Personalized Pricing and Distribution Strategies
Online Appendix (not for publication)

This Online Appendix consists of five sections. In Section A, we provide the proofs of Propositions 3 - 6. In Section B, we analyze the demand pattern where v and s exceed 1 and show that our main insights carry over to this case. In Section C, we extend Proposition 3 to more general demands. In Section D, we show that comparing the profits from personalized pricing and uniform pricing in case of inter-brand competition provides little guidance for the optimal distribution strategy in the case of intra-brand competition. Finally, in Section E, we consider the equilibrium pricing regime in the game in which each firm chooses its pricing policy individually.

A Proofs of Propositions 3, 4, 5, and 6

Proof of Proposition 3

Uniform pricing by B

We first analyze the situation of uniform pricing by both firms. To solve for the subgame-perfect equilibrium, we proceed by backward induction and first determine the reaction functions in the downstream stage. To simplify the exposition, we proceed under the assumption that both demands are positive in equilibrium and verify later that this is in fact true. If both firms are active, they charge retail prices p_A and p_B such that some consumers favor A whereas others favor B. Let $x_{AB} > 0$ denote the consumer indifferent between buying from A or B, and $x_B > 0$ denote the consumer indifferent between buying from B and not buying:

$$x_{AB}(p_A, p_B) = \frac{1 - p_A - v + p_B}{1 - s} \quad \text{and} \quad x_B(p_B) = \frac{v - p_B}{s}.$$

The demands for A and B are, respectively, x_{AB} and $x_B - x_{AB}$. Their profit functions (gross of the fixed fee) are then $\Pi_A = x_{AB}(p_A, p_B)p_A + [x_B(p_B) - x_{AB}(p_A, p_B)]\, w$ and $\Pi_B = [x_B(p_B) - x_{AB}(p_A, p_B)]\,(p_B - w)$. The linearity of the demand functions ensures that firms' profit functions are strictly concave in their prices; hence, firms' reaction functions are characterized by the first-order conditions, which yield:

$$p_A(p_B; w) = \frac{1 - v + p_B + w}{2} \quad \text{and} \quad p_B(p_A; w) = \frac{v + w}{2} - \frac{s\,(1 - p_A)}{2}.$$

Combining these reaction functions yields the equilibrium retail prices, as a function of the wholesale price w:

$$\begin{aligned} p_A(w) &= \frac{2-s+3w-v}{4-s}, \\ p_B(w) &= \frac{v(2-s)+w(2+s)-s}{4-s}. \end{aligned}$$

The associated demands are $D_A(w) = x_{AB}(p_A(w), p_B(w))$ and $D_B(w) = x_B(p_B(w)) - x_{AB}(p_A(w), p_B(w))$.

In the first stage, A and B negotiate over w and F, taking into account that, in the second stage, each firm sets its retail price so as to maximize its own profit. Hence, they set w to maximize the industry profit and use F to share it according to their bargaining powers and outside options.[1] The industry profit is given by:

$$x_{AB}(p_A(w), p_B(w))p_A(w) + [x_B(p_B(w)) - x_{AB}(p_A(w), p_B(w))]\, p_B(w)$$

This profit is again a strictly concave function of w, as its second-order derivative is given by:

$$\Pi''(w) = -\frac{2(4+5s)}{s(4-s)^2} < 0.$$

Hence, the equilibrium wholesale price is characterized by the first-order condition, leading to (the subscript UU stands for *Uniform pricing by A and B*):

$$w^*_{UU} = \frac{s(4(1+v)+s)}{2(4+5s)}.$$

Inserting the equilibrium prices into the demand functions D_A and D_B yields:

$$D^*_A = \frac{2(2-v)+s(3-4v-s)}{2(4+5s)(1-s)} \quad \text{and} \quad D^*_B = \frac{(2+s)(v-s)}{s(4+5s)(1-s)}.$$

The assumption that the two demand functions intersect at a positive valuation (i.e., $v > s$) ensures that both equilibrium demands are positive. Indeed, D^*_A is strictly falling in v and is equal to $(2+s)/(2(4+5s)) > 0$ at the highest possible value of v, which is 1. Direct inspection of D_B shows that it is positive for $v > s$. As w^* constitutes a global maximum in the relevant range, and achieving $D_B = 0$ is feasible with a high enough w, it follows that in equilibrium it is optimal for the firms to generate positive

[1] Specifically, A's outside option is its profit from mono distribution whereas B's outside option is zero.

sales for B. Indeed, the resulting profit, equal to:

$$\Pi_{UU}^{*}=\frac{s\left(5s+4-s^{2}\right)+4v\left(1+s\right)\left(v-2s\right)}{4s\left(1-s\right)\left(4+5s\right)},\tag{12}$$

exceeds the monopoly profit that A can obtain with mono distribution, Π_{U}^{m}:

$$\Pi_{UU}^{*}-\Pi_{U}^{m}=\frac{(1+s)(v-s)^{2}}{s(4+5s)(1-s)}>0.$$

Personalized pricing by B

We next analyze the case in which B charges personalized prices (and A still a uniform one). Given w and p_A, B's price response is such that consumers x with $u_A(x)-p_A>u_B(x)-w$, or:

$$x<\check{x}\left(w,p_{A}\right)=\frac{1-p_{A}-v+w}{1-s},$$

end-up buying from A. Instead, consumers $\check{x}\left(w,p_{A}\right)<x<\tilde{x}_{B}\left(w\right)$, with $\tilde{x}_{B}\left(w\right)=(v-w)/s$, end-up buying from B at price $p_{B}\left(x\right)=u_{B}\left(x\right)-\max\left\{u_{A}\left(x\right)-p_{A},0\right\}$. A's variable profit (gross of the fee F) is therefore given by:

$$p_{A}\check{x}\left(w,p_{A}\right)+w\left[\tilde{x}_{B}\left(w\right)-\check{x}\left(w,p_{A}\right)\right].$$

Optimizing this with respect to p_A yields:

$$p_{A}\left(w\right)=w+\frac{1-v}{2}.$$

We now turn to the wholesale stage. The two firms seek to maximize the industry profit given by:

$$\Pi=p_{A}(w)\check{x}\left(w\right)+\int_{\check{x}(w)}^{\hat{x}(w)}\left[p_{A}(w)+u_{B}\left(x\right)-u_{A}\left(x\right)\right]dx+\int_{\hat{x}(w)}^{\tilde{x}_{B}(w)}u_{B}\left(x\right)dx,\tag{13}$$

where $p_A(w)=w+(1-v)/2$, $\check{x}(w)=\left(1-p_{A}-v+w\right)/\left(1-s\right)$, $\hat{x}(w)=\left(1+v\right)/2-w$, and $\tilde{x}_B(w)=(v-w)/s$. Maximizing this profit with respect to w yields (the subscript UP stands for the pricing regime in which A sets a *Uniform price* and B *Personalized prices*):[2]

$$w_{UP}^{*}=\frac{s(1+v)}{2(1+s)}.$$

Inserting $w=w_{UP}^{*}$ into (13), we obtain that the industry profit is given by:

$$\Pi_{UP}^{*}=\frac{s+2s^{2}-4vs-2vs^{2}+2v^{2}+v^{2}s}{4s(1-s^{2})}.\tag{14}$$

[2]It is straightforward to check that the industry profit is a concave function of w.

As $\Pi^*_{UU} > \Pi^m_U$, to show that $\Pi^*_{UP} > \Pi^m_U$ it suffices to show that $\Pi^*_{UP} > \Pi^*_{UU}$. Indeed, we obtain

$$\Pi^*_{UP} - \Pi^*_{UU} = \frac{(s-v)^2}{4s} \frac{4+6s+s^2}{(4+5s)(1-s^2)} > 0. \tag{15}$$

Proof of Proposition 4

Personalized pricing by B

We start with the situation of personalized pricing by both firms. As noted in the main text, if the wholesale price w is such that $w \geq \hat{u} = (v-s)/(1-s)$, B will be inactive;[3] hence, the industry profit cannot be larger than Π^m_P.

We now focus on $w \leq \hat{u}$. We need to distinguish whether or not B finds it profitable to supply (some) consumers uninterested in A's product. From Figure 6, such consumers exist if and only if $x_B(w) > 1$. The latter inequality can only hold if w is sufficiently low, that is $w < \underline{w} \equiv v - s$. Note that $\underline{w} = (1-s)\hat{u} < \hat{u}$.

Region $w \leq \underline{w}$

In this region, in which $x_B(w) \geq 1$, as shown in the text, the industry profit is given by:

$$\Pi(w) = \int_0^{x_A(w)} \left[w + |u_B(x) - u_A(x)|\right] dx + \int_{x_A(w)}^{x_B(w)} u_B(x) dx.$$

It is strictly concave in w: using $u_A(x_A(w)) = u_B(x_B(w)) = w$, we have:

$$\Pi'(w) = x_A(w) + w\frac{dx_B(w)}{dw} = 1 - w - \frac{w}{s} = 1 - \frac{w(1+s)}{s},$$

and thus:

$$\Pi''(w) = -\frac{1+s}{s} < 0.$$

Region $\underline{w} < w \leq \hat{u}$

If instead $w > \underline{w}$, the industry profit includes an additional term, as illustrated by Figure 8. This term corresponds to consumers in the region $x_B(w) < x \leq 1$: B does not find it profitable to supply these consumers (as $u_B(x) < w$), but they are still willing to buy from A, which can extract their full surplus. The industry profit can then be written as:

$$\Pi(w) = \int_0^{x_A(w)} \left[w + |u_B(x) - u_A(x)|\right] dx + \int_{x_A(w)}^{x_B(w)} u_B(x) dx + \int_{x_B(w)}^{1} u_A(x) dx.$$

[3] Recall that $\hat{u} = u_A(\hat{x}) = u_B(\hat{x})$.

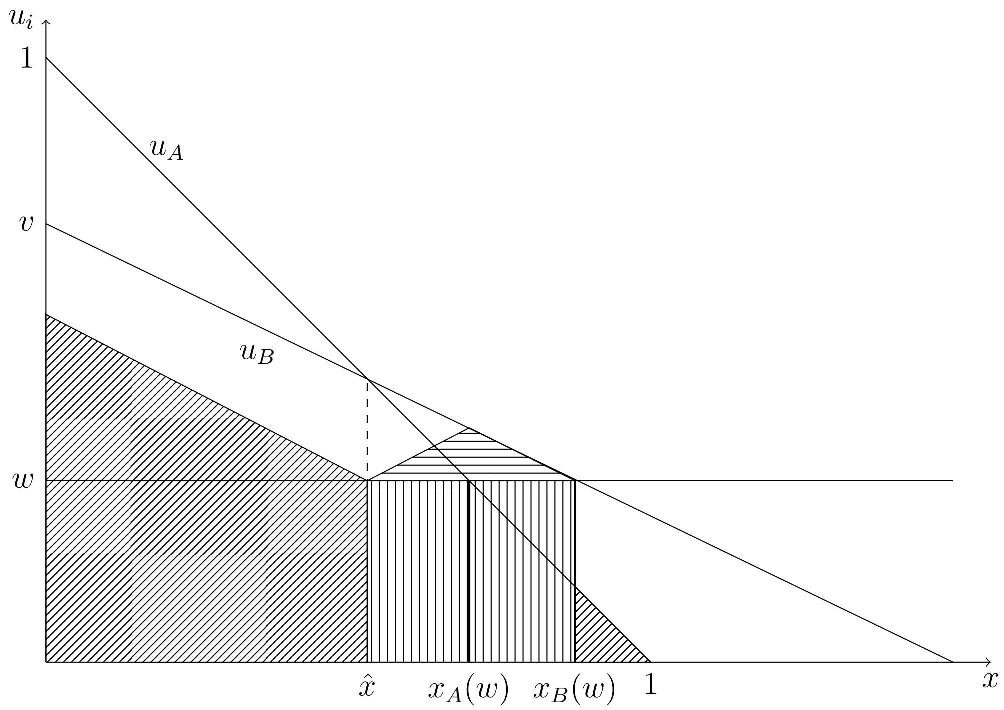

Figure 8: Profits if $w > \underline{w}$

The first-order derivative is equal to:

$$\begin{aligned} \Pi'(w) &= x_A(w) + [w - u_A(x_B(w))] \frac{dx_B(w)}{dw} \\ &= (1-w)\left(1+\frac{1}{s}\right) - (v-w)\frac{1}{s^2} \\ &= \frac{s^2 + s - v + (1 - s - s^2)\, w}{s^2}. \end{aligned}$$

Hence:

$$\begin{aligned} \Pi'_{-}(\hat{u}) &= \frac{1-v}{1-s}, \\ \Pi'_{+}(\underline{w}) &= \frac{1+s}{s}\left(\frac{s(2+s)}{1+s} - v\right), \\ \Pi''(w) &= \frac{1-s-s^2}{s^2}. \end{aligned}$$

It follows that $\Pi(w)$ is strictly concave in w if:

$$s > \hat{s} = \frac{\sqrt{5}-1}{2} \simeq 0.618,$$

and is instead weakly convex if $s \leq \hat{s}$; in addition, $\Pi'(\hat{u}) > 0$ whereas $\Pi'(\underline{w}) \geq 0$ if and only if:

$$v \leq \hat{v}(s) \equiv \frac{s(2+s)}{1+s},$$

where $\hat{v}(s)$ increases with s and exceeds 1 for $s \geq \hat{s}$. Furthermore, not only is the profit function $\Pi(w)$ continuous at $w = \underline{w}$, its derivative $\Pi'(w)$ is also continuous:

$$\Pi'_{-}(\underline{w}) = \left(1 - \frac{1+s}{s}w\right)\Bigg|_{w=v-s} = \frac{1+s}{s}\left(\frac{s(2+s)}{1+s} - v\right) = \Pi'_{+}(\underline{w}).$$

Optimal distribution policy

As long as $w \geq \hat{u}$, B cannot attract any consumer at any profitable price: hence, doing so cannot be more profitable than mono distribution. Furthermore, if $v \leq \hat{v}(s)$, then $\Pi'(\underline{w}) \geq 0$, implying that dual distribution cannot be more profitable than mono distribution:

- in the range $\underline{w} \leq w \leq \hat{u}$, the profit function $\Pi(w)$ is increasing, as it is quadratic and its derivative is non-negative at both ends of the range (namely, $\Pi'(\underline{w}) \geq 0$ and $\Pi'(\hat{u}) > 0$);
- in the range $w \leq \underline{w}$, the profit function $\Pi(w)$ is again increasing, as it is concave and its derivative is non-negative at the upper end of the range (namely, $\Pi'(\underline{w}) \geq 0$);
- it follows that the profit achieved under dual distribution cannot exceed $\Pi(\hat{u})$, which is less profitable than mono distribution.

As already noted, $\hat{v}(s)$ is increasing in s in the range $s \in [0, 1]$, and satisfies $\hat{v}(s) \geq 1$ for $s \geq \hat{s}$. It follows that, if $s \geq \hat{s}$, then dual distribution cannot be more profitable than mono distribution, as we then have $\hat{v}(s) \geq 1\,(> v)$.

If instead $s < \hat{s}$ and $v > \hat{v}(s)$, then $\Pi'(\underline{w}) < 0$. From the analysis for the region $w \leq \underline{w}$ above, the first-order condition $\Pi'(w) = 0$ then determines the candidate optimal wholesale price, which is given by:

$$w = w^*_{PP} = \frac{s}{1+s} \in (0, \underline{w}).$$

The corresponding profit is:

$$\Pi^*_{PP} = \frac{1}{2}\frac{s(1+3s) - 4s(1+s)v + (1+s)^2 v^2}{s(1-s^2)}.$$

Compared with the profit from mono distribution, Π^m_P, dual distribution introduces a change in profit equal to:

$$\frac{1}{2}\frac{s^2(s+3) - 4s(1+s)v + (1+s)^2 v^2}{s(1-s^2)}.$$

The numerator of this expression is a convex quadratic polynomial of v and its roots are:

$$\frac{s\left(2-\sqrt{1-s}\right)}{1+s} \text{ and } \frac{s\left(2+\sqrt{1-s}\right)}{1+s}.$$

Furthermore, $\hat{v}\left(s\right)$ lies between these two roots in the relevant range $s < \hat{s}$:

$$\begin{aligned}
\frac{\frac{s\left(2+\sqrt{1-s}\right)}{1+s}}{\hat{v}\left(s\right)} &= \frac{\frac{s\left(2+\sqrt{1-s}\right)}{1+s}}{\frac{s(2+s)}{1+s}} = \frac{2-\sqrt{1-s}}{2+s} < 1, \\
\frac{\frac{s\left(2+\sqrt{1-s}\right)}{1+s}}{\hat{v}\left(s\right)} &= \frac{\frac{s\left(2+\sqrt{1-s}\right)}{1+s}}{\frac{s(2+s)}{1+s}} = \frac{2+\sqrt{1-s}}{2+s} > 1,
\end{aligned}$$

where the last inequality stems from $\sqrt{1-s} > s$ in the relevant range $s < \hat{s}$. It follows that dual distribution is more profitable than mono distribution if and only if $s < \hat{s}$ and v exceeds the larger root, that is, if:

$$v > \tilde{v}\left(s\right) \equiv \frac{s\left(2+\sqrt{1-s}\right)}{1+s}$$

Note that $\tilde{v}\left(s\right)$ is increasing in s in the range $s \leq \hat{s}$, and exceeds 1 in the range $s \geq \hat{s}$. Hence, as $v < 1$, the condition $v > \tilde{v}\left(s\right)$ implies $s < \hat{s}$.

We next show that the threshold $\tilde{v}\left(s\right)$ translates into a threshold $\beta\left(\alpha\right)$, such that dual distribution is optimal if and only if $\beta > \beta\left(\alpha\right)$. Let:

$$\Omega \equiv \{(v,s) \text{ satisfying } 0 < s < v < 1\}$$

denote the set of admissible values. For any $(v,s) \in \Omega$, the channels' contributions are respectively given by:

$$\alpha = \hat{\alpha}\left(v,s\right) \equiv \int_0^{\frac{1-v}{1-s}} \left[1-x-(v-sx)\right] dx = \frac{(1-v)^2}{2(1-s)}$$

and:

$$\beta = \hat{\beta}\left(v,s\right) \equiv \int_{\frac{1-v}{1-s}}^{1} \left[v-sx-(1-x)\right] dx + \int_1^{\frac{v}{s}} \left[v-sx\right] dx = \frac{(v-s)^2}{2s(1-s)},$$

where:

$$\frac{\partial\hat{\alpha}}{\partial v} = -\frac{1-v}{1-s} < 0 \quad \text{and} \quad \frac{\partial\hat{\alpha}}{\partial s} = \frac{(1-v)^2}{2(1-s)^2} > 0, \tag{16}$$

and:

$$\frac{\partial\hat{\beta}}{\partial v} = \frac{v-s}{s(1-s)} > 0 \quad \text{and} \quad \frac{\partial\hat{\beta}}{\partial s} = -\frac{(v-s)(v+s-2vs)}{2(1-s)^2s^2} < 0. \tag{17}$$

Let:

$$\hat{\Omega} \equiv \left\{ (\alpha, \beta) \mid \exists\, (v, s) \in \Omega \text{ such that } \left(\hat{\alpha}(v, s), \hat{\beta}(v, s) \right) = (\alpha, \beta) \right\}$$

denote the set of admissible values for α and β. We now show that there exists a bijection between Ω and $\hat{\Omega}$. To see this, fix $(\alpha, \beta) \in \hat{\Omega}$ and consider the slopes of the iso-α curve:

$$\hat{\alpha}(v, s) = \alpha$$

and of the iso-β curve:

$$\hat{\beta}(v, s) = \beta$$

in the space (v, s). The slopes of these curves satisfy:

$$\left.\frac{dv}{ds}\right|_{\hat{\beta}(v,s)=\beta} = \left.\frac{-\frac{\partial \hat{\beta}}{\partial s}(v, s)}{\frac{\partial \hat{\beta}}{\partial v}(v, s)}\right|_{\hat{\beta}(v,s)=\beta} = \left.\frac{v + s - 2vs}{2s(1-s)}\right|_{\hat{\beta}(v,s)=\beta} > \frac{1}{2},$$

where the last inequality is due to the fact that $v + s - 2vs > s(1-s)$ for $v \in (0, 1)$,[4] and:

$$\left.\frac{dv}{ds}\right|_{\hat{\alpha}(v,s)=\alpha} < \left.\frac{-\frac{\partial \hat{\alpha}}{\partial s}(v, s)}{\frac{\partial \hat{\alpha}}{\partial v}(v, s)}\right|_{\hat{\alpha}(v,s)=\alpha} = \left.\frac{1-v}{2(1-s)}\right|_{\hat{\alpha}(v,s)=\alpha} < \frac{1}{2}.$$

It follows that

$$\left.\frac{dv}{ds}\right|_{\hat{\beta}(v,s)=\beta} > \left.\frac{dv}{ds}\right|_{\hat{\alpha}(v,s)=\alpha},$$

implying that the iso-α and iso-β curves intersect at most once. Hence, for any $(\alpha, \beta) \in \hat{\Omega}$, there exists a unique $(v, s) \in \Omega$ such that $\left(\hat{\alpha}(v, s), \hat{\beta}(v, s)\right) = (\alpha, \beta)$. Let $\hat{v}(\alpha, \beta)$ and $\hat{s}(\alpha, \beta)$ denote these values.

By construction, we have:

$$\hat{\alpha}\left(\hat{v}(\alpha, \beta), \hat{s}(\alpha, \beta)\right) = \alpha \quad \text{and} \quad \hat{\beta}\left(\hat{v}(\alpha, \beta), \hat{s}(\alpha, \beta)\right) = \beta. \tag{18}$$

Differentiating these equalities with respect to α yields (dropping the arguments for ease of exposition):

$$\begin{aligned} \frac{\partial \hat{\alpha}}{\partial v}\frac{\partial \hat{v}}{\partial \alpha} + \frac{\partial \hat{\alpha}}{\partial s}\frac{\partial \hat{s}}{\partial \alpha} &= 1, \\ \frac{\partial \hat{\beta}}{\partial v}\frac{\partial \hat{v}}{\partial \alpha} + \frac{\partial \hat{\beta}}{\partial s}\frac{\partial \hat{s}}{\partial \alpha} &= 0. \end{aligned}$$

[4] To see this, note that $v + s - 2vs - s(1-s) = v(1-v) + (v-s)^2$.

Solving for $\frac{\partial \hat{v}}{\partial \alpha}$ and $\frac{\partial \hat{s}}{\partial \alpha}$ then yields:

$$\frac{\partial \hat{v}}{\partial \alpha}(\alpha, \beta) = \left.\frac{\frac{\partial \hat{\beta}}{\partial s}(v, s)}{D(v, s)}\right|_{(v,s)=(\hat{v}(\alpha,\beta),\hat{s}(\alpha,\beta))}, \tag{19}$$

$$\frac{\partial \hat{s}}{\partial \alpha}(\alpha, \beta) = \left.\frac{-\frac{\partial \hat{\beta}}{\partial v}(v, s)}{D(v, s)}\right|_{(v,s)=(\hat{v}(\alpha,\beta),\hat{s}(\alpha,\beta))}, \tag{20}$$

where:

$$\begin{aligned} D(v, s) &\equiv \frac{\partial \hat{\alpha}}{\partial v}(v, s)\frac{\partial \hat{\beta}}{\partial s}(v, s) - \frac{\partial \hat{\alpha}}{\partial s}(v, s)\frac{\partial \hat{\beta}}{\partial v}(v, s) \\ &= -\frac{\partial \hat{\alpha}}{\partial v}(v, s)\frac{\partial \hat{\beta}}{\partial v}(v, s)\left[\frac{-\frac{\partial \hat{\beta}}{\partial s}(v, s)}{\frac{\partial \hat{\beta}}{\partial v}(v, s)} - \frac{-\frac{\partial \hat{\alpha}}{\partial s}(v, s)}{\frac{\partial \hat{\alpha}}{\partial v}(v, s)}\right] > 0, \end{aligned}$$

where the inequality stems from $\frac{\partial \hat{\alpha}}{\partial v}(v, s) < 0 < \frac{\partial \hat{\beta}}{\partial v}(v, s)$ and, as already noted:

$$\frac{-\frac{\partial \hat{\beta}}{\partial s}(v, s)}{\frac{\partial \hat{\beta}}{\partial v}(v, s)} > \frac{1}{2} > \frac{-\frac{\partial \hat{\alpha}}{\partial s}(v, s)}{\frac{\partial \hat{\alpha}}{\partial v}(v, s)}.$$

It then follows from $\frac{\partial \hat{\beta}}{\partial s}(v, s) < 0 < \frac{\partial \hat{\beta}}{\partial v}(v, s)$ that:

$$\frac{\partial \hat{v}}{\partial \alpha}(\alpha, \beta) < 0 \quad \text{and} \quad \frac{\partial \hat{s}}{\partial \alpha}(\alpha, \beta) < 0.$$

Likewise, differentiating the equalities in (18) with respect to β yields:

$$\begin{aligned} \frac{\partial \hat{\alpha}}{\partial v}\frac{\partial \hat{v}}{\partial \beta} + \frac{\partial \hat{\alpha}}{\partial s}\frac{\partial \hat{s}}{\partial \beta} &= 0, \\ \frac{\partial \hat{\beta}}{\partial v}\frac{\partial \hat{v}}{\partial \beta} + \frac{\partial \hat{\beta}}{\partial s}\frac{\partial \hat{s}}{\partial \beta} &= 1. \end{aligned}$$

Solving for $\frac{\partial \hat{v}}{\partial \beta}$ and $\frac{\partial \hat{s}}{\partial \beta}$ then yields:

$$\frac{\partial \hat{v}}{\partial \beta}(\alpha, \beta) = \left.\frac{-\frac{\partial \hat{\alpha}}{\partial s}(v, s)}{D(v, s)}\right|_{(v,s)=(\hat{v}(\alpha,\beta),\hat{s}(\alpha,\beta))} < 0, \tag{21}$$

$$\frac{\partial \hat{s}}{\partial \beta}(\alpha, \beta) = \left.\frac{\frac{\partial \hat{\alpha}}{\partial v}(v, s)}{D(v, s)}\right|_{(v,s)=(\hat{y}(\alpha,\beta),\hat{s}(\alpha,\beta))} < 0, \tag{22}$$

where the inequalities stem from $D(v, s) > 0$ and $\frac{\partial \hat{\alpha}}{\partial v}(v, s) < 0 < \frac{\partial \hat{\alpha}}{\partial s}(v, s)$.

The condition:

$$v > \tilde{v}(s) = \frac{s\left(2 + \sqrt{1 - s}\right)}{1 + s}$$

amounts to:

$$\Phi(\alpha,\beta) \equiv \hat{v}(\alpha,\beta) - \tilde{v}(\hat{s}(\alpha,\beta)) > 0,$$

where:

$$\frac{\partial\Phi}{\partial\beta}(\alpha,\beta) = \frac{\partial\hat{v}}{\partial\beta}(\alpha,\beta) - \tilde{v}'(s)\frac{\partial\hat{s}}{\partial\beta}(\alpha,\beta). \tag{23}$$

Using (21), (22) and (23), we have:

$$\frac{\partial\Phi}{\partial\beta}(\alpha,\beta) = \left[\tilde{v}'(s) - \frac{(1-v)}{2(1-s)}\right]\frac{1-v}{(1-s)D(v,s)}\bigg|_{(v,s)=(\hat{v}(\alpha,\beta),\hat{s}(\alpha,\beta))}.$$

To show that $\frac{\partial\Phi}{\partial\beta}(\alpha,\beta) > 0$, it thus suffices to show that:

$$\tilde{v}'(s) - \frac{1-v}{2(1-s)} > 0$$

in the relevant range $v > \tilde{v}(s)$. A sufficient condition is therefore $\tilde{\psi}(s) > 0$, where:

$$\tilde{\psi}(s) \equiv \tilde{v}'(s) - \frac{1-\tilde{v}(s)}{2(1-s)} =$$

$$= \frac{d}{ds}\left(\frac{s\left(2+\sqrt{1-s}\right)}{1+s}\right) - \frac{1-\frac{s\left(2+\sqrt{1-s}\right)}{1+s}}{2(1-s)} = \frac{2(1-s)+(3-s)\sqrt{1-s}}{2(1+s)^2(1-s)},$$

which is indeed positive for $s \in (0,1)$. It follows that there exists a unique value of β, which we denote $\beta_{PP}(\alpha)$, such that dual distribution is strictly optimal if and only if $\beta > \beta_{PP}(\alpha)$.

Uniform pricing by B

We now move to the hybrid regime in which A charges personalized prices and B a uniform one. Again, we solve the game by backward induction. Consider first A's price response to a given w and p_B. Two market share configurations can occur, depending on the value of p_B. When p_B is relatively small, the marginal consumer indifferent between buying from B and not buying, $x_B = (v - p_B)/s$, exceeds 1. A's best response is to serve consumers x with $v_A(x) - w > u_B(x) - p_B$, or:

$$x < x_{AB}(w) \equiv \frac{1-w-v+p_B}{1-s},$$

whereas consumers x with $x_{AB}(w) < x < x_B$ end-up buying from B.

Instead, when p_B is high enough so that $x_B < 1$, a third demand region exists between x_B and 1 in which consumers end-up buying from A. The thresholds for the first two demand regions are the same as in the market share configuration above.

We start with the first case. If B serves consumers x with $x_{AB}(w) < x < x_B$, its profit is:

$$(p_B - w)\left(\frac{v - p_B}{s} - \frac{1 - w - v + p_B}{1 - s}\right).$$

Maximizing with respect to p_B yields:

$$p_B\left(w\right) = \frac{v + w - s(1 - w)}{2}. \tag{24}$$

We now turn to the negotiation at the wholesale stage. Because A charges to each consumer x a personalized price of $1 - x - v + sx + p_B(w)$, the industry profit is:

$$\int_0^{\frac{1-w-v+p_B(w)}{1-s}} \left[1 - x - v + sx + p_B(w)\right] dx + p_B(w)\left(\frac{v - p_B(w)}{s} - \frac{1 - w - v + p_B(w)}{1 - s}\right), \tag{25}$$

with $p_B(w)$ given by (24). Maximizing (25) with respect to w yields (the subscript PU stands for the pricing regime in which A sets *Personalized prices* and B a *Uniform price*)[5]

$$w^*_{PU} = \frac{s\left[2 + s + v\right]}{2 + 5s + s^2}.$$

This market configuration is only valid if $x_B \geq 1$. Comparing the two thresholds at the equilibrium values, we obtain that the inequality is fulfilled if and only if:

$$v \geq \frac{s(2 + 4s + s^2)}{1 + 2s}. \tag{26}$$

As v is bounded above by 1, this inequality can only be satisfied if $s^2(4 + s) - 1 \leq 0$, which, in the relevant range $s \in (0, 1)$, amounts to

$$s \leq \check{s} \simeq 0.473.$$

Inserting $w = w^*_{PU}$ in (25), the resulting industry profit is:

$$\Pi^*_{PU} = \frac{v^2 + s(2 - 4v + 3v^2) + s^2(5 - 8v + 2v^2) - s^3}{2s(1 - s)(2 + 5s + s^2)}. \tag{27}$$

Compared with the profit from mono distribution, Π^m_P, the profit from dual distribution is larger if and only if:[6]

$$v \geq \check{v}(s) \equiv s\frac{2 + \sqrt{\frac{(1-s)(2+5s+s^2)}{1+2s}}}{1 + s}. \tag{28}$$

[5]The maximization problem is strictly concave.

[6]Because Π^*_{PU} is a convex quadratic polynomial in v, the equation $\Pi^*_{PU} - \Pi^m_P = 0$ has two roots. The lower one is below zero, and the larger one is $\check{v}(s)$.

It can be checked that $\check{v}(s)$ is increasing in s in the range $s \in (0, \check{s})$ and exceeds 1 in the range $s \in (\check{s}, 1)$. As $v < 1$, the condition $v \geq \check{v}(s)$ implies $s < \check{s}$. Moreover, $\check{v}(s)$ is indeed larger than the right-hand side of (26) for $s < \check{s}$. Hence, if the first market configuration is valid, both firms are active if and only if (28) holds.

We now turn to the second market configuration. In this case, the industry profit is:

$$\int_0^{\frac{1-w-v+p_B(w)}{1-s}} [1 - x - v + sx + p_B(w)]\, dx \tag{29}$$

$$+p_B(w)\left(\frac{v - p_B}{s} - \frac{1 - w - v + p_B(w)}{1 - s}\right) + \int_{\frac{v-p_B}{s}}^{1} [1 - x]\, dx,$$

with $p_B(w)$ again given by (24). Maximizing with respect to w, we obtain that the second-order condition for an interior solution is fulfilled if and only if $s^2(4+s)-1 > 0$, resulting in a wholesale price of:

$$\frac{s^3 + s^2(3 + v) + s(1 - v) - v}{s^3 + 4s^2 - 1}.$$

However, at this wholesale price, B's demand is negative for $s^2(4+s)-1 > 0$. As a consequence, in case the maximization problem is concave, mono distribution is optimal. Instead, if $s^2(4+s) - 1 \leq 0$, the maximization problem (29) is convex. It follows that w is optimally set either so high that B is not active, which results in mono distribution, or so low that $x_B \geq 1$.

In the latter case, the first market configuration is valid if (26) holds. Instead, if (26) is not fulfilled, the optimal w is set such that x_B exactly equals 1, or $(v - p_B(w))/s = 1$, with $p_B(w)$ given by (24). Solving the last equation for w yields $w = (v - s)/(1 + s)$. The resulting industry profit is:

$$\frac{1 + v^2 - 2sv(1 - v) - s^2(1 + 4v) + 2s^3 + s^4}{2(1 - s)(1 + s)^2}.$$

Compared with Π_P^m, the profit with dual distribution is larger if and only if:

$$v \geq s + (1 + s)\sqrt{\frac{s(1 - s)}{1 + 2s}}. \tag{30}$$

However, the right-hand side of (30) is larger than the right-hand side of (26) for all values of s, with $s^2(4 + s) - 1 \leq 0$. Since the profit function with $x_B = 1$ is only valid if (26) does not hold, (30) cannot be fulfilled for any admissible value of s. Hence, mono distribution is optimal is this case.

As a consequence, dual distribution is more profitable than mono distribution if and only if $v \geq \check{v}(s)$, which can only hold if $s < \check{s}$. In the same way as in the first part of the proof (i.e., personalized pricing by B), we can show that the existence of a unique threshold value $\check{v}(s)$ implies that there is also a unique threshold value for β, denoted by $\beta_{PU}(\alpha)$, such that dual distribution is optimal if and only if $\beta > \beta_{PU}(\alpha)$. Following the same steps as before, a sufficient condition is:

$$\check{\psi}(s) \equiv \check{v}'(s) - \frac{1-\check{v}(s)}{2(1-s)} > 0,$$

which indeed holds:

$$\check{v}'(s) - \frac{1-\check{v}(s)}{2(1-s)} = \frac{d}{ds}\left(s\frac{2+\sqrt{\frac{(1-s)(2+5s+s^2)}{1+2s}}}{1+s}\right) - \frac{1-s\frac{2+\sqrt{\frac{(1-s)(2+5s+s^2)}{1+2s}}}{1+s}}{2(1-s)}$$

$$= \frac{3-s}{2(1+s)^2} + \frac{4+19s+25s^2+8s^3+2s^4}{2(1+s)^2(1+2s)^2\sqrt{\frac{(2+5s+s^2)}{(1-s)(1+2s)}}},$$

which is strictly positive for all $s \in (0,1)$.

Finally, we compare the two thresholds $\tilde{v}(s)$ and $\check{v}(s)$ with each other. Note that $\check{v}(s)$ can be written as $s\left(2+\delta(s)\sqrt{1-s}\right)/(1+s)$ with $\delta(s) = \sqrt{(2+5s+s^2)/(1+2s)}$, whereas $\tilde{v}(s)$ is $s\left(2+\sqrt{1-s}\right)/(1+s)$. Since:

$$\sqrt{\frac{2+5s+s^2}{1+2s}} > 1,$$

mono distribution is optimal for a larger range in the hybrid regime—i.e., the regime in which only B charges a uniform price—compared to the symmetric regime in which both firms charge personalized prices. Due to the one-to-one mapping between the thresholds in the v-s-plane and the thresholds in the α-β-plane, it follows that $\beta_{PU}(\alpha) > \beta_{PP}(\alpha)$.

Proof of Proposition 5

Conditional on opting for mono distribution, firm A obviously favors personalized pricing, which yields the profit $\Pi_P^m = 1/2$. Conditional on opting for dual distribution, firms negotiate over the pricing regime that maximizes the industry profit.

The profits generated by the pricing regimes in which A charges uniform prices are Π_{UU}^* and Π_{UP}^*, respectively given by (12) and (14), which satisfy $\Pi_{UP}^* - \Pi_{UU}^* > 0$ from (15). Therefore, firms never choose the uniform pricing regime.

The profits generated by the two pricing regimes in which A charges personalized prices are Π_{PP} and Π_{PU}, respectively given by (3) and (27), which satisfy:

$$\Pi_{PP}^{*} - \Pi_{PU}^{*} = \frac{\left(1+3s+s^2\right)\left(2s-v-sv\right)^2}{2s(1-s^2)(2+5s+s^2)} > 0.$$

Therefore, firms never choose the hybrid regime.

The relevant options are therefore mono distribution with personalized pricing, yielding Π_P^m, and dual distribution with personalized pricing by B and either personalized pricing or uniform pricing by A, yielding respectively Π_{PP}^{*} and Π_{UP}^{*}.

Comparing the relevant profits from dual distribution yields:

$$\Pi_{PP}^{*} - \Pi_{UP}^{*} = \frac{\left(1-v\right)\left(1+4s-v(3+2s)\right)}{4\left(1-s^2\right)},$$

which is positive if:

$$v < \lambda(s) \equiv \frac{1+4s}{3+2s}.$$

This condition amounts to:

$$\Lambda\left(\alpha,\beta\right) \equiv \hat{v}\left(\alpha,\beta\right) - \lambda\left(\hat{s}\left(\alpha,\beta\right)\right) < 0.$$

Differentiating $\Lambda\left(\alpha,\beta\right)$ with respect to α yields:

$$\frac{\partial\Lambda}{\partial\alpha}\left(\alpha,\beta\right) = \frac{\partial\hat{v}}{\partial\alpha}\left(\alpha,\beta\right) - \lambda'\left(s\right)\frac{\partial\hat{s}}{\partial\alpha}\left(\alpha,\beta\right).$$

Using (16), (17), (19), and (20), this amounts to:

$$\frac{\partial\Lambda}{\partial\alpha}\left(\alpha,\beta\right) = \left[\lambda'\left(s\right) - \frac{v+s-2vs}{2s(1-s)}\right]\frac{v-s}{(1-s)sD\left(v,s\right)}\Bigg|_{(v,s)=(\hat{v}(\alpha,\beta),\hat{s}(\alpha,\beta))}.$$

To show that $\frac{\partial\Lambda}{\partial\alpha}\left(\alpha,\beta\right) < 0$ in the relevant range $v < \lambda(s)$, it again suffices to show that:

$$\lambda'\left(s\right) - \frac{\lambda(s)+s-2s\lambda(s)}{2s(1-s)} < 0.$$

Simplifying this condition yields $-3\left(1+4s^2\right) / \left[2s\left(3+2s\right)^2\right] < 0$, which is indeed holds because $s \in (0,1)$. It follows that there exists a unique solution for α, which we denote $\tilde{\alpha}(\beta)$, such that $\Pi_{PP}^{*} > \Pi_{UP}^{*}$ if and only if $\alpha > \tilde{\alpha}(\beta)$.

From Proposition 4, $\Pi_{PP}^{*} > \Pi_P^m$ if and only if $\beta > \beta_{PP}(\alpha)$. It follows that dual distribution together with personalized pricing by both firms is optimal if and only if $\alpha > \tilde{\alpha}(\beta)$ and $\beta > \beta_{PP}(\alpha)$.

Finally, the comparison of Π^*_{UP} with Π^m_P yields:

$$\Pi^*_{UP} > \Pi^m_P \iff \frac{s + 2s^2 - 4vs - 2vs^2 + 2v^2 + v^2 s}{4s\left(1 - s^2\right)} > \frac{1}{2}.$$

This holds if

$$v > s + \sqrt{\frac{s\left(1 - s^2\right)}{2 + s}} \quad \text{or} \quad v < s - \sqrt{\frac{s\left(1 - s^2\right)}{2 + s}}.$$

Because $v > s$, the only relevant case is:

$$v > \hat{\lambda}(s) \equiv s + \sqrt{\frac{s\left(1 - s^2\right)}{2 + s}}.$$

Following the same steps as above, the existence of a unique threshold value for v as a function of s implies that there exists a unique solution for β, which we denote $\beta_{UP}(\alpha)$, such that $\Pi^*_{UP} > \Pi^m_P$ if and only if $\beta > \beta_{UP}(\alpha)$. In particular, the condition

$$\frac{\partial\left(s + \sqrt{\frac{s(1-s^2)}{2+s}}\right)}{\partial s} - \frac{1 - v}{2\left(1 - s\right)} > 0$$

is fulfilled in the relevant range where $v > s + \sqrt{\frac{s(1-s^2)}{2+s}}$ because the left-hand side is equal to:

$$\frac{1}{2} + \frac{\left(1 - s\right)\left(2 + 4s + s^2\right)}{\left(2 + s\right)^2 \sqrt{\frac{s(1-s^2)}{2+s}}},$$

which is strictly positive for all $s \in (0, 1)$.

It can be checked that, in the relevant range $\alpha \in (0, 1/2)$, $\beta_{UP}(\alpha) \leq \beta_{PP}(\alpha)$ if and only if $\alpha \leq \hat{\alpha} \simeq 0.12$. It follows that mono distribution is optimal if and only if:

$$\beta < \hat{\beta}(\alpha) \equiv \min\left\{\beta_{UP}(\alpha), \beta_{PP}(\alpha)\right\} = \begin{cases} \beta_{UP}(\alpha) & \text{for } \alpha \leq \hat{\alpha}, \\ \beta_{PP}(\alpha) & \text{for } \hat{\alpha} < \alpha < 1/2. \end{cases}$$

Proof of Proposition 6

Independent valuations and negative correlation of valuations

As discussed in Sections 3 and 4, if $s \leq 0$ then, in all pricing regimes, dual distribution enables the firms to obtain the profit of an integrated monopolist controlling both channels. As A adds value to the industry, delegated distribution is never optimal.

Positive correlation of valuations

We start with the analysis of the regime in which both firms set uniform prices. If A's channel is shut down, B sets the monopoly price in the retail market, equal to $v/2$, and the industry profit is $v^2/(4s)$. Comparing this profit with the industry profit under dual distribution, which is given Π^*_{UU}, yields that delegated distribution gives a higher industry profit if and only if:

$$v \geq v^+_{UU}(s) \equiv \frac{4(1+s)-(1-s)\sqrt{4+5s}}{3+5s}.$$

It is straightforward to check that this inequality always holds at $v = 1$ but is never fulfilled at $v = s$. In addition, $v^+_{UU}(s) \in (0,1)$ for all $s \in (0,1)$, that is $v^+_{UU}(s)$ is in the interior of the admissible range.

Second, we analyze the pricing regime UP. Without the presence of A's channel, B extracts the entire surplus in the retail market, which implies that the industry profit is $v^2/(2s)$. Instead, with dual distribution, the industry profit is Π^*_{UP}. Comparing the two profits yields:

$$\frac{v^2}{2s} \geq \Pi^*_{UP} \quad \Leftrightarrow \quad v \geq v^+_{UP} \equiv \frac{2+s-\sqrt{3(1+s)(1-s)}}{1+2s}.$$

This inequality always holds at the upper bound $v = 1$. At the lower bound $v = s$, the inequality is also fulfilled if $s \geq 1/2$. Hence, delegated distribution is optimal in this regime for all admissible values of v if $s \geq 1/2$, and, for $v \geq v^+_{UP}(s)$ if $s < 1/2$.

Third, we turn to the regime PU. Without A's channel, the industry profit is $v^2/(4s)$ because B charges only a uniform price. Instead, if A's channel is open, dual distribution is optimal if and only if:

$$v \geq s\frac{2+\sqrt{\frac{(1-s)(2+5s+s^2)}{1+2s}}}{1+s} \tag{31}$$

and leads to a profit of Π^*_{PU}; otherwise, mono distribution by A is optimal with a profit of $\Pi^m_P = 1/2$. Comparing Π^m_P with the profit from delegated distribution (i.e., $v^2/(4s)$) yields that the latter is larger if and only if $v \geq \sqrt{2s}$. As the upper bound of v equals 1, this inequality can only be fulfilled if $s \leq 0.5$. Because this comparison is only relevant if (31) does not hold, we need to check if $\sqrt{2s}$ is smaller than the right-hand side of (31). Thus is true if and only if $0.357 \lessapprox s$. It follows that for $0.357 \lessapprox s \leq 0.5$, delegated distribution is optimal if $v \geq \sqrt{2s}$, whereas for $s > 0.5$, delegated distribution can never be optimal.

Instead, if dual distribution is optimal in case A's channel is open, we obtain:

$$\frac{v^2}{4s} \geq \Pi^*_{PU} \quad \Leftrightarrow \quad v \geq \frac{4(1+2s) - (1-s)\sqrt{2(2+5s+s^2)}}{3+8s+s^2}. \tag{32}$$

This threshold is larger than the one of the right-hand side of (32) if and only if $0 \leq s < 0.357$, approximately. Hence, delegated distribution is optimal if $v \geq v_{PU}$, with:

$$v^+_{PU}(s) \equiv \begin{cases} \frac{4(1+2s)-(1-s)\sqrt{2(2+5s+s^2)}}{3+8s+s^2} & \text{if} \quad 0 < s \lessapprox 0.357; \\ \sqrt{2s} & \text{if} \quad 0.357 \gtrapprox s < 1; \end{cases}$$

Finally, proceeding in the same way for the regime PP, we obtain that delegated distribution is optimal if:

$$v^+_{PP}(s) \equiv \begin{cases} 1 - \sqrt{\frac{1-s}{2(1+s)}} & \text{if} \quad 0 < s \lessapprox 0.157; \\ \sqrt{s} & \text{if} \quad 0.157 \gtrapprox s < 1; \end{cases}$$

We now compare the ranges for which delegated distribution is optimal in the four pricing regimes. We start with a comparison of the regimes PU with UU. In the latter, delegated distribution is optimal if $v \geq v^+_{UU}(s)$ holds, whereas in the former delegated distribution is optimal if $v \geq v^+_{PU}(s)$. We start with the case $0 \leq s < 0.357$. The difference:

$$v^+_{PU}(s) - v^+_{UU}(s) \quad \Leftrightarrow \quad \frac{4(1+2s) - (1-s)\sqrt{2(2+5s+s^2)}}{3+8s+s^2} - \frac{4(1+s) - (1-s)\sqrt{4+5s}}{3+5s}$$

equals 0 at the lower bound $s = 0$. Instead, at the upper bound it is approximately equal to 0.034. The difference is also increasing in s, which implies that it is positive for all s between 0 and 0.357. In the range, $0.357 < s \leq 1$, the relevant comparison is:

$$\sqrt{2s} - \frac{4(1+s) - (1-s)\sqrt{4+5s}}{3+5s}.$$

It is easy to check that this difference is again equal to 0.034 at the lower bound. At the upper bound, it is equal to $\sqrt{2} - 1 > 0$. It is also increasing for all values of s in the range between 0.357 and 1. It follows that $v^+_{PU}(s) > v^+_{UU}(s)$, which implies that the range in which delegated distribution is optimal in the regime PU is a subset of the one in the regime UU.

Next, we compare $v^+_{UU}(s)$ with $v^+_{PP}(s)$. In the latter regime, we need to distinguish whether s is below or above approximately 0.157. We start again with the former case.

The difference:

$$v_{UU}^{+}(s) - v_{PP}^{+}(s) \quad \Leftrightarrow \quad \frac{4(1+s) - (1-s)\sqrt{4+5s}}{3+5s} - 1 - \sqrt{\frac{1-s}{2(1+s)}}$$

equals $1/\sqrt{2} - 1/3 > 0$ at $s = 0$, it is approximately equal to 0.339 at $s = 0.157$, and increasing for all values of s between 0 and 0.157. Turning to the range $0.157 < s \leq 1$, the relevant comparison is:

$$\frac{4(1+s) - (1-s)\sqrt{4+5s}}{3+5s} - \sqrt{s},$$

which is approximately equal to 0.339 at $s = 0.157$, decreasing in s for $s \in (0.157, 1]$, and equal to 0 at $s = 1$. It follows that $v_{UU}^{+}(s) > v_{PP}^{+}(s)$.

Finally, we compare $v_{PP}^{+}(s)$ with $v_{UP}^{+}(s)$, noting that in the regime UP, delegated distribution maximizes the industry profit for all $s \geq 1/2$. We start with the region $0 \leq s < 0.157$. At the lower bound $s = 0$, the difference $v_{PP}^{+}(s) - v_{UP}^{+}(s)$ equals $\sqrt{3} - 1 - 1/\sqrt{2} = 0.025 > 0$, and the upper bound $s = 0.157$, this difference is approximately 0.056. The difference is also increasing in $s \in [0, 0.157)$, which implies that it is positive in the entire range. Turning to the range $0.157 < s \leq 0.5$, the difference is approximately equal to 0.056 at $s = 0.157$, is decreasing in s for $s < 1$, and equals 0 at $s = 1$; hence the difference is positive for $s \in [0, 0.157)$. It follows that $v_{PP}^{+}(s) > v_{UP}^{+}(s)$.

As a consequence, the ordering of the thresholds is: $v_{UP}^{+}(s) < v_{PP}^{+}(s) < v_{UU}^{+}(s) < v_{PU}^{+}(s)$. In the same way as in the proof of the previous propositions, we can show that for each threshold, there is a corresponding threshold in the α-β-plane, such that delegated distribution is optimal if and only if α is below a threshold in the respective pricing regime. These thresholds, which are are all increasing in β, can be ordered accordingly, that is, $\alpha_{UP}^{+}(\beta) > \alpha_{PP}^{+}(\beta) > \alpha_{UU}^{+}(\beta) > \alpha_{PU}^{+}(\beta)$.

B Positive Correlation of Valuations with v and s being larger than 1

In this appendix, we consider a demand pattern of positive correlation of valuations with $s > 1$. The assumptions $\hat{x} > 0$ and $\hat{u} > 0$ then imply $v \in (1, s)$. This demand pattern corresponds to a situation of vertical differentiation between firms in which B offers the high-end product.

We first provide a proposition that characterizes the optimal distribution strategy and pricing regime (part *(i)*), as well as the optimality of delegated distribution (part *(ii)*). We then explain the intuition behind the results. Finally, we provide the proof of the proposition.

Proposition *In the case of positive correlation of valuations with B offering the high-end product:*
(i) In the absence of delegated distribution: (a) dual distribution is the optimal strategy in all pricing regimes; (b) if firms can contract on their pricing policies, personalized pricing by both firms is optimal.
(ii) If delegated distribution is feasible, in each pricing regime $YZ \in \{UU, UP, PP\}$ there exists a threshold $\alpha_{YZ}^{++}(\beta)$ such that delegated distribution is optimal if $\alpha < \alpha_{YZ}^{++}(\beta)$; by contrast, it is never optimal in the regime PU. The thresholds can be ordered as follows:

$$\alpha_{UP}^{++}(\beta) > \alpha_{PP}^{++}(\beta) > \alpha_{UU}^{++}(\beta) \quad \text{if} \quad \beta > \check{\beta},$$

$$\alpha_{UP}^{++}(\beta) > \alpha_{UU}^{++}(\beta) > \alpha_{PP}^{++}(\beta) \quad \text{if} \quad \beta < \check{\beta},$$

where $\check{\beta} \approx 0.82$.

If valuations are positively correlated and B is the one catering to high-end consumers, dual distribution is optimal, regardless of the pricing regime. To see why, consider first the situation where A charges a uniform price, and suppose that the firms negotiate a wholesale price equal to A's monopoly price under mono distribution, p_U^m. In the continuation equilibrium, A can then secure the monopoly profit Π_U^m by charging p_U^m: this can only increase sales—regardless of the price charged by B—and A obtains the same margin on every sale, regardless of which firm make it. Furthermore, as B can charge a higher price to high-valuation consumers, the industry profit increases compared to mono distribution. The argument carries over when A charges personalized prices, setting the wholesale price to the highest of A's monopoly prices, that is, $w = 1$—in this case, A is strictly better off if B's makes a sale.

In addition, the industry profit is highest in the regime in which both firms charge personalized prices. Indeed, the optimal wholesale price then exceeds $\hat{u}$—i.e., the crossing point of the consumers' utility functions. This prevents B from competing for consumers in A's core segment, as consumers who have a higher utility from A's product have a utility from B's product that is lower than $\hat{u}$. A thus acts as a monopolist towards these consumers, and can extract more surplus with personalized pricing. A wholesale price above $\hat{u}$ also gives A a large revenue when B serves a consumer. This dampens competition in B's core segment, and there as well, charging personalized prices enables B to increases its profit.

The proposition therefore confirms a general theme of the paper: With intra-brand competition, dual distribution is optimal for a large number of cases. This is due to the fact that an appropriately chosen wholesale tariff enables the firms to dampen competition.

Part *(ii)* of the proposition shows that delegated distribution can be optimal in three of the four pricing regimes. The only pricing regime in which this does not occur is PU. In this regime, A can extract the entire consumer surplus from low-valuation consumers whereas B cannot extract the entire surplus from any consumer (due to competition). This implies that using A's direct channel is always profitable. Conversely, A contributes least value to the industry monopoly profit in the regime UP; hence, delegated distribution occurs for the largest parameter range there. Comparing the two symmetric pricing regimes, delegated distribution is optimal for a larger range in the regime PP than in UU if β is large. In that case, A is not a strong competitor to B for high-valuation consumers; shutting down the direct channel is then more profitable if B can extract the surplus from high-valuation consumers through personalized pricing.

Proof of the Proposition

We start with part *(i)*, statement *(a)*—i.e., we show that dual distribution is the optimal strategy in all pricing regimes (in the absence of delegated distribution). We first analyze the situation in which A sets a uniform price and then the one in which A sets personalized prices.

Uniform pricing by A

Suppose that the firms agreed on the wholesale tariff $F = 0$ and $w = p_A^m$.

Assume first that B also charges a uniform price. In the continuation equilibrium, B then charges a price $p_B > w = p_A^m$ on all consumers served and obtains a positive share the market,[7] whereas A charges some price p_A^+.[8] Suppose now that A deviates and charges the mono-distribution price p_A^m. Following this deviation, total demand (weakly) exceeds that of the mono-distribution outcome (as consumers have more choice) and A obtains a margin p_A^m (either directly through p_A or indirectly through w). It follows that B's profit is strictly positive after the deviation, and A's profit is weakly larger. Therefore, in the continuation equilibrium in which A best responds to p_B, the same result must hold. As a consequence, the tariff $F = 0$ and $w = p_A^m$ yields a continuation equilibrium that strictly increases B's profit and weakly increases A's profit.

[7] To see this, suppose instead that B does not attract any consumer. In that case, B prices at cost (i.e., $p_B = w = p_A^m$) and A prices so as to attract consumers with type $x = 0$ (i.e., $p_A \leq p_A^m - (v-1) < p_A^m$), and obtains a profit lower than Π_A^m (as it charges $p_A \neq p_A^m$, and B attracts no additional consumer). But then, A would profitably deviate by charging $p'_A = p_A^m$: compared with the mono distribution outcome, this would (weakly) expand demand (as consumers can now buy from both firms) and A would obtain the same margin on each customer (either directly through p'_A, or indirectly through w); hence, the deviation brings a profit of at least Π_A^m.

[8] Compared with mono distribution, B now charges a "lower" price (the mono distribution outcome can be interpreted as B charging $p_B \geq v$), and in response A also lowers its own price.

The same reasoning applies to the case in which B charges personalized prices. In the continuation equilibrium, after firms agreed on a tariff $F = 0$ and $w = p_A^m$, B's demand is positive, and A charges some price p_A^{++}. A deviation to $p_A = p_A^m$ then gives A a profit that is weakly larger than Π_A^m due to the fact that it obtains the same price p_A^m on all consumers served and demand weakly exceeds that with mono distribution.

Personalized pricing by A

Under mono distribution, A charges each consumer a price $u_A(x)$. Suppose now that the firms agree on the two-part tariff $F = 0$ and $w = 1$. In the continuation equilibrium, B obtains a positive demand from consumers close enough to $x = 0$: these consumers have a net willingness-to-pay for B's product equal to $u_B(x)$, which is larger than B's wholesale price $w = 1$, and A earns a higher margin by letting B serve these consumers, as $w = 1 > u_A(x)$. Therefore, A charges prices larger than or equal to w to the consumers served by B, and obtains a higher margin compared to mono distribution. This holds regardless of whether B charges personalized prices or a uniform price. In addition, B does not offer a positive net utility to consumers it does not serve: it it did, B would serve these consumers because A is better off by letting B serve these consumers and get a margin $w > u_A(x)$ instead of serving the consumers itself. It follows that A can still charge a price of $p_A(x) = u_A(x)$ to these consumers. Hence, A obtains the same profit as under mono distribution from consumers that it serves but a strictly larger profit from consumers served by B. Therefore, the two-part tariff $F = 0$ and $w = 1$ increases the profits of both firms.

Endogenous Pricing Policy

We next prove statement *(b)* of part *(i)*, that is, we show that if firms can contract on their pricing policies, they choose the regime of personalized pricing by both firms. To simplify the exposition, we focus on the case $X > 1$ (i.e., X is large enough that it does not limit A's demand). All our results also hold if this inequality was not fulfilled.

We start with the regime in which both firms set a uniform price. Following the analysis in Section 5 and the proof of Proposition 3, we denote the consumer indifferent between buying from B or A by $x_{BA} > 0$, whereby:

$$x_{BA}(p_A, p_B) = \frac{v - p_B - 1 + p_A}{s - 1},$$

and the consumer indifferent between buying from A and not buying by $x_A > 0$, whereby:

$$x_B(p_A) = v - p_A.$$

The profit functions (gross of the fixed fee) are then $\Pi_A = x_{BA}(p_A, p_B)w + \left[x_A(p_A) - x_{BA}(p_A, p_B)\right]p_A$ and $\Pi_B = x_{BA}(p_A, p_B)(p_B - w)$. Solving for the equilibrium retail prices, as a function of the wholesale price w, yields:

$$p_A(w) = \frac{2-1+3w-v}{4s-1},$$
$$p_B(w) = \frac{v(2s-1)+w(2s+1)-s}{4s-1}.$$

The associated demands are $D_A(w) = x_A(p_A(w)) - x_{BA}(p_A(w), p_B(w))$ and $D_B(w) = x_{BA}(p_A(w), p_B(w))$. In the first stage, firms choose w so as to maximize the industry profit, $\Pi(w) = p_A(w) D_A(w) + p_B(w) D_B(w)$. Inserting the respective demands and prices, we obtain that equilibrium wholesale price is:

$$w^*_{UU} = \frac{s(1+4v+4s)}{2(5+4s)}.$$

Inserting the equilibrium prices into the industry profit yields:

$$\Pi^*_{UU} = \frac{5s+4s^2-1+4v(1+s)(2-v)}{4(s-1)(5+4s)}$$

We next turn to the case in which only B charges personalized prices. Given w and p_A, B's price response is such that consumers x with $v_B(x) - w > v_A(x) - p_A$, or:

$$x < \frac{v-w-1+p_A}{s-1},$$

buy from B. A's maximization problem with respect to p_A is therefore given by:

$$w\left(\frac{v-w-1+p_A}{s-1}\right) + p_A\left(1 - p_A - \frac{v-w-1+p_A}{s-1}\right),$$

leading to an optimal p_A of:

$$p_A(w) = \frac{w}{s}\frac{s-v}{2s}.$$

We now turn to the wholesale stage. The two firms seek to maximize the industry profit given by:

$$\Pi = p_A(w)\left(1 - p_A(w) - \frac{v-w-1+p_A(w)}{s-1}\right) + \int_0^{\frac{v-w-1+p_A(w)}{s-1}} \left[p_A(w) + v_B(x) - v_A(x)\right] dx.$$

Maximizing this profit with respect to w yields $w^*_{UP} = (s+v)/(2(1+s))$. Inserting $w = w^*_{UP}$ into the industry profit yields:

$$\Pi^*_{UP} = \frac{s(2+s) - v(1+2s)(2-v)}{4(1+s)(s-1)}.$$

Comparing Π^*_{UP} and Π^*_{UU}, we obtain:

$$\Pi^*_{UP} - \Pi^*_{UU} = \frac{(v-1)^2(1+4s^2+6s)}{4(5+4s)(1+s)(s-1)} > 0.$$

Third, we determine the industry profit in case both firms charge personalized prices. Following the analysis in Section 5 and the proof of Proposition 4, taking into account that the optimal wholesale price will be between $\hat{u}$ and 1, the industry profit in this case can be written as:

$$\Pi = \int_0^{1-w} [w + v_B(x) - v_A(x)]\,dx + \int_{1-w}^{\frac{v-w}{s}} [v_B(x)(x)]\,dx + \int_{\frac{v-w}{s}}^{1} [v_A(x)(x)]\,dx.$$

The first term is the profit from consumers close $x = 0$, who have a higher valuation than w for the product of each firm, which implies that the two firms compete for these consumers. The second term is the profit from consumers in the middle, whose valuation for B's product is above w but their valuation for A's product is below w. As w is A's opportunity cost, A is better off when B serves these consumers, which implies that B extracts their entire consumer surplus. Finally, the third term is the profit from consumer whose valuation for B's product is below w, which implies that A can extract their consumer surplus. Maximizing with respect to w, we obtain that the optimal wholesale price is:

$$w^*_{PP} = \frac{1+s-v}{s^2-1+s},$$

leading to an industry profit of:

$$\Pi^*_{PP} = \frac{(1+s)^2 - v(2+s)(2-v)}{2(s^2-1+s)}.$$

We next turn to the regime in which only A sets personalized prices. Following the same steps as in the proof of Proposition 2, A's best response to p_B is to serve all consumers x with $v_A(x) - w > v_B(x) - p_B$. Since, as shown above, B is always active in equilibrium, this implies that A serves consumers x, such that $(v - p_B)/s < x \leq 1$. Therefore, B's profit at the retail stage is:

$$(p_B - w)\left(\frac{v - p_B}{s}\right),$$

leading to an optimal retail price of $p_B(w) = (v+w)/w$. Turning to the wholesale stage, firms maximize the industry profit, given by:

$$p_B(w)\left(\frac{v-p_B(w)}{s}\right)+\int_{\frac{v-p_B(w)}{s}}^{1} v_A(x)dx,$$

with respect to w. This yields $w^*_{PU} = (2s-v)\,/\,(2s-1)$. Inserting this into the industry profit yields:

$$\Pi^*_{PU} = \frac{2s+v\,(v-2)}{2\,(2s-1)}.$$

Comparing Π^*_{PP} and Π^*_{PU}, we obtain:

$$\Pi^*_{PP} - \Pi^*_{PU} = \frac{(v-1)^2\,(s^2+2s-1)}{2\,(s^2-1+s)\,(2s-1)} > 0.$$

Finally, to determine the optimal pricing regime in the negotiation, we need to compare Π^*_{PP} with Π^*_{UP}. The sign of the difference between the two profits is given by:

$$\text{sign}\left\{\Pi^*_{PP} - \Pi^*_{PU}\right\} = \text{sign}\left\{\left(s^2-2\right)\left(1+s^2+s\right)+v\,(2-v)\left(3-s^2+s\right)\right\}. \tag{33}$$

At the lower bound of v—i.e., $v = 1$—the right-hand side of (33) equals $(s-1)(s+1)(s^2+s-1)$, which is strictly positive due to the fact that $s > 1$. At the upper bound of v—i.e., $v = s$—the right-hand side of (33) equals $2(s-1)(s^3-s+1)$, which is again strictly positive as $s > 1$. Finally, $\Pi^*_{PP} - \Pi^*_{PU}$ is a concave second-order polynomial in v, which implies that the difference must be positive in the admissible range, given that it is positive at the limits of this range; hence, $\Pi^*_{PP} > \Pi^*_{PU}$ for all v in the admissible range. Therefore, firms choose the regime PP in case they can contract on their pricing policies.

Delegated Distribution

We last prove the statement in part *(ii)* of the proposition. To so so, we proceed in the same way as in the proof of Proposition 6 by comparing the respective profits from dual distribution (i.e., Π^*_{UU}, Π^*_{UP}, Π^*_{PP}, and Π^*_{PU}) with the profits from delegated distribution. The latter are $v^2/(4s)$, in case B sets only a uniform price, and $v^2/(2s)$, in case B sets personalized prices.

Starting with the regime in which both firms set a uniform price, the comparison between Π^*_{UU} and $v^2/(4s)$ yields that $\Pi^*_{UU} - v^2/(4s) = (s-1)/(4s) > 0$ at the lower bound of v (i.e., $v = 1$) and $\Pi^*_{UU} - v^2/(4s) = -\left((s-1)\right)/\left(4(5+4s)\right) < 0$ at the upper bound of v (i.e., $v = s$). In addition, the difference is strictly decreasing in v, and the threshold value of v at which both profits are equal to each other is:

$$v_{UU}^{++}(s) \equiv \frac{4s(1+s)-(s-1)\sqrt{s(5+4s)}}{5+3s}.$$

It is easy to check that the threshold is below s for all $s > 1$. Therefore, delegated distribution is optimal if and only if $v > v_{UU}^{++}(s)$.

Turning to the regime in which only B sets personalized prices, we obtain that $\Pi_{UP}^{*} - v^2/(2s)$ is positive at the lower bound of v if and only if $s > 2$ but strictly negative at the upper bound of v. The difference is again strictly decreasing in v. If a solution to $\Pi_{UP}^{*} = v^2/(4s)$ in the admissible range exists, which is the case if $s > 2$, the threshold value at which both profits are equal to each other is:

$$v_{UP}^{++}(s) \equiv \frac{s\left(2s+1-\sqrt{3(s+1)(s-1)}\right)}{2+s}.$$

This threshold is again below s for $s > 1$, which implies that delegated distribution is optimal if and only if $v > v_{UP}^{++}(s)$.

Proceeding in the same way in the regime in which both firms set personalized prices yields that $\Pi_{PP}^{*} - v^2/(2s)$ is positive at the lower bound of v, negative at the upper bound of v, and strictly decreasing in v. The threshold value at which the two profits are equal to each other is:[9]

$$v_{PP}^{++}(s) \equiv \frac{s(2+s)-\sqrt{s(s^2+s-1)}}{1+s};$$

hence, delegated distribution is optimal if and only if $v > v_{PP}^{++}(s)$.

Finally, in the regime in which only A sets personalized prices, the difference between $\Pi_{PU}^{*} - v^2/(4s)$ is:

$$\frac{(2s-v)^2}{4s\,(2s-1)},$$

which is strictly positive for all parameters in the admissible range.

Therefore, we have shown that, in case of positive correlation of valuations with $v \in (1, s)$ and $s > 1$, delegated distribution is optimal if $v > v_{YZ}^{++}(s)$ in the regimes $\{Y, Z\}\, in\, \{UU, UP, PP\}$, but it is never optimal in the regime PU.

In the same way as in previous proofs, we can show that, for the pricing regimes UU, UP, and PP, there is a unique threshold in the α-β-plane. We denote the respective thresholds by $\alpha_{UU}^{++}(\beta)$, $\alpha_{UP}^{++}(\beta)$, and $\alpha_{PP}^{++}(\beta)$.

We next compare the threshold values in the regimes UU, UP, and PP. We start with a comparison between v_{UU} and v_{UP}. Taking the difference between $v_{UU}^{++}(s)$ and

[9] This threshold is also below s for $s > 1$.

$v_{UP}^{++}(s)$ yields:

$$\frac{s\,(5+3s)\,\sqrt{3\,(s+1)\,(s-1)}-(s-1)\left[s\,(3+2s)-(2+s)\,\sqrt{s\,(5+4s)}\right]}{(2+s)\,(5+3s)},$$

which is equal to 0 at the lower bound of s (i.e., $s=1$)[10] but strictly increasing in s for all $s>1$. It follows that $v_{UU}^{++}(s)>v_{UP}^{++}(s)$, which implies $\alpha_{UU}^{++}(\beta)<\alpha_{UP}^{++}(\beta)$.

Next, we compare $v_{PP}^{++}(s)$ and $v_{UP}^{++}(s)$. Taking the difference between $v_{PP}^{++}(s)$ and $v_{UP}^{++}(s)$ yields:

$$\frac{s\,(1+s)\,\sqrt{3\,(s+1)\,(s-1)}-s\,(s^2-s-3)-(2+s)\,\sqrt{s\,(s^2+s-1)}}{(2+s)\,(1+s)},$$

which is again 0 at $s=1$ but strictly increasing in s for all $s>1$; hence, $v_{PP}^{++}(s)>v_{UP}^{++}(s)$, or $\alpha_{PP}^{++}(\beta)<\alpha_{UP}^{++}(\beta)$.

Lastly, the difference between $v_{UU}^{++}(s)$ and $v_{PP}^{++}(s)$ is given by:

$$\frac{s\,(s^2-3s-6)+s\,(5+3s)\,\sqrt{s\,(s^2+s-1)}-(1+s)\,(1-s)\,\sqrt{s\,(5+4s)}}{(1+s)\,(5+3s)}.$$

This difference is again 0 at $s=1$. It is increasing in s for $s\searrow 1$, which implies that $v_{UU}^{++}(s)>v_{PP}^{++}(s)$ for values of s close to 1. By contrast, the difference is negative as $s\to\infty$, which implies that $v_{UU}^{++}(s)<v_{PP}^{++}(s)$ for very high values of s. Translating this into the respective thresholds values of α implies that $\alpha_{UU}^{++}(\beta)<\alpha_{PP}^{++}(\beta)$ for values of β close to 0 and $\alpha_{UU}^{++}(\beta)<\alpha_{PP}^{++}(\beta)$ for sufficiently large values of β. Solving $\alpha_{UU}^{++}(\beta)=\alpha_{PP}^{++}(\beta)$ yields that there is a unique threshold for β in the admissible range, which is approximately equal to 0.82. This threshold is denoted $\check{\beta}$ in the proposition.

C Generalization of Proposition 3

In this appendix, we generalize part Proposition 3 to an extended setting in which consumers with unit demand obtain values, net of distribution costs, of $u_A(x)$ and $u_B(x)$ for the products of the two firms, where $u_A(\cdot)$ and $u_B(\cdot)$ are both twice continuously differentiable, x is distributed according to a twice continuously differentiable c.d.f. $G(x)$ over $\mathbb{R}_+$ and:

- $\forall x\in\mathbb{R}_+$, $u_A'(x)<u_B'(x)<0$;
- $u_i(\bar{x}_i)=0$ for some $\bar{x}_i>0$; and
- $u_A(\hat{x})=u_B(\hat{x})>0$ for some $\hat{x}>0$.

[10] Recall that $s\geq 1$ if B offers the high-quality product.

This implies that, as in our baseline model, the curves $u_A(\hat{x})$ and $u_B(\hat{x})$ intersect exactly once, and this intersection occurs in the positive quadrant.

Let:

$$D_i^m(p_i) \equiv G\left(u_i^{-1}(p_i)\right),$$

denote the monopolistic demand for firm i's product:

$$p_i^m \equiv \arg\max_{p_i} p_i D_i^m(p_i),$$

denote firm i's monopoly price:

$$x_i^m \equiv u_i^{-1}(p_i^m),$$

denote the location of the associated marginal consumer, and:

$$q_i^m \equiv D_i^m(p_i^m) = G(x_i^m) \quad \text{and} \quad \pi_i^m \equiv p_i^m q_i^m,$$

denote the monopoly output and profit, respectively. Our working assumption is that B would seek to serve more consumers than A in these monopoly situations:

Assumption A: B's monopoly profit function is strictly quasi-concave and $q_B^m > q_A^m$.

Let $w^m \equiv u_B(x_A^m)$. For $w \geq w^m$, there exists a continuation equilibrium in which A charges its monopoly price, p_A^m, and B does not serve any consumer (e.g., by charging $p_B = w^m$). If instead $w < w^m$, both firms can obtain a positive market share: A then faces a demand:

$$D_A(p_A, p_B) \equiv G\left(\Delta^{-1}(p_A - p_B)\right),$$

where:

$$\Delta(x) \equiv u_A(x) - u_B(x),$$

whereas B faces a demand given by:

$$D_B(p_A, p_B) \equiv D_B^m(p_B) - D_A(p_A, p_B).$$

For the sake of exposition, we will assume that there then exists an equilibrium where both firms obtain a positive market share, which is moreover "well-behaved":

Assumptions B: For any $w \leq w^m$, there exists a unique downstream equilibrium, $(p_A^e(w), p_B^e(w))$, where $p_A^e(w)$ and $p_B^e(w)$ are continuous and increasing in w, and such that $p_A^e(w^m) = p_A^m$ and $p_B^e(w^m) = w^m$.

We have:

Proposition: *Under Assumptions A and B, dual distribution is the optimal strategy if valuations are positively correlated and A charges a uniform price.*

Proof: We first consider the regime in which both firms charge a uniform price. Starting from a situation in which the firms negotiate $w = w^m$, and thus A obtains Π_A^m, consider a small reduction in the wholesale price from w^m to $w < w^m$, together with a fixed fee, $F(w)$, designed to appropriate B's profit (or almost all of it, to ensure acceptance). A then obtains (almost all of) the industry profit, which can be expressed as:

$$\Pi(w) = \Pi_A(w) + \Pi_B(w),$$

where:

$$\begin{aligned}\Pi_A(w) &= p_A^e(w) D_A(p_A^e(w), p_B^e(w)) + w D_B(p_A^e(w), p_B^e(w)) + F(w),\\ \Pi_B(w) &= [p_B^e(w) - w] D_B(p_A^e(w), p_B^e(w)) - F(w).\end{aligned}$$

By deviating from the downstream equilibrium and charging:

$$\hat{p}_A(w) = p_B^e(w) - u_B(x_A^m) + u_A(x_A^m) = p_A^m + p_B^e(w) - w^m,$$

A would maintain its output of q_A^m, and generate an output $\hat{q}_B = D_B^m(p_B^e(w)) - q_A^m$ for B. Therefore:

$$\begin{aligned}\Pi_A(w) &\geq \hat{p}_A(w) D_A(\hat{p}_A(w), p_B^e(w)) + w D_B(\hat{p}_A(w), p_B^e(w)) + F(w)\\ &= [p_A^m + p_B^e(w) - w^m] q_A^m + w[D_B^m(p_B^e(w)) - q_A^m] + F(w)\\ &= \pi_A^m + [p_B^e(w) - w - w^m] q_A^m + w D_B^m(p_B^e(w)) + F(w).\end{aligned}$$

Likewise, noting that B could always choose to deviate from the downstream equilibrium and charge $p_B = w$, we have:

$$\Pi_B(w) \geq -F(w).$$

Adding these two inequalities yields (recalling that $\Pi(w) = \Pi_A(w) + \Pi_B(w)$):

$$\Pi(w) - \pi_A^m \geq \phi(w) \equiv [p_B^e(w) - w - w^m] q_A^m + w D_B^m(p_B^e(w)).$$

Note that $\phi(w^m) = 0$ because $p_B^e(w^m) = w^m$ and $D_B^m(w^m) = G(x_A^m) = q_A^m$. Taking the derivative of $\phi(w)$ and evaluating it at $w = w^m$, we obtain (again using $p_B^e(w^m) = w^m$

and $q_A^m = D_B^m(w^m)$):

$$\begin{aligned}\phi'(w^m) &= \left[\frac{dp_B^e}{dw}(w) - 1\right] q_A^m + D_B^m(w^m) + w\frac{dD_B^m}{dp_B^e}(p_B^e(w))\frac{dp_B^e}{dw}(w) \\ &= \frac{dp_B^e}{dw}(w)\left[D_B^m(w^m) + w^m\frac{dD_B^m}{dp_B^e}(w^m)\right],\end{aligned}$$

where the expression within bracket is negative from Assumption A.[11] It follows that a reduction of w below w^m is strictly profitable, implying that dual distribution is the unique optimal mode of distribution.

Turning to the hybrid regime in which B charges personalized prices, the same logic as in the main text can be applied. In particular, setting $p_A = p_A^*$ and $w = p_B^*$, where p_A^* and p_B^* are the equilibrium retail prices under uniform pricing, delivers a higher industry profit than dual distribution with uniform pricing, and therefore also a higher profit than Π_U^m.

D Inter-brand versus intra-brand competition

In this appendix, we analyze a situation of inter-brand competition between two completely independent firms in the scenario of positive correlation of valuations. Specifically, we compare the profits of the two symmetric regimes (i.e., uniform pricing by both firms and personalized pricing by both firms) with each other. We show that the insights from such an analysis are misleading if applied to a situation of intra-brand competition between two channels, which is partly governed by the negotiated wholesale contract.

The case of inter-brand competition is equivalent to a situation in which no wholesale contract exists, that is, the wholesale price w equals zero. We can then apply the same techniques as in the proof of Propositions 3 and 4 to solve for the aggregate profits of the firms in the two pricing regimes via inserting $w = 0$ in the respective equations. This yields:

$$\Pi_{UU}^A(w=0) = \frac{(2-v-s)^2}{(4-s)^2(1-s)} \quad \text{and} \quad \Pi_{UU}^B(w=0) = \frac{(2v-s(1+v))^2}{s(4-s)^2(1-s)}$$

for the uniform pricing regime, and:

$$\Pi_{PP}^A(w=0) = \frac{(1-v)^2}{2(1-s)} \quad \text{and} \quad \Pi_{PP}^B(w=0) = \frac{(v-s)^2}{2s(1-s)}$$

[11] Indeed, Assumption A implies that firm B's optimal monopoly demand is strictly larger than $q_A^m = D_B^m(w^m)$; hence, firm B's monopoly price is below w^m, which implies that the first-order condition evaluated at w^m is negative.

for the personalized pricing regime. Comparing $\Pi^i_{PP}(w=0)$ with $\Pi^i_{UU}(w=0)$, $i=A,B$, we obtain that A benefits from personalized pricing if and only if:

$$v \leq v_A(s) \equiv \frac{\left(\sqrt{2}-1\right)\left(2\sqrt{2}-s\right)}{4-\sqrt{2}-s},$$

and B benefits from personalized pricing if and only if:

$$v \geq v_B(s) \equiv \frac{s\left(\sqrt{2}+1\right)\left(4-\sqrt{2}-s\right)}{2\sqrt{2}+s}.$$

Turning to the joint profits, the difference between the firms' aggregate profits with personalized pricing and their profits with uniform pricing (i.e., $\Pi^A_{PP}(w=0)+\Pi^B_{PP}(w=0)-\Pi^A_{UU}(w=0)-\Pi^B_{UU}(w=0)$) is:

$$\frac{1}{2}\frac{\left(8+14s-9s^2+s^3\right)\left(v^2+s\right)-4vs\left(12-6s+s^2\right)}{(1-s)s(4-s)^2}. \tag{34}$$

The numerator is a convex quadratic polynomial of v with the two roots:

$$\underline{v}(s) \equiv \frac{2s\left(12-6s+s^2\right)-(1-s)(4-s)\sqrt{s(12s-4-s^2)}}{8+14s-9s^2+s^3}$$

and

$$\overline{v}(s) \equiv \frac{2s\left(12-6s+s^2\right)+(1-s)(4-s)\sqrt{s(12s-4-s^2)}}{8+14s-9s^2+s^3}.$$

It is straightforward to check that for $s<2\left(3-2\sqrt{2}\right)\approx 0.343$, the numerator of (34) is always positive (i.e., no root exists in this case), which implies that firms' aggregate profits are larger with personalized prices. In addition, for these values of s, $v_A(s)>v_B(s)$, which implies that both firms benefit from personalized pricing if and only if $v_B(s)\leq v\leq v_A(s)$. Instead, for $s\geq 2\left(3-2\sqrt{2}\right)$, the joint profit with personalized pricing is larger if $v\leq \underline{v}(s)$ or $v\geq \overline{v}(s)$. The threshold $\overline{v}(s)$ is increasing in s and reaches the upper bound for v, which is 1, at $s=5-\sqrt{17}\approx 0.877$. Similarly, $\underline{v}(s)$ is decreasing in s and reaches the lower bound for v, which is s, also at $s=5-\sqrt{17}\approx 0.877$. It follows that for $s>5-\sqrt{17}$, the joint profit from uniform pricing is higher in the admissible range for v.

The result shows that inter-brand competition is less fierce in a situation in which valuations are positively correlated than in one with negative correlation of valuations. As mentioned in the main text, Thisse and Vives (1988) and Shaffer and Zhang (1995) show that in a Hotelling model—which implies a negative correlation of valuations—aggregate profits with personalized pricing are unambiguously lower than those with uniform pricing. Instead, our analysis shows that with positive correlation of valuation, this is not necessarily true and there is indeed a sizable range in which the opposite result occurs.

The comparison between the profits from personalized pricing and from uniform pricing in case of inter-brand competition provides little guidance for the question whether or not dual distribution is optimal in case of intra-brand competition. In particular, as shown in the proof of Proposition 4, mono distribution is optimal under intra-brand competition if v is close to s—i.e., if $v < s\left(2+\sqrt{1-s}\right)/(1+s)$. Instead, as shown above, with inter-brand competition, for all values of $s \leq 0.877$ personalized pricing leads to higher aggregate profits than uniform pricing if v is close to s. Moreover, mono distribution can be optimal even if personalized pricing increases the profits of *both* firms in the case of inter-brand competition. The latter occurs if $v_B(s) \leq v \leq v_A(s)$. From the proof of Proposition 4, mono distribution under intra-brand competition is optimal if and only if $v \geq \tilde{v}(s)$. It is straightforward to check that $\tilde{v}(s) > v_B(s)$ for $s \in (0,1)$ and $\tilde{v}(s) < v_A(s)$ for $s \lessapprox 0.230$; hence, the situation occurs for $s \in (0, 0.230)$ if $v_B(s) \leq v \leq \tilde{v}(s)$.

E Independent Pricing Policy Choices

In this appendix, we analyze the game laid out in the second part of Section 5.3—i.e., A and B independently choose their individual pricing policy (i.e., uniform pricing or personalized pricing).

In Sections 5.1 and 5.2, we determined for each of the four pricing regimes the result of the wholesale contract negotiation between A and B. We now move one stage backwards and analyze stage 2 of the game. If the distribution strategy chosen in the first stage is mono distribution, A chooses personalized pricing, which leads to a profit of 1/2 for A.

Now suppose that, in the first stage, the firms agree to use dual distribution. We start with the pricing policy of B. First, we note that comparing B's profit from personalized pricing with its profit from uniform pricing yields that the difference does not depend on B's bargaining power $1-a$. The reason is that B's outside option equals zero. This implies that in any combination of pricing choices of A and B, B always receives the industry profit from dual distribution minus A's outside option, multiplied by $1-a$; hence, $1-a$ cancels out in any comparison. Therefore, B chooses the pricing policy that maximizes the industry profit, given A's pricing policy.

If A chooses uniform pricing, we know from the proof of Proposition 3 that:

$$\Pi^*_{UP} - \Pi^*_{UU} = \frac{(s-v)^2}{4s}\frac{4+6s+s^2}{(4+5s)\left(1-s^2\right)} > 0,$$

which implies that B's best response is to choose personalized pricing. Similarly, if A chooses personalized pricing, we know from the proof of Proposition 5 that:

$$\Pi^*_{PP} - \Pi^*_{PU} = \frac{(1+3s+s^2)\,(v(1+s)-2s)^2}{2s(1-s^2)(2+5s+s^2)} > 0;$$

hence, choosing personalized pricing is again the best response of B. Therefore, regardless of A's choice, it is optimal for B individually to choose personalized pricing.

We next consider A's optimal individual choice. As personalized pricing is a dominant strategy for B, A needs to compare the profit it receives in the symmetric regime PP with the profit it receives in the hybrid regime UP. In either case, due to the fact that B's outside option is zero, A receives the resulting industry profit, multiplied by a, plus its outside option, multiplied by $1-a$. Therefore, A's profit when choosing uniform pricing is:

$$a\left(\frac{s+2s^2-4vs-2vs^2+2v^2+v^2s}{4s\,(1-s^2)}\right)+(1-a)\frac{1}{4}, \tag{35}$$

and its profit when personalized pricing is:

$$a\left(\frac{s(1+3s)-4vs(1+s)+v^2(1+s)^2}{2s(1-s^2)}\right)+(1-a)\frac{1}{2}. \tag{36}$$

Comparing (35) with (36) yields that A's profit from personalized pricing is higher if and only if:

$$a \le \frac{(1+s)(1-s)}{2v(2+3s)-v^2(3+2s)-s(4+s)}.$$

It is straightforward to check that the right-hand side of this inequality is strictly positive for all $v \in (0,1)$ and $s \in (0,v)$. Moreover, it is smaller than 1 as long as $v \ge (1+4s)/(3+2s)$. If the reverse holds true, the inequality is always fulfilled as $a \in (0,1)$, which implies that A always chooses personalized pricing. Instead, if the inequality holds, A chooses uniform pricing if a is sufficiently large.

Finally, we analyze the first stage of the game in which firms agree on whether or not to use dual distribution. A is only willing to do so if its resulting profit is larger than the one it obtains from mono distribution. As stated above, the latter profit is $1/2$ because A will always choose personalized pricing in that case. Therefore, the equilibrium distribution regime can be determined by comparing the profit in (35) with 1/2, in case $a > (1+s)(1-s)/\left[2v(2+3s)-v^2(3+2s)-s(4+s)\right]$ and comparing the profit in (36) with 1/2, in case $a \le (1+s)(1-s)/\left[2v(2+3s)-v^2(3+2s)-s(4+s)\right]$. Doing so yields that, for a sufficiently high, the equilibrium outcome is qualitatively similar to that of the game in which firms negotiate the pricing regime. In particular, if $a \to 1$, it is evident that the profits in (35) and (36) go to the industry profits in the respective pricing regimes. Therefore, A's comparison of whether to use dual distribution or not is the same as in the game in which the pricing regime is negotiated, which leads to the same thresholds as in Proposition 5. Instead, if a is low, A receives only a small part of the profit from dual distribution, which implies that mono distribution occurs for a larger range in equilibrium.