Online Appendix

A. Proof of All Corollaries

Proof of Corollary 1:

Proof. We prove all the bullet points one by one.

• **Second-period price:** For the benchmark case, Proposition 2 has shown that $r_M^B(v) = (1+\kappa_L)v$. For the main with GDPR, Proposition 1 has shown that $r_M^*(v) = \kappa_L v + \min\{v, p_2\} - c \le r_M^B(v)$.

For the price for the basic good, if the basic good is offered under both cases with and without privacy rights, we notice that

$$\begin{aligned} p_{2M}^* - p_{2M}^B(P_1^B) &= \frac{1}{2} - \frac{2-\delta+2\delta\sigma-(2-\delta+2\delta\mu+2\delta\sigma)c}{2(4-\delta+2\delta\mu+2\delta\sigma)} \\ &> \frac{1}{2} - \frac{2-\delta+2\delta\sigma}{2(4-\delta+2\delta\mu+2\delta\sigma)} = \frac{1+\delta\mu}{4-\delta+2\delta(\mu+\sigma))} > 0. \end{aligned}$$

• **First-period price:** For the benchmark case, Proposition 2 has shown that $p_{1M}^B = P_1^B$ for $c \le (2-\delta+2\delta\sigma)/(2-\delta+2\delta\mu+2\delta\sigma)$ and $p_{1M}^B = P_2^B$ for $(2-\delta+2\delta\sigma)/(2-\delta+2\delta\mu+2\delta\sigma) < c \le 1+\delta/(1+\delta)\cdot\mu$. For the main with GDPR, Proposition 1 has shown that $p_{1M}^* = P_1^*$ for $c \le \delta\sigma/2$ and $p_{1M}^* = 1/2$ for $c > \delta\sigma/2$. We prove the claim in two cases.

In the first case, $c > \delta\sigma/2$. Let's first prove that $P_1^B < 1/2$. In fact, because P_1^B decreases with c, we only need to show that $1/2 - P_1^B|_{c=\delta\sigma/2} > 0$. Notice that,

$$\begin{aligned} &\frac{1}{2} - P_1^B|_{c=\delta\sigma/2} \\ &= \frac{1}{2} - \frac{(2-\delta+2\delta\sigma\kappa)^2 - [4+4\delta-\delta^2+2\delta^2\sigma+2\delta(2+\delta)\mu]\,\frac{\delta\sigma}{2}}{2(4-\delta+2\delta\mu+2\delta\sigma)} \\ &= \frac{\delta\left(6-2\delta-\left(8-12\delta+\delta^2\right)\sigma-2(4-\delta)\delta\sigma^2+\mu(4+2\delta(2+\delta)\sigma)\right)}{16-4\delta+8\delta\mu+8\delta\sigma} \\ &\ge \frac{\delta\left(8-2\delta+2\delta\sigma(9+\delta(5+\delta)\sigma)\right)}{16-4\delta+8\delta\mu+8\delta\sigma} > 0, \end{aligned}$$

where the second last inequality is due to $\mu \ge 1/2 + (2+\delta)\sigma$ as implied by Assumption 1.

Next, we prove that $P_2^B < 1/2$. In fact, by the definition of P_2^B, we only need to show for $c > \delta\sigma/2$, $1/2 > [(1+\delta\sigma)^2 - (1+\delta)(1+\delta+\delta\mu)c]/(2+\delta+\delta\mu+\delta\sigma)$, which decreases with c. Thus, we only need to show that $1/2 > [(1+\delta\sigma)^2 - (1+\delta)(1+\delta+\delta\mu)\delta\sigma/2]/(2+\delta+\delta\mu+\delta\sigma)$. Notice that,

$$\frac{1}{2} - \frac{(1+\delta\sigma)^2 - (1+\delta)(1+\delta+\delta\mu)\delta\sigma/2}{2+\delta+\delta\mu+\delta\sigma} \geq \frac{\delta(3+\delta\sigma(7+3\delta+2\delta(3+\delta)\sigma))}{2+\delta+\delta\mu+\delta\sigma} > 0,$$

where the second last inequality is due to $\mu \geq 1/2 + (2+\delta)\sigma$ as implied by Assumption 1.

So far, we have proved that $p_{1M}^* > p_{1M}^B$ for $c > \delta\sigma/2$. Next, we consider the other with $c \leq \delta\sigma/2$. It is easy to check that both $P_1^* - P_1^B$ and $P_1^* - P_2^B$ increase with c by checking that the coefficient in front of c is positive. This completes the proof.

- **Opt-in and demand:** According to Propositions 1 and 2, we can write down,

$$v_M^B = \begin{cases} \dfrac{2-\delta+2\delta\sigma+2c}{4-\delta+2\delta\mu+2\delta\sigma}, & \text{if } c \leq \dfrac{2-\delta+2\delta\sigma}{2-\delta+2\delta\sigma+2\delta\mu}, \\ c, & \text{if } \dfrac{2-\delta+2\delta\sigma}{2-\delta+2\delta\sigma+2\delta\mu} < c \leq \dfrac{1+\delta\sigma}{1+\delta\mu+\delta\sigma}, \\ \dfrac{1+\delta\sigma+(1+\delta)c}{2+\delta+\delta\mu+\delta\sigma}, & \text{if } \dfrac{1+\delta\sigma}{1+\delta\mu+\delta\sigma} < c \leq 1 + \dfrac{\delta}{1+\delta}\mu, \\ 1, & \text{otherwise.} \end{cases}$$

$$v_M^* = \begin{cases} \dfrac{1+\delta\sigma+(1+\delta)c}{2+\delta+\delta\mu+\delta\sigma}, & \text{if } c \leq \dfrac{(1+\delta\sigma)\delta\sigma}{2+\delta+\delta\mu-\delta^2\sigma}, \\ \dfrac{c}{\delta\sigma}, & \text{if } \dfrac{(1+\delta\sigma)\delta\sigma}{2+\delta+\delta\mu-\delta^2\sigma} < c \leq \delta\sigma, \\ 1, & \text{otherwise.} \end{cases}$$

The number of consumers opted-in with and without privacy rights is $1 - v_M^B$ and $1 - v_M^*$ respectively. Therefore, we need to prove that $v_M^B \leq v_M^*$. Notice that both v_M^B and v_M^* increase with c and are in $[0,1]$. This implies for $c \leq 1 + \delta/(1+\delta)\cdot\mu$, we have

$$v_M^B \leq \frac{1+\delta\sigma+(1+\delta)c}{2+\delta+\delta\mu+\delta\sigma} \leq v_M^*.$$

Moreover, for $c > 1 + \delta/(1+\delta)\cdot\mu$, Assumption 1 implies that $c > \delta\sigma$, and thus $v_M^B = v_M^* = 1$. This completes the proof for the claim regarding to opt-in.

Next, notice that without privacy rights, the first-period demand is equal to the number of consumers who opt-in, $1-v_M^B$; with privacy rights, the first period demand is equal to $1-v_M^*$ for $c<\delta\sigma/2$ and $1/2$ otherwise. Therefore, for $c<\delta\sigma/2$, the first-period demand is higher without privacy rights. Meanwhile, it is straightforward to show that v_M^B is continuous and increasing with c and $v_M^B=1$ for $c>1+\delta/(1+\delta)\cdot\mu$. This implies that for $c\geq\delta\sigma/2$, there exists a threshold, under which, the first-period demand is higher without privacy rights, and above which, the first-period demand is higher with privacy rights. ■

Proof of Corollary 2:

Proof. First, let's calculate consumer welfare C_M^B and the firm's profit π_M^B in the benchmark case, given which, we have the social welfare, $S_M^B=C_M^B+\pi_M^B$.

If $0\leq c\leq(2-\delta+2\delta\sigma)/(2-\delta+2\delta\mu+2\delta\sigma)$,

$$C_M^B=\int_{v_M^B(P_1^B)}^1\left[v-P_1^B-c+\delta((1+\mu)v-r_M^B(v)-c)\right]dv+\delta\int_{r_M^B(P_1^B)+c}^{v_M^B(P_1^B)}(v-r_M^B(P_1^B)-c)dv,$$

$$\pi_M^B=\int_{v_M^B(P_1^B)}^1(P_1^B+\delta r_M^B(v))dv+\delta\left[v_M^B(P_1^B)-(r_M^B(P_1^B)+c)\right]r_M^B(P_1^B).$$

If $(2-\delta+2\delta\sigma)/(2-\delta+2\delta\mu+2\delta\sigma)<c<1+\delta/(1+\delta)\cdot\mu$,

$$C_M^B=\int_{v_2^B}^1\left[v-P_2^B-c+\delta((1+\mu)v-r_M^B(v)-c)\right]dv,$$

$$\pi_M^B=\int_{v_2^B}^1(P_2^B+\delta r_M^B(v))dv.$$

If $c\geq1+\delta/(1+\delta)\cdot\mu$, no consumer purchases in either period.

$$C_M^B=\pi_M^B=0.$$

Next, we calculate consumer surplus C_M^* and firm profit π_M^* with privacy rights, given which, the social welfare $S_M^*=C_M^*+\pi_M^*$. There are four cases to consider as implied by Proposition 1.

First, if $0\leq c\leq\delta\sigma/2$, we have that

$$C_M^*=\int_{v_1^*}^1\left[v-P_1^*-c+\delta\sigma v\right]dv,$$

$$\pi_M^* = \int_{v_1^*}^{1} \left[P_1^* + \delta\left((\mu - \sigma + 1)v - c\right)\right] dv.$$

Second, if $\delta\sigma/2 < c \leq \delta\sigma/\sqrt{2}$, we have that

$$C_M^* = \int_{\frac{c}{\delta\sigma}}^{1} \left(v - \frac{1}{2} - c + \delta\sigma v\right) dv + \int_{\frac{1}{2}}^{\frac{c}{\delta\sigma}} \left(v - \frac{1}{2}\right) dv$$

$$\pi_M^* = \int_{\frac{c}{\delta\sigma}}^{1} \left[\frac{1}{2} + \delta\left((\mu - \sigma + 1)v - c\right)\right] dv + \int_{\frac{1}{2}}^{\frac{c}{\delta\sigma}} \frac{1}{2} dv.$$

Third, if $\delta\sigma/\sqrt{2} < c < \delta\sigma$, we have that

$$C_M^* = \int_{\frac{c}{\delta\sigma}}^{1} \left[v - \frac{1}{2} - c + \delta\left((1+\sigma)v - \frac{1}{2}\right)\right] dv + (1+\delta)\int_{\frac{1}{2}}^{\frac{c}{\delta\sigma}} \left(v - \frac{1}{2}\right) dv$$

$$\pi_M^* = \int_{\frac{c}{\delta\sigma}}^{1} \left[\frac{1}{2} + \delta\left((\mu - \sigma)v + \frac{1}{2} - c\right)\right] dv + (1+\delta)\int_{\frac{1}{2}}^{\frac{c}{\delta\sigma}} \frac{1}{2} dv.$$

Lastly, if $c \geq \delta\sigma$, we have that

$$C_M^* = (1+\delta)\int_{\frac{1}{2}}^{1} \left(v - \frac{1}{2}\right) dv,$$

$$\pi_M^* = (1+\delta)\left(\frac{1}{2}\right)^2,$$

which do not depend on c.

• **Consumer welfare:** First notice that for $c \geq 1 + \delta/(1+\delta) \cdot \mu$, $C_M^* > 0 = C_M^B$. We only need to show that for $c = 0$, $C_M^* < C_M^B$. Then, by the continuity of C_M^* with respect to c except at $c = \delta\sigma/\sqrt{2}$ and continuity of C_M^B with respect to c, we have proved the first half of the original statement in this bullet point. In fact, under $c = 0$, we have that

$$C_M^B - C_M^* = \frac{H(\mu)}{8(2+\delta+\delta\mu+\delta\sigma)^2(4-\delta+2\delta\mu+2\delta\sigma)^2},$$

where
$$\begin{aligned} H(\mu) \equiv{} & \delta\left(48 + 44\delta - 12\delta^3 + \delta^4\right) + 2\delta^2\left(32 + 26\delta + 14\delta^2 - 3\delta^3\right)\sigma + \delta^3\left(20 - 12\delta + 13\delta^2\right)\sigma^2 \\ & + 4(2-3\delta)\delta^4\sigma^3 + 4\delta^5\sigma^4 + \mu^3\left(-8(-4+\delta)\delta^4 + 32\delta^5\sigma\right) \\ & + \mu^2\left(\delta^3\left(116 + 16\delta - 15\delta^2\right) + 8\delta^4(17+5\delta)\sigma + 20\delta^5\sigma^2\right) \end{aligned}$$

$$+\mu\left(2\delta^2\left(64+40\delta-14\delta^2-3\delta^3\right)+2\delta^3\left(76+64\delta+\delta^2\right)\sigma+8\delta^4(2+3\delta)\sigma^2-8\delta^5\sigma^3\right).$$

We only need to prove that $H(\mu)>0$. In fact, notice that $\mu\geq 1/2+(2+\delta)\sigma$ by Assumption 1.

$$H'''(\mu)=48\delta^4(4-\delta+4\delta\sigma)>0,$$

$$H''(1/2+(2+\delta)\sigma)=2\delta^3\left(116+64\delta-27\delta^2\right)+16\delta^4\left(41+17\delta-3\delta^2\right)\sigma+8\delta^5(53+24\delta)\sigma^2>0.$$

These two together imply that $H''(\mu)>0$. Moreover,

$$\begin{aligned}H'(1/2+(2+\delta)\sigma)=&\delta^2\left(128+196\delta+12\delta^2-27\delta^3\right)+2\delta^3\left(308+376\delta+43\delta^2-27\delta^3\right)\sigma\\&+4\delta^4\left(236+239\delta+44\delta^2-6\delta^3\right)\sigma^2+8\delta^5\left(57+53\delta+12\delta^2\right)\sigma^3>0.\end{aligned}$$

This in turn implies that $H'(\mu)>0$. Moreover,

$$\begin{aligned}H(1/2+(2+\delta)\sigma)=&\delta\left(48+108\delta+69\delta^2-18\delta^3-\frac{27\delta^4}{4}\right)+\delta^2\left(320+648\delta+346\delta^2-33\delta^3-27\delta^4\right)\sigma\\&+\delta^3\left(788+1396\delta+690\delta^2+22\delta^3-27\delta^4\right)\sigma^2\\&+4\delta^4\left(210+326\delta+157\delta^2+18\delta^3-2\delta^4\right)\sigma^3\\&+4\delta^5\left(81+114\delta+53\delta^2+8\delta^3\right)\sigma^4>0.\end{aligned}$$

This implies that $H(\mu)>0$.

• **Firm profit:** First notice that for $c\geq 1+\delta/(1+\delta)\cdot\mu$, $\pi_M^*>0=\pi_M^B$. We next show that for for $c=0$, $\pi_M^*>\pi_M^B$. Then, by the continuity of π_M^* and π_M^B with respect to c, we have proved the original statement. In fact, under $c=0$, we have that

$$\begin{aligned}\pi_M^*-\pi_M^B&=\frac{\delta\left(2+2\delta+\delta^2\left(-1+3\sigma-2\sigma^2\right)+\delta\mu(4-\delta+4\delta\sigma)\right)}{(2+\delta(1+\mu+\sigma))(16+\delta(-4+8\mu+8\sigma))}\\&\geq\frac{\delta\left(2+4\delta-\frac{3\delta^2}{2}+\delta\left(8+7\delta-\delta^2\right)\sigma+2\delta^2(3+2\delta)\sigma^2\right)}{(2+\delta(1+\mu+\sigma))(16+\delta(-4+8\mu+8\sigma))}>0.\end{aligned}$$

where the first inequality is due to two observations: (1) $\pi_M^*-\pi_M^B$ increases with μ, (2) $\mu\geq 1/2+(2+\delta)\sigma$ by Assumption 1. Therefore, for c high and low, we have $\pi_M^*>0=\pi_M^B$.

Lastly, let's show that for $c = \delta\sigma$, $\pi_M^B > \pi_M^*$, which implies that for intermediate c, we have $\pi_M^B > \pi_M^*$. There are three cases to consider according to Proposition 2. First notice that $c = \delta\sigma < 1 + \delta\mu/(1+\delta)$ always holds under Assumption 1.

1. $c = \delta\sigma \le (2-\delta+2\delta\sigma)/(2-\delta+2\delta\mu+2\delta\sigma)$, or equivalently, $\mu \le (2-\delta)/(2\delta^2\sigma) + 1/2 - \sigma$.

   In this case, it is easy to show that $\pi_M^B - \pi_M^* = H_1(\mu)/(16-4\delta+8\delta\mu+8\delta\sigma)$, where $H_1(\mu)$ is a second-order polynomial of μ that satisfies that $H_1''(\mu) > 0$, $H_1'(1/2+(2+\delta)\sigma) > 0$ and $H_1(1/2+(2+\delta)\sigma) > 0$. This implies that $H_1(\mu) > 0$ for $\mu \ge 1/2+(2+\delta)\sigma$ and thus $\pi_M^B - \pi_M^* > 0$.

2. $(2-\delta+2\delta\sigma)/(2-\delta+2\delta\mu+2\delta\sigma) < c = \delta\sigma \le (1+\delta\sigma)/(1+\delta\mu+\delta\sigma)$, or equivalently, $(2-\delta)/(2\delta^2\sigma) + 1/2 - \sigma < \mu \le 1/(\delta^2\sigma) - \sigma$.

   By Assumption 1, we have $\mu \ge (3+\delta)\sigma$, which combined with $\mu \le 1/(\delta^2\sigma) - \sigma$, implies that $\delta\sigma \le 1/\sqrt{4+\delta} < 1/2$. This further implies that $\mu > (2-\delta)/(2\delta^2\sigma) + 1/2 - \sigma > 3/(2\delta) - 1/2$. We have $\pi_M^B - \pi_M^* = H_2(\mu)$, where $H_2(\mu)$ is linear of μ that satisfies that $H_2'(\mu) = \delta/2[1-(\delta\sigma)^2] > \delta/2[1-(1/2)^2] > 0$. Moreover, utilizing $\delta\sigma < 1/2$, it is easy to show that $H_2(1/2+(2+\delta)\sigma) > (2\delta-1)/4$ and $H_2(3/(2\delta)-1/2) > -(2\delta-1)/4$. These two together implies that $H_2(\mu) > 0$, for $\mu > \max\{1/2+(2+\delta)\sigma, 3/(2\delta)-1/2\}$. This implies that $\pi_M^B - \pi_M^* > 0$.

3. $(1+\delta\sigma)/(1+\delta\mu+\delta\sigma) < c = \delta\sigma < 1 + \delta\mu/(1+\delta)$, or equivalently, $\mu > 1/(\delta^2\sigma) - \sigma$. In this case, it is easy to show that $\pi_M^B - \pi_M^* = H_3(\mu)/[4(2+\delta+\delta\mu+\delta\sigma)]$, where $H_3(\mu)$ is a second-order polynomial of μ that satisfies that $H_3''(\mu) > 0$, $H_3'(1/2+(2+\delta)\sigma) > 0$ and $H_3(1/2+(2+\delta)\sigma) > 0$. This implies that $H_3(\mu) > 0$ for $\mu \ge 1/2+(2+\delta)\sigma$ and thus $\pi_M^B - \pi_M^* > 0$.

• **Social welfare:** First notice that for $c \ge 1 + \delta/(1+\delta)\cdot\mu$, $S_M^* > 0 = S_M^B$. We only need to show that for for $c = 0$, $S_M^* < S_M^B$. Then, by the continuity of S_M^* and S_M^B with respect to c, we have proved the original statement. In fact, under $c = 0$, we have that

$$S_M^B - S_M^* = \frac{\delta H_4(\mu)}{8(2+\delta+\delta\mu+\delta\sigma)^2(4-\delta+2\delta\mu+2\delta\sigma)^2},$$

where we have $H_4(\mu)$ is a third-order polynomial of μ that satisfies that $H_4'''(\mu) > 0$, $H_4''(1/2+(2+\delta)\sigma) > 0$, $H_4'(1/2+(2+\delta)\sigma) > 0$ and $H_4(1/2+(2+\delta)\sigma) > 0$. This implies that $H_4(\mu) > 0$ for $\mu \ge 1/2+(2+\delta)\sigma$ and thus $S_M^B - S_M^* > 0$ under $c = 0$. ■

Proof of Corollary 4:

Proof. The statement regarding to first-period price and demand is obvious. For the statement regarding to second-period price and opt-in, we prove them below.

- **Second-period price:**

$$r_C^*(v) = \min\{(\mu-\sigma)v - c, s\} \le (\mu-\sigma)v - c \le (\mu-\sigma)v + \min\{v-c, 0\} = r_C^B(v).$$

- **Opt-in:** By comparing $v_C^* = \min\{[\delta s + (1+\delta)c]/(\delta\mu), c/(\delta\sigma)\}$ and $v_C^B = c/(1+\delta\sigma)$, it is straightforward to prove the statement. ■

Proof of Corollary 5:

Proof. For consumer surplus and firm profit with privacy rights, we calculate them in the proof of Corollary 6. The social welfare $S_C^* = C_C^* + n\pi_C^*$. Here, we calculate the welfare of a consumer with valuation v, $c_C^B(v)$, the total consumer welfare C_C^B and the firm's profit π_C^B in the benchmark case, and the social welfare $S_C^B = C_C^B + n\pi_C^B$. In fact,

$$c_C^B(v) = \max\left\{v - c + \delta\left((1+\mu)v - c - r_C^B(v)\right), 0\right\}$$

$$C_C^B = \int_0^1 c_C^B(v)dv = \int_{\frac{c}{1+\delta\sigma}}^1 [v - c + \delta\left((1+\mu)v - c - r_C^B(v)\right)]dv,$$

$$\pi_C^B = \frac{\delta}{n}\int_{\frac{c}{1+\delta\sigma}}^1 r_C^B(v)dv.$$

- **Consumer surplus:** With privacy rights, the welfare of a consumer with valuation v is,

$$c_C^*(v) = \max\{v - c + \delta((1+\mu)v - c - r_C^*(v)), (1+\delta)v\}$$

$$\ge \max\left\{v - c + \delta\left((1+\mu)v - c - r_C^B(v)\right), 0\right\} = c_C^B(v),$$

where we have utilized $r_C^*(v) \le r_C^B(v)$.

- **Firm profit:** When $c = 0$, we have $v_C^B = v_C^* = 0$. Then $r_C^*(v) \le r_C^B(v)$ implies that

$$\pi_C^* = \frac{\delta}{n}\int_0^1 r_C^*(v)dv \le \frac{\delta}{n}\int_0^1 r_C^B(v)dv = \pi_C^B.$$

By continuity of π_C^* and π_C^B respect to c, we have that when c is relatively low, $\pi_C^* \le \pi_C^B$.

When $c \geq 1+\delta\sigma$, we have $\pi_C^B = 0$. Similarly, v_C^* increases linearly with c when c is sufficiently high, $v_C^* = 1$ and $\pi_C^* = 0 = \pi_C^B$.

Lastly, when $s < (\mu-(1+\delta)\sigma)/(\delta\sigma)$ and $1+\delta\sigma \leq c < (\delta\mu-\delta s)/(1+\delta)$, we have that

$$\pi_C^* = \frac{s[\delta\mu-(1+\delta)c-\delta s]}{n\mu} > 0 = \pi_C^B.$$

$1+\delta\sigma \leq c < (\delta\mu-\delta s)/(1+\delta)$ implies that $\mu > 1+1/\delta+(1+\delta)\sigma$.

- **Social welfare:** When $c \leq \min\{\delta\sigma s/[\mu-(1+\delta)\sigma], \delta\sigma, 1\}$, we have that

$$S_C^B = \frac{(1+\delta\sigma-c)((1+\delta\sigma)(1+\delta+\delta\mu)-(1+\delta-\delta\mu+2\delta(1+\delta)\sigma)c)}{2(1+\delta\sigma)^2},$$

$$S_C^* = \frac{1}{2}\left(1+\delta-2c(1+\delta)+\delta\mu+\frac{c^2(2(1+\delta)\sigma-\mu)}{\delta\sigma^2}\right).$$

We have that

$$S_C^B - S_C^* = \frac{c^2(\mu+2\delta\mu\sigma-(1+\delta)\sigma(2+3\delta\sigma))}{2\delta\sigma^2(1+\delta\sigma)^2},$$

which is greater than zero if and only if

$$\mu > \frac{(1+\delta)\sigma(2+3\delta\sigma)}{1+2\delta\sigma}.$$

Notice that under Assumption 1, we have $\mu \geq (3+\delta)\sigma$ and

$$(3+\delta)\sigma > \frac{(1+\delta)\sigma(2+3\delta\sigma)}{1+2\delta\sigma} \Leftrightarrow 1-\delta+(3-\delta)\delta\sigma > 0,$$

which always hold. This proves that $S_C^B > S_C^*$ for $c \leq \min\{\delta\sigma s/[\mu-(1+\delta)\sigma], \delta\sigma, 1\}$.

On the other hand, when c is sufficiently high, we have

$$S_C^* = \frac{1+\delta}{2} > 0 = S_C^B.$$

■

Proof of Corollary 6:

Proof. The statement on opt-in is obvious based on the expression of v_C^*. Below, we prove the statements on consumer surplus and firm profit. Consumer surplus C_C^* and each firm's profit π_C^* are respectively,

$$C_C^* = \int_{v_C^*}^1 [v-c+\delta((1+\mu)v-c-r_C^*(v))]dv + (1+\delta)\int_0^{v_C^*} vdv,$$

$$\pi_C^* = \frac{\delta}{n}\int_{v_C^*}^{1} r_C^*(v)dv.$$

C_C^* and π_C^* depend on v_C^* and $r_C^*(v)$, which take different forms depending on the parameters. Therefore, we have four cases to consider.

First, $s \le [\mu - (1+\delta)\sigma]c/(\delta\sigma)$ and $c < \delta(\mu - s)/(1+\delta)$, under which case, $v_C^* = (\delta s + (1+\delta)c)/(\delta\mu) < 1$ and $r_C^*(v) = (\mu-\sigma)v - c$. We have,

$$\frac{\partial C_C^*}{\partial c} = \frac{(1+\delta)((1+\delta)c + \delta \mathrm{s} - \delta\mu)}{\delta\mu} < 0,$$

where the last inequality is due to $c < \delta(\mu - s)/(1+\delta)$.

$$\frac{\partial \pi_C^*}{\partial c} = -\frac{(1+\delta)s}{n\mu} < 0.$$

Second, $[\mu - (1+\delta)\sigma]c/(\delta\sigma) < s \le \mu - \sigma - c$, which implies that $c < \delta\sigma$. Under this case, $v_C^* = c/(\delta\sigma)$, and $r_C^*(v) = (\mu-\sigma)v - c$ for $v_C^* \le v < (c+s)/(\mu-\sigma)$ and $r_C^*(v) = s$ for $(c+s)/(\mu-\sigma) \le v \le 1$. We have,

$$\frac{\partial C_C^*}{\partial c} = -1 - \delta + \left(\frac{\delta}{\mu-\sigma} + \frac{1}{\delta\sigma}\right)c + \frac{\delta \mathrm{s}}{\mu-\sigma} \le \frac{c - \delta\sigma}{\delta\sigma} < 0,$$

where the second last inequality is due to $s \le \mu - \sigma - c$ and the last inequality is due to $c < \delta\sigma$.

$$\frac{\partial \pi_C^*}{\partial c} = -\frac{\delta^2\sigma^2 s + c(\mu - (1+\delta)\sigma)^2}{n\delta(\mu-\sigma)\sigma^2} < 0.$$

Thirdly, $s > \mu - \sigma - c$ and $c < \delta\sigma$. Under this case, $v_C^* = c/(\delta\sigma)$, and $r_C^*(v) = (\mu-\sigma)v - c$ for $v_C^* \le v \le 1$. We have,

$$\frac{\partial C_C^*}{\partial c} = \frac{c - \delta\sigma}{\delta\sigma} < 0,$$

where the last inequality is due to $c < \delta\sigma$.

$$\frac{\partial \pi_C^*}{\partial c} = -\frac{\delta}{n} - \frac{(\mu - (1+2\delta)\sigma)c}{n\delta\sigma^2} < 0.$$

Lastly, $c \ge \delta(\mu - s)/(1+\delta)$ and $c \ge \delta\sigma$, under which case, $v_C^* \ge 1$. We have $C_C^* = \pi_C^* = 0$.

By continuity of C_C^* and π_C^* with respect to c, we have proved the corollary. ■

B. Analysis of Model Based on Browsing Data

We use subscript T to denote this case.

With Privacy Rights: We consider the equilibrium where consumers with $v \geq v_T^*$ will opt-in for data collection in the first period; those with $v < v_T^*$ will not opt-in. We can follow exactly the same procedure as the main model to analyze the equilibrium except that p_1 is set to zero. We have

$$v_T^* = \frac{c}{\delta\sigma}.$$

The second-period price for the basic good is $p_{2T}^* = 1/2$ if $c \geq \delta\sigma/\sqrt{2}$ and $p_{2T}^* = 1$ otherwise; the second-period price for the personalized good is $r_T^*(v, p_{2T}^*) = r_M^*(v, p_{2T}^*) = (\mu - \sigma)v + \min\{v, p_{2T}^*\} - c$.

Without Privacy Rights: Next, we consider the benchmark case when consumers do not have privacy rights. Following a similar analysis with the monopoly case, one can show that the personalized price is:

$$r_T^B(v) = (1 + \kappa_L)v = (1 + \mu - \sigma)v.$$

Consumers with $v \geq v_T^B$ make a purchase in the first period, where

$$v_T^B = \max\left\{\frac{2+\delta}{2-\delta+2\delta\sigma}, \frac{1+\delta}{1+\delta\sigma}\right\}c.$$

If $\sigma < 1$, the firm offers a basic good in the second period to those with $v < v_T^B$ at the price of $p_{2T}^B = (v_T^B - c)/2$; otherwise, if $\sigma \geq 1$, we have $v_T^B \leq c$ so consumers with $v < v_T^B$ will not make a purchase in the second period.

C. Relaxing Price Bound in Competition

We analyze the equilibrium without privacy rights. Consumers with valuation $v \geq v_c^B$ purchase in the first period, where we use subscript "c" to differentiate with the notation in the main model. We will verify this threshold structure below.

Second Period: Consider a consumer with $v \geq v_c^B$ who has purchased from firm j in the first period. In the second period, the consumer learns her κ. If the consumer continue to purchase from firm j, she pays $r^j(v)$ and expects utility of $(1+\kappa)v - r^j(v) - c$; if she purchases from firm k, she pays p_2^k and expects utility of $v - p_2^k - c$; she can also take the outside option of zero. Therefore, firm j's profit from this consumer is

$$r^j(v)\Pr\left[(1+\kappa)v - r^j(v) - c \geq \max_k \left\{v - p_2^k - c, 0\right\}\right] \\ + p_2^j \Pr\left[v - p_2^j - c > \max_{k \neq j}\left\{v - p_2^k - c, (1+\kappa)v - r^j(v) - c, 0\right\}\right].$$

Notice that consumers who opted-in cannot choose the basic good anymore, so there is no issue of cannibalization. Consequently, competition always drives p_2^k to be $p_{2c}^B = 0$. To determine the equilibrium personalized price $r_c^B(v)$, there are two cases to consider. First, if $v \geq c/(\mu - 3\sigma + 1)$, it is optimal for the firm to satisfy both κ_H and κ_L consumers, and we have $r_c^B(v) = (\mu - \sigma)v + \min\{v - c, 0\}$. Otherwise, if $v < c/(\mu - 3\sigma + 1)$, it is optimal for the firm to only satisfy κ_H consumers, and we have $r_c^B(v) = (\mu + \sigma)v + \min\{v - c, 0\} = (\mu + \sigma + 1)v - c$, where the second equation is due to $v < c/(\mu - 3\sigma + 1) < c$ under Assumption 1.

First Period: Bertrand competition among firms drives their profit to be zero. There are two cases to consider.

First, if $v_c^B \geq c/(\mu - 3\sigma + 1)$, all consumers opted-in will be offered $r_c^B(v) = (\mu - \sigma)v + \min\{v - c, 0\}$, and $p_{1c}^B = -\delta \int_{v_c^B}^1 r_c^B(v)dv$. A consumer of valuation v will purchase in the first period if and only if her expected utility from the two periods, $v - p_{1c}^B - c + \delta \mathrm{E}[(1+\kappa)v - r_c^B(v) - c] \geq 0$, which is equivalent to

$$v \geq \frac{p_{1c}^B + c}{1 + \delta\sigma} = \frac{c - \delta \int_{v_c^B}^1 \left[(\mu - \sigma)v + \min\{v - c, 0\}\right] dv}{1 + \delta\sigma} = v_c^B,$$

where the inequality above verifies the threshold structure and the second equality above is by definition. It is straightforward to calculate consumer surplus:

$$C_c^B = \int_{v_c^B}^1 \left[(1 + \delta\sigma)(v - v_c^B) + \delta \max\{v - c, 0\}\right] dv.$$

Second, if $v_c^B < c/(\mu - 3\sigma + 1)$, consumers with $v \in [v_c^B, c/(\mu - 3\sigma + 1)]$ will be offered $r_c^B(v) = (\mu + \sigma + 1)v - c$ so that only κ_H consumers purchase the personalized good; consumers with $v \in (c/(\mu - 3\sigma + 1), 1]$ will be offered $r_c^B(v) = (\mu - \sigma)v + \min\{v - c, 0\}$ so that all of them purchase the personalized good. Therefore, we have

$$p_{1c}^B = -\frac{\delta}{2}\int_{v_c^B}^{\frac{c}{\mu-3\sigma+1}} [(\mu + \sigma + 1)v - c]\, dv - \delta\int_{\frac{c}{\mu-3\sigma+1}}^{1} [(\mu - \sigma)v + \min\{v - c, 0\}]\, dv.$$

A consumer of valuation v will purchase in the first period if and only if her expected utility from the two periods, $v - p_{1c}^B - c + \delta/2[(1 + \mu + \sigma)v - r_c^B(v) - c] \geq 0$, which is equivalent to

$$v \geq p_{1c}^B + c = c - \frac{\delta}{2}\int_{v_c^B}^{\frac{c}{\mu-3\sigma+1}} [(\mu + \sigma + 1)v - c]\, dv - \delta\int_{\frac{c}{\mu-3\sigma+1}}^{1} [(\mu - \sigma)v + \min\{v - c, 0\}]\, dv = v_c^B,$$

where the inequality above verifies the threshold structure and the second equality above is by definition. If the solution of v_c^B to the above equation is less than zero, we set $v_c^B = 0$. That is, all consumers make a purchase. It is straightforward to calculate consumer surplus:

$$C_c^B = \int_{v_c^B}^{\frac{c}{\mu-3\sigma+1}} (v - v_c^B)dv + \int_{\frac{c}{\mu-3\sigma+1}}^{1} \left[(1 + \delta\sigma)v - v_c^B + \delta\max\{v - c, 0\}\right] dv.$$

D. Competition among Ex-ante Asymmetric/Differentiated Firms

We extend our main model by introducing ex-ante asymmetry/differentiation to the n firms under competition. Particularly, consider n consumer segments. For a consumer in segment i, her valuation of firm j's basic good is $v - g(1 - \delta_{ij})$, where $\delta_{ii} = 1$ and $\delta_{ij} = 0$ for $i \neq j$. That is, consumers in segment i prefers firm i than all other firms, which they value equally. Moreover, her valuation of firm j's personalized good is $(1 + \kappa)v - g(1 - \delta_{ij})$. This implies that consumers' relative preference between any two firms remains the same whether both of them provide personalization or neither does. Lastly, it is assumed that consumer segmentation is common knowledge at the beginning of the game and price discrimination based on consumer segmentation is allowed in both periods.[20] This assumption can be justified by allowing firms to garner consumer segmentation information via traditional market research or loyalty programs. This segment-level information is in contrast

[20] Without this assumption, the equilibrium price is either at monopoly level or only admits mixed strategies.

with the consumer personal data we have considered so far that allows the firm to get individual consumers' preference information (v). This assumption also implies that each firm essentially makes separately independent decisions for each consumer segment so that we can without loss of generality focus on just one consumer segment, i in the subsequent analysis. Given this consumer segment fixed, n firms are ex-ante asymmetric, where firm i is more preferred than the rest of the firms.

To connect this setup to our main models, notice that $g \geq 0$ captures the degree of ex-ante differentiation/asymmetry among firms. Our main model of monopoly corresponds to the special case with g sufficiently high (specifically, $g \geq 1+\mu+\sigma$ suffices), while our main model of competition corresponds to the special case with $g=0$.

The following proposition summarizes the equilibrium under relatively low g that ensures the market being relatively competitive.

PROPOSITION 5. *Assume* $0 \leq g \leq \min\left\{\frac{c}{\delta\sigma}, \frac{\delta s+(1+\delta)c}{\delta(\mu-1/2)}\right\}$ *and define:*

$$v_G^* = \min\left\{\frac{c}{\delta\sigma}, \max\left\{\frac{\delta(s+g)+(1+\delta)c}{\delta(\mu+1/2)}, \frac{\delta s+(1+\delta)c}{\delta\mu}\right\}\right\},$$

$$p_{1G}^* = \min\left\{\frac{1}{2}, g\right\},$$

$$p_{2G}^* = \min\left\{\max\left\{\frac{s+g+c}{1+2(\mu-\sigma)}, \frac{v_G^*}{2}\right\}, \frac{1}{2}, g\right\},$$

$$r_G^*(v) = \min\left\{(\mu-\sigma)v - c + p_{2G}^*, s+g\right\}.$$

In an ex-ante differentiated market with privacy rights (under Assumption 1),

- *consumers in segment i with valuation $v \geq v_G^*$ both purchase and opt-in from firm i in the first period by paying p_{1G}^*. They continue to purchase personalized service from the same firm in the second period by paying $r_G^*(v)$;*
- *consumers in segment i with valuation $v_G^* > v \geq p_{1G}^*$ make a purchase but opt-out of data collection from firm i in the first period by paying p_{1G}^*. They purchase basic service from the same firm by paying p_{2G}^* in the second period;*

- *consumers in segment i with valuation $p_{1G}^* > v \geq p_{2G}^*$ do not make a purchase in the first period. They purchase basic service from firm i by paying p_{2G}^* in the second period;*
- *consumers with valuation $v < p_{2G}^*$ do not purchase in either period.*

Proof. We solve the equilibrium by backward induction. In equilibrium, consumers with valuation $v \geq v_G^*(p_1^i)$ purchase and opt-in during the first period, where we use subscript G to denote "general market setup". We will verify this threshold structure below.

Second Period: Competition drives firms other than i to price both basic and personalized good at zero. This implies firm i's profit from a segment-i consumer is

$$r^i(v)\Pr\left[(1+\kappa)v - r^i(v) - c \geq \max\left\{v-g, (1+\kappa)v - g - c - s, v - p_2^i, 0\right\}\right]$$
$$+ p_2^i \Pr\left[v - p_2^i > \max\left\{v - g, (1+\kappa)v - g - c - s, (1+\kappa)v - r^i(v) - c, 0\right\}\right].$$

Similarly, one can show that firm i's optimal personalized price is:

$$r_G^*(v, p_2^i) = \min\left\{(\mu-\sigma)v - c + \min\left\{p_2^i, g, v\right\}, s+g\right\}.$$

which incentivizes both κ_L and κ_H consumers to continue to purchase the personalized service from him. One can verify that as $g \to 0$, $r_G^*(v, p_2^i) \to r_C^*(v)$, and for g sufficiently large, $r_G^*(v, p_2^i) \to r_M^*(v, p_2^i)$.

Firm i's profit maximization problem in the second period is

$$\max_{0 \leq p_2^i \leq g} \int_{v_G^*(p_1^i)}^1 r_G^*(v, p_2^i)dv + p_2^i \max\left\{v_G^*(p_1) - p_2, 0\right\},$$

where we have made the restriction that $0 \leq p_2^i \leq g$, because $p_2^i = g$ weakly dominates $p_2^i > g$ in terms of the firm's profit. In fact, if $p_2^i > g$, consumers who opted out will choose other firms' basic good at zero price, and for consumers who opted-in, we have $r_G^*(v, p_2^i) = r_G^*(v, g)$ for $p_2^i > g$. To simplify exposition, we will focus on the case with g relatively small—to be more specific, $g \leq v_G^*(p_1)$. We will express this restriction in terms of model primitives after we solve the equilibrium. One can show that the optimal choice of p_2^i is:

$$p_{2G}^*(p_1) = \min\left\{\max\left\{\frac{s+g+c}{1+2(\mu-\sigma)}, \frac{v_G^*(p_1)}{2}\right\}, \frac{1}{2}, g\right\}.$$

First Period: In the first period, a consumer with valuation v will make a purchase and opt-in if and only if,

$$v - p_1 - c + \delta\left((1+\mu)v - r_G^*(v, p_{2G}^*(p_1)) - c\right) \geq \max\{v - p_1, 0\} + \delta\left(v - p_{2G}^*(p_1)\right).$$

We focus on the case with g relatively small so that $v_G^*(p_{1G}^*) \geq p_{1G}^*$, under which, the above inequality can be reduced as $v \geq v_G^*$, where

$$v_G^* = \min\left\{\frac{c}{\delta\sigma}, \max\left\{\frac{\delta(s+g)+(1+\delta)c}{\delta(\mu+1/2)}, \frac{\delta s+(1+\delta)c}{\delta\mu}\right\}\right\}.$$

Competition drives firms other than i to offer a zero price in the first period. This implies that consumers will purchase from firm i if and only if $p_1 \leq g$. The firm's profit maximization problem in the first period is as the following:

$$\max_{g \geq p_1 \geq 0} \int_{v_G^*}^{1} \left[p_1 + \delta r_G^*(v, p_{2G}^*)\right] dv + p_1(v_G^* - p_1) + \delta p_{2G}^*\left(v_G^* - p_{2G}^*\right),$$

which leads to the following optimal first-period price,

$$p_{1G}^* = \min\left\{\frac{1}{2}, g\right\}.$$

Recall that in the above analysis, we have assumed that $v_G^* \geq g$ and $v_G^* \geq p_{1G}^*$. Given the expression of p_{1G}^*, it suffices to ensure that $v_G^* \geq g$, which, given the expression of v_G^*, is equivalent to:

$$g \leq \min\left\{\frac{c}{\delta\sigma}, \frac{\delta s+(1+\delta)c}{\delta(\mu-1/2)}\right\}.$$

■

Proposition 5 implies that consumer opt-in is reduced as the firms become more ex-ante differentiated, because the equilibrium first- and second-period prices are higher. Figure 14 illustrates the market segmentation, which is very similar to that in Case C of Figure 2a except that here there is a new segment of consumers who opt out and delay their purchase to the second period.

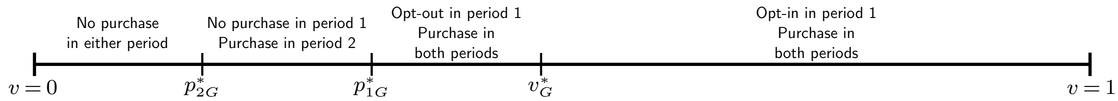

Figure 14 **Equilibrium segmentation for ex-ante differentiated market.**