Appendix A: Sample Task Page for the Experiment

Original instructions in Chinese, English translation below. The task paragraph was in English.

- Please identify three letters that consist of “e-any letter-vowel” (not including any space).
- Write down words that contain such letter strings. For example, egregious contains “egi”, which is “e-any letter-vowel”. If the same word appears multiple times, please write them down in order. Please count the number of times the letter strings appear (including repetitions).

Our SPM methodology is specially tailored to account for the complex patterns of variation and covariation among the observations of a dependent measure from a set of studies that appear in a typical behavioral research paper. In particular, it extends prior hierarchical metaanalytic models to accommodate (i) an arbitrary number of study conditions that result from the variation of one or more experimental factors and give rise to multiple dependent effects of interest (e.g., simple effects and interaction effects); (ii) a mix of study designs (e.g., two condition vs. 2 × 2, between-subjects vs. within-subjects); (iii) the variation (or heterogeneity) resulting from differences in method factors among the observations; and (iv) the covariation induced by the fact that the observations are nested within studies and study conditions (or, when one or more studies follow a within-subjects design, nested within studies, subject groups, and study conditions). To achieve this, our SPM methodology decomposes each observation—that is, a statistic (e.g., the mean, the proportion of successes) that summarizes the individual-level observations in each condition of each study—into three components: (i) an overall condition average component,(ii)a study-condition component that reflects method factors, and (iii) a study-condition component that reflects sampling error.

You can quit the exercise at any time, or you can continue the exercise.

○ Quit (please skip to the end of the survey and submit)
○ Continue

Appendix B: Proofs

Demand function derivation for Monopoly

The consumer will prefer to invest effort κ only if $\beta v + \lambda p - \kappa \geq 0$. This reduces to the condition that:

$$v \geq \frac{\kappa - \lambda p}{\beta} \equiv v_1 \tag{B1}$$

Note that since κ is less than β some consumers will always exert effort when the costs are low. The consumer would exert effort even in the high cost case i.e., when costs are 1 if:

$$v \geq \frac{1 - \lambda p}{\beta} \equiv v_2 \tag{B2}$$

Clearly $v_2 > v_1$. If the consumer only exerts effort when the costs are low then he is willing to buy if:

$$v \geq \frac{p}{\alpha} + \kappa \equiv v_3 \tag{B3}$$

Thus, the consumer buys and uses only under low cost if $v_2 \geq v \geq \max(v_1, v_3)$. If $p > (1-\kappa)\alpha$ then $v_3 > 1$. In other words, no one buys the product. Therefore, in any optimal price $p^* < (1-\kappa)\alpha$.

The consumer who exerts effort regardless of the costs is willing to buy if $v - \alpha\kappa - (1-\alpha) - p \geq 0$. Thus, this consumer buys if:

$$v \geq p + \alpha\kappa + (1-\alpha) \equiv v_4 \tag{B4}$$

Thus, the consumer buys with over-consumption as long as $v \geq \max(v_2, v_4)$. Note that $v_4 > v_3$.

Thus, the demand is given by:

$$\mathrm{D} = \max\left(0, F(v_2) - F(\max(v_1, v_3))\right) + \max\left(0, 1 - F(\max(v_2, v_4))\right) \tag{B5}$$

The first term is the demand from the segment which exerts effort only under low costs and the second term is the demand from the consumers who always exert effort.

We can see that $v_3 > v_1$ if:

$$\frac{p}{\alpha} + \kappa \geq \frac{\kappa - \lambda p}{\beta} \tag{B6}$$

$$\implies p \geq \frac{\kappa(1-\beta)\alpha}{\lambda\alpha + \beta} \equiv \hat{p}_a \tag{B7}$$

Note that if $v_4 > v_2$ then consumers in the segment (v_2, v_4) do not buy even though consumers with a lower valuation purchase. These consumers abstain to avoid over-consumption. The condition that $v_4 > v_2$ which leads to a "hole" in the demand is equivalent to:

$$p + \alpha\kappa + (1-\alpha) \geq \frac{1 - \lambda p}{\beta} \tag{B8}$$

$$\implies p \geq \frac{1 - \beta(\alpha\kappa + 1 - \alpha)}{\beta + \lambda} \equiv \hat{p}_b \tag{B9}$$

Claim B.1. $\hat{p}_b > \hat{p}_a$. *Thus,* $v_4 > v_2 \implies v_3 > v_1$.

Proof: Note that $\hat{p}_a$ is increasing in κ and $\hat{p}_b$ is decreasing in κ. Since $\kappa \leq \beta$. We have:

$$\hat{p}_b - \hat{p}_a \geq \frac{1 - \beta(\alpha\beta + 1 - \alpha)}{\beta + \lambda} - \left(\frac{\beta(1-\beta)\alpha}{\lambda\alpha + \beta}\right) \tag{B10}$$

$$= \frac{(1-\beta)\left(\alpha\lambda(1-(1-\alpha)\beta) + \beta\right)}{(\lambda\alpha + \beta)(\lambda + \beta)} > 0 \tag{B11}$$

Thus, $\hat{p}_b \geq \hat{p}_a$. $\square$

If $v_2 > v_3 > v_1$ then consumers in the region (v_3, v_2) purchase. The condition for $v_2 > v_3$ is $p < \hat{p}_c$ where:

$$\hat{p}_c = \frac{\alpha(1-\kappa\beta)}{\lambda\alpha+\beta} \tag{B12}$$

Claim B.2. *If* $\lambda > \lambda_1 = \frac{1-\beta}{\alpha(1-\kappa)}$ *then* $\hat{p}_c$ *and* $\hat{p}_b$ *are less than* $(1-\kappa)\alpha$*. Furthermore,* $\hat{p}_c > \hat{p}_b$*. Thus,* $v_2 < v_3 \implies v_4 > v_2$*.*

Proof: Note that $(1-\kappa)\alpha - \hat{p}_b$ is increasing in λ and so is $(1-\kappa)\alpha - \hat{p}_c$. Solving for the critical λ we find that if $\lambda > \lambda_1 = \frac{1-\beta}{\alpha(1-\kappa)}$ then $(1-\kappa)\alpha > \hat{p}_b$ and $(1-\kappa)\alpha > \hat{p}_c$. Also:

$$\hat{p}_c - \hat{p}_b = \frac{\beta(1-\alpha)(\alpha\lambda(1-\kappa)-(1-\beta))}{(\lambda+\beta)(\alpha\lambda+\beta)} \tag{B13}$$

This is positive as long as $\lambda > \lambda_1$ and $\alpha < 1$. $\square$

Claim B.3. *If* $\lambda > \lambda_1$ *and* $p \in (\hat{p}_c, (1-\kappa)\alpha)$ *then the demand is* $1 - F(v_4)$*. For* $p > \alpha(1-\kappa)$ *the market demand is zero.*

Proof: In this price range $v_3 > v_2$ and $v_4 > v_2$. Therefore, only consumers with $v > v_4$ buy. Note that $v_4 < 1$ for $p < \alpha(1-\kappa)$ when $\lambda > \lambda_1$. $\square$

Claim B.4. *If* $\lambda > \lambda_1$ *then for* $p \in (\hat{p}_b, \hat{p}_c)$ *the demand will be* $F(v_2) - F(v_3) + 1 - F(v_4)$*.*

Proof: It is sufficient to establish that v_3 at $\hat{p}_b$ is strictly less than 1. Since $p < \hat{p}_c$, we know that $v_2 > v_3$ and $p > \hat{p}_b$ implies that $v_4 > v_2$. Note that $v_4 > v_3$. Thus, $v_4 > v_2 > v_3 > 0$. Therefore, it is sufficient to show that $v_4 < 1$. Note that $v_4 < 1$ for $p < \alpha(1-\kappa)$ when $\lambda > \lambda_1$. $\square$

Claim B.5. *If* $\lambda > \lambda_1$ *and* $p \in (\hat{p}_d, \hat{p}_b)$ *where* $\hat{p}_d = \frac{1-\beta}{\lambda}$ *then consumers in* $(v_2, 1)$ *buy the product and overconsume. Consumers in* (v_3, v_2) *invest only under low costs and buy the product. The total demand is therefore* $1 - F(v_3)$*.*

Proof: Now consider the case when $\lambda > \lambda_1$ and $p < \hat{p}_b$. In this case, over-consumption could happen but $v_2 > v_4$ so all consumers who over-consume will buy as long as $v_2 < 1$. We have:

$$1 - v_2 = \frac{-(1-\beta)+\lambda p}{\beta} \tag{B14}$$

This is positive as long as:

$$p \geq \frac{1-\beta}{\lambda} = \hat{p}_d \tag{B15}$$

Note that at $\hat{p}_b$, $v_2 = v_4$ and by Claim B.2, it follows that $1 - v_4(\hat{p}_b) = 1 - v_2(\hat{p}_b) > 0$ when $\lambda > \lambda_1$. Therefore,

$$1 - v_2(\hat{p}_d) = 0 < 1 - v_2(\hat{p}_b) \tag{B16}$$

$$\implies v_2(\hat{p}_d) > v_2(\hat{p}_b) \tag{B17}$$

However, since v_2 is decreasing in price - it follows that $\hat{p}_d < \hat{p}_b$. Thus, for $p \in (\hat{p}_d, \hat{p}_b)$ consumers in $(v_2, 1)$ overconsume and buy the product. Also,note that:

$$\hat{p}_d - \hat{p}_a = \frac{1-\beta}{\lambda} - \frac{\kappa(1-\beta)\alpha}{\lambda\alpha+\beta} \tag{B18}$$

$$= \frac{(1-\beta)\left[\beta+\alpha\lambda(1-\kappa)\right]}{\lambda(\alpha\lambda+\beta)} > 0 \tag{B19}$$

This implies that for $p \in (\hat{p}_d, \hat{p}_b)$ we have $v_3 > v_1$ and therefore all consumers in the range (v_3, v_2) will invest under low costs and buy. $\square$

Claim B.6. *If* $\lambda > \lambda_1$ *and* $p \in (\hat{p}_a, \hat{p}_d)$ *then the demand is* $1 - F(v_3)$.

Proof: In this case $v_2 > 1$ and $v_3 > v_1$ and therefore the result follows. □

Claim B.7. *If* $\lambda > \lambda_1$ *and* $p < \hat{p}_a$ *then the demand is* $1 - F(v_1)$.

Proof: In this case, $v_1 > v_3$ and $v_2 > 1$ and therefore only consumers in $(v_1, 1)$ buy the product since $v_1 \in (0, 1)$. □

Claim B.8. *If* $\lambda < \lambda_1$ *and* $p \in (\hat{p}_a, (1-\kappa)\alpha)$ *then the demand is* $1 - F(v_3)$.

Proof: First note that in this case $v_2 > 1$. To see this note that:

$$1 - v_2 = \frac{-(1-\beta) + \lambda p}{\beta} \tag{B20}$$

$$\leq \frac{-(1-\beta) + \lambda\alpha(1-\kappa)}{\beta} \tag{B21}$$

which is negative if $\lambda < \lambda_1$. Thus, over-consumption can only happen if $\lambda > \lambda_1$. For $p > \hat{p}_a$ we know that $v_3 > v_1$. Furthermore, as long as $p < (1-\kappa)\alpha$, $v_3 < 1$. The result then follows. □

Claim B.9. *If* $\lambda < \lambda_1$ *and* $p < \hat{p}_a$ *then the demand is* $1 - F(v_1)$.

Proof: In this case $v_1 > v_3$ and the result immediately follows. □

Summarizing, it follows that if $\lambda > \lambda_1$ then the demand function is:

$$\mathrm{D}(p) = \begin{cases} 1 - F(v_1) & \text{if } p \leq \hat{p}_a \\ 1 - F(v_3) & \text{if } p \in (\hat{p}_a, \hat{p}_b] \\ F(v_2) - F(v_3) + 1 - F(v_4) & \text{if } p \in (\hat{p}_b, \hat{p}_c] \\ 1 - F(v_4) & \text{if } p \in (\hat{p}_c, (1-\kappa)\alpha) \\ 0 & \text{if } p \geq (1-\kappa)\alpha \end{cases} \tag{B22}$$

If $\lambda < \lambda_1$ then the demand function is:

$$\mathrm{D}(p) = \begin{cases} 1 - F(v_1) & \text{if } p \leq \hat{p}_a \\ 1 - F(v_3) & \text{if } p \in (\hat{p}_a, (1-\kappa)\alpha) \\ 0 & \text{if } p \geq (1-\kappa)\alpha \end{cases} \tag{B23}$$

Note that if $\alpha = 1$, $\hat{p}_b = \hat{p}_c$ and $v_3 = v_4$ and therefore the two demand functions become identical at $\alpha = 1$. This is intuitive since in this case, the issue of over-consumption does not exist.

Proof of Proposition 1

The demand function has already been derived. We only need to establish how the demand changes with λ and β. Note that v_1 is decreasing in λ and β. Therefore, when $p < \hat{p}_a$ demand is increasing in λ and β. Also, v_3 is independent of β and λ. Therefore, demand weakly increases in λ and β. □

Proof of Proposition 2

This has already been proved in our derivation of the demand function.

Proof of Proposition 3

When the demand function is upward sloping the firm will not charge a price less than $\hat{p}_a$. If the firm can charge p_r and $p_r > \hat{p}_a$ then it is clearly optimal to charge p_r since the usage constraint does not bind. Note that at $\lambda = 0$, $\hat{p}_a(\lambda = 0) = \frac{\kappa\alpha(1-\beta)}{\beta}$. Also $\hat{p}_a$ is decreasing in λ and $\hat{p}_a \to 0$ as $\lambda \to \infty$. If $p_r < \frac{\kappa\alpha(1-\beta)}{\beta}$ then at $\lambda = 0$ the usage constraint binds and the firm will charge at least $\hat{p}_a$. Also, there would exist a λ^* such that $p_r < \hat{p}_a$ for $\lambda \in (0, \lambda^*)$. If $\lambda \in (0, \lambda^*)$ then the firm must charge a price $p^* \geq \hat{p}_a$. Therefore, the firm's problem is to choose p^* such that:

$$p^* = \underset{p \geq \hat{p}_a}{\operatorname{argmax}} \, (1 - F(v_3))(p - c) \tag{B24}$$

However, p_r is the solution when the pricing constraint does not bind. Since $p_r < \hat{p}_a$ and the profit function is concave, it follows that $p^* = \hat{p}_a$ for $\lambda \in (0, \lambda^*)$.

To see the second part note that p_r is increasing in c. If for a given λ and c, $p_r < \hat{p}_a$ then for large enough c this inequality reverses and therefore for $c \in (0, c^*)$, the optimal price is $\hat{p}_a$ which is independent of c.

The firm chooses price to maximize profit under the constraint $p^* \geq \hat{p}_a$. However, as λ increases, the constraint relaxes and the profits increase. If $\lambda > \lambda^*$, profits, social welfare and consumer surplus is unaffected by λ. So let's consider the case when $\lambda < \lambda^*$. Here:

$$\text{SW} = \int_{v_3(\hat{p}_a)}^{1} \alpha(v - \kappa) dF(v) \tag{B25}$$

Therefore:

$$\frac{\partial \text{SW}}{\partial \lambda} = \left[-\frac{\partial v_3(\hat{p}_a)}{\partial \lambda} \right] \cdot \alpha(v_3(\hat{p}_a) - \kappa) f(v_3(\hat{p}_a)) > 0 \tag{B26}$$

where the inequality follows since $v_3(\hat{p}_a)$ is decreasing in λ because $\hat{p}_a$ is decreasing in λ. Similarly consumer surplus is:

$$\text{CS} = \int_{v_3(\hat{p}_a)}^{1} \left(\alpha(v - \kappa) - \hat{p}_a\right) dF(v) \tag{B27}$$

Thus:

$$\frac{\partial \text{CS}}{\partial \lambda} = \left[-\frac{\partial \hat{p}_a}{\partial \lambda} \right] \cdot \alpha \int_{v_3(\hat{p}_a)}^{1} dF(v) > 0 \tag{B28}$$

This completes the proof. □

Proof of Proposition 4

In this case the relevant profit function is:

$$\Pi = (\hat{p}_a - c)\left(1 - F\left(\frac{\hat{p}_a}{\alpha} + \kappa - q\right)\right) - \phi(q) \tag{B29}$$

For the optimal q^* the first order condition is:

$$\begin{aligned} \frac{\partial \Pi}{\partial q} &= \frac{\partial \hat{p}_a}{\partial q}\left(1 - F\left(\frac{\hat{p}_a}{\alpha} + \kappa - q\right)\right) - (\hat{p}_a - c)\frac{\partial v_3}{\partial q} f\left(\frac{\hat{p}_a}{\alpha} + \kappa - q\right) - \phi'(q) && \text{(B30)} \\ &= \frac{\partial \hat{p}_a}{\partial q}\left[1 - F(v_3) - (\hat{p}_a - c)\frac{f(v_3)}{\alpha}\right] + (\hat{p}_a - c) f(v_3) - \phi'(q) && \text{(B31)} \end{aligned}$$

Note that:

$$\left[1 - F(v_3) - (\hat{p}_a - c)\frac{f(v_3)}{\alpha}\right]\bigg|_{p=\hat{p}_a} < 0 \tag{B32}$$

since $\hat{p}_a > p_r$. Thus, the first term in (B31) is positive which implies that $q^* > 0$.

Define:

$$\mathcal{A} \equiv \left[1 - F(v_3) - (\hat{p}_a - c)\frac{f(v_3)}{\alpha}\right] < 0 \tag{B33}$$

Then, using (B31) it follows that

$$\frac{\partial^2 \Pi}{\partial q \partial \lambda} = \frac{\partial^2 \hat{p}_a}{\partial q \partial \lambda}\mathcal{A} + \frac{\partial \hat{p}_a}{\partial q} f(v_3) + (\hat{p}_a - c) f'(v_3)\frac{\partial v_3}{\partial \lambda} + \frac{\partial \mathcal{A}}{\partial \lambda}\frac{\partial \hat{p}_a}{\partial q} \tag{B34}$$

Since $\mathcal{A} < 0$ and $\frac{\partial^2 \hat{p}_a}{\partial q \partial \lambda} > 0$, the first term is negative. Similarly, since $\frac{\partial \hat{p}_a}{\partial q} < 0$, the second term is negative. The third term is also negative since $f' > 0$ and $\frac{\partial v_3}{\partial \lambda} < 0$ for $p = \hat{p}_a$. Therefore, if $\frac{\partial \mathcal{A}}{\partial \lambda} > 0$ then it would follow that q^* is decreasing in λ. We have:

$$\frac{\partial \mathcal{A}}{\partial \lambda} = -f(v_3)\frac{\partial v_3}{\partial \lambda} - \frac{f'(v_3)}{\alpha}\frac{\partial v_3}{\partial \lambda} > 0 \tag{B35}$$

Thus q^* decreases in λ. □.

Case when the firm invests in functionality

In this case:

$$v_1 = \frac{\kappa - \lambda p}{q\beta} \tag{B36}$$

$$v_3 = \frac{p}{\alpha q} + \frac{\kappa}{q} \tag{B37}$$

The critical price $\hat{p}_a$ remains the same i.e., $\hat{p}_a = \frac{\kappa\alpha(1-\beta)}{\lambda\alpha+\beta}$. In this case the relevant profit function is:

$$\Pi = (\hat{p}_a - c)\left(1 - F\left(\frac{\hat{p}_a}{\alpha q} + \frac{\kappa}{q}\right)\right) - \phi(q) \tag{B38}$$

For the optimal q^* the first order condition is:

$$\frac{\partial \Pi}{\partial q} = (\hat{p}_a - c) f(v_3)\frac{v_3}{q} - \phi'(q) \tag{B39}$$

Thus, the first term in (B39) is positive which implies that $q^* > 0$. We have:

$$\frac{\partial^2 \Pi}{\partial q \partial \lambda} = \frac{\partial \hat{p}_a}{\partial \lambda} \cdot f(v_3) \cdot \frac{v_3}{q} + (\hat{p}_a - c) f'(v_3) \cdot \frac{\partial v_3}{\partial \lambda} \cdot \frac{v_3}{q} + (\hat{p}_a - c) \cdot \frac{f(v_3)}{q} \cdot \frac{\partial v_3}{\partial \lambda} < 0 \tag{B40}$$

where the inequality follows since $\frac{\partial \hat{p}_a}{\partial \lambda} < 0$ and $\frac{\partial v_3}{\partial \lambda} < 0$. Thus, the firm invests less in quality as λ increases. □

Proof of Proposition 5

For the case when $F(\cdot)$ is uniform, p_r is given by the equation:

$$1 - \frac{p_r}{\alpha} - \kappa - \frac{p_r - c}{\alpha} = 0 \tag{B41}$$

which reduces to:

$$p_r = \frac{\alpha(1-\kappa) + c}{2} \tag{B42}$$

We establish the proposition via a series of claims.

Claim 5.1. *If $\lambda > \lambda_1$ then $p_r > \hat{p}_a$.*

Proof: We know that $\hat{p}_a = \frac{\kappa\alpha(1-\beta)}{\lambda\alpha+\beta}$. Therefore:

$$\begin{aligned} p_r - \hat{p}_a &= \frac{\alpha\lambda(c + \alpha(1-\kappa)) + \alpha\kappa\beta + \alpha\beta - 2\alpha\kappa + \beta c}{2(\alpha\lambda + \beta)} && \text{(B43)} \\ &\geq \frac{\alpha\lambda\alpha(1-\kappa) + \alpha\kappa\beta + \alpha\beta - 2\alpha\kappa}{2(\alpha\lambda + \beta)} && \text{(B44)} \\ &\geq \frac{\alpha\lambda_1\alpha(1-\kappa) + \alpha\kappa\beta + \alpha\beta - 2\alpha\kappa}{2(\alpha\lambda + \beta)} && \text{(B45)} \\ &= \frac{\alpha(1-\kappa)(1 + \beta\kappa - 2\kappa)}{2(1-\beta\kappa)} && \text{(B46)} \\ &\geq \frac{\alpha(1-\kappa)(1 + \kappa^2 - 2\kappa)}{2(1-\beta\kappa)} && \text{(B47)} \\ &= \frac{\alpha(1-\kappa)^3}{2(1-\beta\kappa)} > 0 && \text{(B48)} \end{aligned}$$

where the first inequality follows since the difference is increasing in c, the second inequality follows since the difference is increasing in λ and $\lambda \geq \lambda_1$ and the third inequality follows since $\beta > \kappa$. □

Claim 5.2. *If $\lambda_1 < \lambda < \frac{2(1-\beta)+\alpha\beta(1-\kappa)-\beta c}{c+\alpha(1-\kappa)} \equiv \lambda_2$ then $p^* = p_r$.*

Proof: First note that $p_r < \hat{p}_b$ iff:

$$\frac{\alpha(1-\kappa) + c}{2} - \frac{1 - \beta(\alpha\kappa + 1 - \alpha)}{\beta + \lambda} < 0 \tag{B49}$$

This reduces to the condition that:

$$\lambda < \frac{2(1-\beta) + \alpha\beta(1-\kappa) - \beta c}{c + \alpha(1-\kappa)} \equiv \lambda_2 \tag{B50}$$

Thus, if $\lambda \in (\lambda_1, \lambda_2)$ then $p_r < \hat{p}_b$. In this case, the profit function coincides with Π_0 for $p \in (\hat{p}_a, \hat{p}_b)$. Since $\Pi_0^* \geq \Pi(p) \forall p$ it follows that $p^* = p_r$. □

Claim 5.3. *If the firm charges a price $p^* > \hat{p}_c$ and $\lambda > \lambda_3 = \frac{(2-(1+\kappa)\beta)\alpha-\beta c}{\alpha(c+\alpha(1-\kappa))}$ then $p^* = p_r$.*

Proof: We have:

$$p_r - \hat{p}_c = \frac{\alpha(1-\kappa)+c}{2} - \frac{\alpha(1-\kappa\beta)}{\lambda\alpha+\beta} \tag{B51}$$

$$= \frac{\alpha\lambda(c+\alpha)(1-\kappa)+\alpha\kappa\beta+\alpha\beta+\beta c-2\alpha}{2(\alpha\lambda+\beta)} \tag{B52}$$

Solving this for λ, we see that for $\lambda > \frac{(2-(1+\kappa)\beta)\alpha-\beta c}{\alpha(c+\alpha(1-\kappa))} \equiv \lambda_3$, $p_r > \hat{p}_c$. If the firm charges $p^* > \hat{p}_c$ then the relevant profit function for the firm is:

$$\Pi_4 = (1-F(v_4))(p-c) \tag{B53}$$

The optimal price is

$$p_4^* = \frac{\alpha(1-\kappa)+c}{2} = p_r \tag{B54}$$

□

Claim 5.4. *If $\lambda > \lambda_2$ then $\hat{p}_b \leq p^* \leq p_r$.*

Proof: Since $\lambda > \lambda_2$, we know that $p_r > \hat{p}_b$. The firm can charge a price $p \in (\hat{p}_b, \hat{p}_c)$ in which case the relevant profit function is:

$$\Pi_3 = (v_2 - v_4 + 1 - v_3)(p-c) \tag{B55}$$

$$\left.\frac{\partial \Pi_3}{\partial p}\right|_{p_r} = (p_r - c)\left[\frac{-\lambda}{\beta} - 1 - \frac{1}{\alpha}\right] + (v_2 - v_4 + 1 - v_3) \tag{B56}$$

$$= \left[\frac{-(p_r-c)}{\alpha} + (1-v_3)\right] + (v_2 - v_4) + (p_r - c)\left[\frac{-\lambda}{\beta} - 1\right] < 0 \tag{B57}$$

where the last equality follows using the first order condition for p_r. This implies that in this case $p^* < p_r$ when $p_r \in (\hat{p}_b, \hat{p}_c)$. We already know that if $p_r > \hat{p}_c$ then $p^* \leq p_r$. Note that profit functions coincide for $p \in (\hat{p}_a, \hat{p}_b)$ and therefore $p_r > \hat{p}_b$ implies that $p^* \geq \hat{p}_b$. □

Claim 5.5. *If $\lambda > \lambda_1$ then p^* is weakly decreasing in λ except possibly at $\lambda_4 > \lambda_3$.*

Proof: In this case, if $\lambda \in (\lambda_1, \lambda_2)$ then $p^* = p_r$ which is independent of λ. If $\lambda > \lambda_2$ then if $p^* > \hat{p}_c$ then as we have seen before $p^* = p_r$ which is independent of λ. The only case remaining is when $p^* \in [p_b, p_c)$. The firm will either charge $\hat{p}_b$ or charge p_3^* when $\hat{p}_b < p_3^*$ which is defined as:

$$p_3^* = \underset{p}{\operatorname{argmax}}\, \Pi_3 = \underset{p}{\operatorname{argmax}} (v_2 - v_4 + 1 - v_3)(p-c) \tag{B58}$$

Note that $\frac{\partial^2 \Pi_3}{\partial p \partial \lambda} < 0$ and therefore p_3^* is decreasing in λ and so is $\hat{p}_b$. Therefore, p^* is weakly decreasing in λ. It is possible however for the price to jump at $\lambda_4 > \lambda_3$ to p_r from $\max(p_3, p_b)$. □

Claim 5.6. *There exists a $\lambda_5 > \lambda_2$ such that $p^* = \hat{p}_b$ for $\lambda \in (\lambda_2, \lambda_5)$*

Proof: Note that at λ_2, $p_r = \hat{p}_b$. Define p_3^* as the price which maximizes Π_3. Therefore, using (B57) it follows that at λ_2, $p_3^* < \hat{p}_b$ and therefore $p^* = \hat{p}_b$. Since both p_3^* and $\hat{p}_b$ are decreasing in λ, the result follows. □

Claim 5.7. *If $\lambda > \lambda_1$ then social welfare increases in λ for $\lambda \in (\lambda_2, \lambda_5)$ and decreases otherwise.*

Proof: Note that if $\lambda > \lambda_1$ and $\lambda \notin (\lambda_2, \lambda_5)$ then demand is weakly decreasing and therefore social welfare also declines. However, when $\lambda \in (\lambda_2, \lambda_5)$ demand increases in λ and therefore social welfare and consumer welfare improves with λ. □

Claim 5.8. *Profits are weakly decreasing in λ for $\lambda > \lambda_1$.*

Proof: If $\lambda \in (\lambda_1, \lambda_2)$ then prices and profits are the same as the unconstrained case and do not depend on λ. Now consider $\lambda > \lambda_2$. If $p^* = \hat{p}_b$ then the demand is increasing in λ but prices are decreasing in λ. However, profits must decline with λ. To see this assume that there exists a λ^* such that profits at $\lambda^* - \epsilon$ are lower than at λ^*, where $\lambda^* > \lambda_2$ is such that the optimal price is $\hat{p}_b$. In other words, assume that $\Pi^*(\lambda^* - \epsilon) < \Pi^*(\lambda^*)$. Since the firm charges $\hat{p}_b(\lambda = \lambda^* - \epsilon) > \hat{p}_b(\lambda^*)$, it must be the case that $\Pi^*(\lambda^* - \epsilon) = \Pi(\hat{p}_b(\lambda^* - \epsilon), \lambda^* - \epsilon) > \Pi(\hat{p}_b(\lambda^*), \lambda^* - \epsilon) > \Pi(\hat{p}_b(\lambda^*), \lambda^*)$, where the first inequality follows from optimality and the second follows since the profits are lower at higher λ at the same prices when $\lambda > \lambda_2$. However, this leads to a contradiction and therefore when the optimal prices are $\hat{p}_b$ then the profit is declining in λ. If $p^* > \hat{p}_b$ then both prices and demand are weakly decreasing in λ (except possibly at some $\lambda_4 > \lambda_3$). However, profits must decrease at λ_4 also since the firm at $\lambda_4 - \epsilon$ could choose p_r and prefers $p^* < p_r$. □

Part a of the proposition follows from Claims 5.2, 5.3 and 5.4. Part b follows from Claim 5.7 by setting $\hat{\lambda} = \lambda_5$ and part c follows from Claim 5.8. □

Proof of Proposition 7

First, consider the case when the firm charges a single price. In this case, for a consumer to use the product in both periods, we would need $\beta v - \kappa + \eta\lambda p \geq 0$. This reduces to the condition that $v \geq v_{1f}$ where:

$$v_{1f} = \frac{\kappa - \eta\lambda p}{\beta} \tag{B59}$$

Similarly, the consumer will at least use the product for one period if $v \geq v_{1p}$ where:

$$v_{1p} = \frac{\kappa - \lambda p}{\beta} \tag{B60}$$

Note that $v_{1p} < v_{1f}$. The consumer who uses the product fully is willing to buy the product only if $2(v - \kappa) - p \geq 0$ i.e., if $v > v_{2f}$ where:

$$v_{2f} = \kappa + \frac{p}{2} \tag{B61}$$

Similarly, the consumer is willing to buy with partial usage only if $v - \kappa - p > 0$ i.e., if $v > v_{2p}$ where:

$$v_{2p} = p + \kappa \tag{B62}$$

Clearly, $v_{2p} > v_{2f}$. Define $\hat{p}_{ap}$ as the price at which $v_{1p} = v_{2p}$ and $\hat{p}_{af}$ as the price at which $v_{1f} = v_{2f}$. We have:

$$\hat{p}_{ap} + \kappa = \frac{\kappa - \lambda\hat{p}_{ap}}{\beta} \tag{B63}$$

$$\hat{p}_{ap} = \frac{\kappa(1 - \beta)}{\beta + \lambda} \tag{B64}$$

Similarly,

$$\frac{\hat{p}_{af}}{2} + \kappa = \frac{\kappa - \eta\lambda\hat{p}_{af}}{\beta} \tag{B65}$$

$$\hat{p}_{af} = \frac{2\kappa(1-\beta)}{\beta + 2\eta\lambda} \tag{B66}$$

Clearly, $\hat{p}_{af} > \hat{p}_{ap}$. Let $\Pi_{2p} = (p-c)(1-F(v_{2p}))$. Define p^*_{rp} as:

$$p^*_{rp} = \underset{p}{\operatorname{argmax}}(p-c)(1-F(v_{2p})) \tag{B67}$$

Similarly, let $\Pi_{2f} = (p-c)(1-F(v_{2f}))$. Define:

$$p^*_{rf} = \underset{p}{\operatorname{argmax}}(p-c)(1-F(v_{2f})) \tag{B68}$$

We prove the proposition using a series of claims.

Claim 7.1. $p^*_{rf} > p^*_{rp} > \frac{p^*_{rf}}{2}$.

Proof: The first order condition for p^*_{rf} is:

$$0 = \frac{-f(v^*_{2f})(p^*_{rf}-c)}{2} + 1 - F(v^*_{2f}) \tag{B69}$$

Define $v_{2p}(p^*_{rf}) = \tilde{v}_{2p}$. Note that $\tilde{v}_{2p} = p^*_{rf} + \kappa > v_{2f}(p^*_f) = \frac{p^*_{rf}}{2} + \kappa$. We have:

$$\left.\frac{\partial \Pi_{2p}}{\partial p}\right|_{p^*_{rf}} = -f(\tilde{v}_{2p})(p^*_{rf}-c) + 1 - F(\tilde{v}_{2p}) \tag{B70}$$

$$= -f(\tilde{v}_{2p})\left[\frac{2(1-F(v^*_{2f}))}{f(v^*_{2f})}\right] + 1 - F(\tilde{v}_{2p}) \tag{B71}$$

$$= f(\tilde{v}_{2p})\left[\frac{-2(1-F(v^*_{2f}))}{f(v^*_{2f})} + \frac{(1-F(\tilde{v}_{2p}))}{f(\tilde{v}_{2p})}\right] < 0 \tag{B72}$$

where the first equality follows using (B69) and the second inequality follows since $\tilde{v}_{2p} > v^*_{2f}$. Thus, $p^*_{rf} > p^*_{rp}$.

Similarly, we have:

$$\left.\frac{\partial \Pi_{2p}}{\partial p}\right|_{\frac{p^*_{rf}}{2}} = -f(v^*_{2f})\left(\frac{p^*_{rf}}{2} - c\right) + 1 - F(v^*_{2f}) \tag{B73}$$

$$= \frac{cf(v^*_{2f})}{2} > 0 \tag{B74}$$

Thus, $p^*_{rp} > \frac{p^*_{rf}}{2}$. □

Claim 7.2. *If $p^*_{rf} < \hat{p}_{af}$ and $p^*_{rp} < \hat{p}_{ap}$ then p^* is either p^*_{ap} or p^*_{af}.*

Proof: We know that Π_{2p} is concave and is maximized at p^*_{rp}. However, since $\hat{p}_{ap} > p^*_{rp}$, the firm will set a price $\hat{p}_{ap}$ when the optimal solution is such that the marginal consumers uses the product partially. An identical argument shows that when the marginal consumer fully uses the product then the optimal price is $\hat{p}_{af}$. The firm then compares the profits $\Pi_{2f}(\hat{p}_{af})$ and $\Pi_{2p}(\hat{p}_{ap})$ and chooses the optimal price. □

Claim 7.3. *If $p^*_{rf} < \hat{p}_{af}$ and $p^*_{rp} < \hat{p}_{ap}$ then the optimal single price is $\hat{p}_{ap}$ for some $\eta < \eta^* < \frac{1}{2}$ and $\hat{p}_{af}$ otherwise.*

Proof: In this case, the firm either chooses p^*_{rf} or p^*_{ap}. Define:

$$\Pi^*_p = (1 - F(\hat{p}_{ap} + \kappa))(\hat{p}_{ap} - c) \tag{B75}$$

$$\Pi^*_f = \left(1 - F\left(\frac{\hat{p}_{af}}{2} + \kappa\right)\right)(\hat{p}_{af} - c) \tag{B76}$$

Thus, the optimal price is $\hat{p}_{af}$ when $\Pi^*_f \geq \Pi^*_p$. Note that Π^*_p is independent of η. Furthermore:

$$\frac{\partial \Pi^*_f}{\partial \eta} = \frac{\partial \hat{p}_{af}}{\partial \eta}\left[-f\left(\frac{\hat{p}_{af}}{2} + \kappa\right) \cdot \frac{\hat{p}_{af} - c}{2} + \left(1 - F\left(\frac{\hat{p}_{af}}{2} + \kappa\right)\right)\right] > 0 \tag{B77}$$

where the inequality follows since $\frac{\partial \hat{p}_{af}}{\partial \eta} < 0$ and the second term is also negative since $\hat{p}_{af} > p^*_{rf}$. Thus, as η increases $\Pi^*_f - \Pi^*_p$ increases. Note that at $\eta = \frac{1}{2}$, $\hat{p}_{ap} = \frac{\hat{p}_{af}}{2}$ and therefore at $\eta = \frac{1}{2}$, $\Pi^*_f > \Pi^*_p$. Thus, there exists some $\eta^* < \frac{1}{2}$ such that the optimal single price is $\hat{p}_a$ for $\eta < \eta^*$ and is $\hat{p}_{af}$ otherwise. □.

Claim 7.4. *If $p^*_{rf} < \hat{p}_{af}$ and $p^*_{rp} < \hat{p}_{ap}$ and optimal single price is $\hat{p}_{af}$ then a payment plan leads to higher profits, lower prices and higher social and consumer welfare. The payment plan is such that $p^*_1 > p^*_2$ and is given by:*

$$p^*_1 = \frac{\max(p^*_{rf}, p_o)}{2 - \eta} \tag{B78}$$

$$p^*_2 = \frac{\max(p^*_{rf}, p_o)(1 - \eta)}{2 - \eta} \tag{B79}$$

where:

$$p_o = \frac{2(1 - \beta)\kappa}{\beta + \frac{2\lambda}{2 - \eta}} \tag{B80}$$

Proof: In this case by assumption the optimal single price is $\hat{p}_{af}$. Let the marginal consumer with the single period price p^* be denoted by v^*. Then we must have:

$$\beta v^* + \lambda \hat{p}_{af} - \kappa > 0 \tag{B81}$$

$$\beta v^* + \eta \lambda \hat{p}_{af} - \kappa = 0 \tag{B82}$$

Note that:

$$v^* = \frac{\kappa - \eta \lambda \hat{p}_{af}}{\beta} \tag{B83}$$

Suppose the firm can offer a payment plan (p_1, p_2) which leads to full usage. Denote $\tilde{p} = p_1 + p_2$. Let the marginal consumer who will still exert effort under this plan be denoted by $\tilde{v}_1$ then we must have:

$$\beta \tilde{v}_1 + \lambda p_1 - \kappa \geq 0 \tag{B84}$$

$$\beta \tilde{v}_1 + \lambda(\eta p_1 + p_2) - \kappa \geq 0 \tag{B85}$$

If we set $p_1 = \frac{\hat{p}_{af}}{2-\eta}$ and $p_2 = \frac{(1-\eta)\hat{p}_{af}}{2-\eta}$ then $\eta p_1 + p_2 = p_1$ and therefore both the constraints are satisfied at $\tilde{v}_1$ which is given by:

$$\tilde{v}_1 = \left(\kappa - \frac{\lambda \tilde{p}}{2 - \eta}\right)\frac{1}{\beta} \tag{B86}$$

The marginal consumer who is willing to purchase under the assumption of full usage is given by $\tilde{v}_2 = \frac{\tilde{p}}{2} + \kappa$. In any optimal plan $\tilde{v}_2 \geq \tilde{v}_1$. This reduces to the condition that:

$$\frac{\tilde{p}}{2} + \kappa > \left(\kappa - \frac{\lambda \tilde{p}}{2-\eta}\right)\frac{1}{\beta} \tag{B87}$$

$$\tilde{p} > \frac{2(1-\beta)\kappa}{\beta + \frac{2\lambda}{2-\eta}} \equiv p_o \tag{B88}$$

Recall that:

$$\hat{p}_{af} = \frac{2(1-\beta)\kappa}{\beta + 2\eta\lambda} \tag{B89}$$

Since $\eta(2-\eta) < 1$ it follows that $p_o < \hat{p}_{af}$. Thus, under the payment plan the firm can offer a lower price and therefore $v^* > \tilde{v}_2$. In other words, the firm can charge a lower price and sell to a larger group of consumers. Thus, social welfare improves. Note however that the firm will not charge a total price which is below p^*_{rf}. Therefore, the optimal prices are:

$$p_1^* = \frac{\max(p^*_{rf}, p_o)}{2-\eta} \tag{B90}$$

$$p_2^* = \frac{\max(p^*_{rf}, p_o)(1-\eta)}{2-\eta} \tag{B91}$$

Note that $p_1^* > p_2^*$ and $p_2^* \to 0$ as $\eta \to 1$. □

Claim 7.5. *If $p^*_{rf} < p_o$ and $p^*_{rp} < \hat{p}_{ap}$ then there exists a η^{**} such that if $\eta \in (\eta^{**}, \eta^*)$ then a payment plan leads to higher profits, higher prices and higher social welfare.*

Proof: Note that if the firm chooses a payment plan then its profits are strictly greater than Π^*_f and are increasing in η. Also, for $\eta < \eta^*$, $\Pi^*_p > \Pi^*_f$ and under the single price plan, the firm chooses $\hat{p}_{ap}$. Furthermore, the firm's profits do not increase under the payment plan if the marginal consumer only partially uses the product. However, using the payment plan, the firm can achieve a profit strictly greater than Π^*_f. Thus, when η is less than but sufficiently close to η^*, i.e., when $\eta \in (\eta^{**}, \eta^*)$ the payment plan would lead to the firm moving to a pricing plan which induces full usage.

Under the payment plan the firm charges $\max(p_o, p^*_{rf})$. First, note that $p_o > \hat{p}_{ap}$. To see this recall that:

$$\hat{p}_{ap} = \frac{\kappa(1-\beta)}{\lambda + \beta} \tag{B92}$$

Note that:

$$2\lambda + 2\beta > \beta + \frac{2\lambda}{2-\eta} \tag{B93}$$

Thus, $\hat{p}_{ap} < p_o$. Thus, the firm charges a total price which is higher than the single price. Since, $\frac{p_o}{2} < \hat{p}_{ap}$, the total demand and usage increases. Consequently, social welfare increases. Now consider the case when $p^*_{rf} > p_o$ and the total payment under the payment plan is therefore p^*_{rf}. From Claim 7.1 we know that $\frac{p^*_{rf}}{2} < p^*_{rp} < \hat{p}_{ap}$. Therefore, the total demand and usage increases in this case too and social welfare increases. □

From Claims 7.4 and 7.5 it follows that regardless of whether the firm serves induces partial or full usage under single price, the use of a payment plan lead to higher profits, sales and welfare. Furthermore, if $\eta \in (\eta^{**}, \eta^*)$ total prices are higher under the payment plan but since optimal single period price induces full usage when $\eta > \eta^*$, from Claim 7.4 it follows that total prices are lower under the payment plan. □

Undifferentiated Competitors

We first show that in equilibrium both firms will charge a price strictly greater than c and make positive profits. To see this, consider the case when firm 1 charges p_1 where $c \leq p_1 < \hat{p}_a$. Firm 2 can either charge $p_2 = p_1$, $p_2 < p_1$ or $p_2 > p_1$. First, note that unless $p_1 = c$, $p_2 = p_1$ is dominated by $p_2 = p_1 - \epsilon$, where ϵ is arbitrarily small positive number. If $p_2 > p_1$ then firm 2's profits are given by $(F(v_1(p_1)) - F(v_3(p_2))(p_2 - c)$ as long as $p_2 \leq \hat{p}_a$. Since this profit function is increasing in p_2 for $p_2 \leq \hat{p}_a$, the firm charges $\hat{p}_a$. Firm 2 cannot do better by charging a price $p_2 > \hat{p}_a$ because only consumers with $v > v_3(\hat{p}_a)$ are willing to purchase. Therefore:

$$\Pi_2^*(p_1) = \max\left((p_1 - \epsilon - c)(1 - F(v_1(p_1)), (F(v_1(p_1)) - F(v_3(\hat{p}_a))(\hat{p}_a - c)\right) \tag{B94}$$

Define:

$$\Omega(p_1) = (p_1 - c)(1 - F(v_1(p_1)) - (F(v_1(p_1)) - F(v_3(\hat{p}_a))(\hat{p}_a - c) \tag{B95}$$

Note that $\Omega(c) < 0$ and $\Omega(\hat{p}_a) > 0$. Also:

$$\Omega'(p_1) = 1 - F(v_1(p_1)) + (p_1 - c)f(v_1(p_1))\frac{\lambda}{\beta} + (\hat{p}_a - c)f(v_1(p_1))\frac{\lambda}{\beta} > 0 \tag{B96}$$

Thus, there exists a unique $p_m \in (c, \hat{p}_a)$ such that firm 2 will charge $\hat{p}_a$ rather than undercut p_m. This implies that no firm will charge a price below p_m. However, since $p_m > c$, both firms will make positive profits. It is clear that a pure strategy equilibrium cannot exist. Using standard arguments, it follows that a mixed strategy equilibrium exists with support $(p_m, \hat{p}_a)$. Furthermore, there can be no holes in the strategy space and also there can be no mass points.

Proof of Proposition 8

We again establish this proposition via a series of claims.

Claim 8.1. *When there are no self-control issues, the equilibrium prices are:*

$$p_o^c = c + \frac{\alpha t}{f(\frac{1}{2})} \tag{B97}$$

Proof: In this case, the marginal consumer is defined by:

$$\theta = \frac{1}{2t}\left(\frac{p_2 - p_1}{\alpha} + t\right) \tag{B98}$$

The profits for firm 1 are given by:

$$\Pi_1 = F(\theta)(p_1 - c) \tag{B99}$$

The first order condition implies that:

$$(p_1 - c)F(\theta)\left(\frac{-1}{2\alpha t}\right) + F(\theta) = 0 \tag{B100}$$

Using symmetry and by noting that $F(\frac{1}{2}) = \frac{1}{2}$, the result follows. □

Claim 8.2. *If* $\lambda \in (\lambda_1^c, \lambda_2^c)$ *then the equilibrium price is* $p_a^c = \frac{\kappa - \beta(u - \frac{t}{2})}{\lambda}$ *where:*

$$\lambda_1^c = \frac{\kappa - \beta(u - \frac{t}{2})}{\alpha(u - \frac{t}{2} - \kappa)} \tag{B101}$$

$$\lambda_2^c = \left\lceil \frac{\kappa - \beta(u - \frac{t}{2})}{c + \frac{\alpha t}{f(\frac{1}{2})}} \right\rceil \tag{B102}$$

Proof: First note that p_o^c is not feasible if the consumers at the middle point do not invest. In other words, if:

$$\beta\left(u - \frac{t}{2}\right) - \kappa + \lambda p_o^c < 0 \tag{B103}$$

This reduce to the condition that:

$$p_o^c < \frac{\kappa - \beta(u - \frac{t}{2})}{\lambda} \tag{B104}$$

Therefore, if $\lambda < \lambda_2^c$ then p_o^c is not an equilibrium where

$$\lambda_2^c = \left\lceil \frac{\kappa - \beta(u - \frac{t}{2})}{c + \frac{\alpha t}{f(\frac{1}{2})}} \right\rceil \tag{B105}$$

For consumers to be willing to buy at p_a^c we need that:

$$\alpha(u - \frac{t}{2} - \kappa) - p_a^c \geq 0 \tag{B106}$$

$$\alpha(u - \frac{t}{2} - \kappa) - \frac{\kappa - \beta(u - \frac{t}{2})}{\lambda} \geq 0 \tag{B107}$$

This reduces to the condition that

$$\lambda \geq \frac{\kappa - \beta(u - \frac{t}{2})}{\alpha(u - \frac{t}{2} - \kappa)} = \lambda_1^c \tag{B108}$$

Note that $\lambda > \lambda_2^c$ implies that $\lambda > \lambda_1^c$. Thus, $\lambda_2^c > \lambda_1^c$. To see this note that $\lambda > \lambda_2^c$ implies $p_o^c > p_a^c$. Since by assumption absent self-control issues consumers in the middle are willing to buy at p_o^c, it follows that for $\lambda > \lambda_1^c$, consumers are also willing to purchase at p_a^c i.e., $\lambda > \lambda_1^c$.

Now we show that if $p = p_a^c$ then there is no over-consumption when costs are high. To see this note that for any consumer to overconsume at p_a^c, we must have:

$$0 < \beta u + \lambda p_a^c - u \tag{B109}$$

$$= -(1-\beta)u + \kappa - \beta(u - \frac{t}{2}) \tag{B110}$$

$$= -\left[u - \kappa - \frac{\beta t}{2}\right] < 0 \tag{B111}$$

To show that if $\lambda \in (\lambda_1^c, \lambda_2^c)$ the equilibrium is (p_a^c, p_a^c) we need to show that no firm would deviate. If a firm deviates to a lower price, it does not sell more and makes lower margin. Therefore, such a deviation is not profitable. If the firm deviates to a higher price, then it loses market share. Since given the equilibrium price, all consumers in $(0, \frac{1}{2})$ only invest under low costs when they buy 1's product, we can consider deviation by firm 1 as $\tilde{p}_1$ to maximize:

$$\widetilde{\Pi}_1 = (\tilde{p}_1 - c)F\left(\tilde{\theta}\right) \tag{B112}$$

where:

$$\tilde{\theta} = \frac{u - \kappa - \frac{\tilde{p}_1}{\alpha}}{t} \tag{B113}$$

Thus:

$$\tilde{p}_1 = \alpha(u - \kappa - t\tilde{\theta}) \tag{B114}$$

The first order condition with respect to $\tilde{\theta}$ is:

$$\frac{\partial \widetilde{\Pi}_1}{\partial \tilde{\theta}} = f(\tilde{\theta})\left[\alpha(u - \kappa - t\tilde{\theta}) - c\right] + F(\tilde{\theta})(-\alpha t) \tag{B115}$$

Since $\tilde{\theta}^* < \frac{1}{2}$ implies that:

$$\begin{aligned}
0 &> f\left(\frac{1}{2}\right)\left[\alpha\left(u - \kappa - \frac{t}{2}\right) - c\right] - \frac{\alpha t}{2} && \text{(B116)}\\
&= f\left(\frac{1}{2}\right)\left[\alpha\left(u - \kappa - \frac{t}{2} - \frac{t}{2f\left(\frac{1}{2}\right)}\right) - c\right] && \text{(B117)}\\
&= f\left(\frac{1}{2}\right)\left[\alpha\left(u - \kappa - \frac{t}{2} + \frac{t}{2f\left(\frac{1}{2}\right)}\right) - p_o^c\right] > 0 && \text{(B118)}
\end{aligned}$$

which is a contradiction. Therefore, the firm will not deviate. Thus, for $\lambda \in (\lambda_1^c, \lambda_2^c)$, the equilibrium is (p_a^c, p_a^c). □

Claim 8.3. *If $\lambda \in (\lambda_2^c, \lambda_4^c)$ then the equilibrium price is p_o^c where:*

$$\lambda_3^c = \frac{(1-\beta)u}{\frac{\alpha t}{f\left(\frac{1}{2}\right)} + c} \tag{B119}$$

$$\lambda_4^c = \frac{\alpha\beta(u-\kappa) + (1-\beta)u}{\frac{\alpha t}{f\left(\frac{1}{2}\right)} + c} - \beta \tag{B120}$$

Proof: If $\lambda > \lambda_2^c$ then we know that $p_o^c > p_a^c$. Therefore, if the firm charges p_o^c then consumers will invest under low costs. This would be an equilibrium as long as no consumers abstain from purchasing. Define θ_o as the marginal consumer who is indifferent between over-consuming and not. This consumer is defined by:

$$\beta(u - t\theta_o) - u + \lambda p = 0 \tag{B121}$$

Therefore:

$$\theta_o = \frac{\lambda p - (1-\beta)u}{\beta t} \tag{B122}$$

If $\theta_o > 0$ then some consumers over-consume. This reduces to the condition that $\lambda > \frac{(1-\beta)u}{p}$. Thus, the relevant condition for over-consumption at p_o^c is:

$$\lambda > \frac{(1-\beta)u}{\frac{\alpha t}{f\left(\frac{1}{2}\right)} + c} = \lambda_3^c \tag{B123}$$

Consumers who overconsume are willing to buy only if $u - t\theta_b - \alpha\kappa - (1-\alpha)u - p \geq 0$. The marginal consumer is therefore:

$$\theta_b = \frac{\alpha(u-\kappa) - p}{t} \tag{B124}$$

Note that if $\theta_o(p_o^c) < \theta_b(p_o^c)$ then consumer who over-consume will continue to buy and therefore the equilibrium would hold. This reduces to the condition:

$$\lambda + \beta \leq \frac{\alpha\beta(u-\kappa) + (1-\beta)u}{p_o^c} \tag{B125}$$

Thus, the equilibrium holds as long as:

$$\lambda \leq \frac{\alpha\beta(u-\kappa) + (1-\beta)u}{\frac{\alpha t}{f(\frac{1}{2})} + c} - \beta \equiv \lambda_4^c \tag{B126}$$

Note that $\lambda_4^c > \lambda_3^c > \lambda_2^c$. To see this note that:

$$\lambda_2^c = \left[\frac{\kappa - \beta(u - \frac{t}{2})}{p_o^c}\right] < \frac{(1-\beta)u}{p_o^c} = \lambda_3^c \tag{B127}$$

Therefore:

$$\lambda_4^c - \lambda_2^c > \frac{\alpha\beta(u-\kappa) + (1-\beta)u}{p_o^c} - \beta - \frac{(1-\beta)u}{p_o^c} \tag{B128}$$

$$= \frac{\beta(\alpha(u-\kappa) - p_o^c)}{p_o^c} > 0 \tag{B129}$$

□

Claim 8.4. *If $\lambda < \lambda_1^c$ then the optimal price is $\hat{p}_a$*

Proof: Recall that the purchasing constraint and the usage constraint bind at $\hat{p}_a$. First, note that at $\theta = \frac{1}{2}$, if the firm charges p_a^c then the consumer will not invest if $\lambda < \lambda_1^c$. We show that $\hat{p}_a = p_a^c$ at λ_1^c. Thus, the usage constraint will not be satisfied at $\theta = \frac{1}{2}$ when $p = \hat{p}_a$ and $\lambda < \lambda_1^c$. To see this, note that at λ_1^c:

$$\lambda_1^c = \frac{\kappa - \beta(u - \frac{t}{2})}{\alpha(u - \frac{t}{2} - \kappa)} \tag{B130}$$

$$p_a^c(\lambda = \lambda_1^c) = \alpha(u - \frac{t}{2} - \kappa) \tag{B131}$$

Rearranging λ_1^c:

$$\kappa - \beta(u - \frac{t}{2}) = \alpha\lambda_1^c\left(u - \frac{t}{2}\right) - \alpha\lambda_1^c\kappa \tag{B132}$$

$$\kappa = (\alpha\lambda_1^c + \beta)\left(u - \frac{t}{2}\right) - \alpha\lambda_1^c\kappa - \beta\kappa + \beta\kappa \tag{B133}$$

$$(1-\beta)\kappa = (\alpha\lambda_1^c + \beta)\left(u - \frac{t}{2}\right) \tag{B134}$$

$$(1-\beta)\alpha\kappa = \alpha(\lambda_1^c + \beta)p_a^c \tag{B135}$$

$$\frac{(1-\beta)\alpha\kappa}{\alpha\lambda_1^c + \beta} = \hat{p}_a = p_a^c \tag{B136}$$

Now we show that for $\lambda < \lambda_1^c$, the usage constraint bind. To see this, assume it is not true and the firm maximizes under the purchase constraint. Let θ be the marginal consumer which is defined by $\theta = \alpha(u - t\theta - \kappa) - p$. Therefore, the firm's first order condition is:

$$\frac{\partial \Pi_1}{\partial \theta} = f(\theta)\left[\alpha(u - \kappa - t\theta) - c\right] + F(\theta)(-\alpha t) \tag{B137}$$

We have:

$$\left.\frac{\partial \Pi}{\partial \theta}\right|_{\theta=\frac{1}{2}} = f\left(\frac{1}{2}\right)\left[\alpha\left(u-\kappa-\frac{t}{2}\right)-c\right]-\frac{\alpha t}{2} \tag{B138}$$

$$= f\left(\frac{1}{2}\right)\left[\alpha\left(u-\kappa-\frac{t}{2}-\frac{t}{2f\left(\frac{1}{2}\right)}\right)-c\right] \tag{B139}$$

$$= f\left(\frac{1}{2}\right)\left[\alpha\left(u-\kappa-\frac{t}{2}+\frac{t}{2f\left(\frac{1}{2}\right)}\right)-p_o^c\right] > 0 \tag{B140}$$

However, we know that the usage constraint is not satisfied at $\theta = \frac{1}{2}$ and therefore the optimal price is $\hat{p}_a$. □

Claim 8.5. *Firm's profits increase in λ for $\lambda \in (0, \lambda_1^c)$ and decrease for $\lambda \in (\lambda_1^c, \lambda_4^c)$.*

Proof: The case when $\lambda < \lambda_1^c$ is the case when the firms are local monopolists and the results there show that profits are increasing when prices are $\hat{p}_a$. When $\lambda \in (\lambda_1^c, \lambda_4^c)$, the price p_a^c is decreasing in λ while the total sales remain the same. Therefore, firm's profits decline with λ. □

Claim 8.6. *Consumer welfare and social welfare weakly increase in λ for $\lambda \in (0, \lambda_3^c)$. Social welfare decreases for $\lambda \in (\lambda_3^c, \lambda_4^c)$.*

Proof: The results from the monopoly case apply when $\lambda < \lambda_1^c$. For $\lambda \in (\lambda_1^c, \lambda_3^c)$, the total sales and usage remains the same and therefore social welfare is not affected by λ. However, consumers pay lower prices and therefore consumer welfare increases. When $\lambda \in (\lambda_3^c, \lambda_4^c)$ some consumers over-consume. Furthermore,

$$\frac{\partial \theta_o^*}{\partial \lambda} = \frac{p_o^c}{\beta t} > 0 \tag{B141}$$

and therefore social welfare declines. □.

Proof of Proposition 9

Now we consider the case when $\lambda > \lambda_3^c$ and therefore over-consumption leads to some consumers abstaining at p_o^c. First, note that if $p > \alpha(u-\kappa)$ then from (B124) it follows that $\theta_b < 0$. However, in this case no consumer will buy and the demand is therefore zero. Thus, if there is sales then $p^* < \alpha(u-\kappa)$. If $p < \alpha(u-\kappa)$ then $\theta_b > 0$. From $\lambda > \lambda_3^c$ it follows that $\theta_o(p_o^c) > \theta_b(p_o^c)$. Since $\theta_o - \theta_b$ is increasing in p, it follows that $\theta_o(p) > \theta_b(p)\ \forall p \geq p_o^c$. The demand in this case is given by $F(\theta_b) + F(\theta_1) - F(\theta_o)$. However, if $p < p_c^c$ then $\theta_b = \theta_o$ and the demand is $F(\theta_1)$ where $\theta_o(p_c^c) = \theta_b(p_c^c)$ and therefore:

$$p_c^c = \frac{\alpha\beta(u-\kappa)+(1-\beta)u}{\lambda+\beta} \tag{B142}$$

Thus, firm 1's demand function is given by:

$$D_1(p_1,p_2) = \begin{cases} F(\theta_1) & \text{if } p \leq p_c^c \\ F(\theta_1) - F(\theta_o) + F(\theta_b) & \text{if } p \in (p_c^c, \alpha(u-\kappa)) \\ 0 & \text{if } p > \alpha(u-\kappa) \end{cases} \tag{B143}$$

where as before:

$$\theta_1 = \frac{1}{2t} \cdot \left(\frac{p_2 - p_1}{\alpha} + t \right) \tag{B144}$$

We will look for symmetric pure strategy equilibrium. We again establish this proposition via a series of claims.

Claim 9.1. *In any symmetric equilibrium* $p^* \geq p_c^c$.

Proof: Suppose not. If firm 2 charges $p < p_c^c$ then the relevant first order condition for firm 1 is given by:

$$\frac{\partial \Pi_1(p_1, p)}{\partial p_1} = \frac{-f(\theta_1)}{2\alpha t}(p_1 - c) + F(\theta_1) \tag{B145}$$

If firm 1 also charges p then we must have:

$$\begin{aligned} 0 &= \frac{-f\left(\frac{1}{2}\right)}{2\alpha t}(p - c) + \frac{1}{2} && \text{(B146)} \\ &= \frac{-(p-c)}{2(p_o^c - c)} + \frac{1}{2} && \text{(B147)} \\ &= \frac{p_o^c - p}{2(p_o^c - c)} > 0 && \text{(B148)} \end{aligned}$$

which is a contradiction. □

Thus, the relevant profit function is given by:

$$\Pi_1(p_1, p_2) = \left[F(\theta_1) - F(\theta_o) + F(\theta_b)\right](p_1 - c) \tag{B149}$$

Note that the function is not differentiable at $p = p_c^c$ but is continuous.

Claim 9.2. *If* $F(\cdot)$ *is uniform then in a symmetric equilibrium, prices are decreasing in* λ.

Proof: There are two possible equilibria $p^* = p_c^c$ or $p^* > p_c^c$. If $p^* = p_c^c$ then prices are clearly decreasing in λ. Let us consider the case when $p^* > p_c^c$. The relevant first order condition is:

$$\frac{\partial \Pi_1(p_1, p_2)}{\partial p_1} = -(p_1 - c)\left[\frac{1}{t} + \frac{1}{2\alpha t} + \frac{\lambda}{\beta t}\right] + \left[F(\theta_1) + F(\theta_b) - F(\theta_o)\right] \tag{B150}$$

Under the symmetric interior equilibrium, we have: $\Omega(p^*) = 0$ where:

$$\Omega(p) = -\left[\frac{\lambda + \beta}{\beta t} + \frac{1}{2\alpha t}\right](p - c) + \left[F(\theta_b) - F(\theta_o) + \frac{1}{2}\right] \tag{B151}$$

We have:

$$\begin{aligned} \Omega'(p) &= -\left[\frac{\lambda + \beta}{\beta t} + \frac{1}{2\alpha t}\right] - \left[\frac{\lambda + \beta}{\beta t}\right] && \text{(B152)} \\ &= -\left[\frac{2(\lambda + \beta)}{\beta t} + \frac{1}{2\alpha t}\right] < 0 && \text{(B153)} \end{aligned}$$

We have:

$$\begin{aligned} \frac{\partial \Omega(p)}{\partial \lambda} &= -\frac{(p - c)}{\beta t} - \frac{p}{\beta t} && \text{(B154)} \\ &= \frac{-(2p - c)}{\beta t} < 0 && \text{(B155)} \end{aligned}$$

Thus, using the implicit function theorem it follows that:

$$\frac{\partial p^*}{\partial \lambda} = \frac{-(2p-c)}{\left[2(\lambda+\beta)+\frac{\beta}{2\alpha}\right]} < 0 \tag{B156}$$

□

Claim 9.3. *For uniform $F(\cdot)$, as λ increases $\theta_o^* - \theta_b^*$ increases i.e., the total demand decreases. Furthermore, profits and social welfare decreases in λ.*

Proof: We have:

$$\begin{aligned}
\frac{\partial(\theta_o^* - \theta_b^*)}{\partial \lambda} &= \frac{p^*}{\beta t} + \left[\frac{\lambda}{\beta t} + \frac{1}{t}\right]\frac{\partial p^*}{\partial \lambda} && \text{(B157)}\\
&= \frac{p^*}{\beta t} + \frac{\lambda+\beta}{\beta t}\cdot\frac{-(2p^*-c)}{\left[2(\lambda+\beta)+\frac{\beta}{2\alpha}\right]} && \text{(B158)}\\
&= \frac{1}{\beta t}\left[p^* - \frac{2p^*-c}{2+\frac{\beta}{2\alpha(\lambda+\beta)}}\right] && \text{(B159)}\\
&> \frac{1}{\beta t}\left[p^* - p^* + \frac{c}{2}\right] = \frac{c}{2\beta t} > 0 && \text{(B160)}
\end{aligned}$$

Also, note that:

$$\frac{\partial \theta_b^*}{\partial \lambda} = -\frac{1}{t}\frac{\partial p^*}{\partial \lambda} > 0 \tag{B161}$$

Thus, as λ increases, more consumers over-consume but the total demand declines. Social welfare therefore declines. Since the prices and total sales are declining in λ profits also decline. □

Appendix C: Analysis when consumers pay in period 0.

Demand function derivation for Monopoly

The analysis is similar to the base case except now the consumer pays the price p in period 0. The consumer will prefer to invest effort κ only if $\beta v + \lambda p - \kappa \geq 0$. This reduces to the condition that:

$$v \geq \frac{\kappa - \lambda p}{\beta} \equiv v_1 \tag{C1}$$

Note that since κ is less than β some consumers will always exert effort when the costs are low. The consumer would exert effort even in the high cost case i.e., when costs are 1 if:

$$v \geq \frac{1 - \lambda p}{\beta} \equiv v_2 \tag{C2}$$

Clearly $v_2 > v_1$. If the consumer only exerts effort when the costs are low then he is willing to buy if:

$$v \geq \frac{p}{\alpha\beta} + \kappa \equiv v_3 \tag{C3}$$

Note that v_3 is higher than in the main model. Thus, the consumer buys and uses only under low cost if $v_2 \geq v \geq \max(v_1, v_3)$. If $p > (1 - \kappa)\alpha\beta$ then $v_3 > 1$. In other words, no one buys the product. Therefore, in any optimal price $p^* < (1 - \kappa)\alpha\beta$.

The consumer who exerts effort regardless of the costs is willing to buy if $\beta(v - \alpha\kappa - (1-\alpha)) - p \geq 0$. Thus, this consumer buys if:

$$v \geq \frac{p}{\beta} + \alpha\kappa + (1 - \alpha) \equiv v_4 \tag{C4}$$

Thus, the consumer buys with over-consumption as long as $v \geq \max(v_2, v_4)$. We can see that compared to the main model, v_4 is higher and therefore over-consumption is less likely. Note that $v_4 > v_3$.

Thus, the demand is given by:

$$\mathrm{D} = \max\left(0, F(v_2) - F(\max(v_1, v_3))\right) + \max\left(0, 1 - F(\max(v_2, v_4))\right) \tag{C5}$$

The first term is the demand from the segment which exerts effort only under low costs and the second term is the demand from the consumers who always exert effort.

We can see that $v_3 > v_1$ if:

$$\frac{p}{\alpha\beta} + \kappa \geq \frac{\kappa - \lambda p}{\beta} \tag{C6}$$

$$\implies p \geq \frac{\kappa(1 - \beta)\alpha}{\lambda\alpha + 1} \equiv \hat{p}_a \tag{C7}$$

Comparing with the main model in the paper, we see that this critical point is lower when consumers pay in period 0. Note that if $v_4 > v_2$ then consumers in the segment (v_2, v_4) do not buy even though consumers with a lower valuation purchase. These consumers abstain to avoid over-consumption. The condition that $v_4 > v_2$ which leads to a “hole” in the demand is equivalent to:

$$\frac{p}{\beta} + \alpha\kappa + (1 - \alpha) \geq \frac{1 - \lambda p}{\beta} \tag{C8}$$

$$\implies p \geq \frac{1 - \beta(\alpha\kappa + 1 - \alpha)}{1 + \lambda} \equiv \hat{p}_b \tag{C9}$$

Claim C.1. *$\hat{p}_b > \hat{p}_a$. Thus, $v_4 > v_2 \implies v_3 > v_1$.*

Proof: Note that $\hat{p}_a$ is increasing in κ and $\hat{p}_b$ is decreasing in κ. Since $\kappa \leq \beta$. We have:

$$\hat{p}_b - \hat{p}_a \geq \frac{1-\beta(\alpha\beta+1-\alpha)}{1+\lambda} - \left(\frac{\beta(1-\beta)\alpha}{\lambda\alpha+1}\right) \tag{C10}$$

$$= \frac{(1-\beta)\left(\alpha\lambda(1-\beta)+1+\alpha^2\beta\lambda\right)}{(\lambda\alpha+1)(\lambda+1)} > 0 \tag{C11}$$

Thus, $\hat{p}_b \geq \hat{p}_a$. □

If $v_2 > v_3 > v_1$ then consumers in the region (v_3, v_2) purchase. The condition for $v_2 > v_3$ is $p < \hat{p}_c$ where:

$$\hat{p}_c = \frac{\alpha(1-\kappa\beta)}{\lambda\alpha+1} \tag{C12}$$

Claim C.2. *If $\lambda > \lambda_1 = \frac{1-\beta}{\alpha\beta(1-\kappa)}$ then $\hat{p}_c$ and $\hat{p}_b$ are less than $(1-\kappa)\alpha\beta$. Furthermore, $\hat{p}_c > \hat{p}_b$. Thus, $v_2 < v_3 \implies v_4 > v_2$.*

Proof: Note that $(1-\kappa)\alpha\beta - \hat{p}_b$ is increasing in λ and so is $(1-\kappa)\alpha\beta - \hat{p}_c$. Solving for the critical λ we find that if $\lambda > \lambda_1 = \frac{1-\beta}{\alpha\beta(1-\kappa)}$ then $(1-\kappa)\alpha\beta > \hat{p}_b$ and $(1-\kappa)\alpha\beta > \hat{p}_c$. Also:

$$\hat{p}_c - \hat{p}_b = \frac{(1-\alpha)(\alpha\beta\lambda(1-\kappa)-(1-\beta))}{(\lambda+1)(\alpha\lambda+1)} \tag{C13}$$

This is positive as long as $\lambda > \lambda_1$ and $\alpha < 1$. □

Claim C.3. *If $\lambda > \lambda_1$ and $p \in (\hat{p}_c, (1-\kappa)\alpha\beta)$ then the demand is $1 - F(v_4)$. For $p > \alpha\beta(1-\kappa)$ the market demand is zero.*

Proof: In this price range $v_3 > v_2$ and $v_4 > v_2$. Therefore, only consumers with $v > v_4$ buy. Note that $v_4 < 1$ for $p < \alpha\beta(1-\kappa)$ when $\lambda > \lambda_1$. □

Claim C.4. *If $\lambda > \lambda_1$ then for $p \in (\hat{p}_b, \hat{p}_c)$ the demand will be $F(v_2) - F(v_3) + 1 - F(v_4)$.*

Proof: It is sufficient to establish that v_3 at $\hat{p}_b$ is strictly less than 1. Since $p < \hat{p}_c$, we know that $v_2 > v_3$ and $p > \hat{p}_b$ implies that $v_4 > v_2$. Note that $v_4 > v_3$. Thus, $v_4 > v_2 > v_3 > 0$. Therefore, it is sufficient to show that $v_4 < 1$. Note that $v_4 < 1$ for $p < \alpha\beta(1-\kappa)$ when $\lambda > \lambda_1$. □

Claim C.5. *If $\lambda > \lambda_1$ and $p \in (\hat{p}_d, \hat{p}_b)$ where $\hat{p}_d = \frac{1-\beta}{\lambda}$ then consumers in $(v_2, 1)$ buy the product and overconsume. Consumers in (v_3, v_2) invest only under low costs and buy the product. The total demand is therefore $1 - F(v_3)$.*

Proof: Now consider the case when $\lambda > \lambda_1$ and $p < \hat{p}_b$. In this case, over-consumption could happen but $v_2 > v_4$ so all consumers who over-consume will buy as long as $v_2 < 1$. We have:

$$1 - v_2 = \frac{-(1-\beta)+\lambda p}{\beta} \tag{C14}$$

This is positive as long as:

$$p \geq \frac{1-\beta}{\lambda} = \hat{p}_d \tag{C15}$$

Note that at $\hat{p}_b$, $v_2 = v_4$ and by Claim C.2, it follows that $1 - v_4(\hat{p}_b) = 1 - v_2(\hat{p}_b) > 0$ when $\lambda > \lambda_1$. Therefore,

$$1 - v_2(\hat{p}_d) = 0 < 1 - v_2(\hat{p}_b) \tag{C16}$$

$$\implies v_2(\hat{p}_d) > v_2(\hat{p}_b) \tag{C17}$$

However, since v_2 is decreasing in price - it follows that $\hat{p}_d < \hat{p}_b$. Thus, for $p \in (\hat{p}_d, \hat{p}_b)$ consumers in $(v_2, 1)$ overconsume and buy the product. Also,note that:

$$\hat{p}_d - \hat{p}_a = \frac{1-\beta}{\lambda} - \frac{\kappa(1-\beta)\alpha}{\lambda\alpha + 1} \tag{C18}$$

$$= \frac{(1-\beta)\left[1 + \alpha\lambda(1-\kappa)\right]}{\lambda(\alpha\lambda + 1)} > 0 \tag{C19}$$

This implies that for $p \in (\hat{p}_d, \hat{p}_b)$ we have $v_3 > v_1$ and therefore all consumers in the range (v_3, v_2) will invest under low costs and buy. □

Claim C.6. *If $\lambda > \lambda_1$ and $p \in (\hat{p}_a, \hat{p}_d)$ then the demand is $1 - F(v_3)$.*

Proof: In this case $v_2 > 1$ and $v_3 > v_1$ and therefore the result follows. □

Claim C.7. *If $\lambda > \lambda_1$ and $p < \hat{p}_a$ then the demand is $1 - F(v_1)$.*

Proof: In this case, $v_1 > v_3$ and $v_2 > 1$ and therefore only consumers in $(v_1, 1)$ buy the product since $v_1 \in (0, 1)$. □

Claim C.8. *If $\lambda < \lambda_1$ and $p \in (\hat{p}_a, (1-\kappa)\alpha\beta)$ then the demand is $1 - F(v_3)$.*

Proof: First note that in this case $v_2 > 1$. To see this note that:

$$1 - v_2 = \frac{-(1-\beta) + \lambda p}{\beta} \tag{C20}$$

$$\leq \frac{-(1-\beta) + \lambda\alpha\beta(1-\kappa)}{\beta} \tag{C21}$$

which is negative if $\lambda < \lambda_1$. Thus, over-consumption can only happen if $\lambda > \lambda_1$. For $p > \hat{p}_a$ we know that $v_3 > v_1$. Furthermore, as long as $p < (1-\kappa)\alpha\beta$, $v_3 < 1$. The result then follows. □

Claim C.9. *If $\lambda < \lambda_1$ and $p < \hat{p}_a$ then the demand is $1 - F(v_1)$.*

Proof: In this case $v_1 > v_3$ and the result immediately follows. □
Summarizing, it follows that if $\lambda > \lambda_1$ then the demand function is:

$$\mathrm{D}(p) = \begin{cases} 1 - F(v_1) & \text{if } p \leq \hat{p}_a \\ 1 - F(v_3) & \text{if } p \in (\hat{p}_a, \hat{p}_b] \\ F(v_2) - F(v_3) + 1 - F(v_4) & \text{if } p \in (\hat{p}_b, \hat{p}_c] \\ 1 - F(v_4) & \text{if } p \in (\hat{p}_c, (1-\kappa)\alpha\beta) \\ 0 & \text{if } p \geq (1-\kappa)\alpha\beta \end{cases} \tag{C22}$$

If $\lambda < \lambda_1$ then the demand function is:

$$\mathrm{D}(p) = \begin{cases} 1 - F(v_1) & \text{if } p \leq \hat{p}_a \\ 1 - F(v_3) & \text{if } p \in (\hat{p}_a, (1-\kappa)\alpha\beta) \\ 0 & \text{if } p \geq (1-\kappa)\alpha\beta \end{cases} \tag{C23}$$

Note that if $\alpha = 1$, $\hat{p}_b = \hat{p}_c$ and $v_3 = v_4$ and therefore the two demand functions become identical at $\alpha = 1$. This is intuitive since in this case, the issue of over-consumption does not exist.

It is useful to compare the base case with the analysis here. It is easy to see that $\hat{p}_a, \hat{p}_b, \hat{p}_c$ are lower when consumers pay in period 0 and $\hat{\lambda}_1$ is higher compared to the base case. Furthermore, the maximum price at which the demand becomes zero is lower. This is intuitive since when consumers pay at period 0, they are more price sensitive. Consequently, there is less over-consumption and also the effort constraint binds for a smaller region. Nevertheless, the basic structure of the demand function remains the same and therefore the key results in the paper still hold.