Online Appendices

Improving Dispute Resolution in Two-sided Platforms The Case of Review Blackmail

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Appendix A: Analysis of the Extended Model with Intermediate Delay

A.1. Centralized Dispute Resolution

We begin from the seller's response to blackmail, which is described in the following proposition. Note that we use the same notation as that in §4.

Proposition 9. *Suppose a malicious consumer enters the system, posts a negative review, and demands a ransom r. Then:*

(1) An equilibrium with $i=n$ exists if and only if $\pi(a_N^n) \geq \max\{\pi(a)-r, \pi(a)-\frac{c}{\delta(1-\mu+\gamma\mu)}\}$.

(2) An equilibrium with $i=s$ exists if and only if $\pi(a_N^s) \leq \min\{\pi(a)-r, \pi(a)+\frac{c-r}{1-\delta(1-\mu+\gamma\mu)}\}$.

(3) An equilibrium with $i=c$ exists if and only if $\pi(a)+\frac{c-r}{1-\delta(1-\mu+\gamma\mu)} \leq \pi(a_N^c) \leq \pi(a)-\frac{c}{\delta(1-\mu+\gamma\mu)}$.

Thus, the result mirrors Proposition 1 (i.e., the corresponding result in the main model), with the difference that γ is replaced by $1-\mu+\gamma\mu$. The same observation holds with regards to the malicious consumer's purchase and ransom strategy.

Proposition 10. *The malicious consumer's equilibrium strategy is described as follows:*

(1) When $c \geq \delta(1-\mu+\gamma\mu)\,[\pi(a)-\pi(a_N^n)]$, the malicious consumer purchases if and only if $p < \pi(a)-\pi(a_N^n)$. He/she then posts a negative review and demands a ransom

$$r^* = \pi(a) - \pi(a_N^n). \tag{11}$$

(2) When $c < \delta(1-\mu+\gamma\mu)\,[\pi(a)-\pi(a_N^n)]$, the malicious consumer purchases if and only if $p < [1-\delta(1-\mu+\gamma\mu)]\,[\pi(a)-\pi(a_N^c)]+c$. He/she then posts a negative review and demands a ransom

$$r^* = [1-\delta(1-\mu+\gamma\mu)]\,[\pi(a)-\pi(a_N^c)]+c. \tag{12}$$

Finally, we note that at the pricing stage, the seller's objective function remains the same as that in the main model (i.e., as given in (3)), with the only difference being that the set of prices where a ransom demand occurs, $\mathcal{P}_C^{pur}$, is now defined by Proposition 10 (instead of Proposition 2). Therefore, Proposition 3 holds unchanged.

A.2. Semi-Decentralized Mechanism.

For the case of the semi-decentralized mechanism, we first present the following result which is analogous to Proposition 4 in the main model.

PROPOSITION 11. *Under the semi-decentralized mechanism, the seller does not remove genuine reviews if and only if* $b \geq \underline{b}_I := (1-\gamma)\mu\,[\pi(a) - \pi\,(a_N^s)]$.

The malicious consumer's equilibrium strategy (which corresponds to Proposition 5 in the main model) is then characterized as follows.

PROPOSITION 12. *The malicious consumer's equilibrium strategy is described as follows:*

(1) When $b \geq \frac{1-(1-\delta)(1-\mu+\gamma\mu)}{1-\delta}\,[\pi(a) - \pi(a_N^n)] > \underline{b}$, *the malicious consumer purchases if and only if* $p < (\pi(a) - \pi(a_N^n))$. *He/she then posts a negative review and demands a ransom*

$$r^* = (\pi(a) - \pi(a_N^n))\,. \tag{13}$$

(2) When $\underline{b} \leq b < \frac{1-(1-\delta)(1-\mu+\gamma\mu)}{1-\delta}\,(\pi(a) - \pi(a_N^n))$, *the malicious consumer purchases if and only if* $p < (1-\delta)\,(\gamma(1-\mu)\,(\pi(a) - \pi(a_N^d)) + b)$. *He/she then posts a negative review and demands a ransom*

$$r^* = (1-\delta)\left[(1-\mu+\gamma\mu)(\pi(a) - \pi(a_N^d)) + b\right]. \tag{14}$$

Next, we note that as in the case of the centralized mechanism, the seller's problem is unchanged with respect to the main model, with the only difference being the definition of the set $\mathcal{P}_D^{pur}$, which is now determined according to Proposition 12. Finally, it is straightforward to show that the platform's optimal penalty is $\underline{b}$, for the same reasons as described in the analysis of the main model.

Appendix B: Supplemental Results

LEMMA 3. *Under the centralized mechanism, the following statements hold:*

(1) If $p_0 \geq \bar{p}_C$, *then* $p^* = p_0$.

(2) If $p_0 < \bar{p}_C$, *then* $p^* \in (0, \bar{p}_C]$.

PROPOSITION 13. *Under the semi-decentralized mechanism, in any equilibrium with* $b < \underline{b}$ *the seller uses the mechanism to remove genuine negative reviews.*

LEMMA 4. *Under the semi-decentralized mechanism, the following statements hold:*

(1) If $p_0 \geq \bar{p}_D$, *then* $p^* = p_0$.

(2) If $p_0 < \bar{p}_D$, *then* $p^* \in (0, \bar{p}_D]$.

Appendix C: Proofs

Proof of Lemma 1

When using the centralized mechanism to remove a non-malicious negative review, the seller incurs cost $c \geq 0$, but the negative review is never removed, since the platform is assumed to never misjudge a genuine review as being malicious). $\square$

Proof of Lemma 2

We calculate the posterior probability using Bayes' Rule. We start with a_P^i for $i = n, c, s$.

$$a_P^i = \frac{\Pr(q=h; R=P; i)}{\Pr(R=P, i)} = \frac{\Pr(R=P \mid q=h, i) \cdot \Pr(q=h)}{\Pr(R=P \mid q=h, i) \cdot \Pr(q=h) + \Pr(R=P \mid q=l, i) \cdot \Pr(q=l)}.$$

As $\Pr(R=P \mid q=l; i) = 0$, we have $a_P^i = 1$ for $i = n, c, s$.

Similarly, for a_0^i, we have:

$$a_0^i = \frac{\Pr(q=h; R=0; i)}{\Pr(R=0, i)} = \frac{\Pr(R=0 \mid q=h, i) \cdot \Pr(q=h)}{\Pr(R=0 \mid q=h, i) \cdot \Pr(q=h) + \Pr(R=0 \mid q=l, i) \cdot \Pr(q=l)}.$$

Note that $R=0$ could occur in two scenarios: when no customer (malicious or regular) purchases, or when a malicious review is removed. In each case, $\Pr(R=0 \mid q, i) = a$ for all q and i. Thus, $a_0^i = a$ for all i.

Finally, for a_N^i, we have:

$$a_N^i = \frac{\Pr(q=h; R=N; i)}{\Pr(R=N, i)} = \frac{\Pr(R=N \mid q=h, i) \cdot \Pr(q=h)}{\Pr(R=N \mid q=h, i) \cdot \Pr(q=h) + \Pr(R=N \mid q=l, i) \cdot \Pr(q=l)}.$$

We have: $\Pr(q=h) = a$ and $\Pr(q=l) = 1-a$. For $\Pr(R=N \mid q, i)$ for $q = h, l$, we consider $i = n$ first:

$$\Pr(R=N \mid q=h, i=n) = \beta \cdot \Pr(R=N \mid q=h, j=M, i=n) + (1-\beta) \cdot \Pr(R=N \mid q=h, j=G, i=n),$$

where $j = M, G$ represents that the customer type malicious or genuine, respectively. If a malicious consumer purchases, leaves a negative review, and the firm does nothing ($i = n$), we have:

$$\Pr(R=N \mid q=h, j=M, i=n) = 1.$$

As for the regular customer ($j = G$), $R = N$ if and only if a regular customer purchases (with probability $1 - \frac{p}{a\theta}$) and has a negative experience (with probability $1-\theta$). Thus,

$$\Pr(R=N \mid q=h, j=G, i=n) = \left(1 - \frac{p}{a\theta}\right)(1-\theta).$$

Then

$$\Pr(R=N \mid q=h, i=n) = \beta(1) + (1-\beta)\left(1 - \frac{p}{a\theta}\right)(1-\theta).$$

Similarly,

$$\Pr(R=N \mid q=l, i=n) = \beta(1) + (1-\beta)\left(1 - \frac{p}{a\theta}\right).$$

Combining the two scenarios, we have

$$a_N^n = \frac{a\left[\beta + (1-\beta)\left(1 - \frac{p}{a\theta}\right)(1-\theta)\right]}{a\left[\beta + (1-\beta)\left(1 - \frac{p}{a\theta}\right)(1-\theta)\right] + (1-a)\left[\beta + (1-\beta)\left(1 - \frac{p}{a\theta}\right)\right]} = \frac{a}{a + (1-a)\frac{\left(\beta + (1-\beta)\left(1 - \frac{p}{a\theta}\right)\right)}{\left(\beta + (1-\beta)\left(1 - \frac{p}{a\theta}\right)(1-\theta)\right)}}.$$

Similarly, for strategy $i=c$, we have

$$a_N^c = \frac{\Pr(R=N \mid q=h, i=c)\cdot \Pr(q=h)}{\Pr(R=N \mid q=h, i=c)\cdot \Pr(q=h)+\Pr(R=N \mid q=l, i=c)\cdot \Pr(q=l)}$$

Note that

$$\begin{aligned}\Pr(R=N \mid q=h, i=c) &= \beta\cdot \Pr(R=N \mid q=h, j=M, i=c)+(1-\beta)\cdot \Pr(R=N \mid q=h, j=G, i=c)\\ &= \beta(1-\gamma\delta)+(1-\beta)\left(1-\frac{p}{a\theta}\right)(1-\theta),\end{aligned}$$

where $(1-\gamma\delta)$ is the probability that the seller's report of a malicious review is not processed correctly and immediately by the platform. Moreover,

$$\Pr(R=N \mid q=l, i=c) = \beta(1-\gamma\delta)+(1-\beta)\left(1-\frac{p}{a\theta}\right).$$

Thus,

$$\begin{aligned}a_N^c &= \frac{a\left[\beta(1-\gamma\delta)+(1-\beta)\left(1-\frac{p}{a\theta}\right)(1-\theta)\right]}{a\left[\beta(1-\gamma\delta)+(1-\beta)\left(1-\frac{p}{a\theta}\right)(1-\theta)\right]+(1-a)\left[\beta(1-\gamma\delta)+(1-\beta)\left(1-\frac{p}{a\theta}\right)\right]}, &(15)\\ &= \frac{a}{a+(1-a)\frac{\left(\beta(1-\gamma\delta)+(1-\beta)\left(1-\frac{p}{a\theta}\right)\right)}{\left(\beta(1-\gamma\delta)+(1-\beta)\left(1-\frac{p}{a\theta}\right)(1-\theta)\right)}}. &(16)\end{aligned}$$

Finally, for strategy $i=s$,

$$a_N^s = \frac{\Pr(R=N \mid q=h, i=s)\cdot \Pr(q=h)}{\Pr(R=N \mid q=h, i=s)\cdot \Pr(q=h)+\Pr(R=N \mid q=l, i=s)\cdot \Pr(q=l)}.$$

Note that

$$\begin{aligned}\Pr(R=N \mid q=h, i=s) &= \beta\cdot \Pr(R=N \mid q=h, j=M, i=s)+(1-\beta)\cdot \Pr(R=N \mid q=h, j=G, i=s)\\ &= \beta(0)+(1-\beta)\left(1-\frac{p}{a\theta}\right)(1-\theta),\end{aligned}$$

where the first term captures the scenario where the firm settles with the malicious customer, so that the negative review is removed by the malicious customer. Similarly,

$$\Pr(R=N \mid q=l, i=s) = \beta(0)+(1-\beta)\left(1-\frac{p}{a\theta}\right).$$

Thus,

$$a_N^s = \frac{a\left[(1-\beta)\left(1-\frac{p}{a\theta}\right)(1-\theta)\right]}{a\left[(1-\beta)\left(1-\frac{p}{a\theta}\right)(1-\theta)\right]+(1-a)\left[(1-\beta)\left(1-\frac{p}{a\theta}\right)\right]} = \frac{a}{a+(1-a)\frac{1}{1-\theta}}.$$

□

Proof of Proposition 1

The conditions under which $i=n$ is an equilibrium are detailed in the discussion before the proposition. In this proof, we focus on the conditions for $i=c$ and $i=s$.

First, $i=s$ is an equilibrium if and only if under the belief that $i=s$, the seller has no incentive to deviate to $i=c$ or $i=n$. We consider these two conditions in turns. First, when the seller faces a negative review and the belief is $i=s$, his net payoff by deviating from $i=s$ to $i=c$ is:

$$\Delta^{c|s} = \gamma\delta\pi(a_0^s)+(1-\gamma\delta)\pi(a_N^s)-c-[\pi(a_0^s)-r].$$

Thus, the seller does not deviate to $i = c$ if and only if $\Delta^{c|s} \leq 0$. As $a_0^s = a$, the condition becomes,

$$\pi(a_N^s) \leq \pi(a) + \frac{c-r}{1-\gamma\delta}. \tag{17}$$

Similarly, the condition that the seller does not deviate to $i = n$ under the belief that $i = s$ is

$$\Delta^{n|s} = \pi(a_N^s) - [\pi(a_0^s) - r] \leq 0,$$

or equivalently,

$$\pi(a_N^s) \leq \pi(a) - r. \tag{18}$$

Combining the two conditions that preclude deviation, that is, (17) and (18), strategy $i = s$ is an equilibrium if and only if

$$\pi(a_N^s) \leq \min\left\{\pi(a) - r,\, \pi(a) - \frac{c}{\gamma\delta}\right\},$$

which corresponds to the second statement in the proposition.

Next, we consider to $i = c$, which is an equilibrium if and only if under this belief, the seller does not have incentive to deviate to $i = n$ and $i = s$. Using the same notation as in the paper, the seller will not deviate to $i = n$ if and only if

$$\Delta^{n|c} = \pi(a_N^c) - [\gamma\delta\pi(a_0^c) + (1-\gamma\delta)\pi(a_N^c) - c] \leq 0.$$

As $a_0^c = a$, the above condition is equivalent to

$$\gamma\delta[\pi(a) - \pi(a_N^c)] - c \geq 0. \tag{19}$$

Similarly, the seller will not deviating to $i = s$ when

$$\Delta^{s|c} = \pi(a_0^c) - r - [\gamma\delta\pi(a_0^c) + (1-\gamma\delta)\pi(a_N^c) - c] \leq 0,$$

that is,

$$(1-\gamma\delta)[\pi(a_N^c) - \pi(a)] - (c-r) \geq 0.$$

Combining this condition with (33), we have that $i = c$ is an equilibrium if and only if:

$$\pi(a_N^c) \in \left[\pi(a) + \frac{c-r}{1-\gamma\delta},\, \pi(a) - \frac{c}{\gamma\delta}\right],$$

which corresponds to the third statement in the proposition. □

Proof of Proposition 2

We prove the result by backward induction. First, assuming the malicious customer has purchased, he will request the equilibrium ransom r^* which is the maximum possible ransom such that $i = s$ is the seller's preferred equilibrium (that is, either $i = s$ is the only equilibrium, or the firm's payoff under $i = s$ is greater than that under $i = c$ or $i = n$ if either is an equilibrium). To identify the relevant conditions, we rearrange Proposition 1 to get the following scenarios:

1. When $c < \gamma\delta(\pi(a) - \pi(a_N^n))$, $i = n$ is not an equilibrium. On the other hand, $i = c$ is an equilibrium if and only if the ransom $r > (1 - \gamma\delta)(\pi(a) - \pi(a_N^c)) + c$. When $i = c$ is the equilibrium, the seller's terminal payoff conditional on a malicious review is

$$\pi^c = (1 - \gamma\delta)\pi(a_N^c) + \gamma\delta\pi(a) - c.$$

On the other hand, $i = s$ is an equilibrium if and only if

$$r \le \min(\pi(a) - \pi(a_N^s),\ (1 - \gamma\delta)(\pi(a) - \pi(a_N^s)) + c).$$

When this condition holds, the seller's terminal payoff conditional on a malicious review is

$$\pi^s = \pi(a) - r.$$

Thus, the sufficient and necessary condition for $i = s$ to be the preferred equilibrium for the seller is that $i = s$ is an equilibrium and $\pi^s \ge \pi^c$, or equivalently,

$$r \le \min\left(\pi(a) - \pi(a_N^s),\ (1 - \gamma\theta)(\pi(a) - \pi(a_N^s)) + c,\ (1 - \gamma\delta)(\pi(a) - \pi(a_N^c)) + c\right).$$

Since $a_N^s < a_N^c < a_N^n$, $(1 - \gamma\theta)(\pi(a) - \pi(a_N^s)) + c > (1 - \gamma\delta)(\pi(a) - \pi(a_N^c)) + c$. Further, when $c < \gamma\delta(\pi(a) - \pi(a_N^n))$, we have $\pi(a) - \pi(a_N^s) > (1 - \gamma\theta)(\pi(a) - \pi(a_N^s)) + c$. Thus, the above condition can be simplified to

$$r \le (1 - \gamma\delta)(\pi(a) - \pi(a_N^c)) + c.$$

This corresponds to the second statement in the proposition.

2. When $c \in [\gamma\delta[\pi(a) - \pi(a_N^n)],\ \gamma\delta(\pi(a) - \pi(a_N^c)))$, both $i = n$ and $i = c$ are equilibria for sufficiently large r. By the first scenario, we know that $i = s$ is an equilibrium and it is preferred by the seller over $i = c$ if and only if

$$r \le (1 - \gamma\delta)(\pi(a) - \pi(a_N^c)) + c. \tag{20}$$

Further, in this scenario, $i = n$ is an equilibrium if and only if $r \ge \pi(a) - \pi(a_N^n)$. In this case, the seller's terminal payoff conditional on a malicious review is $\pi^n = \pi(a_N^n)$, which is less than π^s if and only if $r \le \pi(a) - \pi(a_N^n)$. Thus, $i = s$ is the preferred equilibrium if and only if

$$r \le \min\{(1 - \gamma\delta)(\pi(a) - \pi(a_N^c)) + c,\ \pi(a) - \pi(a_N^n)\},$$

which is equivalent to $r \le \pi(a) - \pi(a_N^n)$ as $a_N^c < a_N^n$ and $c < \gamma\delta(\pi(a) - \pi(a_N^n))$.

Combined, $i = s$ is the preferred equilibrium if and only if (20) holds.

3. When $c \geq \gamma\delta(\pi(a) - \pi(a_N^c))$, $i = c$ is not an equilibrium because $a_N^n > a_N^c$. On the other hand, $i = n$ is an equilibrium if and only if $r > \pi(a) - \pi(a_N^n)$. From the analysis of the previous scenario, it follows that $i = s$ is the preferred equilibrium if and only if

$$r \leq \pi(a) - \pi(a_N^n).$$

Combining this with the second scenario above leads to the equilibrium ransom r^* in the first statement in the proposition.

Next, anticipating that if he purchases the equilibrium ransom will be r^* as described above, the malicious customer makes the purchase if and only if $p < r^*$. Substituting r^* from the first step into this condition leads to the purchase conditions in the proposition. □

Proof of Corollary 1

First, note that from the first statement in Proposition 2, when $\gamma\delta \leq \frac{c}{\pi(a)-\pi(a_N^n)}$ and $p < (1-\gamma\delta)[\pi(a)-\pi(a_N^n)]$, the equilibrium ransom is $r_-^* = \pi(a) - \pi(a_N^n)$.

Next, by the second statement in Proposition 2, when $\gamma\delta > \frac{c}{\pi(a)-\pi(a_N^n)}$, and $p < (1-\gamma\delta)[\pi(a) - \pi(a_N^n)]$, as $a_N^n < a_N^c$, the equilibrium ransom $r_+^* = (1-\gamma\delta)[\pi(a) - \pi(a_N^c)] + c$. Let $\gamma\delta = \frac{c}{\pi(a)-\pi(a_N^n)} + \epsilon$ for $\epsilon > 0$. Thus,

$$r_+^* = \left(1 - \frac{c}{\pi(a) - \pi(a_N^n)} - \epsilon\right)[\pi(a) - \pi(a_N^c)] + c = (1-\epsilon)[\pi(a) - \pi(a_N^c)] + c\left(1 - \frac{\pi(a) - \pi(a_N^c)}{\pi(a) - \pi(a_N^n)}\right).$$

Comparing r_+^* and r_-^*, we have:

$$\begin{aligned} r_+^* - r_-^* &= (1-\epsilon)[\pi(a) - \pi(a_N^c)] + c\left(1 - \frac{\pi(a) - \pi(a_N^c)}{\pi(a) - \pi(a_N^n)}\right) - [\pi(a) - \pi(a_N^n)]. \\ &= [\pi(a_N^n) - \pi(a_N^c)]\left(1 - \frac{c}{\pi(a) - \pi(a_N^n)}\right) - \epsilon[\pi(a) - \pi(a_N^c)]. \end{aligned}$$

By the assumption that $\gamma\delta = \frac{c}{\pi(a)-\pi(a_N^n)} + \epsilon$, we have $\frac{c}{\pi(a)-\pi(a_N^n)} < 1$. In addition, by Lemma 2, $a_N^c > a_N^n$, and hence $\pi(a_N^n) > \pi(a_N^c)$. Therefore, for sufficiently small ϵ, we have $r_+^* - r_-^* > 0$. Put differently, when $\gamma\delta$ increases from $\frac{c}{\pi(a)-\pi(a_N^n)}$ to $\frac{c}{\pi(a)-\pi(a_N^n)} + \epsilon$, r^* increases. Therefore, we have that r^* is not monotonically decreasing in $\gamma\delta$. □

Proof of Proposition 3

We prove the second point first. We show that there exists $\underline{\beta}_C$ such that if $\beta \geq \underline{\beta}_C$, then $p_0 \geq \bar{p}_C$ (and then we have $p^* = p_0$ by Lemma 3). Note first that p_0 does not depend on β. Next, note that a_N^n and a_N^c are both strictly increasing in β (see Lemma 2) and recall that $\pi'(\cdot) > 0$. It follows from Proposition 2 that $\bar{p}_C$ (i.e., the maximum price at which the malicious consumer purchases) is strictly decreasing in β. Therefore, there exists $\underline{\beta}_C \in [0,1]$ such that if $\beta \geq \underline{\beta}_C$ we have $p_0 \geq \bar{p}_C$ and $p^* = p_0$ by Lemma 3.

To prove the first point it suffices to show that if $\beta < \underline{\beta}_C$ the seller's profit function (3) is strictly increasing at any $p < p_0$ and is strictly decreasing at any $p > \bar{p}_C$. Note that according to the proof of Lemma 3, $\Pi_0(p)$ is a concave function which is maximized at p_0. Note first that in the case $\beta < \underline{\beta}_C$, we have $p_0 < \bar{p}_C$ and it follows by concavity of Π_0 that the seller's profit is strictly decreasing at any $p > \bar{p}_C$. Now consider any price $\hat{p} < p_0$. Note that both a_N^n and a_N^c are strictly increasing in p by Lemma 2. Since $\pi'(\cdot) > 0$, it follows by

Proposition 2 that $\mathcal{P}_C^{Pur}$ is a connected set. Thus, for $\hat{p} < p_0$, $\hat{p} \in \mathcal{P}_C^{pur}$ the seller's profit at $\hat{p}$ can be written $\Pi_C(\hat{p}) = \Pi_0(\hat{p}) - \beta(r^*(\hat{p}) - \hat{p})$. The term $\Pi_0(\cdot)$ is strictly increasing at $\hat{p}$ by concavity. Therefore, to show that $\Pi_C(\cdot)$ is strictly increasing at $\hat{p}$ it suffices to show that $\beta(r^*(\hat{p}) - \hat{p})$ is strictly decreasing, and to show the latter it suffices to show that $r^*(\cdot)$ is strictly decreasing at $\hat{p}$. Recall that both a_N^n and a_N^c are strictly increasing in p by Lemma 2. Since $\pi'(\cdot) > 0$, it follows by Proposition 2 that $r^*(\cdot)$ is strictly decreasing at $\hat{p}$, completing the argument.

Proof of Proposition 4

Note that it is straightforward to show that the seller would never use the mechanism to remove genuine positive reviews. In what follows, we consider whether the seller uses the mechanism to remove genuine negative reviews.

For ease of reference, we use ij to represent the potential strategy the seller may follow when facing a malicious customer $i = s, n, d$ and when facing a regular customer $j = n, d$. Extending the definition a_R^i in Lemma 2, let a_R^{ij} be the market's posterior belief about the product when the review is $R \in \{P, 0, N\}$ and the market believes that the seller's strategy is ij. Further, with a slight abuse of notation, we represent cases where the malicious consumer does not purchase using $i = 0$.

There are two types of equilibria to consider: (1) the product price p is such that a malicious consumer does not purchase (this corresponds to $i = 0$) and (2) p is such that a malicious consumer does purchase and demands a ransom. In this case, we only need to consider $i = s$, that is, that the seller settles with the malicious customer by paying the ransom (that is, the other two possibilities, $i = n$ or $i = d$, cannot be part of an equilibrium, because, anticipating such a strategy by the seller, the malicious consumer is better off by choosing not to purchase).

For each scenario, it is sufficient to prove the following two statements: (1) there exists an equilibrium such the seller does not remove genuine negative review ($ij = 0n$ for the first scenario, or $0n$ for the second scenario) if and only if $b \geq \underline{b}$; (2) For $b \geq \underline{b}$, no equilibrium exists such that the seller removes genuine negative reviews ($ij = 0d$ for the first scenario, or $0d$ for the second scenario). We note that the second statement follows directly from the proof of Proposition 13, which shows that pure strategy equilibria where the seller removes genuine negative reviews only exist for $b < \underline{b}_g < \underline{b}$. Thus, in what follows, we focus on proving the first statement.

Consider first the scenario where the malicious consumer purchases (and the seller settles, $i = s$). We establish the conditions under which strategy $ij = sn$ is an equilibrium (i.e., the seller settles with a malicious customer and does not remove a genuine negative review). For $ij = sn$ to be an equilibrium, we require that deviating to $ij = sd$ (when the market belief is that $ij = sn$) is not profitable for the seller. By considering the seller's payoff, such a deviation is not profitable provided

$$(1 - \gamma)\pi(a_0^{sn}) + \gamma\pi(a_N^{sn}) - b \leq \pi(a_N^{sn}), \tag{21}$$

By the definition of a_R^{ij}, we have $a_R^{in} = a_R^i$ for $i = s, n$, as in Lemma 2. Thus, $a_0^{sn} = a$ and $a_N^{sn} = a_N^s$, and hence the above condition is equivalent to $b \geq \underline{b}$.

Next, consider an equilibrium where the malicious customer is deterred from purchasing ($i=0$), we note that the market posterior beliefs are equivalent to the case where the malicious consumer purchases and the seller's strategy is $i=s$, that is, $a_R^{0j}=a_R^{sj}$ for $j=n,d$ and $R=P,0,N$. It follows that the above analysis holds also for cases where the malicious consumer does not purchase, so that the condition with respect to the penalty b continues to hold.

To complete the proof, observe that $\underline{b}$ is independent of the product's price p, which implies that the condition $b\geq\underline{b}$ is necessary and sufficient for existence of an equilibrium with $j=n$. □

Proof of Proposition 5

By Proposition 4, under the assumption that $b\geq\underline{b}$, it suffices to focus on the strategies $ij=sn,dn,nn$. To simplify the notation, in what follows we omit the component $j=n$ and write only $i=s,d,n$. The proof follows a similar structure of that of Propositions 1 and 2 with the centralized mechanism. Specifically, we follows three steps:

1. Establish conditions for $i\in\{s,d,n\}$ to be an equilibrium.
2. Determine the equilibrium ransom r^* given that a malicious customer has purchased.
3. Determine the malicious customer's purchase decision.

Step 1: Conditions for $i\in\{s,d,n\}$ as an equilibrium. First, $i=s$ is an equilibrium if and only if

$$\begin{aligned}&\pi(a_0^s)-r\geq(1-\gamma(1-\delta))\pi(a_0^s)+\gamma(1-\delta)\pi(a_N^s)-(1-\delta)b;\\&\pi(a_0^s)-r\geq\pi(a_N^s).\end{aligned}$$

where the first (second) condition guarantees that the seller has no incentive to deviate to $i=d$ ($i=n$). Since $a_0^s=a$, the above conditions are equivalent to:

$$r\leq\min(\pi(a)-\pi(a_N^s),\ (1-\delta)(\gamma(\pi(a)-\pi(a_N^s))+b)).$$

Similarly, $i=n$ is an equilibrium if and only if

$$\begin{aligned}&\pi(a_N^n)\geq\pi(a_0^n)-r;\\&\pi(a_N^n)\geq(1-\gamma(1-\delta))\pi(a_0^n)+\gamma(1-\delta)\pi(a_N^n)-(1-\delta)b,\end{aligned}$$

where the first (second) condition guarantees that the seller has no incentive to deviate to $i=s$ ($i=d$). We note that $a_0^n=a$, so that the above conditions can be written as

$$\begin{aligned}&r\geq\pi(a)-\pi(a_N^n);\\&b\geq\frac{1-\gamma(1-\delta)}{1-\delta}(\pi(a)-\pi(a_N^n)).\end{aligned}$$

Finally, $i=d$ is an equilibrium if and only if

$$\begin{aligned}&(1-\gamma(1-\delta))\pi(a_0^d)+\gamma(1-\delta)\pi(a_N^d)-(1-\delta)b\geq\pi(a_N^d),\\&(1-\gamma(1-\delta))\pi(a_0^d)+\gamma(1-\delta)\pi(a_N^d)-(1-\delta)b\geq\pi(a_0^d)-r,\end{aligned}$$

which can be simplified to

$$b \le \frac{1-\gamma(1-\delta)}{1-\delta}(\pi(a)-\pi(a_N^d));$$
$$b \le \gamma(\pi(a)-\pi(a_N^d)) - r.$$

Step 2: Equilibrium ransom. To determine the equilibrium ransom, we first compare the magnitudes of the relevant posterior beliefs. In particular, we have $a_0^d = a$, and

$$a_N^d = \frac{a}{a + (1-a)\frac{\beta\gamma(1-\delta)+(1-\beta)\left(1-\frac{p}{a\theta}\right)}{\beta\gamma(1-\delta)+(1-\beta)\left(1-\frac{p}{a\theta}\right)(1-\theta)}}. \tag{22}$$

Thus, we have that $a = a_0^s = a_0^n = a_0^d > a_N^n > a_N^d > a_N^s$. Given this relationship, we determine the equilibrium ransom r^* according to the following three scenarios:

1. When $b \le \frac{1-\gamma(1-\delta)}{1-\delta}(\pi(a)-\pi(a_N^n))$, $i=n$ is not an equilibrium. Thus, for $i=s$ to be the seller's preferred equilibrium, the seller's payoff under $i=s$ must not be less than under $i=d$, that is,

   $$\pi(a_0^s) - r \ge (1-\gamma(1-\delta))\pi(a_0^d) + \gamma(1-\delta)\pi(a_N^d) - (1-\delta)b,$$

   Since $a_0^s = a_0^d = a$, the above condition becomes

   $$r \le (1-\delta)[\gamma(\pi(a)-\pi(a_N^d)) + b].$$

   Thus, the equilibrium ransom is $r^* = (1-\delta)[\gamma(\pi(a)-\pi(a_N^d)) + b]$.

2. When $b \in \left[\frac{1-\gamma(1-\delta)}{1-\delta}(\pi(a)-\pi(a_N^n)), \frac{1-\gamma(1-\delta)}{1-\delta}(\pi(a)-\pi(a_N^d))\right)$, for $i=s$ to be the preferred equilibrium, the seller's payoff under $i=s$ must not be less than that under $i=n$ and $i=d$, that is,

   $$\pi(a_0^s) - r \ge \max\left(\pi(a_0^n),\, (1-\gamma(1-\delta))\pi(a_0^d) + \gamma(1-\delta)\pi(a_N^d) - (1-\delta)b\right),$$

   or, equivalently,

   $$r \le \min\left(\pi(a)-\pi(a_N^n),\, (1-\delta)[\gamma(\pi(a)-\pi(a_N^d)) + b]\right).$$

   Since $b < \frac{1-\gamma(1-\delta)}{1-\delta}(\pi(a)-\pi(a_N^d))$ and $a_N^n < a_N^d$, we have $\pi(a)-\pi(a_N^n) < (1-\delta)[\gamma(\pi(a)-\pi(a_N^d)) + b]$. Thus, the binding constraint is $r \le \pi(a)-\pi(a_N^n)$ and the equilibrium ransom is $r^* = \pi(a)-\pi(a_N^n)$.

3. When $b > \frac{1-\gamma(1-\delta)}{1-\delta}(\pi(a)-\pi(a_N^d))$, $i=d$ cannot be an equilibrium. Thus, the equilibrium ransom is $r^* = \pi(a)-\pi(a_N^n)$, as in the previous scenario. Combining this case with the previous one, we arrive at the equilibrium ransom in the first statement of the proposition.

Step 3: Malicious customer's purchase decision. The malicious customer will purchase if and only if $r^* > p$, so that the purchase decision follows immediately from the equilibrium ransom. □

Proof of Proposition 6

To prove the second point, we show that there exists $\underline{\beta}_D$ such that if $\beta \geq \underline{\beta}_D$ then $p_0 \geq \bar{p}_D$ (and then we have $p^* = p_0$ by Lemma 4). Note first that p_0 does not depend on β. Next, note that a_N^n and a_N^d are both strictly increasing in β (see Lemma 2 and (5)) and recall that $\pi'(\cdot) > 0$. It follows from Proposition 5 that $\bar{p}_D$ (i.e., the maximum price at which the malicious consumer purchases) is strictly decreasing in β. Therefore, there exists $\underline{\beta}_D \in [0,1]$ such that if $\beta \geq \underline{\beta}_D$ we have $p_0 \geq \bar{p}_C$ and $p^* = p_0$ by Lemma 4.

To prove the first point it suffices to show that if $\beta < \underline{\beta}_D$ the seller's profit function (8) is strictly increasing at any $p < p_0$ and is strictly decreasing at any $p > \bar{p}_D$. To show that, note that the function $\Pi_0(p)$ (as defined in Eq. (37) in Lemma 3) is maximized at p_0. In the case $\beta < \underline{\beta}_D$, we have $p_0 < \bar{p}_D$ and it follows by concavity of Π_0 that the seller's profit is strictly decreasing at any $p > \bar{p}_D$. Now consider any price $\hat{p} < p_0$. If $\hat{p} \notin \mathcal{P}_D^{pur}$, then the seller's profit function is strictly increasing at $\hat{p}$ by concavity of Π_0. On the other hand, if $\hat{p} \in \mathcal{P}_D^{pur}$ then the seller's profit at $\hat{p}$ can be written $\Pi_D(\hat{p}) = \Pi_0(\hat{p}) - \beta(r^*(\hat{p}) - \hat{p})$. The term $\Pi_0(\cdot)$ is strictly increasing at $\hat{p}$ by concavity. Therefore, to show that $\Pi_D(\cdot)$ is strictly increasing at $\hat{p}$ it suffices to show that $\beta(r^*(\hat{p}) - \hat{p})$ is strictly decreasing, and to show the latter it suffices to show that $r^*(\cdot)$ is strictly decreasing at $\hat{p}$. Observe that both a_N^n and a_N^d are strictly increasing in p by Lemma 2 and (5). Since $\pi'(\cdot) > 0$, it follows by Proposition 5 that $r^*(\cdot)$ is strictly decreasing at $\hat{p}$. □

Proof of Proposition 7

We prove the result in two steps. First, we show that among $b \leq \frac{1-\gamma(1-\delta)}{1-\delta}(\pi(a) - \pi(a_N^n))$ (the second statement in Proposition 5), the seller's profit is the highest at $b = \underline{b}$. Second, we compare the seller's profit when the platform offers the semi-decentralized mechanism with $b = \underline{b}$ against that when the semi-decentralized mechanism is not offered (or equivalently, when it is offered with a high penalty satisfying $b > \frac{1-\gamma(1-\delta)}{1-\delta}(\pi(a) - \pi(a_N^n))$, such as $b \to \infty$).

For the first step, it suffices to show that under any penalty b' satisfying $\frac{1-\gamma(1-\delta)}{1-\delta}(\pi(a) - \pi(a_N^n)) \geq b' > \underline{b}$, the seller's profit is no greater than that under $\underline{b}$. Let the equilibrium price under b' be p', and let the seller's equilibrium profit be $\Pi'_D := \Pi_D(p'|b')$. Furthermore, let $\Pi_D(p'|\underline{b})$ be the seller profit under the same price p', but under penalty $\underline{b}$. Note that since p' is not necessarily optimal under penalty $\underline{b}$, it follows that $\Pi_D(p'|\underline{b})$ is no greater than the seller's payoff under the optimal price at penalty $\underline{b}$. Therefore, to prove the proposition, it suffices to show that $\Pi_D(p'|b') \leq \Pi_D(p'|\underline{b})$. To show this, let $\bar{p}_D(b')$ and $\bar{p}_D(\underline{b})$ be the highest prices at which the malicious consumer chooses to enter the market under penalty b' and $\underline{b}$ respectively. Note that by Proposition 5, we have $\bar{p}_D(b') \geq \bar{p}_D(\underline{b})$. Next, in comparing $\Pi_D(p'|b')$ to $\Pi_D(p'|\underline{b})$, we have the following three scenarios depending on the relative magnitude between p', $\bar{p}_D(b')$ and $\bar{p}_D(\underline{b})$:

1. When $p' < \bar{p}_D(\underline{b})$, the malicious customer purchases under both b' and $\underline{b}$; thus, according to (8), the seller's profits under b' and $\underline{b}$ are:

$$\Pi_D(p'|b') = \beta[\pi(a) - (r^*(p'|b') - p')] + (1-\beta)\left[\frac{p}{a\theta}\pi(a) + \left(1 - \frac{p}{a\theta}\right)(p + a\theta\pi(1) + (1-a\theta)\pi(a_N^s))\right];$$
$$\Pi_D(p'|\underline{b}) = \beta[\pi(a) - (r^*(p'|\underline{b}) - p')] + (1-\beta)\left[\frac{p}{a\theta}\pi(a) + \left(1 - \frac{p}{a\theta}\right)(p + a\theta\pi(1) + (1-a\theta)\pi(a_N^s))\right].$$

By Proposition 5, we have $r^*(p'|b') > r^*(p'|\underline{b})$, so that $\Pi_D(p'|b') < \Pi_D(p'|\underline{b})$.

2. When $p' \in [\bar{p}_D(\underline{b}),\, \bar{p}_D(b'))$, a malicious customer purchases under b', but not under $\underline{b}$; thus, $\Pi_D(p'|b')$ is the same as in the above scenario, while

$$\Pi_D(p'|\underline{b}) = \beta[\pi(a)] + (1-\beta)\left[\frac{p}{a\theta}\pi(a) + \left(1 - \frac{p}{a\theta}\right)(p + a\theta\pi(1) + (1-a\theta)\pi(a_N^s))\right].$$

Since $r^*(p'|b') \geq p'$, we have $\Pi_D(p'|b') < \Pi_D(p'|\underline{b})$.

3. When $p' > \bar{p}_D(b')$, a malicious customer does not purchase under either b' or $\underline{b}$; thus, $\Pi_D(p'|b') = \Pi_D(p'|\underline{b})$.

Combining the above three scenarios, we have $\Pi_D(p'|\underline{b}) \geq \Pi_D(p'|b')$. This completes the proof of the first step.

For the second step, observe that from Proposition 5 (first statement), when b is sufficiently large (which is equivalent to the platform not offering the semi-decentralized mechanism), the ransom decision, and the seller's profit function are both independent of δ. The optimal profit under this case is Π^*_{no}, as defined in §4.5. On the other hand, at $b = \underline{b}$, while b is independent of δ, under a given price p, the equilibrium ransom r monotonically decreases in δ. In particular, we note that at $\delta = 1$, we have $r^* = 0$. In this extreme, the malicious customer will not purchase (since he cannot extract any ransom), and the seller's profit will be $\beta\pi(a) + (1-\beta)\left[\frac{p}{a\theta}\pi(a) + \left(1 - \frac{p}{a\theta}\right)\left(p + a\theta\pi(1) + (1-a\theta)\pi(a_N^s)\right)\right]$. The maximum profit in this case is Π^*_{opt}, which is achieved at $p^* = p_0$ (i.e., this is the "first best" profit where the malicious consumer does not purchase). Since $\Pi^*_{opt} \geq \Pi^*_{no}$, there are two relevant cases to consider. First, if $\Pi^*_{opt} > \Pi^*_{no}$, it follows by continuity of the seller's profit function in δ that there exists a threshold $\Delta < 1$ such that for any $\delta \geq \Delta$ the semi-decentralized mechanism with $b = \underline{b}$ strictly dominates not offering the mechanism (Statement 1 in the proposition), while for $\delta < \Delta$, the platform either chooses to offer the semi-decentralized mechanism with $b = \underline{b}$ (by the first step of the proof above), or not to offer the mechanism (Statement 2 in the proposition). Second, if $\Pi^*_{no} = \Pi^*_{opt}$, then it follows that not offering the mechanism weakly dominates for all values of δ (Statement 2 in the proposition). □

Proof of Proposition 8

We first consider the centralized mechanism. Note that for any $\delta > \bar{\delta}_C := \frac{c}{\gamma\left(\pi(a) - \pi(a_N^n)\right)}$, the second case of Proposition 2 holds and the equilibrium ransom is

$$r^*(\delta) = (1-\gamma\delta)[\pi(a) - \pi(a_N^c(\delta))] + c. \tag{23}$$

Here, we write r^* and a_N^c explicitly as functions of δ to highlight their dependence on δ. Next, we prove $r^*(\delta)$ decreases in δ for $\delta > \bar{\delta}_C$. To show this, we note:

$$\frac{dr^*(\delta)}{d\delta} = -\gamma[\pi(a) - \pi(a_N^c)] - (1-\gamma\delta)\pi'(a_N^c)\frac{\partial a_c^N}{\partial \delta}. \tag{24}$$

Since $\pi(a) > 0$ is convex and strictly increasing in a and $a > a_N^c$, we have $\pi(a) - \pi(a_N^c) > (a - a_N^c)\pi'(a_N^c)$. Thus, a sufficient condition for $\frac{dr^*(\delta)}{d\delta} < 0$ is

$$\gamma(a - a_N^c) > -(1-\gamma\delta)\frac{\partial a_c^N}{\partial \delta}. \tag{25}$$

By Eq. (16) in the proof of Lemma 2, we have a_c^N as:

$$a_c^N(\delta) = a - a\frac{(1-a)\theta(1-\beta)\left(1-\frac{p}{a\theta}\right)}{\beta(1-\gamma\delta)+\left(1-a\theta(1-\beta)\left(1-\frac{p}{a\theta}\right)\right)}, \tag{26}$$

and,

$$\frac{\partial a_c^N(\delta)}{\partial \delta} = -\frac{\beta\gamma a(1-a)\theta(1-\beta)\left(1-\frac{p}{a\theta}\right)}{\left[\beta(1-\gamma\delta)+\left(1-a\theta(1-\beta)\left(1-\frac{p}{a\theta}\right)\right)\right]^2} < 0. \tag{27}$$

Thus, Eq. (25) is equivalent to:

$$\gamma a\frac{(1-a)\theta(1-\beta)\left(1-\frac{p}{a\theta}\right)}{\beta(1-\gamma\delta)+\left(1-a\theta(1-\beta)\left(1-\frac{p}{a\theta}\right)\right)} > (1-\gamma\delta)\frac{\beta\gamma a(1-a)\theta(1-\beta)\left(1-\frac{p}{a\theta}\right)}{\left[\beta(1-\gamma\delta)+\left(1-a\theta(1-\beta)\left(1-\frac{p}{a\theta}\right)\right)\right]^2}, \tag{28}$$

which always holds because $\left(1-a\theta(1-\beta)\left(1-\frac{p}{a\theta}\right)\right)>0$. This establishes that r^* decreases in δ for $\delta > \bar{\delta}_c$.

Using this result, we next show that the seller's profit $\Pi_c(p)$ (Eq. 3) under any price p is non-decreasing in δ for $\delta \geq \bar{\delta}_c$. Consider any δ_1 and δ_2 such that $\delta_1 > \delta_2 > \bar{\delta}_c$. By the definition of the range of prices p that induce the malicious customer to purchase, $\mathcal{P}_C^{pur}(\delta) = \{p : p < r^*(\delta)\}$, we have, $\mathcal{P}_C^{pur}(\delta_1) \subset \mathcal{P}_C^{pur}(\delta_2)$. There are three possible scenarios depending on the magnitude of p.

1. For any $p \in \mathcal{P}_C^{pur}(\delta_1)$, the seller will pay ransom under both δ_1 and δ_2. Since $r^*(\delta_1) < r^*(\delta_2)$, we have $\Pi_c(p|\delta_1) > \Pi_c(p|\delta_2)$.
2. For any $p \in \mathcal{P}_C^{pur}(\delta_2) - \mathcal{P}_C^{pur}(\delta_1)$, the seller will only pay ransom under δ_2, but not under δ_1. Thus, $\Pi_c(p|\delta_1) > \Pi_c(p|\delta_2)$.
3. For any $p \notin \mathcal{P}_C^{pur}(\delta_2)$, the seller will not pay ransom under either δ_1 or δ_2. Thus, $\Pi_c(p|\delta_1) = \Pi_c(p|\delta_2)$.

Combining the three scenarios, we have $\Pi_c(p|\delta_1) \geq \Pi_c(p|\delta_2)$ for any price p. Therefore, we have that the seller's optimal profit increases in δ for $\delta > \bar{\delta}_c$.

The proof for the case of the semi-decentralized mechanism follows a similar structure. By Proposition 5, define $\bar{\delta}_d = 1 - \left(\gamma + \frac{b}{(\pi(a)-\pi(a_N^n))}\right)^{-1}$. For any $\delta > \bar{\delta}_d$, the second case of Proposition 5 holds and the equilibrium ransom is $r^* = (1-\delta)(\gamma(\pi(a) - \pi(a_N^d)) + b)$. By the definition of a_N^d (Eq. 22 in the proof of Proposition 5) and convexity of $\pi()$, we can show that r^* decreases in δ for $\delta > \bar{\delta}_d$ following the same procedure as above for the centralized case. Finally, following the same argument as above, it can be shown that for any price p, the seller's profit increases in δ. Therefore, we have that the seller's optimal profit under the semi-decentralized mechanism also increases in δ for $\delta > \bar{\delta}_d$.

Combining the results for the two mechanisms, we have that for any $\delta > \bar{\Delta} := \max(\bar{\delta}_c, \bar{\delta}_d)$, the seller's profit increases in δ under both the centralized and decentralized mechanism. □

Proof of Proposition 9

We follow the same approach as in Proposition 1. First, $i = n$ is an equilibrium if and only if under the belief that $i = n$, the seller has no incentive to deviate to $i = s$ or $i = c$. We consider these two conditions in turns. First, when the seller faces a negative review and the belief is $i = n$, his net payoff by deviating from $i = n$ to $i = s$ is:

$$\Delta^{s|n} = \pi(a) - \pi(a_N^n) - r$$

Similarly, if the seller deviates to strategy $i = c$, the difference in payoff gains is

$$\begin{aligned}\Delta^{c|n} &= \delta(1-\mu+\gamma\mu)\pi(a_0^n) + [(1-\delta(1-\mu+\gamma\mu)]\,\pi(a_N^n) - \pi(a_N^n) - c,\\ &= \delta(1-\mu+\gamma\mu)\,[\pi(a) - \pi(a_N^n)] - c.\end{aligned}$$

Then, for $i = n$ to be an equilibrium strategy, we require that both $\Delta^{s|n}$ and $\Delta^{c|n}$ are non-positive. This occurs when

$$\Delta^{s|n} \leq 0 \iff \pi(a_N^n) \geq \pi(a) - r. \tag{29}$$

$$\Delta^{c|n} \leq 0 \iff \pi(a_N^n) \geq \pi(a) - \frac{c}{\delta(1-\mu+\gamma\mu)}. \tag{30}$$

Combining the two conditions that preclude deviation, that is, (29) and (30), strategy $i = n$ is an equilibrium if and only if

$$\pi(a_N^n) \geq \max\left\{\pi(a) - r,\, \pi(a) - \frac{c}{\delta(1-\mu+\gamma\mu)}\right\},$$

which corresponds to the first statement in the proposition.

Next, $i = s$ is an equilibrium if and only if under the belief that $i = s$, the seller has no incentive to deviate to $i = c$ or $i = n$. We consider these two conditions in turns. First, when the seller faces a negative review and the belief is $i = s$, his net payoff by deviating from $i = s$ to $i = c$ is:

$$\Delta^{c|s} = \delta(1-\mu+\gamma\mu)\pi(a_0^s) + [1-\delta(1-\mu+\gamma\mu)]\,\pi(a_N^s) - c - [\pi(a_0^s) - r].$$

Thus, the seller does not deviate to $i = c$ if and only if $\Delta^{c|s} \leq 0$. As $a_0^s = a$, the condition becomes,

$$\pi(a_N^s) \leq \pi(a) + \frac{c-r}{1-\delta(1-\mu+\gamma\mu)}. \tag{31}$$

Similarly, the condition that the seller does not deviate to $i = n$ under the belief that $i = s$ is

$$\Delta^{n|s} = \pi(a_N^s) - [\pi(a_0^s) - r] \leq 0,$$

or equivalently,

$$\pi(a_N^s) \leq \pi(a) - r. \tag{32}$$

Combining the two conditions that preclude deviation, that is, (31) and (32), strategy $i = s$ is an equilibrium if and only if

$$\pi(a_N^s) \leq \min\left\{\pi(a) - r,\, \pi(a) + \frac{c-r}{1-\delta(1-\mu+\gamma\mu)}\right\},$$

which corresponds to the second statement in the proposition.

Finally, we consider to $i = c$, which is an equilibrium if and only if under this belief, the seller does not have incentive to deviate to $i = n$ and $i = s$. Using the same notation as in the paper, the seller will not deviate to $i = n$ if and only if

$$\Delta^{n|c} = \pi(a_N^c) - [\delta(1-\mu+\gamma\mu)\pi(a_0^c) + (1-\delta(1-\mu+\gamma\mu))\pi(a_N^c) - c] \leq 0.$$

As $a_0^c = a$, the above condition is equivalent to

$$\pi(a_N^c) \leq \pi(a) - \frac{c}{\delta(1-\mu+\gamma\mu)}. \tag{33}$$

Similarly, the seller will not deviating to $i = s$ when

$$\Delta^{s|c} = \pi(a_0^c) - r - [\delta(1-\mu+\gamma\mu)\pi(a_0^c) + (1-\delta(1-\mu+\gamma\mu))\pi(a_N^c) - c] \leq 0,$$

that is,

$$\pi(a_N^c) \geq \pi(a) + \frac{c-r}{1-\delta(1-\mu+\gamma\mu)}.$$

Combining this condition with (33), we have that $i = c$ is an equilibrium if and only if:

$$\pi(a_N^c) \in \left[\pi(a) + \frac{c-r}{1-\delta(1-\mu+\gamma\mu)},\, \pi(a) - \frac{c}{\delta(1-\mu+\gamma\mu)}\right],$$

which corresponds to the third statement in the proposition. □

Proof of Proposition 10

The proof is analogous to the proof of Proposition 2. We prove the result by backward induction. First, assuming the malicious customer has purchased, he will request the equilibrium ransom r^* which is the maximum possible ransom such that $i = s$ is the seller's preferred equilibrium (that is, either $i = s$ is the only equilibrium, or the firm's payoff under $i = s$ is greater than that under $i = c$ or $i = n$ if either is an equilibrium). To identify the relevant conditions, we rearrange Proposition 9 to get the following scenarios:

1. When $c < \delta(1-\mu+\gamma\mu)(\pi(a) - \pi(a_N^n))$, $i = n$ is not an equilibrium. On the other hand, $i = c$ is an equilibrium if and only if the ransom $r > (1-\delta(1-\mu+\gamma\mu))(\pi(a) - \pi(a_N^c)) + c$. When $i = c$ is the equilibrium, the seller's terminal payoff conditional on a malicious review is

$$\pi^c = [1-\delta(1-\mu+\gamma\mu))]\pi(a_N^c) + \delta(1-\mu+\gamma\mu)\pi(a) - c.$$

On the other hand, $i = s$ is an equilibrium if and only if

$$r \leq \min(\pi(a) - \pi(a_N^s),\, (1-\delta(1-\mu+\gamma\mu))(\pi(a) - \pi(a_N^s)) + c).$$

When this condition holds, the seller's terminal payoff conditional on a malicious review is

$$\pi^s = \pi(a) - r.$$

Thus, the sufficient and necessary condition for $i = s$ to be the preferred equilibrium for the seller is that $i = s$ is an equilibrium and $\pi^s \geq \pi^c$, or equivalently,

$$r \leq \min\left(\pi(a) - \pi(a_N^s),\, (1-\delta(1-\mu+\gamma\mu))(\pi(a) - \pi(a_N^s)) + c,\, (1-\delta(1-\mu+\gamma\mu))(\pi(a) - \pi(a_N^c)) + c\right).$$

Since $c < \delta(1-\mu+\gamma\mu)(\pi(a) - \pi(a_N^n))$ and $a_N^s < a_N^c$, the above condition can be simplified to

$$r \leq (1-\delta(1-\mu+\gamma\mu))(\pi(a) - \pi(a_N^c)) + c.$$

This corresponds to the equilibrium ransom r^* in the second statement in the proposition.

2. When $c \in [\delta(1-\mu+\gamma\mu)(\pi(a)-\pi(a_N^n)),\ \delta(1-\mu+\gamma\mu)(\pi(a)-\pi(a_N^c)))$, both $i=n$ and $i=c$ are equilibria for sufficiently large r. By the first scenario, we know that $i=s$ is an equilibrium and it is preferred by the seller over $i=c$ if and only if

$$r \leq (1-\delta(1-\mu+\gamma\mu))(\pi(a)-\pi(a_N^c))+c. \tag{34}$$

   Further, in this scenario, $i=n$ is an equilibrium if and only if $r \geq \pi(a)-\pi(a_N^n)$. In this case, the seller's terminal payoff conditional on a malicious review is $\pi^n = \pi(a_N^n)$, which is less than π^s if and only if $r \leq \pi(a)-\pi(a_N^n)$. This condition is tighter than Eq. 34, as $a_N^c < a_N^n$ and $c > \delta(1-\mu+\gamma\mu)(\pi(a)-\pi(a_N^n))$.

3. When $c \geq \delta(1-\mu+\gamma\mu)(\pi(a)-\pi(a_N^c))$, $i=c$ is not an equilibrium because $a_N^n > a_N^c$. On the other hand, $i=n$ is an equilibrium if and only if $r > \pi(a)-\pi(a_N^n)$. From the analysis of the previous scenario, it follows that $i=s$ is the preferred equilibrium if and only if

$$r \leq \pi(a)-\pi(a_N^n).$$

   Combining this with the scenario above leads to the the equilibrium ransom r^* in the first statement in the proposition.

Next, anticipating that if he purchases the equilibrium ransom will be r^* as described above, the malicious customer makes the purchase if and only if $p < r^*$. Substituting r^* from the first step into this condition leads to the purchase conditions in the proposition. □

Proof of Proposition 11

The proof is similar to that of Propositions 4 and 13, and it consists of two parts.

1. There exists an equilibrium such that the seller does not remove genuine review ($j=n$ using the notation in the proof of Proposition 4) if and only if $b \geq \underline{b}_I$.
2. There exists no equilibrium such that the seller removes genuine review ($j=d$) for $b \geq \underline{b}_I$.

For the first part, consider first the scenario where the malicious consumer purchases. We establish the conditions under which strategy $ij=sn$ is an equilibrium (i.e., the seller settles with a malicious customer and does not remove a genuine negative review). For $ij=sn$ to be an equilibrium, we require that deviating to $ij=sd$ (when the market belief is that $ij=sn$) is not profitable for the seller. By considering the seller's payoff, such a deviation is not profitable provided

$$(1-\gamma)\mu\pi(a_0^{sn})+(1-\mu+\gamma\mu)\pi(a_N^{sn})-b \leq \pi(a_N^{sn}), \tag{35}$$

Since $a_0^{sn}=a$ and $a_N^{sn}=a_N^s$, the above condition is equivalent to $b \geq \underline{b}_I$.

Next, consider the scenario where the malicious customer is deterred from purchasing, we note that the market posterior beliefs are equivalent to the case where the malicious consumer purchases and the seller's strategy is $i=s$. In particular, with a slight abuse of the notation, we may represent cases where the malicious consumer does not purchase using $i=0$, and then we have the market posterior beliefs $a_R^{0j}=a_R^{sj}$ for $j=n,d$ and $R=P,0,N$. It follows that the above analysis holds also for cases where the malicious consumer does not purchase, so that the condition with respect to the penalty b continues to hold.

To complete the proof of the first part, observe that $\underline{b}$ is independent of the product's price p, which implies that the condition $b \geq \underline{b}$ is necessary and sufficient for existence of an equilibrium with $j = n$.

To prove the second part, again, we first consider the scenario where the malicious customer purchases. In this case, the condition under which strategy $ij = sd$ is an equilibrium is:

$$b \leq (1-\gamma)\mu(\pi(a_0^{sd}) - \pi(a_N^s) < \underline{b}_I, \tag{36}$$

where the second inequality follows from $a_0^{sd} < a$. Thus, $ij = sd$ could not be an equilibrium for $b \geq \underline{b}_I$.

Similarly, we can also show that under the scenario where the malicious customer does not purchase, $ij = 0d$ also could not be an equilibrium for $b \geq \underline{b}_I$. This completes the proof of part 2. □

Proof of Proposition 12

By Proposition 11, under the assumption that $b \geq \underline{b}$ it suffices to focus on the strategies with $j = n$. To simplify the notation, in what follows we omit the component $j = n$ and write only $i = s, d, n$. The proof follows a similar structure of that of Propositions 9 and 10 with the centralized mechanism. Specifically, we follows three steps:

1. Establish conditions for $i \in \{s, d, n\}$ to be an equilibrium.
2. Determine the equilibrium ransom r^* given that a malicious customer has purchased.
3. Determine the malicious customer's purchase decision.

Step 1: Conditions for $i \in \{s, d, n\}$ as an equilibrium. First, $i = s$ is an equilibrium if and only if

$$\begin{aligned}
&\pi(a_0^s) - r \geq (1-(1-\mu+\gamma\mu)(1-\delta))\pi(a_0^s) + (1-\mu+\gamma\mu)(1-\delta)\pi(a_N^s) - (1-\delta)b;\\
&\pi(a_0^s) - r \geq \pi(a_N^s).
\end{aligned}$$

where the first (second) condition guarantees that the seller has no incentive to deviate to $i = d$ ($i = n$). Since $a_0^s = a$, the above conditions are equivalent to:

$$r \leq \min(\pi(a) - \pi(a_N^s),\ (1-\delta)((1-\mu+\gamma\mu)(\pi(a) - \pi(a_N^s)) + b)).$$

Similarly, $i = n$ is an equilibrium if and only if

$$\begin{aligned}
&\pi(a_N^n) \geq \pi(a_0^n) - r;\\
&\pi(a_N^n) \geq (1-(1-\mu+\gamma\mu)(1-\delta))\pi(a_0^n) + (1-\mu+\gamma\mu)(1-\delta)\pi(a_N^n) - (1-\delta)b,
\end{aligned}$$

where the first (second) condition guarantees that the seller has no incentive to deviate to $i = s$ ($i = d$). We note that $a_0^n = a$, so that the above conditions can be written as

$$\begin{aligned}
&r \geq \pi(a) - \pi(a_N^n);\\
&b \geq \frac{1-(1-\mu+\gamma\mu)(1-\delta)}{1-\delta}(\pi(a) - \pi(a_N^n)).
\end{aligned}$$

Finally, $i = d$ is an equilibrium if and only if

$$(1-(1-\mu+\gamma\mu)(1-\delta))\pi(a_0^d) + (1-\mu+\gamma\mu)(1-\delta)\pi(a_N^d) - (1-\delta)b \geq \pi(a_N^d),$$

$$(1-(1-\mu+\gamma\mu)(1-\delta))\pi(a_0^d)+(1-\mu+\gamma\mu)(1-\delta)\pi(a_N^d)-(1-\delta)b\geq\pi(a_0^d)-r,$$

which can be simplified to

$$b\leq\frac{1-(1-\mu+\gamma\mu)(1-\delta)}{1-\delta}(\pi(a)-\pi(a_N^d));$$
$$r\geq(1-\mu+\gamma\mu)(1-\delta)(\pi(a)-\pi(a_N^d))+(1-\delta)b.$$

Step 2: Equilibrium ransom. Next, we determine the equilibrium ransom r^* according to the following three scenarios:

1. When $b\leq\frac{1-(1-\mu+\gamma\mu)(1-\delta)}{1-\delta}(\pi(a)-\pi(a_N^n))$, $i=n$ is not an equilibrium. Thus, for $i=s$ to be the seller's preferred equilibrium, the seller's payoff under $i=s$ must not be less than under $i=d$, that is,

$$\pi(a_0^s)-r\geq(1-(1-\mu+\gamma\mu)(1-\delta))\pi(a_0^d)+(1-\mu+\gamma\mu)(1-\delta)\pi(a_N^d)-(1-\delta)b,$$

   Since $a_0^s=a_0^d=a$, the above condition becomes

$$r\leq(1-\delta)[(1-\mu+\gamma\mu)(\pi(a)-\pi(a_N^d))+b].$$

   Thus, the equilibrium ransom is $r^*=(1-\delta)[(1-\mu+\gamma\mu)(\pi(a)-\pi(a_N^d))+b]$.

2. When $b\in\left[\frac{1-(1-\mu+\gamma\mu)(1-\delta)}{1-\delta}(\pi(a)-\pi(a_N^n)),\ \frac{1-(1-\mu+\gamma\mu)(1-\delta)}{1-\delta}(\pi(a)-\pi(a_N^d))\right)$, for $i=s$ to be the preferred equilibrium, the seller's payoff under $i=s$ must not be less than that under $i=n$ and $i=d$, that is,

$$\pi(a_0^s)-r\geq\max\left(\pi(a_N^n),\ (1-(1-\mu+\gamma\mu)(1-\delta))\pi(a_0^d)+(1-\mu+\gamma\mu)(1-\delta)\pi(a_N^d)-(1-\delta)b\right),$$

   or, equivalently,

$$r\leq\min\left(\pi(a)-\pi(a_N^n),\ (1-\delta)[(1-\mu+\gamma\mu)(\pi(a)-\pi(a_N^d))+b]\right).$$

   Since $b>\frac{1-(1-\mu+\gamma\mu)(1-\delta)}{1-\delta}(\pi(a)-\pi(a_N^n))$ and $a_N^n>a_N^d$, we have $\pi(a)-\pi(a_N^n)<(1-\delta)[(1-\mu+\gamma\mu)(\pi(a)-\pi(a_N^d))+b]$. Thus, the binding constraint is $r\leq\pi(a)-\pi(a_N^n)$ and the equilibrium ransom is $r^*=\pi(a)-\pi(a_N^n)$.

3. When $b>\frac{1-(1-\mu+\gamma\mu)(1-\delta)}{1-\delta}(\pi(a)-\pi(a_N^d))$, $i=d$ cannot be an equilibrium. Thus, the equilibrium ransom is $r^*=\pi(a)-\pi(a_N^n)$, as in the previous scenario. Combining this case with the previous one, we arrive at the equilibrium ransom in the first statement of the proposition.

Step 3: Malicious customer's purchase decision. The malicious customer will purchase if and only if $r^*>p$, so that the purchase decision follows immediately from the equilibrium ransom. □

Proof of Lemma 3

Define function $\Pi_0(p)$ for $p\in[0,1]$ as

$$\Pi_0(p)=\beta\pi(a)+(1-\beta)\left(\frac{p}{a\theta}\pi(a)+\left(1-\frac{p}{a\theta}\right)[p+a\theta\pi(1)+(1-a\theta)\pi(a_N^s)]\right). \tag{37}$$

which is the seller's profit ignoring the ransom term and recall that this is a concave function which is maximized at p_0. By the definition of p_0, we have that $p_0=\arg\max_{p\in[0,1]}\Pi_0(p)$.

To prove the first statement in the result, we note that by the definition of $\Pi_C(p)$ and $\Pi_0(p)$, $\Pi_0(p) \geq \Pi_C(p)$ for all $p \in [0,1]$. Thus, if $p_0 \geq \bar{p}_C$, by the definition of $\bar{p}_C$, we have $p_0 \notin \mathcal{P}_C^{pur}$. Consequently, $\Pi_C(p^0) = \Pi_0(p_0) \geq \Pi_0(p) \geq \Pi_C(p)$ for all $p \in [0,1]$. Therefore, $p^* = p_0$.

For the second statement, note that as $\Pi_0(p)$ is concave in p and $p_0 < \bar{p}_C$. Thus, $\Pi_0(p) < \Pi_0(\bar{p}_C)$ for all $p > \bar{p}_C$. Further, by the definition of $\bar{p}_C$, we have that $p \notin \mathcal{P}_C^{pur}$ for $p > \bar{p}_C$. In other words, for $p > \bar{p}_C$, the malicious customer does not purchase, and hence $\Pi_0(p) = \Pi_C(p)$ for $p > \bar{p}_C$. Thus, $\Pi_C(p) < \Pi_C(\bar{p}_C)$ for all $p > \bar{p}_C$, and hence $p^* \leq \bar{p}_C$. □

Proof of Proposition 13

Note that it is straightforward to show that the seller would never use the mechanism to remove genuine positive reviews. Thus, in what follows, we consider whether the seller uses the mechanism to remove genuine negative reviews.

For ease of reference, we use ij to represent the potential strategy the seller may follow when facing a malicious customer $i = s, n, d$ and when facing a regular customer $j = n, d$. Extending the definition a_R^i in Lemma 2, let a_R^{ij} be the market's posterior belief about the product when the review is $R \in \{P, 0, N\}$ and the market believes that the seller's strategy is ij. Accordingly, we have $a_R^{in} = a_R^i$ as in Lemma 2, and using the same technique as in the proof of Lemma 2, a_R^{id} (i.e., the posterior belief assuming the seller uses the decentralized mechanism to remove non-malicious reviews) is given by: $a_P^{id} = 1$ for $i = s, n, d$, and

$$\begin{aligned} a_0^{sd} &= \frac{a}{a + (1-a)\frac{\beta + (1-\beta)\left[\frac{p}{a\theta} + (1 - \frac{p}{a\theta})(1-\gamma)\right]}{\beta + (1-\beta)\left[\frac{p}{a\theta} + (1-\frac{p}{a\theta})(1-\gamma)(1-\theta)\right]}} < a; \\ a_N^{sd} &= \frac{a}{a + (1-a)\frac{1}{1-\theta}} = a_N^s; \end{aligned} \tag{38}$$

There are two possible types of equilibria to consider: (1) the product price p is such that a malicious consumer does not purchase (this corresponds to $i = 0$), and (2) p is such that a malicious consumer does purchase and demands a ransom. In the latter case, we only need to consider the strategy $i = s$, that is, that the seller settles with the malicious customer by paying the ransom (that is, the other two possibilities, $i = n$ or $i = d$, cannot be part of an equilibrium, because, anticipating such a strategy by the seller, the malicious consumer is better off by choosing not to purchase).

Now, consider first the scenario in which the malicious consumer purchases (and the seller settles, $i = s$). In this case, we show the following two results: (i) there exists $\underline{b}_g \in (0, \underline{b})$ such that the equilibrium sd exists (that is, the seller uses the mechanism to remove genuine negative reviews) if and only if $b \leq \underline{b}_g$; (ii) if $\underline{b}_g < b < \underline{b}$, no pure strategy equilibrium exists. To show this, we establish the conditions under which strategy $ij = sd$ is an equilibrium. For $ij = sd$ to be an equilibrium, we require that deviating to $ij = sn$ (when the market belief is $ij = sd$) is not profitable for the seller. By considering the seller's payoff, such a deviation is not profitable provided

$$(1-\gamma)\pi(a_0^{sd}) + \gamma\pi(a_N^{sd}) - b \geq \pi(a_N^{sd}),$$

where a_0^{sd}, a_N^{sd} are given in (38). Since $a_N^{sd} = a_N^s$, the last inequality is equivalent to

$$b \leq \underline{b}_g := (1-\gamma)(\pi(a_0^{sd}) - \pi(a_N^s)).$$

It is straightforward to verify that since $a_0^{sd} < a$, we have $\underline{b}_g < \underline{b}$, where $\underline{b}$ is defined in Proposition 4. It follows that, if $b \leq \underline{b}_g$, then there exists an equilibrium in which the malicious consumer purchases (and the seller settles, i.e., $i = s$), while if the non-malicious consumer purchases and leaves a negative review, the seller uses the semi-decentralized mechanism to remove the review (i.e., $j = d$). By contrast, if $\underline{b}_g < b < \underline{b}$, then there is no such equilibrium because the seller has an incentive to deviate from $j = d$ to $j = n$. Furthermore, note that by Proposition 4 we also have that if $\underline{b}_g < b < \underline{b}$ there is also no equilibrium such that $ij = sn$, since in this case the seller has an incentive to deviate from $j = n$ to $j = d$.

Next, consider the scenario where the malicious consumer does not purchase $(i = 0)$. In this case, we note that the market posterior beliefs are equivalent to the case where the malicious consumer purchases and the seller's strategy is $i = s$, that is, $a_R^{0j} = a_R^{sj}$ for $j = n, d$ and $R = P, 0, N$. It then follows that the above analysis holds also for cases where the malicious consumer does not purchase, so that the same conditions with respect to the penalty b hold. □

Proof of Lemma 4

The proof is analogous to that of Lemma 3. To prove the first statement in the result, we note that by the definition of $\Pi_0(p)$ and $\Pi_D(p)$, $\Pi_0(p) \geq \Pi_D(p)$ for all $p \in [0,1]$. Thus, if $p_0 \geq \bar{p}_D$, by the definition of $\bar{p}_D$, we have $p_0 \notin \mathcal{P}_D^{pur}$. Consequently, $\Pi_D(p^0) = \Pi_0(p_0) \geq \Pi_0(p) \geq \Pi_D(p)$ for all $p \in [0,1]$. Therefore, $p^* = p_0$.

For the second statement, note that as $\Pi_0(p)$ is concave in p and $p_0 < \bar{p}_D$. Thus, $\Pi_0(p) < \Pi_0(\bar{p}_D)$ for all $p > \bar{p}_D$. Further, by the definition of $\bar{p}_D$, we have that $p \notin \mathcal{P}_D^{pur}$ for $p > \bar{p}_D$. In other words, for $p > \bar{p}_D$, the malicious customer does not purchase, and hence $\Pi_0(p) = \Pi_D(p)$ for $p > \bar{p}_D$. Thus, $\Pi_D(p) < \Pi_C(\bar{p})$ for all $p > \bar{p}_D$, and hence $p^* \leq \bar{p}_D$. □

Appendix D: Numerical Experiments

In this section, we conduct numerical experiments to evaluate the difference in the seller's payoff under the semi-decentralized versus under the centralized mechanism for dispute resolution. In each experiment, we fix the model parameters θ, a, β, c, $\pi(\cdot)$ and vary parameters δ and γ in the range $[0,1]$ in steps of 0.025. For each of the 1600 δ-γ combinations, we calculate the normalized difference $\mathit{Diff} = (\Pi_D^* - \Pi_C^*)/(\Pi_{opt}^* - \Pi_{no}^*)$ and report summary statistics on this difference. In the experiments presented below, we use $\theta = 0.9$, $a = 0.5$, $\beta = 0.3$, $c = 0$, and $\pi(a) = 50a^2$ as base values (i.e., unless otherwise stated in the table, these are the values at which the parameters are fixed in the experiment).

Across our experiments, we observe the same qualitative pattern as that observed in Figure 7: the centralized mechanism tends to perform (modestly) better only in cases where δ is very low, while the semi-decentralized mechanism tends to perform better at most combinations of δ and γ, with a dominance that becomes more pronounced at higher values of δ and lower values of γ.[16] In addition, as Table 1 suggests, the dominance of the semi-decentralized mechanism over the centralized mechanism is greater when the

[16] Note that maximum values greater than one can be observed in Table 1 in instances where the centralized mechanism results in a lower payoff for the seller as compared to no mechanism being present (see Corollary 1 and Figure 4).

Diff	min	max	median	average
$\theta = 0.3$	-0.1518	0.9470	0.0729	0.1641
$\theta = 0.6$	-0.1536	0.9469	0.0731	0.1642
$\theta = 0.9$	-0.1358	0.9489	0.0961	0.1795
$a = 0.2$	-0.0883	0.9511	0.1357	0.2072
$a = 0.5$	-0.1358	0.9489	0.0961	0.1795
$a = 0.8$	-0.051	0.9402	0.0001	0.1514
$\beta = 0.1$	-0.0331	0.9507	0.1680	0.2318
$\beta = 0.3$	-0.1358	0.9489	0.0961	0.1795
$\beta = 0.5$	-0.2298	0.9399	-0.002	0.1169
$c = 0$	-0.1358	0.9489	0.0961	0.1795
$c = 1$	-0.0788	0.9911	0.1980	0.2656
$c = 2$	-0.0788	1.0086	0.2833	0.3172
$\pi(a) = 50a^2$	-0.1358	0.9489	0.0961	0.1795
$\pi(a) = 50a^3$	-0.0828	0.9513	0.1418	0.2122
$\pi(a) = 50a^4$	-0.0471	0.9520	0.1650	0.2305
$\pi(a) = 50a^2$	-0.1358	0.9489	0.0961	0.1795
$\pi(a) = 5000a^2$	-0.1469	1.0264	0.1039	0.1941
$\pi(a) = 500000a^2$	-0.0766	1.1677	0.1182	0.2322

Table 1 Summary statistics for the normalized difference $(\Pi_D^* - \Pi_C^*)/(\Pi_{opt}^* - \Pi_{no}^*)$.

performance of a high-quality product θ is higher; the prior belief about the product's quality a is lower; the probability that the seller encounters a malicious consumer β is lower; the hassle cost associated with the centralized mechanism c is higher; and the seller's future profit potential $\pi(\cdot)$ is greater and more convex in the market belief a.