Online Appendix to
"Dynamic ESG Equilibrium"

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This Online Appendix presents the proofs and derivations, the summary statistics of the data, the estimation methodology, as well as the supplementary estimation and calibration material discussed in the paper.

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A Derivations

A.1 Proof of Proposition 1 (Euler equation)

The optimization program is formulated as

$$U_t = \max_{C_t, \boldsymbol{\omega}_t} \left((1-\beta)\, \varphi_t A_t^{1-\frac{1}{\psi}} + \beta \mathrm{E}_t \left[U_{t+1}^{1-\gamma} \right]^{\frac{1}{\theta}} \right)^{\frac{1}{1-\frac{1}{\psi}}}, \tag{A.1}$$

$$A_t = \left(C_t^{1-\frac{1}{\phi}} + \delta_t S_t^{1-\frac{1}{\phi}} \right)^{\frac{1}{1-\frac{1}{\phi}}} = C_t \left(1 + \delta_t \left(\frac{S_t}{C_t} \right)^{1-\frac{1}{\phi}} \right)^{\frac{1}{1-\frac{1}{\phi}}}, \tag{A.2}$$

$$S_t = G_{W,t} \left(W_t - C_t \right), \tag{A.3}$$

where $G_{W,t} = \sum_{n=1}^{N} \omega_{n,t} G_{n,t}$ is the aggregate greenness (ESG supply), φ_t is a variable reflecting shocks to the time rate of preference (Albuquerque et al., 2016, Schorfheide et al., 2018), and δ_t represents time-varying preferences for ESG (demand). Note that, in the most general case where ϕ can be finite, the aggregate ESG supply must be nonnegative. In the main text, we develop the analysis considering additive preferences, i.e., $\phi \to +\infty$, in which case $G_{W,t}$ can take any value. The budget constraint states that $W_{t+1} = (W_t - C_t) R_{W,t+1}$, where $R_{W,t+1} = R_{f,t+1} + \sum_{n=1}^{N} \omega_{n,t} (R_{n,t+1} - R_{f,t+1})$. At the optimum, the value function depends on wealth only, that is $U_t = J(W_t)$. The agent then optimizes

$$J(W_t) = \max_{C_t, \boldsymbol{\omega}_t} \left((1-\beta) \varphi_t A_t^{1-\frac{1}{\psi}} + \beta \mathrm{E}_t \left[J(W_{t+1})^{1-\gamma} \right]^{\frac{1}{\theta}} \right)^{\frac{1}{1-\frac{1}{\psi}}}. \tag{A.4}$$

The first order condition with respect to consumption is given by

$$0 = (1-\beta) \varphi_t A_t^{-\frac{1}{\psi}} \frac{\partial A_t}{\partial C_t} - \beta \mathrm{E}_t \left[J(W_{t+1})^{1-\gamma} \right]^{\frac{1}{\theta}-1} \mathrm{E}_t \left[J(W_{t+1})^{-\gamma} \frac{\partial J(W_{t+1})}{\partial W_{t+1}} R_{W,t+1} \right], \tag{A.5}$$

where $\frac{\partial A_t}{\partial C_t} = \left(\frac{A_t}{C_t}\right)^{\frac{1}{\phi}} - \left(\frac{A_t}{S_t}\right)^{\frac{1}{\phi}} \delta_t G_{W,t}$ and $\lim_{\phi \to +\infty} \frac{\partial A_t}{\partial C_t} = 1 - \delta_t G_{W,t}$. Next, the first order condition with respect to $\omega_{n,t}$ is given by

$$\begin{aligned} 0 = {} & \beta \mathrm{E}_t \left[J(W_{t+1})^{1-\gamma} \right]^{\frac{1}{\theta}-1} \mathrm{E}_t \left[J(W_{t+1})^{-\gamma} \frac{\partial J(W_{t+1})}{\partial W_{t+1}} (R_{n,t+1} - R_{f,t+1}) \right] \\ & + (1-\beta) \varphi_t A_t^{-\frac{1}{\psi}} \frac{\partial A_t}{\partial S_t} G_{n,t}. \end{aligned} \tag{A.6}$$

As $\frac{\partial A_t}{\partial S_t} = \left(\frac{A_t}{S_t}\right)^{\frac{1}{\phi}} \delta_t$ and $\lim_{\phi \to +\infty} \frac{\partial A_t}{\partial S_t} = \delta_t$, multiplying (A.6) by $\omega_{n,t}$ and summing across assets yields

$$\begin{aligned} 0 = {} & \beta \mathrm{E}_t \left[J(W_{t+1})^{1-\gamma} \right]^{\frac{1}{\theta}-1} \mathrm{E}_t \left[J(W_{t+1})^{-\gamma} \frac{\partial J(W_{t+1})}{\partial W_{t+1}} R_{W,t+1} \right] \\ & - \beta \mathrm{E}_t \left[J(W_{t+1})^{1-\gamma} \right]^{\frac{1}{\theta}-1} \mathrm{E}_t \left[J(W_{t+1})^{-\gamma} \frac{\partial J(W_{t+1})}{\partial W_{t+1}} R_{f,t+1} \right] \\ & + (1-\beta) \varphi_t A_t^{-\frac{1}{\psi}} \frac{\partial A_t}{\partial S_t} G_{W,t}. \end{aligned} \tag{A.7}$$

Noting that

$$\frac{\partial A_t}{\partial C_t} + \frac{\partial A_t}{\partial S_t} G_{W,t} = \left(1 + \delta_t \left(\frac{S_t}{C_t} \right)^{1-\frac{1}{\phi}} \right)^{\frac{1}{\phi-1}} = \left(\frac{A_t}{C_t} \right)^{\frac{1}{\phi}}, \tag{A.8}$$

which tends to one for $\phi \to +\infty$, and combining (A.5) and (A.7), we obtain

$$\mathrm{E}_t \left[M_{t+1} R_{f,t+1} \right] = 1. \tag{A.9}$$

This is the Euler equation for the risk-free gross return, where the stochastic discount factor

(SDF) is formulated as

$$M_{t+1}=\beta\frac{\mathrm{E}_t\left[J\left(W_{t+1}\right)^{1-\gamma}\right]^{\frac{1}{\theta}-1}J\left(W_{t+1}\right)^{-\gamma}\frac{\partial J(W_{t+1})}{\partial W_{t+1}}}{\left(1-\beta\right)\varphi_t A_t^{-\frac{1}{\psi}}\left(\frac{A_t}{C_t}\right)^{\frac{1}{\phi}}}. \tag{A.10}$$

From (A.6), we can express the Euler equation for excess return on a generic asset as

$$\mathrm{E}_t\left[M_{t+1}\left(R_{n,t+1}-R_{f,t+1}\right)\right]=-\left(\frac{S_t}{C_t}\right)^{-\frac{1}{\phi}}\delta_t G_{n,t}. \tag{A.11}$$

Note that, for $\phi\to+\infty$, the Euler equation becomes $\mathrm{E}_t\left[M_{t+1}\left(R_{n,t+1}-R_{f,t+1}\right)\right]=-\delta_t G_{n,t}$, hence, $\mathrm{E}_t\left[R_{n,t+1}-R_{f,t+1}\right]=-\frac{\mathrm{Cov}_t\left[R_{n,t+1}-R_{f,t+1},M_{t+1}\right]}{\mathrm{E}_t[M_{t+1}]}-\frac{\delta_t G_{n,t}}{\mathrm{E}_t[M_{t+1}]}$. Summing (A.9) and (A.11), we obtain the Euler equation for the gross return on a generic asset:

$$\mathrm{E}_t\left[M_{t+1}R_{n,t+1}\right]=1-\left(\frac{S_t}{C_t}\right)^{-\frac{1}{\phi}}\delta_t G_{n,t}. \tag{A.12}$$

We next derive an explicit solution for the value function. To start, we guess $J\left(W_t\right)=\Phi_t W_t$. Then, equations (A.4) and (A.5) can be expressed as

$$\beta\mathrm{E}_t\left[\Phi_{t+1}^{1-\gamma}W_{t+1}^{1-\gamma}\right]^{\frac{1}{\theta}}=\Phi_t^{1-\frac{1}{\psi}}W_t^{1-\frac{1}{\psi}}-\left(1-\beta\right)\varphi_t A_t^{1-\frac{1}{\psi}} \tag{A.13}$$

and

$$0=\left(1-\beta\right)\varphi_t A_t^{-\frac{1}{\psi}}\frac{\partial A_t}{\partial C_t}\left(W_t-C_t\right)-\beta\mathrm{E}_t\left[\Phi_{t+1}^{1-\gamma}W_{t+1}^{1-\gamma}\right]^{\frac{1}{\theta}}, \tag{A.14}$$

respectively. Combining both equations yields

$$\Phi_t=\left(1-\beta\right)^{\frac{1}{1-\frac{1}{\psi}}}\varphi_t^{\frac{1}{1-\frac{1}{\psi}}}\left(\frac{A_t}{C_t}\right)^{\frac{1}{\phi}\frac{1}{1-\frac{1}{\psi}}}\left(\frac{W_t}{A_t}\right)^{\frac{\frac{1}{\psi}}{1-\frac{1}{\psi}}}. \tag{A.15}$$

Then, M_{t+1} in (A.10) can be developed as

$$M_{t+1}=\beta^{\theta}\left(\frac{\varphi_{t+1}}{\varphi_t}\right)^{\theta}\left(\frac{A_{t+1}}{A_t}\right)^{-\frac{\theta}{\psi}}\left(\frac{\frac{C_{t+1}}{A_{t+1}}}{\frac{C_t}{A_t}}\right)^{-\frac{\theta}{\phi}}\left(\frac{R_{W,t+1}}{1-\left(\frac{S_t}{C_t}\right)^{-\frac{1}{\phi}}\delta_t G_{W,t}}\right)^{\theta-1}, \tag{A.16}$$

where $R_{W,t+1}=\frac{W_{t+1}}{W_t-C_t}$. Defining $\tilde{R}_{W,t+1}=\frac{R_{W,t+1}}{1-\left(\frac{S_t}{C_t}\right)^{-\frac{1}{\phi}}\delta_t G_{W,t}}$ and $\tilde{R}_{n,t+1}=\frac{R_{n,t+1}}{1-\left(\frac{S_t}{C_t}\right)^{-\frac{1}{\phi}}\delta_t G_{n,t}}$, the Euler equation in (A.12) can be equivalently expressed as

$$\mathrm{E}_t\left[M_{t+1}\tilde{R}_{n,t+1}\right]=1, \tag{A.17}$$

where

$$M_{t+1}=\beta^{\theta}\left(\frac{\varphi_{t+1}}{\varphi_t}\right)^{\theta}\left(\frac{A_{t+1}}{A_t}\right)^{-\frac{\theta}{\psi}}\left(\frac{\frac{C_{t+1}}{A_{t+1}}}{\frac{C_t}{A_t}}\right)^{-\frac{\theta}{\phi}}\tilde{R}_{W,t+1}^{\theta-1}. \tag{A.18}$$

Before considering the simplified case discussed in the text, where the time-preference shock φ_t is muted and ϕ approaches infinity, we briefly comment on the incremental contribution of the extended version.

Similar to Albuquerque et al. (2016), the growth in time preference $\frac{\varphi_{t+1}}{\varphi_t}$ establishes an additional multiplicative factor to the SDF in equation (5). That is, allowing time-preference shocks helps matching the market premium by considering a positive *valuation premium* of the market portfolio due to time-varying demand for consumption. It also improves the model ability to generate an upward-sloping term structure of interest rates.

The implications of a finite elasticity of substitution ϕ are twofold. First, as implied by the term $(S_t/C_t)^{-\frac{1}{\phi}}$ in equation (A.12), which diminishes in S_t, the convenience yield of any asset, green or brown, is attenuated when aggregate ESG supply grows larger relative to physical consumption. Second, the factor $\left(\frac{C_{t+1}/A_{t+1}}{C_t/A_t}\right)^{-\frac{\theta}{\phi}}$ appears in the SDF (A.16). To analyze its contribution, consider the case where preferences are time additive ($\theta = 1$), so that the factor depending on the return on wealth can be excluded. The SDF then increases when the share of physical consumption C_t relative to the consumption bundle A_t diminishes. As physical consumption and sustainability are not perfect substitutes, the agent favors assets that deliver higher returns in times when physical consumption is low relative to the value of the consumption bundle. Such assets would deliver lower expected returns in equilibrium. Both the effects described above become more prominent when the elasticity of substitution diminishes.

In what follows, we assume $\varphi_t = 1$ and $\phi \to +\infty$. The Euler equation (A.12) and the corresponding SDF in (A.16) can be expressed as

$$\mathrm{E}_t\left[M_{t+1}\tilde{R}_{n,t+1}\right] = 1, \tag{A.19}$$

$$M_{t+1} = \beta^\theta \left(\frac{A_{t+1}}{A_t}\right)^{-\frac{\theta}{\psi}} \tilde{R}_{W,t+1}^{\theta-1}, \tag{A.20}$$

where $\tilde{R}_{W,t+1} = \frac{R_{W,t+1}}{1-\delta_t G_{W,t}}$ and $\tilde{R}_{n,t+1} = \frac{R_{n,t+1}}{1-\delta_t G_{n,t}}$ are the ESG-adjusted gross returns on the consumption asset and on a generic asset, respectively. The Euler equation undertakes the standard form only when the financial return is replaced by the ESG-adjusted return.

We then calculate the logarithm of the SDF, $m_{t+1} = \log M_{t+1}$:

$$\begin{aligned} m_{t+1} = {} & \theta \log \beta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1)\left(r_{W,t+1} - \log\left(1 - \delta_t G_{W,t}\right)\right) \\ & - \frac{\theta}{\psi} \log\left(\frac{1 + \delta_{t+1}\frac{W_{t+1}-C_{t+1}}{C_{t+1}} G_{W,t+1}}{1 + \delta_t \frac{W_t - C_t}{C_t} G_{W,t}}\right), \end{aligned} \tag{A.21}$$

where $\Delta c_{t+1} = \log \frac{C_{t+1}}{C_t}$ and $r_{W,t+1} = \log \frac{W_{t+1}}{W_t - C_t}$ is the logarithmic return on financial wealth. The expected excess return of a generic asset then satisfies the following relation

$$\mathrm{E}_t\left[r_{n,t+1} - r_{f,t+1}\right] + \frac{1}{2}\mathrm{Var}_t\left[r_{n,t+1}\right] = -\mathrm{Cov}_t\left[m_{t+1}, r_{n,t+1}\right] - y_{n,t}, \tag{A.22}$$

where $r_{n,t+1} = \log R_{n,t+1}$, $r_{f,t+1} = \log R_{f,t+1}$, and $y_{n,t} = -\log\left(1 - \delta_t G_{n,t}\right)$.

Finally, we aim to determine the concavity of the value function with respect to $G_{W,t}$ and

δ_t. We start evaluating the first derivatives:

$$\frac{\partial J(W_t)}{\partial G_{W,t}} = (1-\beta)\left((1-\beta)A_t^{1-\frac{1}{\psi}} + \beta \mathrm{E}_t\left[U_{t+1}^{1-\gamma}\right]^{\frac{1}{\theta}}\right)^{\frac{1}{1-\frac{1}{\psi}}-1} A_t^{-\frac{1}{\psi}}\delta_t (W_t - C_t)$$

$$= (1-\beta) J(W_t)^{\frac{1}{\psi}} A_t^{-\frac{1}{\psi}} \delta_t (W_t - C_t), \tag{A.23}$$

$$\frac{\partial J(W_t)}{\partial \delta_t} = (1-\beta)\left((1-\beta)A_t^{1-\frac{1}{\psi}} + \beta \mathrm{E}_t\left[U_{t+1}^{1-\gamma}\right]^{\frac{1}{\theta}}\right)^{\frac{1}{1-\frac{1}{\psi}}-1} A_t^{-\frac{1}{\psi}} G_{W,t} (W_t - C_t)$$

$$= (1-\beta) J(W_t)^{\frac{1}{\psi}} A_t^{-\frac{1}{\psi}} G_{W,t} (W_t - C_t), \tag{A.24}$$

which, respectively, are positive for $\delta_t > 0$ and $G_{W,t} > 0$.

The second derivatives are

$$\frac{\partial^2 J(W_t)}{\partial G_{W,t}^2} = -\frac{1}{\psi}(1-\beta) J(W_t)^{\frac{1}{\psi}} A_t^{-\frac{1}{\psi}-1} \delta_t^2 (W_t - C_t)^2 \cdot \frac{\beta \mathrm{E}_t\left[J(W_{t+1})^{1-\gamma}\right]^{\frac{1}{\theta}}}{(1-\beta)A_t^{1-\frac{1}{\psi}} + \beta \mathrm{E}_t\left[J(W_{t+1})^{1-\gamma}\right]^{\frac{1}{\theta}}} < 0, \tag{A.25}$$

$$\frac{\partial^2 J(W_t)}{\partial \delta_t^2} = -\frac{1}{\psi}(1-\beta) J(W_t)^{\frac{1}{\psi}} A_t^{-\frac{1}{\psi}-1} G_{W,t}^2 (W_t - C_t)^2 \cdot \frac{\beta \mathrm{E}_t\left[J(W_{t+1})^{1-\gamma}\right]^{\frac{1}{\theta}}}{(1-\beta)A_t^{1-\frac{1}{\psi}} + \beta \mathrm{E}_t\left[J(W_{t+1})^{1-\gamma}\right]^{\frac{1}{\theta}}} < 0, \tag{A.26}$$

which are both negative. The value function is thus concave in both $G_{W,t}$ and δ_t.

A.2 Proof of Proposition 2 (stochastic discount factor)

The time-t investable wealth is $P_t = W_t - C_t$, i.e., wealth after consumption, and is conventionally called *price* of the consumption asset (wealth portfolio). The return on wealth between time t and time $t+1$ is:

$$r_{W,t+1} = \log\left(\frac{W_{t+1}}{W_t - C_t}\right) = \log\left(\frac{P_{t+1} + C_{t+1}}{P_t}\right) = \log\left(\frac{\frac{P_{t+1}}{C_{t+1}} + 1}{\frac{P_t}{C_t}}\frac{C_{t+1}}{C_t}\right)$$

$$= \log\left(1 + e^{pc_{t+1}}\right) - pc_t + \Delta c_{t+1}, \tag{A.27}$$

where $pc_t = \log \frac{P_t}{C_t}$ is the log price/consumption ratio. We perform the Campbell and Shiller (1988) log-linearization by developing the first-order Taylor expansion of the first term around the average model-implied log price-to-consumption ratio $\overline{pc}$:

$$\log\left(1 + e^{pc_{t+1}}\right) \simeq \log\left(1 + e^{pc_{t+1}}\right)\big|_{\overline{pc}} + \frac{\mathrm{d}}{\mathrm{d}pc_{t+1}} \log\left(1 + e^{pc_{t+1}}\right)\bigg|_{\overline{pc}} (pc_{t+1} - \overline{pc})$$

$$= \log\left(1 + e^{\overline{pc}}\right) + \frac{e^{\overline{pc}}}{1 + e^{\overline{pc}}} (pc_{t+1} - \overline{pc}). \tag{A.28}$$

Then,

$$r_{W,t+1} \simeq \kappa_{rW,0} + \kappa_{rW,pc} pc_{t+1} - pc_t + \Delta c_{t+1}, \tag{A.29}$$

where $\kappa_{rW,pc} = \frac{e^{\overline{pc}}}{1+e^{\overline{pc}}}$, $\kappa_{rW,0} = \log\left(1 + e^{\overline{pc}}\right) - \kappa_{rW,pc}\overline{pc}$, with $\overline{pc}$ being determined as the solution of a fixed-point problem. Following the same logic, we perform two additional approximations. The first is given by

$$\begin{aligned}
\log\left(1 + e^{pc_t}\delta_t G_{W,t}\right) &\simeq \log\left(1 + e^{\overline{pc}}\bar{\delta}\bar{G}_W\right) + \frac{e^{\overline{pc}}\bar{G}_W}{1 + e^{\overline{pc}}\bar{\delta}\bar{G}_W}\left(\delta_t - \bar{\delta}\right) \\
&\quad + \frac{e^{\overline{pc}}\bar{\delta}}{1 + e^{\overline{pc}}\bar{\delta}\bar{G}_W}\left(G_{W,t} - \bar{G}_W\right) + \frac{e^{\overline{pc}}\bar{\delta}\bar{G}_W}{1 + e^{\overline{pc}}\bar{\delta}\bar{G}_W}\left(pc_t - \overline{pc}\right) \\
&= \kappa_{m,0} + \kappa_{m,\delta}\delta_t + \kappa_{m,G}G_{W,t} + \kappa_{m,pc}pc_t,
\end{aligned} \tag{A.30}$$

where $\kappa_{m,\delta} = \frac{e^{\overline{pc}}\bar{G}_W}{1+e^{\overline{pc}}\bar{\delta}\bar{G}_W}$, $\kappa_{m,G} = \frac{e^{\overline{pc}}\bar{\delta}}{1+e^{\overline{pc}}\bar{\delta}\bar{G}_W}$, $\kappa_{m,pc} = \frac{e^{\overline{pc}}\bar{\delta}\bar{G}_W}{1+e^{\overline{pc}}\bar{\delta}\bar{G}_W}$, and $\kappa_{m,0} = \log\left(1 + e^{\overline{pc}}\bar{\delta}\bar{G}_W\right) - \kappa_{m,\delta}\bar{\delta} - \kappa_{m,G}\bar{G}_W - \kappa_{m,pc}\overline{pc}$. The second approximation is

$$\begin{aligned}
\log\left(1 - \delta_t G_{W,t}\right) &\simeq \log\left(1 - \bar{\delta}\bar{G}_W\right) - \frac{\bar{G}_W}{1 - \bar{\delta}\bar{G}_W}\left(\delta_t - \bar{\delta}\right) - \frac{\bar{\delta}}{1 - \bar{\delta}\bar{G}_W}\left(G_{W,t} - \bar{G}_W\right) \\
&= \kappa_{W,0} + \kappa_{W,\delta}\delta_t + \kappa_{W,G}G_{W,t},
\end{aligned} \tag{A.31}$$

where $\kappa_{W,\delta} = -\frac{\bar{G}_W}{1-\bar{\delta}\bar{G}_W}$, $\kappa_{W,G} = -\frac{\bar{\delta}}{1-\bar{\delta}\bar{G}_W}$, and $\kappa_{W,0} = \log\left(1 - \bar{\delta}\bar{G}_W\right) - \kappa_{W,\delta}\bar{\delta} - \kappa_{W,G}\bar{G}_W$. Then, we can rewrite the SDF in (A.21) as

$$\begin{aligned}
m_{t+1} &\simeq \theta\log\beta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1)\, r_{W,t+1} - (\theta - 1)\left(\kappa_{W,0} + \kappa_{W,\delta}\delta_t + \kappa_{W,G}G_{W,t}\right) \\
&\quad - \frac{\theta}{\psi}\kappa_{m,\delta}\Delta\delta_{t+1} - \frac{\theta}{\psi}\kappa_{m,G}\Delta G_{W,t+1} - \frac{\theta}{\psi}\kappa_{m,pc}\Delta pc_{t+1}.
\end{aligned} \tag{A.32}$$

As introduced in Section 2.3, we specify four dynamic processes:

$$\Delta c_{t+1} = \mu_c + x_t + \sigma_c\varepsilon_{c,t+1}, \tag{A.33}$$

$$G_{W,t+1} = \mu_G + \rho_G G_{W,t} + \rho_{G,\delta}\delta_t + \sigma_G\varepsilon_{G,t+1}, \tag{A.34}$$

$$\delta_{t+1} = \mu_\delta + \rho_\delta\delta_t + \sigma_\delta\varepsilon_{\delta,t+1}, \tag{A.35}$$

$$x_{t+1} = \rho_x x_t + \sigma_x\varepsilon_{x,t+1}, \tag{A.36}$$

where $G_{W,t+1}$ and δ_{t+1} are mean reverting, $\mu_G = (1 - \rho_G)\,\bar{G}_W - \rho_{G,\delta}\bar{\delta}$, and $\mu_\delta = (1 - \rho_\delta)\,\bar{\delta}$. Based on (A.31), we rewrite the Euler equation as:

$$\mathrm{E}_t\left[e^{m_{t+1}+r_{W,t+1}}\right] = e^{\kappa_{W,0}+\kappa_{W,\delta}\delta_t+\kappa_{W,G}G_{W,t}}. \tag{A.37}$$

To characterize the SDF, we make the following guess on the functional form of the price-to-consumption ratio:

$$pc_t = A_{pc,0} + A_{pc,G}G_{W,t} + A_{pc,\delta}\delta_t + A_{pc,x}x_t. \tag{A.38}$$

Then, substituting in equation (A.29), it follows that

$$\begin{aligned} r_{W,t+1} &\simeq \kappa_{rW,0} + \kappa_{rW,pc} pc_{t+1} - pc_t + \Delta c_{t+1} \\ &= \kappa_{rW,0} + A_{pc,0}\left(\kappa_{rW,pc} - 1\right) + A_{pc,G}\left(\kappa_{rW,pc} G_{W,t+1} - G_{W,t}\right) \\ &\quad + A_{pc,\delta}\left(\kappa_{rW,pc}\delta_{t+1} - \delta_t\right) + A_{pc,x}\left(\kappa_{rW,pc} x_{t+1} - x_t\right) + \Delta c_{t+1}. \end{aligned} \tag{A.39}$$

Summing equations (A.32) and (A.39), we further obtain

$$\begin{aligned} & m_{t+1} + r_{W,t+1} = \\ & \theta \log\beta - (\theta - 1)\kappa_{W,0} + (1-\gamma)\mu_c + \theta\kappa_{rW,0} + \theta A_{pc,0}\left(\kappa_{rW,pc} - 1\right) \\ & + \theta\kappa_{rW,pc}\left(A_{pc,\delta}\mu_\delta + A_{pc,G}\mu_G\right) \\ & - \frac{\theta}{\psi}\left(\left(\kappa_{m,G} + \kappa_{m,pc}A_{pc,G}\right)\mu_G + \left(\kappa_{m,\delta} + \kappa_{m,pc}A_{pc,\delta}\right)\mu_\delta\right) \\ & + \left(\theta A_{pc,G}\left(\kappa_{rW,pc}\rho_G - 1 - \frac{\kappa_{m,pc}}{\psi}\left(\rho_G - 1\right)\right) - (\theta-1)\kappa_{W,G} - \frac{\theta}{\psi}\kappa_{m,G}\left(\rho_G - 1\right)\right) G_{W,t} \\ & + \left(\begin{array}{l} \theta A_{pc,\delta}\left(\kappa_{rW,pc}\rho_\delta - 1 - \frac{\kappa_{m,pc}}{\psi}\left(\rho_\delta - 1\right)\right) - (\theta-1)\kappa_{W,\delta} - \frac{\theta}{\psi}\kappa_{m,\delta}\left(\rho_\delta - 1\right) \\ + \left(\theta A_{pc,G}\left(\kappa_{rW,pc} - \frac{\kappa_{m,pc}}{\psi}\right) - \frac{\theta}{\psi}\kappa_{m,G}\right)\rho_{G,\delta} \end{array}\right)\delta_t \\ & + \left((1-\gamma) + \theta A_{pc,x}\left(\kappa_{rW,pc}\rho_x - 1 - \frac{\kappa_{m,pc}}{\psi}\left(\rho_x - 1\right)\right)\right) x_t \\ & + (1-\gamma)\sigma_c\varepsilon_{c,t+1} \\ & + \left(\theta A_{pc,G}\left(\kappa_{rW,pc} - \frac{\kappa_{m,pc}}{\psi}\right) - \frac{\theta}{\psi}\kappa_{m,G}\right)\sigma_G\varepsilon_{G,t+1} \\ & + \left(\left(\theta A_{pc,\delta}\left(\kappa_{rW,pc} - \frac{\kappa_{m,pc}}{\psi}\right) - \frac{\theta}{\psi}\kappa_{m,\delta}\right)\sigma_\delta\right)\varepsilon_{\delta,t+1} \\ & + \theta A_{pc,x}\left(\kappa_{rW,pc} - \frac{\kappa_{m,pc}}{\psi}\right)\sigma_x\varepsilon_{x,t+1}. \end{aligned} \tag{A.40}$$

As $\mathrm{E}_t\left[e^{m_{t+1}+r_{W,t+1}}\right] = e^{\kappa_{W,0}+\kappa_{W,\delta}\delta_t+\kappa_{W,G}G_{W,t}}$, we can solve for the coefficients:

$$A_{pc,0} = \frac{1}{\theta\left(1-\kappa_{rW,pc}\right)}\left(\begin{array}{l} \theta\log\beta - \theta\kappa_{W,0} + (1-\gamma)\mu_c \\ +\theta\kappa_{rW,0} + \theta\kappa_{rW,pc}\left(A_{pc,G}\mu_G + A_{pc,\delta}\mu_\delta\right) \\ -\frac{\theta}{\psi}\left(\kappa_{m,G} + \kappa_{m,pc}A_{pc,G}\right)\mu_G \\ -\frac{\theta}{\psi}\left(\kappa_{m,\delta} + \kappa_{m,pc}A_{pc,\delta}\right)\mu_\delta \\ +\frac{(1-\gamma)^2\sigma_c^2}{2} + \frac{\left(\theta A_{pc,G}\left(\kappa_{rW,pc} - \frac{\kappa_{m,pc}}{\psi}\right) - \frac{\theta}{\psi}\kappa_{m,G}\right)^2\sigma_G^2}{2} \\ +\frac{\left(\theta A_{pc,\delta}\left(\kappa_{rW,pc} - \frac{\kappa_{m,pc}}{\psi}\right) - \frac{\theta}{\psi}\kappa_{m,\delta}\right)^2\sigma_\delta^2}{2} \\ +\frac{\left(\theta\kappa_{rW,pc} - \frac{\theta}{\psi}\kappa_{m,pc}\right)^2 A_{pc,x}^2\sigma_x^2}{2} \end{array}\right), \tag{A.41}$$

$$A_{pc,G} = \frac{\kappa_{m,G}\left(1-\rho_G\right) - \psi\kappa_{W,G}}{\psi - \kappa_{m,pc} - \left(\psi\kappa_{rW,pc} - \kappa_{m,pc}\right)\rho_G} = \frac{\kappa_{m,G} - \frac{\psi}{1-\rho_G}\kappa_{W,G}}{\psi\frac{1-\kappa_{rW,pc}\rho_G}{1-\rho_G} - \kappa_{m,pc}}, \tag{A.42}$$

$$A_{pc,\delta} = \frac{\kappa_{m,\delta}\left(1-\rho_\delta\right) - \psi\kappa_{W,\delta} + \left(\left(\psi\kappa_{rW,pc} - \kappa_{m,pc}\right)A_{pc,G} - \kappa_{m,G}\right)\rho_{G,\delta}}{\psi - \kappa_{m,pc} - \left(\psi\kappa_{rW,pc} - \kappa_{m,pc}\right)\rho_\delta}$$

$$= \frac{\kappa_{m,\delta} - \frac{\psi}{1-\rho_\delta}\kappa_{W,\delta}}{\psi\frac{1-\kappa_{rW,pc}\rho_\delta}{1-\rho_\delta} - \kappa_{m,pc}} + \frac{\frac{\psi\kappa_{rW,pc}-\kappa_{m,pc}}{1-\rho_\delta}A_{pc,G} - \frac{\kappa_{m,G}}{1-\rho_\delta}}{\psi\frac{1-\kappa_{rW,pc}\rho_\delta}{1-\rho_\delta} - \kappa_{m,pc}}\rho_{G,\delta}, \tag{A.43}$$

$$A_{pc,x} = \frac{\psi - 1}{\psi - \kappa_{m,pc} - (\psi\kappa_{rW,pc} - \kappa_{m,pc})\,\rho_x} = \frac{\frac{\psi-1}{1-\rho_x}}{\psi\frac{1-\kappa_{rW,pc}\rho_x}{1-\rho_x} - \kappa_{m,pc}}. \tag{A.44}$$

A sufficient condition for the coefficients $A_{pc,G}$, $A_{pc,\delta}$, and $A_{pc,x}$ to be positive is that $\rho_{G,\delta} = 0$, $\psi > 1$, $\bar{\delta} \geq 0$, and $\bar{G}_W \geq 0$. m_{t+1} in (A.32) can be then written as

$$\begin{aligned}
m_{t+1} \simeq\, & \theta\log\beta - \gamma\mu_c + (\theta-1)\left(\kappa_{rW,0} - \kappa_{W,0} + A_{pc,0}\left(\kappa_{rW,pc} - 1\right)\right) \\
& + (\theta-1)\,\kappa_{rW,pc}\left(A_{pc,G}\mu_G + A_{pc,\delta}\mu_\delta\right) \\
& - \frac{\theta}{\psi}\left(\left(\kappa_{m,\delta} + \kappa_{m,pc}A_{pc,\delta}\right)\mu_\delta + \left(\kappa_{m,G} + \kappa_{m,pc}A_{pc,G}\right)\mu_G\right) \\
& + \left((\theta-1)\left(A_{pc,G}\left(\kappa_{rW,pc}\rho_G - 1\right) - \kappa_{W,G}\right) - \frac{\theta}{\psi}\left(\kappa_{m,G} + \kappa_{m,pc}A_{pc,G}\right)\left(\rho_G - 1\right)\right)G_{W,t} \\
& + \left(\begin{array}{l} (\theta-1)\left(A_{pc,\delta}\left(\kappa_{rW,pc}\rho_\delta - 1\right) - \kappa_{W,\delta}\right) - \frac{\theta}{\psi}\left(\kappa_{m,\delta} + \kappa_{m,pc}A_{pc,\delta}\right)\left(\rho_\delta - 1\right) \\ + \left((\theta-1)\,A_{pc,G}\kappa_{rW,pc} - \frac{\theta}{\psi}\left(\kappa_{m,G} + \kappa_{m,pc}A_{pc,G}\right)\right)\rho_{G,\delta} \end{array}\right)\delta_t \\
& + \left(-\gamma + \left((\theta-1)\left(\kappa_{rW,pc}\rho_x - 1\right) - \frac{\theta}{\psi}\kappa_{m,pc}\left(\rho_x - 1\right)\right)A_{pc,x}\right)x_t \\
& - \gamma\sigma_c\varepsilon_{c,t+1} \\
& + \left((\theta-1)\,A_{pc,G}\kappa_{rW,pc} - \frac{\theta}{\psi}\left(\kappa_{m,pc}A_{pc,G} + \kappa_{m,G}\right)\right)\sigma_G\varepsilon_{G,t+1} \\
& + \left((\theta-1)\,A_{pc,\delta}\kappa_{rW,pc} - \frac{\theta}{\psi}\left(\kappa_{m,pc}A_{pc,\delta} + \kappa_{m,\delta}\right)\right)\sigma_\delta\varepsilon_{\delta,t+1} \\
& + \left((\theta-1)\,\kappa_{rW,pc} - \frac{\theta}{\psi}\kappa_{m,pc}\right)A_{pc,x}\sigma_x\varepsilon_{x,t+1}.
\end{aligned} \tag{A.45}$$

We can identify the market prices of risk by rewriting m_{t+1} as:

$$m_{t+1} = m_0 + m_G G_{W,t} + m_\delta\delta_t + m_x x_t - \lambda_c\varepsilon_{c,t+1} - \lambda_G\varepsilon_{G,t+1} - \lambda_\delta\varepsilon_{\delta,t+1} - \lambda_x\varepsilon_{x,t+1}, \tag{A.46}$$

where

$$\begin{aligned}
m_0 =\, & \theta\log\beta - \gamma\mu_c + (\theta-1)\left(\kappa_{rW,0} - \kappa_{W,0} + A_{pc,0}\left(\kappa_{rW,pc} - 1\right)\right) \\
& + (\theta-1)\,\kappa_{rW,pc}\left(A_{pc,G}\mu_G + A_{pc,\delta}\mu_\delta\right) \\
& - \frac{\theta}{\psi}\left(\left(\kappa_{m,\delta} + \kappa_{m,pc}A_{pc,\delta}\right)\mu_\delta + \left(\kappa_{m,G} + \kappa_{m,pc}A_{pc,G}\right)\mu_G\right),
\end{aligned} \tag{A.47}$$

$$\begin{aligned}
m_G &= (\theta-1)\left(A_{pc,G}\left(\kappa_{rW,pc}\rho_G - 1\right) - \kappa_{W,G}\right) - \frac{\theta}{\psi}\left(\kappa_{m,G} + \kappa_{m,pc}A_{pc,G}\right)\left(\rho_G - 1\right) \\
&= \frac{\frac{\kappa_{m,G}}{\psi}\left(1-\rho_G\right) - \kappa_{W,G}\frac{\kappa_{m,pc}}{\psi}\frac{1-\rho_G}{1-\kappa_{rW,pc}\rho_G}}{1 - \frac{\kappa_{m,pc}}{\psi}\frac{1-\rho_G}{1-\kappa_{rW,pc}\rho_G}},
\end{aligned} \tag{A.48}$$

$$\begin{aligned}
m_\delta =\, & (\theta-1)\left(A_{pc,\delta}\left(\kappa_{rW,pc}\rho_\delta - 1\right) - \kappa_{W,\delta}\right) - \frac{\theta}{\psi}\left(\kappa_{m,\delta} + \kappa_{m,pc}A_{pc,\delta}\right)\left(\rho_\delta - 1\right) \\
& + \left((\theta-1)\,A_{pc,G}\kappa_{rW,pc} - \frac{\theta}{\psi}\left(\kappa_{m,G} + \kappa_{m,pc}A_{pc,G}\right)\right)\rho_{G,\delta}
\end{aligned}$$

$$= \frac{\frac{\kappa_{m,\delta}}{\psi}\left(1-\rho_{\delta}\right) - \kappa_{W,\delta}\frac{\kappa_{m,pc}}{\psi}\frac{1-\rho_{\delta}}{1-\kappa_{rW,pc}\rho_{\delta}}}{1-\frac{\kappa_{m,pc}}{\psi}\frac{1-\rho_{\delta}}{1-\kappa_{rW,pc}\rho_{\delta}}} + \left(\left(\theta-1\right)A_{pc,G}\kappa_{rW,pc} - \frac{\theta}{\psi}\left(\kappa_{m,G}+\kappa_{m,pc}A_{pc,G}\right)\right)\rho_{G,\delta}, \tag{A.49}$$

$$m_x = -\gamma + \left(\left(\theta-1\right)\left(\kappa_{rW,pc}\rho_x - 1\right) - \frac{\theta}{\psi}\kappa_{m,pc}\left(\rho_x-1\right)\right)A_{pc,x}$$

$$= -\frac{1}{\psi}\frac{1-\kappa_{m,pc}\frac{1-\rho_x}{1-\kappa_{rW,pc}\rho_x}}{1-\frac{\kappa_{m,pc}}{\psi}\frac{1-\rho_x}{1-\kappa_{rW,pc}\rho_x}}, \tag{A.50}$$

$$\lambda_c = \gamma\sigma_c, \tag{A.51}$$

$$\lambda_G = \left(\left(1-\theta\right)\kappa_{rW,pc}A_{pc,G} + \frac{\theta}{\psi}\left(\kappa_{m,G}+\kappa_{m,pc}A_{pc,G}\right)\right)\sigma_G, \tag{A.52}$$

$$\lambda_\delta = \left(\left(1-\theta\right)\kappa_{rW,pc}A_{pc,\delta} + \frac{\theta}{\psi}\left(\kappa_{m,\delta}+\kappa_{m,pc}A_{pc,\delta}\right)\right)\sigma_\delta \tag{A.53}$$

$$\lambda_x = \left(\left(1-\theta\right)\kappa_{rW,pc} + \frac{\theta}{\psi}\kappa_{m,pc}\right)A_{pc,x}\sigma_x$$

$$= \left(\frac{\gamma\psi-1}{\psi-1}\kappa_{rW,pc} - \frac{\gamma-1}{\psi-1}\kappa_{m,pc}\right)A_{pc,x}\sigma_x. \tag{A.54}$$

Assuming that $\left|\bar{\delta}\bar{G}_W\right| < 1$, it follows that $m_x < 0$. Assuming that $\psi > 1$, $\bar{\delta} \geq 0$, and $\bar{G}_W \geq 0$ is a sufficient condition for $m_G, m_\delta > 0$, as well as $-1 < m_x < 0$. As for the market prices of risk, it is useful to notice that $\kappa_{rW,pc} > \kappa_{m,pc}$. It turns out that $\lambda_c, \lambda_x > 0$ at all times. The signs of λ_G and λ_δ depend on both positive and negative contributions. For instance, for $\theta < 0$, they are characterized by positive contributions stemming from the impact of shocks to $G_{W,t}$ and δ_t on the return on aggregate wealth, while negative contributions arise from the effect on the ESG factor. However, the concavity of expected utility with respect to $G_{W,t}$ and δ_t implies a diminishing marginal utility of such variables, and thus positive risk premia.

The return on wealth can be formulated as

$$\begin{aligned} r_{W,t+1} = {} & \underbrace{\kappa_{rW,0} + A_{pc,0}\left(\kappa_{rW,pc}-1\right) + A_{pc,G}\kappa_{rW,pc}\mu_G + A_{pc,\delta}\kappa_{rW,pc}\mu_\delta + \mu_c}_{r_{W,0}} \\ & + A_{pc,G}\left(\kappa_{rW,pc}\rho_G - 1\right)G_{W,t} + A_{pc,\delta}\left(\kappa_{rW,pc}\rho_\delta - 1\right)\delta_t \\ & + \left(A_{pc,x}\left(\kappa_{rW,pc}\rho_x - 1\right) + 1\right)x_t \\ & + A_{pc,G}\kappa_{rW,pc}\sigma_G\varepsilon_{G,t+1} + A_{pc,\delta}\kappa_{rW,pc}\sigma_\delta\varepsilon_{\delta,t+1} \\ & + A_{pc,x}\kappa_{rW,pc}\sigma_x\varepsilon_{x,t+1} + \sigma_c\varepsilon_{c,t+1}. \end{aligned} \tag{A.55}$$

As $A_{pc,G}$, $A_{pc,\delta}$, and $A_{pc,x}$ are positive, the return on wealth is positively correlated with the shocks $\varepsilon_{G,t+1}$, $\varepsilon_{\delta,t+1}$, $\varepsilon_{x,t+1}$, and $\varepsilon_{c,t+1}$. The expected excess return of the consumption asset can be expressed as

$$\mathrm{E}_t\left[r_{W,t+1} - r_{f,t+1}\right] = \underbrace{\sigma_c}_{\mathrm{Cov}_t\left[r_{W,t+1},\varepsilon_{c,t+1}\right]}\lambda_c + \underbrace{\kappa_{rW,pc}A_{pc,G}\sigma_G}_{\mathrm{Cov}_t\left[r_{W,t+1},\varepsilon_{G,t+1}\right]}\lambda_G$$

$$
+ \underbrace{\kappa_{rW,pc} A_{pc,\delta} \sigma_\delta}_{\mathrm{Cov}_t\left[r_{W,t+1}, \varepsilon_{\delta,t+1}\right]} \lambda_\delta + \underbrace{\kappa_{rW,pc} A_{pc,x} \sigma_x}_{\mathrm{Cov}_t\left[r_{W,t+1}, \varepsilon_{x,t+1}\right]} \lambda_x - \frac{1}{2}\mathrm{Var}_t\left[r_{W,t+1}\right]
$$

$$
\underbrace{+ \kappa_{W,0} - \frac{\bar{\delta}}{1 - \bar{\delta}\bar{G}_W} G_{W,t} - \frac{\bar{G}_W}{1 - \bar{\delta}\bar{G}_W} \delta_t}_{-y_{W,t}}. \tag{A.56}
$$

To determine the risk-free rate of return, we express the Euler equation as

$$
\mathrm{E}_t\left[e^{m_{t+1} + r_{f,t+1}}\right] = 1. \tag{A.57}
$$

As $r_{f,t+1}$ is known at time t, it follows that $\mathrm{E}_t\left[e^{r_{f,t+1}}\right] = e^{\mathrm{E}_t\left[r_{f,t+1}\right]}$. Thus, the risk-free rate of return is given by

$$
r_{f,t+1} = -\log \mathrm{E}_t\left[e^{m_{t+1}}\right]. \tag{A.58}
$$

Using (A.46):

$$
\begin{aligned}
\mathrm{E}_t\left[e^{m_{t+1}}\right] &= \mathrm{E}_t\left[e^{m_0 + m_G G_{W,t} + m_\delta \delta_t + m_x x_t - \lambda_c \varepsilon_{c,t+1} - \lambda_G \varepsilon_{G,t+1} - \lambda_\delta \varepsilon_{\delta,t+1} - \lambda_x \varepsilon_{x,t+1}}\right] \\
&= e^{m_0 + m_G G_{W,t} + m_\delta \delta_t + m_x x_t + \frac{\lambda_c^2}{2} + \frac{\lambda_G^2}{2} + \frac{\lambda_\delta^2}{2} + \frac{\lambda_x^2}{2}}.
\end{aligned} \tag{A.59}
$$

Then, using (A.38) yields

$$
r_{f,t+1} = \underbrace{-m_0 - \frac{\lambda_c^2}{2} - \frac{\lambda_G^2}{2} - \frac{\lambda_\delta^2}{2} - \frac{\lambda_x^2}{2}}_{r_{f,0}} \underbrace{-m_G}_{r_{f,G}} G_{W,t} \underbrace{-m_\delta}_{r_{f,\delta}} \delta_t \underbrace{-m_x}_{r_{f,x}} x_t. \tag{A.60}
$$

A.3 Proof of Proposition 3 (risky asset return)

For an arbitrary risky asset, the Euler equation reads:

$$
\mathrm{E}_t\left[M_{t+1} R_{n,t+1}\right] = 1 - \delta_t G_{n,t}. \tag{A.61}
$$

Similarly to (A.31), we can write

$$
\log\left(1 - \delta_t G_{n,t}\right) \simeq \kappa_{n,0} + \kappa_{n,Gn} G_{n,t} + \kappa_{n,\delta} \delta_t, \tag{A.62}
$$

where $\kappa_{n,Gn} = -\frac{\bar{\delta}}{1 - \bar{\delta}\bar{G}_n}$, $\kappa_{n,\delta} = -\frac{\bar{G}_n}{1 - \bar{\delta}\bar{G}_n}$, and $\kappa_{n,0} = \log\left(1 - \bar{\delta}\bar{G}_n\right) - \kappa_{n,Gn}\bar{G}_n - \kappa_{n,\delta}\bar{\delta}$.

Similar to the approach followed in Online Appendix A.2 for the return on aggregate wealth, we adopt the log-linearization technique introduced by Campbell and Shiller (1988) to express the logarithmic return on the risky asset as follows:

$$
r_{n,t+1} \simeq \kappa_{rn,0} + \kappa_{rn,pd} pd_{n,t+1} - pd_{n,t} + \Delta d_{n,t+1}, \tag{A.63}
$$

where $\kappa_{rn,pd} = \frac{e^{\overline{pd}_n}}{1 + e^{\overline{pd}_n}}$ and $\kappa_{rn,0} = \log\left(1 + e^{\overline{pd}_n}\right) - \kappa_{rn,pd}\overline{pd}_n$. Consider the following dynamics:

$$
G_{n,t+1} = \mu_{Gn} + \rho_{Gn} G_{n,t} + \rho_{Gn,\delta} \delta_t + \sigma_{Gn,G} \varepsilon_{G,t+1} + \sigma_{Gn} \varepsilon_{Gn,t+1}, \tag{A.64}
$$

$$\Delta d_{n,t+1} = \mu_{dn} + \rho_{dn,x} x_t + \rho_{dn,\delta}\delta_t + \sigma_{dn,c}\varepsilon_{c,t+1} + \sigma_{dn,G}\varepsilon_{G,t+1} + \sigma_{dn,\delta}\varepsilon_{\delta,t+1} + \sigma_{dn,x}\varepsilon_{x,t+1} + \sigma_{dn,Gn}\varepsilon_{Gn,t+1} + \sigma_{dn,dM}\varepsilon_{dM,t+1} + \sigma_{dn}\varepsilon_{dn,t+1}, \tag{A.65}$$

where $\mu_{Gn} = (1-\rho_{Gn})\,\bar{G}_n - \rho_{Gn,\delta}\bar{\bar{\delta}}$. We make the guess:

$$pd_{n,t} = A_{n,0} + A_{n,G}G_{W,t} + A_{n,\delta}\delta_t + A_{n,x}x_t + A_{n,Gn}G_{n,t}. \tag{A.66}$$

We then write the log asset return as:

$$\begin{aligned} r_{n,t+1} \simeq\ & \underbrace{\kappa_{rn,0} + \kappa_{rn,pd}\left(A_{n,0} + A_{n,G}\mu_G + A_{n,\delta}\mu_\delta + A_{n,Gn}\mu_{Gn}\right) - A_{n,0} + \mu_{dn}}_{r_{n,0}} \\ & + \underbrace{\left(\kappa_{rn,pd}\rho_G - 1\right)A_{n,G}}_{r_{n,G}} G_{W,t} \\ & + \underbrace{\left(\left(\kappa_{rn,pd}\rho_\delta - 1\right)A_{n,\delta} + \kappa_{rn,pd}\left(A_{n,G}\rho_{G,\delta} + A_{n,Gn}\rho_{Gn,\delta}\right) + \rho_{dn,\delta}\right)}_{r_{n,\delta}} \delta_t \\ & + \underbrace{\left(\left(\kappa_{rn,pd}\rho_x - 1\right)A_{n,x} + \rho_{dn,x}\right)}_{r_{n,x}} x_t + \underbrace{\left(\kappa_{rn,pd}\rho_{Gn} - 1\right)A_{n,Gn}}_{r_{n,Gn}} G_{n,t} \\ & + \underbrace{\sigma_{dn,c}}_{\sigma_{rn,c}} \varepsilon_{c,t+1} + \underbrace{\left(\kappa_{rn,pd}\left(A_{n,G}\sigma_G + A_{n,Gn}\sigma_{Gn,G}\right) + \sigma_{dn,G}\right)}_{\sigma_{rn,G}} \varepsilon_{G,t+1} \\ & + \underbrace{\left(\kappa_{rn,pd}A_{n,\delta}\sigma_\delta + \sigma_{dn,\delta}\right)}_{\sigma_{rn,\delta}} \varepsilon_{\delta,t+1} \\ & + \underbrace{\left(\kappa_{rn,pd}A_{n,x}\sigma_x + \sigma_{dn,x}\right)}_{\sigma_{rn,x}} \varepsilon_{x,t+1} + \underbrace{\left(\kappa_{rn,pd}A_{n,Gn}\sigma_{Gn} + \sigma_{dn,Gn}\right)}_{\sigma_{rn,Gn}} \varepsilon_{Gn,t+1} \\ & + \underbrace{\sigma_{dn,dM}}_{\sigma_{rn,dM}} \varepsilon_{dM,t+1} + \underbrace{\sigma_{dn}}_{\sigma_{rn,dn}} \varepsilon_{dn,t+1}. \end{aligned} \tag{A.67}$$

We apply the Euler condition

$$\mathrm{E}_t\left[e^{m_{t+1}+r_{n,t+1}}\right] = e^{\kappa_{n,0}+\kappa_{n,Gn}G_{n,t}+\kappa_{n,\delta}\delta_t}, \tag{A.68}$$

where

$$\begin{aligned} m_{t+1} + r_{n,t+1} \simeq\ & m_0 + \kappa_{rn,0} + \left(\kappa_{rn,pd} - 1\right)A_{n,0} + \kappa_{rn,pd}\left(A_{n,G}\mu_G + A_{n,\delta}\mu_\delta + A_{n,Gn}\mu_{Gn}\right) + \mu_{dn} \\ & + \left(m_G + \left(\kappa_{rn,pd}\rho_G - 1\right)A_{n,G}\right)G_{W,t} \\ & + \left(m_\delta + \left(\kappa_{rn,pd}\rho_\delta - 1\right)A_{n,\delta} + \kappa_{rn,pd}\left(A_{n,G}\rho_{G,\delta} + A_{n,Gn}\rho_{Gn,\delta}\right) + \rho_{dn,\delta}\right)\delta_t \\ & + \left(m_x + \left(\kappa_{rn,pd}\rho_x - 1\right)A_{n,x} + \rho_{dn,x}\right)x_t \\ & + \left(\kappa_{rn,pd}\rho_{Gn} - 1\right)A_{n,Gn}G_{n,t} \\ & + \left(-\lambda_c + \sigma_{dn,c}\right)\varepsilon_{c,t+1} \\ & + \left(-\lambda_G + \kappa_{rn,pd}\left(A_{n,G}\sigma_G + A_{n,Gn}\sigma_{Gn,G}\right) + \sigma_{dn,G}\right)\varepsilon_{G,t+1} \\ & + \left(-\lambda_\delta + \kappa_{rn,pd}A_{n,\delta}\sigma_\delta + \sigma_{dn,\delta}\right)\varepsilon_{\delta,t+1} \\ & + \left(-\lambda_x + \kappa_{rn,pd}A_{n,x}\sigma_x + \sigma_{dn,x}\right)\varepsilon_{x,t+1} \end{aligned}$$

$$+\left(\kappa_{rn,pd}A_{n,Gn}\sigma_{Gn}+\sigma_{dn,Gn}\right)\varepsilon_{Gn,t+1}$$
$$+\sigma_{dn,dM}\varepsilon_{dM,t+1}+\sigma_{dn}\varepsilon_{dn,t+1}. \tag{A.69}$$

Therefore

$$\begin{aligned}0 = {} & m_0+\kappa_{rn,0}+\left(\kappa_{rn,pd}-1\right)A_{n,0}+\kappa_{rn,pd}\left(A_{n,G}\mu_G+A_{n,\delta}\mu_\delta+A_{n,Gn}\mu_{Gn}\right)+\mu_{dn}\\ & -\kappa_{n,0}+\frac{\left(-\lambda_c+\sigma_{dn,c}\right)^2}{2}+\frac{\left(-\lambda_G+\kappa_{rn,pd}\left(A_{n,G}\sigma_G+A_{n,Gn}\sigma_{Gn,G}\right)+\sigma_{dn,G}\right)^2}{2}\\ & +\frac{\left(-\lambda_\delta+\kappa_{rn,pd}A_{n,\delta}\sigma_\delta+\sigma_{dn,\delta}\right)^2}{2}+\frac{\left(-\lambda_x+\kappa_{rn,pd}A_{n,x}\sigma_x+\sigma_{dn,x}\right)^2}{2}\\ & +\frac{\left(\kappa_{rn,pd}A_{n,Gn}\sigma_{Gn}+\sigma_{dn,Gn}\right)^2}{2}+\frac{\sigma_{dn,dM}^2}{2}+\frac{\sigma_{dn}^2}{2}\\ & +\left(m_G+\left(\kappa_{rn,pd}\rho_G-1\right)A_{n,G}\right)G_{W,t}\\ & +\left(m_\delta+\left(\kappa_{rn,pd}\rho_\delta-1\right)A_{n,\delta}+\kappa_{rn,pd}\left(A_{n,G}\rho_{G,\delta}+A_{n,Gn}\rho_{Gn,\delta}\right)+\rho_{dn,\delta}-\kappa_{n,\delta}\right)\delta_t\\ & +\left(m_x+\left(\kappa_{rn,pd}\rho_x-1\right)A_{n,x}+\rho_{dn,x}\right)x_t\\ & +\left(\left(\kappa_{rn,pd}\rho_{Gn}-1\right)A_{n,Gn}-\kappa_{n,Gn}\right)G_{n,t}.\end{aligned} \tag{A.70}$$

Finally, the coefficients in (A.66) are

$$A_{n,0}=\frac{1}{1-\kappa_{rn,pd}}\left(\begin{array}{l} m_0+\kappa_{rn,0}+\kappa_{rn,pd}\left(A_{n,G}\mu_G+A_{n,\delta}\mu_\delta+A_{n,Gn}\mu_{Gn}\right)\\ +\mu_{dn}-\kappa_{n,0}\\ +\frac{\left(-\lambda_c+\sigma_{dn,c}\right)^2}{2}+\frac{\left(-\lambda_G+\kappa_{rn,pd}\left(A_{n,G}\sigma_G+A_{n,Gn}\sigma_{Gn,G}\right)+\sigma_{dn,G}\right)^2}{2}\\ +\frac{\left(-\lambda_\delta+\kappa_{rn,pd}A_{n,\delta}\sigma_\delta+\sigma_{dn,\delta}\right)^2}{2}+\frac{\left(-\lambda_x+\kappa_{rn,pd}A_{n,x}\sigma_x+\sigma_{dn,x}\right)^2}{2}\\ +\frac{\left(\kappa_{rn,pd}A_{n,Gn}\sigma_{Gn}+\sigma_{dn,Gn}\right)^2}{2}+\frac{\sigma_{dn,dM}^2}{2}+\frac{\sigma_{dn}^2}{2}\end{array}\right), \tag{A.71}$$

$$A_{n,G}=\frac{m_G}{1-\kappa_{rn,pd}\rho_G}, \tag{A.72}$$

$$A_{n,\delta}=\frac{m_\delta+\kappa_{rn,pd}\left(A_{n,G}\rho_{G,\delta}+A_{n,Gn}\rho_{Gn,\delta}\right)+\rho_{dn,\delta}-\kappa_{n,\delta}}{1-\kappa_{rn,pd}\rho_\delta}, \tag{A.73}$$

$$A_{n,x}=\frac{m_x+\rho_{dn,x}}{1-\kappa_{rn,pd}\rho_x}, \tag{A.74}$$

$$A_{n,Gn}=\frac{-\kappa_{n,Gn}}{1-\kappa_{rn,pd}\rho_{Gn}}. \tag{A.75}$$

Note that $A_{n,Gn}>0$ and that the return coefficient on $G_{n,t}$ is $r_{n,Gn}=\kappa_{n,Gn}<0$. Furthermore, $A_{n,G}>0$, and thus $r_{n,G}<0$, when $\psi>1$. Finally, $A_{n,\delta}$ is positive when $\bar{G}_n > -\frac{m_\delta+\kappa_{rn,pd}\left(A_{n,G}\rho_{G,\delta}+A_{n,Gn}\rho_{Gn,\delta}\right)+\rho_{dn,\delta}}{1-\left(m_\delta+\kappa_{rn,pd}\left(A_{n,G}\rho_{G,\delta}+A_{n,Gn}\rho_{Gn,\delta}\right)+\rho_{dn,\delta}\right)\bar{\delta}}$. We can also rewrite the return on an asset as follows

$$
r_{n,t+1} \simeq \underbrace{\begin{pmatrix} -m_0 + \kappa_{n,0} \\ -\frac{\left(-\lambda_c+\sigma_{dn,c}\right)^2}{2} - \frac{\left(-\lambda_G+\kappa_{rn,pd}\left(A_{n,G}\sigma_G+A_{n,Gn}\sigma_{Gn,G}\right)+\sigma_{dn,G}\right)^2}{2} \\ -\frac{\left(-\lambda_\delta+\kappa_{rn,pd}A_{n,\delta}\sigma_\delta+\sigma_{dn,\delta}\right)^2}{2} - \frac{\left(-\lambda_x+\kappa_{rn,pd}A_{n,x}\sigma_x+\sigma_{dn,x}\right)^2}{2} \\ -\frac{\left(\kappa_{rn,pd}A_{n,Gn}\sigma_{Gn}+\sigma_{dn,Gn}\right)^2}{2} - \frac{\sigma_{dn,dM}^2}{2} - \frac{\sigma_{dn}^2}{2} \end{pmatrix}}_{r_{n,0}}
$$

$$
\underbrace{-m_G}_{r_{n,G}} G_{W,t} + \underbrace{\left(\kappa_{n,\delta} - m_\delta\right)}_{r_{n,\delta}} \delta_t \underbrace{-m_x}_{r_{n,x}} x_t + \underbrace{\kappa_{n,Gn}}_{r_{n,Gn}} G_{n,t}
$$

$$
+ \underbrace{\sigma_{dn,c}}_{\sigma_{rn,c}} \varepsilon_{c,t+1} + \underbrace{\left(\kappa_{rn,pd}A_{n,G}\sigma_G + \kappa_{rn,pd}A_{n,Gn}\sigma_{Gn,G} + \sigma_{dn,G}\right)}_{\sigma_{rn,G}} \varepsilon_{G,t+1}
$$

$$
+ \underbrace{\left(\kappa_{rn,pd}A_{n,\delta}\sigma_\delta + \sigma_{dn,\delta}\right)}_{\sigma_{rn,\delta}} \varepsilon_{\delta,t+1} + \underbrace{\left(\kappa_{rn,pd}A_{n,x}\sigma_x + \sigma_{dn,x}\right)}_{\sigma_{rn,x}} \varepsilon_{x,t+1}
$$

$$
+ \underbrace{\left(\kappa_{rn,pd}A_{n,Gn}\sigma_{Gn} + \sigma_{dn,Gn}\right)}_{\sigma_{rn,Gn}} \varepsilon_{Gn,t+1} + \underbrace{\sigma_{dn,dM}}_{\sigma_{rn,dM}} \varepsilon_{dM,t+1} + \underbrace{\sigma_{dn}}_{\sigma_{rn,dn}} \varepsilon_{dn,t+1}. \tag{A.76}
$$

Recalling (12), the excess return $\hat{r}_{n,t+1} = r_{n,t+1} - r_{f,t+1}$ can be expressed as

$$
\hat{r}_{n,t+1} \simeq \underbrace{\begin{pmatrix} \kappa_{n,0} - \frac{\left(-\lambda_c+\sigma_{dn,c}\right)^2}{2} - \frac{\left(-\lambda_G+\kappa_{rn,pd}\left(A_{n,G}\sigma_G+A_{n,Gn}\sigma_{Gn,G}\right)+\sigma_{dn,G}\right)^2}{2} \\ -\frac{\left(-\lambda_\delta+\kappa_{rn,pd}A_{n,\delta}\sigma_\delta+\sigma_{dn,\delta}\right)^2}{2} - \frac{\left(-\lambda_x+\kappa_{rn,pd}A_{n,x}\sigma_x+\sigma_{dn,x}\right)^2}{2} \\ -\frac{\left(\kappa_{rn,pd}A_{n,Gn}\sigma_{Gn}+\sigma_{dn,Gn}\right)^2}{2} - \frac{\sigma_{dn,dM}^2}{2} - \frac{\sigma_{dn}^2}{2} + \frac{\lambda_c^2}{2} + \frac{\lambda_G^2}{2} + \frac{\lambda_\delta^2}{2} + \frac{\lambda_x^2}{2} \end{pmatrix}}_{\hat{r}_{n,0}}
$$

$$
+ \underbrace{\kappa_{n,\delta}}_{\hat{r}_{n,\delta}} \delta_t + \underbrace{\kappa_{n,Gn}}_{\hat{r}_{n,Gn}} G_{n,t}
$$

$$
+ \underbrace{\sigma_{dn,c}}_{\sigma_{rn,c}} \varepsilon_{c,t+1} + \underbrace{\left(\kappa_{rn,pd}A_{n,G}\sigma_G + \kappa_{rn,pd}A_{n,Gn}\sigma_{Gn,G} + \sigma_{dn,G}\right)}_{\sigma_{rn,G}} \varepsilon_{G,t+1}
$$

$$
+ \underbrace{\left(\kappa_{rn,pd}A_{n,\delta}\sigma_\delta + \sigma_{dn,\delta}\right)}_{\sigma_{rn,\delta}} \varepsilon_{\delta,t+1} + \underbrace{\left(\kappa_{rn,pd}A_{n,x}\sigma_x + \sigma_{dn,x}\right)}_{\sigma_{rn,x}} \varepsilon_{x,t+1}
$$

$$
+ \underbrace{\left(\kappa_{rn,pd}A_{n,Gn}\sigma_{Gn} + \sigma_{dn,Gn}\right)}_{\sigma_{rn,Gn}} \varepsilon_{Gn,t+1} + \underbrace{\sigma_{dn,dM}}_{\sigma_{rn,dM}} \varepsilon_{dM,t+1} + \underbrace{\sigma_{dn}}_{\sigma_{rn,dn}} \varepsilon_{dn,t+1}. \tag{A.77}
$$

The first two lines represent the conditional expected excess return, $\mathrm{E}_t\left[\hat{r}_{n,t+1}\right]$, which can be further developed as

$$
\begin{aligned}
\mathrm{E}_t\left[\hat{r}_{n,t+1}\right] = {} & \sigma_{dn,c}\lambda_c + \left(\kappa_{rn,pd}A_{n,G}\sigma_G + \kappa_{rn,pd}A_{n,Gn}\sigma_{Gn,G} + \sigma_{dn,G}\right)\lambda_G \\
& + \left(\kappa_{rn,pd}A_{n,\delta}\sigma_\delta + \sigma_{dn,\delta}\right)\lambda_\delta + \kappa_{rn,pd}A_{n,x}\sigma_x\lambda_x - \frac{1}{2}\mathrm{Var}_t\left[\hat{r}_{n,t+1}\right] \\
& + \underbrace{\log\left(1 - \bar{\delta}\bar{G}_n\right) - \frac{\bar{G}_n\left(\delta_t - \bar{\delta}\right)}{1 - \bar{\delta}\bar{G}_n} - \frac{\bar{\delta}\left(G_{n,t} - \bar{G}_n\right)}{1 - \bar{\delta}\bar{G}_n}}_{-y_{n,t}},
\end{aligned} \tag{A.78}
$$

where

$$\begin{aligned}\mathrm{Var}_t\left[\hat{r}_{n,t+1}\right] = &\sigma^2_{dn,c} + \left(\kappa_{rn,pd}\left(A_{n,G}\sigma_G + A_{n,Gn}\sigma_{Gn,G}\right) + \sigma_{dn,G}\right)^2 \\ &+ \left(\kappa_{rn,pd}A_{n,\delta}\sigma_\delta + \sigma_{dn,\delta}\right)^2 + \left(\kappa_{rn,pd}A_{n,x}\sigma_x + \sigma_{dn,x}\right)^2 \\ &+ \left(\kappa_{rn,pd}A_{n,Gn}\sigma_{Gn} + \sigma_{dn,Gn}\right)^2 + \sigma^2_{dn,dM} + \sigma^2_{dn}.\end{aligned} \tag{A.79}$$

Proposition 3 is obtained imposing $\rho_{dn,\delta} = \sigma_{dn,G} = \sigma_{dn,\delta} = \sigma_{dn,x} = \sigma_{dn,Gn} = 0$.

A.4 Proof of Proposition 3 (market return)

For the market portfolio, we assume $G_{M,t} = G_{W,t}$. Then, we can express the Euler condition (A.12) as

$$\mathrm{E}_t\left[M_{t+1}R_{M,t+1}\right] = 1 - \delta_t G_{W,t}. \tag{A.80}$$

Recalling (A.31), we can write $\log\left(1 - \delta_t G_{W,t}\right) \simeq \kappa_{W,0} + \kappa_{W,G}G_{W,t} + \kappa_{W,\delta}\delta_t$.

We use the following log-linearization for the return of the market portfolio:

$$r_{M,t+1} \simeq \kappa_{rM,0} + \kappa_{rM,pd}pd_{M,t+1} - pd_{M,t} + \Delta d_{M,t+1}, \tag{A.81}$$

where $\kappa_{rM,pd} = \frac{e^{\overline{pd}_M}}{1+e^{\overline{pd}_M}}$ and $\kappa_{rM,0} = \log\left(1 + e^{\overline{pd}_M}\right) - \kappa_{rM,pd}\overline{pd}_M$. Consider the following dynamics:

$$\begin{aligned}\Delta d_{M,t+1} = &\mu_{dM} + \rho_{dM,x}x_t + \rho_{dM,\delta}\delta_t \\ &+ \sigma_{dM,c}\varepsilon_{c,t+1} + \sigma_{dM,G}\varepsilon_{G,t+1} + \sigma_{dM,\delta}\varepsilon_{\delta,t+1} + \sigma_{dM,x}\varepsilon_{x,t+1} + \sigma_{dM}\varepsilon_{dM,t+1}.\end{aligned} \tag{A.82}$$

We make the guess:

$$pd_{M,t} = A_{M,0} + A_{M,G}G_{W,t} + A_{M,\delta}\delta_t + A_{M,x}x_t. \tag{A.83}$$

We then write the log market return as:

$$\begin{aligned}r_{M,t+1} \simeq &\underbrace{\kappa_{rM,0} + \kappa_{rM,pd}\left(A_{M,0} + A_{M,G}\mu_G + A_{M,\delta}\mu_\delta\right) - A_{M,0} + \mu_{dM}}_{r_{M,0}} \\ &+ \underbrace{\left(\kappa_{rM,pd}\rho_G - 1\right)A_{M,G}}_{r_{M,G}}G_{W,t} + \underbrace{\left(\left(\kappa_{rM,pd}\rho_\delta - 1\right)A_{M,\delta} + \rho_{dM,\delta} + \kappa_{rM,pd}A_{M,G}\rho_{G,\delta}\right)}_{r_{M,\delta}}\delta_t \\ &+ \underbrace{\left(\left(\kappa_{rM,pd}\rho_x - 1\right)A_{M,x} + \rho_{dM,x}\right)}_{r_{M,x}}x_t \\ &+ \underbrace{\sigma_{dM,c}}_{\sigma_{rM,c}}\varepsilon_{c,t+1} + \underbrace{\left(\kappa_{rM,pd}A_{M,G}\sigma_G + \sigma_{dM,G}\right)}_{\sigma_{rM,G}}\varepsilon_{G,t+1} \\ &+ \underbrace{\left(\kappa_{rM,pd}A_{M,\delta}\sigma_\delta + \sigma_{dM,\delta}\right)}_{\sigma_{rM,\delta}}\varepsilon_{\delta,t+1} \\ &+ \underbrace{\left(\kappa_{rM,pd}A_{M,x}\sigma_x + \sigma_{dM,x}\right)}_{\sigma_{rM,x}}\varepsilon_{x,t+1} + \underbrace{\sigma_{dM}}_{\sigma_{rM,dM}}\varepsilon_{dM,t+1}.\end{aligned} \tag{A.84}$$

We apply the Euler condition

$$\mathrm{E}_t\left[e^{m_{t+1}+r_{M,t+1}}\right] \simeq e^{\kappa_{W,0}+\kappa_{W,G}G_{W,t}+\kappa_{W,\delta}\delta_t}, \tag{A.85}$$

where

$$\begin{aligned} m_{t+1}+r_{M,t+1} \simeq\, & \kappa_{rM,0}+\kappa_{rM,pd}\left(A_{M,0}+A_{M,G}\mu_G+A_{M,\delta}\mu_\delta\right)-A_{M,0}+\mu_{dM}+m_0 \\ & +\left(\left(\kappa_{rM,pd}\rho_G-1\right)A_{M,G}+m_G\right)G_{W,t} \\ & +\left(\left(\kappa_{rM,pd}\rho_\delta-1\right)A_{M,\delta}+\rho_{dM,\delta}+\kappa_{rM,pd}A_{M,G}\rho_{G,\delta}+m_\delta\right)\delta_t \\ & +\left(\left(\kappa_{rM,pd}\rho_x-1\right)A_{M,x}+\rho_{dM,x}+m_x\right)x_t \\ & +\left(\sigma_{dM,c}-\lambda_c\right)\varepsilon_{c,t+1}+\left(\kappa_{rM,pd}A_{M,G}\sigma_G+\sigma_{dM,G}-\lambda_G\right)\varepsilon_{G,t+1} \\ & +\left(\kappa_{rM,pd}A_{M,\delta}\sigma_\delta+\sigma_{dM,\delta}-\lambda_\delta\right)\varepsilon_{\delta,t+1} \\ & +\left(\kappa_{rM,pd}A_{M,x}\sigma_x+\sigma_{dM,x}-\lambda_x\right)\varepsilon_{x,t+1}+\sigma_{dM}\varepsilon_{dM,t+1}. \end{aligned} \tag{A.86}$$

Therefore,

$$\begin{aligned} 0 \simeq\, & \kappa_{rM,0}+\kappa_{rM,pd}\left(A_{M,G}\mu_G+A_{M,\delta}\mu_\delta\right)+\left(\kappa_{rM,pd}-1\right)A_{M,0}+\mu_{dM}+m_0 \\ & -\kappa_{W,0}+\frac{\left(\sigma_{dM,c}-\lambda_c\right)^2}{2}+\frac{\left(\kappa_{rM,pd}A_{M,G}\sigma_G+\sigma_{dM,G}-\lambda_G\right)^2}{2} \\ & +\frac{\left(\kappa_{rM,pd}A_{M,\delta}\sigma_\delta+\sigma_{dM,\delta}-\lambda_\delta\right)^2}{2} \\ & +\frac{\left(\kappa_{rM,pd}A_{M,x}\sigma_x+\sigma_{dM,x}-\lambda_x\right)^2}{2}+\frac{\sigma_{dM}^2}{2} \\ & +\left(\left(\kappa_{rM,pd}\rho_G-1\right)A_{M,G}+m_G-\kappa_{W,G}\right)G_{W,t} \\ & +\left(\left(\kappa_{rM,pd}\rho_\delta-1\right)A_{M,\delta}+\rho_{dM,\delta}+\kappa_{rM,pd}A_{M,G}\rho_{G,\delta}+m_\delta-\kappa_{W,\delta}\right)\delta_t \\ & +\left(\left(\kappa_{rM,pd}\rho_x-1\right)A_{M,x}+\rho_{dM,x}+m_x\right)x_t. \end{aligned} \tag{A.87}$$

Finally, the coefficients in (A.83) are

$$A_{M,0}=\frac{1}{1-\kappa_{rM,pd}}\left(\begin{array}{c} \kappa_{rM,0}+\kappa_{rM,pd}\left(A_{M,G}\mu_G+A_{M,\delta}\mu_\delta\right)+\mu_{dM}+m_0 \\ -\kappa_{W,0}+\frac{\sigma_{dM}^2}{2}+\frac{\left(\sigma_{dM,c}-\lambda_c\right)^2}{2}+\frac{\left(\kappa_{rM,pd}A_{M,G}\sigma_G+\sigma_{dM,G}-\lambda_G\right)^2}{2} \\ +\frac{\left(\kappa_{rM,pd}A_{M,\delta}\sigma_\delta+\sigma_{dM,\delta}-\lambda_\delta\right)^2}{2}+\frac{\left(\kappa_{rM,pd}A_{M,x}\sigma_x+\sigma_{dM,x}-\lambda_x\right)^2}{2} \end{array}\right), \tag{A.88}$$

$$A_{M,G}=\frac{m_G-\kappa_{W,G}}{1-\kappa_{rM,pd}\rho_G}, \tag{A.89}$$

$$A_{M,\delta}=\frac{m_\delta+\rho_{dM,\delta}-\kappa_{W,\delta}+\kappa_{rM,pd}A_{M,G}\rho_{G,\delta}}{1-\kappa_{rM,pd}\rho_\delta}, \tag{A.90}$$

$$A_{M,x}=\frac{m_x+\rho_{dM,x}}{1-\kappa_{rM,pd}\rho_x}. \tag{A.91}$$

The return can be then rewritten as

$$
\begin{aligned}
r_{M,t+1} \simeq & \underbrace{\begin{pmatrix} -m_0 + \kappa_{W,0} - \frac{\left(\sigma_{dM,c}-\lambda_c\right)^2}{2} - \frac{\left(\kappa_{rM,pd}A_{M,G}\sigma_G+\sigma_{dM,G}-\lambda_G\right)^2}{2} \\ -\frac{\left(\kappa_{rM,pd}A_{M,\delta}\sigma_\delta+\sigma_{dM,\delta}-\lambda_\delta\right)^2}{2} - \frac{\left(\kappa_{rM,pd}A_{M,x}\sigma_x+\sigma_{dM,x}-\lambda_x\right)^2}{2} - \frac{\sigma_{dM}^2}{2} \end{pmatrix}}_{r_{M,0}} \\
& + \underbrace{\left(\kappa_{W,G} - m_G\right)}_{r_{M,G}} G_{W,t} + \underbrace{\left(\kappa_{W,\delta} - m_\delta\right)}_{r_{M,\delta}} \delta_t \underbrace{-m_x}_{r_{M,x}} x_t \\
& + \underbrace{\sigma_{dM,c}}_{\sigma_{rM,c}} \varepsilon_{c,t+1} + \underbrace{\left(\kappa_{rM,pd}A_{M,G}\sigma_G + \sigma_{dM,G}\right)}_{\sigma_{rM,G}} \varepsilon_{G,t+1} \\
& + \underbrace{\left(\kappa_{rM,pd}A_{M,\delta}\sigma_\delta + \sigma_{dM,\delta}\right)}_{\sigma_{rM,\delta}} \varepsilon_{\delta,t+1} \\
& + \underbrace{\left(\kappa_{rM,pd}A_{M,x}\sigma_x + \sigma_{dM,x}\right)}_{\sigma_{rM,x}} \varepsilon_{x,t+1} + \underbrace{\sigma_{dM}}_{\sigma_{rM,dM}} \varepsilon_{dM,t+1}.
\end{aligned} \tag{A.92}
$$

Recalling (12), the excess return $\hat{r}_{M,t+1} = r_{M,t+1} - r_{f,t+1}$ is thus

$$
\begin{aligned}
\hat{r}_{M,t+1} \simeq & \underbrace{\begin{pmatrix} \kappa_{W,0} - \frac{\left(\sigma_{dM,c}-\lambda_c\right)^2}{2} - \frac{\left(\kappa_{rM,pd}A_{M,G}\sigma_G+\sigma_{dM,G}-\lambda_G\right)^2}{2} \\ -\frac{\left(\kappa_{rM,pd}A_{M,\delta}\sigma_\delta+\sigma_{dM,\delta}-\lambda_\delta\right)^2}{2} - \frac{\left(\kappa_{rM,pd}A_{M,x}\sigma_x+\sigma_{dM,x}-\lambda_x\right)^2}{2} \\ -\frac{\sigma_{dM}^2}{2} + \frac{\lambda_c^2}{2} + \frac{\lambda_G^2}{2} + \frac{\lambda_\delta^2}{2} + \frac{\lambda_x^2}{2} \end{pmatrix}}_{\hat{r}_{M,0}} \\
& + \underbrace{\kappa_{W,G}}_{\hat{r}_{M,G}} G_{W,t} + \underbrace{\kappa_{W,\delta}}_{\hat{r}_{M,\delta}} \delta_t \\
& + \underbrace{\sigma_{dM,c}}_{\sigma_{rM,c}} \varepsilon_{c,t+1} + \underbrace{\left(\kappa_{rM,pd}A_{M,G}\sigma_G + \sigma_{dM,G}\right)}_{\sigma_{rM,G}} \varepsilon_{G,t+1} \\
& + \underbrace{\left(\kappa_{rM,pd}A_{M,\delta}\sigma_\delta + \sigma_{dM,\delta}\right)}_{\sigma_{rM,\delta}} \varepsilon_{\delta,t+1} \\
& + \underbrace{\left(\kappa_{rM,pd}A_{M,x}\sigma_x + \sigma_{dM,x}\right)}_{\sigma_{rM,x}} \varepsilon_{x,t+1} + \underbrace{\sigma_{dM}}_{\sigma_{rM,dM}} \varepsilon_{dM,t+1}.
\end{aligned} \tag{A.93}
$$

$\psi > 1$ is a sufficient condition for $A_{M,G} > 0$. When $\rho_{G,\delta} = 0$, if in addition $\bar{G}_W > -\frac{m_\delta+\rho_{dM,\delta}}{1-\bar{\delta}\left(m_\delta+\rho_{dM,\delta}\right)}$, then $A_{M,\delta} > 0$ ($\bar{G}_W > 0$ is a sufficient condition for the positivity of $A_{M,\delta}$). In this case, expected returns are negatively correlated with G_t and δ_t, as $r_{M,G}, r_{M,\delta} < 0$. The market portfolio in Proposition 3 is characterized imposing $\rho_{G,\delta} = \rho_{dM,\delta} = \sigma_{dM,G} = \sigma_{dM,\delta} = \sigma_{dM,x} = 0$.

B Summary statistics

Table A.1 tabulates the summary statistics of the value-weighted portfolios sorted by prior ESG and environmental scores, constructed following the methodology outlined in Section 3. Panel (a) refers to portfolios constructed using the entire universe, while Panel (b) is based on the universe excluding stocks in the technology sector.

Next, we analyze the weighted-average ESG profile by industry of the stocks in our sample. To do so, we adopt the Fama-French five-industry definitions and identify stocks in our sample based on their SIC codes. The time series of capitalization-weighted ESG and environmental-pillar scores for each of the industry sectors are shown in Figure A.1. As displayed in Panel (a), no specific industry sector exhibits an ESG score that is consistently above or below the others. Furthermore, the industry-level ESG scores are always comprised between -0.1 and 0.2. These observations imply that, according to the ESG scores used for the analysis, no industry sector is specifically green or brown.

We also assess the prominence of each industry sector in the market portfolio, as well as in the ESG and environmental-pillar sorted portfolios. The left graphs in Figure A.2 illustrate, for each portfolio, the time series of the ratios between the total capitalization of each industry sector and the overall market capitalization. Similarly, the right graphs display the time series of the ratios between the number of stocks within a specific sector and the total number of stocks in the universe.

C Estimation methodology

To perform the estimation, we use the Kalman filter (Hamilton, 1994) to write a likelihood function that is then numerically maximized relative to the parameter space. We first develop the state space representation, jointly considering the equations representing the dynamics of consumption growth in (6), aggregate ESG supply and demand in (8) and (9), long-run risk in (7), the grenness of portfolio j ($j = \{br, neu, gr\}$) in (13), market excess return in (A.93), and individual portfolio excess returns in (A.77):

$$\Delta c_{t+1} = \mu_c + x_t + \sigma_c \varepsilon_{c,t+1}, \tag{C.1}$$

$$G_{W,t+1} = \mu_G + \rho_G G_{W,t} + \rho_{G,\delta}\delta_t + \sigma_G \varepsilon_{G,t+1}, \tag{C.2}$$

$$\delta_{t+1} = \mu_\delta + \rho_\delta \delta_t + \sigma_\delta \varepsilon_{\delta,t+1}, \tag{C.3}$$

$$x_{t+1} = \rho_x x_t + \sigma_x \varepsilon_{x,t+1}, \tag{C.4}$$

$$G_{j,t+1} = \mu_{Gj} + \rho_{Gj,\delta}\delta_t + \rho_{Gj} G_{j,t} + \sigma_{Gj,G}\varepsilon_{G,t+1} + \sigma_{Gj}\varepsilon_{Gj,t+1}, \tag{C.5}$$

$$\begin{aligned} \hat{r}_{M,t+1} \simeq\ & \hat{r}_{M,0} + \hat{r}_{M,G} G_{W,t} + \hat{r}_{M,\delta}\delta_t + \hat{r}_{M,x} x_t \\ & + \sigma_{rM,c}\varepsilon_{c,t+1} + \sigma_{rM,G}\varepsilon_{G,t+1} + \sigma_{rM,\delta}\varepsilon_{\delta,t+1} \\ & + \sigma_{rM,x}\varepsilon_{x,t+1} + \sigma_{rM,dM}\varepsilon_{dM,t+1}, \end{aligned} \tag{C.6}$$

$$\begin{aligned} \hat{r}_{j,t+1} \simeq\ & \hat{r}_{j,0} + \hat{r}_{j,G} G_{W,t} + \hat{r}_{j,\delta}\delta_t + \hat{r}_{j,x} x_t + \hat{r}_{j,Gj} G_{j,t} \\ & + \sigma_{rj,c}\varepsilon_{c,t+1} + \sigma_{rj,G}\varepsilon_{G,t+1} + \sigma_{rj,\delta}\varepsilon_{\delta,t+1} + \sigma_{rj,x}\varepsilon_{x,t+1} \\ & + \sigma_{rj,Gj}\varepsilon_{Gj,t+1} + \sigma_{rj,dM}\varepsilon_{dM,t+1} + \sigma_{rj,dj}\varepsilon_{dj,t+1}. \end{aligned} \tag{C.7}$$

Note that the right-hand side depends on the current value of the state variables $G_{W,t}$, δ_t, x_t, and $G_{j,t}$, as well as on the innovations $\varepsilon_{c,t+1}$, $\varepsilon_{G,t+1}$, $\varepsilon_{\delta,t+1}$, $\varepsilon_{x,t+1}$, $\varepsilon_{Gj,t+1}$, $\varepsilon_{dM,t+1}$, and $\varepsilon_{dj,t+1}$. The equations can be stacked through a VAR representation:

$$\boldsymbol{X}_{t+1} = \boldsymbol{A_X} + \boldsymbol{B_X}\boldsymbol{X}_t + \boldsymbol{\sigma_X}\boldsymbol{\varepsilon}_{t+1}, \tag{C.8}$$

where:

$$\boldsymbol{X}_t = \begin{bmatrix} \Delta c_t \\ G_{W,t} \\ \delta_t \\ x_t \\ \vdots \\ G_{j,t} \\ \vdots \\ \hat{r}_{M,t+1} \\ \vdots \\ \hat{r}_{j,t+1} \\ \vdots \end{bmatrix}, \qquad \boldsymbol{A}_{\boldsymbol{X}} = \begin{bmatrix} \mu_c \\ \mu_G \\ \mu_\delta \\ 0 \\ \vdots \\ \mu_{Gj} \\ \vdots \\ \hat{r}_{M,0} \\ \vdots \\ \hat{r}_{j,0} \\ \vdots \end{bmatrix}, \qquad \boldsymbol{\varepsilon}_{t+1} = \begin{bmatrix} \varepsilon_{c,t+1} \\ \varepsilon_{G,t+1} \\ \varepsilon_{\delta,t+1} \\ \varepsilon_{x,t+1} \\ \vdots \\ \varepsilon_{Gj,t+1} \\ \vdots \\ \varepsilon_{dM,t+1} \\ \vdots \\ \varepsilon_{dj,t+1} \\ \vdots \end{bmatrix}, \tag{C.9}$$

$$\boldsymbol{B}_{\boldsymbol{X}} = \begin{bmatrix} 0 & 0 & 0 & 1 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots \\ 0 & \rho_G & \rho_{G,\delta} & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots \\ 0 & 0 & \rho_\delta & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots \\ 0 & 0 & 0 & \rho_x & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 & 0 & 0 & 0 & \cdots & 0 & \cdots \\ 0 & 0 & \rho_{Gj,\delta} & 0 & 0 & \rho_{Gj} & 0 & 0 & 0 & \cdots & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & 0 & 0 & \ddots & 0 & 0 & \cdots & 0 & \cdots \\ 0 & \hat{r}_{M,G} & \hat{r}_{M,\delta} & \hat{r}_{M,x} & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 & 0 & 0 & 0 & \cdots & 0 & \cdots \\ 0 & \hat{r}_{j,G} & \hat{r}_{j,\delta} & \hat{r}_{j,x} & 0 & \hat{r}_{j,Gj} & 0 & 0 & 0 & \cdots & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & 0 & 0 & \ddots & 0 & 0 & \cdots & 0 & \cdots \end{bmatrix}, \tag{C.10}$$

$$\boldsymbol{\sigma}_{\boldsymbol{X}} = \begin{bmatrix} \sigma_c & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & \cdots \\ 0 & \sigma_G & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & \cdots \\ 0 & 0 & \sigma_\delta & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & \cdots \\ 0 & 0 & 0 & \sigma_x & 0 & 0 & 0 & 0 & \cdots & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 & 0 & \vdots & \cdots & 0 & \cdots \\ 0 & \sigma_{Gj,G} & 0 & 0 & 0 & \sigma_{Gj} & 0 & 0 & \cdots & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & 0 & 0 & \ddots & \vdots & \cdots & 0 & \cdots \\ \sigma_{rM,c} & \sigma_{rM,G} & \sigma_{rM,\delta} & \sigma_{rM,x} & \cdots & 0 & \cdots & \sigma_{rM,dM} & \cdots & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 & 0 & \vdots & \ddots & 0 & 0 \\ \sigma_{rj,c} & \sigma_{rj,G} & \sigma_{rj,\delta} & \sigma_{rj,x} & 0 & \sigma_{rj,Gj} & 0 & \sigma_{rj,dM} & 0 & \sigma_{rj,dj} & 0 \\ \vdots & \vdots & \vdots & \vdots & 0 & 0 & \ddots & \vdots & 0 & 0 & \ddots \end{bmatrix}. \tag{C.11}$$

We consider as observables the real monthly consumption growth, the ESG scores of the market (proxying for the greenness of the aggregate wealth portfolio) and its excess return, as well as the ESG scores of the portfolios and their monthly returns. We stack these variables in the vector $\boldsymbol{Y}_t$:

$$\boldsymbol{Y}_t = \begin{bmatrix} \Delta c_t & G_{W,t} & \cdots & G_{j,t} & \cdots & \hat{r}_{M,t} & \cdots & \hat{r}_{j,t} & \cdots \end{bmatrix}'. \tag{C.12}$$

The observation equation of the Kalman filter (with zero observation errors) is given by

$$\boldsymbol{Y}_t = \boldsymbol{H}\boldsymbol{X}_t, \tag{C.13}$$

and $\boldsymbol{H}$ is a sparse matrix loading with unit weights the elements of $\boldsymbol{X}_t$ that belong to $\boldsymbol{Y}_t$:

$$\boldsymbol{H} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{C.14}$$

The prediction stage is described by the following transition equations, which provide the time-t conditional expectation and covariances of the state variables in $t+1$:

$$\boldsymbol{X}_{t+1|t} = \boldsymbol{A}_{\boldsymbol{X}} + \boldsymbol{B}_{\boldsymbol{X}}\boldsymbol{X}_{t|t}, \tag{C.15}$$

$$\boldsymbol{\Sigma}^X_{t+1|t} = \boldsymbol{B}_{\boldsymbol{X}}\boldsymbol{\Sigma}^X_{t|t}\boldsymbol{B}'_{\boldsymbol{X}} + \boldsymbol{\sigma}_{\boldsymbol{X}}\boldsymbol{\sigma}'_{\boldsymbol{X}}, \tag{C.16}$$

$\boldsymbol{X}_{1|0}$ is initialized considering the initial values of the observable variables, complemented by δ_0 and x_0, which belong to the parameter space and represent the unobservable initial values of the processes δ_t and x_t. $\boldsymbol{\Sigma}^X_{1|0}$ is initialized at $\boldsymbol{\sigma}_{\boldsymbol{X}}\boldsymbol{\sigma}'_{\boldsymbol{X}}$. The predicted vector of observables is thus $\boldsymbol{Y}_{t+1|t} = \boldsymbol{H}\boldsymbol{X}_{t+1|t}$. The updating equations, which consider the $t+1$ observed values $\boldsymbol{Y}_{t+1}$, are then

$$\boldsymbol{X}_{t+1|t+1} = \boldsymbol{X}_{t+1|t} + \boldsymbol{K}_{t+1}\left(\boldsymbol{Y}_{t+1} - \boldsymbol{H}\boldsymbol{X}_{t+1|t}\right), \tag{C.17}$$

$$\boldsymbol{\Sigma}^X_{t+1|t+1} = \boldsymbol{\Sigma}^X_{t+1|t} - \boldsymbol{K}_{t+1}\left(\boldsymbol{H}\boldsymbol{\Sigma}^X_{t+1|t}\boldsymbol{H}'\right)\boldsymbol{K}'_{t+1}, \tag{C.18}$$

where $\boldsymbol{K}_{t+1} = \boldsymbol{\Sigma}^X_{t+1|t}\boldsymbol{H}'\left(\boldsymbol{H}\boldsymbol{\Sigma}^X_{t+1|t}\boldsymbol{H}'\right)^{-1}$ is the Kalman gain. Given a candidate set of model parameters Θ, equations (C.15) through (C.18) are evaluated recursively. Then, for each time step, the following log-likelihood function is evaluated

$$\ell_{t+1}(\Theta) = -\frac{n}{2}\log(2\pi) - \frac{1}{2}\log\left|\boldsymbol{H}\boldsymbol{\Sigma}^X_{t+1|t}\boldsymbol{H}'\right| - \frac{1}{2}\left(\boldsymbol{Y}_t - \boldsymbol{H}\boldsymbol{X}_{t+1|t}\right)'\left(\boldsymbol{H}\boldsymbol{\Sigma}^X_{t+1|t}\boldsymbol{H}'\right)^{-1}\left(\boldsymbol{Y}_t - \boldsymbol{H}\boldsymbol{X}_{t+1|t}\right). \tag{C.19}$$

The total log-likelihood, $\ell(\Theta) = \sum_{t=1}^{T} \ell_t(\Theta)$, is numerically maximized with respect to the parameter space Θ to obtain the model estimates. In the optimization, we impose the long-run means of the aggregate ESG score, $\bar{G}_W$, and the individual asset scores, $\bar{G}_n$, to be equal to their sample means. Similarly, $\bar{\delta}$ is equal the sample mean of the filtered state variable δ_t. We further set the long-run means of the model-implied price-to-dividend ratios of the market portfolios and individual assets to match the sample average of the observed price-to-dividend ratios. Finally, δ_t is restricted to be nonnegative.

D Supplementary empirical findings

D.1 Supplementary empirical findings based on environmental-pillar scores

Figure A.3 displays time-series findings obtained by estimating the model using environmental-pillar scores. In particular, the figure shows the expected consumption growth, as well as environmental scores, expected excess returns, convenience yields, total ESG premium, and price-to-dividend ratios of the market portfolio and of the portfolios constructed by sorting stocks based on their environmental-pillar scores.

D.2 Supplementary empirical findings excluding stocks in the technology sector

We report the empirical results obtained by estimating the model when stocks in the technology sector are excluded from the investable universe. In each of the tables and figures mentioned below, the findings are obtained observing ESG scores in Panel (a) and environmental-pillar scores in Panel (b).

Figure A.4 displays the time series of aggregate demand and supply for ESG and environmental attributes. ESG demand exhibits a very similar pattern to that obtained using the entire universe (Figure 2a), while demand for environmental attributes reaches even higher values than for the baseline findings in Figure 2b.

Table A.2 reports the parameter estimates, with findings overall in line with those obtained considering the entire universe (Table 1). For instance, risk aversion is insignificantly different from 10, the prices of risk are all positive and significant, and the brown portfolio is more exposed to long-run risk shocks than the green portfolio ($\rho_{dbr} > \rho_{dgr}$).

Table A.3 reports the model-implied decomposition of excess returns. Similar to the baseline findings in Table 2, the green-minus-brown displays a positive and significant ESG demand risk premium, which partially offsets the negative average convenience yield premium. The positive and significant average unexpected return induced by shocks to ESG demand adds to the negative conditional expected return and is essential to match the positive return observed in the data.

Finally, Figures A.5 and A.6 display time-series evidence that is consistent with that obtained considering the entire universe, shown in Figures 3, A.3, and 4.

E Supplementary calibration exercises

E.1 Shocks to ESG demand and ESG score in the presence of correlated cashflows

In this section, we perform a set of supplementary analyses studying the impact of dividend growth rates that are correlated with shocks to ESG demand or ESG scores.

In the first analysis, dividend growth is allowed to be correlated with innovations of ESG demand and of the asset's ESG score. This implies relaxing the hypothesis that the coefficients $\sigma_{dn,\delta}$ and $\sigma_{dn,Gn}$, appearing in equation (A.65), are equal to zero.

To allow for a conditional correlation between dividend growth and ESG demand, we consider the baseline parameter values reported in Section 4.2 and replace $\sigma_{dn,\delta}$ (which baseline value is zero) and σ_{dn} with $\tilde{\sigma}_{dn,\delta}$ and $\tilde{\sigma}_{dn}$, respectively, such that i) the conditional correlation between dividend growth and ESG demand equals the value we aim to impose, $\text{Corr}_t\left[\Delta d_{n,t+1}, \delta_{t+1}\right]$, and ii) the total dividend growth volatility, $\sigma_{dn,tot}$, is the same as the estimated one:

$$\sigma_{dn,tot} = \sqrt{\sigma^2_{dn,c} + \sigma^2_{dn,G} + \sigma^2_{dn,\delta} + \sigma^2_{dn,x} + \sigma^2_{dn,Gn} + \sigma^2_{dn,dM} + \sigma^2_{dn}}, \tag{E.1}$$

$$\tilde{\sigma}_{dn,\delta} = \sigma_{dn,tot} \cdot \text{Corr}_t\left[\Delta d_{n,t+1}, \delta_{t+1}\right], \tag{E.2}$$

$$\tilde{\sigma}_{dn} = \sqrt{\sigma^2_{dn,tot} - \left(\sigma^2_{dn,c} + \sigma^2_{dn,G} + \tilde{\sigma}^2_{dn,\delta} + \sigma^2_{dn,x} + \sigma^2_{dn,Gn} + \sigma^2_{dn,dM}\right)}. \tag{E.3}$$

The graphs in Figure A.7 show that, when dividend growth is positively correlated with ESG demand, the expected return of the green-minus-brown spread portfolio increases relative to the zero correlation case, while a negative correlation implies a lower expected return. This is because the positive correlation implies a return contribution that is also positively correlated with ESG demand, and thus a higher loading on the positive price of risk of ESG demand. If the green (brown) asset's dividend growth is positively (negatively) correlated with ESG demand, during a positive ESG demand shock the positive realized return gap in favor of the green asset widens, while the equilibrium expected return gap in favor of the brown asset shrinks. In this case, the positive effect on the cumulative return of the green-minus-brown portfolio is stronger and vanishes over a longer period relative to the zero correlation case.

Similarly, to allow for a conditional correlation between dividend growth and the asset's ESG score, $\text{Corr}_t\left[\Delta d_{n,t+1}, G_{n,t+1}\right]$, we determine $\tilde{\sigma}_{dn,Gn}$ and $\tilde{\sigma}_{dn}$ such that:

$$\sigma_{dn,tot} = \sqrt{\sigma^2_{dn,c} + \sigma^2_{dn,G} + \sigma^2_{dn,\delta} + \sigma^2_{dn,x} + \sigma^2_{dn,Gn} + \sigma^2_{dn,dM} + \sigma^2_{dn}}, \tag{E.4}$$

$$\tilde{\sigma}_{dn,Gn} = \sigma_{dn,tot} \cdot \text{Corr}_t\left[\Delta d_{n,t+1}, G_{n,t+1}\right], \tag{E.5}$$

$$\tilde{\sigma}_{dn} = \sqrt{\sigma^2_{dn,tot} - \left(\sigma^2_{dn,c} + \sigma^2_{dn,G} + \sigma^2_{dn,\delta} + \sigma^2_{dn,x} + \tilde{\sigma}^2_{dn,Gn} + \sigma^2_{dn,dM}\right)}. \tag{E.6}$$

The graphs in Figure A.8 show the response to an annual shock to the ESG score of the green asset. When the correlation between the dividend growth and the ESG score is zero, the effects on returns are qualitatively similar to those described for an aggregate ESG demand shock, while the impact on both expected and realized returns is slightly weaker. This is because a positive shock to the ESG score triggers only an increase in the convenience yield, but does not imply a reducing risk-free rate.

A positive correlation between dividend growth and ESG score is plausible if an improvement of the firm's ESG profile triggers a higher demand for goods and services and thus higher cashflows. Then, the realized return corresponding to the positive ESG score shock can be significantly higher than that in the baseline case. The correlation between dividend growth and ESG score could yet be negative. For instance, this could result from increasing costs incurred for the improvement of the firm's sustainability profile. Then, the negative cashflow effect could imply a lower, even negative, realized return due to the unexpected ESG score improvement.

E.2 Effect of ESG demand on ESG supply

The graphs in Figure A.9 show the responses to a one-standard deviation positive annual shock applied to ESG demand when the parameters $\rho_{G,\delta}$ and $\rho_{Gn,\delta}$ in equations (A.34) and (A.64) are allowed to be nonzero. Positive values would capture the endogenous increase in ESG supply upon increasing ESG preferences. For instance, a value $\rho_{Ggr,\delta} = 5$ (10) implies that, following a positive one-standard deviation annual shock to δ_t, the long-run ESG scores of the green asset increases by about 8 (16) percentile points.[1]

In our calibration, when the drift of ESG scores is positively related to ESG demand ($\rho_{G,\delta}, \rho_{Gn,\delta} > 0$), the realized return contemporaneous to the shock of both the green and brown assets are higher than in the base case ($\rho_{G,\delta} = \rho_{Gn,\delta} = 0$). A similar effect is observed on the valuation of the assets following the shock, as the nonpecuniary benefits are higher than in the base case for both assets. As the correlation between the nonpecuniary benefits of the green (brown) asset and the nonpecuniary benefits of the wealth portfolio is higher (lower) than in the base case, the expected return of the green (brown) asset is also higher (lower).

Remarkably, while the values considered for the parameters $\rho_{G,\delta}$ and $\rho_{Gn,\delta}$ imply sizable effects of an increase in ESG demand on ESG scores, the impulse response exercise highlights that the key model implications on expected and realized returns are qualitatively unchanged relative to the baseline specification.

E.3 Effect of long-run aggregate ESG supply

In our setup, according to equation (2), for a given aggregate ESG score $G_{W,t}$, a variation in ESG demand $\Delta\delta_t$ implies a variation in aggregate nonpecuniary benefits proportional to $G_{W,t}\Delta\delta_t$. The representative agent's utility in equation (1) depends on these benefits. Then, when the aggregate ESG score increases, the sensitivity of the agent's utility to ESG demand shocks also increases, leading to a higher market price of ESG demand risk. Conversely, when $G_{W,t}$ turns negative, the sensitivity to ESG demand shocks flips sign, i.e., an increased ESG demand implies lower aggregate nonpecuniary benefits, as the agent perceives a brown aggregate portfolio as more harmful, and the market price of ESG demand risk turns negative.

To assess this effect in the context of the log-linearized model, we take the long-run aggregate ESG score $\bar{G}_W$, estimated at about 0.05, and consider higher (0.15) and lower (-0.05) values, corresponding to improved and worsened ESG profiles by one decile in the normalized scale

[1] The variation of the long-run ESG score of asset n corresponding to an increase in ESG preferences $\Delta\delta$ is given by $\frac{\rho_{Gn,\delta}\Delta\delta}{1-\rho_{Gn}}$.

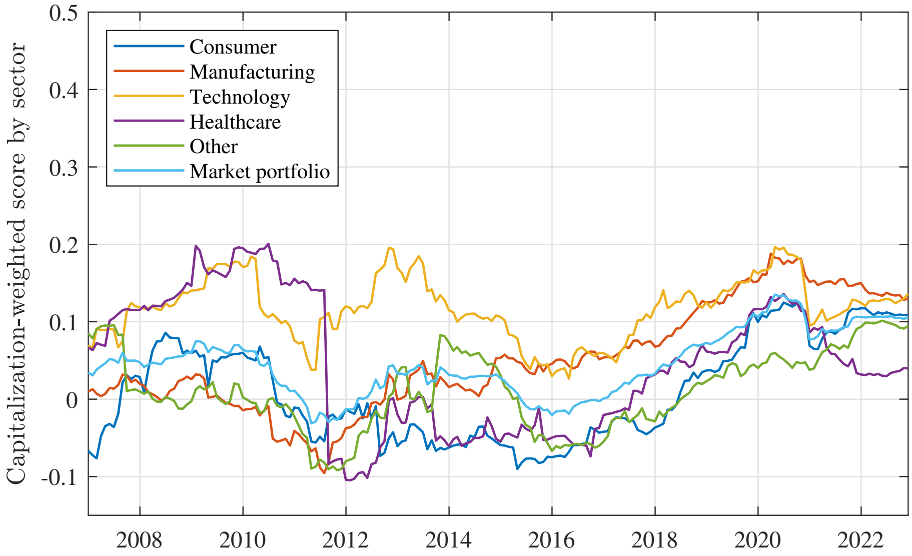
$[-0.50, 0.50]$. Increasing $\bar{G}_W$ results in a threefold increase in the market price of ESG demand risk, λ_δ, rising from 0.0107 to 0.0342. Consequently, the ESG demand risk premium for the green-minus-brown portfolio increases from 22 to 70 basis points per month. Conversely, when $\bar{G}_W$ is decreased to -0.05, λ_δ flips sign, diminishing from 0.0107 to -0.0128. Then, the ESG demand risk premium for the green-minus-brown portfolio decreases from 22 to -26 basis points per month. Although these values correspond to extreme cases, as 0.15 and -0.05 exceed the maximum and minimum observed values for the market ESG score, the resulting ESG demand risk premia are not unrealistically large. For comparison, the convenience yield premium of the green-minus-brown portfolio is -37 basis points on average and -126 basis points in 2022.

To explore the impact of a higher or lower market ESG profile on the response to an unexpected ESG demand shock, we conduct an additional comparative statics impulse response exercise by varying the long-run mean of the aggregate ESG score. As represented in Figure A.10, an elevated long-run market greenness reduces the gap in expected returns between brown and green portfolios, reflecting an increase in the ESG demand risk premium for the green-minus-brown portfolio. Conversely, the gap widens for a lower aggregate greenness. Corresponding to an annual one-standard deviation shock to ESG demand, the gap widens approximately by the same amount irrespective of the value of the long-run market ESG score. The unexpected return contemporaneous to the ESG demand shock, which adds to the expected return to determine the realized return, as well as the valuation ratios, are also approximately unaffected by the value of $\bar{G}_W$. Overall, the effect of an increased long-run market ESG score for the green-minus-brown portfolio is primarily concentrated on the ESG demand risk premium component, influencing the time required for the cumulative realized return to absorb the shock. For the baseline value of $\bar{G}_W$, the recovery time is approximately four years after the shock, while a long-run market greenness increased by one decile extends the recovery time to about nine years, and a decreased value shortens it to three years.

Figure A.1: Capitalization-weighted ESG and environmental scores by industry sector.

The figure shows the time series of the capitalization-weighted ESG and environmental scores by industrial sector, as well as of the market portfolio.

(a) ESG scores by industry sector



(b) Environmental scores by industry sector

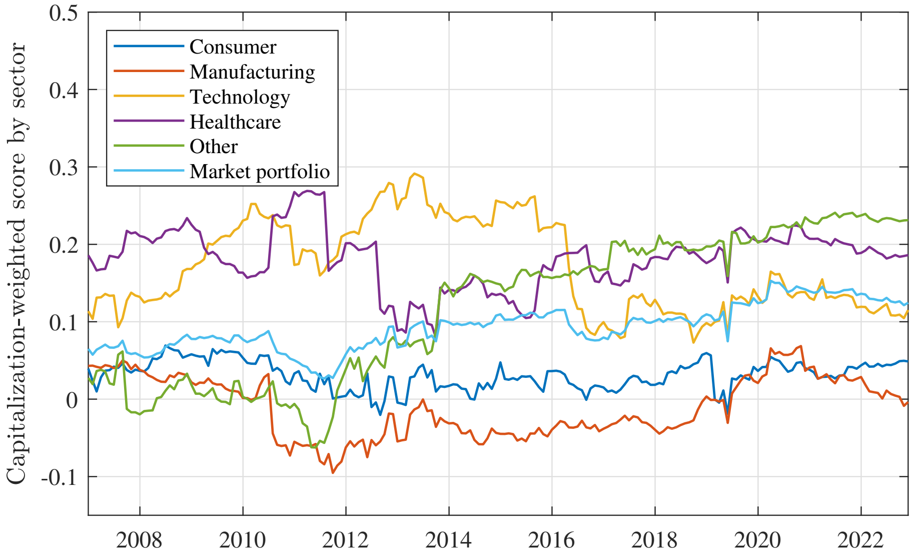


Figure A.2: Relative capitalization and number of stocks in ESG and environmental score-sorted portfolios by industry sector.

The graphs show the time series of the relative capitalization and number of stocks of the five industry sectors within the market and score-sorted portfolios. Panel (a) is based on ESG scores, Panel (b) is based on environmental scores.

(a) Portfolios based on ESG scores

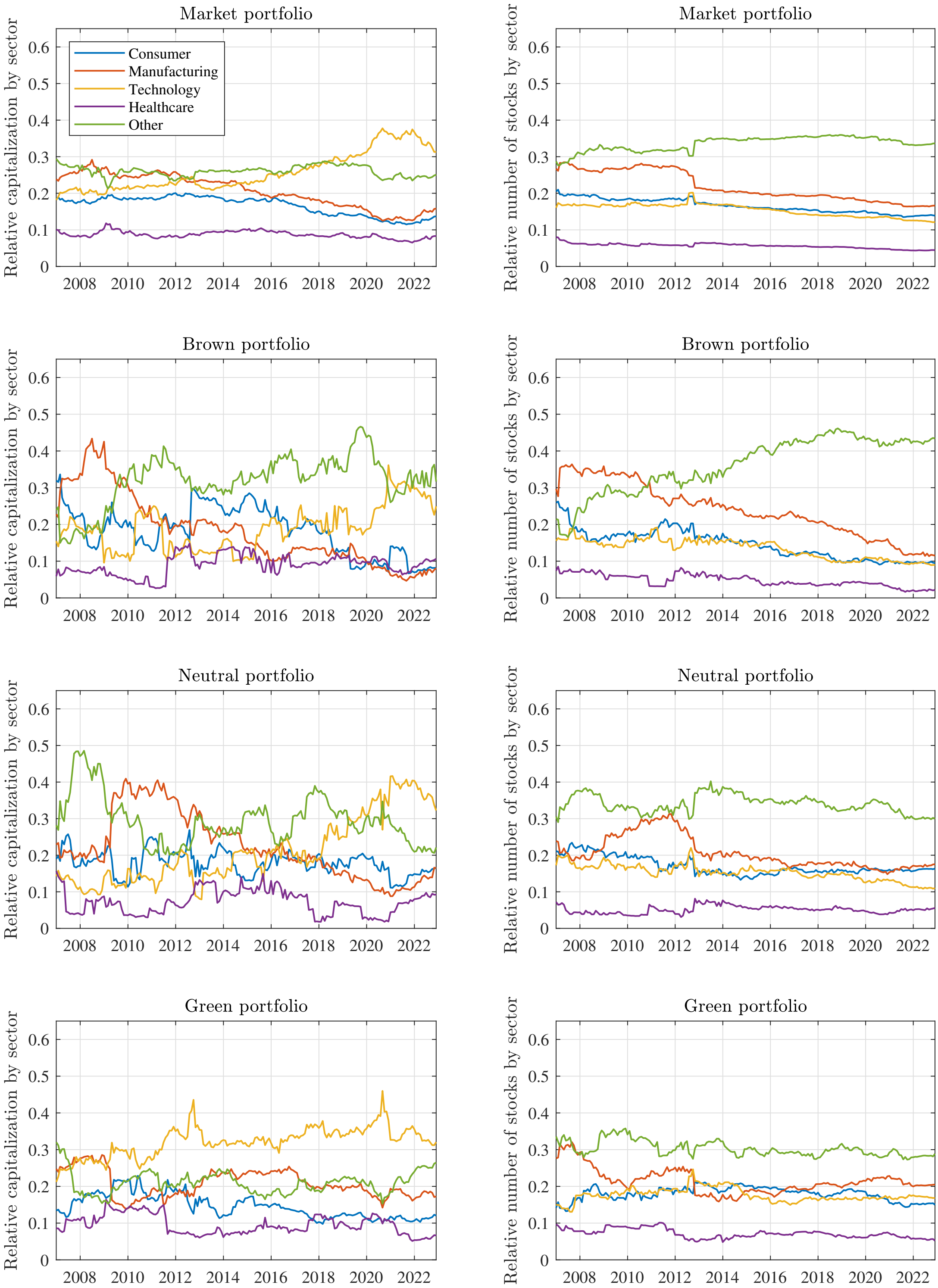


Figure A.2 (continued)

(b) Portfolios based on environmental scores

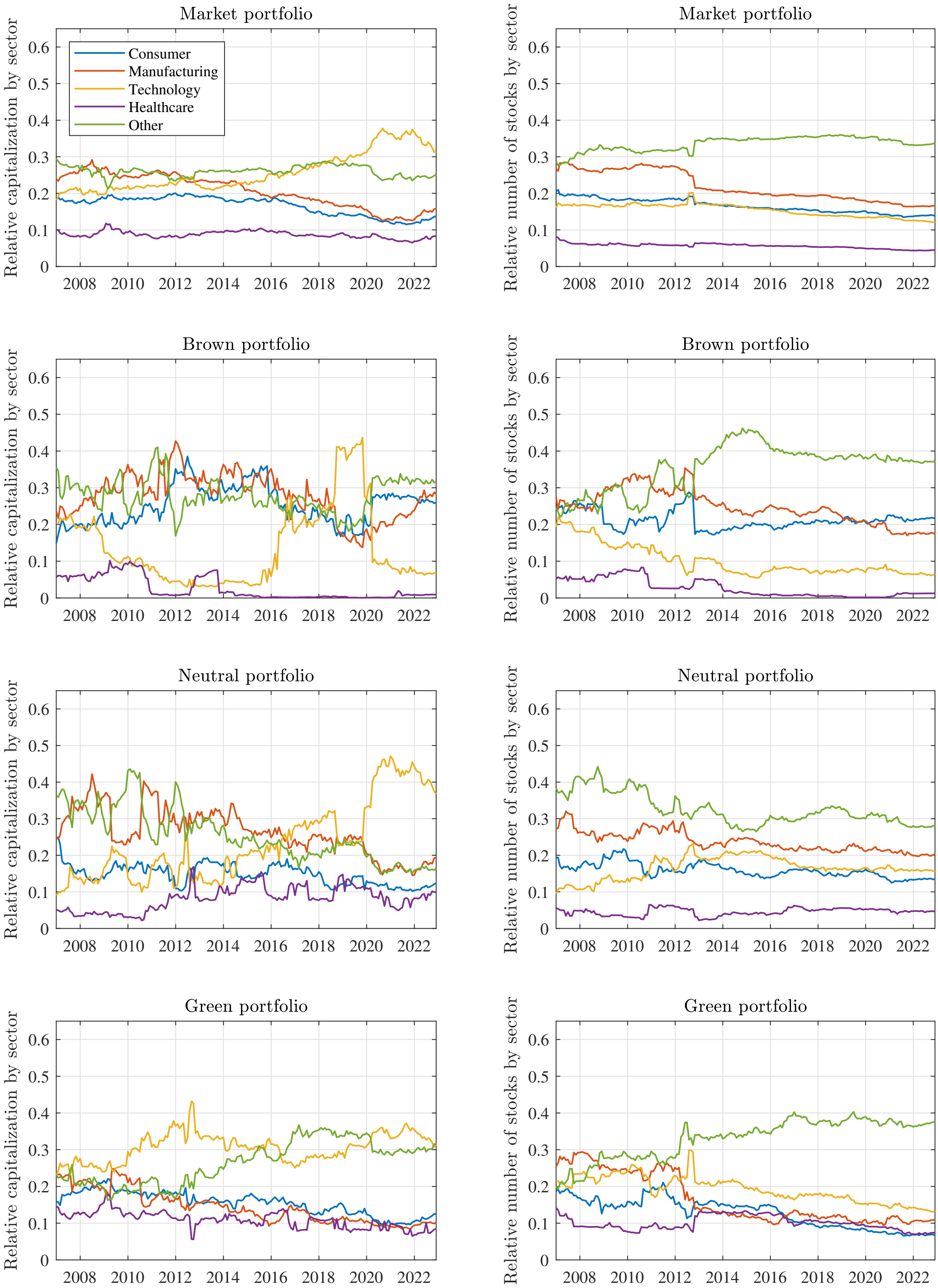


Figure A.3: Time series of expected consumption growth, environmental scores, expected excess returns, and price-to-dividend ratios.

The figure shows the estimated time series of expected consumption growth, environmental scores, expected market and portfolio excess returns, convenience yields from environmental ESG investing, total environmental ESG premia, as well as price-to-annual dividend ratios. All quantities are annualized. The green, neutral, and brown portfolios are obtained sorting stocks by environmental scores. The estimation is performed by maximum likelihood, observing the time series of market and portfolio returns, environmental scores, and consumption growth, as well as average price-to-dividend ratios. The sample runs from January 2007 to December 2022.

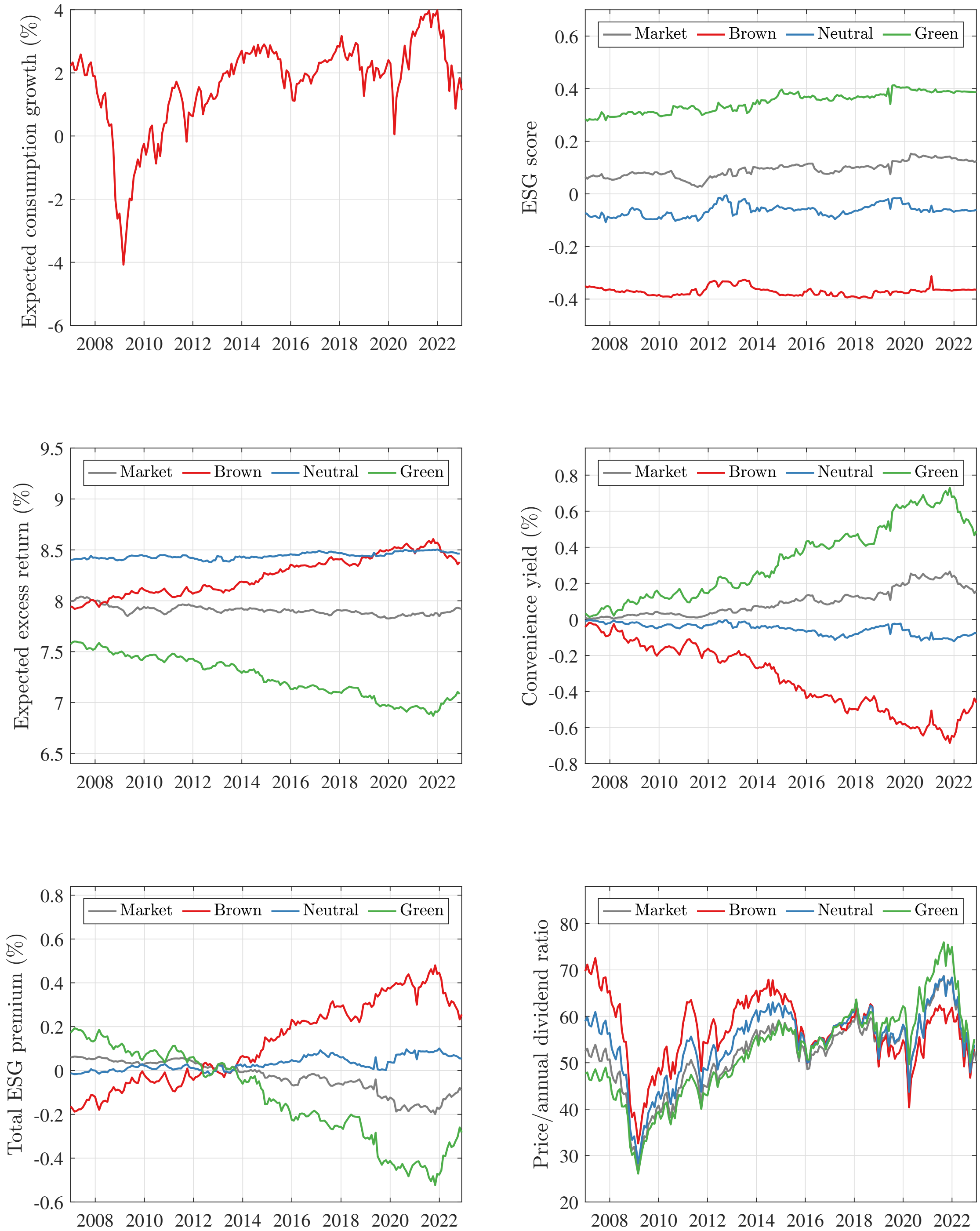


Figure A.4: Time series of aggregate demand and supply for ESG and environmental attributes (excluding stocks in the technology sector).

The figure shows the estimated time series of aggregate ESG demand, δ_t, and supply, $G_{W,t}$. The sample runs from January 2007 to December 2022.

(a) Estimation based on ESG scores

(b) Estimation based on environmental scores

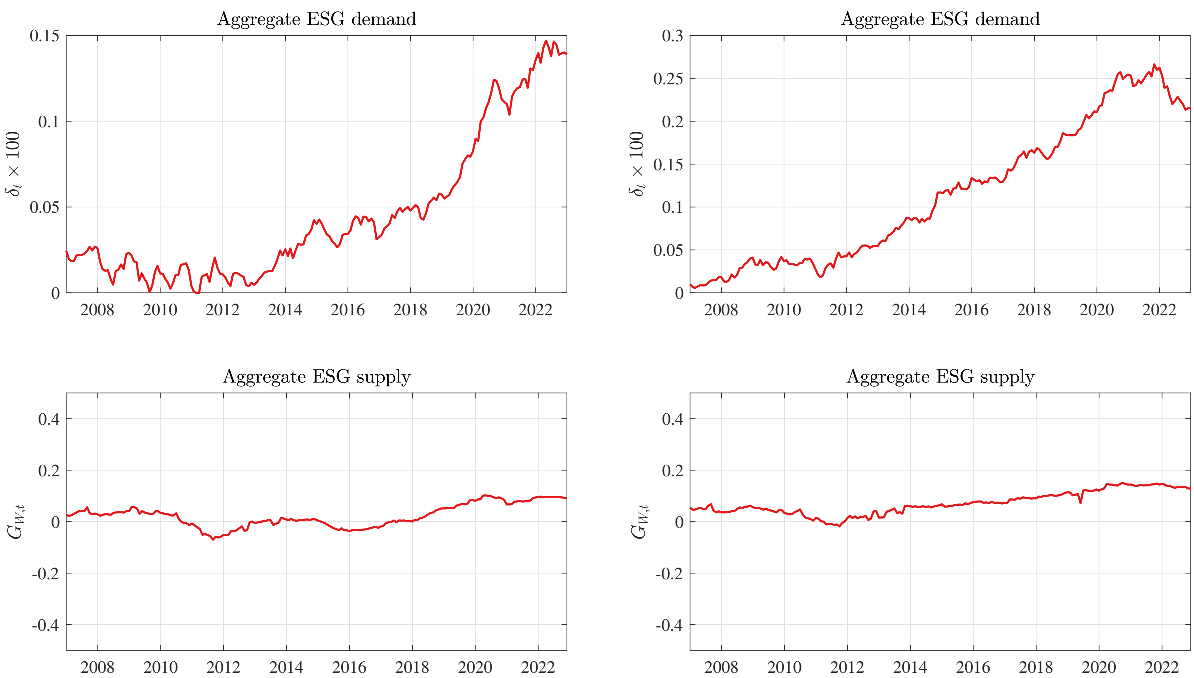


Figure A.5: Time series of expected consumption growth, ESG scores, expected excess returns, and price-to-dividend ratios (excluding stocks in the technology sector).

The figure shows the estimated time series of expected consumption growth, ESG scores, expected market and portfolio excess returns, convenience yields from ESG investing, total ESG premia, as well as price-to-annual dividend ratios. All quantities are annualized. The green, neutral, and brown portfolios are obtained sorting stocks by ESG score. The portfolios are obtained excluding the technology sector and value-weighting stocks sorted by their ESG scores in Panel (a) and environmental scores in Panel (b). The estimation is performed by maximum likelihood, observing the time series of market and portfolio returns, ESG or environmental ratings, and consumption growth, as well as average price-to-dividend ratios. The sample runs from January 2007 to December 2022.

(a) Estimation based on ESG scores

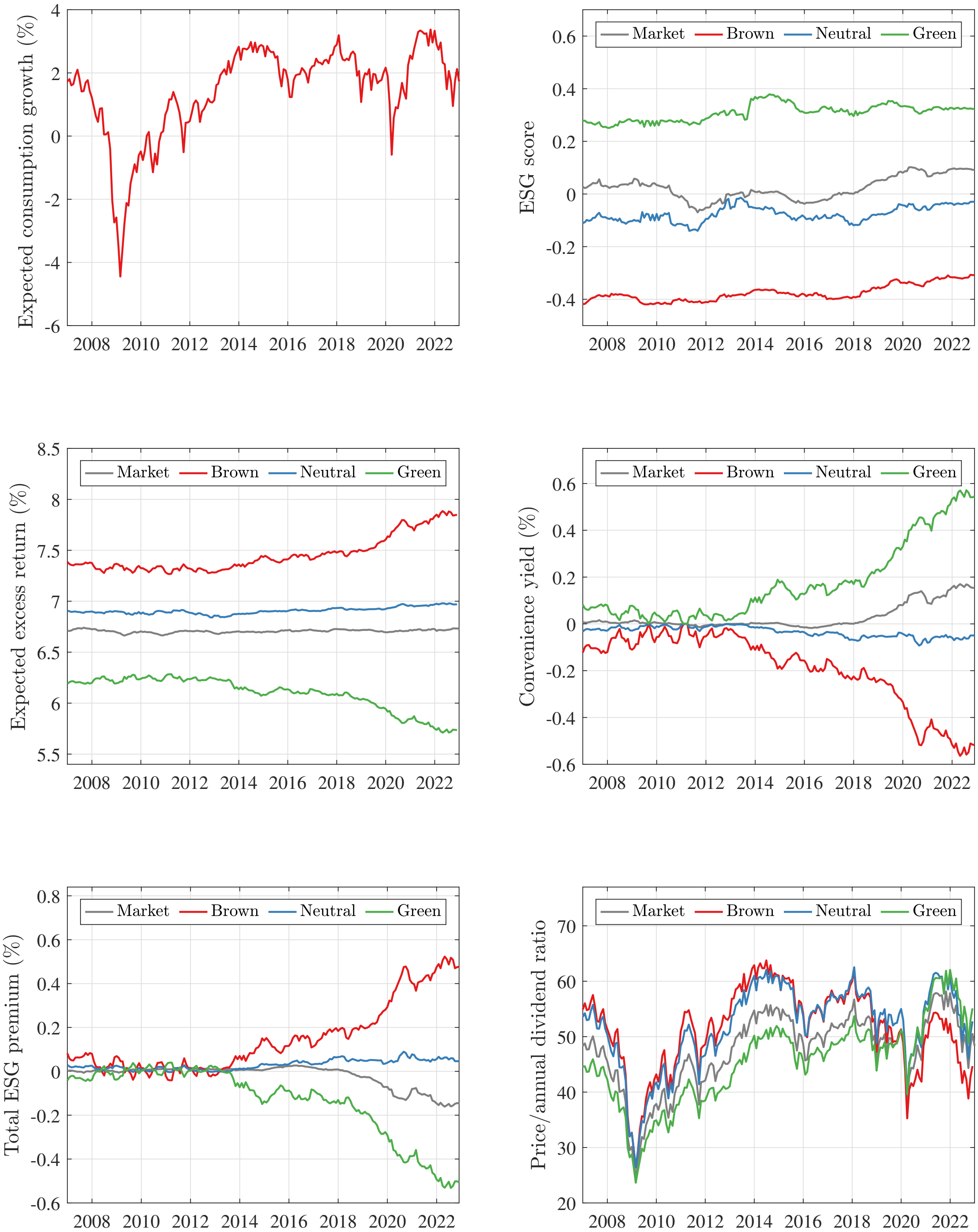


Figure A.5 (continued)

(b) Estimation based on environmental scores

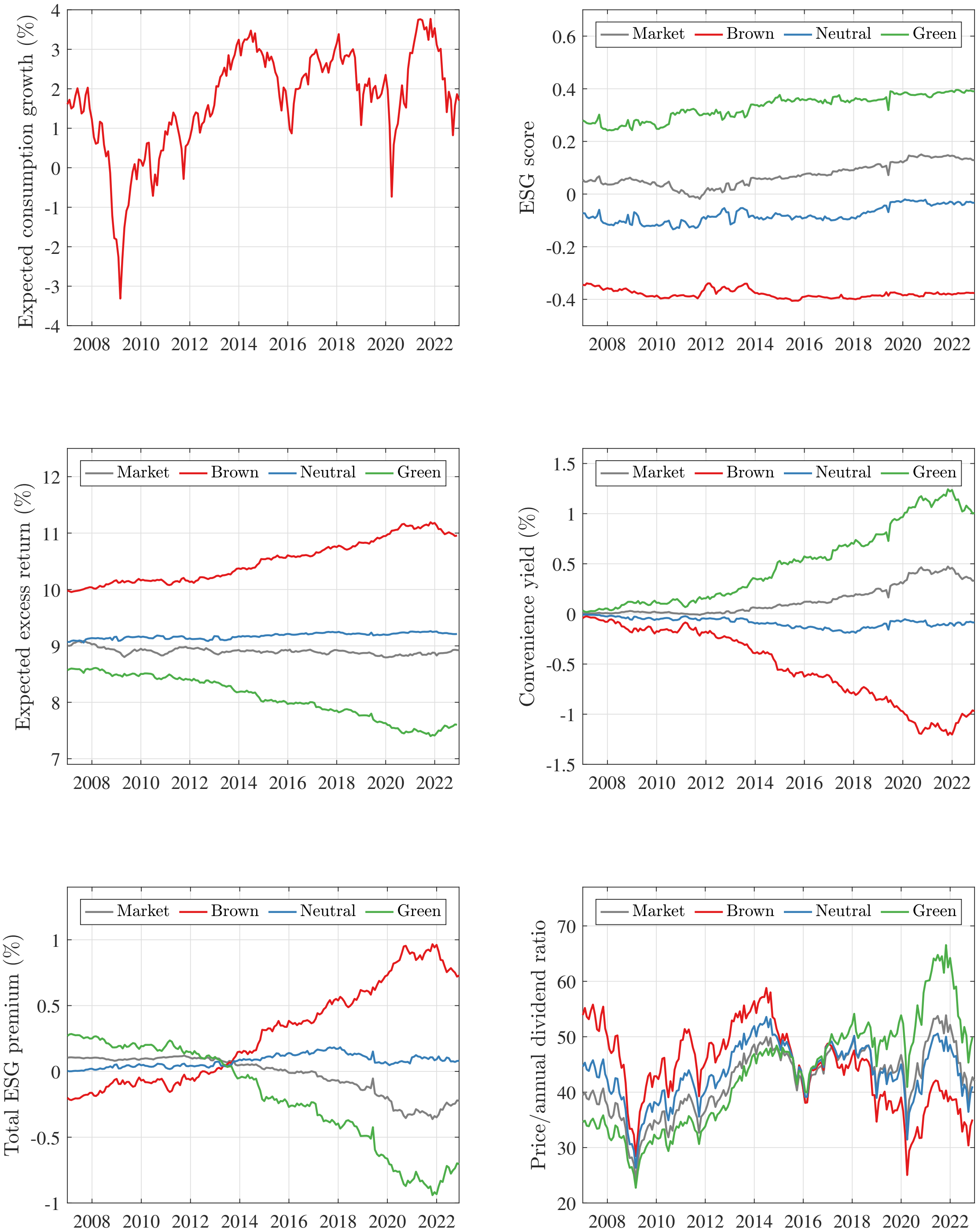
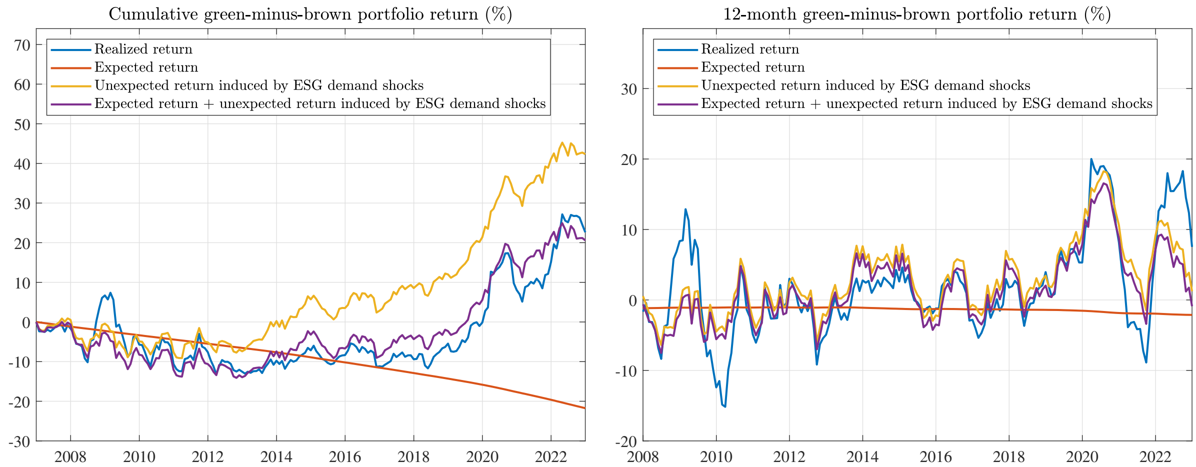


Figure A.6: Returns of green-minus-brown portfolio (excluding stocks in the technology sector).

The left graphs show the realized cumulative logarithmic return of the green-minus-brown portfolio, as well as the model-implied expected return, the unexpected return contribution attributable to ESG demand shocks, and the model-implied expected return augmented by the unexpected return contribution attributable to ESG demand shocks. The right graphs show the 12-month rolling logarithmic return of the same portfolio. The portfolios are obtained excluding the technology sector and value-weighting stocks sorted by their ESG scores in Panel (a) and environmental scores in Panel (b). The estimation is performed by maximum likelihood, observing the time series of market and portfolio returns, ESG or environmental ratings, and consumption growth, as well as average price-to-dividend ratios. The sample runs from January 2007 to December 2022.

(a) Green and brown portfolios based on ESG-scores



(b) Green and brown portfolios based on environmental scores

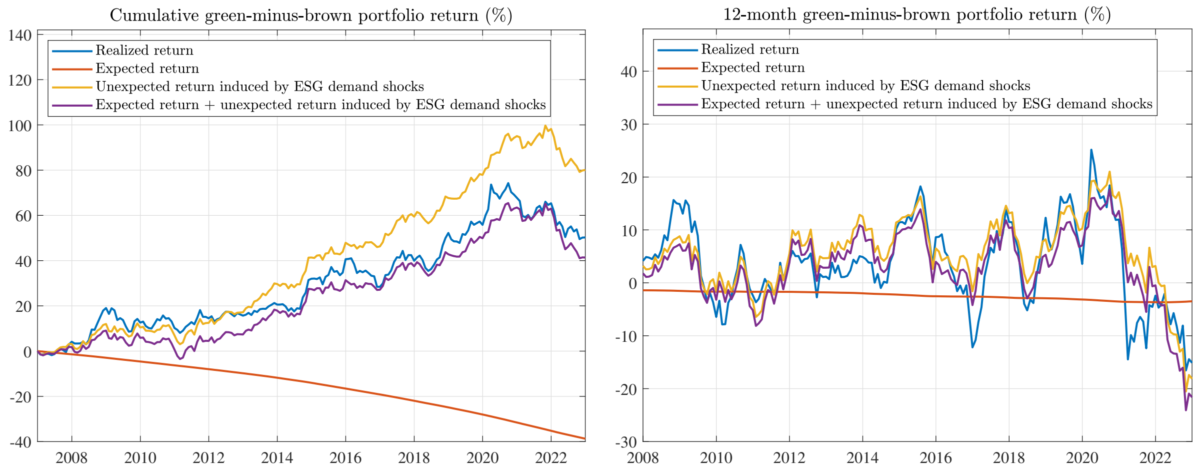


Figure A.7: Impact of cashflows' correlation with ESG demand.

The graphs show responses to a one-standard deviation positive annual shock applied to δ_t. Solid lines correspond to zero correlations, $\text{Corr}_t[\Delta d_{gr,t+1}, \delta_{t+1}]$ and $\text{Corr}_t[\Delta d_{br,t+1}, \delta_{t+1}]$, between portfolio dividend growth and δ_t. Dashed lines correspond to a negative (positive) correlation for the green (brown) asset. Dotted lines correspond to a positive (negative) correlation for the green (brown) asset. The expected and realized excess returns of the brown and green portfolios, the cumulative return of the green-minus-brown portfolio, and the price-to-annual dividend ratios of the brown and green portfolios are shown. The state variables are initially set at their average values and the shocks are equally distributed throughout 12 consecutive months. Online Appendix E.1 provides detailed information on the mapping of correlations on the parameter space.

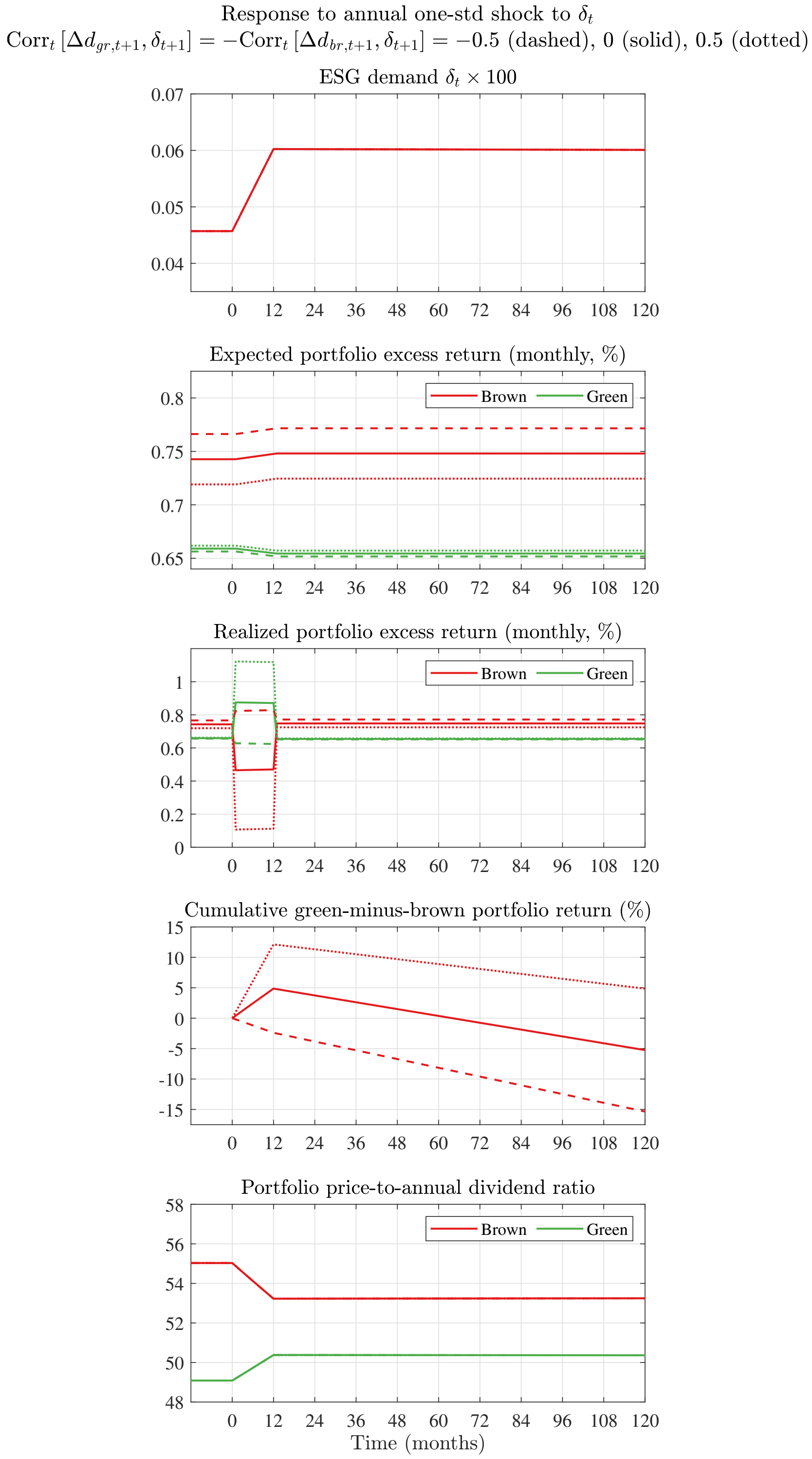


Figure A.8: Impact of cashflows' correlation with ESG score.

The graphs show responses to a +0.1 annual shock to the ESG score of the green portfolio. Solid lines correspond to a zero correlation $\text{Corr}_t[\Delta d_{gr,t+1}, G_{gr,t+1}]$, dashed (dotted) lines to a negative (positive) correlation. The expected and realized excess returns of the brown and green portfolios, the cumulative return of the green-minus-brown portfolio, and the price-to-annual dividend ratios of the brown and green portfolios are shown. The state variables are initially set at their average values and the shocks are equally distributed throughout 12 consecutive months. Online Appendix E.1 provides detailed information on the mapping of correlations on the parameter space.

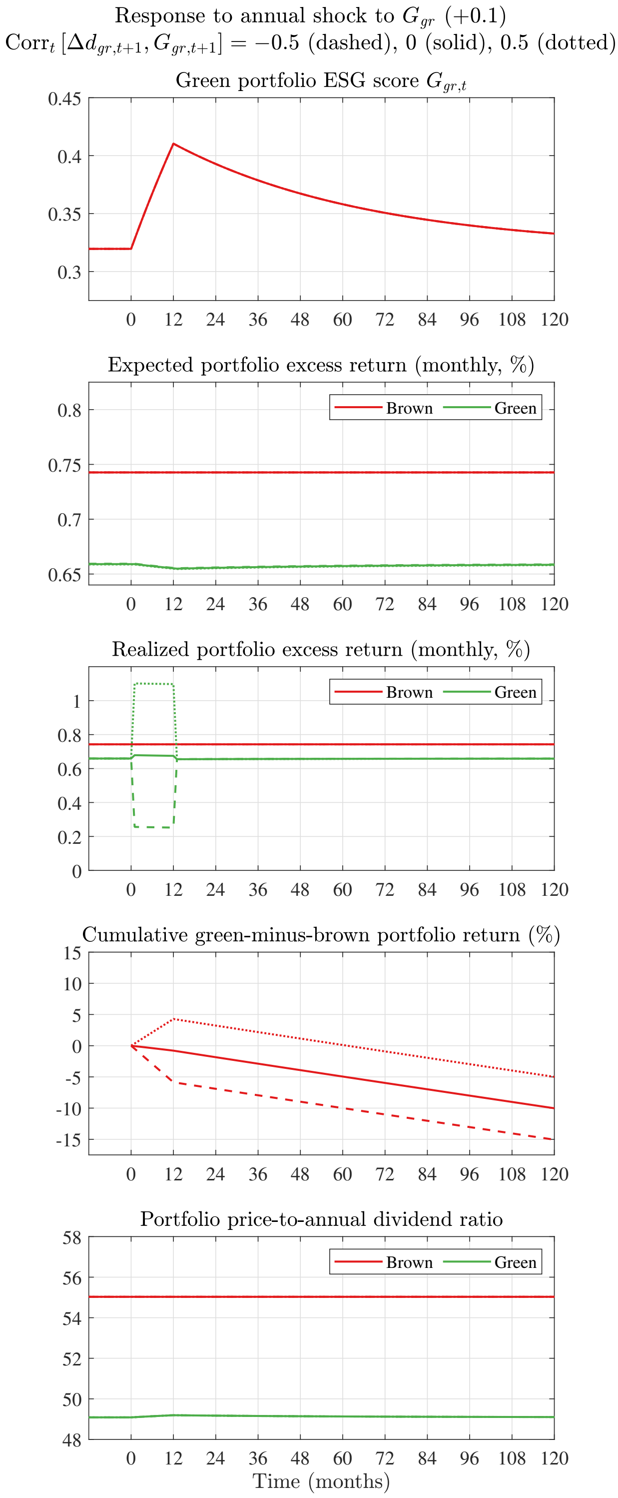


Figure A.9: Impact of dependence of ESG scores over ESG demand.

The graphs show responses to a one-standard deviation positive annual shock applied to δ_t. Solid lines correspond to the baseline specification, where the dynamics of ESG scores in Equations (A.34) and (A.64) is independent of ESG demand ($\rho_{G,\delta} = \rho_{Gn,\delta} = 0$). Dashed and dotted lines correspond to a positive dependence ($\rho_{G,\delta} = \rho_{Gn,\delta} = 5$ and $\rho_{G,\delta} = \rho_{Gn,\delta} = 10$, respectively). The expected and realized excess returns of the brown and green portfolios, the cumulative return of the green-minus-brown portfolio, and the price-to-annual dividend ratios of the brown and green portfolios are shown. The state variables are initially set at their average values and the shocks are equally distributed throughout 12 consecutive months.

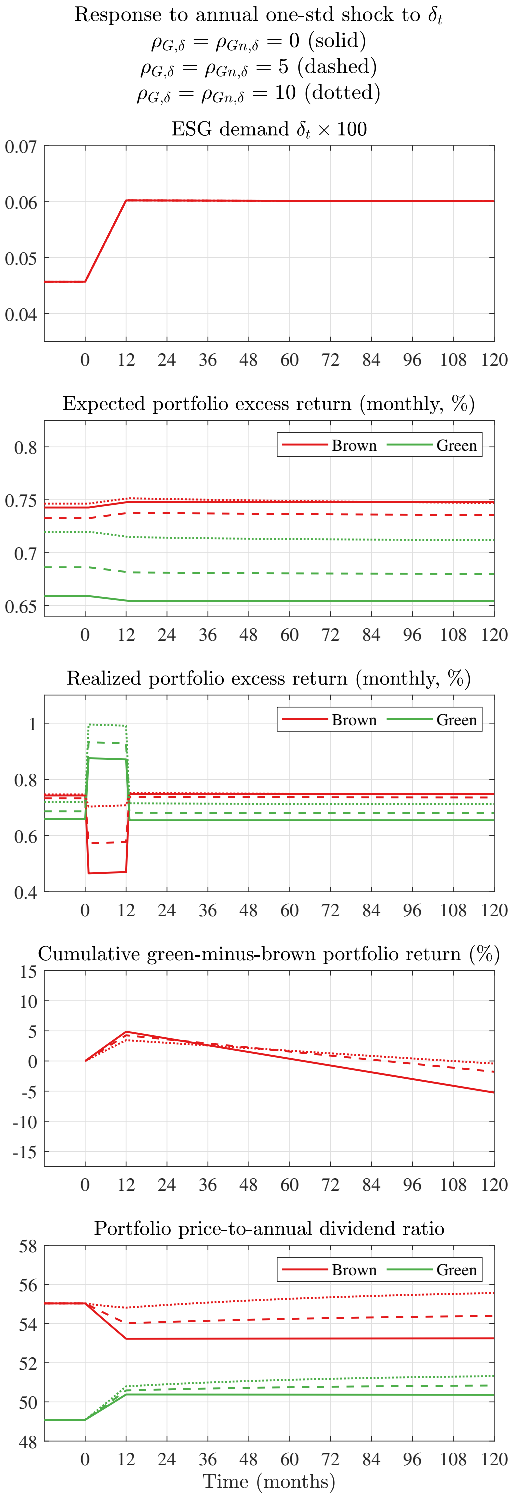


Figure A.10: Impact of ESG demand shock for different values of long-run aggregate ESG score.

The graphs show responses to a one-standard deviation positive annual shock applied to δ_t. Solid lines correspond to the estimated value of the long-run market ESG score, i.e., $\bar{G}_W = 0.0456$. Dashed and dotted lines correspond to values of $\bar{G}_W$ that are respectively increased (0.1456) and decreased (-0.0544) by one decile within the scale of ESG scores $[-0.50, 0.50]$. The expected and realized excess returns of the brown and green portfolios, the cumulative return of the green-minus-brown portfolio, and the price-to-annual dividend ratios of the brown and green portfolios are shown. The state variables are initially set at their average values and the shocks are equally distributed throughout 12 consecutive months.

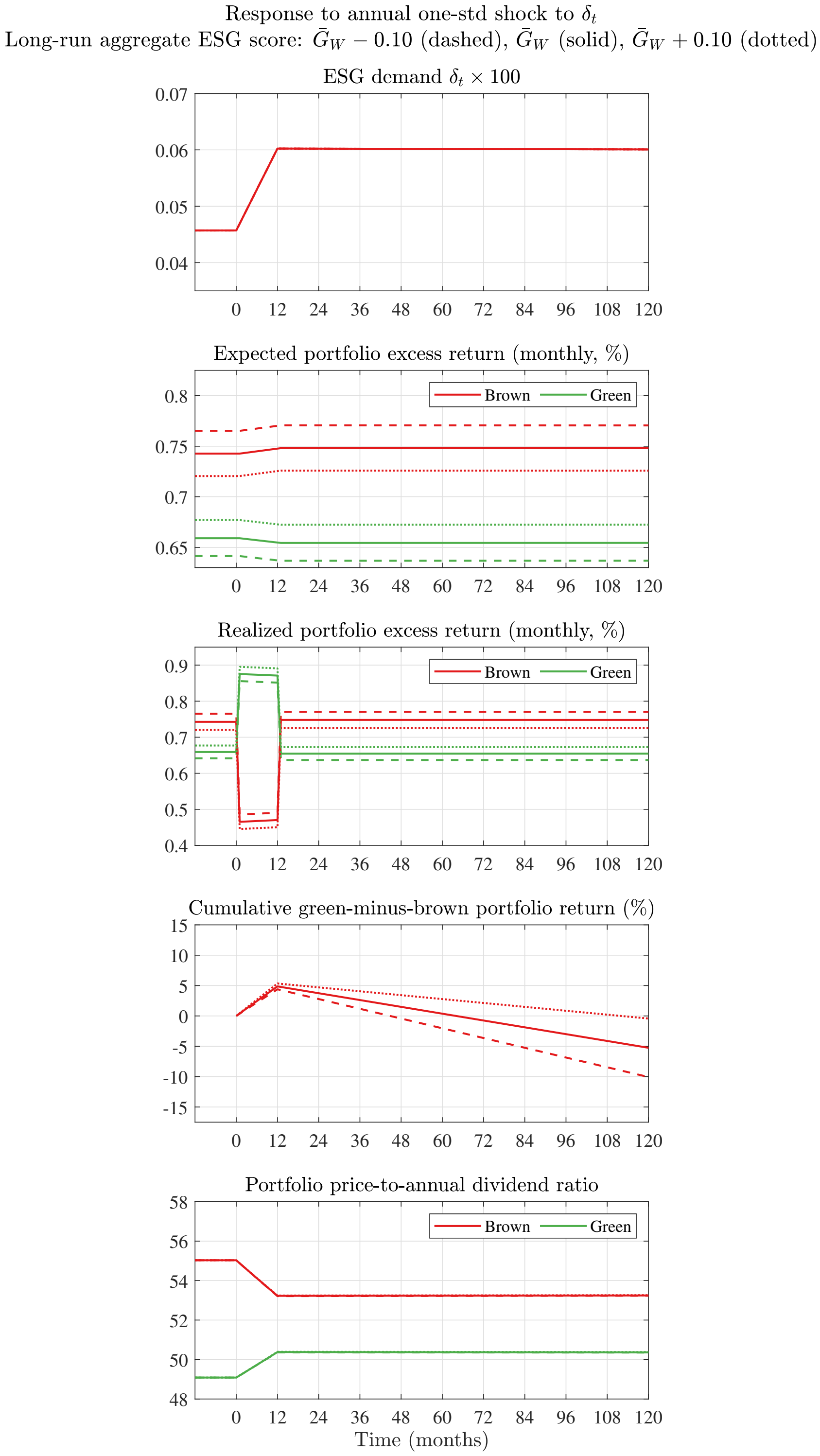


Table A.1: Summary statistics

The table reports the summary statistics of the value-weighted portfolios based on prior ESG and environmental scores. Portfolios in Panel (a) are constructed considering the entire universe of stocks, portfolios in Panel (b) are constructed excluding stocks in the technology sector. The portfolio construction methodology is described in Section 3.

(a) All stocks

	ESG-score portfolios				Environmental-score portfolios			
	Market	Brown	Neutral	Green	Market	Brown	Neutral	Green
Average monthly return (%)	0.84	0.71	0.82	0.88	0.84	0.77	0.85	0.87
Monthly return standard deviation (%)	4.57	5.13	4.69	4.47	4.57	5.13	4.86	4.34
Average score	0.05	-0.37	-0.07	0.32	0.09	-0.37	-0.06	0.35
Score standard deviation	0.04	0.03	0.03	0.03	0.03	0.02	0.02	0.04
Min score	-0.03	-0.41	-0.13	0.25	0.03	-0.40	-0.10	0.28
Max score	0.14	-0.30	0.00	0.37	0.15	-0.33	-0.01	0.41
Average market capitalization (US$ trillion)	20.63	3.99	7.39	9.25	20.63	2.87	7.61	10.15
Average number of stocks	1491	448	596	447	1491	448	596	447

(b) All stocks excluding technology sector

	ESG-score portfolios				Environmental-score portfolios			
	Market	Brown	Neutral	Green	Market	Brown	Neutral	Green
Average monthly return (%)	0.75	0.70	0.76	0.77	0.75	0.62	0.73	0.83
Monthly return standard deviation (%)	4.52	5.19	4.78	4.26	4.52	5.30	4.87	4.24
Average score	0.02	-0.38	-0.08	0.31	0.07	-0.38	-0.08	0.33
Score standard deviation	0.04	0.03	0.03	0.03	0.04	0.02	0.03	0.05
Min score	-0.07	-0.42	-0.14	0.25	-0.02	-0.41	-0.13	0.24
Max score	0.10	-0.31	-0.01	0.38	0.15	-0.34	-0.02	0.40
Average market capitalization (US$ trillion)	14.80	3.11	5.34	6.35	14.80	2.22	5.38	7.20
Average number of stocks	1270	381	508	381	1270	381	508	380

Table A.2: Parameter estimates excluding stocks in the technology sector.

The table reports the estimated parameters for the baseline model specification. The subjective discount rate β is set at 0.998, the intertemporal elasticity of substitution ψ at 1.5, the long-run risk persistence ρ_x at 0.979, its volatility σ_x at 0.00034, and the persistence of ESG demand ρ_δ at 0.9999. The brown, neutral, and green portfolios are obtained by value-weighting stocks sorted by their ESG scores in Panel (a) and environmental scores in Panel (b). The estimation procedure is described in Section 4.1. The sample runs from January 2007 to December 2022.

(a) Estimation based on ESG scores

Economy-wide parameters (Θ_E) and market prices of risk

γ	μ_c	σ_c	x_0	μ_G	ρ_G	σ_G	δ_0	$\bar{\delta}$	σ_δ
10.27107	0.00113	0.01212	0.00031	0.00032	0.98600	0.00942	0.00024	0.00045	0.00004
(2.53918)	(0.00131)	(0.00061)	(0.00365)	(0.00020)	(0.00901)	(0.00047)	(0.00012)	(0.00014)	(0.00000)
λ_c	λ_G	λ_δ	λ_x						
0.12446	0.00384	0.00415	0.14215						
(0.03127)	(0.00126)	(0.00119)	(0.03673)						

Market portfolio parameters (Θ_M)

μ_{dM}	$\rho_{dM,x}$	$\sigma_{dM,c}$	σ_{dM}
0.00498	3.69382	0.00125	0.00291
(0.00130)	(0.69363)	(0.00348)	(0.01047)

Brown portfolio parameters (Θ_{br})

μ_{dbr}	$\rho_{dbr,x}$	$\sigma_{dbr,c}$	$\sigma_{dbr,dM}$	σ_{dbr}	μ_{Gbr}	ρ_{Gbr}	σ_{GbrG}	σ_{Gbr}
0.00574	3.97858	0.00259	0.01145	0.00466	-0.00351	0.99066	-0.01501	0.01701
(0.00156)	(0.79657)	(0.00398)	(0.01456)	(0.00087)	(0.00389)	(0.01035)	(0.00145)	(0.00084)

Neutral portfolio parameters (Θ_{neu})

μ_{dneu}	$\rho_{dneu,x}$	$\sigma_{dneu,c}$	$\sigma_{dneu,dM}$	σ_{dneu}	μ_{Gneu}	ρ_{Gneu}	σ_{GneuG}	σ_{Gneu}
0.00530	3.81903	0.00075	-0.00610	0.00001	-0.00338	0.95578	0.00197	0.00973
(0.00139)	(0.75475)	(0.00368)	(0.01300)	(0.00076)	(0.00184)	(0.02414)	(0.00070)	(0.00049)

Green portfolio parameters (Θ_{gr})

μ_{dgr}	$\rho_{dgr,x}$	$\sigma_{dgr,c}$	$\sigma_{dgr,dM}$	σ_{dgr}	μ_{Ggr}	ρ_{Ggr}	σ_{GgrG}	σ_{Ggr}
0.00439	3.46856	0.00111	0.00653	0.00093	0.00422	0.98635	0.01759	0.01742
(0.00130)	(0.64990)	(0.00327)	(0.01060)	(0.00053)	(0.00278)	(0.00900)	(0.00155)	(0.00084)

(b) Estimation based on environmental scores

Economy-wide parameters (Θ_E) and market prices of risk

γ	μ_c	σ_c	x_0	μ_G	ρ_G	σ_G	δ_0	$\bar{\delta}$	σ_δ
13.23035	0.00224	0.01208	-0.00090	0.00246	0.96554	0.01215	0.00010	0.00114	0.00005
(2.51290)	(0.00136)	(0.00061)	(0.00362)	(0.00078)	(0.01094)	(0.00061)	(0.00013)	(0.00017)	(0.00000)
λ_c	λ_G	λ_δ	λ_x						
0.15978	0.00966	0.02312	0.18772						
(0.03115)	(0.00200)	(0.00661)	(0.03688)						

Market portfolio parameters (Θ_M)

μ_{dM}	$\rho_{dM,x}$	$\sigma_{dM,c}$	σ_{dM}
0.00700	3.58461	0.00114	0.01184
(0.00174)	(0.57199)	(0.00345)	(0.01166)

Brown portfolio parameters (Θ_{br})

μ_{dbr}	$\rho_{dbr,x}$	$\sigma_{dbr,c}$	$\sigma_{dbr,dM}$	σ_{dbr}	μ_{Gbr}	ρ_{Gbr}	σ_{GbrG}	σ_{Gbr}
0.00846	4.08316	0.00138	0.00535	0.00431	-0.05435	0.85650	-0.01971	0.01930
(0.00162)	(0.58987)	(0.00409)	(0.01416)	(0.00168)	(0.01078)	(0.02846)	(0.00173)	(0.00095)

Neutral portfolio parameters (Θ_{neu})

μ_{dneu}	$\rho_{dneu,x}$	$\sigma_{dneu,c}$	$\sigma_{dneu,dM}$	σ_{dneu}	μ_{Gneu}	ρ_{Gneu}	σ_{GneuG}	σ_{Gneu}
0.00731	3.67717	0.00147	0.01888	0.00174	-0.00569	0.92769	0.00136	0.01071
(0.00209)	(0.70049)	(0.00372)	(0.01400)	(0.00098)	(0.00230)	(0.02924)	(0.00077)	(0.00054)

Green portfolio parameters (Θ_{gr})

μ_{dgr}	$\rho_{dgr,x}$	$\sigma_{dgr,c}$	$\sigma_{dgr,dM}$	σ_{dgr}	μ_{Ggr}	ρ_{Ggr}	σ_{GgrG}	σ_{Ggr}
0.00633	3.36876	0.00079	0.00929	0.00006	0.01492	0.95491	0.02564	0.01579
(0.00164)	(0.50490)	(0.00321)	(0.01056)	(0.00046)	(0.00451)	(0.01364)	(0.00174)	(0.00076)

Table A.3: Decomposition of model-implied excess returns and average observed excess returns excluding stocks in the technology sector.

The table reports the observed and model-implied annualized excess returns, as well as the decomposition of model-implied excess returns. The brown, neutral, and green portfolios are obtained by value-weighting stocks sorted by their ESG scores in Panel (a) and environmental scores in Panel (b). The estimation procedure is described in Section 4.1. The sample runs from January 2007 to December 2022.

(a) Estimation based on ESG scores

Portfolio	Market	Brown	Neutral	Green	Green-brown
Model-implied short-run consumption risk premium	0.19%	0.39%	0.11%	0.17%	-0.22%
	(0.55%)	(0.62%)	(0.59%)	(0.51%)	(0.21%)
Model-implied long-run consumption risk premium	7.78%	8.56%	8.15%	7.17%	-1.39%
	(1.66%)	(1.98%)	(1.84%)	(1.58%)	(0.49%)
Model-implied ESG supply risk premium	0.01%	0.00%	0.01%	0.01%	0.01%
	(0.01%)	(0.01%)	(0.01%)	(0.01%)	(0.00%)
Model-implied ESG demand risk premium	0.00%	-0.04%	-0.01%	0.03%	0.08%
	(0.00%)	(0.01%)	(0.00%)	(0.01%)	(0.02%)
Average model-implied convenience yield premium	-0.03%	0.19%	0.03%	-0.17%	-0.36%
	(0.00%)	(0.06%)	(0.01%)	(0.05%)	(0.11%)
Average model-implied expected excess return	6.69%	7.44%	6.90%	6.09%	-1.35%
	(1.64%)	(1.96%)	(1.81%)	(1.60%)	(0.50%)
Average ESG demand shock-induced return (δ-induced return)	0.07%	-1.54%	-0.34%	1.09%	2.63%
	(0.03%)	(0.20%)	(0.04%)	(0.16%)	(0.36%)
Average model-implied expected excess return + δ-induced return	6.76%	5.90%	6.56%	7.18%	1.28%
	(1.63%)	(1.98%)	(1.80%)	(1.57%)	(0.57%)
Average observed excess return	6.97%	5.91%	6.85%	7.33%	1.41%

(b) Estimation based on environmental scores

Portfolio	Market	Brown	Neutral	Green	Green-brown
Model-implied short-run consumption risk premium	0.22%	0.27%	0.28%	0.15%	-0.11%
	(0.64%)	(0.76%)	(0.69%)	(0.60%)	(0.34%)
Model-implied long-run consumption risk premium	9.93%	11.68%	10.27%	9.20%	-2.48%
	(1.90%)	(1.95%)	(2.37%)	(1.68%)	(0.62%)
Model-implied ESG supply risk premium	0.06%	0.05%	0.05%	0.06%	0.01%
	(0.03%)	(0.03%)	(0.03%)	(0.03%)	(0.00%)
Model-implied ESG demand risk premium	0.05%	-0.29%	-0.06%	0.24%	0.54%
	(0.02%)	(0.08%)	(0.02%)	(0.07%)	(0.15%)
Average model-implied convenience yield premium	-0.14%	0.53%	0.08%	-0.49%	-1.02%
	(0.02%)	(0.08%)	(0.02%)	(0.07%)	(0.15%)
Average model-implied expected excess return	8.85%	10.51%	9.15%	8.04%	-2.46%
	(1.84%)	(1.74%)	(2.28%)	(1.69%)	(0.59%)
Average ESG demand shock-induced return (δ-induced return)	0.49%	-2.77%	-0.56%	2.26%	5.04%
	(0.06%)	(0.23%)	(0.05%)	(0.21%)	(0.44%)
Average model-implied expected excess return + δ-induced return	9.34%	7.73%	8.60%	10.31%	2.57%
	(1.81%)	(1.82%)	(2.28%)	(1.60%)	(0.61%)
Average observed excess return	6.97%	4.89%	6.46%	8.03%	3.14%