

A. Online Appendix – Further Extensions

The following extensions are also analyzed in a setting with $T = 2$, $V(\eta, w) = \eta w$, and $c(e) = e^3/3$.

A.1. Negative Reciprocity

I have abstracted from any potential “dark side” of reciprocal preferences in the sense that if an agent is granted a lower payment than expected, he wants to actively harm the principal. The potential consequences of negative reciprocity have been explored by, for example, Dufwenberg and Kirchsteiger (2004), Dohmen et al. (2009), and Netzer and Schmutzler (2014). In this section, I introduce negative reciprocity and show that it leads to the same results as in Section 2.2, even if s does not switch to SI after a deviation by the principal. Thus, as previously shown by Macleod (2003), the potential for organizational conflict can increase the efficiency of employment relationships in the face of contracting frictions. I use the approach introduced by Hart and Moore (2008), who assume that the terms of a contract provide reference points and determine a party’s ex post performance. If someone receives less than they feel entitled to, they shade on performance, thereby causing a deadweight loss that has to be borne by the other party. I adapt the setting of Hart and Moore (2008) or Halonen-Akatwijuka and Hart (2020) to my environment and assume that the relational contract also determines the agent’s reference point.

Therefore, the agent feels entitled to the equilibrium bonus \hat{b}_1 . If he receives a lower bonus, his period-1 utility decreases by $\underline{\eta}(\hat{b}_1 - b_1)$, where $\underline{\eta} \geq 0$. Moreover, the agent can reduce this utility loss via shading (e.g., by sabotaging the principal), by an amount ρ at the agent’s discretion. I assume that the agent still has to be employed by the principal to shade and the principal can fire the agent before making the choice whether to pay the bonus. Hence, she can escape the shading costs ρ but would then also sacrifice potential future profits.¹⁵

All this implies that the utility stream of the agent, conditional on not being fired, amounts to

$$U_1 = b_1 + w_1 - c(e_1) + \eta w_1 e_1 - \max \{ [\underline{\eta}(\hat{b}_1 - b_1) - \rho], 0 \} \\ + \delta (w_2 - c(e_2) + \eta w_2 e_2).$$

¹⁵Thus, a bonus is still not feasible in the last period of the game.

The principal's payoff stream if she does not fire the agent before paying the bonus amounts to

$$\begin{aligned}\Pi_1 = & e_1 - w_1 - b_1 - \rho \\ & + \delta(e_2 - w_2).\end{aligned}$$

Since shading is not costly for the agent, it is optimal to set $\rho = \underline{\eta}(\hat{b}_1 - b_1)$ (for $b_1 \leq \hat{b}_1$). Furthermore, the second-period effort and wage equal $w_2 = \eta/4$ and $e_2 = \eta/2$, respectively; hence, the second-period profits are $\pi_2 = \eta/4$ (as in Section 2).

The principal faces two decisions. First, which bonus $b_1 \in [0, \hat{b}_1]$ to pay, and second whether to fire the agent. Concerning the first decision, if the principal decides to pay a bonus $b_1 \leq \hat{b}_1$ and not to fire the agent, her profits amount to

$$\begin{aligned}\Pi_1 = & e_1 - w_1 + (\underline{\eta} - 1)b_1 - \underline{\eta}\hat{b}_1 \\ & + \delta\frac{\eta}{4}.\end{aligned}$$

This immediately reveals that $b_1 = 0$ is optimal for $\underline{\eta} < 1$, whereas $b_1 = \hat{b}_1$ for $\underline{\eta} \geq 1$. Since $b_1 = \hat{b}_1$ in equilibrium, $\underline{\eta} < 1$ also implies $\hat{b}_1 = b_1 = 0$, and that only reciprocity spot contracts are feasible.

Now assume $\underline{\eta} \geq 1$. Then, the principal sets $b_1 = \hat{b}_1$ if she does not fire the agent. She will terminate the relationship, however, if the bonus is larger than the period-2 profits, i.e., if $\hat{b}_1 > \delta\pi_2$.

The principal's optimization problem becomes maximizing $\pi_1 = e_1 - \hat{b}_1 - w_1$, subject to the agent's binding (IC) constraint, which yields $\hat{b}_1 = e_1^3/3 - \eta w_1 e_1 + 2/3(\sqrt{\eta w_1})^3$, as well as subject to $\hat{b}_1 \leq \delta\pi_2$. The last condition is equivalent to the (DE) constraint, and thus the problem in this section is the same as the optimization problem in Section 2.2.

These results are collected in Lemma 5.

Lemma 5 *The profit-maximizing equilibrium with negative reciprocity and a constant norm function $s_t(h^{t-1}) = GW \forall h^{t-1}$, has the following characteristics:*

- If $\underline{\eta} < 1$, $\hat{b}_1 = 0$. Moreover, $e_1 = e_2 = \eta/2$ and $w_1 = w_2 = \eta/4$.
- If $\underline{\eta} \geq 1$, $\hat{b}_1 > 0$, and outcomes are as characterized in Section 2.2, with $w_1 < w_2 = \eta/4$ and $e_1 > e_2 = \eta/2$.

A.2. Adjustment of the Reference Wage

Some evidence points toward a declining effect of gifts in long-term interactions. Gneezy and List (2006) conduct a field experiment in which they permanently increase the wages of recruited workers. Although workers respond with an immediate effort increase, this is only temporary, and effort falls to an amount only slightly above the initial level. Jayaraman et al. (2016) explore the effects of a mandated 30% wage increase for tea pluckers in India. They find that productivity substantially increases immediately after the wage raise. However, it starts falling again in the second month after the change and returns to its initial level after four months. Sliwka and Werner (2017) examine how reciprocal effort is affected by the timing of wage increases. They find that a permanent wage raise only temporarily increases effort and that the only way to permanently benefit from an individual's reciprocal behavior is to constantly raise wages.

This evidence suggests that individuals adapt to wage increases and update their reference wages.¹⁶ In the following, I incorporate this evidence and assume that the agent starts with a reference wage of zero. In the second period, the first-period wage w_1 becomes the new reference wage. Hence, the agent's utilities are

$$u_1 = b_1 + w_1 - c(e_1) + \eta w_1 e_1$$

$$u_2 = b_2 + w_2 - c(e_2) + \max\{0, \eta(w_2 - w_1)e_2\}.$$

First, I compute the profit-maximizing spot reciprocity contract in the last period. Then, no bonus is paid, and – taking into account that setting $w_2 \geq w_1$ is optimal – effort maximizes $-e_2^3/3 + \eta(w_2 - w_1)e_2$. As shown in Lemma 4, effort is unaffected by the higher reference wage; hence, $e_2 = \eta/2$ and $w_2 = \eta/4 + w_1$.

The outcomes for an optimal relational contract are given in Lemma A.2.

Lemma 6 *Assume the second-period reference wage is equal to w_1 . Then, $w_1 < w_2$. Moreover, the (DE) constraint might or might not bind.*

- *If it does not bind, $de_1/d\eta < de_2/d\eta$. Furthermore, there exists a $\bar{\eta} > 0$ such that the optimal wage is zero for $\eta \leq \bar{\eta}$. In this case, $e_1 > e_2$. For $\eta > \bar{\eta}$, setting a strictly positive wage is optimal, and e_1 can be smaller or larger than e_2 .*
- *If it binds, there exists a $\hat{\eta} > 0$ such that the optimal wage equals zero for $\eta \leq \hat{\eta}$, whereas it is*

¹⁶Also see Eliaz and Spiegler (2018) who assume that the reference wage is a weighted average of past salaries.

strictly positive for $\eta \geq \tilde{\eta}$. In both cases, e_1 can be smaller or larger than e_2 .

$\bar{\eta}$ can be smaller or larger than $\tilde{\eta}$, and both are larger than if the second-period reference wage equals zero independent of w_1 .

The proof can be found in Online Appendix C.

The principal is reluctant to trigger the agent's reciprocal preferences in the first period. In particular, if δ is large, she wants to maintain this opportunity until later when relational contracts are no longer feasible. Therefore, the threshold for η above which a positive first-period wage is paid is larger than that in Section 2.2 – implying that the backloading of reciprocity-based incentives is more pronounced than with a constant reference wage. A higher w_1 also does not necessarily relax the (DE) constraint anymore (which implies that $\tilde{\eta}$ does not have to be smaller than $\bar{\eta}$). This is because a positive first-period wage has two effects on the tightness of the (DE) constraint. On the one hand, the necessary bonus to implement a certain effort level is reduced, which relaxes the constraint. On the other hand, future profits are reduced via the adjustment of the reference wage, which tightens the constraint.

A.3. Reciprocity Triggered by Rents

Finally, I explore the implications of reciprocity being triggered by the agent's material rent, in contrast to only by monetary payments. Thus, I assume that the agent's per-period utilities are

$$\begin{aligned} u_1 &= (b_1 + w_1 - c(e_1))(1 + \eta e_1) \\ u_2 &= (w_2 - c(e_2))(1 + \eta e_2). \end{aligned}$$

Importantly, when choosing his effort level, the agent also reciprocates on the equilibrium bonus of this period *before* it is paid. Hence, the principal is less inclined to pay a positive fixed wage in the first period. Only if a sufficiently tight (DE) constraint considerably restrains the bonus is w_1 positive.

Formally, effort in the second period is given by the agent's first order condition,

$$-e_2^2 - \frac{4}{3}e_2^3\eta + w_2\eta = 0.$$

This is taken into account by the principal who sets w_2 to maximize $\pi_2 = e_2 - w_2$.

In the first period, the principal's (DE) constraint still equals $-b_1 + \delta\pi_2 \geq 0$, whereas the agent's (IC)

constraint becomes

$$\left(b_1 + w_1 - \frac{e_1^3}{3}\right)(1 + \eta e_1) \geq \left(w_1 - \frac{\tilde{e}_1^3}{3}\right)(1 + \eta \tilde{e}_1). \quad (\text{IC})$$

Here, \tilde{e}_1 is characterized by $-\tilde{e}_1^2 - \frac{4}{3}\tilde{e}_1^3\eta + w_1\eta = 0$, and $e_1 > \tilde{e}_1$ if $b_1 > 0$.

Lemma 7 *Assume that the agent's preferences for reciprocity are triggered by his material rent. Then, the (DE) constraint binds given $T = 2$ and $\delta \leq 1$. Moreover, there exists a $\tilde{\eta} > 0$ such that the optimal wage equals zero for $\eta \leq \tilde{\eta}$, whereas it is strictly positive for $\eta \geq \tilde{\eta}$.*

In any case, $e_1 > e_2$ and $w_1 < w_2$.

The proof can be found in Online Appendix C.

With $T = 2$, second-period profits cannot be sufficiently large for a non-binding (DE) constraint given $\delta \leq 1$. However, in a more general setting with more than two periods, (DE) might indeed be slack. In this case, the proof to Lemma 7 reveals that paying a positive wage would not be optimal because the purpose of a positive wage – triggering the agent's reciprocal inclinations – can equivalently be achieved by a bonus, which additionally allows for higher effort via the use of relational incentives. With a binding (DE) constraint, the principal might pay a fixed wage in the first period, but only if η is large enough.

B. Online Appendix – General Model

Here, I introduce a more general version of the model and provide a formal definition of the nature of the agent's reciprocal preferences. As I have already provided the results for a positive reference wage above, I stick to the assumption that the agent reciprocates to any positive wage.

B.1. Setup

There is one risk-neutral principal (“she”) and one risk-neutral agent (“he”). At the beginning of every period $t \in \{1, \dots, T\}$, with $2 \leq T < \infty$, the principal decides whether to make an employment offer to the agent or not ($d_t^P \in \{0, 1\}$). In case an offer is made ($d_t^P = 1$), it specifies an upfront wage $w_t \in \mathbb{R}_+$. The agent's acceptance/rejection decision is described by $d_t^A \in \{0, 1\}$. Upon acceptance ($d_t^A = 1$), the agent receives w_t and chooses an effort level $e_t \in \mathbb{R}_+$, which is associated with effort costs $c(e)$, with $c', c'', c''' > 0$ and $c(0) = 0$.¹⁷ Effort generates an expected output e_t , which is subsequently consumed by the principal (note that the linearity of output in effort is without loss of generality as long as output and

¹⁷A positive third derivative is needed for an interior solution in Section B.3.

effort cost are additively separable). Afterwards, the principal can pay a discretionary bonus $b_t \in \mathbb{R}_+$. If the principal refrains from making an offer ($d_t^P = 0$) or if the agent rejects an offer made by the principal ($d_t^A = 0$), both consume their outside option utilities which are set to zero. Moreover, the principal and agent share a discount factor $\delta \in (0, 1]$.

B.2. Relational Contract, Preferences, and the Norm of Reciprocity

Because effort and output are only observable but not verifiable, no formal incentive contract can be used to motivate the agent. A relational contract is generally feasible, though, which is a self-enforcing agreement that constitutes a subgame perfect equilibrium of the game. In addition to the standard components of a game – players, information, action spaces, preferences, and equilibrium concept – I incorporate a norm function that activates the norm of reciprocity and maps the game’s history into the agent’s preferences. Before introducing this norm, I formally describe histories and feasible strategies.

Histories and feasible strategies The events in period t are denoted by $h_t = (d_t^P, w_t, d_t^A, e_t, b_t)$, with h_t being public information. A *history* of length $t - 1$, h^{t-1} (for $t \geq 2$) collects the events up to, and including, time $t - 1$, i.e. $h^{t-1} := (h_\tau)_{\tau=1}^{t-1}$. The set of histories of length $t - 1$ is denoted by \mathcal{H}^{t-1} (and $\mathcal{H}^0 = \{\emptyset\}$). I focus on pure strategies. For the agent, a pure strategy specifies what wage offers to accept in each period as a function of the previous history, and what level of effort to exert as a function of the previous history and current-period wages. Formally, it is a sequence of mappings $\{\sigma_t^A\}_{t=1}^T$ where, for each $t \leq T$, $\sigma_t^A = (d_t^A, e_t)$, and $d_t^A : \mathcal{H}^{t-1} \times \{0, 1\} \times \mathbb{R}_+ \rightarrow \{0, 1\}$, $(h^{t-1}, d_t^P, w_t) \mapsto d_t^A(h^{t-1}, d_t^P, w_t)$ and $e_t : \mathcal{H}^{t-1} \times \{0, 1\} \times \mathbb{R}_+ \times \{0, 1\} \rightarrow \mathbb{R}_+$, $(h^{t-1}, d_t^P, w_t, d_t) \mapsto e_t(h^{t-1}, d_t^P, w_t, d_t)$.

In each period, a pure strategy for the principal specifies her wage offer as a function of the previous history as well as the bonus payment as a function of the previous history, current-period wages and effort. Formally, it is a sequence of mappings $\{\sigma_t^P\}_{t=1}^T$, where, for each $t \leq T$, $\sigma_t^P = (d_t^P, w_t, b_t)$, and $d_t^P : \mathcal{H}^{t-1} \rightarrow \{0, 1\}$, $(h^{t-1}) \mapsto d_t^P(h^{t-1})$, $w_t : \mathcal{H}^{t-1} \times \{0, 1\} \rightarrow \mathbb{R}_+$, $(h^{t-1}, d_t^P) \mapsto w_t(h^{t-1})$, $b_t : \mathcal{H}^{t-1} \times \{0, 1\} \times \mathbb{R}_+ \times \{0, 1\} \times \mathbb{R}_+ \rightarrow \mathbb{R}_+$, $(h^{t-1}, d_t^P, w_t, d_t^A, e_t) \mapsto b_t(h^{t-1}, d_t^P, w_t, d_t^A, e_t)$.

Relational Contract and the Norm of Reciprocity The relational contract is agreed upon at the beginning of the game. It “activates” the norm of reciprocity and stipulates *reference functions* which specify history-dependent actions players are *supposed* to take. For the agent, the relational contract determines an acceptance function $\hat{d}_t^A(h^{t-1}, d_t^P, w_t)$ as well as an effort function $\hat{e}_t(h^{t-1}, d_t^P, w_t, d_t^A)$, with $(\hat{d}_t^A, \hat{e}_t) \in \sigma_t^A$. For the principal, the relational contract determines an offer function $\hat{d}_t^P(h^{t-1})$, a wage

function $\hat{w}_t(h^{t-1}, d_t^P)$ and a bonus function $\hat{b}_t(h^{t-1}, w_t, d_t^A, e_t)$, with $(\hat{d}_t^P, \hat{w}_t, \hat{b}_t) \in \sigma_t^P$.

The *norm of reciprocity* states how the agent is supposed to reciprocate against “non-discretionary” upfront wages, that is, wages that are not paid as a reward for past effort. To incorporate this notion, the total wage $\hat{w}_t(h^{t-1}, d_t^P)$ is split into a discretionary part $\hat{w}_t^d(h^{t-1}, d_t^P)$ and a non-discretionary component, which is defined as $\hat{w}_t^{nd}(h^{t-1} \setminus \{e^{t-1}, d^{A,t-1}\}, d_t^P)$, where $e^{t-1} := (e_\tau)_{\tau=1}^{t-1}$ and $d^{A,t-1} := (d_\tau^A)_{\tau=1}^{t-1}$. Note that this specification does not rule out an indirect relationship between the agent’s effort and w_t^{nd} , since the latter is a function of the principal’s previous actions which itself are affected by the agent’s behavior. Whereas the bonus and discretionary wage constitute the “direct” incentive system that grants payments as a reward for previously exerted effort, w_t^{nd} stipulates subsequent effort by the agent who adheres to the norm of reciprocity.

For the following, I follow the approach introduced by Abreu (1988) dividing histories into events corresponding to whether someone performed or breached and adapt it to my setting. Therefore, I define the following sets of histories. First, $\mathcal{H}^{t,A*}$ is the set of histories where the agent has not deviated, i.e., where the following holds for all all $\tau \leq t$:

$$\begin{aligned} d_\tau^A &= \hat{d}_\tau^A(h^{\tau-1}, d_\tau^P, w_\tau) \\ e_\tau &\geq \hat{e}_\tau(h^{\tau-1}, d_\tau^P, w_\tau, d_\tau^A). \end{aligned}$$

$\mathcal{H}^{t,A-breach}$ is the set of histories including a deviation by the agent, hence where above conditions are violated for at least one $\tau \leq t$.

Second, $\mathcal{H}^{t,P*}$ is the set of histories where the principal has not deviated in the subsequent dimensions, in that the following holds for all $\tau \leq t$:

$$\begin{aligned} d_\tau^P &= \hat{d}_\tau^P(h^{\tau-1}) \\ b_\tau &\geq \hat{b}(h^{\tau-1}, d_\tau^P, w_\tau, d_\tau^A, e_\tau) \\ w_\tau &\geq \hat{w}^d(h^{\tau-1}, d_\tau^P). \end{aligned}$$

$\mathcal{H}^{t,P-breach}$ is the set of histories including a deviation by the principal, i.e., where above conditions are violated for at least one $\tau \leq t$. Importantly, $\mathcal{H}^{t,P-breach}$ does *not* include deviations in $\hat{w}_t^{nd}(\cdot)$. Therefore, my approach differs from Abreu (1988) in that not all observable deviations from equilibrium

actions trigger a punishment. Because of the definition of w_t^{nd} , the principal does not “breach” if she does not pay the equilibrium amount.

Finally $\mathcal{H}^t \equiv \mathcal{H}^{t,P*} \cup \mathcal{H}^{t,A*}$ is the set of histories after which the relational contract does not specify a punishment.

The agent’s utility function contains his material payoff, which in period t is $b_t + w_t - c(e_t)$. On top, his utility depends on the state $s_t \in \{GW, SI\}$ which indicates whether he is in a state of goodwill or a in a state of self interest. I assume that $s_1 = GW$; in all periods $t \geq 2$,

$$\mathbf{s}_t = \begin{cases} SI & \text{if } h^{t-1} \subseteq \mathcal{H}^{t-1,P-breath} \\ GW & \text{otherwise.} \end{cases}$$

s_t does *not* switch to zero after a deviation by the agent (and if no bonus is paid in response). Hence, the agent’s reciprocal inclinations towards the principal disappear once the latter refuses to make an offer she was supposed to make, or if she does not compensate the agent accordingly.

The agent’s period- t utility equals

$$u_t = \begin{cases} d_t^A (b_t + w_t - c(e_t) + e_t V(\eta, w_t^{nd})), & s_t = GW \\ d_t^A (b_t + w_t - c(e_t)), & s_t = SI. \end{cases}$$

with $V(0, w_t^{nd}) = V(\eta_t, 0) = 0$, $V_\eta, V_w, V_{\eta w} > 0$ and $V_{ww} \leq 0$.

The term $\eta \in [0, \infty)$ captures the agent’s responsiveness to the norm of reciprocity and lets the principal’s output enters the agent’s utility whenever and $w_t^{nd} > 0$.¹⁸ Note that this definition can equivalently be applied to settings in which the principal does not observe effort and output is not verifiable. Then, the bonus could be a function of output, and s_t would switch to SI if the principal reneged on paying it.

The principal only cares about her material payoffs,

$$\pi_t = d_t (e_t - b_t - w_t).$$

A subgame perfect equilibrium determines equilibrium functions $d_t^P(h^{t-1}), w_t(h^{t-1}, d_t^P), d_t^A(h^{t-1}, d_t^P, w_t), e_t(h^{t-1}, d_t^P, w_t, d_t^A)$, and $b_t(h^{t-1}, d_t^P, w_t, d_t^A, e_t)$. In addition, for every history, I impose the consistency

¹⁸In a more general setting, the norm of reciprocity would be activated if w_t^{nd} exceeded some reference wage. Here, such a reference wage would equal zero; in Section 3.2, I consider positive reference wages.

Furthermore, one could argue that, if performance pay was very generous in relation to the agent’s effort cost, it should be regarded as a gift. However, such a payment could be split into a part that compensates the agent for his effort costs and one that grants him a rent. Paying this rent up front (and anticipating that the agent exerts effort accordingly) would then be equivalent to paying a non-discretionary wage in my setting.

requirements $\hat{d}_t^P = d_t^P$, $\hat{w}_t = w_t$, $\hat{d}_t^A = d_t^A$, $\hat{e}_t = e_t$, and $\hat{b}_t = b_t$. The equilibrium concept is subgame perfect equilibrium, incorporating these consistency requirements as additional restrictions on equilibrium strategies.

In an equilibrium with $d_t^P = d_t^A = 1 \forall t$, the following recursive relationships hold in all periods $t \in \{1, \dots, T\}$ for the principal's profits Π_t and the agent's utility U_t , where I set $\Pi_{T+1} = U_{T+1} = 0$:

$$\begin{aligned}\Pi_t &= e_t - b_t - w_t + \delta \Pi_{t+1} \\ U_t &= b_t + w_t - c(e_t) + e_t V(\eta_t, w_t^{nd}) + \delta U_{t+1}.\end{aligned}$$

In what follows, the objective is to characterize a subgame perfect equilibrium that maximizes the principal's profits at the beginning of the game, Π_1 .

B.3. Reciprocity Spot Contract

First, I discuss a profit-maximizing spot contract. Since $w = w^{nd}$, I omit the "nd" superscript in this section. As stated in the main part of the paper, such a contract will also be offered in the final period, T (this follows from Lemma 9 below). Moreover, I have shown that $b = 0$ in a spot contract, and the only means to incentivize the agent is a positive non-discretionary wage if and only if $s = GW$ (if $s = SI$, the agent chooses zero effort for any wage). Given his per-period utility $u = w - c(e) + eV(\eta, w)$ in this case, effort e^{GW} is characterized by

$$-c'(e^{GW}) + V(\eta, w) = 0. \quad (1)$$

The principal sets w to maximize her expected per-period profits $\pi = e^{GW} - w$. Here, she has to take into account that accepting the contract must be optimal for the agent, hence $w - c(e^{GW}) + e^{GW}V(\eta, w) \geq 0$ must hold. It is immediate that this is satisfied for any $w \geq 0$. Thus, the principal's problem is to

$$\max_w e^{GW} - w,$$

subject to the non-negativity constraint $w \geq 0$, and that the agent chooses effort according to 1.

Lemma 8 *If $s = GW$, wage and effort in the profit-maximizing reciprocity spot contract are characterized by $-c'(e^{GW}) + V(\eta, w^{GW}) = 0$ and $-c''(e^{GW}) + V_w = 0$. Moreover, $\pi^{GW}, u^{GW} > 0$, and $de^{GW}/d\eta, d\pi^{GW}/d\eta > 0$.*

Proof: I maximize profits $\pi = e - w$, taking into account that effort e^{GW} is characterized by $-c'(e^{GW}) + V(\eta, w) = 0$, and that the agent's (IR) constraint, $u = w - c(e^{GW}) + e^{GW}V(\eta, w) \geq 0$, must be satisfied. Naturally, the latter holds for any $w \geq 0$ because the agent can always secure $u = w$ by choosing zero effort.

The principal's first order condition equals

$$\frac{d\pi}{dw} = \frac{de^{GW}}{dw} - 1 = 0,$$

where

$$\frac{de^{GW}}{dw} = \frac{V_w}{c''}.$$

Hence, the optimal levels of e^{GW} and w^{GW} are characterized by

$$\begin{aligned} -c'(e^{GW}) + V(\eta, w^{GW}) &= 0 \\ -c''(e^{GW}) + V_w(\eta, w^{GW}) &= 0. \end{aligned}$$

Therefore,

$$\begin{aligned} \frac{de^{GW}}{d\eta} &= \frac{\begin{vmatrix} -V_\eta & V_w \\ -V_{w\eta} & V_{ww} \end{vmatrix}}{\begin{vmatrix} -c'' & V_w \\ -c''' & V_{ww} \end{vmatrix}} \\ &= \frac{-V_\eta V_{ww} + V_{w\eta} V_w}{-c'' V_{ww} + c''' V_w} > 0 \\ \frac{dw}{d\eta} &= \frac{\begin{vmatrix} -c'' & -V_\eta \\ -c''' & -V_{w\eta} \end{vmatrix}}{\begin{vmatrix} -c'' & V_w \\ -c''' & V_{ww} \end{vmatrix}} \\ &= \frac{V_{w\eta} c'' - c''' V_\eta}{-c'' V_{ww} + c''' V_w}, \end{aligned}$$

and

$$\begin{aligned}\frac{d\pi^{GW}}{d\eta} &= \frac{de^{GW}}{d\eta} - \frac{dw^{GW}}{d\eta} \\ &= \frac{-V_\eta V_{ww} + c''' V_\eta}{-c'' V_{ww} + c''' V_w} > 0.\end{aligned}$$

Finally, $\pi^{GW}, u^{GW} > 0$ follows from $c(0) = c'(0)$. ■

Intuitively, a positive wage lets the agent partially internalize the principal's payoff, which is why he reciprocates and selects a positive effort level. Because this interaction is stronger for a more reciprocal agent, a higher η induces larger values of e^{GW} and π^{GW} .

B.4. Relational Contract

Now, I analyze how a relational contract is used to incentivize the agent. Two aspects are of particular interest, namely the enforceability of the relational contract and how the norm of reciprocity affects outcomes.

B.4.1. Preliminaries and Optimization Problem

The relational contract determines payment functions, and the promise to make these payments must be credible. This is captured by dynamic enforcement (DE) constraints for each period t . Assuming $s_t = GW$,

$$-b_t + \delta \Pi_{t+1}^{GW} \geq \delta \Pi_{t+1}^{SI}. \quad (\text{DE})$$

Moreover, individual rationality (IR) constraints, $\Pi_t^{GW} \geq 0$, must hold. Because $b_t \geq 0$, (IR) are implied by (DE) constraints and can hence be omitted. Generally, relational contracts require a (potentially) infinite time horizon because of a standard unraveling argument that can be applied once a predetermined last period exists: If the equilibrium outcome in the last period is unique, the same holds for all preceding periods. In my case, however, s_{t+1} switches to SI if the principal reneges on paying b_t once the principal refuses to make a specified payment. Moreover, the "standard" grim trigger punishment is imposed afterward and relational contracts are no longer feasible (adapting Abreu (1988) to my setting, as laid out above). Thus, the principal's continuation profits are $\Pi_{t+1}^{SI} = 0$ if a punishable deviation has occurred in any $\tau \leq t$, and her behavior in period $t < T$ indeed affects her future profits.

Hence, not only does the relational contract determine whether a given payment “activates” the agent’s reciprocal preferences, but the latter are also a prerequisite for the relational contract to work. Finally, all this implies that no positive bonus can be promised in periods $\tau \geq t$ once $s_t = SI$.

In the next step, I explore the agent’s incentives to exert equilibrium effort. Those are determined by a combination of reciprocity-based incentives (via a positive w_t^{nd}) and relational incentives (via b_t and w_t^d). Recall that my specification of the norm function implies that after a deviation by the agent, the state remains in GW . This indicates that the agent does not necessarily deviate to an effort level of zero. Moreover, since effort is public information, it is without loss to only specify a positive bonus $b_t \geq 0$ if the agent has exerted equilibrium effort and no bonus otherwise. Thus, the agent’s (IC) constraint in state GW – which must hold in every period t – equals

$$\begin{aligned} & -c(e_t) + e_t V(\eta_t, w_t^{nd}) + b_t + \delta U_{t+1}(\mathcal{H}^{t,A*}) \\ \geq & -c(\tilde{e}_t) + \tilde{e}_t V(\eta_t, w_t^{nd}) + \delta U_{t+1}(\mathcal{H}^{t,A-breach}). \end{aligned} \quad (\text{IC})$$

There, I omit the GW -superscript because the state is not affected by the agent’s actions. Note that, if the agent deviates, he will choose an effort level \tilde{e}_t characterized by $-c'(\tilde{e}_t) + V(\eta, w_t^{nd}) = 0$. \tilde{e}_t is the effort the agent would select if he only responded to the norm of reciprocity. Relational incentives using subsequent discretionary payments are needed to motivate the agent to exert additional effort $e_t - \tilde{e}_t$.

An (IR) constraint $U_t(\mathcal{H}^{t-1,A*}) \geq 0$ must also hold in every period but is implied by (IC) because payments are assumed to be non-negative and because the right-hand side of (IC) cannot be smaller than zero.

Concluding, the principal’s problem is to maximize

$$\Pi_1 = \sum_{t=1}^T \delta^{t-1} \pi_t,$$

subject to a (DE) and (IC) constraint for every period t .

First, I derive a number of preliminary results, which simplify the problem and are collected in Lemma 9.

Lemma 9 *There exists a profit-maximizing equilibrium which has the following properties in all periods t :*

- $w_t = w_t^{nd}$

- (IC) holds as an equality
- $U_t(\mathcal{H}^{t-1,A^*}) = U_t(\mathcal{H}^{t-1,A-breach})$
- the equilibrium is sequentially optimal, hence the problem is equivalent to maximizing each π_t

The proof can be found in Online Appendix C.

First, it is without loss of generality to only use bonus payments for the provision of relational incentives. Thus, $w_t = w_t^{nd}$ from now on, and all upfront wages are non-discretionary.

Second, the (IC) constraint binds in every period, thus the agent does not receive a rent for relational incentives. However, he enjoys a rent whenever $w_t (= w_t^{nd}) > 0$, i.e., when reciprocity-based incentives are provided. Importantly, though, these “warm-glow” rents cannot be used to provide relational incentives in earlier periods: If the agent was bound to lose them after a deviation (for example because of a firing threat as with efficiency wages), the upfront wage would not be non-discretionary anymore, and the agent would not reciprocate.¹⁹

Third, $U_t(\mathcal{H}^{t,A^*}) = U_t(\mathcal{H}^{t-1,A-breach})$ follows because it is without loss of generality to provide relational incentives only with a current bonus. Thus, continuation play is not affected by the agent’s actions, which finally implies that the profit-maximizing relational contract is sequentially optimal and the optimization problem has a recursive structure.

Collecting all results, binding (IC) constraints as well as $U_t(\mathcal{H}^{t,A^*}) = U_t(\mathcal{H}^{t-1,A-breach})$ yield

$b_t = c(e_t) - c(\tilde{e}_t) - V(\eta, w_t)(e_t - \tilde{e}_t)$. Plugging this into the principal’s profits and (DE) constraints, the optimization problem becomes to maximize

$$\pi_t = e_t - b_t - w_t = e_t - c(e_t) + c(\tilde{e}_t) + V(\eta, w_t)(e_t - \tilde{e}_t) - w_t$$

in every period t , subject to

$$c(e_t) - e_t V(\eta, w_t) \leq \delta \Pi_{t+1}^{GW} + c(\tilde{e}_t) - \tilde{e}_t V(\eta, w_t). \quad (\text{DE})$$

First, I show that Lemma 1 also holds in this more general setting

Lemma 10 *Assume the (DE) constraint does not bind in period $t < T$. Then, there exists a $\bar{\eta} > 0$ such that setting a strictly positive wage is optimal for $\eta > \bar{\eta}$, whereas the optimal wage is zero for $\eta \leq \bar{\eta}$.*

¹⁹This would be different if either bonus or discretionary wages also triggered direct reciprocal responses by the agent. Then, the respective payments would merely assume a larger relative weight in the optimal incentive scheme (see Sections 4.1 and A.3).

The proof can be found in Online Appendix C.

Second, I explicitly take the (DE) constraint into account and relate Proposition 1 to the general setting.

Lemma 11 *Assume the (DE) constraint binds in period $t < T$. Then, equilibrium effort is smaller than with a non-binding (DE) constraint. Moreover, if paying a fixed wage is optimal in the situation with a non-binding (DE) constraint (i.e., if $\eta > \bar{\eta}$), the fixed wage now is strictly larger. Otherwise (i.e., if $\eta \leq \bar{\eta}$), there exists a $\tilde{\eta}_t < \bar{\eta}$ such that setting a strictly positive wage is optimal for $\eta > \tilde{\eta}_t$, whereas the optimal wage equals zero for $\eta \leq \tilde{\eta}_t$. Finally, $\tilde{\eta}_t$ is increasing in δ .*

The proof can be found in in Online Appendix C.

This implies that reciprocity-based incentives can improve the power of relational incentives for a given value of η , and vice versa (this complementarity between the two means to provide incentives is further fueled by a positive effect of η on future profits, see Proposition 6).

Generally, the (DE) constraint might or might not bind in any period $t < T$ depending on discount factor δ , reciprocity parameter η , and the function $V(\eta, w)$. Moreover, the constraint becomes “tighter” over time since on-path profits decline. I have displayed the resulting consequences for equilibrium effort, compensation, and payoffs above, in Proposition 3 and Lemma 3.

Finally, also in the general model η has a positive effect on equilibrium profits and effort.

Proposition 6 *Equilibrium profits Π_t and effort e_t increase in η .*

The proof can be found in Online Appendix C.

C. Online Appendix – Proofs

Proof of Lemma 1 See the proof to Lemma 10 below. ■

Proof of Proposition 1 To prove existence of $\bar{\delta}(\eta)$, I plug first-best values into the (DE) constraint and assess the conditions for it to hold.

For $\eta \leq \bar{\eta}$, the (DE) constraint becomes $c(e^{FB}) \leq \delta \pi_2(\eta)$, with $c'(e^{FB}) = 1$, thus $\delta \geq \bar{\delta}(\eta) \equiv c(e^{FB})/\pi_2(\eta)$.

For $\eta > \bar{\eta}$, the (DE) constraint becomes

$$\begin{aligned} \delta \pi_2(\eta) + c(\tilde{e}) - c(e^{FB}) + V(\eta, w^{FB})(e^{FB} - \tilde{e}) &\geq 0 \\ \Leftrightarrow \delta \geq \bar{\delta}(\eta) &\equiv \frac{c(e^{FB}) - c(\tilde{e}) - V(\eta, w^{FB})(e^{FB} - \tilde{e})}{\pi_2(\eta)}. \end{aligned}$$

For the remainder of the proof, see the proof to Proposition 3 below. ■

Proof of Proposition 2 For effort and wage dynamics, see the proof to Proposition 3 below; for payoff dynamics, see the proof to Lemma 3 below. ■

Proof of Lemma 2 See the proof of Proposition 6 below. ■

Proof of Proposition 3 Note that this proof makes use of some results proven later; it is put here to preserve the same order as in the main text. Before proving the Proposition 3, though, I confirm the following Lemma:

Lemma 12 *For every $\delta > 0$, the (DE) constraint in period $T - 1$ may or may not hold for first-best effort and wage levels, depending on the size of η and the shape of $V(\eta, w)$. For any value η , the (DE) constraint in period $T - 1$ does not hold for first-best effort and wage levels if the discount factor is sufficiently small.*

Furthermore, $\Pi_{t-1} > \Pi_t$ for all $t \leq T$.

Proof: The (DE) constraint for period $T - 1$ equals

$$\delta \Pi_T + c(\tilde{e}_{T-1}) - \tilde{e}_{T-1} V(\eta, w_{T-1}) - c(e_{T-1}) + e_{T-1} V(\eta, w_{T-1}) \geq 0,$$

with

$c'(\tilde{e}_{T-1}) - V(\eta, w_{T-1}) = 0$. Moreover, $d\Pi_T/d\eta > 0$ (see the proof to Lemma 8), and first-best effort and wage are characterized by

$$1 - c'(e) + V(\eta, w) = 0$$

$$V_w(\eta, w)(e - \tilde{e}) - 1 + \lambda_w = 0.$$

First, note that because \tilde{e} maximizes $eV(\eta, w) - c(e)$ and $e_{T-1} > \tilde{e}_{T-1}$, $c(\tilde{e}_{T-1}) - \tilde{e}_{T-1}V(\eta, w_{T-1}) - c(e_{T-1}) + e_{T-1}V(\eta, w_{T-1}) < 0$ for all $w_{T-1} \geq 0$. Therefore first-best effort cannot be implemented for $\delta \rightarrow 0$.

To show that first-best values in period $T - 1$ may be implementable for certain values of η and forms of $V(\eta, w)$, assume $V(\eta, w) = \eta w$. If $\eta > \bar{\eta}$, first-best values are

$$w^{FB} = \frac{c'(e^{FB}) - 1}{\eta} > 0$$

$$e^{FB} - \tilde{e}^{FB} = \frac{1}{\eta}.$$

The latter implies that $e^{FB} - \tilde{e}^{FB}$ decreases in η , and $e \rightarrow \tilde{e}$ for $\eta \rightarrow \infty$. Moreover, plugging $\eta(e - \tilde{e}) = 1$ into (DE) yields

$$\delta \left(e_T - \frac{c'(e_T)}{\eta} \right) + c(\tilde{e}) - c(e) + w \geq 0.$$

Since $e^{FB} \rightarrow \tilde{e}^{FB}$ for $\eta \rightarrow \infty$ implies $c(\tilde{e}^{FB}) - c(e^{FB}) \rightarrow 0$ in that case, first-best values satisfy the (DE) constraint for $\eta \rightarrow \infty$.

Concerning the second part of the Lemma, again note that the (DE) constraint for period t equals $\delta \Pi_{t+1} + c(\tilde{e}_t) - \tilde{e}_t V(\eta, w_t) - c(e_t) + e_t V(\eta, w_t) \geq 0$. It follows that, *for a given* w_t , the maximum implementable effort in period t is strictly increasing in Π_{t+1} , therefore per-period profits π_t are weakly increasing in Π_{t+1} . This implies that per-period profits in periods $t < T$ can be expressed as functions of Π_{t+1} , i.e. $\pi_t(\Pi_{t+1})$, with $\pi_t' \geq 0$.

The profit-maximizing spot reciprocity contract is the principal's optimal choice in the last period T , hence $\pi_T = e_T - w_T$, with

$$-c'(e) + V(\eta, w) = 0$$

$$-c''(e) + V_w(\eta, w) = 0.$$

In all previous periods, the principal still has the option to implement the spot reciprocity contract (by setting $b_t = 0$ and paying the optimal spot wage), therefore $\pi_t \geq \pi_T \forall t$.

Now, I apply proof by induction to verify that $\Pi_{t-1} > \Pi_t$. First, $\Pi_{T-1} > \Pi_T$ because

$$\Pi_{T-1} = \pi_{T-1} + \delta\Pi_T \geq \pi_T + \delta\Pi_T = \Pi_T(1 + \delta) > \Pi_T.$$

For the induction step, assume that $\Pi_t > \Pi_{t+1}$. Since $\pi'_t(\Pi_{t+1}) \geq 0$, $\pi_{t-1} \geq \pi_t$. Therefore, $\Pi_{t-1} = \pi_{t-1} + \delta\Pi_t \geq \pi_t + \delta\Pi_t > \pi_t + \delta\Pi_{t+1} = \Pi_t$, which completes the proof. ■

Now I am ready to deliver the

Proof of Proposition 3 Take an arbitrary period $t > T$ and recall first-order conditions

$$\begin{aligned} 1 + (V(\eta, w_t) - c'(e_t))(1 + \lambda_{DE_t}) &= 0 \\ V_w(\eta, w_t)(e_t - \tilde{e}_t)(1 + \lambda_{DE_t}) - 1 + \lambda_{w_t} &= 0, \end{aligned}$$

Hence, $w_t = w_{t-1}$ and $e_t = e_{t-1}$ if $\lambda_{DE_t} = \lambda_{DE_{t-1}} = 0$. By Lemma 11, if $\lambda_{DE_{t-1}} = 0$ but $\lambda_{DE_t} > 0$, then $w_t \geq w_{t-1}$ and $e_t < e_{t-1}$. Now, assume that $\lambda_{DE_{t-1}} > 0$. It follows that in this case also $\lambda_{DE_t} > 0$ because $\Pi_{t+1} > \Pi_t$. By the same argument, if $\lambda_{DE_t} = 0$ in a period t , this also holds for all previous periods.

To show that the wage schedule is increasing in periods $t < T$ and the effort path is decreasing, I first assume $\lambda_{w_t} = 0$, thus first-order conditions become

$$\begin{aligned} 1 + (V(\eta, w_t) - c'(e_t))(1 + \lambda_{DE_t}) &= 0 \\ V_w(\eta, w_t)(e_t - \tilde{e}_t)(1 + \lambda_{DE_t}) - 1 &= 0. \end{aligned}$$

The bordered Hessian equals

$$\frac{c''(e_t) - V_w(\eta, w_t) - V_w(\eta, w_t) \frac{c''(\tilde{e}) - V_w(\eta, w)}{c''(\tilde{e})} - V_{ww}(\eta, w_t)(e_t - \tilde{e}_t)}{(1 + \lambda_{DE_t})}$$

and must be positive to guarantee a maximum. This is relevant for the following steps.

Moreover, combining both first-order conditions yields

$$V_w(\eta, w_t)(e_t - \tilde{e}_t) + V(\eta, w_t) - c'(e_t) = 0$$

which, together with

$$\delta\Pi_{t+1} + c(\tilde{e}_t) - \tilde{e}_t V(\eta, w_t) - c(e_t) + e_t V(\eta, w_t) = 0,$$

determines e_t and w_t if (DEt) binds (i.e., if it is not satisfied for first best effort and wage).

Therefore,

$$\begin{aligned} \frac{de_t}{d\delta\Pi_{t+1}} &= \frac{\begin{vmatrix} 0 & V_{ww}(e_t - \tilde{e}_t) - V_w \frac{d\tilde{e}_t}{dw_t} + V_w \\ -1 & V_w(e_t - \tilde{e}_t) \end{vmatrix}}{\begin{vmatrix} V_w - c''(e_t) & V_{ww}(e_t - \tilde{e}_t) - V_w \frac{d\tilde{e}_t}{dw_t} + V_w \\ -c'(e_t) + V(\eta, w_t) & V_w(e_t - \tilde{e}_t) \end{vmatrix}} \\ &= \frac{V_{ww}(\eta, w_t)(e_t - \tilde{e}_t) + V_w(\eta, w_t) \frac{c''(\tilde{e}) - V_w(\eta, w)}{c''(\tilde{e})}}{V_w(e_t - \tilde{e}_t) \left[V_w - c''(e_t) + V_w \frac{c''(\tilde{e}) - V_w}{c''(\tilde{e})} + V_{ww}(e_t - \tilde{e}_t) \right]} \\ \frac{dw_t}{d\delta\Pi_{t+1}} &= \frac{\begin{vmatrix} V_w - c''(e_t) & 0 \\ -c'(e_t) + V(\eta, w_t) & -1 \end{vmatrix}}{\begin{vmatrix} V_w - c''(e_t) & V_{ww}(e_t - \tilde{e}_t) - V_w \frac{d\tilde{e}_t}{dw_t} + V_w \\ -c'(e_t) + V(\eta, w_t) & V_w(e_t - \tilde{e}_t) \end{vmatrix}} \\ &= \frac{c''(e_t) - V_w}{V_w(e_t - \tilde{e}_t) \left[V_w - c''(e_t) + V_w \frac{c''(\tilde{e}) - V_w}{c''(\tilde{e})} + V_{ww}(e_t - \tilde{e}_t) \right]}, \end{aligned}$$

where the denominators are negative (because of the positive bordered Hessian).

I now show that $c''(e_t) - V_w > 0$ and $c''(\tilde{e}) - V_w < 0$ (which implies $V_w \frac{c''(\tilde{e}) - V_w}{c''(\tilde{e})} + V_{ww}(e_t - \tilde{e}_t) < 0$). First, note that if these conditions hold for a given $\delta\Pi_{t+1}$, they also hold for all higher levels of continuation profits. The reason is that, if $c''(e_t) - V_w > 0$ and $c''(\tilde{e}) - V_w < 0$ for some $\delta\Pi_{t+1}$, then e_t is increasing and w_t decreasing in $\delta\Pi_{t+1}$, where the latter reduces \tilde{e}_t and increases V_w . Second, assume $\delta\Pi_{t+1} \rightarrow 0$. Then, the outcome approaches the optimum of the one-period contract, in which effort and wage are characterized by $-c'(e) + V(\eta, w) = 0$ and $-c''(e) + V_w = 0$ (see the proof to Lemma 8). Now, assume to the contrary that $c''(e_t) - V_w \leq 0$ for $\delta\Pi_{t+1} \rightarrow 0$. Then, $dw_t/d\delta\Pi_{t+1} \geq 0$ and thus $c''(\tilde{e}) - V_w \geq 0$ for $\delta\Pi_{t+1} \rightarrow 0$, which contradicts $e_t > \tilde{e}_t$. Therefore, $c''(e_t) - V_w > 0$ for $\delta\Pi_{t+1} \rightarrow 0$, which implies $dw_t/d\delta\Pi_{t+1} < 0$ and $c''(\tilde{e}) - V_w < 0$.

All this implies that

$$\frac{de_t}{d\delta\Pi_{t+1}} > 0 \text{ and}$$

$$\frac{dw_t}{d\delta\Pi_{t+1}} < 0$$

if the (DEt) constraint binds in period t (otherwise, $de_t/d\delta\Pi_{t+1} = dw_t/d\delta\Pi_{t+1} = 0$) and $w_t > 0$. Moreover, with a binding (DEt) constraint and taking into account that $b_t = c(e_t) - c(\tilde{e}_t) - V(\eta, w_t)(e_t - \tilde{e}_t)$,

$$\begin{aligned} \frac{db_t}{d\delta\Pi_{t+1}} &= c'(e_t) \frac{de_t}{d\delta\Pi_{t+1}} - V(\eta, w_t) \frac{de_t}{d\delta\Pi_{t+1}} - V_w \frac{dw_t}{d\delta\Pi_{t+1}} (e_t - \tilde{e}_t) \\ &= V_w (e_t - \tilde{e}_t) \left(\frac{de_t}{d\delta\Pi_{t+1}} \theta - \frac{dw_t}{d\delta\Pi_{t+1}} \right) > 0, \end{aligned}$$

whereas

$$\frac{d(w_t + b_t)}{d\delta\Pi_{t+1}} \geq 0.$$

With $w_t = 0$, effort e_t is constrained by

$$\delta\Pi_{t+1} - c(e_t) \geq 0.$$

If this constraint binds, equilibrium effort and bonus strictly increase in Π_{t+1} . ■

Proof of Lemma 3. For the following comparative statics, I again compute the consequences of a higher continuation profit. Moreover, I assume that (DEt) binds (otherwise, a higher $\delta\Pi_{t+1}$ clearly has no effect on π_t and u_t)

First, I consider the case $w_t > 0$.

Then,

$$\begin{aligned} u_t &= w_t + b_t - c(e_t) + e_t V(\eta, w_t) \\ &= w_t - c(\tilde{e}_t) + \tilde{e}_t V(\eta, w_t), \text{ and} \end{aligned}$$

$$\frac{du_t}{d\delta\Pi_{t+1}} = \frac{dw_t}{d\delta\Pi_{t+1}} (1 + \tilde{e}_t V_w(\eta, w_t)) < 0$$

Moreover,

$$\begin{aligned}
\pi_t &= e_t - w_t - b_t \\
&= e_t - c(e_t) - w_t + c(\tilde{e}_t) + V(\eta, w_t)(e_t - \tilde{e}_t) \text{ and} \\
\frac{d\pi_t}{d\delta\Pi_{t+1}} &= (1 - c'(e_t)) \frac{de_t}{d\delta\Pi_{t+1}} - \frac{dw_t}{d\delta\Pi_{t+1}} \\
&\quad + V_w(\eta, w_t) \frac{dw_t}{d\delta\Pi_{t+1}} (e_t - \tilde{e}_t) + V(\eta, w_t) \frac{de_t}{d\delta\Pi_{t+1}} \\
&= \left(\frac{de_t}{d\delta\Pi_{t+1}} - \frac{dw_t}{d\delta\Pi_{t+1}} \right) [1 - V_w(\eta, w_t)(e_t - \tilde{e}_t)] > 0
\end{aligned}$$

The latter follows from the first-order condition, $V_w(\eta, w_t)(1 + \lambda_{DE_t}) - 1 = 0$, which implies $1 > V_w(\eta, w_t)(e_t - \tilde{e}_t)$ if $\lambda_{DE_t} > 0$.

Second, I consider the case $w_t = 0$.

Then

$$\begin{aligned}
u_t &= 0, \\
\pi_t &= e_t - c(e_t), \text{ and} \\
\frac{d\pi_t}{d\delta\Pi_{t+1}} &= (1 - c'(e_t)) \frac{de_t}{d\delta\Pi_{t+1}} > 0,
\end{aligned}$$

because $1 - c'(e_t) > 0$ with a binding (DEt) constraint and $w_t = 0$. ■

Proof of Lemma 4. For a given $w \geq \bar{w}$, the agent chooses an effort level that maximizes $u = w + eV(\eta, w - \bar{w}) - c(e)$, hence effort is characterized by $V(\eta, w - \bar{w}) - c'(e) = 0$, with

$$\frac{de}{dw} = \frac{V_w(\eta, w - \bar{w})}{c''(e)} > 0.$$

Taking this into account, the principal maximizes her profits $\pi = e - w$, subject to $w \geq \bar{w}$. First ignoring the latter constraint, the principal's first-order condition equals

$$V_w(\eta, w - \bar{w}) - c''(e) = 0.$$

Thus, wage and effort are characterized by this condition as well as $V(\eta, w - \bar{w}) - c'(e) = 0$, and

$$\frac{de}{d\bar{w}} = \frac{\begin{vmatrix} V_w & V_w \\ V_{ww} & V_{ww} \end{vmatrix}}{\begin{vmatrix} -c''(e) & V_w \\ -c'''(e) & V_{ww} \end{vmatrix}} = 0$$

$$\frac{dw}{d\bar{w}} = \frac{\begin{vmatrix} -c''(e) & V_w \\ -c'''(e) & V_{ww} \end{vmatrix}}{\begin{vmatrix} -c''(e) & V_w \\ -c'''(e) & V_{ww} \end{vmatrix}} = 1.$$

These two results also imply that $d\pi/d\bar{w} = -1$. ■

Proof of Proposition 4. In any period t , the principal maximizes

$\pi_t = e_t - c(e_t) + V(\eta, w_t - \bar{w})(e_t - \tilde{e}_t) + c(\tilde{e}_t) - w_t$, with $V(\eta, w_t - \bar{w}) - c'(\tilde{e}_t) = 0$, subject to (DE) and $w_t \geq \bar{w}$. First, I assume that (DE) does not bind (which is possible if η and/or δ are sufficiently large and the upper bound of $V(\eta, w - \bar{w})$ is not “too small” – see the proof to Lemma 12 which can readily be adapted to the present setting). Then, the Lagrange function equals

$$L_t = e_t - c(e_t) + V(\eta, w_t - \bar{w})(e_t - \tilde{e}_t) \\ + c(\tilde{e}_t) - w_t + \lambda_{w_t}(w_t - \bar{w}),$$

with first order conditions (which already take into account that $-V(\eta, w_t - \bar{w}) + c'(\tilde{e}_t) = 0$)

$$\frac{\partial L_t}{\partial e_t} = 1 - c'(e_t) + V(\eta, w_t - \bar{w}) = 0 \\ \frac{\partial L_t}{\partial w_t} = V_w(\eta, w_t - \bar{w})(e_t - \tilde{e}_t) - 1 + \lambda_{w_t} = 0.$$

I start with $\lambda_{w_t} = 0$. Then,

$$\frac{de}{d\bar{w}} = \frac{\begin{vmatrix} V_w(\eta, w_t - \bar{w}) & V_w(\eta, w_t - \bar{w}) \\ -V_w(\eta, w_t - \bar{w}) \frac{d\tilde{e}_t}{dw_t} & -V_w(\eta, w_t - \bar{w}) \frac{d\tilde{e}_t}{dw_t} \end{vmatrix}}{\begin{vmatrix} -c''(e_t) & V_w(\eta, w_t - \bar{w}) \\ V_w(\eta, w_t - \bar{w}) & -V_w(\eta, w_t - \bar{w}) \frac{d\tilde{e}_t}{dw_t} \end{vmatrix}} = 0$$

$$\frac{dw}{d\bar{w}} = \frac{\begin{vmatrix} -c''(e_t) & V_w(\eta, w_t - \bar{w}) \\ V_w(\eta, w_t - \bar{w}) & -V_w(\eta, w_t - \bar{w}) \frac{d\tilde{e}_t}{dw_t} \end{vmatrix}}{\begin{vmatrix} -c''(e_t) & V_w(\eta, w_t - \bar{w}) \\ V_w(\eta, w_t - \bar{w}) & -V_w(\eta, w_t - \bar{w}) \frac{d\tilde{e}_t}{dw_t} \end{vmatrix}} = 1.$$

If $\lambda_{w_t} > 0$, effort is characterized by $1 - c'(e_t)$ and clearly independent of \bar{w} .

Moreover, the threshold $\bar{\eta}$ above which a wage $w_t > \bar{w}$ is optimal is independent of \bar{w} :

$$\frac{\partial \pi_t}{\partial w_t} \Big|_{w_t = \bar{w}} = V_w(\eta, 0)e_t - 1$$

where e_t is characterized by $1 - c'(e_t) = 0$. $V_w(\eta, 0)e_t - 1$ is increasing in η ; either it is positive for all η (if $V_w(\eta, 0)$ is sufficiently large for any $\eta > 0$), or it is positive for large η and negative for small η , which confirms that $\bar{\eta}$ exists and is independent of \bar{w} .

Now, I include the respective (DE) constraints, which yields the Lagrange function of the principal's maximization problem in a period t ,

$$\begin{aligned} L_t &= e_t - c(e_t) + c(\tilde{e}_t) + V(\eta, w_t - \bar{w})(e_t - \tilde{e}_t) - w_t \\ &\quad + \lambda_{DE_t} [\delta \Pi_{t+1} + c(\tilde{e}_t) - c(e_t) + V(\eta, w_t - \bar{w})(e_t - \tilde{e}_t)] \\ &\quad + \lambda_{w_t} (w_t - \bar{w}). \end{aligned}$$

First-order conditions are

$$\begin{aligned} \frac{\partial L}{\partial e_t} &= 1 + (V(\eta, w_t - \bar{w}) - c'(e_t))(1 + \lambda_{DE_t}) = 0 \\ \frac{\partial L}{\partial w_t} &= V_w(\eta, w_t - \bar{w})(e_t - \tilde{e}_t)(1 + \lambda_{DE_t}) - 1 + \lambda_{w_t} = 0. \end{aligned}$$

Thus, with a binding (DEt) constraint, effort and wage are characterized by

$$\begin{aligned} & V_w(\eta, w_t - \bar{w})(e_t - \tilde{e}_t) - c'(e_t) + V(\eta, w_t - \bar{w}) \\ & + \lambda_{w_t} [c'(e_t) - V(\eta, w_t - \bar{w})] = 0 \\ & \delta \Pi_{t+1} + c(\tilde{e}_t) - c(e_t) + V(\eta, w_t - \bar{w})(e_t - \tilde{e}_t) = 0. \end{aligned}$$

For the following, I take into account that Π_t decreases in \bar{w} : I have already shown (in the proof to Lemma 4) that $\Pi_T = \pi_T$ is decreasing in \bar{w} . Therefore, (DE) in period $T - 1$ is tightened by a larger \bar{w} ; moreover, there is a direct negative effect of a higher \bar{w} on profits (equilibrium effort for given payments goes down). All this implies that profits π_{T-1} and Π_{T-1} go down. This effect carries over to all earlier periods.

Assume $w_t > 0$, thus $\lambda_{w_t} = 0$. Then,

$$\begin{aligned} \frac{de}{d\bar{w}} &= \frac{\begin{vmatrix} V_{ww}(e_t - \tilde{e}_t) + V_w \left(1 + \frac{d\tilde{e}_t}{d\bar{w}}\right) & V_{ww}(e_t - \tilde{e}_t) + V_w \left(1 - \frac{d\tilde{e}_t}{dw_t}\right) \\ -\frac{d\delta\Pi_{t+1}}{d\bar{w}} + V_w(e_t - \tilde{e}_t) & V_w(e_t - \tilde{e}_t) \end{vmatrix}}{\begin{vmatrix} V_w - c''(e_t) & V_{ww}(e_t - \tilde{e}_t) + V_w \left(1 - \frac{d\tilde{e}_t}{dw_t}\right) \\ -c'(e_t) + V(\eta, w_t - \bar{w}) & V_w(e_t - \tilde{e}_t) \end{vmatrix}} \\ &= \frac{\frac{d\delta\Pi_{t+1}}{d\bar{w}} \left(V_{ww}(e_t - \tilde{e}_t) + V_w \frac{c''(\tilde{e}) - V_w}{c''(\tilde{e})} \right)}{V_w(e_t - \tilde{e}_t) \left[V_w - c''(e_t) + V_w \left(\frac{c''(\tilde{e}^{FB}) - V_w}{c''(\tilde{e}^{FB})} \right) + V_{ww}(e_t - \tilde{e}_t) \right]} < 0 \\ \frac{dw}{d\bar{w}} &= \frac{\begin{vmatrix} V_w - c''(e_t) & V_{ww}(e_t - \tilde{e}_t) + V_w \left(1 + \frac{d\tilde{e}_t}{d\bar{w}}\right) \\ -c'(e_t) + V(\eta, w_t - \bar{w}) & -\frac{d\delta\Pi_{t+1}}{d\bar{w}} + V_w(e_t - \tilde{e}_t) \end{vmatrix}}{\begin{vmatrix} V_w - c''(e_t) & V_{ww}(e_t - \tilde{e}_t) + V_w \left(1 - \frac{d\tilde{e}_t}{dw_t}\right) \\ -c'(e_t) + V(\eta, w_t - \bar{w}) & V_w(e_t - \tilde{e}_t) \end{vmatrix}} \\ &= 1 + \frac{-\frac{d\delta\Pi_{t+1}}{d\bar{w}} (V_w - c''(e_t))}{V_w(e_t - \tilde{e}_t) \left[V_w - c''(e_t) + V_w \left(\frac{c''(\tilde{e}^{FB}) - V_w}{c''(\tilde{e}^{FB})} \right) + V_{ww}(e_t - \tilde{e}_t) \right]} > 1. \end{aligned}$$

If $\lambda_{w_t} > 0$, effort is characterized by $\delta\Pi_{t+1} - c(e_t) = 0$, with $de_t/d\bar{w} < 0$.

$$V_w(\eta, w_t - \bar{w})(e_t - \tilde{e}_t) - c'(e_t) + V(\eta, w_t - \bar{w})$$

$$\delta\Pi_{t+1} + c(\tilde{e}_t) - c(e_t) + V(\eta, w_t - \bar{w})(e_t - \tilde{e}_t) = 0.$$

To derive the threshold $\tilde{\eta}$ above which a positive w_t is optimal note that

$$\lambda_{w_t} = \frac{c'(e_t) - V(\eta, w_t - \bar{w}) - V_w(e_t - \tilde{e}_t)}{c'(e_t) - V(\eta, w_t - \bar{w})}$$

Thus,

$$\lambda_{w_t} > 0 \Leftrightarrow c'(e_t) > e_t V_w(\eta, 0).$$

Take any e_t that is weakly smaller than the effort characterized by $1 - c'(e_t) = 0$, and define $\tilde{\eta}$ as $c'(e_t) - e_t V_w(\tilde{\eta}, 0) = 0$. Then,

$$\frac{d\tilde{\eta}}{de_t} = \frac{c''(e_t) - V_w(\tilde{\eta}, 0)}{e_t V_{w\eta}(\tilde{\eta}, 0)} = \frac{c''(e_t) - \frac{c'(e_t)}{e_t}}{e_t V_{w\eta}(\tilde{\eta}, 0)} > 0.$$

where the positive numerator follows from $c''' > 0$. ■

Proof of Proposition 5. First, I show that, for $p \rightarrow 1$, a separating contract yields higher profits than a pooling contract. There, note that, in any profit-maximizing equilibrium, (ICS), the selfish type's (IC) constraint, is tighter than (ICR), the reciprocal type's (IC) constraint:

$$-\frac{e_1^3}{3} + \delta w_2 \geq 0 \tag{ICS}$$

$$-\frac{e_1^3}{3} + \eta w_1 e_1 + \delta \left[w_2 + \frac{2(\sqrt{\eta w_2})^3}{3} \right]$$

$$\geq \frac{2}{3} (\sqrt{\eta w_1})^3. \tag{ICR}$$

With $w_1 = 0$, (ICS) is tighter than (ICR) for any second-period wage w_2 because second-period utilities are larger for the reciprocal type. A strictly positive w_1 can only possibly be optimal for the principal if it further relaxes (ICR) ((ICS) is unaffected by w_1), which confirms that (ICS) is tighter than (ICR) in

any profit-maximizing equilibrium. This implies that a strictly higher effort level can be implemented with a separating contract (then however only exerted by the reciprocal type) than with a pooling contract (then exerted by both). For $p \rightarrow 1$, profits under both regimes approach $e_1 - w_1 + \delta(\sqrt{w_2\eta} - w_2)$, which is larger with a separating contract because of the higher effort implemented in this case for any given set of wages.

To show that a pooling contract yields higher profits than a separating contract for $p \rightarrow 0$, I first assume that the principal offers a pooling contract and explore its properties. Then, I do the same with a separating contract, and finally compare both alternatives.

Pooling contract In any profit-maximizing equilibrium, (ICS) is tighter than (ICR). Therefore, (ICS) determines feasible effort in a pooling contract. This also implies that $w_1 = 0$, because a positive w_1 might only relax (ICR).

Now, the principal maximizes Π_1 , subject to her own (DE) constraint, $pe_2 - w_2 \geq 0$, as well as the selfish agent's (IC) constraint, $-\frac{e_1^3}{3} + \delta w_2 \geq 0$. This will bind because, otherwise, the principal could ask for a higher first-period effort level without violating any constraint. Moreover, the reciprocal type exerts an effort level $e_2 = \sqrt{w_2\eta}$ in the second period, whereas the selfish type's second period effort amounts to zero, hence $\Pi_1 = e_1 + \delta(p\sqrt{w_2\eta} - w_2)$.

Taking all this into account, the Lagrange function becomes

$$L = e_1 + \delta \left[p\sqrt{\frac{e_1^3}{3\delta}\eta} - \frac{e_1^3}{3\delta} \right] + \lambda_{DE} \left[p\sqrt{\frac{e_1^3}{3\delta}\eta} - \frac{e_1^3}{3\delta} \right],$$

and the first order condition

$$\frac{\partial L}{\partial e_1} = 1 + \left[\frac{p\eta}{2\sqrt{\frac{e_1^3}{3\delta}\eta}} - 1 \right] \frac{e_1^2}{\delta} (\delta + \lambda_{DE}) = 0.$$

First, assume $\lambda_{DE} = 0$. Then, e_1 is characterized by

$$2\sqrt{\frac{\eta}{3\delta}}(1 - e_1^2) + p\eta\sqrt{e_1} = 0. \quad (2)$$

Second, assume $\lambda_{DE} > 0$. Then, e_1 is determined by the binding (DE) constraint,

$$e_1 = \sqrt[3]{3\delta p^2\eta}.$$

To compute the condition for when (DE) binds, I plug $e_1 = \sqrt[3]{3\delta p^2 \eta}$ into the first order condition,

$$\begin{aligned}
& 1 + \left[\frac{p\eta}{2\sqrt{\frac{e_1^3}{3\delta}\eta}} - 1 \right] \frac{e_1^2}{\delta} (\delta + \lambda_{DE}) \\
& \quad = 1 - \frac{1}{2} \frac{e_1^2}{\delta} (\delta + \lambda_{DE}) \\
& \quad = 1 - \frac{1}{2} \frac{\left(\sqrt[3]{3\delta p^2 \eta}\right)^2}{\delta} (\delta + \lambda_{DE}) = 0 \\
& \Leftrightarrow \lambda_{DE} = 2\delta \frac{\left[1 - \frac{1}{2} \left(\sqrt[3]{3\delta p^2 \eta}\right)^2\right]}{\left(\sqrt[3]{3\delta p^2 \eta}\right)^2}
\end{aligned}$$

Therefore, (DE) binds if $1 - \frac{1}{2} \left(\sqrt[3]{3\delta p^2 \eta}\right)^2 \geq 0$, or

$$p^2 \leq \frac{(\sqrt{2})^3}{3\delta\eta}.$$

In this case, which is the relevant one for $p \rightarrow 0$, the principal's profits with a pooling equilibrium are

$$\Pi_1^P = e_1 = \sqrt[3]{3\delta p^2 \eta}.$$

Otherwise, $\Pi_1^P = e_1 + \delta \left[p\sqrt{\frac{e_1^3}{3\delta}\eta} - \frac{e_1^3}{3\delta} \right] = e_1 \left[1 + \frac{e_1^2 - 2}{3} \right]$, where e_1 is characterized by (2).

Separating contract In case she offers a separating contract, the principal maximizes $\Pi_1 = p[e_1 + \delta(e_2 - w_2)] - w_1$, where $e_2 = \sqrt{w_2 \eta}$, subject to her own (DE) constraint, $e_2 - w_2 \geq 0$ (which is relevant in case the agent turns out to be reciprocal), the non-negativity constraint $w_1 \geq 0$, as well as the reciprocal agent's binding (IC) constraint,

$$\begin{aligned}
& -\frac{e_1^3}{3} + \eta w_1 e_1 + \delta \left[w_2 + \frac{2(\sqrt{\eta w_2})^3}{3} \right] \\
& = \frac{2(\sqrt{\eta w_1})^3}{3}. \tag{IC}
\end{aligned}$$

There, note that

$$\frac{de_1}{dw_1} = \eta \frac{e_1 - \sqrt{\eta w_1}}{e_1^2 - \eta w_1} = \frac{\eta}{e_1 + \sqrt{\eta w_1}}$$

$$\frac{de_1}{dw_2} = \frac{\delta [1 + \sqrt{\eta w_2 \eta}]}{e_1^2 - \eta w_1}.$$

Therefore, the Lagrange function becomes $L = p[e_1 + \delta(\sqrt{w_2 \eta} - w_2) + \lambda_{DE}(\sqrt{w_2 \eta} - w_2)] - w_1 + \lambda_{w_1} w_1$, with first order conditions

$$\frac{\partial L}{\partial w_1} = p \frac{\eta}{e_1 + \sqrt{\eta w_1}} - 1 + \lambda_{w_1} = 0$$

$$\frac{\partial L}{\partial w_2} = p \left[\frac{\delta [1 + \sqrt{\eta w_2 \eta}]}{e_1^2 - \eta w_1} + \delta \left(\frac{\eta}{2\sqrt{w_2 \eta}} - 1 \right) + \lambda_{DE} \left(\frac{\eta}{2\sqrt{w_2 \eta}} - 1 \right) \right] = 0$$

For later use, note that the first condition implies that $w_1 = 0$ for $p \rightarrow 0$ (because e_1 is bounded away from zero for any strictly positive δ).

First, assume $\lambda_{DE} = 0$, hence

$$\frac{1 + \sqrt{\eta w_2 \eta}}{e_1^2 - \eta w_1} + \frac{\eta}{2\sqrt{w_2 \eta}} - 1 = 0.$$

This, together with the reciprocal agent's (IC) constraint, determines outcomes if $w_1 = 0$.

If $w_1 > 0$, outcomes are further specified by

$$p \frac{\eta}{e_1 + \sqrt{\eta w_1}} - 1 = 0.$$

Now, assume $\lambda_{DE} > 0$. Then, a binding (DE) constraint implies $w_2 = \eta$.

If $w_1 = 0$, (IC) yields

$$e_1 = \sqrt[3]{3\delta \left[\eta + \frac{2\eta^3}{3} \right]}$$

To compute the condition for when $w_1 = 0$ (if (DE) binds), I plug these values into the first first order condition, $p \frac{\eta}{\sqrt[3]{3\delta \left[\eta + \frac{2\eta^3}{3} \right]}} - 1 + \lambda_{w_1} = 0$. Therefore, $w_1 = 0$ if

$$p \frac{\eta}{\sqrt[3]{3\delta \left[\eta + \frac{2\eta^3}{3} \right]}} - 1 \leq 0$$

$$\Leftrightarrow p^3 \leq \frac{3\delta \left[\eta + \frac{2\eta^3}{3} \right]}{\eta^3}.$$

To compute the condition for when (DE) binds (if $w_1 = 0$), I plug these values into the second first-order-condition. Therefore, (DE) binds for

$$\frac{\delta [1 + \eta^2]}{\left(\sqrt[3]{3\delta \left[\eta + \frac{2\eta^3}{3} \right]} \right)^2} - \frac{1}{2} \delta \geq 0,$$

or

$$\delta^2 \leq \frac{8 [1 + \eta^2]^3}{9 \left[\eta + \frac{2\eta^3}{3} \right]^2}.$$

The right hand side of this condition is larger than 1, and (DE) *always* binds if $w_1 = 0$. Therefore, (DE) always binds if $p \rightarrow 0$ because then, $w_1 = 0$ (see above). Note, though, that this might change in a more general setup with a longer time horizon.

All this implies that, for $p \rightarrow 0$, profits with a separating contract are

$$\Pi_1^S = p e_1 = p \sqrt[3]{3\delta \left[\eta + \frac{2\eta^3}{3} \right]}.$$

Comparison For $p \rightarrow 0$, profits with a pooling contract are $\Pi_1^P = \sqrt[3]{3\delta p^2 \eta}$, and $\Pi_1^S = p \sqrt[3]{3\delta \left[\eta + \frac{2\eta^3}{3} \right]}$ for a separating contract. Therefore,

$$\Pi_1^P > \Pi_1^S$$

$$\Leftrightarrow \sqrt[3]{3\delta p^2 \eta} \geq p \sqrt[3]{3\delta \left[\eta + \frac{2\eta^3}{3} \right]}$$

$$\Leftrightarrow 1 \geq p \left(1 + \frac{2\eta^2}{3} \right),$$

which holds for $p \rightarrow 0$. ■

Proof of Lemma 6. The principal maximizes

$$\Pi_1 = e_1 - e_1^3/3 + \eta w_1 e_1 - 2/3(\sqrt{\eta w_1})^3 - w_1 + \delta \left(\frac{\eta}{4} - w_1 \right),$$

subject to $w_1 \geq 0$ and

$$\frac{e_1^3}{3} - \eta w_1 e_1 + \frac{2}{3}(\sqrt{\eta w_1})^3 \leq \delta \left(\frac{\eta}{4} - w_1 \right). \quad (\text{DE})$$

This yields the Lagrange function

$$\begin{aligned} L = & e_1 - (e_1)^3/3 + \eta w_1 e_1 - 2/3(\sqrt{\eta w_1})^3 - w_1 + \delta \left(\frac{\eta}{4} - w_1 \right) \\ & + \lambda_{w_1} w_1 + \lambda_{DE} \left[\eta w_1 e_1 + \delta \left(\frac{\eta}{4} - w_1 \right) - \frac{2}{3}(\sqrt{\eta w_1})^3 - \frac{e_1^3}{3} \right], \end{aligned}$$

where $\lambda_{w_1} \geq 0$ represents the Lagrange multiplier for the agent's limited liability constraint, and $\lambda_{DE} \geq 0$ the Lagrange multiplier for the principal's dynamic enforcement constraint.

First order conditions are

$$\begin{aligned} \frac{\partial L}{\partial e_1} = & 1 - e_1^2 + \eta w_1 + \lambda_{DE} [\eta w_1 - e_1^2] = 0 \\ \frac{\partial L}{\partial w_1} = & \eta e_1 - \eta \sqrt{\eta w_1} - 1 - \delta + \lambda_{w_1} \\ & + \lambda_{DE} [\eta e_1 - \delta - \eta \sqrt{\eta w_1}] = 0. \end{aligned}$$

First, assume $\lambda_{DE} = 0$. Then, I have to consider the two cases $w_1 = 0$ and $w_1 > 0$.

If $w_1 = 0$, $e_1 = 1$ and $\Pi_1 = 2/3 + \delta\eta/4$. Moreover, $d\Pi_1/dw_1|_{w_1=0} = \sqrt{\eta^2} - 1 - \delta$, therefore $w_1 = 0$ for $\eta^2 \leq (1 + \delta)^2$, whereas $w_1 > 0$ for $\eta^2 > (1 + \delta)^2$. Recall that the condition for a positive wage in case (DE) is not binding in the main part (i.e., without an adjustment of the reference wage) equals $\eta > 1$.

Furthermore, $e_1 > e_2 \Leftrightarrow \eta^2 < 4$, which holds because $\eta^2 < (1 + \delta)^2$. Moreover, $0 = w_1 < w_2 = \frac{\eta}{4}$, and $\frac{de_1}{d\eta} = 0 < \frac{de_2}{d\eta}$.

To check the feasibility of the case $\lambda_{DE} = 0$ and $w_1 = 0$, I plug the respective values into the (DE) constraint, and obtain

$$\frac{16}{9\delta^2} \leq \eta^2.$$

This is consistent with $\eta^2 \leq (1 + \delta)^2$ if $3\delta(1 + \delta) \geq 4$. Now, assume $\eta^2 > (1 + \delta)^2$ and $\lambda_{DE} = 0$. Hence, $\lambda_{w_1} = 0$, and the first order conditions yield $e_1 = \frac{(1+\delta)^2 + \eta^2}{2\eta(1+\delta)}$ and $w_1 = \frac{[\eta^2 - (1+\delta)^2]^2}{4(1+\delta)^2\eta^3}$. Moreover, $e_1 > e_2 \Leftrightarrow \delta\eta^2 < (1 + \delta)^2$, which only is consistent with $\eta^2 > (1 + \delta)^2$ if δ is sufficiently small. In any case, $w_1 < w_2$.

To check the feasibility of the case $\lambda_{DE} = 0$ and $w_1 > 0$, I plug the respective values into the (DE) constraint, and obtain

$$2 \leq \delta \left(\frac{(1 + \delta)^2 \eta^2 - 1}{(1 + \delta)^2} \right) + (1 + \delta)^2 \frac{1}{3} \frac{(2 - \delta)}{\eta^2}.$$

The right hand side is increasing in η^2 if δ is large enough. Since $\eta^2 > (1 + \delta)^2$, this condition holds if it is satisfied for $\eta^2 = (1 + \delta)^2$. For this case, it becomes

$$\frac{4}{3} \leq \delta^2 (2 + \delta) + \frac{2}{3} \delta - \frac{\delta}{(1 + \delta)^2}.$$

There, the right hand side is increasing in δ and, for $\delta \rightarrow 1$, approaches $3 + \frac{5}{12} > \frac{4}{3}$. Hence, this case is feasible if η and/or δ are large enough.

Now, assume that the (DE) constraint binds, hence $\lambda_{DE} > 0$.

First, I assume that $\lambda_{w_1} > 0$, hence $w_1 = 0$ and $e_1 = \sqrt{1/(1 + \lambda_{DE})}$. To establish the existence of $\tilde{\eta}$, note that $\partial L / \partial w_1 |_{w_1=0} = \left(\eta \sqrt{1/(1 + \lambda_{DE})} - \delta \right) (1 + \lambda_{DE}) - 1$, which is positive for $\eta^2 > (1 + \delta(1 + \lambda_{DE}))^2 / (1 + \lambda_{DE})$. This threshold is larger than with a non-binding (DE) if $\lambda_{DE} > (1 - \delta^2) / \delta^2$, which might or might not hold. Moreover, provided $\eta^2 \leq (1 + \delta(1 + \lambda_{DE}))^2 / (1 + \lambda_{DE})$, $e_1 > e_2 \Leftrightarrow \eta^2 (1 + \lambda_{DE}) < 4$, which might or might not hold.

Second, I assume $\eta^2 > (1 + \delta(1 + \lambda_{DE}))^2 / (1 + \lambda_{DE})$, hence $\lambda_{w_1} = 0$. Then, the first order conditions yield

$$e_1 = \frac{\eta^2 (1 + \lambda_{DE}) + (1 + \delta(1 + \lambda_{DE}))^2}{2\eta (1 + \lambda_{DE}) (1 + \delta(1 + \lambda_{DE}))}$$

$$w_1 = \frac{\left[\eta^2 (1 + \lambda_{DE}) - (1 + \delta(1 + \lambda_{DE}))^2 \right]^2}{4\eta^3 (1 + \lambda_{DE})^2 (1 + \delta(1 + \lambda_{DE}))^2}.$$

Now, $e_1 > e_2 \Leftrightarrow \frac{(1 + \delta(1 + \lambda_{DE}))^2}{\delta(1 + \lambda_{DE})^2} > \eta^2$, which might or might not be consistent with $\eta^2 > (1 + \delta(1 + \lambda_{DE}))^2 / (1 + \lambda_{DE})$.

■

Proof of Lemma 7. In the second period, the principal maximizes $\pi_2 = e_2 - w_2$, where e_2 is given by

$$-e_2^2 - \frac{4}{3}e_2^3\eta + w_2\eta = 0.$$

This yields

$$\begin{aligned} e_2 &= \frac{\sqrt{1+4\eta^2}-1}{4\eta} \\ w_2 &= \frac{e_2^2 + \frac{4}{3}e_2^3\eta}{\eta} \\ \pi_2 &= e_2 \left(\frac{8\eta^2 + 1 - \sqrt{1+4\eta^2}}{12\eta^2} \right) \\ u_2 &= \frac{e_2^2(1+\eta e_2)^2}{\eta}. \end{aligned}$$

Recall that last-period profits in the main setup are $\eta/4$, which is larger than the amount obtained here.

In the first period, at e_1 , u_1 is decreasing in e_1 . If it were increasing, the agent would further raise his effort level. This implies that (IC) is binding in a profit-maximizing equilibrium. If it were not binding, the principal could ask for a higher effort level without paying more.

Plugging the binding (IC) constraint,

$$b_1 = \frac{e_1^3}{3} - w_1 + \left(w_1 - \frac{\tilde{e}_1^3}{3} \right) \frac{(1+\eta\tilde{e}_1)}{(1+\eta e_1)},$$

into profits and the (DE) constraint yields the Lagrange function

$$\begin{aligned} L &= e_1 - \frac{e_1^3}{3} - \left(w_1 - \frac{\tilde{e}_1^3}{3} \right) \frac{(1+\eta\tilde{e}_1)}{(1+\eta e_1)} \\ &\quad + \lambda_{w_1} w_1 + \lambda_{DE} \left[-\frac{e_1^3}{3} + w_1 - \left(w_1 - \frac{\tilde{e}_1^3}{3} \right) \frac{(1+\eta\tilde{e}_1)}{(1+\eta e_1)} + \delta\pi_2 \right], \end{aligned}$$

where $\lambda_{w_1} \geq 0$ represents the Lagrange multiplier for the agent's limited liability constraint, and $\lambda_{DE} \geq 0$ the Lagrange multiplier for the principal's dynamic enforcement constraint.

First order conditions are

$$\begin{aligned}\frac{\partial L}{\partial e_1} &= 1 - e_1^2 + \left(w_1 - \frac{\tilde{e}_1^3}{3}\right) \frac{\eta(1 + \eta\tilde{e}_1)}{(1 + \eta e_1)^2} \\ &\quad + \lambda_{DE} \left[-e_1^2 + \left(w_1 - \frac{\tilde{e}_1^3}{3}\right) \frac{\eta(1 + \eta\tilde{e}_1)}{(1 + \eta e_1)^2} \right] = 0 \\ \frac{\partial L}{\partial w_1} &= - \left(1 - \tilde{e}_1^2 \frac{d\tilde{e}}{dw_1}\right) \frac{(1 + \eta\tilde{e}_1)}{(1 + \eta e_1)} - \left(w_1 - \frac{\tilde{e}_1^3}{3}\right) \frac{\eta}{(1 + \eta e_1)} \frac{d\tilde{e}_1}{dw_1} \\ &\quad + \lambda_{DE} \left[1 - \left(1 - \tilde{e}_1^2 \frac{d\tilde{e}_1}{dw_1}\right) \frac{(1 + \eta\tilde{e}_1)}{(1 + \eta e_1)} - \left(w_1 - \frac{\tilde{e}_1^3}{3}\right) \frac{\eta}{(1 + \eta e_1)} \frac{d\tilde{e}_1}{dw_1} \right] \\ &\quad + \lambda_{w_1} = 0\end{aligned}$$

Using $-\tilde{e}_1^2 - 4/3\tilde{e}_1^3\eta + w_1\eta = 0$, which implies $w_1 = \tilde{e}_1^2/\eta + 4/3\tilde{e}_1^3$, those conditions become

$$\begin{aligned}\frac{\partial L}{\partial e_1} &: 1 - \left(e_1^2 - \tilde{e}_1^2 \frac{(1 + \eta\tilde{e}_1)^2}{(1 + \eta e_1)^2}\right) (1 + \lambda_{DE}) = 0 \\ \frac{\partial L}{\partial w_1} &: - \frac{(1 + \eta\tilde{e})}{(1 + \eta e_1)} (1 + \lambda_{DE}) + \lambda_{w_1} + \lambda_{DE} = 0\end{aligned}$$

First, assume $\lambda_{DE} = 0$. Then, I have to consider the two cases $w_1 = 0$ and $w_1 > 0$. However, $w_1 > 0$ and consequently $\lambda_{w_1} = 0$ cannot be optimal, since in this case, the second condition would become $-(1 + \eta\tilde{e})/(1 + \eta e_1) = 0$.

Therefore, $\lambda_{DE} = 0$ implies $w_1 = 0$; hence $\tilde{e} = 0$ and

$$e_1 = 1.$$

Moreover $e_1 = 1 > \left(\sqrt{1 + 4\eta^2} - 1\right)/(4\eta) = e_2$ and $w_1 = 0 < (e_2^2 + \frac{4}{3}e_2^3\eta)/\eta = w_2$.

However, note that for two periods and $\delta \leq 1$, $\lambda_{DE} = 0$ is not feasible: For $w_1 = 0$, $b_1 = 1/3$ and $e_1 = 1$, the (DE) constraint becomes

$$-\frac{1}{3} + \delta \frac{(1 + 4\eta^2) \sqrt{1 + 4\eta^2} - 6\eta^2 - 1}{24\eta^3} \geq 0.$$

There, the second term increases in η and approaches $\delta 2/9$ for $\eta \rightarrow \infty$. Therefore, the constraint does not hold for any η if $\delta \leq 1$.

Now, assume that (DE) binds. Again, I start with $w_1 = 0$. Then, $e_1 = \sqrt{1/(1 + \lambda_{DE})}$, and

$$\begin{aligned}\lim_{w_1 \rightarrow 0} \frac{\partial L}{\partial w_1} &= -\frac{(1 + \lambda_{DE})}{(1 + \eta e_1)} + \lambda_{DE} \\ &= -\frac{(1 + \lambda_{DE})}{\left(1 + \sqrt{\frac{\eta^2}{(1 + \lambda_{DE})}}\right)} + \lambda_{DE},\end{aligned}$$

which is positive for

$$\eta^2 > \frac{1 + \lambda_{DE}}{\lambda_{DE}^2}.$$

Put differently,

$$e_1 = \sqrt{\frac{1}{(1 + \lambda_{DE})}},$$

if $\eta^2 \lambda_{DE}^2 - \lambda_{DE} - 1 \leq 0$, hence if $\lambda_{DE} \leq (1 + \sqrt{1 + 4\eta^2}) / (2\eta^2)$. In this case,

$$\begin{aligned}e_1 &\geq \sqrt{\frac{1}{\left(1 + \frac{1 + \sqrt{1 + 4\eta^2}}{2\eta^2}\right)}} \\ &= \sqrt{\frac{2\eta^2}{2\eta^2 + 1 + \sqrt{1 + 4\eta^2}}}\end{aligned}$$

This is larger than $e_2 = (\sqrt{1 + 4\eta^2} - 1) / (4\eta)$, if

$$12\eta^4 > 0.$$

Therefore, $e_1 > e_2$ and $w_1 < w_2$.

Now, assume that $\lambda_{DE} > (1 + \sqrt{1 + 4\eta^2}) / (2\eta^2)$, hence $w_1 > 0$. Solving the first first order condition for λ_{DE} and plugging it into the second yields

$$\eta - \frac{[1 + \eta\theta(e_1 + \tilde{e}_1)][e_1(1 + e_1\eta) + \tilde{e}_1(1 + \eta\tilde{e}_1)]}{(1 + \eta e_1)} = 0,$$

which, together with the binding (DE) constraint, determines e_1 as well as \tilde{e}_1 (and consequently w_1).

Making use of $-\tilde{e}_1^2 - \frac{4}{3}\tilde{e}_1^3\eta + w_1\eta = 0 \Rightarrow w_1 = \frac{\tilde{e}_1^2}{\eta} + \frac{4}{3}\tilde{e}_1^3$, the latter becomes

$$-\frac{(e_1^3 - \tilde{e}_1^3)}{3} + \tilde{e}_1^2(e_1 - \tilde{e}_1) \frac{(1 + \tilde{e}_1\eta)}{(1 + \eta e_1)} + \delta\pi_2 = 0,$$

In order to prove $e_1 > e_2$ and $w_1 < w_2$, I first show that e_1 is increasing and \tilde{e}_1 is decreasing in $\delta\pi_2$:

$$\frac{de_1}{d(\delta\pi_2)} = \frac{\begin{vmatrix} 0 & -\frac{\eta\{e_1^2\eta + \eta\tilde{e}_1^2 + 2(e_1 + \tilde{e}_1)(1 + \eta\tilde{e}_1)\} + (1 + 2\eta\tilde{e}_1)}{(1 + \eta e_1)} \\ -1 & -\frac{(e_1^2 + e_1\tilde{e}_1 + \tilde{e}_1^2) + (e_1 - \tilde{e}_1)(e_1 + 2\tilde{e}_1)}{3} + \frac{2\tilde{e}_1 e_1 + 3e_1\tilde{e}_1^2\eta - 3\tilde{e}_1^2 - 4\tilde{e}_1^3\eta}{(1 + \eta e_1)} \end{vmatrix}}{\begin{vmatrix} \frac{\tilde{e}_1\eta(1 + \eta\tilde{e}_1)^2 - [(1 + \eta\tilde{e}_1)(1 + e_1\eta) + \eta e_1(1 + 2e_1\eta)](1 + \eta e_1)}{(1 + \eta)^2} & -\frac{\eta\theta\{e_1^2\eta + \tilde{e}_1^2\eta + 2(e_1 + \tilde{e}_1)(1 + \eta\tilde{e}_1)\} + (1 + 2\eta\tilde{e}_1)}{(1 + \eta e_1)} \\ -e_1^2 + \tilde{e}_1^2 \frac{(1 + \eta\tilde{e}_1)^2}{(1 + \eta e_1)^2} & \tilde{e}_1^2 + \frac{2\tilde{e}_1 e_1 + 3e_1\tilde{e}_1^2\eta - 3\tilde{e}_1^2 - 4\tilde{e}_1^3\eta}{(1 + \eta e_1)} \end{vmatrix}}$$

There, the numerator equals

$$-\frac{\eta\{e_1^2\eta + \eta\tilde{e}_1^2 + 2(e_1 + \tilde{e}_1)(1 + \eta\tilde{e}_1)\} + (1 + 2\eta\tilde{e}_1)}{(1 + \eta e_1)} < 0,$$

and the denominator

$$\frac{\tilde{e}_1\eta(1 + \eta\tilde{e}_1)^2 - (\eta\tilde{e}_1 + 1)(1 + e_1\eta)^2 - \eta e_1(1 + 2e_1\eta)(1 + \eta e_1)}{(1 + \eta e_1)^2} \left[2(e_1 - \tilde{e}_1)\tilde{e}_1 \frac{1 + 2\eta\tilde{e}_1}{(1 + \eta e_1)} \right] + \left(-e_1^2 + \tilde{e}_1^2 \frac{(1 + \eta\tilde{e}_1)^2}{(1 + \eta e_1)^2} \right) \frac{\eta\theta\{e_1^2\eta + \tilde{e}_1^2\eta + 2(e_1 + \tilde{e}_1)(1 + \eta\tilde{e}_1)\} + (1 + 2\eta\tilde{e}_1)}{(1 + \eta e_1)},$$

which is negative because of $e_1 > \tilde{e}_1$. Therefore,

$$\frac{de_1}{d(\delta\pi_2)} > 0.$$

If $\delta\pi_1 = 0$, $b_1 = 0$, and π_1 is maximized by setting $w_1 = w_2$, implying $e_1 = e_2$. Therefore, $e_1 > e_2$ given $\delta\pi_1 > 0$.

Moreover,

$$\frac{d\tilde{e}_1}{d(\delta\pi_2)} = \frac{\begin{vmatrix} \frac{\tilde{e}_1\eta(1 + \eta\tilde{e}_1)^2 - [(1 + \eta\tilde{e}_1)(1 + \eta e_1) + \eta e_1(1 + 2\eta e_1)](1 + \eta e_1)}{(1 + \eta e_1)^2} & 0 \\ -e_1^2 + \tilde{e}_1^2 \frac{(1 + \eta\tilde{e}_1)^2}{(1 + \eta e_1)^2} & -1 \end{vmatrix}}{\begin{vmatrix} \frac{\eta\tilde{e}_1(1 + \eta\tilde{e}_1)^2 - [(1 + \eta\tilde{e}_1)(1 + \eta e_1) + \eta e_1(1 + 2\eta e_1)](1 + \eta e_1)}{(1 + \eta e_1)^2} & -\frac{\eta\theta\{e_1^2\eta + \tilde{e}_1^2\eta + 2(e_1 + \tilde{e}_1)(1 + \eta\tilde{e}_1)\} + (1 + 2\eta\tilde{e}_1)}{(1 + \eta e_1)} \\ -e_1^2 + \tilde{e}_1^2 \frac{(1 + \eta\tilde{e}_1)^2}{(1 + \eta e_1)^2} & \frac{2\tilde{e}_1(e_1 - \tilde{e}_1)(1 + 2\tilde{e}_1)}{(1 + \eta e_1)} \end{vmatrix}}$$

This is negative, since the denominator is negative and the numerator, which equals

$$\frac{[(1 + \eta\tilde{e}_1)(1 + \eta e_1) + \eta e_1(1 + 2\eta e_1)](1 + \eta e_1) - \eta\tilde{e}_1(1 + \eta\tilde{e}_1)^2}{(1 + \eta e_1)^2},$$

is positive.

Therefore,

$$\frac{dw_1}{d(\delta\pi_2)} < 0.$$

If $\delta\pi_1 = 0$, $b_1 = 0$, and π_1 is maximized by setting $w_1 = w_2$. Therefore, $w_1 < w_2$ given $\delta\pi_1 > 0$. ■

Proof of Lemma 9 The principal maximizes Π_1 , subject to (IC) and (DE) constraints for every period, $U_t(\mathcal{H}^{t,A*}) = U_t(\mathcal{H}^{t-1,A-cheat})$

$$\begin{aligned} & b_t - c(e_t) + e_t V(\eta, w_t^{nd}) + \delta U_{t+1}(\mathcal{H}^{t,A*}) \\ & \geq -c(\tilde{e}_t) + \tilde{e}_t V(\eta, w_t^{nd}) + \delta U_{t+1}(\mathcal{H}^{t,A-cheat}) \end{aligned} \quad (\text{IC})$$

$$-b_t + \delta \Pi_{t+1}(\mathcal{H}^{t,P*}) \geq 0. \quad (\text{DE})$$

After the principal deviates (downwards), η drops to zero, and continuation payoffs of principal and agent are zero. For this proof, I focus on deviations by the agent, hence assume (DE) constraints hold on and off the equilibrium path and η remains constant. I only have to take care of the possibility of upward deviations by the principal (i.e., increasing future payments of w_t^{nd} after a deviation by the agent), which do not reduce η . If these upward deviations are optimal off the equilibrium path, they affect the size of $U_{t+1}(\mathcal{H}^{t,A-cheat})$ and hence the agent's incentives to provide equilibrium effort.

First, it is without loss of generality to set $w_t^d = 0$ after all histories: Assume there is a profit-maximizing equilibrium with $w_t^d > 0$. Reducing it to zero and increasing b_{t-1} by δw_t^d leaves all payoffs and constraints unaffected, thus there exists another profit-maximizing equilibrium with $w_t^d = 0$ in all periods t and for all histories. In the following, I consider such an equilibrium.

Second, w_t^{nd} is – by definition – independent of the agent's past effort choices. Thus, I need to prove consistency in the sense that it is indeed optimal for the principal to not change w_t^{nd} after a deviation by the agent. In this respect, I assume that all w_t^{nd} in a profit-maximizing relational contract are (weakly) smaller than the wage in a reciprocity spot contract and later verify that this is optimal (see Proposition 3). Then, firing the agent after he did not perform accordingly is *not* subgame perfect and can thus be ruled out as a potential response of the principal. The reason is that, with non-discretionary wages not exceeding the profit-maximizing wage in a reciprocity spot contract, the principal makes positive profits even if no relational incentives are provided on top (if some w_t^{nd} were larger, I would have to check whether spot contracts with “too high” wages still generate positive profits). This also implies that the principal does not lower non-discretionary wages after a deviation by the agent.

Third, I need to verify that increasing w^{nd} after a deviation by the agent is not optimal (otherwise, a deviation might increase the agent's continuation utility and consequently his incentives to deviate). But

this follows from the definition of w^{nd} , which states that the agent only reciprocates to wage components that are independent of the agent's past effort, hence increasing the wage after a deviation of the agent would not induce a stronger reciprocal reaction.

All this implies that $U_{t+1}(\mathcal{H}^{t,A-cheat})$ contains the same non-discretionary wage stream $\{w_\tau^{nd}\}_{\tau=t+1}^T$ as $U_{t+1}(\mathcal{H}^{t,A^*})$. Moreover, I can set $U_{t+1}(\mathcal{H}^{t,A-cheat}) = \sum_{\tau=t+1}^T \delta^{\tau-t} (w_\tau^{nd} - c(\tilde{e}_\tau) + \tilde{e}_\tau V(\eta, w_\tau^{nd}))$, which corresponds to the agent's minmax payoff given $\{w_\tau^{nd}\}_{\tau=t}^T$ (and provided $w_t \geq 0$, which rules out negative upfront payments to extract the agent's "reciprocity rent"). The reason is that, using standard arguments, a series of spot contracts always constitutes an equilibrium of such a finitely repeated game.

Fourth, I show that for given values w_t^{nd} and e_t , it is (weakly) optimal for the principal to set $b_t - c(e_t) + e_t V(\eta, w_t^{nd}) = -c(\tilde{e}_t) + \tilde{e}_t V(\eta, w_t^{nd})$ in all periods. To do so, I proceed sequentially and start with period $t = 1$:

- Assume $b_1 - c(e_1) + e_1 V(\eta, w_1^{nd}) > -c(\tilde{e}_1) + \tilde{e}_1 V(\eta, w_1^{nd})$. Reduce b_1 by a small $\varepsilon > 0$, which increases Π_1 and relaxes the first-period (DE) constraint.
- Assume $b_1 - c(e_1) + e_1 V(\eta, w_1^{nd}) < -c(\tilde{e}_1) + \tilde{e}_1 V(\eta, w_1^{nd})$. Because $w^d = 0 \forall t$, the (IC) constraint for period $t = 1$ requires at least one period $\tau > 1$ in which $b_\tau - c(e_\tau) + e_\tau V(\eta, w_\tau^{nd}) > -c(\tilde{e}_\tau) + \tilde{e}_\tau V(\eta, w_\tau^{nd})$. Assume τ_1 is the first of these periods. Reduce b_{τ_1} by $\varepsilon > 0$ and increase b_1 by $\delta^{\tau_1-1} \varepsilon$. Proceed until either $b_1 - c(e_1) + e_1 V(\eta, w_1^{nd}) = -c(\tilde{e}_1) + \tilde{e}_1 V(\eta, w_1^{nd})$ or $b_{\tau_1} - c(e_{\tau_1}) + e_{\tau_1} V(\eta, w_{\tau_1}^{nd}) = -c(\tilde{e}_{\tau_1}) + \tilde{e}_{\tau_1} V(\eta, w_{\tau_1}^{nd})$. In the latter case, move to the second period $\tau_2 > \tau_1$ in which $b_{\tau_2} - c(e_{\tau_2}) + e_{\tau_2} V(\eta, w_{\tau_2}^{nd}) > -c(\tilde{e}_{\tau_2}) + \tilde{e}_{\tau_2} V(\eta, w_{\tau_2}^{nd})$ (such a period must exist as long as $b_1 - c(e_1) + e_1 V(\eta, w_1^{nd}) < -c(\tilde{e}_1) + \tilde{e}_1 V(\eta, w_1^{nd})$), reduce b_{τ_2} by $\varepsilon > 0$ and increase b_1 by $\delta^{\tau_2-1} \varepsilon$, and so on. Continue until $b_1 - c(e_1) + e_1 V(\eta, w_1^{nd}) = -c(\tilde{e}_1) + \tilde{e}_1 V(\eta, w_1^{nd})$.

In period $t = 2$, proceed accordingly if $b_2 - c(e_2) + e_2 V(\eta, w_2^{nd}) \neq 0$, as well as in all subsequent periods.

It follows that $u_t = w_t^{nd} + b_t - c(e_t) + e_t V(\eta, w_t^{nd}) = w_t^{nd} - c(\tilde{e}_t) + \tilde{e}_t V(\eta, w_t^{nd})$, and consequently that $U_t(\mathcal{H}^{t-1,A^*}) = U_t(\mathcal{H}^{t-1,A-cheat})$ in all periods t , and that all (IC) constraints hold as equalities.

Finally, these results imply that there exists a sequentially optimal profit-maximizing equilibrium, in the sense that maximizing Π_1 is equivalent to maximizing each per-period profit π_t , subject to (DE) and binding (IC) constraints. This is because the agent's incentives to exert effort in any period t are solely determined by payments made in period t , w_t^{nd} and b_t . There, b_t is bounded by the principal's future payoff stream, thus maximizing each π_t ceteris paribus maximizes Π_1 , but also yields the largest maximum feasible value of b_t without adverse effect on (IC) constraints. ■

Proof of Lemma 10 If the (DE) constraint does not bind in a period t , the principal maximizes profits $\pi_t = e_t - c(e_t) + c(\tilde{e}_t) + V(\eta, w_t)(e_t - \tilde{e}_t) - w_t$, subject to $w_t \geq 0$, where \tilde{e}_t is characterized by $-c'(\tilde{e}_t) + V(\eta, w_t) = 0$.

Ignoring the non-negativity constraint on w_t , first order conditions are

$$\begin{aligned}\frac{\partial \pi_t}{\partial e_t} &= 1 - c'(e_t) + V(\eta, w_t) = 0 \\ \frac{\partial \pi_t}{\partial w_t} &= (c'(\tilde{e}_t) - V(\eta, w_t)) \frac{d\tilde{e}_t}{dw_t} + V_w(\eta, w_t)(e_t - \tilde{e}_t) - 1 \\ &= V_w(\eta, w_t)(e_t - \tilde{e}_t) - 1 = 0\end{aligned}$$

Since the second-order partial derivatives, $\partial^2 \pi_t / \partial e_t^2$ and $\partial^2 \pi_t / \partial w_t^2$, are negative, this problem cannot have a minimum. Therefore, if $\frac{\partial \pi_t}{\partial w_t} |_{w_t=0}$ (evaluated at the effort level obtained by the first-order condition) is non-positive, the optimal wage is zero; if it is strictly positive, the optimal wage is positive as well:

$$\frac{\partial \pi_t}{\partial w_t} |_{w_t=0} = V_w(\eta, 0)e_t - 1$$

where e_t is characterized by $1 - c'(e_t) = 0$. Because $V_{w\eta} > 0$, $V_w(\eta, 0)e_t - 1$ is increasing in η holding e_t constant. Since $\lim_{\eta \rightarrow \infty} V_w(\eta, 0) > 1/e_t$ there exists a $\bar{\eta}$ as stated in the Lemma. Note that, if $\lim_{w \rightarrow 0} V_w(\eta, 0) = \infty$ for all $\eta > 0$, $\bar{\eta} = 0$. ■

Proof of Lemma 11 Including the respective (DE) constraints, the Lagrange function of the principal's maximization problem in a period t becomes

$$\begin{aligned}L_t &= e_t - c(e_t) + c(\tilde{e}_t) + V(\eta, w_t)(e_t - \tilde{e}_t) - w_t \\ &\quad + \lambda_{DE_t} [\delta \Pi_{t+1} + c(\tilde{e}_t) - c(e_t) + V(\eta, w_t)(e_t - \tilde{e}_t)] + w_t \lambda_{w_t},\end{aligned}$$

where $\lambda_{w_t} \geq 0$ represents the Lagrange multiplier for the agent's limited liability constraint and $\lambda_{DE_t} \geq 0$ represents the Lagrange multiplier for the principal's dynamic enforcement constraint.

First-order conditions are

$$1 + (V(\eta, w_t) - c'(e_t))(1 + \lambda_{DE_t}) = 0$$

$$V_w(\eta, w_t)(e_t - \tilde{e}_t)(1 + \lambda_{DE_t}) - 1 + \lambda_{w_t} = 0,$$

where the latter yields $e_t > \tilde{e}_t$ (unless $e_t = \tilde{e}_t = 0$).

Solving the first condition for $1 + \lambda_{DE_t}$ and plugging this into the second condition gives

$$\lambda_{w_t} = \frac{(c'(e_t) - V(\eta, w_t)) - V_w(\eta, w_t)(e_t - \tilde{e}_t)}{(c'(e_t) - V(\eta, w_t))}$$

The denominator is positive because $e_t > \tilde{e}_t$ (and \tilde{e}_t is characterized by $c'(\tilde{e}_t) - V(\eta, w_t)$); taking $V(\eta, 0) = \tilde{e}_t(w_t = 0) = 0$ into account,

$$\lambda_{w_t} > 0 \Leftrightarrow c'(e_t) > e_t V_w(\eta, 0).$$

Take any e_t that is weakly smaller than the effort characterized by $1 - c'(e_t) = 0$, and define $\tilde{\eta}$ as $c'(e_t) - e_t V_w(\tilde{\eta}, 0) = 0$. Then,

$$\frac{d\tilde{\eta}}{de_t} = \frac{c''(e_t) - V_w(\tilde{\eta}, 0)}{e_t V_{w\eta}(\tilde{\eta}, 0)} = \frac{c''(e_t) - \frac{c'(e_t)}{e_t}}{e_t V_{w\eta}(\tilde{\eta}, 0)} > 0.$$

where the positive numerator follows from $c''' > 0$.

If $w_t = 0$, profits are constrained by $\delta\Pi_{t+1} - c(e_t) \geq 0$, with $1 - c'(e_t) = 0$ if the constraint does not bind and $1 - c'(e_t) > 0$ if it does. In the latter case e_t is strictly increasing in $\delta\Pi_{t+1}$. Therefore, $\tilde{\eta} < \bar{\eta}$, and $\tilde{\eta}$ increases in continuation profits $\delta\Pi_{t+1}$. ■

Proof of Proposition 6. First, I demonstrate that profits are increasing in η . In Lemma 8, I have shown that $d\pi_T/d\eta > 0$. In any earlier period, profits are $\pi_t = e_t - c(e_t) - w_t + c(\tilde{e}_t) + V(\eta, w_t)(e_t - \tilde{e}_t)$. Keeping wage, bonus, and equilibrium effort fixed, an increase in η would yield higher profits:

$$\frac{\partial \pi_t}{\partial \eta} = (c'(\tilde{e}_t) - V(\eta, w_t)) \frac{\partial \tilde{e}_t}{\partial \eta} + V_\eta(e_t - \tilde{e}_t) = V_\eta(e_t - \tilde{e}_t) \geq 0.$$

Thus, also if wage, bonus, and/or effort are adapted in response, profits (weakly) have to go up. Moreover, higher future profits relax the (DE) constraint for period t . Further taking into account the results

of Lemmas 10 and 11, the following holds:

$$\frac{d\pi_t}{d\eta} \begin{cases} = 0 & \text{if } \eta < \bar{\eta} \text{ and (DEt) is slack} \\ > & \text{otherwise.} \end{cases}$$

Therefore, in all periods t ,

$$\frac{d\Pi_t}{d\eta} > 0.$$

This has the following consequences for equilibrium effort. If $\eta < \bar{\eta}$ and (DEt) is slack, $de_t/d\eta = 0$. If $w_t = 0$ and (DEt) binds, effort is characterized by $\delta\Pi_{t+1} - c(e_t) = 0$ and strictly increases in η . If $w_t > 0$ and (DEt) is slack, effort and wage are characterized by

$$\begin{aligned} 1 - c'(e^{FB}) + V(\eta, w^{FB}) &= 0 \\ V_w(\eta, w^{FB})(e^{FB} - \tilde{e}^{FB}) - 1 &= 0. \end{aligned}$$

Then,

$$\begin{aligned} \frac{de^{FB}}{d\eta} &= \frac{\begin{vmatrix} -V_\eta & V_w \\ -[V_{w\eta}(e^{FB} - \tilde{e}^{FB}) - V_w \frac{d\tilde{e}^{FB}}{d\eta}] & V_{ww}(e^{FB} - \tilde{e}^{FB}) - V_w \frac{d\tilde{e}}{dw} \end{vmatrix}}{\begin{vmatrix} -c''(e^{FB}) & V_w \\ V_w & V_{ww}(e^{FB} - \tilde{e}^{FB}) - V_w \frac{d\tilde{e}}{dw} \end{vmatrix}} \\ &= \frac{-V_\eta \frac{V_{ww}}{V_w} + V_{w\eta}}{-c''(e^{FB})V_{ww}(e^{FB} - \tilde{e}^{FB}) + V_w^2 \frac{c''(e^{FB}) - c''(\tilde{e}^{FB})}{c''(\tilde{e}^{FB})}} > 0 \end{aligned}$$

If $w_t > 0$ and (DEt) binds, effort and wage are characterized by

$$\begin{aligned} V_w(\eta, w_t)(e_t - \tilde{e}_t) - c'(e_t) + V(\eta, w_t) &= 0 \\ \delta\Pi_{t+1} + c(\tilde{e}_t) - c(e_t) + V(\eta, w_t)(e_t - \tilde{e}_t) &= 0. \end{aligned}$$

Then,

$$\begin{aligned}
\frac{de_t}{d\eta} &= \frac{\partial e_t}{\partial \eta} + \frac{de_t}{d\delta\Pi_{t+1}} \frac{d\delta\Pi_{t+1}}{d\eta} \\
&= \frac{\begin{vmatrix} -\left[V_{w\eta}(e_t - \tilde{e}_t) - V_w \frac{d\tilde{e}}{d\eta} + V_\eta\right] & V_{ww}(e_t - \tilde{e}_t) - V_w \frac{d\tilde{e}_t}{dw_t} + V_w \\ -V_\eta(e_t - \tilde{e}_t) & V_w(e_t - \tilde{e}_t) \end{vmatrix}}{\begin{vmatrix} V_w - c''(e_t) & V_{ww}(e_t - \tilde{e}_t) - V_w \frac{d\tilde{e}_t}{dw_t} + V_w \\ -c'(e_t) + V(\eta, w_t) & V_w(e_t - \tilde{e}_t) \end{vmatrix}} \\
&\quad + \frac{de_t}{d\delta\Pi_{t+1}} \frac{d\delta\Pi_{t+1}}{d\eta} \\
&= \frac{-V_w V_{w\eta} + V_\eta V_{ww}}{-V_w \left[c''(e_t) - V_w - V_w \left(\frac{c''(\tilde{e}^{FB}) - V_w}{c''(\tilde{e}^{FB})} \right) - V_{ww}(e_t - \tilde{e}_t) \right]} (e_t - \tilde{e}_t) \\
&\quad + \frac{de_t}{d\delta\Pi_{t+1}} \frac{d\delta\Pi_{t+1}}{d\eta} > 0.
\end{aligned}$$

■