

# Web Appendix of “Optimal Contracts for Time-Inconsistent Consumers with Heterogeneous Beliefs”

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## Perfect Competition with Heterogeneous Consumer Beliefs

In this appendix, we consider a perfectly competitive market. As in the main model, the consumers are time-inconsistent and can be either sophisticated or naive. The firms cannot observe the consumers’ type. In this perfectly competitive market, the consumers contract with the firm that gives them the highest perceived utility. A firm’s problem is to maximize the perceived utility for each consumer type subject to their participation constraints and incentive compatibility constraints, as well as the zero-profit constraints. The zero-profit constraints require that  $\mu_s = T(p_s)$  and  $\mu_n = T(p_n)$ . It follows that the perceived utility is  $T(p_s)$  for the sophisticated type and  $T(p_n) + I(p_n)$  for the naive type. Based on the above analysis, each firm’s problem is to choose  $p_s$  and  $p_n$  that solve

$$\max \gamma_s T(p_s) + \gamma_n [T(p_n) + I(p_n)]$$

s.t.

$$\begin{aligned} T(p_s) &\geq 0 && (PC_s) \\ T(p_n) + I(p_n) &\geq 0 && (PC_n) \\ T(p_s) &\geq T(p_n) && (IC_s) \\ T(p_n) + I(p_n) &\geq T(p_s) + I(p_s) && (IC_n) \end{aligned}$$

$PC_j$  is the participation constraint for type  $j$ , which requires that the type- $j$  consumers perceive nonnegative utility from contracting with the firm.  $IC_j$  is the incentive compatibility constraint for type  $j$ , which requires that the type- $j$  consumers perceive a weakly higher utility from choosing their contract than choosing the contract for the other type. We can first ignore the participation constraints and solve that the optimal prices are  $p_s^{FB}$  and  $p_n^{FB}$ . This is because the first-best prices, by definition, maximize the perceived utility for each consumer type and automatically satisfy the incentive constraints. We can verify that the participation constraints are satisfied at  $p_s^{FB}$  and  $p_n^{FB}$ , making the two prices optimal in the firm’s original problem. It follows that asymmetric information on consumer beliefs does not generate any distortion under perfect competition. The optimal prices are again above marginal cost for temptation goods.