Internet Appendix to "Machine-Learning-Based Return Predictors and the Spanning Controversy in Macro-Finance"

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IA.A Inferring Higher-Order Yield Principal Components

This section examines the relation between PCs of observed yields and those of "true" yields to provide justification for the use of filtered PCs in tests of Spanning Hypothesis II (H_0^{S2}).

It is known that due to the negligible role of higher-order PCs in the cross section, it is difficult to disentangle them from noise in yields. Panel A of Table IA.A1 illustrates the (limited) effectiveness of direct Principal Component Analysis (PCA) in recovering information in true yields. Column 2 reports population correlations between true yield factors ($PC_{1-5,t}$) and PCs of the observed yield curve ($PC^o_{1-5,t}$), where the correlations are all computed from Monte Carlo simulations based on an estimated five-factor yields-only (Gaussian) term structure model (YTSM). Note that while $\text{corr}(PC^o_{i,t}, PC_{i,t})\,\forall i = 1, 2, 3$ is very high (> 0.96), it is 0.72 for $i = 4$ and 0.21 for $i = 5$, suggesting the inability of PCA to accurately infer $PC_{4,t}$ and $PC_{5,t}$ from yield data. This finding is not surprising given the magnitude of yield loadings on the higher-order PCs: untabulated results indicate that a one-standard-deviation shock to $PC_{4,t}$ or $PC_{5,t}$ does not change any yield by more than seven basis points (bps). On the other hand, the estimated standard deviation of the measurement error in yields is about six bps, which is enough to obscure the cross-sectional effects of $PC_{4,t}$ and $PC_{5,t}$.

In the real data sample, true yield factors are not observable. Duffee (2011) shows that filtering techniques, while no substitute for direct observation, are helpful in retrieving information in those higher-order factors. We find from an unreported simulation analysis that model-implied correlations between true and filtered factors are higher than 0.85 for both the fourth and fifth factors. As a result, we use filtered PCs in our empirical tests of H_0^{S2}.

Will it make a difference at all if we ignore the "hidden" nature of higher-order factors? Column 3 in panel A of Table IA.A1 shows that for these factors, the filtered and PCA-based estimates are significantly different in our 1964–2014 sample.[IA.1] More importantly, replacing the former with the latter leads to underestimation of the predictive power possessed by the historical yield dynamics.

Panel B of Table IA.A1 presents results from regressions of excess bond returns on $PC^o_{1-5,t}$ for two- through five-year maturities. Comparing the panel with Table IA.B5 (columns 10–13)

[IA.1] Joslin, Singleton, and Zhu (2011; Section 6) document very similar results over the sample period 1990–2007: the model-implied filtered $PC_{1-3,t}$ are nearly identical to $PC^o_{1-3,t}$ regardless of the model dimension, but $PC^o_{4-5,t}$ do not closely correspond to their model-implied counterparts. Especially, the authors notice that filtered high-order factors appear to be a smoothed version of $PC^o_{4-5,t}$.

reveals that replacing (filtered) $PC_{4-5,t}$ by $PC^o_{4-5,t}$ results in lower R^2 values, regardless of the bond maturity, where for convenience those four R^2 values from Table 2 are shown in the row labeled "R^2 (Table 2)" in panel B. Note that the decline in R^2 ranges from 0.014 for the four-year bond to 0.018 for the three-year bond (the second last row of panel B), and the percentage decrease in R^2 ranges from 5.49% for the four-year bond to 7.73% for the three-year bond (the last row). Given that the first three factors alone can have an R^2 of 16–19% (columns 2–5 of Table IA.B5), the aforementioned amount of information loss in the fourth and fifth factors is far from trivial. As such, using $PC^o_{1-5,t}$ in regression tests of H_0^{S2} would make the hypothesis overrejected.

IA.B More on Properties of the SAGLasso Macro Factor

This appendix further examines the properties of the SAGLasso macro factor ($\widehat{G}$). Section IA.B.1 investigates the predictive power of the three group factors which constitute the SAGLasso factor. Section IA.B.2 examines whether the three group factors are spanned or not. Section IA.B.3 compares the predictive power of the $\widehat{G}$ and Ludvigson and Ng (2009; LN09 hereafter) macro factors. Section IA.B.4 investigates the potential impact of data revision and publication lags on return predictability. Section IA.B.5 presents in-sample tests of the spanning hypotheses, H_0^{S1} and H_0^{S2}. Lastly, Section IA.B.6 implements the SAGLasso algorithm using 131 macro variables along with different numbers of their lags.

IA.B.1 Predictive Power of the Three Group Factors

The SAGLasso macro factor $\widehat{G}_t$ consists of three group factors: the employment ($\widehat{g}_{1t}$), housing ($\widehat{g}_{2t}$), and inflation ($\widehat{g}_{3t}$) factors. To better understand the information content of factor $\widehat{G}_t$, we examine properties of these three group factors in this subsection. Let $\{\widehat{g}_{it}\}_{1\leq i\leq 3}$ denote $\{\widehat{g}_{it}, 1 \leq i \leq 3\}$.

IA.B.1.1 Sample Period 1964–2014

Table IA.B1 reports the Pearson correlation matrix of $\widehat{G}$, $\{\hat{g}_{it}\}_{1\leq i\leq 3}$, and five yield curve factors. The five yield curve factors include the first three principal components (PCs) of *observed* bond yields, $\{PC^o_{i,t}, i = 1, 2, 3\}$, and the filtered higher-order PCs of noise-uncontaminated yields, $PC_{4,t}$ and $PC_{5,t}$. As expected, $\widehat{g}_{1t}$, $\widehat{g}_{2t}$, and $\widehat{g}_{3t}$ all have low correlations with the yield curve factors. In particular, the novel housing factor $\widehat{g}_{2t}$ has a correlation of -0.167 with $PC^o_{1,t}$, -0.073 with $PC^o_{2,t}$, and 0.222 with $PC^o_{3,t}$. As a result, $\widehat{G}$ is weakly correlated with $PC^o_{1-3,t}$ and hardly correlated with $PC_{4,t}$ and $PC_{5,t}$. Recall that by construction the $\widehat{G}$ factor and its three component factors control for the Treasury and FX variables (group 5) out of the 131 macro series. The results shown in the table verify that $\widehat{G}$ and $\{\hat{g}_{it}\}_{1\leq i\leq 3}$ are all weakly correlated with the yield curve. Nonetheless, as shown below these group factors have strong predictive power as a result of the SAGLasso procedure used for model selection.

We now examine the predictive power of $\widehat{g}_{1t}$, $\widehat{g}_{2t}$, and $\widehat{g}_{3t}$, both individually and jointly. Panel A of Table IA.B2 presents results from predictive regressions of excess bond returns on normalized $\widehat{g}_{1t}$, $\widehat{g}_{2t}$, and $\widehat{g}_{3t}$, for 2-, 3-, 4-, and 5-year bonds in the full sample period. Panel A1 reports coefficient estimates, t-statistics, and R-squared of univariate regressions on each of the three group factors. Note that these factors all exhibit significant unconditional predictive power, with an R^2 of 21–22% for $\widehat{g}_{1t}$, about 14–15% for $\widehat{g}_{2t}$, and 17–18% for $\widehat{g}_{3t}$. Results from multivariate regressions, reported in panel A2, show that the three group factors are still all significant and together have an (adjusted) R^2 ranging from about 40% for the 2-year bond to 43% for the 3-year bond.

As shown in Joslin, Priebsch, and Singleton (2014; JPS hereafter), the impact of macro risk factors on bond risk premia depends on horizons. Panel A of Table IA.B2 illustrates the relative importance of the three group factors across bond maturity. The univariate regression results indicate that the regression coefficient on $\widehat{g}_{1t}$ is the largest, followed by the one on $\widehat{g}_{3t}$, and the coefficient on $\widehat{g}_{2t}$ is the lowest, regardless of the bond maturity. The univariate regression R^2 values exhibit the same pattern. In the multivarite regressions, the regression coefficients on the three group factors maintain the same ranking, regardless of the bond maturity. These results indicate that relatively speaking, among the three group factors, the employment factor ($\widehat{g}_{1t}$) is the most important, followed by the inflation factor ($\widehat{g}_{3t}$), and then by the housing factor ($\widehat{g}_{2t}$). Note, however, that these group factors are trained on the aggregate bond market returns rather than returns on bonds with a specific maturity. Bianchi, Büchner, and Tamoni (2021) consider more categories of macro variables and find that variables related to the stock and labor market (the output & income and orders & inventories) are more important for the short-end (long-end) of the yield curve.

IA.B.1.2 Sample Period 1952–2014

The full sample used in this study is 1964–2014. However, it is known that the relationship between interest rate and real activity changed significantly around 1964. This raises one concern about the robustness of our evidence for the predictive power of $\widehat{g}_{it}, i = 1, 2, 3$ and $\widehat{G}_t$ based on the 1964–2014 sample: If we extend the sample to several years earlier, that may significantly change the results. To address this concern, we reexamine the predictive power of these macro factors using the sample extended to 1952, the year from which the data coverage of the original Fama-Bliss yields starts.[IA.2] However, some macro series, especially those related to housing, are not available going back that far; thus, we reconstruct the employment factor only in this robustness check, and denote the factor constructed in-sample by $\widehat{g}^*_{1t}$ and its out-of-sample version by $\widetilde{g}^*_{1t}$. Recall from Section 3 that the "labor" group includes 32 series. As two of these series are no longer available when the sample extends back to 1952, $\widehat{g}^*_{1t}$ or $\widetilde{g}^*_{1t}$ is constructed using the remaining 30 macro series.

Results from in-sample regressions, reported in panel B1 of Table IA.B2, indicate that the predictive power of the employment factor is robust to the extended sample. Comparing panels A1 and B1, we see that the predictive power of $\widehat{g}^*_{1t}$ is slightly weaker than that of $\widehat{g}_{1t}$ in terms of the magnitude of regression coefficients or R^2 value but the coefficient on $\widehat{g}^*_{1t}$ has greater t-value than that on $\widehat{g}_{1t}$, regardless of the bond maturity.

In the out-of-sample tests, the training period is 20 years, which is close to the 21-year period length adopted in our full-sample (1964–2014) analysis. In other words, the employment factor is reconstructed every month after December 1971 using Adaptive Lasso within a given group, and the predictive regression is also reestimated recursively. As before, we consider the following three out-of-sample statistics: the "ENC-REG" (Ericsson 1992), the "ENC-NEW" (Clark and McCracken 2001), and the out-of-sample R-squared "R^2_{oos}" (Campbell and Thompson 2008) statistics. The results shown in panel B2 of Table IA.B2 indicate that $\widetilde{g}^*_{1t}$ has significant out-of-sample predictive power for every bond considered. Additionally, R^2_{oos} increases in the bond maturity, ranging from 0.155 for the 2-year bond to 0.169 for the 5-year bond.

Overall, the above results provide evidence that the predictive power of the employment factor (one main component of the SAGLasso single factor $\widehat{G}_t$) is robust to the longer sample 1952–2014.

[IA.2] The supplement to Cochrane and Piazzesi (2005), available at `http://www.stanford.edu/~piazzesi/cp.zip`, suggests that Fama-Bliss yield data prior to 1964 is unreliable.

IA.B.2 Spanning Properties of the Group Factors

Having examined the predictive power of the three group factors, we explore, to what extent, each of the three factors is spanned or unspanned in this subsection.

Recall from Table IA.B1 that $\widehat{g}_{1t}$, $\widehat{g}_{2t}$, and $\widehat{g}_{3t}$ all have low correlations with the yield curve factors. In an untabulate analysis, we find that the three group factors are not highly correlated with *GRO* (growth) and *INF* (inflation) either, two standard single macro variables used in the literature. Unsurprisingly, the largest correlation (0.497) occurs between the two inflation factors, $\widehat{g}_{3t}$ and INF_t. The correlations between INF_t and the other two group factors are 0.237 for $\widehat{g}_{1t}$ and 0.144 for $\widehat{g}_{2t}$. The growth variable *GRO* has a correlation of -0.013, 0.167, and -0.015 with $\widehat{g}_{1t}$, $\widehat{g}_{2t}$, and $\widehat{g}_{3t}$, respectively. These findings suggest that the three group factors are viable candidates for unspanned macro variables.

We examine whether the three group factors are spanned by the yield curve, following Section 5.4 that conducts a similar analysis for $\widehat{G}_t$. That is, for each of the three group factors, we first regress the factor on the first $\mathcal{R}$ PCs of the yield curve ($PC^o_{1-\mathcal{R},t}$), where $\mathcal{R} = 3$ or 6; we then evaluate the regression R^2 against its distribution implied from a constrained and spanned model; we also estimate the first-order correlation of residuals from the regression to see if the residuals are serially uncorrelated. The model used here to generate distributions of R^2 is model $CSM(3,6)_{group}$, whose state vector includes three yield curve factors (the first three PCs) and three macro factors, $\widehat{g}_{1t}$, $\widehat{g}_{2t}$, and $\widehat{g}_{3t}$. The estimation of the model is done under the assumption that the three macro factors are measured either with or without errors.

Table IA.B3 reports the regression results for each of the group factors with $\mathcal{R} = 3$ (panel A) or 6 (panel B). Column 2 indicates whether the three macro variables are assumed to be measured with errors in the estimation of model. Columns 3 (panel A) and 5 (panel B) show the regression R^2s, and in brackets beneath are reported 95% confidence intervals based on 5,000 artificial samples simulated from model $CSM(3,6)_{group}$. Columns 4 (panel A) and 6 (panel B) report the first-order serial correlation of regression residuals. Clearly, the regression R^2 is outside of the 95% confidence intervals for each of the group factors in either panel. Moreover, even the smallest estimated first-order serial correlation is around 90%, suggesting that much of the persistent component is mistakenly treated as white-noise shocks. All of the evidence indicates that the three SAGLasso group macro factors $\{\widehat{g}_{it}, 1 \leq i \leq 3\}$ are not spanned by the yield curve factors.

IA.B.3 Comparison with the Ludvigson and Ng (2009) Factor

The LN09 single factor, constructed through dynamic factor analysis and BIC-based stepwise predictive regression, is $\overrightarrow{F6}_t = (\widehat{F}_{1t}, \widehat{F}_{1t}^3, \widehat{F}_{2t}, \widehat{F}_{3t}, \widehat{F}_{4t}, \widehat{F}_{8t})$, the particular polynomial function of LN09's eight dynamic factors that minimizes the BIC over the sample period 1964–2003. However, using our panel of 131 "real-time" macro series over 1964–2014, we find that the selected subset includes $\overrightarrow{\mathcal{F}7}_t = (\widehat{F}_{1t}, \widehat{F}_{1t}^3, \widehat{F}_{2t}, \widehat{F}_{5t}, \widehat{F}_{5t}^2, \widehat{F}_{8t}, \widehat{F}_{8t}^2)$,[IA.3] whose R^2 value is 0.256, higher than 0.214 of $\overrightarrow{F6}_t$'s. Hence, we let $\widehat{LN}_t^m$ (the modified LN factor) be $\overrightarrow{\mathcal{F}7}_t$ in our empirical analysis.

Although both $\widehat{G}$ and $\widehat{LN}^m$ are extracted from the same set of 131 macro series, they differ in several aspects. First, whereas $\widehat{LN}^m$ includes all 131 series and squares and cubes of these macro variables, $\widehat{G}$ is a *linear* combination of 19 series and some of their lagged variables, and consists of three easy-to-interpret macro group factors. Second, in terms of economic interpretation, $\widehat{G}$ includes a housing factor that contributes little to $\widehat{LN}^m$, whose important components are the "real activity" (highly correlated with measures of employment and production such as IP growth), "inflation," and "stock market" factors. Also, $\widehat{G}$ includes no variables from the "bond and FX" group and thus is much less correlated with the yield curve than $\widehat{LN}^m$ is. Lastly, by construction $\widehat{G}$ takes into account the dynamic response of bond risk premia to macroeconomic innovations. In contrast, information on term premia does not enter $\widehat{LN}^m$ until the last step of its construction.

Panel A of Table IA.B4 reports the prediction results based on $\widehat{LN}_t^m$ for the full sample. Results from the in-sample analysis reported in panel A1 show that $\widehat{LN}_t^m$ is significant, regardless of the bond maturity, and that the R^2 increases in the bond maturity, ranging from 0.168 for the 2-year bond to 0.250 for the 5-year bond. Recall from panel A1 of Table 1 that the R^2 from regressions on $\widehat{G}_t$ ranges from 0.352 for the 2-year bond to 0.392 for the 5-year bond. The difference between this R^2 and that of $\widehat{LN}_t^m$ is 0.18, 0.16, 0.15, and 0.14 for the 2-, 3-, 4-, and 5-year bonds, respectively. These results indicate that $\widehat{G}_t$ has a greater predictive power than $\widehat{LN}_t^m$ for excess bond returns.[IA.4] Results from the out-of-sample analysis also support this conclusion, as can be seen from evidence shown in panel A2 of Table IA.B4 for $\widehat{LN}_t^m$ and that in panel A of Table 2 for $\widehat{G}_t$. To summarize,

[IA.3] The variable $\widehat{F}_{5t}^2$ is also selected by Ludvigson and Ng (2011), who consider the sample period 1964–2008.

[IA.4] This finding is robust in the post-1984 sample period (untabulated).

even though $\widehat{G}_t$ is linear and much more parsimonious than $\widehat{LN}_t^m$, the former predictor shows stronger predictive ability than the latter in both in-sample and out-of-sample analyses.

IA.B.4 Data Revisions, Publication Lags, and Return Predictability

The SAGLasso macro factor $\widehat{G}$ (as well as $\widehat{LN}^m$ considered before) is constructed based on the set of 131 macro series compiled in this study that adjust for both data revisions and publication lags. This subsection examines the impact of these two adjustments on bond return predictability.

To this end, we construct two new macro factors using the same SAGLasso procedure as described before in Section 4 but with different macro data. The first factor, denoted $\widehat{G}_t^{rev}$, is constructed based on the set of the same 131 macro series that, however, adjust for publication lags only (and not data revisions). The other new macro factor, denoted $\widehat{G}_t^{rev,lag}$, is constructed based on the set of the 131 macro series that does not adjust for either data revisions or publication lags—namely, the original set of macro series used in LN09 less the one series no longer available.

Panel B of Table IA.B4 reports the results from predictive regressions of excess bond returns on $\widehat{G}_t^{rev}$ from both in-sample (panel B1) and out-of-sample (panel B2) analyses. Comparing panel B1 with panel A1 of Table 1 reveals that both the regression coefficient on $\widehat{G}_t^{rev}$ and its in-sample R^2 are slightly larger than those for $\widehat{G}_t$ except for the 2-year bond. Similarly, the out-of-sample R^2_{oos} of $\widehat{G}_t^{rev}$ (panel B2 of Table IA.B4) is slightly higher than that of $\widehat{G}_t$ (panel A of Table 2), regardless of the bond maturity. These findings indicate that the predictive ability of $\widehat{G}_t$ is slightly inferior to that of $\widehat{G}_t^{rev}$. In other words, data revisions inflate the predictability only slightly in our sample.

Conversely, results reported in panel C of Table IA.B4 show that return predictability is substantially exaggerated if publication lags are not adjusted. For instance, the in-sample R^2 of $\widehat{G}_t^{rev,lag}$ is 0.414, 0.441, 0.453, and 0.464 for the 2- through 5-year bonds, respectively (panel C1) and is much higher than that of $\widehat{G}_t$ (panel A1 of Table 1). The increase in the R^2 ranges from 6.2% for the 2-year bond to 7.2% for the 5-year bond. That is, the inflated predictability is especially notable in the in-sample regressions. The out-of-sample evidence shown in panel C2 of Table IA.B4 (based on $\widehat{G}_t^{rev,lag}$) and panel A of Table 2 (based on $\widehat{G}_t$) also indicates that ignoring publication lags inflates the predictability, albeit to a lesser degree.

To summarize, we find that publication lags pose much greater "danger" than data revisions in forecasting future bond returns based on macro variables, at least in our sample. This problem can be mitigated straightforwardly, however, since in practice it is easier to make an adjustment for publication lags than to figure out preliminary macro data releases and adjust for data revisions.

Note that the main finding of this subsection is consistent with Ghysels, Horan, and Moench (2018), who document that using revised macro series inflates the predictive power of macro variables. However, while they focus on a particular macro variable—"total non-farm payroll employment" (#33 on our list of 131 series)—and find that both data revisions and publication lags are highly important, we examine the impact of these two elements on a large panel of macro time series and find that the predictive power of the SAGLasso (aggregate) macro variable is robust to the use of vintage data. Namely, the importance of revision/delay biases depends on specific macro series, especially given that variable #33 itself is not included in $\widehat{G}$ (see Table A.1 in the paper). This implication is consistent with Ghysels et al. (2018) too. In a robustness analysis, they consider the Chicago Fed National Activity Index (an unsmoothed version of macro variable GRO) and find that the combined effect of publication lags and data revisions on these two aggregate macro variables is small. Also, Barillas (2012) finds that the bond return predictability is robust to the use of real time series for 16 macro variables (7 inflation and 9 real growth measures) considered in his study.

IA.B.5 In-Sample Spanning Tests

This subsection tests the spanning hypotheses, H_0^{S1} and H_0^{S2}, by examining the incremental predictive power of $\widehat{G}$ over the yield curve. As before, we focus mainly on the test statistics based on the HH or NW standard errors in the discussion of test results that follows.

Table IA.B5 presents the results based on the full sample. Results from regressions on $PC^o_{1-3,t}$, reported in columns 2–5, indicate that only $PC^o_{2,t}$ is significant and that the R^2 ranges from 0.156 for the 3-year bond to 0.194 for the 5-year bond. Results from each of the above regressions augmented with $\widehat{G}_t$, reported in columns 6–9, show that $\widehat{G}$ is significant regardless of the bond maturity. The incremental R-squared due to $\widehat{G}$, ΔR^2, ranges from 0.243 for the 5-year bond to 0.262 for the 3-year bond. These results provide strong evidence against H_0^{S1}.

Results from regressions on $PC_{1-5,t}$, shown in columns 10 through 13, indicate that in addition to $PC_{2,t}$, the higher-order $PC_{4,t}$ and $PC_{5,t}$ are also significant for most bonds.[IA.5] The R^2 ranges

[IA.5] Internet Appendix IA.A presents empirical evidence that the PCA of the observed yields is unable to effectively from 0.221 for the 2-year bond to 0.255 for the 4-year bond. Augmenting these regressions with $\widehat{G}_t$ yields a ΔR^2 ranging from 0.232 for the 5-year bond (column 17) to 0.253 for the 3-year bond (column 15). Importantly, $\widehat{G}_t$ is significantly different from zero regardless of the bond maturity and standard errors used, indicating a rejection of H_0^{S2}. In addition, $PC_{2,t}$ and $PC_{4,t}$ become less significant (and insignificant for the 2- and 3-year bonds) in the presence of $\widehat{G}_t$.

The results for the post-1984 sample, reported in Table IA.B6, are qualitatively the same as those for the full sample. Particularly, $\widehat{G}_t$ is significant, regardless of the bond maturity and standard errors used, conditional on either $PC^o_{1-3,t}$ (columns 6–9) or $PC_{1-5,t}$ (columns 14–17); namely, H_0^{S1} and H_0^{S2} are strongly rejected by the post-1984 sample too. Compared with its counterparts for the full sample (Table IA.B5), ΔR^2 due to $\widehat{G}_t$ is actually higher except for the 2-year bond. For instance, ΔR^2 from regression tests of H_0^{S1} for the 5-year bond is 0.297 for the post-1984 sample (column 7) and 0.243 for the full sample (column 9 in Table IA.B5). Regarding the impact of PCs in the presence of $\widehat{G}_t$, $PC^o_{1,t}$ ($PC^o_{2,t}$) remains significant for the 2- and 5-year bonds (10-year bond) in the tests of H_0^{S1}. For regression tests of H_0^{S2} (columns 14–17), $PC_{1,t}$ is significant regardless of the bond maturity, $PC_{2,t}$ is significant for the 7- and 10-year bonds, and $PC_{5,t}$ for the 2-year bond only.

We also consider test statistics based on Hodrick 1B standard errors in an earlier version of the paper. We find that $\widehat{G}$ remains significant regardless of the bond maturity, whereas some of the PCs become insignificant. For instance, $PC^o_{2,t}$ remains significant only for the 4- and 5-year bonds and is subsumed by $\widehat{G}_t$ regardless of the bond maturity.

In summary, when factor $\widehat{G}$ is used as the macro-based return predictor, our in-sample test results show that this new macro variable has predictive power above and beyond the contemporaneous yield curve or yield dynamics, and thereby reject both Spanning Hypotheses I and II.

IA.B.6 Tests Using Macro Variables with Different Lags

So far the SAGLasso algorithm has been implemented using 131 macro variables along with six of their lags. In this subsection we address the following two questions: (1) Are lags of macro variables are essential to maintain the predictive performance as documented in Section 4, given

disentangle higher-order PCs from noise in yields. Filtered higher-order PCs ($PC_{4-5,t}$) contain more information about bond risk premia than higher-order observed PCs ($PC^o_{4-5,t}$).

that 21 constituent variables (out of 30) of G are lagged? (2) If so, what is the optimal number of lags to be included in our supervised learning?

These are nontrivial questions as a panel of macro data with no lags or a small number of lags has a denser structure and might deliver better out-of-sample performance given the limited length of the training period. To see this, recall that tuning parameters are selected using cross-validations in the SAGLasso algorithm (see Appendix C). Therefore, as we include more and more lags, the estimation process is inevitably subject to more "noise", which could overweigh benefits of incorporating more historical information in the construction of the SAGLasso factor.

In what follows, we repeat the analysis in Section 4.1 using 131 macro variables along with N_L of their lags, where $N_L = 0, 3, 9, 12$. To be more specific, for each value of N_L, we first reconstruct the SAGLasso factor following the procedures described in Appendix C and then examine the predictive power of the reconstructed SAGLasso factor.

Figure IA.B1 depicts the unconditional predictive power of the SAGLasso factor constructed using the macro data with $N_L = 0, 3, 6, 9, 12$. For brevity, we report the results for 2-year and 5-year bonds only. Panel A shows that including lags clearly enhances the in-sample predictive power of the SAGLasso macro factor.[IA.6] However, using more lags does not necessarily raise the R^2 value: it is the highest with $N_L = 3$ for the 2-year bond and with $N_L = 6$ for the 5-year bond.

As discussed above, including more than 6 lags may induce nontrivial sampling variability of the SAGLasso estimates that is sufficiently large to offset the gains from using more data. This conjecture is confirmed by the results for the out-of-sample R^2 shown in Panel B. Since the SAGLasso factor is estimated recursively (with a rolling 20-year window) in the out-of-sample analysis, we face greater uncertainty compared to the in-sample estimation. As a result, we find that the SAGLasso factor with $N_L = 9$ or 12 hardly outperforms the SAGLasso factor with $N_L = 0$ (no lag) in terms of the out-of-sample R^2.

Overall, the results shown in Figure IA.B1 suggest that the SAGLasso factor constructed using the 131 macro variables along with 3 or 6 of their lags has the best performance in both the in-sample and out-of-sample predictions. This finding reflects a trade-off between including more information

[IA.6] Note that including lags into the SAGLasso algorithm does not simply lead to an expansion in the set of selected macro variables. Instead, the coefficients of some previously selected (contemporaneous) variables are shrunk to zero, "crowded out" by more powerful lagged variables. For example, 29 macro variables are selected with $N_L = 0$, but only 9 of them have nonzero coefficients with $N_L = 3$.

in the supervised learning and imposing a denser data structure to enhance the estimation stability. While the baseline SAGLasso factor (with $N_L = 6$) seems to capture more information on long-term bond premiums, the alternative SAGLasso factor with $N_L = 3$ outperforms for short-term bonds.

Next, we examine whether or not the choice of lag length affects our inferences with respect to Spanning Hypotheses I and II. We test these two hypotheses using the above SAGLasso factor with different values of N_L and report the test results in panels A and B of Table IA.B7, respectively. As before, the test statistics used include the Hansen-Hodrick one, the Newey-West statistic, and ΔR^2 (the incremental in-sample R^2) for the in-sample tests, as well as the ENC-REG statistic, the ENC-NEW statistic, and ΔR^2_{oos} (the incremental out-of-sample R^2). Note that both of the spanning hypotheses are overwhelmingly rejected in both the in-sample and out-of-sample tests, regardless of the value of N_L considered. In particular, the two hypotheses are strongly rejected when no lags ($N_L = 0$) are used in the construction of the SAGLasso factor.

Finally, we perform the finite-sample analysis based on the SAGLasso factor with $N_L = 3$. Untabulated results show that the finite-sample critical values of the aforementioned six statistics are fairly close to their counterparts as reported in Table 4. It follows that this newly formed macro factor still results in a rejection of the two spanning hypotheses. Therefore, an alteration to the lag length does not change the conclusion on the finite-sample tests.

To summarize, our test results indicate that the choice of lag length hardly affects the our inferences with respect to the two spanning hypotheses.

IA.C Estimation and Selection of MTSMs

It is mentioned in Section 5.2.2 that in our estimation of MTSMs we use the canonical form of Gaussian MTSMs developed by Joslin, Le, and Singleton (2013; hereinafter JLS). This section reviews the JLS canonical form first. We then discuss restrictions on risk premium parameters.

IA.C.1 The Joslin, Le, and Singleton (2013) Canonical Form

We follow the JPS framework for MTSMs in Section 5.1. However, for the purpose of estimation, it is convenient to use a slightly different parameterization that is consistent with the JLS canonical form (following JPS and Duffee 2013a). Building on the Joslin, Singleton, and Zhu (2011) canonical form for YTSMs, the JLS canonical form defines the most general admissible Gaussian MTSM for a given dimension of the state vector.

Denote the state vector satisfying the JLS canonical form is denoted by X_t^*. Its $\mathbb{Q}$-measure dynamics and the resulting bond pricing formula are

$$\begin{aligned} r_t &= r_\infty^{\mathbb{Q}} + \imath \cdot X_t^*, && \text{(IA.C1)} \\ X_t^* &= \Phi_x^{*\mathbb{Q}} X_{t-1}^* + \Sigma_x^* \epsilon_t^{\mathbb{Q}}, && \text{(IA.C2)} \\ y_t^{(m)} &= A_m^*(\Theta_Y^{\mathbb{Q}}) + B_m^*(\Theta_Y^{\mathbb{Q}})' X_t^* && \text{(IA.C3)} \end{aligned}$$

where $r_\infty^{\mathbb{Q}}$ denotes the long-run mean of the short rate under $\mathbb{Q}$,[IA.7] $\imath$ is a vector of ones, $\Phi_x^{*\mathbb{Q}} - I$ has the real Jordan form determined by the eigenvalue vector $\gamma^{\mathbb{Q}}$, and Σ_x^* is lower triangular. Under this representation, $\Theta_Y^{\mathbb{Q}} \equiv \{\gamma^{\mathbb{Q}}, r_\infty^{\mathbb{Q}}, \Sigma_x^*\}$ governs X_t^*'s $\mathbb{Q}$-dynamics and thus fully determines bond pricing. Coefficients A_m^* and B_m^* are given by

$$\begin{aligned} B_m^* &= \frac{1}{m}\left(I - \Phi_x^{*\mathbb{Q}\prime}\right)^{-1}\left(I - (\Phi_x^{*\mathbb{Q}\prime})^m\right)\imath, \\ A_m^* &= r_\infty^{\mathbb{Q}} - \frac{1}{2m}\sum_{i=1}^{m-1} B_i^{*\prime}\Sigma_x^*\Sigma_x^{*\prime}B_i^*. \end{aligned}$$

While the state vector X_t^* defines the minimum number of parameters shaping the risk-neutral distribution of bond yields, it keeps silent about the role of macro factors F_t in bond pricing. Unless the macro-unspanning restrictions, as specified in Eq. (12), are imposed, F_t are included in MTSMs as pricing factors, i.e., there is a linear mapping between F_t and X_t^* as follows:

$$F_t = A_f + B_f X_t^*.$$

For ease of notation, in the discussion that follows in this this subsection, we drop the subscript/superscript $\mathcal{M}$ from $Y_t^{\mathcal{M}}$ and $\{\mathcal{A}_{\mathcal{M}}^*, \mathcal{B}_{\mathcal{M}}^*\}$, where $\mathcal{M}$ denotes the maturities of zero yields to be considered. Suppose that the yield-curve factors in X_t are defined by a full-rank loading matrix

[IA.7] In the JSZ canonical form there is no constant term in the short-rate equation (IA.C1). Instead, there is a constant term in the transition equation:

$$X_t^* = \mu_x^{*\mathbb{Q}} + \Phi_x^{*\mathbb{Q}} X_{t-1}^* + \Sigma_x^* \epsilon_t^{\mathbb{Q}},$$

where $\mu_x^{*\mathbb{Q}} = (u_\infty^{\mathbb{Q}}, 0_{1\times(\mathcal{N}-1)})'$. However, as long as X_t^* is stationary under the risk-neutral measure and the first element of $\gamma^{\mathbb{Q}}$ is non-repeated, $r_\infty^{\mathbb{Q}}$ and $u_\infty^{\mathbb{Q}}$ are interchangeable in defining the canonical form: $r_\infty^{\mathbb{Q}} = -u_\infty^{\mathbb{Q}}/\gamma_1^{\mathbb{Q}}$.

$W_{\mathcal{L}} \in \mathbb{R}^{\mathcal{L}\times\mathcal{L}}$, i.e., $\mathcal{P}_t = W_{\mathcal{L}} Y_t$. It follows that the latent state vector X_t^* can be rotated to X_t[IA.8]

$$X_t = \Gamma_0 + \Gamma_1 X_t^*,$$

where

$$\Gamma_0 = \begin{bmatrix} W_{\mathcal{L}}\mathcal{A}^* \\ A_f \end{bmatrix}, \Gamma_1 = \begin{bmatrix} W_{\mathcal{L}}\mathcal{B}^* \\ B_f \end{bmatrix}.$$

The resultant bond-pricing coefficients for the rotated state vector X_t are

$$\begin{aligned} B_m &= \Gamma_1^{-1\prime} B_m^*, \\ A_m &= A_m^* - B_m \Gamma_0. \end{aligned}$$

This leads to a closed-form expression for the probability density function of observed yields conditional on X_t, which completes the maximum likelihood estimation.

Note that $\{A_m, B_m\}$, defined in Eq. (8), depend on $\Theta_M^{\mathbb{Q}} \equiv \{\gamma^{\mathbb{Q}}, r_\infty^{\mathbb{Q}}, A_f, B_f, \Sigma_x^*\} \supset \Theta_Y^{\mathbb{Q}}$. As such, adding macro factors to DTSMs allows for greater flexibility in fitting the conditional distribution of bond yields, as evidenced by the $(\mathcal{N}-\mathcal{L})(\mathcal{N}+1)$ additional free parameters in MTSMs.[IA.9]

Even if we ignore the additional flexibility offered by F_t, it is preferable to factorize the conditional likelihood function in terms of $X_t = (\mathcal{P}_t', F_t')$, as opposed to latent factors X_t^*. First, if the yield portfolios as represented by $\mathcal{P}_t$ are assumed to be priced perfectly (JSZ; JPS), the $\mathbb{P}$-measure conditional density of state variables, $l(X_t|X_{t-1}, \mu_x^{\mathbb{P}}, \Phi_x^{\mathbb{P}}, \Sigma_x)$, can be assessed with standard linear projection; JSZ show that the OLS leads to ML estimators of $\{\mu_x^{\mathbb{P}}, \Phi_x^{\mathbb{P}}\}$. Second, even if we allow all yields to be measured with error, an OLS regression of X_t^o $(= X_t + \eta_t)$ provides fairly reasonable starting values in the estimation of $\{\mu_x^{\mathbb{P}}, \Phi_x^{\mathbb{P}}, \Sigma_x\}$.

IA.C.2 Selection of MTSMs

In Sections IA.G.2 and IA.G.3 of the paper, we follow JPS and conduct a large-scale search for the best set of zero restrictions on risk premium parameters in constrained models $CSM(\mathcal{L},\mathcal{N})$ and $CUSM(\mathcal{L},\mathcal{N})$. This section provides details of this analysis.

[IA.8]The invariant transformation from X_t^* to X_t calls for the loading matrix $W_{\mathcal{L}}$. As the number of yield factors $\mathcal{L} \leq 5$ in models considered in Sections 5.2 and IA.G, $W_{\mathcal{L}}$ is estimated based on model $YTSM(5)$ (see Internet Appendix IA.A for details). Unreported results show that the first three rows of W_5 are almost identical to those of W_5^o (as well as the loading matrix implied from model $YTSM(3)$), but there is substantial difference in the remaining rows.

[IA.9]Therefore, model $SM(\mathcal{L},\mathcal{N})$ has $2.5\mathcal{N}^2 + 3.5\mathcal{N} - \mathcal{N}\mathcal{L} - \mathcal{L} + 2$ parameters in total to estimate.

Recall from Section IA.G.1 that $CSM(\mathcal{L},\mathcal{N})$ and $CUSM(\mathcal{L},\mathcal{N})$ denote $\mathcal{N}$-factor constrained, spanned and unspanned MTSMs, respectively, where the underlying state vector $X_t = (PC_{1-\mathcal{L},t}, \widehat{G}_t)$ and $PC_{1-\mathcal{L}} = (PC_1, \ldots, PC_{\mathcal{L}})$ denotes the first $\mathcal{L}$ PCs of bond yields. The one-period risk premium is as specified in Eq. (13):

$$\Sigma\Lambda_t = \lambda_0 + \lambda_1 X_t \;=\; \lambda_0 + \lambda_1 \cdot (PC_{1-\mathcal{L},t},\; \widehat{G}_t)',$$

where risk premium parameters λ_0 and λ_1 are an $\mathcal{N}$-dimensional vector and an $\mathcal{N} \times \mathcal{N}$ matrix, respectively. In the discussion below, we focus on the selection of spanned models $CSM(\mathcal{L},\mathcal{N})$. The selection of unspanned models is done similarly.

Table IA.C1 shows the maximum likelihood estimates of λ_0 and λ_1 in models selected by BIC, under $CSM(3,4)$ (panel A), $CSM(4,5)$ (panel B), and $CSM(5,6)$ (panel C), respectively. Note from the three panels that while the estimates are model dependent, they show three robust properties that hold regardless of the model dimension $\mathcal{N}$.[IA.10] First, both $\lambda_1(1,\mathcal{N})$ and $\lambda_1(2,\mathcal{N})$ are negative and statistically significant, $\forall \mathcal{N} = 4, 5, 6$. For instance, $\lambda_1(1,4) = -6.11\text{e-}4$ and $\lambda_1(2,4) = -1.45\text{e-}4$ (panel A); and $\lambda_1(1,6) = -6.47\text{e-}4$ and $\lambda_1(2,6) = -2.60\text{e-}4$ (panel C). This finding suggests that $\widehat{G}_t$ drives time variations in both expected excess returns to PC_1 and PC_2. In addition, note that the ratio, $\lambda_1(1,\mathcal{N})/\lambda_1(2,\mathcal{N})$, ranges from 2.5 ($\mathcal{N} = 6$) to 6.2 ($\mathcal{N} = 5$), suggesting that $\widehat{G}_t$ influences excess bond returns mainly through its impact on the "level" risk premium.

Second, in all three models $\{CSM(\mathcal{N}\text{-}1,\mathcal{N}),\ \mathcal{N} = 4, 5, 6\}$, the risk premium driving factors include the first two factors that govern the market prices of "level" and "slope" risks, and the first one appears more important in shaping the unconditional bond risk premia (Koijen et al., 2010). More specifically, persistent contributors to the first risk-premium factor include $PC_{1,t}$, $PC_{2,t}$ and $\widehat{G}_t$; those to the second risk-premium factor include $PC_{3,t}$ and $\widehat{G}_t$. Furthermore, if a model, say, model $CSM(5,6)$, allows for hidden yield factors, then the level risk premium significantly varies with the fifth PC as well (row 1 in panel C). Note that conditioning only on yield curve information (and not on macro variables), the models of Cochrane and Piazzesi (2008) and Duffee (2011) suggest that variations in expected excess bond returns are driven by a single factor.

Third, rows corresponding to $\{PC_{i,t}, i \geq 3\}$ in both λ_0 and λ_1 are uniformly zero in every panel. Hence, among yield PCs only the level and slope risks are priced. This result coincides with

[IA.10] Unreported results indicate that these three properties also emerge in our model selections for unspanned models.

JPS's finding. Duffee (2010) also documents that there are two factors driving the variation in risk premium and presents evidence that this is a robust property of models with the Sharpe ratio constraints. These findings in turn help explain why the restrictions placed on λ_0 and λ_1 make model-implied Sharpe ratios consistent with ones observed in data.

Note that while all three models, $CSM(\mathcal{N}\text{-}1, \mathcal{N})$ with $4 \leq \mathcal{N} \leq 6$, imply non-zero compensation for exposure to the macro risk, the loadings of relevant risk premium on state variables are not robust across models as shown in the last row in each panel. One implication of this result is that the loadings on macro state variables may be difficult to estimate robustly via yield factors in a spanned model. On the other hand, unspanned models are not subject to this problem as λ_0 and λ_1 include no such rows on unspanned macro factors (untabulated).

IA.D Data-Generating Processes Based on VARs

Section 5.2 of the paper presents a finite-sample analysis of Spanning Hypotheses I & II using MTSMs as data-generating processes (DGPs). This section examines an alternative, VAR-based DGP, generated using an approach proposed by Bauer and Hamilton (2018) to address small-sample issues in testing Spanning Hypothesis I (H_0^{S1}).

We first illustrate that the parametric bootstrap design proposed in Bauer and Hamilton (2018; BH hereinafter) is actually more suitable for testing the unconditional predictive power of macro variables than testing H_0^{S1}. We then show that the spanned MTSM specified in Section 5.2.2 provides a more robust test of H_0^{S1} in finite sample analysis than does the VAR-based DGP.

IA.D.1 VAR-based DGPs

BH model the joint dynamics of bond yields and a j-dimensional macroeconomic vector F_t using the following restricted VAR system:

$$Y_t^o = U_{\mathcal{M}} \cdot PC_{1-3,t}^o + \varepsilon_t, \tag{IA.D4}$$

$$\begin{bmatrix} PC_{1-3,t}^o \\ F_t \end{bmatrix} = \begin{bmatrix} \mu_p \\ \mu_f \end{bmatrix} + \begin{bmatrix} \Phi_{pp} & 0_{3\times j} \\ 0_{j\times 3} & \Phi_{ff} \end{bmatrix} \begin{bmatrix} PC_{1-3,t-1}^o \\ F_{t-1} \end{bmatrix} + \begin{bmatrix} \Sigma_p & 0_{3\times j} \\ 0_{j\times 3} & \Sigma_f \end{bmatrix} \begin{bmatrix} \epsilon_t^{\mathcal{P}} \\ \epsilon_t^F \end{bmatrix} \tag{IA.D5}$$

where Y_t^o denotes the time-t observed yields of k zero-coupon bonds with maturities $\mathcal{M} = \{m_1, \ldots, m_k\}$, $U_{\mathcal{M}}$ is a $k \times 3$ matrix with columns equal to the first three eigenvectors of the variance matrix of Y_t^o, and the diagonal matrix ε_t represents fitting errors.

We aim to show that the parameter restrictions specified in Eq. (IA.D5) have a close affinity to the restrictions required for the MTSM in Section 5.1 to satisfy the hypothesis that macro variables have no predictive power for excess bond returns unconditionally (under the $\mathbb{P}$-measure). Following Duffee (2007), we refer to this hypothesis as the "general" null hypothesis (GNH). To proceed, we first introduce such restrictions, termed "macro-independence" restrictions in this study.

IA.D.2 "Macro-Independence" Restrictions

Consider the MTSM in Section 5.1. Given that the expected excess return on an m-period bond from t to $t+j$ is

$$\begin{aligned} E_t\left(rx_{t,t+j}^{(m)}\right) &= \text{constant} + \psi'_{m,j} X_t, \\ \text{where} \quad \psi_{m,j} &= mB'_m - (m-j)B'_{m-j}(\Phi^{\mathbb{P}})^j - jB'_j, \end{aligned} \tag{IA.D6}$$

the GNH implies that the last $\mathcal{N}$-$\mathcal{L}$ columns of the model-implied matrix $\psi_{m,j}$ are entirely zero, regardless of bond maturity m or return horizon j. How to implement such restrictions in the model depends on j. Recall that, in our empirical analysis, predictive regressions use annual excess returns sampled at the monthly frequency, while MTSMs are estimated with monthly observations.

Let $\lambda_1 = [\lambda_{1p}, \lambda_{1f}]$ in Eq. (9). If $j = 1$ (month), then setting λ_{1f} to zero prevents macro factors from affecting expected one-period excess returns. Without loss of generality, we allow all $\mathcal{L}$ yield curve factors $\mathcal{P}_t$ to drive variations in bond risk premia. As a result,

$$\psi_{m,1} = -(m-1)B'_{m-1}\lambda_1 \; = \; -(m-1)B'_{m-1}\left[\lambda_{1p}\, , \, 0_{\mathcal{N}\times(\mathcal{N}-\mathcal{L})}\right]. \tag{IA.D7}$$

Under this specification, $E_t\left(rx_{t,t+1}\right)$ is orthogonal to the macro state vector F_t. However, F_t can still affect longer-horizon ($j > 1$) excess returns because future monthly returns, $\{E_t\left(rx_{t+i,t+i+1}\right)\}_{i\geq 1}$, are not orthogonal to F_t. For instance, note that $E_{t+1}\left(rx_{t+1,t+2}\right)$ is determined by $\mathcal{P}_{t+1}$ and $E_t(\mathcal{P}_{t+1})$ depends on F_t. Consequently, F_t contains information about future excess annual returns.

As a result, when $j > 1$, to ensure the state variables determining term premia to vary independently of the macro factors, we specify the following $\mathbb{P}$-measure dynamics of X_t:

$$X_t = \begin{bmatrix} \mathcal{P}_t \\ F_t \end{bmatrix} = \begin{bmatrix} \mu_p^{\mathbb{P}} \\ \mu_f^{\mathbb{P}} \end{bmatrix} + \begin{bmatrix} \Phi_{pp}^{\mathbb{P}} & 0_{\mathcal{L}\times(\mathcal{N}-\mathcal{L})} \\ 0_{(\mathcal{N}-\mathcal{L})\times\mathcal{L}} & \Phi_{ff}^{\mathbb{P}} \end{bmatrix} \begin{bmatrix} \mathcal{P}_{t-1} \\ F_{t-1} \end{bmatrix} + \Sigma_x \, \epsilon_{x,t}^{\mathbb{Q}}. \tag{IA.D8}$$

That is, the variation in F_t is independent of expected monthly bond returns at all leads and lags; thus, even for annual excess returns, the last $\mathcal{N}$-$\mathcal{L}$ columns of $\psi_{m,12}$ are constrained to be zero.

Eqs. (IA.D7) and (IA.D8) together lead to the following conditions, termed "macro-independence" restrictions and denoted by H_0^{MI}, for the model to satisfy the GNH:

$$H_0^{MI}: \ \Phi_{fp}^{\mathbb{P}} = 0, \quad \Phi_{pf}^{\mathbb{Q}} = \Phi_{pf}^{\mathbb{P}} = 0, \ \text{and} \ \Phi_{ff}^{\mathbb{Q}} = \Phi_{ff}^{\mathbb{P}}. \tag{IA.D9}$$

Let $MIM(\mathcal{L}, \mathcal{N})$ denote the model subject to these restrictions. Unless specified otherwise, we focus on MTSMs with $\mathcal{N} = \mathcal{L} + 1$ and $F_t = G_t$ in the analysis that follows. For instance, model $MIM(3,4)$ is used below to conduct the finite-sample inference about the GNH.

IA.D.3 VAR-based DGPs and Tests of the GNH

Note that the parameter restrictions specified in Eq. (IA.D5) are very close to the "macro-independence" restrictions given in Eq. (IA.D9) under model $MIM(3,4)$. The only fundamental difference between the VAR-based model in Eqs. (IA.D4) and (IA.D5) and model $MIM(3,4)$ is that the former does not rely on the Duffie and Kan (1996) restrictions for an affine mapping from bond yields to the yield-curve factors. However, empirically this difference is expected to have little impact on the dynamics of expected excess returns, as matrix $U_{\mathcal{M}}$ obtained from the PCA does not significantly deviate from the loading matrix $\mathcal{B}_{\mathcal{M}}$ in Eq. (11).[IA.11] Therefore, like model $MIM(3,4)$, the above VAR-based model implies that term premia are time-varying and driven by yield PCs only; that is, by construction, the macro factors F_t have no predictive power for future yields and bond returns. As such, the VAR-based model in Eqs. (IA.D4) and (IA.D5) satisfies the GNH rather than H_0^{S1} stated in Section 2.2. Put differently, as macro risks are not priced at all in this VAR-based DGP, it is not suitable for conducting tests of evidence for unspanned macro risks.

To further illustrate this point, we generate bootstrap samples using the VAR-based model and

[IA.11] To see this, another equivalent approach to estimating Eq. (11) is regressing the bond yields on yield PCs. While the Duffie-Kan restrictions are not imposed in this estimation (unless the number of factors equals $k-1$), the small magnitude of measurement errors ensures that the OLS-implied loading matrix for $PC_{1-3,t}$ is very close to $\mathcal{B}_{\mathcal{M}}$ if the term structure is truly described by a no-arbitrage dynamic term structure model (Duffee 2010a).

investigate the properties of regression statistics under the same DGP. To proceed, letting F_t be the single SAGLasso factor G_t in the model, we estimate $\mu_p, \mu_f, \Phi_{pp}, \Phi_{ff}, \Sigma_p$, and Σ_f with MLE as in Section 5. Next, we generate bootstrap samples from Eqs. (IA.D4) and (IA.D5) and use a residual bootstrap to resample the PCs and SAGLasso factor based on Eq. (IA.D5). We construct bootstrapped yields, Y_t^b, as follows:

$$Y_t^b = U_{\mathcal{M}} \cdot PC_{1-3,t}^b + \eta_t^b,$$

where $PC_{1-3,t}^b$ denotes the vector of three bootstrapped PCs. Following BH, η_t^b is generated from $MVN(0, \sigma_\eta^2 I)$, where σ_η is set to the sample standard deviation of the fitting errors $\hat{\varepsilon}_t$ (pooled across maturities).[IA.12] Finally, excess bond returns are calculated using bootstrapped yields.

IA.D.4 Finite Sample Analysis Using the VAR-based DGP

What if the above VAR bootstrap design is used to examine the finite-sample properties of the regression in Eq. (1) in tests of H_0^{S1}? To answer this question, we examine finite-sample distributions of regression statistics in testing H_0^{S1}:

$$rx_{t,t+12}^{(12n)} = \alpha + \beta_p' PC_{1-3,t}^o + \beta_g G_t + e_{t+12}. \tag{IA.D10}$$

Table IA.D1 reports the results. A comparison of panels A1–B2 of the table with their counterparts in Table 3 based on model $SM(2,3)$ reveals that the VAR-based bootstrap still understates the size distortions in the regression in Eq. (IA.D10). Indeed, the 5% critical values implied by model $SM(2,3)$ are more than twice as great as those implied by the VAR-based model for most statistics/maturities. The discrepancy between these two DGPs is substantial in both in-sample and out-of-sample analyses and especially glaring in the coefficients of determination. For instance, panel A1 of Table IA.D1 indicates that the upper bound of the 95% confidence interval for ΔR^2 is around 3.3%, but this upper bound is merely comparable to the median of the $SM(2,3)$-implied distributions. More precisely, the VAR-based 5% critical value has a true size of up to 46%, implying that the finite-sample test based on the VAR bootstrap design would reject the null more than eight times as often as it should.

For completeness, panels A3–B4 of Table IA.D1 report the finite-sample distributions implied

[IA.12] We find that replacing these simulated measurement errors with the ones bootstrapped from the actual (maturity-specific) fitting errors has only marginal impact on the finite-sample distributions.

by the macro-independent model $MIM(3,4)$. As expected, they closely resemble their VAR-based counterparts illustrated in panels A1–B2 of the table. Namely, both $MIM(3,4)$ and the VAR-based DGP differ sharply from spanned MTSMs and lead to inflated rejection rates in tests of H_0^{S1}.

To summarize, in our case, finite-sample tests of H_0^{S1} using the VAR-based DGP is actually oversized and thus biased against the null hypothesis. In contrast, the spanned MTSM specified in Section 5.2.2 provides a more relevant and robust test of H_0^{S1} in finite sample analysis.

IA.E Ibragimov-Müller Tests of Spanning Hypotheses I and II

This section conducts an alternative and robust test of H_0^{S1} and H_0^{S2}, drawing an inference about the hypotheses based on the test developed by Ibragimov and Müller (2010; IM hereinafter).

It is known that standard heteroscedasticity and autocorrelation consistent (HAC) corrections perform poorly in small samples. The IM test can improve the performance of these procedures by not relying on consistency of the given variance estimator. In IM's approach, regression coefficients β are estimated q times on q subsets of the whole sample. IM prove that, for each coefficient β_i, the t-statistic computed from the q estimates of $\hat{\beta}_i$ has approximately the same distribution as a standard t-statistic computed from independent and zero-mean Gaussian variables. Müller (2014a) finds that the IM test has outstanding size and power properties in the presence of strongly autocorrelated of regression disturbances. Müller (2014b) further notes that the IM test is an "attractive choice" for predictive regression problem and is also robust to structural breaks.

Following Müller (2014a), we divide the whole sample into q nonoverlapping consecutive blocks of (approximately) equal length, with $q = 8$ or 16. Table IA.E1 reports the p-values of the resultant t-tests of both H_0^{S1} and H_0^{S2}, for both the full and post-1984 samples. As the IM test assumes the independence of blocks, we insert 12-month gaps between adjacent blocks in the full-sample analysis. As such, the regression coefficients estimated from different blocks of data are arguably independent from each other. For brevity, we report the testing results for the average excess bond return only, which is over two- through four-year (ten-year) maturities for the 1964–2014 (1985–2014) sample, as maturity-specific estimates for each of q sample subsets are rather noisy. While the evidence on $PC^o_{2,t}$ (the "slope" factor) is consist with BH, $PC^o_{1,t}$ (the "level" factor) becomes insignificant in the post-1984 sample when $Z_t = \widehat{G}_t$ (the SAGLasso factor). However, even the strong evidence for the predictive power of $PC^o_{2,t}$ is tempered when we consider H_0^{S2}: its p-value skyrockets to 0.33 and 0.38 in the full and post-1984 samples, respectively. In contrast, the p-values of $\widehat{G}_t$ are uniformly lower than 0.05 for both H_0^{S1} and H_0^{S2}, regardless of the choice of q.

Overall, the IM tests indicate that among the five yield curve factors and the macro factor $\widehat{G}_t$, the latter is the only robust predictor of future excess bond returns at the 5% significance level.

IA.F An Alternative Version of Spanning Hypothesis II

In the tests of H_0^{S2} conducted so far, the yield-curve factors used in the hypothesis are the first five principal components (PCs) of the noise-uncontaminated yield curve. As mentioned in Section 2.2, including the higher-order PCs is motivated by the notion of hidden factors à la Duffee (2011). This section introduces and tests another version of H_0^{S2} that is based on an alternative set of the yield-curve factors, the "cycle" factor ($\widehat{cf}$) of Cieslak and Povala (2015). As noted in Cieslak and Povala (2015), the cycle factor is spanned (see also Cieslak 2018), as well as analogous to the single risk premium factor in Duffee (2011) that contains a hidden component.

Cieslak and Povala (2015) propose an illustrative three-factor dynamic term-structure model (DTSM) in which $\widehat{cf}$ corresponds to a single "risk premium factor" denoted by x_t, where x_t captures all of forecastable variation in one-year expected excess returns for bonds of all maturities. While the Cochrane and Piazzesi (2005) factor ($\widehat{CP}$) plays a similar role in the DTSM proposed in Cochrane and Piazzesi (2008), Cieslak and Povala (2015) demonstrate that their methodology (based on linear projections of yields on trend inflation) is more effective in recovering the variation in risk premiums from noise-contaminated yields and, as a result, $\widehat{cf}$ subsumes $\widehat{CP}$ in predicting excess bond returns. In other words, x_t is analogous to Duffee (2011)'s single risk premium factor, RP_t, that determines the *one-month-ahead* risk premia on all bonds.[IA.13] In particular, x_t contains a hidden component that cannot be detected using the cross-section of yields and that needs to be inferred, say, with a proxy for trend inflation as done in Cieslak and Povala (2015). In this sense, x_t can be regarded as an "annual" version of RP_t and, accordingly, $\widehat{cf}$ maps to the smoothed estimate of RP_t obtained in Duffee (2011). That is, as an estimate of x_t, $\widehat{cf}$ summarizes all information on *one-year-ahead* risk premia.

[IA.13] The state vector underlying the five-factor DTSM in Duffee (2011) consists of the first five PCs of yield innovations. As a result, RP_t is a linear combination of these five PCs.

It follows that we can formulate an alternative version of H_0^{S2} using $\widehat{cf}$ as the conditioning variable:

> $H_0^{S2,cf}$: The SAGLasso macro factor $\widehat{G}$ has no additional predictive power for bond risk premia in the presence of $\widehat{cf}$.

One way to test $H_0^{S2,cf}$ is based on the following predictive regression of excess bond returns:

$$rx_{t,t+12}^{(12n)} = \alpha + \beta_c' \widehat{cf}_t + \beta_g' \widehat{G}_t + e_{t+12}. \tag{IA.F11}$$

As mentioned in Section 4.4.4, we find that in this setting $\widehat{\beta}_g$ is highly significant—based on asymptotic distributions of test statistics. See Table IA.F1 for the results from both in-sample (panel A) and out-of-sample (panel B) tests of $H_0^{S2,cf}$.[IA.14]

To understand finite-sample properties of the regression in Eq. (IA.F11), we extend Cieslak and Povala (2015)'s three-factor DTSM to include the macro factor G_t, and then use this extended model as the DGP for simulation. Note that the structure of this DGP is the same as that presented in Section 5.3.1, except that the state vector here is rotated to $\mathbb{X}_t = (\tau_t, r_t^r, x_t, G_t)$, where τ_t denotes trend inflation and r_t^r the real short rate. Following Cieslak and Povala (2015), we measure τ_t and r_t^r by τ_t^{CPI} and $c_t^{(1)}$, respectively when estimating the model, where τ_t^{CPI} is a discounted moving average of the past 10-year realized core CPI with the gain parameter of 0.99 and $c_t^{(1)}$ the fitted residual from univariate regressions of yields $y_t^{(12n)}$ on τ_t^{CPI}.

The bootstrap design adopted for the regression in Eq. (IA.F11), however, departs from that discussed in Section 5.2 in two aspects. First, the entire model is presented at the annual frequency. As such, we adopt Cieslak and Povala (2015)'s specification of one-period-ahead risk premia:

$$\Sigma\Lambda_t = \begin{bmatrix} \lambda_{0\tau} \\ \lambda_{0\mathfrak{r}} \\ 0_{2\times 1} \end{bmatrix} + \begin{bmatrix} 0 & 0 & \lambda_{1\tau} & 0 \\ 0 & 0 & \lambda_{1\mathfrak{r}} & 0 \\ & & 0_{2\times 4} & \end{bmatrix} \mathbb{X}_t \tag{IA.F12}$$

This specification guarantees that x_t fully determines variations in expected excess *annual* returns. Second, as in Cieslak and Povala (2015), τ_t, r_t^r, and x_t are assumed to evolve independently of each

[IA.14] As is the case of our out-of-sample exercises for the post-1984 sample, as described in Section 4.4.2, we use the initial 15 years as our training sample. That is, the initial coefficient estimates are obtained based on the period from November 1971 to October 1986. Both $\widetilde{cf}$ and $\tilde{G}$ are constructed recursively afterwards.

other. It follows that the $\mathbb{P}$-measure dynamics of the state variables are

$$\mathbb{X}_t = \mu_x^{\mathbb{P}} + \begin{bmatrix} \phi_{\tau\tau}^{\mathbb{P}} & 0 & 0 & \phi_{\tau g}^{\mathbb{P}} \\ 0 & \phi_{\mathfrak{r}\mathfrak{r}}^{\mathbb{P}} & 0 & \phi_{\mathfrak{r}g}^{\mathbb{P}} \\ 0 & 0 & \phi_{xx}^{\mathbb{P}} & \phi_{xg}^{\mathbb{P}} \\ \phi_{x\tau}^{\mathbb{P}} & \phi_{g\mathfrak{r}}^{\mathbb{P}} & \phi_{gx}^{\mathbb{P}} & \phi_{gg}^{\mathbb{P}} \end{bmatrix} \mathbb{X}_{t-1} + \begin{bmatrix} \sigma_{\tau\tau} & 0 & 0 & \sigma_{\tau g} \\ 0 & \sigma_{\mathfrak{r}\mathfrak{r}} & 0 & \sigma_{\mathfrak{r}g} \\ 0 & 0 & \sigma_{xx} & \sigma_{xg} \\ \sigma_{x\tau} & \sigma_{g\mathfrak{r}} & \sigma_{gx} & \sigma_{gg} \end{bmatrix} \epsilon_{x,t}^{\mathbb{P}}. \tag{IA.F13}$$

Note that the parameter $\phi_{xg}^{\mathbb{P}}$ is important as it determines whether G_t has unconditional predictive power for excess bond returns; if $\phi_{xg}^{\mathbb{P}} = 0$, the model in Eq. (IA.F13) degenerates into conformity with Duffee (2007)'s GNH. Under the specification in Eq. (IA.F12), G_t contains no *conditional* predictive power when x_t is controlled for, regardless of the value of $\phi_{xg}^{\mathbb{P}}$.

As, by definition, the risk-premium factor x_t does not affect the short rate, we specify the following equation to complete the model:

$$r_t = \delta_0 + \delta_r r_t^r + \delta_\tau \tau_t, \tag{IA.F14}$$

where r_t denotes the one-year yield with $\delta_r > 0$ and $\delta_\tau > 0$.

The MTSM as represented by Eqs. (IA.F12)–(IA.F14) is estimated using zero yields with maturities of one through ten years over the full sample period 1971.11–2014.12 (matching the beginning of the sample used in Cieslak and Povala 2015). The estimated model is then used to generate 5,000 bootstrapped data samples.

Table IA.F1 summarizes the finite-sample properties of the six test statistics used that are based on the 5,000 bootstrapped data samples, including the 95th percentile of the bootstrap distribution (underlined as the 5% critical value in the table) and the p-value (in angle brackets) for each test statistic. A comparison of these finite-sample critical values with those (under H_0^{S2}) reported in panels B3 and B4 of Table 3 reveals that the small-sample bias is less severe in the regression in Eq. (IA.F11) than that in Eq. (1) specified for testing H_0^{S2}. Consistent with the conclusion drawn from their asymptotic distributions, the bootstrap distributions of all statistics shown in Table IA.F1 overwhelmingly reject the null hypothesis $H_0^{S2,cf}$—that $\widehat{G}_t$ contains no predictive power conditioned on $\widehat{cf}_t$—at the 5% significance level, with the only exception of the ENC-REG test for the 7-year bond for which the small-sample p-value is 6.6%.

To summarize, the above results of tests of the spanning hypothesis $H_0^{S2,cf}$ provide further evidence that the SAGLasso macro factor has significant, additional predictive power for excess bond returns conditioning on the yield curve information.

IA.G Unspanning Tests and Applications of Unspanned Models

This section focuses on unspanned MTSMs with the SAGLasso factor as the sole macro risk factor. We formally describe the macro-unspanning hypothesis (MUH) in Section 5.3.1 and then investigate its statistical significance as well as its economic importance in Sections 5.3.2 through IA.G.2. Lastly, Section IA.G.3 quantifies the information content of the SAGLasso factor.

While the above test results demonstrate the empirical relevance of G_t as an unspanned macro risk, the tests of the MUH per se are more interesting statistically than economically. On the one hand, to be statistically legitimate, the MUH has to be formulated as Eq. (12). On the other hand, the consensus is that, in general, macro variables hold greater promise in helping to improve a term structure model's time-series accuracy than its goodness-of-fit (e.g., Duffee (2011) finds that a $YTSM(5)$ is adequate for producing fitting errors of 6 bps). Given this insight, a more relevant question to ask is whether using G_t as a pricing factor has any economic benefits. Put differently, does an unspanned MTSM with G_t as its sole macro factor provide any added *economic* value over an otherwise spanned model? As shown below, the answer to these questions depends on whether MTSMs are subject to certain constraints on their model-implied Sharpe ratios.

IA.G.1 Model-Implied Sharpe Ratios

One issue not addressed in the likelihood-ratio tests considered in Section 5.3.2 (as well as in BR) is that the MTSMs under scrutiny impose no constraints on the Sharpe ratio (SR) of bond returns and that such "unconstrained" models may imply unrealistic SRs, as noted in Duffee (2010) and Joslin, Singleton, and Zhu (2011; JSZ hereafter). Specifically, Duffee documents that while the empirical benchmark for the unconditional maximum SR is 0.15∼0.20, SRs implied from unconstrained Gaussian dynamic term structure models in his analysis are much higher than the benchmark.

Untabulated results indicate that among the three spanned MTSMs, $\{SM(\mathcal{L}, \mathcal{N})\}_{3 \leq \mathcal{L} \leq 5}$, considered in panel A of Table 4, even the most "reasonable" model-implied sample mean (population mean) of conditional maximum SRs is 0.715 (0.825) when SRs are computed with log returns; the sample mean increases to 1.309 when SRs are computed with simple returns. Consistent with Duffee (2010), we find that the model-implied SRs increase with the model dimension. For model $SM(5,6)$, the sample mean of the maximum conditional SRs could even be higher than 10^{35} (for simple returns), an obviously unplausible level.[IA.15] As such, though statistically appealing, the test results presented in panel A of Table 4 are based on misspecified models.

One way to ensure that an MTSM generates plausible SRs is to directly impose restrictions on risk premia, say, that only the level and slope risks be priced (a restriction suggested by Duffee 2010 and implemented in Duffee 2011). Another way is to let the data decide what restrictions are empirically relevant. We implement the latter approach by following JPS to search for the best zero restrictions on risk premium parameters $\{\lambda_0, \lambda_1\}$ that minimize the Bayesian information criterion (note that Λ_t essentially represents SRs of bond portfolios with payoffs that track the pricing factors). The resultant models selected by this approach (see Internet Appendix IA.C.2) all possess the following two properties: (a) variations in expected excess bond returns are mainly driven by two factors and, (b) the SAGLasso factor plays a significant role in both term-premium factors. Importantly, conditional maximum SRs implied by these selected models are generally in line with those observed empirically. For convenience, the MTSMs with the selected zero restrictions on $\{\lambda_0, \lambda_1\}$ are referred to as constrained MTSMs and denoted by $CSM(\mathcal{L},\mathcal{N})$ ($CUSM(\mathcal{L},\mathcal{N})$) for spanned (unspanned) models, with $\mathcal{L}$ being the number of yield factors included in the model.

With model selections performed on market prices of risk, unspanned and spanned models are no longer nested, however, and as a result, the LR test-based statistical inference made in Section 5.3.2 no longer applies. Nonetheless, as shown below we can still measure the economic values of the macro-unspanning restrictions imposed on constrained models.

IA.G.2 Out-of-Sample Forecasts of Bond Yields

This subsection investigates whether it is beneficial to include the SAGLasso factor as unspanned by the yield curve in an MTSM. We consider MTSMs with and without the macro-unspanning restrictions and examine whether these restrictions help to forecast future bond yields. We seek to

[IA.15] Untabulated results indicate that this problem persists in MTSMs tested by BR, in which our SAGLasso factor is replaced with (GRO, INF), two macro factors often used in this literature.

quantify the effectiveness of these restrictions as forecasting tools.

We focus on six-factor models in this analysis given Duffee's (2011) argument that five yield factors summarize all information (in both the time series and cross section) in the yield curve. Regardless, including at least five yield factors instead of three makes it harder to see the importance of the macro-unspanning restrictions.

The procedure is similar to the out-of-sample analysis in Section 4.4.2. However, since recursive estimation of MTSMs is computationally very costly (especially for models $CSM(5,6)$ and $CUSM(5,6)$, which require model selection for the risk premium), our yield forecasts are formed based on model estimates for the 1985–2007 sample. With the model parameters fixed, we refilter yield factors using observations up to month t ($\geq$ 2007.12) and then construct forecasts of the T-year bond yield in month-$(t+h)$, where $T = 0.5, 1, 3, 5, 7, 10$ and $h = 1, 3, 6, 12$ in our empirical analysis. The out-of-sample (test) period extends from January 2008 to December 2013.

Panel A of Table IA.G1 reports the root mean squared forecast error (RMSE) produced by unconstrained models $SM(5,6)$ and $USM(5,6)$ for each of 24 combinations of T and h. Note that the models deliver closely comparable forecasting performance, especially at short horizons. This finding is not surprising, given that they should produce identical yield forecasts if $PC_{1-5,t}$ is assumed to be observed without error (see Section 4.2 of JSZ). Although our assumption that all bonds (and portfolios) are priced imperfectly prevents us from exploiting the JSZ-type separation of parameters in the likelihood function, the assumption allows the macro-unspanning restrictions to affect the filtering process and thus the model estimations. As indicated by our empirical results, this impact makes a sizable difference only at the one-year horizon, where $USM(5,6)$ provides more accurate forecasts at the short end of the yield curve but is outperformed by $SM(5,6)$ at the long end. Nonetheless, recall that both $SM(5,6)$ and $USM(5,6)$ generate unrealistic model-implied SRs.

Panel B of Table IA.G1 shows the results from constrained models $CSM(5,6)$ and $CUSM(5,6)$. They too have similar forecasting performance when the forecast horizon is short with h=1,3 (month). However, when h=6 or 12, $CUSM(5,6)$ significantly outperforms $CSM(5,6)$, especially for the 1-year and longer maturity yields. For example, when h=12, the unspanning restrictions reduce the forecast error by as much as 30 bps for the 3-year yield or 40 bps for the 7-year yield. That is, the improvements in forecasting performance owing to an unspanned $\widehat{G}$ are much more robust once certain zero restrictions on Λ_t are imposed. To decipher the discrepancy between these two pairs of models, we examine the model-implied $\mathbb{P}$-dynamics. As discussed by JPS (in their Section IV.B), enforcing zero restrictions on their risk premium parameters increases the persistence of state variables. We confirm this finding by noting that the eigenvalues of $\Phi_x^{\mathbb{P}}$ in $CUSM(5,6)$ are substantially larger than their counterparts in $CSM(5,6)$. This increase prevents variations in risk premia from completely dominating short-rate expectations and makes model-implied long-dated yield expectations more reasonable and potentially closer to the "true" yield expectations.

Taking the above findings together with the LR test results presented in Section 5.3.2, we conclude that by making the model more parsimonious, the macro-unspanning restrictions do not hurt the in-sample fitting and thus boost the out-of-sample performance.

IA.G.3 Forecastable Variations in Excess Returns Attributable to G_t

Having explored the unspanned nature of G_t, we quantify the information content in G_t within a (G-based) MTSM. Specifically, we examine how much of the predictable variations in excess bond returns can be captured by G_t and the potential role of hidden yield factors in the model. Put differently, we examine how much information related to the bond risk premium may be lost by excluding unspanned macro risks from term structure modeling. Note that this exercise represents an MTSM-based version of the regression analysis conducted in Section IA.B.5.

To this end, we consider the constrained models only (because this exercise requires reasonable model-implied moments of risk premia), and focus on the unspanned models.[IA.16] We implement models $CUSM(\mathcal{L},\mathcal{N})$ for $\mathcal{L}=3,4,5$.

IA.G.3.1 Variance Decomposition for Excess Bond Returns

We discuss the population properties of annual excess bond returns. Results reported in Table IA.G2 cover the five-year bond only as it is closely related to the "in-four-years-for-one-year" forward premium, as shown in the following:

$$E_t\left(rx_{t+12}^{(60)}\right) = FP_t^{4,1} - 4E_t\left(\Delta y_{t+12}^{48}\right) + \left(E_t(y_{t+48}^{(12)}) - y_t^{(12)}\right).$$

But the results for other maturities are broadly similar.

[IA.16] Although the macro-unspanning restrictions tend to grant macro factors the "privilege" of retaining their contributions to term premia, this is less of an issue here given that G_t is constructed after controlling for the yield curve information. Regardless, the spanned models generate qualitatively similar results (untabulated).

Consider the model $CUSM(3,4)$ first. Its model-implied unconditional mean of excess bond returns is 2.85% (column 2), consistent with its data counterpart of 2.73% (untabulated). The unconditional variance is 58.25 (column 3) and calculated using the following formula:

$$\begin{aligned} \mathrm{Var}\left(rx_{t,t+12}^{(60)}\right) &= \psi' \,\mathrm{Var}(X_t)\psi + 48^2 \left[\sum_{i=0}^{11} B_{48}'\Phi^i\Sigma\Sigma'\Phi^{i\prime}B_{48}\right] + (5^2+4^2+1)\sigma_{\eta_y}^2, \quad \text{(IA.G15)} \\ \text{where} \quad \psi &= 60B_{60}' - 48B_{48}'\Phi^{12} - 12B_{12}'. \end{aligned}$$

Among the three terms on the right-hand side (RHS) of Eq. (IA.G15), the first one represents the unconditional variance of the conditional expectation, which quantifies forecastable variation in the excess bond return; the second term denotes the variance of shocks to the "true" excess return; and the last term is the variance of the measurement error's contribution to the observed return shocks. Since this last term is typically small in models with $\mathcal{N} \geq 3$, the predictability of bond returns is mainly determined by the relative magnitudes of the first two terms on the RHS.

Furthermore, how much of $\mathrm{Var}\left(rx_{t,t+12}^{(60)}\right)$ is forecastable depends on the conditioning information used to forecast. If the state vector X_t itself is used, then the full-information R^2 implied by $CUSM(3,4)$ is 0.463 (the ratio of 27.01 in column 4 to 58.25), comparable to the regression R^2 of 0.439 reported in Table IA.B6 (column 7). The full-information R^2, however, cannot be achieved when the conditioning variables consist of the first $\mathcal{R}\,(\leq \mathcal{L})$ PCs of observed yields only. An effective measure for the gap between the information contained in X_t and that in $PC_{1-\mathcal{R},t}^o$ is the following ratio of variances of these two relevant forecasts:

$$VR_{\mathcal{R}}^o = \frac{\psi' \,\mathrm{Var}(X_t|PC_{1-\mathcal{R},t}^o)\psi}{\psi' \,\mathrm{Var}(X_t)\psi}, \quad \mathcal{R} \leq \mathcal{L}. \quad \text{(IA.G16)}$$

If $\mathcal{R} = 3$, then $CUSM(3,4)$ implies a $VR_{\mathcal{R}}^o$ of 71.4% (Table IA.G2, column 5 in braces); that is, almost 30% of the information in X_t is lost if we ignore G_t and rely solely on $PC_{1-3,t}^o$ to infer term premia.[IA.17]

What happens if the first $\mathcal{R}\,(\leq \mathcal{L})$ PCs of the *true* yields are used as the conditioning variables? We can repeat the above analysis using the following variant of Eq. (IA.G16):

$$VR_{\mathcal{R}} = \frac{\psi' \,\mathrm{Var}(X_t|PC_{1-\mathcal{R},t})\psi}{\psi' \,\mathrm{Var}(X_t)\psi}. \quad \text{(IA.G17)}$$

Column 6 shows that VR_3 is 72.9% (in braces), only slightly greater than VR_3^o (71.4%). This is

[IA.17] Duffee (2011) uses $VR_{\mathcal{R}}^o$ to evaluate the importance of yield factors hidden from the contemporaneous term structure and finds that $PC_{1-3,t}^o$ recover only 70% of the information on expected excess returns on the five-year bond, consistent with the notion of hidden factors.

not surprising as the cross-sectional effect of $PC_{1-3,t}$ is supposedly large enough to dominate the measurement error. Obviously, if the conditioning variables are X_t, the variance ratio is 100% (column 9). Notice that because $\mathcal{R} = \mathcal{L} = 3$, results from $CUSM(3,4)$ in columns 7 and 8 are the same as those in columns 5 and 6.

Given that model $CUSM(3,4)$ leaves no room for hidden yield factors, we consider the higher-dimensional models ($\mathcal{L} > 3$) next. As expected, in such cases the information loss will be higher (than with $\mathcal{L} = 3$) if the conditioning information consists of $PC^o_{1-3,t}$ only. As shown in column 5, VR^o_3 is about 71% under $CUSM(4,5)$ and 65% under $CUSM(5,6)$ (a model that is supposed to encompass both unspanned yield and macro factors). That is, about one-third of the information in X_t is lost if only $PC^o_{1-3,t}$ are used to infer term premia under $CUSM(5,6)$. As before, replacing $PC^o_{1-3,t}$ with $PC_{1-3,t}$ hardly reduces the information lost, with VR_3 equal to 71.3% under $CUSM(4,5)$ and 67.2% under $CUSM(5,6)$ (column 6). Note from column 7 that including higher-order PCs of the *observed* yield curve, $PC^o_{4,t}$ and $PC^o_{5,t}$, in the conditioning variables hardly helps to dig up more information on risk premia. For instance, for $CUSM(5,6)$, $VR^o_5 = 65.1\%$ (column 7), only slightly higher than $VR^o_3 = 64.9\%$ (column 5). Again, this is because the cross-sectional effect of higher-order PCs is too small to overwhelm the measurement error.

If we can *perfectly* infer the hidden factors by extracting information from yield dynamics as well as in the cross section, we can estimate risk premia more accurately. For instance, under model $CUSM(5,6)$, $VR_5 = 75.8\%$ (column 8), much higher than either $VR^0_5 = 65.1\%$ (column 7) or $VR_3 = 67.2\%$ (column 6). In fact, this difference between VR^o_5 and VR_5 suggests a wedge between the information in observed yields and that in "true" yields, whereas there is no evidence for a similar gap for the first three PCs, as indicated by columns 5 and 6. Nonetheless, the VR_5 of 75.8% still implies an information loss of almost 25% even in this ideal case. Given that under model $CUSM(5,6)$, G_t is not spanned by $PC_{1-6,t}$ and that the five yield factors presumably summarize all (time-series and cross-sectional) information on the yield side (Duffee 2011), a more reasonable implication of the result that $VR_5 = 75.8\%$ is the following: The information loss is *at least* about one-quarter when G_t is excluded from return predictors, even though they include $PC_{1-5,t}$.

We use the phrase "at least" for two reasons: First, the variance ratio is computed under the assumption that $PC_{1-5,t}$ are perfectly observable. In practice, however, econometricians have to perform filtering analysis to infer $PC_{4-5,t}$. Duffee (2011) documents that the Kalman filter recovers only two-thirds of the information in the true state vector for monthly excess returns (and about 82% of that for annual excess returns in an earlier version of the paper). In contrast, measurement error has little impact on factor G_t. Second, the period 1985–2007 sample is special in the sense that the fraction of the total variance attributable to macro-driven variations is particularly low. If the estimation sample is extended to either 1964 or 2014, the model-implied variance ratio would drop below 67% (untabulated). Once these two facts are taken into account, the results from model-based risk premium decomposition are expected to be close to the test results of H_0^{S2} (columns 14–17 in Table IA.B6)—namely, with respect to the state vector $X_t = (PC_{1-5,t}, G_t)$, the SAGLasso factor accounts for almost half of the predictable variations in excess bond returns.

It is worth emphasizing that risk premium accounting based on variance ratios is analogous to the variance decomposition (in the context of reduced-form VARs), of which the results are sensitive to the order of state factors chosen for identification. The projection of G_t on $PC_{1-\mathcal{L},t}$ in $VR_{\mathcal{L}}$ maximizes the explanatory power of yield PCs (Bikbov and Chernov 2010). This point can be illustrated by changing the order of state factors and calculating the following variance ratio:

$$VR_{3+G} = \frac{\psi' \operatorname{Var}(X_t | X_t^{\setminus H})\psi}{\psi' \operatorname{Var}(X_t)\psi}, \tag{IA.G18}$$

where $X_t^{\setminus H} = (PC_{1-3,t}, G_t)$. Results in column 9 indicate that under $CUSM(5,6)$, the first three PCs plus the SAGLasso factor capture 97.9% of forecastable variation in excess bond returns. Although this finding does not necessarily mean that hidden factors are unimportant in return prediction in this case, it does imply that, compared to ignoring hidden yield factors (as shown in column 9), excluding unspanned macro risks (associated with G_t, as shown in column 8) bears more serious economic consequences in the inference of term premia.

IA.G.3.2 Calculations of Variance Ratios

This subsection provides details on the calculations of variance ratios used in Section IA.G.3.1. All the calculations are based on MTSMs specified in either Section 5.2 (for spanned models) or Section IA.G (for unspanned ones).

Consider $VR^o_{\mathcal{R}}$, the variance ratio defined in Eq. (IA.G16) that focuses on the forecast of excess bond returns based on the first $\mathcal{R}\,(\leq \mathcal{L})$ PCs of observed yields. Recall that by definition, the first

$\mathcal{L}$ PCs are given by

$$PC^o_{1-\mathcal{L},t} = W_{\mathcal{R},\mathcal{M}} Y^o_t = W_{\mathcal{R},\mathcal{M}} \mathcal{A}_{\mathcal{M}} + W_{\mathcal{R},\mathcal{M}} \mathcal{B}'_{\mathcal{M}} X_t + W_{\mathcal{R},\mathcal{M}}\, \eta_t,$$

where $W_{\mathcal{R},\mathcal{M}}$ is an $\mathcal{R} \times k$ loading matrix, which is equal to the transpose of $U_{\mathcal{R},\mathcal{M}}$ in Eq. (IA.D4). Below subscripts are suppressed for simplicity of notations. It follows that the expectation of the true state factor X_t conditioned on these PCs equals

$$E(X_t|PC^o_{1-\mathcal{R},t}) = E(X_t) + \mathrm{Var}(X_t)\mathcal{B}W' \,\mathrm{Var}(PC^o_{1-\mathcal{R},t})^{-1} PC^o_{1-\mathcal{R},t},$$

where the variance of $PC^o_{1-\mathcal{R},t}$ is

$$\mathrm{Var}(PC^o_{1-\mathcal{R},t}) = W\mathcal{B}' \,\mathrm{Var}(X_t)\mathcal{B}W + WW'\sigma^2_\eta.$$

The variance of $E(X_t|PC^o_{1-\mathcal{R},t})$ is

$$\mathrm{Var}(X_t|PC^o_{1-\mathcal{R},t}) = \mathrm{Var}(X_t)\mathcal{B}W' \,\mathrm{Var}(PC^o_{1-\mathcal{R},t})^{-1} W\mathcal{B}' \,\mathrm{Var}(X_t).$$

Next, consider $VR_{\mathcal{R}}$, the variance ratio specified in Eq. (IA.G17) that concerns the inference of risk premium based on the first $\mathcal{R}$ PCs of true yields. Recall that these PCs, $PC_{1-\mathcal{R},t}$, constitute a segment of the state vector X_t. Denoting the remaining $\mathcal{N}$-$\mathcal{R}$ state factors by $X^{\backslash\mathcal{R}}_t$, we have

$$\begin{aligned} E(X_t|PC_{1-\mathcal{R},t}) &= \begin{bmatrix} PC_{1-\mathcal{R},t} \\ E(X^{\backslash\mathcal{R}}_t) + \mathfrak{C}\mathfrak{V}^{-1} PC_{1-\mathcal{R},t} \end{bmatrix}, \quad \text{and} \\ \mathrm{Var}(X_t|PC_{1-\mathcal{R},t}) &= \begin{bmatrix} \mathfrak{V} & \mathfrak{C}' \\ \mathfrak{C} & \mathfrak{C}\mathfrak{V}^{-1}\mathfrak{C}' \end{bmatrix}, \end{aligned}$$

where $\mathfrak{V} = \mathrm{Var}(PC_{1-\mathcal{R},t})$ and $\mathfrak{C} = \mathrm{Cov}(X^{\backslash\mathcal{R}}_t, PC_{1-\mathcal{R},t})$.

Table IA.A1: Properties of Principal Components of Observed Yield Curves

Panel A reports correlations between principal components (PCs) of observed yields and filtered estimates of yield PCs, denoted by PC_i^o and PC_i, respectively, where $i = 1, \ldots, 5$. Population correlations are computed by simulating 100,000 months of bond yields. The 95% confidence intervals for the sample correlations, as displayed in parentheses, are derived from 5,000 simulations with the same number of observations as in the data sample. The yield maturities in all simulations are three months and one through five years. Panel B reports results from regressions of the return to an n-year zero-coupon bond from month t to month $t+12$ less the month-t yield on a one-year bond on the first five PCs of observed yields, $PC_{1-5}^o = (PC_1^o, \ldots, PC_5^o)$. Test statistics are computed using either the Hansen and Hodrick (1980) GMM covariance estimator (in parentheses) or the Newey and West (1987) HAC covariance estimator (in brackets). The row labeled "R^2 (Table 2)" copies the R^2 values from regressions of excess returns on filtered estimates of the first five PCs, reported in Table IA.B5 (columns 10 through 13). The ΔR^2 measure represents the differences between R^2 values in Panel B and R^2 (Table 2). The last row in panel B reports the percentage decrease in the R^2. The sample spans the period January 1964 to December 2014.

(1)	(2)	(3)		(4)	(5)	(6)	(7)
	Panel A: Corr($PC_{i,t}$,$PC_{i,t}^o$)			Panel B: Predictive regressions of excess returns to an n-year zero-coupon bond on $PC_{1-5,t}^o$			
				Bond maturity n			
	Population	Sample		2	3	4	5
$PC_{1,t}^o$	0.9999	0.9998		3.526	3.031	2.080	0.422
		[0.9998 0.9999]		(1.116)	(0.529)	(0.264)	(0.043)
				[1.266]	[0.601]	[0.300]	[0.048]
$PC_{2,t}^o$	0.9905	0.9902		-0.688	-1.330	-2.038	-2.621
		[0.9885 0.9916]		(-3.650)	(-3.674)	(-3.961)	(-4.105)
				[-4.021]	[-4.092]	[-4.431]	[-4.601]
$PC_{3,t}^o$	0.9612	0.9818		0.784	1.011	1.485	1.688
		[0.9787 0.9845]		(1.233)	(0.920)	(1.041)	(0.978)
				[1.384]	[1.034]	[1.156]	[1.079]
$PC_{4,t}^o$	0.7233	0.7595		-1.956	-2.836	-2.798	-0.961
		[0.7238 0.7912]		(-1.702)	(-1.295)	(-0.910)	(-0.244)
				[-1.842]	[-1.418]	[-1.005]	[-0.271]
$PC_{5,t}^o$	0.2125	0.6107		4.060	10.521	15.004	14.677
		[0.5584 0.6581]		(2.693)	(4.536)	(5.636)	(4.358)
				[2.363]	[3.620]	[4.074]	[3.151]
			R^2	0.205	0.215	0.241	0.228
			R^2 (Table IA.B5)	0.221	0.233	0.255	0.245
			ΔR^2	-0.016	-0.018	-0.014	-0.017
			Percentage decrease in R^2	-7.24%	-7.73%	-5.49%	-6.94%

Table IA.B1: Correlation between Yield Curve and New Macro Factors

This table reports the Pearson correlation coefficients between four newly constructed macroeconomic factors and five yield-curve factors. The four macroeconomic factors include employment ($\hat{g}_{1t}$), housing ($\hat{g}_{2t}$), inflation ($\hat{g}_{3t}$), and the aggregate SAGLasso factor ($\widehat{G}_t$) constructed in Section 4.2. The five yield curve factors include the first three principal components (PCs) of *observed* bond yields, $\{PC^o_{i,t}, i = 1,2,3\}$, and the filtered higher-order PCs of noise-uncontaminated yields, $PC_{4,t}$ and $PC_{5,t}$. The sample spans the period January 1964 to December 2014.

	$\widehat{G}_t$	$\widehat{G}_{1t}$	$\widehat{G}_{2t}$	$\widehat{G}_{3t}$	$PC^o_{1,t}$	$PC^o_{2,t}$	$PC^o_{3,t}$	$PC_{4,t}$
$\widehat{G}_{1t}$	0.620							
$\widehat{G}_{2t}$	0.527	0.577						
$\widehat{G}_{3t}$	0.524	0.467	0.351					
$PC^o_{1,t}$	−0.100	−0.226	−0.167	−0.199				
$PC^o_{2,t}$	−0.352	−0.222	−0.073	−0.270	−0.006			
$PC^o_{3,t}$	0.167	0.239	0.222	0.196	0.018	0.003		
$PC_{4,t}$	−0.094	−0.031	−0.106	0.021	−0.000	0.013	0.044	
$PC_{5,t}$	−0.021	−0.027	−0.282	0.284	0.024	0.008	−0.011	0.092

Table IA.B2: Predictive Power of Three SAGLasso Group Factors

The return to an n-year zero-coupon bond from month t to month $t+12$ less the month-t yield on a one-year bond is regressed on $\widehat{g}_{1t}$, $\widehat{g}_{2t}$, and $\widehat{g}_{3t}$, the group factors, constructed in Section 4.2, that represent employment, housing and inflation, respectively, for $n = 2, \ldots, 5$. Results reported in panel A are based on the January 1964–December 2014 sample, including those from both univariate (panel A1) and multivariate predictive regressions (panel A2). Results reported in panel B are based on the January 1952–December 2014 sample, where only the employment factor is considered (because of data limitations) and constructed using this longer sample ($\widehat{g}^*_{1t}$). In-sample results from regressions on $\widehat{g}^*_{1t}$ are shown in panel B1 and out-of-sample (OOS) results based on $\widetilde{g}^*_{1t}$ are reported in panel B2. Test statistics are computed using either the Hansen and Hodrick (1980) GMM covariance estimator (in parentheses) or the Newey and West (1987) HAC covariance estimator (in brackets). The ENC-REG statistic denotes the OOS t-statistic proposed by Ericsson (1992), whose 95th percentile of the asymptotic distribution is $\Phi^{-1} = 1.645$. The row labeled "ENC-NEW" reports a variant of the ENC-REG statistic proposed by Clark and McCracken (2001); their simulation shows that the 95% critical value is around 1.584 for testing one additional predictor. Both tests share the same null hypothesis that the benchmark model encompasses the unrestricted model with excess predictors. The R^2_{oos} statistic denotes the OOS R^2 of Campbell and Thompson (2008).

maturity n (year)	2	3	4	5		2	3	4	5
					Panel A: Sample period 1964–2014				
	Panel A1: Univariate Regressions					Panel A2: Multivariate Regressions			
$\widehat{g}_{1t}$	0.828	1.526	2.082	2.568		0.942	1.401	1.744	2.062
	(3.796)	(4.149)	(4.272)	(4.545)		(2.850)	(3.040)	(3.038)	(3.165)
	[4.237]	[4.594]	[4.689]	[4.948]		[3.107]	[3.301]	[3.280]	[3.401]
R^2	0.220	0.222	0.213	0.211					
$\widehat{g}_{2t}$	0.643	1.162	1.631	2.051		0.722	0.968	1.223	1.461
	(2.608)	(2.861)	(3.178)	(3.364)		(1.965)	(2.052)	(2.215)	(2.321)
	[2.969]	[3.249]	[3.589]	[3.786]		[2.141]	[2.236]	[2.413]	[2.528]
R^2	0.143	0.139	0.141	0.149					
$\widehat{g}_{3t}$	0.723	1.358	1.872	2.222		0.847	1.249	1.574	1.763
	(3.096)	(3.075)	(3.023)	(2.875)		(2.544)	(2.534)	(2.486)	(2.358)
	[3.449]	[3.431]	[3.380]	[3.215]		[2.754]	[2.748]	[2.698]	[2.555]
R^2	0.168	0.176	0.172	0.173		0.404	0.431	0.420	0.417
					Panel B: Sample period 1952–2014				
	Panel B1: In-Sample Regressions					Panel B2: $\widetilde{g}^*_{1t}$ vs. constant (OOS)			
$\widehat{g}^*_{1t}$	0.751	1.397	1.932	2.390					
	(4.161)	(4.524)	(4.767)	(5.084)	ENC-REG	3.329	3.600	3.719	3.991
	[4.665]	[5.025]	[5.227]	[5.525]	ENC-NEW	136.93	134.56	127.99	125.341
R^2	0.206	0.213	0.211	0.211	R^2_{oos}	0.155	0.164	0.166	0.169

Table IA.B3: Unspanned Variation in SAGLasso Group Factors

This table reports results from linear projections of each of the three SAGLasso group macro factors $\{\widehat{g}_{it}, 1 \leq i \leq 3\}$ onto the first $\mathcal{R}$ principal components (PCs) of the yield curve ($PC^o_{1-\mathcal{R},t}$), where $\mathcal{R} = 3$ (panel A) or 6 (panel B) and the group factors are the employment ($\widehat{g}_{1t}$), housing ($\widehat{g}_{2t}$), and inflation ($\widehat{g}_{3t}$) factors. Columns 3 and 5 show the regression R^2s, and in brackets beneath are reported 95% confidence intervals based on 5,000 artificial samples simulated from a six-factor constrained term structure model with spanned macro risks. The state vector of the model, denoted by $CSM(3,6)_{group}$ and specified in Section IA.G.1, includes three yield curve factors (the first three PCs) and three macro factors, $\widehat{g}_{1t}$, $\widehat{g}_{2t}$, and $\widehat{g}_{3t}$. Column 2 indicates whether the three macro variables are assumed to be measured with errors in the estimation of model $CSM(3,6)_{group}$. Columns 4 and 6 report the first-order serial correlation of regression residuals.

(1)	(2)	(3)	(4)	(5)	(6)
		Panel A: Regressions of $\widehat{g}_{it}$ on $PC^o_{1-3,t}$		Panel B: Regressions of $\widehat{g}_{it}$ on $PC^o_{1-6,t}$	
Dependent variable	Macro Measurement Error	R^2	AR(1) of residuals	R^2	AR(1) of residuals
$\widehat{g}_{1t}$		0.116		0.125	
	No	[0.148 0.770]		[0.206 0.809]	
	Yes	[0.140 0.773]	0.951	[0.201 0.814]	0.948
$\widehat{g}_{2t}$		0.082		0.119	
	No	[0.144 0.816]		[0.171 0.845]	
	Yes	[0.137 0.802]	0.960	[0.164 0.829]	0.946
$\widehat{g}_{3t}$		0.106		0.123	
	No	[0.152 0.708]		[0.219 0.766]	
	Yes	[0.145 0.697]	0.903	[0.216 0.753]	0.893

Table IA.B4: Predictive Power of Alternative Macroeconomic Factors for Excess Bond Returns

The return to an n-year zero-coupon bond from month t to month $t+12$ less the month-t yield on a one-year bond is regressed on a macro factor (F_t), for $n = 2, \ldots, 5$. The three macro factors considered include the modified Ludvigson and Ng (2011) factor, $\widehat{LN}_t^m$ (panel A); a factor denoted $\widehat{G}_t^{rev}$ (panel B) and constructed using the SAGLasso procedure (Section 4.1) albeit with the set of 131 macro series that are not adjusted for data revisions; and a factor denoted $\widehat{G}_t^{rev,lag}$ (panel C) and constructed using the SAGLasso procedure albeit with the set of 131 macro series that are not adjusted for either data revisions or publication lags. In the in-sample analysis, t-statistics are computed using the Hansen and Hodrick (1980) GMM covariance estimator (in parentheses) and the Newey and West (1987) HAC covariance estimator (in brackets), respectively. In the out-of-sample analysis, the row labeled "ENC-REG" reports the out-of-sample t-statistics proposed by Ericsson (1992), whose 95th percentile of the asymptotic distribution is $\Phi^{-1} = 1.645$. The row labeled "ENC-NEW" reports a variant of the ENC-REG statistic proposed by Clark and McCracken (2001); their simulation shows that the 95% critical value is around 1.584 for testing one additional predictor. Both tests share the same null hypothesis that the benchmark model encompasses the unrestricted model with excess predictors. The row labeled "R^2_{oos}" denotes the out-of-sample R^2 of Campbell and Thompson (2008). The sample spans the period January 1964 to December 2014.

	Predictive regressions of excess returns to an n-year zero-coupon bond on alternative macro factors (F_t)											
	Panel A: $F_t = \widehat{LN}_t^m$				Panel B: $F_t = \widehat{G}_t^{rev}$				Panel C: $F_t = \widehat{G}_t^{rev,lag}$			
maturity n (year)	2	3	4	5	2	3	4	5	2	3	4	5
	Panel A1: In-sample				Panel B1: In-sample				Panel C1: In-sample			
Coeff. on F_t	0.724	1.473	2.181	2.792	1.039	1.993	2.832	3.542	1.135	2.150	3.037	3.807
	(4.227)	(4.932)	(5.804)	(6.554)	(5.780)	(6.357)	(6.715)	(7.009)	(6.023)	(6.412)	(6.681)	(7.102)
	[4.641]	[5.390]	[6.319]	[7.074]	[6.215]	[6.802]	[7.191]	[7.513]	[6.547]	[6.946]	[7.183]	[7.595]
R^2	0.168	0.207	0.234	0.250	0.347	0.379	0.394	0.402	0.414	0.441	0.453	0.464
	Panel A2: Out-of-sample				Panel B2: Out-of-sample				Panel C2: Out-of-sample			
ENC-REG	2.660	3.336	4.136	5.104	3.867	4.975	5.933	7.266	4.308	5.217	5.647	6.558
ENC-NEW	115.96	137.06	162.59	173.71	159.78	182.11	207.94	219.67	204.64	235.34	253.81	262.21
R^2_{oos}	0.051	0.112	0.186	0.225	0.118	0.200	0.274	0.314	0.123	0.238	0.294	0.326

Table IA.B5: In-Sample Tests of Spanning Hypotheses I and II: 1964–2014

The return to an n-year zero-coupon bond from month t to month $t+12$ less the month-t yield on a one-year bond is regressed respectively on (i) the first three principal components (PCs) of observed bond yields $PC^o_{1-3,t}$ (columns 2-5); (ii) $PC^o_{1-3,t}$ and the SAGLasso macro factor $\widehat{G}_t$ (columns 6-9); (iii) the filtered first five PCs of noise-uncontaminated yields $PC_{1-5,t}$ (columns 10-13); and (iv) $PC_{1-5,t}$ and $\widehat{G}_t$ (columns 14-17). Test statistics are computed using the Hansen and Hodrick (1980) GMM covariance estimator (in parentheses), or the Newey and West (1987) HAC covariance estimator (in brackets). The sample spans the period January 1964 to December 2014.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
					Spanning Hypothesis I								Spanning Hypothesis II			
maturity n (year)	2	3	4	5	2	3	4	5	2	3	4	5	2	3	4	5
$\mathcal{P}1_t$	3.636	3.286	2.426	0.738	5.512	6.774	7.176	6.535	2.193	2.163	1.918	1.216	3.034	3.713	4.020	3.777
	(1.106)	(0.533)	(0.285)	(0.070)	(2.810)	(1.814)	(1.364)	(1.010)	(1.280)	(0.692)	(0.443)	(0.226)	(3.091)	(2.058)	(1.527)	(1.147)
	[1.257]	[0.608]	[0.325]	[0.079]	[3.097]	[2.014]	[1.521]	[1.126]	[1.451]	[0.786]	[0.504]	[0.257]	[3.378]	[2.272]	[1.704]	[1.280]
$\mathcal{P}2_t$	−0.688	−1.328	−2.036	−2.619	−0.236	−0.489	−0.893	−1.224	−0.441	−0.838	−1.279	−1.636	−0.151	−0.303	−0.553	−0.752
	(−3.351)	(−3.291)	(−3.559)	(−3.789)	(−1.338)	(−1.413)	(−1.808)	(−1.994)	(−3.720)	(−3.657)	(−3.924)	(−4.071)	(−1.424)	(−1.477)	(−1.900)	(−2.048)
	[−3.732]	[−3.709]	[−4.028]	[−4.288]	[−1.456]	[−1.565]	[−2.019]	[−2.231]	[−4.124]	[−4.098]	[−4.413]	[−4.580]	[−1.554]	[−1.638]	[−2.116]	[−2.284]
$\mathcal{P}3_t$	0.782	1.000	1.466	1.665	0.118	−0.233	−0.213	−0.385	0.480	0.571	0.811	0.857	0.053	−0.217	−0.258	−0.444
	(1.174)	(0.849)	(0.958)	(0.917)	(0.214)	(−0.236)	(−0.166)	(−0.258)	(1.065)	(0.742)	(0.799)	(0.694)	(0.150)	(−0.372)	(−0.336)	(−0.474)
	[1.319]	[0.956]	[1.065]	[1.015]	[0.236]	[−0.260]	[−0.179]	[−0.274]	[1.167]	[0.812]	[0.866]	[0.749]	[0.160]	[−0.394]	[−0.349]	[−0.486]
$\mathcal{P}4_t$									−0.766	−2.467	−4.004	−5.112	−0.205	−1.431	−2.600	−3.402
									(−0.988)	(−1.880)	(−2.396)	(−2.384)	(−0.312)	(−1.338)	(−1.925)	(−1.982)
									[−0.999]	[−1.871]	[−2.337]	[−2.330]	[−0.331]	[−1.397]	[−1.960]	[−2.007]
$\mathcal{P}5_t$									4.375	9.333	12.097	11.581	4.424	9.423	12.220	11.730
									(2.185)	(2.573)	(2.405)	(1.858)	(2.967)	(3.534)	(3.322)	(2.540)
									[2.241]	[2.629]	[2.472]	[1.914]	[3.104]	[3.691]	[3.489]	[2.681]
$\widehat{G}_t$					1.007	1.872	2.548	3.110					1.005	1.853	2.513	3.061
					(5.958)	(6.370)	(6.649)	(6.542)					(6.575)	(7.100)	(7.135)	(6.892)
					[6.268]	[6.599]	[6.847]	[6.809]					[7.042]	[7.573]	[7.628]	[7.401]
R^2	0.167	0.156	0.183	0.194	0.422	0.418	0.432	0.436	0.221	0.233	0.255	0.245	0.472	0.486	0.494	0.477
ΔR^2 due to $\widehat{G}_t$					0.255	0.262	0.250	0.243					0.251	0.253	0.240	0.232

Table IA.B6: In-Sample Tests of Spanning Hypotheses I and II: 1985–2014

The return to an n-year zero-coupon bond from month t to month $t+12$ less the month-t yield on a one-year bond is regressed respectively on (i) the first three principal components (PCs) of observed bond yields $PC^o_{1-3,t}$ (columns 2-5); (ii) $PC^o_{1-3,t}$ and the SAGLasso macro factor $\widehat{G}_t$ (columns 6-9); (iii) the filtered first five PCs of noise-uncontaminated yields $PC_{1-5,t}$ (columns 10-13); and (iv) $PC_{1-5,t}$ and $\widehat{G}_t$ (columns 14-17). Test statistics are computed using the Hansen and Hodrick (1980) GMM covariance estimator (in parentheses), or the Newey and West (1987) HAC covariance estimator (in brackets). The sample spans the period January 1985 to December 2014.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
					Spanning Hypothesis I								Spanning Hypothesis II			
maturity n (year)	2	5	7	10	2	5	7	10	2	5	7	10	2	5	7	10
$\mathcal{P}^o_{1,t}(\mathcal{P}_{1,t})$	6.268	11.951	15.082	19.997	6.160	11.544	14.522	19.305	2.877	7.345	10.466	14.210	2.482	5.863	8.469	11.699
	(2.172)	(1.168)	(0.979)	(1.001)	(3.629)	(2.066)	(1.564)	(1.511)	(2.385)	(1.701)	(1.618)	(1.631)	(3.939)	(2.794)	(2.270)	(2.193)
	[2.439]	[1.314]	[1.101]	[1.119]	[3.881]	[2.224]	[1.720]	[1.672]	[2.668]	[1.915]	[1.827]	[1.834]	[4.058]	[2.909]	[2.491]	[2.435]
$\mathcal{P}^o_{2,t}(\mathcal{P}_{2,t})$	−0.274	−1.450	−2.255	−3.210	−0.039	−0.568	−1.043	−1.711	−0.107	−0.709	−1.281	−1.662	0.003	−0.294	−0.722	−0.960
	(−1.279)	(−2.013)	(−2.331)	(−2.850)	(−0.223)	(−1.024)	(−1.466)	(−2.194)	(−1.269)	(−2.369)	(−3.020)	(−3.101)	(0.041)	(−1.247)	(−2.188)	(−2.532)
	[−1.468]	[−2.292]	[−2.626]	[−3.179]	[−0.253]	[−1.160]	[−1.635]	[−2.410]	[−1.416]	[−2.646]	[−3.359]	[−3.407]	[0.046]	[−1.408]	[−2.463]	[−2.819]
$\mathcal{P}^o_{3,t}(\mathcal{P}_{3,t})$	0.187	−0.525	−1.867	−2.318	0.244	−0.311	−1.573	−1.954	−0.022	−0.295	−1.253	−0.885	0.003	−0.203	−1.129	−0.729
	(0.524)	(−0.439)	(−1.146)	(−1.054)	(1.096)	(−0.396)	(−1.457)	(−1.445)	(−0.100)	(−0.428)	(−1.483)	(−0.783)	(0.016)	(−0.356)	(−1.626)	(−0.952)
	[0.561]	[−0.469]	[−1.203]	[−1.123]	[1.215]	[−0.446]	[−1.601]	[−1.625]	[−0.103]	[−0.439]	[−1.481]	[−0.776]	[0.017]	[−0.389]	[−1.725]	[−1.031]
$\mathcal{P}_{4,t}$									−0.222	−0.305	−0.388	−0.297	−0.152	−0.041	−0.033	0.150
									(−0.757)	(−0.313)	(−0.291)	(−0.183)	(−0.647)	(−0.054)	(−0.031)	(0.122)
									[−0.835]	[−0.345]	[−0.320]	[−0.201]	[−0.711]	[−0.059]	[−0.034]	[0.134]
$\mathcal{P}_{5,t}$									1.023	2.230	0.790	0.356	0.870	1.657	0.018	−0.615
									(2.101)	(1.573)	(0.396)	(0.136)	(2.038)	(1.456)	(0.011)	(−0.308)
									[2.164]	[1.451]	[0.359]	[0.122]	[2.199]	[1.391]	[0.010]	[−0.280]
$\widehat{G}_t$					0.713	2.677	3.679	4.553					0.693	2.605	3.512	4.414
					(6.419)	(6.330)	(5.576)	(5.221)					(6.480)	(6.225)	(5.551)	(5.278)
					[6.822]	[6.841]	[6.014]	[5.636]					[6.819]	[6.674]	[5.945]	[5.676]
R^2	0.140	0.142	0.170	0.206	0.393	0.439	0.448	0.471	0.184	0.176	0.225	0.240	0.417	0.450	0.472	0.483
ΔR^2 due to $\widehat{G}_t$					0.253	0.297	0.278	0.265					0.232	0.274	0.247	0.243

Table IA.B7: Tests of Spanning Hypotheses Using Macroeconomic Variables with Different Lags

This table reports both the in-sample and out-of-sample tests of spanning hypotheses, using macroeconomic variables with 0, 3, 6, 9, or 12 lags. The benchmark model in tests of Spanning Hypothesis I is based on the first three principal components (PCs) of observed bond yields and, in tests of Spanning Hypothesis II, is based on the filtered first five PCs of noise-uncontaminated yields. The columns labeled "HH" report test statistics based on the Hansen and Hodrick (1980) GMM covariance estimator, and the columns labeled "NW" report their counterparts based on the Newey and West (1987) HAC covariance estimator. "ENC-REG" denotes the out-of-sample t-statistics proposed by Ericsson (1992), and "ENC-NEW" represents a variant of the ENC-REG statistic proposed by Clark and McCracken (2001). The rows labeled "ΔR^2" (ΔR^2_{oos}) represent the incremental in-sample (out-of-sample) R^2 due to $\tilde{G}$. The sample spans the period January 1964 to December 2014.

Lags	In-sample tests						Out-of-sample tests					
	2-year bond			5-year bond			2-year bond			5-year bond		
	HH	NW	ΔR^2	HH	NW	ΔR^2	ENC-REG	ENC-NEW	ΔR^2_{oos}	ENC-REG	ENC-NEW	ΔR^2_{oos}
Panel A: Tests of Spanning Hypotheses I												
0	6.711	7.021	0.240	6.136	6.394	0.194	5.168	151.150	0.299	4.548	103.117	0.205
3	6.425	6.845	0.280	6.414	6.716	0.239	4.570	221.703	0.331	3.761	156.355	0.228
6	5.958	6.268	0.255	6.542	6.809	0.243	4.764	191.912	0.349	4.871	147.103	0.271
9	7.449	7.634	0.289	7.104	7.033	0.225	3.481	163.430	0.211	2.935	98.113	0.104
12	6.947	7.367	0.305	6.795	7.170	0.244	5.420	236.068	0.325	4.606	151.385	0.206
Panel B: Tests of Spanning Hypotheses II												
0	7.392	7.918	0.226	6.758	7.220	0.176	4.928	135.506	0.296	4.389	83.981	0.186
3	7.059	7.617	0.265	7.004	7.455	0.223	5.286	243.125	0.387	3.861	150.602	0.239
6	6.575	7.042	0.251	6.892	7.401	0.232	4.781	180.940	0.353	4.526	130.102	0.256
9	8.931	9.081	0.274	7.982	8.048	0.210	3.656	152.386	0.204	2.835	82.606	0.073
12	8.444	8.863	0.289	7.748	8.314	0.227	4.916	230.903	0.330	3.717	136.416	0.186

Table IA.C1: Estimates of Parameters on the Market Price of Risk

This table reports the maximum likelihood estimates of parameters λ_0 and λ_1 that govern bond risk premia in an $\mathcal{N}$-factor constrained, spanned macro-finance term structure model (MTSM) as specified and denoted $CSM(\mathcal{L}, \mathcal{N})$ in Section IA.G.1. The underlying state variables include the SAGLasso macro factor, $\widehat{G}$, constructed in Section 4.1 and the first $\mathcal{L}$ principal components (PCs) of bond yields, $PC_{1-\mathcal{L}} = (PC_1, \ldots, PC_{\mathcal{L}})$. The three MTSMs considered include $CSM(3,4)$ (panel A), $CSM(4,5)$ (panel B), and $CSM(5,6)$ (panel C). The one-period risk premium is as specified in Eq. (13): $\Sigma\Lambda_t = \lambda_0 + \lambda_1 X_t = \lambda_0 + \lambda_1 \cdot (PC_{1-\mathcal{L},t},\ \widehat{G}_t)'$. Zero entries of λ_0 and λ_1 reflect our model selection outcome. Values in parentheses are standard errors computed using Monte Carlo simulations.

State variables	λ_0	λ_1 ($\mathcal{N} \times \mathcal{N}$)					
		$\lambda_1(\cdot, \mathcal{N})$	$\lambda_1(\cdot, 1)$		$\cdots$		$\lambda_1(\cdot, 5)$
		$\widehat{G}_t$	$PC_{1,t}$	$PC_{2,t}$	$PC_{3,t}$	$PC_{4,t}$	$PC_{5,t}$
Panel A: Model $CSM(3,4)$							
$PC_{1,t}$	0.013	-6.11e-04	-0.054	-0.313	0		
	(0.002)	(8.97e-05)	(0.016)	(0.087)			
$PC_{2,t}$	0.002	-1.45e-04	0	0	-0.458		
	(9.31e-04)	(7.12e-05)			(0.0143)		
$PC_{3,t}$	0	0	0	0	0		
$\widehat{G}_t$	-0.278	-0.159	0.093	0	0		
	(0.152)	(0.081)	(0.035)				
Panel B: Model $CSM(4,5)$							
$PC_{1,t}$	0.018	-8.13e-04	-0.049	-0.152	0	0	
	(0.003)	(9.04e-05)	(0.007)	(0.060)			
$PC_{2,t}$	0.002	-1.32e-04	0	-0.031	-0.129	0.140	
	(0.001)	(9.13e-05)		(0.035)	(0.139)	(0.148)	
$PC_{3,t}$	0	0	0	0	0	0	
$PC_{4,t}$	0	0	0	0	0	0	
$\widehat{G}_t$	0	0	-0.633	0	0	-8.77	
			(0.243)			(4.871)	
Panel C: Model $CSM(5,6)$							
$PC_{1,t}$	0.029	-6.47e-04	-0.048	-0.173	0	0	-0.708
	(0.003)	(8.76e-05)	(0.009)	(0.076)			(0.259)
$PC_{2,t}$	0	-2.60e-04	0	0	-0.207	0.098	0
		(9.26e-05)			(0.102)	(0.115)	
$PC_{3,t}$	0	0	0	0	0	0	0
$PC_{4,t}$	0	0	0	0	0	0	0
$PC_{5,t}$	0	0	0	0	0	0	0
$\widehat{G}_t$	-0.646	0	0	0	0	0	0
	(0.084)						

Table IA.D1: Finite-Sample Properties of Statistics in Testing Spanning Hypothesis I under a VAR-based Data-Generating Process

This table presents results based on finite-sample distributions of the statistics that are involved in tests of Spanning Hypotheses I stated in Section 2.2. The analysis is based on 5,000 bootstrapped samples generated from the reduced-form VAR described in Eqs. (IA.D4) and (IA.D5) (panels A1 through B2) or from the macro-finance term structure model *MIM*(3, 4) (panels A3 through B4) that satisfies the "macro-independence restrictions" given in Eq. (IA.D9). The length of each bootstrapped sample is set to be consistent with either the full sample (panel A) or the post-1984 subsample (panel B). Test statistics considered include those computed using the Hansen and Hodrick (1980) GMM covariance estimator (HH), the Newey and West (1987) HAC covariance estimator (NW) with 18 lags, the out-of-sample ENC-REG test of Ericsson (1992), or the out-of-sample ENC-NEW test of Clark and McCracken (2001). For each test statistics, the 95th percentile of the bootstrap distribution is reported as the 5% critical value, and the *p*-values (in angle brackets) are the frequency of bootstrap replications in which the test statistics are at least as large as the statistic in the data. The "ΔR^2" and "ΔR^2_{oos}" measures denote the incremental R^2 and out-of-sample R^2 of Campbell and Thompson (2008), respectively.

	Panel A: Full sample, 1964–2014				Panel B: Subsample, 1985–2014			
maturity (year)	2	3	4	5	2	5	7	10
	Panel A1: In-sample based on VAR				Panel B1: In-sample based on VAR			
HH	1.872	1.889	1.883	1.885	1.999	1.965	2.013	2.055
	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩
NW	1.984	1.997	1.986	1.997	2.031	2.015	2.039	2.059
	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩
ΔR^2	0.032	0.033	0.033	0.034	0.036	0.037	0.037	0.035
	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩
	Panel A2: Out-of-sample based on VAR				Panel B2: Out-of-sample based on VAR			
ENC-REG	1.784	1.747	1.752	1.753	2.002	2.102	2.019	2.013
	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.008⟩	⟨0.001⟩	⟨0.001⟩	⟨0.001⟩
ENC-NEW	10.711	11.286	11.676	11.950	6.688	6.758	6.604	6.506
	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩
ΔR^2_{oos}	0.031	0.031	0.029	0.029	0.047	0.048	0.048	0.047
	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩
	Panel A3: In-sample based on *MIM*(3, 4)				Panel B3: In-sample based on *MIM*(3, 4)			
HH	1.888	1.921	1.912	1.939	1.994	2.001	2.030	2.003
	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩
NW	1.999	2.021	2.043	2.041	2.046	2.027	2.041	2.039
	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩
ΔR^2	0.032	0.033	0.033	0.033	0.039	0.039	0.038	0.039
	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩
	Panel A4: Out-of-sample based on *MIM*(3, 4)				Panel B4: Out-of-sample based on *MIM*(3, 4)			
ENC-REG	1.749	1.716	1.729	1.756	1.870	1.995	2.057	2.033
	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.007⟩	⟨0.002⟩	⟨0.003⟩	⟨0.001⟩
ENC-NEW	11.236	11.217	11.101	11.424	6.313	6.326	6.361	6.313
	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩
ΔR^2_{oos}	0.030	0.030	0.030	0.030	0.048	0.051	0.053	0.052
	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩

Table IA.E1: Ibragimov-Müller Test of Spanning Hypotheses I and II

The average return to zero-coupon bonds from month t to month $t+12$ less the month-t yield on a one-year bond is regressed on either $PC^o_{1-3,t}$ and $\widehat{G}_t$ for Spanning Hypothesis I (H_0^{S1}) or $PC_{1-5,t}$ and G_t for Spanning Hypothesis II (H_0^{S2}), where $PC^o_{1-3,t}$ denotes the first three principal components (PCs) of observed bond yields; $PC_{1-5,t}$ the filtered first five PCs of noise-uncontaminated yields; and $\widehat{G}_t$ the SAGLasso single factor. All reported quantities are the p-values for the Ibragimov-Müller (2010) test of the individual significance of the coefficients. The dependent variable is the excess return averaged over 2- through 5-year (10-year) bond maturities in regressions over the full sample period 1964–2014 (the post-1984 subsample). In the full-sample analysis, each block is constructed such that they are 12 months apart from each other.

	Spanning Hypotheses Tested							
	H_0^{S1}		H_0^{S2}		H_0^{S1}		H_0^{S2}	
	Full sample, 1964–2014				Subsample, 1985–2014			
q (# of blocks)	$q=8$	$q=16$	$q=8$	$q=16$	$q=8$	$q=16$	$q=8$	$q=16$
$PC^o_{1,t}(PC_{1,t})$	0.039	0.001	0.006	0.003	0.219	0.417	0.001	0.001
$PC^o_{2,t}(PC_{2,t})$	0.016	0.009	0.327	0.047	0.020	0.006	0.036	0.379
$PC^o_{3,t}(PC_{3,t})$	0.162	0.354	0.309	0.615	0.037	0.647	0.536	0.743
$PC_{4,t}$			0.186	0.961			0.278	0.942
$PC_{5,t}$			0.170	0.107			0.002	0.002
$\widehat{G}_t$	0.009	0.018	0.004	0.014	0.044	0.049	0.015	0.019

Table IA.F1: Tests of An Alternative Version of Spanning Hypotheses II

This table presents asymptotic and finite-sample results from tests of an alternative version of Spanning Hypothesis II, denoted $H_0^{S2,cf}$, that states that the SAGLasso macro factor (Section 4.2) has no additional predictive power for future excess bond returns, conditional on the "cycle" factor of Cieslak and Povala (2015). The tests of $H_0^{S2,cf}$ are based on the following regression, as specified in Eq. (IA.F11):

$$rx_{t,t+12}^{(12n)} = \alpha + \beta_c' \widehat{cf}_t + \beta_g' \widehat{G}_t + e_{t+12},$$

where $rx_{t,t+12}^{(12n)}$ is the excess return to an n-year zero-coupon bond from month t to month $t+12$, for $n = 2, 5, 7, 10$; $\widehat{cf}_t$ denotes the cycle factor; and $\widehat{G}_t$ the SAGLasso macro factor. For the in-sample results (panel A), t-statistics are computed using either the Hansen and Hodrick (1980) GMM covariance estimator (in parentheses) or the Newey and West (1987) HAC covariance estimator (in brackets). Out-of-sample tests considered (panel B) include the "ENC-REG" test of Ericsson (1992) and the "ENC-NEW" test proposed by Clark and McCracken (2001). The ΔR^2 and 'ΔR^2_{oos} measures denote the incremental R^2 and out-of-sample R^2 of Campbell and Thompson (2008), respectively, due to augmenting univariate regressions of $rx_{t,t+12}^{(12n)}$ on $\widehat{cf}_t$ with $\widehat{G}_t$ as in the above equation. The sample spans the period November 1971–December 2014. To obtain the finite-sample distributions of the aforementioned six statistics, 5,000 bootstrapped samples are generated from the term structure model specified in Eqs. (IA.F12)–(IA.F14) in Section IA.F. For each set of test statistics, the 95th percentile of the bootstrap distribution is reported and underlined as the 5% critical value, and the p-values (in angle brackets) are the frequency of bootstrap replications in which the test statistics are at least as large (small) as the statistic in the data.

maturity (year)	2	5	7	10		2	5	7	10
	Panel A: In-sample under $H_0^{S2,cf}$					Panel B: Out-of-sample under $H_0^{S2,cf}$			
$\widehat{G}_t$	0.688	2.491	3.376	4.153					
HH	(4.257)	(4.994)	(4.537)	(4.367)	ENC-REG	1.917	3.099	3.620	4.513
	($\underline{1.964}$)	($\underline{2.121}$)	($\underline{2.601}$)	($\underline{2.890}$)		($\underline{1.617}$)	($\underline{2.675}$)	($\underline{3.837}$)	($\underline{4.067}$)
	⟨0.001⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩		⟨0.033⟩	⟨0.026⟩	⟨0.066⟩	⟨0.029⟩
NW	[4.684]	[5.506]	[4.953]	[4.735]	ENC-NEW	41.321	77.762	80.203	77.102
	[$\underline{1.872}$]	[$\underline{2.120}$]	[$\underline{2.588}$]	[$\underline{2.857}$]		[$\underline{1.374}$]	[$\underline{4.429}$]	[$\underline{9.080}$]	[$\underline{10.746}$]
	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩		⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩
ΔR^2	0.132	0.144	0.131	0.124	ΔR^2_{oos}	0.053	0.143	0.132	0.103
	($\underline{0.011}$)	($\underline{0.032}$)	($\underline{0.062}$)	($\underline{0.072}$)		($\underline{0.010}$)	($\underline{0.033}$)	($\underline{0.065}$)	($\underline{0.075}$)
	⟨0.000⟩	⟨0.000⟩	⟨0.000⟩	⟨0.001⟩		⟨0.000⟩	⟨0.000⟩	⟨0.001⟩	⟨0.009⟩

Table IA.G1: Out-of-sample Forecasting Performance of Macro-Finance Term Structure Models

This table reports the root mean square errors (RMSEs) for out-of-sample yield forecasts formed from four different macro-finance term structure models (MTSMs). Forecasted yields include the 0.5-, 1-, 3-, 5-, 7-, and 10-year yields. The forecast horizons considered are 1, 3, 6, and 12 months. The four MTSMs—denoted by $SM(\mathcal{L},\mathcal{N})$, $USM(\mathcal{L},\mathcal{N})$, $CSM(\mathcal{L},\mathcal{N})$, and $CUSM(\mathcal{L},\mathcal{N})$—are all driven by six factors ($\mathcal{N}$=6), including five yield curve factors ($\mathcal{L}$=5) and the sole SAGLasso macro factor ($\widehat{G}_t$). Among the four models, $SM(5,6)$ and $USM(5,6)$ are unconstrained models (panel A), and $CSM(5,6)$ and $CUSM(5,6)$ are constrained models (panel B), in which selected zero restrictions are placed on parameters governing bond risk premia. Additionally, while models $SM(5,6)$ and $CSM(5,6)$ are spanned, the other two are unspanned models that satisfy the macro-unspanning conditions (H_0^{US}) specified in Eq. (12). Each of the four MTSMs is estimated once using the 1985-2007 sample. The Kalman filter is implemented recursively with observations from 1985:1 to the time that the forecast is made, beginning in 2008:1 and extending through 2014:12. Errors are reported in basis points of annualized yields.

Panel A: Out-of-sample RMSEs for unconstrained models

Forecast	$y^{(6)}$		$y^{(12)}$		$y^{(36)}$		$y^{(60)}$		$y^{(84)}$		$y^{(120)}$	
horizon	$SM(5,6)$	$USM(5,6)$	$SM(5,6)$	$USM(5,6)$	$SM(5,6)$	$USM(5,6)$	$SM(5,6)$	$USM(5,6)$	$SM(5,6)$	$USM(5,6)$	$SM(5,6)$	$USM(5,6)$
1	25.40	24.32	21.08	20.71	25.62	25.72	27.50	26.79	30.05	29.26	32.29	31.66
3	48.19	46.31	48.30	45.96	54.07	54.98	53.50	54.37	51.95	50.65	48.77	51.97
6	80.27	74.93	81.44	76.26	84.47	85.86	81.61	84.27	76.98	78.20	69.34	73.90
12	139.51	128.08	142.04	130.69	133.32	128.10	114.04	112.76	99.78	99.73	82.13	89.99

Panel B: Out-of-sample RMSEs for constrained models

Forecast	$y^{(6)}$		$y^{(12)}$		$y^{(36)}$		$y^{(60)}$		$y^{(84)}$		$y^{(120)}$	
horizon	$CSM(5,6)$	$CUSM(5,6)$	$CSM(5,6)$	$CUSM(5,6)$	$CSM(5,6)$	$CUSM(5,6)$	$CSM(5,6)$	$CUSM(5,6)$	$CSM(5,6)$	$CUSM(5,6)$	$CSM(5,6)$	$CUSM(5,6)$
1	24.15	27.73	20.00	20.63	25.01	24.86	26.08	26.08	28.71	43.32	30.54	30.63
3	45.77	46.85	45.89	43.75	51.84	48.78	50.92	49.41	48.77	52.60	46.23	46.15
6	74.83	69.99	74.63	63.74	79.10	68.63	79.22	72.83	74.17	63.69	67.11	59.98
12	102.13	95.51	104.02	81.35	116.20	87.04	115.87	91.30	109.84	69.34	92.13	66.01

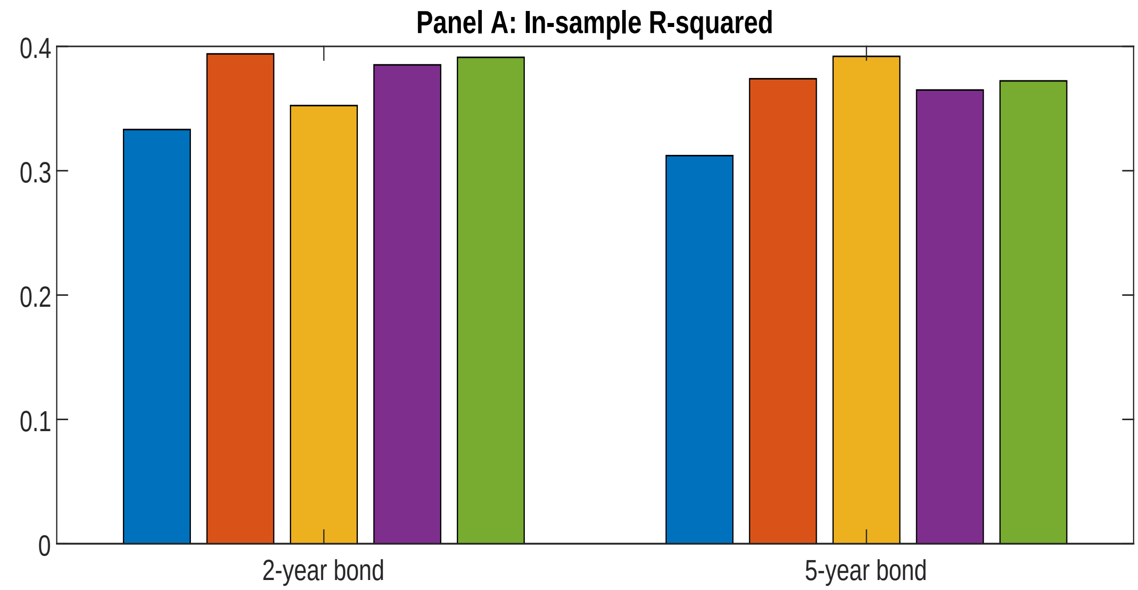
Table IA.G2: Properties of Annual Excess Returns for Five-Year Bonds Implied by Term Structure Models with Unspanned Macro Risks

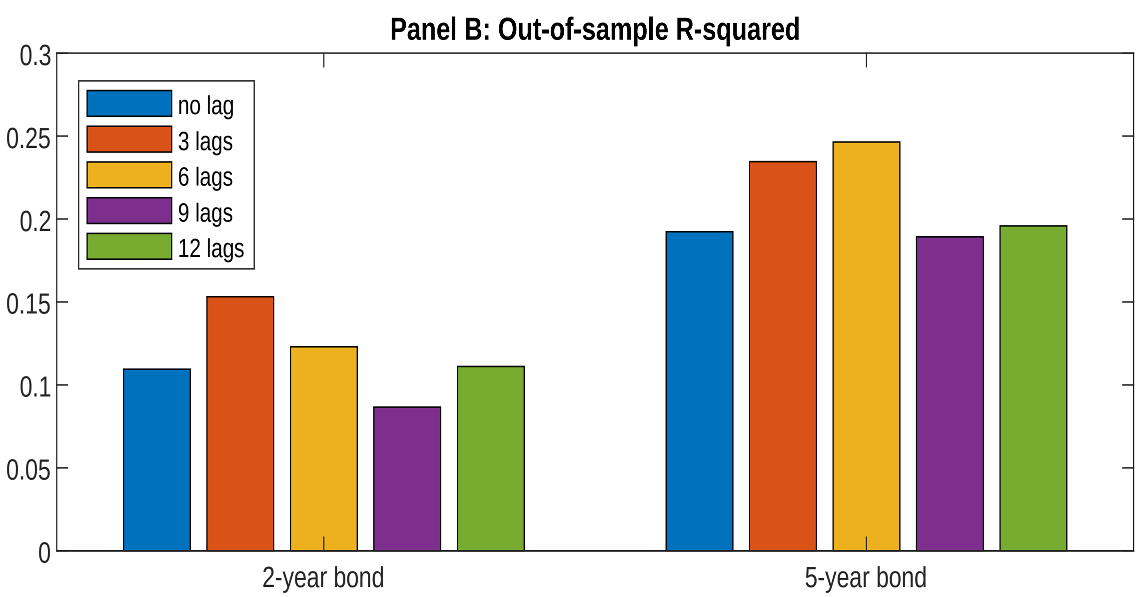
This table presents the model-implied population moments of unconditional and conditional excess returns on a five-year bond, based on the macro-finance term structure model $CUSM(\mathcal{L},\mathcal{N})$ specified in Section IA.G.1. Model $CUSM(\mathcal{L},\mathcal{N})$ is an $\mathcal{N}$-factor model with unspanned macro risks and "zero restrictions" imposed on risk premium parameters, whose underlying state vector $X_t = (PC_{1-\mathcal{L},t}, G_t)$, where $PC_{1-\mathcal{L},t}$ represent the first $\mathcal{L}$ principal components (PCs) of the *true* yields and G_t the (unspanned) SAGLasso macro factor. The last six columns quantify the variance of true conditional expected excess returns attributable to time variation in the true state vector X_t (column 4), the first three PCs of *observed* yields (column 5), the first three PCs of *true* yields (uncontaminated by measurement errors) (column 6), the first $\mathcal{L}$ PCs of *observed* yields (column 7), the first $\mathcal{L}$ PCs of *true* yields (column 8), and the first three yield PCs plus the SAGLasso factor G_t (column 9), respectively. For each of the last five columns, their ratios to the full-information variance (column 4)—the variance ratios "*VR*"—are reported in braces. The R^2 reported for each of the last three columns is their ratios to the total variance (column 3). The sample period extends from January 1985 to December 2007.

(1)	(2)	(3)		(4)	(5)	(6)	(7)	(8)	(9)
$\mathcal{L}$	Mean	Total Variance		Variance of conditional expectation based on					
				Full info	$PC^o_{1-3,t}$	$PC_{1-3,t}$	$PC^o_{1-\mathcal{L},t}$	$PC_{1-\mathcal{L},t}$	$PC_{1-3,t}+G_t$
3	2.85	58.25		27.01	19.26	19.70	19.26	19.70	27.01
			VR		{0.714}	{0.729}	{0.714}	{0.729}	{1.000}
			R^2				33.1%	33.8%	46.4%
4	2.53	63.13		30.34	21.47	21.62	22.41	23.29	29.76
			VR		{0.708}	{0.713}	{0.739}	{0.768}	{0.965}
			R^2				35.5%	36.9%	47.2%
5	2.50	67.37		33.05	21.45	22.21	21.50	25.05	32.36
			VR		{0.649}	{0.672}	{0.651}	{0.758}	{0.979}
			R^2				31.9%	37.2%	48.1%

Figure IA.B1: Predictive R^2 of Macroeconomic Factors Based on Different Lags

This figure depicts the in-sample and out-of-sample R^2 from bond return predictions with single macroeconomic factors. Macroeconomic factors are constructed from 131 macro variables, along with 0, 3, 6, 9, or 12 of their lags. The sample spans the period January 1964 to December 2014.





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