The Interplay Between Product Variety and Customer Retention: Theory and Evidence
Online Appendix

This Online Appendix consists of six appendices (A-F).

Online Appendix A presents details on the analyses of the annual reports of NASDAQ 100 firms (pp. 2-9). It first explains the keyword analysis (Online Appendix A.1) and then the text analysis (Online Appendix A.2).

Online Appendix B presents the proofs of Results 1-4 (pp. 10-14).

Online Appendix C provides technical results (pp. 15-23). First, it shows the optimality of symmetric prices and discusses potential multiplicity of equilibria (Online Appendix C.1). Second, it demonstrates that the first-order conditions are sufficient (Online Appendix C.2).

Online Appendix D presents several model extensions (pp. 24-42). Online Appendix D.1 considers an extended model in which the customer base of each firm is endogenous. Online Appendix D.2 analyzes the case of asymmetric firms. Online Appendix D.3 extends the model to the situation in which investment in customer retention not only has a positive effect on the repurchase probability but also on attracting switching consumers. Online Appendix D.4 considers the case in which unsatisfied consumers do not buy from the same firm. Online Appendix D.5 presents a model with sequential instead of simultaneous decisions where investments in product variety and customer retention precede the pricing decision. Online Appendix D.6 considers a concrete example with specific cost functions, which allows to obtain a closed-form solution.

Online Appendix E provides details on the data and the descriptives (pp. 43-45). It first presents more details on the data (Online Appendix E.1) and then explains details on the descriptives (Online Appendix E.2).

Finally, Online Appendix F provides details on the estimation results (pp. 46-53). It presents the OLS estimation table without variable transformation (Online Appendix F.1), an explanation of the transformation (Online Appendix F.2), additional robustness tables (Online Appendix F.3), details on robustness checks (Online Appendix F.4), the regression sensitivity analysis (Online Appendix F.5), and details on the instruments (Online Appendix F.6).

Online Appendix A: Details on Text Analyses

The goal of the text analyses is to understand systematically whether there is evidence for both the importance of the two strategic variables product variety and customer retention as well as their interplay being mentioned and talked about specifically in firms' annual reports.

Sample: In the following text analyses, we consider the annual reports filed by firms publicly listed in the NASDAQ 100 as of the beginning 2024. These reports are usually called "10-K". The stock symbols assigned to the above companies are retrieved from NASDAQ's website and then matched to the central index key (CIK) uniquely identifying companies in the documents filed on the EDGAR platform provided by the US Securities and Exchange Commission (SEC).[33]

Online Appendix A.1: Keyword Analysis

The data for the keyword analysis is based on the 10-X[34] Document Dictionaries provided by the Software Repository for Accounting and Finance at the University of Notre Dame.[35] The corresponding file contains counts of words from a dictionary for each 10-X filing of the last 30 years and allows a simple keyword analysis without going through the annual reports themselves. We take the so-called Loughran-McDonald Master dictionary and infer our words of interest "retention" (ID: 64349) and "variety" (ID: 82824) along with alternative strategic variables of a company as a benchmark ("quality": 60038, "version": 83137).[36]

As outlined above, we restrict attention to NASDAQ 100 firms, of which 95 are included in the database.[37] Additionally, we restrict attention to the last 20 years, as this gives us sufficient coverage with at least 60 firms in each period. For each firm and period, we look for the count of the respective word. Accordingly, we can measure both the share of annual reports containing the word at least once as well as how often the word comes up on average across the annual reports. Figure 1 shows the corresponding graphs over time and aggregated across firms. The upper panel shows that "quality" is always mentioned, while the words "variety" and "retention" experience a growth in importance over the years as opposed to "version", which roughly remained at its initial level that was comparable to "retention" at the beginning of the observation period. The lower panel displays the average amount of

[33] See `https://www.nasdaq.com/market-activity/quotes/nasdaq-ndx-index` and `https://raw.githubusercontent.com/jadchaar/sec-cik-mapper/main/mappings/stocks/ticker_to_cik.json`.

[34] The "X" in 10-X represents all different form types a company needs to file, e.g., 10-K, 10-Q. We focus on 10-K as these are the most comprehensive reports about a company.

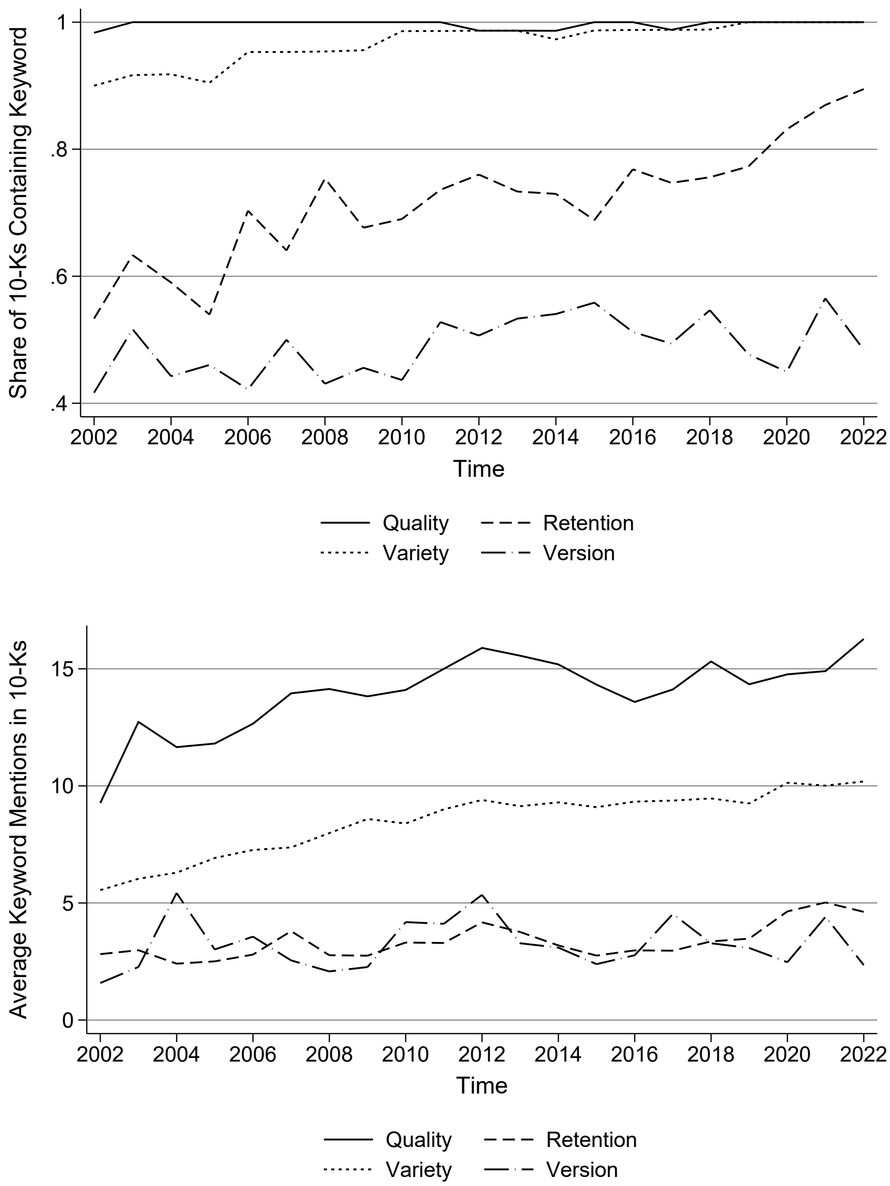
[35] See `https://sraf.nd.edu/sec-edgar-data/10x-document-dictionaries/`.

[36] See `https://sraf.nd.edu/loughranmcdonald-master-dictionary/`.

[37] The CIKs of AstraZeneca, ASML Holding, Coca-Cola, GlobalFoundries, and PDD Holdings cannot be found.

mentions. It shows a similar dynamic with "variety" and "retention" roughly doubling the corresponding frequency, whereas this does not hold for "quality" and "version".

Figure 1: Frequency of Words in 10-K Reports of NASDAQ 100 Firms



Notes: Based on 10-K reports from 2002-2022 of NASDAQ 100 firms, N = 1,564.

Despite the illustrative evidence, the analysis does neither pick up synonyms nor paraphrases of specific concepts and does not elaborate on where it was mentioned. While this is a drawback, the words "variety" and "retention" are quite specific on its own and the reports predominantly revolve around the firm.

More importantly, however, the analysis remains silent about the context of the mentions, which is necessary to classify it as a firm's strategy and whether the mentions of the specific variables go hand in hand, thereby displaying an interplay. Hence, the next subsection digs deeper into an actual text analysis of the annual reports.

Online Appendix A.2: Text Analysis

The data for this text analysis is based on the Stage One 10-X Parse Data provided by the Software Repository for Accounting and Finance at the University of Notre Dame.[38] This data base contains all 10-X filings available on the EDGAR platform provided by the U.S. SEC as raw documents and is only cleaned of all extraneous materials (e.g., HTML code, PDF's, jpg's etc.).

As outlined above, we restrict attention to NASDAQ 100 firms and annual reports ("10-K"). Moreover, we focus on the section "PART I Item 1 - Business" within a 10-K as this has been found indicative for describing a firm's strategy, also through the previously mentioned anecdotes.[39] As these descriptions vary in size, we focus on roughly the first three pages (i.e., first 15,000 characters) and on the last 13 years (2011-2023) to ease the computational burden of the analysis.

This prepared text of the company's business description serves as the "input" for the query process via OpenAI's GPT-4[40] in order to classify instances in the text, where information describing product variety and/or customer retention is contained.

In the following, the query process via GPT-4 is described, which involves both the prompt (including the instruction) as well as technical details (that allow replicability):

1. Preparation of the following prompt given to GPT-4 (turbo) without a specific role assigned:

    - Please review the company description provided below. Your task is to analyze the text and compile two separate lists. Each list should contain the top two sentences that best support a classification of the company's business strategy into one of two categories: "product variety" and "customer retention".
    - Instructions:

        (a) Identify and list the top two sentences that support the classification of the company's business strategy as "product variety".

[38] See `https://sraf.nd.edu/data/stage-one-10-x-parse-data/`.

[39] The extraction of the specific section 'PART I Item 1 - Business' is done through the use of regular expressions.

[40] Running a newer version of OpenAI's GPT does not lead to qualitatively different results.

   (b) Identify and list the top two sentences that support the classification of the company's business strategy as "customer retention".
   (c) If no relevant sentences can be found for a category, provide an empty list for that category.

2. Dispatch requests to OpenAI API in batch sizes of 20, employ settings as in Gilardi et al. (2023), and catch different API errors through a maximum of three retries. Parse "answers" in a json format.

The "output" by GPT-4 is thus for each input up to two sentences classified being about product variety and customer retention, respectively. We focus on the first (i.e., top) sentence for each measure and look at the position of each sentence in the initial text. To illustrate the process from starting with the text and classifications to ending up with our measure of an interplay between variety and retention, we use the annual report from 2023 by Netflix (i.e., its section "PART I Item 1 - Business") and the corresponding first five sentences:

- about us netflix, inc. **netflix, the company, registrant, we, or us is one of the world's leading entertainment services with approximately 231 million paid memberships in over 190 countries enjoying tv series, films and games across a wide variety of genres and languages.** members can play, pause and resume to watch as much as they want, anytime, anywhere, and can change their plans at any time. our core strategy is to grow our business globally within the parameters of our operating margin target. *we strive to continuously improve our members' experience by offering compelling content that delights them and attracts new members.* we seek to drive conversation around our content to further enhance member joy, and we are continuously enhancing our user interface to help our members more easily choose content that they will find enjoyable.

In the example, OpenAI's GPT-4 classified the "product variety" sentence (highlighted in bold) starting at character position 23 and the "customer retention" sentence (highlighted in italics) starting at character position 503 out of the whole text comprising 15,000 characters. We then compute the difference between the two positions to measure the distance in characters between the sentences. Hence, it is 0 for the very same sentence and increases the further away the sentences are.

As our goal is to assess the interplay between the two measures, we want to know whether the sentences describing them appear close to each other. For this, we define sentences being close if the respective sentences are not further apart than 500 characters (in the space of

15,000 characters). This can be thought of as one paragraph.[41] The Netflix example above can be thus considered as one with relatively distant sentences but still making it into our definition. As a result, we can first look for how many texts describe the two measures. If both are described, we can then count how often the respective parts are "close" to each other to assume an interplay.

Getting to the data, the input are 977[42] texts, of which GPT-4 classified 99.9 (98.7) percent as containing sentences that reflect product variety (customer retention). Hence, apparently almost all companies talk about the two measures somewhere in the first three pages of their company description. This may indicate that keywords themselves only serve as a lower bound because they do not pick up synonyms or paraphrases.

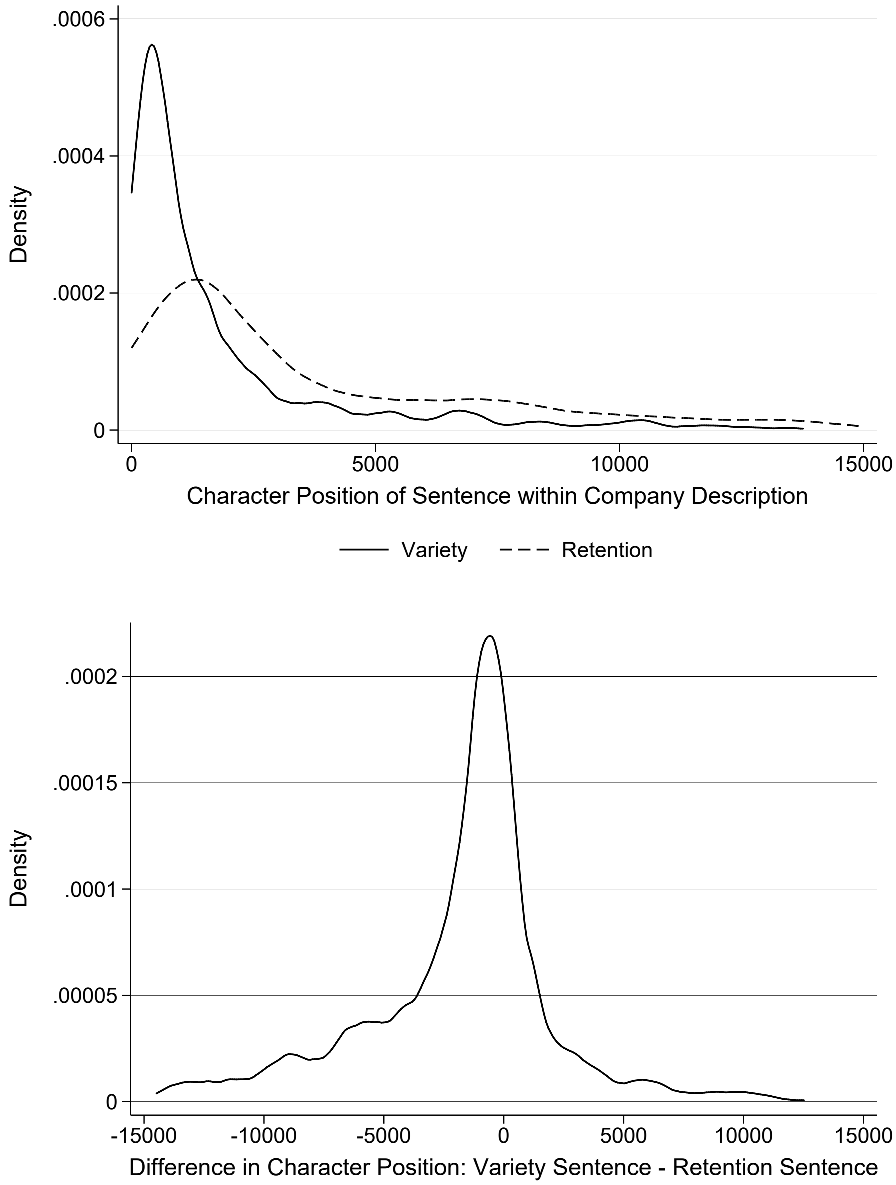
More interestingly, we turn toward the positioning of the respective sentences. The upper panel in Figure 2 shows that both measures are described in the beginning of the company description, with product variety having an especially early (and high-level) peak compared to customer retention. While these patterns may already hint at an overlap, the lower panel in Figure 2 shows the difference where the respective sentences come up. It shows a concentration around 0, with the majority being negative (i.e., "variety" appearing before "retention"), which resonates well with the upper panel. In numbers, 19.3 percent (or 189 of 977) of texts describe product variety and customer retention within one paragraph (+/- 500 characters). The figure also shows that the threshold at 500 characters is not crucial.

Robustness of the classification: In our previous analysis, OpenAI's GPT-4 identified a very high number of annual reports that contain sentences reflecting product variety (customer retention). A concern could therefore be that our instructions given to GPT-4 are too restrictive, i.e., they require to find sentences that are only tangentially related to these measures but are in general quite distant from the concepts. To mitigate these concerns, we add a second stage to our GPT-4-based classification funnel. This additional layer starts from the previously identified sentences in isolation as input. The task is then, based on definitions of the measures, to classify whether each sentence indeed corresponds to product variety and customer retention, respectively. The exact prompt is given at the end of this section. We also leverage the new reasoning capabilities by OpenAI's GPT-4, introduced in September 2024, to manually assess the thought process behind the classification. The results show that 69 and 38 percent and, hence, a large share of the previously identified sentences are

[41] There is no unique definition for how many sentences or words constitute a paragraph. However, estimates range between 100 to 300 words (see `https://libguides.hull.ac.uk/writing/paras` and `https://warwick.ac.uk/fac/soc/al/globalpad-rip/openhouse/academicenglishskills/writing/paragraphing/`). Thus, we take the lower bound and translate 100 words into 500 characters.

[42] We depart from 1,014 texts, but in 37 instances, the texts may violate the terms and services according to OpenAI and hence there is no output.

Figure 2: Distribution and Distance of Sentence Positions



Notes: Difference (in the lower panel) means starting position in characters (variety) minus starting position in characters (retention). Based on 'PART I Item 1 - Business' within a 10-K from 2011-2023 of NASDAQ 100 firms, N = 977.

confirmed, i.e., classified again, to be about product variety and customer retention, respectively. If we restrict to the subset of sentences confirmed by the second stage, the pattern of product variety and customer retention being mentioned closely together remains, suggesting that the previous results are not due to a possible misclassification.

Finally, to assess the validity of the classification "externally," a subsample of 100 observations was randomly drawn to be manually classified. Specifically, we revisited the sentences to re-classify them as product variety and customer retention, using the previously mentioned definitions. In general, there has been a confirmation in the majority of sentences, i.e., 77 (51) percent were classified again as reflecting product variety (customer retention). This resonates well with the results from the second stage from the GPT, thereby showing that the two measures are mentioned very frequently in annual reports. The agreement between the manual classifications and the second stage of GPT-4 is 88 (78) percent for product variety (customer retention). Restricting to the subset of sentences that were re-classified successfully shows, similarly to before, that the pattern of product variety and customer retention coming up together (i.e., within 500 characters) remains.

Second stage prompt to GPT-4: You will be provided with sentences from descriptions of a company from 10-K forms. Your task is to determine whether each sentence supports the classification of the company's business strategy into 'X'. For each sentence, return your answer in JSON format according to the following guideline:

1 if the sentence supports the business strategy 'X', defined as:

- *if X is product variety: A broader product variety increase demand from consumers who are willing to switch. By supplying a broader product portfolio, a firm is more likely to offer a match with a consumer's preference and therefore attracts more new customers.*
- *if X is customer retention: An investment in customer retention increases the repurchase probability from consumers in the firm's customer base, as these consumers are then less likely to switch to an alternative*

0 if the sentence:

- Does not contain enough information to support 'X' as a business strategy.
- Is too vague or general, or is closer related to different business strategies.

Provide your reasoning in one or two sentences.

OUTPUT FORMAT: 'classification': 1 or 0, 'reasoning': '...'

SENTENCE: sentence

Online Appendix B: Proofs of Results 1 - 4

Proof of Result 1

Using the Implicit-Function Theorem to (3) yields

$$\frac{dm_i}{dr_j} = -\frac{\frac{\partial^2 \Pi_i(\mathbf{m},\mathbf{r},\mathbf{p})}{\partial m_i \partial r_j}}{\frac{\partial^2 \Pi_i(\mathbf{m},\mathbf{r},\mathbf{p})}{\partial (m_i)^2}}.$$

Due to the fact that $\partial^2\Pi_i(\mathbf{m},\mathbf{r},\mathbf{p})/\partial\,(m_i)^2 < 0$—i.e., second-order conditions are satisfied—the sign of dm_i/dr_j is determined by the sign of $\partial^2\Pi_i(\mathbf{m},\mathbf{r},\mathbf{p})/\partial m_i \partial r_j$. Differentiating $\partial\Pi_i(\mathbf{m},\mathbf{s},\mathbf{p})/\partial m_i$ with respect to r_j, we obtain

$$\text{sign}\left\{\frac{dm_i}{dr_j}\right\} = \text{sign}\left\{-\frac{p_i\,(v-p_i)^{\frac{1}{\beta}}\left(\sum_{j=1,\,i\neq j}^{M} m_j(v-p_j)^{\frac{1}{\beta}}\right)}{M\left(\sum_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}\right)^2}\right\} < 0. \tag{9}$$

Similarly, the sign of dm_i/dr_i is also determined by the sign of the term in curly brackets in (9), which implies that $dm_i/dr_i < 0$ as well. Therefore, holding all other variables constant, an increase in r_j and r_i both lead to a decrease in m_i.

Proof of Result 2

The symmetric equilibrium is implicitly characterized by the three equations given in (6) and (7) of the main text. Totally differentiating (6) and (7) with respect to m^*, r^*, p^*, and v, we obtain

$$-\left(\frac{p^*(M-1)(1-r^*)}{M^2\,(m^*)^2} + f''(m^*)\right)dm^* - \left(\frac{p^*(M-1)}{M^2 m^*}\right)dr^* + \left(\frac{(M-1)(1-r^*)}{M^2 m^*}\right)dp^* = 0,$$

$$-c''(r^*)dr^* + \left(\frac{(M-1)}{M^2}\right)dp^* = 0,$$

and

$$M\beta dv + (M-1)p^*dr^* - (M\beta + (M-1)(1-r^*))\,dp^* = 0.$$

From these expressions, we can solve for dm^*/dv, dr^*/dv, and dp^*/dv. Doing so and focusing on dm^*/dv (as we are interested in how the equilibrium number of products changes with

v), we obtain

$$\frac{dm^*}{dv} = \frac{\beta m^* M(M-1)\left(M^2(1-r^*)c''(r^*) - p^*(M-1)\right)}{\left(c''(r^*)M\left(\beta M + (M-1)(1-r^*)\right) - p^*\left(M-1\right)^2\right)\left(p^*(M-1)(1-r^*) + M^2\left(m^*\right)^2 f''(m^*)\right)}. \quad (10)$$

Solving (7) for p^* yields

$$p^* = \frac{v\beta M}{\beta M + (M-1)(1-r^*)}.$$

Inserting the last expression into (10) and simplifying yields

$$\frac{dm^*}{dv} = \frac{(M-1)m^*\beta\mu\left(c''(r^*)M(1-r^*)\mu - v\beta(M-1)\right)}{\left(c''(r^*)M\mu^2 - v\beta\left(M-1\right)^2\right)\left(v\beta(M-1)(1-r^*) + M\left(m^*\right)^2\mu f''(m^*)\right)}, \quad (11)$$

with $\mu \equiv \beta M + (M-1)(1-r^*) > 0$.

The assumption $c''(\cdot) > v\left(M-1\right)^2 / \left(\beta M^3\right)$ implies that the term in the first parentheses of the denominator of the right-hand side of (11)—i.e., $c''(s^*)M\mu^2 - v\beta\left(M-1\right)^2$ is strictly positive. As the term in the second parentheses is also strictly positive, the denominator is strictly positive.

Therefore, the sign of dm^*/dv depends on the sign of the numerator, which is determined by the sign of $c''(r^*)M(1-r^*)\mu - v\beta(M-1)$. This term is positive if and only if

$$v < \frac{c''(r^*)M(1-r^*)\mu}{\beta(M-1)}. \quad (12)$$

For $v \to 0$, this inequality holds, as the left-hand side goes to zero whereas the right hand side is strictly positive. As v increases, the slope of the left-hand side of (12) equals 1. We next determine the slope of the right-hand side. Taking the derivative of the right-hand side with respect to v yields

$$\frac{-c''(r^*)M\left(\beta M + 2(M-2)(1-r^*)\right) + c'''(r^*)M(1-r^*)\mu}{\beta(M-1)}\frac{dr^*}{dv}, \quad (13)$$

where dr^*/dv can be determined in the same way as dm^*/dv above and is given by

$$\frac{dr^*}{dv} = \frac{(M-1)\beta\mu}{c''(r^*)M\mu^2 - v\beta\left(M-1\right)^2} > 0.$$

Due to the assumption that $c'''(\cdot)$ is either negative or, in case it is positive, small relative to $c''(\cdot)$, it follows that the expression in (13) is negative. Therefore, the right-hand side of (12) is negative, which implies that the inequality in (12) does no longer hold if v is sufficiently large. Therefore, the numerator of dm^*/dv is negative for v large enough.

It remains to show that the assumption $c''(\cdot) > v\,(M-1)^2 / (\beta M^3)$ allows for a v large enough so that the numerator can be negative. Inserting the upper bound for v that results from the assumption above into the numerator and simplifying yields that the sign of the numerator equals the sign of the expression

$$\frac{-c''(r^*)M\left(\beta^2 M^2 - \beta M(M-1) + r^*(M-1)\left(\beta M + (M-1)(2-r^*)\right)\right)}{M-1},$$

which is negative for all $\beta > 1$. Therefore, the numerator is indeed negative for v large enough but still in the admissible bounds. It follows that $dm^*/dv > 0$ for v below a threshold, but $dm^*/dv < 0$ for v above the threshold.

Proof of Result 3

In the same way as in the proof of Result 2, we can totally differentiate (6) and (7) with respect to m^*, r^*, p^*, and β, and solve for $dm^*/d\beta$, $dr^*/d\beta$, and $dp^*/d\beta$. Focusing on $dm^*/d\beta$ and using $p^* = (v\beta M) / (\beta M + (M-1)(1-r^*))$, we obtain

$$\frac{dm^*}{d\beta} = \frac{vm^*(1-r^*)\mu(M-1)^2\left(v\beta(M-1) + c''(r^*)M(1-r^*)\mu\right)}{\left(c''(r^*)M\mu^2 - v\beta\,(M-1)^2\right)\left(v\beta(M-1)(1-r^*) + M\,(m^*)^2\,\mu f''(m^*)\right)}.$$

By the same argument as in the proof of Result 2, the term in the first parentheses in the denominator is strictly positive. As all other terms are strictly positive as well, $dm^*/d\beta > 0$.

Proof of Result 4

To determine how dm_i/dr_j is affected by v and β in equilibrium, we first use that, at a symmetric equilibrium, $m_1^* = \cdots = m_M^* = m^*$, $r_1^* = \cdots = r_M^* = r^*$, and $p_1^* = \cdots = p_M^* = p^*$. Plugging this into the expression for dm_i/dr_j, which is given by

$$\frac{dm_i}{dr_j} = -\frac{\frac{\partial^2 \Pi_i(\mathbf{m},\mathbf{r},\mathbf{p})}{\partial m_i \partial r_j}}{\frac{\partial^2 \Pi_i(\mathbf{m},\mathbf{r},\mathbf{p})}{\partial (m_i)^2}},$$

and simplifying yields

$$\frac{dm_i^*}{dr_j^*} = -\frac{p^* m^*(M-1)}{2p^*(1-r^*)(M-1) + f''(m^*)\,(m^*)^2\,M^3}.$$

Inserting $p^* = v\beta M/\left(\beta M + (M-1)(1-r^*)\right)$ into the last equation, we obtain

$$\frac{dm_i^*}{dr_j^*} = -\frac{\beta v m^*(M-1)}{2\beta v(1-r^*)(M-1) + f''(m^*)\,(m^*)^2\,M^2\left(\beta M + (M-1)(1-r^*)\right)}. \tag{14}$$

We first take the derivative of the right-hand side of (14) with respect to v. This yields

$$-\frac{\beta(M-1)}{\left(2\beta v(M-1)(1-r^*)+f''(m^*)\left(m^*\right)^2 M^2(\beta M+(M-1)(1-r^*)\right)^2}\times \tag{15}$$

$$\times\frac{\left\{f''(m^*)\left(m^*\right)^3 M^2\left(\beta M+(M-1)(1-r^*)\right)+\left(2\beta v(M-1)+f''(m^*)\left(m^*\right)^2 M^2\right)vm^*(M-1)\frac{dr^*}{dv}\right.}{\left.\left(2\beta v^2(M-1)(1-r^*)-\left(f''(m^*)+f'''(m^*)vm^*\right)\left(m^*\right)^2 M^2(\beta M+(M-1)(1-r^*)\right)\frac{dm^*}{dv}\right\}}.$$

Inserting dr^*/dv and dv^*/dv from Result 2, we obtain that the sign of (15) depends on the sign of

$$-\Big[\left(\left(f''(m^*)\right)^2\left(m^*\right)^4 M^3\left(\beta M+(M-1)(1-r^*)\right)^2+2\beta^2v^2(M-1)^2(1-r^*)^2\right)c''(r^*)$$

$$+f''(m^*)\beta^2v^2(m^*)^2(M+2)(M-1)^2-$$

$$-f'''(m^*)\beta v(m^*)^3M(M-1)\left(\beta v(M-1)-c''(r^*M(1-r^*)(\beta M+(M-1)(1-r^*))\right.\Big]$$

The terms in the first two lines of the square bracket are all positive due to the fact that $c''(\cdot)>0$ and $f''(\cdot)\geq 0$. Moreover, since $f'''(\cdot)$ is negative or, if positive, rather small compared to $f''(\cdot)$, the sign of the term in the last line is also positive or only slightly negative. Therefore, the expression in the square brackets is positive, which implies that the entire term is negative. This implies that

$$\frac{\partial\left(\frac{dm_i^*}{dr_j^*}\right)}{\partial v}<0.$$

Hence, an increase in v amplifies the negative effect of consumer retention on variety.

Proceeding in the same way for β yields that the sign of

$$\frac{\partial\left(\frac{dm_i^*}{dr_j^*}\right)}{\partial \beta}$$

is given by the sign of

$$-\Big[\left(\left(f''(m^*)\right)^2 v\beta m^* M^2+2(M-1)^2(1-r^*)^2\left(\beta M+(M-1)(1-r^*)\right)\right)c''(r^*)$$

$$+f''(m^*)(m^*)^2(M+2)(M-1)\left(\beta M+(M-1)(1-r^*)\right)^2-$$

$$-f'''(m^*)\beta v(m^*)^3M(M-1)\left(\beta v(M-1)-c''(r^*M(1-r^*)(\beta M+(M-1)(1-r^*))\right.\Big]$$

By the same arguments as above, this term is strictly negative, which implies that an increase in β also amplifies the negative effect of consumer retention on variety. Finally, since dm_i^*/dr_i^* is determined by the same term as dm_i^*/dr_j^*, the result also holds for dm_i^*/dr_i^*.

Online Appendix C: Heterogeneous Prices and Second-Order Conditions

Online Appendix C.1: Optimality of Symmetric Prices and Discussion of Equilibrium Symmetry

In the model of the main text, we assumed that each firm sets the same price for all its products. In this appendix, we first show that this pricing strategy is indeed optimal even if a firm could set different prices. As stated in footnote 21 of the main text, if a firm could set different prices for its products, the probability with which a switching consumer buys a product of firm i is

$$\frac{\int_0^{m_i} (v - p_{\ell,i})^{\frac{1}{\beta}} d\ell}{\sum_{j=1}^{M} \int_0^{m_j} (v - p_{\ell,j})^{\frac{1}{\beta}} d\ell},$$

and the probability with which a retained consumer of firm i buys product k is

$$\frac{(v - p_{k,i})^{\frac{1}{\beta}}}{\int_0^{m_i} (v - p_{\ell,i})^{\frac{1}{\beta}} d\ell}.$$

The maximization problem of firm i is then given by

$$\max_{m_i, r_i, p_{k,i} \forall k \in [0, m_i]} \frac{r_i}{M} \left(\frac{\int_0^{m_i} p_{\ell,i}(v - p_{\ell,i})^{\frac{1}{\beta}} d\ell}{\int_0^{m_i} (v - p_{\ell,i})^{\frac{1}{\beta}} d\ell} \right) + \frac{\sum_{j=1}^{M} (1 - r_j)}{M} \left(\frac{\int_0^{m_i} p_{\ell,i}(v - p_{\ell,i})^{\frac{1}{\beta}} d\ell}{\sum_{j=1}^{M} \int_0^{m_j} (v - p_{\ell,j})^{\frac{1}{\beta}} d\ell} \right) - f(m_i) - c(r_i). \tag{16}$$

From Leibniz's rule, the derivative of $\int_0^{m_i} p_{\ell,i}(v - p_{\ell,i})^{\frac{1}{\beta}} d\ell$ with respect to m_i is $p_{m_i,i}(v - p_{m_i,i})^{\frac{1}{\beta}}$. We can use this in the derivative of (16) with respect to m_i to obtain that the first-order condition with respect to m_i is

$$\frac{(v - p_{m_i,i})^{\frac{1}{\beta}}}{M} \left(\frac{r_i \left(p_{m_i,i} \int_0^{m_i} (v - p_{\ell,i})^{\frac{1}{\beta}} d\ell - \int_0^{m_i} p_{\ell,i}(v - p_{\ell,i})^{\frac{1}{\beta}} d\ell \right)}{\left(\int_0^{m_i} (v - p_{\ell,i})^{\frac{1}{\beta}} d\ell \right)^2} + \right.$$

$$\left. + \frac{\sum_{j=1}^{M} (1 - r_j) p_{m_i,i} \left(\sum_{j=1, i \neq j}^{M} \int_0^{m_j} (v - p_{\ell,j})^{\frac{1}{\beta}} d\ell \right)}{\left(\sum_{j=1}^{M} \int_0^{m_j} (v - p_{\ell,j})^{\frac{1}{\beta}} d\ell \right)^2} \right) - f'(m_i) = 0. \tag{17}$$

Second, the first-order condition with respect to r_i is given by

$$\left(\frac{\int\limits_0^{m_i} p_{\ell,i}(v-p_{\ell,i})^{\frac{1}{\beta}}d\ell}{M}\right)\left(\frac{\sum\limits_{j=1,\,i\neq j}^{M}\int\limits_0^{m_j}(v-p_{\ell,j})^{\frac{1}{\beta}}d\ell}{\int\limits_0^{m_i}(v-p_{\ell,i})^{\frac{1}{\beta}}d\ell\left(\sum\limits_{j=1}^{M}\int\limits_0^{m_j}(v-p_{\ell,j})^{\frac{1}{\beta}}d\ell\right)}\right)-c'(r_i)=0. \tag{18}$$

Third, we take the derivative of (16) with respect to $p_{k,i}$. This can be done by considering that firm i charges $p_{k,i}$ for a set $n>0$ out of its m_i products, and then let $n\to 0$. To do this, we can write the function to be maximized as

$$\frac{r_i}{M}\left(\frac{np_{k,i}(v-p_{k,i})^{\frac{1}{\beta}}+\int\limits_n^{m_i}p_{\ell,i}(v-p_{\ell,i})^{\frac{1}{\beta}}d\ell}{n(v-p_{k,i})^{\frac{1}{\beta}}+\int\limits_n^{m_i}(v-p_{\ell,i})^{\frac{1}{\beta}}d\ell}\right)+$$

$$+\frac{\sum\limits_{j=1}^{M}(1-r_j)}{M}\left(\frac{np_{k,i}(v-p_{k,i})^{\frac{1}{\beta}}+\int\limits_n^{m_i}p_{\ell,i}(v-p_{\ell,i})^{\frac{1}{\beta}}d\ell}{\sum\limits_{j=1,i\neq j}^{M}\int\limits_0^{m_j}(v-p_{\ell,j})^{\frac{1}{\beta}}d\ell+n(v-p_{k,i})^{\frac{1}{\beta}}+\int\limits_n^{m_i}(v-p_{\ell,i})^{\frac{1}{\beta}}d\ell}\right)-f\left(m_i\right)-c\left(r_i\right).$$

Taking the derivative with respect to $p_{k,i}$, setting the result equal to zero, and letting n go to zero then yields

$$r_i\frac{(\beta(v-p_{k,i})-p_{k,i})\int\limits_0^{m_i}(v-p_{\ell,i})^{\frac{1}{\beta}}d\ell+\int\limits_0^{m_i}p_{\ell,i}(v-p_{\ell,i})^{\frac{1}{\beta}}d\ell}{\left(\int\limits_0^{m_i}(v-p_{\ell,i})^{\frac{1}{\beta}}d\ell\right)^2}+ \tag{19}$$

$$+\sum_{j=1}^{M}(1-r_j)\frac{(\beta(v-p_{k,i})-p_{k,i})\sum\limits_{j=1}^{M}\int\limits_0^{m_j}(v-p_{\ell,j})^{\frac{1}{\beta}}d\ell+\int\limits_0^{m_i}p_{\ell,i}(v-p_{\ell,i})^{\frac{1}{\beta}}d\ell}{\left(\sum\limits_{j=1}^{M}\int\limits_0^{m_j}(v-p_{\ell,j})^{\frac{1}{\beta}}d\ell\right)^2}=0,\quad\forall k\in[0,m_i].$$

Using (19), we can now show that it is optimal for each firm i to set the same prices for all its products, given m_i, r_i, and the choices of competitors. To see this, we can rewrite (19) as

$$(\beta(v-p_{k,i})-p_{k,i})\left[\frac{r_i}{\int\limits_0^{m_i}(v-p_{\ell,i})^{\frac{1}{\beta}}d\ell}+\frac{\sum\limits_{j=1}^{M}(1-r_j)}{\sum\limits_{j=1}^{M}\int\limits_0^{m_j}(v-p_{\ell,j})^{\frac{1}{\beta}}d\ell}\right]+ \tag{20}$$

$$+\left[\frac{r_i \int_0^{m_i} p_{\ell,i}(v-p_{\ell,i})^{\frac{1}{\beta}} d\ell}{\left(\int_0^{m_i} (v-p_{\ell,i})^{\frac{1}{\beta}} d\ell\right)^2} + \frac{\sum_{j=1}^{M}(1-r_j) \int_0^{m_i} p_{\ell,i}(v-p_{\ell,i})^{\frac{1}{\beta}} d\ell}{\left(\sum_{j=1}^{M} \int_0^{m_j} (v-p_{\ell,j})^{\frac{1}{\beta}} d\ell\right)^2}\right] = 0, \quad \forall k \in [0, m_i].$$

Denoting

$$X \equiv \left[\frac{r_i}{\int_0^{m_i} (v-p_{\ell,i})^{\frac{1}{\beta}} d\ell} + \frac{\sum_{j=1}^{M}(1-r_j)}{\sum_{j=1}^{M} \int_0^{m_j} (v-p_{\ell,j})^{\frac{1}{\beta}} d\ell}\right]$$

and

$$Y \equiv \frac{r_i \int_0^{m_i} p_{\ell,i}(v-p_{\ell,i})^{\frac{1}{\beta}} d\ell}{\left(\int_0^{m_i} (v-p_{\ell,i})^{\frac{1}{\beta}} d\ell\right)^2} + \frac{\sum_{j=1}^{M}(1-r_j) \int_0^{m_i} p_{\ell,i}(v-p_{\ell,i})^{\frac{1}{\beta}} d\ell}{\left(\sum_{j=1}^{M} \int_0^{m_j} (v-p_{\ell,j})^{\frac{1}{\beta}} d\ell\right)^2},$$

it follows from (20), that the first-order conditions for all $p_{k,i}$, $k \in [0, m_i]$, have the structure $(\beta(v - p_{k,i}) - p_{k,i})\, X + Y = 0$. Specifically, X and Y are the same in all first-order conditions for $p_{k,i}, k \in [0, m_i]$. As the term in parentheses is linear in $p_{k,i}$, the solution to (20) must be the same for all $p_{k,i}$. Therefore, firm i optimally sets the prices of all its products equal to each other, that is, $p_{k,i} = p_i$ for all $k \in [0, m_i]$. Using this for all firms, the first-order conditions just derived—i.e., (17), (18), and (19)—can be simplified to (3), (4), and (5).

We next discuss equilibrium symmetry and the possibility of an asymmetric equilibrium to exist. In a symmetric equilibrium, where $m_1^* = \cdots = m_M^* \equiv m^*$, $r_1^* = \cdots = r_M^* \equiv r^*$, and $p_1^* = \cdots p_M^* \equiv p^*$, conditions (3), (4), and (5) can then be simplified to obtain (6) and (7). Solving (7) for p^* yields

$$p^* = \frac{\beta v M}{\beta M + (M-1)(1-r^*)}.$$

Inserting this into the second equation of (6) yields

$$\frac{\beta v(M-1)}{M\left(\beta M + (M-1)(1-r^*)\right)} = c'(r^*) \tag{21}$$

It is easy to see from (21) that the assumption $c'(1) > v\,(M-1)\,/M^2$ ensures that the solution for r^* is below 1.

Moreover, our assumptions also ensure uniqueness of a symmetric equilibrium. First, uniqueness of r^* follows from (21): The left-hand side is increasing in r^* and strictly convex. Instead, due to the assumption that $c'''(\cdot)$ is negative or only slightly positive, the right-hand side, although also increasing in r^*, is concave or only slightly convex. As the left-hand side is larger than the right-hand side at $r = 0$ and the reverse holds for $r = 1$, there is a unique intersection point between the two. Similarly, inserting $p^* = \beta vM/\left(\beta M + (M-1)(1-r^*)\right)$ into the first equation of (6) yields

$$\frac{\beta v(M-1)(1-r^*)}{Mm^*\left(\beta M + (M-1)(1-r^*)\right)} = f'(m^*). \tag{22}$$

Since (21) does not depend on m^*, the left-hand side is strictly decreasing in m^*, whereas the right-hand side is weakly increasing. This again implies that there is a unique value for m^* that satisfies (22). Therefore, there exists a unique symmetric equilibrium.

Finally, we turn to the question whether an asymmetric equilibrium in which e.g. some firms set a relatively high product variety but invest only a small amount in customer retention, whereas for others the opposite holds, can exist. The first observation is that such an equilibrium can only occur if different firms charge different prices. This can be seen from (3). Potential differences in customer retention levels of firms affect all firms in the same way as the first term of (3) depends on $\sum_{j=1}^{M}(1-r_j)$. Therefore, any difference in the number of products for firms can only be due to different prices. It follows that, if prices of all firms are the same, then also $m_1^* = \cdots = m_M^*$. Using this in (4), the first term in this equation is then also the same for all firms, which implies that $r_1^* = \cdots = r_M^*$.

A consequence of this result is that for $\beta \to \infty$, only a symmetric equilibrium exists. The reason is that the optimal prices for all firms are $p_j^* = v$, $j \in \{1, ..., M\}$, as $\beta \to \infty$. Since prices are the same for all firms in that case, the arguments above imply that also m_j^* and r_j^* must be the same for all firms.

Instead, if prices of firms were different, the equilibrium conditions (3) and (4) can in principle be satisfied with different values for m_i and r_i for firms that charge different prices. Condition (5) can then also be satisfied for all firms although firms set different strategic variables. We performed several numerical simulation with different parameters and cost functions to check whether asymmetric equilibria can arise. However, in all of the simulations we found that the only equilibrium is the symmetric one. Although we could not show analytically that no asymmetric equilibrium exists, the simulations strongly point to the uniqueness of the symmetric equilibrium.

There are indeed forces in the model that push toward a symmetric equilibrium. To see this, consider (3)—i.e., the first-order condition for m_i—and suppose there are two firms, j and k, that set a different number of products in equilibrium—e.g., $m_j > m_k$. For both firms, (3) must hold. Since $f''(\cdot) \geq 0$ and $m_j > m_k$, it follows that $f'(m_j) \geq f'(m_k)$. Therefore, the first term of (3) must be larger in the condition for firm j than in the condition for firm k. Because $\sum_{j=1}^{M}(1-r_j)$ and also the denominator is the same in both conditions, it must hold that

$$p_j\,(v-p_j)^{\frac{1}{\beta}} \times \left(\sum_{i=1,\,i\neq j}^{M} m_i(v-p_i)^{\frac{1}{\beta}}\right) \geq p_k\,(v-p_k)^{\frac{1}{\beta}} \times \left(\sum_{i=1,\,i\neq k}^{M} m_i(v-p_i)^{\frac{1}{\beta}}\right).$$

The first term on each side of the inequality is $p_h\,(v-p_h)^{\frac{1}{\beta}}$, $h\in\{j,k\}$. The sign of the derivative of $p_h\,(v-p_h)^{\frac{1}{\beta}}$ is equal to the sign of $\beta(v-p_h)-p_h$, which is strictly negative, as otherwise (5) cannot be satisfied. Therefore, the first term points to $p_j \leq p_k$ to fulfill (3) for both firms. However, for the second term, the opposite holds. Since $m_j \geq m_k$, if p_j would be larger than p_k, then

$$\left(\sum_{i=1,\,i\neq j}^{M} m_i(v-p_i)^{\frac{1}{\beta}}\right) \leq \left(\sum_{i=1,\,i\neq k}^{M} m_i(v-p_i)^{\frac{1}{\beta}}\right).$$

This demonstrates that there are balancing forces in the conditions (3) and (5) that make it difficult to satisfy (3) for different values of m_i.[43]

We note that we focused on potential asymmetric equilibria that are interior—i.e., equilibria where the first-order conditions are satisfied. This is indeed sufficient because we show in the next online appendix that, given our assumptions, it cannot not be optimal for a firm to set some of its strategic variables at extremal values.

Online Appendix C.2: Sufficiency of First-Order Conditions

In this appendix, we determine the conditions so that the first-order conditions (3), (4), and (5) of the main text are indeed sufficient for a maximum. To do so, we first show that the Hessian matrix given by[44]

$$\begin{pmatrix} \frac{\partial^2\Pi_i}{\partial(m_i)^2} & \frac{\partial^2\Pi_i}{\partial m_i\partial r_i} & \frac{\partial^2\Pi_i}{\partial m_i\partial p_i} \\ \frac{\partial^2\Pi_i}{\partial r_i\partial m_i} & \frac{\partial^2\Pi_i}{\partial(r_i)^2} & \frac{\partial^2\Pi_i}{\partial r_i\partial p_i} \\ \frac{\partial^2\Pi_i}{\partial p_i\partial m_i} & \frac{\partial^2\Pi_i}{\partial p_i\partial r_i} & \frac{\partial^2\Pi_i}{\partial(p_i)^2} \end{pmatrix}$$

is negative definite at the symmetric equilibrium.

We start with the entries on the diagonal to determine the first-order principal minors.

[43] This argument holds independently of the values of r_i as these values enter (3) for all firms in the same way.

[44] To reduce the notational burden, we omit the arguments of the function Π_i.

Taking the derivative of the right-hand side of (3) with respect to m_i yields

$$\frac{\partial^2 \Pi_i}{\partial (m_i)^2} = -\left(\frac{2\,(v-p_i)^{\frac{2}{\beta}}}{M}\right)\left(\frac{\sum_{j=1}^{M}(1-r_j)p_i\left(\sum_{j=1,\, i\neq j}^{M} m_j(v-p_j)^{\frac{1}{\beta}}\right)}{\left(\sum_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}\right)^3}\right) - f''(m_i) < 0, \tag{23}$$

where the inequality is due to the fact that the first term is negative and $f''(m) \geq 0$.

Next, taking the derivative of the right-hand side of (4) with respect to r_i yields $\partial^2 \Pi_i / \partial (r_i)^2 = -c''(r_i) < 0$.

Proceeding in the same way for the derivative with respect to p_i yields that $\partial^2 \Pi_i / \partial (p_i)^2$ evaluated at a symmetric equilibrium is

$$\frac{p(M-1)(1-r) - \beta(2Mv - p(M+1+M(1-r)))}{M^2 m \beta^2 (v-p)^2}.$$

This term is negative if and only if

$$\beta > \frac{p(M-1)(1-r)}{2Mv - p(M+1+M(1-r))} \tag{24}$$

This threshold value of β is increasing in p and decreasing in r. As p can never be above v in any equilibrium, the highest value of p equals v. As r is a probability, the lowest value of r equals 0. Inserting $p = v$ and $r = 0$ into the threshold value and simplifying, we obtain that the right-hand side of (24) is 1. Therefore, for β larger than 1, also $\partial \Pi_i / \partial p_i^2 < 0$. It follows that all first-order principal minors are negative.

We next consider the second-order principal minors. We start with the one for m_i and r_i, where the respective determinant is given by $\left(\partial^2 \Pi_i / \partial (m_i)^2\right)\left(\partial^2 \Pi_i / \partial (r_i)^2\right) - \left(\partial^2 \Pi_i / (\partial m_i \partial r_i)\right)^2$. Evaluating this determinant at a symmetric equilibrium, we obtain

$$\frac{c''(r)\left(2M^3(M-1)p(1-r) + M^6 m^2 f''(m)\right) - (M-1)^2 p^2}{M^6 m^2}.$$

Inserting the equilibrium value of p given by $p = \beta M v / (\beta M + (M-1)(1-r))$ into this expression and simplifying yields

$$\frac{c''(r)(\beta M + (M-1)(1-r))\left(2\beta v M^2 (M-1)(1-r) + M^4 m^2 f''(m)(\beta M + (M-1)(1-r))\right)}{M^4 m^2 (\beta M + (M-1)(1-r))^2}$$
$$- \frac{\beta^2 v^2 (M-1)^2}{M^4 m^2 (\beta M + (M-1)(1-r))^2}. \tag{25}$$

This term is increasing in $f''(m)$. Therefore, if it is positive for $f''(\cdot) = 0$, it is also positive if $f''(\cdot) \geq 0$. Inserting $f''(m) = 0$ and solving for $c''(r)$ yields that (25) is positive if

$$c''(r) > \frac{\beta v(M-1)}{2(1-r)M^2(\beta M + (M-1)(1-r))}.$$

From the first-order conditions, r is implicitly given by $r = (\beta M + M - 1)/(M-1) - \beta v/c'(r)$. Inserting this into the last inequality and simplifying yields

$$c''(r) > \frac{v(M-1)\left(c'(r)\right)^2}{2\beta(M-1+M^2c'(r))}.$$

For any value of $c'(r) \in (0, \infty)$, the condition $c''(r) > v(M-1)^2/(\beta M^3)$ ensures that the inequality holds. It follows that the determinant of the Hessian matrix for m_i and r_i is positive.

We next turn to the determinant of the second-order principal minor with respect to m_i and p_i, which is given by $\left(\partial^2\Pi_i/\partial\left(m_i\right)^2\right)\left(\partial^2\Pi_i/\partial\left(p_i\right)^2\right) - \left(\partial^2\Pi_i/\left(\partial m_i \partial p_i\right)\right)^2$. At a symmetric equilibrium, it is equal to

$$\frac{[f''(m)M^2m^2 + 2p(M-1)(1-s)]\,[\beta(2Mv - p(M+1+M(1-s))) - p(M-1)(1-s)]}{M^5m^3\beta^2(v-p)^2}$$

As $f''(m) \geq 0$, the sign of this determinant is positive if (24) holds. It is therefore fulfilled for $\beta > 1$.

Similarly, the determinant of the second-order principal minor with respect to r_i and p_i—i.e., $\left(\partial^2\Pi_i/\partial\left(r_i\right)^2\right)\left(\partial^2\Pi_i/\partial\left(p_i\right)^2\right) - \left(\partial^2\Pi_i/\left(\partial r_i \partial p_i\right)\right)^2$—evaluated at a symmetric equilibrium is

$$\frac{c''(r)\,[\beta(2Mv - p(M+1+M(1-s))) - p(M-1)(1-s)]}{M^2m\beta^2(v-p)^2}.$$

Due to the fact that $c''(r) > 0$, this expression is positive if the term in square brackets in the numerator is positive. This is again true if (24) holds. It is therefore fulfilled for $\beta > 1$.

Finally, we turn to the determinant of the third-order principal minor, which is given by

$$\begin{aligned}
&\left(\frac{\partial^2\Pi_i}{\partial\left(p_i\right)^2}\right)\left[\left(\frac{\partial^2\Pi_i}{\partial\left(m_i\right)^2}\right)\left(\frac{\partial^2\Pi_i}{\partial\left(r_i\right)^2}\right) - \left(\frac{\partial^2\Pi_i}{\partial m_i \partial r_i}\right)^2\right] + \\
&+ \left(\frac{\partial^2\Pi_i}{\partial p_i \partial r_i}\right)\left[\left(\frac{\partial^2\Pi_i}{\partial\left(m_i\right)^2}\right)\left(\frac{\partial^2\Pi_i}{\partial p_i \partial r_i}\right) - \left(\frac{\partial^2\Pi_i}{\partial m_i \partial r_i}\right)\left(\frac{\partial^2\Pi_i}{\partial m_i \partial p_i}\right)\right] + \\
&+ \left(\frac{\partial^2\Pi_i}{\partial p_i \partial m_i}\right)\left[\left(\frac{\partial^2\Pi_i}{\partial m_i \partial r_i}\right)\left(\frac{\partial^2\Pi_i}{\partial p_i \partial r_i}\right) - \left(\frac{\partial^2\Pi_i}{\partial (r_i)^2}\right)\left(\frac{\partial^2\Pi_i}{\partial m_i \partial p_i}\right)\right].
\end{aligned} \tag{26}$$

Inserting the relevant second-order derivatives into this term and evaluating it at the symmetric equilibrium yields that the sign of this term is given by the sign of

$$-\left[c''(r)\left(2M^3p(M-1)(1-s)+f''(m)M^6m^2\right)+p^2(M-1)^2\right]\times$$
$$\times\left[2M\beta v-p\left(\beta(M(1-s)+M+1)+(M-1)(1-s)\right)\right]. \tag{27}$$

The first term in square brackets is positive due to the fact that $c''(r) > 0$ and $f''(m) \geq 0$. The second term is strictly decreasing in p. Inserting the highest possible value for p in any equilibrium, which is v, into the second term and simplifying, we obtain that it is given by $v(M-1)(1-s)(\beta-1)$, which is strictly positive as $\beta > 1$. Therefore, (26) is negative.

As a consequence, if the assumptions of Section 2 are fulfilled, the first-order principal minors are negative, the second-order principal minors are positive, and the third-order principal minor is again negative. As a consequence, the Hessian matrix is negative definite, which implies that the first-order conditions constitute an interior local maximum. Therefore, if all firms $j \neq i$ choose its strategic variables as prescribed by the first-order conditions, doing so as well constitutes a local maximum for firm i.[45]

We next check under which conditions this interior local maximum is also a global maximum. First, our assumptions on $c(r_i)$ ensure that the optimal value of r_i is interior. Second, the structure of the maximization problem implies that for any value of r_i and p_i, there is a single value of m_i that constitutes a local maximum. This is due to the fact that $\partial^2\Pi_i/\partial(m_i)^2 < 0$—i.e., the demand function is strictly concave in m_i and $f'' \geq 0$. Third, if firm i sets the price for all products equal to v, the derivative of $\Pi_i(\mathbf{m}, \mathbf{r}, \mathbf{p})$ with respect to m_i is negative. In fact, the first term in (3) is zero, whereas the second term is negative as $f''(\cdot) \geq 0$.

Taking these arguments together, if the local maximum does not constitute a global maximum, this global maximum cannot be interior, but must occur at a point at which some of the variables take extreme values. Since the optimal value of r_i is necessarily interior, the only candidate for the potential global maximum is the one in which firm i focuses only on consumer retention but does not invest in product variety. It then optimally sets $m_i = 0$ and $p_i = v$, that is, m_i and p_i are set at extreme values. The resulting optimization problem

[45] If a firm could set different prices for its products, the second-order conditions are more complicated, as a firm has a continuum of choice variables. However, the structure of the first derivative for any two prices is the same. It is then possible to check the second-order conditions by focusing on the derivatives of two prices—e.g., $p_{k,i}$ and $p_{k',i}$—where $p_{k,i}$ applies to a set n of products and $p_{k',i}$ to a set n' of products. The resulting Hessian is then a 4x4 matrix. Following the same procedure as above, and letting n and n' go to zero after derivatives are taken, we can show that the Hessian is negative definite if the assumptions of Section 2 are fulfilled.

of firm i is

$$\max_{r_i} \frac{r_i v}{M} - c(r_i),$$

yielding an optimal customer retention level, denoted by r_i^{ext}, that is implicitly defined by

$$r_i^{ext} = (c')^{-1}\left(\frac{v}{M}\right). \tag{28}$$

Inserting this optimal solution into the profit function, we obtain that firm i's profit is then given by

$$(c')^{-1}(v/M)\left(\frac{v}{M}\right) - c\left((c')^{-1}(v/M)\right).$$

Instead, firm i's profit at the symmetric equilibrium can be written as

$$\frac{\beta v}{\beta M + (M-1)(1-r^*)} - c(r^*) - f(m^*).$$

Since firm i charges the maximal price to retained consumers and sets $m_i = 0$ at the extremal solution, $r_i^{ext} > r^*$. This implies that its cost of investment in customer retention are larger in the extremal solution compared to the interior local maximum. It therefore follows that the extremal solution cannot be a global maximum if the cost at this solution—i.e., $c\left((c')^{-1}(v/M)\right)$—are sufficiently large compared to the costs at lower values of r_i.

Expressing this via the primitives of the model, we can first determine the implicit solution for r^*. Solving (7) for p^* and inserting the respective value in the second equation of (6), we obtain that r^* is implicitly given by

$$\frac{v\beta(M-1)}{M(\beta M + (M-1)(1-r^*))} = c'(r^*).$$

The left-hand side is increasing in r^* and is equal to $v\beta(M-1)/\left(M(\beta M + (M-1))\right)$ at $r^* = 0$. Therefore, a sufficient (but not necessary) condition for the interior local maximum to be a global maximum is that

$$c\left((c')^{-1}\left(\frac{v}{M}\right)\right) >> c\left((c')^{-1}\left(\frac{v\beta(M-1)}{M(\beta M + M - 1)}\right)\right).$$

In words, the costs at the extremal solution are substantially larger than the lowest possible cost at the interior solution, which implies that a firm is better off by choosing a low value of r_i than choosing $r_i = r_i^{ext}$. Therefore, the extremal solution is not optimal.

Online Appendix D: Extensions

Online Appendix D.1: Endogenous Customer Base

In the main model, we considered a scenario in which the customer base of each firm is exogenous—i.e., each firm has the same base of size $1/M$. In this appendix, we analyze an extended scenario in which each firm can influence its customer base via setting the prices of its existing products.

To incorporate this in a simple way, suppose that each firm $i = 1, ..., M$ has a mass $\tilde{m}$ of existing products and sets a price $\tilde{p}_i$ for these products.[46] Consistent with the main model, the resulting demand of firm i for its existing products is then

$$\frac{\tilde{m}(v-\tilde{p}_i)^{\frac{1}{\beta}}}{\sum_{j=1}^{M}\tilde{m}(v-\tilde{p}_j)^{\frac{1}{\beta}}} = \frac{(v-\tilde{p}_i)^{\frac{1}{\beta}}}{\sum_{j=1}^{M}(v-\tilde{p}_j)^{\frac{1}{\beta}}}.$$

Each firm obtains profits for its existing products, which are given by the respective price times the demand. The aggregate demand of firm i then determines firm i's customer base. As in the main model, each firm can then invest to retain consumers who are in the customer base, but can also invest in product variety to attract potential switching consumers.

Following the notation of the main model, the vector of all firms' prices for the existing products is denoted by $\tilde{\mathbf{p}}$, whereas the respective vectors for the other strategic variables are still denoted by $\mathbf{m}$, $\mathbf{r}$, and $\mathbf{p}$, respectively. Firm i's profit function in this extended model can then be written as

$$\Pi_i(\mathbf{m},\mathbf{r},\tilde{\mathbf{p}},\mathbf{p}) = \frac{\tilde{p}_i(v-\tilde{p}_i)^{\frac{1}{\beta}}}{\sum_{j=1}^{M}(v-\tilde{p}_j)^{\frac{1}{\beta}}} + r_i p_i \left(\frac{(v-\tilde{p}_i)^{\frac{1}{\beta}}}{\sum_{j=1}^{M}(v-\tilde{p}_j)^{\frac{1}{\beta}}}\right) +$$

$$+\left(\frac{\sum_{j=1}^{M}(1-r_j)(v-\tilde{p}_j)^{\frac{1}{\beta}}}{\sum_{j=1}^{M}(v-\tilde{p}_j)^{\frac{1}{\beta}}}\right)\left(\frac{p_i m_i (v-p_i)^{\frac{1}{\beta}}}{\sum_{j=1}^{M} m_j (v-p_j)^{\frac{1}{\beta}}}\right) - f\left(m_i\right) - c\left(r_i\right).$$

In contrast to the main model, the profit function now consists of an additional term—i.e., the first term in $\Pi_i(\mathbf{m},\mathbf{r},\tilde{\mathbf{p}},\mathbf{p})$—which gives the profit that firm i obtains from the existing products. Moreover, in the second and the third term, the customer base is now no longer

[46]We focus on the case in which firms are symmetric with respect to the mass of their existing products. Considering asymmetric masses would complicate the analysis, but would not change the main insights, as the analysis follows similar lines as considered in Online Appendix D.2.

$1/M$ but

$$\frac{(v-\tilde{p}_i)^{\frac{1}{\beta}}}{\sum_{j=1}^{M}(v-\tilde{p}_j)^{\frac{1}{\beta}}}.$$

Firm i maximizes this profit function with respect to $\tilde{p}_i$, m_i, r_i, and p_i. To simplify the exposition, we consider a game in which firms choose their maximization variables at the same time.[47]

The first-order condition with respect to $\tilde{p}_i$ is

$$(\beta(v-\tilde{p}_i)-\tilde{p}_i)\left(\sum_{j=1}^{M}(v-\tilde{p}_j)^{\frac{1}{\beta}}\right)+\tilde{p}_i(v-\tilde{p}_i)^{\frac{1}{\beta}}-r_ip_i\sum_{j=1,\jmath\neq i}^{M}(v-\tilde{p}_j)^{\frac{1}{\beta}}$$

$$+\frac{p_im_i(v-p_i)^{\frac{1}{\beta}}}{\sum_{j=1}^{M}m_j(v-p_j)^{\frac{1}{\beta}}}\left(\sum_{j=1}^{M}(r_i-r_j)(v-\tilde{p}_j)^{\frac{1}{\beta}}\right)=0.$$

Turning to the variables that are also present in the main model, we obtain that the respective first-order conditions need to be slightly modified. Specifically, the first-order condition with respect to m_i is

$$\left(\frac{\sum_{j=1}^{M}(1-r_j)(v-\tilde{p}_i)^{\frac{1}{\beta}}}{\sum_{j=1}^{M}(v-\tilde{p}_j)^{\frac{1}{\beta}}}\right)\frac{p_i\,(v-p_i)^{\frac{1}{\beta}}\left(\sum_{j=1,\,i\neq j}^{M}m_j(v-p_j)^{\frac{1}{\beta}}\right)}{M\left(\sum_{j=1}^{M}m_j(v-p_j)^{\frac{1}{\beta}}\right)^2}-f'(m_i)=0,$$

the one with respect to r_i is

$$p_i\left(\frac{(v-\tilde{p}_i)^{\frac{1}{\beta}}}{\sum_{j=1}^{M}(v-\tilde{p}_j)^{\frac{1}{\beta}}}\right)\left(1-\frac{m_i(v-p_i)^{\frac{1}{\beta}}}{\sum_{j=1}^{M}m_j(v-p_j)^{\frac{1}{\beta}}}\right)-c'(r_i)=0,$$

and the one with respect to p_i is

$$r_i\left(\frac{(v-\tilde{p}_i)^{\frac{1}{\beta}}}{\sum_{j=1}^{M}(v-\tilde{p}_j)^{\frac{1}{\beta}}}\right)-$$

[47] The analysis of sequential decisions in which e.g. $\tilde{p}_i$ is chosen before the other variables, is considerably more complicated. However, it proceeds along the same lines as in Online Appendix D.5. We numerically analyzed this case and obtained that our main results continue to hold.

$$+\frac{\sum_{j=1}^{M}(1-r_j)}{\beta}\left(\frac{(v-\tilde{p}_i)^{\frac{1}{\beta}}}{\sum_{j=1}^{M}(v-\tilde{p}_j)^{\frac{1}{\beta}}}\right)\left(\frac{m_i\,(v-p_i)^{\frac{1-\beta}{\beta}}}{\sum_{j=1}^{M}m_j(v-p_j)^{\frac{1}{\beta}}}\right)\left(\beta(v-p_i)-p_i+\frac{p_i m_i\,(v-p_i)^{\frac{1}{\beta}}}{\sum_{j=1}^{M}m_j(v-p_j)^{\frac{1}{\beta}}}\right)=0.$$

In a symmetric equilibrium, where we denote the equilibrium variables by $\tilde{p}^*$, p^*, m^*, and r^*, these conditions can be simplified to

$$M\beta\,(v-\tilde{p}^*)-(M-1)\tilde{p}^*-(M-1)p^*r^*=0, \tag{29}$$

$$\frac{(1-r^*)(M-1)p^*}{m^*M^2}-f'(m^*)=0, \quad \frac{(M-1)p^*}{M^2}-c'(r^*)=0, \tag{30}$$

$$\text{and} \quad M\beta\,(v-p^*)-(M-1)p^*\,(1-r^*)=0. \tag{31}$$

The new condition (29) is the simplified first-order condition with respect to $\tilde{p}_i$ and determines $\tilde{p}^*$. However, it is straightforward to see that (30) and (31) are equivalent to the conditions in the main model. Therefore, all results of the main model carry over to the baseline model.

The intuition behind this result is that in a symmetric equilibrium, all firms charge the same prices for their existing products, which implies that in equilibrium, the customer base of each firm is $1/M$. The maximization with respect to m_i, r_i, and p_i then yields the same results as in the main model.

Although this intuition is simple, the analysis shows that the main model can be extended to allow for an endogenous customer base. For instance, a similar result would apply if firms can influence their customer base not only via setting prices for their existing products, but also if the mass of existing products was a strategic variable of each firm. Therefore, our results are robust to these extensions.

Online Appendix D.2: Firms are Asymmetric with respect to Customer Base

In the main model, we considered the case in which all firms have the same customer base, that is, each firm's customer base is $1/M$. We now demonstrate that our results carry over to the situation in which firms have asymmetric customer bases. To simplify the exposition, we consider a situation with two different types of firms, where one type of firms has a smaller customer base than the other.[48] As will become clear, the situation with M different firms can be tackled in the same way and delivers qualitatively the same results.

[48] Firms may also differ in other dimensions. For example, they may have different costs to offer a larger product variety (i.e. a different $f(\cdot)$-function) or differ in their investment cost of customer retention (i.e., a different $c(\cdot)$-function). Solving for the equilibrium in these situations can be done in the same way as in the case in which firms differ in their customer bases.

Suppose that, among the M firms, $K < M$ firms have a customer base of $\underline{b}/\left(K\underline{b} + (M-K)\overline{b}\right)$, whereas $M-K$ firms have a customer base of $\overline{b}/\left(K\underline{b} + (M-K)\overline{b}\right)$, with $\underline{b} \neq \overline{b}$. The profit function of firm i, with $b_i \in \left\{\underline{b}, \overline{b}\right\}$ is then given by

$$\Pi_i(\mathbf{m}, \mathbf{r}, \mathbf{p}) = \frac{b_i r_i p_i}{K\underline{b} + (M-K)\overline{b}} + \frac{\sum_{j=1}^{M} b_j(1-r_j)}{K\underline{b} + (M-K)\overline{b}} \left(\frac{p_i m_i (v-p_i)^{\frac{1}{\beta}}}{\sum_{j=1}^{M} m_j (v - p_{\ell,j})^{\frac{1}{\beta}}} \right) - f\left(m_i\right) - c\left(r_i\right).$$

Determining the first-order conditions, we obtain[49]

$$\frac{\partial \Pi_i(\mathbf{m}, \mathbf{r}, \mathbf{p})}{\partial m_i} = \frac{\sum_{j=1}^{M} b_j(1-r_j) p_i \left(v-p_i\right)^{\frac{1}{\beta}} \left(\sum_{j=1,\, i\neq j}^{M} m_j (v-p_j)^{\frac{1}{\beta}} \right)}{\left(K\underline{b} + (M-K)\overline{b}\right) \left(\sum_{j=1}^{M} m_j (v-p_j)^{\frac{1}{\beta}} \right)^2} - f'(m_i) = 0,$$

$$\frac{\partial \Pi_i(\mathbf{m}, \mathbf{r}, \mathbf{p})}{\partial r_i} = \frac{p_i b_i}{K\underline{b} + (M-K)\overline{b}} \left(1 - \frac{m_i (v-p_i)^{\frac{1}{\beta}}}{\sum_{j=1}^{M} m_j (v-p_j)^{\frac{1}{\beta}}} \right) - c'(r_i) = 0,$$

and

$$\frac{\partial \Pi_i(\mathbf{m}, \mathbf{r}, \mathbf{p})}{\partial p_i} = \frac{b_i r_i}{\left(K\underline{b} + (M-K)\overline{b}\right)} -$$

$$+ \frac{\sum_{j=1}^{M} b_j(1-r_j)}{\left(K\underline{b} + (M-K)\overline{b}\right)\beta} \frac{m_i \left(v-p_i\right)^{\frac{1-\beta}{\beta}}}{\sum_{j=1}^{M} m_j (v-p_j)^{\frac{1}{\beta}}} \left(\beta(v-p_i) - p_i + \frac{m_i p_i \left(v-p_i\right)^{\frac{1}{\beta}}}{\sum_{j=1}^{M} m_j (v-p_j)^{\frac{1}{\beta}}} \right) = 0.$$

It is straightforward to check that the signs of the cross-derivatives dm_i/dr_j and dm_i/dr_i are the same as in the main model. This is because the terms in the first-order conditions (with the exception of the cost functions that do not affect the cross derivatives) are only multiplied by a different parameter compared to the main model (i.e., $b_j/\left(K\underline{b} + (M-K)\overline{b}\right)$ instead of $1/M$), but are not affected otherwise. Therefore, Result 1 also holds with asymmetric firms.

Proceeding in the same way as in the main model, we obtain the equilibrium conditions from these first-order conditions. In the unique symmetric equilibrium, all firms of the same type set the same equilibrium values. Denoting by $\underline{m}^*$, $\underline{r}^*$, and $\underline{p}^*$ the equilibrium levels of firms with a smaller product variety and by $\overline{m}^*$, $\overline{r}^*$, and $\overline{p}^*$ as those of firms with a larger

[49]As in the main model, it is optimal for each firm to set the same price for all its products.

product variety, we can write the equilibrium conditions for the latter three variables as

$$\left(K\underline{b}\,(1-\underline{r}^*)+(M-K)\overline{b}\,(1-\overline{r}^*)\right)\times$$

$$\times\frac{\overline{p}^*\,(v-\overline{p}^*)^{\frac{1}{\beta}}\left(\underline{m}^*K\left(v-\underline{p}^*\right)^{\frac{1}{\beta}}+\overline{m}^*(M-K-1)\,(v-\overline{p}^*)^{\frac{1}{\beta}}\right)}{\left(K\underline{b}+(M-K)\overline{b}\right)\left(\underline{m}^*K\left(v-\underline{p}^*\right)^{\frac{1}{\beta}}+\overline{m}^*(M-K)\,(v-\overline{p}^*)^{\frac{1}{\beta}}\right)^2}-f'\left(\overline{m}^*\right)=0,$$

$$\frac{\overline{b}\overline{p}^*\left(\underline{m}^*K\left(v-\underline{p}^*\right)^{\frac{1}{\beta}}+\overline{m}^*(M-K-1)\,(v-\overline{p}^*)^{\frac{1}{\beta}}\right)}{\left(K\underline{b}+(M-K)\overline{b}\right)\left(\underline{m}^*K\left(v-\underline{p}^*\right)^{\frac{1}{\beta}}+\overline{m}^*(M-K)\,(v-\overline{p}^*)^{\frac{1}{\beta}}\right)}-c'\left(\overline{r}^*\right)=0,$$

and

$$\frac{\overline{r}^*\overline{b}\beta\,(v-\overline{p}^*)^{\frac{1}{\beta}}}{\overline{m}^*}-\frac{K\underline{b}\,(1-\underline{r}^*)+(M-K)\overline{b}\,(1-\overline{r}^*)}{\underline{m}^*K\left(v-\underline{p}^*\right)^{\frac{1}{\beta}}+\overline{m}^*(M-K)\,(v-\overline{p}^*)^{\frac{1}{\beta}}}\times$$

$$\times\left(\overline{p}^*-\beta(v-\overline{p}^*)-\frac{\overline{m}^*\overline{p}^*\,(v-\overline{p}^*)^{\frac{1}{\beta}}}{\underline{m}^*K\left(v-\underline{p}^*\right)^{\frac{1}{\beta}}+\overline{m}^*(M-K)\,(v-\overline{p}^*)^{\frac{1}{\beta}}}\right)=0.$$

In a similar way, we can write the equilibrium conditions for $\underline{m}^*$, $\underline{r}^*$, and $\underline{p}^*$. This provides us with six equations for the six equilibrium values.

Proceeding in the same way as in the proofs of Results 2, and 3 we can totally differentiate these equations with respect to the six equilibrium variables as well as v and β, which allows us to determine $d\overline{m}^*/dv$, $d\underline{m}^*/dv$, $d\overline{m}^*/d\beta$, and $d\underline{m}^*/d\beta$. The resulting expressions are very long compared to the main model due to the additional parameters $\underline{b}$, $\overline{b}$, and K. However, we can show that the results are akin to those of the main model. In particular, $d\overline{m}^*$ and $d\underline{m}^*$ change non-monotonically with v, that is, $d\overline{m}^*/dv$ and $d\underline{m}^*/dv$ are both positive for small values of v, but negative for large values of v and there is a unique value of v at which both derivatives are zero. Instead, $d\overline{m}^*/d\beta$, and $d\underline{m}^*/d\beta$ are positive for all values of v. Therefore, Results 2 and 3 also hold with asymmetric firms.

In addition, evaluating dm_i/dr_j and dm_i/dr_i at the equilibrium, and differentiating with respect to v and β yields that the respective derivatives are negative, regardless of whether firm i or firm j are those with a high or a low customer base. This implies that Result 4 holds as well.

Finally, we note that considering more than two types of firms leads to similar results. The model becomes more complicated to solve, as, given that there then are k different types of firms, with $2 \leq k \leq M$, there are $3 \times k$ unknowns. However, the method is the same and the results are qualitatively similar.

Online Appendix D.3: Investment in Customer Retention has Positive Effects on Switching Consumers

In the main text, we considered the situation in which investment in customer retention affects the probability that a consumer of firm i repurchases from firm i. This is consistent with the findings of many papers in the Marketing literature. However, since a firm can achieve a larger customer retention e.g. by a rise in the quality of its products, it may therefore also affect the demand from switching consumers, that is, an increase r_i may also help firm i to attract a larger mass of switching consumers.

In this appendix, we consider the above scenario. To model this effect in a simple way, suppose that the probability that a switching consumer buys a product of firm i is

$$\frac{m_i(v+\psi(r_i)-p_i)^{\frac{1}{\beta}}}{\sum_{j=1}^{M} m_j(v+\psi(r_j)-p_j)^{\frac{1}{\beta}}}, \tag{32}$$

where $\psi'(\cdot) > 0$ and $\psi''(\cdot) < 0$. This formulation implies that an investment in r_i does not only increase firm i's retention rate, but also the value that consumers attribute to firm i's products. Because $\psi(\cdot)$ is increasing but concave, a consumer who buys a product from firm i benefits if r_i is larger, but at a decreasing rate. This assumption also ensures that second-order conditions are satisfied.

Firm i's profit function can then be written as

$$\Pi_i(\mathbf{m},\mathbf{r},\mathbf{p}) = \frac{r_i p_i}{M} + \frac{\sum_{j=1}^{M}(1-r_j)}{M}\left(\frac{p_i m_i(v+\psi(r_i)-p_i)^{\frac{1}{\beta}}}{\sum_{j=1}^{M} m_j(v+\psi(r_j)-p_j)^{\frac{1}{\beta}}}\right) - f\left(m_i\right) - c\left(r_i\right).$$

The resulting the first-order conditions are (using again that each firm i sets the same price for all of its products)

$$\frac{\partial \Pi_i(\mathbf{m},\mathbf{r},\mathbf{p})}{\partial m_i} = \frac{\sum_{j=1}^{M}(1-r_j)p_i\left(v+\psi(r_i)-p_i\right)^{\frac{1}{\beta}}\left(\sum_{j=1,\, i\neq j}^{M} m_j(v+\psi(r_j)-p_j)^{\frac{1}{\beta}}\right)}{M\left(\sum_{j=1}^{M} m_j(v+\psi(r_j)-p_j)^{\frac{1}{\beta}}\right)^2} - f'(m_i) = 0,$$

$$\frac{\partial \Pi_i(\mathbf{m},\mathbf{r},\mathbf{p})}{\partial r_i} = \frac{p_i}{M}\left(1-\frac{m_i(v+\psi(r_i)-p_i)^{\frac{1}{\beta}}}{\sum_{j=1}^{M} m_j(v+\psi(r_j)-p_j)^{\frac{1}{\beta}}}\right) + \sum_{j=1}^{M}(1-r_j)\times$$

$$\times \frac{m_i p_i \psi'(r_i)(v+\psi(r_i)-p_i)^{\frac{1-\beta}{\beta}} \left(\sum_{j=1,\, i\neq j}^{M} m_j (v+\psi(r_j)-p_j)^{\frac{1}{\beta}} \right)}{\beta M \left(\sum_{j=1}^{M} m_j (v+\psi(r_j)-p_j)^{\frac{1}{\beta}} \right)^2} - c'(r_i) = 0,$$

and

$$\frac{\partial \Pi_i(\mathbf{m},\mathbf{s},\mathbf{p})}{\partial p_i} = \frac{r_i}{M} + \frac{\sum_{j=1}^{M}(1-r_j)}{M\beta} \frac{m_i\,(v+\psi(r_i)-p_i)^{\frac{1-\beta}{\beta}}}{\sum_{j=1}^{M} m_j (v+\psi(r_j)-p_j)^{\frac{1}{\beta}}} \times$$

$$\times \left(\beta\,(v+\psi(r_i)-p_i) - p_i + \frac{p_i m_i\,(v+\psi(r_i)-p_i)^{\frac{1}{\beta}}}{\sum_{j=1}^{M} m_j (v+\psi(r_j)-p_j)^{\frac{1}{\beta}}} \right) = 0.$$

From the first-order conditions for m_i and r_i, we now determine the relation between m_i and r_j as well as m_i and r_i, as in Result 1 of the main model. Using the first-order condition for m_i, we obtain that the sign of dm_i/dr_j is given by the sign of

$$-\frac{p_i\,(v-p_i)^{\frac{1}{\beta}} \left(\sum_{j=1,\, i\neq j}^{M} m_j (v-p_j)^{\frac{1}{\beta}} \right)}{M \left(\sum_{j=1}^{M} m_j (v-p_j)^{\frac{1}{\beta}} \right)^2} -$$

$$-\sum_{j=1}^{M}(1-r_j) \frac{p_i \psi'(r_i)(v+\psi(r_i)-p_i)^{\frac{1}{\beta}} m_j (v+\psi(r_j)-p_j)^{\frac{1-\beta}{\beta}} \sum_{j=1}^{M} m_j (v+\psi(r_j)-p_j)^{\frac{1}{\beta}}}{M\beta \left(\sum_{j=1}^{M} m_j (v+\psi(r_j)-p_j)^{\frac{1}{\beta}} \right)^4} \times$$

$$\times \left(2 \sum_{j=1,\, i\neq j}^{M} m_j (v+\psi(r_j)-p_j)^{\frac{1}{\beta}} - \sum_{j=1}^{M} m_j (v+\psi(r_j)-p_j)^{\frac{1}{\beta}} \right).$$

The term in the first line is equivalent to that in the main model. Instead, the term in the second and third line is new and arises because of $\psi'(r_i) > 0$. Its sign depends again on the sign of the term in parentheses in the third line. For symmetric firms, this term is positive, which implies that the second term is negative overall and therefore the entire expression is strictly negative. The effect of the main model regarding dm_i/dr_j is then reinforced. In particular, if a rival firm j increases r_j, this does not only imply that fewer consumers switch, but also that the rival's products become more attractive. Investing in product portfolio size then becomes less profitable for firm i, as the firm can attract fewer consumers with each of its products.

Turning to dm_i/dr_i, we obtain that the sign of this term is given by the sign of

$$-\frac{p_i\left(v+\psi(r_i)-p_i\right)^{\frac{1}{\beta}}\left(\sum\limits_{j=1,\,i\neq j}^{M} m_j(v+\psi(r_i)-p_j)^{\frac{1}{\beta}}\right)}{M\left(\sum\limits_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}\right)^2}+$$

$$+\sum_{j=1}^{M}(1-r_j)\frac{p_i\psi'(r_i)\sum\limits_{j=1,\,i\neq j}^{M} m_j(v+\psi(r_j)-p_j)^{\frac{1}{\beta}}\sum\limits_{j=1}^{M} m_j(v+\psi(r_j)-p_j)^{\frac{1}{\beta}}(v+\psi(r_i)-p_i)^{\frac{1-\beta}{\beta}}}{M\beta\left(\sum\limits_{j=1}^{M} m_j(v+\psi(r_j)-p_j)^{\frac{1}{\beta}}\right)^4}\times$$

$$\times\left(\sum_{j=1,\,i\neq j}^{M} m_j(v+\psi(r_j)-p_j)^{\frac{1}{\beta}}-m_i(v+\psi(r_i)-p_i)^{\frac{1}{\beta}}\right).$$

Again, the term in the first line is the same as that in the main model, whereas the new term is that in the second and third line. The sign of this term is again determined by the sign of the term in parentheses in the third line. If, for instance, there are more than two firms and firms are symmetric, this term is positive. It follows that the sign of dm_i/dr_i is no longer clear. The reason is that offering a larger product portfolio size becomes more valuable for firm i if r_i increases the value of each product. However, for this effect to dominate the effect of the main model, which is represented by the first line, $\psi'(r_i) > 0$ must be sufficiently large. If $\psi'(r_i)$ is rather small, $dm_i/dr_i < 0$ as in the main model.

Overall, this analysis shows that Result 1 of the main model tends to carry over to this extension. Although the sign of dm_i/dr_i, which is certainly negative in the main model, is now no longer clear, the term dm_i/dr_j remains negative and is even exacerbated. If $\psi'(r_i)$ is rather small, the sign of sign of dm_i/dr_i remains negative and so we get the same results as in the main model. However, even if it is large, the negative effect is likely to be dominating overall, as dm_i/dr_j applies to all competitors whereas dm_i/dr_i applies only to firm i. Hence, there is a strong indication that the relation between product variety and customer retention is negative.

We now turn to the equilibrium of the game. Denoting the equilibrium values again by m^*, r^*, and p^*, as in the main model, the three conditions determining the symmetric equilibrium can be written as

$$\frac{(1-r^*)(M-1)p^*}{m^*M^2}-f'(m^*)=0, \quad \frac{(M-1)p^*\beta\left(v+\psi\left(r^*\right)-p^*\right)+\psi'\left(r^*\right)\left(1-r^*\right)}{M^2\beta\left(v+\psi\left(r^*\right)-p^*\right)}-c'(r^*)=0,$$

and

$$M\beta\left(v+\psi\left(r^*\right)-p^*\right)-(M-1)p^*\left(1-r^*\right)=0.$$

Following the proof of Result 2 and totally differentiating these first-order conditions with respect to m^*, r^*, p^*, and v, we can determine $dm^\star/dv$. Tedious but otherwise routine calculations show that

$$\text{sign}\left\{\frac{dm^\star}{dv}\right\} = \text{sign}\left\{\beta m^*(M-1)\mu\left[c''\left(r^*\right)M(1-r^*)\mu - \beta(M-1)\left(v+\psi\left(r^*\right)\right) + \psi''\left(r^*\right)\mu(1-r^*)\right]\right\},$$

with $\mu \equiv (M-1)(1-r^*) + \beta M$, as above. In the same way as in the proof of Result 2, we can show that this expression is positive if $v \to 0$, decreasing in v, and turning negative as v gets large. Hence, Result 2 of the main model also holds in this extension.

Similarly, totally differentiating these first-order conditions with respect to m^*, r^*, p^*, and β, we can solve for $dm^\star/d\beta$ to get

$$\text{sign}\left\{\frac{dm^\star}{d\beta}\right\} = \text{sign}\Big\{\left(v+\psi\left(r^*\right)\right)m^*\mu(M-1)^2 \times$$

$$\times\left[\beta(M-1)\left(v+\psi\left(r^*\right) + \psi''\left(r^*\right)\mu(1-r^*) + c''\left(r^*\right)M(1-r^*)\mu\right)\right]\Big\},$$

which is strictly positive.

Finally, to determine how dm_i/dr_j and dm_i/dr_i are affected by v and β in equilibrium, we can proceed in the same way as in the proof of Result 4 of the main model. This yields that $\partial\left(dm_i^*/dr_j^*\right)/\partial v < 0$ and $\partial\left(dm_i^*/dr_j^*\right)/\partial\beta < 0$. Therefore, the result that v and β reinforce the negative effect of r_j^* on m_i^* carries over to this extended model. For dm_i^*/dr_i^*, we obtain that $\partial\left(dm_i^*/dr_i^*\right)/\partial v < 0$ and $\partial\left(dm_i^*/dr_i^*\right)/\partial\beta < 0$ if $\psi'(r^*)$ is sufficiently small. This is in line with the previous result that $dm_i/dr_i < 0$ also only holds if $\psi'(r^*)$ rather small.

Online Appendix D.4: Switching Consumers do not Buy from the Same Firm

In the main model, we assumed that switching consumers make their choice among the products of all firms. This implies that a consumer who belongs to the customer base of firm i but is not retained by firm i, nevertheless takes the products of firm i into account and may buy one of its products. In this extension, we now consider a modification of our main model in which switching consumers do not buy from their respective original firm—e.g., because they were not satisfied with the product of that firm and therefore do not consider this firm's products in their choice set.

The profit function of firm i then takes the following form:

$$\Pi_i(\mathbf{m},\mathbf{r},\mathbf{p}) = \frac{r_i p_i}{M} + \frac{\sum\limits_{j=1,\,j\neq i}^{M}(1-r_j)}{M}\left(\frac{p_i m_i(v-p_{\ell,i})^{\frac{1}{\beta}}}{\sum\limits_{j=1}^{M} m_j(v-p_{\ell,j})^{\frac{1}{\beta}}}\right) - f\left(m_i\right) - c\left(r_i\right). \tag{33}$$

The difference to the main model is only in the second term of the profit function, where the sum over the unsatisfied consumers now only considers firms $j = 1, ..., i-1, i+1, ..., M$, but no longer firm i. Otherwise, the profit function is unchanged.

In the same way as in Online Appendix C.1, we can show that a firm optimally charges the same prices for all of its products. The counterpart to the first-order conditions of the main text are the first-order conditions resulting from the maximization of (33). They are given by

$$\frac{\sum\limits_{j=1,\,j\neq i}^{M}(1-r_j)p_i\left(v-p_i\right)^{\frac{1}{\beta}}\left(\sum\limits_{j=1,\,i\neq j}^{M} m_j(v-p_j)^{\frac{1}{\beta}}\right)}{M\left(\sum\limits_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}\right)^2} - f'(m_i) = 0,$$

$$\frac{p_i}{M} - c'(r_i) = 0,$$

and

$$r_i + \frac{\sum\limits_{j=1,\,j\neq i}^{M}(1-r_j)}{\beta}\frac{m_i\left(v-p_i\right)^{\frac{1-\beta}{\beta}}}{\sum\limits_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}}\left(\beta(v-p_i) - p_i + \frac{p_i m_i\left(v-p_i\right)^{\frac{1}{\beta}}}{\sum\limits_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}}\right) = 0.$$

It is easy to see that $dm_i/dr_j < 0$ still holds. However, as mentioned in the main text, $dm_i/dr_i = 0$ now. As consumers who switch away from firm i do not consider firm i's product in their choices, firm i cannot gain any of these consumers when offering a larger product variety. This implies that the choices m_i and r_i are no longer interdependent.

As in the main model, in a symmetric equilibrium, the three first-order conditions can be simplified to get

$$\frac{(1-r^*)(M-2)p^*}{m^*M(M-1)} - f'(m^*) = 0, \quad \frac{p^*}{M} - c'(r^*) = 0,$$

and

$$\frac{(M-1)\beta\left(v-p^*\right) - (M-1)p^*\left(1-r^*\right)}{m^*M(M-1)\beta\left(v-p^*\right)} = 0.$$

We can now follow the same procedure as in the proof of Results 2 and 3. First, determining dm^*/dv yields

$$\frac{dm^*}{dv} = \frac{(M-2)m^*\beta\hat{\mu}\left(c''(r^*)M(1-r^*)\hat{\mu} - v\beta(M-1)\right)}{\left(c''(r^*)M\hat{\mu}^2 - v\beta(M-1)(M-2)\right)\left(v\beta(M-2)(1-r^*) + M\left(m^*\right)^2\hat{\mu}f''(m^*)\right)},$$

with $\hat{\mu} \equiv (M-2)(1-r^*) + \beta(M-1) > 0$. In the same way as in Online Appendix B, we can show that it is non-monotonic in v, that is, the sign of dm^*/dv is determined by the sign of the numerator, which is positive for v small, but negative if v is above a threshold value. Second, determining $dm^*/d\beta$, we obtain

$$\frac{dm^*}{d\beta} = \frac{vm^*(1-r^*)\hat{\mu}(M-1)(M-2)\left(v\beta(M-1) + c''(r^*)M(1-r^*)\hat{\mu}\right)}{\left(c''(r^*)M\hat{\mu}^2 - v\beta(M-1)(M-2)\right)\left(v\beta(M-2)(1-r^*) + M\left(m^*\right)^2\hat{\mu}f''(m^*)\right)}.$$

which is strictly positive, as the numerator is strictly positive.

Finally, we turn to Result 4. Evaluating dm_i/dr_j at the equilibrium yields

$$\frac{dm_i^*}{dr_j^*} = -\frac{\beta v m^*(M-2)}{2\beta v(1-r^*)(M-2) + f''(m^*)\left(m^*\right)^2 M(M-1)\left(\beta(M-1) + (M-2)(1-r^*)\right)}.$$

Taking the derivative with respect to v, we obtain

$$-\Big[\Big(\left(f''(m^*)\right)^2\left(m^*\right)^4 M^2(M-1)\left(M(\beta-1) + (M-2)(1-r^*)\right)^2 + 2\beta^2v^2(M-2)^2(1-r^*)^2\Big)c''(r^*)$$
$$+f''(m^*)\beta^2v^2(m^*)^2(M+1)(M-1)(M-2)-$$
$$-f'''(m^*)\beta v(m^*)^3(M-1)(M-2)\left(\beta v(M-1) - c''(r^*M(1-r^*)(M(\beta-1) + (M-2)(1-r^*))\right)\Big].$$

Similarly, taking the derivative with respect to β, we obtain

$$-\Big[\Big(\left(f''(m^*)\right)^2 v\beta m^* M^2(M-1) + 2(M-2)^2(1-r^*)^2\left(M(\beta-1) + (M-2)(1-r^*)\right)\Big)c''(r^*)$$
$$+f''(m^*)(m^*)^2(M+1)M\left(M(\beta-1) + (M-2)(1-r^*)\right)^2 -$$
$$-f'''(m^*)\beta v(m^*)^3(M-1)(M-2)\left(\beta v(M-1) - c''(r^*M(1-r^*)(M(\beta-1) + (M-2)(1-r^*))\right)\Big].$$

Because $c''(\cdot) > 0$, $f''(\cdot) \geq 0$, and $f'''(\cdot)$ is negative, or if positive, then small compared to the second derivatives, both of these expressions are negative. This shows that also Result 4 holds in case switching consumers do not buy from the same firm.

Online Appendix D.5: Customer Retention and Product Variety Choices Precede Price Choices

In the main model, we considered the case in which firms simultaneously choose product variety, customer retention, and product prices. In this section, we analyze the case in which the first two variables—i.e., product variety and customer retention—are chosen before product prices are set. A natural reason for such a sequential timing could be that product variety and customer retention are more long-term decisions than product prices. In particular, in some industries product prices can be changed at a relatively fast speed, whereas changing the product portfolio size or improving the functionality of products may take longer.[50]

The sequential game therefore unfolds as follows. In the first, stage, each firm $i = 1, ..., M$ chooses the mass of its products, m_i, and the customer retention level, r_i. Given these choices, in the second stage, each firm i sets prices for its products p_i. We analyze the game by backward induction and solve for the subgame-perfect equilibrium of the game.

In the second stage, the first-order condition with respect to p_i is the same as in the main model—i.e., (5)—taking m_i and r_i, $i = 1, ..., M$, from the previous stage as given.

In the first stage, envoking the Envelope-Theorem, the two first-order conditions for m_i and r_i can be written as

$$\frac{\partial \Pi_i}{\partial m_i} = \left.\frac{\partial \Pi_i}{\partial m_i}\right|_{sim} + \sum_{j=1,\, j\neq i}^{M} m_j \frac{\partial \Pi_i}{\partial p_j}\left(-\frac{\frac{\partial^2 \Pi_j}{\partial p_j \partial m_i}}{\frac{\partial^2 \Pi_i}{\partial (p_j)^2}}\right) = 0 \tag{34}$$

and

$$\frac{\partial \Pi_i}{\partial r_i} = \left.\frac{\partial \Pi_i}{\partial r_i}\right|_{sim} + \sum_{j=1,\, j\neq i}^{M} m_j \frac{\partial \Pi_i}{\partial p_j}\left(-\frac{\frac{\partial^2 \Pi_j}{\partial p_j \partial r_i}}{\frac{\partial^2 \Pi_i}{\partial (p_j)^2}}\right) = 0, \tag{35}$$

respectively, where we used that $dp_j/dm_i = -\left(\frac{\partial^2 \Pi_j}{\partial p_j \partial m_i}\right) / \left(\frac{\partial^2 \Pi_i}{\partial (p_j)^2}\right)$ and $dp_j/dr_i = -\left(\frac{\partial^2 \Pi_j}{\partial p_j \partial r_i}\right) / \left(\frac{\partial^2 \Pi_i}{\partial (p_j)^2}\right)$. In these equations, $\left.\frac{\partial \Pi_i}{\partial m_i}\right|_{sim}$ and $\left.\frac{\partial \Pi_i}{\partial r_i}\right|_{sim}$ are the respective first-order conditions from the simultaneous game and are given by (3) and (4). In addition to the first-order conditions of the simultaneous game, those of the sequential game also consider the effect that a change in m_i and r_i has on the prices chosen by rival firms in the second stage. This is represented by the second term in the two conditions (34) and (35).

[50] Nevertheless, as mentioned in the main text, in many digital markets, it is relatively simple and takes little time to add or withdraw products, and changes in the software code to improve the functionality can also be introduced at a fast rate. Therefore—and also to bring out our effects in the simplest way—we consider a simultaneous timing in the main model.

From firm i's profit function, we obtain

$$\frac{\partial \Pi_i}{\partial p_j} = \frac{\sum_{j=1}^{M}(1-r_j)}{M}\left(\frac{m_i p_i (v-p_i)^{\frac{1}{\beta}}(v-p_j)^{\frac{1-\beta}{\beta}}}{\beta\left(\sum_{j=1}^{M} m_j (v-p_j)^{\frac{1}{\beta}}\right)^2}\right) > 0. \tag{36}$$

We now turn to the signs of the terms for dp_j/dm_i and dp_j/dr_i. The terms in the respective numerators are the cross derivatives of firm j's profit function with respect to p_j and m_i in (34) and with respect to p_j and r_i in (35). These terms are given by

$$\frac{\partial^2 \Pi_j}{\partial p_j \partial m_i} = -\frac{\sum_{j=1}^{M}(1-r_j)}{M}\left(\frac{(v-p_j)^{\frac{1-\beta}{\beta}}(v-p_i)^{\frac{1}{\beta}}}{\beta\left(\sum_{j=1}^{M} m_j (v-p_j)^{\frac{1}{\beta}}\right)^2}\right)\left(2\frac{m_j p_j (v-p_j)^{\frac{1}{\beta}}}{\sum_{j=1}^{M} m_j (v-p_j)^{\frac{1}{\beta}}} - (p_j - \beta(v-p_j))\right) \tag{37}$$

and

$$\frac{\partial^2 \Pi_j}{\partial p_j \partial r_i} = -\left(\frac{(v-p_j)^{\frac{1-\beta}{\beta}}}{M\beta \sum_{j=1}^{M} m_j (v-p_j)^{\frac{1}{\beta}}}\right)\left(\frac{m_j p_j (v-p_j)^{\frac{1}{\beta}}}{\sum_{j=1}^{M} m_j (v-p_j)^{\frac{1}{\beta}}} - (p_j - \beta(v-p_j))\right) > 0, \tag{38}$$

respectively. While we cannot determine the sign of (37), the sign of (38) is strictly positive. This is due to the fact that the first term in parentheses is positive, while the second term in parentheses is negative. The latter follows from the first-order condition for the prices, as given by (5). This first-order condition can only be satisfied if the following inequality holds:

$$p_j - \beta(v-p_j) > \frac{m_j p_j (v-p_j)^{\frac{1}{\beta}}}{\sum_{j=1}^{M} m_j (v-p_j)^{\frac{1}{\beta}}}. \tag{39}$$

The reason is that the first term in (5)—i.e., $r_j \beta (v-p_j)^{(\beta-1)/\beta}/m_j$—is strictly positive, which implies that the second term must be negative. This is only true if (39) holds. The implication of this argument is that, in (38), the second term in parentheses is negative; hence, $\partial^2 \Pi_j / (\partial p_j \partial r_i)$ is positive. Finally, turning to (35) again, the denominator of the term in the large parentheses $\partial^2 \Pi_i / \partial (p_j)^2$, which is strictly negative because of the second-order condition. As a consequence, $dp_j/dr_i > 0$ then implies that the second term in (35) is positive.

Taken these results together, it follows that investment in customer retention is larger in the sequential timing as compared to the simultaneous timing. Due to the fact that the second term of (35) is positive, at the point of r_i at which the first-order condition in the simultaneous timing is fulfilled (i.e., the equilibrium value of r_i in the simultaneous timing), the first-order condition in the sequential timing is positive. It follows that the maximum in the sequential timing must lie to the right of the maximum of the simultaneous timing, which implies that the equilibrium investment in customer retention is larger in the sequential timing. The intuition is that investing more in customer retention by firm i induces fewer consumers to switch, which implies that competition for switching consumers is reduced. Hence, the pricing pressure on products is lower. This induces all rival firms to increase their prices, which is beneficial for firm i. Therefore, firm i has a stronger incentive to raise r_i.

Instead, for the optimal number of products, the direction of the change between sequential and simultaneous timing is not clear. This is because the sign of $\partial^2\Pi_j / (\partial p_j \partial m_i)$ is not clear-cut. It depends on the sign of the last term in parentheses. In contrast to the term in (38), this term has two times the positive expression $\left(p_j(v-p_j)^{\frac{1}{\beta}}\right) / \left(\sum_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}\right)$ instead of only once. Therefore the term can either be positive or negative. The intuition is that an increase in m_i has a twofold effect on pricing incentives of rivals. First, each rival will serve fewer consumers with its products, which, similar to the effect outlined in the previous paragraph, induces rivals to increase prices. Second, a larger number of products by firm i enhances competition for switching consumers, which leads to downward pressure in prices. Therefore, the overall effect is ambiguous.

We now turn to the interaction between the number of products and customer retention, both between firms and within a firm. To determine these effects, we need to take the derivative of the first-order condition (34) with respect to r_j and r_i. Unfortunately, the resulting expressions are rather unwieldy without clear results. However, we performed numerous numerical simulations with different functions and verified that in almost all of them, the direction of the interaction between the number of products and customer retention is the same as in Result 1.[51]

We provide two reasons for the findings that we obtained in our simulations: First, in (34), the first term is the same as in the first-order condition for the simultaneous case, which by itself is responsible for Result 1. In the simulations, we obtain that for many parametrizations, the effect resulting from the first term is dominating the effect resulting from the second term. Second, the effect of the second term often also goes in the same direction as that of the first term. To see this, note that the second term in (34) is the

[51] The numerical simulations are available from the authors.

fraction between $\partial^2\Pi_j/\left(\partial p_j \partial m_i\right)$ and $\partial^2\Pi_i/\partial\left(p_j\right)^2$. In both terms, the levels of customer retention show up only in the term $\sum_{j=1}^{M}(1-r_j)$, which implies that they cancel out in the fraction. Therefore, the derivative of the second term of (34) with respect to r_j and r_i is equal to

$$\sum_{j=1,\, j\neq i}^{M} m_j \frac{\partial^2\Pi_i}{\partial p_j \partial r_j}\left(-\frac{\frac{\partial^2\Pi_j}{\partial p_j \partial m_i}}{\frac{\partial^2\Pi_i}{\partial (p_j)^2}}\right) \quad \text{and} \quad \sum_{j=1,\, j\neq i}^{M} m_j \frac{\partial^2\Pi_i}{\partial p_j \partial r_i}\left(-\frac{\frac{\partial^2\Pi_j}{\partial p_j \partial m_i}}{\frac{\partial^2\Pi_i}{\partial (p_j)^2}}\right),$$

respectively. From (36), it is easy to see that $\partial^2\Pi_i/\left(\partial p_j \partial r_j\right) < 0$ and $\partial^2\Pi_i/\left(\partial p_j \partial r_i\right) < 0$. Moreover, as explained above, the sign of $\partial^2\Pi_j/\left(\partial p_j \partial m_i\right)$ in these expressions is determined by the sign of

$$\beta\left(v-p_j\right) - p_j + 2\frac{p_j(v-p_j)^{\frac{1}{\beta}}}{\sum\limits_{j=1}^{M} m_j(v-p_j)^{\frac{1}{\beta}}}. \tag{40}$$

Inspection of (5) shows that for small values of r_i, the first term of (5) is small, which implies that the second term in the large parentheses in (5) must be close to zero, so that (5) holds. This, however, implies that the expression in (40) is positive. Taken together, for small values of customer retention, the derivative of the second term in (34) with respect to r_j and r_i, respectively, is negative, and therefore goes in the same direction as the first term, which amplifies Result 1.

We now turn to Results 2 and 3. In the symmetric equilibrium, all firms choose the same number of products, denoted by $m^\star$, and set the same customer retention level, denoted by $r^\star$, in the first stage, and, in the second stage, set the same price $p^\star$ for their products. As the first-order conditions in the second stage are the same as in the simultaneous game, the equation for the equilibrium price is the same as that shown in the proof of Result 2 and is given by

$$p^* = \frac{v\beta M}{\beta M + (M-1)(1-r^*)}.$$

Using it in the two first-order conditions (34) and (35) and simplifying, these first-order conditions are given by

$$\frac{v\beta(1-r^*)(M-1)}{Mm^\star\mu} - f'(m^\star) - \frac{v\beta(1-r^*)\left(r^*(M-1)-1\right)}{M\mu\left(Mm^\star(1+\beta)-2r^*(M-1)\right)} = 0, \tag{41}$$

and

$$\frac{v\beta(M-1)}{M\mu} - c'(r^\star) - \frac{v\beta m^\star r^*(M-1)}{M\mu\left(Mm^\star(1+\beta)-2s^*(M-1)\right)} = 0, \tag{42}$$

where, as above, $\mu \equiv (M-1)(1-r^*) + \beta M$.

Totally differentiating (41) and (42) with respect to m^*, r^*, p^*, and v, is tedious but standard calculations show that the sign of $m^\star/dv$ is given by the sign of

$$c''(s^\star)M(1-r^*)\mu\eta\left(m^\star(\beta M(M-1)+M^2-(M-1)(1-r^*))-2r^*(M-1)^2\right)-$$

$$-v\beta(M-1)\Big\{(m^\star)^2\left[\beta^2M^2(M-1)+\beta\left(2r^\star M(M-1)+2M^2(M-1)-M\right)+r^\star(M-1)(2M-r^\star)+\right.$$

$$\left.+(M^2-1)(M-1)\right]-2m^\star(M-1)\left(2\beta r^\star(M-1)+2r^\star(M-1)(M-r^\star)-1\right)+4\left(r^*\right)^2(M-1)^3\Big\}.$$

with $\eta \equiv Mm^\star(1+\beta)-2r^*(M-1)$. Following the method of the proof of Result 2, this expression is positive for $v \to 0$, but strictly decreasing in v and becomes negative as v gets large. Therefore, $m^\star$ changes non-monotonically with v—i.e., it is increasing in v if v is below a certain level, but decreasing for v above this level. Proceeding in the same way for $m^\star/d\beta$ yields that its sign is strictly positive. Therefore, Results 2 and 3 of the simultaneous model also hold with sequential decisions.

As we could show Result 1 on the interaction between the number of products and customer retention only numerically, we can determine Result 4 only numerically as well. Considering several concrete functions for $f(m_i)$ and $c(r_i)$, we numerically solve (41) and (42) for m^* and r^* and plug it into the expressions for dm_i/dr_j and dm_i/dr_i. Taking the respective derivatives with respect to v and β, we obtain that the result is negative, that is, an increase in the value of the market as well as in the degree of consumer heterogeneity strengthens the negative effect of consumer retention on a firm's product portfolio.

Online Appendix D.6: Concrete Example

In this appendix, we provide a concrete example that allows for closed-form solutions. Consider the following functional forms for the firms' cost functions: $f\left(m_i\right) = fm_i$ and $c\left(r_i\right) = cr_i^2$, that is, marginal costs for investment in product variety are constant and marginal costs for investment in customer retention are increasing. These assumptions seem reasonable in many industries: once a firm has entered a market segment (and incurred the respective fixed costs), the cost for launching an additional set of products in this segment is usually independent of the number of products. Instead, raising customer retention becomes increasingly costly as different instruments to do so have different costs (e.g., providing an update is usually cheaper than general improvements in product quality).

With this formulation, the three conditions that determine a symmetric equilibrium, are given by

$$\frac{(1-r^*)(M-1)p^*}{m^*M^2}-f=0, \quad \frac{(M-1)p^*}{M^2}-2cr^*=0,$$

and

$$M\beta\left(v-p^{*}\right)-(M-1)p^{*}\left(1-r^{*}\right)=0.$$

Solving the last condition for p^* yields

$$p^{*}=\frac{M\beta v}{(M-1)(1-r^{*})+\beta M}. \tag{43}$$

Inserting this into the first condition and solving for m^*, we obtain

$$m^{*}=\frac{\beta v\left((M-1)(1-r^{*})\right)}{fM\left((M-1)(1-r^{*})+\beta M\right)}. \tag{44}$$

Inserting (43) and (44) into the second condition and solving for r^*, we obtain two solutions:

$$\frac{cM\left(M-1+\beta M\right)+\sqrt{\psi}}{2cM(M-1)} \quad \text{and} \quad \frac{cM\left(M-1+\beta M\right)-\sqrt{\psi}}{2cM(M-1)},$$

with $\psi\equiv cM\left(cM\left(M-1+\beta M\right)^{2}-2v\beta(M-1)^{2}\right)$. It is easy to check that only the second solution is in the admissible range as the first solution is above 1 for all admissible values, which is not possible due to the fact that r^* is a probability. Inserting the resulting solution for r^* back into the equations for m^* and p^*, the resulting equilibrium expressions for the three variables are

$$m^{*}=\frac{v\beta\left(\sqrt{\psi}+cM\left(M-1-\beta M\right)\right)}{fM\left(cM\left(M-1+\beta M\right)+\sqrt{\psi}\right)}, \quad r^{*}=\frac{cM\left(M-1+\beta M\right)-\sqrt{\psi}}{2cM(M-1)}, \tag{45}$$

and

$$p^{*}=\frac{2cM^{2}\beta v}{cM\left(M-1+\beta M\right)+\sqrt{\psi}}.$$

The resulting equilibrium profit in a symmetric equilibrium is

$$\frac{\beta v\left(cM^{2}(1+\beta)+3cM-\sqrt{\psi}\right)}{2M\left(cM^{2}(1+\beta)-cM+\sqrt{\psi}\right)}. \tag{46}$$

We can now determine the conditions so that our assumptions spelled out at the end of Section 2 are satisfied. First, from the second expression of (45), we obtain that $r^*<1$ if $c<v(M-1)/(2M^2)$.[52] Second, because $c''(\cdot)=2c$, the assumption that guarantees that the Hessian is negative definite is $c<v(M-1)^2/(2\beta M^3)$. Finally, to ensure that no global deviation exists, the profit in (46) must be larger than the profit from setting all prices equal

[52] This is also obtained from the assumption given as the end of Section 2, as $c'(1)=2c$.

to v and not investing in product variety (i.e., $m_i = 0$). If a firm would follow this strategy, its optimal value of customer retention would be $v/(2cM)$, resulting in a profit of $v^2/(4cM^2)$. Subtracting this profit from the one in (46), we obtain that the difference is positive if

$$cM\left(2c\beta M(3+M+\beta M)-v(M-1+\beta M)\right)-(2c\beta M+v)\sqrt{\psi}>0.$$

It is easy to show numerically that this inequality is satisfied as long as c is large enough. For instance, considering the values $M = 7$, $v = 5$, and $\beta = 2$, the inequality is satisfied for c approximately larger than 0.965. If β gets larger, which implies that product variety becomes more important, the inequality naturally becomes less tight—e.g. if $\beta = 4$, it is satisfied for c approximately larger than 0.556.

We now show Results 2-4 with the concrete example. Taking the derivative of m^* with respect to v yields that it is positive if and only if

$$v<\frac{cM\left(M-1+2\beta M\right)}{2\beta(M-1)},$$

thereby confirming Result 2.

Taking the derivative of m^* with respect to M yields

$$2c^2M^2(M-1)^2\left[cM\left(M(1+\beta)-1\right)+v\beta\left(\beta M+2(M-1)\right)\right]+$$
$$+c^2M^2\left((M-1)^2-\beta M\left(\beta M-2(M-1)\right)\right)\sqrt{\psi}+(\psi)^{\frac{3}{2}},$$

which is strictly positive, due to the assumption that $c > v\left(M-1\right)^2/\left(2\beta M^3\right)$—i.e., the assumption that guarantees an interior solution. This confirms Result 3.

Finally, dm_i/dr_j (and dm_i/dr_i) are given by (14). Using the concrete example, we can set $f''(\cdot)$ equal to 0, and insert m^* and r^* determined above into (14). This yields

$$-\frac{cv\beta(M-1)}{f\left(cM\left(M-1+\beta M\right)+\sqrt{\psi}\right)}. \tag{47}$$

Taking the derivative of (47) with respect to v and simplifying, we obtain that the sign of this derivative is given by the sign of

$$-\left((\beta M+M-1)\sqrt{\psi}+c\beta^2M^3+cM(M-1)^2+(M-1)(2cM^2-v(M-1))\right),$$

which is strictly negative for all $c > v(M-1)^2/(2\beta M^3)$. Similarly, differentiating (47) with respect to β and simplifying yields that the sign of this derivative is given by the sign of

$$-\left(\sqrt{\psi} + cM(M-1) + \beta(2cM^2 - v(M-1))\right),$$

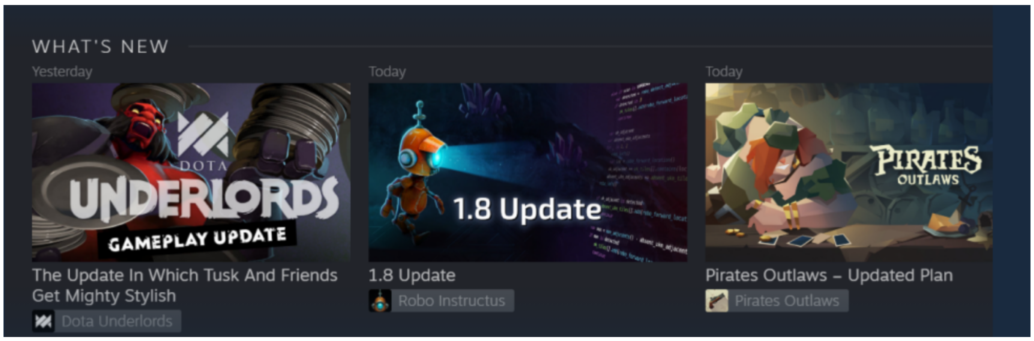
which is again strictly negative for all $c > v(M-1)^2/(2\beta M^3)$.

Online Appendix E: Details on Data & Descriptives

Online Appendix E.1: Details on Data

Prominence of updates to a game owner: The screenshot in Figure 3 shows how prominently updates of games are displayed to a user who already owns the respective game. Users not having the game do not see this.

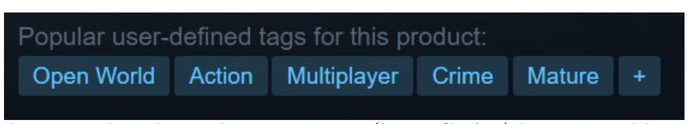
Figure 3: Updates in User Library



Notes: See `https://store.steampowered.com/libraryupdate`

Measuring customer heterogeneity: Users can characterize games with tags. An example can be seen in Figure 4. In total, a game can have up to 20 tags that describe a game beyond the genre and characterize a game in more depth—according to the users. In our sample, there are 435 unique tags.

Figure 4: User-defined Tags of Games on Steam



Notes: The screenshot shows the tags a game (here: GTA 5) has received by users. The first five are prominently shown and correspond to the game's most popular tags, but the total can be up to 20. For an overview, see `https://store.steampowered.com/tag/browse/#global_492`

Our implementation involves the following process. First, we look for the most popular tag in a segment-year (excluding the segment itself). For this, we count the occurrence of tags within each segment-year to end up with a list of the most prominent ones. The number of distinct tags per segment-year varies considerably, with 16 (min) to 427 (max), which shows a varying degree of diversity. In turn, it already suggests that in some markets, games often share certain tags, thus reflecting homogeneity, whereas in other markets, games have many different tags, thereby characterizing them as more heterogeneous. We play the very 'top' tag back to the games via the segment-year, which enables a comparison of the individual tags a game has and the most popular tag of the segment-year. Specifically, we can measure whether the most popular tag of the segment-year is among the game's individual tags. This enables us to define customer heterogeneity as the share of games within a segment-year which do not have the most popular tag among their first five (and thereby most important) user-defined tags.

To illustrate this process with an example, we take the game GTA 5 and its user-defined tags, as displayed in Figure 4. GTA 5 was released in 2015 and primarily classified as an Action genre game. The most popular tag in this segment-year has been Indie (followed by Adventure and Singleplayer). If we compare this with the five most important user-defined tags of GTA 5, we see that none of them match the most popular tag (and even neither the second nor third most popular one). Hence, this game gets assigned a value of one for a dummy variable indicating whether the most popular tag in the respective segment-year is not among the game's user tags. We repeat this process for all the games in our sample and then compute averages per segment-year. A higher average corresponds to a more heterogeneous market as games are more diverse with respect to their user-defined tags, which reflect users' preferences. We did the same for the two most prominent and three most prominent tags and received very similar patterns.

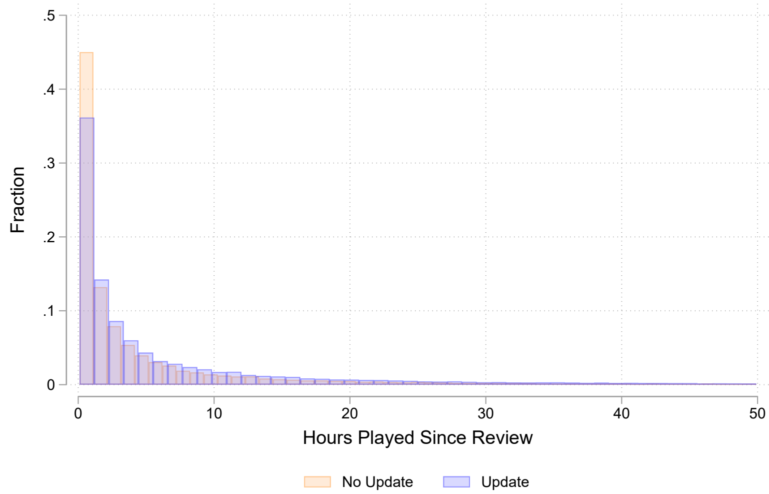
Online Appendix E.2: Details on Descriptives

The following Figures 5 and 6 provide evidence complementing subsection 4.3, which outlines that updates lead to retention by combining information on hours played of reviewers for games on Steam and game updates.

Figure 5 shows the distribution of hours played since writing a review, censored at 50 hours. It suggests that players not experiencing an update of a game after writing a review play less compared to those whose game received an update. It also provides evidence that for many of the players whose game did not receive an update, the additional playing time amounts to less than 1 hour.

In Figure 6, the decision to play at all after writing the review (i.e., share of players returning) and hours played after writing the review are compared within levels of game quality (left column) and by recommendation or non-recommendation of a reviewer (right column). Specifically, for game quality, we divide games into quartiles with respect to the share of positive reviews. We then take the likelihood that players return as well as their hours played after writing the review and compare values by the presence of an update between the time of writing the review and our data collection. We find a statistically significant difference for both outcomes, suggesting that irrespective of game quality, an update in

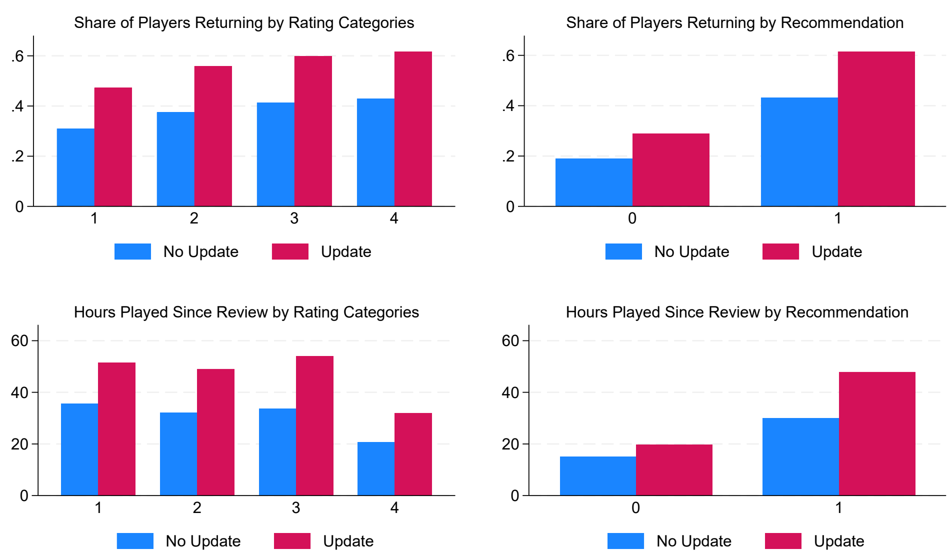
Figure 5: Distribution of Hours Played by Update Presence



Notes: The distribution is capped at 50 hours to ease the interpretation.

between leads to more retention. We repeat this exercise by distinguishing reviews by the binary assessment into a positive and negative judgment. Similarly, we find a statistically significant increase in retention when an update is being made after the review, irrespective of the reviewer's assessment.

Figure 6: Retention Measures by Game and Review Quality



Notes: On the left panel, the 4 cutoffs for the game quality at 25 % - 50 % - 75 % - 100 % are <66 - 66-80 - 80-90 - > 90 as the share of positive reviews. For the right panel, 0 denotes negative reviews, while 1 corresponds to positive ones.

Online Appendix F: Details on Estimation Results

Online Appendix F.1: Table OLS Estimation without Transformation

Table 6: Baseline Estimations without Transformations

	# Own Products				
# Updates	-0.001***			-0.001***	-0.002***
	(0.000)			(0.000)	(0.000)
Price		0.006***		0.007***	0.005***
		(0.001)		(0.001)	(0.001)
$Price^2$		-0.000***		-0.000***	-0.000***
		(0.000)		(0.000)	(0.000)
% w/o Popular Tag 1			0.200***	0.213***	0.418***
			(0.076)	(0.076)	(0.123)
Constant	1.106***	1.039***	0.949***	0.913***	0.677***
	(0.051)	(0.051)	(0.074)	(0.074)	(0.109)
Segment FE	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes
Month FE	Yes	Yes	Yes	Yes	Yes
No. of Obs.	38353	38387	38387	38353	17717

Notes: Column 5 only includes publishers with more than one observation. Heteroskedasticity-robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Online Appendix F.2: Details on Regressions

We are estimating ihs-transformed variables on ihs-transformed variables. For large values of the untransformed variable, the log-transformation and the ihs-transformation are equivalent. Scaling (and hence the size of the untransformed variable) matter as demonstrated by Aihounton and Henningsen (2021), which is why we calculate exact elasticities to gauge size effects.

Our estimating equation, written in terms of the non–IHS transformed variables, is

$$ASinh(m_{ijt}) = \alpha_0+\alpha_1 ASinh(r_{ijt})+\alpha_2 ASinh(v_{ijt})+\alpha_3 ASinh(v_{ijt}^2)+\alpha_4 ASinh(\beta_{jt})+X_{ijt}\theta+\eta_{ijt},$$

where $ASinh$ denotes the arcus sinus hyperbolicus. Omitting subscripts for simplicity and taking the inverse function of $ASinh$ leads to

$$m = Sinh\left[\alpha_0 + \alpha_1 ASinh(r) + \alpha_2 ASinh(v) + \alpha_3 ASinh(v^2) + \alpha_4 ASinh(\beta) + X\theta + \eta\right],$$

where $Sinh$ denotes the sinus hyperbolicus. Taking the partial derivative of m with respect to r (and, similarly, with respect to β) and multiplying by the ratio of r to m we get the elasticity of the number of games with respect to r yields

$$\frac{\partial m}{\partial r}\frac{r}{m} = \frac{r}{\sqrt{1+r^2}}\alpha_1 \coth\left(\alpha_0 + \alpha_1 ASinh(r) + \alpha_2 ASinh(v) + \alpha_3 ASinh(v^2) + \alpha_4 ASinh(\beta) + X\delta + \eta\right),$$

where coth denotes the kotangens hyperbolicus. This formula is equivalent to that provided by Bellemare and Wichmann (2020).

The elasticity of m with respect to v, which involves v's squared term, is

$$\frac{\partial m}{\partial v}\frac{v}{m} = v\left(\frac{\alpha_2}{\sqrt{1+v^2}} + \frac{2v\alpha_3}{\sqrt{1+v^4}}\right)\times$$

$$\times \coth\left(\alpha_0 + \alpha_1 ASinh(r) + \alpha_2 ASinh(v) + \alpha_3 ASinh(v^2) + \alpha_4 ASinh(\beta) + X\delta + \eta\right).$$

To calculate the corresponding elasticities, we use the parameter estimates from Table 3, column (4). The main text displays mean elasticities.

Online Appendix F.3: Further Robustness Tables

Table 7: Baseline Estimations with More Controls

	# Own Products	
# Updates (ihs)	-0.020***	-0.030***
	(0.002)	(0.003)
Price (ihs)	0.288***	0.390***
	(0.020)	(0.033)
$Price^2$ (ihs)	-0.147***	-0.199***
	(0.011)	(0.018)
% w/o Popular Tag 1 (ihs)	0.068**	0.098**
	(0.034)	(0.048)
Prev. # Own Products	0.029***	0.026***
	(0.004)	(0.004)
% Positive Ratings	-0.000***	-0.001***
	(0.000)	(0.000)
# Ratings	0.000	0.000
	(0.000)	(0.000)
Website Dummy	-0.043***	-0.077***
	(0.005)	(0.008)
Size (in KB)	0.000**	0.000***
	(0.000)	(0.000)
Constant	0.920***	0.920***
	(0.033)	(0.044)
Segment FE	Yes	Yes
Year FE	Yes	Yes
Month FE	Yes	Yes
No. of Obs.	21959	12252

Notes: Column 2 only include publishers with more than one observation. The number of observations is lower due to missing information for ratings. Heteroskedasticity-robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Table 8: Baseline Estimations with Other FE

	# Own Products		
# Updates (ihs)	-0.030***	-0.001	-0.031***
	(0.002)	(0.004)	(0.006)
Price (ihs)	0.267***	0.367***	0.373***
	(0.021)	(0.065)	(0.074)
$Price^2$ (ihs)	-0.128***	-0.192***	-0.194***
	(0.012)	(0.036)	(0.041)
% w/o Popular Tag 1 (ihs)	-0.136	0.064	0.042
	(0.513)	(0.056)	(0.062)
% Positive Ratings	-0.000**	0.000	0.001
	(0.000)	(0.000)	(0.000)
# Ratings	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)
Website Dummy	-0.048***	-0.035***	-0.017
	(0.005)	(0.013)	(0.014)
Size (in KB)	0.000***	0.000	0.000
	(0.000)	(0.000)	(0.000)
Constant	1.035**	0.801***	0.877***
	(0.409)	(0.059)	(0.074)
Segment FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
Month FE	Yes	Yes	Yes
Segment x Year FE	Yes	No	No
Publisher FE	No	Yes	Yes
No. of Obs.	21959	11078	8821

Notes: Column 1 additionally includes Segment x Year FE. Columns 2 and 3 include publisher FE. Column 3 excludes video games without an update. Heteroskedasticity-robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Online Appendix F.4: Additional Robustness Checks

Addressing Survivorship Bias: Having data from one period in time may lead to missing firms (and products) that were active in the past but exited until that date. However, for video games the costs to stay in the market are very low as there are e.g. no recurring store fees or maintenance costs. One can see this, for instance, by comparing video games with a release date from 2014 in our data (1,522) to the total of video games published on Steam in 2014 (1,772).[53] This means that the majority of video games from 2014 (about 85 percent) are still in the market. Furthermore, we crawled all games available on Steam exactly one year after the initial data collection in the same way. It shows that more than 94 percent of the games in the estimation sample are still available in August 2023.

Of course, there are shocks affecting the market environment by removals of firms or increasing costs/decreasing revenues through new policies, thus creating larger exits. However, given the low costs to stay in the market, one can presume that only very low-value video games leave the market. These games do not play a crucial role for the market. Finally, we also account for year-specific effects by including release year fixed effects in our regressional analyses.

Publisher vs. Developer: An important distinction is the one between developers and publishers in digital markets, especially for video games where the development and distribution is more expensive. A (too) simple way of describing the roles would be that a developer programs a software, while the publisher is in charge of selling it. However, the actual relationship between the two parties is more complicated and the degree of influence varies. Following the classification on Steam, we take the field "Publisher" on each page of a video game to identify a producer. In case this information is missing, we take the field "Developer". In two-third of the cases for our video games, the two fields coincide and thus it is the same entity. Re-running our analysis with developers instead of publishers leads to qualitatively similar results.

[53] See `https://www.polygon.com/platform/amp/2016/12/1/13807904/steam-releases-2016-growth`.

Online Appendix F.5: Regression Sensitivity Analysis

Motivation: The last explanatory variable motivated by our theoretical model is consumer heterogeneity, approximated by the share of games without the most popular tag. This measure is not endogenous in the theoretical model, but potentially endogenous unobserved heterogeneity might affect both this variable and the error term of our equation of interest. An obvious approach would be to include all relevant factors, and, indeed, we account for segment-specific effects already through segment fixed effects in the baseline regressions along with other fixed effects specifications in the robustness checks (see subsection 5.2). Still, there might be unobserved factors partly determining the customer base. To assess the degree to which this possible correlation might influence our parameter estimates, we use the kinky least square estimator (KLS, Kripfganz and Kiviet, 2021).[54]

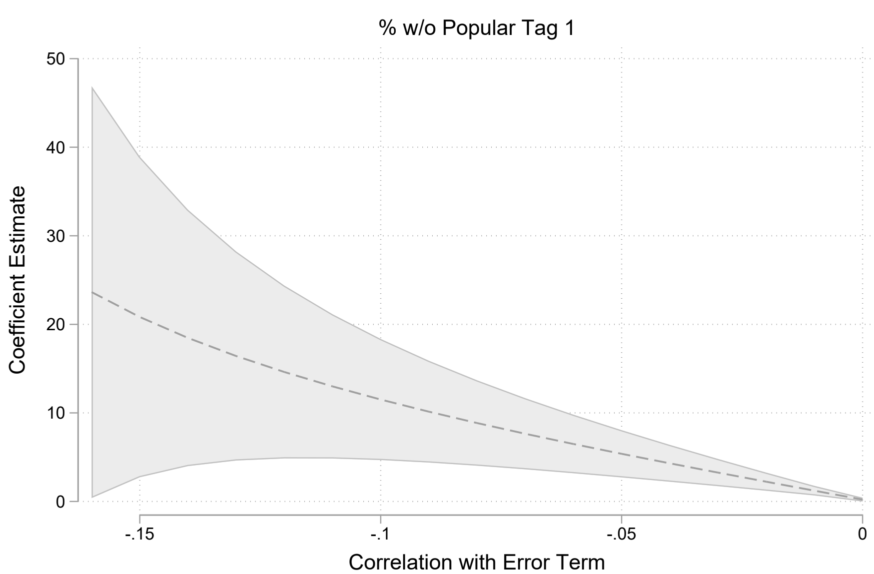
The basic idea behind this estimator is to allow for a range of correlations between the potentially endogenous variable and the error term of the equation of interest. Since KLS assesses the possible bias of the OLS estimates conditional on a given degree of correlation on a grid, this bias is best analyzed by a plot of coefficient estimates generated under varying degrees of correlation. The corresponding search grid over these correlations in principle ranges from -1 to 1, but both extreme values make little intuitive sense in real world applications. Kripfganz and Kiviet (2021) indeed consider an interval of [-.75 and .75] in their empirical examples. Moreover, as Oster (2019, p. 188) as well as Kripfganz and Kiviet (2021) explicitly point out, economic theory should be used to determine whether the correlation of the error term and the endogenous variable is likely to be positive or negative. We reason that the correlation should be assumed to be negative since firms focusing on homogeneous consumer bases may exploit economies of scale more effectively than those with more heterogeneous consumers. Similarly, publishers with lower managerial or operational capability might be less able to handle heterogeneity in consumer preferences and maintain smaller product portfolios. In both cases, these unobserved variables would induce a negative correlation between the error term of our estimating equation and consumer heterogeneity. In our regression sensitivity results we therefore restrict attention to a possible negative correlation.

Results: Combinations of the possible correlations of the endogenous variable "heterogeneity" and the error term as well as the resulting estimate on the coefficient of interest are shown in Figure 7, along with their corresponding confidence bounds.

[54] Another approach to bound the causal effect of an endogenous variable are 'Oster (2019) bounds', which are a special case of the KLS model (i.e, they are nested within KLS). Oster (2019) bounds make, however, strong assumptions on the relative correlation of unobserved and observed heterogeneity, which is why we resort to KLS instead.

The figure displays the coefficient estimates on the heterogeneity variable if we let the correlation with the error term vary in the range [-.16,0]. The dotted line denotes the coefficient estimate from our baseline results in Table 3, column (4), which is the specification used in this robustness check. Following our discussion above, we assume that the correlation is negative and we consider a maximum correlation of -.16 since the coefficient estimate is no longer statistically significant at the 5% level, a direct consequence of the relative imprecision with which the coefficient is estimated given its low variation within and across observations. It is clear, however, that our parameter of interest is positive over the whole range of negative correlations we deem to be economically meaningful, and also statistically significant for modest negative correlations. Clearly, the estimated coefficients for the variable become excessively large even for small values of correlations. The economic effects of the theory-motivated variables are, however, of secondary order since the theoretical model only predicts effect signs and is uninformative regarding effect sizes. Overall, the KLS robustness check indicates that the sign of the coefficient remains positive as in the main model even for higher values of possible negative correlations, indicating that the true coefficient estimates indeed are positive, which is consistent with our theoretical model.

Figure 7: Regression Sensitivity of Heterogeneity



Online Appendix F.6: Details on Instruments

Procedure to match supported languages on Steam with exchange rates:

1. Map language data from Steam to the official names of the main national languages[55] and those that are not official national languages directly to the ISO country codes.
2. Assign currencies (from Western Union) and languages to the ISO country codes and thus have a mapping between currencies and languages.
3. Find and take the most relevant currencies, i.e. currencies of the largest economic areas (based on total GDP, World Bank).
4. Some manual corrections are made, as Steam has some peculiarities for specific languages: For "Spanish - Spain", only Spain is taken into account; for "Spanish - Latin America", only the largest Latin American countries are taken into account; similar procedures for "Portuguese - Portugal" & "Portuguese - Brazil".
5. Correct country code (if unique) or the 3 most relevant country codes (if not unique) are assigned to the language information from Steam.
6. Currency rates (scraped via Yahoo Finance API) are then assigned via these country codes. Currency pair X - USD in the period 2012-2022 are queried.

Details and descriptive statistics on SteamWorks: The diffusion process of the SteamWorks SDK can be roughly described as follows: SteamWorks was already released in 2008. However, at the beginning, it only saw a modest uptake as has been documented by the previously mentioned entries in developer forums. This resonates well with our data, where only 6 publishers had their whole game portfolio equipped with the respective SDK. Instead, this number rose to 803 publishers in the year 2022 alone. The distribution across segments is rather equal with above-average (below average) exceptions of Massively Multiplayer (Sports) that can be due to the smaller size, thus ruling out an industry-specific selection.

[55] See `https://plos.figshare.com/articles/dataset/List_of_official_and_most_spoken_languages_for_each_country_in_the_world_/19622821/1`.

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