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E-Companion to When Worse Is Better: Strategic Choice of Vendors with Differentiated Capabilities in a Complex Co-Creation Environment

In this E-Companion, we first provide the table containing the list of key variables in Section EC.1, followed by the analytical proofs of the Lemmas, Propositions, and Corollaries from Sections 4 and 5 based on the main model in Section EC.2. We then proceed to provide in Section EC.3, the detailed analysis of the various model extensions showcased in Section 6.

EC.1. Tables

Table EC.1 List of Key Variables and Parameters

Symbol	Definition
Variables	
X	Level of effort exerted by the client
Y_A	Level of effort exerted by the primary vendor (vendor A)
Y_B	Level of effort exerted by the secondary vendor (vendor B)
t_A	Price per unit of effort exerted by vendor A
t_B	Price per unit of effort exerted by vendor B'
Π	Client's profit
π_A	Vendor A's profit
π_B	Vendor B's profit
Parameters	
μ	value per unit of output of the project
α	Sensitivity of the output to the client's effort (Client's efficiency)
β_A	Sensitivity of the output to vendor A's effort (Vendor A's efficiency)
β_B	Sensitivity of the output to vendor B's effort (Vendor B's efficiency)
$c_{R_{1V}} = c_R$	Cost multiplier for the client's effort in one-vendor co-creation
$c_{A_{1V}} = c_A$	Cost multiplier for vendor A's effort in one-vendor co-creation
$c_{R_{2V}} = \rho_R c_R$	Cost multiplier for the client's effort in two-vendor co-creation
$c_{A_{2V}} = \rho_A c_A$	Cost multiplier for vendor A's effort in two-vendor co-creation
c_B	Cost multiplier for vendor B's effort
ρ_R	Coordination cost multiplier for the client in two-vendor co-creation
ρ_A	Coordination cost multiplier for vendor A in two-vendor co-creation

EC.2. Proofs of Lemmas, Propositions, and Corollaries

In this section, we provide the proofs for the Lemmas, Propositions, and Corollaries presented in the paper in Sections 4 and 5.

EC.2.1. Proof of Lemmas 1, 2, and 3 in the Two-Vendor Scenario

Proof of Lemma 1. We solve the model using backward induction. In Stage 2, in response to the client's price per unit offering t_A to vendor A and t_B to vendor B from Stage 1, the client and the two vendors A and B simultaneously determine their effort levels $X^*(t_A, t_B)$, $Y_A^*(t_A, t_B)$, and $Y_B^*(t_A, t_B)$, which will maximize their respective profits, given by Π_{2V} from (2), and $\pi_{A_{2V}}, \pi_{B_{2V}} > 0$ from (3), respectively. Observe that the profit functions are concave in the respective effort levels.

Applying first-order conditions (FOC) simultaneously on (2) and (3), we obtain the efforts exerted by the client and the vendors in response to (t_A, t_B), given by (EC.1a) and (EC.1b).

$$X^*_{2V}(t_A,t_B) = \left(\left(\frac{\mu}{k}\right)\left(\frac{\alpha}{\rho_R c_R}\right)\left(\frac{t_A}{k\rho_A c_A}\right)^{\frac{\beta_A}{k-1}}\left(\frac{t_B}{kc_B}\right)^{\frac{\beta_B}{k-1}}\right)^{\frac{1}{k-\alpha}}, \tag{EC.1a}$$

$$Y^*_{A_{2V}}(t_A,t_B) = \left(\frac{t_A}{k\rho_A c_A}\right)^{\frac{1}{k-1}}, \text{ and } Y^*_{B_{2V}}(t_B,t_B) = \left(\frac{t_B}{kc_B}\right)^{\frac{1}{k-1}}. \tag{EC.1b}$$

Anticipating these responses in Stage 2, the client determines the prices t^*_A and t^*_B in Stage 1, which maximize its total profit from (2), rewritten as follows:

$$\Pi_{2V}(t_A,t_B) = \mu Q\left(X_{2V}(t_A,t_B), Y_{A_{2V}}(t_A,t_B), Y_{B_{2V}}(t_A,t_B)\right) - \rho_R c_R \left(X_{2V}(t_A,t_B)\right)^k - \sum_{i\in\{A,B\}} t_i Y_{i_{2V}}(t_A,t_B). \tag{EC.2}$$

Substituting (EC.1a) and (EC.1b) in (EC.2), we note that the client's total profit is jointly concave in (t_A, t_B). Simultaneously applying FOC with respect to t_A and t_B, we obtain:

$$t^*_{A_{2V}} = (k\rho_A c_A)^{\frac{1}{k}} (\beta_A \Psi_{2V})^{\frac{k-1}{k}} \text{ and } t^*_{B_{2V}} = (kc_B)^{\frac{1}{k}} (\beta_B \Psi_{2V})^{\frac{k-1}{k}}, \text{ where} \tag{EC.3}$$

$$\Psi_{2V} \stackrel{def}{=} \left(\left(\frac{\mu}{k}\right)^k \left(\frac{\alpha}{\rho_R c_R}\right)^{\alpha} \left(\frac{\beta_A}{k\rho_A c_A}\right)^{\beta_A} \left(\frac{\beta_B}{kc_B}\right)^{\beta_B}\right)^{\frac{1}{k-\alpha-\beta_A-\beta_B}}, \tag{EC.4}$$

which concludes the proof of Lemma 1. □

Proof of Lemma 2. Substituting (EC.3) in (EC.1a) and (EC.1b), we obtain the equilibrium efforts:

$$X^*_{2V} = \left(\frac{\alpha\Psi_{2V}}{\rho_R c_R}\right)^{\frac{1}{k}}, \; Y^*_{A_{2V}} = \left(\frac{\beta_A\Psi_{2V}}{k\rho_A c_A}\right)^{\frac{1}{k}}, \text{ and } Y^*_{B_{2V}} = \left(\frac{\beta_B\Psi_{2V}}{kc_B}\right)^{\frac{1}{k}}, \tag{EC.5}$$

where Ψ_{2V} is given by (EC.4). This concludes the proof of Lemma 2. □

Proof of Lemma 3. First, by substituting X^*_{2V}, $Y^*_{A_{2V}}$, and $Y^*_{B_{2V}}$ from (EC.5) in (1), we obtain the equilibrium output in the two-vendor co-creation environment:

$$Q^*_{2V} = \left(\frac{k}{\mu}\right)\Psi_{2V}, \tag{EC.6}$$

where Ψ_{2V} is given by (EC.4).

Substituting the equilibrium efforts and prices from (EC.5) and (EC.3) in (2) and (3), we obtain the equilibrium profits of the client, vendor A, and vendor B, respectively:

$$\Pi^*_{2V} = (k-\alpha-\beta_A-\beta_B)\,\Psi_{2V}, \tag{EC.7}$$

$$\pi^*_{A_{2V}} = \left(\frac{k-1}{k}\right)\beta_A\Psi_{2V}, \text{ and } \pi^*_{B_{2V}} = \left(\frac{k-1}{k}\right)\beta_B\Psi_{2V}, \tag{EC.8}$$

where Ψ_{2V} is given by (EC.4). This concludes the proof of Lemma 3. □

EC.2.2. Proof of Lemmas 4 and 5 in the One-Vendor Scenario

Proof of Lemma 4. In response to the client's offer t_A from Stage 1, the client and the vendor simultaneously determine their optimal effort levels $X^*(t_A)$ and $Y_A^*(t_A)$, respectively, which maximizes their respective profits Π_{1V} and $\pi_{A_{1V}}$ in Stage 2, given by (4). Observe that the profit functions are concave in the respective effort levels. Applying first-order conditions (FOC) simultaneously on Π_{1V} and $\pi_{A_{1V}}$ from (4), we obtain the efforts exerted by the client and vendor A in response to t_A, given by (EC.9):

$$X_{1V}^*(t_A)=\left(\left(\frac{\mu}{k}\right)\left(\frac{\alpha}{c_R}\right)\left(\frac{t_A}{kc_A}\right)^{\frac{\beta_A}{k-1}}\right)^{\frac{1}{k-\alpha}} \text{ and } Y_{A_{1V}}^*(t_A)=\left(\frac{t_A}{kc_A}\right)^{\frac{1}{k-1}}. \tag{EC.9}$$

Anticipating these responses in Stage 2, the client determines the total profit maximizing price t_A^* in Stage 1. Substituting (EC.9) in Π_{1V} from (4), we note that the client's total profit is concave in t_A. Applying FOC with respect to t_A, we obtain:

$$t_{A_{1V}}^*=(kc_A)^{\frac{1}{k}}\left(\beta_A\Psi_{1V}\right)^{\frac{k-1}{k}}, \text{ where} \tag{EC.10}$$

$$\Psi_{1V}\overset{def}{=}\left(\left(\frac{\mu}{k}\right)^k\left(\frac{\alpha}{c_R}\right)^{\alpha}\left(\frac{\beta_A}{kc_A}\right)^{\beta_A}\right)^{\frac{1}{k-\alpha-\beta_A}}. \tag{EC.11}$$

Substituting (EC.10) in (EC.9), we obtain the equilibrium levels of effort exerted by the client and the primary vendor in the one-vendor co-creation environment, as given by:

$$X_{1V}^*=\left(\frac{\alpha\Psi_{1V}}{c_R}\right)^{\frac{1}{k}} \text{ and } Y_{A_{1V}}^*=\left(\frac{\beta_A\Psi_{1V}}{kc_A}\right)^{\frac{1}{k}}, \text{ respectively,} \tag{EC.12}$$

where Ψ_{1V} is given by (EC.11). This concludes the proof of Lemma 4. □

Proof of Lemma 5. First, by substituting X_{1V}^* and $Y_{A_{1V}}^*$ from (EC.12) in $Q_{1V}(X,Y_A)$ from (4), we obtain the equilibrium output in the two-vendor co-creation environment:

$$Q_{1V}^*=\left(\frac{k}{\mu}\right)\Psi_{1V}, \tag{EC.13}$$

where Ψ_{1V} is given by (EC.11).

Substituting the equilibrium efforts and price from (EC.12) and (EC.10) in Π_{1V} and $\pi_{A_{1V}}$ from (4) we obtain the equilibrium profits of the client and vendor A, respectively:

$$\Pi_{1V}^*=(k-\alpha-\beta_A)\,\Psi_{1V}, \text{ and} \tag{EC.14}$$

$$\pi_{A_{1V}}^*=\left(\frac{k-1}{k}\right)\beta_A\Psi_{1V}, \tag{EC.15}$$

where Ψ_{1V} is given by (EC.11). This concludes the proof of Lemma 5. □

EC.2.3. Proof of Lemmas 6 and 7 in the Comparison of the Two-Vendor and One-Vendor Scenarios

Proof of Lemma 6. From (EC.7) and (EC.14), we obtain the following:

$$\frac{\Pi_{2V}^*}{\Pi_{1V}^*} = \xi\left(\frac{\Psi_{2V}}{\Psi_{1V}}\right), \text{ where} \tag{EC.16}$$

$$\xi = \frac{k-\alpha-\beta_A-\beta_B}{k-\alpha-\beta_A}, \text{ and} \tag{EC.17}$$

Ψ_{2V} and Ψ_{1V} are defined in (EC.4) and (EC.11), respectively, yielding

$$\begin{aligned}\frac{\Psi_{2V}}{\Psi_{1V}} &= \left(\left(\left(\frac{\mu}{k}\right)^k\left(\frac{\alpha}{c_R}\right)^\alpha\left(\frac{\beta_A}{kc_A}\right)^{\beta_A}\right)^{\frac{\beta_B}{k-\alpha-\beta_A}}\left(\frac{1}{\rho_R}\right)^\alpha\left(\frac{1}{\rho_A}\right)^{\beta_A}\left(\frac{\beta_B}{kc_B}\right)^{\beta_B}\right)^{\frac{1}{k-\alpha-\beta_A-\beta_B}} \\ &= \Gamma\mu^{\frac{\xi k\beta_B}{(k-\alpha-\beta_A-\beta_B)^2}},\end{aligned} \tag{EC.18}$$

where

$$\Gamma = \left(\left(\left(\frac{1}{k}\right)^k\left(\frac{\alpha}{c_R}\right)^\alpha\left(\frac{\beta_A}{kc_A}\right)^{\beta_A}\right)^{\frac{\beta_B}{k-\alpha-\beta_A}}\left(\frac{1}{\rho_R}\right)^\alpha\left(\frac{1}{\rho_A}\right)^{\beta_A}\left(\frac{\beta_B}{kc_B}\right)^{\beta_B}\right)^{\frac{1}{k-\alpha-\beta_A-\beta_B}} \tag{EC.19}$$

is independent of μ. Note that

$$\frac{\partial}{\partial\mu}\left(\frac{\Psi_{2V}}{\Psi_{1V}}\right) = \frac{\xi k\beta_B}{\mu(k-\alpha-\beta_A-\beta_B)^2}\left(\frac{\Psi_{2V}}{\Psi_{1V}}\right). \tag{EC.20}$$

We differentiate (EC.16) with respect to μ, and substitute (EC.20) to obtain:

$$\frac{\partial}{\partial\mu}\left(\frac{\Pi_{2V}^*}{\Pi_{1V}^*}\right) = \xi\frac{\partial}{\partial\mu}\left(\frac{\Psi_{2V}}{\Psi_{1V}}\right) = \frac{k\beta_B}{\mu(k-\alpha-\beta_A)^2}\left(\frac{\Psi_{2V}}{\Psi_{1V}}\right). \tag{EC.21}$$

Since $\alpha+\beta_A+\beta_B<k$ and $\alpha,\beta_A,\beta_B>0$, therefore $k-\alpha-\beta_A>k-\alpha-\beta_A-\beta_B>0$, hence $0<\xi<1$. Further, $\mu, c_R, c_A, c_B>0$. Therefore, from (EC.20) and (EC.21) respectively, we can see that

$$\frac{\partial}{\partial\mu}\left(\frac{\Psi_{2V}}{\Psi_{1V}}\right)>0, \text{ and } \frac{\partial}{\partial\mu}\left(\frac{\Pi_{2V}^*}{\Pi_{1V}^*}\right)>0, \tag{EC.22}$$

implying that $\frac{\Pi_{2V}^*}{\Pi_{1V}^*}$ is increasing in μ.

Solving $\frac{\Pi_{2V}^*}{\Pi_{1V}^*}=1$ for μ using (EC.16), we obtain the threshold of value per unit of output for the client's profit, as given by (EC.23):

$$\overline{\mu}_\Pi = \left(\frac{1}{\xi\Gamma}\right)^{\frac{(k-\alpha-\beta_A-\beta_B)^2}{\xi k\beta_B}}, \tag{EC.23}$$

where ξ and Γ are given by (EC.17) and (EC.19), respectively.

Therefore, $\mu>\overline{\mu}_\Pi \iff \Pi_{2V}^*>\Pi_{1V}^*$. This concludes the proof of Lemma 6. □

Proof of Lemma 7. We provide the proof in two parts.

(a) From (EC.6) and (EC.13), we obtain the following:

$$\frac{Q_{2V}^*}{Q_{1V}^*} = \frac{\Psi_{2V}}{\Psi_{1V}}, \tag{EC.24}$$

where Ψ_{2V} and Ψ_{1V} are defined in (EC.4) and (EC.11), respectively. From (EC.22), we know that $\frac{\partial}{\partial\mu}\left(\frac{\Psi_{2V}}{\Psi_{1V}}\right) > 0$, implying that $\frac{Q_{2V}^*}{Q_{1V}^*}$ is increasing in μ.

Using (EC.24) and solving for $\frac{Q_{2V}^*}{Q_{1V}^*} = 1$, we obtain the threshold of value per unit of output for the co-creation output, as given by (EC.25):

$$\overline{\mu}_Q = \Gamma^{-\frac{(k-\alpha-\beta_A-\beta_B)^2}{\xi k \beta_B}}, \tag{EC.25}$$

where ξ and Γ are given by (EC.17) and (EC.19), respectively, such that $\mu > \overline{\mu}_Q \iff Q_{2V}^* > Q_{1V}^*$.

(b) Since $0 < \xi < 1$, dividing (EC.23) by (EC.25), we obtain

$$\frac{\overline{\mu}_\Pi}{\overline{\mu}_Q} = \left(\frac{1}{\xi}\right)^{\frac{(k-\alpha-\beta_A-\beta_B)^2}{\xi k \beta_B}} > 1,$$

hence $\overline{\mu}_Q < \overline{\mu}_\Pi$. This concludes the proof of Lemma 7. □

EC.2.4. Proof of Proposition 1

Proof of Proposition 1. We provide the proof in two steps and omit the complex expressions for expositional clarity.

(a) First, from (EC.16), it can be shown that $\frac{\Pi_{2V}^*}{\Pi_{1V}^*}$ is convex in β_B. We solve $\frac{\Pi_{2V}^*}{\Pi_{1V}^*} = 1$ for β_B to obtain two roots: the first is $\beta_B = 0$ (the secondary vendor is not present), and the second (positive) root is the threshold value of efficiency of the new vendor (i.e., $\overline{\beta}_B$). All else being equal, if the efficiency of the secondary vendor (i.e., β_B) is greater than $\overline{\beta}_B$, then the client's profit in the two-vendor scenario (i.e., Π_{2V}) is greater than the client's profit in the one-vendor scenario (i.e., Π_{1V}). Thus, in this case, the client is better off adding a second vendor to the project, and vice versa.

Now, from Lemma 6, we know that all else being equal, the client will be more willing to add a second vendor as the value per unit of output μ increases. Conversely, as μ increases, the client becomes better off adding a second vendor for a lower β_B at which the client would otherwise have preferred to have a single vendor, if μ were lower. Consequently, $\overline{\beta}_B$ decreases with μ.

(b) Then, by replacing β_B with β_A in (EC.23), we obtain the threshold value of value per unit of output for the secondary vendor's efficiency (i.e., $\overline{\mu}_{\beta_B}$) where the efficiency of two vendors is equal (i.e., $\beta_A = \beta_B$). In other words, $\overline{\mu}_{\beta_B}$ is the solution to $\left.\frac{\Pi_{2V}^*}{\Pi_{1V}^*}\right|_{\beta_B=\beta_A} = 1$, where

$$\left.\frac{\Pi_{2V}^*}{\Pi_{1V}^*}\right|_{\beta_B=\beta_A} = \frac{k-\alpha-2\beta_A}{k-\alpha-\beta_A}\left(\left.\frac{\Psi_{2V}}{\Psi_{1V}}\right|_{\beta_B=\beta_A}\right), \text{ and}$$

$$\left.\frac{\Psi_{2V}}{\Psi_{1V}}\right|_{\beta_B=\beta_A} = \left(\left(\left(\left(\frac{\mu}{k}\right)^k \left(\frac{\alpha}{c_R}\right)^\alpha \left(\frac{\beta_A}{kc_A}\right)^{\beta_A}\right)^{\frac{\beta_A}{k-\alpha-\beta_A}} \left(\frac{1}{\rho_R}\right)^\alpha \left(\frac{1}{\rho_A}\right)^{\beta_A} \left(\frac{\beta_A}{kc_B}\right)^{\beta_A}\right)^{\frac{1}{k-\alpha-2\beta_A}}.$$

We know from part (a) above that as μ increases, $\overline{\beta}_B$ decreases. Therefore, at $\mu=\overline{\mu}_{\beta_B}$, we have $\overline{\beta}_B=\beta_A$, and as $\mu>\overline{\mu}_{\beta_B}$, we have $\overline{\beta}_B<\beta_A$. Since the client is willing to add the secondary vendor whose efficiency is greater than $\overline{\beta}_B$, the client will add a new vendor even if $\overline{\beta}_B<\beta_B<\beta_A$ when $\mu>\overline{\mu}_{\beta_B}$. It can be similarly proven that as $\mu<\overline{\mu}_{\beta_B}$, we have $\overline{\beta}_B>\beta_A$. Since the client only adds the secondary vendor if $\beta_B>\overline{\beta}_B$, the client would not add the secondary vendor if $\beta_A<\beta_B<\overline{\beta}_B$ when $\mu<\overline{\mu}_{\beta_B}$. This concludes the proof of Proposition 1. □

EC.2.5. Proof of Proposition 2, and Corollaries 1 and 2

Proof of Proposition 2. To prove Proposition 2, we demonstrate that $\overline{\mu}_\Pi$ is (concave) increasing in c_A. We take the first and second derivatives of $\overline{\mu}_\Pi$ in (EC.23) with respect to the cost multiplier of vendor A (i.e., c_A) as follows:

$$\begin{aligned}
\frac{\partial\overline{\mu}_\Pi}{\partial c_A} &= \left(\frac{k-\alpha-\beta_A}{k-\alpha-\beta_A-\beta_B}\right)^{\frac{(k-\alpha-\beta_A-\beta_B)(k-\alpha-\beta_A)}{k\beta_B}} \frac{\partial}{\partial c_A}\Gamma^{-\frac{(k-\alpha-\beta_A-\beta_B)(k-\alpha-\beta_A)}{k\beta_B}} \\
&= \left(\frac{\beta_A}{kc_A}\right)\left(\frac{k-\alpha-\beta_A}{\Gamma(k-\alpha-\beta_A-\beta_B)}\right)^{\frac{(k-\alpha-\beta_A-\beta_B)(k-\alpha-\beta_A)}{k\beta_B}} = \left(\frac{\beta_A}{kc_A}\right)\overline{\mu}_\Pi, \text{ and} \\
\frac{\partial^2\overline{\mu}_\Pi}{\partial c_A^2} &= \left(1-\frac{k}{\beta_A}\right)\left(\frac{\beta_A}{kc_A}\right)^2\overline{\mu}_\Pi.
\end{aligned}$$

Since $\alpha+\beta_A+\beta_B<k$ and $\alpha,\beta_A,\beta_B>0$, therefore $k-\alpha-\beta_A>k-\alpha-\beta_A-\beta_B>0$. Further, $\mu,c_R,c_A,c_B>0$. Therefore, $\frac{\partial\overline{\mu}_\Pi}{\partial c_A}>0$, while $\frac{\partial^2\overline{\mu}_\Pi}{\partial c_A^2}<0$, which implies that $\overline{\mu}_\Pi$ is concave increasing with c_A. This concludes the proof of Proposition 2. □

Proof of Corollary 1. We provide the proof in two steps.

(a) First, using (EC.16), we show that $\frac{\Pi^*_{2V}}{\Pi^*_{1V}}$ is decreasing in c_A. We differentiate (EC.16) with respect to c_A to obtain:

$$\begin{aligned}
\frac{\partial}{\partial c_A}\left(\frac{\Pi^*_{2V}}{\Pi^*_{1V}}\right) &= \left(\frac{k-\alpha-\beta_A-\beta_B}{k-\alpha-\beta_A}\right)\frac{\partial}{\partial c_A}\left(\frac{\Psi_{2V}}{\Psi_{1V}}\right) \\
&= \left(\frac{k-\alpha-\beta_A-\beta_B}{k-\alpha-\beta_A}\right)\left(-\frac{\beta_A\beta_B}{c_A(k-\alpha-\beta_A-\beta_B)(k-\alpha-\beta_A)}\right)\left(\frac{\Psi_{2V}}{\Psi_{1V}}\right) \\
&= -\frac{\beta_A\beta_B}{c_A(k-\alpha-\beta_A)^2}\left(\frac{\Psi_{2V}}{\Psi_{1V}}\right),
\end{aligned} \tag{EC.26}$$

which is negative, implying that $\frac{\Pi^*_{2V}}{\Pi^*_{1V}}$ is decreasing in c_A.

We then use (EC.16) to solve $\frac{\Pi^*_{2V}}{\Pi^*_{1V}}=1$ for c_A in order to obtain the threshold of cost per unit effort of existing vendor (i.e., $\overline{c}_A$) given by:

$$\overline{c}_A=\Omega\left(\frac{k-\alpha-\beta_A-\beta_B}{k-\alpha-\beta_A}\right)^{\frac{(k-\alpha-\beta_A)(k-\alpha-\beta_A-\beta_B)}{\beta_A\beta_B}}, \text{ where}$$

$$\Omega = \left(\frac{\mu}{k}\right)^{\frac{k}{\beta_A}} \left(\frac{\alpha}{c_R}\right)^{\frac{\alpha}{\beta_A}} \left(\frac{\beta_A}{k}\right) \left(\left(\frac{1}{\rho_R}\right)^{\alpha} \left(\frac{1}{\rho_A}\right)^{\beta_A} \left(\frac{\beta_B}{kc_B}\right)^{\beta_B}\right)^{\frac{k-\alpha-\beta_A}{\beta_A\beta_B}}, \tag{EC.27}$$

such that all else bbeingequal, $c_A < \overline{c}_A \iff \Pi^*_{2V} > \Pi^*_{1V}$.

(b) Now we show that $\overline{c}_A$ is increasing in μ and β_B, but is decreasing in c_B. First, we take the first and second derivative of $\overline{c}_A$ with respect to μ as follows:

$$\begin{aligned}
\frac{\partial \overline{c}_A}{\partial \mu} =& \frac{1}{\mu}\frac{k}{\beta_A}\Omega \left(\frac{k-\alpha-\beta_A-\beta_B}{k-\alpha-\beta_A}\right)^{\frac{(k-\alpha-\beta_A)(k-\alpha-\beta_A-\beta_B)}{\beta_A\beta_B}}, \\
\frac{\partial \overline{c}_A}{\partial \mu^2} =& \frac{1}{\mu^2}\frac{k}{\beta_A}\left(\frac{k}{\beta_A}-1\right)\Omega \left(\frac{k-\alpha-\beta_A-\beta_B}{k-\alpha-\beta_A}\right)^{\frac{(k-\alpha-\beta_A)(k-\alpha-\beta_A-\beta_B)}{\beta_A\beta_B}},
\end{aligned}$$

where Ω is given by (EC.27) and is positive. Since $\alpha + \beta_A + \beta_B < k$ and $\alpha, \beta_A, \beta_B > 0$, therefore $k - \alpha - \beta_A > k - \alpha - \beta_A - \beta_B > 0$. Further, $\mu, c_R, c_A, c_B > 0$. Therefore, $\frac{\partial \overline{c}_A}{\partial \mu} > 0$ and $\frac{\partial \overline{c}_A}{\partial \mu^2} > 0$, implying that $\overline{c}_A$ is convex increasing in μ.

Similarly, the first derivative of $\overline{c}_A$ with respect to β_B is:

$$\begin{aligned}
\frac{\partial \overline{c}_A}{\partial \beta_B} =& \left(\frac{(k-\alpha-\beta_A-\beta_B)^{(k-\alpha-\beta_A)(k-\alpha-\beta_A-\beta_B)}}{(k-\alpha-\beta_A)^{(k-\alpha-\beta_A)^2-(k-\alpha)\beta_B}}\right)^{\frac{1}{\beta_A\beta_B}} \left(\frac{\alpha}{c_R}\right)^{\frac{\alpha}{\beta_A}} \left(\frac{\mu^k\beta_B^{k-\alpha-3\beta_A}}{k^{2k-\alpha}c_B^{k-\alpha-\beta_A}}\right)^{\frac{1}{\beta_A}} \left(\frac{1}{\rho_A}\right)^{\frac{k-\alpha-\beta_A}{\beta_B}} \\
&\left(\frac{1}{\rho_R}\right)^{\frac{\alpha(k-\alpha-\beta_A)}{\beta_A\beta_B}} \left((k-\alpha-\beta_A)\log\left(\frac{k-\alpha-\beta_A}{k-\alpha-\beta_A-\beta_B}\right) + \log\left(\rho_R^{\alpha}\rho_A^{\beta_A}\right)\right) > 0.
\end{aligned}$$

Thus, $\overline{c}_A$ is increasing in β_B.

Finally, the first derivative of $\overline{c}_A$ with respect to c_B is:

$$\frac{\partial \overline{c}_A}{\partial c_B} = -\frac{1}{c_B}\frac{k-\alpha-\beta_A}{\beta_A}\Omega\left(\frac{k-\alpha-\beta_A-\beta_B}{k-\alpha-\beta_A}\right)^{\frac{(k-\alpha-\beta_A)(k-\alpha-\beta_A-\beta_B)}{\beta_A\beta_B}} < 0.$$

Thus, $\overline{c}_A$ is decreasing in c_B. This concludes the proof of Corollary 1. □

Proof of Corollary 2. First, using (EC.16), we show that $\frac{\Pi^*_{2V}}{\Pi^*_{1V}}$ is decreasing in both ρ_R and ρ_A. We differentiate (EC.16) with respect to ρ_R and ρ_A respectively, to obtain:

$$\begin{aligned}
\frac{\partial}{\partial \rho_R}\left(\frac{\Pi^*_{2V}}{\Pi^*_{1V}}\right) &= \left(\frac{k-\alpha-\beta_A-\beta_B}{k-\alpha-\beta_A}\right)\frac{\partial}{\partial \rho_R}\left(\frac{\Psi_{2V}}{\Psi_{1V}}\right) \\
&= \left(\frac{k-\alpha-\beta_A-\beta_B}{k-\alpha-\beta_A}\right)\left(-\frac{\alpha}{\rho_R(k-\alpha-\beta_A-\beta_B)}\right)\left(\frac{\Psi_{2V}}{\Psi_{1V}}\right) \\
&= -\frac{\alpha}{\rho_R(k-\alpha-\beta_A)}\left(\frac{\Psi_{2V}}{\Psi_{1V}}\right) < 0, && \text{(EC.28a)} \\
\frac{\partial}{\partial \rho_A}\left(\frac{\Pi^*_{2V}}{\Pi^*_{1V}}\right) &= \left(\frac{k-\alpha-\beta_A-\beta_B}{k-\alpha-\beta_A}\right)\frac{\partial}{\partial \rho_A}\left(\frac{\Psi_{2V}}{\Psi_{1V}}\right) \\
&= \left(\frac{k-\alpha-\beta_A-\beta_B}{k-\alpha-\beta_A}\right)\left(-\frac{\beta_A}{\rho_A(k-\alpha-\beta_A-\beta_B)}\right)\left(\frac{\Psi_{2V}}{\Psi_{1V}}\right) \\
&= -\frac{\beta_A}{\rho_A(k-\alpha-\beta_A)}\left(\frac{\Psi_{2V}}{\Psi_{1V}}\right) < 0, && \text{(EC.28b)}
\end{aligned}$$

implying that $\frac{\Pi^*_{2V}}{\Pi^*_{1V}}$ is decreasing in both ρ_R and ρ_A.

From (EC.16), (EC.17), and (EC.18), we obtain:

$$\begin{aligned}
\frac{\Pi^*_{2V}}{\Pi^*_{1V}} &= \xi\Theta\left(\left(\frac{1}{\rho_R}\right)^{\alpha}\left(\frac{1}{\rho_A}\right)^{\beta_A}\right)^{\frac{1}{k-\alpha-\beta_A-\beta_B}}\mu^{\frac{\xi k\beta_B}{(k-\alpha-\beta_A-\beta_B)^2}}, \text{ where}\\
\xi &= \frac{k-\alpha-\beta_A-\beta_B}{k-\alpha-\beta_A}, \text{ and}\\
\Theta &= \left(\left(\left(\frac{1}{k}\right)^{k}\left(\frac{\alpha}{c_R}\right)^{\alpha}\left(\frac{\beta_A}{kc_A}\right)^{\beta_A}\right)^{\frac{\beta_B}{k-\alpha-\beta_A}}\left(\frac{\beta_B}{kc_B}\right)^{\beta_B}\right)^{\frac{1}{k-\alpha-\beta_A-\beta_B}}.
\end{aligned} \tag{EC.29}$$

Since $\frac{\Pi^*_{2V}}{\Pi^*_{1V}}$ is decreasing in both ρ_R and ρ_A, we use (EC.29) to solve $\frac{\Pi^*_{2V}}{\Pi^*_{1V}} = 1$ for both ρ_R and ρ_A to obtain the two thresholds of the coordination cost multipliers, given by:

$$\overline{\rho}_R = (\xi\Theta)^{\frac{k-\alpha-\beta_A-\beta_B}{\alpha}}\left(\frac{1}{\rho_A}\right)^{\frac{\beta_A}{\alpha}}\mu^{\frac{k\beta_B}{\alpha(k-\alpha-\beta_A)}}, \text{ and} \tag{EC.30}$$

$$\overline{\rho}_A = (\xi\Theta)^{\frac{k-\alpha-\beta_A-\beta_B}{\beta_A}}\left(\frac{1}{\rho_R}\right)^{\frac{\alpha}{\beta_A}}\mu^{\frac{k\beta_B}{\beta_A(k-\alpha-\beta_A)}}, \tag{EC.31}$$

such that all else being equal, (a) $\rho_R < \overline{\rho}_R \iff \Pi^*_{2V} > \Pi^*_{1V}$, and (b) $\rho_A < \overline{\rho}_A \iff \Pi^*_{2V} > \Pi^*_{1V}$.

Now, we take differentiate $\overline{\rho}_R$ and $\overline{\rho}_A$ from (EC.30) and (EC.31) respectively, with respect to μ, and observe that:

$$\frac{\partial\overline{\rho}_R}{\partial\mu} = \frac{k\beta_B}{\mu\alpha(k-\alpha-\beta_A)}(\xi\Theta)^{\frac{k-\alpha-\beta_A-\beta_B}{\alpha}}\left(\frac{1}{\rho_A}\right)^{\frac{\beta_A}{\alpha}}\mu^{\frac{k\beta_B}{\alpha(k-\alpha-\beta_A)}} > 0, \text{ and} \tag{EC.32}$$

$$\frac{\partial\overline{\rho}_A}{\partial\mu} = \frac{k\beta_B}{\mu\beta_A(k-\alpha-\beta_A)}(\xi\Theta)^{\frac{k-\alpha-\beta_A-\beta_B}{\beta_A}}\left(\frac{1}{\rho_R}\right)^{\frac{\alpha}{\beta_A}}\mu^{\frac{k\beta_B}{\beta_A(k-\alpha-\beta_A)}} > 0. \tag{EC.33}$$

Thus, $\overline{\rho}_R$ and $\overline{\rho}_A$ increase as μ increases. This concludes the proof of Corollary 2. □

EC.2.6. Proof of Propositions 3 and 4

Proof of Proposition 3. We provide the proof in two parts.

(a) From (EC.8) and (EC.15), we obtain the following:

$$\frac{\pi^*_{A_{2V}}}{\pi^*_{A_{1V}}} = \frac{\Psi_{2V}}{\Psi_{1V}}, \tag{EC.34}$$

where Ψ_{2V} and Ψ_{1V} are defined in (EC.4) and (EC.11), respectively.

From (EC.22), we know that $\frac{\partial}{\partial\mu}\left(\frac{\Psi_{2V}}{\Psi_{1V}}\right) > 0$, implying that $\frac{\pi^*_{A_{2V}}}{\pi^*_{A_{1V}}}$ is increasing in μ. Using (EC.34) and solving for $\frac{\pi^*_{A_{2V}}}{\pi^*_{A_{1V}}} = 1$, we obtain the threshold of value per unit of output for the primary vendor's profit, as given by (EC.35):

$$\overline{\mu}_{\pi_A} = \Gamma^{-\frac{(k-\alpha-\beta_A-\beta_B)(k-\alpha-\beta_A)}{k\beta_B}}, \tag{EC.35}$$

where Γ is given by (EC.19), such that $\mu > \overline{\mu}_{\pi_A} \iff \pi^*_{A_{2V}} > \pi^*_{A_{1V}}$.

(b) Since $\beta_B > 0$, therefore $\frac{k-\alpha-\beta_A}{k-\alpha-\beta_A-\beta_B} > 1$. Thus, dividing (EC.23) by (EC.35), we obtain

$$\frac{\overline{\mu}_\Pi}{\overline{\mu}_{\pi_A}} = \left(\frac{k-\alpha-\beta_A}{k-\alpha-\beta_A-\beta_B}\right)^{\frac{(k-\alpha-\beta_A-\beta_B)(k-\alpha-\beta_A)}{k\beta_B}} > 1,$$

hence $\overline{\mu}_{\pi_A} < \overline{\mu}_\Pi$. This concludes the proof of Proposition 3. □

Proof of Proposition 4. We provide the proof in two parts.

(a) From (EC.5) and (EC.12), we obtain the following:

$$\frac{X^*_{2V}}{X^*_{1V}} = \left(\frac{\Psi_{2V}}{\rho_R \Psi_{1V}}\right)^{\frac{1}{k}}, \text{ and } \frac{Y^*_{A_{2V}}}{Y^*_{A_{1V}}} = \left(\frac{\Psi_{2V}}{\rho_A \Psi_{1V}}\right)^{\frac{1}{k}}, \tag{EC.36}$$

where Ψ_{2V} and Ψ_{1V} are defined in (EC.4) and (EC.11), respectively. From (EC.22), we know that $\frac{\partial}{\partial\mu}\left(\frac{\Psi_{2V}}{\Psi_{1V}}\right) > 0$, implying that $\frac{X^*_{2V}}{X^*_{1V}}$ and $\frac{Y^*_{A_{2V}}}{Y^*_{A_{1V}}}$ are increasing in μ. Using (EC.36) and solving for $\frac{X^*_{2V}}{X^*_{1V}} = 1$ and $\frac{Y^*_{A_{2V}}}{Y^*_{A_{1V}}} = 1$ respectively, we obtain the threshold of the value per unit of output for the client's and the primary vendor's efforts, as given by (EC.37):

$$\overline{\mu}_X = \left(\frac{\rho_R}{\Gamma}\right)^{\frac{(k-\alpha-\beta_A-\beta_B)(k-\alpha-\beta_A)}{k\beta_B}}, \text{ and } \overline{\mu}_{Y_A} = \left(\frac{\rho_A}{\Gamma}\right)^{\frac{(k-\alpha-\beta_A-\beta_B)(k-\alpha-\beta_A)}{k\beta_B}}, \tag{EC.37}$$

where Γ is given by (EC.19), such that $\mu > \overline{\mu}_X \iff X^*_{2V} > X^*_{1V}$, and $\mu > \overline{\mu}_{Y_A} \iff Y^*_{A_{2V}} > Y^*_{A_{1V}}$. (or $\overline{\mu}_{Y_A}$),

(b) Since $\beta_B > 0$, therefore $\frac{k-\alpha-\beta_A}{k-\alpha-\beta_A-\beta_B} > 1$. Further, $\rho_R, \rho_A > 1$. Thus, from (EC.23) and (EC.37), we obtain

$$\frac{\overline{\mu}_\Pi}{\overline{\mu}_X} = \left(\frac{k-\alpha-\beta_A}{\rho_R(k-\alpha-\beta_A-\beta_B)}\right)^{\frac{(k-\alpha-\beta_A-\beta_B)(k-\alpha-\beta_A)}{k\beta_B}} > 1, \text{ and}$$
$$\frac{\overline{\mu}_\Pi}{\overline{\mu}_{Y_A}} = \left(\frac{k-\alpha-\beta_A}{\rho_A(k-\alpha-\beta_A-\beta_B)}\right)^{\frac{(k-\alpha-\beta_A-\beta_B)(k-\alpha-\beta_A)}{k\beta_B}} > 1, \text{ if}$$
$$\rho_R < \overline{\rho} \stackrel{def}{=} \frac{k-\alpha-\beta_A}{k-\alpha-\beta_A-\beta_B} = \frac{1}{\xi}, \text{ and } \rho_A < \overline{\rho}, \text{ respectively.}$$

This concludes the proof of Proposition 4. □

EC.3. Detailed Analysis of Extensions

In this section, we provide a detailed analysis of the five extensions presented in the paper in Section 6, and prove the key results in each of them.

EC.3.1. Analysis of Section 6.1

In this section, we provide a detailed analysis of the extension in Section 6.1: Adding a new vendor to an existing multi-vendor co-creation project. Extending the collaborative output to a co-creation environment with j vendors, we define $Q_{jV} = X^{\alpha} \prod_{i=1}^{j} \left(Y_i^{\beta_i} \right)$, where α and β_i are the efficiencies of the client and vendor i. The cost multipliers are c_R for the client, and c_i for vendor i. Without loss of generality, let the vendors be introduced in the co-creation environment in a sequential manner: vendor 1 is the first to be added, followed by vendor 2, and so on. We continue to restrict our attention to co-creation environments with diseconomies of scale, thus $k - \alpha - \sum_{i=1}^{j} \beta_i > 0$ for all j in the consideration set.

For the sake of analytical tractability, we consider that for every new vendor, the client and each existing vendor i will incur coordination costs given by ρ_R and ρ_i, respectively. Therefore, the client's cost multipliers are c_R when producing alone, $\rho_R c_R$ when co-creating with a single vendor, $\rho_R^2 c_R$ when co-creating with two vendors, and $\rho_R^j c_R$ when co-creating with j-vendors. In a j-vendor co-creation, the cost multiplier is $\rho_1^{j-1} c_1$ for vendor 1, $\rho_i^{j-i} c_i$ for vendor i, $i < j$, and c_j for vendor j.[18]

The model is qualitatively identical to that described in Section 3. Analogous to the analysis in Section 4, using induction, we define the function Ψ_{jV} below for the j-vendor co-creation.

$$\Psi_{jV} \overset{def}{=} \left(\left(\frac{\mu}{k}\right)^k \left(\frac{\alpha}{\rho_R^j c_R}\right)^{\alpha} \left(\frac{\beta_j}{k c_j}\right)^{\beta_j} \prod_{i=1}^{j-1} \left(\frac{\beta_i}{k \rho_i^{j-i} c_i}\right)^{\beta_i} \right)^{\frac{1}{k-\alpha-\sum_{i=1}^{j}\beta_i}}, \tag{EC.38a}$$

$$= \mu^{\frac{k}{k-\alpha-\sum_{i=1}^{j}\beta_i}} \psi_{jV}, \text{ where} \tag{EC.38b}$$

$$\psi_{jV} \overset{def}{=} \left(\left(\frac{1}{k}\right)^k \left(\frac{\alpha}{\rho_R^j c_R}\right)^{\alpha} \left(\frac{\beta_j}{k c_j}\right)^{\beta_j} \prod_{i=1}^{j-1} \left(\frac{\beta_i}{k \rho_i^{j-i} c_i}\right)^{\beta_i} \right)^{\frac{1}{k-\alpha-\sum_{i=1}^{j}\beta_i}}, \tag{EC.38c}$$

that represents the combined effect of the relative contribution from each firm in the j-vendor co-creation process, and the client's profit is $\Pi_{jV}^* = \left(k - \alpha - \sum_{i=1}^{j} \beta_i\right) \Psi_{jV}$.

We utilize (EC.38a) to develop two functions $\Psi_{(n+1)V}$ and Ψ_{nV} below, corresponding to the $n+1$-vendor and the n-vendor scenarios respectively:

$$\Psi_{(n+1)V} \overset{def}{=} \left(\left(\frac{\mu}{k}\right)^k \left(\frac{\alpha}{\rho_R^n c_R}\right)^{\alpha} \left(\frac{\beta_{n+1}}{k c_{n+1}}\right)^{\beta_{n+1}} \prod_{i=1}^{n} \left(\frac{\beta_i}{k \rho_i^{n+1-i} c_i}\right)^{\beta_i} \right)^{\frac{1}{k-\alpha-\sum_{i=1}^{n+1}\beta_i}}, \text{ and} \tag{EC.39a}$$

$$\Psi_{nV} \overset{def}{=} \left(\left(\frac{\mu}{k}\right)^k \left(\frac{\alpha}{\rho_R^{n-1} c_R}\right)^{\alpha} \left(\frac{\beta_n}{k c_n}\right)^{\beta_n} \prod_{i=1}^{n-1} \left(\frac{\beta_i}{k \rho_i^{n-i} c_i}\right)^{\beta_i} \right)^{\frac{1}{k-\alpha-\sum_{i=1}^{n}\beta_i}}. \tag{EC.39b}$$

Let us denote the client's profit as Π_{nV}^* when the client engages with the n vendors, and as $\Pi_{(n+1)V}^*$ when the client includes the new vendor in this project. Following the analysis in Section 4, we compare $\Pi_{(n+1)V}^* = \left(k - \alpha - \sum_{i=1}^{n+1} \beta_i\right) \Psi_{(n+1)V}$, and $\Pi_{nV}^* = \left(k - \alpha - \sum_{i=1}^{n} \beta_i\right) \Psi_{nV}$, in order to generalize the result of Lemma 6 in Lemma EC.1 below.

[18] In the base model described in Section 3, we modeled the coordination cost of the client in the one-vendor scenario as part of the client's cost multiplier. In this extension, we consider the coordination cost at each new vendor inclusion.

LEMMA EC.1. *There exists a threshold $\left(\overline{\mu}_{\Pi_n}\right)$ on the value per unit of output (μ), such that the client benefits from including the new vendor in the co-creation environment consisting of n existing primary vendors, if and only if the value per unit of output received by the client exceeds this threshold (i.e., $\Pi^*_{2V} > \Pi^*_{1V} \iff \mu > \overline{\mu}_{\Pi_n}$).*

Proof of Lemma EC.1. We utilize the above definitions of Π^*_{jV}, Ψ_{jV} and ψ_{jV}. Since

$$\begin{aligned}
\Pi^*_{(n+1)V} &= \left(k-\alpha-\sum_{i=1}^{n+1}\beta_i\right)\Psi_{(n+1)V}, \text{ and}\\
\Pi^*_{nV} &= \left(k-\alpha-\sum_{i=1}^{n}\beta_i\right)\Psi_{nV}, \text{ therefore}\\
\frac{\Pi^*_{(n+1)V}}{\Pi^*_{nV}} &= \left(\frac{k-\alpha-\sum_{i=1}^{n+1}\beta_i}{k-\alpha-\sum_{i=1}^{n}\beta_i}\right)\left(\frac{\Psi_{(n+1)V}}{\Psi_{nV}}\right), \text{ where}\\
\frac{\Psi_{(n+1)V}}{\Psi_{nV}} &= \mu^{\frac{k\beta_{n+1}}{\left(k-\alpha-\sum_{i=1}^{n+1}\beta_i\right)\left(k-\alpha-\sum_{i=1}^{n}\beta_i\right)}}\left(\frac{\psi_{(n+1)V}}{\psi_{nV}}\right).
\end{aligned} \tag{EC.40}$$

Note that $\frac{\psi_{(n+1)V}}{\psi_{nV}}$ is independent of μ. Taking derivative of $\frac{\Psi_{(n+1)V}}{\Psi_{nV}}$ from (EC.40) with respect to μ, we get:

$$\begin{aligned}
\frac{\partial}{\partial\mu}\left(\frac{\Psi_{(n+1)V}}{\Psi_{nV}}\right) &= \frac{k\beta_{n+1}}{\mu\left(k-\alpha-\sum_{i=1}^{n+1}\beta_i\right)\left(k-\alpha-\sum_{i=1}^{n}\beta_i\right)}\mu^{\frac{k\beta_{n+1}}{\left(k-\alpha-\sum_{i=1}^{n+1}\beta_i\right)\left(k-\alpha-\sum_{i=1}^{n}\beta_i\right)}}\left(\frac{\psi_{(n+1)V}}{\psi_{nV}}\right),\\
&> 0.
\end{aligned} \tag{EC.41}$$

Taking derivative of $\frac{\Pi^*_{(n+1)V}}{\Pi^*_{nV}}$ from (EC.40) with respect to μ, and substituting (EC.41), we get:

$$\begin{aligned}
\frac{\partial}{\partial\mu}\left(\frac{\Pi^*_{(n+1)V}}{\Pi^*_{nV}}\right) &= \left(\frac{k-\alpha-\sum_{i=1}^{n+1}\beta_i}{k-\alpha-\sum_{i=1}^{n}\beta_i}\right)\frac{\partial}{\partial\mu}\left(\frac{\Psi_{(n+1)V}}{\Psi_{nV}}\right),\\
&= \frac{k\beta_{n+1}}{\mu\left(k-\alpha-\sum_{i=1}^{n}\beta_i\right)^2}\mu^{\frac{k\beta_{n+1}}{\left(k-\alpha-\sum_{i=1}^{n+1}\beta_i\right)\left(k-\alpha-\sum_{i=1}^{n}\beta_i\right)}}\left(\frac{\psi_{(n+1)V}}{\psi_{nV}}\right) > 0,
\end{aligned} \tag{EC.42}$$

which implies that $\frac{\Pi^*_{(n+1)V}}{\Pi^*_{nV}}$ is increasing in μ.

Solving $\frac{\Pi^*_{(n+1)V}}{\Pi^*_{nV}} = 1$ for μ in (EC.40), we obtain the threshold of value per unit of output for the client's profit, as given by (EC.43):

$$\overline{\mu}_{\Pi_n} = \left(\left(\frac{k-\alpha-\sum_{i=1}^{n}\beta_i}{k-\alpha-\sum_{i=1}^{n+1}\beta_i}\right)\left(\frac{\psi_{nV}}{\psi_{(n+1)V}}\right)\right)^{\frac{\left(k-\alpha-\sum_{i=1}^{n+1}\beta_i\right)\left(k-\alpha-\sum_{i=1}^{n}\beta_i\right)}{k\beta_{n+1}}}, \tag{EC.43}$$

such that $\Pi^*_{2V} > \Pi^*_{1V} \iff \mu > \overline{\mu}_{\Pi_n}$. This concludes the proof of Lemma EC.1. □

Parallels to the results shown in Section 5 can be similarly drawn in the n-vendor general case. Our results therefore hold qualitatively for general n-vendor co-creation environments. We omit the details for the sake of brevity.

EC.3.2. Analysis of Section 6.2

In this section, we provide a detailed analysis of the extension in Section 6.2: Impact of the value per unit of output changing with the number of vendors. The sequence of events is the same as in Section 3, and we follow the same backward induction as in Section 4 to determine the equilibrium in the two-vendor and one-vendor co-creation scenarios. Analogous to the analysis with $\mu_1 = \mu_2 = \mu$ in Section 4, we define two functions $\Psi_{2V}(\mu_2)$ and $\Psi_{1V}(\mu_1)$ below, corresponding to the two-vendor and the one-vendor scenarios respectively, that represent the combined effect of the relative contribution from each firm in the respective co-creation process:

$$\Psi_{2V}(\mu_2) \stackrel{def}{=} \left(\left(\frac{\mu_2}{k}\right)^k \left(\frac{\alpha}{\rho_R c_R}\right)^\alpha \left(\frac{\beta_A}{k\rho_A c_A}\right)^{\beta_A} \left(\frac{\beta_B}{k c_B}\right)^{\beta_B} \right)^{\frac{1}{k-\alpha-\beta_A-\beta_B}}, \text{ and} \tag{EC.44a}$$

$$\Psi_{1V}(\mu_1) \stackrel{def}{=} \left(\left(\frac{\mu_1}{k}\right)^k \left(\frac{\alpha}{c_R}\right)^\alpha \left(\frac{\beta_A}{k c_A}\right)^{\beta_A} \right)^{\frac{1}{k-\alpha-\beta_A}}. \tag{EC.44b}$$

Since $\mu_1, \mu_2 > 0$, observe that the sensitivity of $\Psi_{2V}(\mu_2)$ and $\Psi_{1V}(\mu_1)$, given by (EC.44a) and (EC.44b) respectively, with respect to model parameters such as μ, k, α, β_A, β_B, c_A, etc., remain directionally the same as in Section 4. The equilibrium results in the two-vendor scenario can be found by replacing μ with μ_2 in Lemmas 1, 2, and 3. Similarly, the equilibrium results in the one-vendor scenario can be found by replacing μ with μ_1 in Lemmas 4, and 5. We specifically focus on the client's profits under the two-vendor and one-vendor scenarios, given by: $\Pi^*_{2V}(\mu_2) = (k - \alpha - \beta_A - \beta_B)\,\Psi_{2V}(\mu_2)$, and $\Pi^*_{1V}(\mu_1) = (k - \alpha - \beta_A)\,\Psi_{1V}(\mu_1)$, respectively, in order to reach the following conclusion.

LEMMA EC.2. *When the value per unit of output in the two-vendor co-creation environment (i.e., μ_2) is different than that in the one-vendor co-creation environment (i.e., μ_1),*

(a) there exists a threshold ($\overline{\mu}_2$) on the value per unit of output in the two-vendor co-creation environment (μ_2), such that the client benefits from including the secondary vendor in the co-creation environment if and only if the value per unit of output received by the client exceeds this threshold (i.e., $\Pi^*_{2V}(\mu_2) > \Pi^*_{1V}(\mu_1) \iff \mu_2 > \overline{\mu}_2$), and

(b) compared to the threshold ($\overline{\mu}_\Pi$) when the value per unit of output is identical for both one-vendor and two-vendor scenarios, this threshold ($\overline{\mu}_2$) is higher (conversely, lower) if the value per unit of output is less (conversely, more) under the two-vendor scenario than the one-vendor scenario (i.e., $\overline{\mu}_2 > \overline{\mu}_\Pi \iff \mu_2 < \mu_1$).

Proof of Lemma EC.2. From (EC.16),we obtain the following:

$$\frac{\Pi^*_{2V}(\mu_2)}{\Pi^*_{1V}(\mu_1)} = \xi \left(\frac{\Psi_{2V}(\mu_2)}{\Psi_{1V}(\mu_1)} \right), \text{ where} \tag{EC.45}$$

ξ, $\Psi_{2V}(\mu_2)$, and $\Psi_{1V}(\mu_1)$ are defined in (EC.17), (EC.44a), and (EC.44b), respectively, yielding

$$\frac{\Psi_{2V}(\mu_2)}{\Psi_{1V}(\mu_1)} = \eta^{\frac{k}{k-\alpha-\beta_A-\beta_B}} \Gamma \mu_1^{\frac{\xi k \beta_B}{(k-\alpha-\beta_A-\beta_B)^2}}, \tag{EC.46}$$

where $\Gamma > 0$ defined in (EC.19) is independent of μ, and $\eta = \mu_2/\mu_1 > 0$. Note that

$$\frac{\partial}{\partial \mu_1}\left(\frac{\Psi_{2V}(\mu_2)}{\Psi_{1V}(\mu_1)}\right) = \eta^{\frac{k}{k-\alpha-\beta_A-\beta_B}} \frac{\xi k \beta_B}{\mu_1(k-\alpha-\beta_A-\beta_B)^2}\left(\frac{\Psi_{2V}(\mu_2)}{\Psi_{1V}(\mu_1)}\right), \tag{EC.47}$$

which implies that:

$$\frac{\partial}{\partial \mu_1}\left(\frac{\Pi^*_{2V}(\mu_2)}{\Pi^*_{1V}(\mu_1)}\right) = \xi \frac{\partial}{\partial \mu_1}\left(\frac{\Psi_{2V}(\mu_2)}{\Psi_{1V}(\mu_1)}\right) = \eta^{\frac{k}{k-\alpha-\beta_A-\beta_B}} \frac{k \beta_B}{\mu_1(k-\alpha-\beta_A)^2}\left(\frac{\Psi_{2V}(\mu_2)}{\Psi_{1V}(\mu_1)}\right). \tag{EC.48}$$

Since $\alpha + \beta_A + \beta_B < k$ and $\alpha, \beta_A, \beta_B > 0$, therefore $k - \alpha - \beta_A > k - \alpha - \beta_A - \beta_B > 0$, hence $0 < \xi < 1$. Further, μ, c_R, c_A, c_B and $\eta > 0$. Therefore, from (EC.47) and (EC.48) respectively, we can see that $\frac{\partial}{\partial \mu_1}\left(\frac{\Psi_{2V}\mu_2}{\Psi_{1V}(\mu_1)}\right)$ and $\frac{\partial}{\partial \mu}\left(\frac{\Pi^*_{2V}(\mu_2)}{\Pi^*_{1V}(\mu_1)}\right)$ are both positive, implying that $\Pi^*_{2V}(\mu_2)/\Pi^*_{1V}(\mu_1)$ is increasing in μ. Using (EC.45) and solving $\Pi^*_{2V}(\mu_2)/\Pi^*_{1V}(\mu_1) = 1$ for μ_1, we obtain the threshold of value per unit of output for client's profit, as given by (EC.49a):

$$\overline{\mu}_1 = \left(\frac{1}{\eta}\left(\frac{1}{\xi\Gamma}\right)^{\frac{k-\alpha-\beta_A-\beta_B}{k}}\right)^{\frac{k-\alpha-\beta_A}{\beta_B}} = \eta^{-\frac{k-\alpha-\beta_A}{\beta_B}} \overline{\mu}_\Pi, \tag{EC.49a}$$

$$\overline{\mu}_2 = \eta\overline{\mu}_1 = \eta^{1-\frac{k-\alpha-\beta_A}{\beta_B}}\left(\frac{1}{\xi\Gamma}\right)^{\frac{(k-\alpha-\beta_A-\beta_B)(k-\alpha-\beta_A)}{k\beta_B}} = \eta^{1-\frac{k-\alpha-\beta_A}{\beta_B}} \overline{\mu}_\Pi, \tag{EC.49b}$$

where ξ, Γ, and $\overline{\mu}_\Pi$ are given by (EC.17), (EC.19), and (EC.23), respectively. Hence, $\mu_1 > \overline{\mu}_1 \iff \mu_2 > \overline{\mu}_2 = \eta\overline{\mu}_1 \iff \Pi^*_{2V}(\mu_2) > \Pi^*_{1V}(\mu_1)$. Since $k - \alpha - \beta_A - \beta_B > 0$, $1 - \frac{k-\alpha-\beta_A}{\beta_B} < 0$, therefore from (EC.49b), $\overline{\mu}_2 > \overline{\mu}_\Pi \iff \eta < 1$. This concludes the proof of Lemma EC.2. □

In other words, when the client obtains an identical value per unit of output μ from both one-vendor and two-vendor co-creation scenarios, the condition $\mu > \overline{\mu}_\Pi$ is sufficient for the client to include the secondary vendor. However, if the client obtains different values per unit output, then the bar to include the secondary vendor shifts. For instance, if $\mu_2 < \mu_1$, then for the secondary vendor to be included, the client needs $\mu_2 > \overline{\mu}_2 > \overline{\mu}_\Pi$, and vice-versa (see Lemma EC.2 in Section EC.3.2). That is, if the inclusion of the secondary vendor results in a decrease (conversely, increase) in the value per unit of output for the client μ, and the client would have included (not included) the secondary vendor had the value remained the same, the client may still include (not include) the secondary vendor if the change in μ is not significant.

In a similar fashion, we establish that the results outlined in Section 5 continue to hold qualitatively even when the value per unit of output is different under different number of vendors. We omit the details for the sake of brevity.

EC.3.3. Analysis of Section 6.3

In this section, we provide a detailed analysis of the extension in Section 6.3: Impact of the value per unit of output decreasing with output. The sequence of events is the same as in Section 3, and we follow the same backward induction as in Section 4 to determine the equilibrium in the two-vendor and one-vendor co-creation scenarios. Analogous to the analysis with constant μ in Section 4, we define two functions $\Psi_{2V}(b)$ and $\Psi_{1V}(b)$ below, corresponding to the two-vendor and one-vendor scenarios respectively, that represent the combined effect of the relative contribution from each firm in the respective co-creation process:

$$\Psi_{2V}(b) \stackrel{def}{=} \left(\left(\frac{\mu(1-b)}{k}\right)^{k/(1-b)}\left(\frac{\alpha}{\rho_R c_R}\right)^{\alpha}\left(\frac{\beta_A}{k\rho_A c_A}\right)^{\beta_A}\left(\frac{\beta_B}{kc_B}\right)^{\beta_B}\right)^{\frac{1-b}{k-(1-b)(\alpha+\beta_A+\beta_B)}}, \text{ and} \quad \text{(EC.50a)}$$

$$\Psi_{1V}(b) \stackrel{def}{=} \left(\left(\frac{\mu(1-b)}{k}\right)^{k/(1-b)}\left(\frac{\alpha}{c_R}\right)^{\alpha}\left(\frac{\beta_A}{kc_A}\right)^{\beta_A}\right)^{\frac{1-b}{k-(1-b)(\alpha+\beta_A)}}. \quad \text{(EC.50b)}$$

Since $0 \le b < 1$, observe that the sensitivity of $\Psi_{2V}(b)$ and $\Psi_{1V}(b)$, given by (EC.50a) and (EC.50b) respectively, with respect to model parameters such as μ, k, α, β_A, β_B, c_A, etc., remain directionally the same as in Section 4. We derive the equilibrium results in the two-vendor scenario under decreasing output-dependent value per unit of output, along the same lines as Lemmas 1, 2, and 3, and present them in Lemma EC.3 below.

Lemma EC.3. *When the client co-creates with two vendors and the value per unit of output is decreasing in the output of the co-creation process, then in equilibrium,*

(a) the price per unit of effort offered by the client to vendors A and B are: $t^*_{A_{2V}}(b) = (k\rho_A c_A)^{\frac{1}{k}}(\beta_A\Psi_{2V}(b))^{\frac{k-1}{k}}$, *and* $t^*_{B_{2V}}(b) = (kc_B)^{\frac{1}{k}}(\beta_B\Psi_{2V}(b))^{\frac{k-1}{k}}$, *respectively;*

(b) the efforts exerted by the client, vendor A, and vendor B are: $X^*_{2V}(b) = \left(\frac{\alpha\Psi_{2V}(b)}{\rho_R c_R}\right)^{\frac{1}{k}}$, $Y^*_{A_{2V}}(b) = \left(\frac{\beta_A\Psi_{2V}(b)}{k\rho_A c_A}\right)^{\frac{1}{k}}$, *and* $Y^*_{B_{2V}}(b) = \left(\frac{\beta_B\Psi_{2V}(b)}{kc_B}\right)^{\frac{1}{k}}$, *respectively;*

(c) the output from the co-creation environment is: $Q^*_{2V}(b) = \left(\frac{k}{\mu(1-b)}\Psi_{2V}(b)\right)^{\frac{1}{1-b}}$;

(d) the profit earned by the client is $\Pi^*_{2V}(b) = \frac{k-(1-b)(\alpha+\beta_A+\beta_B)}{1-b}\Psi_{2V}(b)$; *and*

(e) the profits earned by vendor A and vendor B are: $\pi^*_{A_{2V}}(b) = \left(\frac{k-1}{k}\right)\beta_A\Psi_{2V}(b)$, *and* $\pi^*_{B_{2V}}(b) = \left(\frac{k-1}{k}\right)\beta_B\Psi_{2V}(b)$, *respectively; where* $\Psi_{2V}(b)$ *is defined in (EC.50a).*

The proof of Lemma EC.3 follows the same backward induction as shown in the proofs of Lemmas 1, 2, and 3. We omit the details for the sake of brevity, and now derive the equilibrium results in the one-vendor scenario under decreasing output-dependent value per unit of output, similar to Lemmas 4, and 5, and present them in Lemma EC.4 below.

Lemma EC.4. *When the client co-creates solely with the primary vendor (vendor A) and the value per unit of output is decreasing in the output of the co-creation process, then in equilibrium,* *(a) the price per unit of effort offered by the client to vendor A is:* $t^*_{A_{1V}}(b) = (kc_A)^{\frac{1}{k}} (\beta_A \Psi_{1V}(b))^{\frac{k-1}{k}}$;

(b) the efforts exerted by the client and vendor A are: $X^*_{1V}(b) = \left(\frac{\alpha \Psi_{1V}(b)}{c_R}\right)^{\frac{1}{k}}$, *and* $Y^*_{A_{1V}}(b) = \left(\frac{\beta_A \Psi_{1V}(b)}{kc_A}\right)^{\frac{1}{k}}$, *respectively;*

(c) the output from the co-creation environment is: $Q^*_{1V}(b) = \left(\frac{k}{\mu(1-b)} \Psi_{1V}(b)\right)^{\frac{1}{1-b}}$

(d) the profits earned by the client and vendor A are: $\Pi^*_{1V}(b) = \frac{k-(1-b)(\alpha+\beta_A)}{1-b} \Psi_{1V}(b)$, *and* $\pi^*_{A_{1V}}(b) = \left(\frac{k-1}{k}\right) \beta_A \Psi_{1V}(b)$, *respectively; where* $\Psi_{1V}(b)$ *is defined in (EC.50b).*

The proof of Lemma EC.4 follows the same arguments as the proofs of Lemmas 4, and 5. We omit the details and note that since $\Psi_{2V}(b)$ and $\Psi_{1V}(b)$ retain the qualitative properties of Ψ_{2V} and Ψ_{1V} as long as $b < 1$, the results outlined in Lemmas EC.3 and EC.4 are qualitatively the same as those shown in Section 4. We now utilize the Lemmas EC.3 and EC.4 to draw parallels to the key result that is highlighted in Lemma 6. It can be observed that $\Pi^*_{2V}(b)/\Pi^*_{1V}(b)$ is increasing in μ if and only if $0 \le b < 1$, which assists us to restate Lemma 6 to include the effect of a decreasing output-dependent value function.

LEMMA EC.5. *When the value per unit of output decreases with the output of the co-creation process, there exists a threshold ($\overline{\mu}_\Pi(b)$) on the value per unit of output (μ) if and only if the total value obtained by the client is increasing with the output (i.e., $0 \le b < 1$), such that the client benefits from including the secondary vendor in the co-creation environment if and only if the value per unit of output received by the client exceeds this threshold, that is,* $\Pi^*_{2V}(b) > \Pi^*_{1V}(b) \iff \mu > \overline{\mu}_\Pi(b)$.

The proof of Lemma EC.5 follows the same methodology as that of Lemmas 6, 7, EC.1, etc. We observe that as long as the total value obtained by the client is increasing in the output produced by the co-creation process, the client may benefit from expanding the output by including the secondary vendor even when the value per unit of output is not constant and may decrease with the output produced, as long as the constant term in the value function μ is sufficiently high (see Lemma EC.5 in Section EC.3.3). In a similar fashion, we establish that the results outlined in Section 5 hold even when the value per unit of output is not a constant, as long as the total value obtained by the client is increasing in the output produced by the co-creation process. We omit the details for the sake of brevity.

EC.3.4. Analysis of Section 6.4

In this section, we provide a detailed analysis of the extension in Section 6.4: Impact of changes to client's and primary vendor's efficiencies. Recall from Section 3 that the efficiencies of the client and the primary vendor are given by α and β_A, respectively. We allow for the possibility that the introduction of the secondary vendor can either decrease or increase the effective efficiencies of the existing firms in a co-creation project. For example, on the one hand, the primary vendor's efficiency could go up due to the ability to focus on its core competence. On the other hand, the changes in inter-firm dynamics may cause additional friction, resulting in a decrease in the primary vendor's efficiency.

To address such changing dynamics, in this extension of our model, we define the efficiencies of the client and the primary vendor in the two-vendor co-creation as $p\alpha$ and $m\beta_A$, respectively, where $p, m > 0$, and those in the one-vendor co-creation continues to be α and β_A. Let us denote the client's profit as $\Pi^*_{2V_{pm}}$ when the client adds a secondary vendor and the addition of this secondary vendor has an impact on the client's and the primary vendor's efficiency. Similar to the analysis in Lemma 6, we analyze the ratio $\frac{\Pi^*_{2V_{pm}}}{\Pi^*_{1V}}$ to reach the following conclusion.

LEMMA EC.6. *There exists a threshold $\overline{\mu}_{pm}$ on the value per unit of output μ if $m > \underline{m}(p)$, where*

$$\underline{m}(p) = \begin{cases} \frac{\alpha - p\alpha + \beta_A - \beta_B}{\beta_A} & \text{if } 0 < p \leq \frac{\alpha + \beta_A - \beta_B}{\alpha}, \\ 0 & \text{if } p > \frac{\alpha + \beta_A - \beta_B}{\alpha}, \end{cases}$$

such that the client benefits from including the secondary vendor (i.e., $\Pi^*_{2V_{pm}} > \Pi^*_{1V}$) only if the value per unit of output exceeds this threshold (i.e., $\mu > \overline{\mu}_{pm}$).

Proof of Lemma EC.6. Extending the result in Lemma 3, we determine the two-vendor profit in equilibrium to be

$$\Pi^*_{2V_{pm}} = (k - p\alpha - m\beta_A - \beta_B)\,\Psi_{2V_{pm}}, \text{ where} \tag{EC.51}$$

$$\Psi_{2V_{pm}} \stackrel{def}{=} \left(\left(\frac{\mu}{k}\right)^k \left(\frac{p\alpha}{\rho_R c_R}\right)^{p\alpha} \left(\frac{m\beta_A}{k\rho_A c_A}\right)^{m\beta_A} \left(\frac{\beta_B}{k c_B}\right)^{\beta_B} \right)^{\frac{1}{k - p\alpha - m\beta_A - \beta_B}}. \tag{EC.52}$$

Recall that Π^*_{1V} is given by (EC.14). Dividing (EC.51) by (EC.14), we obtain

$$\frac{\Pi^*_{2V_{pm}}}{\Pi^*_{1V}} = \xi_{pm} \left(\frac{\Psi_{2V_{pm}}}{\Psi_{1V}}\right), \text{ where} \tag{EC.53}$$

$$\xi_{pm} = \frac{k - p\alpha - m\beta_A - \beta_B}{k - \alpha - \beta_A}, \text{ and} \tag{EC.54}$$

$\Psi_{2V_{pm}}$ and Ψ_{1V} are defined in (EC.52) and (EC.11), respectively, yielding

$$\frac{\Psi_{2V_{pm}}}{\Psi_{1V}} = (\Gamma_{pm})\,\mu^{\frac{k(\beta_B - (1-p)\alpha - (1-m)\beta_A)}{(k - p\alpha - m\beta_A - \beta_B)(k - \alpha - \beta_A)}}, \tag{EC.55}$$

where

$$\Gamma_{pm} = \frac{\left(\left(\frac{1}{k}\right)^k \left(\frac{p\alpha}{\rho_R c_R}\right)^{p\alpha} \left(\frac{m\beta_A}{k\rho_A c_A}\right)^{m\beta_A} \left(\frac{\beta_B}{k c_B}\right)^{\beta_B} \right)^{\frac{1}{k - p\alpha - m\beta_A - \beta_B}}}{\left(\left(\frac{1}{k}\right)^k \left(\frac{\alpha}{c_R}\right)^{\alpha} \left(\frac{\beta_A}{k c_A}\right)^{\beta_A} \right)^{\frac{1}{k - \alpha - \beta_A}}} \tag{EC.56}$$

is independent of μ, and $\frac{\Psi_{2V_{pm}}}{\Psi_{1V}} > 0$. Taking derivative of $\frac{\Psi_{2V_{pm}}}{\Psi_{1V}}$ from (EC.55) with respect to μ, we obtain

$$\frac{\partial}{\partial \mu}\left(\frac{\Psi_{2V_{pm}}}{\Psi_{1V}}\right) = \frac{k\left(\beta_B - (1-p)\alpha - (1-m)\beta_A\right)}{\mu\left(k - p\alpha - m\beta_A - \beta_B\right)\left(k - \alpha - \beta_A\right)}\left(\frac{\Psi_{2V_{pm}}}{\Psi_{1V}}\right). \tag{EC.57}$$

Taking derivative of $\frac{\Pi^*_{2V_{pm}}}{\Pi^*_{1V}}$ from (EC.53) with respect to μ, and substituting (EC.57), we obtain

$$\begin{aligned}\frac{\partial}{\partial \mu}\left(\frac{\Pi^*_{2V_{pm}}}{\Pi^*_{1V}}\right) &= \xi_{pm}\frac{\partial}{\partial \mu}\left(\frac{\Psi_{2V_{pm}}}{\Psi_{1V}}\right) \\ &= \frac{k\left(\beta_B - (1-p)\alpha - (1-m)\beta_A\right)}{\mu\left(k - \alpha - \beta_A\right)^2}\left(\frac{\Psi_{2V_{pm}}}{\Psi_{1V}}\right).\end{aligned} \tag{EC.58}$$

For $\frac{\Pi^*_{2V_{pm}}}{\Pi^*_{1V}}$ to be increasing in μ, we need $\beta_B - (1-p)\alpha - (1-m)\beta_A > 0$, which holds true if $m > \underline{m}(p)$, where

$$\underline{m}(p) = \begin{cases} \frac{\alpha - p\alpha + \beta_A - \beta_B}{\beta_A} & \text{if } 0 < p \le \frac{\alpha + \beta_A - \beta_B}{\alpha}, \\ 0 & \text{if } p > \frac{\alpha + \beta_A - \beta_B}{\alpha}. \end{cases}$$

When $m > \underline{m}(p)$, $\frac{\Pi^*_{2V_{pm}}}{\Pi^*_{1V}}$ is increasing in μ. Solving $\frac{\Pi^*_{2V_{pm}}}{\Pi^*_{1V}} = 1$ from (EC.53) for μ, we obtain the threshold

$$\overline{\mu}_{pm} = \left(\frac{1}{\xi_{pm}\Gamma_{pm}}\right)^{\frac{(k - p\alpha - m\beta_A - \beta_B)(k - \alpha - \beta_A)}{k(\beta_B - (1-p)\alpha - (1-m)\beta_A)}}, \tag{EC.59}$$

such that the client benefits from including the secondary vendor (i.e., $\Pi^*_{2V_{pm}} > \Pi^*_{1V}$) only if the value per unit of output exceeds this threshold (i.e., $\mu > \overline{\mu}_{pm}$). This concludes the proof of Lemma EC.6. □

We make two observations. First, the client's decision to add a secondary vendor is affected by the impact on the efficiencies of the client and the primary vendor. While the client still benefits from including the vendor only if $\mu > \overline{\mu}_{pm}$ (see Lemma EC.6 in Section EC.3.4), the change in the efficiencies of the client (α) and the primary vendor (β_A) dictate whether a high μ alone is sufficient for the client to make that decision. Secondly, the condition $\mu > \overline{\mu}_{pm}$ is both necessary and sufficient for the client to add the secondary vendor for a large set of values of p and m. When α decreases significantly, the client may still benefit from involving the secondary vendor as long as β_A is impacted less negatively.

EC.3.5. Analysis of Section 6.5

In this section, we provide a detailed analysis of the extension in Section 6.5: Output-based payment contract. In the two-vendor co-creation environment, the client offers payments per unit output $t_A > 0$ and $t_B > 0$ to vendors A and B, respectively, such that $t_A + t_B < \mu$. In response to these payment terms, the two vendors exert effort levels Y_A and Y_B, while the client exerts effort

X to jointly produce the output $Q_{2V} = X^{\alpha} Y_A^{\beta_A} Y_B^{\beta_B}$ given by (1), which yields a total value of μQ_{2V}. The profits of the client and the two vendors are then given by:

$$\Pi_{2V} = (\mu - t_A - t_B)\left(X^{\alpha} Y_A^{\beta_A} Y_B^{\beta_B}\right) - \rho_R c_R X^k, \tag{EC.60a}$$

$$\pi_{A_{2V}} = t_A\left(X^{\alpha} Y_A^{\beta_A} Y_B^{\beta_B}\right) - \rho_A c_A Y_A^k, \text{ and } \pi_{B_{2V}} = t_B\left(X^{\alpha} Y_A^{\beta_A} Y_B^{\beta_B}\right) - c_B Y_B^k. \tag{EC.60b}$$

Similarly, the profit functions of the client and vendor A in the single-vendor co-creation environment are, respectively:

$$\Pi_{1V} = (\mu - t_A)\left(X^{\alpha} Y_A^{\beta_A}\right) - c_R X^k, \text{ and } \pi_{A_{1V}} = t_A\left(X^{\alpha} Y_A^{\beta_A}\right) - c_A Y_A^k. \tag{EC.61}$$

The sequence of events is the same as in Section 3, and we follow the same backward induction as in Section 4 to determine the equilibrium in the two-vendor and one-vendor co-creation environments. In Stage 2 in the two-vendor scenario, in response to the client's payment per unit output offering t_A to vendor A and t_B to vendor B from Stage 1, the client and the two vendors A and B simultaneously determine their effort levels $X^*(t_A, t_B)$, $Y_A^*(t_A, t_B)$, and $Y_B^*(t_A, t_B)$, which will maximize their respective profits, given by Π_{2V} from (EC.60a), and $\pi_{A_{2V}}, \pi_{B_{2V}} > 0$ from (EC.60b), respectively. These effort levels are given by:

$$X_{2V}^*(t_A, t_B) = \left(\frac{\alpha(\mu - t_A - t_B)}{\rho_R c_R} \zeta_{2V}^{OB}(t_A, t_B)\right)^{\frac{1}{k}}, \tag{EC.62a}$$

$$Y_{A_{2V}}^*(t_A, t_B) = \left(\frac{t_A \beta_A}{\rho_A c_A} \zeta_{2V}^{OB}(t_A, t_B)\right)^{\frac{1}{k}}, \text{ and} \tag{EC.62b}$$

$$Y_{B_{2V}}^*(t_B, t_B) = \left(\frac{t_B \beta_B}{c_B} \zeta_{2V}^{OB}(t_A, t_B)\right)^{\frac{1}{k}}, \text{ respectively, where} \tag{EC.62c}$$

$$\zeta_{2V}^{OB}(t_A, t_B) = \left(\left(\frac{1}{k}\right)^k \left(\frac{\alpha(\mu - t_A - t_B)}{\rho_R c_R}\right)^{\alpha} \left(\frac{t_A \beta_A}{\rho_A c_A}\right)^{\beta_A} \left(\frac{t_B \beta_B}{c_B}\right)^{\beta_B}\right)^{\frac{1}{k - \alpha - \beta_A - \beta_B}}, \tag{EC.62d}$$

anticipating which, the client chooses t_A and t_B in Stage 1 to maximize:

$$\pi_{2V}^*(t_A, t_B) = (k - \alpha)(\mu - t_A - t_B)\zeta_{2V}^{OB}(t_A, t_B). \tag{EC.63}$$

Similarly, in the one-vendor scenario, in response to the client's payment per unit output offering t_A to vendor A from Stage 1, the client and vendor A simultaneously determine their optimal effort levels $X^*(t_A)$ and $Y_A^*(t_A)$. These effort levels are given by:

$$X_{1V}^*(t_A) = \left(\frac{\alpha(\mu - t_A)}{c_R} \zeta_{1V}^{OB}(t_A)\right)^{\frac{1}{k}}, \text{ and} \tag{EC.64a}$$

$$Y_{A_{1V}}^*(t_A) = \left(\frac{t_A \beta_A}{c_A} \zeta_{1V}^{OB}(t_A)\right)^{\frac{1}{k}}, \text{ respectively, where} \tag{EC.64b}$$

$$\zeta_{1V}^{OB}(t_A) = \left(\left(\frac{1}{k}\right)^k \left(\frac{\alpha(\mu - t_A)}{c_R}\right)^{\alpha} \left(\frac{t_A \beta_A}{c_A}\right)^{\beta_A}\right)^{\frac{1}{k - \alpha - \beta_A}}, \tag{EC.64c}$$

anticipating which, the client chooses t_A in Stage 1 to maximize:

$$\pi^*_{1V}(t_A) = (k-\alpha)(\mu - t_A)\,\zeta^{OB}_{1V}(t_A). \tag{EC.65}$$

Analogous to the analysis under the effort-based payment contract in Section 4, we define two functions Ψ^{OB}_{2V} and Ψ^{OB}_{1V} under the output-based payment contract, corresponding to the two-vendor and one-vendor scenarios respectively, that represent the combined effect of the relative contribution from each firm in the respective co-creation process:

$$\Psi^{OB}_{2V} \stackrel{def}{=} \left(\left(\frac{\mu}{k^2}\right)^k \left(\frac{\alpha(k-\beta_A-\beta_B)}{\rho_R c_R}\right)^\alpha \left(\frac{\beta_A^2}{\rho_A c_A}\right)^{\beta_A} \left(\frac{\beta_B^2}{c_B}\right)^{\beta_B}\right)^{\frac{1}{k-\alpha-\beta_A-\beta_B}}, \text{ and} \tag{EC.66a}$$

$$\Psi^{OB}_{1V} \stackrel{def}{=} \left(\left(\frac{\mu}{k^2}\right)^k \left(\frac{\alpha(k-\beta_A)}{c_R}\right)^\alpha \left(\frac{\beta_A^2}{c_A}\right)^{\beta_A}\right)^{\frac{1}{k-\alpha-\beta_A}}. \tag{EC.66b}$$

Observe that the sensitivity of Ψ^{OB}_{2V} and Ψ^{OB}_{1V}, given by (EC.66a) and (EC.66b) respectively, with respect to model parameters such as μ, k, α, β_A, β_B, c_A, etc., remain directionally the same as in Section 4. We derive the equilibrium results in the two-vendor scenario under the output-based payment contract, along the same lines as Lemmas 1, 2, and 3, and present them in Lemma EC.7 below.

Lemma EC.7. *When the client co-creates with two vendors and offers each of them a payment per unit of the total co-creation output, then in equilibrium,*

(a) the payment per unit output offered by the client to vendors A and B are: $t^{OB*}_{A_{2V}} = \mu\frac{\beta_A}{k}$, *and* $t^{OB*}_{B_{2V}} = \mu\frac{\beta_B}{k}$, *respectively;*

(b) the efforts exerted by the client, vendor A, and vendor B are: $X^{OB*}_{2V} = \left(\frac{\alpha(k-\beta_A-\beta_B)\Psi^{OB}_{2V}}{\rho_R c_R}\right)^{\frac{1}{k}}$, $Y^{OB*}_{A_{2V}} = \left(\frac{\beta_A^2\Psi^{OB}_{2V}}{\rho_A c_A}\right)^{\frac{1}{k}}$, *and* $Y^{OB*}_{B_{2V}} = \left(\frac{\beta_B^2\Psi^{OB}_{2V}}{c_B}\right)^{\frac{1}{k}}$, *respectively;*

(c) the output from the co-creation environment is: $Q^{OB*}_{2V} = \frac{k^2}{\mu}\Psi^{OB}_{2V}$;

(d) the profit earned by the client is $\Pi^{OB*}_{2V} = (k-\alpha)(k-\beta_A-\beta_B)\Psi^{OB}_{2V}$; *and*

(e) the profits earned by vendor A and vendor B are: $\pi^{OB*}_{A_{2V}} = (k-\beta_A)\beta_A\Psi^{OB}_{2V}$, *and* $\pi^{OB*}_{B_{2V}} = (k-\beta_B)\beta_B\Psi^{OB}_{2V}$, *respectively; where* Ψ^{OB}_{2V} *is defined in (EC.66a).*

The proof of Lemma EC.7 follows the same backward induction as shown in the proofs of Lemmas 1, 2, and 3. We omit the details for the sake of brevity, and now derive the equilibrium results in the one-vendor scenario under the output-based payment contract, similar to Lemmas 4, and 5, and present them in Lemma EC.8 below.

Lemma EC.8. *When the client co-creates solely with the primary vendor (vendor A) and offers a payment per unit of the total co-creation output, then in equilibrium,*

(a) the payment per unit output offered by the client to vendor A is: $t^{OB*}_{A_{1V}} = \mu\frac{\beta_A}{k}$;

(b) the efforts exerted by the client and vendor A are: $X_{1V}^{OB*} = \left(\frac{\alpha(k-\beta_A)\Psi_{1V}^{OB}}{c_R}\right)^{\frac{1}{k}}$*, and* $Y_{A_{1V}}^{OB*} = \left(\frac{\beta_A^2 \Psi_{1V}^{OB}}{c_A}\right)^{\frac{1}{k}}$*, respectively;*

(c) the output from the co-creation environment is: $Q_{1V}^{OB*} = \frac{k^2}{\mu}\Psi_{1V}^{OB}$

(d) the profits earned by the client and vendor A are: $\Pi_{1V}^{OB*} = (k-\alpha)(k-\beta_A)\Psi_{1V}^{OB}$*, and* $\pi_{A_{1V}}^{OB*} = (k-\beta_A)\beta_A\Psi_{1V}^{OB}$*, respectively; where* Ψ_{1V}^{OB} *is defined in (EC.66b).*

The proof of Lemma EC.8 follows the same arguments as the proofs of Lemmas 4 and 5. We omit the details and note that since Ψ_{2V}^{OB} and Ψ_{1V}^{OB} retain the qualitative properties of Ψ_{2V} and Ψ_{1V}, the results outlined in Lemmas EC.7 and EC.8 are qualitatively the same as those shown in Section 4. We now utilize the Lemmas EC.7 and EC.8 to draw parallels to the key result that is highlighted in Lemma 6. It can be observed that $\Pi_{2V}^{OB*}/\Pi_{1V}^{OB*}$ is increasing in μ, which assists us to restate Lemma 6 to include the effect of the output-based payment contract.

Lemma EC.9. *When the client offers an output-based payment contract to the vendors, there exists a threshold* $\left(\overline{\mu}_{\Pi}^{OB}\right)$ *on the value per unit of output* (μ)*, such that the client benefits from including the secondary vendor in the co-creation environment if and only if the value per unit of output received by the client exceeds this threshold, that is,* $\Pi_{2V}^{OB*} > \Pi_{1V}^{OB*} \iff \mu > \overline{\mu}_{\Pi}^{OB}$*.*

The proof of Lemma EC.12 follows the same methodology as that of Lemmas 6, 7, EC.1, etc. In a similar fashion, we establish that the results outlined in Section 5 hold even when the client offers the vendor(s) an output-based contract instead of an effort-based contract. We omit the details for the sake of brevity. This demonstrates that our results continue to hold qualitatively irrespective of whether the client offers an effort-based contract or an output-based contract, and that they are driven solely by the collaborative nature of the interaction.

EC.3.6. Analysis of Section 6.6

In this section, we provide a detailed analysis of the extension in Section 6.6: Vendors offer the price per unit of effort. When vendors offer their respective prices per unit of effort, and the client responds by determining the optimal level of effort to be exerted by each firm, it results in a change in the sequence of the game. In the rest of this section, we demonstrate that our results continue to hold even if we allow the sequence of the contracting game to change. For analytical tractability, we consider $\alpha + \beta_A + \beta_B < 1$, which allows the output to be a concave function of the efforts (Roels et al. 2010, Demirezen et al. 2016), and a general convex cost $k \geq 1$. Note that when the vendor offers the price per unit of effort, we can consider a more general convex cost function, which includes the linear cost function (i.e., $k = 1$). This is because when the vendor offers the price (unlike when the client offers it), the client's concave profit function implies that the client will not choose an infinite amount of effort from the vendor for any positive price.

When the client co-creates with vendors A and B, the vendors simultaneously offer prices t_A and t_B per unit of effort, respectively, in Stage 1. The client's optimization problem in Stage 2 involves jointly determining its own effort X_A and the vendors' respective efforts Y_A and Y_B, in order to maximize Π_{2V} given by (2), which yields:

$$X^*_{2V}(t_A,t_B)=\left(\mu\left(\frac{\alpha}{k\rho_R c_R}\right)^{1-\beta_A-\beta_B}\left(\frac{\beta_A}{t_A}\right)^{\beta_A}\left(\frac{\beta_B}{t_B}\right)^{\beta_B}\right)^{\frac{1}{k(1-\beta_A-\beta_B)-\alpha}}, \tag{EC.67a}$$

$$Y^*_{A_{2V}}(t_A,t_B)=\left(\mu^k\left(\frac{\alpha}{k\rho_R c_R}\right)^{\alpha}\left(\frac{\beta_A}{t_A}\right)^{k(1-\beta_B)-\alpha}\left(\frac{\beta_B}{t_B}\right)^{k\beta_B}\right)^{\frac{1}{k(1-\beta_A-\beta_B)-\alpha}}, \text{ and} \tag{EC.67b}$$

$$Y^*_{B_{2V}}(t_A,t_B)=\left(\mu^k\left(\frac{\alpha}{k\rho_R c_R}\right)^{\alpha}\left(\frac{\beta_A}{t_A}\right)^{k\beta_A}\left(\frac{\beta_B}{t_B}\right)^{k(1-\beta_A)-\alpha}\right)^{\frac{1}{k(1-\beta_A-\beta_B)-\alpha}}. \tag{EC.67c}$$

In Stage 1, both vendor A and vendor B anticipate the client's reaction given by (EC.67a)-(EC.67c) to their price offers t_A and t_B. Accordingly, they simultaneously determine t^*_A and t^*_B in order to maximize $\pi_{A_{2V}}$ and $\pi_{B_{2V}}$, given by (3). Substituting this t^*_A and t^*_B in (EC.67a)-(EC.67c), we obtain the client's equilibrium choice of effort levels in Stage 2, given by X^*_{2V}, $Y^*_{A_{2V}}$, and $Y^*_{B_{2V}}$. Substituting these equilibrium prices and effort levels in (1), (2) and (3), we obtain the equilibrium output and profits for the client and the two vendors under the two-vendor co-creation with vendor-offered price.

When the client co-creates solely with vendor A, vendor A offers price t_A per unit of effort in Stage 1. The client's optimization problem in Stage 2 involves determining its own effort X_A and vendor A's effort Y_A in order to maximize Π_{1V} given by (4), which yields:

$$X^*_{1V}(t_A)=\left(\mu\left(\frac{\alpha}{kc_R}\right)^{1-\beta_A}\left(\frac{\beta_A}{t_A}\right)^{\beta_A}\right)^{\frac{1}{k(1-\beta_A)-\alpha}}, \text{ and} \tag{EC.68a}$$

$$Y^*_{A_{1V}}(t_A)=\left(\mu^k\left(\frac{\beta_A}{t_A}\right)^{k-\alpha}\left(\frac{\alpha}{kc_R}\right)^{\alpha}\right)^{\frac{1}{k(1-\beta_A)-\alpha}}. \tag{EC.68b}$$

In Stage 1, vendor A anticipates the client's reaction given by (EC.68a)-(EC.68b) to his price offer t_A, and determines t^*_A in order to maximize $\pi_{A_{1V}}$ given by (4). Substituting t^*_A in (EC.68a)-(EC.68b), we obtain the client's equilibrium choice of effort levels in Stage 2, given by X^*_{1V} and $Y_{A_{1V}}$. Finally, substituting these equilibrium prices and effort levels in (4), we obtain the equilibrium output and profits for the client and vendor A under the single-vendor co-creation with vendor pricing.

Analogous to the analysis with constant μ in Section 4, we define two functions Ψ^{VP}_{2V} and Ψ^{VP}_{1V} below, corresponding to the two-vendor and one-vendor scenarios respectively, that represent the combined effect of the relative contribution from each firm in the respective co-creation process:

$$\Psi^{VP}_{2V}\stackrel{def}{=}\left(\mu^k\left(\frac{\alpha}{k\rho_R c_R}\right)^{\alpha}\left(\left(\frac{\beta_A}{\rho_A c_A}\right)\left(\frac{\beta_A}{k(1-\beta_B)-\alpha}\right)\right)^{\beta_A}\left(\left(\frac{\beta_B}{c_B}\right)\left(\frac{\beta_B}{k(1-\beta_A)-\alpha}\right)\right)^{\beta_B}\right)^{\frac{1}{k\Delta_2}}, \text{ and} \tag{EC.69a}$$

$$\Psi_{1V}^{VP} \stackrel{def}{=} \left(\mu^k \left(\frac{\alpha}{kc_R} \right)^{\alpha} \left(\left(\frac{\beta_A}{c_A} \right) \left(\frac{\beta_A}{k-\alpha} \right) \right)^{\beta_A} \right)^{\frac{1}{k\Delta_1}}, \tag{EC.69b}$$

where $\Delta_1 = k - \alpha - \beta_A > 0$ and $\Delta_2 = k - \alpha - \beta_A - \beta_B > 0$. Now, we first present the equilibrium results in the two-vendor scenario under vendor-led pricing.

Lemma EC.10. *When the client co-creates with two vendors, and the vendors offer their respective prices per unit of effort, then in equilibrium,*

(a) the price per unit of effort offered by vendors A and B to the client are: $t^*_{A_{2V}} = \beta_A \left(\left(\frac{\rho_A c_A}{\beta_A} \right) \left(\frac{k(1-\beta_B)-\alpha}{\beta_A} \right) \right)^{\frac{1}{k}} (\Psi_{2V}^{VP})^{k-1}$, *and* $t^*_{B_{2V}} = \beta_B \left(\left(\frac{c_B}{\beta_B} \right) \left(\frac{k(1-\beta_A)-\alpha}{\beta_B} \right) \right)^{\frac{1}{k}} (\Psi_{2V}^{VP})^{k-1}$, *respectively;*

(b) the efforts exerted by the client, vendor A, and vendor B are: $X^*_{2V} = \left(\frac{\alpha}{k\rho_R c_R} \right)^{\frac{1}{k}} \Psi_{2V}^{VP}$ $Y^*_{A_{2V}} = \left(\left(\frac{\beta_A}{\rho_A c_A} \right) \left(\frac{\beta_A}{k(1-\beta_B)-\alpha} \right) \right)^{\frac{1}{k}} \Psi_{2V}^{VP}$, *and* $Y^*_{B_{2V}} = \left(\left(\frac{\beta_B}{c_B} \right) \left(\frac{\beta_B}{k(1-\beta_A)-\alpha} \right) \right)^{\frac{1}{k}} \Psi_{2V}^{VP}$, *respectively;*

(c) the output from the co-creation process is given by: $Q^*_{2V} = \left(\frac{1}{\mu} \right) (\Psi_{2V}^{VP})^k$*;*

(d) the profit earned by the client is: $\Pi^*_{2V} = \left(1 - \beta_A - \beta_B - \frac{\alpha}{k} \right) (\Psi_{2V}^{VP})^k$*; and*

(e) the profits earned by vendor A and vendor B are: $\pi^*_{A_{2V}} = \beta_A \left(1 - \frac{\beta_A}{k(1-\beta_B)-\alpha} \right) (\Psi_{2V}^{VP})^k$, *and* $\pi^*_{B_{2V}} = \beta_B \left(1 - \frac{\beta_B}{k(1-\beta_A)-\alpha} \right) (\Psi_{2V}^{VP})^k$, *respectively; where* Ψ_{2V}^{VP} *is defined in (EC.69a).*

Next, we present the equilibrium results in the one-vendor scenario when the vendor offers his price per unit of effort.

Lemma EC.11. *When the client co-creates solely with the primary vendor (vendor A) and the vendor offers his price per unit of effort, then in equilibrium,*

(a) the price per unit of effort offered by the client to vendor A is: $t^*_{A_{1V}} = \beta_A \left(\left(\frac{c_A}{\beta_A} \right) \left(\frac{k-\alpha}{\beta_A} \right) \right)^{\frac{1}{k}} (\Psi_{1V}^{VP})^{k-1}$*;*

(b) the efforts exerted by the client and vendor A are: $X^*_{1V} = \left(\frac{\alpha}{kc_R} \right)^{\frac{1}{k}} \Psi_{1V}^{VP}$, *and* $Y^*_{A_{1V}} = \left(\left(\frac{\beta_A}{c_A} \right) \left(\frac{\beta_A}{k-\alpha} \right) \right)^{\frac{1}{k}} \Psi_{1V}^{VP}$, *respectively;*

(c) the output from the co-creation process is given by: $Q^*_{1V} = \left(\frac{1}{\mu} \right) (\Psi_{1V}^{VP})^k$*;*

(d) the profits earned by the client and vendor A are: $\Pi^*_{1V} = \left(1 - \beta_A - \frac{\alpha}{k} \right) (\Psi_{1V}^{VP})^k$, *and* $\pi^*_{A_{1V}} = \beta_A \left(1 - \frac{\beta_A}{k-\alpha} \right) (\Psi_{1V}^{VP})^k$, *respectively; where* Ψ_{1V}^{VP} *is defined in (EC.69b).*

Since $\alpha + \beta_A + \beta_B < 1$ and $k > 1$, the equilibrium decisions and outputs retain the directional properties of their counterparts in 4. Therefore, we analyze the ratio Π^*_{2V}/Π^*_{1V} from Lemma EC.10 and Lemma EC.11, and note that it is concave increasing in the value per unit of output μ. Now, we draw parallels to Lemma 6 by including the effect of vendor pricing.

Lemma EC.12. *When the vendors offer their respective prices per unit of effort, there exists a threshold* $(\overline{\mu}_{\Pi}^{VP})$ *on the value per unit of output* (μ)*, such that the client benefits from including* *the secondary vendor in the co-creation environment if and only if the value per unit of output received by the client exceeds this threshold, that is,* $\Pi^*_{2V} > \Pi^*_{1V} \iff \mu > \overline{\mu}^{VP}_{\Pi}$.

The proof of Lemma EC.12 follows the same methodology as that of Lemmas 6, 7, EC.1, etc. In a similar fashion, we establish that the results outlined in Section 5 hold even when the price per unit of effort is offered by the vendor and the client responds by determining the level of effort to be exerted by each firm. We omit the details for the sake of brevity. This demonstrates that our results continue to hold qualitatively irrespective of whether the client or the vendor determines the price(s) per unit effort, and that they are driven solely by the collaborative nature of the interaction.