ONLINE E-COMPANION:

An Economic Analysis of Online Ad Fraud Deterrence

Min Chen
Costello College of Business, George Mason University, Fairfax, Virginia 22030, mchen15@gmu.edu

Subodha Kumar
Fox School of Business, Temple University, Philadelphia, Pennsylvania 19122, subodha@temple.edu

Abhishek Ray
Costello College of Business, George Mason University, Fairfax, Virginia 22030, aray8@gmu.edu

EC.1. Table of Notations

Symbol	Definition
i	Index of the publisher's *genuine* or *true type* $i \in \{M, N\}$, where M and N indicate malicious and nonmalicious publishers, respectively
j	Index of the publisher's *classified type* $j \in \{M, N\}$ by the ad network, where M and N indicate publishers classified as malicious (*type-M*) and nonmalicious (*type-N*), respectively
Parameters	
T_i	The number of publishers of true type $i \in \{M, N\}$ in the ad network
α	The accuracy of classifying publisher's type by the ad network, $\alpha \in [\frac{1}{2}, 1)$
b	The ratio of valid ad traffic generated by a malicious publisher over a nonmalicious publisher
σ_W	The ratio of whitelisted nonmalicious publishers over those in the unknown group
w	The cost parameter for the ad network's penalty on malicious publishers for the fraudulent ad traffic detected
θ	The proportion of penalty received by the ad network, $\theta \in (0, 1)$
c	The malicious publishers' fraud generation efficiency. Small values of c indicate high fraud generation efficiency, i.e., low cost for producing fraudulent ad traffic
k	The parameter for the ad network's mislabeling cost on nonmalicious publishers' ad traffic
r	The *raw quality* (or *quality* for short) of the ad network's ad fraud detection technology
$\tilde{r}$	The *effective quality* of the ad fraud detection technology, $\tilde{r} = \phi c r$, where ϕ is a scaling factor
v	The advertisers' valuation of a unit of valid traffic (e.g., click, impression, acquisition, etc.)
Decision Variables	
p_j	The ad network's payment for each charged ad traffic to type-j publishers, $j \in \{M, N\}$
p_W	The ad network's payment for each charged ad traffic to the whitelisted publishers
s_j	The sensitivity, or true positive rate (TPR), of the ad network's technology configuration for type-j publishers, $j \in \{M, N\}$
y_j	The respective false positive rate (FPR) given the ad network's technology configuration (s_j)
x_j	The amount of fraud ad traffic from malicious publishers of type-j, $j \in \{M, N\}$
Equilibrium Outcomes	
u_{ij}	The expected utility from participating in the ad network for publishers of true type i and classified type j, $i, j \in \{M, N\}$, in the unknown group
n_{gij}	The amount of good/valid ad traffic from type-j publishers with true type i, $i, j \in \{M, N\}$
n_{gj}	The amount of good/valid ad traffic from type-j publishers, $j \in \{M, N\}$, $n_{gj} = \sum_{i=M,N} n_{gij}$
n_{gW}	The amount of good/valid traffic from publishers in the whitelisted group
n_{fj}	The amount of fraudulent ad traffic from type-j publishers, $j \in \{M, N\}$
n_g	The total amount of good/valid ad traffic, i.e, $n_g = n_{gW} + n_{gN} + n_{gM}$
n_f	The total amount of fraudulent ad traffic, i.e., $n_f = n_{fM} + n_{fN} = (\alpha x_M + (1-\alpha) x_N) T_M$
n_c	The total amount of charged ad traffic, i.e., $n_c = n_{gW} + \sum_{j=M,N} (n_{gj}(1 - y_j) + n_{fj}(1 - s_j))$
n_t	The total amount of ad traffic (valid and fraud) in the ad network, i.e., $n_t = n_g + n_f$
$\tilde{v}$	The advertisers' updated ad valuation for each charged ad traffic, i.e., $\tilde{v} = v \left(\frac{n_g}{n_c}\right)$
π	The ad network's expected profit

Table EC.1 List of Key Parameters, Variables, and Equilibrium Outcomes

EC.2. Contextual Background of Online Advertising and Ad Fraud

In this section, we elaborate on a few critical elements of the fast-evolving online advertising ecosystem to provide important contextual information on (1) the important coordination role of ad networks, (2) the common payment models adopted by ad networks, (3) the related ad fraud, (4) the challenges in deterring online ad fraud, (5) the ad network's revenue sharing arrangement with publishers, (6) identification of different publishers' types, (7) the ad network's methods of detection and deterrence of online ad fraud, (8) how to set different sensitivities for ad fraud detection system, and (9) performance tracking of ad campaigns and publishers.

EC.2.1. The Important Coordination Role of Ad Networks

Ad networks serve as intermediaries that connect advertisers and publishers, facilitating the exchange of ad inventory between the two entities. In the subsequent paragraphs, we elaborate on the ad placement approach on publisher websites via ad networks. The essential steps are as follows:

1. **Advertiser Onboarding:** Advertisers take the lead by registering with the ad network, submitting their ad materials, establishing budgets, modifying frequency limits, and providing targeting criteria. This level of control allows advertisers to tailor their ad campaigns to their specific needs and goals.
2. **Publisher Onboarding:** Publishers register with the ad network and provide details about their ad inventory, including the types of ads they accept, the dimensions of their ad space, and occasionally their pricing.
3. **Matching:** Advertiser-publisher matching involves the ad network using algorithms to automatically pair the advertiser's ads with the most suitable publisher websites. This pairing is determined by content relevancy, audience targeting, and previous performance statistics.
4. **Ad placement and optimization:** The ads are strategically positioned on the publisher's website. The advertising network is committed to continuously enhancing the positioning of advertisements in real-time, using measures such as click-through rates and conversions.
5. **Performance tracking**: This allows advertisers and publishers to monitor the effectiveness of their ads via the ad network's dashboard. This dashboard gives metrics such as impressions, clicks, and conversions.

Figure EC.1 provides an overview of how an ad network facilitates ad placement and matching between advertisers and publishers. As explained above, the ad network determines the pairing of an ad with its target audience based on several factors, including content relevance and previous performance statistics. One thing worth noting is that advertisers generally have little control over the targeting of their ads. For example, Google's optimized targeting "is automatically turned on for all campaigns. Optimized targeting looks at information like keywords on the landing page or in your creative assets, then finds audiences that can meet the campaign's goals."[21] While advertisers can add criteria like keywords to optimized targeting, the overall control of the ad matching and placement falls in the hands of ad networks, and advertisers generally have minimal control over where their advertisements are displayed.

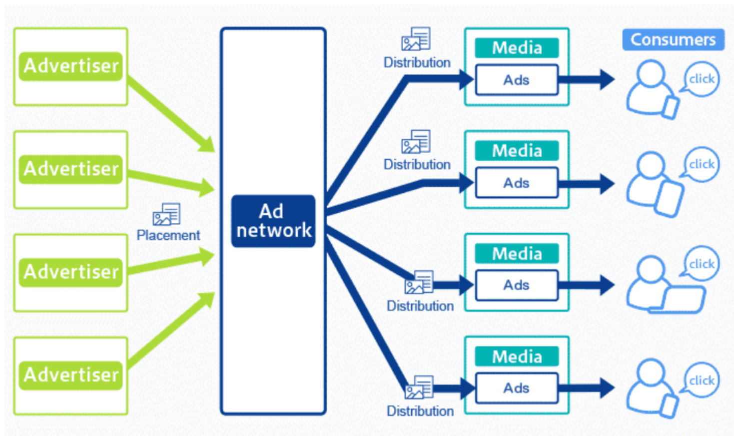


Figure EC.1 Ad Network Overview (Parti 2020)

Because of the dynamic nature of publisher-ads matching and ad placement, as described above, it is technically infeasible for advertisers to customize their willingness-to-pay (WTP) or prices at the publisher level. Thus, ad networks generally provide little flexibility to advertisers regarding their payment for ad traffic. For example, Google Ads offers two payment strategies to advertisers.[22] The first is an automated strategy, which uses Google AI to set ad payments based on "that ad's likelihood to result in a click or conversion" to help advertisers achieve a specific business goal.[23] The second is a manual strategy, which lets the advertisers manage their maximum payment themselves, e.g., a maximum price on the cost of someone clicking on your ads.[24]

Thus, advertisers cannot typically tailor their prices or WTP at the publisher level. Although an ad network may allow it, tracking ad performance across multiple publishers might be technically impractical or excessively expensive, hindering advertisers' ability to make educated judgments regarding ad pricing tailored to specific publishers.

EC.2.2. Online Advertising Payment Models

Online advertising payment mechanisms (Hu et al. 2016, Choi et al. 2020) refer to the models advertisers employ to remunerate publishers or platforms for showcasing their advertisements. Below are the three most prevalent payment models adopted in online advertising.

[21]See https://support.google.com/google-ads/answer/10537509

[22]See https://support.google.com/google-ads/answer/2472725

[23]See https://support.google.com/google-ads/answer/6325042

[24]See https://support.google.com/google-ads/answer/2464960

1. **CPM**: A standard payment mechanism used in advertising is the *cost per mille* (CPM) model, where advertisers pay a set sum for every one thousand ad *impressions*, independent of user interaction. This strategy offers advertisers extensive coverage and brand visibility, as they get charged based on their ad frequency. Nevertheless, it fails to ensure engagement or conversions, rendering it more appropriate for brand awareness efforts than direct response objectives.
2. **CPC**: Another often employed payment mechanism is the *cost per click* (CPC) model, in which advertisers are only charged when users click on their advertisements. This strategy provides a more precise metric for user engagement. It enables advertisers to only pay for real interactions with their advertisements, making it ideal for increasing website traffic or generating leads.
3. **CPA**: The *cost per action* (CPA) model is a payment model in which advertisers are only charged when certain predetermined activities are successfully carried out, such as completing a purchase, subscribing to a newsletter, or filling out a form. This methodology ensures that advertiser fees are directly linked to the specific objectives they want to achieve, as they are only charged for quantifiable conversions. CPA is commonly employed in performance-based advertising campaigns, wherein advertisers aim to accomplish specified objectives, such as attracting new consumers or generating revenue. Nevertheless, CPA campaigns necessitate meticulous monitoring and fine-tuning to guarantee that the cost per acquisition stays under acceptable thresholds and provides advertisers with a favorable return on investment.

An ad network may adopt one or several of the payment models above. In the ever-changing world of online advertising, advertisers can select the payment method(s) that aligns with their campaign goals, financial resources, and intended results.

EC.2.3. Online Ad Fraud

Online ad fraud involves deceptive tactics that produce fraudulent clicks, impressions, or conversions, deceiving advertisers and wasting their money (Zhu et al. 2017b). Corresponding to the payment models described above, three common types of ad fraud are as follows.

- **Impression fraud** is a deceitful method of ad fraud where fraudulent websites or networks create false impressions by displaying advertising in regions that cannot be seen or employing invisible web-frames (Jackson 2025). This strategy employs deception to mislead advertisers into thinking that authentic users are viewing their advertisements while they are not. Consequently, this leads to a waste of advertising funds and a reduction in the returns on investment. Examples of impression fraud are ad stacking, where multiple ads are stacked on each other, inflating impressions and clicks artificially, and pixel stuffing, where multiple ads are placed in a single pixel, misleadingly increasing impressions.

- **Click fraud** is a common form of ad fraud in which automated bots or human actors deliberately increase ad clicks without genuine interest in the promoted content (Zhu et al. 2017a). These deceitful clicks deplete marketers' financial resources and distort campaign effectiveness measurements, resulting in unreliable data for campaign improvement. Examples of click fraud include click farms, where low-paid workers are used to click on ads, fraudulently boosting click-through rates, and cookie stuffing, which involves placing affiliate cookies on users' browsers without their knowledge, leading to false clicks and commissions.
- **Action fraud** is a fraudster's deliberate manipulation of a user's activity to generate personal profit. Instances of such acts include using fake devices or automated bots to install the software inflating the number of installations.

Figure EC.2 Ad Fraud Overview (Zezelj 2017)

Ad fraud poses a significant threat to online advertising ecosystems, where unscrupulous publishers use unethical techniques to claim undeserved commissions. In Figure EC.2, we also illustrate some standard methods for conducting ad fraud. These various types of online ad fraud erode advertisers' trust in digital advertising platforms and undermine the effectiveness and integrity of the entire online advertising ecosystem, highlighting the urgent need for stricter regulations to combat such unethical practices (Kahn 2023).

EC.2.4. Why has Online Ad Fraud Persisted?

Ad fraud persists in online advertising despite the use of fraud detection methods and multiple payment models. This is primarily due to several obstacles within the digital ecosystem (Davies 2025).

An essential factor is the constant advancement in the expertise of fraudsters, who consistently develop novel methods to circumvent detection systems (Zhu et al. 2017b). As advancements in fraud detection technologies occur, fraudsters also enhance their techniques, resulting in an ongoing game of evasion between fraudsters and security measures.

Furthermore, the vast magnitude and intricate nature of the online advertising environment contribute to the continued existence of ad fraud (Kahn 2024). Given the vast number of advertisements displayed on various platforms and websites daily, effectively monitoring and identifying every fraudulent behavior case becomes complicated. Moreover, the anonymity and worldwide accessibility of the internet facilitate the activities of fraudsters, enabling them to operate across national boundaries and elude law authorities (Fulgoni 2016).

Moreover, the incentive system inherent in online advertising worsens the situation. Advertisers frequently prioritize metrics such as clicks, impressions, or conversions, emphasizing numbers rather than quality (Pooranian et al. 2021). The focus on performance metrics provides openings for fraudsters to exploit weaknesses in the system to generate deceptive interactions and illegal income. Furthermore, the absence of transparency and accountability in certain aspects of the digital advertising supply chain exacerbates the prevalence of ad fraud (Gordon et al. 2021). Advertisers may lack comprehensive visibility over the placement of their ads or the allocation of their funds, which increases the likelihood of undetected fraud.

Overall, the multifaceted nature of online ad fraud, coupled with the challenges inherent in the digital advertising ecosystem, ensures its persistence despite ongoing efforts to combat it.

EC.2.5. Ad Network's Revenue Sharing with Publishers

Given its central role in the ecosystem, ad networks are often in control of the payment to participating publishers. For example, both Google and Microsoft decide on the payment to publishers, which can track their earning balance and estimated earnings in their accounts.[25] In terms of payment arrangement, ad networks commonly share a fixed portion of the revenue with publishers. For example, Google discloses that it shares a fixed percentage of revenue with publishers in its AdSense and that the "percentages are consistent, regardless of a publisher's geographic location, and are not in any way averaged between publishers."[26] In addition to AdSense, which is best for publishers "who want more automation for their ad solutions and have a small dedicated ad management team," another key advertising product of Google is Ad Manager, which is "an ad management platform for large publishers."[27] Similar to AdSense, Ad Manager also shares a portion of ad revenue with publishers, although the percentage is different.[28]

[25] See https://help.ads.microsoft.com/#apex/pcv4/en/07506/-1

[26] See https://support.google.com/adsense/answer/180195

[27] See https://support.google.com/admanager/answer/9234653

[28] See https://blog.google/products/admanager/display-buying-share-revenue-publishers/

While Google had attempted to implement publisher-specific "smart pricing" where the payment to publishers is adjusted based on their performance, its effectiveness is limited for a variety of reasons (Graham 2019). Specifically, smart pricing applies at the account level, which can negatively impact publishers with a mix of high and low-performing sites under one account. Furthermore, for low-performing accounts, smart pricing can result in lower payouts to publishers, as Google adjusts the cost to advertisers based on their perceived value. Overall, advertisers may end up paying too much for low-value traffic and too little for high-value traffic across a network, creating inefficiencies in the marketplace (Lolk 2024). This helps explain why Google did not widely adopt the system despite the pilot implementation and introduction of smart pricing over a decade ago.

We employ the perspectives above to establish the pricing mechanism for our model, wherein the ad network primarily focuses on ad traffic (e.g., impressions or clicks) and advertisers' valuation of ad traffic (their ad payments) rather than performance-based characteristics at the publisher level to determine the revenue shared with publishers. While the ad network may further categorize publishers into different types (e.g., AdSense and Ad Manager for Google) and apply different revenue-sharing percentages accordingly, these percentages remain consistent within each type. In other words, based on classified ad traffic and predefined publisher types, the ad network establishes incentives that influence publishers' participation and potential engagement in ad fraud. As outlined above, this framework generally aligns with anecdotal evidence and prevailing industry practices.

EC.2.6. Identification of Publishers' Type

Ad networks typically serve a diverse range of publishers, including large and reputable publishers, as well as small or new websites seeking to monetize through ad revenue. Although it is generally difficult to determine the true type (malicious or nonmalicious) of a publisher, there are ways for ad networks to make informed inferences about some publishers' types. From an economic perspective, larger, well-known publishers generally have a lower risk of ad fraud due to prohibitively high moral or reputational costs (Fulgoni 2016). For example, digital journalism by well-known publishers such as Forbes and The Wall Street Journal has negligible reports of ad fraud (Braun and Eklund 2019). From a technical perspective, Google can leverage the vast amount of data on publishers to assess their risk levels and identify those with minimal or no risk. Correspondingly, we consider that an ad network may designate some of these publishers to a "whitelist" in the paper and that their ad traffic is not subject to the ad fraud detection system's classifications.

Conversely, certain publishers may have strong incentives to engage in ad fraud due to the minimal moral and reputational costs. Examples include relatively unknown or newly established websites with questionable credibility. Ad networks typically institute strict policies to prevent their participation. For example, Google's AdSense program requires that publishers be "activated" before participating in the network and serving ads. To activate, Google will "review your entire site to check it complies with the AdSense Program policies."[29] Another method to stop these "bad apples" from joining is the use of minimum traffic criteria when considering publisher approval (see, e.g., Clapperton 2024). Some recent examples of such thresholds are Raptive (minimum of 100,000 pageviews in the past 30 days) and Mediavine (50,000 sessions in the last 30 days).

In addition to preventive measures, ad networks enforce strict policies to penalize or remove malicious publishers upon detection. Google's AdSense policy states that if a publisher fails to comply with these policies, Google will "disable ad serving to your site and/or disable your AdSense account at any time. If your account is disabled, you will not be eligible for further participation in the AdSense program."[30] These measures collectively facilitate the identification of publishers with strong malicious intent and a history of recurrent fraudulent activities, thereby safeguarding the reputation of ad networks. Accordingly, the ad network may designate these publishers to a "blacklist." As "blacklisted" publishers are either prohibited from participating or removed from the network upon detection of violating the ad networks' program policies, we consider this "blacklist" group to be effectively barred from participating in the network to serve ads and exclude them from explicit consideration in the model.

To summarize, the identification of whitelisted and blacklisted publishers typically draws on a multifaceted set of criteria, including the nature and legitimacy of their business, length of business establishment, sizes and content quality, current and historical performance metrics, and other relevant contextual factors. In contrast, the classification and detection of fraudulent ad traffic is primarily data-driven, relying on granular ad stream information, such as impressions and clicks, and the application of statistical analysis or data mining techniques to identify anomalous or suspicious behavioral patterns.

For the remaining publishers whose true types are difficult to predict, ad networks can utilize a collection of measures, such as publisher size, past performance, and advertisers' reports of suspicious activities, to make inferences about their malicious intent. Intuitively, the classifications of publishers' types based on such inferences are imperfect and can mistakenly classify nonmalicious publishers as malicious and vice versa. We refer to this group of publishers as the "unknown" group in the paper. The "unknown" publishers' true type is denoted by a subscript $i \in \{M, N\}$, where M and N indicate malicious and nonmalicious, respectively. Their classified or inferred type is denoted by a subscript $j \in \{M, N\}$. The ad network can correctly infer a publisher's true type with probability α. We let $\alpha \in [\frac{1}{2}, 1)$ to indicate this classifier is at least as good as a random classifier.

[29]See https://support.google.com/adsense/answer/10162

[30]See https://support.google.com/adsense/answer/48182

EC.2.7. Detection and Deterrence of Online Ad Fraud

Given the importance of ad fraud and its profound implications, ad networks have developed and deployed a sophisticated suite of tools to filter and detect fraudulent ad traffic. For example, Google has a dedicated Ad Traffic Quality Team that "uses live reviewers, automatic filters, machine learning, and deep research to detect and filter as much invalid and fraudulent activity as possible."[31] Technically, Google's defense against invalid activity comprises the following three components.[32]

- **Automated detection:** It employs machine learning and advanced online filters to filter and detect fraudulent ad traffic in real-time or soon after. As such, fraudulent ad traffic is filtered out before it is charged to advertisers. The filters are continuously monitored and updated by specialists to ensure their efficiency.
- **Manual review:** Because these online filters are usually effective against simple forms of known fraudulent activities, Google also uses a manual detection system to manually review issues flagged by advertisers or its automated system using more sophisticated tools and data gathered over a much longer period (e.g., weeks). This helps identify more complicated forms of fraudulent activities and new threats which helps update and improve the filters used in the automated detection.
- **Advanced research:** In addition to the automated and manual systems above, Google continuously researches emerging types of fraudulent activities that its filters have yet to detect and investigates unique or smaller cases that might otherwise go unnoticed.

In addition to detecting and filtering fraudulent activities at the traffic level, another effective way to curb ad fraud is to suspend or disable a publisher's account if it generates excessive fraudulent ad traffic. However, if the invalid traffic is not excessive or appears unintentional, the ad network may temporarily suspend the account and withhold payments to the publisher until the issue is resolved.[33] Publishers with disabled accounts have the right to appeal. For example, Google carefully reviews all appeals that provide sufficient information for the investigation, and it will restore the account if there is a valid reason or evidence for reactivation.[34] However, if a publisher's account is disabled, it is ineligible for further participation in the AdSense program.

EC.2.8. Setting Different Sensitivities for Detection of Ad Fraud

As mentioned in the paper, the quality profile of a fraud detection system can be typically measured or characterized using an ROC curve, which can be derived experimentally or analytically

[31] See https://www.google.com/ads/adtrafficquality/

[32] See https://www.google.com/ads/adtrafficquality/how-we-prevent-it/

[33] See https://support.google.com/google-ads/answer/9841640

[34] See https://support.google.com/admanager/answer/2848628

(Cavusoglu et al. 2009). Generally, the fraud detection system classifies ad traffic based on whether a numerical score computed from the ad traffic exceeds a threshold value or whether it satisfies a set of rules or a combination of both.

Consider the case where an ad fraud detection system uses a numerical score x computed from ad traffic and a threshold value t to detect fraud ad traffic. Let the system classify ad traffic as fraudulent if $x > t$. Further, we denote $f_G(x)$ and $f_F(x)$ the probability density functions of x for valid/good and fraudulent ad traffic, respectively, as illustrated in Figure EC.3. Intuitively, when these two distributions completely overlap, the detection system cannot distinguish between fraudulent and valid ad traffic at all. On the contrary, if they are completely separated, then the ad fraud detection system can perfectly discern fraud traffic from valid ones.

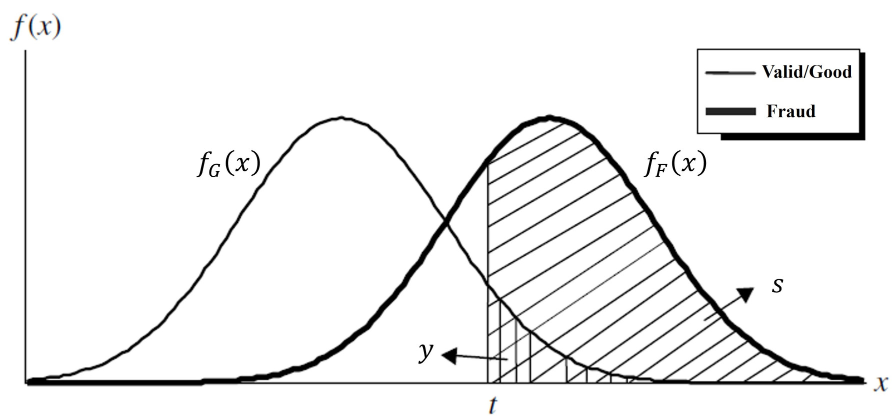


Figure EC.3 Illustration of ROC Configuration, adapted from Figure 1 in Cavusoglu et al. (2009)

In reality, the two probability density functions have an overlapping region, indicating that when configuring the technology, i.e., selecting a threshold value of t, it results in the detection of some ad fraud but also the misdetection of some valid traffic as fraud. For the example illustrated in Figure EC.3, increasing the threshold t results in less fraud traffic being detected (i.e., a lower true positive rate) and also less misdetection of valid traffic (i.e., a lower false positive rate). On the other hand, lowering the threshold t has the opposite effect. The shape of the ROC curve depends on these two probability density functions. In general, the ROC curve is "steeper" when the distributions are more separated, indicating a better ability of the detection system to distinguish between fraud and valid ad traffic as it can achieve a lower false positive rate for the same true positive rate.

EC.2.9. Tracking Performance of Ad Campaigns and Publishers

To assist their advertiser clients in tracking ad performance, ad networks provide a suite of analytics tools to track different metrics of ad campaign performance, including how much ad traffic was charged to the ad network and their respective costs. For instance, Google provides its publishers with ad-performance monitoring tools to help them make informed investment decisions. As such, advertisers often know the amount of charged ad traffic. Figure EC.4 provides an example of ad campaign reporting in Google Analytics.

In addition, the ad campaign performance is determined by the amount of truly valid ad traffic, which can be estimated by the ad network or advertisers using various analytic tools and data sources. For example, as shown in Figure EC.4, Google Analytics provides a variety of ad campaign metrics (e.g., pages per session, goal conversion rate, goal completion, and goal values) to help advertisers estimate campaign performance. Thus, even though advertisers may assess and estimate the overall amount of valid ad traffic, they cannot do so at each publisher level because ad networks generally provide performance tracking at the aggregate level instead of for individual publishers.

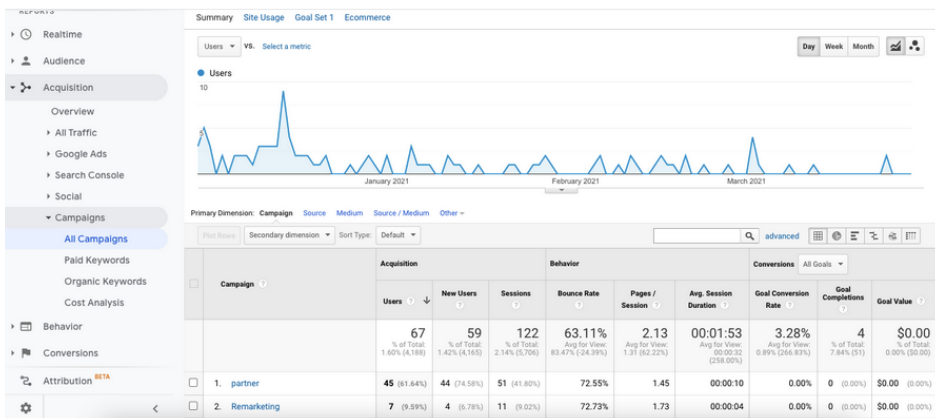


Figure EC.4 An Example of Campaign Performance Reporting in Google Analytics (Thompson 2021)

Similarly, the ad network cannot accurately evaluate the true performance of individual publishers. While this is partly due to the inherent complexities of predicting the true nature of ad traffic, another important reason is that the ad network generally has access to the traffic data before the events, such as clicks or impressions, which limits its ability to predict the true nature of the ad traffic. On the other hand, advertisers may have access to exclusive information which helps them better assess the true performance. For example, advertisers have access to the "post-click" stream data - a sequence of actions and pages a user navigates after clicking on an ad, providing valuable insights into the true nature of the ad traffic. Additionally, advertisers may utilize vanity URLs, branded hashtags, or post-campaign analysis to assess the actual campaign performance (Lucid 2021).

References

Braun JA, Eklund JL (2019) Fake news, real money: Ad tech platforms, profit-driven hoaxes, and the business of journalism. *Digital Journalism* 7(1):1–21.

Cavusoglu H, Raghunathan S, Cavusoglu H (2009) Configuration of and interaction between information security technologies: The case of firewalls and intrusion detection systems. *Information Systems Research* 20(2):198–217.

Choi H, Mela CF, Balseiro SR, Leary A (2020) Online display advertising markets: A literature review and future directions. *Information Systems Research* 31(2):556–575.

Clapperton N (2024) Raptive vs Mediavine: Which is the better ad network? URL https://sheknowsseo.co/raptive-vs-mediavine/.

Davies R (2025) The state of digital advertising fraud in 2024. URL https://adsdax.com/the-state-of-digital-advertising-fraud-in-2024/.

Fulgoni GM (2016) Fraud in digital advertising: A multi-billion-dollar black hole: How marketers can minimize losses caused by bogus web traffic. *Journal of Advertising Research* 56(2):122–125.

Gordon BR, Jerath K, Katona Z, Narayanan S, Shin J, Wilbur KC (2021) Inefficiencies in digital advertising markets. *Journal of Marketing* 85(1):7–25.

Graham K (2019) How can publishers avoid smart-pricing on AdSense. URL https://www.monetizemore.com/blog/can-publishers-avoid-smart-pricing-adsense/.

Hu Y, Shin J, Tang Z (2016) Incentive problems in performance-based online advertising pricing: Cost per click vs. cost per action. *Management Science* 62(7):2022–2038.

Jackson J (2025) Click fraud uncovered: Ultimate guide to protecting ad Spend. URL https://hitprobe.com/blog/click-fraud-complete-guide#types-of-click-fraud.

Kahn R (2023) Why aren’t there laws to stop ad fraud? URL https://www.anura.io/blog/why-arent-there-laws-to-stop-ad-fraud.

Kahn R (2024) Ad fraud cost advertisers $125 billion in 2023. URL https://www.anura.io/blog/ad-fraud-cost-advertisers-125-billion-in-2023.

Lolk A (2024) What every CMO should know about conversion lag and smart bidding. URL https://savvyrevenue.com/blog/conversion-lag/.

Lucid (2021) Exploring the latest updates to impact measurement. https://www.cint.com/products/lucid-measurement/.

Parti A (2020) What is an ad network and how does it work? URL https://rocketium.com/academy/what-is-an-ad-network-and-how-does-it-work/.

Pooranian Z, Conti M, Haddadi H, Tafazolli R (2021) Online advertising security: Issues, taxonomy, and future directions. *IEEE Communications Surveys & Tutorials* 23(4):2494–2524.

Thompson E (2021) How to use Google Analytics to track campaigns. URL https://getjess.com/how-to-use-google-analytics-to-track-campaigns/.

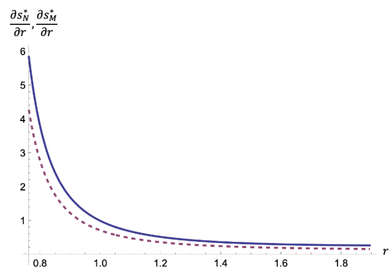
Zezelj V (2017) 7 types of ad fraud that can hurt your business. URL https://adcumulus.com/blog/ad-fraud-types/.

Zhu X, Tao H, Wu Z, Cao J, Kalish K, Kayne J (2017a) Ad fraud categorization and detection methods. *Fraud Prevention in Online Digital Advertising*, 25–38 (Springer International Publishing).

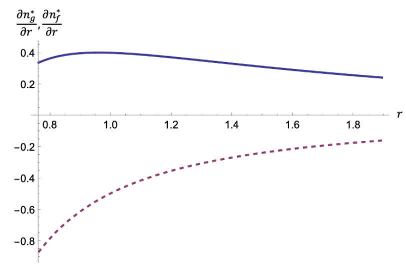
Zhu X, Tao H, Wu Z, Cao J, Kalish K, Kayne J (2017b) Ad fraud detection tools and systems. *Fraud Prevention in Online Digital Advertising*, 45–49 (Springer International Publishing).

EC.3. Illustration of Additional Comparative Statics Results

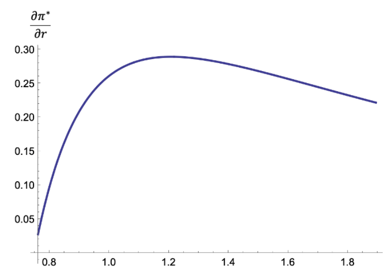
In the following subsections, we illustrate the additional comparative statics analysis numerically to help understand the intuitions of the results in the paper.



(a) Illustrating $\frac{\partial s_N^*}{\partial r}$ (solid), $\frac{\partial s_M^*}{\partial r}$ (dashed)

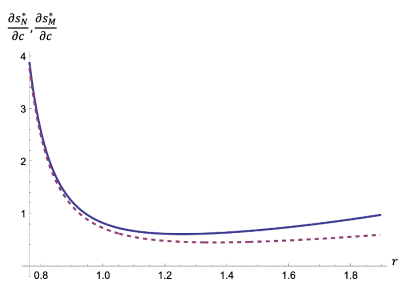


(b) Illustrating $\frac{\partial n_g^*}{\partial r}$ (solid), $\frac{\partial n_f^*}{\partial r}$ (dashed)

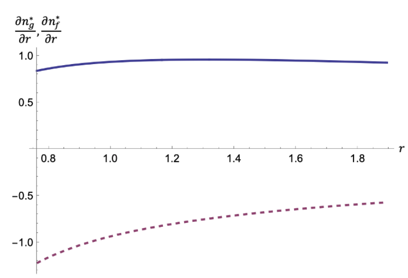


(c) Illustrating $\frac{\partial \pi^*}{\partial r}$

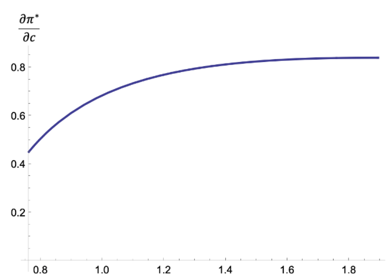
Figure EC.5 Additional Comparative Statics Analysis for Proposition 1 - Effect of Improved Technology Quality (r). Values of Parameters: $\{v=2, k=0.2, c=0.5, w=0.05, \alpha=0.65, T_N=2, \phi=1, b=1, \theta=0.25, \sigma_W=0.2\}$.



(a) Illustrating $\frac{\partial s_N^*}{\partial c}$ (solid), $\frac{\partial s_M^*}{\partial c}$ (dashed)

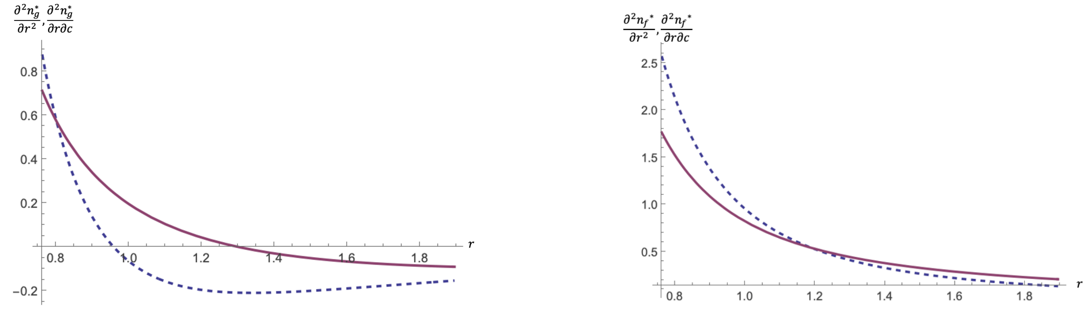


(b) Illustrating $\frac{\partial n_g^*}{\partial c}$ (solid), $\frac{\partial n_f^*}{\partial c}$ (dashed)



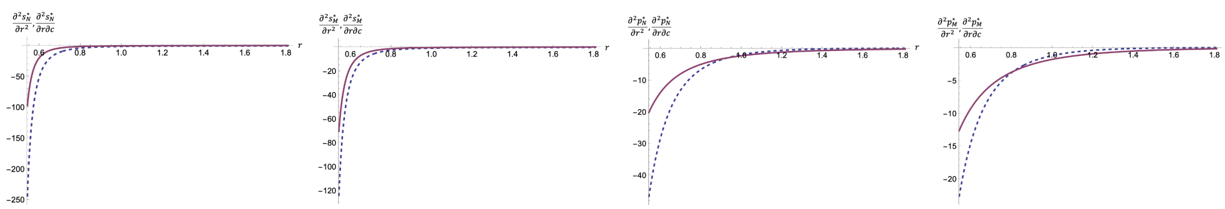
(c) Illustrating $\frac{\partial \pi^*}{\partial c}$

Figure EC.6 Additional Comparative Statics Analysis for Proposition 2 - Effect of Decreased Fraud Generation Efficiency (c). Values of Parameters: $\{v=2, k=0.2, c=0.5, w=0.05, \alpha=0.65, T_N=2, \phi=1, b=1, \theta=0.25, \sigma_W=0.2\}$.



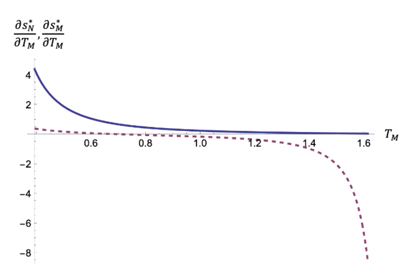
(a) Illustrating $\frac{\partial^2 n_g^*}{\partial r^2}$ (Solid) and $\frac{\partial^2 n_g^*}{\partial r \partial c}$ (Dashed) **(b)** Illustrating $\frac{\partial^2 n_f^*}{\partial r^2}$ (Solid) and $\frac{\partial^2 n_f^*}{\partial r \partial c}$ (Dashed)

Figure EC.7 Additional Comparative Statics Analysis for Illustrating Proposition 3 - Part 1. Values of Parameters: $\{v=2, k=0.2, c=0.5, w=0.05, \alpha=0.65, T_N=2, \phi=r=1, b=1, \theta=0.25, \sigma_W=0.2\}$.

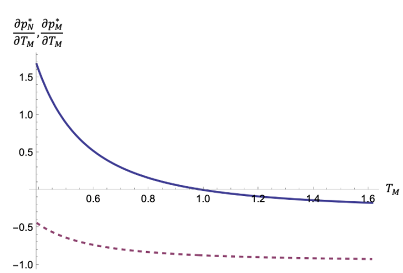


(a) Illustrating $\frac{\partial^2 s_N^*}{\partial r^2}$ (Solid) and $\frac{\partial^2 s_N^*}{\partial r \partial c}$ (Dashed) **(b)** Illustrating $\frac{\partial^2 s_M^*}{\partial r^2}$ (Solid) and $\frac{\partial^2 s_M^*}{\partial r \partial c}$ (Dashed) **(c)** Illustrating $\frac{\partial^2 p_N^*}{\partial r^2}$ (Solid) and $\frac{\partial^2 p_N^*}{\partial r \partial c}$ (Dashed) **(d)** Illustrating $\frac{\partial^2 p_M^*}{\partial r^2}$ (Solid) and $\frac{\partial^2 p_M^*}{\partial r \partial c}$ (Dashed)

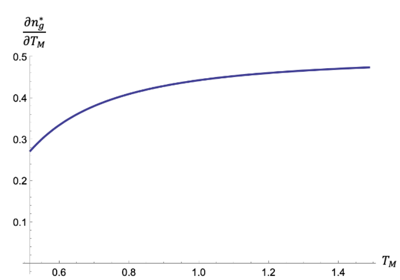
Figure EC.8 Additional Comparative Statics Analysis for Proposition 3 - Part 2. Values of Parameters: $\{v=2, k=0.2, c=0.5, w=0.05, \alpha=0.65, T_N=2, \phi=r=1, b=1, \theta=0.25, \sigma_W=0.2\}$.



(a) Illustrating $\frac{\partial s_N^*}{\partial T_M}$ (solid), $\frac{\partial s_M^*}{\partial T_M}$ (dashed) with $b=1$

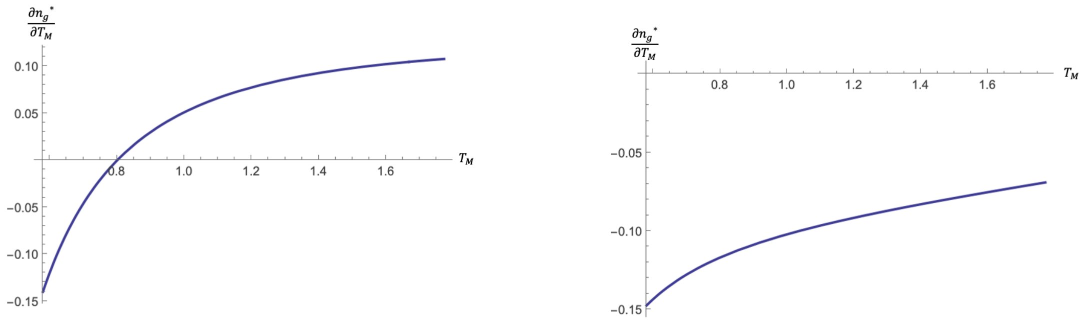


(b) Illustrating $\frac{\partial p_N^*}{\partial T_M}$ (solid), $\frac{\partial p_M^*}{\partial T_M}$ (dashed) with $b=1$



(c) Illustrating $\frac{\partial n_g^*}{\partial T_M}$ with $b=0.5$

Figure EC.9 Additional Comparative Statics Analysis for Illustrating Proposition 4. Values of Parameters: $\{v=2, k=0.15, c=0.5, w=0.2, \alpha=0.65, T_N=2, \phi=r=1, \theta=0.25, \sigma_W=0.3\}$.



(a) Illustrating $\frac{\partial n_g^*}{\partial T_M}$ **(b)** Illustrating $\frac{\partial n_f^*}{\partial T_M}$.

Figure EC.10 Additional Comparative Statics Analysis for Illustrating Proposition 5. Values of Parameters: $\{v = 2, k = 0.15, c = 0.5, w = 0.2, \alpha = 0.65, T_N = 2, \phi = r = 1, b = 0.25, \theta = 0.25, \sigma_W = 0.3\}$**.**

EC.4. Proofs of Lemma and Propositions in the Main Model

In the following subsections, we present the proofs for Lemma 1 and Propositions 1, 2, 3, 4, and 5.

EC.4.1. Proof of Lemma 1

Proof. We solve the game using backward induction. In Stage 3, advertisers' ad valuation for each charged ad traffic by the ad network $\tilde{v} = v(\frac{n_g}{n_c})$ is updated, given the *charged* traffic (n_c) and the valid traffic (n_g). The charged traffic, i.e., $n_c = n_{gW} + \sum_{j=M,N}(n_{gj}(1-y_j) + n_{fj}(1-s_j))$, where n_{gW}, n_{gj}, and n_{fj}, $j \in \{M, N\}$ are given in the paper, is impacted by the corresponding technology configuration (s_j). Malicious publishers generate fraudulent traffic in Stage 2 given the technology configuration or true positive rate (s_j) and payment (p_j) set in Stage 1 by the ad network. Similarly, the valid traffic is determined by the publisher's participation decisions in Stage 2, given the ad network's decisions in Stage 1. Hence, the advertiser valuation update is impacted by the ad network's previous stage actions (Stage 1) and the malicious publisher (Stage 2). Specifically, the following is the valuation update in Stage 3:

$$\tilde{v} = \frac{n_{gM} + n_{gN} + n_{gW}}{n_c} = \frac{bT_M + T_N(\alpha p_N(1 - wy_N - y_N) + (1-\alpha)p_M(1 - wy_M - y_M) + p_W\sigma_W)}{\sum_{j=M,N}(n_{gj}(1-y_j) + n_{fj}(1-s_j)) + n_{gW}}.$$

In Stage 2, malicious publishers set the amount of fraudulent traffic, given the technology configuration s_j and the payment p_j set in Stage 1. That is, the malicious publishers solve the following problem of maximizing their utility u_M:

$$\max_{x_j \geq 0} \quad u_{Mj} = p_j(1-y_j)D_M + p_j x_j(1-s_j) - w x_j s_j p_j - c x_j^2. \tag{EC.1}$$

It follows from the Second Order Condition test that this objective function u_{Mj} is concave in x_j, as shown in the condition L_1 below:

$$L_1 : \frac{\partial^2 u_{Mj}}{\partial x_j^2} = -2c < 0, \ \forall \ x_j \geq 0.$$

So, using the first order condition $\frac{\partial u_{Mj}}{\partial x_j} = 0$ yields the optimal amount of fraud traffic produced by malicious publishers $x_j^* = \frac{p_j(1-s_j(w+1))}{2c}$. Since we consider non-negative fraud traffic from malicious publishers, $x_j \geq 0$, we have that $s_j \leq \frac{1}{1+w}$.

Next, moving on to Stage 1, we have that the ad network sets its technology configuration (s_j) and payment (p_j) to maximize its profit (π), as defined below:

$$\begin{aligned} \max_{p_j, s_j, p_W} \quad \pi = (\tilde{v} - p_W)n_{gW} &+ \sum_{j=M,N}(\tilde{v} - p_j)(n_{gj}(1-y_j) + n_{fj}(1-s_j)) \\ &+ \theta w \sum_{j=M,N} p_j(s_j n_{fj} + y_j \sum_{i=M,N} n_{gij}) - k \sum_{j=M,N} y_j n_{gj} p_j, \end{aligned} \tag{EC.2}$$

$$\text{s.t.} \quad y_j = (1-\tilde{r})s_j, \\ 0 \le s_j \le \frac{1}{1+w}.$$

where $\tilde{r} = cr\phi$ as mentioned in Table EC.1.

We use the Lagrangian method to formulate the problem as follows:

$$\mathbb{L} = \pi + \lambda_1\left(\frac{1}{1+w_N} - s_N\right) + \lambda_2(s_N) + \lambda_3\left(\frac{1}{1+w_M} - s_M\right) + \lambda_4(s_M), \tag{EC.3}$$

where $\lambda_1, \cdots, \lambda_4$ are the Lagrangian multipliers. Assuming the case where $\lambda_1, \cdots, \lambda_4 = 0$, we get the following solution:

$$\begin{aligned}
p_W^* &= \frac{v}{2}, \\
p_M^* &= \frac{A_1(\alpha b T_M(1+w\theta-k) + (1-\alpha)(w+1)T_N v)}{A_2(4(1-\alpha)^2 c^2 T_N^2 (w(1-\theta)+k)^2)}, \\
p_N^* &= \frac{A_3((1-\alpha)wT_M(1-\theta-(\theta+1)\tilde{r}) + 2\alpha c w(1-\theta)T_N(1-\tilde{r})^2)}{A_4(\alpha k^2 T_N(1-\tilde{r})^2 - 2(1-\alpha)krT_M\phi(1-\tilde{r}) - 2(1-\alpha)cT_M\tilde{r}^2)}, \\
s_M^* &= \frac{A_5(w(1-\theta)((1-\alpha)T_N v + \alpha b T_M) + 2\alpha b k T_M)}{A_6(\alpha b T_M(k+1-\theta(1-k)) + (1-\alpha)(1-\theta)T_N v)}, \\
s_N^* &= \frac{A_7(2\alpha c w(1-\theta)T_N(1-\tilde{r}) + 2\alpha c k T_N(1-\tilde{r}) - 2(1-\alpha)\tilde{r}T_M + (1-\alpha)wT_M(1-2\tilde{r}-\theta))}{A_8(1+k+\tilde{r}(\theta(3-k)+1-k) - \theta(1-k))}.
\end{aligned}$$

where the terms $A_1, \cdots, A_8$ are defined as follows:

$$\begin{aligned}
A_1 = 2c\Bigg(&\frac{\alpha\tilde{r}T_M(w(1+\theta)+2)\Big(\alpha b T_M(w\theta-k+1) + (\alpha-1)(w+1)T_N v\Big)}{\alpha b T_M(-w\theta+k-1) + (\alpha-1)(w+1)T_N v} \\
&+ \alpha T_M\left(\frac{2\alpha b(w+1)kT_M}{\alpha b T_M(w\theta-k+1) + (\alpha-1)(w+1)T_N v} + w\theta - w\right) + 2(\alpha-1)cT_N\tilde{r}^2(w(1-\theta)+k) \\
&- 4(\alpha-1)cT_N\tilde{r}(w(1-\theta)+k) + 2(\alpha-1)cT_N(w(1-\theta)+k)\Bigg),
\end{aligned}$$

$$\begin{aligned}
A_2 = &\frac{\alpha^2 w^2(\theta-1)^2 T_M^2}{4(\alpha-1)^2 c^2 T_N^2 (w(1-\theta)+k)^2} + \tilde{r}^2 \\
&+ \frac{2\tilde{r}\Big(\alpha(w+1)rT_M\phi(w\theta-k+1) - (\alpha-1)T_N(w(1-\theta)+k)^2\Big)}{(\alpha-1)T_N(w(1-\theta)+k)^2} + \frac{\alpha w(\theta-1)T_M}{(\alpha-1)cT_N(w(1-\theta)+k)} \\
&+ \frac{\alpha r T_M\phi\Big(w^2(\theta-1)^2 - w(\theta-3)k + 2k\Big) + (\alpha-1)T_N(w(1-\theta)+k)^2}{(\alpha-1)T_N(w(1-\theta)+k)^2},
\end{aligned}$$

$$A_3 = 2c\Bigg((\alpha-1)bT_M\Bigg(\frac{2\Big(\alpha ck^2T_N(\tilde{r}-1)^2 - k(\tilde{r}-1)\Big(\alpha cT_N(\tilde{r}-1)+(\alpha-1)T_M\Big)+(\alpha-1)\tilde{r}T_M\Big)}{(1-\alpha)wT_M(-\tilde{r}(1+\theta)-\theta+1)+2\alpha cw(1-\theta)T_N(1-\tilde{r})^2}$$
$$+\frac{(\alpha-1)wT_M\Big(-\tilde{r}(\theta(k-3)+k-1)-\theta+\theta k+k+1\Big)-2\alpha cwT_N(-\theta+(2\theta-1)k+1)(\tilde{r}-1)^2}{(1-\alpha)wT_M(-\tilde{r}(1+\theta)-\theta+1)+2\alpha cw(1-\theta)T_N(1-\tilde{r})^2}$$
$$+\frac{w\theta\Big((\alpha-1)wT_M(\tilde{r}(1+\theta)-\theta+1)+2\alpha cw(\theta-1)T_N(\tilde{r}-1)^2\Big)}{(1-\alpha)wT_M(-\tilde{r}(1+\theta)-\theta+1)+2\alpha cw(1-\theta)T_N(1-\tilde{r})^2}\Bigg)$$
$$+\alpha(w+1)T_Nv\Bigg(\frac{2\alpha ckT_N(\tilde{r}-1)^2+2(\alpha-1)\tilde{r}T_M}{(1-\alpha)wT_M(-\tilde{r}(1+\theta)-\theta+1)+2\alpha cw(1-\theta)T_N(1-\tilde{r})^2}+1\Bigg)\Bigg),$$

$$A_4 = \frac{4\alpha cwT_N\Big(2(\alpha-1)(\theta+1)T_M\tilde{r}^2-k(\tilde{r}-1)\Big((\alpha-1)T_M(2\tilde{r}+\theta-1)+2\alpha c(\theta-1)T_N(\tilde{r}-1)\Big)\Big)}{\alpha k^2T_N(\tilde{r}-1)^2-2(\alpha-1)krT_M\phi(\tilde{r}-1)+2(\alpha-1)rT_M\phi\tilde{r}}$$
$$+4\alpha c^2T_N+\frac{w^2(4\alpha cT_N\tilde{r}^2\Big(\alpha c(\theta-1)^2T_N+2(\alpha-1)\theta T_M\Big))}{\alpha k^2T_N(\tilde{r}-1)^2-2(\alpha-1)krT_M\phi(\tilde{r}-1)+2(\alpha-1)rT_M\phi\tilde{r}}$$
$$-\frac{4\alpha c^2(\theta-1)^2rT_N\phi(2\alpha cT_N-\alpha T_M+T_M)}{\alpha k^2T_N(\tilde{r}-1)^2-2(\alpha-1)krT_M\phi(\tilde{r}-1)+2(\alpha-1)rT_M\phi\tilde{r}}$$
$$+\frac{(\theta-1)^2(2\alpha cT_N-\alpha T_M+T_M)^2}{\alpha k^2T_N(\tilde{r}-1)^2-2(\alpha-1)krT_M\phi(\tilde{r}-1)+2(\alpha-1)rT_M\phi\tilde{r}},$$

$$A_5=\alpha T_M-\frac{2(\alpha-1)cT_N\tilde{r}(w(1-\theta)+k)\Big((\alpha-1)T_Nv-\alpha bT_M\Big)}{w(\theta-1)\Big((\alpha-1)T_Nv-\alpha bT_M\Big)+2\alpha bkT_M}$$
$$+\frac{2c(\alpha rT_M\phi\Big(\alpha bT_M(w\theta-k+1)+(\alpha-1)(w+1)T_Nv\Big))}{w(\theta-1)\Big((\alpha-1)T_Nv-\alpha bT_M\Big)+2\alpha bkT_M}$$
$$+\frac{2c((\alpha-1)T_N(w(1-\theta)+k)\Big((\alpha-1)T_Nv-\alpha bT_M\Big))}{w(\theta-1)\Big((\alpha-1)T_Nv-\alpha bT_M\Big)+2\alpha bkT_M},$$

$$A_6=\frac{2(\alpha-1)cT_N\tilde{r}^2(w(\theta-1)-k)(\alpha bT_M(w\theta-k+1)-(\alpha-1)(w+1)T_Nv)}{\alpha bT_M(\theta(k-1)+k+1)+(\alpha-1)(\theta-1)T_Nv}$$
$$-\frac{4(\alpha-1)cT_N\tilde{r}(w(\theta-1)-k)(\alpha bT_M(w\theta-k+1)-(\alpha-1)(w+1)T_Nv)}{\alpha bT_M(\theta(k-1)+k+1)+(\alpha-1)(\theta-1)T_Nv}$$
$$+\frac{\alpha\tilde{r}T_M(w(1+\theta)+2)(\alpha bT_M(w\theta-k+1)+(\alpha-1)(w+1)T_Nv)}{\alpha bT_M(\theta(k-1)+k+1)+(\alpha-1)(\theta-1)T_Nv}$$
$$+\frac{2(\alpha-1)cT_N(w(1-\theta)+k)(\alpha bT_M(-w\theta+k-1)+(\alpha-1)(w+1)T_Nv)}{\alpha bT_M(\theta(k-1)+k+1)+(\alpha-1)(\theta-1)T_Nv}$$
$$+\frac{\alpha T_M\left(w^2(1-\theta)(\alpha b\theta T_M+T_N(v-\alpha v))\right)}{\alpha bT_M(\theta(k-1)+k+1)+(\alpha-1)(\theta-1)T_Nv}$$
$$+\frac{\alpha T_M(w(\alpha bT_M(\theta(k-1)+k+1)+(\alpha-1)(\theta-1)T_Nv)+2\alpha bkT_M)}{\alpha bT_M(\theta(k-1)+k+1)+(\alpha-1)(\theta-1)T_Nv},$$

$$A_7 = \frac{(\alpha-1)bT_M(w\Big((\alpha-1)T_M(\theta(2\tilde{r}-1)+1)+2\alpha c(1-\theta)T_N(\tilde{r}-1)\Big))}{(\alpha-1)wT_M(2\tilde{r}+\theta-1)+2\alpha cw(\theta-1)T_N(\tilde{r}-1)-2\alpha ckT_N(\tilde{r}-1)+2(\alpha-1)\tilde{r}T_M} + \frac{(\alpha-1)bT_M\Big(2k(1-\tilde{r})\Big((\alpha-1)T_M-\alpha cT_N\Big)+2(\alpha-1)\tilde{r}T_M\Big)}{(\alpha-1)wT_M(2\tilde{r}+\theta-1)+2\alpha cw(\theta-1)T_N(\tilde{r}-1)-2\alpha ckT_N(\tilde{r}-1)+2(\alpha-1)\tilde{r}T_M} + \alpha T_N v,$$

$$A_8 = (\alpha-1)bT_M(\frac{w^2\theta\left((\alpha-1)T_M(\tilde{r}(1+\theta)-\theta+1)+2\alpha c(\theta-1)T_N(\tilde{r}-1)^2\right)}{-\tilde{r}(\theta(k-3)+k-1)-\theta+\theta k+k+1} + \frac{w\left((\alpha-1)T_M(-\tilde{r}(\theta(k-3)+k-1)-\theta+\theta k+k+1)-2\alpha cT_N(-\theta+(2\theta-1)k+1)(\tilde{r}-1)^2\right)}{-\tilde{r}(\theta(k-3)+k-1)-\theta+\theta k+k+1} + \frac{2\left(\alpha ck^2T_N(\tilde{r}-1)^2-k(\tilde{r}-1)(\alpha cT_N(\tilde{r}-1)+(\alpha-1)T_M)+(\alpha-1)\tilde{r}T_M\right)}{-\tilde{r}(\theta(k-3)+k-1)-\theta+\theta k+k+1}) + \frac{\alpha(w+1)T_Nv((\alpha-1)wT_M(\tilde{r}(1+\theta)+\theta-1)-2\alpha cw(\theta-1)T_N(\tilde{r}-1)^2)}{-\tilde{r}(\theta(k-3)+k-1)-\theta+\theta k+k+1} + \frac{\alpha(w+1)T_Nv(2\alpha ckT_N(\tilde{r}-1)^2+2(\alpha-1)\tilde{r}T_M)}{-\tilde{r}(\theta(k-3)+k-1)-\theta+\theta k+k+1}$$

This solution is valid in the following region χ which is defined by:

$\chi =$ Concavity condition L_2 for the ad networks profit π, along with

C1: $0 < s_j^* < 1/(1+w)$,

C2: $x_j^* > 0$,

C3: $p_j^* > 0$,

C4: $\pi^* \geq 0$,

C5: $0 < u_{Nj}^* < 1$, in which some, but not all, of the nonmalicious publishers participate.

From the structure of π, it follows that since $(\tilde{v}-p_W)n_{gW}$ is concave in p_W, the concavity of π, composed of the non-whitelisted publishers' $j \in \{M, N\}$ revenue and mislabeling costs, is sufficient to ensure that the entire function is concave. This follows from Kannai (1977), where it is shown that the sum of concave functions is concave. Therefore, we define the concavity condition using L_2 as defined with the Hessian matrix $\mathbb{H}(p_j, s_j)$ as follows. Using standard notation, since there are $\binom{n}{k}$ principal minors of order k, we denote Δ_k for the *leading* principal minors of order k.

$$L_2 : \mathbb{H}(p_j, s_j) \text{ is negative semi-definite } \Leftrightarrow (-1)^k\Delta_k \geq 0, \forall\ 1 \leq k \leq 4, j \in \{M, N\}. \tag{EC.4}$$

where the Hessian is constructed as follows:

$$\mathbb{H}(p_j, s_j) = \begin{bmatrix} \frac{\partial^2\pi}{\partial p_N^2} & \frac{\partial^2\pi}{\partial p_N \partial p_M} & \frac{\partial^2\pi}{\partial p_N \partial s_N} & \frac{\partial^2\pi}{\partial p_N \partial s_M} \\ \frac{\partial^2\pi}{\partial p_N \partial p_M} & \frac{\partial^2\pi}{\partial p_M^2} & \frac{\partial^2\pi}{\partial p_M \partial s_N} & \frac{\partial^2\pi}{\partial p_M \partial s_M} \\ \frac{\partial^2\pi}{\partial p_N \partial s_N} & \frac{\partial^2\pi}{\partial p_M \partial s_N} & \frac{\partial^2\pi}{\partial s_N^2} & \frac{\partial^2\pi}{\partial s_M \partial s_N} \\ \frac{\partial^2\pi}{\partial p_N \partial s_M} & \frac{\partial^2\pi}{\partial p_M \partial s_M} & \frac{\partial^2\pi}{\partial s_M \partial s_N} & \frac{\partial^2\pi}{\partial s_M^2} \end{bmatrix}$$

The leading principal minors are expressed as follows. First, for $k=1$, the First-Order Leading Principal Minor is

$$\Delta_1 = \frac{\partial^2 \pi}{\partial p_N^2}.$$

Then, for $k=2$ the Second-Order Leading Principal Minor is

$$\Delta_2 = \begin{vmatrix} \frac{\partial^2 \pi}{\partial p_N^2} & \frac{\partial^2 \pi}{\partial p_N \partial p_M} \\ \frac{\partial^2 \pi}{\partial p_N \partial p_M} & \frac{\partial^2 \pi}{\partial p_M^2} \end{vmatrix} = \frac{\partial^2 \pi}{\partial p_N^2} \cdot \frac{\partial^2 \pi}{\partial p_M^2} - \left(\frac{\partial^2 \pi}{\partial p_N \partial p_M} \right)^2.$$

Further, for $k=3$, the Third-Order Leading Principal Minor is

$$\Delta_3 = \begin{vmatrix} \frac{\partial^2 \pi}{\partial p_N^2} & \frac{\partial^2 \pi}{\partial p_N \partial p_M} & \frac{\partial^2 \pi}{\partial p_N \partial s_N} \\ \frac{\partial^2 \pi}{\partial p_N \partial p_M} & \frac{\partial^2 \pi}{\partial p_M^2} & \frac{\partial^2 \pi}{\partial p_M \partial s_N} \\ \frac{\partial^2 \pi}{\partial p_N \partial s_N} & \frac{\partial^2 \pi}{\partial p_M \partial s_N} & \frac{\partial^2 \pi}{\partial s_N^2} \end{vmatrix},$$

which, when expanded along the first row:

$$\Delta_3 = \frac{\partial^2 \pi}{\partial p_N^2} \begin{vmatrix} \frac{\partial^2 \pi}{\partial p_M^2} & \frac{\partial^2 \pi}{\partial p_M \partial s_N} \\ \frac{\partial^2 \pi}{\partial p_M \partial s_N} & \frac{\partial^2 \pi}{\partial s_N^2} \end{vmatrix} - \frac{\partial^2 \pi}{\partial p_N \partial p_M} \begin{vmatrix} \frac{\partial^2 \pi}{\partial p_N \partial p_M} & \frac{\partial^2 \pi}{\partial p_M \partial s_N} \\ \frac{\partial^2 \pi}{\partial p_N \partial s_N} & \frac{\partial^2 \pi}{\partial s_N^2} \end{vmatrix} + \frac{\partial^2 \pi}{\partial p_N \partial s_N} \begin{vmatrix} \frac{\partial^2 \pi}{\partial p_N \partial p_M} & \frac{\partial^2 \pi}{\partial p_M^2} \\ \frac{\partial^2 \pi}{\partial p_N \partial s_N} & \frac{\partial^2 \pi}{\partial p_M \partial s_N}. \end{vmatrix}$$

which, on simplification, yields

$$\Rightarrow \frac{\partial^2 \pi}{\partial p_N^2} \left[\frac{\partial^2 \pi}{\partial p_M^2} \cdot \frac{\partial^2 \pi}{\partial s_N^2} - \left(\frac{\partial^2 \pi}{\partial p_M \partial s_N} \right)^2 \right] - \frac{\partial^2 \pi}{\partial p_N \partial p_M} \left[\frac{\partial^2 \pi}{\partial p_N \partial p_M} \cdot \frac{\partial^2 \pi}{\partial s_N^2} - \frac{\partial^2 \pi}{\partial p_M \partial s_N} \cdot \frac{\partial^2 \pi}{\partial p_N \partial s_N} \right] + \frac{\partial^2 \pi}{\partial p_N \partial s_N} \left[\frac{\partial^2 \pi}{\partial p_N \partial p_M} \cdot \frac{\partial^2 \pi}{\partial p_M \partial s_N} - \frac{\partial^2 \pi}{\partial p_M^2} \cdot \frac{\partial^2 \pi}{\partial p_N \partial s_N} \right]$$

Finally, for $k=4$, the Fourth-Order Leading Principal Minor is

$$\Delta_4 = \begin{vmatrix} \frac{\partial^2 \pi}{\partial p_N^2} & \frac{\partial^2 \pi}{\partial p_N \partial p_M} & \frac{\partial^2 \pi}{\partial p_N \partial s_N} & \frac{\partial^2 \pi}{\partial p_N \partial s_M} \\ \frac{\partial^2 \pi}{\partial p_N \partial p_M} & \frac{\partial^2 \pi}{\partial p_M^2} & \frac{\partial^2 \pi}{\partial p_M \partial s_N} & \frac{\partial^2 \pi}{\partial p_M \partial s_M} \\ \frac{\partial^2 \pi}{\partial p_N \partial s_N} & \frac{\partial^2 \pi}{\partial p_M \partial s_N} & \frac{\partial^2 \pi}{\partial s_N^2} & \frac{\partial^2 \pi}{\partial s_M \partial s_N} \\ \frac{\partial^2 \pi}{\partial p_N \partial s_M} & \frac{\partial^2 \pi}{\partial p_M \partial s_M} & \frac{\partial^2 \pi}{\partial s_M \partial s_N} & \frac{\partial^2 \pi}{\partial s_M^2} \end{vmatrix} = \det(\mathbb{H})$$

For the sake of brevity, we do not present the exact expression for the above logical expression. However, we verify that the parameter space denoted by the above logical expression is non-empty and that all our results are valid in this parameter space. For example, at $\{w = 0.05, c = 0.5, k = 0.2, \phi = 1, T_N = 2, T_M = 1, \alpha = 0.65, b = 1, v = 2, \theta = 0.2, \sigma_W = 0.3, r = 1.33\}$, conditions C1, C2, C3, C4, C5, and L_2 are satisfied, and the following equilibrium is realized

$\{p_N^* = 1.0532, p_M^* = 0.6150, p_W^* = 1, s_N^* = 0.7601, s_M^* = 0.8340, \pi^* = 3.6064, u_{NN}^* = 0.7716, u_{MN}^* = 0.7949, u_{NM}^* = 0.4346, u_{MM}^* = 0.4321, x_M^* = 0.0764, x_N^* = 0.2127\}$.

We now consider other regions for which equilibrium outcomes exist. Specifically, we note that several equilibria are possible, but we focus on those that can be practically realized. First, we analyze the possibility where the ad network treats all ad traffic from type-N publishers as valid and uses the fraud detection system on ad traffic from type-M publishers only, i.e., $s_N = y_N = 0$. We do not provide the expressions for those regions but define the region χ' for which the region having $s_N = y_N = 0$ holds, as follows:

$\chi' =$ Concavity condition L_3 for the ad networks profit π, along with

C1$'$: $0 < s_{M,\chi'}^* < 1/(1+w)$, $s_{N,\chi'}^* = y_{N,\chi'}^* = 0$,

C2$'$: $x_{j,\chi'}^* > 0$,

C3$'$: $p_{j,\chi'}^* > 0$,

C4$'$: $\pi_{\chi'}^* \geq 0$,

C5$'$: $0 < u_{Nj,\chi'}^* < 1$, in which some, but not all, of the nonmalicious publishers participate.

and,

$$\begin{aligned}\lambda_2 = &\frac{c((1-\alpha)bT_M + \alpha T_N v)}{2((\alpha-1)T_M - 2\alpha c T_N)^2} + \frac{\alpha T_N v((1-\alpha)wT_M(2\tilde{r}+\theta-1) - 2\alpha c w(\theta-1)T_N(\tilde{r}-1))}{2((\alpha-1)T_M - 2\alpha c T_N)^2}\\ &+ \frac{(\alpha T_N v(2\alpha c k T_N(\tilde{r}-1) - 2(\alpha-1)\tilde{r}T_M)}{2((\alpha-1)T_M - 2\alpha c T_N)^2}\\ &+ \frac{(1-\alpha)bT_M(w\Big((\alpha-1)T_M(\theta(2\tilde{r}-1)+1) + 2\alpha c(1-\theta)T_N(\tilde{r}-1)\Big))}{2((\alpha-1)T_M - 2\alpha c T_N)^2}\\ &+ \frac{(1-\alpha)bT_M\Big(2k(1-\tilde{r})((\alpha-1)T_M - \alpha c T_N) + 2(\alpha-1)\tilde{r}T_M\Big))}{2((\alpha-1)T_M - 2\alpha c T_N)^2}\\ &\geq 0,\end{aligned}$$

As previously derived, the objective function in this scenario consists of $p_j, j \in \{M, N\}, p_W$, and s_M. Since concave functions for the whitelisted and non-whitelisted publishers are still separable, the Hessian for this scenario is defined for three decision variables and is, therefore, expressed as

$$L_3 : \mathbb{H}(p_j, s_M) \text{ is negative semi-definite } \Leftrightarrow (-1)^k \Delta_k \geq 0, \forall\ 1 \leq k \leq 3, j \in \{M, N\}. \quad \text{(EC.5)}$$

where the Hessian is

$$\mathbb{H}(p_j, s_j) = \begin{bmatrix} \frac{\partial^2 \pi}{\partial p_N^2} & \frac{\partial^2 \pi}{\partial p_N \partial p_M} & \frac{\partial^2 \pi}{\partial p_N \partial s_M} \\ \frac{\partial^2 \pi}{\partial p_N \partial p_M} & \frac{\partial^2 \pi}{\partial p_M^2} & \frac{\partial^2 \pi}{\partial p_M \partial s_M} \\ \frac{\partial^2 \pi}{\partial p_N \partial s_M} & \frac{\partial^2 \pi}{\partial p_M \partial s_M} & \frac{\partial^2 \pi}{\partial s_M^2} \end{bmatrix}$$

Using standard notation, we denote Δ_k for the leading principal minors, which are expressed as follows. First, for $k=1$, the First-Order Leading Principal Minor is

$$\Delta_1 = \frac{\partial^2 \pi}{\partial p_N^2}.$$

Then, for $k=2$ the Second-Order Leading Principal Minor is

$$\Delta_2 = \begin{vmatrix} \frac{\partial^2\pi}{\partial p_N^2} & \frac{\partial^2\pi}{\partial p_N \partial p_M} \\ \frac{\partial^2\pi}{\partial p_N \partial p_M} & \frac{\partial^2\pi}{\partial p_M^2} \end{vmatrix} = \frac{\partial^2\pi}{\partial p_N^2}\cdot\frac{\partial^2\pi}{\partial p_M^2} - \left(\frac{\partial^2\pi}{\partial p_N \partial p_M}\right)^2.$$

Further, for $k=3$, the Third-Order Leading Principal Minor is

$$\Delta_3 = \begin{vmatrix} \frac{\partial^2\pi}{\partial p_N^2} & \frac{\partial^2\pi}{\partial p_N \partial p_M} & \frac{\partial^2\pi}{\partial p_N \partial s_M} \\ \frac{\partial^2\pi}{\partial p_N \partial p_M} & \frac{\partial^2\pi}{\partial p_M^2} & \frac{\partial^2\pi}{\partial p_M \partial s_M} \\ \frac{\partial^2\pi}{\partial p_N \partial s_M} & \frac{\partial^2\pi}{\partial p_M \partial s_M} & \frac{\partial^2\pi}{\partial s_M^2} \end{vmatrix} = \det(\mathbb{H}).$$

Again, for the sake of brevity, we do not present the exact expressions for the equations mentioned above. However, we verify that the parameter space denoted by these expressions is non-empty and that all our results are valid in this parameter space. For example, at $\{w = 0.05, c = 0.5, k = 0.2, \phi = 1, T_N = 2, T_M = 1, \alpha = 0.65, b = 1, v = 2, \theta = 0.2, \sigma_W = 0.3, r = 0.745\}$, conditions C1$'$, C2$'$, C3$'$, C4$'$, C5$'$, and L_3 are satisfied, and the following equilibrium is realized: $\{p^*_{N,\chi'} = 0.6818, p^*_{M,\chi'} = 0.3212, p^*_{W,\chi'} = 1, s^*_{N,\chi'} = 0, s^*_{M,\chi'} = 0.1693, \pi^*_{\chi'} = 3.4714, u^*_{NN,\chi'} = 0.6818, u^*_{MN,\chi'} = 0.9143, u^*_{NM,\chi'} = 0.2854, u^*_{MM,\chi'} = 0.3166, x^*_{M,\chi'} = 0.2641, x^*_{N,\chi'} = 0.6818\}$.

Finally, we explore the case where ad network treats all ad traffic from type-N publishers as valid and all ad traffic from type-M publishers as fraudulent, i.e., $s_N = y_N = 0, s_M = y_M = 1$. Since all ad traffic by type-M publishers is considered fraudulent and blocked, i.e., $s_M = y_M = 1$, it follows that $x^*_{M,\chi''} = 0$. We do not provide the expressions for those regions but define the region $\chi'' \in \overline{(\chi \cup \chi')}$ for which the region having $s_N = y_N = 0, s_M = y_M = 1$ holds, as follows.

χ'' = Concavity condition L_4 for the ad networks profit π, along with

C1$''$: $\chi'' \in \overline{(\chi \cup \chi')}$,

C2$''$: $x^*_{N,\chi''} > 0$,

C3$''$: $p^*_{N,\chi''} > 0$,

C4$''$: $\pi_{\chi''} \geq 0$,

C5$''$: $0 < u^*_{NN,\chi''} < 1$, in which some, but not all, of the type-N nonmalicious publishers participate.

Here, we note that the profit of the ad network can be succinctly expressed as follows:

$$\pi_{\chi''} = (\alpha-1)bT_M(p_N - v) + \frac{(\alpha-1)p_N^2 T_M}{2c} + \alpha p_N T_N(v - p_N) + \underbrace{p_W \sigma \mathrm{w} T_N(v - p_W)}_{\text{Whitelisted advertisers}}$$

Since the whitelisted advertisers component of the profit function is concave, we test for the concavity of the rest of the function through the simple uni-variate concavity test as follows:

$$L_4 : \frac{\partial^2 \pi_{\chi''}}{\partial p_N^2} \leq 0$$

On conducting the L_4 test, we obtain the following expression:

$$-\frac{(1-\alpha)T_M}{c} - 2\alpha T_N < 0$$

which satisfies the L_4 test. Hence, we obtain the following expressions for equilibrium p_N^*, p_W^*:

$$\begin{aligned} p^*_{N,\chi''} =& p^*_{N,\chi'} = \frac{c(\alpha T_N v - (1-\alpha)bT_M)}{2\alpha c T_N + T_M(1-\alpha)}, \\ p_{W,\chi''} =& p^*_{W,\chi'} = p^*_W = \frac{v}{2} \end{aligned}$$

For example, at $\{w = 0.05, c = 0.6, k = 0.709, \phi = 1, T_N = 2, T_M = 1, \alpha = 0.65, b = 1, v = 2, \theta = 0.2, \sigma_W = 0.3, r = 0.875\}$, conditions C1$''$, C2$''$, C3$''$, C4$''$, C5$''$, and L_4 are satisfied, and the following equilibrium is realized: $\{p^*_{N,\chi''} = 0.7068, p^*_{W,\chi''} = 1, s^*_{N,\chi''} = y^*_{N,\chi''} = 0, s^*_{M,\chi''} = y^*_{M,\chi''} = 1, \pi^*_{\chi''} = 2.0952, u^*_{NN,\chi''} = 0.7068, u^*_{MN,\chi''} = 0.9150, x^*_{N,\chi''} = 0.5890\}$.

Thus, we can show the existence of the solution at a valid interior, at a corner, and in a separate region outside the valid region defined by Equation EC.3.

EC.4.2. Proof of Proposition 1

Proof. The proof follows from Lemma EC.4.1. It suffices to show that $\frac{\partial p_j^*}{\partial r}, j \in \{M, N\}$ can be positive. Consider the expressions for $\frac{\partial p_j^*}{\partial r}$:

$$\begin{aligned} \frac{\partial p_N^*}{\partial r} &= \frac{BC1 + BC2r + BC3r^2}{BC4 + BC5r + BC6r^2 + BC7r^3 + BC8r^4} = \hat{G}_1(\alpha, c, r, k, T_N, T_M, w, \phi, \theta), \\ \frac{\partial p_M^*}{\partial r} &= \frac{BC9 + BC10r + BC11r^2}{BC12 + BC13r + BC14r^2 + BC15r^3 + BC16r^4} = \hat{G}_2(\alpha, c, r, k, T_N, T_M, w, \phi, \theta), \end{aligned}$$

where for $\frac{\partial p_N^*}{\partial r}$ the terms are as follows:

$$BC1 = 2(\alpha-1)c^2T_M\phi\Bigg((\alpha-1)bT_M\Bigg(2w^2\Bigg(2\alpha^2c^2T_N^2\Big(\theta\left(k^2-4\right)+\theta^2\Big(k(3k-8)+2\Big)+2k(k+4)+2\Big)$$

$$-2(\alpha-1)\alpha cT_MT_N\Big(-4\theta+\theta^2\Big((k-5)k+2\Big)+2\theta k+3k(k+1)+2\Big)\Bigg)-(\alpha-1)^2(\theta-1)^2(k-1)T_M^2$$

$$+w^4(\theta-1)^2\theta(\theta+1)\Big(2\alpha cT_N-\alpha T_M+T_M\Big)^2$$

$$-w^3(\theta-1)\Big((\alpha-1)T_M-2\alpha cT_N\Big)\Bigg((\alpha-1)(\theta-1)T_M\Big(\theta(k-3)+k-1\Big)-2\alpha cT_N\Big(3\theta^2(k-1)+2\theta(k+1)+3k+1\Big)\Bigg)$$

$$-4\alpha cwkT_N\Bigg(\alpha cT_N\Big(8(\theta-1)+k\Big(\theta(k-7)+k-5\Big)\Big)$$

$$+2(\alpha-1)T_M\Big(-2\theta+(\theta+3)k+2\Big)\Bigg)-8\alpha ck^2T_N\Big(\alpha c(k-3)T_N+2(\alpha-1)T_M\Big)\Bigg)$$

$$+\alpha(w+1)T_Nv\Bigg(w(\theta-1)\Big((\alpha-1)T_M-2\alpha cT_N\Big)+2\alpha ckT_N\Bigg)\Bigg(w^2\Big(\theta^2-1\Big)\Big((\alpha-1)T_M-2\alpha cT_N\Big)$$

$$+2w\Big(\alpha cT_N\Big(\theta(k-2)-3k+2\Big)+(\alpha-1)(\theta-1)T_M\Big)-4\alpha ckT_N\Bigg)\Bigg).$$

$$BC2 = 8(\alpha-1)\,\alpha\,c^4\,T_M\,T_N\,\phi^2\Bigg((\alpha-1)bT_M\Bigg(w^4(\theta-1)\,\theta\,(\theta+1)^2\Big((\alpha-1)T_M-2\alpha cT_N\Big)$$

$$+\ w^3\Bigg(2\alpha cT_N\Big(\Big(3(\theta+1)\theta^2+\theta-3\Big)k-(\theta-1)(\theta+1)(5\theta+1)\Big)$$

$$-(\alpha-1)T_M\Big(\Big((\theta(\theta(2\theta+3)+4)-1)\Big)k-(\theta-1)(\theta+1)(5\theta+1)\Big)\Bigg)$$

$$+\ w^2\Bigg((\alpha-1)T_M\Big(-4(\theta+1)+\theta^2(k-8)(k-1)+2\theta(k-7)k+k(5k-1)\Big)$$

$$-2\alpha cT_N\Big(\theta^2\Big(3(k-4)k+8\Big)+\theta\Big(k(3k-8)-4\Big)+2k(k+4)-4\Big)\Bigg)$$

$$+2w\Bigg(\alpha cT_N\Big(-4\theta+k\Big(16\theta+k\Big(\theta(k-9)+k-7\Big)-4\Big)+4\Big)+2(\alpha-1)T_M\Big(\theta+k\Big(-4\theta+(\theta+3)k-2\Big)-1\Big)\Bigg)$$

$$+\ 4k\Big(\alpha c\Big((k-4)k+2\Big)T_N+2(\alpha-1)(k-1)T_M\Big)\Bigg)$$

$$-\alpha(w+1)T_Nv\Bigg(w\Big(w(\theta+1)^2+4(\theta+1)-(\theta+3)k\Big)-4k+4\Bigg)\Bigg(w(\theta-1)\Big((\alpha-1)T_M-2\alpha cT_N\Big)$$

$$+2\alpha ckT_N\Bigg)\Bigg).$$

$$BC3 = 8(1-\alpha)\,\alpha\, c^5 T_M T_N \phi^3 \Big(w\theta + w + 2\Big)\Bigg((\alpha-1)bT_M\Big(-w\theta+k-1\Big)\Bigg(\alpha c T_N\Big(w(-\theta)+w+k\Big)\Big(-w(\theta+3)+k-4\Big) - 2(\alpha-1)(w+1)T_M\Big(w\theta-k+1\Big)\Bigg) + \alpha(w+1)T_N v\Bigg(\alpha c T_N\Big(w(\theta-1)-k\Big)\Big(3w\theta+w-3k+4\Big) + 2(\alpha-1)(w+1)T_M\Big(w\theta-k+1\Big)\Bigg)\Bigg).$$

and

$$BC4 = (w(\theta-1)((\alpha-1)T_M - 2\alpha c T_N) + 2\alpha c k T_N)^4$$

$$BC5 = 8\,\alpha\, c^2 T_N\,\phi\Bigg(w(\theta-1)\Big((\alpha-1)T_M - 2\alpha c T_N\Big) + 2\alpha c k T_N\Bigg)^2\Bigg(w^2(\theta-1)^2\Big((\alpha-1)T_M - 2\alpha c T_N\Big) + wk\Big(4\alpha c(\theta-1)T_N - (\alpha-1)(\theta-3)T_M\Big) - 2k\Big(\alpha c k T_N - \alpha T_M + T_M\Big)\Bigg).$$

$$BC6 = 8\,\alpha\, c^3 T_N\,\phi^2\Bigg(2w^3(\theta-1)\Big((\alpha-1)T_M - 2\alpha c T_N\Big)\Bigg(12\alpha^2c^2(\theta-1)^2 k T_N^2 - \alpha(\alpha-1)cT_MT_N\Big(\theta^2(3k+2) - 16\theta k + 9k - 2\Big) + (\alpha-1)^2(\theta-1)T_M^2(\theta-k+1)\Bigg) - 4\alpha c\, w\, k T_N\Bigg(12\alpha^2c^2(\theta-1)k^2T_N^2 - \alpha(\alpha-1)cT_MT_N\Big(-4\theta + k\Big(14\theta+3\theta k - 9k - 10\Big)+4\Big) + 2(\alpha-1)^2T_M^2\Big(-\theta+2(\theta-2)k+1\Big)\Bigg) + 4\alpha c\, k^2 T_N\Bigg(3\alpha^2c^2k^2T_N^2 + 2\alpha c T_M T_N\Big(\alpha - 3\alpha k + 3k - 1\Big) + 2(\alpha-1)^2T_M^2\Bigg) + w^2\Bigg(72\alpha^3c^3(\theta-1)^2k^2T_N^3 - 4\alpha^2(\alpha-1)c^2T_MT_N^2\Big(-2(\theta-1)^2 + (\theta(9\theta-32)+21)k^2 + 2(\theta-1)(5\theta-1)k\Big) + \alpha(\alpha-1)^2cT_M^2T_N\Big(-8(\theta-1)^2 + (\theta(3\theta-22)+27)k^2 + 8(\theta-1)(3\theta-1)k\Big) - 2(\alpha-1)^3(\theta-1)^2(k-1)T_M^3\Bigg) + w^4(\theta-1)^2\Big(2\alpha c T_N - \alpha T_M + T_M\Big)^2\Big(3\alpha c(\theta-1)^2T_N + 2(\alpha-1)\theta T_M\Big)\Bigg)$$

$$BC7 = 32\alpha^2 c^5 T_N^2 \phi^3 \left(w^2(\theta-1)^2((\alpha-1)T_M - 2\alpha c T_N) + wk((1-\alpha)(\theta-3)T_M + 4\alpha c(\theta-1)T_N) + 2k(\alpha c k T_N - \alpha T_M + T_M)\right)\left(\alpha c T_N(w(1-\theta)+k)^2 + 2(\alpha-1)(w+1)T_M(w\theta - k + 1)\right)$$

$$BC8 = 16\alpha^2 c^6 T_N^2 \phi^4 \left(\alpha c T_N(w(1-\theta)+k)^2 + 2(\alpha-1)(w+1)T_M(w\theta-k+1)\right)^2$$

Additionally, for $\frac{\partial p_M^*}{\partial r}$, the terms are as follows:

$$\begin{aligned}BC9 = 2\alpha c^2 T_M \phi \Bigg(& 4(\alpha-1)^2 c^2 T_N^2 \Big(w(\theta-1)-k\Big)\Bigg(w^3(\theta^2-1)\Big(\alpha b\theta T_M + (\alpha-1)T_N v\Big) \\ & - w^2\Big(\alpha b T_M\Big(\theta(2-3\theta) + \Big(2\theta^2+\theta+3\Big)k+1\Big) + (\alpha-1)T_N v\Big((\theta-3)k - (\theta-1)(\theta+3)\Big)\Big) \\ & + w\Big(\alpha b T_M\Big(2(\theta-1) + k\Big(\theta(k-5)+k-7\Big)\Big) - (\alpha-1)T_N v\Big(\theta(k-2)-5k+2\Big)\Big) \\ & + 2k\Big(\alpha b(k-3)T_M + (\alpha-1)T_N v\Big)\Bigg) \\ & - 4\alpha(\alpha-1)cT_M T_N\Bigg(w^4(\theta-1)^2(\theta+1)\Big(\alpha b\theta T_M + (\alpha-1)T_N v\Big) \\ & - w^3(\theta-1)\Big(\alpha b T_M\Big(\theta^2(2k-3) + \theta(k+2) + k + 1\Big) + (\alpha-1)(\theta-1)T_N v\Big(-\theta+k-3\Big)\Big) \\ & + w^2\Big(\alpha b T_M\Big(-4\theta + \theta^2((k-5)k+2) + 2\theta k + 3k(k+1) + 2\Big) - (\alpha-1)(\theta-1)^2(k-2)T_N v\Big) \\ & + 2\alpha b w k T_M\Big(-2\theta + (\theta+3)k + 2\Big) + 4\alpha b k^2 T_M\Bigg) \\ & + \alpha^2 w^2(\theta-1)^2 T_M^2(w\theta+w+2)\Big(\alpha b T_M(w\theta-k+1) + (\alpha-1)(w+1)T_N v\Big)\Bigg).\end{aligned}$$

$$BC10 = 8(\alpha-1)\,\alpha\, c^4 T_M T_N \phi^2 \Bigg(\alpha T_M \Bigg(w^4(\theta-1)(\theta+1)^2 \Big(\alpha b\,\theta\, T_M + T_N(v-\alpha v)\Big) + w^3\Big(\alpha b T_M\Big((\theta-1)(\theta+1)(5\theta+1) - \theta\Big(\theta(2\theta+3)+4\Big)k + k\Big) - (\alpha-1)(\theta-1)T_N v\Big((\theta+1)(\theta+5) - (\theta+3)k\Big)\Big) + w^2\Big(\alpha b T_M\Big(-4(\theta+1) + \theta^2(k-8)(k-1) + 2\theta(k-7)k + k(5k-1)\Big) + (\alpha-1)(\theta-1)T_N v\Big(\theta(k-4) + 7k - 8\Big)\Big) + 4w\Big(\alpha b T_M\Big(\theta + k\Big(\theta(k-4)+3k-2\Big) - 1\Big) + (\alpha-1)(\theta-1)(k-1)T_N v\Big) + 8\alpha b(k-1)kT_M \Bigg) - 2(\alpha-1)cT_N\Bigg(\Big(w(\theta-1)-k\Big)\Big(\alpha b T_M\Big(w^3\theta(\theta+1)^2 + w^2\Big((\theta+1)(5\theta+1) - \Big(\theta(2\theta+3)+3\Big)k\Big) + w\Big(8\theta + k\Big(\theta(k-7)+k-9\Big)+4\Big) + 2(k-4)k + 4\Big) - (\alpha-1)(w+1)T_N v\Big(w\Big(w(\theta+1)^2 + 4(\theta+1) - (\theta+3)k\Big) - 4k + 4\Big)\Big)\Bigg)\Bigg).$$

$$BC11 = 8(\alpha-1)\,\alpha\, c^3 T_M T_N \phi \tilde{r}^2 (w\theta + w + 2)\Bigg((\alpha-1)cT_N\Big(w(\theta-1) - k\Big)\Big(\alpha b T_M(w\theta - k + 1)(w(\theta+3) - k + 4) - (\alpha-1)(w+1)T_N v(3w\theta + w - 3k + 4)\Big) - 2\alpha(w+1)T_M(w\theta - k + 1)\Big(\alpha b T_M(w\theta - k + 1) + (\alpha-1)(w+1)T_N v\Big)\Bigg).$$

and

$$\begin{aligned} BC12 = {} & 16(\alpha-1)^4 c^4 T_N^4 (w(1-\theta)+k)^4 + 32(\alpha-1)^3 \alpha c^3 w(\theta-1) T_M T_N^3 (w(1-\theta)+k)^3 \\ & + 24(\alpha-1)^2\alpha^2 c^2 w^2(\theta-1)^2 T_M^2 T_N^2 (w(1-\theta)+k)^2 \\ & + 8(\alpha-1)\alpha^3 c w^3 (\theta-1)^3 T_M^3 T_N (w(1-\theta)+k) + \alpha^4 w^4 (\theta-1)^4 T_M^4 \end{aligned}$$

$$\begin{aligned} BC13 = {} & 64(1-\alpha)^4 c^5 T_N^4 \phi (w(-\theta)+w+k)^4 \\ & + 32(\alpha-1)^3 \alpha c^4 T_M T_N^3 \phi \left(w^2(\theta-1)^2 - w(\theta-3)k + 2k\right)(w(-\theta)+w+k)^2 \\ & - 64(\alpha-1)^3 \alpha c^4 w(\theta-1) T_M T_N^3 \phi (w(-\theta)+w+k)^3 \\ & - 16(\alpha-1)^2 \alpha^2 c^3 w^2 (\theta-1)^2 T_M^2 T_N^2 \phi (w(-\theta)+w+k)^2 \\ & + 32(\alpha-1)^2 \alpha^2 c^3 w(\theta-1) T_M^2 T_N^2 \phi \left(w^2(\theta-1)^2 - w(\theta-3)k + 2k\right)(w(-\theta)+w+k) \\ & + 8(\alpha-1)\alpha^3 c^2 w^2 (\theta-1)^2 T_M^3 T_N \phi \left(w^2(\theta-1)^2 - w(\theta-3)k + 2k\right) \end{aligned}$$

$$BC14 = 8(\alpha-1)c^3T_N\phi^2\Bigg(12(\alpha-1)^3c^3T_N^3(w(1-\theta)+k)^4$$
$$-4(\alpha-1)^2\alpha c^2T_MT_N^2\left(w\left(w(\theta(3\theta-8)+3)-2\theta-3\theta k+9k-2\right)+6k-2\right)(w(1-\theta)+k)^2$$
$$+(\alpha-1)\alpha^2cT_M^2T_N\Big(w^4(\theta-1)^2(\theta(3\theta-14)+3)-2w^3(\theta-1)\left(4\theta^2+(3(\theta-6)\theta+11)k-4\right)$$
$$+w^2\left(-8(\theta-1)^2+(\theta(3\theta-22)+27)k^2+8(\theta-1)(3\theta-1)k\right)$$
$$+8wk(\theta-2(\theta-2)k-1)+8k^2\Big)$$
$$+2\alpha^3w^2(w+1)(\theta-1)^2T_M^3(w\theta-k+1)\Bigg)$$

$$BC15 = 32(\alpha-1)^2c^5T_N^2\phi^3\Bigg(-2(\alpha-1)^2c^2T_N^2(w(1-\theta)+k)^4$$
$$+(\alpha-1)\alpha cT_MT_N\left(w(w(\theta-6)\theta+w-\theta(k+4)+7k-4)+6k-4\right)(w(1-\theta)+k)^2$$
$$+2\alpha^2(w+1)T_M^2\left(-w^2(\theta-1)^2+w(\theta-3)k-2k\right)(-w\theta+k-1)\Bigg)$$

$$BC16 = 16(\alpha-1)^2c^6T_N^2\phi^4\left((\alpha-1)^2c^2T_N^2(w(1-\theta)+k)^4\right.$$
$$\left.+4(\alpha-1)\alpha c(w+1)T_MT_N(w\theta-k+1)(w(1-\theta)+k)^2+4\alpha^2(w+1)^2T_M^2(w\theta-k+1)^2\right)$$

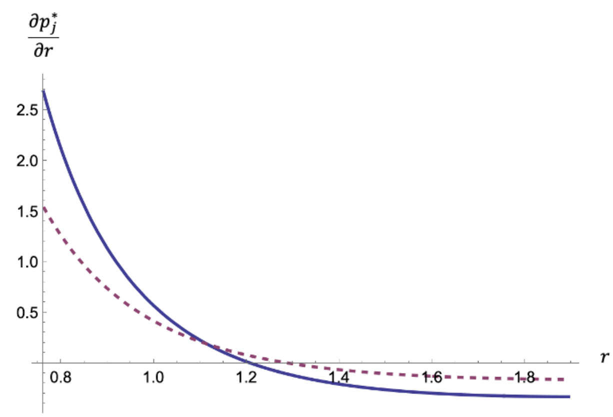


Figure EC.11 Illustrating $\frac{\partial p_j^*}{\partial r}$ with parameter values $\{v=2, k=0.1, c=0.5, w=0.05, \alpha=0.65, T_N=2, T_M=1, \phi=1, b=1, \theta=0.2, \sigma_W=0.3\}$.

Using extensive numerical simulations, we show that there exist regions of $r \in (0, \frac{1}{c\phi})$ beyond which $\frac{\partial p_N^*}{\partial r}$ and $\frac{\partial p_M^*}{\partial r}$ are negative. Fig. EC.11 demonstrates existence.

EC.4.3. Proof of Proposition 2

Proof. The proof follows from Lemma 1. It suffices to show that $\frac{\partial p_j^*}{\partial c}, j \in \{N, M\}$ can be negative. Consider the expression for $\frac{\partial p_j^*}{\partial c}$ as follows:

$$\frac{\partial p_N^*}{\partial c} = \frac{BC17 + BC18r + BC19r^2 + BC20r^3 + BC21r^4}{BC22 + BC23r + BC24r^2 + BC25r^3 + BC26r^4} = \hat{G}_4(\alpha, c, r, k, T_N, T_M, w, \phi, \theta, \sigma_W),$$
$$\frac{\partial p_M^*}{\partial c} = \frac{BC27 + BC28r + BC29r^2 + BC30r^3}{BC31 + BC32r + BC33r^2 + BC34r^3 + BC35r^4} = \hat{G}_3(\alpha, c, r, k, T_N, T_M, w, \phi, \theta, \sigma_W),$$

where for $\frac{\partial p_N^*}{\partial c}$ we have:

$$BC17 = 2(1-\alpha)T_M\left(w(\theta-1)\left((\alpha-1)T_M - 2\alpha c T_N\right) + 2\alpha c k T_N\right)\Bigg((\alpha-1)bT_M\Big(w^3(\theta - 1)^2\theta\left((\alpha-1)T_M - 2\alpha c T_N\right) - w^2(\theta-1)\Big(2\alpha c T_N(\theta - 2\theta k + 3k - 1) + (\alpha-1)T_M(\theta(k-1)+k+1)\Big) - 2wk\left(\alpha c T_N(\theta + (\theta-3)k - 1) + (\alpha-1)(\theta-1)T_M\right) + 4\alpha c k^2 T_N\Big) - \alpha w(w+1)(\theta-1)T_N v\left(w(\theta-1)\left((\alpha-1)T_M - 2\alpha c T_N\right) + 2\alpha c k T_N\right)\Bigg),$$

$$BC18 = 4(\alpha-1)cT_M\phi\Bigg((\alpha-1)bT_M\Big(-w^3(\theta-1)\Big(8\alpha^2c^2(\theta-1)^2(3k-1)T_N^2 - 2\alpha(\alpha-1)cT_MT_N\left(\theta(6-5\theta) + (\theta(6\theta-5)+3)k - 1\right) + (\alpha-1)^2(\theta-1)T_M^2(\theta(k-3)+k-1)\Big) - 2w^2\Big(-12\alpha^2c^2(\theta-2)(\theta-1)k^2T_N^2 + (\alpha-1)\alpha c T_M T_N\left(2(\theta-1)^2 + (\theta(3\theta-4)+5)k^2 + \theta(6-7\theta)k + k\right) + (\alpha-1)^2(\theta-1)^2(k-1)T_M^2\Big) + w^4(\theta-1)^2\theta\left((\alpha-1)T_M - 2\alpha c T_N\right)\left((\alpha-1)(\theta+1)T_M - 4\alpha c(\theta-1)T_N\right) - 4\alpha c w k T_N\Big(2\alpha c k T_N(\theta(k+3) - 3(k+1)) + (\alpha-1)T_M(-2\theta + (\theta+3)k + 2)\Big) + 8\alpha c k^2 T_N(2\alpha c k T_N - \alpha T_M + T_M)\Big) + \alpha w(w+1)(\theta-1)T_N v\Big(2w(\theta-1)\left(8\alpha^2c^2kT_N^2 - \alpha(\alpha-1)c(k+2)T_MT_N + (\alpha-1)^2T_M^2\right) - 8\alpha^2c^2k^2T_N^2 + w^2(\theta-1)\left((\alpha-1)T_M - 2\alpha c T_N\right)\left(4\alpha c(\theta-1)T_N + (\alpha-1)(\theta+1)T_M\right)\Big)\Bigg),$$

$$BC19 = 8(\alpha-1)\alpha c^3 T_M T_N \phi^2 \Bigg((\alpha-1) b T_M \Big(w^4(\theta-1)\theta \left(\alpha c(-5(\theta-2)\theta-9)T_N + 2(\alpha-1)(\theta^2+1)T_M\right)$$

$$+ w^3\Big(\alpha c T_N \left((7-5\theta)\theta^2 - 11\theta + (\theta(\theta(15\theta-49)+53)-15)k+9\right)$$

$$- 2(\alpha-1)T_M\left(-3\theta^3+\theta^2+\theta+(\theta(\theta(2\theta-1)+4)-1)k+1\right)\Big)$$

$$+ w^2\Big(\alpha c T_N\left(-34k^2-\theta^2(k(15k+4)+4)+\theta(k(53k+8)-4)+8k+8\right)$$

$$+ 2(\alpha-1)T_M\left(-2(\theta+1)+\theta^2(k-4)(k-1)-6\theta k+k(3k-1)\right)\Big)$$

$$+ w\Big(\alpha c T_N(-4\theta+k(k(23\theta+5\theta k-19k-15)+12)+4)$$

$$+4(\alpha-1)T_M(\theta+k(\theta(k-4)+3k-2)-1)\Big) + 2k\left(\alpha c(k(2-7k)+2)T_N + 4(\alpha-1)(k-1)T_M\right)\Big)$$

$$- \alpha(w+1)T_N v\Big(w^3(\theta-1)\left(\alpha c((10-9\theta)\theta-5)T_N + 2(\alpha-1)(\theta^2+1)T_M\right)$$

$$- 2w^2\Big(\alpha c T_N(4(\theta-1)\theta + ((14-9\theta)\theta-7)k) + (\alpha-1)(\theta^2-1)(k-2)T_M\Big)$$

$$+ w\Big(\alpha c T_N\left(-4\theta-9(\theta-1)k^2+8\theta k+4\right) - 4(\alpha-1)(\theta-1)(k-1)T_M\Big) + 4\alpha c k T_N\Big)\Bigg),$$

$$BC20 = 16(1-\alpha)\alpha c^4 r^3 T_M T_N \phi^3 \left((\alpha-1)bT_M(w\theta-k+1)((\alpha-1)(w+1)T_M(w(1+\theta)+2)(w\theta-k+1)\right.$$

$$- \alpha c T_N(w(\theta-1)-k)(w(w(\theta-3)(\theta-1)-\theta(k+2)+7k+2)+6k))$$

$$+ \alpha(w+1)T_N v\left(\alpha c T_N(w(\theta-1)-k)\left(w\left(3w\theta^2-4w(1+\theta)+2\theta-3\theta k+5k-2\right)+2k\right)\right.$$

$$\left.\left. + (\alpha-1)(w+1)T_M(w(1+\theta)+2)(w\theta-k+1)\right)\right),$$

$$BC21 = 32(\alpha-1)\alpha^2 c^6 (w+1) r^4 T_M T_N^2 \phi^4 (-w\theta+k-1)(w(1-\theta)+k)(-(\alpha-1)bT_M(-w\theta+k-1)$$

$$- (\alpha(w+1)T_N v))$$

Additionally, for the denominator of $\frac{\partial p_N^*}{\partial c}$, we have:

$$BC22 = (w(\theta-1)((\alpha-1)T_M - 2\alpha c T_N) + 2\alpha c k T_N)^4$$

$$BC23 = 8\alpha c^2 T_N \phi (w(\theta-1)((\alpha-1)T_M - 2\alpha c T_N) + 2\alpha c k T_N)^2 \left(w^2(\theta-1)^2((\alpha-1)T_M - 2\alpha c T_N)\right.$$

$$\left. + wk(4\alpha c(\theta-1)T_N - (\alpha-1)(\theta-3)T_M) - 2k(\alpha c k T_N - \alpha T_M + T_M)\right)$$

$$
\begin{aligned}
BC24 = 8\alpha c^3 T_N \phi^2 \Bigg(& 2w^3(\theta-1)((\alpha-1)T_M - 2\alpha c T_N)\left(12\alpha^2 c^2(\theta-1)^2 k T_N^2\right. \\
& \left. - \alpha(\alpha-1)cT_M T_N\left(\theta^2(3k+2) - 16\theta k + 9k - 2\right) + (\alpha-1)^2(\theta-1)(-T_M^2)(-\theta+k-1)\right) \\
& - 4\alpha c w k T_N\left(12\alpha^2c^2(\theta-1)k^2T_N^2 - \alpha(\alpha-1)cT_M T_N(-4\theta + k(14\theta+3\theta k - 9k - 10)+4)\right. \\
& \left. + 2(\alpha-1)^2T_M^2(-\theta+2(\theta-2)k+1)\right) \\
& + 4\alpha c k^2 T_N\left(3\alpha^2c^2k^2T_N^2 + 2\alpha c T_M T_N(\alpha - 3\alpha k + 3k - 1) + 2(\alpha-1)^2T_M^2\right) \\
& + w^2\Big(72\alpha^3c^3(\theta-1)^2k^2T_N^3 \\
& - 4\alpha^2(\alpha-1)c^2T_M T_N^2\left(-2(\theta-1)^2 + (\theta(9\theta-32)+21)k^2 + 2(\theta-1)(5\theta-1)k\right) \\
& + \alpha(\alpha-1)^2cT_M^2T_N\left(-8(\theta-1)^2 + (\theta(3\theta-22)+27)k^2 + 8(\theta-1)(3\theta-1)k\right) \\
& - 2(\alpha-1)^3(\theta-1)^2(k-1)T_M^3\Big) \\
& + w^4(\theta-1)^2(2\alpha c T_N - \alpha T_M + T_M)^2\left(3\alpha c(\theta-1)^2T_N + 2(\alpha-1)\theta T_M\right)\Bigg),
\end{aligned}
$$

$$
\begin{aligned}
BC25 = 32\alpha^2c^5T_N^2\phi^3 & \left(w^2(\theta-1)^2((\alpha-1)T_M - 2\alpha c T_N) + wk((\alpha-1)(\theta-3)T_M - 4\alpha c(\theta-1)T_N)\right. \\
& \left. + 2k(\alpha c k T_N - \alpha T_M + T_M)\right)\left(\alpha c T_N(w(1-\theta)+k)^2 + 2(\alpha-1)(w+1)T_M(w\theta - k + 1)\right)
\end{aligned}
$$

$$
BC26 = 16\alpha^2c^6T_N^2\phi^4\left(\alpha c T_N(w(1-\theta)+k)^2 + 2(\alpha-1)(w+1)T_M(w\theta-k+1)\right)^2
$$

Having illustrated $\frac{\partial p_N^*}{\partial c}$ as a ratio of polynomials, we now move on to $\frac{\partial p_M^*}{\partial c}$, with the coefficients as follows.

$$
\begin{aligned}
BC27 = 2\Bigg(& 4(\alpha-1)^2\alpha c^2 T_M T_N^2(w(-\theta)+w+k)^2\left(w^2(\theta-1)(\alpha b\theta T_M + T_N(v-\alpha v))\right. \\
& \left. - w(\alpha b T_M(\theta(k-1)+k+1) + (\alpha-1)(\theta-1)T_N v) - 2\alpha b k T_M\right) \\
& + 8(\alpha-1)^2\alpha c^2 w(\theta-1)T_M T_N^2(w(-\theta)+w+k)^2(\alpha b T_M(-w\theta+k-1) + (\alpha-1)(w+1)T_N v) \\
& + 4(\alpha-1)\alpha^2 c w^2(\theta-1)^2T_M^2T_N(w(-\theta)+w+k)(\alpha b T_M(-w\theta+k-1) + (\alpha-1)(w+1)T_N v) \\
& + \alpha^3(-w^2)(\theta-1)^2T_M^3\left(w^2(\theta-1)(\alpha b\theta T_M + T_N(v-\alpha v))\right. \\
& \left. - w(\alpha b T_M(\theta(k-1)+k+1) + (\alpha-1)(\theta-1)T_N v) - 2\alpha b k T_M\right)\Bigg)
\end{aligned}
$$

$$
\begin{aligned}
BC28 = {} & 16(1-\alpha)^3c^4T_N^3\phi(w(\theta-1)-k)(w(1-\theta)+k)^2(\alpha b T_M(w\theta-k+1) - (\alpha-1)(w+1)T_N v) \\
& + 16(\alpha-1)^3c^4T_N^3\phi(w(1-\theta)+k)^3(\alpha b T_M(-w\theta+k-1) + (\alpha-1)(w+1)T_N v) \\
& - 16(\alpha-1)^2\alpha c^3T_M T_N^2\phi(w(1-\theta)+k)^2\left(w^2(\theta-1)(\alpha b\theta T_M + T_N(v-\alpha v))\right. \\
& \left. - w(\alpha b T_M(\theta(k-1)+k+1) + (\alpha-1)(\theta-1)T_N v) - 2\alpha b k T_M\right) - 32(\alpha-1)^2\alpha c^3 w(\theta \\
& - 1)T_M T_N^2\phi(w(\theta-1)-k)(w(1-\theta)+k)(\alpha b T_M(w\theta-k+1) - (\alpha-1)(w+1)T_N v) \\
& - 12(\alpha-1)\alpha^2c^2w^2(\theta-1)^2T_M^2T_N\phi(w(\theta-1)-k)(\alpha b T_M(w\theta-k+1) - (\alpha-1)(w+1)T_N v) \\
& + 4(\alpha-1)\alpha^2c^2T_M^2T_N\phi\left(w^2(\theta-1)^2 - w(\theta-3)k + 2k\right)\left(w^2(\theta-1)(\alpha b\theta T_M + T_N(v-\alpha v))\right. \\
& \left. - w(\alpha b T_M(\theta(k-1)+k+1) + (\alpha-1)(\theta-1)T_N v) - 2\alpha b k T_M\right) + 4(\alpha-1)\alpha^2c^2w(\theta \\
& - 1)T_M^2T_N\phi(w\theta+w+2)(w(1-\theta)+k)(\alpha b T_M(w\theta-k+1) + (\alpha-1)(w+1)T_N v) \\
& + 2\alpha^3 c w^2(\theta-1)^2T_M^3\phi(w\theta+w+2)(\alpha b T_M(w\theta-k+1) + (\alpha-1)(w+1)T_N v)
\end{aligned}
$$

$$
\begin{aligned}
BC29 = {} & 16(\alpha-1)^3c^5T_N^3\phi^2(w(\theta-1)-k)(w(1-\theta)+k)^2(\alpha bT_M(w\theta-k+1)-(\alpha-1)(w+1)T_Nv) \\
& -16(\alpha-1)^3c^5T_N^3\phi^2(w(1-\theta)+k)^3(\alpha bT_M(-w\theta+k-1)+(\alpha-1)(w+1)T_Nv) \\
& -16(\alpha-1)^2\alpha c^4T_MT_N^2\phi^2(w(\theta-1)-k)(w^2(\theta-1)^2-w(\theta-3)k+2k)(\alpha bT_M(w\theta-k+1) \\
& \qquad -(\alpha-1)(w+1)T_Nv) \\
& +12(\alpha-1)^2\alpha c^4T_MT_N^2\phi^2(w(1-\theta)+k)^2\left(w^2(\theta-1)(\alpha b\theta T_M+T_N(v-\alpha v))\right. \\
& \qquad \left.-w(\alpha bT_M(\theta(k-1)+k+1)+(\alpha-1)(\theta-1)T_Nv)-2\alpha bkT_M\right) \\
& +8(\alpha-1)^2\alpha c^4T_MT_N^2\phi^2(w\theta+w+2)(w(1-\theta)+k)^2(\alpha bT_M(w\theta-k+1)+(\alpha-1)(w+1)T_Nv) \\
& +24(\alpha-1)^2\alpha c^4w(\theta-1)T_MT_N^2\phi^2(w(\theta-1)-k)(w(1-\theta)+k)(\alpha bT_M(w\theta-k+1) \\
& \qquad -(\alpha-1)(w+1)T_Nv)-16(\alpha-1)^2\alpha c^4(w+1)T_MT_N^2\phi^2(w(1-\theta)+k)(w\theta-k \\
& \qquad +1)(\alpha bT_M(-w\theta+k-1)+(\alpha-1)(w+1)T_Nv) \\
& +8(\alpha-1)\alpha^2c^3w^2(\theta-1)^2T_M^2T_N\phi^2(w(\theta-1)-k)(\alpha bT_M(w\theta-k+1)-(\alpha-1)(w+1)T_Nv) \\
& +16(\alpha-1)\alpha^2c^3(w+1)T_M^2T_N\phi^2(w\theta-k+1)\left(w^2(\theta-1)(\alpha b\theta T_M+T_N(v-\alpha v))\right. \\
& \qquad \left.-w(\alpha bT_M(\theta(k-1)+k+1)+(\alpha-1)(\theta-1)T_Nv)-2\alpha bkT_M\right)
\end{aligned}
$$

$$
\begin{aligned}
BC30 = {} & 16(1-\alpha)^2\alpha c^5T_MT_N^2\phi^3(w(\theta-1)-k)\left(w^2(\theta-1)^2-w(\theta-3)k+2k\right)(\alpha bT_M(w\theta-k+1) \\
& \qquad -(\alpha-1)(w+1)T_Nv) \\
& -8(\alpha-1)^2\alpha c^5T_MT_N^2\phi^3(w(1+\theta)+2)(w(1-\theta)+k)^2(\alpha bT_M(w\theta-k+1)+(\alpha-1)(w+1)T_Nv) \\
& -8(\alpha-1)\alpha^2c^4(w+1)T_M^2T_N\phi^3(w(1+\theta)+2)(w\theta-k+1)(\alpha bT_M(w\theta-k+1) \\
& \qquad +(\alpha-1)(w+1)T_Nv)
\end{aligned}
$$

and for the denominator we have the following:

$$
\begin{aligned}
BC31 = {} & 16(1-\alpha)^4c^4T_N^4(w(1-\theta)+k)^4+32(\alpha-1)^3\alpha c^3w(\theta-1)T_MT_N^3(w(1-\theta)+k)^3 \\
& +24(\alpha-1)^2\alpha^2c^2w^2(\theta-1)^2T_M^2T_N^2(w(1-\theta)+k)^2 \\
& +8(\alpha-1)\alpha^3cw^3(\theta-1)^3T_M^3T_N(w(1-\theta)+k)+\alpha^4w^4(\theta-1)^4T_M^4
\end{aligned}
$$

$$
\begin{aligned}
BC32 = {} & 64(\alpha-1)^4c^5T_N^4\phi(w\theta-w-k)^4 \\
& +32(\alpha-1)^3\alpha c^4T_MT_N^3\phi\left(w^2(\theta-1)^2-w(\theta-3)k+2k\right)(w(1-\theta)+k)^2 \\
& -64(\alpha-1)^3\alpha c^4w(\theta-1)T_MT_N^3\phi(w(1-\theta)+k)^3 \\
& -16(\alpha-1)^2\alpha^2c^3w^2(\theta-1)^2T_M^2T_N^2\phi(w(1-\theta)+k)^2 \\
& +32(\alpha-1)^2\alpha^2c^3w(\theta-1)T_M^2T_N^2\phi\left(w^2(\theta-1)^2-w(\theta-3)k+2k\right)(w(1-\theta)+k) \\
& +8(\alpha-1)\alpha^3c^2w^2(\theta-1)^2T_M^3T_N\phi\left(w^2(\theta-1)^2-w(\theta-3)k+2k\right)
\end{aligned}
$$

$$
\begin{aligned}
BC33 = {} & 96(\alpha-1)^4c^6T_N^4\phi^2(w(1-\theta)+k)^4 \\
& -64(\alpha-1)^3\alpha c^5T_MT_N^3\phi^2\left(w^2(\theta-1)^2-w(\theta-3)k+2k\right)(w(1-\theta)+k)^2 \\
& +32(\alpha-1)^3\alpha c^5w(\theta-1)T_MT_N^3\phi^2(w(1-\theta)+k)^3 \\
& +64(\alpha-1)^3\alpha c^5(w+1)T_MT_N^3\phi^2(w\theta-k+1)(w(1-\theta)+k)^2 \\
& +16(\alpha-1)^2\alpha^2c^4T_M^2T_N^2\phi^2\left(w^2(\theta-1)^2-w(\theta-3)k+2k\right)^2 \\
& +8(\alpha-1)^2\alpha^2c^4w^2(\theta-1)^2T_M^2T_N^2\phi^2(w(1-\theta)+k)^2 \\
& +64(\alpha-1)^2\alpha^2c^4w(w+1)(\theta-1)T_M^2T_N^2\phi^2(w\theta-k+1)(w(1-\theta)+k) \\
& +16(\alpha-1)\alpha^3c^3w^2(w+1)(\theta-1)^2T_M^3T_N\phi^2(w\theta-k+1)
\end{aligned}
$$

$$\begin{aligned}BC34 = &-64(\alpha-1)^4c^7T_N^4\phi^3(w(1-\theta)+k)^4\\&+32(\alpha-1)^3\alpha c^6T_MT_N^3\phi^3\left(w^2(\theta-1)^2-w(\theta-3)k+2k\right)(w(1-\theta)+k)^2\\&-128(\alpha-1)^3\alpha c^6(w+1)T_MT_N^3\phi^3(w(1-\theta)+k)^2(w\theta-k+1)\\&+64(\alpha-1)^2\alpha^2c^5(w+1)T_M^2T_N^2\phi^3\left(w^2(\theta-1)^2-w(\theta-3)k+2k\right)(w\theta-k+1)\end{aligned}$$

$$\begin{aligned}BC35 = &16(\alpha-1)^4c^8T_N^4\phi^4(w(1-\theta)+k)^4+64(\alpha-1)^3\alpha c^7(w+1)T_MT_N^3\phi^4(w\theta-k+1)(w(1-\theta)+k)^2\\&+64(\alpha-1)^2\alpha^2c^6(w+1)^2T_M^2T_N^2\phi^4(w\theta-k+1)^2\end{aligned}$$

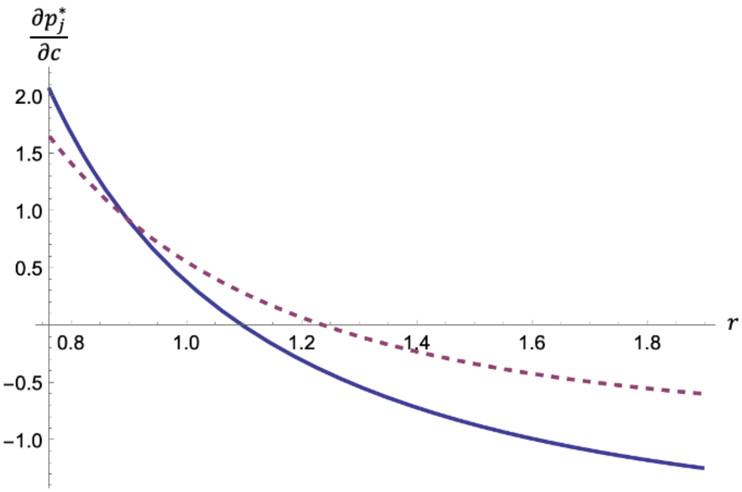


Figure EC.12 Effect of Change in c on $p_j^*, j\in\{N,M\}$ for values of Parameters: $\{v=2,k=0.2,c=0.5,\alpha=0.65,w=0.05,T_N=2,T_M=1,\phi=b=1,\theta=0.2,\sigma_W=0.3\}$: $\frac{\partial p_N^*}{\partial c}$ **(Dashed) and** $\frac{\partial p_M^*}{\partial c}$ **(Solid).**

Using extensive numerical simulations we show that there exist regions of $r\in(0,\frac{1}{c\phi})$ in which $\frac{\partial p_N^*}{\partial c}$ and $\frac{\partial p_M^*}{\partial c}$ are negative. The Figures EC.12 demonstrates existence.

EC.4.4. Proof of Proposition 3

Proof. The proof follows from Lemma 1 and is derived using the first order expressions for revenue and mislabeling costs. Consider the following representation of π^*:

$$\begin{aligned}\pi^* = &bT_M((1-\alpha)p_N^*(s_N^*(\tilde r-1)(-w\theta+k-1)-1)+\alpha p_M^*(s_M^*(\tilde r-1)(-w\theta+k-1)-1)+v)\\&+(\alpha-1)c^2(w+1)(p_M^*)^2r^2T_N(s_M^*)^2\phi^2(w\theta-k+1)\\&-(\alpha-1)cp_M^*rT_Ns_M^*\phi\left(p_M^*\left(2w^2\theta s_M^*-w(\theta+2s_M^*(-\theta+k-1)+1)-2ks_M^*+k+2s_M^*-2\right)+wv+v\right)\\&+\frac{1}{2}(p_N^*)^2\left(\frac{(\alpha-1)T_M(ws_N^*+s_N^*-1)(w\theta s_N^*+s_N^*-1)}{c}\right.\\&\qquad\left.-2\alpha T_N((w+1)s_N^*(\tilde r-1)+1)(1-s_N^*(\tilde r-1)(-w\theta+k-1))\right)\\&+\alpha p_N^*T_Nv((w+1)s_N^*(\tilde r-1)+1)-\frac{\alpha(p_M^*)^2T_M(ws_M^*+s_M^*-1)(w\theta s_M^*+s_M^*-1)}{2c}\\&+T_N\left((\alpha-1)w^2\theta(p_M^*)^2(s_M^*)^2-(\alpha-1)wp_M^*s_M^*(p_M^*(\theta+s_M^*(-\theta+k-1)+1)-v)\right.\\&\qquad+p_M^*(kp_M^*s_M^*(\alpha-\alpha s_M^*+s_M^*-1)+\alpha(s_M^*-1)(p_M^*(s_M^*-1)+v)-s_M^*(p_M^*s_M^*+v))\\&\qquad\left.+p_W\sigma_W(v-p_W)\right)+p_M^*T_N(p_M^*(2s_M^*-1)+v)\end{aligned}$$

where following Lemma 1, each of the equilibrium outcomes are expressed as follows:

$$\begin{aligned}
p_W^* &= \frac{v}{2},\\
p_M^* &= \frac{A_1(\alpha b T_M(1+w\theta-k)+(1-\alpha)(w+1)T_N v)}{A_2(4(1-\alpha)^2c^2T_N^2(w(1-\theta)+k)^2)},\\
p_N^* &= \frac{A_3((1-\alpha)wT_M(1-\theta-(\theta+1)\tilde{r})+2\alpha cw(1-\theta)T_N(1-\tilde{r})^2)}{A_4(\alpha k^2 T_N(1-\tilde{r})^2-2(1-\alpha)krT_M\phi(1-\tilde{r})-2(1-\alpha)cT_M\tilde{r}^2)},\\
s_M^* &= \frac{A_5(w(1-\theta)((1-\alpha)T_N v+\alpha bT_M)+2\alpha bkT_M)}{A_6(\alpha bT_M(k+1-\theta(1-k))+(1-\alpha)(1-\theta)T_N v)},\\
s_N^* &= \frac{A_7(2\alpha cw(1-\theta)T_N(1-\tilde{r})+2\alpha ckT_N(1-\tilde{r})-2(1-\alpha)\tilde{r}T_M+(1-\alpha)wT_M(1-2\tilde{r}-\theta))}{A_8(1+k+\tilde{r}(\theta(3-k)+1-k)-\theta(1-k))}.
\end{aligned}$$

with the terms $A_1,\cdots,A_8$ as mentioned in Lemma 1 proof.

Considering the polynomial expression for π^* following are the compact representations of the expressions

$$\begin{aligned}
\frac{\partial^2\pi^*}{\partial r^2} &= \hat{G}_5(\alpha,c,r,k,T_N,T_M,w,\phi,\theta),\\
\frac{\partial^2\pi^*}{\partial r\partial c} &= \hat{G}_6(\alpha,c,r,k,T_N,T_M,w,\phi,\theta)
\end{aligned}$$

where, for the sake of brevity, we do not provide expressions here but show the numerical analysis of $\hat{G}_5(\alpha,c,r,k,T_N,T_M,w,\phi,\theta)$ and $\hat{G}_6(\alpha,c,r,k,T_N,T_M,w,\phi,\theta)$ as follows.

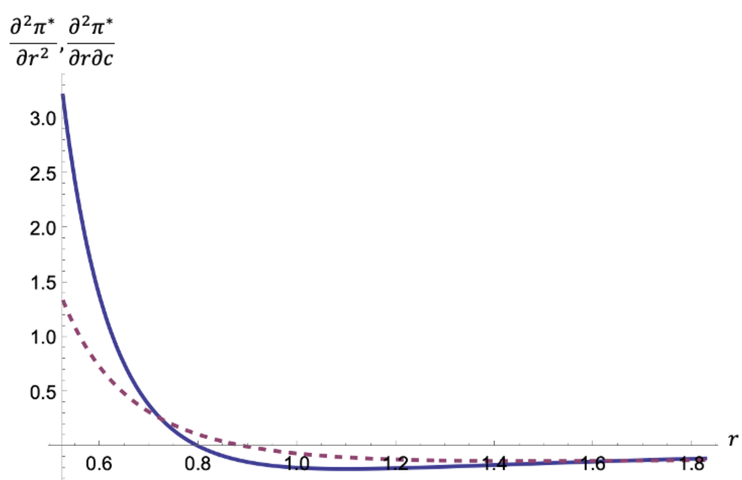


Figure EC.13 Sensitivity Analysis of $\frac{\partial^2\pi^*}{\partial r^2}$ **and** $\frac{\partial^2\pi^*}{\partial r\partial c}$ **w.r.t** r **at** $\{v=2,k=0.1,c=0.5,w=0.05,\alpha=0.65,T_N=2,T_M=1,\phi=1,b=1,\theta=0.2,\sigma_W=0.3\}$**. In this figure,** $\frac{\partial^2\pi^*}{\partial r^2}$ **(Solid) and** $\frac{\partial^2\pi^*}{\partial r\partial c}$ **(Dashed)**

EC.4.5. Proof of Proposition 4

Proof. The proof follows from Lemma 1. It is sufficient to show that $\frac{\partial n_f^*}{\partial T_M} < 0$. Consider the expression for n_f^* as follows:

$$n_f^* = n_{f,MM}^* + n_{f,MN}^*$$

where

$$n_{fMM} = \frac{BC37}{BC38},$$
$$n_{fMN} = \frac{BC39}{BC40}$$

Similar to past proofs, here too we list the terms in the numerator and denominator separately, as follows.

$$\begin{aligned} BC37 = T_M\alpha \big(&-2r^2T_N(\alpha-1)(w(\theta-1)-k)(bT_M\alpha(-k+w\theta+1)-(w+1)T_Nv(\alpha-1))\phi^2c^3 \\ &+4rT_N(\alpha-1)(w(\theta-1)-k)(bT_M\alpha(-k+w\theta+1)-(w+1)T_Nv(\alpha-1))\phi c^2 \\ &-2T_N(\alpha-1)(-\theta w+w+k)((w+1)T_Nv(\alpha-1)+bT_M\alpha(k-w\theta-1))c \\ &-rT_M\alpha(\theta w+w+2)((w+1)T_Nv(\alpha-1)+bT_M\alpha(-k+w\theta+1))\phi c \\ &+T_M\alpha\big((\theta-1)(T_N(v-v\alpha)+bT_M\alpha\theta)w^2 \\ &-(T_Nv(\alpha-1)(\theta-1)+bT_M\alpha(k+(k-1)\theta+1))w-2bkT_M\alpha\big)\big)\left(\frac{BCN1}{BCD1}+\frac{BCN2}{BCD2}-1\right) \end{aligned}$$

where

$$\begin{aligned} BCN1 = &-2rT_N(\alpha-1)(T_Nv(\alpha-1)-bT_M\alpha)(-\theta w+w+k)\phi c^2 \\ &+2(T_N(\alpha-1)(T_Nv(\alpha-1)-bT_M\alpha)(-\theta w+w+k) \\ &+rT_M\alpha((w+1)T_Nv(\alpha-1)+bT_M\alpha(-k+w\theta+1))\phi)c \\ &+T_M\alpha(2bkT_M\alpha+w(T_Nv(\alpha-1)-bT_M\alpha)(\theta-1)), \end{aligned}$$

$$\begin{aligned} BCD1 = &\,2r^2T_N(\alpha-1)(w(\theta-1)-k)(bT_M\alpha(-k+w\theta+1)-(w+1)T_Nv(\alpha-1))\phi^2c^3 \\ &-4rT_N(\alpha-1)(w(\theta-1)-k)(bT_M\alpha(-k+w\theta+1)-(w+1)T_Nv(\alpha-1))\phi c^2 \\ &+2T_N(\alpha-1)(-\theta w+w+k)((w+1)T_Nv(\alpha-1)+bT_M\alpha(k-w\theta-1))c \\ &+rT_M\alpha(\theta w+w+2)((w+1)T_Nv(\alpha-1)+bT_M\alpha(-k+w\theta+1))\phi c \\ &+T_M\alpha\big(-\big((\theta-1)(T_N(v-v\alpha)+bT_M\alpha\theta)w^2\big) \\ &+(T_Nv(\alpha-1)(\theta-1)+bT_M\alpha(k+(k-1)\theta+1))w+2bkT_M\alpha\big), \end{aligned}$$

and

$$\begin{aligned} BCN2 = w\big(&-2rT_N(\alpha-1)(T_Nv(\alpha-1)-bT_M\alpha)(-\theta w+w+k)\phi c^2 \\ &+2(T_N(\alpha-1)(T_Nv(\alpha-1)-bT_M\alpha)(-\theta w+w+k) \\ &+rT_M\alpha((w+1)T_Nv(\alpha-1)+bT_M\alpha(-k+w\theta+1))\phi)c \\ &+T_M\alpha(2bkT_M\alpha+w(T_Nv(\alpha-1)-bT_M\alpha)(\theta-1))\big), \end{aligned}$$

$$\begin{aligned} BCD2 = &\,2r^2T_N(\alpha-1)(w(\theta-1)-k)(bT_M\alpha(-k+w\theta+1)-(w+1)T_Nv(\alpha-1))\phi^2c^3 \\ &-4rT_N(\alpha-1)(w(\theta-1)-k)(bT_M\alpha(-k+w\theta+1)-(w+1)T_Nv(\alpha-1))\phi c^2 \\ &+2T_N(\alpha-1)(-\theta w+w+k)((w+1)T_Nv(\alpha-1)+bT_M\alpha(k-w\theta-1))c \\ &+rT_M\alpha(\theta w+w+2)((w+1)T_Nv(\alpha-1)+bT_M\alpha(-k+w\theta+1))\phi c \\ &+T_M\alpha\big(-\big((\theta-1)(T_N(v-v\alpha)+bT_M\alpha\theta)w^2\big) \\ &+(T_Nv(\alpha-1)(\theta-1)+bT_M\alpha(k+(k-1)\theta+1))w+2bkT_M\alpha\big). \end{aligned}$$

Additionally, the denominator for $n_{f,MM}$ is,

$$\begin{aligned}BC38 =\ & 4(\alpha-1)^2c^4r^2T_N^2\phi^2(w(1-\theta)+k)^2\\&+8(\alpha-1)c^3rT_N\phi\left(\alpha(w+1)rT_M\phi(w\theta-k+1)-(\alpha-1)T_N(w(1-\theta)+k)^2\right)\\&+4(\alpha-1)c^2T_N\left(\alpha rT_M\phi\left(w^2(\theta-1)^2-w(\theta-3)k+2k\right)+(\alpha-1)T_N(w(1-\theta)+k)^2\right)\\&+4(\alpha-1)\alpha cw(\theta-1)T_MT_N(w(1-\theta)+k)+\alpha^2w^2(\theta-1)^2T_M^2\end{aligned}$$

Now, considering $n_{f,MN}$ we have,

$$\begin{aligned}BC39 = T_M(\alpha-1)\Big(&(w+1)T_Nv\alpha\left(2ckT_N\alpha(\tilde r-1)^2-2cwT_N\alpha(\theta-1)(\tilde r-1)^2+2crT_M(\alpha-1)\phi\right.\\&\left.+wT_M(\alpha-1)(\theta+cr(\theta+1)\phi-1)\right)\\&+bT_M(\alpha-1)\left(\theta\left(2cT_N\alpha(\theta-1)(\tilde r-1)^2+T_M(\alpha-1)(-\theta+cr(\theta+1)\phi+1)\right)w^2\right.\\&+\left(T_M(\alpha-1)(\theta k+k-\theta-cr(k+(k-3)\theta-1)\phi+1)-2cT_N\alpha(-\theta+k(2\theta-1)+1)(\tilde r-1)^2\right)w\\&\left.+2\left(ck^2T_N\alpha(\tilde r-1)^2-k(T_M(\alpha-1)+cT_N\alpha(\tilde r-1))(\tilde r-1)+crT_M(\alpha-1)\phi\right)\right)\Big)\left(\frac{BCN3}{BCD3}\right.\\&\left.+\frac{BCN4}{BCD4}-1\right)\end{aligned}$$

where

$$\begin{aligned}BCN3 =\ & T_Nv\alpha(2crT_M(\alpha-1)\phi-2ckT_N\alpha(\tilde r-1)+2cwT_N\alpha(\theta-1)(\tilde r-1)+wT_M(\alpha-1)(\theta+2\tilde r-1))\\&+bT_M(\alpha-1)(2crT_M(\alpha-1)\phi-2k(T_M(\alpha-1)-cT_N\alpha)(\tilde r-1)\\&+w(T_M(\alpha-1)(\theta(2\tilde r-1)+1)-2cT_N\alpha(\theta-1)(\tilde r-1))),\end{aligned}$$

$$\begin{aligned}BCD3 =\ & (w+1)T_Nv\alpha\left(2ckT_N\alpha(\tilde r-1)^2-2cwT_N\alpha(\theta-1)(\tilde r-1)^2+2crT_M(\alpha-1)\phi\right.\\&\left.+wT_M(\alpha-1)(\theta+cr(\theta+1)\phi-1)\right)\\&+bT_M(\alpha-1)\left(\theta\left(2cT_N\alpha(\theta-1)(\tilde r-1)^2+T_M(\alpha-1)(-\theta+cr(\theta+1)\phi+1)\right)w^2\right.\\&+\left(T_M(\alpha-1)(\theta k+k-\theta-cr(k+(k-3)\theta-1)\phi+1)-2cT_N\alpha(-\theta+k(2\theta-1)+1)(\tilde r-1)^2\right)w\\&\left.+2\left(ck^2T_N\alpha(\tilde r-1)^2-k(T_M(\alpha-1)+cT_N\alpha(\tilde r-1))(\tilde r-1)+crT_M(\alpha-1)\phi\right)\right),\end{aligned}$$

and

$$\begin{aligned}BCN4 =\ & w(T_Nv\alpha(2crT_M(\alpha-1)\phi-2ckT_N\alpha(\tilde r-1)+2cwT_N\alpha(\theta-1)(\tilde r-1)+wT_M(\alpha-1)(\theta+2\tilde r-1))\\&+bT_M(\alpha-1)(2crT_M(\alpha-1)\phi-2k(T_M(\alpha-1)-cT_N\alpha)(\tilde r-1)\\&+w(T_M(\alpha-1)(\theta(2\tilde r-1)+1)-2cT_N\alpha(\theta-1)(\tilde r-1)))),\end{aligned}$$

$$\begin{aligned}BCD4 =\ & (w+1)T_Nv\alpha\left(2ckT_N\alpha(\tilde r-1)^2-2cwT_N\alpha(\theta-1)(\tilde r-1)^2+2crT_M(\alpha-1)\phi\right.\\&\left.+wT_M(\alpha-1)(\theta+cr(\theta+1)\phi-1)\right)\\&+bT_M(\alpha-1)\left(\theta\left(2cT_N\alpha(\theta-1)(\tilde r-1)^2+T_M(\alpha-1)(-\theta+cr(\theta+1)\phi+1)\right)w^2\right.\\&+\left(T_M(\alpha-1)(\theta k+k-\theta-cr(k+(k-3)\theta-1)\phi+1)-2cT_N\alpha(-\theta+k(2\theta-1)+1)(\tilde r-1)^2\right)w\\&\left.+2\left(ck^2T_N\alpha(\tilde r-1)^2-k(T_M(\alpha-1)+cT_N\alpha(\tilde r-1))(\tilde r-1)+crT_M(\alpha-1)\phi\right)\right).\end{aligned}$$

Additionally, denominator for $n^*_{f,MN}$ is,

$$\begin{aligned}BC40 =\ & 4\alpha cwT_N\left(2(\alpha-1)c^2(\theta+1)r^2T_M\phi^2-k(\tilde r-1)((\alpha-1)T_M(2\tilde r+\theta-1)+2\alpha c(\theta-1)T_N(\tilde r-1))\right)\\&+4\alpha c^2T_N\left(\alpha k^2T_N(\tilde r-1)^2-2(\alpha-1)krT_M\phi(\tilde r-1)+2(\alpha-1)cr^2T_M\phi^2\right)\\&+w^2\left(4\alpha cT_N\tilde r^2\left(\alpha c(\theta-1)^2T_N+2(\alpha-1)\theta T_M\right)-4\alpha c^2(\theta-1)^2rT_N\phi(2\alpha cT_N-\alpha T_M+T_M)\right.\\&\left.+(\theta-1)^2(2\alpha cT_N-\alpha T_M+T_M)^2\right)\end{aligned}$$

We use these expressions to derive the $\frac{\partial n_f^*}{\partial T_M}$. Given the complicated structure of the derivative expressions, we do not present here for brevity. However, we denote this expression as $\hat{nf}_{MM}(\alpha, c, r, k, T_N, T_M, w, \phi, \theta)$ and $\hat{nf}_{MN}(\alpha, c, r, k, T_N, T_M, w, \phi, \theta, \sigma_W)$. Further, we establish the conditions that if $\hat{G}_7(\alpha, c, r, k, T_N, T_M, w, \phi, \theta, \sigma_W) = \hat{nf}_{w_N}(\alpha, c, r, k, T_N, T_M, w, \phi, \theta, \sigma_W) + \hat{nf}_{w_M}(\alpha, c, r, k, T_N, T_M, w, \phi, \theta, \sigma_W) < 0$ then it follows that $\frac{\partial n_f^*}{\partial T_M} < 0$. We further demonstrate the nature of the fraud clicks behavior w.r.t T_M using the following diagram.

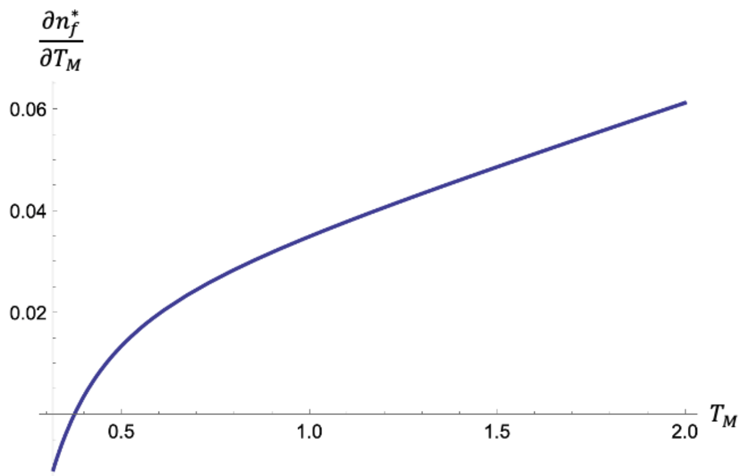


Figure EC.14 Sensitivity Analysis of $\frac{\partial n_f^*}{\partial T_M}$ w.r.t T_M at:
$\{v = 2, k = 0.15, c = 0.5, w = 0.05, \alpha = 0.65, T_N = 2, \phi = b = r = 1, \theta = 0.2, \sigma_W = 0.3\}$

EC.4.6. Proof of Proposition 5

Proof. We consider the expression for π^*:

$$\begin{aligned}
\pi^* = {} & bT_M((1-\alpha)p_N^*(s_N^*(\tilde{r}-1)(-w\theta+k-1)-1)+\alpha p_M^*(s_M^*(\tilde{r}-1)(-w\theta+k-1)-1)+v) \\
& + (\alpha-1)c^2(w+1)(p_M^*)^2 r^2 T_N (s_M^*)^2 \phi^2 (w\theta-k+1) \\
& - (\alpha-1)cp_M^* r T_N s_M^* \phi \left(p_M^* \left(2w^2\theta s_M^* - w(\theta+2s_M^*(-\theta+k-1)+1) - 2ks_M^* + k + 2s_M^* - 2\right) + wv + v\right) \\
& + \frac{1}{2}(p_N^*)^2 \Bigg(\frac{(\alpha-1)T_M(ws_N^*+s_N^*-1)(w\theta s_N^*+s_N^*-1)}{c} \\
& \qquad - 2\alpha T_N((w+1)s_N^*(\tilde{r}-1)+1)(1-s_N^*(\tilde{r}-1)(-w\theta+k-1)) \Bigg) \\
& + \alpha p_N^* T_N v((w+1)s_N^*(\tilde{r}-1)+1) - \frac{\alpha (p_M^*)^2 T_M (ws_M^*+s_M^*-1)(w\theta s_M^*+s_M^*-1)}{2c} \\
& + T_N \big((\alpha-1)w^2\theta(p_M^*)^2(s_M^*)^2 - (\alpha-1)wp_M^* s_M^*(p_M^*(\theta+s_M^*(-\theta+k-1)+1)-v) \\
& \qquad + p_M^*(kp_M^* s_M^*(\alpha-\alpha s_M^*+s_M^*-1) + \alpha(s_M^*-1)(p_M^*(s_M^*-1)+v) - s_M^*(p_M^* s_M^*+v)) \\
& \qquad + p_W \sigma_W (v-p_W)\big) + p_M^* T_N (p_M^*(2s_M^*-1)+v)
\end{aligned}$$

where, from the proof of Lemma 1 we can restate the equilibrium outcomes as,

$$p_W^* = \frac{v}{2},$$

$$p_M^* = \frac{A_1(\alpha b T_M(1+w\theta-k)+(1-\alpha)(w+1)T_N v)}{A_2(4(1-\alpha)^2c^2T_N^2(w(1-\theta)+k)^2)},$$
$$p_N^* = \frac{A_3((1-\alpha)wT_M(1-\theta-(\theta+1)\tilde{r})+2\alpha cw(1-\theta)T_N(1-\tilde{r})^2)}{A_4(\alpha k^2T_N(1-\tilde{r})^2-2(1-\alpha)krT_M\phi(1-\tilde{r})-2(1-\alpha)cT_M\tilde{r}^2)},$$
$$s_M^* = \frac{A_5(w(1-\theta)((1-\alpha)T_N v+\alpha bT_M)+2\alpha bkT_M)}{A_6(\alpha bT_M(k+1-\theta(1-k))+(1-\alpha)(1-\theta)T_N v)},$$
$$s_N^* = \frac{A_7(2\alpha cw(1-\theta)T_N(1-\tilde{r})+2\alpha ckT_N(1-\tilde{r})-2(1-\alpha)\tilde{r}T_M+(1-\alpha)wT_M(1-2\tilde{r}-\theta))}{A_8(1+k+\tilde{r}(\theta(3-k)+1-k)-\theta(1-k))}.$$

with the terms $A_1,\cdots,A_8$ as mentioned in Lemma 1 proof.

Considering the polynomial expression for π^* following is the compact representations of the expression $\frac{\partial\pi^*}{\partial T_M}$.

$$\frac{\partial\pi^*}{\partial T_M} = \hat{G}_8(\alpha,c,r,k,T_N,T_M,w,\phi,\theta)$$

Hence, the condition that should be satisfied for $\frac{\partial\pi^*}{\partial T_M} > 0$ is given as $\hat{G}_8(\alpha,c,r,k,T_N,T_M,w_N,w_M,\phi) > 0$ where for brevity we do not provide the full expression here. Figure EC.15 represents the numerical analysis of the expression as stated above.

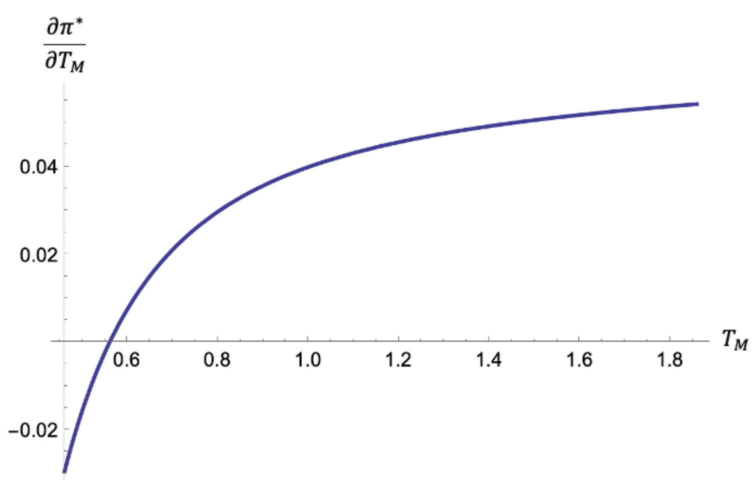


Figure EC.15 Illustrating Proposition 5. Values of Parameters:
$\{v=1,k=0.15,c=0.5,w=0.2,\alpha=0.65,T_N=2,\phi=r=1,b=0.1,\theta=0.2,\sigma_W=0.3\}$.

EC.5. Proofs of Extensions

In this section, we present the proofs of the lemmas and propositions for the three extensions considered in the paper, as presented in Sections 5.1 to 5.3.

EC.5.1. Extension 1 – Imperfect Valuation Update

EC.5.1.1. Proof of Lemma 2

Proof. The proof is similar to the proof of Lemma 1. For the sake of brevity, we mention the main aspects of the proof as follows. First, note that for the equilibrium to exist, given $\gamma > 0$, the following should hold $\forall j \in \{N, M\}$.

- $p_j^* \geq 0$,
- $s_j^* \in (0,1)$,
- $y_j^* \in (0,1)$
- $x_j^* > 0$,
- $u_{NN}^*, u_{NM}^* \in (0,1)$ which implies some but not all of the nonmalicious publishers participate,
- $\pi^* > 0$,
- Concavity conditions as derived in proof of Lemma 1:

$$L_2 : \mathbb{H}(p_j, s_j) \text{ is negative semi-definite } \Leftrightarrow (-1)^k \Delta_k \geq 0, \forall\ 1 \leq k \leq 4, j \in \{M, N\}. \qquad \text{(EC.6)}$$

where the Hessian is constructed as follows:

$$\mathbb{H}(p_j, s_j) = \begin{bmatrix} \frac{\partial^2 \pi}{\partial p_N^2} & \frac{\partial^2 \pi}{\partial p_N \partial p_M} & \frac{\partial^2 \pi}{\partial p_N \partial s_N} & \frac{\partial^2 \pi}{\partial p_N \partial s_M} \\ \frac{\partial^2 \pi}{\partial p_N \partial p_M} & \frac{\partial^2 \pi}{\partial p_M^2} & \frac{\partial^2 \pi}{\partial p_M \partial s_N} & \frac{\partial^2 \pi}{\partial p_M \partial s_M} \\ \frac{\partial^2 \pi}{\partial p_N \partial s_N} & \frac{\partial^2 \pi}{\partial p_M \partial s_N} & \frac{\partial^2 \pi}{\partial s_N^2} & \frac{\partial^2 \pi}{\partial s_M \partial s_N} \\ \frac{\partial^2 \pi}{\partial p_N \partial s_M} & \frac{\partial^2 \pi}{\partial p_M \partial s_M} & \frac{\partial^2 \pi}{\partial s_M \partial s_N} & \frac{\partial^2 \pi}{\partial s_M^2} \end{bmatrix}$$

From the Hessian's expression, it follows that there are 15 principal minors to examine. These principal minors are as follows:

$$\Delta_1 \equiv \left\{ \frac{\partial^2 \pi}{\partial p_N^2}, \frac{\partial^2 \pi}{\partial p_M^2}, \frac{\partial^2 \pi}{\partial s_N^2}, \frac{\partial^2 \pi}{\partial s_M^2} \right\},$$
$$\Delta_2 \equiv \left\{ [\mathbb{H}]_{1,2}, [\mathbb{H}]_{1,3}, [\mathbb{H}]_{1,4}, [\mathbb{H}]_{2,3}, [\mathbb{H}]_{2,4}, [\mathbb{H}]_{3,4} \right\},$$
$$\Delta_3 \equiv \left\{ [\mathbb{H}]_{1,2,3}, [\mathbb{H}]_{1,2,4}, [\mathbb{H}]_{1,3,4}, [\mathbb{H}]_{2,3,4} \right\},$$
$$\Delta_4 \equiv [\mathbb{H}]$$

where the expression $[\mathbb{H}]_{i,j}$ or $[\mathbb{H}]_{i,j,k}$ represents principal minors after removing row and columns i, j (or i, j, k). Since the expressions for the Hessian and the equilibrium decision variables for the ad network are substantially large, we provide an existence proof using the following values. However, below are the expressions for $p_{j,I}^*, s_{j,I}^*$.

$$p_{W,I}^* = \gamma p_W^*,$$
$$p_{M,I}^* = p_M^* + (\gamma - 1) B_1,$$

$$p^*_{N,I} = p^*_N + (\gamma - 1)B_2,$$
$$s^*_{M,I} = B_3(\gamma),$$
$$s^*_{N,I} = B_4(\gamma),$$

where the expressions for $B_1, \cdots, B_4$ are as follows:

$$B_1 = \left(\frac{2(1-\alpha)c(w+1)T_N v(-2(\alpha-1)cT_N\tilde{r}^2(w(1-\theta)+k) + 4(\alpha-1)cT_N\tilde{r}(w(1-\theta)+k)}{BD_1}\right) + \left(\frac{2(1-\alpha)cT_N(w(1-\theta)+k) - \alpha\tilde{r}T_M(w(1+\theta)+2) - \alpha w(\theta-1)T_M)}{BD_1}\right)$$

where

$$\begin{aligned} BD_1 = {} & 4(\alpha-1)^2c^4r^2T_N^2\phi^2(w(1-\theta)+k)^2 \\ & + 8(\alpha-1)c^3rT_N\phi\left(\alpha(w+1)rT_M\phi(w\theta-k+1) - (\alpha-1)T_N(w(1-\theta)+k)^2\right) \\ & + 4(\alpha-1)c^2T_N\left(\alpha rT_M\phi\left(w^2(\theta-1)^2 - w(\theta-3)k + 2k\right) + (\alpha-1)T_N(w(1-\theta)+k)^2\right) \\ & + 4(\alpha-1)\alpha cw(\theta-1)T_MT_N(w(1-\theta)+k) + \alpha^2w^2(\theta-1)^2T_M^2, \end{aligned}$$

and

$$B_2 = \frac{2\alpha c(w+1)T_N v((\alpha-1)wT_M(\tilde{r}(1+\theta)+\theta-1) - 2\alpha cw(\theta-1)T_N(\tilde{r}-1)^2)}{BD_2} + \frac{2\alpha c(w+1)T_N v(2\alpha ckT_N(\tilde{r}-1)^2 + 2(\alpha-1)\tilde{r}T_M)}{BD_2},$$

where

$$\begin{aligned} BD_2 = {} & 4\alpha cwT_N\left(2(\alpha-1)(\theta+1)T_M\tilde{r}^2 - k(\tilde{r}-1)\left((\alpha-1)T_M(2\tilde{r}+\theta-1) + 2\alpha c(\theta-1)T_N(\tilde{r}-1)\right)\right) \\ & + 4\alpha c^2T_N\left(\alpha k^2T_N(\tilde{r}-1)^2 - 2(\alpha-1)krT_M\phi(\tilde{r}-1) + 2(\alpha-1)rT_M\phi\tilde{r}\right) \\ & + w^2\left(4\alpha cT_N\tilde{r}^2\left(\alpha c(\theta-1)^2T_N + 2(\alpha-1)\theta T_M\right) - 4\alpha c^2(\theta-1)^2rT_N\phi(2\alpha cT_N - \alpha T_M + T_M)\right. \\ & \left. + (\theta-1)^2(2\alpha cT_N - \alpha T_M + T_M)^2\right) \end{aligned}$$

In addition, the expressions for technology configurations are as follows:

$$s^*_{N,I} = B_3(\gamma) = \frac{BN_3}{BD_3}$$

where

$$\begin{aligned} BN_3 = {} & (\alpha-1)bT_M(w((\alpha-1)T_M(\theta(2\tilde{r}-1)+1) - 2\alpha c(\theta-1)T_N(\tilde{r}-1)) \\ & - 2k(\tilde{r}-1)((\alpha-1)T_M - \alpha cT_N) + 2(\alpha-1)\tilde{r}T_M) \\ & + \alpha\gamma T_N v((\alpha-1)wT_M(2\tilde{r}+\theta-1) + 2\alpha cw(\theta-1)T_N(\tilde{r}-1) - 2\alpha ckT_N(\tilde{r}-1) + 2(\alpha-1)\tilde{r}T_M), \end{aligned}$$

$$\begin{aligned} BD_3 = {} & (\alpha-1)bT_M\left(w^2\theta\left((\alpha-1)T_M(\tilde{r}(1+\theta)-\theta+1) + 2\alpha c(\theta-1)T_N(\tilde{r}-1)^2\right)\right. \\ & + w\left((\alpha-1)T_M(-\tilde{r}(\theta(k-3)+k-1) - \theta + \theta k + k + 1) - 2\alpha cT_N(-\theta + (2\theta-1)k + 1)(\tilde{r}-1)^2\right) \\ & \left. + 2\left(\alpha ck^2T_N(\tilde{r}-1)^2 - k(\tilde{r}-1)(\alpha cT_N(\tilde{r}-1) + (\alpha-1)T_M) + (\alpha-1)\tilde{r}T_M\right)\right) \\ & + \alpha\gamma(w+1)T_N v\left((\alpha-1)wT_M(\tilde{r}(1+\theta)+\theta-1) - 2\alpha cw(\theta-1)T_N(\tilde{r}-1)^2 + 2\alpha ckT_N(\tilde{r}-1)^2\right. \\ & \left. + 2(\alpha-1)\tilde{r}T_M\right) \end{aligned}$$

and,

$$s_{M,I}^{*} = B_4(\gamma) = \frac{BN_4}{BD_4}$$

where

$$\begin{aligned} BN_4 = \alpha b T_M \left(2(\alpha-1)cT_N\tilde{r}(w(1-\theta)+k) + 2\alpha\tilde{r}T_M(w\theta-k+1) - 2(\alpha-1)cT_N(w(1-\theta)+k)\right. \\ + \alpha T_M(w(-\theta)+w+2k)) + (\alpha-1)\gamma T_N v\left(-2(\alpha-1)cT_N\tilde{r}(w(1-\theta)+k)\right. \\ \left. + 2c((\alpha-1)T_N(w(1-\theta)+k) + \alpha(w+1)rT_M\phi) + \alpha w(\theta-1)T_M\right), \end{aligned}$$

$$\begin{aligned} BD_4 = \alpha b T_M \big(2(\alpha-1)cT_N\tilde{r}^2(-w\theta+k-1)(w(1-\theta)+k) - 4(\alpha-1)cT_N\tilde{r}(-w\theta+k-1)(w(1-\theta)+k) \\ + c(w\theta-k+1)(\alpha r T_M\phi(w(1+\theta)+2) - 2(\alpha-1)T_N(w(1-\theta)+k)) \\ + \alpha T_M\left(w\left(-w\theta^2+\theta(w+k-1)+k+1\right)+2k\right)\big) \\ - (\alpha-1)\gamma(w+1)T_N v\big(-2(\alpha-1)cT_N\tilde{r}^2(w(1-\theta)+k) + 4(\alpha-1)cT_N\tilde{r}(w(1-\theta)+k) \\ - 2(\alpha-1)cT_N(w(1-\theta)+k) - \alpha\tilde{r}T_M(w(1+\theta)+2) - \alpha w(\theta-1)T_M\big) \end{aligned}$$

First, we consider the case when advertisers underestimate valuation and specifically consider $\gamma = 0.5$. Using the following parameter values that satisfy the above-stated feasibility constraints: $\{w = 0.05, c = 0.75, k = 0.05, \phi = 1, T_N = 2, T_M = 1, \alpha = 0.65, b = 1, v = 2, \gamma = 0.5, r = 1.15\}$ we can solve for the interior solution and have the following values at equilibrium for the ad network's profit and technology configurations as follows:

$$\begin{aligned} &\pi_I^{*} = 1.32056, \\ &p_{N,I}^{*} = 0.410709, p_{M,I}^{*} = 0.0325062, p_{W,I}^{*} = 0.5, \\ &s_{N,I}^{*} = 0.917494, s_{M,I}^{*} = 0.676483. \end{aligned}$$

Using the following parameter values that satisfy the above-stated feasibility constraints: $\{w = 0.8, c = 0.8, k = 0.2, \phi = 1, T_N = 2, T_M = 1, \alpha = 0.65, b = 1, v = 2, \gamma = 2, r = 1.15\}$, we can solve for the interior solution and have the following values at equilibrium for ad network's profit and technology configurations as follows:

$$\begin{aligned} &\pi_I^{*} = 12.4786, \\ &p_{N,I}^{*} = 2.11044, p_{M,I}^{*} = 1.73797, p_{W,I}^{*} = 2, \\ &s_{N,I}^{*} = 0.935717, s_{M,I}^{*} = 0.956084. \end{aligned}$$

Having demonstrated numerically the existence of the feasible solutions, we conclude that equilibrium exists in this scenario.

EC.5.1.2. Proof of Proposition 6 The proof follows from Lemma 1 and Lemma 2. Specifically, from Lemma 1, the derived equilibrium technology configuration s_j^* and payment p_j^* are used to derive the ad network profit π^* under perfect valuation update ($\gamma = 1$). It is then sufficient to show that for $\gamma > 1$, profit under imperfect valuation update dominates the profit under perfect valuation update. In contrast, profit under $\gamma < 1$ is dominated under imperfect valuation update. Consider the expression for profit under imperfect valuation update, derived from equilibrium payment p_j^* and technology configuration s_j^* in Lemma 2:

$$
\begin{aligned}
\pi_I^* = bT_M\Bigg(& (1-\alpha)p_{N,I}^*\left(s_{N,I}^*(\tilde{r}-1)(-w\theta+k-1)-1\right) + \alpha p_{M,I}^*\left(s_{M,I}^*(\tilde{r}-1)(-w\theta+k-1)-1\right) \\
& + \gamma v\Bigg) + (\alpha-1)c^2(w+1)(p_{M,I}^*)^2 r^2 T_N (s_{M,I}^*)^2\phi^2(w\theta-k+1) \\
& -(\alpha-1)cp_{M,I}^* r T_N s_{M,I}^*\phi\left(p_{M,I}^*\left(2w^2\theta s_{M,I}^* - w\left(\theta+2s_{M,I}^*(-\theta+k-1)+1\right) - 2ks_{M,I}^* + k + 2s_{M,I}^* - 2\right)\right. \\
& \left. + \gamma(w+1)v\right) \\
& - \frac{\alpha w^2\theta(p_{M,I}^*)^2 T_M (s_{M,I}^*)^2}{2c} + \frac{1}{2c}(p_{N,I}^*)^2((\alpha-1)T_M(ws_{N,I}^* + s_{N,I}^* - 1)(w\theta s_{N,I}^* + s_{N,I}^* - 1) \\
& - 2\alpha c T_N\left((w+1)s_{N,I}^*(\tilde{r}-1)+1\right)\left(1 - s_{N,I}^*(\tilde{r}-1)(-w\theta+k-1)\right)) \\
& + \alpha\gamma p_{N,I}^* T_N v\left((w+1)s_{N,I}^*(\tilde{r}-1)+1\right) - \frac{\alpha w\theta(p_{M,I}^*)^2 T_M (s_{M,I}^*)^2}{2c} - \frac{\alpha w(p_{M,I}^*)^2 T_M (s_{M,I}^*)^2}{2c} \\
& + \frac{\alpha w\theta(p_{M,I}^*)^2 T_M s_{M,I}^*}{2c} + \frac{\alpha w(p_{M,I}^*)^2 T_M s_{M,I}^*}{2c} - \frac{\alpha(p_{M,I}^*)^2 T_M}{2c} - \frac{\alpha(p_{M,I}^*)^2 T_M (s_{M,I}^*)^2}{2c} \\
& + \frac{\alpha(p_{M,I}^*)^2 T_M s_{M,I}^*}{c} + \alpha w^2\theta(p_{M,I}^*)^2 T_N (s_{M,I}^*)^2 - w^2\theta(p_{M,I}^*)^2 T_N (s_{M,I}^*)^2 - \alpha wk(p_{M,I}^*)^2 T_N (s_{M,I}^*)^2 \\
& + wk(p_{M,I}^*)^2 T_N (s_{M,I}^*)^2 + \alpha w\theta(p_{M,I}^*)^2 T_N (s_{M,I}^*)^2 + \alpha w(p_{M,I}^*)^2 T_N (s_{M,I}^*)^2 - w\theta(p_{M,I}^*)^2 T_N (s_{M,I}^*)^2 \\
& - w(p_{M,I}^*)^2 T_N (s_{M,I}^*)^2 - \alpha w\theta(p_{M,I}^*)^2 T_N s_{M,I}^* - \alpha w(p_{M,I}^*)^2 T_N s_{M,I}^* + w\theta(p_{M,I}^*)^2 T_N s_{M,I}^* \\
& + w(p_{M,I}^*)^2 T_N s_{M,I}^* + \alpha\gamma w p_{M,I}^* T_N v s_{M,I}^* - \gamma w p_{M,I}^* T_N v s_{M,I}^* - \alpha k(p_{M,I}^*)^2 T_N (s_{M,I}^*)^2 \\
& + k(p_{M,I}^*)^2 T_N (s_{M,I}^*)^2 + \alpha k(p_{M,I}^*)^2 T_N s_{M,I}^* - k(p_{M,I}^*)^2 T_N s_{M,I}^* + \alpha(p_{M,I}^*)^2 T_N \\
& + \alpha(p_{M,I}^*)^2 T_N (s_{M,I}^*)^2 - (p_{M,I}^*)^2 T_N (s_{M,I}^*)^2 - 2\alpha(p_{M,I}^*)^2 T_N s_{M,I}^* + (p_{M,I}^*)^2 T_N (2s_{M,I}^* - 1) \\
& - \alpha\gamma p_{M,I}^* T_N v + \gamma p_{M,I}^* T_N v + \alpha\gamma p_{M,I}^* T_N v s_{M,I}^* - \gamma p_{M,I}^* T_N v s_{M,I}^* - (p_{W,I}^*)^2\sigma_W T_N + \gamma p_{W,I}^*\sigma_W T_N v
\end{aligned}
$$

where the equilibrium outcomes are:

$$
\begin{aligned}
p_{W,I}^* &= \gamma p_W^*, \\
p_{M,I}^* &= p_M^* + (\gamma-1)B_1, \\
p_{N,I}^* &= p_N^* + (\gamma-1)B_2, \\
s_{M,I}^* &= B_3(\gamma), \\
s_{N,I}^* &= B_4(\gamma),
\end{aligned}
$$

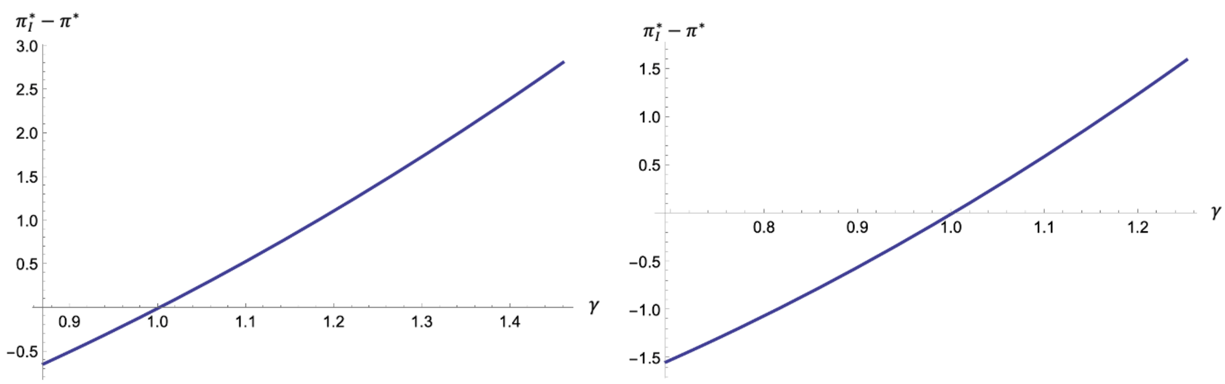
where the terms $B_1, \cdots, B_4$ are mentioned in the proof of Lemma 2.

Further, the expression for π^* is as per the proof in Appendix EC.4.4. We do not repeat it here for the sake of brevity. In order to compare dominance, it follows that we first compute the difference between the above equation and the profit from the base model. Essentially, we obtain the following:

$$\hat{G}_9(\alpha, c, k, r, T_M, T_N, w_M, w_N, \phi, \theta, \gamma) = \pi_I^* - \pi^*$$

For tractability, we show (numerical) existence of the dominance of π_I^* over π^* when $\gamma > 1$, and otherwise, if $0 < \gamma < 1$. The following figure plots the function $\hat{G}_9(\alpha, c, k, r, T_M, T_N, w_M, w_N, \phi, \gamma) = \pi_I^* - \pi^*$ and shows the existence for the following assumed parametric values that satisfy the stated feasibility constraints in Lemma 2: $\{w = 0.05, c = 0.5, k = 0.2, \phi = 1, T_N = 2, T_M = 0.6, \alpha = 0.55, b = 1, v = 1, \gamma = 0.5, r = 1.467\}$

Figure EC.15 also demonstrates the existence of regions $\gamma < 1$ and $\gamma > 1$ such that $\pi_I^* < \pi^*$ and $\pi_I^* > \pi^*$, respectively.



(a) Illustrating $\pi_I^* - \pi^*$ vs. γ when $r = 1.16$ **(b)** Illustrating $\pi_I^* - \pi^*$ vs. γ when $r = 0.75$

Figure EC.16 Impact of change in γ on dominance of ad network profit in base model vs. imperfect valuation update model using parameter values:
$\{k = 0.2, c = 0.4, w = 0.05, \alpha = 0.65, T_N = 2, T_M = 1, \phi = b = 1, v = 2, \theta = 0.2, \sigma_W = 0.3\}$

We now consider the payoffs of publishers under imperfect valuation update and compare it to the payoff under the base case, i.e., perfect valuation update. Following are the expressions for $u_{Nj,I}^* - u_{Nj}^*$:

$$u_{NN,I}^* - u_{NN} = \frac{CN_1}{CD_1},$$
$$u_{NM,I}^* - u_{NM} = \frac{CN_2}{CD_2},$$

where

$$CN_1 = 4(\alpha - 1)\alpha(\gamma - 1)c^2(w + 1)rT_M T_N v\phi(c(w + 1)r\phi + w(\theta - 1)),$$

$$\begin{aligned}CD_1 = {}& 4\alpha c w T_N \left(2(\alpha-1)c^2(\theta+1)r^2T_M\phi^2 - k(\tilde{r}-1)((\alpha-1)T_M(2\tilde{r}+\theta-1)+2\alpha c(\theta-1)T_N(\tilde{r}-1))\right)\\ &+ 4\alpha c^2 T_N\left(\alpha k^2 T_N(\tilde{r}-1)^2 - 2(\alpha-1)krT_M\phi(\tilde{r}-1) + 2(\alpha-1)cr^2T_M\phi^2\right)\\ &+ w^2\left(4\alpha c T_N\tilde{r}^2\left(\alpha c(\theta-1)^2T_N + 2(\alpha-1)\theta T_M\right) - 4\alpha c^2(\theta-1)^2 r T_N\phi(2\alpha c T_N - \alpha T_M + T_M)\right.\\ &\left.\qquad + (\theta-1)^2(2\alpha c T_N - \alpha T_M + T_M)^2\right),\end{aligned}$$

and

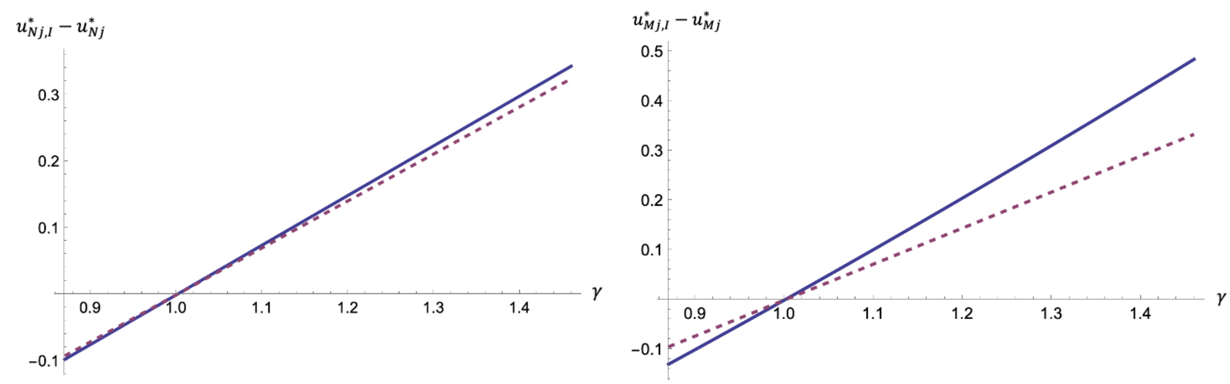
$$CN_2 = 4(\alpha-1)\alpha(\gamma-1)c^2(w+1)rT_MT_Nv\phi(c(w+1)r\phi + w(\theta-1)),$$

$$\begin{aligned}CD_2 = {}& 4(\alpha-1)^2c^4r^2T_N^2\phi^2(w(1-\theta)+k)^2\\ &+ 8(\alpha-1)c^3rT_N\phi\left(\alpha(w+1)rT_M\phi(w\theta-k+1) - (\alpha-1)T_N(w(1-\theta)+k)^2\right)\\ &+ 4(\alpha-1)c^2T_N\left(\alpha r T_M\phi\left(w^2(\theta-1)^2 - w(\theta-3)k + 2k\right) + (\alpha-1)T_N(w(1-\theta)+k)^2\right)\\ &+ 4(\alpha-1)\alpha c w(\theta-1)T_MT_N(w(1-\theta)+k) + \alpha^2w^2(\theta-1)^2T_M^2\end{aligned}$$

Further, the expressions for $u^*_{Mj,I} - u^*_{Mj}$ are represented using the following representation:

$$u^*_{Mj,I} - u^*_{Mj} = \begin{cases}\hat{G}_{12}(\alpha,c,k,r,T_M,T_N,w,\phi,\gamma,\theta) \ , j = M,\\ \hat{G}_{13}(\alpha,c,k,r,T_M,T_N,w,\phi,\gamma,\theta) \ \ , j = N\end{cases}$$

For ease of exposition, we present an existence proof through numerical analysis. Essentially, we denote $u^*_{Nj,I} - u^*_{Nj} = \begin{cases}\hat{G}_{10}(\alpha,c,k,r,T_M,T_N,w,\phi,\gamma,\theta) \ , j = N,\\ \hat{G}_{11}(\alpha,c,k,r,T_M,T_N,w,\phi,\gamma,\theta) \ , j = M\end{cases}$ and use the above expressions for $u^*_{Mj,I} - u^*_{Mj}$. In the following graphs, we show how the payoffs for malicious and nonmalicious publishers change when γ varies from below 1 to greater than 1. Specifically, as valuation goes from under to over-estimation, publishers gain payoff, similar to the ad network.



(a) Illustrating $u^*_{Mj,I} - u^*_{Mj}$ vs. γ when $r = 1.16$ and $j \in \{M, N\}$

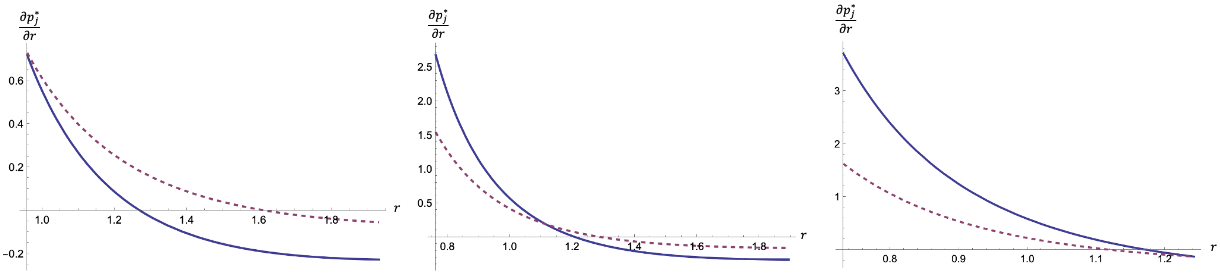
(b) Illustrating $u^*_{Nj,I} - u^*_{Nj}$ vs. γ when $r = 1.16$ and $j \in \{M, N\}$

Figure EC.17 Impact of change in γ on dominance of publisher payoffs in base model vs. imperfect valuation update model using parameter values:

$\{k = 0.2, c = 0.4, w = 0.05, \alpha = 0.65, T_N = 2, T_M = 1, \phi = b = 1, v = 2, \theta = 0.2, \sigma_W = 0.3\}$

EC.5.1.3. Robustness Checks of Propositions 1 - 5 In this section, we numerically examine the robustness of the main insights as discussed in Propositions 1, 2, 3, 4 and Proposition 5, under the imperfect valuation update extension, as discussed in Section 5.1. In each figure, the dotted $(\cdots)$ line and bolded line are mentioned as required.

Following Figures EC.18a and EC.18b demonstrate that the qualitative insights remain similar for Proposition 1.



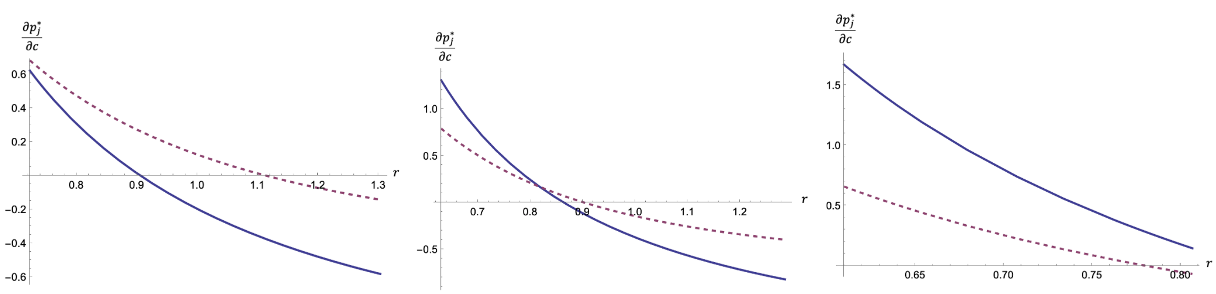
(a) Illustrating $\frac{\partial p_j^*}{\partial r}$ vs. r when $\gamma = 0.75$, $j \in \{M, N\}$

(b) Illustrating $\frac{\partial p_j^*}{\partial r}$ vs. r when $\gamma = 1$, $j \in \{M, N\}$

(c) Illustrating $\frac{\partial p_j^*}{\partial r}$ vs. r when $\gamma = 1.25$, $j \in \{M, N\}$

Figure EC.18 Illustrating Proposition 1 for imperfect valuation update. Values of Parameters: $\{k = 0.2, c = 0.5, w = 0.05, \alpha = 0.65, T_N = 2, T_M = 1, \phi = b = 1, v = 2, \theta = 0.2, \sigma_W = 0.3\}$**. In each figure dotted line represents** $\frac{\partial p_M^*}{\partial r}$ **and solid line represents** $\frac{\partial p_N^*}{\partial r}$

Following Figures EC.19a and EC.19b demonstrate that the qualitative insights remain similar for Proposition 2.



(a) Illustrating $\frac{\partial p_j^*}{\partial c}$ vs. r when $\gamma = 0.75$, $j \in \{M, N\}$

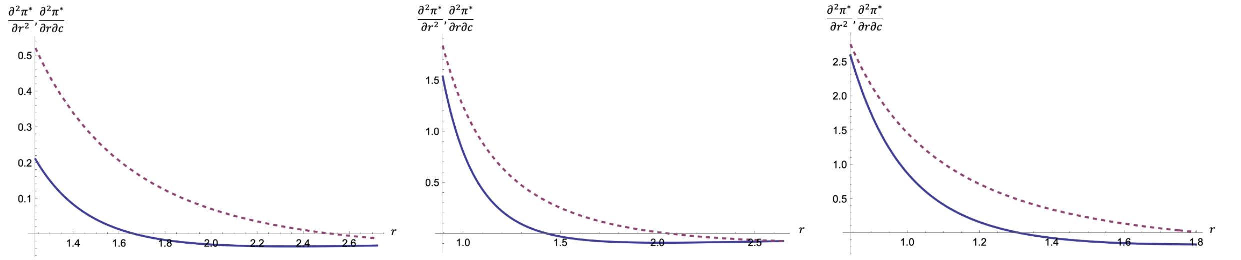
(b) Illustrating $\frac{\partial p_j^*}{\partial c}$ vs. r when $\gamma = 1$, $j \in \{M, N\}$

(c) Illustrating $\frac{\partial p_j^*}{\partial c}$ vs. r when $\gamma = 1.25$, $j \in \{M, N\}$

Figure EC.19 Illustrating Proposition 2 for imperfect valuation update. Values of Parameters: $\{k = 0.2, c = 0.5, w = 0.05, \alpha = 0.65, T_N = 2, T_M = 1, \phi = b = 1, v = 2, \theta = 0.2, \sigma_W = 0.3\}$**. In each figure dotted line represents** $\frac{\partial p_M^*}{\partial c}$ **and solid line represents** $\frac{\partial p_N^*}{\partial c}$

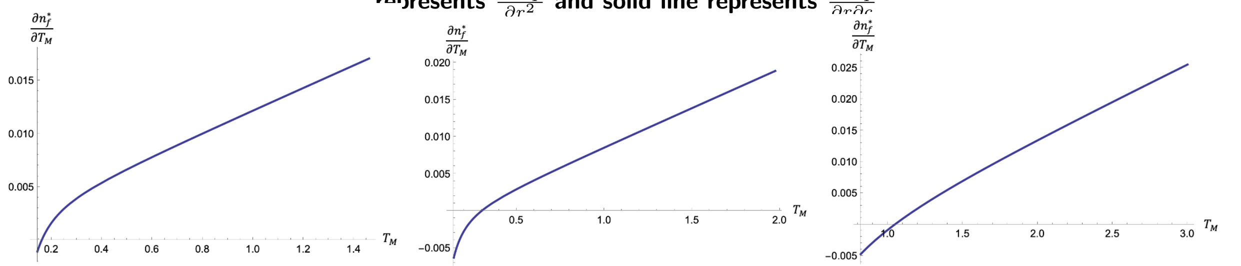
Following Figures EC.20a and EC.20b demonstrate that the qualitative insights remain similar for Proposition 3.

Following Figure EC.21 demonstrates that the qualitative insights remain similar for Proposition 4.



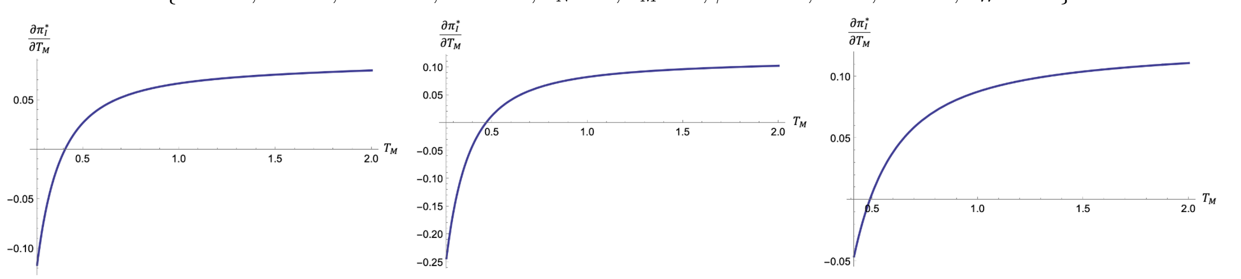
(a) Illustrating $\frac{\partial^2 \pi_I^*}{\partial r^2}$ and $\frac{\partial^2 \pi_I^*}{\partial r \partial c}$ vs. r when $\gamma = 0.75$, $j \in \{M, N\}$ **(b)** Illustrating $\frac{\partial^2 \pi_I^*}{\partial r^2}$ and $\frac{\partial^2 \pi_I^*}{\partial r \partial c}$ vs. r when $\gamma = 1$, $j \in \{M, N\}$ **(c)** Illustrating $\frac{\partial^2 \pi_I^*}{\partial r^2}$ and $\frac{\partial^2 \pi_I^*}{\partial r \partial c}$ vs. r when $\gamma = 1.25$, $j \in \{M, N\}$

Figure EC.20 Illustrating Proposition 3 for imperfect valuation update. Values of Parameters: $\{k = 0.2, c = 0.5, w = 0.05, \alpha = 0.65, T_N = 2, T_M = 1, \phi = b = 1, v = 2, \theta = 0.2, \sigma_W = 0.3\}$**. In each figure dotted line represents** $\frac{\partial^2 \pi_I^*}{\partial r^2}$ **and solid line represents** $\frac{\partial^2 \pi_I^*}{\partial r \partial c}$



(a) Illustrating $\frac{\partial n_F^*}{\partial T_M}$ vs. T_M when $\gamma = 0.75$, $j \in \{M, N\}$ **(b)** Illustrating $\frac{\partial n_F^*}{\partial T_M}$ vs. T_M when $\gamma = 1$, $j \in \{M, N\}$ **(c)** Illustrating $\frac{\partial n_F^*}{\partial T_M}$ vs. T_M when $\gamma = 1.25$, $j \in \{M, N\}$

Figure EC.21 Illustrating Proposition 4 for imperfect valuation update. Values of Parameters: $\{k = 0.2, c = 0.5, w = 0.05, \alpha = 0.65, T_N = 2, T_M = 1, \phi = b = 1, v = 2, \theta = 0.2, \sigma_W = 0.3\}$**.**



(a) Illustrating $\frac{\partial \pi_I^*}{\partial T_M}$ vs. T_M when $\gamma = 0.75$, $j \in \{M, N\}$ **(b)** Illustrating $\frac{\partial \pi_I^*}{\partial T_M}$ vs. T_M when $\gamma = 1$, $j \in \{M, N\}$ **(c)** Illustrating $\frac{\partial \pi_I^*}{\partial T_M}$ vs. T_M when $\gamma = 1.25$, $j \in \{M, N\}$

Figure EC.22 Illustrating Proposition 5 for imperfect valuation update. Values of Parameters: $\{k = 0.2, c = 0.5, w = 0.05, \alpha = 0.65, T_N = 2, T_M = 1, \phi = b = 1, v = 2, \theta = 0.2, \sigma_W = 0.3\}$**.**

Following Figure EC.22 demonstrates that the qualitative insights remain similar for Proposition 5. We, therefore, have established that even under imperfect valuation updates, our base model insights remain qualitatively similar.

EC.5.2. Extension 2 – Only Fraud Traffic from Malicious Publishers

EC.5.2.1. Proof of Lemma 3

Proof We begin the proof by noting that the only change in the model is that now $b=0$ and hence, the following equations are now the utilities of the malicious publishers.

$$u_{Mj} = \underbrace{p_j x_j(1-s_j) - w s_j x_j p_j - c x_j^2}_{\text{Payoff from ad fraud}}. \tag{EC.7}$$

Therefore the derivation is similar to Lemma 1, and for brevity we present the following conditions that are satisfied for the existence of an interior equilibrium $\forall j \in \{N, M\}$.

- $p^*_{j,f} \geq 0$,
- $s^*_{j,f} \in (0,1)$,
- $y^*_{j,f} \in (0,1)$
- $x^*_{j,f} > 0$,
- $u^*_{NN}, u^*_{NM} \in (0,1)$,
- $\pi^* > 0$,
- Concavity conditions as derived in proof of Lemma 1:

$$L_2 : \mathbb{H}(p_{j,f}, s_{j,f}) \text{ is negative semi-definite } \Leftrightarrow (-1)^k \Delta_k \geq 0, \forall\ 1 \leq k \leq 4, j \in \{M, N\}. \tag{EC.8}$$

where the Hessian is constructed as follows:

$$\mathbb{H}(p_j, s_j) = \begin{bmatrix} \frac{\partial^2 \pi}{\partial p_{N,f}^2} & \frac{\partial^2 \pi}{\partial p_{N,f} \partial p_{M,f}} & \frac{\partial^2 \pi}{\partial p_{N,f} \partial s_{N,f}} & \frac{\partial^2 \pi}{\partial p_{N,f} \partial s_{M,f}} \\ \frac{\partial^2 \pi}{\partial p_{N,f} \partial p_{M,f}} & \frac{\partial^2 \pi}{\partial p_{M,f}^2} & \frac{\partial^2 \pi}{\partial p_{M,f} \partial s_{N,f}} & \frac{\partial^2 \pi}{\partial p_{M,f} \partial s_{M,f}} \\ \frac{\partial^2 \pi}{\partial p_{N,f} \partial s_{N,f}} & \frac{\partial^2 \pi}{\partial p_{M,f} \partial s_{N,f}} & \frac{\partial^2 \pi}{\partial s_{N,f}^2} & \frac{\partial^2 \pi}{\partial s_{M,f} \partial s_{N,f}} \\ \frac{\partial^2 \pi}{\partial p_{N,f} \partial s_{M,f}} & \frac{\partial^2 \pi}{\partial p_{M,f} \partial s_{M,f}} & \frac{\partial^2 \pi}{\partial s_{M,f} \partial s_{N,f}} & \frac{\partial^2 \pi}{\partial s_{M,f}^2} \end{bmatrix}$$

Since these conditions hold, we present the analytical expressions of p^*_j, s^*_j, which we derive following similar steps as in Lemma EC.1.

$$p^*_{W,f} = p^*_W,$$
$$p^*_{M,f} = \frac{A_9(\alpha b T_M(1 + w\theta - k) + (1-\alpha)(w+1)T_N v)}{A_{10}(4(1-\alpha)^2 c^2 T_N^2 (w(1-\theta) + k)^2)},$$
$$p^*_{N,f} = \frac{A_{11}((1-\alpha) w T_M (1 - \theta - (\theta+1)\tilde{r}) + 2\alpha c w (1-\theta) T_N (1-\tilde{r})^2)}{A_{12}(\alpha k^2 T_N (1-\tilde{r})^2 - 2(1-\alpha) k r T_M \phi (1-\tilde{r}) - 2(1-\alpha) c T_M \tilde{r}^2)},$$
$$s^*_{M,f} = \frac{A_{13}(w(1-\theta)((1-\alpha)T_N v + \alpha b T_M) + 2\alpha b k T_M)}{A_{14}(\alpha b T_M (k + 1 - \theta(1-k)) + (1-\alpha)(1-\theta) T_N v)},$$
$$s^*_{N,f} = \frac{A_{15}(2\alpha c w(1-\theta) T_N (1-\tilde{r}) + 2\alpha c k T_N (1-\tilde{r}) - 2(1-\alpha)\tilde{r} T_M + (1-\alpha) w T_M (1 - 2\tilde{r} - \theta))}{A_{16}(1 + k + \tilde{r}(\theta(3-k) + 1 - k) - \theta(1-k))},$$

where $A_9, A_{10}, \cdots, A_{16}$ are as follows:

$$A_9 = \frac{2c}{(w+1)v}\big(2(\alpha-1)cT_N\tilde{r}^2\left(w\theta(-w+k-2)+(w+1)v(w+k)\right) - 4(\alpha-1)cT_N\tilde{r}\left(w\theta(-w+k-2)+(w+1)v(w+k)\right) + 2(\alpha-1)cT_N\left(w\theta(-w+k-2)+(w+1)v(w+k)\right)+\alpha\tilde{r}T_M\left(w\theta+w+2\right)\left(-w\theta+wv+v\right) + \alpha wT_M\left(\theta(w\theta+w+2)+(w+1)(\theta-1)v\right)\big),$$

$$A_{10} = \frac{1}{4(\alpha-1)^2c^2T_N^2\left(w(1-\theta)+k\right)^2}\left(4(\alpha-1)^2c^4r^2T_N^2\phi^2(w+k)^2 + 8(\alpha-1)c^3rT_N\phi\left((\alpha-1)T_N\left(-(w+k)^2\right)-\alpha(w+1)(k-1)rT_M\phi\right) + 4(\alpha-1)c^2T_N\left(\alpha rT_M\phi\left(w\left(\theta(w-k+2)+w+3k\right)+2k\right)+(\alpha-1)T_N(w+k)^2\right) + 4(\alpha-1)\alpha cw(\theta-1)T_MT_N(w+k)+\alpha^2w^2(\theta-1)^2T_M^2\right)$$

$$A_{11} = \frac{1}{AN_{11}}\Bigg(2\alpha cT_N\left(w^2((\alpha-1)T_M(c(\theta+1)r\phi(v-\theta)+\theta^2+\theta(1+v)-v)+2\alpha cT_N(v-\theta)(\tilde{r}-1)^2\right) + w(2\alpha cT_N(\tilde{r}-1)^2(-2\theta+\theta k+kv+v)+(\alpha-1)T_M\left(\tilde{r}(\theta(v-2)+3v)+2\theta+(\theta-1)v\right)) + 2cv(\alpha kT_N(\tilde{r}-1)^2+(\alpha-1)rT_M\phi))\Bigg),$$

where $AN_{11} = ((1-\alpha)wT_M(1-\theta-(\theta+1)\tilde{r})+2\alpha cw(1-\theta)T_N(1-\tilde{r})^2)$. Further,

$$A_{12} = \frac{1}{AD_{12}}(w^2\left(4\alpha^2c^2T_N^2(\tilde{r}-1)^2+4(\alpha-1)\alpha cT_MT_N(c(\theta+1)r\phi+\theta-1)+(\alpha-1)^2(\theta-1)^2T_M^2\right) + 4\alpha c^2T_N\left(\alpha k^2T_N(\tilde{r}-1)^2-2(\alpha-1)krT_M\phi(\tilde{r}-1)+2(\alpha-1)cr^2T_M\phi^2\right) + 4\alpha cwT_N\left(k(\tilde{r}-1)\left(2\alpha cT_N(\tilde{r}-1)-(\alpha-1)T_M(2\tilde{r}+\theta-1)\right)+2(\alpha-1)\tilde{r}T_M(\tilde{r}+\theta)\right)),$$

where $AD_{12} = (\alpha k^2T_N(1-\tilde{r})^2-2(1-\alpha)krT_M\phi(1-\tilde{r})-2(1-\alpha)cT_M\tilde{r}^2)$.

$$A_{13} = \frac{1}{(1-\alpha)w(1-\theta)T_Nv}(2(1-\alpha)cT_N\tilde{r}(v(w+k)-2w\theta) + 2c((\alpha-1)T_N(v(w+k)-2w\theta)+\alpha rT_M\phi(-w(1+\theta)v+v))+\alpha wT_M(2\theta+(\theta-1)v)),$$

$$A_{14} = \frac{1}{(1-\alpha)(1-\theta)T_Nv}(2(\alpha-1)cT_N\tilde{r}^2(w\theta(-w+k-2)+(w+1)v(w+k)) - 4(\alpha-1)cT_N\tilde{r}(w\theta(-w+k-2)+(w+1)v(w+k)) + 2(\alpha-1)cT_N(w\theta(-w+k-2)+(w+1)v(w+k))+\alpha\tilde{r}T_M(w(1+\theta)+2)(-w\theta+wv+v) + \alpha wT_M(\theta(w(1+\theta)+2)+(w+1)(\theta-1)v)),$$

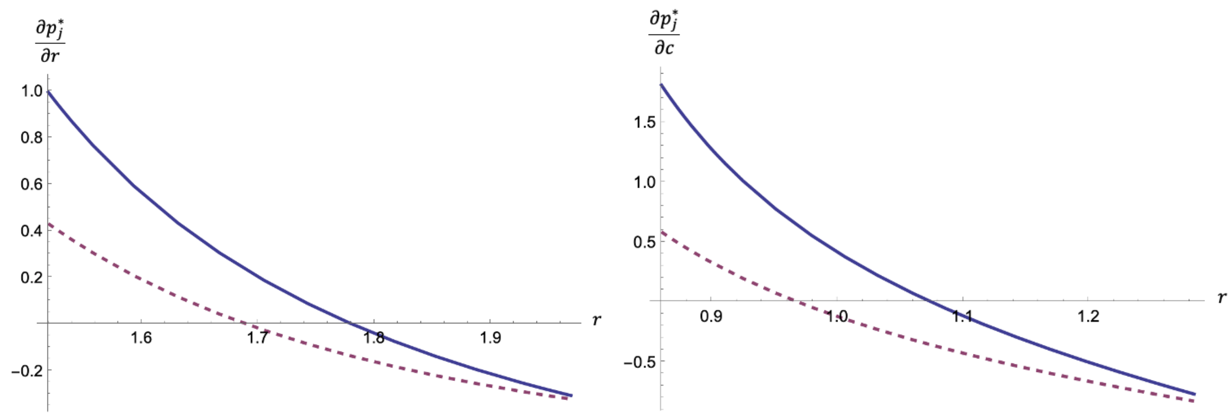
$$A_{15} = \frac{1}{AN_{15}}(w((\alpha-1)T_M(2\tilde{r}(v-\theta)+2\theta+(\theta-1)v)-2\alpha cT_N(v-2\theta)(\tilde{r}-1)) + 2cv(\alpha kT_N(1-\tilde{r})+(\alpha-1)rT_M\phi)),$$

where $AN_{15} = (\alpha-1)wT_M(2\tilde{r}+\theta-1)+2\alpha cw(\theta-1)T_N(\tilde{r}-1)-2\alpha ckT_N(\tilde{r}-1)+2(\alpha-1)\tilde{r}T_M$ and finally,

$$A_{16} = \frac{1}{AD_{16}}(w^2\left((\alpha-1)T_M\left(\tilde{r}(1+\theta)(v-\theta)+\theta^2+\theta+\theta v-v\right)+2\alpha cT_N(v-\theta)(\tilde{r}-1)^2\right) + w\left(2\alpha cT_N(\tilde{r}-1)^2(-2\theta+\theta k+kv+v)+(\alpha-1)T_M(\tilde{r}(\theta(v-2)+3v)+2\theta+(\theta-1)v)\right) + 2cv\left(\alpha kT_N(\tilde{r}-1)^2+(\alpha-1)rT_M\phi\right)),$$

where $AD_{16} = (1 + k + \tilde{r}(\theta(3-k) + 1 - k) - \theta(1-k))$.

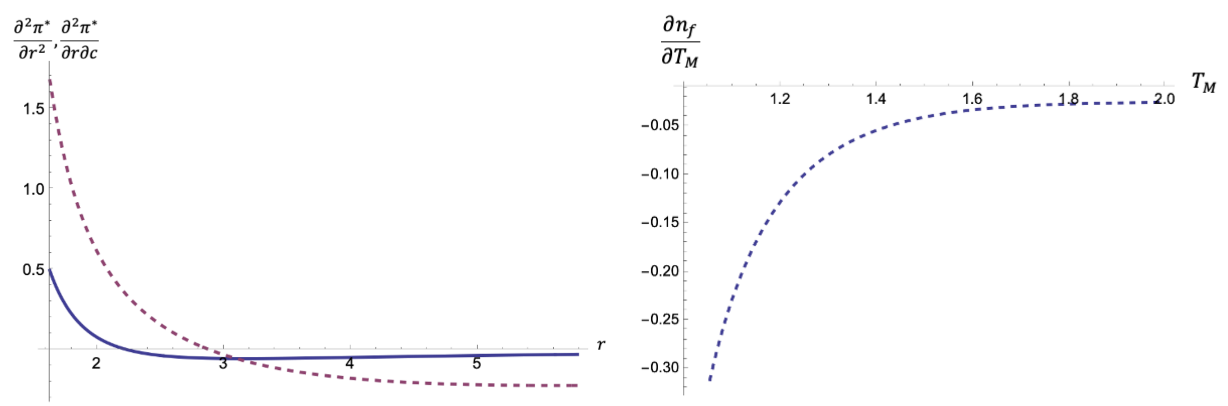
EC.5.2.2. Robustness Checks of Propositions 1 - 5 In this section, we numerically examine the robustness of the main insights as discussed in Propositions 1, 2, 3, 4 and Proposition 5, as discussed in Section 4. Figures EC.23a and EC.23b demonstrate that the qualitative insights remain similar for Propositions 1 and 2, respectively.



(**a**) Illustrating $\frac{\partial p_j^*}{\partial r}$ vs. r (**b**) Illustrating $\frac{\partial p_j^*}{\partial c}$ vs. r

Figure EC.23 Impact of change in r on p_j^* using parameter values:
$\{w = 0.05, c = 0.5, k = 0.2, \phi = 1, T_N = 1, T_M = 0.5, \alpha = 0.65, v = 2, \theta = 0.2, \sigma_W = 0.3\}$

Following Figures EC.24a and EC.24b demonstrate that the qualitative insights remain similar for Proposition 3 and Proposition 4.



(**a**) Illustrating $\frac{\partial^2 \pi^*}{\partial r^2}$ and $\frac{\partial^2 \pi^*}{\partial r \partial c}$ vs. r using parameter values: $\{w = 0.05, c = 0.5, k = 0.2, \phi = 1, T_N = 1, T_M = 0.5, \alpha = 0.65, v = 2, \sigma_W = 0.3\}$

(**b**) Illustrating $\frac{\partial n_f^*}{\partial T_M}$ vs. T_M using parameter values: $\{w = 0.05, c = 0.5, k = 0.2, \phi = 1, T_N = 1, r = 0.8, \alpha = 0.65, b = 1, v = 2, \theta = 0.2, \sigma_W = 0.3\}$

Figure EC.24 Impact of change in c on p_j^* using parameter values:
$\{w = 0.05, c = 0.5, k = 0.2, \phi = 1, T_N = 1, T_M = 0.5, \alpha = 0.65, v = 2, \theta = 0.2, \sigma_W = 0.3\}$

Following Figure EC.25 demonstrates that the qualitative insights remain similar for Proposition 5.

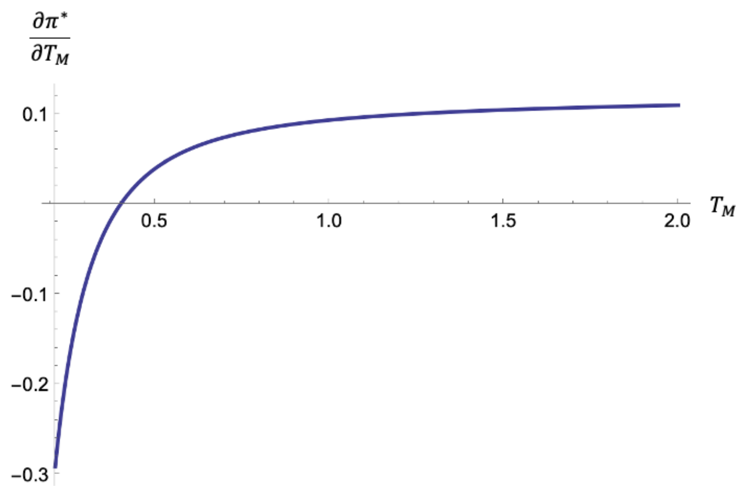


Figure EC.25 Illustrating $\frac{\partial \pi^*}{\partial T_M}$ vs. T_M using parameter values:
$\{w = 0.05, c = 0.5, k = 0.2, \phi = 1, T_N = 1, r = 0.8, \alpha = 0.65, v = 2, \theta = 0.2, \sigma_W = 0.3\}$

We, therefore, have established that when malicious publishers generate only fraud traffic, our base model insights remain qualitatively similar.

EC.5.3. Extension 3 – Fraud Ad Traffic Impacts ROC Curve

EC.5.3.1. Equilibrium Existence In this section, we provide numerical analysis to prove the existence of an interior equilibrium. Specifically, the following conditions are satisfied when solving the Stage 1 problem of ad network, $\forall j \in \{N, M\}$.

$$p_j^* \geq 0,$$
$$x_j^* \geq 0,$$
$$u_{NN}^*, u_{NM}^* \in (0,1),$$
$$s_j^* \in (0,1),$$
$$\pi^* > 0,$$

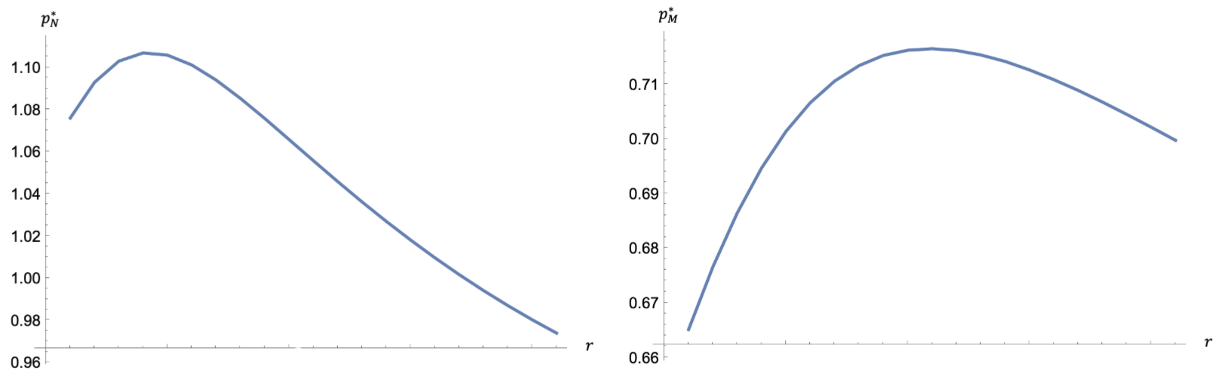
Note that since we solve this model numerically due to analytical intractability, we verify the concavity of π^* using the numerical values of p_j^*, s_j^*. The following numerical solutions obtained satisfy all said constraints and concavity using parameter values: $\{w = c = 0.1, k = 0.3, \phi = 2, T_N = 1, T_M = 0.2, r = 1.6, \alpha = 0.75, b = 1, v = 2, \theta = 0.6, \sigma_W = 0.3\}$:

$$p_N^* = 1.06, p_M^* = 0.66, p_W^* = 1,$$
$$s_N^* = 0.79, s_M^* = 0.76,$$
$$\pi^* = 1.334.$$

We further generate a range of solutions for parameter values r, T_M, which are the main parameters of interest across Propositions 1-5. We demonstrate the results from these simulations in the following subsections.

EC.5.3.2. Robustness Checks of Propositions 1 - 5 In this section, we numerically examine the robustness of the insights from our base model.

We, therefore, have established that even under the case where ROC is impacted by the amount of fraud traffic, qualitatively, our base model insights remain qualitatively similar.



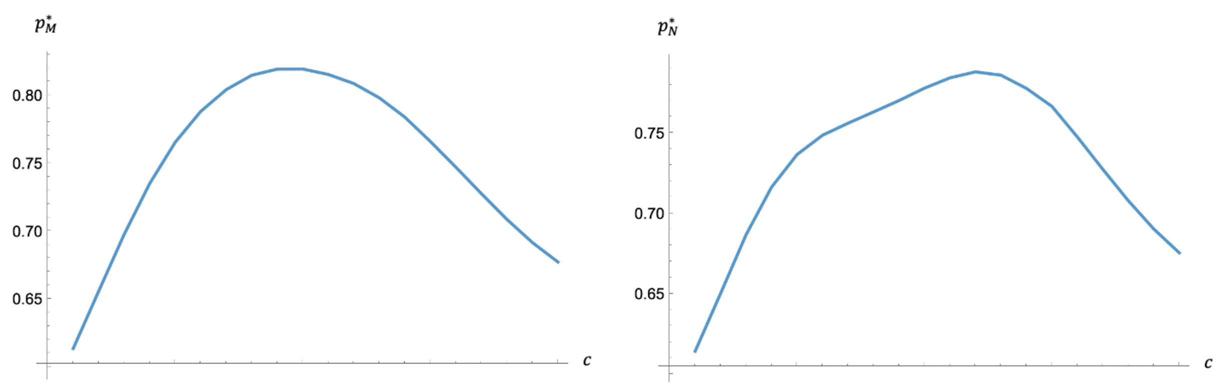
(a) Illustrating $\frac{\partial p_N^*}{\partial r}$ vs. r using the changing slope of the p_N^* vs. r

(b) Illustrating $\frac{\partial p_M^*}{\partial r}$ vs. r using the changing slope of the p_M^* vs. r

Figure EC.26 Impact of change in r on p_j^*. Parameter values are:

$\{w = c = 0.1, k = 0.3, \phi = 2, T_N = 1, T_M = 0.2, \alpha = 0.75, b = 1, v = 2, \theta = 0.6, \sigma_W = 0.3\}$

Figures EC.26a and EC.26b demonstrate that the qualitative insights remain similar for Proposition 1.



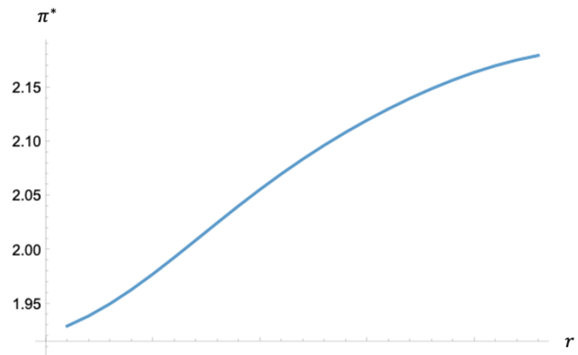
(a) Illustrating $\frac{\partial p_M^*}{\partial c}$ using the changing slope of the p_M^* vs. c

(b) Illustrating $\frac{\partial p_N^*}{\partial c}$ using the changing slope of the p_N^* vs. c

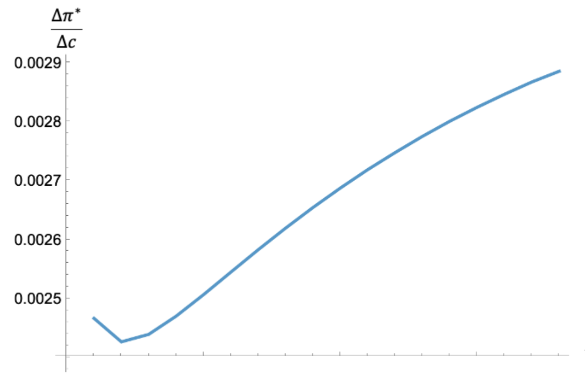
Figure EC.27 Impact of change in c on p_j^*. Parameter values are:

$\{w = 0.1, r = 1.12, k = 0.3, \phi = 2, T_N = 1, T_M = 0.2, \alpha = 0.75, b = 1, v = 2, \theta = 0.6, \sigma_W = 0.3\}$

Figures EC.27a and EC.27b demonstrate that the qualitative insights remain similar for Proposition 2.



(a) Illustrating $\frac{\partial^2 \pi^*}{\partial r^2}$ vs. r using the changing slope of the π^* vs. r



(b) Illustrating $\frac{\partial^2 \pi^*}{\partial r \partial c}$ vs. r using the changing slope of the $\frac{\Delta \pi^*}{\Delta c}$ vs. r

Figure EC.28 Impact of change in c, r on π^*. Parameter values are: $\{w = 0.01, k = 0.2, \phi = 1, T_N = 2, T_M = 1, \alpha = 0.75, b = 1, v = 2, \theta = 0.2, \sigma_W = 0.3\}$

Figures EC.28a and EC.28b demonstrate that the qualitative insights remain similar for Proposition 3 and Proposition 4. When conducting simulations for this analysis, we used a range of c, r such that $r \in (0.2, \frac{1}{c\phi})$ and $c \in (0.15, 1)$.

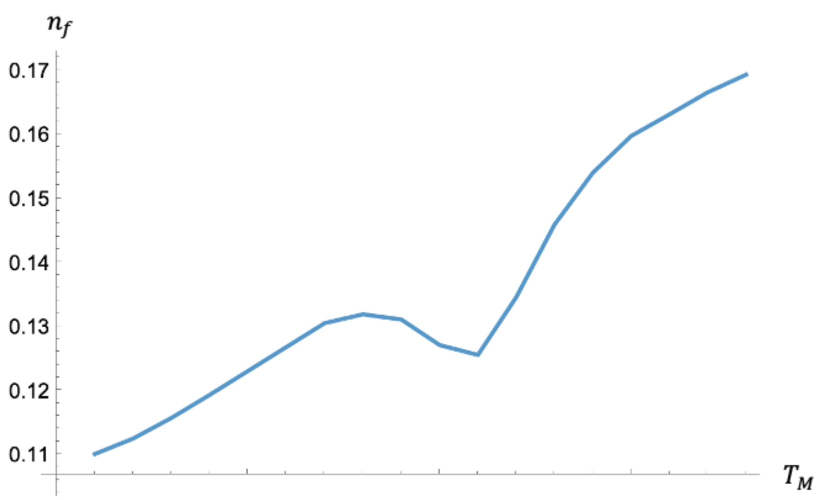


Figure EC.29 Illustrating $\frac{\partial n_f^*}{\partial T_M}$ vs. T_M using the changing slope of the n_f^* vs. T_M. Parameter values are: $\{w = c = 0.05, k = 0.2, \phi = 3, T_N = 20, r = 6, \alpha = 0.6, b = 1, v = 2, \theta = 0.5, \sigma_W = 0.3\}$

Figure EC.29 demonstrates that the qualitative insights remain similar for Proposition 4.

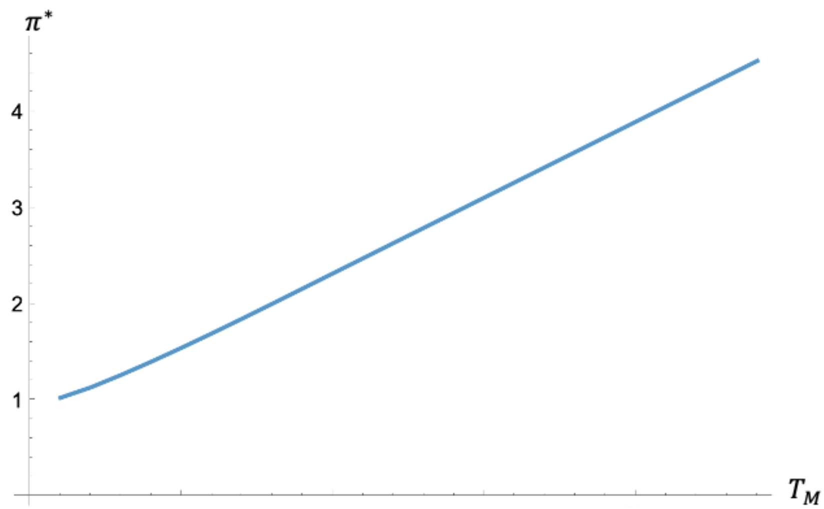


Figure EC.30 Illustrating $\frac{\partial \pi^*}{\partial T_M}$ vs. T_M using the changing slope of the π^* vs. T_M. Parameter values are: $\{w = c = 0.05, k = 0.2, \phi = 3, T_N = 20, r = 6, \alpha = 0.6, b = 1, v = 2, \theta = 0.5, \sigma_W = 0.3\}$

Figure EC.30 demonstrates that the qualitative insights remain similar for Proposition 5.

References

Kannai Y (1977) Concavifiability and constructions of concave utility functions. *Journal of Mathematical Economics* 4(1):1–56.