)$ accompanied by agent l's randomization $q^* \in (0,1)$ and agent h's shirking as a stage game equilibrium is optimal.

c) If $2+\sqrt{2} < x$ and $\delta_h > \underline{\delta}^2(x)$, the optimal contract is $w_d^{h*} = \frac{c(1-\delta_h)}{f(2)-f(1)}$, $w_d^{l**} = \frac{c\delta_l}{f(2)-f(1)}\left(1-\frac{1-\delta_h}{\delta_h}\right)$ for $\delta_l > \frac{\delta_h}{3\delta_h-1}$ and $\delta^m > \delta_h$. In this case, $\delta_l > \frac{\delta_h}{3\delta_h-1}$ implies $\delta_h > \frac{2}{3}$. If $\delta_l \leq \frac{\delta_h}{3\delta_h-1}$, the optimal contract is $w_d^{i*} = \frac{c(1-\delta_i)}{f(2)-f(1)}$ for $\delta_i < \delta^m$, $i = h, l$. If $\delta^m < \delta_l < \delta_h$, then S_2 remains to be optimal provided that $x \leq \frac{\delta_h+\delta_l}{\delta_h\delta_l}+1$; if $x > \frac{\delta_h+\delta_l}{\delta_h\delta_l}+1$, $w_d^{l**} = \frac{c}{f(1)-f(0)}$, $w_d^{h*\prime} = \frac{c}{f(2)-f(1)}\left(\delta_h - \frac{\delta_h}{\delta_l}\frac{1}{x-1}\right)$ accompanied by agent l's randomization $q^* \in (0,1)$ and agent h's shirking as a stage game equilibrium is optimal, where

$S_2 =$

$$\begin{cases} w_d^{i*} = \frac{c}{f(2)-f(1)}(1-\delta_i), \text{ if } \delta_l < \delta_h < \delta^m \\ w_d^{i*} = \frac{c}{f(2)-f(1)} \cdot \frac{1}{1+\delta_i(x-1)} \text{ if} \frac{1}{2} \leq \delta_l < \delta_h < \underline{\delta}(\delta_h) \text{ or } \delta_l < \underline{\delta}(\delta_h) < \delta_h \\ w_d^{h*} = \frac{c}{f(2)-f(1)} \cdot \frac{1}{1+\delta_h(x-1)}, w_d^{l**} = \frac{c\delta_l}{f(2)-f(1)}\left(1 - \frac{1}{\delta_h(1+\delta_h(x-1))}\right) \text{if } \underline{\delta}(\delta_h) < \delta_l < \delta_h, \\ \qquad \text{and } \delta_l \leq \delta^S(\delta_h) \\ w_d^{h*} = \frac{c}{f(2)-f(1)}\left(\delta_h - \frac{\delta_h}{\delta_l} \cdot \frac{1}{x-1}\right), w_d^{l**} = \frac{c}{f(1)-f(0)} \text{if } \delta_l > \delta^S(\delta_h). \end{cases}$$

As in homogenous assignment, strong substitutability makes the briber agent h better off.

Optimality: We now claim that with this change, which assignment dominates the other in our main setting remains qualitatively the same. First, observe that cooperative wages are not qualitatively affected by the magnitude of x, thus when the (Pareto) constraints do not bind across all teams, our result that the two assignments perform equally remains the same. Thus, our proof is to check whether diverse assignment is optimal whenever collusion is a pressing concern. First, for homogenous assignment, the total collusion-proof wages are $\frac{c\delta_i}{f(2)-f(1)}$ for each team. For diverse assignment, the total collusion-proof wages are

$$\begin{cases} a) \; \frac{2c}{f(2)-f(1)}\left(\delta_l + \left(1-\frac{\delta_l}{\delta_h}\right)\frac{1}{1+\delta_h(x-1)}\right) \text{ for} \frac{3+\sqrt{5}}{2} < x \leq 2+\sqrt{2} \text{ and } \delta_l \leq \delta^S(\delta_h), \\ \qquad \text{or } 2+\sqrt{2} < x \leq \frac{\delta_h+\delta_l}{\delta_h\delta_l} + 1 \text{ and } \delta_h \leq \underline{\delta}^2(x); \text{or} \\ b) \; \frac{2c}{f(2)-f(1)}\left(\delta_h - \left(\frac{\delta_h}{\delta_l} - 1\right)\frac{1}{x-1}\right) \text{ for } \frac{3+\sqrt{5}}{2} < x \leq 2+\sqrt{2} \text{ and } \delta_l > \delta^S(\delta_h), \\ \qquad \text{or } x > \frac{\delta_h+\delta_l}{\delta_h\delta_l} + 1. \\ c) \; \frac{2c}{f(2)-f(1)}\left((1-\delta_h)\left(1-\frac{\delta_l}{\delta_h}\right) + \delta_l\right) \text{ for } 2+\sqrt{2} < x, \underline{\delta}^2(x) < \delta_h < \delta^m \text{ and } \delta_l > \frac{\delta_h}{3\delta_h-1}. \end{cases}$$

If none of these total collusion-proof wages are greater than $\frac{c(\delta_h+\delta_l)}{f(2)-f(1)}$, then diverse assignment is optimal. To show this, note that if a) is the total wages, we already show in the main setting that it is always less than $\frac{c(\delta_h+\delta_l)}{f(2)-f(1)}$. If b) is the total wages:

$$W_d < W_s \Rightarrow \frac{2c}{f(2)-f(1)}\left(\delta_h - \left(\frac{\delta_h}{\delta_l} - 1\right)\frac{1}{x-1}\right) < \frac{c(\delta_h+\delta_l)}{f(2)-f(1)} \Rightarrow \delta_l < \frac{2}{x-1}.$$

The above inequality is always satisfied given $\frac{2}{x-1} \geq \delta_h$ since $\delta_l \leq \delta_h$. Lastly, if c) is the total wages:

$$W_d < W_s \Rightarrow \frac{2c}{f(2)-f(1)}\left((1-\delta_h)\left(1-\frac{\delta_l}{\delta_h}\right)+\delta_l\right) < \frac{c(\delta_h+\delta_l)}{f(2)-f(1)} \Rightarrow \frac{2}{3} < \delta_h,$$

which we already show that $\delta_h > \frac{2}{3}$ is satisfied whenever $\frac{2c}{f(2)-f(1)}\left((1-\delta_h)\left(1-\frac{\delta_l}{\delta_h}\right)+\delta_l\right)$ is the optimal total collusion-proof wages. Therefore, whenever the (Pareto) constraints bind across all teams, then diverse assignment is optimal. As in the main analysis, there are cases where the (Pareto) constraints bind 1) only under the team of two agent hs or 2) under the team of two agent hs and two teams of diverse assignment, but not under the team of two agent ls. In these cases, diverse assignment continues to be optimal. To see this, for case 1), adding more expensive collusion-proof wages for the team of two agent hs makes W_s higher than W_d. For case 2), given $W_s > W_d$ when (Pareto) constraints bind across all teams, paying more expensive cooperative wages for two agent ls ensures the inequality $W_s > W_d$.

To summarize, strong substitutability requires the principal adjust the contract for a high discount factor to ensure the collusion-proof wage does not upset the stage game equilibrium (*shirk, shirk*). If substitutability is too strong, then there can be a collusion problem when the stage game equilibria are (*work, shirk*) and (*shirk, work*), or there is no pure strategy stage game equilibrium that can be used as a punishment, in which case one agent's randomization (p^* under homogenous assignment and q^* under diverse assignment) is used as a stage game equilibrium. Nonetheless, diverse teams are still optimal whenever collusion is a pressing concern. ■