Online Appendix (Not for Publication)

Contents

OA.1: Model Extension to Risk-Averse Informed Traders

In this extension, we assume that informed traders are risk averse. Trader i chooses y_i to maximize a standard mean-variance objective function, which results in the following expression for the optimal demand:

$$y_i = \frac{\mathbb{E}\left[v - p|\Omega_i\right]}{g\mathrm{Var}\left(v - p|\Omega_i\right)} \tag{OA.1.1}$$

where $g > 0$ denotes the traders' coefficient of risk aversion, $\Omega_i = \epsilon$ for $\epsilon-$informed, and $\Omega_i = \beta$ for $\beta-$informed.

As in the main model, the payoff is given by:

$$v = \epsilon + \beta\theta \tag{OA.1.2}$$

and the equilibrium stock price is given by:

$$p = \mathbb{E}[v|X] + \frac{\gamma}{2}X\mathrm{Var}(v|X). \tag{OA.1.3}$$

It follows that we get the following four realizations for y_i. To ease the exposition, we express the conditioning sets as $\{HH, HL, LH, LL\}$, where HH corresponds to $\epsilon = \Delta_\epsilon, \beta = \overline{\beta} + \Delta_\beta$, HL corresponds to $\epsilon = \Delta_\epsilon, \beta = \overline{\beta} - \Delta_\beta$, LH corresponds to $\epsilon = -\Delta_\epsilon, \beta = \overline{\beta} + \Delta_\beta$, and LL corresponds to $\epsilon = -\Delta_\epsilon, \beta = \overline{\beta} - \Delta_\beta$.

1. $\epsilon = \Delta_\epsilon$:

$$y(H\emptyset) = \frac{\mathbb{E}[v-p|HH] + \mathbb{E}[v-p|HL]}{g\left[\mathbb{E}[(v-p)^2|HH] + \mathbb{E}[(v-p)^2|HL] - \frac{1}{2}\left(\mathbb{E}[v-p|HH] + \mathbb{E}[v-p|HL]\right)^2\right]} \tag{OA.1.4}$$

2. $\epsilon = -\Delta_\epsilon$:

$$y(L\emptyset) = \frac{\mathbb{E}[v-p|LH] + \mathbb{E}[v-p|LL]}{g\left[\mathbb{E}[(v-p)^2|LH] + \mathbb{E}[(v-p)^2|LL] - \frac{1}{2}\left(\mathbb{E}[v-p|LH] + \mathbb{E}[v-p|LL]\right)^2\right]} \tag{OA.1.5}$$

3. $\beta = \overline{\beta} + \Delta_\beta$:

$$y(\emptyset H) = \frac{\mathbb{E}[v-p|HH] + \mathbb{E}[v-p|LH]}{g\left[\mathbb{E}[(v-p)^2|HH] + \mathbb{E}[(v-p)^2|LH] - \frac{1}{2}\left(\mathbb{E}[v-p|HH] + \mathbb{E}[v-p|LH]\right)^2\right]} \tag{OA.1.6}$$

4. $\beta = \overline{\beta} - \Delta_\beta$:

$$y(\emptyset L) = \frac{\mathbb{E}[v-p|HL] + \mathbb{E}[v-p|LL]}{g\left[\mathbb{E}[(v-p)^2|HL] + \mathbb{E}[(v-p)^2|LL] - \frac{1}{2}\left(\mathbb{E}[v-p|HL] + \mathbb{E}[v-p|LL]\right)^2\right]}. \tag{OA.1.7}$$

Next, we focus on the case with $\overline{\theta} > 0$ and provide sufficient conditions such that $y(H\emptyset), y(\emptyset H) \geq 1$ and that $y(L\emptyset), y(\emptyset L) \leq -1$. If $\overline{\theta} < 0$, then the sufficient conditions will ensure that $y(L\emptyset), y(\emptyset L) \geq 1$ and that $y(H\emptyset), y(\emptyset H) \leq -1$. Under these conditions, informed traders will continue to trade up to their position limits and all of our positive results in Section 3 continue to hold.

$$\begin{align}
g &< \frac{\mathbb{E}[v-p|HH] + \mathbb{E}[v-p|HL]}{\mathbb{E}[(v-p)^2|HH] + \mathbb{E}[(v-p)^2|HL] - \frac{1}{2}\left(\mathbb{E}[v-p|HH] + \mathbb{E}[v-p|HL]\right)^2} \tag{OA.1.8}\\
g &< \frac{-\left(\mathbb{E}[v-p|LH] + \mathbb{E}[v-p|LL]\right)}{\mathbb{E}[(v-p)^2|LH] + \mathbb{E}[(v-p)^2|LL] - \frac{1}{2}\left(\mathbb{E}[v-p|LH] + \mathbb{E}[v-p|LL]\right)^2} \tag{OA.1.9}\\
g &< \frac{\mathbb{E}[v-p|HH] + \mathbb{E}[v-p|LH]}{\mathbb{E}[(v-p)^2|HH] + \mathbb{E}[(v-p)^2|LH] - \frac{1}{2}\left(\mathbb{E}[v-p|HH] + \mathbb{E}[v-p|LH]\right)^2} \tag{OA.1.10}\\
g &< \frac{-\left(\mathbb{E}[v-p|HL] + \mathbb{E}[v-p|LL]\right)}{\mathbb{E}[(v-p)^2|HL] + \mathbb{E}[(v-p)^2|LL] - \frac{1}{2}\left(\mathbb{E}[v-p|HL] + \mathbb{E}[v-p|LL]\right)^2}. \tag{OA.1.11}
\end{align}$$

We have already shown in Appendix A.1 that the numerators in these four conditions are positive. Furthermore, the denominator is proportional to the conditional return variance and thus also positive. Importantly, the right-hand sides do not depend on the informed traders' coefficient of risk-aversion, which implies that g has to be lower than a positive constant.

Note that we can rewrite $\mathbb{E}[v-p]$ and $\mathbb{E}[(v-p)^2]$ as:

$$\begin{align}
\mathbb{E}[v-p|HH] &= \Delta_\epsilon + (\overline{\beta} + \Delta_\beta)\overline{\theta} - \mathbb{E}[p|HH] \tag{OA.1.12}\\
\mathbb{E}[v-p|HL] &= \Delta_\epsilon + (\overline{\beta} - \Delta_\beta)\overline{\theta} - \mathbb{E}[p|HL] \tag{OA.1.13}\\
\mathbb{E}[v-p|LH] &= -\Delta_\epsilon + (\overline{\beta} + \Delta_\beta)\overline{\theta} - \mathbb{E}[p|LH] \tag{OA.1.14}\\
\mathbb{E}[v-p|LL] &= -\Delta_\epsilon + (\overline{\beta} - \Delta_\beta)\overline{\theta} - \mathbb{E}[p|LL] \tag{OA.1.15}\\
\mathbb{E}[(v-p)^2|HH] &= \left(\Delta_\epsilon + (\overline{\beta} + \Delta_\beta)\overline{\theta}\right)^2 - 2\left(\Delta_\epsilon + (\overline{\beta} + \Delta_\beta)\overline{\theta}\right)\mathbb{E}[p|HH] + \mathbb{E}[p^2|HH] \tag{OA.1.16}\\
\mathbb{E}[(v-p)^2|HL] &= \left(\Delta_\epsilon + (\overline{\beta} - \Delta_\beta)\overline{\theta}\right)^2 - 2\left(\Delta_\epsilon + (\overline{\beta} - \Delta_\beta)\overline{\theta}\right)\mathbb{E}[p|HL] + \mathbb{E}[p^2|HL] \tag{OA.1.17}\\
\mathbb{E}[(v-p)^2|LH] &= \left((\overline{\beta} + \Delta_\beta)\overline{\theta} - \Delta_\epsilon\right)^2 - 2\left(-\Delta_\epsilon + (\overline{\beta} + \Delta_\beta)\overline{\theta}\right)\mathbb{E}[p|LH] + \mathbb{E}[p^2|LH] \tag{OA.1.18}\\
\mathbb{E}[(v-p)^2|LL] &= \left((\overline{\beta} - \Delta_\beta)\overline{\theta} - \Delta_\epsilon\right)^2 - 2\left(-\Delta_\epsilon + (\overline{\beta} - \Delta_\beta)\overline{\theta}\right)\mathbb{E}[p|LL] + \mathbb{E}[p^2|LL] \tag{OA.1.19}
\end{align}$$

Finally, we can express $\mathbb{E}[p|\cdot\cdot]$ and $\mathbb{E}[p^2|\cdot\cdot]$ in terms of model parameters. First, if $\chi_\epsilon > \chi_\beta$, then we get the following set of expressions:

$$\begin{aligned}\mathbb{E}[p|HH] = \frac{1}{36}\Big(&2\chi_\epsilon\Big(2\gamma\chi_\epsilon\Big(\Delta_\beta\Big(2\overline{\theta}^2\Delta_\beta - 3\overline{\beta}\sigma_\theta^2\Big) + 2\Delta_\epsilon^2\Big) - 6\gamma\overline{\beta}\Delta_\beta\sigma_\theta^2 + \overline{\theta}\Delta_\beta\Big(\gamma\overline{\theta}\Delta_\beta - 12\Big) - 8\gamma\Delta_\epsilon^2 + 6\Delta_\epsilon\Big)\\
&+ 3\gamma\sigma_\theta^2\Big(8\overline{\beta}\Delta_\beta + 3\overline{\beta}^2 + 3\Delta_\beta^2\Big) + \overline{\theta}\Delta_\beta\Big(24 - \gamma\overline{\theta}\Delta_\beta\Big) + 18\overline{\beta}\overline{\theta} + 8\gamma\Delta_\epsilon^2 + 6\Delta_\epsilon\Big)\end{aligned}$$

$$\begin{aligned}\mathbb{E}[p|HL] = \frac{1}{12}\Big(&2\chi_\epsilon\Big(2\gamma\overline{\beta}\chi_\epsilon\Delta_\beta\sigma_\theta^2 + \gamma\sigma_\theta^2\Big(-4\overline{\beta}\Delta_\beta + 3\overline{\beta}^2 + 3\Delta_\beta^2\Big) + 3\gamma\overline{\theta}^2\Delta_\beta^2 + 6\Delta_\epsilon\Big)\\
&+ \gamma\sigma_\theta^2\Big(4\overline{\beta}\Delta_\beta - 3\overline{\beta}^2 - 3\Delta_\beta^2\Big) - 3\gamma\overline{\theta}^2\Delta_\beta^2 + 6\overline{\beta}\overline{\theta} - 6\Delta_\epsilon\Big)\end{aligned}$$

$$\begin{aligned}\mathbb{E}[p|LH] = \frac{1}{12}\Big(&-2\chi_\epsilon\Big(-2\gamma\overline{\beta}\chi_\epsilon\Delta_\beta\sigma_\theta^2 + \gamma\sigma_\theta^2\Big(4\overline{\beta}\Delta_\beta + 3\overline{\beta}^2 + 3\Delta_\beta^2\Big) + 3\gamma\overline{\theta}^2\Delta_\beta^2 + 6\Delta_\epsilon\Big)\\
&+ \gamma\sigma_\theta^2\Big(4\overline{\beta}\Delta_\beta + 3\overline{\beta}^2 + 3\Delta_\beta^2\Big) + 3\overline{\theta}\Big(\gamma\overline{\theta}\Delta_\beta^2 + 2\overline{\beta}\Big) + 6\Delta_\epsilon\Big)\end{aligned}$$

$$\begin{aligned}\mathbb{E}[p|LL] = \frac{1}{36}\Big(&-4\gamma\chi_\epsilon^2\Big(\Delta_\beta\Big(2\overline{\theta}^2\Delta_\beta + 3\overline{\beta}\sigma_\theta^2\Big) + 2\Delta_\epsilon^2\Big) - 2\chi_\epsilon\Big(6\gamma\overline{\beta}\Delta_\beta\sigma_\theta^2 + \overline{\theta}\Delta_\beta\Big(\gamma\overline{\theta}\Delta_\beta - 12\Big) - 8\gamma\Delta_\epsilon^2 + 6\Delta_\epsilon\Big)\\
&- 3\gamma\sigma_\theta^2\Big(-8\overline{\beta}\Delta_\beta + 3\overline{\beta}^2 + 3\Delta_\beta^2\Big) + \overline{\theta}\Delta_\beta\Big(\gamma\overline{\theta}\Delta_\beta - 24\Big) + 18\overline{\beta}\overline{\theta} - 8\gamma\Delta_\epsilon^2 - 6\Delta_\epsilon\Big)\end{aligned}$$

$$\begin{aligned}\mathbb{E}[p^2|HH] = \frac{1}{486}\Big(&-64\gamma^2\chi_\epsilon^3\Delta_\epsilon^4 + 64\gamma^2\Delta_\epsilon^4 + 192\gamma^2\chi_\epsilon^2\Delta_\epsilon^4 - 192\gamma^2\chi_\epsilon\Delta_\epsilon^4 + 72\gamma\chi_\epsilon^2\Delta_\epsilon^3 + 72\gamma\Delta_\epsilon^3 - 144\gamma\chi_\epsilon\Delta_\epsilon^3 - 128\gamma^2\overline{\theta}^2\Delta_\beta^2\chi_\epsilon^3\Delta_\epsilon^2\\
&- 144\gamma^2\overline{\beta}^2\sigma_\theta^2\chi_\epsilon^3\Delta_\epsilon^2 - 144\gamma^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^3\Delta_\epsilon^2 - 96\gamma^2\overline{\beta}\Delta_\beta\sigma_\theta^2\chi_\epsilon^3\Delta_\epsilon^2 + 128\gamma^2\overline{\theta}^2\Delta_\beta^2\Delta_\epsilon^2 + 144\gamma^2\overline{\beta}^2\sigma_\theta^2\Delta_\epsilon^2 + 144\gamma^2\Delta_\beta^2\sigma_\theta^2\Delta_\epsilon^2 + 96\gamma^2\overline{\beta}\Delta_\beta\sigma_\theta^2\Delta_\epsilon^2\\
&+ 384\gamma^2\overline{\theta}^2\Delta_\beta^2\chi_\epsilon^2\Delta_\epsilon^2 + 432\gamma^2\overline{\beta}^2\sigma_\theta^2\chi_\epsilon^2\Delta_\epsilon^2 + 432\gamma^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^2\Delta_\epsilon^2 + 288\gamma^2\overline{\beta}\Delta_\beta\sigma_\theta^2\chi_\epsilon^2\Delta_\epsilon^2 + 216\gamma\overline{\beta}\overline{\theta}\chi_\epsilon^2\Delta_\epsilon^2 + 72\gamma\overline{\theta}\Delta_\beta\chi_\epsilon^2\Delta_\epsilon^2 + 216\gamma\overline{\beta}\overline{\theta}\Delta_\epsilon^2\\
&+ 72\gamma\overline{\theta}\Delta_\beta\Delta_\epsilon^2 - 384\gamma^2\overline{\theta}^2\Delta_\beta^2\chi_\epsilon\Delta_\epsilon^2 - 432\gamma^2\overline{\beta}^2\sigma_\theta^2\chi_\epsilon\Delta_\epsilon^2 - 432\gamma^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon\Delta_\epsilon^2 - 288\gamma^2\overline{\beta}\Delta_\beta\sigma_\theta^2\chi_\epsilon\Delta_\epsilon^2 - 432\gamma\overline{\beta}\overline{\theta}\chi_\epsilon\Delta_\epsilon^2 - 144\gamma\overline{\theta}\Delta_\beta\chi_\epsilon\Delta_\epsilon^2\\
&+ 216\chi_\epsilon\Delta_\epsilon^2 + 27\Delta_\epsilon^2 - 171\gamma\overline{\theta}^2\Delta_\beta^2\Delta_\epsilon + 81\gamma\overline{\beta}^2\sigma_\theta^2\Delta_\epsilon + 81\gamma\Delta_\beta^2\sigma_\theta^2\Delta_\epsilon + 540\gamma\overline{\beta}\Delta_\beta\sigma_\theta^2\Delta_\epsilon + 72\gamma\overline{\theta}^2\Delta_\beta^2\chi_\epsilon^2\Delta_\epsilon - 162\gamma\overline{\beta}^2\sigma_\theta^2\chi_\epsilon^2\Delta_\epsilon\\
&- 162\gamma\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^2\Delta_\epsilon - 432\gamma\overline{\beta}\Delta_\beta\sigma_\theta^2\chi_\epsilon^2\Delta_\epsilon + 162\overline{\beta}\overline{\theta}\Delta_\epsilon + 540\overline{\theta}\Delta_\beta\Delta_\epsilon + 342\gamma\overline{\theta}^2\Delta_\beta^2\chi_\epsilon\Delta_\epsilon + 324\gamma\overline{\beta}^2\sigma_\theta^2\chi_\epsilon\Delta_\epsilon + 324\gamma\Delta_\beta^2\sigma_\theta^2\chi_\epsilon\Delta_\epsilon\\
&- 108\gamma\overline{\beta}\Delta_\beta\sigma_\theta^2\chi_\epsilon\Delta_\epsilon + 324\overline{\beta}\overline{\theta}\chi_\epsilon\Delta_\epsilon - 540\overline{\theta}\Delta_\beta\chi_\epsilon\Delta_\epsilon - 17\gamma^2\overline{\theta}^4\Delta_\beta^4 + 81\gamma^2\overline{\beta}^4\sigma_\theta^4 + 81\gamma^2\Delta_\beta^4\sigma_\theta^4 + 432\gamma^2\overline{\beta}\Delta_\beta^3\sigma_\theta^4 + 522\gamma^2\overline{\beta}^2\Delta_\beta^2\sigma_\theta^4\\
&+ 432\gamma^2\overline{\beta}^3\Delta_\beta\sigma_\theta^4 + 72\gamma\overline{\theta}^3\Delta_\beta^3 + 98\gamma^2\overline{\theta}^4\Delta_\beta^4\chi_\epsilon^3 - 432\gamma^2\overline{\beta}\Delta_\beta^3\sigma_\theta^4\chi_\epsilon^3 - 360\gamma^2\overline{\beta}^2\Delta_\beta^2\sigma_\theta^4\chi_\epsilon^3 - 432\gamma^2\overline{\beta}^3\Delta_\beta\sigma_\theta^4\chi_\epsilon^3 + 180\gamma^2\overline{\theta}^2\Delta_\beta^4\sigma_\theta^2\chi_\epsilon^3\\
&- 96\gamma^2\overline{\beta}\overline{\theta}^2\Delta_\beta^3\sigma_\theta^2\chi_\epsilon^3 + 180\gamma^2\overline{\beta}^2\overline{\theta}^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^3 + 243\overline{\beta}^2\overline{\theta}^2 - 27\gamma\overline{\beta}\overline{\theta}^3\Delta_\beta^2 + 270\overline{\theta}^2\Delta_\beta^2 - 18\gamma^2\overline{\theta}^2\Delta_\beta^4\sigma_\theta^2 + 96\gamma^2\overline{\beta}\overline{\theta}^2\Delta_\beta^3\sigma_\theta^2 + 324\gamma\overline{\theta}\Delta_\beta^3\sigma_\theta^2\\
&- 18\gamma^2\overline{\beta}^2\overline{\theta}^2\Delta_\beta^2\sigma_\theta^2 + 783\gamma\overline{\beta}\overline{\theta}\Delta_\beta^2\sigma_\theta^2 + 243\gamma\overline{\beta}^3\overline{\theta}\sigma_\theta^2 + 972\gamma\overline{\beta}^2\overline{\theta}\Delta_\beta\sigma_\theta^2 - 51\gamma^2\overline{\theta}^4\Delta_\beta^4\chi_\epsilon^2 + 324\gamma^2\overline{\beta}\Delta_\beta^3\sigma_\theta^4\chi_\epsilon^2 + 108\gamma^2\overline{\beta}^2\Delta_\beta^2\sigma_\theta^4\chi_\epsilon^2\\
&+ 324\gamma^2\overline{\beta}^3\Delta_\beta\sigma_\theta^4\chi_\epsilon^2 + 72\gamma\overline{\theta}^3\Delta_\beta^3\chi_\epsilon^2 + 216\gamma\overline{\beta}\overline{\theta}^3\Delta_\beta^2\chi_\epsilon^2 - 54\gamma^2\overline{\theta}^2\Delta_\beta^4\sigma_\theta^2\chi_\epsilon^2 + 288\gamma^2\overline{\beta}\overline{\theta}^2\Delta_\beta^3\sigma_\theta^2\chi_\epsilon^2 - 162\gamma\overline{\theta}\Delta_\beta^3\sigma_\theta^2\chi_\epsilon^2 - 54\gamma^2\overline{\beta}^2\overline{\theta}^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^2\\
&- 432\gamma\overline{\beta}\overline{\theta}\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^2 - 486\gamma\overline{\beta}^2\overline{\theta}\Delta_\beta\sigma_\theta^2\chi_\epsilon^2 + 648\overline{\beta}\overline{\theta}^2\Delta_\beta + 51\gamma^2\overline{\theta}^4\Delta_\beta^4\chi_\epsilon - 324\gamma^2\overline{\beta}\Delta_\beta^3\sigma_\theta^4\chi_\epsilon - 108\gamma^2\overline{\beta}^2\Delta_\beta^2\sigma_\theta^4\chi_\epsilon - 324\gamma^2\overline{\beta}^3\Delta_\beta\sigma_\theta^4\chi_\epsilon - 144\gamma\overline{\theta}^3\Delta_\beta^3\chi_\epsilon\\
&+ 54\gamma\overline{\beta}\overline{\theta}^3\Delta_\beta^2\chi_\epsilon - 270\overline{\theta}^2\Delta_\beta^2\chi_\epsilon + 54\gamma^2\overline{\theta}^2\Delta_\beta^4\sigma_\theta^2\chi_\epsilon - 288\gamma^2\overline{\beta}\overline{\theta}^2\Delta_\beta^3\sigma_\theta^2\chi_\epsilon - 162\gamma\overline{\theta}\Delta_\beta^3\sigma_\theta^2\chi_\epsilon + 54\gamma^2\overline{\beta}^2\overline{\theta}^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon - 108\gamma\overline{\beta}\overline{\theta}\Delta_\beta^2\sigma_\theta^2\chi_\epsilon - 486\gamma\overline{\beta}^2\overline{\theta}\Delta_\beta\sigma_\theta^2\chi_\epsilon\\
&- 648\overline{\beta}\overline{\theta}^2\Delta_\beta\chi_\epsilon\Big)\end{aligned}$$

$$
\begin{aligned}
\mathbb{E}[p^2|HL] = \frac{1}{486}\Big(&-128\gamma^2\chi_\epsilon^3\Delta_\epsilon^4 + 128\gamma^2\Delta_\epsilon^4 + 384\gamma^2\chi_\epsilon^2\Delta_\epsilon^4 - 384\gamma^2\chi_\epsilon\Delta_\epsilon^4 + 144\gamma\chi_\epsilon^2\Delta_\epsilon^3 + 144\gamma\Delta_\epsilon^3 - 288\gamma\chi_\epsilon\Delta_\epsilon^3 - 256\gamma^2\overline{\theta}^2\Delta_\beta^2\chi_\epsilon^3\Delta_\epsilon^2 \\
&- 288\gamma^2\overline{\beta}^2\sigma_\theta^2\chi_\epsilon^3\Delta_\epsilon^2 - 288\gamma^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^3\Delta_\epsilon^2 + 256\gamma^2\overline{\theta}^2\Delta_\beta^2\Delta_\epsilon^2 + 288\gamma^2\overline{\beta}^2\sigma_\theta^2\Delta_\epsilon^2 + 288\gamma^2\Delta_\beta^2\sigma_\theta^2\Delta_\epsilon^2 + 768\gamma^2\overline{\theta}^2\Delta_\beta^2\chi_\epsilon^2\Delta_\epsilon^2 \\
&+ 864\gamma^2\overline{\beta}^2\sigma_\theta^2\chi_\epsilon^2\Delta_\epsilon^2 + 864\gamma^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^2\Delta_\epsilon^2 + 144\gamma\overline{\theta}\Delta_\beta\chi_\epsilon^2\Delta_\epsilon^2 + 144\gamma\overline{\theta}\Delta_\beta\Delta_\epsilon^2 - 768\gamma^2\overline{\theta}^2\Delta_\beta^2\chi_\epsilon\Delta_\epsilon^2 - 864\gamma^2\overline{\beta}^2\sigma_\theta^2\chi_\epsilon\Delta_\epsilon^2 \\
&- 864\gamma^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon\Delta_\epsilon^2 - 288\gamma\overline{\theta}\Delta_\beta\chi_\epsilon\Delta_\epsilon^2 + 432\chi_\epsilon\Delta_\epsilon^2 - 189\Delta_\epsilon^2 - 99\gamma\overline{\theta}^2\Delta_\beta^2\Delta_\epsilon - 81\gamma\overline{\beta}^2\sigma_\theta^2\Delta_\epsilon - 81\gamma\Delta_\beta^2\sigma_\theta^2\Delta_\epsilon + 144\gamma\overline{\theta}^2\Delta_\beta^2\chi_\epsilon^2\Delta_\epsilon \\
&+ 162\gamma\overline{\beta}^2\sigma_\theta^2\chi_\epsilon^2\Delta_\epsilon + 162\gamma\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^2\Delta_\epsilon - 486\overline{\beta}\,\overline{\theta}\Delta_\epsilon + 108\overline{\theta}\Delta_\beta\Delta_\epsilon + 198\gamma\overline{\theta}^2\Delta_\beta^2\chi_\epsilon\Delta_\epsilon + 162\gamma\overline{\beta}^2\sigma_\theta^2\chi_\epsilon\Delta_\epsilon + 162\gamma\Delta_\beta^2\sigma_\theta^2\chi_\epsilon\Delta_\epsilon \\
&+ 972\overline{\beta}\,\overline{\theta}\chi_\epsilon\Delta_\epsilon - 108\overline{\theta}\Delta_\beta\chi_\epsilon\Delta_\epsilon + 47\gamma^2\overline{\theta}^4\Delta_\beta^4 + 81\gamma^2\overline{\beta}^4\sigma_\theta^4 + 81\gamma^2\Delta_\beta^4\sigma_\theta^4 + 234\gamma^2\overline{\beta}^2\Delta_\beta^2\sigma_\theta^4 + 144\gamma\overline{\theta}^3\Delta_\beta^3 + 34\gamma^2\overline{\theta}^4\Delta_\beta^4\chi_\epsilon^3 \\
&- 72\gamma^2\overline{\beta}^2\Delta_\beta^2\sigma_\theta^4\chi_\epsilon^3 + 36\gamma^2\overline{\theta}^2\Delta_\beta^4\sigma_\theta^2\chi_\epsilon^3 + 36\gamma^2\overline{\beta}^2\overline{\theta}^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^3 + 243\overline{\beta}^2\overline{\theta}^2 - 243\gamma\overline{\beta}\,\overline{\theta}^3\Delta_\beta^2 + 54\overline{\theta}^2\Delta_\beta^2 + 126\gamma^2\overline{\theta}^2\Delta_\beta^4\sigma_\theta^2 + 162\gamma\overline{\theta}\Delta_\beta^3\sigma_\theta^2 \\
&+ 126\gamma^2\overline{\beta}^2\overline{\theta}^2\Delta_\beta^2\sigma_\theta^2 - 243\gamma\overline{\beta}\,\overline{\theta}\Delta_\beta^2\sigma_\theta^2 - 243\gamma\overline{\beta}^3\overline{\theta}\sigma_\theta^2 + 486\gamma\overline{\beta}^2\overline{\theta}\Delta_\beta\sigma_\theta^2 + 141\gamma^2\overline{\theta}^4\Delta_\beta^4\chi_\epsilon^2 + 243\gamma^2\overline{\beta}^4\sigma_\theta^4\chi_\epsilon^2 + 243\gamma^2\Delta_\beta^4\sigma_\theta^4\chi_\epsilon^2 \\
&+ 702\gamma^2\overline{\beta}^2\Delta_\beta^2\sigma_\theta^4\chi_\epsilon^2 + 144\gamma\overline{\theta}^3\Delta_\beta^3\chi_\epsilon^2 + 378\gamma^2\overline{\theta}^2\Delta_\beta^4\sigma_\theta^2\chi_\epsilon^2 + 162\gamma\overline{\theta}\Delta_\beta^3\sigma_\theta^2\chi_\epsilon^2 + 378\gamma^2\overline{\beta}^2\overline{\theta}^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^2 + 486\gamma\overline{\beta}^2\overline{\theta}\Delta_\beta\sigma_\theta^2\chi_\epsilon^2 \\
&- 141\gamma^2\overline{\theta}^4\Delta_\beta^4\chi_\epsilon - 243\gamma^2\overline{\beta}^4\sigma_\theta^4\chi_\epsilon - 243\gamma^2\Delta_\beta^4\sigma_\theta^4\chi_\epsilon - 702\gamma^2\overline{\beta}^2\Delta_\beta^2\sigma_\theta^4\chi_\epsilon - 288\gamma\overline{\theta}^3\Delta_\beta^3\chi_\epsilon + 486\gamma\overline{\beta}\,\overline{\theta}^3\Delta_\beta^2\chi_\epsilon - 54\overline{\theta}^2\Delta_\beta^2\chi_\epsilon \\
&- 378\gamma^2\overline{\theta}^2\Delta_\beta^4\sigma_\theta^2\chi_\epsilon - 324\gamma\overline{\theta}\Delta_\beta^3\sigma_\theta^2\chi_\epsilon - 378\gamma^2\overline{\beta}^2\overline{\theta}^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon + 486\gamma\overline{\beta}\,\overline{\theta}\Delta_\beta^2\sigma_\theta^2\chi_\epsilon + 486\gamma\overline{\beta}^3\overline{\theta}\sigma_\theta^2\chi_\epsilon - 972\gamma\overline{\beta}^2\overline{\theta}\Delta_\beta\sigma_\theta^2\chi_\epsilon\Big)
\end{aligned}
$$

$$
\begin{aligned}
\mathbb{E}[p^2|LH] = \frac{1}{486}\Big(&-128\gamma^2\chi_\epsilon^3\Delta_\epsilon^4 + 128\gamma^2\Delta_\epsilon^4 + 384\gamma^2\chi_\epsilon^2\Delta_\epsilon^4 - 384\gamma^2\chi_\epsilon\Delta_\epsilon^4 + 144\gamma\chi_\epsilon^2\Delta_\epsilon^3 + 144\gamma\Delta_\epsilon^3 - 288\gamma\chi_\epsilon\Delta_\epsilon^3 - 256\gamma^2\overline{\theta}^2\Delta_\beta^2\chi_\epsilon^3\Delta_\epsilon^2 \\
&- 288\gamma^2\overline{\beta}^2\sigma_\theta^2\chi_\epsilon^3\Delta_\epsilon^2 - 288\gamma^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^3\Delta_\epsilon^2 + 256\gamma^2\overline{\theta}^2\Delta_\beta^2\Delta_\epsilon^2 + 288\gamma^2\overline{\beta}^2\sigma_\theta^2\Delta_\epsilon^2 + 288\gamma^2\Delta_\beta^2\sigma_\theta^2\Delta_\epsilon^2 + 768\gamma^2\overline{\theta}^2\Delta_\beta^2\chi_\epsilon^2\Delta_\epsilon^2 \\
&+ 864\gamma^2\overline{\beta}^2\sigma_\theta^2\chi_\epsilon^2\Delta_\epsilon^2 + 864\gamma^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^2\Delta_\epsilon^2 + 144\gamma\overline{\theta}\Delta_\beta\chi_\epsilon^2\Delta_\epsilon^2 + 144\gamma\overline{\theta}\Delta_\beta\Delta_\epsilon^2 - 768\gamma^2\overline{\theta}^2\Delta_\beta^2\chi_\epsilon\Delta_\epsilon^2 - 864\gamma^2\overline{\beta}^2\sigma_\theta^2\chi_\epsilon\Delta_\epsilon^2 \\
&- 864\gamma^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon\Delta_\epsilon^2 - 288\gamma\overline{\theta}\Delta_\beta\chi_\epsilon\Delta_\epsilon^2 + 432\chi_\epsilon\Delta_\epsilon^2 - 189\Delta_\epsilon^2 - 99\gamma\overline{\theta}^2\Delta_\beta^2\Delta_\epsilon - 81\gamma\overline{\beta}^2\sigma_\theta^2\Delta_\epsilon - 81\gamma\Delta_\beta^2\sigma_\theta^2\Delta_\epsilon + 144\gamma\overline{\theta}^2\Delta_\beta^2\chi_\epsilon^2\Delta_\epsilon \\
&+ 162\gamma\overline{\beta}^2\sigma_\theta^2\chi_\epsilon^2\Delta_\epsilon + 162\gamma\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^2\Delta_\epsilon + 486\overline{\beta}\,\overline{\theta}\Delta_\epsilon + 108\overline{\theta}\Delta_\beta\Delta_\epsilon + 198\gamma\overline{\theta}^2\Delta_\beta^2\chi_\epsilon\Delta_\epsilon + 162\gamma\overline{\beta}^2\sigma_\theta^2\chi_\epsilon\Delta_\epsilon + 162\gamma\Delta_\beta^2\sigma_\theta^2\chi_\epsilon\Delta_\epsilon \\
&- 972\overline{\beta}\,\overline{\theta}\chi_\epsilon\Delta_\epsilon - 108\overline{\theta}\Delta_\beta\chi_\epsilon\Delta_\epsilon + 47\gamma^2\overline{\theta}^4\Delta_\beta^4 + 81\gamma^2\overline{\beta}^4\sigma_\theta^4 + 81\gamma^2\Delta_\beta^4\sigma_\theta^4 + 234\gamma^2\overline{\beta}^2\Delta_\beta^2\sigma_\theta^4 + 144\gamma\overline{\theta}^3\Delta_\beta^3 + 34\gamma^2\overline{\theta}^4\Delta_\beta^4\chi_\epsilon^3 \\
&- 72\gamma^2\overline{\beta}^2\Delta_\beta^2\sigma_\theta^4\chi_\epsilon^3 + 36\gamma^2\overline{\theta}^2\Delta_\beta^4\sigma_\theta^2\chi_\epsilon^3 + 36\gamma^2\overline{\beta}^2\overline{\theta}^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^3 + 243\overline{\beta}^2\overline{\theta}^2 + 243\gamma\overline{\beta}\,\overline{\theta}^3\Delta_\beta^2 + 54\overline{\theta}^2\Delta_\beta^2 + 126\gamma^2\overline{\theta}^2\Delta_\beta^4\sigma_\theta^2 + 162\gamma\overline{\theta}\Delta_\beta^3\sigma_\theta^2 \\
&+ 126\gamma^2\overline{\beta}^2\overline{\theta}^2\Delta_\beta^2\sigma_\theta^2 + 243\gamma\overline{\beta}\,\overline{\theta}\Delta_\beta^2\sigma_\theta^2 + 243\gamma\overline{\beta}^3\overline{\theta}\sigma_\theta^2 + 486\gamma\overline{\beta}^2\overline{\theta}\Delta_\beta\sigma_\theta^2 + 141\gamma^2\overline{\theta}^4\Delta_\beta^4\chi_\epsilon^2 + 243\gamma^2\overline{\beta}^4\sigma_\theta^4\chi_\epsilon^2 + 243\gamma^2\Delta_\beta^4\sigma_\theta^4\chi_\epsilon^2 \\
&+ 702\gamma^2\overline{\beta}^2\Delta_\beta^2\sigma_\theta^4\chi_\epsilon^2 + 144\gamma\overline{\theta}^3\Delta_\beta^3\chi_\epsilon^2 + 378\gamma^2\overline{\theta}^2\Delta_\beta^4\sigma_\theta^2\chi_\epsilon^2 + 162\gamma\overline{\theta}\Delta_\beta^3\sigma_\theta^2\chi_\epsilon^2 + 378\gamma^2\overline{\beta}^2\overline{\theta}^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^2 + 486\gamma\overline{\beta}^2\overline{\theta}\Delta_\beta\sigma_\theta^2\chi_\epsilon^2 \\
&- 141\gamma^2\overline{\theta}^4\Delta_\beta^4\chi_\epsilon - 243\gamma^2\overline{\beta}^4\sigma_\theta^4\chi_\epsilon - 243\gamma^2\Delta_\beta^4\sigma_\theta^4\chi_\epsilon - 702\gamma^2\overline{\beta}^2\Delta_\beta^2\sigma_\theta^4\chi_\epsilon - 288\gamma\overline{\theta}^3\Delta_\beta^3\chi_\epsilon - 486\gamma\overline{\beta}\,\overline{\theta}^3\Delta_\beta^2\chi_\epsilon - 54\overline{\theta}^2\Delta_\beta^2\chi_\epsilon \\
&- 378\gamma^2\overline{\theta}^2\Delta_\beta^4\sigma_\theta^2\chi_\epsilon - 324\gamma\overline{\theta}\Delta_\beta^3\sigma_\theta^2\chi_\epsilon - 378\gamma^2\overline{\beta}^2\overline{\theta}^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon - 486\gamma\overline{\beta}\,\overline{\theta}\Delta_\beta^2\sigma_\theta^2\chi_\epsilon - 486\gamma\overline{\beta}^3\overline{\theta}\sigma_\theta^2\chi_\epsilon - 972\gamma\overline{\beta}^2\overline{\theta}\Delta_\beta\sigma_\theta^2\chi_\epsilon\Big)
\end{aligned}
$$

$$
\begin{aligned}
\mathbb{E}[p^2|LL] = \frac{1}{486} \Big(& -64\gamma^2\chi_\epsilon^3\Delta_\epsilon^4 + 64\gamma^2\Delta_\epsilon^4 + 192\gamma^2\chi_\epsilon^2\Delta_\epsilon^4 - 192\gamma^2\chi_\epsilon\Delta_\epsilon^4 + 72\gamma\chi_\epsilon^2\Delta_\epsilon^3 + 72\gamma\Delta_\epsilon^3 - 144\gamma\chi_\epsilon\Delta_\epsilon^3 - 128\gamma^2\bar{\theta}^2\Delta_\beta^2\chi_\epsilon^3\Delta_\epsilon^2 \\
& - 144\gamma^2\bar{\beta}^2\sigma_\theta^2\chi_\epsilon^3\Delta_\epsilon^2 - 144\gamma^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^3\Delta_\epsilon^2 + 96\gamma^2\bar{\beta}\Delta_\beta\sigma_\theta^2\chi_\epsilon^3\Delta_\epsilon^2 + 128\gamma^2\bar{\theta}^2\Delta_\beta^2\Delta_\epsilon^2 + 144\gamma^2\bar{\beta}^2\sigma_\theta^2\Delta_\epsilon^2 + 144\gamma^2\Delta_\beta^2\sigma_\theta^2\Delta_\epsilon^2 - 96\gamma^2\bar{\beta}\Delta_\beta\sigma_\theta^2\Delta_\epsilon^2 \\
& + 384\gamma^2\bar{\theta}^2\Delta_\beta^2\chi_\epsilon^2\Delta_\epsilon^2 + 432\gamma^2\bar{\beta}^2\sigma_\theta^2\chi_\epsilon^2\Delta_\epsilon^2 + 432\gamma^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^2\Delta_\epsilon^2 - 288\gamma^2\bar{\beta}\Delta_\beta\sigma_\theta^2\chi_\epsilon^2\Delta_\epsilon^2 - 216\gamma\bar{\beta}\bar{\theta}\chi_\epsilon^2\Delta_\epsilon^2 + 72\gamma\bar{\theta}\Delta_\beta\chi_\epsilon^2\Delta_\epsilon^2 - 216\gamma\bar{\beta}\bar{\theta}\Delta_\epsilon^2 \\
& + 72\gamma\bar{\theta}\Delta_\beta\Delta_\epsilon^2 - 384\gamma^2\bar{\theta}^2\Delta_\beta^2\chi_\epsilon\Delta_\epsilon^2 - 432\gamma^2\bar{\beta}^2\sigma_\theta^2\chi_\epsilon\Delta_\epsilon^2 - 432\gamma^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon\Delta_\epsilon^2 + 288\gamma^2\bar{\beta}\Delta_\beta\sigma_\theta^2\chi_\epsilon\Delta_\epsilon^2 + 432\gamma\bar{\beta}\bar{\theta}\chi_\epsilon\Delta_\epsilon^2 - 144\gamma\bar{\theta}\Delta_\beta\chi_\epsilon\Delta_\epsilon^2 \\
& + 216\chi_\epsilon\Delta_\epsilon^2 + 27\Delta_\epsilon^2 - 171\gamma\bar{\theta}^2\Delta_\beta^2\Delta_\epsilon + 81\gamma\bar{\beta}^2\sigma_\theta^2\Delta_\epsilon + 81\gamma\Delta_\beta^2\sigma_\theta^2\Delta_\epsilon - 540\gamma\bar{\beta}\Delta_\beta\sigma_\theta^2\Delta_\epsilon + 72\gamma\bar{\theta}^2\Delta_\beta^2\chi_\epsilon^2\Delta_\epsilon - 162\gamma\bar{\beta}^2\sigma_\theta^2\chi_\epsilon^2\Delta_\epsilon \\
& - 162\gamma\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^2\Delta_\epsilon + 432\gamma\bar{\beta}\Delta_\beta\sigma_\theta^2\chi_\epsilon^2\Delta_\epsilon - 162\bar{\beta}\bar{\theta}\Delta_\epsilon + 540\bar{\theta}\Delta_\beta\Delta_\epsilon + 342\gamma\bar{\theta}^2\Delta_\beta^2\chi_\epsilon\Delta_\epsilon + 324\gamma\bar{\beta}^2\sigma_\theta^2\chi_\epsilon\Delta_\epsilon + 324\gamma\Delta_\beta^2\sigma_\theta^2\chi_\epsilon\Delta_\epsilon \\
& + 108\gamma\bar{\beta}\Delta_\beta\sigma_\theta^2\chi_\epsilon\Delta_\epsilon - 324\bar{\beta}\bar{\theta}\chi_\epsilon\Delta_\epsilon - 540\bar{\theta}\Delta_\beta\chi_\epsilon\Delta_\epsilon - 17\gamma^2\bar{\theta}^4\Delta_\beta^4 + 81\gamma^2\bar{\beta}^4\sigma_\theta^4 + 81\gamma^2\Delta_\beta^4\sigma_\theta^4 - 432\gamma^2\bar{\beta}\Delta_\beta^3\sigma_\theta^4 + 522\gamma^2\bar{\beta}^2\Delta_\beta^2\sigma_\theta^4 \\
& - 432\gamma^2\bar{\beta}^3\Delta_\beta\sigma_\theta^4 + 72\gamma\bar{\theta}^3\Delta_\beta^3 + 98\gamma^2\bar{\theta}^4\Delta_\beta^4\chi_\epsilon^3 + 432\gamma^2\bar{\beta}\Delta_\beta^3\sigma_\theta^4\chi_\epsilon^3 - 360\gamma^2\bar{\beta}^2\Delta_\beta^2\sigma_\theta^4\chi_\epsilon^3 + 432\gamma^2\bar{\beta}^3\Delta_\beta\sigma_\theta^4\chi_\epsilon^3 + 180\gamma^2\bar{\theta}^2\Delta_\beta^4\sigma_\theta^2\chi_\epsilon^3 \\
& + 96\gamma^2\bar{\beta}\bar{\theta}^2\Delta_\beta^3\sigma_\theta^2\chi_\epsilon^3 + 180\gamma^2\bar{\beta}^2\bar{\theta}^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^3 + 243\bar{\beta}^2\bar{\theta}^2 + 27\gamma\bar{\beta}\bar{\theta}^3\Delta_\beta^2 + 270\bar{\theta}^2\Delta_\beta^2 - 18\gamma^2\bar{\theta}^2\Delta_\beta^4\sigma_\theta^2 - 96\gamma^2\bar{\beta}\bar{\theta}^2\Delta_\beta^3\sigma_\theta^2 + 324\gamma\bar{\theta}\Delta_\beta^3\sigma_\theta^2 \\
& - 18\gamma^2\bar{\beta}^2\bar{\theta}^2\Delta_\beta^2\sigma_\theta^2 - 783\gamma\bar{\beta}\bar{\theta}\Delta_\beta^2\sigma_\theta^2 - 243\gamma\bar{\beta}^3\bar{\theta}\sigma_\theta^2 + 972\gamma\bar{\beta}^2\bar{\theta}\Delta_\beta\sigma_\theta^2 - 51\gamma^2\bar{\theta}^4\Delta_\beta^4\chi_\epsilon^2 - 324\gamma^2\bar{\beta}\Delta_\beta^3\sigma_\theta^4\chi_\epsilon^2 + 108\gamma^2\bar{\beta}^2\Delta_\beta^2\sigma_\theta^4\chi_\epsilon^2 \\
& - 324\gamma^2\bar{\beta}^3\Delta_\beta\sigma_\theta^4\chi_\epsilon^2 + 72\gamma\bar{\theta}^3\Delta_\beta^3\chi_\epsilon^2 - 216\gamma\bar{\beta}\bar{\theta}^3\Delta_\beta^2\chi_\epsilon^2 - 54\gamma^2\bar{\theta}^2\Delta_\beta^4\sigma_\theta^2\chi_\epsilon^2 - 288\gamma^2\bar{\beta}\bar{\theta}^2\Delta_\beta^3\sigma_\theta^2\chi_\epsilon^2 - 162\gamma\bar{\theta}\Delta_\beta^3\sigma_\theta^2\chi_\epsilon^2 - 54\gamma^2\bar{\beta}^2\bar{\theta}^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^2 \\
& + 432\gamma\bar{\beta}\bar{\theta}\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^2 - 486\gamma\bar{\beta}^2\bar{\theta}\Delta_\beta\sigma_\theta^2\chi_\epsilon^2 - 648\bar{\beta}\bar{\theta}^2\Delta_\beta + 51\gamma^2\bar{\theta}^4\Delta_\beta^4\chi_\epsilon + 324\gamma^2\bar{\beta}\Delta_\beta^3\sigma_\theta^4\chi_\epsilon - 108\gamma^2\bar{\beta}^2\Delta_\beta^2\sigma_\theta^4\chi_\epsilon + 324\gamma^2\bar{\beta}^3\Delta_\beta\sigma_\theta^4\chi_\epsilon - 144\gamma\bar{\theta}^3\Delta_\beta^3\chi_\epsilon \\
& - 54\gamma\bar{\beta}\bar{\theta}^3\Delta_\beta^2\chi_\epsilon - 270\bar{\theta}^2\Delta_\beta^2\chi_\epsilon + 54\gamma^2\bar{\theta}^2\Delta_\beta^4\sigma_\theta^2\chi_\epsilon + 288\gamma^2\bar{\beta}\bar{\theta}^2\Delta_\beta^3\sigma_\theta^2\chi_\epsilon - 162\gamma\bar{\theta}\Delta_\beta^3\sigma_\theta^2\chi_\epsilon + 54\gamma^2\bar{\beta}^2\bar{\theta}^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon + 108\gamma\bar{\beta}\bar{\theta}\Delta_\beta^2\sigma_\theta^2\chi_\epsilon - 486\gamma\bar{\beta}^2\bar{\theta}\Delta_\beta\sigma_\theta^2\chi_\epsilon \\
& + 648\bar{\beta}\bar{\theta}^2\Delta_\beta\chi_\epsilon \Big)
\end{aligned}
$$

Second, if $\chi_\epsilon \leq \chi_\beta$, then we get the following set of expressions:

$$
\begin{aligned}
\mathbb{E}[p|HH] = \frac{1}{36} \Big(& -12\gamma\bar{\beta}\chi_\epsilon^2\Delta_\beta\sigma_\theta^2 + 36\gamma\bar{\beta}\chi_\epsilon\Delta_\beta\sigma_\theta^2 + 8\gamma\bar{\theta}^2\chi_\epsilon^2\Delta_\beta^2 - 18\gamma\bar{\theta}^2\chi_\epsilon\Delta_\beta^2 \\
& + 24\bar{\theta}\chi_\epsilon\Delta_\beta + 9\gamma\bar{\theta}^2\Delta_\beta^2 + 9\gamma\bar{\beta}^2\sigma_\theta^2 + 18\bar{\beta}\bar{\theta} + 8\gamma\Delta_\epsilon^2\chi_\epsilon^2 - 12\Delta_\epsilon\chi_\epsilon + 18\Delta_\epsilon + 9\gamma\Delta_\beta^2\sigma_\theta^2 \Big)
\end{aligned}
$$

$$
\mathbb{E}[p|HL] = \frac{1}{12} \Big(4\gamma\bar{\beta}\chi_\epsilon^2\Delta_\beta\sigma_\theta^2 + 6\gamma\bar{\theta}^2\chi_\epsilon\Delta_\beta^2 + 6\gamma\bar{\beta}^2\chi_\epsilon\sigma_\theta^2 - 3\gamma\bar{\theta}^2\Delta_\beta^2 - 3\gamma\bar{\beta}^2\sigma_\theta^2 + 6\bar{\beta}\bar{\theta} + 6\gamma\chi_\epsilon\Delta_\beta^2\sigma_\theta^2 + 12\Delta_\epsilon\chi_\epsilon - 6\Delta_\epsilon - 3\gamma\Delta_\beta^2\sigma_\theta^2 \Big)
$$

$$
\mathbb{E}[p|LH] = \frac{1}{12} \Big(4\gamma\bar{\beta}\chi_\epsilon^2\Delta_\beta\sigma_\theta^2 - 6\gamma\bar{\theta}^2\chi_\epsilon\Delta_\beta^2 - 6\gamma\bar{\beta}^2\chi_\epsilon\sigma_\theta^2 + 3\gamma\bar{\theta}^2\Delta_\beta^2 + 3\gamma\bar{\beta}^2\sigma_\theta^2 + 6\bar{\beta}\bar{\theta} - 6\gamma\chi_\epsilon\Delta_\beta^2\sigma_\theta^2 - 12\Delta_\epsilon\chi_\epsilon + 6\Delta_\epsilon + 3\gamma\Delta_\beta^2\sigma_\theta^2 \Big)
$$

$$
\begin{aligned}
\mathbb{E}[p|LL] = \frac{1}{36} \Big(& -12\gamma\bar{\beta}\chi_\epsilon^2\Delta_\beta\sigma_\theta^2 + 36\gamma\bar{\beta}\chi_\epsilon\Delta_\beta\sigma_\theta^2 - 8\gamma\bar{\theta}^2\chi_\epsilon^2\Delta_\beta^2 + 18\gamma\bar{\theta}^2\chi_\epsilon\Delta_\beta^2 \\
& - 24\bar{\theta}\chi_\epsilon\Delta_\beta - 9\gamma\bar{\theta}^2\Delta_\beta^2 - 9\gamma\bar{\beta}^2\sigma_\theta^2 + 18\bar{\beta}\bar{\theta} - 8\gamma\Delta_\epsilon^2\chi_\epsilon^2 + 12\Delta_\epsilon\chi_\epsilon - 18\Delta_\epsilon - 9\gamma\Delta_\beta^2\sigma_\theta^2 \Big)
\end{aligned}
$$

$$
\begin{aligned}
\mathbb{E}[p^2|HH] = \frac{1}{486}\Big(& 64\gamma^2\chi_\epsilon^3\Delta_\epsilon^4 + 72\gamma\chi_\epsilon^2\Delta_\epsilon^3 + 128\gamma^2\overline{\theta}^2\Delta_\beta^2\chi_\epsilon^3\Delta_\epsilon^2 + 144\gamma^2\overline{\beta}^2\sigma_\theta^2\chi_\epsilon^3\Delta_\epsilon^2 + 144\gamma^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^3\Delta_\epsilon^2 \\
& + 96\gamma^2\overline{\beta}\Delta_\beta\sigma_\theta^2\chi_\epsilon^3\Delta_\epsilon^2 + 216\gamma\overline{\beta}\overline{\theta}\chi_\epsilon^2\Delta_\epsilon^2 + 72\gamma\overline{\theta}\Delta_\beta\chi_\epsilon^2\Delta_\epsilon^2 - 216\chi_\epsilon\Delta_\epsilon^2 + 243\Delta_\epsilon^2 + 243\gamma\overline{\theta}^2\Delta_\beta^2\Delta_\epsilon + 243\gamma\overline{\beta}^2\sigma_\theta^2\Delta_\epsilon \\
& + 243\gamma\Delta_\beta^2\sigma_\theta^2\Delta_\epsilon + 72\gamma\overline{\theta}^2\Delta_\beta^2\chi_\epsilon^2\Delta_\epsilon - 162\gamma\overline{\beta}^2\sigma_\theta^2\chi_\epsilon^2\Delta_\epsilon - 162\gamma\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^2\Delta_\epsilon - 432\gamma\overline{\beta}\Delta_\beta\sigma_\theta^2\chi_\epsilon^2\Delta_\epsilon + 486\overline{\beta}\overline{\theta}\Delta_\epsilon \\
& - 486\gamma\overline{\theta}^2\Delta_\beta^2\chi_\epsilon\Delta_\epsilon + 972\gamma\overline{\beta}\Delta_\beta\sigma_\theta^2\chi_\epsilon\Delta_\epsilon - 324\overline{\beta}\overline{\theta}\chi_\epsilon\Delta_\epsilon + 540\overline{\theta}\Delta_\beta\chi_\epsilon\Delta_\epsilon + 81\gamma^2\overline{\theta}^4\Delta_\beta^4 + 81\gamma^2\overline{\beta}^4\sigma_\theta^4 \\
& + 81\gamma^2\Delta_\beta^4\sigma_\theta^4 + 162\gamma^2\overline{\beta}^2\Delta_\beta^2\sigma_\theta^4 - 98\gamma^2\overline{\theta}^4\Delta_\beta^4\chi_\epsilon^3 + 432\gamma^2\overline{\beta}\Delta_\beta^3\sigma_\theta^4\chi_\epsilon^3 + 360\gamma^2\overline{\beta}^2\Delta_\beta^2\sigma_\theta^4\chi_\epsilon^3 + 432\gamma^2\overline{\beta}^3\Delta_\beta\sigma_\theta^4\chi_\epsilon^3 \\
& - 180\gamma^2\overline{\theta}^2\Delta_\beta^4\sigma_\theta^2\chi_\epsilon^3 + 96\gamma^2\overline{\beta}\overline{\theta}^2\Delta_\beta^3\sigma_\theta^2\chi_\epsilon^3 - 180\gamma^2\overline{\beta}^2\overline{\theta}^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^3 + 243\overline{\beta}^2\overline{\theta}^2 + 243\gamma\overline{\beta}\overline{\theta}^3\Delta_\beta^2 + 162\gamma^2\overline{\theta}^2\Delta_\beta^4\sigma_\theta^2 \\
& + 162\gamma^2\overline{\beta}^2\overline{\theta}^2\Delta_\beta^2\sigma_\theta^2 + 243\gamma\overline{\beta}\overline{\theta}\Delta_\beta^2\sigma_\theta^2 + 243\gamma\overline{\beta}^3\overline{\theta}\sigma_\theta^2 + 243\gamma^2\overline{\theta}^4\Delta_\beta^4\chi_\epsilon^2 - 972\gamma^2\overline{\beta}\Delta_\beta^3\sigma_\theta^4\chi_\epsilon^2 - 972\gamma^2\overline{\beta}^2\Delta_\beta^2\sigma_\theta^4\chi_\epsilon^2 \\
& - 972\gamma^2\overline{\beta}^3\Delta_\beta\sigma_\theta^4\chi_\epsilon^2 + 72\gamma\overline{\theta}^3\Delta_\beta^3\chi_\epsilon^2 + 216\gamma\overline{\beta}\overline{\theta}^3\Delta_\beta^2\chi_\epsilon^2 + 486\gamma^2\overline{\theta}^2\Delta_\beta^4\sigma_\theta^2\chi_\epsilon^2 - 162\gamma\overline{\theta}\Delta_\beta^3\sigma_\theta^2\chi_\epsilon^2 \\
& + 486\gamma^2\overline{\beta}^2\overline{\theta}^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^2 - 432\gamma\overline{\beta}\overline{\theta}\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^2 - 486\gamma\overline{\beta}^2\overline{\theta}\Delta_\beta\sigma_\theta^2\chi_\epsilon^2 - 243\gamma^2\overline{\theta}^4\Delta_\beta^4\chi_\epsilon + 972\gamma^2\overline{\beta}\Delta_\beta^3\sigma_\theta^4\chi_\epsilon \\
& + 972\gamma^2\overline{\beta}^2\Delta_\beta^2\sigma_\theta^4\chi_\epsilon + 972\gamma^2\overline{\beta}^3\Delta_\beta\sigma_\theta^4\chi_\epsilon - 486\gamma\overline{\beta}\overline{\theta}^3\Delta_\beta^2\chi_\epsilon + 270\overline{\theta}^2\Delta_\beta^2\chi_\epsilon - 486\gamma^2\overline{\theta}^2\Delta_\beta^4\sigma_\theta^2\chi_\epsilon \\
& + 486\gamma\overline{\theta}\Delta_\beta^3\sigma_\theta^2\chi_\epsilon - 486\gamma^2\overline{\beta}^2\overline{\theta}^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon + 972\gamma\overline{\beta}\overline{\theta}\Delta_\beta^2\sigma_\theta^2\chi_\epsilon + 1458\gamma\overline{\beta}^2\overline{\theta}\Delta_\beta\sigma_\theta^2\chi_\epsilon + 648\overline{\beta}\overline{\theta}^2\Delta_\beta\chi_\epsilon\Big)
\end{aligned}
$$

$$
\begin{aligned}
\mathbb{E}[p^2|HL] = \frac{1}{486}\Big(& 128\gamma^2\chi_\epsilon^3\Delta_\epsilon^4 + 144\gamma\chi_\epsilon^2\Delta_\epsilon^3 + 256\gamma^2\overline{\theta}^2\Delta_\beta^2\chi_\epsilon^3\Delta_\epsilon^2 + 288\gamma^2\overline{\beta}^2\sigma_\theta^2\chi_\epsilon^3\Delta_\epsilon^2 + 288\gamma^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^3\Delta_\epsilon^2 \\
& + 144\gamma\overline{\theta}\Delta_\beta\chi_\epsilon^2\Delta_\epsilon^2 - 432\chi_\epsilon\Delta_\epsilon^2 + 243\Delta_\epsilon^2 + 243\gamma\overline{\theta}^2\Delta_\beta^2\Delta_\epsilon + 243\gamma\overline{\beta}^2\sigma_\theta^2\Delta_\epsilon + 243\gamma\Delta_\beta^2\sigma_\theta^2\Delta_\epsilon + 144\gamma\overline{\theta}^2\Delta_\beta^2\chi_\epsilon^2\Delta_\epsilon \\
& + 162\gamma\overline{\beta}^2\sigma_\theta^2\chi_\epsilon^2\Delta_\epsilon + 162\gamma\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^2\Delta_\epsilon - 486\overline{\beta}\overline{\theta}\Delta_\epsilon - 486\gamma\overline{\theta}^2\Delta_\beta^2\chi_\epsilon\Delta_\epsilon - 486\gamma\overline{\beta}^2\sigma_\theta^2\chi_\epsilon\Delta_\epsilon - 486\gamma\Delta_\beta^2\sigma_\theta^2\chi_\epsilon\Delta_\epsilon \\
& + 972\overline{\beta}\overline{\theta}\chi_\epsilon\Delta_\epsilon + 108\overline{\theta}\Delta_\beta\chi_\epsilon\Delta_\epsilon + 81\gamma^2\overline{\theta}^4\Delta_\beta^4 + 81\gamma^2\overline{\beta}^4\sigma_\theta^4 + 81\gamma^2\Delta_\beta^4\sigma_\theta^4 + 162\gamma^2\overline{\beta}^2\Delta_\beta^2\sigma_\theta^4 - 34\gamma^2\overline{\theta}^4\Delta_\beta^4\chi_\epsilon^3 \\
& + 72\gamma^2\overline{\beta}^2\Delta_\beta^2\sigma_\theta^4\chi_\epsilon^3 - 36\gamma^2\overline{\theta}^2\Delta_\beta^4\sigma_\theta^2\chi_\epsilon^3 - 36\gamma^2\overline{\beta}^2\overline{\theta}^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^3 + 243\overline{\beta}^2\overline{\theta}^2 - 243\gamma\overline{\beta}\overline{\theta}^3\Delta_\beta^2 + 162\gamma^2\overline{\theta}^2\Delta_\beta^4\sigma_\theta^2 \\
& + 162\gamma^2\overline{\beta}^2\overline{\theta}^2\Delta_\beta^2\sigma_\theta^2 - 243\gamma\overline{\beta}\overline{\theta}\Delta_\beta^2\sigma_\theta^2 - 243\gamma\overline{\beta}^3\overline{\theta}\sigma_\theta^2 + 243\gamma^2\overline{\theta}^4\Delta_\beta^4\chi_\epsilon^2 + 243\gamma^2\overline{\beta}^4\sigma_\theta^4\chi_\epsilon^2 + 243\gamma^2\Delta_\beta^4\sigma_\theta^4\chi_\epsilon^2 \\
& + 486\gamma^2\overline{\beta}^2\Delta_\beta^2\sigma_\theta^4\chi_\epsilon^2 + 144\gamma\overline{\theta}^3\Delta_\beta^3\chi_\epsilon^2 + 486\gamma^2\overline{\theta}^2\Delta_\beta^4\sigma_\theta^2\chi_\epsilon^2 + 162\gamma\overline{\theta}\Delta_\beta^3\sigma_\theta^2\chi_\epsilon^2 + 486\gamma^2\overline{\beta}^2\overline{\theta}^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^2 \\
& + 486\gamma\overline{\beta}^2\overline{\theta}\Delta_\beta\sigma_\theta^2\chi_\epsilon^2 - 243\gamma^2\overline{\theta}^4\Delta_\beta^4\chi_\epsilon - 243\gamma^2\overline{\beta}^4\sigma_\theta^4\chi_\epsilon - 243\gamma^2\Delta_\beta^4\sigma_\theta^4\chi_\epsilon - 486\gamma^2\overline{\beta}^2\Delta_\beta^2\sigma_\theta^4\chi_\epsilon + 486\gamma\overline{\beta}\overline{\theta}^3\Delta_\beta^2\chi_\epsilon \\
& + 54\overline{\theta}^2\Delta_\beta^2\chi_\epsilon - 486\gamma^2\overline{\theta}^2\Delta_\beta^4\sigma_\theta^2\chi_\epsilon - 486\gamma^2\overline{\beta}^2\overline{\theta}^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon + 486\gamma\overline{\beta}\overline{\theta}\Delta_\beta^2\sigma_\theta^2\chi_\epsilon + 486\gamma\overline{\beta}^3\overline{\theta}\sigma_\theta^2\chi_\epsilon\Big)
\end{aligned}
$$

$$
\begin{aligned}
\mathbb{E}[p^2|LH] = \frac{1}{486}\Big(& 128\gamma^2\chi_\epsilon^3\Delta_\epsilon^4 + 144\gamma\chi_\epsilon^2\Delta_\epsilon^3 + 256\gamma^2\overline{\theta}^2\Delta_\beta^2\chi_\epsilon^3\Delta_\epsilon^2 + 288\gamma^2\overline{\beta}^2\sigma_\theta^2\chi_\epsilon^3\Delta_\epsilon^2 + 288\gamma^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^3\Delta_\epsilon^2 \\
& + 144\gamma\overline{\theta}\Delta_\beta\chi_\epsilon^2\Delta_\epsilon^2 - 432\chi_\epsilon\Delta_\epsilon^2 + 243\Delta_\epsilon^2 + 243\gamma\overline{\theta}^2\Delta_\beta^2\Delta_\epsilon + 243\gamma\overline{\beta}^2\sigma_\theta^2\Delta_\epsilon + 243\gamma\Delta_\beta^2\sigma_\theta^2\Delta_\epsilon + 144\gamma\overline{\theta}^2\Delta_\beta^2\chi_\epsilon^2\Delta_\epsilon \\
& + 162\gamma\overline{\beta}^2\sigma_\theta^2\chi_\epsilon^2\Delta_\epsilon + 162\gamma\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^2\Delta_\epsilon + 486\overline{\beta}\overline{\theta}\Delta_\epsilon - 486\gamma\overline{\theta}^2\Delta_\beta^2\chi_\epsilon\Delta_\epsilon - 486\gamma\overline{\beta}^2\sigma_\theta^2\chi_\epsilon\Delta_\epsilon - 486\gamma\Delta_\beta^2\sigma_\theta^2\chi_\epsilon\Delta_\epsilon \\
& - 972\overline{\beta}\overline{\theta}\chi_\epsilon\Delta_\epsilon + 108\overline{\theta}\Delta_\beta\chi_\epsilon\Delta_\epsilon + 81\gamma^2\overline{\theta}^4\Delta_\beta^4 + 81\gamma^2\overline{\beta}^4\sigma_\theta^4 + 81\gamma^2\Delta_\beta^4\sigma_\theta^4 + 162\gamma^2\overline{\beta}^2\Delta_\beta^2\sigma_\theta^4 - 34\gamma^2\overline{\theta}^4\Delta_\beta^4\chi_\epsilon^3 \\
& + 72\gamma^2\overline{\beta}^2\Delta_\beta^2\sigma_\theta^4\chi_\epsilon^3 - 36\gamma^2\overline{\theta}^2\Delta_\beta^4\sigma_\theta^2\chi_\epsilon^3 - 36\gamma^2\overline{\beta}^2\overline{\theta}^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^3 + 243\overline{\beta}^2\overline{\theta}^2 + 243\gamma\overline{\beta}\overline{\theta}^3\Delta_\beta^2 + 162\gamma^2\overline{\theta}^2\Delta_\beta^4\sigma_\theta^2 \\
& + 162\gamma^2\overline{\beta}^2\overline{\theta}^2\Delta_\beta^2\sigma_\theta^2 + 243\gamma\overline{\beta}\overline{\theta}\Delta_\beta^2\sigma_\theta^2 + 243\gamma\overline{\beta}^3\overline{\theta}\sigma_\theta^2 + 243\gamma^2\overline{\theta}^4\Delta_\beta^4\chi_\epsilon^2 + 243\gamma^2\overline{\beta}^4\sigma_\theta^4\chi_\epsilon^2 + 243\gamma^2\Delta_\beta^4\sigma_\theta^4\chi_\epsilon^2 \\
& + 486\gamma^2\overline{\beta}^2\Delta_\beta^2\sigma_\theta^4\chi_\epsilon^2 + 144\gamma\overline{\theta}^3\Delta_\beta^3\chi_\epsilon^2 + 486\gamma^2\overline{\theta}^2\Delta_\beta^4\sigma_\theta^2\chi_\epsilon^2 + 162\gamma\overline{\theta}\Delta_\beta^3\sigma_\theta^2\chi_\epsilon^2 + 486\gamma^2\overline{\beta}^2\overline{\theta}^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^2 \\
& + 486\gamma\overline{\beta}^2\overline{\theta}\Delta_\beta\sigma_\theta^2\chi_\epsilon^2 - 243\gamma^2\overline{\theta}^4\Delta_\beta^4\chi_\epsilon - 243\gamma^2\overline{\beta}^4\sigma_\theta^4\chi_\epsilon - 243\gamma^2\Delta_\beta^4\sigma_\theta^4\chi_\epsilon - 486\gamma^2\overline{\beta}^2\Delta_\beta^2\sigma_\theta^4\chi_\epsilon - 486\gamma\overline{\beta}\overline{\theta}^3\Delta_\beta^2\chi_\epsilon \\
& + 54\overline{\theta}^2\Delta_\beta^2\chi_\epsilon - 486\gamma^2\overline{\theta}^2\Delta_\beta^4\sigma_\theta^2\chi_\epsilon - 486\gamma^2\overline{\beta}^2\overline{\theta}^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon - 486\gamma\overline{\beta}\overline{\theta}\Delta_\beta^2\sigma_\theta^2\chi_\epsilon - 486\gamma\overline{\beta}^3\overline{\theta}\sigma_\theta^2\chi_\epsilon\Big)
\end{aligned}
$$

$$
\begin{aligned}
\mathbb{E}[p^2|LL] = \frac{1}{486} \Big(& 64\gamma^2\chi_\epsilon^3\Delta_\epsilon^4 + 72\gamma\chi_\epsilon^2\Delta_\epsilon^3 + 128\gamma^2\overline{\theta}^2\Delta_\beta^2\chi_\epsilon^3\Delta_\epsilon^2 + 144\gamma^2\overline{\beta}^2\sigma_\theta^2\chi_\epsilon^3\Delta_\epsilon^2 + 144\gamma^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^3\Delta_\epsilon^2 \\
& - 96\gamma^2\overline{\beta}\Delta_\beta\sigma_\theta^2\chi_\epsilon^3\Delta_\epsilon^2 - 216\gamma\overline{\beta\theta}\chi_\epsilon^2\Delta_\epsilon^2 + 72\gamma\overline{\theta}\Delta_\beta\chi_\epsilon^2\Delta_\epsilon^2 - 216\chi_\epsilon\Delta_\epsilon^2 + 243\Delta_\epsilon^2 + 243\gamma\overline{\theta}^2\Delta_\beta^2\Delta_\epsilon + 243\gamma\overline{\beta}^2\sigma_\theta^2\Delta_\epsilon \\
& + 243\gamma\Delta_\beta^2\sigma_\theta^2\Delta_\epsilon + 72\gamma\overline{\theta}^2\Delta_\beta^2\chi_\epsilon^2\Delta_\epsilon - 162\gamma\overline{\beta}^2\sigma_\theta^2\chi_\epsilon^2\Delta_\epsilon - 162\gamma\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^2\Delta_\epsilon + 432\gamma\overline{\beta}\Delta_\beta\sigma_\theta^2\chi_\epsilon^2\Delta_\epsilon - 486\overline{\beta\theta}\Delta_\epsilon \\
& - 486\gamma\overline{\theta}^2\Delta_\beta^2\chi_\epsilon\Delta_\epsilon - 972\gamma\overline{\beta}\Delta_\beta\sigma_\theta^2\chi_\epsilon\Delta_\epsilon + 324\overline{\beta\theta}\chi_\epsilon\Delta_\epsilon + 540\overline{\theta}\Delta_\beta\chi_\epsilon\Delta_\epsilon + 81\gamma^2\overline{\theta}^4\Delta_\beta^4 + 81\gamma^2\overline{\beta}^4\sigma_\theta^4 \\
& + 81\gamma^2\Delta_\beta^4\sigma_\theta^4 + 162\gamma^2\overline{\beta}^2\Delta_\beta^2\sigma_\theta^4 - 98\gamma^2\overline{\theta}^4\Delta_\beta^4\chi_\epsilon^3 - 432\gamma^2\overline{\beta}\Delta_\beta^3\sigma_\theta^4\chi_\epsilon^3 + 360\gamma^2\overline{\beta}^2\Delta_\beta^2\sigma_\theta^4\chi_\epsilon^3 - 432\gamma^2\overline{\beta}^3\Delta_\beta\sigma_\theta^4\chi_\epsilon^3 \\
& - 180\gamma^2\overline{\theta}^2\Delta_\beta^4\sigma_\theta^2\chi_\epsilon^3 - 96\gamma^2\overline{\beta\theta}^2\Delta_\beta^3\sigma_\theta^2\chi_\epsilon^3 - 180\gamma^2\overline{\beta}^2\overline{\theta}^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^3 + 243\overline{\beta}^2\overline{\theta}^2 - 243\gamma\overline{\beta\theta}^3\Delta_\beta^2 + 162\gamma^2\overline{\theta}^2\Delta_\beta^4\sigma_\theta^2 \\
& + 162\gamma^2\overline{\beta}^2\overline{\theta}^2\Delta_\beta^2\sigma_\theta^2 - 243\gamma\overline{\beta\theta}\Delta_\beta^2\sigma_\theta^2 - 243\gamma\overline{\beta}^3\overline{\theta}\sigma_\theta^2 + 243\gamma^2\overline{\theta}^4\Delta_\beta^4\chi_\epsilon^2 + 972\gamma^2\overline{\beta}\Delta_\beta^3\sigma_\theta^4\chi_\epsilon^2 - 972\gamma^2\overline{\beta}^2\Delta_\beta^2\sigma_\theta^4\chi_\epsilon^2 \\
& + 972\gamma^2\overline{\beta}^3\Delta_\beta\sigma_\theta^4\chi_\epsilon^2 + 72\gamma\overline{\theta}^3\Delta_\beta^3\chi_\epsilon^2 - 216\gamma\overline{\beta\theta}^3\Delta_\beta^2\chi_\epsilon^2 + 486\gamma^2\overline{\theta}^2\Delta_\beta^4\sigma_\theta^2\chi_\epsilon^2 - 162\gamma\overline{\theta}\Delta_\beta^3\sigma_\theta^2\chi_\epsilon^2 \\
& + 486\gamma^2\overline{\beta}^2\overline{\theta}^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^2 + 432\gamma\overline{\beta\theta}\Delta_\beta^2\sigma_\theta^2\chi_\epsilon^2 - 486\gamma\overline{\beta}^2\overline{\theta}\Delta_\beta\sigma_\theta^2\chi_\epsilon^2 - 243\gamma^2\overline{\theta}^4\Delta_\beta^4\chi_\epsilon - 972\gamma^2\overline{\beta}\Delta_\beta^3\sigma_\theta^4\chi_\epsilon \\
& + 972\gamma^2\overline{\beta}^2\Delta_\beta^2\sigma_\theta^4\chi_\epsilon - 972\gamma^2\overline{\beta}^3\Delta_\beta\sigma_\theta^4\chi_\epsilon + 486\gamma\overline{\beta\theta}^3\Delta_\beta^2\chi_\epsilon + 270\overline{\theta}^2\Delta_\beta^2\chi_\epsilon - 486\gamma^2\overline{\theta}^2\Delta_\beta^4\sigma_\theta^2\chi_\epsilon \\
& + 486\gamma\overline{\theta}\Delta_\beta^3\sigma_\theta^2\chi_\epsilon - 486\gamma^2\overline{\beta}^2\overline{\theta}^2\Delta_\beta^2\sigma_\theta^2\chi_\epsilon - 972\gamma\overline{\beta\theta}\Delta_\beta^2\sigma_\theta^2\chi_\epsilon + 1458\gamma\overline{\beta}^2\overline{\theta}\Delta_\beta\sigma_\theta^2\chi_\epsilon - 648\overline{\beta\theta}^2\Delta_\beta\chi_\epsilon \Big)
\end{aligned}
$$

For $\overline{\theta} < 0$, we can derive equivalent conditions to ensure that $\beta-$informed traders continue to sell upon observing $\beta = \overline{\beta} + \Delta_\beta$ and to buy upon observing $\beta = \overline{\beta} - \Delta_\beta$.

OA.2: Model Extensions with Multiple Assets

OA.2.1 Two Assets and a Single Market Maker

Now we consider a model with two assets and the same asset payoff structure as in Eq. (1), within a model where the market maker makes markets in *both assets*. We continue to assume that $(\theta, \epsilon, \beta)$ are mutually independent with ϵ and β being independent across stocks and θ being a common factor. We denote the mass of ϵ- and β-informed investors in asset j by $\chi_{j,\epsilon}$ and $\chi_{j,\beta}$, where $j \in \{1, 2\}$. Noise trader demand in asset j is given by z_j. For simplicity, we assume that noise trader demand is uncorrelated across assets. The total demand in market j is:

$$X_j = \chi_{j,\epsilon} y_{j,\epsilon} + \chi_{j,\beta} y_{j,\beta} + z_j. \tag{OA.2.1}$$

We can write the market maker's expected utility over the two assets as:

$$\mathbb{E}\left[U_{MM}\right] = \mathbb{E}\left[X_1\left(p_1 - v_1\right) + X_2\left(p_2 - v_2\right)|X_1, X_2\right] - \frac{\gamma}{2}\mathrm{Var}[X_1(p_1 - v_1) + X_2(p_2 - v_2)|X_1, X_2], \tag{OA.2.2}$$

where $\gamma \geq 0$ represents the market maker's coefficient of risk aversion. In the equilibrium, we need to impose $\mathbb{E}\left[U_{MM}\right] = 0$, which gives one equation for two endogenous prices p_1 and p_2. The complication additionally induced in this setting is that order flows have non-zero covariance because of the common factor θ. We consider equilibria under two approaches.

Covariance Terms Allocated To First Asset

In this version of the model, we allocate the covariance terms to the first asset. It immediately follows that the equilibrium in the second asset is identical to the asset considered in the main model. In particular, we have:

$$\mathbb{E}\left[X_1\left(p_1 - v_1\right)|X_1, X_2\right] - \frac{\gamma}{2}\mathrm{Var}[X_1(p_1 - v_1)|X_1, X_2] - \gamma\mathrm{Cov}[X_1(p_1 - v_1), X_2(p_2 - v_2|X_1, X_2] = 0, \tag{OA.2.3}$$

and

$$\mathbb{E}\left[X_2\left(p_2 - v_2\right)|X_1, X_2\right] - \frac{\gamma}{2}\mathrm{Var}[X_2(p_2 - v_2)|X_1, X_2] = 0. \tag{OA.2.4}$$

The solution to the above equation system is:

$$p_1 = \mathbb{E}\left[v_1|X_1\right] + \frac{\gamma}{2}X_1\mathrm{Var}(v_1|X_1) + \gamma X_2\mathrm{Cov}(v_1, v_2|X_1, X_2), \tag{OA.2.5}$$

and

$$p_2 = \mathbb{E}\left[v_2|X_2\right] + \frac{\gamma}{2} X_2 \mathrm{Var}(v_2|X_2). \tag{OA.2.6}$$

Since the expression of p_2 is identical to that in Theorem 1, we focus on p_1. In the expression for p_1, the novel term is $\mathrm{Cov}(v_1, v_2|X_1, X_2)$, which can be written as:

$$\begin{aligned}
\mathrm{Cov}(v_1, v_2|X_1, X_2) &= \mathrm{Cov}(\beta_1\theta, \beta_2\theta|X_1, X_2) \\
&= \mathbb{E}\left[\beta_1\beta_2\theta^2|X_1, X_2\right] - \mathbb{E}\left[\beta_1\theta|X_1, X_2\right]\mathbb{E}\left[\beta_2\theta|X_1, X_2\right] \\
&= \mathbb{E}\left(\beta_1|X_1\right)\mathbb{E}\left(\beta_2|X_2\right)(\overline{\theta}^2 + \sigma_\theta^2) - \mathbb{E}\left(\beta_1|X_1\right)\mathbb{E}\left(\beta_2|X_2\right)\overline{\theta}^2 \\
&= \mathbb{E}\left(\beta_1|X_1\right)\mathbb{E}\left(\beta_2|X_2\right)\sigma_\theta^2. \qquad \text{(OA.2.7)}
\end{aligned}$$

We then have the following proposition.

Proposition 1 (Financial Market Equilibrium - Extension 1) *Given* $(\chi_{j,\epsilon}, \chi_{j,\beta})$ *and the conditions mentioned above, there is a financial market equilibrium in which:*

1. $\epsilon-$*informed traders buy on a high signal* ($\epsilon_j = \Delta_\epsilon$) *and sell on a low signal* ($\epsilon_j = -\Delta_\epsilon$); $\beta-$*informed traders buy (sell) on a high signal* ($\beta_j = \overline{\beta} + \Delta_\beta$) *and sell (buy) on a low signal* ($\beta_j = \overline{\beta} - \Delta_\beta$) *if* $\overline{\theta} > 0$ ($\overline{\theta} < 0$).

2. *The equilibrium stock price depends on total order flow and is given by:*

$$p_1 = \mathbb{E}\left[v_1|X_1\right] + \frac{\gamma}{2} X_1 \mathrm{Var}(v_1|X_1) + \frac{\gamma}{2} X_2 \mathrm{Cov}(v_1, v_2|X_1, X_2),$$

where $\mathbb{E}\left[v_1|X_1\right]$, $\mathrm{Var}(v_1|X_1)$, *and* $\mathrm{Cov}(v_1, v_2|X_1, X_2)$ *are as follows.*

   (a) $\mathbb{E}\left[v_1|X_1\right]$:

If $\chi_{1,\epsilon} > \chi_{1,\beta}$*:*

$$\mathbb{E}\left[v_1|X_1\right] = \begin{cases} \Delta_\epsilon + \left(\overline{\beta} + \text{sign}(\overline{\theta})\Delta_\beta\right)\overline{\theta} & \textit{if } X_1 \in (1 + X_{HL}, 2) \\ \Delta_\epsilon + \overline{\beta}\,\overline{\theta} & \textit{if } X_1 \in (1 + X_{LH}, 1 + X_{HL}) \\ \frac{\Delta_\epsilon}{3} + \left(\overline{\beta} + \text{sign}(\overline{\theta})\frac{\Delta_\beta}{3}\right)\overline{\theta} & \textit{if } X_1 \in (0, 1 + X_{LH}) \\ \frac{-\Delta_\epsilon}{3} + \left(\overline{\beta} - \text{sign}(\overline{\theta})\frac{\Delta_\beta}{3}\right)\overline{\theta} & \textit{if } X_1 \in (-1 + X_{HL}, 0) \\ -\Delta_\epsilon + \overline{\beta}\,\overline{\theta} & \textit{if } X_1 \in (-1 + X_{LH}, -1 + X_{HL}) \\ -\Delta_\epsilon + \left(\overline{\beta} - \text{sign}(\overline{\theta})\Delta_\beta\right)\overline{\theta} & \textit{if } X_1 \in (-2, -1 + X_{LH}) \end{cases}$$

If $\chi_{1,\epsilon} < \chi_{1,\beta}$*:*

$$\mathbb{E}\left[v_1|X_1\right] = \begin{cases} \Delta_\epsilon + \left(\overline{\beta} + \text{sign}(\overline{\theta})\Delta_\beta\right)\overline{\theta} & \textit{if } X_1 \in (1 + X_{LH}, 2) \\ \left(\overline{\beta} + \text{sign}(\overline{\theta})\Delta_\beta\right)\overline{\theta} & \textit{if } X_1 \in (1 + X_{HL}, 1 + X_{LH}) \\ \frac{\Delta_\epsilon}{3} + \left(\overline{\beta} + \text{sign}(\overline{\theta})\frac{\Delta_\beta}{3}\right)\overline{\theta} & \textit{if } X_1 \in (0, 1 + X_{HL}) \\ \frac{-\Delta_\epsilon}{3} + \left(\overline{\beta} - \text{sign}(\overline{\theta})\frac{\Delta_\beta}{3}\right)\overline{\theta} & \textit{if } X_1 \in (-1 + X_{LH}, 0) \\ \left(\overline{\beta} - \text{sign}(\overline{\theta})\Delta_\beta\right)\overline{\theta} & \textit{if } X_1 \in (-1 + X_{HL}, -1 + X_{LH}) \\ -\Delta_\epsilon + \left(\overline{\beta} - \text{sign}(\overline{\theta})\Delta_\beta\right)\overline{\theta} & \textit{if } X_1 \in (-2, -1 + X_{HL}) \end{cases}$$

(b) $\text{Var}(v_1|X_1)$

If $\chi_{1,\epsilon} > \chi_{1,\beta}$*:*

$$\text{Var}(v_1|X_1) = \begin{cases} \left(\overline{\beta} + \text{sign}(\overline{\theta})\Delta_\beta\right)^2 \sigma_\theta^2 & \textit{if } X_1 \in (1 + X_{HL}, 2) \\ \overline{\beta}^2 \sigma_\theta^2 + \Delta_\beta^2 \left(\overline{\theta}^2 + \sigma_\theta^2\right) & \textit{if } X_1 \in (1 + X_{LH}, 1 + X_{HL}) \\ \frac{8}{9}\Delta_\epsilon^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2 + \left(\Delta_\beta^2 + \overline{\beta}^2 + \text{sign}(\overline{\theta})\frac{2}{3}\overline{\beta}\Delta_\beta\right)\sigma_\theta^2 & \textit{if } X_1 \in (0, 1 + X_{LH}) \\ \frac{8}{9}\Delta_\epsilon^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2 + \left(\Delta_\beta^2 + \overline{\beta}^2 - \text{sign}(\overline{\theta})\frac{2}{3}\overline{\beta}\Delta_\beta\right)\sigma_\theta^2 & \textit{if } X_1 \in (-1 + X_{HL}, 0) \\ \overline{\beta}^2 \sigma_\theta^2 + \Delta_\beta^2 \left(\overline{\theta}^2 + \sigma_\theta^2\right) & \textit{if } X_1 \in (-1 + X_{LH}, -1 + X_{HL}) \\ \left(\overline{\beta} - \text{sign}(\overline{\theta})\Delta_\beta\right)^2 \sigma_\theta^2 & \textit{if } X_1 \in (-2, -1 + X_{LH}) \end{cases}$$

If $\chi_{1,\epsilon} < \chi_{1,\beta}$*:*

$$\mathrm{Var}(v_1|X_1) = \begin{cases} \left(\overline{\beta} + \mathrm{sign}(\overline{\theta})\Delta_\beta\right)^2 \sigma_\theta^2 & \textit{if } X_1 \in (1 + X_{LH}, 2) \\ \Delta_\epsilon^2 + \left(\overline{\beta} + \mathrm{sign}(\overline{\theta})\Delta_\beta\right)^2 \sigma_\theta^2 & \textit{if } X_1 \in (1 + X_{HL}, 1 + X_{LH}) \\ \frac{8}{9}\Delta_\epsilon^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2 + \left(\Delta_\beta^2 + \overline{\beta}^2 + \mathrm{sign}(\overline{\theta})\frac{2}{3}\overline{\beta}\Delta_\beta\right) \sigma_\theta^2 & \textit{if } X_1 \in (0, 1 + X_{HL}) \\ \frac{8}{9}\Delta_\epsilon^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2 + \left(\Delta_\beta^2 + \overline{\beta}^2 - \mathrm{sign}(\overline{\theta})\frac{2}{3}\overline{\beta}\Delta_\beta\right) \sigma_\theta^2 & \textit{if } X_1 \in (-1 + X_{LH}, 0) \\ \Delta_\epsilon^2 + \left(\overline{\beta} - \mathrm{sign}(\overline{\theta})\Delta_\beta\right)^2 \sigma_\theta^2 & \textit{if } X_1 \in (-1 + X_{HL}, -1 + X_{LH}) \\ \left(\overline{\beta} - \mathrm{sign}(\overline{\theta})\Delta_\beta\right)^2 \sigma_\theta^2 & \textit{if } X_1 \in (-2, -1 + X_{HL}) \end{cases}$$

(c) $\mathrm{Cov}(v_1, v_2|X_1, X_2) = \mathbb{E}\left(\beta_1|X_1\right)\mathbb{E}\left(\beta_2|X_2\right)\sigma_\theta^2$*, where* $\mathbb{E}\left(\beta_1|X_1\right)$ *and* $\mathbb{E}\left(\beta_2|X_2\right)$ *are as follows:*

If $\chi_{j,\epsilon} > \chi_{j,\beta}$*:*

$$\mathbb{E}\left(\beta_j|X_j\right) = \begin{cases} \overline{\beta} + \mathrm{sign}(\overline{\theta})\Delta_\beta & \textit{if } X_j \in (1 + X_{HL}, 2) \\ \overline{\beta} & \textit{if } X_j \in (1 + X_{LH}, 1 + X_{HL}) \\ \overline{\beta} + \mathrm{sign}(\overline{\theta})\frac{\Delta_\beta}{3} & \textit{if } X_j \in (0, 1 + X_{LH}) \\ \overline{\beta} - \mathrm{sign}(\overline{\theta})\frac{\Delta_\beta}{3} & \textit{if } X_j \in (-1 + X_{HL}, 0) \\ \overline{\beta} & \textit{if } X_j \in (-1 + X_{LH}, -1 + X_{HL}) \\ \overline{\beta} - \mathrm{sign}(\overline{\theta})\Delta_\beta & \textit{if } X_j \in (-2, -1 + X_{LH}) \end{cases}$$

If $\chi_{j,\epsilon} < \chi_{j,\beta}$*:*

$$\mathbb{E}\left(\beta_j|X_j\right) = \begin{cases} \overline{\beta} + \mathrm{sign}(\overline{\theta})\Delta_\beta & \textit{if } X_j \in (1 + X_{LH}, 2) \\ \overline{\beta} + \mathrm{sign}(\overline{\theta})\Delta_\beta & \textit{if } X_j \in (1 + X_{HL}, 1 + X_{LH}) \\ \overline{\beta} + \mathrm{sign}(\overline{\theta})\frac{\Delta_\beta}{3} & \textit{if } X_j \in (0, 1 + X_{HL}) \\ \overline{\beta} - \mathrm{sign}(\overline{\theta})\frac{\Delta_\beta}{3} & \textit{if } X_j \in (-1 + X_{LH}, 0) \\ \overline{\beta} - \mathrm{sign}(\overline{\theta})\Delta_\beta & \textit{if } X_j \in (-1 + X_{HL}, -1 + X_{LH}) \\ \overline{\beta} - \mathrm{sign}(\overline{\theta})\Delta_\beta & \textit{if } X_j \in (-2, -1 + X_{HL}) \end{cases}$$

where $X_{HL} \equiv \chi_\epsilon - \chi_\beta$*,* $X_{LH} \equiv -\chi_\epsilon + \chi_\beta$*, and* $\mathrm{sign}(\overline{\theta}) = 1$ *if* $\overline{\theta} > 0$ *and* $\mathrm{sign}(\overline{\theta}) = -1$ *otherwise.*

As is evident from the above, the conditional variance terms are unaltered relative to Theorem 1. Indeed, the covariance terms only influence the intercept but not the slope (specifically, the order flow in asset 2 does not influence the conditional variance, i.e., the slope of the liquidity function, of asset 1). Therefore, the unimodality result, as well as the comparative statics on (the moments of) δ associated with Theorem 1, are unaltered in this equilibrium.

Covariance Terms Evenly Split

In this alternative setting, the covariance term is evenly split across the two assets. Thus, we have

$$\mathbb{E}\left[X_1\left(p_1-v_1\right)|X_1,X_2\right]-\frac{\gamma}{2}\text{Var}[X_1(p_1-v_1)|X_1,X_2]-\frac{\gamma}{2}\text{Cov}[X_1(p_1-v_1),X_2(p_2-v_2|X_1,X_2]=0, \tag{OA.2.8}$$

and

$$\mathbb{E}\left[X_2\left(p_2-v_2\right)|X_1,X_2\right]-\frac{\gamma}{2}\text{Var}[X_2(p_2-v_2)|X_1,X_2]-\frac{\gamma}{2}\text{Cov}[X_1(p_1-v_1),X_2(p_2-v_2|X_1,X_2]=0. \tag{OA.2.9}$$

The solution to the above equation system is:

$$p_1=\mathbb{E}\left[v_1|X_1\right]+\frac{\gamma}{2}X_1\text{Var}(v_1|X_1)+\frac{\gamma}{2}X_2\text{Cov}(v_1,v_2|X_1,X_2), \tag{OA.2.10}$$

and

$$p_2=\mathbb{E}\left[v_2|X_2\right]+\frac{\gamma}{2}X_2\text{Var}(v_2|X_2)+\frac{\gamma}{2}X_1\text{Cov}(v_1,v_2|X_1,X_2). \tag{OA.2.11}$$

where

$$\text{Cov}(v_1,v_2|X_1,X_2)=\mathbb{E}\left(\beta_1|X_1\right)\mathbb{E}\left(\beta_2|X_2\right)\sigma_\theta^2. \tag{OA.2.12}$$

The financial market equilibrium follows the same steps as in the previous equilibrium. Once again, the conditional variance of either asset is not affected by the covariance term. Hence all of the results related to δ survive in this alternative equilibrium as well.

OA.2.2 A Finite Economy with Multiple Assets and Multiple Market Makers

We now consider a model variant with several assets and market makers, and show that our central results remain unchanged in this alternative setting. Each asset's payoff structure is given by Eq. (1) with ϵ and β being independent across stocks. The number of ϵ-informed investors,

β-informed investors, and risk-averse market makers in market $j \in \{1, \ldots, J\}$ are $N_{j,\epsilon}$, $N_{j,\beta}$, and N_M, respectively. As a consequence, the total number of informed traders and market makers is equal to $N \geq 1$ with $N = N_{j,\epsilon} + N_{j,\beta} + N_M$. All investors behave competitively. For ease of exposition, we denote the masses (fractions) of ϵ-informed investors, β-informed investors, and market makers in market j by $\Lambda_{j,\epsilon}$, $\Lambda_{j,\beta}$, and Λ_M, respectively. It follows that $\Lambda_{j,\epsilon} + \Lambda_{j,\beta} + \Lambda_M = 1$. Among informed investors, the fractions of ϵ- and β-informed investors are respectively $\chi_{j,\epsilon}$ and $\chi_{j,\beta}$, where $\chi_{j,\epsilon} + \chi_{j,\beta} = 1$. Noise trader demand in market j is given by Z_j. Z_j is in the same order of magnitude as the number of informed investors, so that $Z_j = (N_{j,\epsilon} + N_{j,\beta}) \cdot z_j$, where the distributional assumption on z_j follows our main model.[27] We assume that market makers can bid for portions of the order flow, as opposed to a single market maker being constrained to absorb the entire order flow.

The total order flow in market j is given by

$$\begin{aligned} X_j &= N_{j,\epsilon} y_{j,\epsilon} + N_{j,\beta} y_{j,\beta} + Z_j \\ &= N(1 - \Lambda_M) \cdot x_j, \end{aligned} \tag{OA.2.13}$$

where

$$x_j = \chi_{j,\epsilon} y_{j,\epsilon} + \chi_{j,\beta} y_{j,\beta} + z_j.$$

We can write the expected utility for an individual market maker m as:

$$\mathbb{E}\left[U_m\right] = \mathbb{E}\left[\sum_{j=1}^{J} X_{m,j}\left(p_j - v_j\right)|\vec{X}\right] - \frac{\gamma}{2}\mathrm{Var}\left[\sum_{j=1}^{J} X_{m,j}\left(p_j - v_j\right)|\vec{X}\right], \tag{OA.2.14}$$

where $\gamma \geq 0$ represents the market maker's coefficient of risk aversion and $\sum_{m=1}^{N_M} X_{m,j} = X_j$. For ease of exposition, we define $\vec{X} \equiv (X_1, \ldots, X_J)$.

We can thus interpret $C_m = \mathbb{E}\left[\sum_{j=1}^{J} X_{m,j} v_j |\vec{X}\right] + \frac{\gamma}{2}\mathrm{Var}\left[\sum_{j=1}^{J} X_{m,j} v_j |\vec{X}\right]$ as the market maker's cost, and $\sum_{j=1}^{J} X_{m,j} p_j$ as the market maker's profit. In the equilibrium with Bertrand competition, the pricing rule for each market maker that ensures no rents at the margin implies a price equal to

---

[27] The assumption that the noise trader demand is of the same order of magnitude as the number of informed investors is to simplify the analysis without loss of economic insight. This assumption closely follows our main model where the total population of informed investors is 1 and the total noise trader demand is uniformly distributed between −1 and 1. Alternatively, if we use different values for the noise trader demand, the results are qualitatively unchanged.

marginal cost, i.e., $p_j = \frac{dC_m}{dX_{m,j}}$. As a consequence, we obtain the following price:

$$p_j = \mathbb{E}\left[v_j|\vec{X}\right] + \gamma X_{m,j}\text{Var}[v_j|\vec{X}] + \gamma \sum_{l\neq j} X_{m,l}\text{Cov}(v_l, v_j|\vec{X}). \tag{OA.2.15}$$

We consider a symmetric equilibrium for each asset. In this equilibrium, each market maker absorbs the same quantity of the asset, i.e., $X_{m,j} = X_{k,j}$ where $m \neq k$. It follows that $X_{m,j} = \frac{X_j}{N_M}$. Inserting $\frac{X_j}{N_M}$ into the above expression for the equilibrium stock price p_j yields:

$$\begin{aligned} p_j &= \mathbb{E}\left[v_j|\vec{X}\right] + \gamma\frac{X_j}{M}\text{Var}[v_j|\vec{X}] + \gamma\sum_{l\neq j}\frac{X_l}{M}\text{Cov}(v_l, v_j|\vec{X}) \\ &= \mathbb{E}\left[v_j|\vec{X}\right] + \overline{\gamma}x_j\text{Var}[v_j|\vec{X}] + \overline{\gamma}\sum_{l\neq j}x_l\text{Cov}(v_l, v_j|\vec{X}), \end{aligned} \tag{OA.2.16}$$

where $\overline{\gamma} \equiv \gamma\frac{N(1-\Lambda_M)}{N\Lambda_M} = \gamma\frac{(1-\Lambda_M)}{\Lambda_M}$. It is important to note that as $N \to \infty$ (so that $N_M \to \infty$), $\overline{\gamma}$ is constant and each market maker absorbs a finite number of the order flow x_j. For ease of exposition, we define $\vec{x} = (x_1, \ldots, x_J)$, which is informationally equivalent to $\vec{X}$. It follows that $\mathbb{E}\left[v_j|\vec{X}\right] = \mathbb{E}\left[v_j|\vec{x}\right]$, $\text{Var}[v_j|\vec{X}] = \text{Var}[v_j|\vec{x}]$, and $\text{Cov}(v_l, v_j|\vec{X}) = \text{Cov}(v_l, v_j|\vec{x})$.

The key quantity in the price function is $\text{Cov}(v_l, v_j|\vec{x})$, which is given by:

$$\begin{aligned} \text{Cov}(v_l, v_j|\vec{x}) &= \text{Cov}(\beta_l\theta, \beta_j\theta|\vec{x}) \\ &= \mathbb{E}\left[\beta_l\beta_j\theta^2|\vec{x}\right] - \mathbb{E}\left[\beta_l\theta|\vec{X}\right]\mathbb{E}\left[\beta_j\theta|\vec{x}\right] \\ &= \mathbb{E}\left(\beta_l|x_l\right)\mathbb{E}\left(\beta_j|x_j\right)(\overline{\theta}^2 + \sigma_\theta^2) - \mathbb{E}\left(\beta_l|x_l\right)\mathbb{E}\left(\beta_j|x_j\right)\overline{\theta}^2 \\ &= \mathbb{E}\left(\beta_l|x_l\right)\mathbb{E}\left(\beta_j|x_j\right)\sigma_\theta^2 \end{aligned} \tag{OA.2.17}$$

We fully characterize the financial equilibrium prices in the following proposition.

Proposition 2 (Financial Market Equilibrium - Extension 2) *Given* $(\chi_{j,\epsilon}, \chi_{j,\beta})$ *and the conditions mentioned above, there is a financial market equilibrium in which:*

1. $\epsilon-$*informed traders buy on a high signal* ($\epsilon_j = \Delta_\epsilon$) *and sell on a low signal* ($\epsilon_j = -\Delta_\epsilon$); $\beta-$*informed traders buy (sell) on a high signal* ($\beta_j = \overline{\beta} + \Delta_\beta$) *and sell (buy) on a low signal* ($\beta_j = \overline{\beta} - \Delta_\beta$) *if* $\overline{\theta} > 0$ ($\overline{\theta} < 0$).

2. *The equilibrium stock prices depend on total order flow and are given by:*

$$p_j = \mathbb{E}\left[v_j|x_j\right] + \overline{\gamma}x_j\text{Var}[v_j|x_j] + \overline{\gamma}\sum_{l\neq j}x_l\text{Cov}(v_l, v_j|x_l, x_j),$$

where $\mathbb{E}\left[v_j|x_j\right]$, $\mathrm{Var}(v_j|x_j)$, *and* $\mathrm{Cov}(v_l, v_j|x_l, x_j)$ *are as follows.*

(a) $\mathbb{E}\left[v_j|x_j\right]$ *is given as follows:*

If $\chi_{j,\epsilon} > \chi_{j,\beta}$*:*

$$\mathbb{E}\left[v_j|x_j\right] = \begin{cases} \Delta_\epsilon + \left(\overline{\beta} + \mathrm{sign}(\overline{\theta})\Delta_\beta\right)\overline{\theta} & \textit{if } x_j \in (1 + x_{HL}, 2) \\ \Delta_\epsilon + \overline{\beta\theta} & \textit{if } x_j \in (1 + x_{LH}, 1 + x_{HL}) \\ \frac{\Delta_\epsilon}{3} + \left(\overline{\beta} + \mathrm{sign}(\overline{\theta})\frac{\Delta_\beta}{3}\right)\overline{\theta} & \textit{if } x_j \in (0, 1 + x_{LH}) \\ \frac{-\Delta_\epsilon}{3} + \left(\overline{\beta} - \mathrm{sign}(\overline{\theta})\frac{\Delta_\beta}{3}\right)\overline{\theta} & \textit{if } x_j \in (-1 + x_{HL}, 0) \\ -\Delta_\epsilon + \overline{\beta\theta} & \textit{if } x_j \in (-1 + x_{LH}, -1 + x_{HL}) \\ -\Delta_\epsilon + \left(\overline{\beta} - \mathrm{sign}(\overline{\theta})\Delta_\beta\right)\overline{\theta} & \textit{if } x_j \in (-2, -1 + x_{LH}) \end{cases}$$

If $\chi_{j,\epsilon} < \chi_{j,\beta}$*:*

$$\mathbb{E}\left[v_j|x_j\right] = \begin{cases} \Delta_\epsilon + \left(\overline{\beta} + \mathrm{sign}(\overline{\theta})\Delta_\beta\right)\overline{\theta} & \textit{if } x_j \in (1 + x_{LH}, 2) \\ \left(\overline{\beta} + \mathrm{sign}(\overline{\theta})\Delta_\beta\right)\overline{\theta} & \textit{if } x_j \in (1 + x_{HL}, 1 + x_{LH}) \\ \frac{\Delta_\epsilon}{3} + \left(\overline{\beta} + \mathrm{sign}(\overline{\theta})\frac{\Delta_\beta}{3}\right)\overline{\theta} & \textit{if } x_j \in (0, 1 + x_{HL}) \\ \frac{-\Delta_\epsilon}{3} + \left(\overline{\beta} - \mathrm{sign}(\overline{\theta})\frac{\Delta_\beta}{3}\right)\overline{\theta} & \textit{if } x_j \in (-1 + x_{LH}, 0) \\ \left(\overline{\beta} - \mathrm{sign}(\overline{\theta})\Delta_\beta\right)\overline{\theta} & \textit{if } x_j \in (-1 + x_{HL}, -1 + x_{LH}) \\ -\Delta_\epsilon + \left(\overline{\beta} - \mathrm{sign}(\overline{\theta})\Delta_\beta\right)\overline{\theta} & \textit{if } x_j \in (-2, -1 + x_{HL}) \end{cases}$$

(b) $\mathrm{Var}(v_j|x_j)$ *is given as follows:*

If $\chi_{j,\epsilon} > \chi_{j,\beta}$*:*

$$\text{Var}(v_j|x_j) = \begin{cases} \left(\overline{\beta} + \text{sign}(\overline{\theta})\Delta_\beta\right)^2 \sigma_\theta^2 & \textit{if } x_j \in (1 + x_{HL}, 2) \\ \overline{\beta}^2 \sigma_\theta^2 + \Delta_\beta^2 \left(\overline{\theta}^2 + \sigma_\theta^2\right) & \textit{if } x_j \in (1 + x_{LH}, 1 + x_{HL}) \\ \frac{8}{9}\Delta_\epsilon^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2 + \left(\Delta_\beta^2 + \overline{\beta}^2 + \text{sign}(\overline{\theta})\frac{2}{3}\overline{\beta}\Delta_\beta\right)\sigma_\theta^2 & \textit{if } x_j \in (0, 1 + x_{LH}) \\ \frac{8}{9}\Delta_\epsilon^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2 + \left(\Delta_\beta^2 + \overline{\beta}^2 - \text{sign}(\overline{\theta})\frac{2}{3}\overline{\beta}\Delta_\beta\right)\sigma_\theta^2 & \textit{if } x_j \in (-1 + x_{HL}, 0) \\ \overline{\beta}^2 \sigma_\theta^2 + \Delta_\beta^2 \left(\overline{\theta}^2 + \sigma_\theta^2\right) & \textit{if } x_j \in (-1 + x_{LH}, -1 + x_{HL}) \\ \left(\overline{\beta} - \text{sign}(\overline{\theta})\Delta_\beta\right)^2 \sigma_\theta^2 & \textit{if } x_j \in (-2, -1 + x_{LH}) \end{cases}$$

If $\chi_{j,\epsilon} > \chi_{j,\beta}$*:*

$$\text{Var}(v_j|x_j) = \begin{cases} \left(\overline{\beta} + \text{sign}(\overline{\theta})\Delta_\beta\right)^2 \sigma_\theta^2 & \textit{if } x_j \in (1 + x_{LH}, 2) \\ \Delta_\epsilon^2 + \left(\overline{\beta} + \text{sign}(\overline{\theta})\Delta_\beta\right)^2 \sigma_\theta^2 & \textit{if } x_j \in (1 + x_{HL}, 1 + x_{LH}) \\ \frac{8}{9}\Delta_\epsilon^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2 + \left(\Delta_\beta^2 + \overline{\beta}^2 + \text{sign}(\overline{\theta})\frac{2}{3}\overline{\beta}\Delta_\beta\right)\sigma_\theta^2 & \textit{if } x_j \in (0, 1 + x_{HL}) \\ \frac{8}{9}\Delta_\epsilon^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2 + \left(\Delta_\beta^2 + \overline{\beta}^2 - \text{sign}(\overline{\theta})\frac{2}{3}\overline{\beta}\Delta_\beta\right)\sigma_\theta^2 & \textit{if } x_j \in (-1 + x_{LH}, 0) \\ \Delta_\epsilon^2 + \left(\overline{\beta} - \text{sign}(\overline{\theta})\Delta_\beta\right)^2 \sigma_\theta^2 & \textit{if } x_j \in (-1 + x_{HL}, -1 + x_{LH}) \\ \left(\overline{\beta} - \text{sign}(\overline{\theta})\Delta_\beta\right)^2 \sigma_\theta^2 & \textit{if } x_j \in (-2, -1 + x_{HL}) \end{cases}$$

(c) $\text{Cov}(v_l, v_j|x_l, x_j) = \mathbb{E}\left(\beta_l|x_l\right)\mathbb{E}\left(\beta_j|x_j\right)\sigma_\theta^2$*, where* $\mathbb{E}\left(\beta_k|x_k\right)$ *(*$k \in \{1, \ldots, J\}$*) are as follows:*

If $\chi_{k,\epsilon} > \chi_{k,\beta}$*:*

$$\mathbb{E}\left(\beta_k|x_k\right) = \begin{cases} \overline{\beta} + \text{sign}(\overline{\theta})\Delta_\beta & \textit{if } x_k \in (1 + x_{HL}, 2) \\ \overline{\beta} & \textit{if } x_k \in (1 + x_{LH}, 1 + x_{HL}) \\ \overline{\beta} + \text{sign}(\overline{\theta})\frac{\Delta_\beta}{3} & \textit{if } x_k \in (0, 1 + Xx_{LH}) \\ \overline{\beta} - \text{sign}(\overline{\theta})\frac{\Delta_\beta}{3} & \textit{if } x_k \in (-1 + x_{HL}, 0) \\ \overline{\beta} & \textit{if } x_k \in (-1 + x_{LH}, -1 + x_{HL}) \\ \overline{\beta} - \text{sign}(\overline{\theta})\Delta_\beta & \textit{if } x_k \in (-2, -1 + x_{LH}) \end{cases}$$

If $\chi_{k,\epsilon} < \chi_{k,\beta}$*:*

$$\mathbb{E}\left(\beta_j | x_j\right) = \begin{cases} \overline{\beta} + \operatorname{sign}(\overline{\theta})\Delta_\beta & \textit{if } x_k \in (1 + x_{LH}, 2) \\ \overline{\beta} + \operatorname{sign}(\overline{\theta})\Delta_\beta & \textit{if } x_k \in (1 + x_{HL}, 1 + x_{LH}) \\ \overline{\beta} + \operatorname{sign}(\overline{\theta})\frac{\Delta_\beta}{3} & \textit{if } x_k \in (0, 1 + x_{HL}) \\ \overline{\beta} - \operatorname{sign}(\overline{\theta})\frac{\Delta_\beta}{3} & \textit{if } x_k \in (-1 + x_{LH}, 0) \\ \overline{\beta} - \operatorname{sign}(\overline{\theta})\Delta_\beta & \textit{if } x_k \in (-1 + x_{HL}, -1 + x_{LH}) \\ \overline{\beta} - \operatorname{sign}(\overline{\theta})\Delta_\beta & \textit{if } x_k \in (-2, -1 + x_{HL}) \end{cases}$$

where $x_{HL} \equiv \chi_\epsilon - \chi_\beta$ *and* $x_{LH} \equiv -\chi_\epsilon + \chi_\beta$.

Proposition 2 characterizes the financial market equilibrium in this extension. The key insight is that the conditional variance terms, and thus the slope of the liquidity function, are unchanged relative to the main model. More specifically, the impact of the conditional payoff variance on the price now depends on the *adjusted* risk-aversion coefficient $\overline{\gamma}$. This coefficient depends on the market maker's risk aversion and the proportion of market makers in the economy. Compared to the main model, the price of asset j also depends on the conditional covariance with asset l. However, these terms influence the intercept but not the slope of the liquidity function. Therefore, the unimodality of price impact as a function of order flow, as well as the comparative statics with respect to (the moments of) δ, are unchanged in this extension with multiple assets and market makers.

OA.3: Negative Mean Betas

In this section, we discuss our central model implications under the assumption that the mean beta is negative. We first re-write the asset cash flow in Eq. (1) as:

$$v = \beta\theta + \epsilon = (-\beta)(-\theta) + \epsilon = \tilde{\beta}\tilde{\theta} + \epsilon \tag{OA.3.1}$$

where $\tilde{\beta}$ has mean $\mathbb{E}[\tilde{\beta}] = -\overline{\beta}$ and takes on values $-(\overline{\beta} + \Delta_\beta) = -\overline{\beta} - \Delta_\beta < 0$ and $-(\overline{\beta} - \Delta_\beta) = -\overline{\beta} + \Delta_\beta$, which might be positive or negative, with equal probability. Note that $\tilde{\theta}$ is Normally distributed with mean $\mathbb{E}[\tilde{\theta}] = -\overline{\theta}$ and variance $\mathrm{Var}(\tilde{\theta}) = \sigma_\theta^2$.

We can therefore see that the results in Theorem 1 extend to settings with negative mean betas if we re-define the factor to be $\tilde{\theta} = -\theta$. Using the results from the baseline model and re-defining the β and θ, we get the following equilibrium trading strategies. First, ϵ-informed investors continue to buy on a high signal and to sell on a low signal. If $\mathbb{E}[\tilde{\theta}]$ is positive (i.e., if $\overline{\theta} < 0$ in the baseline model), then β-informed investors buy in response to $\tilde{\beta} = -\overline{\beta} + \Delta_\beta$ (i.e., $\beta = \beta_L$ in the baseline model) and sell in response to $\tilde{\beta} = -\overline{\beta} - \Delta_\beta$ (i.e., $\beta = \beta_H$ in the baseline model). Vice versa, if $\mathbb{E}[\tilde{\theta}]$ is negative, then they use the opposite trading strategy.

Next, we discuss the implications for our central empirical predictions related to the covariance between order flow X and δ. This covariance is still negatively related to the expected return, because the market maker's pricing strategy implies that $\mathbb{E}[p] = \mathbb{E}[v] + \mathrm{Cov}(\delta, X)$, as before.

We find that the covariance between δ and X is now equal to:

$$\mathrm{Cov}(\delta, X) = \begin{cases} +\gamma\Delta_\beta\overline{\beta}\sigma_\theta^2\chi_\beta = -\gamma\Delta_\beta\mathbb{E}[\tilde{\beta}]\sigma_\theta^2\chi_\beta > 0 & \text{if } \mathbb{E}[\tilde{\theta}] < 0 \\ -\gamma\Delta_\beta\overline{\beta}\sigma_\theta^2\chi_\beta = +\gamma\Delta_\beta\mathbb{E}[\tilde{\beta}]\sigma_\theta^2\chi_\beta < 0 & \text{if } \mathbb{E}[\tilde{\theta}] > 0 \end{cases} \tag{OA.3.2}$$

Therefore, the covariance between δ and order flow is positive if the mean of the factor loading is *negative* and it is negative otherwise. This prediction is therefore flipped, compared to the baseline model with positive mean betas. Furthermore, it directly follows from the expression above that $\mathrm{Cov}(\delta, X)$ increases in the mean beta if and only if $\mathbb{E}[\tilde{\theta}] > 0$, which is identical to the corresponding prediction in the baseline model.

To sum up, the central difference is that negative mean betas lead to a negative covariance between δ and order flow for a positive factor mean, and vice versa. However, we still find that the expected return (i) decreases in $\text{Cov}(\delta, X)$ and (ii) decreases (increases) in the expected beta if the factor mean is positive (negative).

OA.4: Positive Asset Supply

In this extension, we consider a variation of our main model in which the asset is in positive supply $e_0 \geq 0$ for each asset. We discuss how the positive net supply affects our results in Corollaries 1, 2, 3, and 4. Briefly, we find that Corollaries 1, 2, and 3 hold when informed investors use the same trading policy as in Theorem 1. We also find that when introducing a positive net supply, the expected return $\mathbb{E}(v - p)$ includes an additional term representing the standard risk premium, and our asset-pricing predictions continue to hold for a robust set of parameter values.

To solve the equilibrium with a positive net supply, we start with the market maker's maximization problem. With the positive net supply e_0, the market maker's expected utility is then given as:

$$\mathbb{E}[U_{MM}] = \mathbb{E}\left[(X - e_0)(p - v)|X\right] - \frac{\gamma}{2}\mathrm{Var}\left((X - e_0)(p - v)|X\right). \tag{OA.4.1}$$

Imposing $\mathbb{E}[U_{MM}] = 0$ and solving for p yields:

$$p = \mathbb{E}[v|X] + \frac{\gamma}{2}(X - e_0)\,\mathrm{Var}(v|X). \tag{OA.4.2}$$

Hence, the equilibrium stock price in this extension is equal to that in our main model plus an extra term $-\frac{\gamma}{2}e_0\mathrm{Var}(v|X)$. It immediately follows that δ is equal to $\frac{\gamma}{2}\mathrm{Var}(v|X)$ and thus identical to the measure in the main model. As a result, all of our main results regarding δ and its moments continue to hold with $e_0 \neq 0$, as long as informed traders use the same trading policy as in Theorem 1. Next, in subsections OA.4.1 and OA.4.2, we derive sufficient conditions for this to hold. In subsection OA.4.3, we show that the asset-pricing predictions in Corollary 4 continue to hold for a robust set of parameter values.

For ease of exposition, let $A_1 = \mathbb{E}(v|X)$, $A_2 = \mathrm{Var}(v|X)X$, and $A_3 = \mathrm{Var}(v|X)$.

OA.4.1 Trading Profits for ϵ-informed investors

In this subsection, we calculate the trading profit of ϵ-informed investors and characterize the sufficient conditions to ensure that ϵ-informed investors use the same trading policy as in Theorem 1. We discuss the cases with $\chi_\epsilon > \chi_\beta$ and $\chi_\epsilon \leq \chi_\beta$ separately as follows. We first focus on the case with $\overline{\theta} > 0$.

Case 1: $\chi_\epsilon > \chi_\beta$

Trading profits depend on $\mathbb{E}(p|\epsilon)$ and thus $\mathbb{E}(A_1|\epsilon)$, $\mathbb{E}(A_2|\epsilon)$, and $\mathbb{E}(A_3|\epsilon)$, which are given as:

(i) When $\epsilon = \epsilon_H$, $X \in (-1 + \chi_\epsilon - \chi_\beta, 1 + \chi_\epsilon + \chi_\beta)$.

$$\begin{aligned}\mathbb{E}(A_1|\epsilon_H) &= \frac{1}{2}\chi_\beta\left[\Delta_\epsilon + (\overline{\beta} + \Delta_\beta)\overline{\theta}\right] + (\chi_\epsilon - \chi_\beta)\left[\Delta_\epsilon + \overline{\beta}\overline{\theta}\ \right] + \chi_\beta\left[\frac{1}{3}\Delta + (\overline{\beta} + \frac{1}{3}\Delta_\beta)\overline{\theta}\ \right] \\ &\quad + (1 - \chi_\epsilon - \chi_\beta)\overline{\beta}\overline{\theta} + \frac{1}{2}\chi_\beta\left[-\frac{1}{3}\Delta_\epsilon + (\overline{\beta} - \frac{1}{3}\Delta_\beta)\overline{\theta}\right] = \Delta_\epsilon(\chi_\epsilon - \frac{\chi_\beta}{3}) + \overline{\beta}\overline{\theta} + \frac{2}{3}\chi_\beta\Delta_\beta\overline{\theta},\end{aligned}$$

$$\begin{aligned}\mathbb{E}(A_2|\epsilon_H) &= \frac{1}{2}\chi_\beta \cdot \left[\beta_H^2\sigma_\theta^2\right] \cdot (1 + \chi_\epsilon) + (\chi_\epsilon - \chi_\beta) \cdot \left[\frac{1}{2}(\beta_H^2 + \beta_L^2)\sigma_\theta^2 + \Delta_\beta^2\overline{\theta}^2\ \right] \\ &\quad + \chi_\beta \cdot \left[\frac{8}{9}\Delta_\epsilon^2 + (\frac{2}{3}\beta_H^2 + \frac{1}{3}\beta_L^2)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2\right] \cdot (1 - \chi_\epsilon) \\ &\quad + \frac{1}{2}\chi_\beta \cdot \left[\frac{8}{9}\Delta_\epsilon^2 + (\frac{1}{3}\beta_H^2 + \frac{2}{3}\beta_L^2)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2\right] \cdot (-1 + \chi_\epsilon) \\ &= \frac{4}{9}\chi_\beta \cdot (1 - \chi_\epsilon)\Delta_\epsilon^2 + \frac{1}{2}\sigma_\theta^2\left[\beta_H^2(\chi_\epsilon + \chi_\beta) + \beta_L^2(\chi_\epsilon - \chi_\beta)\right] + \Delta_\beta^2\overline{\theta}^2\left[(\chi_\epsilon - \chi_\beta) + \frac{4}{9}\chi_\beta(1 - \chi_\epsilon)\right],\end{aligned}$$

$$\begin{aligned}\mathbb{E}(A_3|\epsilon_H) &= \frac{1}{2}\chi_\beta\left[\beta_H^2\sigma_\theta^2\right] + (\chi_\epsilon - \chi_\beta)\left[\frac{1}{2}(\beta_H^2 + \beta_L^2)\sigma_\theta^2 + \Delta_\beta^2\overline{\theta}^2\ \right] + \chi_\beta\left[\frac{8}{9}\Delta_\epsilon^2 + (\frac{2}{3}\beta_H^2 + \frac{1}{3}\beta_L^2)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2\right] \\ &\quad + (1 - \chi_\epsilon - \chi_\beta)\left[\Delta_\epsilon^2 + \frac{1}{2}(\beta_H^2 + \beta_L^2)\sigma_\theta^2 + \Delta_\beta^2\overline{\theta}\right] + \frac{1}{2}\chi_\beta\left[\frac{8}{9}\Delta_\epsilon^2 + (\frac{1}{3}\beta_H^2 + \frac{2}{3}\beta_L^2)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2\right] \\ &= \sigma_\theta^2\left[\overline{\beta}^2 + \Delta_\beta^2 + \frac{4\chi_\beta}{3}\overline{\beta}\Delta_\beta\right] + \Delta_\epsilon^2\left[1 - \chi_\epsilon + \chi_\beta/3\right] + \Delta_\beta^2\overline{\theta}^2\left[1 - 2\chi_\beta/3\right].\end{aligned}$$

It follows that we can write the expected trading profits for $\epsilon-$informed traders conditional on $\epsilon = \epsilon_H$ as:

$$\begin{aligned}\pi_{\epsilon_H} &= \Delta_\epsilon + \overline{\beta}\overline{\theta} - \Delta_\epsilon(\chi_\epsilon - \frac{\chi_\beta}{3}) - \overline{\beta}\overline{\theta} - \frac{2}{3}\chi_\beta\Delta_\beta\overline{\theta} - \frac{\gamma}{2}\mathbb{E}(A_2|\Delta_\epsilon) + \frac{\gamma}{2}\mathbb{E}(A_3|\Delta_\epsilon) \\ &= \Delta_\epsilon(1 - \chi_\epsilon + \frac{\chi_\beta}{3}) - \frac{2}{3}\chi_\beta\Delta_\beta\overline{\theta} - \frac{2\gamma}{9}\chi_\beta \cdot (1 - \chi_\epsilon)\Delta_\epsilon^2 \\ &\quad - \frac{\gamma}{4}\sigma_\theta^2\left[\beta_H^2(\chi_\epsilon + \chi_\beta) + \beta_L^2(\chi_\epsilon - \chi_\beta)\right] - \frac{\gamma}{2}\Delta_\beta^2\overline{\theta}^2\left[(\chi_\epsilon - \chi_\beta) + \frac{4}{9}\chi_\beta(1 - \chi_\epsilon)\right] + \frac{\gamma}{2}\mathbb{E}(A_3|\Delta_\epsilon)e_0.\end{aligned}$$

(ii) When $\epsilon = \epsilon_L$, $X \in (-1 - \chi_\epsilon - \chi_\beta, 1 - \chi_\epsilon + \chi_\beta)$.

$$\begin{aligned}\mathbb{E}(A_1|\epsilon_L) &= \frac{1}{2}\chi_\beta\left[\frac{1}{3}\Delta_\epsilon + (\overline{\beta} + \frac{1}{3}\Delta_\beta)\overline{\theta}\right] + (1 - \chi_\epsilon - \chi_\beta)\overline{\beta}\overline{\theta} + \chi_\beta\left[-\frac{1}{3}\Delta_\epsilon + (\overline{\beta} - \frac{1}{3}\Delta_\beta)\overline{\theta}\right] \\ &\quad + (\chi_\epsilon - \chi_\beta)\left[-\Delta_\epsilon + \overline{\beta}\overline{\theta}\ \right] + \frac{1}{2}\chi_\beta\left[-\Delta_\epsilon + (\overline{\beta} - \Delta_\beta)\overline{\theta}\right] = -\Delta_\epsilon(\chi_\epsilon - \frac{\chi_\beta}{3}) + \overline{\beta}\overline{\theta} - \frac{2}{3}\chi_\beta\Delta_\beta\overline{\theta},\end{aligned}$$

$$\begin{aligned}
\mathbb{E}(A_2|\epsilon_L) &= \frac{\chi_\beta}{2}\left[\frac{8}{9}\Delta_\epsilon^2+(\frac{2}{3}\beta_H^2+\frac{1}{3}\beta_L^2)\sigma_\theta^2+\frac{8}{9}\Delta_\beta^2\overline{\theta}^2\right](1-\chi_\epsilon)\\
&-\chi_\beta\left[\frac{8}{9}\Delta_\epsilon^2+(\frac{1}{3}\beta_H^2+\frac{2}{3}\beta_L^2)\sigma_\theta^2+\frac{8}{9}\Delta_\beta^2\overline{\theta}^2\right](1-\chi_\epsilon)\\
&-(\chi_\epsilon-\chi_\beta)\left[\frac{1}{2}(\beta_H^2+\beta_L^2)\sigma_\theta^2+\Delta_\beta^2\overline{\theta}^2\right]+\frac{1}{2}\chi_\beta\left[\beta_L^2\sigma_\theta^2\ \right](-1-\chi_\epsilon)\\
&= -\frac{4}{9}(1-\chi_\epsilon)\chi_\beta\Delta_\epsilon^2-\frac{1}{2}\sigma_\theta^2\left[\beta_H^2(\chi_\epsilon-\chi_\beta)+\beta_L^2(\chi_\epsilon+\chi_\beta)\right]-\Delta_\beta^2\overline{\theta}^2\left[\chi_\epsilon-\chi_\beta+\frac{4}{9}(1-\chi_\epsilon)\chi_\beta\right],
\end{aligned}$$

$$\begin{aligned}
\mathbb{E}(A_3|\epsilon_L) &= \frac{1}{2}\chi_\beta\left[\frac{8}{9}\Delta_\epsilon^2+(\frac{2}{3}\beta_H^2+\frac{1}{3}\beta_L^2)\sigma_\theta^2+\frac{8}{9}\Delta_\beta^2\overline{\theta}^2\right]+(\chi_\epsilon-\chi_\beta)\left[\frac{1}{2}(\beta_H^2+\beta_L^2)\sigma_\theta^2+\Delta_\beta^2\overline{\theta}^2\right]+\frac{1}{2}\chi_\beta\beta_L^2\sigma_\theta^2\\
&+(1-\chi_\epsilon-\chi_\beta)\left[\Delta_\epsilon^2+\frac{1}{2}(\beta_H^2+\beta_L^2)\sigma_\theta^2+\Delta_\beta^2\overline{\theta}\right]+\chi_\beta\left[\frac{8}{9}\Delta_\epsilon^2+(\frac{1}{3}\beta_H^2+\frac{2}{3}\beta_L^2)\sigma_\theta^2+\frac{8}{9}\Delta_\beta^2\overline{\theta}^2\right]\\
&= \sigma_\theta^2\left[\overline{\beta}^2+\Delta_\beta^2-\frac{4\chi_\beta}{3}\overline{\beta}\Delta_\beta\right]+\Delta_\epsilon^2\left[1-\chi_\epsilon+\chi_\beta/3\right]+\Delta_\beta^2\overline{\theta}^2\left[1-2\chi_\beta/3\right].
\end{aligned}$$

It follows that we can write the expected trading profits for $\epsilon-$informed traders conditional on $\epsilon=\epsilon_L$ as:

$$\begin{aligned}
\pi_{\epsilon_L} &= \Delta_\epsilon-\overline{\beta}\overline{\theta}-\Delta_\epsilon(\chi_\epsilon-\frac{\chi_\beta}{3})+\overline{\beta}\overline{\theta}-\frac{2}{3}\chi_\beta\Delta_\beta\overline{\theta}+\frac{\gamma}{2}\mathbb{E}(A_2|-\Delta_\epsilon)-\frac{\gamma}{2}\mathbb{E}(A_3|-\Delta_\epsilon)\\
&= \Delta_\epsilon(1-\chi_\epsilon+\frac{\chi_\beta}{3})-\frac{2}{3}\chi_\beta\Delta_\beta\overline{\theta}-\frac{2\gamma}{9}\chi_\beta.(1-\chi_\epsilon)\Delta_\epsilon^2-\frac{\gamma}{4}\sigma_\theta^2\left[\beta_H^2(\chi_\epsilon-\chi_\beta)+\beta_L^2(\chi_\epsilon+\chi_\beta)\right]\\
&-\frac{\gamma}{2}\Delta_\beta^2\overline{\theta}^2\left[(\chi_\epsilon-\chi_\beta)+\frac{4}{9}\chi_\beta(1-\chi_\epsilon)\right]-\frac{\gamma}{2}E(A_3|-\Delta_\epsilon)e_0.
\end{aligned}$$

The sufficient condition to ensure positive conditional trading profits of ϵ-informed investors is:

$$\chi_\beta\frac{2}{3}\frac{\Delta_\beta\overline{\theta}}{\Delta_\epsilon}-\frac{\chi_\beta}{3}+\chi_\epsilon<1 \text{ and } \gamma \text{ is sufficiently low.}$$

For the case $\overline{\theta}<0$, we can show that the equivalent condition is:

$$\chi_\beta\frac{2}{3}\frac{\Delta_\beta|\overline{\theta}|}{\Delta_\epsilon}-\frac{\chi_\beta}{3}+\chi_\epsilon<1 \text{ and } \gamma \text{ is sufficiently low.}$$

Case 2: $\chi_\epsilon\leq\chi_\beta$

Trading profits depend on $\mathbb{E}(p|\epsilon)$ and thus $\mathbb{E}(A_1|\epsilon)$, $\mathbb{E}(A_2|\epsilon)$, and $\mathbb{E}(A_3|\epsilon)$, which are given as:

(i) When $\epsilon = \epsilon_H$, $X \in (-1 + \chi_\epsilon - \chi_\beta, 1 + \chi_\epsilon + \chi_\beta)$.

$$\begin{aligned}
\mathbb{E}(A_1|\epsilon_H) &= \frac{1}{2}\chi_\epsilon\left[\Delta_\epsilon + (\overline{\beta} + \Delta_\beta)\overline{\theta}\right] + \frac{1}{2}(\chi_\beta - \chi_\epsilon)\left[(\overline{\beta} + \Delta_\beta)\overline{\theta}\right] \\
&\quad + \chi_\epsilon[\frac{1}{3}\Delta_\epsilon + (\overline{\beta} + \frac{1}{3}\Delta_\beta)\overline{\theta}] + (1 - \chi_\epsilon - \chi_\beta)\overline{\beta}\overline{\theta} + \frac{1}{2}\chi_\epsilon(-\frac{1}{3}\Delta_\epsilon + (\overline{\beta} - \frac{1}{3}\Delta_\beta)\overline{\theta}) \\
&\quad + \frac{1}{2}(\chi_\beta - \chi_\epsilon)(\overline{\beta} - \Delta_\beta)\overline{\theta} = \frac{2}{3}\chi_\epsilon\Delta_\epsilon + \overline{\theta}\overline{\beta} + \frac{2}{3}\chi_\epsilon\Delta_\beta\overline{\theta},
\end{aligned}$$

$$\begin{aligned}
\mathbb{E}(A_2|\epsilon_H) &= \frac{1}{2}\chi_\epsilon\beta_H^2\sigma_\theta^2(1 + \chi_\beta) + \frac{1}{2}(\chi_\beta - \chi_\epsilon)\left[\Delta_\epsilon^2 + \beta_H^2\sigma_\theta^2\right] \\
&\quad + \chi_\epsilon[\frac{8}{9}\Delta_\epsilon^2 + (\frac{2}{3}\beta_H^2 + \frac{1}{3}\beta_L^2)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2](1 - \chi_\beta) \\
&\quad + \frac{1}{2}\chi_\epsilon\left[\frac{8}{9}\Delta_\epsilon^2 + (\frac{1}{3}\beta_H^2 + \frac{2}{3}\beta_L^2)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2\right](-1 + \chi_\beta) - \frac{1}{2}(\chi_\beta - \chi_\epsilon)\left[\Delta_\epsilon^2 + \beta_L^2\sigma_\theta^2\right] \\
&= \frac{4}{9}\Delta_\epsilon^2\chi_\epsilon(1 - \chi_\beta) + \frac{1}{2}\sigma_\theta^2\left[(\chi_\beta + \chi_\epsilon)\beta_H^2 - (\chi_\beta - \chi_\epsilon)\beta_L^2\right] + \frac{4}{9}\chi_\epsilon(1 - \chi_\beta)\Delta_\beta^2\overline{\theta}^2,
\end{aligned}$$

$$\begin{aligned}
\mathbb{E}(A_3|\epsilon_H) &= \frac{1}{2}\chi_\epsilon\left[\beta_H^2\sigma_\theta^2\right] + \frac{1}{2}(\chi_\beta - \chi_\epsilon)\left[\Delta_\epsilon^2 + \beta_H^2\sigma_\theta^2\right] \\
&\quad + \chi_\epsilon[\frac{8}{9}\Delta_\epsilon^2 + (\frac{2}{3}\beta_H^2 + \frac{1}{3}\beta_L^2)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2] + (1 - \chi_\epsilon - \chi_\beta)\left[\Delta_\epsilon^2 + \frac{1}{2}(\beta_H^2 + \beta_L^2)\sigma_\theta^2 + \Delta_\beta^2\overline{\theta}^2\right] \\
&\quad + \frac{1}{2}\chi_\epsilon\left[\frac{8}{9}\Delta_\epsilon^2 + (\frac{1}{3}\beta_H^2 + \frac{2}{3}\beta_L^2)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2\right] + \frac{1}{2}(\chi_\beta - \chi_\epsilon)\left[\Delta_\epsilon^2 + \beta_L^2\sigma_\theta^2\right].
\end{aligned}$$

It follows that we can write the expected trading profits for $\epsilon-$informed traders conditional on $\epsilon = \epsilon_H$ as:

$$\begin{aligned}
\pi_{\epsilon_H} &= \Delta_\epsilon + \overline{\beta}\overline{\theta} - \frac{2}{3}\chi_\epsilon\Delta_\epsilon - \overline{\theta}\overline{\beta} - \frac{2}{3}\chi_\epsilon\Delta_\beta\overline{\theta} - \frac{\gamma}{2}\mathbb{E}(A_2|\Delta_\epsilon) + \frac{\gamma}{2}\mathbb{E}(A_3|\Delta_\epsilon)e_0 \\
&= \Delta_\epsilon(1 - \frac{2}{3}\chi_\epsilon) - \frac{2}{3}\chi_\epsilon\Delta_\beta\overline{\theta} - \frac{2\gamma}{9}\chi_\epsilon(1 - \chi_\beta)\Delta_\epsilon^2 - \frac{\gamma}{4}\sigma_\theta^2\left[(\chi_\beta + \chi_\epsilon)\beta_H^2 - (\chi_\beta - \chi_\epsilon)\beta_L^2\right] \\
&\quad - \frac{2\gamma}{9}\chi_\epsilon(1 - \chi_\beta)\Delta_\beta^2\overline{\theta}^2 + \frac{\gamma}{2}\mathbb{E}(A_3|\Delta_\epsilon)e_0.
\end{aligned}$$

(ii) When $\epsilon = \epsilon_L$, $X \in (-1 - \chi_\epsilon - \chi_\beta, 1 - \chi_\epsilon + \chi_\beta)$.

$$\begin{aligned}
\mathbb{E}(A_1|\epsilon_L) &= \frac{1}{2}(\chi_\beta - \chi_\epsilon)\left[(\overline{\beta} + \Delta_\beta)\overline{\theta}\right] + \frac{1}{2}\chi_\epsilon[\frac{1}{3}\Delta_\epsilon + (\overline{\beta} + \frac{1}{3}\Delta_\beta)\overline{\theta}] + (1 - \chi_\epsilon - \chi_\beta)\overline{\beta}\overline{\theta} \\
&\quad + \chi_\epsilon[-\frac{1}{3}\Delta_\epsilon + (\overline{\beta} - \frac{1}{3}\Delta_\beta)\overline{\theta}] + \frac{1}{2}(\chi_\beta - \chi_\epsilon)(\overline{\beta} - \Delta_\beta)\overline{\theta} + \frac{1}{2}\chi_\epsilon\left[-\Delta_\epsilon + (\overline{\beta} - \Delta_\beta)\overline{\theta}\right] \\
&= -\frac{2}{3}\chi_\epsilon\Delta_\epsilon + \overline{\beta}\overline{\theta} - \frac{2}{3}\chi_\epsilon\Delta_\beta\overline{\theta},
\end{aligned}$$

$$\begin{aligned}
E(A_2|\epsilon_L) &= \frac{1}{2}(\chi_\beta - \chi_\epsilon)\left[\Delta_\epsilon^2 + \beta_H^2\sigma_\theta^2\right] + \frac{1}{2}\chi_\epsilon[\frac{8}{9}\Delta_\epsilon^2 + (\frac{2}{3}\beta_H^2 + \frac{1}{3}\beta_L^2)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2] \cdot (1 - \chi_\beta) \\
&\quad + \chi_\epsilon[\frac{8}{9}\Delta_\epsilon^2 + (\frac{1}{3}\beta_H^2 + \frac{2}{3}\beta_L^2)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2] \cdot (-1 + \chi_\beta) \\
&\quad - \frac{1}{2}(\chi_\beta - \chi_\epsilon)\left[\Delta_\epsilon^2 + \beta_L^2\sigma_\theta^2\right] + \frac{1}{2}\chi_\epsilon\left[\beta_L^2\sigma_\theta^2\right] \cdot (-1 - \chi_\beta) \\
&= -\frac{4}{9}\Delta_\epsilon\chi_\epsilon(1 - \chi_\beta) - \frac{1}{2}\sigma_\theta^2\left[(\chi_\beta + \chi_\epsilon)\beta_L^2 - (\chi_\beta - \chi_\epsilon)\beta_H^2\right] - \frac{4}{9}\chi_\epsilon(1 - \chi_\beta)\Delta_\beta^2\overline{\theta}^2,
\end{aligned}$$

$$\begin{aligned}
E(A_3|\epsilon_L) &= \frac{1}{2}(\chi_\beta - \chi_\epsilon)\left[\Delta_\epsilon^2 + \beta_H^2\sigma_\theta^2\right] + \frac{1}{2}\chi_\epsilon[\frac{8}{9}\Delta_\epsilon^2 + (\frac{2}{3}\beta_H^2 + \frac{1}{3}\beta_L^2)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2] \\
&\quad + (1 - \chi_\epsilon - \chi_\beta)\left[\Delta_\epsilon^2 + \frac{1}{2}(\beta_H^2 + \beta_L^2)\sigma_\theta^2 + \Delta_\beta^2\overline{\theta}^2\right] + \chi_\epsilon[\frac{8}{9}\Delta_\epsilon^2 + (\frac{1}{3}\beta_H^2 + \frac{2}{3}\beta_L^2)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2] \\
&\quad + \frac{1}{2}(\chi_\beta - \chi_\epsilon)\left[\Delta_\epsilon^2 + \beta_L^2\sigma_\theta^2\right] + \frac{1}{2}\chi_\epsilon\left[\beta_L^2\sigma_\theta^2\right].
\end{aligned}$$

It follows that we can write the expected trading profits for $\epsilon-$informed traders conditional on $\epsilon = \epsilon_L$ as:

$$\begin{aligned}
\pi_{\epsilon_L} &= \Delta_\epsilon - \overline{\beta}\overline{\theta} - \frac{2}{3}\chi_\epsilon\Delta_\epsilon + \overline{\beta}\overline{\theta} - \frac{2}{3}\chi_\epsilon\Delta_\beta\overline{\theta} + \frac{\gamma}{2}\mathbb{E}(A_2|-\Delta_\epsilon) - \frac{\gamma}{2}\mathbb{E}(A_3|-\Delta_\epsilon)e_0 \\
&= \Delta_\epsilon(1 - \frac{2}{3}\chi_\epsilon) - \frac{2}{3}\chi_\epsilon\Delta_\beta\overline{\theta} - \frac{2\gamma}{9}\Delta_\epsilon^2\chi_\epsilon(1 - \chi_\beta) - \frac{\gamma}{4}\sigma_\theta^2\left[(\chi_\beta + \chi_\epsilon)\beta_L^2 - (\chi_\beta - \chi_\epsilon)\beta_H^2\right] \\
&\quad - \frac{2\gamma}{9}\chi_\epsilon(1 - \chi_\beta)\Delta_\beta^2\overline{\theta}^2 - \frac{\gamma}{2}\mathbb{E}(A_3|-\Delta_\epsilon)e_0.
\end{aligned}$$

The sufficient condition to ensure positive conditional trading profits of ϵ-informed investors is:

$$\chi_\epsilon\frac{2}{3}\frac{\Delta_\beta\overline{\theta}}{\Delta_\epsilon} + \frac{2}{3}\chi_\epsilon < 1 \text{ and } \gamma \text{ is sufficiently low.}$$

For the case $\overline{\theta} < 0$, we can show that the equivalent condition is:

$$\chi_\epsilon\frac{2}{3}\frac{\Delta_\beta|\overline{\theta}|}{\Delta_\epsilon} + \frac{2}{3}\chi_\epsilon < 1 \text{ and } \gamma \text{ is sufficiently low.}$$

OA.4.2 Trading Profits for β-informed investors

In this subsection, we calculate the trading profit of β-informed investors and characterize the sufficient conditions to ensure that β-informed investors use the same trading policy as in Theorem 1. We discuss the cases with $\chi_\epsilon > \chi_\beta$ and $\chi_\epsilon \leq \chi_\beta$ separately as follows. We first focus on the case with $\overline{\theta} > 0$.

Case 1: $\chi_\epsilon > \chi_\beta$

Trading profits depend on $\mathbb{E}(p|\beta)$ and thus $\mathbb{E}(A_1|\beta)$, $\mathbb{E}(A_2|\beta)$, and $\mathbb{E}(A_3|\beta)$, which are given as:

(i) When $\beta = \beta_H$, $X \in (-1 - \chi_\epsilon + \chi_\beta, 1 + \chi_\epsilon + \chi_\beta)$.

$$\begin{aligned}
\mathbb{E}(A_1|\beta_H) &= \frac{1}{2}\chi_\beta \left[\Delta_\epsilon + (\overline{\beta} + \Delta_\beta)\overline{\theta}\right] + \frac{1}{2}(\chi_\epsilon - \chi_\beta)\left[\Delta_\epsilon + \overline{\beta}\overline{\theta}\ \right] + \chi_\beta\left[\frac{1}{3}\Delta_\epsilon + (\overline{\beta} + \frac{1}{3}\Delta_\beta)\overline{\theta}\ \right] \\
&\quad + (1 - \chi_\epsilon - \chi_\beta)\overline{\beta}\overline{\theta} + \frac{1}{2}\chi_\beta\left[-\frac{1}{3}\Delta_\epsilon + (\overline{\beta} - \frac{1}{3}\Delta_\beta)\overline{\theta}\right] + \frac{1}{2}(\chi_\epsilon - \chi_\beta)\left[-\Delta_\epsilon + \overline{\beta}\overline{\theta}\ \ \right] \\
&= \frac{2}{3}\chi_\beta\Delta_\epsilon + \overline{\beta}\overline{\theta} + \frac{2}{3}\chi_\beta\Delta_\beta\overline{\theta},
\end{aligned}$$

$$\begin{aligned}
\mathbb{E}(A_2|\beta_H) &= \frac{1}{2}\chi_\beta \cdot \left[\beta_H^2\sigma_\theta^2\right] \cdot (1 + \chi_\epsilon) + \frac{1}{2}(\chi_\epsilon - \chi_\beta) \cdot \left[\frac{1}{2}(\beta_H^2 + \beta_L^2)\sigma_\theta^2 + \Delta_\beta^2\overline{\theta}^2\ \right] \\
&\quad + \chi_\beta \cdot \left[\frac{8}{9}\Delta_\epsilon^2 + (\frac{2}{3}\beta_H^2 + \frac{1}{3}\beta_L^2)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2\right] \cdot (1 - \chi_\epsilon) \\
&\quad + \frac{1}{2}\chi_\beta \cdot \left[\frac{8}{9}\Delta_\epsilon^2 + (\frac{1}{3}\beta_H^2 + \frac{2}{3}\beta_L^2)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2\right] \cdot (-1 + \chi_\epsilon) \\
&\quad - \frac{1}{2}(\chi_\epsilon - \chi_\beta) \cdot \left[\frac{1}{2}(\beta_H^2 + \beta_L^2)\sigma_\theta^2 + \Delta_\beta^2\overline{\theta}^2\ \right] = \frac{4}{9}\chi_\beta \cdot (1 - \chi_\epsilon)\Delta_\epsilon^2 + \sigma_\theta^2\beta_H^2\chi_\beta + \frac{4}{9}\Delta_\beta^2\overline{\theta}^2\chi_\beta(1 - \chi_\epsilon),
\end{aligned}$$

$$\begin{aligned}
\mathbb{E}(A_3|\beta_H) &= \frac{1}{2}\chi_\beta \cdot \left[\beta_H^2\sigma_\theta^2\right] + \frac{1}{2}(\chi_\epsilon - \chi_\beta) \cdot \left[\frac{1}{2}(\beta_H^2 + \beta_L^2)\sigma_\theta^2 + \Delta_\beta^2\overline{\theta}^2\ \right] \\
&\quad + \chi_\beta \cdot \left[\frac{8}{9}\Delta_\epsilon^2 + (\frac{2}{3}\beta_H^2 + \frac{1}{3}\beta_L^2)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2\right] + (1 - \chi_\epsilon - \chi_\beta) \cdot \left[\Delta_\epsilon^2 + \frac{1}{2}(\beta_H^2 + \beta_L^2)\sigma_\theta^2 + \Delta_\beta^2\overline{\theta}^2\right] \\
&\quad + \frac{1}{2}\chi_\beta \cdot \left[\frac{8}{9}\Delta_\epsilon^2 + (\frac{1}{3}\beta_H^2 + \frac{2}{3}\beta_L^2)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2\right] + \frac{1}{2}(\chi_\epsilon - \chi_\beta) \cdot \left[\frac{1}{2}(\beta_H^2 + \beta_L^2)\sigma_\theta^2 + \Delta_\beta^2\overline{\theta}^2\ \right].
\end{aligned}$$

It follows that we can write the expected trading profits for $\beta-$informed traders conditional on $\beta = \beta_H$ as:

$$\begin{aligned}
\pi_{\beta_H} &= \beta_H\overline{\theta} - \frac{2}{3}\chi_\beta\Delta_\epsilon + \overline{\beta}\overline{\theta} + \frac{2}{3}\chi_\beta\Delta_\beta\overline{\theta} - \frac{\gamma}{2}\mathbb{E}(A_2|\beta_H) + \frac{\gamma}{2}\mathbb{E}(A_3|\beta_H)e_0 \\
&= \Delta_\beta\overline{\theta}(1 - \frac{2\chi_\beta}{3}) - \frac{2}{3}\chi_\beta\Delta_\epsilon - \frac{2\gamma}{9}\chi_\beta \cdot (1 - \chi_\epsilon)\Delta_\epsilon^2 - \frac{\gamma}{2}\sigma_\theta^2\beta_H^2\chi_\beta - \frac{2\gamma}{9}\Delta_\beta^2\overline{\theta}^2\chi_\beta(1 - \chi_\epsilon) \\
&\quad + \frac{\gamma}{2}\mathbb{E}(A_3|\beta_H)e_0.
\end{aligned}$$

(ii) When $\beta = \beta_L$, $X \in (-1 - \chi_\epsilon - \chi_\beta, 1 + \chi_\epsilon - \chi_\beta)$.

$$\begin{aligned}\mathbb{E}(A_1|\beta_L) &= \frac{1}{2}(\chi_\epsilon - \chi_\beta)\left[\Delta_\epsilon + \overline{\beta}\overline{\theta}\right] + \frac{1}{2}\chi_\beta\left[\frac{1}{3}\Delta_\epsilon + (\overline{\beta} + \frac{1}{3}\Delta_\beta)\overline{\theta}\right] + (1 - \chi_\epsilon - \chi_\beta)\overline{\beta}\overline{\theta} \\ &\quad + \chi_\beta\left[-\frac{1}{3}\Delta_\epsilon + (\overline{\beta} - \frac{1}{3}\Delta_\beta)\overline{\theta}\right] + \frac{1}{2}(\chi_\epsilon - \chi_\beta)\left[-\Delta_\epsilon + \overline{\beta}\overline{\theta}\ \right] + \frac{1}{2}\chi_\beta\left[-\Delta_\epsilon + (\overline{\beta} - \Delta_\beta)\overline{\theta}\right] \\ &= -\frac{2}{3}\chi_\beta\Delta_\epsilon + \overline{\beta}\overline{\theta} - \frac{2}{3}\chi_\beta\Delta_\beta\overline{\theta},\end{aligned}$$

$$\begin{aligned}\mathbb{E}(A_2|\beta_L) &= \frac{1}{2}(\chi_\epsilon - \chi_\beta)[\frac{1}{2}(\beta_H^2 + \beta_L^2)\sigma_\theta^2 + \Delta_\beta^2\overline{\theta}^2] + \frac{1}{2}\chi_\beta \cdot \left[\frac{8}{9}\Delta_\epsilon^2 + (\frac{2}{3}\beta_H^2 + \frac{1}{3}\beta_L^2)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2\right] \cdot (1 - \chi_\epsilon) \\ &\quad + \chi_\beta \cdot \left[\frac{8}{9}\Delta_\epsilon^2 + (\frac{1}{3}\beta_H^2 + \frac{2}{3}\beta_L^2)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2\right] \cdot (-1 + \chi_\epsilon) - \frac{1}{2}(\chi_\epsilon - \chi_\beta) \cdot \left[\frac{1}{2}(\beta_H^2 + \beta_L^2)\sigma_\theta^2 + \Delta_\beta^2\overline{\theta}^2\right] \\ &\quad + \frac{1}{2}\chi_\beta \cdot \left[\beta_L^2\sigma_\theta^2\ \right] \cdot (-1 - \chi_\epsilon) = -\frac{4}{9}(1 - \chi_\epsilon)\chi_\beta\Delta_\epsilon^2 - \sigma_\theta^2\beta_L^2\chi_\beta - \frac{4}{9}\Delta_\beta^2\overline{\theta}^2\chi_\beta(1 - \chi_\epsilon),\end{aligned}$$

$$\begin{aligned}\mathbb{E}(A_3|\beta_L) &= \frac{1}{2}(\chi_\epsilon - \chi_\beta)[\frac{1}{2}(\beta_H^2 + \beta_L^2)\sigma_\theta^2 + \Delta_\beta^2\overline{\theta}^2] + \frac{1}{2}\chi_\beta \cdot \left[\frac{8}{9}\Delta_\epsilon^2 + (\frac{2}{3}\beta_H^2 + \frac{1}{3}\beta_L^2)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2\right] \\ &\quad + (1 - \chi_\epsilon - \chi_\beta) \cdot \left[\Delta_\epsilon^2 + \frac{1}{2}(\beta_H^2 + \beta_L^2)\sigma_\theta^2 + \Delta_\beta^2\overline{\theta}^2\right] + \chi_\beta \cdot \left[\frac{8}{9}\Delta_\epsilon^2 + (\frac{1}{3}\beta_H^2 + \frac{2}{3}\beta_L^2)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2\right] \\ &\quad + \frac{1}{2}(\chi_\epsilon - \chi_\beta) \cdot \left[\frac{1}{2}(\beta_H^2 + \beta_L^2)\sigma_\theta^2 + \Delta_\beta^2\overline{\theta}^2\right] + \frac{1}{2}\chi_\beta \cdot \left[\beta_L^2\sigma_\theta^2\ \right].\end{aligned}$$

It follows that we can write the expected trading profits for $\beta-$informed traders conditional on $\beta = \beta_L$ as:

$$\begin{aligned}\pi_{\beta_L} &= -\beta_L\overline{\theta} - \frac{2}{3}\chi_\beta\Delta_\epsilon + \overline{\beta}\overline{\theta} - \frac{2}{3}\chi_\beta\Delta_\beta\overline{\theta} + \frac{\gamma}{2}\mathbb{E}(A_2|\beta_L) - \frac{\gamma}{2}\mathbb{E}(A_3|\beta_L)e_0 \\ &= (1 - \frac{2}{3}\chi_\beta)\Delta_\beta\overline{\theta} - \frac{2}{3}\chi_\beta\Delta_\epsilon - \frac{2\gamma}{9}(1 - \chi_\epsilon)\chi_\beta\Delta_\epsilon^2 - \frac{\gamma}{2}\chi_\beta\sigma_\theta^2\beta_L^2 - \frac{2\gamma}{9}\Delta_\beta^2\overline{\theta}^2\chi_\beta(1 - \chi_\epsilon) - \frac{\gamma}{2}\mathbb{E}(A_3|\beta_L)e_0.\end{aligned}$$

The sufficient condition to ensure positive conditional trading profits of β-informed investors is:

$$\chi_\beta\frac{2}{3}\frac{\Delta_\epsilon}{\Delta_\beta\overline{\theta}} + \frac{2}{3}\chi_\beta < 1 \text{ and } \gamma \text{ is sufficient low.}$$

For the case $\overline{\theta} < 0$, we can show that the equivalent condition is:

$$\chi_\beta\frac{2}{3}\frac{\Delta_\epsilon}{\Delta_\beta|\overline{\theta}|} + \frac{2}{3}\chi_\beta < 1 \text{ and } \gamma \text{ is sufficient low.}$$

Case 2: $\chi_\epsilon \leq \chi_\beta$

Trading profits depend on $\mathbb{E}(p|\beta)$ and thus $\mathbb{E}(A_1|\beta)$, $\mathbb{E}(A_2|\beta)$ and $\mathbb{E}(A_3|\beta)$, which are given as:

(i) When $\beta = \beta_H$, $X \in (-1 - \chi_\epsilon + \chi_\beta, 1 + \chi_\epsilon + \chi_\beta)$.

$$\begin{aligned}
\mathbb{E}(A_1|\beta_H) &= \frac{1}{2}\chi_\epsilon\left[\Delta_\epsilon + (\bar{\beta} + \Delta_\beta)\bar{\theta}\right] + (\chi_\beta - \chi_\epsilon)\left[(\bar{\beta} + \Delta_\beta)\bar{\theta}\right] + \chi_\epsilon\left[\frac{1}{3}\Delta_\epsilon + (\bar{\beta} + \frac{1}{3}\Delta_\beta)\bar{\theta}\ \right] \\
&+(1 - \chi_\epsilon - \chi_\beta)\bar{\beta}\bar{\theta} + \frac{1}{2}\chi_\epsilon\left[-\frac{1}{3}\Delta_\epsilon + (\bar{\beta} - \frac{1}{3}\Delta_\beta)\bar{\theta}\right] = \frac{2}{3}\chi_\epsilon\Delta_\epsilon + \bar{\beta}\bar{\theta} + (\chi_\beta - \frac{1}{3}\chi_\epsilon)\Delta_\beta\bar{\theta},
\end{aligned}$$

$$\begin{aligned}
\mathbb{E}(A_2|\beta_H) &= \frac{1}{2}\chi_\epsilon\left[\beta_H^2\sigma_\theta^2\right]\cdot(1 + \chi_\beta) + (\chi_\beta - \chi_\epsilon)\left[\Delta_\epsilon^2 + \beta_H^2\sigma_\theta^2\ \right] \\
&+\chi_\epsilon\left[\frac{8}{9}\Delta_\epsilon^2 + (\frac{2}{3}\beta_H^2 + \frac{1}{3}\beta_L^2)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\bar{\theta}^2\ \right]\cdot(1 - \chi_\beta) \\
&+\frac{1}{2}\chi_\epsilon\left[\frac{8}{9}\Delta_\epsilon^2 + (\frac{1}{3}\beta_H^2 + \frac{2}{3}\beta_L^2)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\bar{\theta}^2\right](-1 + \chi_\beta) \\
&= \Delta_\epsilon^2\left[(\chi_\beta - \chi_\epsilon) + \frac{4}{9}\chi_\epsilon(1 - \chi_\beta)\right] + \chi_\beta\beta_H^2\sigma_\theta^2 + \frac{4}{9}\chi_\epsilon(1 - \chi_\beta)\Delta_\beta^2\bar{\theta}^2,
\end{aligned}$$

$$\begin{aligned}
\mathbb{E}(A_3|\beta_H) &= \frac{1}{2}\chi_\epsilon\left[\beta_H^2\sigma_\theta^2\right] + (\chi_\beta - \chi_\epsilon)\left[\Delta_\epsilon^2 + \beta_H^2\sigma_\theta^2\ \right] + \chi_\epsilon\left[\frac{8}{9}\Delta_\epsilon^2 + (\frac{2}{3}\beta_H^2 + \frac{1}{3}\beta_L^2)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\bar{\theta}^2\ \right] \\
&+(1 - \chi_\epsilon - \chi_\beta)\left[\Delta_\epsilon^2 + \frac{1}{2}(\beta_H^2 + \beta_L^2)\sigma_\theta^2 + \Delta_\beta^2\bar{\theta}^2\right] + \frac{1}{2}\chi_\epsilon\left[\frac{8}{9}\Delta_\epsilon^2 + (\frac{1}{3}\beta_H^2 + \frac{2}{3}\beta_L^2)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\bar{\theta}^2\right].
\end{aligned}$$

It follows that we can write the expected trading profits for $\beta-$informed traders conditional on $\beta = \beta_H$ as:

$$\begin{aligned}
\pi_{\beta_H} &= \beta_H\bar{\theta} - \frac{2}{3}\chi_\epsilon\Delta_\epsilon - \bar{\beta}\bar{\theta} - (\chi_\beta - \frac{1}{3}\chi_\epsilon)\Delta_\beta\bar{\theta} - \frac{\gamma}{2}E(A_2|\beta_H) + \frac{\gamma}{2}E(A_3|\beta_H)e_0 \\
&= \Delta_\beta\bar{\theta}(1 - \chi_\beta + \frac{1}{3}\chi_\epsilon) - \frac{2}{3}\chi_\epsilon\Delta_\epsilon - \frac{\gamma}{2}\Delta_\epsilon^2\left[(\chi_\beta - \chi_\epsilon) + \frac{4}{9}\chi_\epsilon(1 - \chi_\beta)\right] \\
&\quad -\frac{\gamma}{2}\chi_\beta\beta_H^2\sigma_\theta^2 - \frac{2\gamma}{9}\chi_\epsilon(1 - \chi_\beta)\Delta_\beta^2\bar{\theta}^2 + \frac{\gamma}{2}\mathbb{E}(A_3|\beta_H)e_0.
\end{aligned}$$

(ii) When $\beta = \beta_L$, $X \in (-1 - \chi_\epsilon - \chi_\beta, 1 + \chi_\epsilon - \chi_\beta)$.

$$\begin{aligned}\mathbb{E}(A_1|\beta_L) &= \frac{1}{2}\chi_\epsilon\left[\frac{1}{3}\Delta_\epsilon + (\overline{\beta} + \frac{1}{3}\Delta_\beta)\overline{\theta}\right] + (1 - \chi_\epsilon - \chi_\beta)\overline{\beta}\overline{\theta} \\ &+\chi_\epsilon\left[-\frac{1}{3}\Delta_\epsilon + (\overline{\beta} - \frac{1}{3}\Delta_\beta)\overline{\theta}\right] + (\chi_\beta - \chi_\epsilon)\left[(\overline{\beta} - \beta_\Delta)\overline{\theta}\right] \\ &+\frac{1}{2}\chi_\epsilon\left[-\Delta_\epsilon + (\overline{\beta} - \Delta_\beta)\overline{\theta}\right] = -\frac{2\chi_\epsilon}{3}\Delta_\epsilon + \overline{\beta}\overline{\theta} - (\chi_\beta - \frac{1}{3}\chi_\epsilon)\Delta_\beta\overline{\theta},\end{aligned}$$

$$\begin{aligned}\mathbb{E}(A_2|\beta_L) &= \frac{1}{2}\chi_\epsilon\left[\frac{8}{9}\Delta_\epsilon^2 + (\frac{2}{3}\beta_H^2 + \frac{1}{3}\beta_L^2)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2\right]\cdot(1 - \chi_\beta) \\ &+\chi_\epsilon\left[\frac{8}{9}\Delta_\epsilon^2 + (\frac{1}{3}\beta_H^2 + \frac{2}{3}\beta_L^2)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2\right]\cdot(-1 + \chi_\beta) \\ &-(\chi_\beta - \chi_\epsilon)\left[\Delta_\epsilon^2 + \beta_L^2\sigma_\theta^2\right] + \frac{1}{2}\chi_\epsilon\beta_L^2\sigma_\theta^2 \cdot (-1 - \chi_\beta) \\ &= -\Delta_\epsilon^2\left[(\chi_\beta - \chi_\epsilon) + \frac{4}{9}\chi_\epsilon(1 - \chi_\beta)\right] - \chi_\beta\beta_L^2\sigma_\theta^2 - \frac{4}{9}\chi_\epsilon(1 - \chi_\beta)\Delta_\beta^2\overline{\theta}^2,\end{aligned}$$

$$\begin{aligned}\mathbb{E}(A_3|\beta_L) &= \frac{1}{2}\chi_\epsilon\left[\frac{8}{9}\Delta_\epsilon^2 + (\frac{2}{3}\beta_H^2 + \frac{1}{3}\beta_L^2)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2\right] + (1 - \chi_\epsilon - \chi_\beta)\left[\Delta_\epsilon^2 + \frac{1}{2}(\beta_H^2 + \beta_L^2)\sigma_\theta^2 + \Delta_\beta^2\overline{\theta}^2\right] \\ &+\chi_\epsilon\left[\frac{8}{9}\Delta_\epsilon^2 + (\frac{1}{3}\beta_H^2 + \frac{2}{3}\beta_L^2)\sigma_\theta^2 + \frac{8}{9}\Delta_\beta^2\overline{\theta}^2\right] + (\chi_\beta - \chi_\epsilon)\left[\Delta_\epsilon^2 + \beta_L^2\sigma_\theta^2\right] + \frac{1}{2}\chi_\epsilon\beta_L^2\sigma_\theta^2 .\end{aligned}$$

It follows that we can write the expected trading profits for $\beta-$informed traders conditional on $\beta = \beta_L$ as:

$$\begin{aligned}\pi_{\beta_L} &= -\beta_L\overline{\theta} - \frac{2\chi_\epsilon}{3}\Delta_\epsilon + \overline{\beta}\overline{\theta} - (\chi_\beta - \frac{1}{3}\chi_\epsilon)\Delta_\beta\overline{\theta} + \frac{\gamma}{2}\mathbb{E}(A_2|\beta_L) - \frac{\gamma}{2}\mathbb{E}(A_3|\beta_L)e_0 \\ &= \Delta_\beta\overline{\theta}[1 - \chi_\beta + \frac{1}{3}\chi_\epsilon)] - \frac{2}{3}\chi_\epsilon\Delta_\epsilon - \frac{2\gamma}{9}\chi_\epsilon(1 - \chi_\beta)\Delta_\beta^2\overline{\theta}^2 - \frac{\gamma}{2}\chi_\beta\beta_L^2\sigma_\theta^2 \\ &-\frac{\gamma}{2}\Delta_\epsilon^2\left[(\chi_\beta - \chi_\epsilon) + \frac{4}{9}\chi_\epsilon(1 - \chi_\beta)\right] - \frac{\gamma}{2}\mathbb{E}(A_3|\beta_L)e_0.\end{aligned}$$

The sufficient condition to ensure positive conditional trading profits of β-informed investors is:

$$\chi_\epsilon\frac{2}{3}\frac{\Delta_\epsilon}{\Delta_\beta\overline{\theta}} - \frac{1}{3}\chi_\epsilon + \chi_\beta < 1 \text{ and } \gamma \text{ is sufficiently low.}$$

For the case $\overline{\theta} < 0$, we can show that the equivalent condition is:

$$\chi_\epsilon\frac{2}{3}\frac{\Delta_\epsilon}{\Delta_\beta|\overline{\theta}|} - \frac{1}{3}\chi_\epsilon + \chi_\beta < 1 \text{ and } \gamma \text{ is sufficiently low.}$$

OA.4.3 Expected Asset Price

In this subsection, we discuss how introducing the positive net supply affects the expected asset price.

The expected asset price in this extension is given as:

$$\mathbb{E}[p] = \mathbb{E}[v] + \frac{\gamma}{2}\left(X - e_0\right)\mathbb{E}\left[\operatorname{Var}(v|X)\right]. \tag{OA.4.3}$$

Compared to our main model, $\mathbb{E}[p]$ contains an additional term, $-\frac{\gamma}{2}e_0\mathbb{E}\left[\operatorname{Var}(v|X)\right]$. So, we have:

$$\mathbb{E}[v-p] = -\operatorname{Cov}(\delta, X) + e_0\mathbb{E}\left[\delta\right]. \tag{OA.4.4}$$

Taking the results in Corollaries 2 and 3 together, we have

$$\mathbb{E}\left[v-p\right] = \begin{cases} \left[\left(\overline{\beta}^2 + \Delta_\beta^2\right)\sigma_\theta^2 + \Delta_\epsilon^2(1-\chi_\epsilon+\frac{\chi_\beta}{3}) + \Delta_\beta^2\overline{\theta}^2(1-\frac{2\chi_\beta}{3})\right]e_0 - \operatorname{sign}(\overline{\theta})\gamma\Delta_\beta\overline{\beta}\sigma_\theta^2\chi_\beta & \text{if } \chi_\epsilon > \chi_\beta \\ \left[\left(\overline{\beta}^2 + \Delta_\beta^2\right)\sigma_\theta^2 + \Delta_\epsilon^2\left(1-\frac{2\chi_\epsilon}{3}\right) + \Delta_\beta^2\overline{\theta}^2\left(1+\frac{\chi_\epsilon}{3}-\chi_\beta\right)\right]e_0 - \operatorname{sign}(\overline{\theta})\gamma\Delta_\beta\overline{\beta}\sigma_\theta^2\chi_\beta & \text{if } \chi_\epsilon \leq \chi_\beta. \end{cases} \tag{OA.4.5}$$

Given the expression of $\mathbb{E}\left[v-p\right]$ above, we can compute the effect of $\overline{\beta}$ on $\mathbb{E}\left[v-p\right]$ as follows:

$$\frac{d\mathbb{E}\left[v-p\right]}{d\overline{\beta}} = \gamma e_0\sigma_\theta^2\overline{\beta} - \operatorname{sign}(\overline{\theta})\gamma\Delta_\beta\sigma_\theta^2\chi_\beta. \tag{OA.4.6}$$

Examining Eq. (OA.4.6) shows that when $e_0\overline{\beta} - \Delta_\beta\chi_\beta < 0$, $\frac{d\mathbb{E}[v-p]}{d\overline{\beta}}$ is still negative for $\overline{\theta} > 0$. Otherwise, $\frac{d\mathbb{E}[v-p]}{d\overline{\beta}}$ becomes positive. In this sense, the results in Corollary 4 continue to hold when the net supply e_0 or $\overline{\beta}$ is sufficiently small.

OA.5: Smooth Pricing Function: A Numerical Analysis

Note that our equilibrium pricing function is stepwise linear, and the steps in the price schedule follow from binary payoff components and a uniform distribution of noise trading. To show the robustness of the key theoretical results, we consider an alternative specification which smooths out the step-wise pricing function by assuming a normal distribution for noise trades. Unfortunately this extension can only be solved numerically, but we show that it gives similar results as in the main model.[28]

We relax the assumption that noise trader demand z is uniformly distributed on $[-1, 1]$, and instead assume that it is drawn from a normal distribution with zero mean and standard deviation σ_z. As shown below, the fact that z is drawn from an unbounded distribution leads to a smooth pricing function with similar properties as in our baseline model. As before, the market maker sets the price equal to:

$$p(X) = \mathbb{E}[v|X] + \frac{\gamma}{2}\text{Var}(v|X)X. \tag{OA.5.1}$$

We denote the conditional probabilities for the event (ϵ, β) by $\phi_{HH} \equiv \Pr(\epsilon = \epsilon_H, \beta = \beta_H|X)$, $\phi_{HL} \equiv \Pr(\epsilon = \epsilon_H, \beta = \beta_L|X)$, $\phi_{LH} \equiv \Pr(\epsilon = \epsilon_L, \beta = \beta_H|X)$, and $\phi_{LL} \equiv \Pr(\epsilon = \epsilon_L, \beta = \beta_L|X)$. We can then use the Law of Iterated Expectations and write the market maker's expected payoff given X as:

$$\begin{aligned}\mathbb{E}[v|X] &= \mathbb{E}[\mathbb{E}[v|\epsilon, \beta, X]|X] = \mathbb{E}[\epsilon + \beta\overline{\theta}|X] \\ &= \phi_{HH}\left(\epsilon_H + \beta_H\overline{\theta}\right) + \phi_{HL}\left(\epsilon_H + \beta_L\overline{\theta}\right) + \phi_{LH}\left(\epsilon_L + \beta_H\overline{\theta}\right) + \phi_{LL}\left(\epsilon_L + \beta_L\overline{\theta}\right).\end{aligned}$$

---

[28] In Appendix OA.6, we present another extension where we endogenize traders' information acquisition of ϵ- and β-information within the model of Section 2. This setting can be interpreted as capturing long-run responses to changes in primitive model parameters, after traders are able to adjust how they gather information. In that setting, we assume that traders are ex ante identical. They can decide to acquire either ϵ-information or β-information, but not both. This learning technology is common in the literature (e.g., Goldstein and Yang, 2015; Brunnermeier et al., 2022).

Next, we apply the Law of Total Variance to express the conditional payoff variance as:

$$\begin{aligned}
\text{Var}(v|X) &= \mathbb{E}\left[\text{Var}(v|\epsilon,\beta,X)|X\right] + \text{Var}\left(\mathbb{E}[v|\epsilon,\beta,X]|X\right) \\
&= \mathbb{E}\left[\beta^2\sigma_\theta^2|X\right] + \text{Var}\left(\epsilon + \beta\overline{\theta}|X\right) \\
&= \left[(\phi_{HH}+\phi_{LH})\beta_H^2 + (\phi_{HL}+\phi_{LL})\beta_L^2\right]\sigma_\theta^2 + \\
&+ \phi_{HH}\left(\epsilon_H+\beta_H\overline{\theta}\right)^2 + \phi_{HL}\left(\epsilon_H+\beta_L\overline{\theta}\right)^2 + \phi_{LH}\left(\epsilon_L+\beta_H\overline{\theta}\right)^2 + \phi_{LL}\left(\epsilon_L+\beta_L\overline{\theta}\right)^2 - \mathbb{E}[v|X]^2.
\end{aligned}$$

In an equilibrium in which informed traders buy on a high and sell on a low signal, the conditional probabilities follow from Bayes' rule:

$$\begin{aligned}
\phi_{HH} &= \frac{\exp\left(-\frac{\left(X-(\chi_\epsilon+\chi_\beta)\right)^2}{2\sigma_z^2}\right)}{D}, \quad \phi_{HL} = \frac{\exp\left(-\frac{\left(X-(\chi_\epsilon-\chi_\beta)\right)^2}{2\sigma_z^2}\right)}{D}, \\
\phi_{LH} &= \frac{\exp\left(-\frac{\left(X-(-\chi_\epsilon+\chi_\beta)\right)^2}{2\sigma_z^2}\right)}{D}, \quad \phi_{LL} = \frac{\exp\left(-\frac{\left(X-(-\chi_\epsilon-\chi_\beta)\right)^2}{2\sigma_z^2}\right)}{D},
\end{aligned}$$

where

$$\begin{aligned}
D &\equiv \exp\left(-\frac{\left(X-(\chi_\epsilon+\chi_\beta)\right)^2}{2\sigma_z^2}\right) + \exp\left(-\frac{\left(X-(\chi_\epsilon-\chi_\beta)\right)^2}{2\sigma_z^2}\right) + \exp\left(-\frac{\left(X-(-\chi_\epsilon+\chi_\beta)\right)^2}{2\sigma_z^2}\right) \\
&+ \exp\left(-\frac{\left(X-(-\chi_\epsilon-\chi_\beta)\right)^2}{2\sigma_z^2}\right). \qquad \text{(OA.5.2)}
\end{aligned}$$

The conditional trading profits for informed traders are equal to the following.

1. Expected trading profits for ϵ-informed given $\epsilon = \epsilon_H$:

$$\pi_{\epsilon_H} = \epsilon_H + \overline{\overline{\theta}\beta} - \frac{1}{2}\mathbb{E}[p(X)|X = \chi_\epsilon + \chi_\beta + z] - \frac{1}{2}\mathbb{E}[p(X)|X = \chi_\epsilon - \chi_\beta + z]; \qquad \text{(OA.5.3)}$$

2. Expected trading profits for ϵ-informed given $\epsilon = \epsilon_L$:

$$\pi_{\epsilon_L} = \frac{1}{2}\mathbb{E}[p(X)|X = -\chi_\epsilon + \chi_\beta + z] + \frac{1}{2}\mathbb{E}[p(X)|X = -\chi_\epsilon - \chi_\beta + z] - \epsilon_L - \overline{\overline{\theta}\beta}; \qquad \text{(OA.5.4)}$$

3. Expected trading profits for β-informed given $\beta = \beta_H$:

$$\pi_{\beta_H} = \overline{\theta}\beta_H - \frac{1}{2}\mathbb{E}[p(X)|X = \chi_\epsilon + \chi_\beta + z] - \frac{1}{2}\mathbb{E}[p(X)|X = -\chi_\epsilon + \chi_\beta + z]; \qquad \text{(OA.5.5)}$$

4. Expected trading profits for β-informed given $\beta = \beta_L$:

$$\pi_{\beta_L} = \frac{1}{2}\mathbb{E}[p(X)|X = \chi_\epsilon - \chi_\beta + z] + \frac{1}{2}\mathbb{E}[p(X)|X = -\chi_\epsilon - \chi_\beta + z] - \beta_L\overline{\theta}. \qquad \text{(OA.5.6)}$$

We can then plug in the equilibrium stock price as a function of total order flow X and use numerical integration (over the distribution for z) to ensure that π_{ϵ_H}, π_{ϵ_L}, π_{β_H}, and π_{β_L} are positive for a given set of parameters. In this case, buying on a high and selling on a low signal is an equilibrium because it does not permit profitable deviations by an individual trader. For instance, selling on $\epsilon = \epsilon_H$ would lead to expected profits of:

$$-1 \times \left(\epsilon_H + \overline{\theta\beta} - \frac{1}{2}\mathbb{E}[p(X)|X = \chi_\epsilon + \chi_\beta + z] - \frac{1}{2}\mathbb{E}[p(X)|X = \chi_\epsilon - \chi_\beta + z]\right) = -\pi_{\epsilon_H} < 0. \quad \text{(OA.5.7)}$$

Following the same logic, the deviations of buying in response to ϵ_L and β_L, and selling in response to β_H lead to negative expected profits of $-\pi_{\epsilon_L}$, $-\pi_{\beta_L}$, and $-\pi_{\beta_H}$, respectively.

Similarly, if β-informed investors buy on a low signal and sell on a high signal, the conditional probabilities are equal to:

$$\begin{aligned}
\phi_{HL} &= \frac{\exp\left(-\frac{\left(X-(\chi_\epsilon+\chi_\beta)\right)^2}{2\sigma_z^2}\right)}{D}, \quad \phi_{HH} = \frac{\exp\left(-\frac{\left(X-(\chi_\epsilon-\chi_\beta)\right)^2}{2\sigma_z^2}\right)}{D}, \\
\phi_{LL} &= \frac{\exp\left(-\frac{\left(X-(-\chi_\epsilon+\chi_\beta)\right)^2}{2\sigma_z^2}\right)}{D}, \quad \phi_{LH} = \frac{\exp\left(-\frac{\left(X-(-\chi_\epsilon-\chi_\beta)\right)^2}{2\sigma_z^2}\right)}{D},
\end{aligned}$$

where D is again as defined in Eq. (OA.5.2).

As before, the conditional trading profits for informed traders are equal to the following.

1. Expected trading profits for ϵ-informed given $\epsilon = \epsilon_H$:

$$\pi_{\epsilon_H} = \epsilon_H + \overline{\theta\beta} - \frac{1}{2}\mathbb{E}[p(X)|X = \chi_\epsilon + \chi_\beta + z] - \frac{1}{2}\mathbb{E}[p(X)|X = \chi_\epsilon - \chi_\beta + z]; \quad \text{(OA.5.8)}$$

2. Expected trading profits for ϵ-informed given $\epsilon = \epsilon_L$:

$$\pi_{\epsilon_L} = \frac{1}{2}\mathbb{E}[p(X)|X = -\chi_\epsilon + \chi_\beta + z] + \frac{1}{2}\mathbb{E}[p(X)|X = -\chi_\epsilon - \chi_\beta + z] - \epsilon_L - \overline{\theta\beta}; \quad \text{(OA.5.9)}$$

3. Expected trading profits for β-informed given $\beta = \beta_H$:

$$\pi_{\beta_H} = \frac{1}{2}\mathbb{E}[p(X)|X = \chi_\epsilon - \chi_\beta + z] - \frac{1}{2}\mathbb{E}[p(X)|X = -\chi_\epsilon - \chi_\beta + z] - \overline{\theta}\beta_H; \quad \text{(OA.5.10)}$$

4. Expected trading profits for β-informed given $\beta = \beta_L$:

$$\pi_{\beta_L} = \beta_L\overline{\theta} - \frac{1}{2}\mathbb{E}[p(X)|X = \chi_\epsilon + \chi_\beta + z] - \frac{1}{2}\mathbb{E}[p(X)|X = -\chi_\epsilon + \chi_\beta + z]. \quad \text{(OA.5.11)}$$

We can use numerical integration (over the distribution for z) to ensure that π_{ϵ_H}, π_{ϵ_L}, π_{β_H}, and π_{β_L} are positive for a given set of parameters. As before, this implies that there are no profitable deviations for an individual trader and confirms that the conjectured trading strategies are optimal.

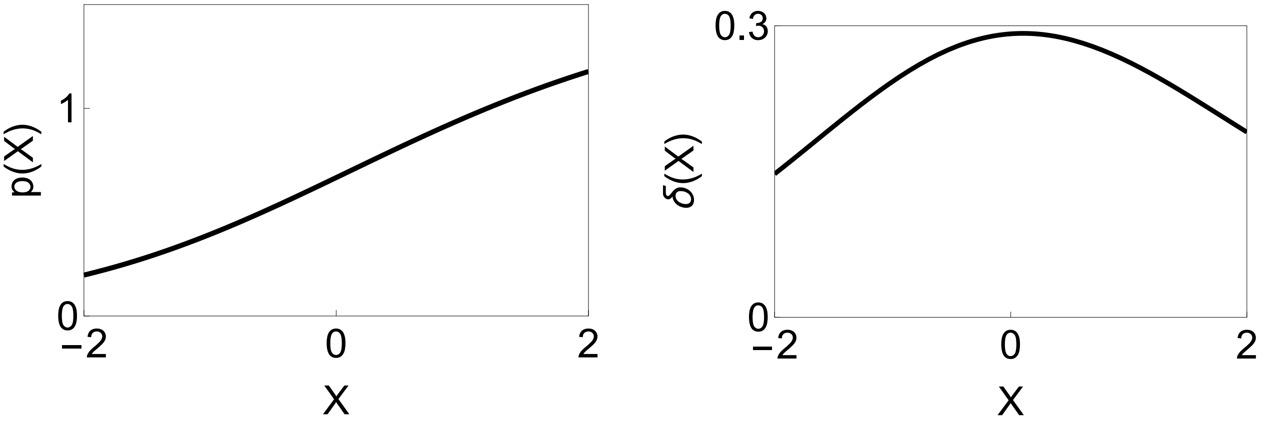
Figure OA.5.1 displays the equilibrium stock price $p(X)$ and the slope $\delta(X) \equiv dp(X)/dX$ as a function of total order flow X. Panel (a) displays a setting where β-informed buy on a high signal and sell on a low signal because the factor mean $\overline{\theta}$ is positive. In this case, we can confirm the result from before that $\delta(X)$ is higher for $X > 0$. Panel (b) considers the alternative setting where $\overline{\theta}$ is negative so that $\beta-$informed investors buy on a low signal and sell on a high signal. In this case, $\delta(X)$ is higher for $X < 0$.[29] Figure OA.5.2 plots $\text{Cov}\left(\delta, X\right)$ against different values of σ_z. The figure supports our central result in Corollary 3 that the covariance between δ and X is positive (negative) if the factor mean $\bar{\theta}$ is positive (negative).

---

[29] For both scenarios, we have verified that expected trading profits for both types of informed traders are positive.

Figure OA.5.1: This figure plots the equilibrium price function and the slope of the price schedule as functions of total order flow X under the assumption that noise trader demand is drawn from a Normal distribution with mean zero and unit variance. Parameters: $\overline{\beta} = 1, \Delta_\beta = \frac{1}{2}, \Delta_\epsilon = \frac{1}{4}, \chi_\epsilon = \frac{2}{3}, \sigma_\theta = \frac{1}{4}$, and $\gamma = \frac{3}{4}$. We set $\sigma_z = 1$ to keep the first two moments of z identical to the baseline model.

(a) $\overline{\theta} = \frac{2}{3}$



(b) $\overline{\theta} = -\frac{2}{3}$

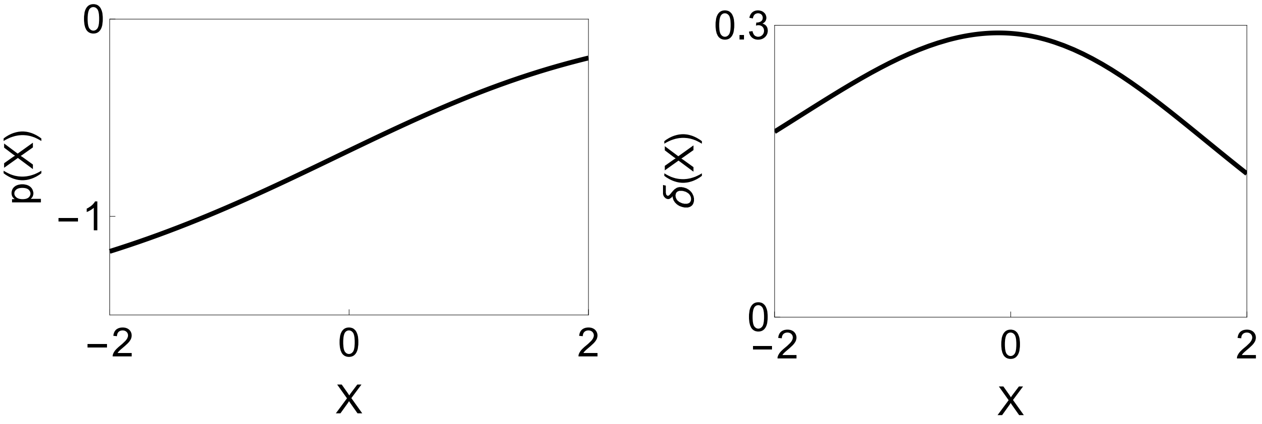
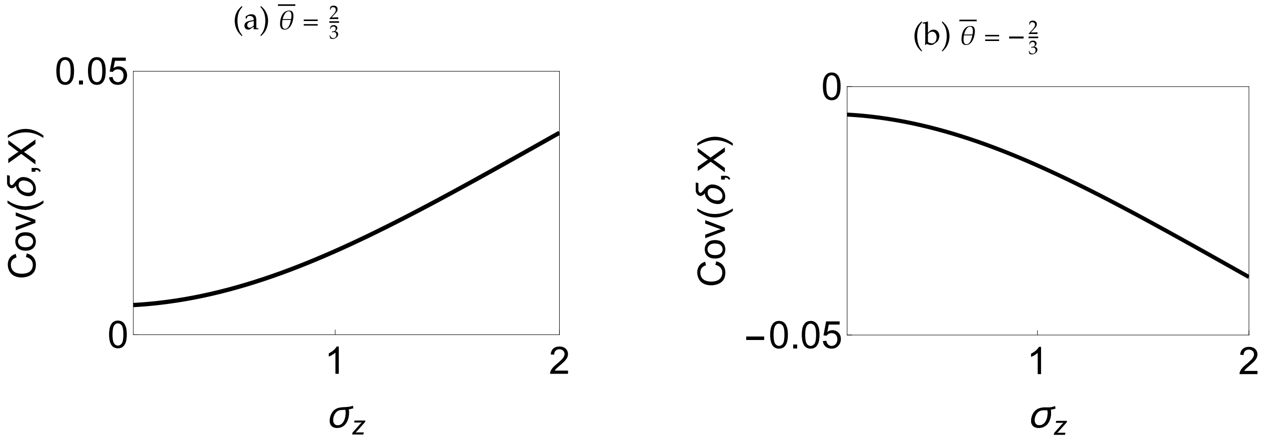


Figure OA.5.2: This figure plots the covariance between the slope of the price schedule δ and total order flow X against the standard deviation of noise trading. Parameters: $\overline{\beta} = 1, \Delta_\beta = \frac{1}{2}, \Delta_\epsilon = \frac{1}{4}, \chi_\epsilon = \frac{2}{3}, \sigma_\theta = \frac{1}{4}$, and $\gamma = \frac{3}{4}$.



(a) $\overline{\theta} = \frac{2}{3}$

(b) $\overline{\theta} = -\frac{2}{3}$

OA.6: Endogenous Information Acquisition

We endogenize traders' information acquisition of ϵ- and β-information within the model of Section 2. We assume that traders ex ante decide to acquire either ϵ-information or β-information. The idea is that investors are attention constrained and can only specialize in one particular type of information. We consider an interior equilibrium in which the mass χ_ϵ^* of ϵ-informed traders is positive but smaller than 1. In equilibrium, a trader must be indifferent between acquiring ϵ-information and acquiring β-information, which determines the equilibrium value of χ_ϵ^*.

The expected trading profits of ϵ- and β-informed traders are $\Pi_\epsilon \equiv \mathbb{E}\left[y_\epsilon(v-p)\right]$ and $\Pi_\beta \equiv \mathbb{E}\left[y_\beta(v-p)\right]$. Using Theorem 1, we have

$$\Pi_\epsilon = \begin{cases} \Delta_\epsilon\left(1-\chi_\epsilon+\frac{\chi_\beta}{3}\right)-\frac{2}{3}\chi_\beta\Delta_\beta|\overline{\theta}| \\ -\frac{2\gamma}{9}\chi_\beta(1-\chi_\epsilon)\Delta_\epsilon^2-\frac{\gamma}{2}\sigma_\theta^2\left(\overline{\beta}^2+\Delta_\beta^2\right)\chi_\epsilon-\frac{\gamma}{2}\Delta_\beta^2\overline{\theta}^2\left[(\chi_\epsilon-\chi_\beta)+\frac{4}{9}\chi_\beta(1-\chi_\epsilon)\right], & \text{if } \chi_\epsilon>\chi_\beta \\ \Delta_\epsilon\left(1-\frac{2}{3}\chi_\epsilon\right)-\frac{2}{3}\chi_\epsilon\Delta_\beta|\overline{\theta}| \\ -\frac{2\gamma}{9}\Delta_\epsilon^2\chi_\epsilon(1-\chi_\beta)-\frac{\gamma}{2}\sigma_\theta^2\left(\overline{\beta}^2+\Delta_\beta^2\right)\chi_\epsilon-\frac{2\gamma}{9}\chi_\epsilon(1-\chi_\beta)\Delta_\beta^2\overline{\theta}^2, & \text{if } \chi_\epsilon\le\chi_\beta \end{cases} \tag{OA.6.1}$$

and

$$\Pi_\beta = \begin{cases} \Delta_\beta|\overline{\theta}|\left(1-\frac{2\chi_\beta}{3}\right)-\frac{2}{3}\chi_\beta\Delta_\epsilon \\ -\frac{2\gamma}{9}(1-\chi_\epsilon)\chi_\beta\Delta_\epsilon^2-\frac{\gamma}{2}\sigma_\theta^2\left(\overline{\beta}^2+\Delta_\beta^2\right)\chi_\beta-\frac{2\gamma}{9}\Delta_\beta^2\overline{\theta}^2\chi_\beta(1-\chi_\epsilon), & \text{if } \chi_\epsilon>\chi_\beta \\ \Delta_\beta|\overline{\theta}|\left(1-\chi_\beta+\frac{1}{3}\chi_\epsilon\right)-\frac{2}{3}\chi_\epsilon\Delta_\epsilon \\ -\frac{\gamma}{2}\Delta_\epsilon^2\left[(\chi_\beta-\chi_\epsilon)+\frac{4}{9}\chi_\epsilon(1-\chi_\beta)\right]-\frac{\gamma}{2}\sigma_\theta^2\left(\overline{\beta}^2+\Delta_\beta^2\right)\chi_\beta-\frac{2\gamma}{9}\chi_\epsilon(1-\chi_\beta)\Delta_\beta^2\overline{\theta}^2. & \text{if } \chi_\epsilon\le\chi_\beta \end{cases}. \tag{OA.6.2}$$

Eqs. (OA.6.1) and (OA.6.2) show that the expected profits of informed traders are comprised of three components: (i) information rent earned by their own information; (ii) the externality effect due to the other type of information; (iii) the premium charged by risk-averse market makers. Intuitively, competition among peers implies that the profit of each type of trader decreases in the masses of their own type populations. Due to the cross-type externality (i.e., the second component in Eqs. (OA.6.1) and (OA.6.2)), the expected profit on each type of information decreases in the strength of the other type of information. For instance, Π_ϵ decreases in $\overline{\beta}$ and Δ_β, and Π_β decreases in Δ_ϵ. Finally, due to the stochastic feature of market liquidity (i.e., the third component in Eqs. (OA.6.1) and (OA.6.2)), the expected trading profits of both types of traders decrease in the volatility σ_θ of market factor.

Proposition 3 (Information Acquisition) *In the financial market equilibrium characterized by Theorem 1 (Part 1), the endogenous mass χ_ϵ^* of ϵ-informed traders is given by:*

1. *If $|\overline{\theta}| < \frac{\Delta_\epsilon}{\Delta_\beta}$:*
$$\chi_\epsilon^* = \frac{1}{2} + \frac{\Delta_\epsilon - \Delta_\beta|\overline{\theta}|}{\Delta_\beta^2\gamma\overline{\theta}^2 + \gamma\sigma_\theta^2\left[\Delta_\beta^2 + \overline{\beta}^2\right] + 2\Delta_\epsilon} \in \left(\frac{1}{2}, 1\right);$$
2. *If $|\overline{\theta}| > \frac{\Delta_\epsilon}{\Delta_\beta}$:*
$$\chi_\epsilon^* = \frac{1}{2} - \frac{\Delta_\beta|\overline{\theta}| - \Delta_\epsilon}{\gamma\left[\sigma_\theta^2\left(\Delta_\beta^2 + \overline{\beta}^2\right) + \Delta_\epsilon^2\right] + 2\Delta_\beta|\overline{\theta}|} \in \left(0, \frac{1}{2}\right).$$

The comparative statics are given by:

$$\begin{aligned}
\frac{\partial \chi_\epsilon^*}{\partial \Delta_\epsilon} &> 0, \frac{\partial \chi_\epsilon^*}{\partial \Delta_\beta} < 0, \frac{\partial \chi_\epsilon^*}{\partial |\overline{\theta}|} < 0;\\
\text{If } |\overline{\theta}| &< \frac{\Delta_\epsilon}{\Delta_\beta}, \text{ then } \frac{\partial \chi_\epsilon^*}{\partial \overline{\beta}} < 0 \text{ and } \frac{\partial \chi_\epsilon^*}{\partial \sigma_\theta} < 0;\\
\text{If } |\overline{\theta}| &> \frac{\Delta_\epsilon}{\Delta_\beta}, \text{ then } \frac{\partial \chi_\epsilon^*}{\partial \overline{\beta}} > 0 \text{ and } \frac{\partial \chi_\epsilon^*}{\partial \sigma_\theta} > 0.
\end{aligned}$$

We can then replace χ_ϵ in Section 3 by χ_ϵ^* described in Proposition 3, and similarly χ_β by $1 - \chi_\epsilon^*$. We formally prove the results in Proposition 3 next. In an interior equilibrium, the expected trading profits of ϵ-informed investors equal that of β-informed investors, i.e., $\Pi_\epsilon = \Pi_\beta$.

1. If $\chi_\epsilon > \chi_\beta$, then setting $\Pi_\epsilon = \Pi_\beta$ yields the following solution for χ_ϵ^*:
$$\chi_\epsilon^* = \frac{1}{2} + \frac{\Delta_\epsilon - \Delta_\beta|\overline{\theta}|}{\Delta_\beta^2\gamma\overline{\theta}^2 + \gamma\sigma_\theta^2\left(\Delta_\beta^2 + \overline{\beta}^2\right) + 2\Delta_\epsilon}. \tag{OA.6.3}$$
It immediately follows that $\chi_\epsilon > \chi_\beta$ if and only if $\Delta_\epsilon > \Delta_\beta|\overline{\theta}|$.

2. If $\chi_\epsilon \leq \chi_\beta$, then setting $\Pi_\epsilon = \Pi_\beta$ yields the following solution for χ_ϵ^*:
$$\chi_\epsilon^* = \frac{1}{2} - \frac{\Delta_\beta|\overline{\theta}| - \Delta_\epsilon}{\gamma\left[\sigma_\theta^2\left(\Delta_\beta^2 + \overline{\beta}^2\right) + \Delta_\epsilon^2\right] + 2\Delta_\beta|\overline{\theta}|}. \tag{OA.6.4}$$

It immediately follows that $\chi_\epsilon \leq \chi_\beta$ if and only if $\Delta_\epsilon < \Delta_\beta|\overline{\theta}|$.

Comparative Statics for χ_ϵ^*:

1. Comparative statics with respect to Δ_ϵ:
   (a) If $|\overline{\theta}| < \frac{\Delta_\epsilon}{\Delta_\beta}$, then $\frac{\partial \chi_\epsilon^*}{\partial \Delta_\epsilon} = \frac{\gamma\sigma_\theta^2\left(\overline{\beta}^2+\Delta_\beta^2\right)+|\overline{\theta}|\Delta_\beta\left(\gamma|\overline{\theta}|\Delta_\beta+2\right)}{\left[\gamma\sigma_\theta^2\left(\overline{\beta}^2+\Delta_\beta^2\right)+\gamma\overline{\theta}^2\Delta_\beta^2+2\Delta_\epsilon\right]^2} > 0.$

   (b) If $|\overline{\theta}| > \frac{\Delta_\epsilon}{\Delta_\beta}$, then $\frac{\partial \chi_\epsilon^*}{\partial \Delta_\epsilon} = \frac{\gamma\Delta_\epsilon\left(2|\overline{\theta}|\Delta_\beta-\Delta_\epsilon\right)+\gamma\sigma_\theta^2\left(\overline{\beta}^2+\Delta_\beta^2\right)+2|\overline{\theta}|\Delta_\beta}{\left[\gamma\sigma_\theta^2\left(\overline{\beta}^2+\Delta_\beta^2\right)+2|\overline{\theta}|\Delta_\beta+\gamma\Delta_\epsilon^2\right]^2} > 0.$

2. Comparative statics with respect to Δ_β:
   (a) If $|\overline{\theta}| < \frac{\Delta_\epsilon}{\Delta_\beta}$, then $\frac{\partial \chi_\epsilon^*}{\partial \Delta_\beta} = \frac{-\gamma\sigma_\theta^2\left(|\overline{\theta}|\left(\overline{\beta}^2-\Delta_\beta^2\right)+2\Delta_\epsilon\Delta_\beta\right)-2|\overline{\theta}|\Delta_\epsilon-\gamma\Delta_\beta\overline{\theta}^2\left(2\Delta_\epsilon-|\overline{\theta}|\Delta_\beta\right)}{\left[\gamma\sigma_\theta^2\left(\overline{\beta}^2+\Delta_\beta^2\right)+\gamma\overline{\theta}^2\Delta_\beta^2+2\Delta_\epsilon\right]^2} < 0.$

   (b) If $|\overline{\theta}| > \frac{\Delta_\epsilon}{\Delta_\beta}$, then $\frac{\partial \chi_\epsilon^*}{\partial \Delta_\beta} = \frac{-\gamma\sigma_\theta^2\left(|\overline{\theta}|\left(\overline{\beta}^2-\Delta_\beta^2\right)+2\Delta_\epsilon\Delta_\beta\right)-|\overline{\theta}|\Delta_\epsilon(\gamma\Delta_\epsilon+2)}{\left[\gamma\sigma_\theta^2\left(\overline{\beta}^2+\Delta_\beta^2\right)+2|\overline{\theta}|\Delta_\beta+\gamma\Delta_\epsilon^2\right]^2} < 0$ if γ is sufficiently small.

3. Comparative statics with respect to $|\overline{\theta}|$:
   (a) If $|\overline{\theta}| < \frac{\Delta_\epsilon}{\Delta_\beta}$, then $\frac{\partial \chi_\epsilon^*}{\partial |\overline{\theta}|} = \frac{-\Delta_\beta\left(2\Delta_\epsilon+\gamma\sigma_\theta^2\left(\overline{\beta}^2+\Delta_\beta^2\right)+\gamma\Delta_\beta|\overline{\theta}|\left(2\Delta_\epsilon-|\overline{\theta}|\Delta_\beta\right)\right)}{\left[\gamma\sigma_\theta^2\left(\overline{\beta}^2+\Delta_\beta^2\right)+\gamma\overline{\theta}^2\Delta_\beta^2+2\Delta_\epsilon\right]^2} < 0.$

   (b) If $|\overline{\theta}| > \frac{\Delta_\epsilon}{\Delta_\beta}$, then $\frac{\partial \chi_\epsilon^*}{\partial |\overline{\theta}|} = \frac{-\Delta_\beta\left(\gamma\sigma_\theta^2\left(\overline{\beta}^2+\Delta_\beta^2\right)+\Delta_\epsilon(\gamma\Delta_\epsilon+2)\right)}{\left[\gamma\sigma_\theta^2\left(\overline{\beta}^2+\Delta_\beta^2\right)+2|\overline{\theta}|\Delta_\beta+\gamma\Delta_\epsilon^2\right]^2} < 0.$

4. Comparative statics with respect to $\overline{\beta}$:
   (a) If $|\overline{\theta}| < \frac{\Delta_\epsilon}{\Delta_\beta}$, then $\frac{\partial \chi_\epsilon^*}{\partial \overline{\beta}} = \frac{2\gamma\overline{\beta}\sigma_\theta^2\left(|\overline{\theta}|\Delta_\beta-\Delta_\epsilon\right)}{\left[\gamma\sigma_\theta^2\left(\overline{\beta}^2+\Delta_\beta^2\right)+\gamma\overline{\theta}^2\Delta_\beta^2+2\Delta_\epsilon\right]^2} < 0.$

   (b) If $|\overline{\theta}| > \frac{\Delta_\epsilon}{\Delta_\beta}$, then $\frac{\partial \chi_\epsilon^*}{\partial \overline{\beta}} = \frac{2\gamma\overline{\beta}\sigma_\theta^2\left(|\overline{\theta}|\Delta_\beta-\Delta_\epsilon\right)}{\left[\gamma\sigma_\theta^2\left(\overline{\beta}^2+\Delta_\beta^2\right)+2|\overline{\theta}|\Delta_\beta+\gamma\Delta_\epsilon^2\right]^2} > 0.$

5. Comparative statics with respect to σ_θ^2:
   (a) If $|\overline{\theta}| < \frac{\Delta_\epsilon}{\Delta_\beta}$, then $\frac{\partial \chi_\epsilon^*}{\partial \sigma_\theta} = -\frac{\left(\Delta_\epsilon-|\overline{\theta}|\Delta_\beta\right)\left(2\gamma\overline{\beta}^2\sigma_\theta+2\gamma\Delta_\beta^2\sigma_\theta\right)}{\left(\gamma\Delta_\beta^2\left(\overline{\theta}^2+\sigma_\theta^2\right)+\gamma\overline{\beta}^2\sigma_\theta^2+2\Delta_\epsilon\right)^2} < 0.$

   (b) If $|\overline{\theta}| > \frac{\Delta_\epsilon}{\Delta_\beta}$, then $\frac{\partial \chi_\epsilon^*}{\partial \sigma_\theta} = -\frac{2\gamma\sigma_\theta\left(\overline{\beta}^2+\Delta_\beta^2\right)\left(\Delta_\epsilon-|\overline{\theta}|\Delta_\beta\right)}{\left(\gamma\left(\sigma_\theta^2\left(\overline{\beta}^2+\Delta_\beta^2\right)+\Delta_\epsilon^2\right)+2|\overline{\theta}|\Delta_\beta\right)^2} > 0.$

Expected δ:

We use the expression for $\mathbb{E}[\delta]$ derived in the main model and replace χ_ϵ by χ_ϵ^*.

1. Comparative statics with respect to σ_θ: we have shown in Corollary 2 that $\frac{\partial \mathbb{E}[\delta]}{\partial \sigma_\theta} > 0$. The indirect effect is analyzed below.

   (a) If $|\overline{\theta}| < \frac{\Delta_\epsilon}{\Delta_\beta}$ we obtain:

   $$\frac{\partial \mathbb{E}[\delta]}{\partial \chi_\epsilon}\frac{\partial \chi_\epsilon}{\partial \sigma_\theta} = \frac{\gamma}{3}\left(2\Delta_\varepsilon^2 - \overline{\theta}^2\Delta_\beta^2\right)\frac{\left(\Delta_\epsilon - |\overline{\theta}|\Delta_\beta\right)\left(2\gamma\overline{\beta}^2\sigma_\theta + 2\gamma\Delta_\beta^2\sigma_\theta\right)}{\left(\gamma\Delta_\beta^2\left(\overline{\theta}^2 + \sigma_\theta^2\right) + \gamma\overline{\beta}^2\sigma_\theta^2 + 2\Delta_\epsilon\right)^2} > 0. \tag{OA.6.5}$$

   (b) If $|\overline{\theta}| > \frac{\Delta_\epsilon}{\Delta_\beta}$ we obtain:

   $$\frac{\partial \mathbb{E}[\delta]}{\partial \chi_\epsilon}\frac{\partial \chi_\epsilon}{\partial \sigma_\theta} = \frac{\gamma}{3}\left(2\overline{\theta}^2\Delta_\beta^2 - \Delta_\epsilon^2\right)\frac{2\gamma\sigma_\theta\left(\overline{\beta}^2 + \Delta_\beta^2\right)\left(|\overline{\theta}|\Delta_\beta - \Delta_\epsilon\right)}{\left(\gamma\left(\sigma_\theta^2\left(\overline{\beta}^2 + \Delta_\beta^2\right) + \Delta_\epsilon^2\right) + 2|\overline{\theta}|\Delta_\beta\right)^2} > 0. \tag{OA.6.6}$$

2. Comparative statics with respect to $\overline{\beta}$: we have shown in Corollary 2 that $\frac{\partial \mathbb{E}[\delta]}{\partial \overline{\beta}} > 0$. The indirect effect is analyzed below.

   (a) If $|\overline{\theta}| < \frac{\Delta_\epsilon}{\Delta_\beta}$ we obtain:

   $$\frac{\partial \mathbb{E}[\delta]}{\partial \chi_\epsilon}\frac{\partial \chi_\epsilon}{\partial \overline{\beta}} = \frac{2\gamma^2\overline{\beta}\sigma_\theta^2\left(\Delta_\epsilon - |\overline{\theta}|\Delta_\beta\right)\left(2\Delta_\varepsilon^2 - \overline{\theta}^2\Delta_\beta^2\right)}{3\left(\gamma\Delta_\beta^2\left(\overline{\theta}^2 + \sigma_\theta^2\right) + \gamma\overline{\beta}^2\sigma_\theta^2 + 2\Delta_\epsilon\right)^2} > 0. \tag{OA.6.7}$$

   (b) If $|\overline{\theta}| > \frac{\Delta_\epsilon}{\Delta_\beta}$ we obtain:

   $$\frac{\partial \mathbb{E}[\delta]}{\partial \chi_\epsilon}\frac{\partial \chi_\epsilon}{\partial \overline{\beta}} = \frac{2\gamma^2\overline{\beta}\sigma_\theta^2\left(|\overline{\theta}|\Delta_\beta - \Delta_\epsilon\right)\left(2\overline{\theta}^2\Delta_\beta^2 - \Delta_\epsilon^2\right)}{3\left(\gamma\left(\sigma_\theta^2\left(\overline{\beta}^2 + \Delta_\beta^2\right) + \Delta_\epsilon^2\right) + 2|\overline{\theta}|\Delta_\beta\right)^2} > 0. \tag{OA.6.8}$$

3. Comparative statics with respect to $|\overline{\theta}|$:

   (a) If $|\overline{\theta}| < \frac{\Delta_\epsilon}{\Delta_\beta}$ we obtain:

   $$\begin{aligned}\frac{d\mathbb{E}[\delta]}{d|\overline{\theta}|} &= \frac{1}{3}\gamma\Delta_\beta\left(\frac{2\gamma|\overline{\theta}|\Delta_\beta\left(|\overline{\theta}|\Delta_\beta - \Delta_\epsilon\right)\left(\overline{\theta}^2\Delta_\beta^2 - 2\Delta_\varepsilon^2\right)}{\left(\gamma\Delta_\beta^2\left(\overline{\theta}^2 + \sigma_\theta^2\right) + \gamma\overline{\beta}^2\sigma_\theta^2 + 2\Delta_\epsilon\right)^2}\right) \\ &+ \frac{1}{3}\gamma\Delta_\beta\left(\frac{-3\overline{\theta}^2\Delta_\beta^2 + 2|\overline{\theta}|\Delta_\beta\Delta_\epsilon + 2\Delta_\epsilon^2}{\gamma\Delta_\beta^2\left(\overline{\theta}^2 + \sigma_\theta^2\right) + \gamma\overline{\beta}^2\sigma_\theta^2 + 2\Delta_\epsilon} + 2|\overline{\theta}|\Delta_\beta\right) > 0.\end{aligned} \tag{OA.6.9}$$

   The indirect effect of $|\overline{\theta}|$ on $\mathbb{E}[\delta]$ is

   $$\frac{\partial \mathbb{E}[\delta]}{\partial \chi_\epsilon}\frac{\partial \chi_\epsilon}{\partial |\overline{\theta}|} = -\left(\frac{4}{3}\Delta_\varepsilon^2 - \frac{2}{3}\Delta_\beta^2\overline{\theta}^2\right)\frac{\partial \chi_\epsilon}{\partial |\overline{\theta}|} > 0.$$

(b) If $|\overline{\theta}|> \frac{\Delta_\epsilon}{\Delta_\beta}$ we obtain:

$$\frac{d\mathbb{E}[\delta]}{d|\overline{\theta}|} = \tag{OA.6.10}$$
$$\frac{\left(\gamma\sigma_\theta^2\left(\overline{\beta}^2+\Delta_\beta^2\right)+\Delta_\epsilon\left(\gamma\Delta_\epsilon+2\right)\right)\left(2|\overline{\theta}|\Delta_\beta\left(\gamma\sigma_\theta^2\left(\overline{\beta}^2+\Delta_\beta^2\right)+|\overline{\theta}|\Delta_\beta\right)+\Delta_\epsilon^2\left(2\gamma|\overline{\theta}|\Delta_\beta+1\right)\right)}{3\left(\gamma\Delta_\beta\right)^{-1}\left(\gamma\left(\sigma_\theta^2\left(\overline{\beta}^2+\Delta_\beta^2\right)+\Delta_\epsilon^2\right)+2|\overline{\theta}|\Delta_\beta\right)^2} > 0.$$

The indirect effect of $|\overline{\theta}|$ on $\mathbb{E}[\delta]$ is

$$\frac{\partial\mathbb{E}[\delta]}{\partial\chi_\epsilon}\frac{\partial\chi_\epsilon}{\partial|\overline{\theta}|} = \left(\frac{4}{3}\Delta_\beta^2\overline{\theta}^2-\frac{2}{3}\Delta_\varepsilon^2\right)\frac{\partial\chi_\epsilon}{\partial|\overline{\theta}|} < 0.$$

4. Comparative statics with respect to Δ_ϵ:

(a) If $|\overline{\theta}|< \frac{\Delta_\epsilon}{\Delta_\beta}$ we obtain:

$$\frac{d\mathbb{E}[\delta]}{d\Delta_\epsilon} = \tag{OA.6.11}$$
$$\frac{\gamma\left(\gamma\sigma_\theta^2\left(\overline{\beta}^2+\Delta_\beta^2\right)+|\overline{\theta}|\Delta_\beta\left(\gamma|\overline{\theta}|\Delta_\beta+2\right)\right)\left(2\Delta_\epsilon\left(\gamma\Delta_\beta^2\left(\overline{\theta}^2+\sigma_\theta^2\right)+\gamma\overline{\beta}^2\sigma_\theta^2+\Delta_\epsilon\right)+\overline{\theta}^2\Delta_\beta^2\right)}{3\left(\gamma\Delta_\beta^2\left(\overline{\theta}^2+\sigma_\theta^2\right)+\gamma\overline{\beta}^2\sigma_\theta^2+2\Delta_\epsilon\right)^2} > 0.$$

The indirect effect of Δ_ϵ on $\mathbb{E}[\delta]$ is

$$\frac{\partial\mathbb{E}[\delta]}{\partial\chi_\epsilon}\frac{\partial\chi_\epsilon}{\partial\Delta_\epsilon} = -\left(\frac{4}{3}\Delta_\varepsilon^2-\frac{2}{3}\Delta_\beta^2\overline{\theta}^2\right)\frac{\partial\chi_\epsilon}{\partial\Delta_\epsilon} < 0.$$

(b) If $|\overline{\theta}|> \frac{\Delta_\epsilon}{\Delta_\beta}$ we obtain:

$$\frac{d\mathbb{E}[\delta]}{d\Delta_\epsilon} = \frac{1}{3}\gamma\left(\frac{2\gamma\Delta_\epsilon\left(\Delta_\epsilon-|\overline{\theta}|\Delta_\beta\right)\left(\Delta_\epsilon^2-2\overline{\theta}^2\Delta_\beta^2\right)}{\left(\gamma\left(\sigma_\theta^2\left(\overline{\beta}^2+\Delta_\beta^2\right)+\Delta_\epsilon^2\right)+2|\overline{\theta}|\Delta_\beta\right)^2}\right) \tag{OA.6.12}$$
$$+ \frac{1}{3}\gamma\left(\frac{2\overline{\theta}^2\Delta_\beta^2+2|\overline{\theta}|\Delta_\beta\Delta_\epsilon-3\Delta_\epsilon^2}{\gamma\left(\sigma_\theta^2\left(\overline{\beta}^2+\Delta_\beta^2\right)+\Delta_\epsilon^2\right)+2|\overline{\theta}|\Delta_\beta}+2\Delta_\epsilon\right) > 0. \tag{OA.6.13}$$

The indirect effect of Δ_ϵ on $\mathbb{E}[\delta]$ is

$$\frac{\partial\mathbb{E}[\delta]}{\partial\chi_\epsilon}\frac{\partial\chi_\epsilon}{\partial\Delta_\epsilon} = \left(\frac{4}{3}\Delta_\beta^2\overline{\theta}^2-\frac{2}{3}\Delta_\varepsilon^2\right)\frac{\partial\chi_\epsilon}{\partial\Delta_\epsilon} > 0.$$

5. Comparative statics with respect to Δ_β:

(a) If $|\overline{\theta}| < \frac{\Delta_\epsilon}{\Delta_\beta}$ we obtain:

$$\begin{aligned}\frac{d\mathbb{E}[\delta]}{d\Delta_\beta} &= \frac{\gamma\left(2\Delta_\epsilon^2 - \overline{\theta}^2\Delta_\beta^2\right)\left(\gamma|\overline{\theta}|\left(\overline{\beta}^2\sigma_\theta^2 - \Delta_\beta^2\left(\overline{\theta}^2 + \sigma_\theta^2\right)\right) + 2\Delta_\epsilon\left(\gamma\overline{\theta}^2\Delta_\beta + |\overline{\theta}| + \gamma\Delta_\beta\sigma_\theta^2\right)\right)}{3\left(\gamma\Delta_\beta^2\left(\overline{\theta}^2 + \sigma_\theta^2\right) + \gamma\overline{\beta}^2\sigma_\theta^2 + 2\Delta_\epsilon\right)^2} \\ &+ \frac{\gamma}{6}\left(2\overline{\theta}^2\Delta_\beta + 6\Delta_\beta\sigma_\theta^2 + 4\overline{\theta}^2\Delta_\beta\left(\frac{\Delta_\epsilon - |\overline{\theta}|\Delta_\beta}{\gamma\Delta_\beta^2\left(\overline{\theta}^2 + \sigma_\theta^2\right) + \gamma\overline{\beta}^2\sigma_\theta^2 + 2\Delta_\epsilon} + \frac{1}{2}\right)\right) > 0.\end{aligned}$$

The indirect effect of Δ_β on $\mathbb{E}[\delta]$ is

$$\frac{\partial\mathbb{E}[\delta]}{\partial\chi_\epsilon}\frac{\partial\chi_\epsilon}{\partial\Delta_\beta} = -\left(\frac{4}{3}\Delta_\varepsilon^2 - \frac{2}{3}\Delta_\beta^2\overline{\theta}^2\right)\frac{\partial\chi_\epsilon}{\partial\Delta_\beta} > 0.$$

(b) If $|\overline{\theta}| > \frac{\Delta_\epsilon}{\Delta_\beta}$ we obtain:

$$\begin{aligned}\frac{d\mathbb{E}[\delta]}{d\Delta_\beta} &= \frac{\gamma\left(\Delta_\epsilon^2 - 2\overline{\theta}^2\Delta_\beta^2\right)\left(\gamma\sigma_\theta^2\left(-|\overline{\theta}|\Delta_\beta^2 + \overline{\beta}^2|\overline{\theta}| + 2\Delta_\beta\Delta_\epsilon\right) + |\overline{\theta}|\Delta_\epsilon\left(\gamma\Delta_\epsilon + 2\right)\right)}{3\left(\gamma\left(\sigma_\theta^2\left(\overline{\beta}^2 + \Delta_\beta^2\right) + \Delta_\epsilon^2\right) + 2|\overline{\theta}|\Delta_\beta\right)^2} \\ &+ \frac{\gamma}{6}\left(6\Delta_\beta\sigma_\theta^2 + 8\overline{\theta}^2\Delta_\beta\left(\frac{\Delta_\epsilon - |\overline{\theta}|\Delta_\beta}{\gamma\left(\sigma_\theta^2\left(\overline{\beta}^2 + \Delta_\beta^2\right) + \Delta_\epsilon^2\right) + 2|\overline{\theta}|\Delta_\beta} + \frac{1}{2}\right)\right) > 0.\end{aligned}$$

The indirect effect of Δ_β on $\mathbb{E}[\delta]$ is

$$\frac{\partial\mathbb{E}[\delta]}{\partial\chi_\epsilon}\frac{\partial\chi_\epsilon}{\partial\Delta_\beta} = \left(\frac{4}{3}\Delta_\beta^2\overline{\theta}^2 - \frac{2}{3}\Delta_\varepsilon^2\right)\frac{\partial\chi_\epsilon}{\partial\Delta_\beta} > 0.$$

Liquidity Risk:

We use the expression for $\sigma^2(\delta)$ derived in the main model and replace χ_ϵ by χ_ϵ^*.

1. Comparative statics with respect to σ_θ:

$$\frac{d\sigma^2(\delta)}{d\sigma_\theta} = \frac{\partial\sigma^2(\delta)}{\partial\sigma_\theta} + \frac{\partial\sigma^2(\delta)}{\partial\chi_\epsilon}\frac{\partial\chi_\epsilon}{\partial\sigma_\theta}. \qquad \text{(OA.6.14)}$$

We have shown in Corollary 2 that $\frac{\partial\sigma^2(\delta)}{\partial\sigma_\theta} > 0$. The indirect effect is analyzed below.

(a) If $|\overline{\theta}| < \frac{\Delta_\epsilon}{\Delta_\beta}$, then we obtain that $\frac{\partial\sigma^2(\delta)}{\partial\chi_\epsilon}\frac{\partial\chi_\epsilon}{\partial\sigma_\theta}$ is equal to

$$\frac{\gamma^3\sigma_\theta\left(\overline{\beta}^2 + \Delta_\beta^2\right)\left(\Delta_\epsilon - |\overline{\theta}|\Delta_\beta\right)\left(\overline{\theta}^4\left(12\chi_\epsilon - 5\right)\Delta_\beta^4 + 4\overline{\theta}^2\left(11 - 12\chi_\epsilon\right)\Delta_\beta^2\Delta_\epsilon^2 + 36\overline{\beta}^2\Delta_\beta^2\sigma_\theta^4 + 16\left(3\chi_\epsilon - 2\right)\Delta_\epsilon^4\right)}{27\left(\gamma\sigma_\theta^2\left(\overline{\beta}^2 + \Delta_\beta^2\right) + \gamma\overline{\theta}^2\Delta_\beta^2 + 2\Delta_\epsilon\right)^2}$$

It follows that the indirect is positive if χ_ϵ is sufficiently large. We have shown before that χ_ϵ decreases in γ. It follows that the indirect effect, and hence the total effect, are positive if γ is sufficiently small.

(b) If $|\overline{\theta}| > \frac{\Delta_\epsilon}{\Delta_\beta}$, then we obtain that $\frac{\partial\sigma^2[\delta]}{\partial\chi_\epsilon}\frac{\partial\chi_\epsilon}{\partial\sigma_\theta}$ is equal to

$$\frac{\gamma^3\sigma_\theta\left(\overline{\beta}^2+\Delta_\beta^2\right)\left(|\overline{\theta}|\Delta_\beta-\Delta_\epsilon\right)\left(16\overline{\theta}^4\left(1-3\chi_\epsilon\right)\Delta_\beta^4+4\overline{\theta}^2\left(12\chi_\epsilon-1\right)\Delta_\beta^2\Delta_\epsilon^2-72\overline{\beta}^2\Delta_\beta^2\sigma_\theta^4+\left(7-12\chi_\epsilon\right)\Delta_\epsilon^4\right)}{27\left(\gamma\left(\sigma_\theta^2\left(\overline{\beta}^2+\Delta_\beta^2\right)+\Delta_\epsilon^2\right)+2|\overline{\theta}|\Delta_\beta\right)^2}$$

The sign of the indirect effect is equal to the sign of:

$$16\overline{\theta}^4\left(1-3\chi_\epsilon\right)\Delta_\beta^4+4\overline{\theta}^2\left(12\chi_\epsilon-1\right)\Delta_\beta^2\Delta_\epsilon^2-72\overline{\beta}^2\Delta_\beta^2\sigma_\theta^4+\left(7-12\chi_\epsilon\right)\Delta_\epsilon^4 \qquad \text{(OA.6.15)}$$

which is decreasing in χ_ϵ. In this case, χ_ϵ is increasing in γ. This term can be positive or negative. For instance, consider the limit $|\overline{\theta}| = \frac{\Delta_\epsilon}{\Delta_\beta}$ in which case it is equal to $\frac{13\Delta_\epsilon^4}{\Delta_\beta^2} - 72\overline{\beta}^2\sigma_\theta^4$, which can be positive or negative depending on σ_θ for example.

Next, we show that the total effect of σ_θ on $\sigma^2(\delta)$ is positive. We substitute χ_ϵ^* into $\frac{d\sigma^2(\delta)}{d\overline{\beta}}$ and write the total effect as follows:

$$\begin{aligned}&36\gamma^2\overline{\beta}^2\sigma_\theta^6\left(\overline{\beta}^2+\Delta_\beta^2\right)^2+\frac{12\gamma\overline{\theta}^6\Delta_\beta^4\left(\overline{\beta}^2+\Delta_\beta^2\right)\left(\frac{\Delta_\epsilon}{|\overline{\theta}|\Delta_\beta}-1\right)^2\left(\frac{\Delta_\epsilon^2}{\overline{\theta}^2\Delta_\beta^2}-2\right)^2}{\gamma\sigma_\theta^2\left(\overline{\beta}^2+\Delta_\beta^2\right)+|\overline{\theta}|\Delta_\beta\left(\frac{\gamma\Delta_\epsilon^2}{|\overline{\theta}|\Delta_\beta}+2\right)}\\&+36\overline{\beta}^2\overline{\theta}^2\Delta_\beta^2\sigma_\theta^2\left(\frac{\gamma\Delta_\epsilon^2}{|\overline{\theta}|\Delta_\beta}+2\right)\left(\frac{\Delta_\epsilon\left(\gamma\Delta_\epsilon-4\right)}{|\overline{\theta}|\Delta_\beta}+6\right)+72\gamma\overline{\beta}^2|\overline{\theta}|\Delta_\beta\sigma_\theta^4\left(\overline{\beta}^2+\Delta_\beta^2\right)\left(\frac{\Delta_\epsilon\left(\gamma\Delta_\epsilon-1\right)}{|\overline{\theta}|\Delta_\beta}+3\right)\\&+\gamma|\overline{\theta}|^5\Delta_\beta^3\left(\overline{\beta}^2+\Delta_\beta^2\right)\left(1-\frac{\Delta_\epsilon}{|\overline{\theta}|\Delta_\beta}\right)\left(\frac{\Delta_\epsilon^4}{\overline{\theta}^4\Delta_\beta^4}+\frac{20\Delta_\epsilon^2}{\overline{\theta}^2\Delta_\beta^2}-8\right) \qquad \text{(OA.6.16)}\end{aligned}$$

which is positive given that $|\overline{\theta}| > \frac{\Delta_\epsilon}{\Delta_\beta}$ and γ sufficiently small.

2. Comparative statics with respect to $\overline{\beta}$:

$$\frac{d\sigma^2(\delta)}{d\overline{\beta}} = \frac{\partial\sigma^2(\delta)}{\partial\overline{\beta}} + \frac{\partial\sigma^2(\delta)}{\partial\chi_\epsilon}\frac{\partial\chi_\epsilon}{\partial\overline{\beta}}. \qquad \text{(OA.6.17)}$$

We have shown in Corollary 2 that $\frac{\partial\sigma^2(\delta)}{\partial\overline{\beta}} > 0$. The indirect effect is analyzed below.

(a) If $|\overline{\theta}| < \frac{\Delta_\epsilon}{\Delta_\beta}$, then we obtain that $\frac{\partial \sigma^2(\delta)}{\partial \chi_\epsilon}\frac{\partial \chi_\epsilon}{\partial \overline{\beta}}$ is equal to

$$\frac{\gamma^3 \overline{\beta} \sigma_\theta^2 \left(\Delta_\epsilon - |\overline{\theta}|\Delta_\beta\right)\left(36\overline{\beta}^2 \Delta_\beta^2 \sigma_\theta^4 - 5\overline{\theta}^4 \Delta_\beta^4 + 12\chi_\epsilon \left(\overline{\theta}^2 \Delta_\beta^2 - 2\Delta_\epsilon^2\right)^2 + 44\overline{\theta}^2 \Delta_\beta^2 \Delta_\epsilon^2 - 32\Delta_\epsilon^4\right)}{27\left(\gamma \Delta_\beta^2 \left(\overline{\theta}^2 + \sigma_\theta^2\right) + \gamma \overline{\beta}^2 \sigma_\theta^2 + 2\Delta_\epsilon\right)^2}$$

It follows that the indirect is positive if χ_ϵ is sufficiently large. We have shown before that χ_ϵ decreases in γ. It follows that the indirect effect, and hence the total effect, are positive if γ is sufficiently small.

(b) If $|\overline{\theta}| > \frac{\Delta_\epsilon}{\Delta_\beta}$, then we obtain that $\frac{\partial \sigma^2(\delta)}{\partial \chi_\epsilon}\frac{\partial \chi_\epsilon}{\partial \overline{\beta}}$ is equal to

$$\frac{\gamma^3 \overline{\beta} \sigma_\theta^2 \left(\Delta_\epsilon - |\overline{\theta}|\Delta_\beta\right)\left(72\overline{\beta}^2 \Delta_\beta^2 \sigma_\theta^4 - 16\overline{\theta}^4 \Delta_\beta^4 + 12\chi_\epsilon \left(\Delta_\epsilon^2 - 2\overline{\theta}^2 \Delta_\beta^2\right)^2 + 4\overline{\theta}^2 \Delta_\beta^2 \Delta_\epsilon^2 - 7\Delta_\epsilon^4\right)}{27\left(\gamma \left(\sigma_\theta^2 \left(\overline{\beta}^2 + \Delta_\beta^2\right) + \Delta_\epsilon^2\right) + 2|\overline{\theta}|\Delta_\beta\right)^2}$$

The sign of the indirect effect is equal to the sign of:

$$-72\overline{\beta}^2 \Delta_\beta^2 \sigma_\theta^4 + 16\overline{\theta}^4 \Delta_\beta^4 - 12\chi_\epsilon \left(\Delta_\epsilon^2 - 2\overline{\theta}^2 \Delta_\beta^2\right)^2 - 4\overline{\theta}^2 \Delta_\beta^2 \Delta_\epsilon^2 + 7\Delta_\epsilon^4 \qquad \text{(OA.6.18)}$$

which is decreasing in χ_ϵ. In this case, χ_ϵ is increasing in γ. This term can be positive or negative. For instance, consider the limit $|\overline{\theta}| = \frac{\Delta_\epsilon}{\Delta_\beta}$ in which case it is equal to $13\Delta_\epsilon^4 - 72\overline{\beta}^2 \Delta_\beta^2 \sigma_\theta^4$, which can be positive or negative depending on σ_θ.

Finally, we show that the total effect of $\overline{\beta}$ on $\sigma^2(\delta)$ is positive. We substitute χ_ϵ^* into $\frac{d\sigma^2(\delta)}{d\overline{\beta}}$ and write the total effect as follows:

$$\begin{aligned} & 18\gamma^2\sigma_\theta^6 \left(\overline{\beta}^2 + \Delta_\beta^2\right)^2 + \frac{12\gamma\overline{\theta}^6 \Delta_\beta^4 \left(\frac{\Delta_\epsilon}{\overline{\theta}\Delta_\beta} - 1\right)^2 \left(\frac{\Delta_\epsilon^2}{\overline{\theta}^2 \Delta_\beta^2} - 2\right)^2}{\gamma\sigma_\theta^2 \left(\overline{\beta}^2 + \Delta_\beta^2\right) + |\overline{\theta}|\Delta_\beta \left(\frac{\gamma\Delta_\epsilon^2}{\overline{\theta}\Delta_\beta} + 2\right)} \\ + \quad & 18\overline{\theta}^2 \Delta_\beta^2 \sigma_\theta^2 \left(\frac{\gamma\Delta_\epsilon^2}{|\overline{\theta}|\Delta_\beta} + 2\right)\left(\frac{\Delta_\epsilon \left(\gamma\Delta_\epsilon - 4\right)}{|\overline{\theta}|\Delta_\beta} + 6\right) + \gamma|\overline{\theta}|^5 \Delta_\beta^3 \left(1 - \frac{\Delta_\epsilon}{|\overline{\theta}|\Delta_\beta}\right)\left(\frac{\Delta_\epsilon^4}{\overline{\theta}^4 \Delta_\beta^4} + \frac{20\Delta_\epsilon^2}{\overline{\theta}^2 \Delta_\beta^2} - 8\right) \\ + \quad & 36\gamma|\overline{\theta}|\Delta_\beta \sigma_\theta^4 \left(\overline{\beta}^2 \left(\frac{\gamma\Delta_\epsilon^2}{|\overline{\theta}|\Delta_\beta} + 2\right) + \Delta_\beta^2 \left(\frac{\Delta_\epsilon\left(\gamma\Delta_\epsilon - 2\right)}{|\overline{\theta}|\Delta_\beta} + 4\right)\right) \qquad \text{(OA.6.19)} \end{aligned}$$

which is positive given that $|\overline{\theta}| > \frac{\Delta_\epsilon}{\Delta_\beta}$.

Finally, we note that under endogenous information acquisition, our main results concerning expected asset returns continue to hold. In particular, via Eq. (11), the expected asset return is still inversely related to Cov(δ, X). Moreover, endogenizing information acquisition only changes the level of Cov(δ, X) but not its sign. For these reasons, the relation of expected asset returns with mean beta is unchanged relative to that in Corollary 4 in the setting with endogenous information acquisition.

OA.7: Additional Empirical Results Conditioned on Firm Size

Table OA.7.1 Performance of *Cov*-Size Sorted Portfolios. This table reports the performance of stock portfolios double-sorted by *Cov* and size. At each month-end, we sort stocks into two groups based on the median value of market capitalization in this month-end. Then, we independently sort the stocks into deciles by *Cov*. We hold the equal-weighted portfolios over the next month. We report the average monthly portfolio alpha relative to the Fama-French five-factor model augmented with a momentum factor. Stocks with a share price below $5 or market capitalization below NYSE 10th percentile value at month t end are excluded at portfolio formation. Panel A reports the performance of the portfolios double-sorted by *Cov1* and size. Panel B reports the performance of the portfolios double-sorted by *Cov2* and size. t-statistics in parentheses are computed based on standard errors with Newey-West correction of 5 lags.

Panel A: Portfolios Sorted by *Cov1* and Size

	Small		Large	
Cov1 Decile	Alpha	t-stat	Alpha	t-stat
1	0.30	(3.06)	0.16	(1.10)
2	0.23	(3.58)	0.22	(1.63)
3	0.18	(1.67)	0.11	(1.70)
4	0.28	(2.03)	0.20	(3.02)
5	0.10	(0.74)	0.18	(3.05)
6	0.25	(1.86)	0.19	(2.83)
7	0.48	(3.21)	0.22	(3.01)
8	0.09	(0.94)	0.08	(0.87)
9	0.08	(1.00)	0.22	(1.87)
10	0.11	(1.36)	0.15	(0.59)
1-10	0.19	(2.31)	0.01	(0.05)

Panel B: Portfolios Sorted by *Cov2* and Size

	Small		Large	
Cov2 Decile	Alpha	t-stat	Alpha	t-stat
1	0.30	(3.04)	0.16	(1.08)
2	0.24	(3.90)	0.20	(1.47)
3	0.18	(1.74)	0.13	(2.01)
4	0.24	(1.71)	0.19	(2.97)
5	0.22	(1.57)	0.19	(3.10)
6	0.30	(1.82)	0.18	(2.75)
7	0.37	(2.71)	0.23	(3.13)
8	0.11	(1.14)	0.08	(0.95)
9	0.06	(0.88)	0.20	(1.71)
10	0.12	(1.44)	0.14	(0.55)
1-10	0.18	(2.17)	0.02	(0.07)

Table OA.7.2 Fama-MacBeth Regressions of Cov on Beta in Size-Subsamples. This table reports results from Fama-MacBeth regressions of *Cov* on the contemporaneous market beta in subsamples. The dependent variable is the covariance between order imbalance and δ (*Cov1* or *Cov1*) measured at month t end. The key independent variables are market beta estimated at month t's end following the methodology of Frazzini and Pedersen (2014) and the decile ranking of market beta at month t end. Control variables include natural logarithm of market capitalization (Size) at month t end, book-to-market ratio at month t end, cumulative stock returns from month $t-6$ to month $t-1$ (PRET), asset growth (AG) as of month t end, and gross profitability (GP) as of month t end. Stocks with a share price below \$5 or market capitalization below NYSE 10th percentile value at month $t-1$ end are excluded. To form the sub-samples, we first split the sample period into recession and non-recession periods based on whether a month falls into NBER-defined recession periods. Then, in each month, we sort stocks into small firms and large firms based on the median value of stock market capitalization at the previous month-end. Panel A.1 (Panel A.2) reports the regression results in the subsample of small firms (large firms) within the non-recession subsample. Panel B.1 (B.2) reports the regression results in the subsample of small firms (large firms) within the recession subsample. t-statistics in parentheses are computed based on standard errors with Newey-West correction of 5 lags.

Panel A: Non-Recession Sample

Panel A.1: Small Firms

	(1)	(2)	(3)	(4)
DepVar:	*Cov1*		*Cov2*	
Beta	2.439***		2.481***	
	(2.88)		(2.91)	
Rank_Beta		0.306***		0.310***
		(3.31)		(3.33)
Size	0.658	0.692	0.814*	0.848*
	(1.43)	(1.52)	(1.73)	(1.83)
BM	−0.300	−0.260	−0.304	−0.264
	(−0.65)	(−0.57)	(−0.65)	(−0.57)
PRET	−3.609***	−3.599***	−3.602***	−3.594***
	(−5.15)	(−5.18)	(−5.14)	(−5.17)
AG	−0.314	−0.310	−0.306	−0.303
	(−1.22)	(−1.22)	(−1.19)	(−1.20)
GP	0.894*	0.866*	0.947*	0.919*
	(1.83)	(1.76)	(1.94)	(1.87)
Adj. R^2	0.010	0.011	0.010	0.011

Panel A.2: Large Firms

	(1)	(2)	(3)	(4)
DepVar:	*Cov1*		*Cov2*	
Beta	0.710***		0.718***	
	(2.98)		(2.99)	
Rank_Beta		0.092***		0.093***
		(3.30)		(3.30)
Size	−0.177***	−0.184***	−0.168**	−0.175**
	(−2.63)	(−2.64)	(−2.52)	(−2.54)
BM	−0.706***	−0.684***	−0.708***	−0.686***
	(−3.13)	(−3.15)	(−3.16)	(−3.18)
PRET	−1.069***	−1.078***	−1.068***	−1.078***
	(−4.27)	(−4.34)	(−4.28)	(−4.35)
AG	0.124	0.129	0.125	0.130
	(1.32)	(1.36)	(1.33)	(1.37)
GP	0.359*	0.349*	0.367*	0.358*
	(1.70)	(1.65)	(1.73)	(1.68)
Adj. R^2	0.006	0.007	0.006	0.007

Table OA.7.2 Continued

Panel B: Recession Sample

Panel B.1: Small Firms

	(1)	(2)	(3)	(4)
DepVar:	*Cov1*		*Cov2*	
Beta	−0.402		-0.501	
	(-0.18)		(-0.22)	
Rank_Beta		-0.046		-0.059
		(-0.17)		(-0.23)
Size	3.189^{**}	3.178^{**}	3.657^{**}	3.645^{**}
	(2.22)	(2.21)	(2.43)	(2.42)
BM	0.960	0.996	0.993	1.028
	(1.22)	(1.22)	(1.26)	(1.26)
PRET	-6.675^{***}	-6.895^{***}	-6.793^{***}	-7.024^{***}
	(-3.58)	(-3.37)	(-3.61)	(-3.38)
AG	-0.522	-0.531	-0.505	-0.512
	(-0.87)	(-0.86)	(-0.84)	(-0.83)
GP	0.198	-0.009	0.481	0.277
	(0.13)	(-0.01)	(0.33)	(0.20)
Adj. R^2	0.007	0.007	0.007	0.008

Panel B.2: Large Firms

	(1)	(2)	(3)	(4)
DepVar:	*Cov1*		*Cov2*	
Beta	-0.048		-0.066	
	(-0.11)		(-0.15)	
Rank_Beta		-0.006		-0.007
		(-0.21)		(-0.22)
Size	-0.121*	-0.123	-0.109	-0.111
	(-1.72)	(-1.73)	(-1.57)	(-1.59)
BM	-0.148	-0.157	-0.157	-0.166
	(-0.75)	(-0.80)	(-0.79)	(-0.85)
PRET	-0.694^{*}	-0.747^{*}	-0.703^{*}	-0.753^{*}
	(-1.94)	(-1.97)	(-1.95)	(-1.98)
AG	-0.043	-0.039	-0.038	-0.035
	(-1.19)	(-1.11)	(-1.02)	(-0.96)
GP	-0.060	-0.054	-0.049	-0.042
	(-0.39)	(-0.36)	(-0.32)	(-0.27)
Adj. R^2	0.005	0.005	0.005	0.005

Table OA.7.3 Asymmetry of Price Impact. This table demonstrates asymmetry in the price impact of order imbalance. The regression sample consists of stock-by-date observations from 1993 to 2019. The dependent variable is daily stock returns. Independent variables are contemporaneous stock-by-date level order imbalance-related variables. OIB is defined as buy dollar volume minus sell dollar volume, scaled by the sum of buy dollar volume plus sell dollar volume. We split OIB into OIB_Pos and OIB_Neg. Specifically, OIB_Pos equals OIB if $OIB \geq 0$, and it equals zero elsewhere. OIB_Neg equals OIB if $OIB < 0$, and it equals zero elsewhere. We include positive and negative OIB and their square root terms in the regression to test the asymmetry of price impact between positive and negative order imbalance. Stocks with share prices below $5 or market cap below NYSE 10th percentile value at the previous month-end are excluded. Date and firm fixed effects are included. Standard errors are double clustered by date and firm. *, **, and *** denote the 10%, 5%, and 1% significance level, respectively.

DepVar: Ret	Test of Asymmetry
OIB_Pos	-0.007^{***}
$\sqrt{OIB\_Pos}$	0.019^{***}
OIB_Neg	-0.004^{***}
$\sqrt{-OIB\_Neg}$	-0.016^{***}
Date FE	Y
Firm FE	Y
No. Obs.	18,173,621
Adj. R^2	0.193

Table OA.7.4 Price Impact of Order Imbalance. This table shows the price impact of order imbalance. The regression sample consists of stock-by-date observations from 1993 to 2019. The dependent variable is daily stock returns. Independent variables are contemporaneous stock-by-date level order imbalance-related variables. OIB is defined as buy dollar volume minus sell dollar volume, scaled by the sum of buy and sell dollar volume for a stock-date. We split OIB into OIB_Top, OIB_Med, and OIB_Bot based on the 25^{th} and 75^{th} percentile value of OIB in full sample. Specifically, OIB_Top equals OIB if $OIB \geq 0.15$, and it equals zero elsewhere; OIB_Med equals OIB if $0.15 > OIB \geq -0.14$, and it equals zero elsewhere; OIB_Bot equals OIB if $OIB < -0.14$, and it equals zero elsewhere. Stocks with share prices below $5 or market cap below NYSE 10th percentile value at the previous month-end are excluded. Date and firm fixed effects are included. Standard errors are double clustered by date and firm. *, **, and *** denote the 10%, 5%, and 1% significance level, respectively.

DepVar: Ret	OIB
OIB_Top	0.019***
OIB_Med	0.042***
OIB_Bot	0.017***
Date FE	Y
Firm FE	Y
No. Obs.	18,173,621
Adj. R^2	0.187