Online Appendix to Dynamic Unravelling

B Policy, Price Discovery, and Welfare

In this Appendix, I show how short-selling constraints and position limits affect price discovery. I moreover derive the model's implied welfare metric and show how preventing low-volume crashes and improving price discovery emerge endogenously as desiderata on the part of the policymaker, justifying my earlier taking of these as reduced-form goals.

B.1 Policy and Price Discovery

Having shown the impact of short-selling constraints and position limits on the likelihood of market crashes, it is further instructive and important for policy to consider the impact of these policies on price discovery.

For the case of short-selling constraints, my analysis simply echoes the result of Diamond and Verrecchia (1987) in my setting: short-selling constraints harm price discovery. In addition to this, I also show that position limits harm price discovery in my model. The intuition for both results is simple: short-selling constraints and position limits both impede the flow of information from informed traders into prices.

I define price discovery analogously to Diamond and Verrecchia (1987). Consider two price thresholds $\underline{P} < P^*_{-1} = p < \overline{P}$. Define $\overline{T}$ as the earliest time at which $P^*_t \geq \overline{P}$ or $P^*_t \leq \underline{P}$. More formally:

$$\overline{T} = \begin{cases} \min\{\ t \mid P^*_t \geq \overline{P} \text{ or } P^*_t \leq \underline{P}\}, \text{ if this exists,} \\ T, \text{ otherwise.} \end{cases} \tag{108}$$

We will call $\overline{T}$ the price discovery time. We are now ready to define formally our meaning of faster (slower) price discovery under various parameterizations of the model:

Definition 3 (Price Discovery)**.** *Given* $(\underline{P}, \overline{P})$*, under two parameterizations of the model* Λ*,* Λ'*, price discovery is slower under* Λ' *than* Λ *in state of the world* $V = i$ *if:*

$$T_i^{\Lambda'} \geq T_i^{\Lambda} \tag{109}$$

where $T_i^{\Lambda} = \mathbb{E}[\overline{T}|V = i]$.

We are now in a position to ask how short-selling costs and position limits affect price discovery in the model. Toward this, recall the likelihood ratio property established earlier.

$$\Omega_{\overline{T}} = \Omega_{\overline{T}-1} \frac{\mathbb{P}[X|V=1, h^{t-1}]}{\mathbb{P}[X|V=0, h^{t-1}]} \tag{110}$$

where $\Omega_t \equiv \frac{P_t^*}{1-P_t^*}$ is the odds ratio. Rearranging this expression yields:

$$\log \Omega_{\overline{T}} = \log \Omega_{\overline{T}-1} + \log \frac{\mathbb{P}[X|V=1, h^{t-1}]}{\mathbb{P}[X|V=0, h^{t-1}]} \tag{111}$$

This is instructive as it tells us that the log-odds ratio follows a random walk, where innovations are given by the relative log ratio of the probability that the signal received at date t was generated in the state of the world where V is high versus the state of the world where V is low. We can therefore iterate Equation (111) to obtain the following expression for the log-odds ratio:

$$\log \Omega_{\overline{T}} = \sum_{t=0}^{\overline{T}} \log \frac{\mathbb{P}[X|V=1, h^{t-1}]}{\mathbb{P}[X|V=0, h^{t-1}]} \tag{112}$$

Given that $\overline{T}$ is a stopping time (see Equation (108)), one can apply Wald's equation to yield:

$$\mathbb{E}[\log \Omega_{\overline{T}}] = \mathbb{E}\Big[\sum_{t=0}^{\overline{T}} \mathbb{E}\Big[\log \frac{\mathbb{P}[X|V=1, h^{t-1}]}{\mathbb{P}[X|V=0, h^{t-1}]} \Big] \Big] \tag{113}$$

To operationalize the use of Wald's equation, it is further necessary to remove the history dependence from the above expression. To this end, I make the following assumption which is necessary for this result only:

Assumption 1. *The following hold:*

1. $G(\cdot|h^{t-1}) \equiv G$ *– book capacity is history independent*
2. $c = \overline{c} > \max\{1, |\underline{\eta}|\}$ *with probability* λ *and* $c = 0$ *with probability* $1-\lambda$ *– with probability* λ *short-selling is prohibitively costly and free otherwise*
3. $\eta = \underline{\eta} < -1$ *with probability* $1-\gamma$ *and* $\eta = \overline{\eta} > 1$ *with probability* γ *– noise traders mechanically always wish to either buy or sell, independently of the current bid-ask spread.*

Under Assumption 1, this expression becomes history independent and we have the following tractable expression for the average price discovery time:

$$\mathbb{E}[\overline{T}] = \frac{\mathbb{E}[\log \Omega_{\overline{T}}]}{\mathbb{E}\Big[\log \frac{\mathbb{P}[X|V=1]}{\mathbb{P}[X|V=0]} \Big]} \tag{114}$$

To perform comparative statics on price discovery, it therefore suffices to perform comparative statics on Equation 114. Performing this exercise for the distribution of position limits and short-selling constraints yields Proposition 7:

Proposition 7. *If Assumption 1 holds, then up to integer issues:*[22]

1. *If two short-selling cost distributions are such that $J' \succ_{FOSD} J$, then under J' price discovery is slower than under J in both states of the world:*

$$T_1' \geq T_1 \quad \textit{and} \quad T_0' \geq T_0 \tag{115}$$

2. *If two position limit distributions are such that $G' \succ_{FOSD} G$. Under G' price discovery is faster than under G in both states of the world:*

$$T_1' \leq T_1 \quad \textit{and} \quad T_0' \leq T_0 \tag{116}$$

Proof. Recall the odds-ratio property of prices established earlier:

$$\Omega_{\overline{T}} = \Omega_{\overline{T}-1} \frac{\mathbb{P}[X|V=1, h^{t-1}]}{\mathbb{P}[X|V=0, h^{t-1}]} \tag{117}$$

where $\Omega_t \equiv \frac{P_t^*}{1-P_t^*}$ is the odds ratio. Rearranging this expression yields:

$$\log \Omega_{\overline{T}} = \log \Omega_{\overline{T}-1} + \log \frac{\mathbb{P}[X|V=1, h^{t-1}]}{\mathbb{P}[X|V=0, h^{t-1}]} \tag{118}$$

Iterating:

$$\log \Omega_{\overline{T}} = \sum_{t=0}^{\overline{T}} \log \frac{\mathbb{P}[X|V=1, h^{t-1}]}{\mathbb{P}[X|V=0, h^{t-1}]} \tag{119}$$

Given that $\overline{T}$ is a stopping time (see Equation (108)), one can apply Wald's equation to yield:

$$\mathbb{E}[\log \Omega_{\overline{T}}] = \mathbb{E}\Big[\sum_{t=0}^{\overline{T}} \mathbb{E}\Big[\log \frac{\mathbb{P}[X|V=1, h^{t-1}]}{\mathbb{P}[X|V=0, h^{t-1}]} \Big] \Big] \tag{120}$$

If one further imposes Assumption 1, the probabilities derived in Proposition 4 can be

[22] See Footnote 23 in the proof.

simplified:

$$\begin{aligned}
\mathbb{P}[B|V=1,h^{t-1}] &= (1-\delta(h^{t-1}))\left[\pi+(1-\pi)(1-K(A(h^{t-1})-P^*_{t-1}))\right] \\
&= (1-\delta)(\pi+(1-\pi)\gamma) \\
\mathbb{P}[B|V=0,h^{t-1}] &= (1-\delta(h^{t-1}))\left[(1-\pi)(1-K(A(h^{t-1})-P^*_{t-1}))\right] \\
&= (1-\delta)(1-\pi)\gamma \\
\mathbb{P}[S|V=1,h^{t-1}] &= (1-\delta(h^{t-1}))\left[(1-\pi)\mathbb{E}_c[K(B(h^{t-1})-P^*_{t-1}-c)]\right] \\
&= (1-\delta)(1-\pi)(1-\gamma)(1-\lambda) \\
\mathbb{P}[S|V=0,h^{t-1}] &= (1-\delta(h^{t-1}))\Big[\pi J(B(h^{t-1}))+(1-\pi)\mathbb{E}_c[K(B(h^{t-1})-P^*_{t-1}-c)]\Big] \\
&= (1-\delta)(\pi(1-\lambda)+(1-\pi)(1-\gamma)(1-\lambda))
\end{aligned} \tag{121}$$

Which are now all history independent. The stopping time equation can therefore be simplified:

$$\mathbb{E}[\overline{T}] = \frac{\mathbb{E}[\log \Omega_{\overline{T}}]}{\mathbb{E}\Big[\log \frac{\mathbb{P}[X|V=1]}{\mathbb{P}[X|V=0]}\Big]} \tag{122}$$

To establish comparative statics in T_i, see also that:

$$T_i = \frac{\mathbb{E}[\log \Omega_{\overline{T}}|V=i]}{\mathbb{E}\Big[\log \frac{\mathbb{P}[X|V=1]}{\mathbb{P}[X|V=0]}|V=i\Big]} \tag{123}$$

As noted by Diamond and Verrecchia (1987), the numerator can be computed given the observation that we are performing a Wald sequential likelihood test of $H_0 : V = 0$ versus $H_1 : V = 1$ with decision thresholds given by:

$$\begin{aligned}
\log \Omega_t \geq \log \frac{\overline{P}}{1-\overline{P}} \equiv \log B \implies H_1 \text{ True} \\
\log \Omega_t \leq \log \frac{\overline{P}}{1-\overline{P}} \equiv \log A \implies H_0 \text{ True}
\end{aligned} \tag{124}$$

Now we wish to compute the conditional expectation of the log-likelihood ratio conditional on stopping, which can be shown to be well-approximated by:[23]

$$\begin{aligned}
\mathbb{E}[\log \Omega_{\overline{T}}|V=0] &\approx \mathbb{P}[\text{Type I Error}]\log B + (1-\mathbb{P}[\text{Type I Error}])\log A \\
\mathbb{E}[\log \Omega_{\overline{T}}|V=1] &\approx (1-\mathbb{P}[\text{Type II Error}])\log B + \mathbb{P}[\text{Type II Error}]\log A
\end{aligned} \tag{125}$$

[23] The approximation stems from crossing the barrier rather than hitting the barrier in discrete time.

Letting $\alpha \equiv \mathbb{P}[\text{Type I Error}]$ and $\beta \equiv \mathbb{P}[\text{Type II Error}]$, we can further compute:

$$\begin{aligned} A &= \frac{\beta}{1-\alpha} \\ B &= \frac{1-\beta}{\alpha} \end{aligned} \tag{126}$$

Solving this system and plugging into the above yields:

$$\begin{aligned} \mathbb{E}[\log \Omega_{\overline{T}} | V = 0] &\approx \frac{1-A}{B-A} \log B + \frac{B-1}{B-A} \log A \\ \mathbb{E}[\log \Omega_{\overline{T}} | V = 1] &\approx \frac{B(1-A)}{B-A} \log B + \frac{A(B-1)}{B-A} \log A \end{aligned} \tag{127}$$

Hence, in both cases, the denominator of T_i depends only on $\underline{P}$ and $\overline{P}$. Hence, to consider comparative statics on T_i it suffices to consider only the denominator:

$$D_i \equiv \mathbb{E}\Big[\log \frac{\mathbb{P}[X|V=1]}{\mathbb{P}[X|V=0]} | V = i \Big] \tag{128}$$

In this context, proving the proposition reduces to showing (i) for $J' \succ_{FOSD} J$ that $D_i^{J'} < D_i^J$ and (ii) for $G' \succ_{FOSD} G$ that $D_i^{G'} < D_i^G$. To this end, see that D_i can be written as:

$$D_i = \mathbb{P}[B|V=i] \log \frac{\mathbb{P}[B|V=1]}{\mathbb{P}[B|V=0]} + \mathbb{P}[S|V=i] \log \frac{\mathbb{P}[S|V=1]}{\mathbb{P}[S|V=0]} + \mathbb{P}[N|V=i] \log \frac{\mathbb{P}[N|V=1]}{\mathbb{P}[N|V=0]} \tag{129}$$

This equation is simply a function of the six numbers pinned down in Proposition 4. See that $\delta_{G'} \leq \delta_G$ and $\lambda_{J'} \geq \lambda_J$. The comparative statics follow from plugging in, differentiating and checking that $\frac{\partial D_i}{\partial \delta} \leq 0$ and $\frac{\partial D_i}{\partial \lambda} \leq 0$.

□

I first provide intuition for the fact that tighter short-selling constraints worsen price discovery. This result echoes the one from Diamond and Verrecchia (1987). In particular, when short-selling constraints are tighter there is naturally slower price discovery when the fundamental value of the security is low as informed traders are prevented from selling the security. When the fundamental value of the security is high, tighter short-selling constraints both reduce the fraction of noise traders who sell and worsen inference conditional on observations of no-trade. However, it can be shown that the latter effect dominates so that price discovery is slower. For the positive impact of the position limit distribution on price discovery, see that when limits are looser, no-trade events become less common which aids discovery. This is because the chance an agent makes an order in excess of the market

maker's capacity is diminished. At the same time, no-trade events constitute a less negative signal. This second effect aids discovery when the value is high and harms it when the value is low. This notwithstanding, it can be shown that the first effect dominates the latter as the latter is infra-marginal. Thus, increases in position limits and corresponding liquidity injections speed price discovery in the model.

B.2 Policy and Welfare

So far my analysis of policy has taken policymaker objectives to reduce the likelihood of crashes and improve price discovery as given. However, the formalism of the model admits a natural welfare metric whereby the planner's objectives can be endogenized. This allows an analysis of optimal policy, both static and dynamic, from the perspective of the welfare of market participants. Price discovery and preventing crashes emerge as natural desiderata when one uses the implied welfare metric, providing a microfoundation for the earlier, qualitative analysis of policy.

B.2.1 The Welfare Metric

As all trades on the basis of information are transfers, welfare in the model stems from the idiosyncratic valuations of noise traders and therefore takes seriously that their wish to trade is optimal. Toward deriving the endogenous welfare metric in the model, I first assume that the policymaker is utilitarian and places equal welfare weights on the expected utility of all market participants. Implicit in this is the assumption that prices or activity in this market have no effect on other (unmodelled) markets or the real economy at large. Formally, the per-period payoff of the designer $W_t : \mathcal{H}^{t-1} \to \mathbb{R}$ is given by:

$$W_t(h^{t-1}) = (1-\pi)\mathbb{E}[U^U|h^{t-1}] + \pi\mathbb{E}[U^I|h^{t-1}] + \mathbb{E}[U^M|h^{t-1}] \tag{130}$$

where U^U, U^I and U^M are the utilities of uninformed traders, informed traders and market makers respectively.

Using the previously established structure of equilibrium and the optimal demand policies of traders, this welfare metric can be expressed as a function of model primitives and the history. This is stated formally in Proposition 8.

Proposition 8. *The welfare function of the policymaker is given by:*

$$W_t(h^{t-1}) = (1-\pi)(1-\delta(h^{t-1}))|x^*(h^{t-1})|\Bigg[\int_{\underline{c}}^{\overline{c}}\int_{\underline{\eta}}^{B(h^{t-1})-c-P^*_{t-1}} -\eta dK(\eta)dJ(c) + \int_{\underline{c}}^{\overline{c}}\int_{A(h^{t-1})-P^*_{t-1}}^{\overline{\eta}} \eta dK(\eta)dJ(c)\Bigg] \tag{131}$$

Proof. We have that the payoff function of the policymaker is given by:

$$W_t(h^{t-1}) = (1-\pi)\mathbb{E}[U^U|h^{t-1}] + \pi\mathbb{E}[U^I|h^{t-1}] + \mathbb{E}[U^M|h^{t-1}] \tag{132}$$

Consider state by state any trades that occur. There are two possibilities. First, an informed trader can trade with a market maker. In this case an informed trader transacts x units of the security at price p when the true value is V. The informed trader gets utility $x(V-p-c\mathbb{I}[x<0]c)$ and the market maker gets utility $x(p-V)$, which nets to zero, modulo short-selling cost. Moreover, the short-selling cost (if incurred) is again just a transfer. Thus, conditional on an informed trade, welfare is zero. Second, an uninformed trader can trade with a market maker. The uninformed trader gets expected utility $x(P^*_{t-1}+\eta-p-c\mathbb{I}[x<0])$ and the market maker gets expected utility $x(p-P^*_{t-1})$. Again the short-selling cost is simply a transfer. Thus welfare is given by ηx conditional on trade.

Now note that the uninformed agent will wish to sell whenever $\eta \in [\underline{\eta}, B(h^{t-1})-c-P^*_{t-1}]$ and buy whenever $\eta \in [A(h^{t-1})-P^*_{t-1}, \overline{\eta}]$. Moreover, conditional on the sign of trade, their demand solves the FOC:

$$(1-G(|x_t|^*)) \times \text{sign}(x^*_t) - g(|x^*_t|) = 0 \tag{133}$$

Thus, as is shown in the proof of Theorem 1, the magnitude of x_t is not a function of η. The probability of the agent failing to trade is then given by $\delta(h^{t-1}) = G(|x_t|^*)$, which again does not depend on η. Thus, the agent's expected utility is separable in x and η up to the sign of x. Noting that the chance of an uninformed trader arriving to the market is $(1-\pi)$, it follows that conditional on a given short-selling cost welfare is given by:

$$W(h^{t-1},c) = (1-\pi)(1-\delta(h^{t-1}))|x^*(h^{t-1})|\Bigg[\int_{\underline{\eta}}^{B(h^{t-1})-c-P^*_{t-1}} -\eta dK(\eta) + \int_{A(h^{t-1})-P^*_{t-1}}^{\overline{\eta}} \eta dK(\eta)\Bigg] \tag{134}$$

Averaging over the distribution of short-selling costs then yields welfare:

$$W_t(h^{t-1}) = (1-\pi)(1-\delta(h^{t-1}))|x^*(h^{t-1})|\Bigg[\int_{\underline{c}}^{\overline{c}}\int_{\underline{\eta}}^{B(h^{t-1})-c-P^*_{t-1}} -\eta dK(\eta)dJ(c) + \int_{\underline{c}}^{\overline{c}}\int_{A(h^{t-1})-P^*_{t-1}}^{\overline{\eta}} \eta dK(\eta)dJ(c)\Bigg] \tag{135}$$

□

The intuition for this form is that all short-selling costs and trades are simply transfers between agents except for trades undertaken by noise traders on the basis of idiosyncratic valuation. As a result, the welfare metric is simply the expected quantity transacted by noise traders multiplied by the expected idiosyncratic valuation conditional on trade. In the following analysis, I use this welfare metric to perform analytical static welfare analysis and numerical dynamic welfare analysis.

B.2.2 Static Welfare Analysis

With the welfare metric in hand, we can now analyze which features of the environment will give rise to high or low welfare. Toward this, I first provide a simple closed form example for the welfare metric in the case of uniform noise trader demand to illustrate the key trade-offs involved in the policymaker's welfare function.

Example 1 (Welfare with Uniform Valuations)**.** Suppose that $\eta \sim U[\underline{\eta}, \overline{\eta}]$ with $\underline{\eta} \leq -1-\overline{c}$ and $\overline{\eta} \geq 1$. Integrating with respect to this density yields:

$$W_t(h^{t-1}) = \underbrace{(1-\pi)(1-\delta(h^{t-1}))x^*(h^{t-1})}_{\text{Expected Quantity Transacted by Noise Traders}} \Bigg[\underbrace{\frac{\overline{\eta}^2+\underline{\eta}^2}{2(\overline{\eta}-\underline{\eta})}}_{\text{Total Value If All Traded}} - \underbrace{\frac{(A(h^{t-1})-P^*_{t-1})^2+\mathbb{E}[(B(h^{t-1})-c-P^*_{t-1})^2]}{2(\overline{\eta}-\underline{\eta})}}_{\text{Lost Trades Because of Bid-Ask Spread}}\Bigg] \tag{136}$$

The term on the second line clarifies that welfare is driven primarily by trade probabilities and the width of bid-ask spreads, with the former being beneficial for welfare and the latter harmful as more noise traders are unwilling to pay to cross the spread, creating unrealized gains from trade and harming welfare. To show this function numerically, I moreover assume that market maker book capacity is Gaussian and $\bar{x}_t = \bar{x} + \varepsilon_t$ with $\varepsilon_t \sim N(0, \sigma^2_\varepsilon)$. Figure 6 Shows welfare as a function of the previous shadow price and the average market maker

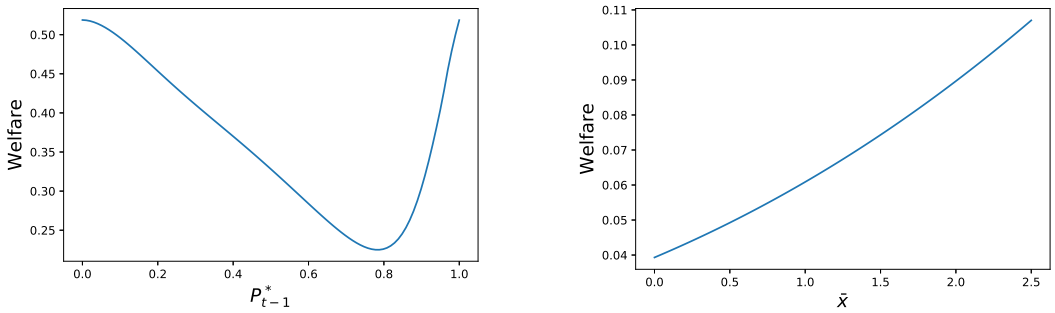

Figure 6: Left: Welfare as a Function of P^*_{t-1}, Right: Welfare as a function of $\bar{x}$. Parameters: $p = 0.7, \pi = 0.5, \eta \sim U[-1, 1], c \sim U[0, 0.8], \varepsilon \sim N(0, 9)$.

book capacity. As the formula shows, welfare is lowest close to the prior as that is where bid-ask spreads are maximized and the unrealized gains from trade are highest. △

These core trade-offs naturally extend to the welfare metric under alternative specifications of noise trader demand. This is stated formally in the following proposition:

Proposition 9. *If (i) $|A(h^{t-1}) - P^*_{t-1}|$ rises (ii) $|P^*_{t-1} - B(h^{t-1})|$ rises or (iii) G_t changes to G'_t such that $G_t \succ_{FOSD} G'_t$, then $W_t(h^{t-1})$ falls*

Proof. The claims all follow by direct inspection of the welfare function in the appropriate objects. I show the claims case-by-case.

1. If $|A(h^{t-1}) - P^*_{t-1}|$ rises, then $W_t(h^{t-1})$ falls:

$$\frac{\partial W_t(h^{t-1})}{\partial |A(h^{t-1}) - P^*_{t-1}|} = -(1-\pi)(1-\delta(h^{t-1}))|x^*(h^{t-1})|k(A(h^{t-1}) - P^*_{t-1}) < 0 \tag{137}$$

2. if $|P^*_{t-1} - B(h^{t-1})|$ rises, then $W_t(h^{t-1})$ falls:

$$\frac{\partial W_t(h^{t-1})}{\partial |P^*_{t-1} - B(h^{t-1})|} = -(1-\pi)(1-\delta(h^{t-1}))|x^*(h^{t-1})| \int_{\underline{c}}^{\overline{c}} k(B(h^{t-1}) - c - P^*_{t-1}) dJ(c) < 0 \tag{138}$$

3. If $G_t \succ_{FOSD} G'_t$, then $W_t(h^{t-1})$ falls. Consider the value function of the noise trader:

$$O(h^{t-1}) \propto (1-\delta(h^{t-1}))|x^*(h^{t-1})| \tag{139}$$

   This must strictly fall when $G_t \succ_{FOSD} G'_t$. Moreover, the bid and ask are unchanged. Thus, $W_t(h^{t-1})$ must fall.

□

This result clarifies that large spreads and small position limits naturally reduce welfare as they limit the volume of welfare-improving trades that can occur. As a result, it provides a natural microfoundation for the earlier qualitative analysis that took it as given that the policymaker wants to reduce the probability of crashes and improve price discovery, as both policies actively reduce spreads and thereby improve welfare.

B.2.3 Dynamic Welfare Analysis

Having understood the static motives for policy, the analysis can be extended to understand how policy affects welfare dynamically. This is interesting as policy in any period t affects the transition probabilities and therefore both future rationing probabilities and the size of the bid-ask spread. I therefore describe a method for computing welfare in a dynamic setting and illustrate the dynamic welfare criterion in a quantitative example.

Mapping to the dynamic context tractably does, however, involve one departure from the model as studied so far. Because the state has so far been persistent, welfare is a function of calendar time – a wholly undesirable feature for a dynamic analysis. I therefore make the problem stationary by assuming that the underlying state V is drawn from $\{0, 1\}$ at date 0 with probability of p that $V = 1$ and that V is then reset at Poisson rate ω and that each reset of V is a commonly known event. Concretely, at the end of each period t, with probability ω, V is drawn from $\{0, 1\}$ with probability p that $V = 1$ and the common prior resets so that $P^*_{t-1} = p$. The policymaker is assumed to have dicount factor ρ and dynamic preferences given by:

$$V(P^*_{t-1}, \bar{x}_t) = \mathbb{E}\left[\sum_{i=0}^{\infty} \rho^t W(P^*_{t+i-1}, \bar{x}_{t+i}) | P^*_{t-1}, \bar{x}_t\right] \tag{140}$$

This can of course be written recursively as:

$$V(P^*_{t-1}, \bar{x}_t) = W(P^*_{t-1}, \bar{x}_t) + (1-\omega)\rho\mathbb{E}[V(P^*_t, \bar{x}_{t+1})|P^*_{t-1}, \bar{x}_t] + \omega\rho\mathbb{E}[V(p, \bar{x}_{t+1})|P^*_{t-1}, \bar{x}_t] \tag{141}$$

Naturally, closed-form computation of this objective is infeasible even in the simplest of circumstances. However, as I have solved for the entire Markov chain describing the model in Proposition 2, computation of V via value function iteration is numerically simple. In particular, one need only specify: i) the short-selling cost distribution J ii) the idiosyncratic valuation distribution K and iii) the Markov process for position limits. In the following example I show one such parameterization and study the implications for the effectiveness of liquidity interventions.

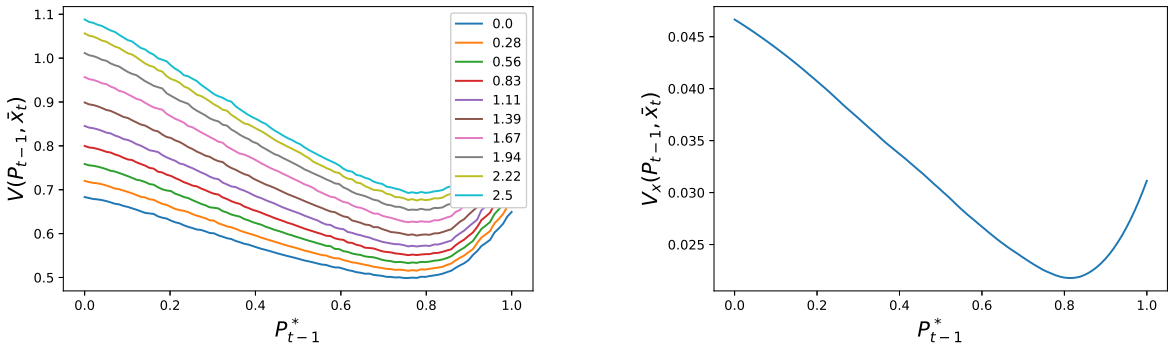

Figure 7: Left: Contour Plot of Value Function (Contours for Differing Values of $\bar{x}_t$), Right: Numerical Derivative of the Value Function as a Function of $\bar{x}_t$ (Averaged over $\bar{x}_t$). Parameters: Parameters: $p = 0.7, \pi = 0.5, \eta \sim U[-1, 1], c \sim U[0, 0.8], \varepsilon \sim N(0, 9), \omega = 0.1, \rho = 0.9$.

Example 2 (Dynamic Policy Analysis)**.** Suppose that trader valuations are uniform $\eta \sim U[\underline{\eta}, \overline{\eta}]$, short-selling costs are uniform $c \sim U[0, \overline{c}]$ and that book capacity follows a random walk:

$$\bar{x}_t = \bar{x}_{t-1} - |x_{t-1}| + \varepsilon_t \tag{142}$$

where $\varepsilon_t \sim N(0, \sigma^2_\varepsilon)$. The left panel Figure 7 plots welfare as a function of the current price and the size of market maker book capacity. Intuitively, this dynamic analysis echoes the static analysis: welfare is highest when we are close to price discovery and lowest around the prior where adverse selection is worst, spreads are widest and gains from trade are forgone. Welfare is also naturally increasing in the level of book capacity $\bar{x}_t$ as this increases the potential for welfare-improving trade. Moreover, the right panel of Figure 7 shows that the marginal value of a position limit intervention is greatest closest to price discovery and lowest around the prior. This is intuitive as there are more noise traders in the market who can utilize any liquidity injection when bid-ask spreads are smallest, which occurs around full price discovery. This provides a rationale for micro-prudential policy targeting liquidity interventions in the early stages of a low volume crash to arrest the dynamic unravelling mechanism and ensure tight bid-ask spreads. △

The ability to perform such dynamic welfare analysis may be of interest other researchers in the context of these models. One can additionally use the Markov chain structure of the model to compute the stationary distribution of prices and compute counterfactual welfare analyses, of which the position limit intervention given in Example 2 is just one possibility.