Online Appendix

for "The Distribution of Investor Beliefs, Stock Ownership and Stock Returns"

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A Proofs

Proof of Proposition 2.1. Suppose $\lambda(n) = \lambda$ for all n. Setting

$$S_i \equiv b_i\bar{D} + \bar{D}_i - b_i\lambda + a\theta_i\sigma_i^2\phi_i, \tag{A.1}$$

we can write (2.8) as (2.10). Equation (2.10) has a unique solution because when the integrand is positive, it is decreasing in ϕ_i. Integrating $\lambda(n) = a\sigma^2\sum_{j=1}^I b_j x_j(n)$ over n and using (2.7) and $\lambda(n) = \lambda$ for all n, we find

$$\lambda = a\sigma^2\sum_{j=1}^I b_j\theta_j. \tag{A.2}$$

Substituting (A.2) into (A.1), we find (2.9). Substituting (A.1) into (2.6) and using $\lambda(n) = \lambda$ for all n, we find (2.11). □

Proposition A.1 derives conditions for the factor premia $\{\lambda(n)\}_{n\in[0,1]}$ to be equal across investors

Proposition A.1. *The factor premia $\{\lambda(n)\}_{n\in[0,1]}$ are equal across investors under either one of the following sufficient conditions:*

- (A) *In addition to trading assets $i = 0, 1, .., I$, investors can trade an asset $I + 1$ that is in zero supply and pays dividend D per share in period 1. Agents can trade asset $I + 1$ without short-sale constraints.*
- (B) *For each stock i, the following conditions hold:*
  - (i) *The function $n \to \epsilon_i(n)$ takes values in a finite set $Z_i = \{z_{i1}, .., z_{ik_i}\}$.*
  - (ii) *For each permutation P of the set $\{1, .., k_i\}$, there exists one stock i' such that $(\theta_i, b_i, \sigma_i) = (\theta_{i'}, b_{i'}, \sigma_{i'})$ and $\{n \in [0, 1] : \epsilon_i(n) = z_{ik}\} = \{n \in [0, 1] : \epsilon_{i'}(n) = z_{iP(k)}\}$.*

Proof of Proposition A.1. Under Condition (A), we can write the maximization problem of investor n as

$$\max_{\substack{\{x_i(n)\}_{i=1,..,I},x(n)\\ x_i(n)\geq 0}} \sum_{i=1}^{I}\left(b_i\bar{D}+\bar{D}_i+\epsilon_i(n)-S_i\right)x_i(n)+(\bar{D}-S)x(n)$$
$$-\frac{a}{2}\left[\sigma^2\left(\sum_{i=1}^{I}b_ix_i(n)+x(n)\right)^2+\sum_{i=1}^{I}\sigma_i^2x_i(n)^2\right],$$

where S denotes the price of asset $I+1$ and $x(n)$ denotes the number of shares of the asset that the investor holds. The first-order condition for stock i is (2.4) and (2.5), where $\lambda(n) \equiv a\sigma^2\left(\sum_{j=1}^{I} b_jx_j(n)+x(n)\right)$. The first-order condition for asset $I+1$ is

$$\bar{D}-S-\lambda(n)=0$$

and implies that $\lambda(n)$ is equal to the common value $\bar{D}-S$ for all n. Following the same steps as in Proposition 2.1 and using that asset $I+1$ is in zero supply, we can then show that equilibrium prices of stocks $i=1,..,I$ are given by (2.9) and equilibrium holdings of these stocks by investors are given by (2.11).

Under Condition (B), (2.6) and $\lambda(n)=a\sum_{i=1}^{I}b_ix_i(n)$ imply

$$\begin{aligned}\lambda(n)&=\sum_{i=1}^{I}b_j\max\left\{\frac{b_i\bar{D}+\bar{D}_i+\epsilon_i(n)-S_i-b_i\lambda(n)}{\sigma_i^2},0\right\}\\&=\sum_{\mathcal{I}}\sum_{i\in\mathcal{I}}b_i\max\left\{\frac{b_i\bar{D}+\bar{D}_i+\epsilon_i(n)-S_i-b_i\lambda(n)}{\sigma_i^2},0\right\},\end{aligned}\tag{A.3}$$

where $\mathcal{I}$ denotes a set formed all stocks i with the same characteristics $(\theta_i, b_i, \sigma_i^2)$. Suppose that S_i is given by (2.9) for all $i=1,..,I$, in which case Condition (B) implies that $\bar{D}_i-S_i$ is equal across $i\in\mathcal{I}$. Suppose also, proceeding by contradiction, that $\lambda(n)>\lambda(n')$ for some $n,n'\in[0,1]$. Condition (B), equality of $\bar{D}_i-S_i$ across $i\in\mathcal{I}$, and $b_i\geq 0$ for all $i\in\mathcal{I}$, imply

$$\sum_{i\in\mathcal{I}}b_i\max\left\{\frac{b_i\bar{D}+\bar{D}_i+\epsilon_i(n)-S_i-b_i\lambda(n)}{\sigma_i^2},0\right\}\leq\sum_{i\in\mathcal{I}}b_i\max\left\{\frac{b_i\bar{D}+\bar{D}_i+\epsilon_i(n')-S_i-b_i\lambda(n')}{\sigma_i^2},0\right\}.\tag{A.4}$$

Summing (A.4) over $\mathcal{I}$ and using (A.3), we find $\lambda(n)\leq\lambda(n')$, which contradicts $\lambda(n)>\lambda(n')$. Therefore, $\lambda(n)$ is equal across n. $\square$

Proof of Lemma 2.1. We denote the supremum and the infimum of the support of the distribution of investor beliefs for stock i by $\overline{\epsilon}_i$ and $\underline{\epsilon}_i$, respectively. We first consider comparative static (I). Setting ϵ such that

$$\overline{\epsilon}_i = \mu_i + \frac{1}{\chi}(\epsilon - \mu_i) \Leftrightarrow \epsilon = \mu_i + \chi(\overline{\epsilon}_i - \mu_i)$$

in (2.12), we find that the supremum of the support of the distribution for stock i' is not larger than $\mu_i + \chi(\overline{\epsilon}_i - \mu_i)$. Setting $\epsilon + \Delta\epsilon$ such that

$$\overline{\epsilon}_i = \mu_i + \frac{1}{\chi}(\epsilon + \Delta\epsilon - \mu_i) \Leftrightarrow \epsilon + \Delta\epsilon = \mu_i + \chi(\overline{\epsilon}_i - \mu_i)$$

in (2.12), we find that the supremum is not smaller than $\mu_i + \chi(\overline{\epsilon}_i - \mu_i)$. Therefore, the supremum is $\mu_i + \chi(\overline{\epsilon}_i - \mu_i)$. The same argument implies that the infimum of the support of the distribution for stock i' is $\mu_i + \chi(\underline{\epsilon}_i - \mu_i)$. Therefore, the range for stock i' is $\chi(\overline{\epsilon}_i - \underline{\epsilon}_i)$, which is χ times the range $\overline{\epsilon}_i - \underline{\epsilon}_i$ for stock i. The variance for stock i' is

$$\int_{\mu_i+\chi(\underline{\epsilon}_i-\mu_i)}^{\mu_i+\chi(\overline{\epsilon}_i-\mu_i)} (\epsilon - \mu_i)^2 dF_{i'}(\epsilon).$$

Using (2.12) and then making the change of variable $\hat{\epsilon} = \mu_i + \frac{1}{\chi}(\epsilon - \mu_i)$, we can write that variance as

$$\begin{aligned}&\int_{\mu_i+\chi(\underline{\epsilon}_i-\mu_i)}^{\mu_i+\chi(\overline{\epsilon}_i-\mu_i)} (\epsilon - \mu_i)^2 dF_i\left(\mu_i + \frac{1}{\chi}(\epsilon - \mu_i)\right)\\ &= \chi^2 \int_{\underline{\epsilon}_i}^{\overline{\epsilon}_i} (\hat{\epsilon} - \mu_i)^2 dF_i\left(\hat{\epsilon}\right),\end{aligned}$$

which is χ^2 times the variance for stock i. Therefore, the standard deviation for stock i' is χ times that for stock i. The kurtosis for stock i' is

$$\frac{\int_{\mu_i+\chi(\underline{\epsilon}_i-\mu_i)}^{\mu_i+\chi(\overline{\epsilon}_i-\mu_i)} (\epsilon - \mu_i)^4 dF_{i'}(\epsilon)}{\left[\int_{\mu_i+\chi(\underline{\epsilon}_i-\mu_i)}^{\mu_i+\chi(\overline{\epsilon}_i-\mu_i)} (\epsilon - \mu_i)^2 dF_{i'}(\epsilon)\right]^2}.$$

Using (2.12) and then making the change of variable $\hat{\epsilon} = \mu_i + \frac{1}{\chi}(\epsilon - \mu_i)$, we can write that kurtosis as

$$\frac{\chi^4 \int_{\underline{\epsilon}_i}^{\overline{\epsilon}_i} (\hat{\epsilon} - \mu_i)^4 dF_i\left(\hat{\epsilon}\right)}{\left[\chi^2 \int_{\underline{\epsilon}_i}^{\overline{\epsilon}_i} (\hat{\epsilon} - \mu_i)^2 dF_i\left(\hat{\epsilon}\right)\right]^2} = \frac{\int_{\underline{\epsilon}_i}^{\overline{\epsilon}_i} (\hat{\epsilon} - \mu_i)^4 dF_i\left(\hat{\epsilon}\right)}{\left[\int_{\underline{\epsilon}_i}^{\overline{\epsilon}_i} (\hat{\epsilon} - \mu_i)^2 dF_i\left(\hat{\epsilon}\right)\right]^2},$$

which is the kurtosis for stock i.

We next consider comparative static (P). The same argument as for comparative static (I) implies that the supremum of the support of the distribution for stock i' is $\overline{\epsilon}_i$ and the infimum is $\underline{\epsilon}_i$. Therefore, the range for stock i' is $\overline{\epsilon}_i - \underline{\epsilon}_i$, which is the range for stock i. Using (2.13), we can write the variance for stock i' as

$$\psi \int_{\overline{\epsilon}_i}^{\underline{\epsilon}_i} (\epsilon - \mu_i)^2 dF_i(\epsilon),$$

which is ψ times the variance for stock i. Therefore, the standard deviation for stock i' is $\sqrt{\psi}$ times that for stock i. Using (2.13), we can write the kurtosis for stock i' as

$$\frac{\psi \int_{\underline{\epsilon}_i}^{\overline{\epsilon}_i} (\epsilon - \mu_i)^4 dF_i(\epsilon)}{\left[\psi \int_{\underline{\epsilon}_i}^{\overline{\epsilon}_i} (\epsilon - \mu_i)^2 dF_i(\epsilon)\right]^2} = \frac{1}{\psi} \frac{\int_{\underline{\epsilon}_i}^{\overline{\epsilon}_i} (\epsilon - \mu_i)^4 dF_i(\epsilon)}{\left[\int_{\underline{\epsilon}_i}^{\overline{\epsilon}_i} (\epsilon - \mu_i)^2 dF_i(\epsilon)\right]^2},$$

which is $\frac{1}{\psi}$ times the kurtosis for stock i. □

Proof of Proposition 2.2. We first consider comparative static (I). When (2.14) is violated, the argument that follows the proposition's statement implies that the short-sale constraint for stock i' is not binding for any investor, and the unique solution of (2.10) for stock i' satisfies

$$\phi_{i'} = \frac{\mu_i}{a\theta_i \sigma_i^2} - 1.$$

When instead (2.14) holds, the short-sale constraint for stock i' is binding for some investors, and the unique solution of (2.10) for stock i' satisfies

$$\phi_{i'} > \frac{\mu_i}{a\theta_i \sigma_i^2} - 1$$

and is given by

$$\int_{a\theta_i \sigma_i^2 \phi_{i'}}^{\overline{\epsilon}_{i'}} \left(\frac{\epsilon}{a\theta_i \sigma_i^2} - \phi_{i'} \right) dF_{i'}(\epsilon) = 1. \tag{A.5}$$

Using (2.12) and then making the change of variable $\hat{\epsilon} = \mu_i + \frac{1}{\chi}(\epsilon - \mu_i)$, we can write (A.5) as

$$\begin{aligned} &\int_{a\theta_i \sigma_i^2 \phi_{i'}}^{\overline{\epsilon}_{i'}} \left(\frac{\epsilon}{a\theta_i \sigma_i^2} - \phi_{i'} \right) dF_i\left(\mu_i + \frac{1}{\chi}(\epsilon - \mu_i) \right) = 1 \\ &= \int_{\mu_i + \frac{1}{\chi}(a\theta_i \sigma_i^2 \phi_{i'} - \mu_i)}^{\overline{\epsilon}_i} \left(\frac{\mu_i + \chi(\hat{\epsilon} - \mu_i)}{a\theta_i \sigma_i^2} - \phi_{i'} \right) dF_i(\hat{\epsilon}) = 1. \end{aligned} \tag{A.6}$$

Differentiating implicitly (A.6) with respect to χ holding $(\mu_i, \theta_i, b_i, \sigma_i, \underline{\epsilon}_i, \overline{\epsilon}_i)$ constant, we find

$$\begin{aligned} &\int_{\mu_i+\frac{1}{\chi}(a\theta_i\sigma_i^2\phi_{i'}-\mu_i)}^{\overline{\epsilon}_i} \left(\frac{\hat{\epsilon}-\mu_i}{a\theta_i\sigma_i^2} - \frac{\partial \phi_{i'}}{\partial \chi} \right) dF_i\left(\hat{\epsilon}\right) = 0 \\ \Rightarrow\ & \frac{\partial \phi_{i'}}{\partial \chi} = \frac{\int_{\mu_i+\frac{1}{\chi}(a\theta_i\sigma_i^2\phi_{i'}-\mu_i)}^{\overline{\epsilon}_i} \frac{\hat{\epsilon}-\mu_i}{a\theta_i\sigma_i^2} dF_i\left(\hat{\epsilon}\right)}{\int_{\mu_i+\frac{1}{\chi}(a\theta_i\sigma_i^2\phi_{i'}-\mu_i)}^{\overline{\epsilon}_i} dF_i\left(\hat{\epsilon}\right)} = \frac{1}{a\theta_i\sigma_i^2} \left[\mathbb{E}\left(\hat{\epsilon} | \hat{\epsilon} \in \left[\mu_i + \frac{1}{\chi}(a\theta_i\sigma_i^2\phi_{i'} - \mu_i), \overline{\epsilon}_i \right] \right) - \mu_i \right] > 0, \end{aligned} \tag{A.7}$$

where the positive sign follows because $\mu_i + \frac{1}{\chi}(a\theta_i\sigma_i^2\phi_{i'} - \mu_i) > \underline{\epsilon}_i$. Therefore, $\phi_{i'}$, viewed as function of χ holding $(\mu_i, \theta_i, b_i, \sigma_i, \underline{\epsilon}_i, \overline{\epsilon}_i)$ constant, increases in χ when (2.14) holds and is independent of χ when (2.14) is violated. Since there exists a threshold χ^* such that (2.14), viewed as function of χ holding $(\mu_i, \theta_i, b_i, \sigma_i, \underline{\epsilon}_i, \overline{\epsilon}_i)$ constant, holds for all $\chi > \chi^*$ and is violated for all $\chi \leq \chi^*$, $\phi_{i'} > \phi_i$ when (2.14) holds and $\phi_{i'} = \phi_i$ when (2.14) is violated. The statement in the proposition for comparative static (I) then follows from (2.9) and because expected return $\frac{b_i\bar{D}+\bar{D}-S_i}{S_i}$ is decreasing in the price.

We next consider comparative static (P). The same argument as for comparative static (I) implies that when (2.14) is violated, the short-sale constraint for stock i' is not binding for any investor, and $\phi_{i'} = \frac{\mu_i}{a\theta_i\sigma_i^2} - 1$. When instead (2.14) holds, the short-sale constraint for stock i' is binding for some investors, and $\phi_{i'}$ satisfies

$$\phi_{i'} > \frac{\mu_i}{a\theta_i\sigma_i^2} - 1$$

and is given by

$$\int_{a\theta_i\sigma_i^2\phi_{i'}}^{\overline{\epsilon}_i} \left(\frac{\epsilon}{a\theta_i\sigma_i^2} - \phi_{i'} \right) dF_{i'}(\epsilon) = 1. \tag{A.8}$$

Using (2.13), we can write (A.8) as

$$\psi \int_{a\theta_i\sigma_i^2\phi_{i'}}^{\overline{\epsilon}_i} \left(\frac{\epsilon}{a\theta_i\sigma_i^2} - \phi_{i'} \right) dF_i(\epsilon) + (1-\psi) \left(\frac{\mu_i}{a\theta_i\sigma_i^2} - \phi_{i'} \right) = 1 \tag{A.9}$$

when $\phi_{i'} < \frac{\mu_i}{a\theta_i\sigma_i^2}$, and as

$$\psi \int_{a\theta_i\sigma_i^2\phi_{i'}}^{\overline{\epsilon}_i} \left(\frac{\epsilon}{a\theta_i\sigma_i^2} - \phi_{i'} \right) dF_i(\epsilon) = 1 \tag{A.10}$$

when $\phi_{i'} \geq \frac{\mu_i}{a\theta_i\sigma_i^2}$. Differentiating implicitly (A.9) with respect to ψ holding $(\mu_i, \theta_i, b_i, \sigma_i, \underline{\epsilon}_i, \overline{\epsilon}_i)$

constant, we find

$$\int_{a\theta_i\sigma_i^2\phi_{i'}}^{\bar{\epsilon}_i}\left(\frac{\epsilon}{a\theta_i\sigma_i^2}-\phi_{i'}\right)dF_i(\epsilon)-\left(\frac{\mu_i}{a\theta_i\sigma_i^2}-\phi_{i'}\right)-\frac{\partial\phi_{i'}}{\partial\psi}\left(\psi\int_{a\theta_i\sigma_i^2\phi_{i'}}^{\bar{\epsilon}_{i'}}dF_i(\epsilon)+(1-\psi)\right)=0$$

$$\Rightarrow\frac{\partial\phi_{i'}}{\partial\psi}=-\frac{\int_{\underline{\epsilon}_i}^{a\theta_i\sigma_i^2\phi_{i'}}\left(\frac{\epsilon}{a\theta_i\sigma_i^2}-\phi_{i'}\right)dF_i(\epsilon)}{\psi\int_{a\theta_i\sigma_i^2\phi_{i'}}^{\bar{\epsilon}_{i'}}dF_i(\epsilon)+(1-\psi)}>0. \tag{A.11}$$

Differentiating implicitly (A.10) with respect to ψ holding $(\mu_i,\theta_i,b_i,\sigma_i,\underline{\epsilon}_i,\bar{\epsilon}_i)$ constant, we find

$$\int_{a\theta_i\sigma_i^2\phi_{i'}}^{\bar{\epsilon}_i}\left(\frac{\epsilon}{a\theta_i\sigma_i^2}-\phi_{i'}\right)dF_i(\epsilon)-\frac{\partial\phi_{i'}}{\partial\psi}\psi\int_{a\theta_i\sigma_i^2\phi_{i'}}^{\bar{\epsilon}_{i'}}dF_i(\epsilon)=0$$

$$\Rightarrow\frac{\partial\phi_{i'}}{\partial\psi}=\frac{\int_{a\theta_i\sigma_i^2\phi_{i'}}^{\bar{\epsilon}_i}\left(\frac{\epsilon}{a\theta_i\sigma_i^2}-\phi_{i'}\right)dF_i(\epsilon)}{\psi\int_{a\theta_i\sigma_i^2\phi_{i'}}^{\bar{\epsilon}_{i'}}dF_i(\epsilon)}>0. \tag{A.12}$$

Therefore, $\phi_{i'}$, viewed as function of ψ holding $(\mu_i,\theta_i,b_i,\sigma_i,\underline{\epsilon}_i,\bar{\epsilon}_i)$ constant, increases in ψ when (2.14) holds and is independent of ψ when (2.14) is violated. Since (2.14), viewed as function of ψ holding $(\mu_i,\theta_i,b_i,\sigma_i,\underline{\epsilon}_i,\bar{\epsilon}_i)$ constant, is independent of ψ, $\phi_{i'}<\phi_i$ when (2.14) holds and $\phi_{i'}=\phi_i$ when (2.14) is violated. The statement in the proposition for comparative static (P) then follows from (2.9) and because expected return is decreasing in the price. □

Proof of Proposition 2.3. We first consider comparative static (I). When (2.14) is violated, the argument that follows the statement of Proposition 2.2 implies that the short-sale constraint for stock i' is not binding for any investor. Therefore, $B_{i'}=1$. When instead (2.14) holds, the short-sale constraint for stock i' is binding for some investors and

$$\begin{aligned}B_{i'}&=\int_{a\theta_i\sigma_i^2\phi_{i'}}^{\bar{\epsilon}_{i'}}dF_{i'}(\epsilon)\\&=F_{i'}(\bar{\epsilon}_{i'})-F_{i'}\left(a\theta_i\sigma_i^2\phi_{i'}\right)\\&=F_i(\bar{\epsilon}_i)-F_i\left(\mu_i+\frac{1}{\chi}\left(a\theta_i\sigma_i^2\phi_{i'}-\mu_i\right)\right)\\&=1-F_i\left(\mu_i+\frac{1}{\chi}\left(a\theta_i\sigma_i^2\phi_{i'}-\mu_i\right)\right),\end{aligned} \tag{A.13}$$

where the third step follows from (2.12) and $\bar{\epsilon}_{i'}=\mu_i+\chi(\bar{\epsilon}_i-\mu_i)$. Differentiating $\mu_i+\frac{1}{\chi}\left(a\theta_i\sigma_i^2\phi_{i'}-\mu_i\right)$

with respect to χ holding $(\mu_i, \theta_i, b_i, \sigma_i, \underline{\epsilon}_i, \bar{\epsilon}_i)$ constant, and using (A.7), we find

$$
\begin{aligned}
&\frac{\partial}{\partial \chi}\left(\mu_i + \frac{1}{\chi}\left(a\theta_i\sigma_i^2\phi_{i'} - \mu_i\right)\right) \\
&= \frac{1}{\chi}\left(-\frac{1}{\chi}\left(a\theta_i\sigma_i^2\phi_{i'} - \mu_i\right) + \left[\mathbb{E}\left(\hat{\epsilon}|\hat{\epsilon} \in \left[\mu_i + \frac{1}{\chi}(a\theta_i\sigma_i^2\phi_{i'} - \mu_i), \bar{\epsilon}_i\right]\right) - \mu_i\right]\right) \\
&> \frac{1}{\chi}\left(-\frac{1}{\chi}\left(a\theta_i\sigma_i^2\phi_{i'} - \mu_i\right) + \left[\left[\mu_i + \frac{1}{\chi}(a\theta_i\sigma_i^2\phi_{i'} - \mu_i), \bar{\epsilon}_i\right] - \mu_i\right]\right) = 0,
\end{aligned}
$$

where the inequality is strict because (A.13) implies $\mu_i + \frac{1}{\chi}\left(a\theta_i\sigma_i^2\phi_{i'} - \mu_i\right) < \bar{\epsilon}_i$. Therefore, $\mu_i + \frac{1}{\chi}\left(a\theta_i\sigma_i^2\phi_{i'} - \mu_i\right)$ increases in χ and (A.13) implies that $B_{i'}$, viewed as function of χ holding $(\mu_i, \theta_i, b_i, \sigma_i, \underline{\epsilon}_i, \bar{\epsilon}_i)$ constant, is non-increasing in χ. Since $B_{i'}$ is non-increasing in χ when (2.14) holds and is independent of χ when (2.14) is violated, and since there exists a threshold χ^* such that (2.14) holds for all $\chi > \chi^*$ and is violated for all $\chi \leq \chi^*$, $B_{i'} \leq B_i$ when (2.14) holds and $B_{i'} = B_i$ when (2.14) is violated. When $F_i(\epsilon)$ increases in ϵ, (A.13) implies that $B_{i'}$ decreases in χ. Therefore, $B_{i'} < B_i$ when (2.14) holds and $B_{i'} = B_i$ when (2.14) is violated.

We next consider comparative static (P). When (2.14) is violated, the argument that follows the statement of Proposition 2.2 implies that the short-sale constraint for stock i' is not binding for any investor. Therefore, $B_{i'} = 1$. Since (2.14), viewed as function of ψ holding $(\mu_i, \theta_i, b_i, \sigma_i, \underline{\epsilon}_i, \bar{\epsilon}_i)$ constant, is independent of ψ, $B_{i'} = B_i$. When instead (2.14) holds, the short-sale constraint for stock i' is binding for some investors and

$$
B_{i'} = \int_{a\theta_i\sigma_i^2\phi_{i'}}^{\bar{\epsilon}_{i'}} dF_{i'}(\epsilon) = F_{i'}(\bar{\epsilon}_{i'}) - F_{i'}\left(a\theta_i\sigma_i^2\phi_{i'}\right). \tag{A.14}
$$

Using (2.13) and $\bar{\epsilon}_{i'} = \bar{\epsilon}_i$, we can write (A.14) as

$$
B_{i'} = \psi\left[F_i(\bar{\epsilon}_i) - F_i\left(a\theta_i\sigma_i^2\phi_{i'}\right)\right] + (1 - \psi) = 1 - \psi F_i\left(a\theta_i\sigma_i^2\phi_{i'}\right) \tag{A.15}
$$

when (2.18) is violated, and as

$$
B_{i'} = \psi\left[F_i(\bar{\epsilon}_i) - F_i\left(a\theta_i\sigma_i^2\phi_{i'}\right)\right] = \psi\left[1 - F_i\left(a\theta_i\sigma_i^2\phi_{i'}\right)\right] \tag{A.16}
$$

when (2.18) holds. We first consider the case where (2.19) holds. Since (A.11) implies that $\phi_{i'}$ increases in ψ, (2.18) is violated for all $\hat{\psi} \in [\psi, 1]$ and (A.15) implies that $B_{i'}$, viewed as function of ψ holding $(\mu_i, \theta_i, b_i, \sigma_i, \underline{\epsilon}_i, \bar{\epsilon}_i)$ constant, decreases in ψ. Therefore, $B_{i'} > B_i$. We next consider the case where (2.18) holds and $F_i(\epsilon)$ has a density $f_i(\epsilon)$ in $[\mu_i, \bar{\epsilon}_i]$ that is positive and non-decreasing in ϵ. Differentiating (A.16) with respect to ψ holding $(\mu_i, \theta_i, b_i, \sigma_i, \underline{\epsilon}_i, \bar{\epsilon}_i)$ constant, and using (A.12), we find

$$\begin{aligned}
\frac{\partial B_{i'}}{\partial \psi} &= 1 - F_i\left(a\theta_i\sigma_i^2\phi_{i'}\right) - \psi a\theta_i\sigma_i^2 f_i\left(a\theta_i\sigma_i^2\phi_{i'}\right)\frac{\int_{a\theta_i\sigma_i^2\phi_{i'}}^{\bar\epsilon_i}\left(\frac{\epsilon}{a\theta_i\sigma_i^2}-\phi_{i'}\right)dF_i(\epsilon)}{\psi\int_{a\theta_i\sigma_i^2\phi_{i'}}^{\bar\epsilon_{i'}}dF_i(\epsilon)} \\
&= 1 - F_i\left(a\theta_i\sigma_i^2\phi_{i'}\right) - \frac{a\theta_i\sigma_i^2 f_i\left(a\theta_i\sigma_i^2\phi_{i'}\right)}{\psi\left[1-F_i\left(a\theta_i\sigma_i^2\phi_{i'}\right)\right]} \\
&\geq 1 - F_i\left(a\theta_i\sigma_i^2\phi_{i'}\right) - \frac{a\theta_i\sigma_i^2}{\psi\left(\bar\epsilon_i - a\theta_i\sigma_i^2\phi_{i'}\right)} \\
&= \frac{a\theta_i\sigma_i^2}{\psi\left(\bar\epsilon_i - a\theta_i\sigma_i^2\phi_{i'}\right)}\left[\psi\left(\frac{\bar\epsilon_i}{a\theta_i\sigma_i^2}-\phi_{i'}\right)\left[1-F_i\left(a\theta_i\sigma_i^2\phi_{i'}\right)\right]-1\right] \\
&> \frac{a\theta_i\sigma_i^2}{\psi\left(\bar\epsilon_i - a\theta_i\sigma_i^2\phi_{i'}\right)}\left[\psi\int_{a\theta_i\sigma_i^2\phi_{i'}}^{\bar\epsilon_i}\left(\frac{\epsilon}{a\theta_i\sigma_i^2}-\phi_{i'}\right)dF_i(\epsilon)-1\right]=0,
\end{aligned}$$

where the second step follows from (A.10), the third step because $f_i(\epsilon)$ is non-decreasing in ϵ, the fifth step because the positive density $f_i(\epsilon)$ implies that $F_i(\epsilon)$ increases in $\epsilon \in [\mu_i, \bar\epsilon_i]$, and the sixth step from (A.10). Therefore, $B_{i'}$ increases in ψ. Since (A.11) implies that $\phi_{i'}$ increases in ψ, (2.18) holds for all $\hat\psi \in [\psi, 1]$, and $B_{i'} < B_i$.

Writing (A.8) as

$$\int_{a\theta_i\sigma_i^2\phi_{i'}}^{\bar\epsilon_i}\left(\epsilon - a\theta_i\sigma_i^2\phi_{i'}\right)dF_{i'}(\epsilon) = a\theta_i\sigma_i^2,$$

and differentiating implicitly with respect to $\theta_i\sigma_i^2$ holding $(\mu_i, b_i, \underline{\epsilon}_i, \bar\epsilon_i)$ constant, we find

$$-a\frac{\partial\left(\theta_i\sigma_i^2\phi_{i'}\right)}{\partial\left(\theta_i\sigma_i^2\right)}\int_{a\theta_i\sigma_i^2\phi_{i'}}^{\bar\epsilon_i}dF_{i'}(\epsilon) = a.$$

Therefore, $\theta_i\sigma_i^2\phi_{i'}$ decreases in $\theta_i\sigma_i^2$. Since (2.18) can be written as

$$\theta_i\sigma_i^2\phi_{i'} \geq \frac{\mu_i}{a},$$

there exists a threshold Θ_ψ^* such that (2.14) holds for $\theta_i\sigma_i^2 \leq \Theta_\psi^*$. Likewise, since (2.19) can be written as

$$\theta_i\sigma_i^2\phi_i < \frac{\mu_i}{a},$$

there exists a threshold Θ_1^* such that (2.14) holds for $\theta_i\sigma_i^2 > \Theta_1^*$. Since $\phi_{i'} < \phi_i$, these thresholds satisfy $\Theta_\psi^* < \Theta_1^*$. □

Proof of Proposition 2.4. When

$$\frac{\mu_i - \underline{\epsilon}_i}{a\theta_{i'}\sigma_i^2} > 1 \tag{A.17}$$

is violated, the argument that follows the statement of Proposition 2.2 implies that the short-sale constraint for stock i' is not binding for any investor. Therefore, $B_{i'} = 1$. When instead (A.17) holds, the short-sale constraint for stock i' is binding for some investors and

$$\begin{aligned} B_{i'} &= \int_{a\theta_{i'}\sigma_i^2\phi_{i'}}^{\bar{\epsilon}_i} dF_i(\epsilon) \\ &= F_i(\bar{\epsilon}_i) - F_i\left(a\theta_{i'}\sigma_i^2\phi_{i'}\right) \\ &= 1 - F_i\left(a\theta_{i'}\sigma_i^2\phi_{i'}\right). \end{aligned} \tag{A.18}$$

Since $\theta_{i'}\sigma_i^2\phi_{i'}$ decreases in $\theta_{i'}\sigma_i^2$, as shown in the proof of Proposition 2.3, (A.18) implies that $B_{i'}$, viewed as function of $\theta_{i'}$ holding $(b_i, \bar{D}_i, \sigma_i, F_i(\epsilon))$ constant is non-decreasing in $\theta_{i'}$. Since $B_{i'}$ is non-decreasing in $\theta_{i'}$ when (A.17) holds and is independent of $\theta_{i'}$ when (A.17) is violated, and since there exists a threshold θ^* such that (A.17) holds for all $\theta_{i'} < \theta^*$ and is violated for all $\theta_{i'} \geq \theta^*$, $B_{i'} \geq B_i$ when (2.20) holds and $B_{i'} = B_i$ when (2.20) is violated. When $F_i(\epsilon)$ increases in ϵ, (A.18) implies that $B_{i'}$ increases in $\theta_{i'}$. Therefore, $B_{i'} > B_i$ when (2.20) holds and $B_{i'} = B_i$ when (2.20) is violated. □

B Investment Styles of 13-F Investors by Thomson Reuters

Table B.I presents the 32 investment styles in which Thompson Reuters (TR) classifies 13-F investors.

TR classifies 13-F investors into styles based on the characteristics of the stocks that they hold, their historical investment behavior, their current transactions and their general business type. TR first classifies each stock into a certain group or style based on its price-earnings ratio, dividend yield, and the three- to five-year projected earnings-per-share growth relative to the corresponding S&P500 or sector averages. For each 13-F investor, TR then calculates the weights of the different groups or styles of stocks. The group with the biggest weight generally characterizes the investor's style.

Some classifications are more mechanical. 13-F investors whose portfolios follow the composition of certain indices (e.g. S&P 500, Russell 1000/2000/3000, etc) are classified into the Index style. Styles such as "Broker Dealer," "Hedge Funds" and "VC/Private Equity" are assigned mainly based on the business type of the corresponding investors. Finally, some 13-F investors

Table B.I: **The 32 investment styles in which Thomson Reuters classifies 13-F investors.**

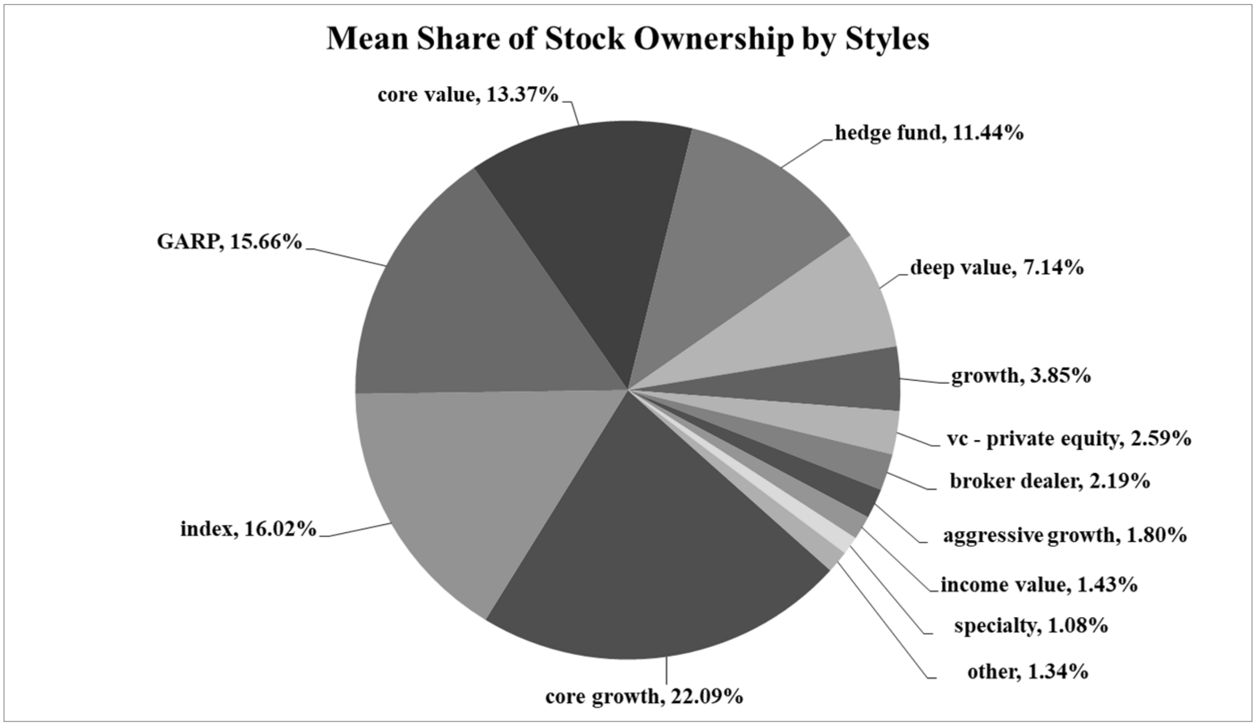
General Styles	Hedge Fund Styles
Aggressive Growth, Broker Dealer, Core Growth, Core Value, Deep Value, Emerging Markets, GARP(Growth at Reasonable Price), Growth, Hedge Fund, Income Value, Index, Mixed Style, Momentum, Sector Specific, Specialty, VC(Venture Capital)/Private Equity, Yield	Capital Structure Arbitrage, Convertible Arbitrage, CTA(Commodity Trading Advisors) Managed Futures, Distressed Securities, Emerging Markets (Hedge), Equity Hedge, Event Driven (Merger/ Risk Arbitrage), Fixed Income Arbitrage, Funds of Funds, Global Macro, Long Bias, Long-Short, Market Neutral, Multi-Strategy (Hedge), Quantitative/Statistical Arbitrage

Note: The 32 investment styles in which Thompson Reuters (TR) classifies 13-F investors. The left column reports the seventeen general styles and the right column reports the fifteen hedge fund styles. The styles are reported alphabetically in each column. The information is available on `http://banker.thomsonib.com/ta/help/webhelp/Ownership_Glossary.htm`

are classified into hedge-fund styles depending on their exact investment strategy (e.g. “Convertible Arbitrage,” “Quantitative-Statistical Arbitrage,” “Emerging Markets,” “Fund of funds”). The relative importance of hedge-fund styles is small.

The pie chart in Figure B.1 shows the size of each of the 32 styles in our sample, defined as the asset value attributed to the style over the total asset value of all styles. There are twelve styles with size above 1%. The combined size of the remaining twenty styles is 1.34%.

Figure B.1: **Mean share of stock ownership by style**



Note: Mean percentage shares in our sample of the 32 investment styles in which Thompson Reuters (TR) classifies 13-F investors. The average shares above 1% are reported separately (twelve styles) and the average shares below 1% are reported together as "other" (twenty styles).