Online Appendix: Endogenizing the Commission Rate

A Model and Results

In the main model, we treat the platforms choice of its commission rate, α, as fixed. When the platform can also adjust α, the platform has two strategic choices.

For tractability, we focus on the case where, for any commission rate α chosen by the platform, the assumptions in our main model continue to hold. We further assume that there is no competition or network effect, and that the platform's profit function in the non-scalable scenario when the platform service is provided, denoted as $\pi_N(\alpha, p)$, is quasi-concave in α and p.

We show that the main results of the paper remain. In terms of output level, the total output increases upon provision of the platform service in the non-scalable scenario, but may decrease in the scalable scenario. In terms of service provision, the platform is less likely to offer the ancillary service in the scalable scenario.

Lemma A.1. *Suppose $\pi_N(\alpha, p)$ is super-modular in α and p. When the platform provides the service, the following holds:*

a. In the non-scalable scenario, the commission rate chosen by the platform decreases.

b. In the scalable scenario, the commission rate chosen by the platform may increase or decrease.

The super-modularity condition ensures that the optimal commission rate moves in the same direction as the change in the service price or marginal cost faced by the sellers. In the non-scalable scenario, the optimal platform service price always falls compared to the market service price $\bar{c}$, and so does the optimal commission rate. In the scalable scenario, sellers may choose to scale and enjoy a lower marginal cost $\underline{c}$. The optimal platform service price may be higher or lower than $\underline{c}$, and thus the optimal commission rate may increase or decrease accordingly.

Proposition A.1. *Suppose $\pi_N(\alpha, p)$ is super-modular in α and p. When the platform provides the service, the following holds:*

a. In the non-scalable scenario, the total output produced by the sellers increases.

b. In the scalable scenario, the total output produced by the sellers can decrease.

A higher commission rate has the same effect on the total output level as a higher price. Therefore, the co-movement of the optimal commission rate and the service price as shown in Lemma A.1 amplifies our results in Proposition 1 from the main model.

Proposition A.2. *If $\pi_S(\alpha, p)$ is increasing in p for any $p \in [\bar{p}(\alpha), \bar{c}]$, and there are sufficiently many sellers of high capacities, then the platform is less likely to provide the platform service in the scalable scenario than in the non-scalable scenario.*

Proposition A.2 follows the same logic as Proposition 2 from the main model. If the platform charges a low price in the scalable scenario, the platform can replicate it in the non-scalable scenario. While if the platform charges a high price in the scalable scenario, our condition that there are sufficiently many sellers of high capacities ensures that there are not enough sellers switching to the platform service, resulting in a lower profit margin than in the non-scalable scenario where all sellers switch. Either way, the extensive margin result in our main model continues to hold when the platform can choose the commission rate.

B Proofs

Proof of Lemma A.1. Denote

$$TR(\alpha, p) = \int_{q_e(\alpha,p)}^{q^*(\alpha,p)} R(k)dG(k) + \int_{q^*(\alpha,p)}^{\bar{k}} R(q^*(\alpha, p))dG(k)$$

and

$$Q(\alpha, p) = \int_{q_e(\alpha,p)}^{q^*(\alpha,p)} kdG(k) + \int_{q^*(\alpha,p)}^{\bar{k}} q^*(\alpha, p)dG(k)$$

as the total revenue and total output level on the platform in the non-scalable scenario. All relevant functions, including $R(q)$, $G(k)$, $q_e(\alpha, p)$ and $q^*(\alpha, p)$, are defined similarly as in the main model, and it is straightforward to show that $q_e(\alpha, p)$ increases in α and p, while $q^*(\alpha, p)$ decreases in α and p.

For part a, in the non-scalable scenario, when the platform service is not provided, the platform chooses a commission rate to maximize

$$\pi_n(\alpha) = \alpha TR(\alpha, \bar{c}).$$

Denote the optimal commission rate in this case as α_n^*.

When the platform service is provided, the platform chooses both a commission rate and a price for the platform service to maximize

$$\pi_N(\alpha, p) = \alpha TR(\alpha, p) + (p - c)Q(\alpha, p).$$

Denote the optimal commission rate and optimal price in this case as α_N^* and p_N^* respectively.

Consider an auxiliary optimization problem, where the platform service price is fixed at $\bar{c}$, and the platform chooses a commission rate α to maximize

$$\pi_N(\alpha, \bar{c}) = \alpha TR(\alpha, \bar{c}) + (\bar{c} - c)Q(\alpha, \bar{c}).$$

Denote the optimal commission rate as α_a^*. Since α_n^* maximizes π_n, and

$$\frac{\partial Q(\alpha, p)}{\partial \alpha} = -q_e(\alpha, p)g(q_e(\alpha, p))\frac{\partial q_e(\alpha, p)}{\partial \alpha} + \int_{q^*(\alpha,p)}^{\bar{k}} \frac{\partial q^*(\alpha, p)}{\partial \alpha} dG(k) \leq 0,$$

by comparing the FOCs, we must have $\alpha_a^* \leq \alpha_n^*$.

Meanwhile, α_N^* can be equivalently regarded as the solution to another auxiliary optimization problem

$$\max_{\alpha} \pi_N(\alpha, p_N^*) = \alpha TR(\alpha, p_N^*) + (p_N^* - c)Q(\alpha, p_N^*).$$

Since $p_N^* \leq c$ and $\pi_N(\alpha, p)$ is super-modular in α and p, we must have

$$\alpha_N^* \leq \alpha_a^* \leq \alpha_n^*,$$

which completes our proof.

For part b, in the scalable scenario, let

$$Q^* = \max_{\alpha} q^*(\alpha, \underline{c})$$

and consider a truncated distribution where $G(Q^*) = 0$. In other words, all sellers are of sufficiently high capacities. We show that for this truncated distribution, part b holds. Part b will then also hold for distributions sufficiently close to this truncated distribution.

Since all sellers are of sufficiently high capacities, when the platform service is not provided, or is provided at a price $p > \bar{p}(\alpha)$, the platform chooses a commission rate to maximize

$$\pi_s(\alpha) = \alpha R(q^*(\alpha, \underline{c})).$$

Denote the optimal commission rate in this case as α_s^*.

When the platform service is provided at a price $p \leq \bar{p}(\alpha)$, the platform chooses both a commission rate and a price for the platform service to maximize

$$\pi_S(\alpha, p) = \alpha R(q^*(\alpha, p)) + (p - c)q^*(\alpha, p).$$

Denote the optimal commission rate and optimal price in this case as α_S^* and p_S^*, respectively.

Suppose $c = \underline{c}$. Therefore, α_s^* can be equivalently regarded as the solution to the auxiliary optimization problem

$$\max_{\alpha} \pi_S(\alpha, \underline{c}) = \alpha R(q^*(\alpha, \underline{c})) + (\underline{c} - \underline{c})q^*(\alpha, \underline{c}) = \alpha R(q^*(\alpha, \underline{c})).$$

Similarly, α_S^* can be equivalently regarded as the solution to another auxiliary optimization problem

$$\max_{\alpha} \pi_S(\alpha, p_S^*) = \alpha R(q^*(\alpha, p_S^*)) + (p_S^* - \underline{c})q^*(\alpha, p_S^*).$$

Note that, as in our main model, $\pi_S(\alpha, p) = \pi_N(\alpha, p)$ for $p \leq \bar{p}(\alpha)$. Therefore, due to the super-modularity of $\pi_N(\alpha, p)$, we have $\alpha_S^* \geq \alpha_s^*$ if $p_S^* > \underline{c}$, and vice versa. □

Proof of Proposition A.1. For part a, since $p_N^* \leq \bar{c}$ and $\alpha_N^* \leq \alpha_n^*$, we have

$$Q(\alpha_N^*, p_N^*) \geq Q(\alpha_n^*, \bar{c}).$$

For part b, if $p_S^* > \underline{c}$, then $\alpha_S^* \geq \alpha_s^*$, and thus

$$q^*(\alpha_S^*, p_S^*) \leq q^*(\alpha_s^*, \underline{c}).$$

□

Proof of Proposition A.2. When the platform can choose the commission rate, the cutoff investment level in the non-scalable scenario becomes

$$I_N = \max_{\alpha,p} \pi_N(\alpha, p) - \max_{\alpha} \pi_n(\alpha),$$

and the cutoff investment level in the scalable scenario becomes:

$$I_S = \max_{\alpha,p} \pi_S(\alpha,p) - \max_{\alpha} \pi_s(\alpha).$$

The platform is less likely to provide the platform service in the scalable scenario than in the non-scalable scenario if

$$I_N - I_S \geq 0 \Leftrightarrow [\max_{\alpha,p} \pi_N(\alpha,p) - \max_{\alpha} \pi_n(\alpha)] - [\max_{\alpha,p} \pi_S(\alpha,p) - \max_{\alpha} \pi_s(\alpha)] \geq 0.$$

Consider the platform's optimal commission rate and platform service price in the scalable scenario when the platform service is provided, denoted as α_S^* and p_S^*, respectively.

If $p_S^* \leq \bar{p}(\alpha_S^*)$, then the platform can choose the same commission rate and platform service price in the non-scalable scenario and achieve the same profits

$$\max_{\alpha,p} \pi_S(\alpha,p) = \pi_S(\alpha_S^*, p_S^*) = \pi_N(\alpha_S^*, p_S^*) \leq \max_{\alpha,p} \pi_N(\alpha,p).$$

Meanwhile, since $\pi_s(\alpha) > \pi_n(\alpha)$ for any α, we have

$$\max_{\alpha} \pi_s(\alpha) > \max_{\alpha} \pi_n(\alpha).$$

Therefore,

$$I_N - I_S \geq \pi_N(\alpha_S^*, p_S^*) - \pi_S(\alpha_S^*, p_S^*) + \max_{\alpha} \pi_s(\alpha) - \max_{\alpha} \pi_n(\alpha) > 0.$$

If $p_S^* > \bar{p}(\alpha_S^*)$, since the platform's profit function $\pi_S(\alpha,p)$ increases in p, we must have $p_S^* = \bar{c}$, and the platform earns

$$\max_{\alpha,p} \pi_S(\alpha,p) = \pi_S(\alpha_S^*, \bar{c}).$$

Suppose in the non-scalable scenario, when the platform service is not provided, the platform's optimal choice of commission rate is α_n^* and earns

$$\max_{\alpha} \pi_n(\alpha) = \pi_n(\alpha_n^*).$$

When the platform service is provided, the platform can choose α_n^* and $\bar{c}$ and guarantee profits

$$\pi_N(\alpha_n^*, \bar{c}) \leq \max_{\alpha,p} \pi_N(\alpha,p).$$

Therefore,

$$I_N - I_S \geq \pi_N(\alpha_n^*, \bar{c}) - \pi_n(\alpha_n^*) - \pi_S(\alpha_S^*, p_S^*) + \pi_s(\alpha_S^*) = (\bar{c} - c)(Q_N^p(\alpha_n^*, \bar{c}) - Q_S^p(\alpha_S^*, \bar{c})),$$

where Q_N^p is the total output level using the platform service in the non-scalable scenario, and Q_S^p in the scalable scenario.

Suppose $\bar{\alpha}$ and $\underline{\alpha}$ solve respectively

$$q_e(\bar{\alpha}, \bar{c}) = q^*(\bar{\alpha}, \bar{c})$$

and

$$q_s(\underline{\alpha}, \bar{c}) = q^*(\underline{\alpha}, \underline{c}).$$

Any possible α such that our assumptions continue to hold must fall between $\underline{\alpha}$ and $\bar{\alpha}$. Therefore, an lower bound for $Q_N^p(\alpha_n^*, \bar{c})$ is

$$Q_N^p(\alpha_n^*, \bar{c}) = \int_{q_e(\alpha_n^*, \bar{c})}^{\bar{k}} \min\{k, q^*(\alpha_n^*, \bar{c})\} dG(k) \geq q^*(\bar{\alpha}, \bar{c})[1 - G(q^*(\bar{\alpha}, \bar{c}))] \triangleq \underline{Q}_N^p,$$

and a upper bound for $Q_S^p(\alpha_S^*, \bar{c})$ is

$$Q_S^p(\alpha_S^*, \bar{c}) = \int_{q_e(\alpha_S^*, \bar{c})}^{q_s(\alpha_S^*, \bar{c})} \min\{k, q^*(\alpha_S^*, \bar{c})\} dG(k) \leq \int_{q_e(\underline{\alpha}, \bar{c})}^{q_s(\bar{\alpha}, \bar{c})} \min\{k, q^*(\underline{\alpha}, \bar{c})\} dG(k) \triangleq \bar{Q}_S^p.$$

It is thus sufficient to show that $\underline{Q}_N^p \geq \bar{Q}_S^p$, which is true if we have a truncated distribution with

$$G(q_s(\bar{\alpha}, \bar{c})) = 0,$$

where we have sufficiently many sellers of high capacities. The result therefore continues to hold for distributions sufficiently close to this truncated distribution. □