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Supplementary Materials for “Network Effects and Multi-Network Sellers’ Dynamic Pricing in the US Smartphone Market”

A Seasonality of Smartphone User Increases

We remove the seasonality in the new smartphone users following the method in Gowrisankaran and Rysman (2012). We first regress the monthly log increases on month dummies. We then divide each monthly increase by the exponentiated fixed effect for the month of the year. The geometric mean of adjusted sales is the same across months of the year, and the total increase of smartphone users from Sep 2009 to Jan 2014 is equal to that in the original data. In Figure 1, the red curve plots the monthly increases in the data. The green curve shows the adjusted monthly increases of smartphone subscribers. Their differences indicate that demand is high during the holiday seasons. The adjusted data remove some of the month effects.

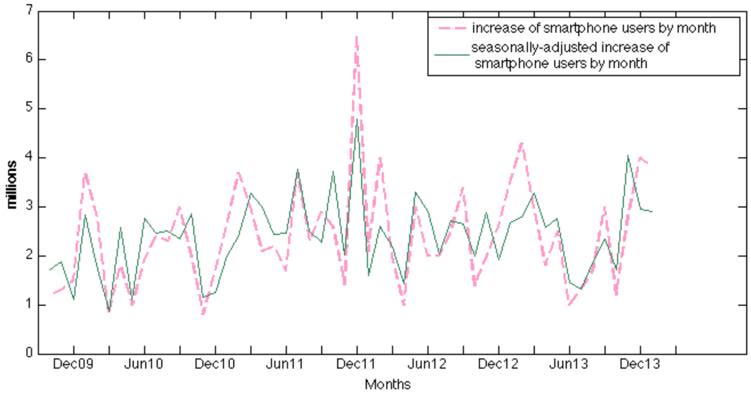

Figure 1: Monthly Increases of Smartphone Users

B Solving for the Model Markups

The FOCs for the carrier prices are:

$$M_t s_{sct} + M_t \sum_{(s')\in\Omega_{ct}} (p^c_{sct} - \omega_s p^m_{sct} + f_{ct} - 24\kappa_{sc} - \lambda_{sct}) \frac{\partial s_{s'ct}}{\partial p^c_{sct}} + \beta^d \frac{\partial V_c(n_{t+1}, t+1)}{\partial p^c_{sct}} = 0. \quad \text{(B.1)}$$

Next, we derive the partial derivatives in equation (B.1), using the smartphone market share equation in the paper. Let consumer i's price coefficient be $\alpha_i = \alpha + \phi_\alpha y_i + \sigma_\alpha \nu_{i\alpha}$.

If $(s', c') = (s, c)$, then the partial derivative is:

$$\frac{\partial s_{s'c't}}{\partial p^c_{sct}} = -\frac{1}{N^{sim}_t} \sum_{i=1}^{N^{sim}_t} \alpha_i s_{isct}(1 - s_{isct}), \quad \text{(B.2)}$$

where N^{sim}_t is the number of simulated consumers who enter the market in period t. If $(s', c') \neq (s, c)$, then

$$\frac{\partial s_{s'c't}}{\partial p^c_{sct}} = \frac{1}{N^{sim}_t} \sum_{i=1}^{N^{sim}_t} \alpha_i s_{is'c't} s_{isct}. \quad \text{(B.3)}$$

Second, the partial derivative of the carriers' value function is

$$\frac{\partial V_c(n_{t+1}, t+1)}{\partial p^c_{sct}} = \sum_{l=1}^{S} \frac{\partial V_c(n_{t+1}, t+1)}{\partial n_{lt+1}} \frac{\partial n_{lt+1}}{\partial p^c_{sct}}. \quad \text{(B.4)}$$

With an approximate of $\hat{V}_c(n_{t+1}, t+1)$, we derive $\frac{\partial V_c(n_{t+1},t+1)}{\partial n_{s't+1}}$. We use the OS network size transition rule to derive $\frac{\partial n_{s't+1}}{\partial p^c_{sct}}$. It also depends on whether $s = s'$ or not. The network size transition rule of OS s' is:

$$n_{s't+1}(\boldsymbol{n}_t, \boldsymbol{p}^c_t, \boldsymbol{\xi}_t) = (1 - q_{s't}) n_{s't} + \frac{M_t}{M} \sum_{c\in\Omega_{s't}} s_{s'ct}(\boldsymbol{p}^c_t, \boldsymbol{\xi}_t), \quad \text{(B.5)}$$

where $q_{s't}$ is the share of OS s' users who re-enter the market, $\Omega_{s't}$ is the set of products with OS s', and $M_t = (n_{0t} + \sum_s q_{st} n_{st})M$. Thus, the partial derivative of the OS shares with respect to the carrier price is:

$$\frac{\partial n_{s't+1}}{\partial p^c_{sct}} = \frac{M_t}{M} \sum_{c \in \Omega_{s't}} \frac{\partial s_{s'ct}(\boldsymbol{p}^c_t, \boldsymbol{\xi}_t)}{\partial p^c_{sct}},$$

where the partial derivatives $\frac{\partial s_{s't}}{\partial p^c_{sct}}$ are

$$\frac{\partial s_{s'ct}}{\partial p^c_{sct}} = \frac{1}{N^{sim}_t} \sum_i \frac{\partial s_{is'c't}}{\partial p^c_{sct}}.$$

When $(s, c) = (s', c')$, we have

$$\frac{\partial s_{is'c't}}{\partial p^c_{sct}} = \alpha_i s_{isct}(1 - s_{isct}).$$

When $(s, c) \neq (s', c')$, the derivative becomes

$$\frac{\partial s_{is'c't}}{\partial p^c_{sct}} = \alpha_i s_{is'c't} s_{isct}.$$

Plug equations (B.2)-(B.5) into equation (B.1), and define the markup as $m_{sct} = p^c_{sct} + f_{ct} - \omega_s p^m_{sct} - \kappa_{sc} - \lambda_{sct}$. We have:

$$s_{sct} + \sum_{s' \in \Omega_{ct}} m_{s'ct} \frac{1}{N^{sim}_t} \sum_{i=1}^{N^{sim}_t} \alpha_i s_{ijsct} s_{ij's'ct} - \frac{1}{N^{sim}_t} \sum_{i=1}^{N^{sim}_t} \alpha_i s_{ijsct} m_{jsct} + \frac{\beta^d}{M_t} \frac{\partial V_c(n_{t+1}, t+1)}{\partial p^c_{jsct}} = 0. \tag{B.6}$$

Given the individual choice probabilities and the carriers' value functions, the only unknowns are the markups m_{sct}s in equation (B.6). There are 16 equations and 16 unknown m_{sct}s. These equations are linear in the markups. Thus, we can solve the markups (m_{sct}'s) using the matrix inversion. With the markups, we can calculate the unobserved cost shock:

$$\lambda_{sct} = p^c_{sct} + f_{ct} - \omega_s p^m_{sct} - \kappa_{sc} - m_{sct}. \tag{B.7}$$

C Computation Details

C.1 Iteration Algorithm to Solve for the Demand Shocks.

We apply an iterative procedure to solve for the unobserved shocks $\xi(\theta_d)$.[1] Let the observed carrier-OS shares be s^{data}_{sct}. Given an initial guess of the unobserved demand shocks, $\xi^0 = \{\xi^0_{sct}\}$, we calculate the predicted market shares $\tilde{s}_{sct}(\theta_d, \xi^0)$. We then compare the predicted shares with data. The updating rule is to increase ξ_{sct} if the predicted share is less than the observed share and decrease it otherwise. Repeat this updating process until the vector ξ^k converges. The iteration equation is

$$\xi^{k+1}_{sct}(\theta_d) = \xi^k_{sct}(\theta_d) + \chi(s^{data}_{sct} - \tilde{s}_{sct}(\theta_d, \xi^k(\theta_d))),$$

where χ is a relaxation parameter, set to be 0.9.

C.2 Solving the Carriers' Cost Shocks and Value Function Approximation

In the GMM estimation, we use the carriers' FOCs to solve for the unobserved cost shocks. The FOCs in period $t < T$ and period T are

$$M_t s_{sct}(\boldsymbol{p}^c_t, \boldsymbol{\xi}_t) + M_t \sum_{s' \in \Omega_{ct}} m_{s'ct} \frac{\partial s_{s'ct}}{\partial p^c_{sct}} + \beta^d \frac{\partial V_c(n_{t+1}(\boldsymbol{n}_t, \boldsymbol{p}^c_t), t+1)}{\partial p^c_{sct}} = 0, \text{ and} \tag{C.1}$$

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[1] This method is from Berry, Levinsohn, and Pakes (1995) and Nevo (2001)

$$M_T s_{scT}(\boldsymbol{p}^c_T, \boldsymbol{\xi}_T) + M_T \sum_{s' \in \Omega_{ct}} m_{s'cT} \frac{\partial s_{s'cT}}{\partial p^c_{scT}} = 0. \tag{C.2}$$

The first two terms in the FOCs use the expressions of $s_{sct}(\boldsymbol{\xi}_t; \boldsymbol{\theta}_d)$s and their derivatives. The last term in equation (C.1) requires an explicit functional form for $\frac{\partial V_c(\boldsymbol{n}_{t+1}, t+1)}{\partial p^c_{sct}}$. Since the value function, $V_c(\boldsymbol{n}, t)$, changes with the carrier and the time periods, we approximate them with Chebyshev polynomials. For each (c, t), $V_c(\boldsymbol{n}_t, t)$ or $V_{ct}(\boldsymbol{n}_t)$ is approximated with order three Chebyshev polynomials for each of the four OS shares in $\boldsymbol{n}_t$. Solving for $\lambda_{sct}(\hat{\boldsymbol{\theta}}_d, \boldsymbol{\theta}_s)$ backward consists of five steps.

1. Solve for λ in period T. Given $(\hat{\boldsymbol{\theta}}_d, \boldsymbol{\theta}_s)$, equation (C.2) can be used to solve for the markups in period T, $m_{scT}(\hat{\boldsymbol{\theta}}_d, \boldsymbol{\theta}_s)$. We then calculate the cost shocks using

$$\lambda_{scT}(\boldsymbol{\theta}_d, \boldsymbol{\theta}_s) = p^c_{scT} + f_{cT} - m_{scT}(\hat{\boldsymbol{\theta}}_d, \boldsymbol{\theta}_s) - \omega_s p^m_{scT} - 24\kappa_{sc}. \tag{C.3}$$

2. Approximate $V_c(\boldsymbol{n}_T, T)$. We draw the order three Chebyshev nodes of the four OS shares $\boldsymbol{n}$. For each OS s, the Chebyshev nodes are drawn from the range of n_{st} between period $T-1$ and T. Thus, there are $81 (= 3^4)$ simulated vectors of $\boldsymbol{n}$ in period T. For each simulated state, $(\tilde{\boldsymbol{n}}_T, T)$, we compute the carriers' contract prices of the smartphones using the estimated pricing functions in a separate regression.[2] Each carrier's profits in period T are the sum of profits from its four OS sales. The value of $V_c(\tilde{\boldsymbol{n}}_T, T)$ is assumed to be the total discounted profits, assuming that the game repeats for 12 periods after the game ends. We then get the Chebyshev approximation parameters using the simulated Chebyshev nodes of $(\tilde{\boldsymbol{n}}_T, T)$ and the corresponding values of $V_c(\tilde{\boldsymbol{n}}_T, T)$.

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[2] We regress p^c_{sct} on the second-order polynomials of $\boldsymbol{n}_t$ and the time trend. In this regression, we have monthly prices for each carrier-OS group for the 32 months from 2011.08 to 2014.03. The regression is separate for each carrier-OS group. In each regression, there are 16 explanatory variables: 15 polynomials up to order two and time. We use Lasso to determine the most significant explanatory variables in each price regression. See the results in the Appendix.

3. Solve for λ in period $t (\leq T-1)$. Given the Chebyshev approximant of $V_c(\boldsymbol{n}_{t+1}, t+1)$, we get its partial derivatives for $\boldsymbol{n}_{t+1}$. Combining these partial derivatives with the transition rule of $\boldsymbol{n}$ and the partial derivatives of $\frac{\partial s_{sct}}{\partial p^c_{sct}}$, we get the functional forms of the price FOCs in equation (C.1). Since the FOCs are linear in the markups given the observed prices, we can directly invert the FOCs to solve for the markups, $m_{sct}(\hat{\boldsymbol{\theta}}_d, \boldsymbol{\theta}_s)$, which leads to the solution of the cost shocks as in equation (C.3).

4. Approximate $V_c(\boldsymbol{n}_t, t)$ for $t \leq T-1$. This step is similar to step 2, except for getting the value of $V_c(\tilde{\boldsymbol{n}}_t, t)$. We first draw the Chebyshev nodes of the four OS shares. For each simulated state, $(\tilde{\boldsymbol{n}}_t, t)$, we get the carriers' contract prices using the price regression results and calculate the next period's OS shares, $(\tilde{\boldsymbol{n}}_{t+1}, t+1)$, using the transition rule of n_{st}. We then obtain the carriers' period profits in period t. The value of $V_c(\tilde{\boldsymbol{n}}_t, t)$ is equal to the sum of the period profits and the discounted value of $V_c(\tilde{\boldsymbol{n}}_{t+1}, t+1)$, in which we update $\tilde{\boldsymbol{n}}_{t+1}$ using $\tilde{\boldsymbol{n}}_t$ and the transition rule. These values and the simulated states are used to compute the Chebyshev polynomial approximation parameters for each carrier in period t.

5. Repeat steps 3 and 4 until all periods' cost shocks are solved.

C.3 Number of Draws of (ξ, λ)

We compare the carriers' value functions and the computation time when using $R = 50$, 100, and 1000 draws of the shocks, (ξ, λ), using the estimated values of the parameters. We evaluate the value functions at the 81 $(= 3^4)$ Chebyshev OS share nodes for each of the 24 periods for one time. The value functions are the average over the draws of the costs. See Table OA1 below for the comparison results.

In the first panel, the table shows the average values of $V_c(n_t, t)$ in billion dollars and differences for the numbers of draws. Columns (1) - (3) show the averages across all simulated OS share nodes and all time periods. Columns (4) and (5) show the percentage differences when increasing the number of draws. The values increase with the number of draws for AT&T and T-Mobile, but drops for Verizon and Sprint when increasing R from 50 to 1000. Thus, using 50 draws of the shocks can cause overestimation of the cost parameters for AT&T and T-Mobile and underestimation of the parameters for Verizon and Sprint.

Table OA1: Comparing the Number of Simulations of (ξ, λ)

	(1) $R = 50$	(2) $R = 100$	(3) $R = 1000$	(4) (100 v.s. 50)	(5) (1000 v.s. 100)
$\bar{V}_c(n_t, t)$ Verizon	189.59	190.84	183.08	+0.84%	-4.22%
$\bar{V}_c(n_t, t)$ AT&T	124.68	128.37	132.00	+3.24%	+2.99%
$\bar{V}_c(n_t, t)$ Sprint	102.40	93.27	94.22	-9.27%	+0.95%
$\bar{V}_c(n_t, t)$ T-Mobile	39.76	42.28	45.31	+6.54%	+7.78%
Time	1	2.21	22.53		

The last row shows the normalized computation time. We normalize the computation time for obtaining the carriers' profits using $R = 50$ draws of the shocks to be 1. When R increases to 100, the computation time more than doubles. When R increases to 1000, the computation time increases more than 20 times. Due to the dramatic computation time increase, we do not use 1000 draws in the estimation.

References

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