

The Engineering of Consumer Experiences under Affect Assimilation and Quality Contrast

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Electronic Companion

This electronic companion (referred with ‘EC’) contains proofs and supplementary analytical results (§EC.1), descriptive statistics (§EC.2), specifications of attribute dissimilarity across contexts (§EC.3), details on the 2-stage estimation procedure (§EC.4), estimation without first-stage correction and controls (§EC.5), and robustness checks (§EC.6).

EC.1. Proofs and Supplementary Analytical Results

Proof of Lemma 1. The proof proceeds by backward induction. To initialize the induction, note that by (8), $V_{T-1}(a_{T-1}, b_{T-1}, \mathcal{J}_{T-1})$ is linear in (a_{T-1}, b_{T-1}) , so that $A_{T-1} = \alpha$, $B_{T-1} = \beta$, and $C_{T-1}(\mathcal{J}_{T-1}) = \max_{j \in \mathcal{J}_{T-1}} q_j$.

Fix $t \leq T - 2$ and suppose that $V_{t+1}(a_{t+1}, b_{t+1}, \mathcal{J}_{t+1}) = A_{t+1}a_{t+1} + B_{t+1}b_{t+1} + C_{t+1}(\mathcal{J}_{t+1})$. Given that $\Phi(u) = u$ and using (4)-(5), we can write (7) as:

$$\begin{aligned} V_t(b_t, a_t, \mathcal{J}_t) &= \max_{j \in \mathcal{J}_t} q_j + \alpha a_t + \beta b_t \\ &\quad + (q_j + \alpha a_t + \beta b_t) \delta A_{t+1} \lambda_a \\ &\quad + \delta A_{t+1} (1 - \lambda_a) a_t \\ &\quad + \delta B_{t+1} (\lambda_b q_j + (1 - \lambda_b) b_t) \\ &\quad + \delta C_{t+1}(\mathcal{J}_t \setminus \{j\}) \\ &= \max_{j \in \mathcal{J}_t} \{q_j (1 + \delta \lambda_a A_{t+1} + \delta \lambda_b B_{t+1}) + \delta C_{t+1}(\mathcal{J}_t \setminus \{j\})\} \\ &\quad + [\alpha + \delta \gamma A_{t+1}] a_t + [\beta + \delta \beta \lambda_a A_{t+1} + \delta (1 - \lambda_b) B_{t+1}] b_t \end{aligned}$$

in which $\gamma := \alpha \lambda_a + (1 - \lambda_a)$. Hence, $V_t(a_t, b_t, \mathcal{J}_t) = A_t a_t + B_t b_t + C_t(\mathcal{J}_t)$, in which

$$A_t = \alpha + \delta \gamma A_{t+1},$$

$$B_t = \beta + \delta \beta \lambda_a A_{t+1} + \delta (1 - \lambda_b) B_{t+1}, \text{ and}$$

$$C_t(\mathcal{J}_t) = \max_{j \in \mathcal{J}_t} \left\{ \delta C_{t+1}(\mathcal{J}_t \setminus \{j\}) + \delta^{T-t} w_t q_j \right\},$$

with $w_t := \delta^{-(T-t)} [1 + \delta \lambda_a A_{t+1} + \delta \lambda_b B_{t+1}]$. Solving the first two recursive equations, we obtain that, for $t = 0, \dots, T - 1$, A_t and B_t are as defined in (12) and w_t is as defined in (13). $\square$

Proof of Theorem 1. By Lemma 1, $V_t(a_t, b_t, \mathcal{J}_t)$ can be expressed as $A_t a_t + B_t b_t + C_t(\mathcal{J}_t)$. Because only the constant term $C_t(\mathcal{J}_t)$ depends on the choice set, the optimal sequence is independent of the state (a_t, b_t)

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for all t , including the initial state (a_0, b_0) . Furthermore, by (12), $C_0(\mathcal{J})$ can be expressed directly as a linear function of the qualities of the selected activities. In particular,

$$C_0(\mathcal{J}) = \delta^T \max_{\{j_0, \dots, j_{T-1}\} \subseteq \mathcal{J}} \sum_{t=0}^{T-1} w_t q_{j_t}. \quad (\text{EC.1})$$

□

Proof of Proposition 1. Using (13), we extend w_t to a continuous function $w(t) := F\delta^{-(T-t)} + G(1 - \lambda_b)^{T-t} - H\gamma^{T-t}$, for some constants $F := \frac{1-\delta(1-\lambda_a)}{1-\delta\gamma} \left(1 + \frac{\beta\delta\lambda_b}{1-\delta(1-\lambda_b)}\right)$, $G := \frac{\beta\lambda_b(\lambda_a-\lambda_b)}{(1-\lambda_b)(1-\delta(1-\lambda_b))(\gamma+\lambda_b-1)}$, and $H := \frac{\alpha\lambda_a}{\gamma(1-\delta\gamma)} \left(1 + \frac{\beta\lambda_b}{\gamma+\lambda_b-1}\right)$. For any interior stationary point ($w'(t) = 0$), the second derivative equals

$$\begin{aligned} w''(t) \Big|_{w'(t)=0} &= F\delta^{-(T-t)}(\ln \delta)^2 + G(1 - \lambda_b)^{T-t}(\ln(1 - \lambda_b))^2 - H\gamma^{T-t}(\ln \gamma)^2 \Big|_{w'(t)=0} \\ &= G(1 - \lambda_b)^{T-t} \ln(1 - \lambda_b) \ln(\delta(1 - \lambda_b)) - H\gamma^{T-t} \ln \gamma \ln(\delta\gamma) := K(t). \end{aligned}$$

We show that $K(t)$ switches sign at most once as t increases, and if it does, the change is from negative to positive if and only if $\gamma + \lambda_b - 1 > -\beta\lambda_b$. Its derivative, $K'(t)|_{K(t)=0}$, equal to $G(1 - \lambda_b)^{T-t}(\ln(1 - \lambda_b))^2 \ln(\delta(1 - \lambda_b)) + H\gamma^{T-t}(\ln \gamma)^2 \ln(\delta\gamma)|_{K(t)=0} = H\gamma^{T-t} \ln \gamma \ln(\delta\gamma) \ln(\gamma/(1 - \lambda_b))$, is indeed positive if and only if $\gamma + \lambda_b - 1 > -\beta\lambda_b$. Hence, when $w'(t) = 0$, $w''(t)$ crosses zero at most once, and if it does, the crossing is from below if and only if $\gamma + \lambda_b - 1 > -\beta\lambda_b$. In other words, $w(t)$ is first pseudo-concave and then pseudo-convex if $\gamma + \lambda_b - 1 > -\beta\lambda_b$ and *vice versa* otherwise. Consequently, $(w_0, \dots, w_{T-1})$ has at most one interior maximum and one interior minimum, and the former precedes the latter if and only if $\gamma + \lambda_b - 1 > -\beta\lambda_b$, or equivalently, if and only if $\lambda_b(1 + \beta) \geq \lambda_a(1 - \alpha)$. Finally, by Theorem 1, $(q_{j_0^*}, \dots, q_{j_{T-1}^*})$ has the same shape as $(w_0, \dots, w_{T-1})$. □

Proof of Corollary 1. When $\alpha = 0$, γ simplifies to $\gamma = (1 - \lambda_a)$. Accordingly, the constants in Proposition 1 simplify to $F = 1 + \frac{\beta\delta\lambda_b}{1-\delta(1-\lambda_b)}$, $G = -\frac{\beta\lambda_b}{(1-\lambda_b)(1-\delta(1-\lambda_b))}$, and $H = 0$. Similar to Proposition 1, we extend w_t to a continuous function. When $w'(t) = 0$,

$$\begin{aligned} w''(t) \Big|_{w'(t)=0} &= F\delta^{-(T-t)}(\ln \delta)^2 + G(1 - \lambda_b)^{T-t}(\ln(1 - \lambda_b))^2 \Big|_{w'(t)=0} \\ &= G(1 - \lambda_b)^{T-t} \ln(1 - \lambda_b) [\ln(\delta(1 - \lambda_b))] \\ &= (1 - \lambda_b)^{T-t} \frac{\beta\lambda_b \ln(1 - \lambda_b)}{(1 - \lambda_b)} \left(\frac{-\ln(\delta(1 - \lambda_b))}{1 - \delta(1 - \lambda_b)} \right) \geq 0. \end{aligned}$$

Accordingly, $w(t)$ is pseudo-convex, which implies that $(w_0, \dots, w_{T-1})$ has no interior maximum, i.e., $(w_0, \dots, w_{T-1})$ is U-shaped. By Theorem 1, $(q_{j_0^*}, \dots, q_{j_{T-1}^*})$ has the same shape as $(w_0, \dots, w_{T-1})$.

Moreover, $w'(t) \geq 0$ if and only if

$$\left(1 + \frac{\beta\delta\lambda_b}{1 - \delta(1 - \lambda_b)}\right) \ln \delta \delta^{-(T-t)} + \left(\frac{\beta\lambda_b}{(1 - \lambda_b)(1 - \delta(1 - \lambda_b))}\right) \ln(1 - \lambda_b)(1 - \lambda_b)^{T-t} \geq 0. \quad (\text{EC.2})$$

Consider the following three cases, conditioned on the value of δ :

- When $\delta \in [\min\{1, (1 - \lambda_b(1 + \beta))^{-1}\}, \max\{1, (1 - \lambda_b(1 + \beta))^{-1}\}]$, both terms in the left-hand side of (EC.2) are positive. In this case, $w'(t) \geq 0$, i.e., the optimal sequence is a crescendo.
- Consider now the case where $\delta < \min\{1, (1 - \lambda_b(1 + \beta))^{-1}\}$. Because $w'(T-1)(1 - \delta(1 - \lambda_b))$, equal to $\ln \delta / \delta - \ln \delta(1 - \lambda_b(1 + \beta)) + \beta\lambda_b \ln(1 - \lambda_b)$, is increasing in δ in this case and positive when $\delta = \min\{1, (1 - \lambda_b(1 + \beta))^{-1}\}$, there exists a threshold value $\delta_{\alpha=0}^{\cup}$ such that $w'(T-1)(1 - \delta(1 - \lambda_b)) \leq 0$ if $\delta \leq \delta_{\alpha=0}^{\cup}$ and

$w'(T-1)(1-\delta(1-\lambda_b)) > 0$ if $\delta \in (\delta_{\alpha=0}^U, \min\{1, (1-\lambda_b(1+\beta))^{-1}\})$. Thus, the optimal sequence is a decrescendo if $\delta \leq \delta_{\alpha=0}^U$ and a U-shape if $\delta \in (\delta_{\alpha=0}^U, \min\{1, (1-\lambda_b(1+\beta))^{-1}\})$; given that the stationary point of the U-shape only depends on the time until the end of the horizon per Proposition 3, the minimum of the U-shape is attained only if the experience is long enough (greater than, say, $T_{\alpha=0}^U(\delta)$ periods); otherwise, the U-shape degenerates into a crescendo.

• Finally, consider the case where $\delta > \max\{1, (1-\lambda_b(1+\beta))^{-1}\}$. Using a similar argument to the one above, one can show that $w'(T-1)(1-\lambda_b)$ is decreasing in δ in this case and equal to zero when $\delta = (1-\delta(1-\lambda_b))^{-1}$, therefore establishing that $w'(T-1) \geq 0$ in this case. Hence, when $\delta \geq \max\{1, (1-\lambda_b(1+\beta))^{-1}\}$, the optimal sequence is a U-shape, which degenerates into a crescendo only if the time horizon is too short.

By Theorem 1, $(q_{j_0}^*, \dots, q_{j_{T-1}}^*)$ has the same shape as $(w_0, \dots, w_{T-1})$. $\square$

Proof of Corollary 2. When $\beta = 0$, the constants in Proposition 1 simplify to $F = \frac{1-\delta(1-\lambda_a)}{1-\delta\gamma}$, $G = 0$, and $H = \frac{\alpha\lambda_a}{\gamma(1-\delta\gamma)}$. Similar to Proposition 1, we extend w_t to a continuous function. When $w'(t) = 0$,

$$\begin{aligned} w''(t) \Big|_{w'(t)=0} &= F\delta^{-(T-t)}(\ln \delta)^2 - H\gamma^{T-t}(\ln \gamma)^2 \Big|_{w'(t)=0} = -H\gamma^{T-t} \ln \gamma [\ln(\delta\gamma)] \\ &= \gamma^{T-t} \frac{\alpha\lambda_a \ln \gamma}{\gamma} \left(\frac{-\ln(\delta\gamma)}{1-\delta\gamma} \right), \end{aligned}$$

which is nonpositive if and only if $\gamma \leq 1$, i.e., if and only if $\alpha \leq 1$. Accordingly, $w(t)$ is pseudo-convex if $\alpha > 1$ and pseudo-concave if $\alpha < 1$. This implies that $(w_0, \dots, w_{T-1})$ has no interior maximum if $\alpha > 1$ and no interior minimum if $\alpha < 1$.

Moreover, $w'(t) \geq 0$ if and only if

$$\left(\frac{1-\delta(1-\lambda_a)}{1-\delta\gamma} \right) \ln \delta \delta^{-(T-t)} + \left(\frac{\alpha\lambda_a}{\gamma(1-\delta\gamma)} \right) \ln \gamma \gamma^{T-t} \geq 0. \quad (\text{EC.3})$$

Consider first the case where $\alpha < 1$. Note that $1 \leq (1-\lambda_a(1-\alpha))^{-1} \leq (1-\lambda_a)^{-1}$. Consider the following cases, conditioned on the value of δ .

- When $\delta \leq 1$, both terms in the left-hand side of (EC.3) are nonpositive; hence $w'(t) \leq 0$ for all t .
- When $\delta \geq (1-\lambda_a)^{-1}$, both terms in the left-hand side of (EC.3) are nonnegative; hence $w'(t) \geq 0$ for all t .

• Consider next the case where $\delta \in (1, (1-\lambda_a)^{-1})$. The derivative of $w'(T-1)(1-\delta\gamma)$ with respect to δ , equal to $(1-\delta(1-\lambda_a)-\ln \delta)/\delta^2$, is decreasing in δ , positive when $\delta = 1$, and negative when $\delta = (1-\lambda_a)^{-1}$; it thus crosses zero once as δ increases from 1 to $(1-\lambda_a)^{-1}$, and the crossing is from above. Accordingly, $w'(T-1)(1-\delta\gamma)$ is first increasing and then decreasing in δ and thus crosses zero at most twice as δ increases from 1 to $(1-\lambda_a)^{-1}$; one such crossing turns out to be when $\delta = 1/\gamma$. Therefore, there exists a threshold value $\delta_{\beta=0}^\cap \in (1, (1-\lambda_a)^{-1})$ such that $w'(T-1)$ is negative when $\delta \in (1, \delta_{\beta=0}^\cap)$ and nonnegative when $\delta \in [\delta_{\beta=0}^\cap, (1-\lambda_a)^{-1})$. Accordingly, the optimal sequence is an inverse U-shape when $\delta \in (1, \delta_{\beta=0}^\cap)$, which degenerates into a decrescendo only if the time horizon is too short (say, shorter than a threshold $T_{\beta=0}^\cap(\delta)$), and it is a crescendo when $\delta \in [\delta_{\beta=0}^\cap, (1-\lambda_a)^{-1})$.

Consider next the case where $\alpha > 1$. Note that $(1-\lambda_a(1-\alpha))^{-1} \leq 1 \leq (1-\lambda_a)^{-1}$. Consider the following cases, conditioned on the value of δ :

• When $1 \leq \delta \leq (1 - \lambda_a)^{-1}$, both terms in the left-hand side of (EC.3) are nonpositive; hence $w'(t) \leq 0$ for all t .

• Consider next the case where $\delta < 1$. Because $w'(T-1)(1 - \delta\gamma)$, equal to $\ln \delta / \delta - \ln \delta(1 - \lambda_a) + \alpha \lambda_a \ln \gamma$, is increasing in δ when $\delta \leq 1$, positive when $\delta = 1$, and tending towards $-\infty$ when $\delta \rightarrow 0$, it crosses zero once when δ increases from 0 to 1, and the crossing is from below; the crossing point turns out to be when $\delta = 1/\gamma$. Therefore, $w'(T-1)$ is nonpositive for all $\delta \in [0, 1]$. Accordingly, the optimal sequence is a decrescendo when $\delta < 1$.

• Finally, consider the case where $\delta > (1 - \lambda_a)^{-1}$. Because $w'(T-1)(1 - \delta\gamma)$ is decreasing in δ when $\delta \geq (1 - \lambda_a)^{-1}$, positive when $\delta = (1 - \lambda_a)^{-1}$, and tending towards $-\infty$ when $\delta \rightarrow \infty$, there exists a threshold value $\delta_{\beta=0}^U > (1 - \lambda_a)^{-1}$ such that $w'(T-1)$ is nonpositive when $\delta \in ((1 - \lambda_a)^{-1}, \delta_{\beta=0}^U]$ and positive when $\delta > \delta_{\beta=0}^U$. Accordingly, the optimal sequence is a decrescendo when $\delta \in ((1 - \lambda_a)^{-1}, \delta_{\beta=0}^U]$ and a U-shape when $\delta > \delta_{\beta=0}^U$, which degenerates into a crescendo only if the time horizon is too short (say, shorter than a threshold $T_{\beta=0}^U(\delta)$).

By Theorem 1, $(q_{j_0}^*, \dots, q_{j_{T-1}}^*)$ has the same shape as $(w_0, \dots, w_{T-1})$. $\square$

LEMMA EC.1. *For any $0 \leq a \leq 1$ and b , the function $F(\gamma) = \ln(\gamma)(ab - (1 - \gamma)^2)/(1 - \gamma) + (\gamma - 1 + a)(\gamma - 1 + b)/\gamma$ is convex over $\gamma \in [1 - a, 1 - b]$.*

Proof. We have:

$$F''(\gamma) = \frac{1}{(\gamma - 1)^3} \left(\frac{\gamma - 1}{\gamma^3} (-2a(\gamma - 1)^2 + ab(2 + 5\gamma(\gamma - 1) + (\gamma - 1)^2(2 - 2b + \gamma + \gamma^2)) - 2ab \ln(\gamma)) \right).$$

We seek to show that $F''(\gamma) \geq 0$ for all $\gamma \geq 0$, $\max\{0, 1 - \gamma\} \leq a \leq 1$ and $b \leq 1 - \gamma$. Because $F''(\gamma)$ is linear in b , it is minimized when either $b = 1 - \gamma$ or when $b \rightarrow -\infty$. That is, $F''(\gamma) \geq \min\{\lim_{b \rightarrow -\infty} b f_1(a, \gamma), f_2(a, \gamma)\}$ in which

$$\begin{aligned} f_1(a, \gamma) &= \frac{1}{(\gamma - 1)^3} \left(\frac{\gamma - 1}{\gamma^3} (a(2 + 5\gamma(\gamma - 1) - 2(\gamma - 1)^2) - 2a \ln(\gamma)) \right), \\ f_2(a, \gamma) &= \frac{1}{(\gamma - 1)^2} \left(\frac{\gamma - 1}{\gamma^2} (a(3 - 5\gamma) + (\gamma - 1)(3 + \gamma)) + 2a \ln(\gamma) \right). \end{aligned}$$

Because both $f_1(a, \gamma)$ and $f_2(a, \gamma)$ are linear in a , they are minimized when either $a = 0$ or $a = 1$. That is, $F''(\gamma) \geq \min\{\lim_{b \rightarrow -\infty} b f_1(0, \gamma), \lim_{b \rightarrow -\infty} b f_1(1, \gamma), f_2(0, \gamma), f_2(1, \gamma)\}$. Finally, observe that $f_1(0, \gamma) = -2/\gamma^3 \leq 0$, $f_1(1, \gamma) = (1 - 4\gamma + 3\gamma^2 - 2\gamma^2 \ln(\gamma))/((\gamma - 1)^3 \gamma^2) \leq 0$, $f_2(0, \gamma) = (3 + \gamma)/\gamma^2 \geq 0$, and $f_2(1, \gamma) = (3 - 4\gamma + \gamma^2 + 2\gamma \ln(\gamma))/((\gamma - 1)^2 \gamma) \geq 0$. Combining these results shows that $F''(\gamma) \geq 0$. $\square$

Proof of Corollary 3. When $\delta = 1$, the constants in Proposition 1 simplify to $F = \frac{\lambda_a}{1 - \gamma}(1 + \beta)$, $G = \frac{\beta(\lambda_a - \lambda_b)}{(1 - \lambda_b)(\gamma + \lambda_b - 1)}$, and $H = \frac{\alpha \lambda_a}{\gamma(1 - \gamma)} \left(1 + \frac{\beta \lambda_b}{\gamma + \lambda_b - 1} \right)$. Similar to Proposition 1, we extend w_t to a continuous function $w(t) = F + G(1 - \lambda_b)^{T-t} - H\gamma^{T-t}$. When $w'(t) = 0$,

$$\begin{aligned} w''(t) \Big|_{w'(t)=0} &= G(1 - \lambda_b)^{T-t} (\ln(1 - \lambda_b))^2 - H\gamma^{T-t} (\ln \gamma)^2 \Big|_{w'(t)=0} \\ &= G(1 - \lambda_b)^{T-t} \ln(1 - \lambda_b) [\ln(1 - \lambda_b) - \ln \gamma] \\ &= (1 - \lambda_b)^{T-t} \frac{\beta \ln(1 - \lambda_b)}{(1 - \lambda_b)} \left(\frac{\ln(1 - \lambda_b) - \ln \gamma}{1 - \lambda_b - \gamma} \right) (\lambda_b - \lambda_a), \end{aligned}$$

which is nonnegative if and only if $\lambda_b \geq \lambda_a$. Hence, $w(t)$ is pseudo-concave when $\lambda_a > \lambda_b$ and pseudo-convex when $\lambda_a < \lambda_b$. This implies that $(w_0, \dots, w_{T-1})$ has no interior minimum if $\lambda_a > \lambda_b$ and no interior maximum if $\lambda_a < \lambda_b$. By Theorem 1, $(q_{j_0^*}, \dots, q_{j_{T-1}^*})$ has the same shape as $(w_0, \dots, w_{T-1})$.

Moreover, $w'(t) \geq 0$ if and only if

$$\left(\frac{\alpha \lambda_a}{\gamma(1-\gamma)} \left(1 + \frac{\beta \lambda_b}{\gamma + \lambda_b - 1} \right) \right) \gamma^{T-t} \ln \gamma - \left(\frac{\beta(\lambda_a - \lambda_b)}{(1-\lambda_b)(\gamma + \lambda_b - 1)} \right) (1-\lambda_b)^{T-t} \ln(1-\lambda_b) \geq 0. \quad (\text{EC.4})$$

Consider first the case where $\lambda_a > \lambda_b$. Note that $1 - \lambda_a < 1 - \lambda_b \leq 1 - \lambda_b(1 + \beta)$ and $\gamma \geq 1 - \lambda_a$. Consider the following cases, conditioned on the value of γ :

- When $\gamma \geq 1 - \lambda_b(1 + \beta)$, both terms in the left-hand side of (EC.4) are nonpositive; hence, $w'(t) \leq 0$ for all t .

- Consider next the case where $\gamma < 1 - \lambda_b(1 + \beta)$. Consider the function $w'(T-1)(\gamma - 1 + \lambda_b)$, equal to $\ln(1 - \lambda_b)\beta(\lambda_b - \lambda_a) + (\gamma - 1 + \lambda_a)(\gamma - 1 + \lambda_b(1 + \beta)) \ln \gamma / (1 - \gamma)$. Its derivative with respect to γ , equal to $f(\gamma) := \ln \gamma (\lambda_a \lambda_b (1 + \beta) - (1 - \gamma)^2) / (1 - \gamma)^2 + (\gamma - 1 + \lambda_a)(\gamma - 1 + \lambda_b(1 + \beta)) / (\gamma(1 - \gamma))$, is positive when $\gamma = 1 - \lambda_a$ and negative when $\gamma = 1 - \lambda_b(1 + \beta)$. By Lemma EC.1, $f(\gamma)(1 - \gamma)$ is convex when $\gamma \in [1 - \lambda_a, 1 - \lambda_b(1 + \beta)]$, which implies that it crosses zero at most twice, first from above and then from below as γ increases. Because $\gamma = 1$ is one root, $f(\gamma)(1 - \gamma)$ crosses zero at least once. Therefore, $f(\gamma)$ crosses at most zero once, and the crossing, if any, is from above as γ increases. That is, $w'(T-1)(\gamma - 1 + \lambda_b)$ is either monotone or first increasing and then decreasing in γ when $\gamma \in [1 - \lambda_a, 1 - \lambda_b(1 + \beta)]$. Because $w'(T-1)(\gamma - 1 + \lambda_b)$ takes the same negative value when $\gamma = 1 - \lambda_a$ and $\gamma = 1 - \lambda_b(1 + \beta)$, it cannot be monotone between these values, i.e., it is first increasing and then decreasing. As a result, $w'(T-1)(\gamma - 1 + \lambda_b)$ is either always negative or it crosses zero twice as γ increases, first from below and then from above. Because $w'(T-1)(\gamma - 1 + \lambda_b)$ has a root at $\gamma = 1 - \lambda_b$, which lies in the range $[1 - \lambda_a, 1 - \lambda_b(1 + \beta)]$, $w'(T-1)(\gamma - 1 + \lambda_b)$ cannot always be negative, i.e., it crosses zero twice, first from below and then from above as γ increases on $[1 - \lambda_a, 1 - \lambda_b(1 + \beta)]$. Therefore, $w'(T-1)$ crosses zero once, and the crossing is from above as γ increases on $[1 - \lambda_a, 1 - \lambda_b(1 + \beta)]$. Accordingly, there exists a threshold $\gamma_{\delta=1}^\cap$ such that $w'(T-1) \geq 0$ for $\gamma \in [1 - \lambda_a, \gamma_{\delta=1}^\cap)$ and $w'(T-1) < 0$ for $\gamma \in (\gamma_{\delta=1}^\cap, 1 - \lambda_b(1 + \beta))$. Therefore, the optimal sequence is a crescendo when $\gamma \in (1 - \lambda_a, \gamma_{\delta=1}^\cap]$ and an inverted U-shape, which degenerates into a decrescendo if the time horizon is too short (say, shorter than a threshold $T_{\delta=1}^\cap(\gamma)$), and it is a crescendo when $\gamma \in (\gamma_{\delta=1}^\cap, 1 - \lambda_b(1 + \beta))$.

Consider next the case where $\lambda_a < \lambda_b$. Note that $1 - \lambda_b \leq \min\{1 - \lambda_a, 1 - \lambda_b(1 + \beta)\}$. Because $\lambda_a < \lambda_b$, $\gamma > 1 - \lambda_b$. Consider the following cases, conditioned on the value of γ :

- When $\gamma \leq 1 - \lambda_b(1 + \beta)$, both terms in the left-hand side of (EC.4) are nonnegative; hence, $w'(t) \geq 0$ for all t .

- Consider next the case where $\gamma > 1 - \lambda_b(1 + \beta)$. Because $w'(T-1)(\gamma - 1 + \lambda_b)(1 - \gamma) / \ln \gamma$, equal to $\ln(1 - \lambda_b)\beta(\lambda_b - \lambda_a)(1 - \gamma) / \ln \gamma + (\gamma - 1 + \lambda_a)(\gamma - 1 + \lambda_b(1 + \beta))$, is convex in γ (being the sum of two convex functions), it crosses zero at most twice, first from above and then from below as γ increases. One root is in fact $\gamma = 1 - \lambda_b$. Moreover, the function is nonpositive given that $\gamma \in [\min\{1 - \lambda_a, 1 - \lambda_b(1 + \beta)\}, \max\{1 - \lambda_a, 1 - \lambda_b(1 + \beta)\}]$. Accordingly, when $\gamma > 1 - \lambda_b$, $w'(T-1)$ crosses zero at most twice, first from below and then from above, and it is nonnegative when $\gamma \in [\min\{1 - \lambda_a, 1 - \lambda_b(1 + \beta)\}, \max\{1 - \lambda_a, 1 - \lambda_b(1 + \beta)\}]$.

Because $\lim_{\gamma \rightarrow \infty} \gamma^2 \ln \gamma / (1 - \gamma) = -\infty$, there exists a threshold $\gamma_{\delta=1}^\cup > \max\{1 - \lambda_a, 1 - \lambda_b(1 + \beta)\}$ such that $w'(T - 1)$ is positive for $\gamma \in (1 - \lambda_b(1 + \beta), \gamma_{\delta=1}^\cup)$ and nonpositive when $\gamma \geq \gamma_{\delta=1}^\cup$. Accordingly, the optimal sequence is a U-shape when $\gamma \in (1 - \lambda_b(1 + \beta), \gamma_{\delta=1}^\cup)$, which degenerates into a crescendo if the time horizon is too short (say, shorter than a threshold $T_{\delta=1}^\cup(\gamma)$), and it is a decrescendo when $\gamma \geq \gamma_{\delta=1}^\cup$.

By Theorem 1, $(q_{j_0^*}, \dots, q_{j_{T-1}^*})$ has the same shape as $(w_0, \dots, w_{T-1})$. $\square$

LEMMA EC.2. $\hat{V}_t(a_t, b_t, \mathbf{q}_t)$ is increasing in a_t , decreasing in b_t , concave in a_t and in b_t with increasing differences in (a_t, b_t) if $\phi(x)$ is decreasing and convex in a_t and in b_t with decreasing differences in (a_t, b_t) if $\phi(x)$ is increasing.

Proof. The proof is by induction. Fix the sequence $\mathbf{q}_0$. To initialize the induction, note that, because $\alpha \geq 0 \geq \beta$, $\hat{V}_{T-1}(a_{T-1}, b_{T-1}, \mathbf{q}_{T-1}) = q_{T-1} + \alpha a_{T-1} + \beta b_{T-1}$ is linear increasing in $(a_{T-1}, -b_{T-1})$. Suppose that, for any (a_{t+1}, b_{t+1}) , $\hat{V}_{t+1}(a_{t+1}, b_{t+1}, \mathbf{q}_{t+1})$ is increasing in a_{t+1} , decreasing in b_{t+1} , concave in (a_{t+1}, b_{t+1}) with increasing differences if $\phi'(x) \leq 0$ for all x and convex in (a_{t+1}, b_{t+1}) with decreasing differences if $\phi'(x) \geq 0$ for all x .

We first show that $V_t(a_t, b_t, \mathbf{q}_t)$ is increasing in a_t :

$$\begin{aligned} & \frac{\partial \hat{V}_t(a_t, b_t, \mathbf{q}_t)}{\partial a_t} \\ &= \alpha + \delta \Phi(q_t + \alpha a_t + \beta b_t)(1 - \lambda_a) \frac{\partial \hat{V}_{t+1}\left((1 - \lambda_a)a_t + \lambda_a, (1 - \lambda_b)b_t + \lambda_b q_t, \mathbf{q}_{t+1}\right)}{\partial a_{t+1}} \\ & \quad + \bar{\Phi}(q_t + \alpha a_t + \beta b_t)(1 - \lambda_a) \frac{\partial \hat{V}_{t+1}\left((1 - \lambda_a)a_t, (1 - \lambda_b)b_t + \lambda_b q_t, \mathbf{q}_{t+1}\right)}{\partial a_{t+1}} \\ & \quad + \delta \alpha \phi(q_t + \alpha a_t + \beta b_t) \\ & \quad \times \left(\hat{V}_{t+1}\left((1 - \lambda_a)a_t + \lambda_a, (1 - \lambda_b)b_t + \lambda_b q_t, \mathbf{q}_{t+1}\right) - \hat{V}_{t+1}\left((1 - \lambda_a)a_t, (1 - \lambda_b)b_t + \lambda_b q_t, \mathbf{q}_{t+1}\right) \right) \\ & \geq 0, \end{aligned}$$

in which the inequality is by the induction hypothesis.

Next, we show that $\hat{V}_t(a_t, b_t, \mathbf{q}_t)$ is decreasing in b_t :

$$\begin{aligned} & \frac{\partial \hat{V}_t(a_t, b_t, \mathbf{q}_t)}{\partial b_t} \\ &= \beta + \delta \Phi(q_t + \alpha a_t + \beta b_t)(1 - \lambda_b) \frac{\partial \hat{V}_{t+1}\left((1 - \lambda_a)a_t + \lambda_a, (1 - \lambda_b)b_t + \lambda_b q_t, \mathbf{q}_{t+1}\right)}{\partial b_{t+1}} \\ & \quad + \delta \bar{\Phi}(q_t + \alpha a_t + \beta b_t)(1 - \lambda_b) \frac{\partial \hat{V}_{t+1}\left((1 - \lambda_a)a_t, (1 - \lambda_b)b_t + \lambda_b q_t, \mathbf{q}_{t+1}\right)}{\partial b_{t+1}} \\ & \quad + \delta \beta \phi(q_t + \alpha a_t + \beta b_t) \\ & \quad \times \left(\hat{V}_{t+1}\left((1 - \lambda_a)a_t + \lambda_a, (1 - \lambda_b)b_t + \lambda_b q_t, \mathbf{q}_{t+1}\right) - \hat{V}_{t+1}\left((1 - \lambda_a)a_t, (1 - \lambda_b)b_t + \lambda_b q_t, \mathbf{q}_{t+1}\right) \right) \\ & \leq 0, \end{aligned}$$

in which the inequality is by the induction hypothesis and because $V_{t+1}(a_{t+1}, b_{t+1}, \mathbf{q}_{t+1})$ is increasing in a_{t+1} .

Next, we show that $\hat{V}_t(a_t, b_t, \mathbf{q}_t)$ is concave in a_t if $\phi'(x) \leq 0$ for all x and convex if $\phi'(x) \geq 0$ for all x :

$$\begin{aligned}
& \frac{\partial^2 \hat{V}_t(a_t, b_t, \mathbf{q}_t)}{\partial a_t^2} \\
&= \delta \Phi(q_t + \alpha a_t + \beta b_t)(1 - \lambda_a)^2 \frac{\partial^2 \hat{V}_{t+1}\left((1 - \lambda_a)a_t + \lambda_a, (1 - \lambda_b)b_t + \lambda_b q_t, \mathbf{q}_{t+1}\right)}{\partial a_{t+1}^2} \\
&+ \delta \bar{\Phi}(q_t + \alpha a_t + \beta b_t)(1 - \lambda_a)^2 \frac{\partial^2 \hat{V}_{t+1}\left((1 - \lambda_a)a_t, (1 - \lambda_b)b_t + \lambda_b q_t, \mathbf{q}_{t+1}\right)}{\partial a_{t+1}^2} \\
&+ \delta \alpha^2 \phi'(q_t + \alpha a_t + \beta b_t) \\
&\quad \times \left(\hat{V}_{t+1}\left((1 - \lambda_a)a_t + \lambda_a, (1 - \lambda_b)b_t + \lambda_b q_t, \mathbf{q}_{t+1}\right) - \hat{V}_{t+1}\left((1 - \lambda_a)a_t, (1 - \lambda_b)b_t + \lambda_b q_t, \mathbf{q}_{t+1}\right) \right) \\
&+ 2\delta \alpha(1 - \lambda_a)\phi(q_t + \alpha a_t + \beta b_t) \\
&\quad \times \left(\frac{\partial \hat{V}_{t+1}\left((1 - \lambda_a)a_t + \lambda_a, (1 - \lambda_b)b_t + \lambda_b q_t, \mathbf{q}_{t+1}\right)}{\partial a_{t+1}} - \frac{\partial \hat{V}_{t+1}\left((1 - \lambda_a)a_t, (1 - \lambda_b)b_t + \lambda_b q_t, \mathbf{q}_{t+1}\right)}{\partial a_{t+1}} \right) \\
&\leq 0
\end{aligned}$$

if $\phi'(x) \leq 0$ for all x and the inequality is reversed if $\phi'(x) \geq 0$ for all x , in which we used the induction hypothesis and the fact that $V_{t+1}(a_{t+1}, b_{t+1}, \mathbf{q}_{t+1})$ is increasing in a_{t+1} .

Next, we show that $\hat{V}_t(a_t, b_t, \mathbf{q}_t)$ has increasing differences in (a_t, b_t) if $\phi'(x) \leq 0$ for all x and decreasing differences if $\phi'(x) \geq 0$ for all x :

$$\begin{aligned}
& \frac{\partial \hat{V}_t(a_t, b_t, \mathbf{q}_t)}{\partial a_t \partial b_t} \\
&= \delta \Phi(q_t + \alpha a_t + \beta b_t)(1 - \lambda_a)(1 - \lambda_b) \frac{\partial^2 \hat{V}_{t+1}\left((1 - \lambda_a)a_t + \lambda_a, (1 - \lambda_b)b_t + \lambda_b q_t, \mathbf{q}_{t+1}\right)}{\partial a_{t+1} \partial b_{t+1}} \\
&+ \delta \bar{\Phi}(q_t + \alpha a_t + \beta b_t)(1 - \lambda_a)(1 - \lambda_b) \frac{\partial^2 \hat{V}_{t+1}\left((1 - \lambda_a)a_t, (1 - \lambda_b)b_t + \lambda_b q_t, \mathbf{q}_{t+1}\right)}{\partial a_{t+1} \partial b_{t+1}} \\
&+ 2\delta \alpha \phi(q_t + \alpha a_t + \beta b_t)(1 - \lambda_b) \\
&\quad \times \left(\frac{\partial \hat{V}_{t+1}\left((1 - \lambda_a)a_t + \lambda_a, (1 - \lambda_b)b_t + \lambda_b q_t, \mathbf{q}_{t+1}\right)}{\partial b_{t+1}} - \frac{\partial \hat{V}_{t+1}\left((1 - \lambda_a)a_t, (1 - \lambda_b)b_t + \lambda_b q_t, \mathbf{q}_{t+1}\right)}{\partial b_{t+1}} \right) \\
&+ \delta \alpha \beta \phi'(q_t + \alpha a_t + \beta b_t) \\
&\quad \times \left(\hat{V}_{t+1}\left((1 - \lambda_a)a_t + \lambda_a, (1 - \lambda_b)b_t + \lambda_b q_t, \mathbf{q}_{t+1}\right) - \hat{V}_{t+1}\left((1 - \lambda_a)a_t, (1 - \lambda_b)b_t + \lambda_b q_t, \mathbf{q}_{t+1}\right) \right) \\
&\geq 0
\end{aligned}$$

if $\phi'(x) \leq 0$ for all x and the inequality is reversed if $\phi'(x) \geq 0$ for all x , in which we used the induction hypothesis and the fact that $V_{t+1}(a_{t+1}, b_{t+1}, \mathbf{q}_{t+1})$ is increasing in a_{t+1} .

Finally, we show that $\hat{V}_t(a_t, b_t, \mathbf{q}_t)$ is concave in b_t if $\phi'(x) \leq 0$ for all x and convex if $\phi'(x) \geq 0$ for all x :

$$\begin{aligned}
& \frac{\partial^2 \hat{V}_t(a_t, b_t, \mathbf{q}_t)}{\partial b_t^2} \\
&= \delta \Phi(q_t + \alpha a_t + \beta b_t)(1 - \lambda_b)^2 \frac{\partial^2 \hat{V}_{t+1}\left((1 - \lambda_a)a_t + \lambda_a, (1 - \lambda_b)b_t + \lambda_b q_t, \mathbf{q}_{t+1}\right)}{\partial b_{t+1}^2}
\end{aligned}$$

$$\begin{aligned}
& + \delta \bar{\Phi}(q_t + \alpha a_t + \beta b_t)(1 - \lambda_b)^2 \frac{\partial^2 \hat{V}_{t+1}((1 - \lambda_a)a_t, (1 - \lambda_b)b_t + \lambda_b q_t, \mathbf{q}_{t+1})}{\partial b_{t+1}^2} \\
& + \delta \beta^2 \phi'(q_t + \alpha a_t + \beta b_t) \\
& \quad \times \left(\hat{V}_{t+1}((1 - \lambda_a)a_t + \lambda_a, (1 - \lambda_b)b_t + \lambda_b q_t, \mathbf{q}_{t+1}) - \hat{V}_{t+1}((1 - \lambda_a)a_t, (1 - \lambda_b)b_t + \lambda_b q_t, \mathbf{q}_{t+1}) \right) \\
& + 2\delta\beta(1 - \lambda_b)\phi(q_t + \alpha a_t + \beta b_t) \\
& \quad \times \left(\frac{\partial \hat{V}_{t+1}((1 - \lambda_a)a_t + \lambda_a, (1 - \lambda_b)b_t + \lambda_b q_t, \mathbf{q}_{t+1})}{\partial b_{t+1}} - \frac{\partial \hat{V}_{t+1}((1 - \lambda_a)a_t, (1 - \lambda_b)b_t + \lambda_b q_t, \mathbf{q}_{t+1})}{\partial b_{t+1}} \right) \\
& \leq 0
\end{aligned}$$

if $\phi'(x) \leq 0$ for all x and the inequality is reversed if $\phi'(x) \geq 0$ for all x , in which we used the induction hypothesis and the fact that $V_{t+1}(a_{t+1}, b_{t+1}, \mathbf{q}_{t+1})$ is increasing in a_{t+1} and has increasing differences in (a_{t+1}, b_{t+1}) if $\phi'(x) \leq 0$ for all x and decreasing differences if $\phi'(x) \geq 0$ for all x . $\square$

Proof of Proposition 2. Consider the cross-derivative with respect to (a_t, q_t) :

$$\begin{aligned}
\frac{\partial^2 \hat{V}_t(a_t, \mathbf{q}_t)}{\partial a_t \partial q_t} &= \delta \phi(q_t + \alpha a_t)(1 - \lambda_a) \left(\frac{\partial \hat{V}_{t+1}((1 - \lambda_a)a_t + \lambda_a, \mathbf{q}_{t+1})}{\partial a_{t+1}} - \frac{\partial \hat{V}_{t+1}((1 - \lambda_a)a_t, \mathbf{q}_{t+1})}{\partial a_{t+1}} \right) \\
& \quad + \delta \alpha \phi'(q_t + \alpha a_t) \left(\hat{V}_{t+1}((1 - \lambda_a)a_t + \lambda_a, \mathbf{q}_{t+1}) - \hat{V}_{t+1}((1 - \lambda_a)a_t, \mathbf{q}_{t+1}) \right) \\
& \leq 0,
\end{aligned}$$

if $\phi'(x) \leq 0$ for all x and the inequality is reversed if $\phi'(x) \geq 0$ for all x , in which the inequality follows because $\hat{V}_{t+1}(a_{t+1}, \mathbf{q}_{t+1})$ is increasing in a_{t+1} and concave in a_{t+1} if $\phi'(x) \leq 0$ for all x and convex if $\phi'(x) \geq 0$ for all x by Lemma EC.2.

We next show by induction that $\hat{V}_t(a_t, \mathbf{q}_t)$ has decreasing differences in $(a_t, q_t, \dots, q_{T-1})$ if $\phi'(x) \leq 0$ for all x and increasing differences in $(a_t, q_t, \dots, q_{T-1})$ if $\phi'(x) \geq 0$ for all x , for any $k \geq 1, \dots, T-1-t$. The hypothesis trivially holds when $t = T-1$ by (10). Fix $t < T-1$ and suppose that $\frac{\partial^2 \hat{V}_t(a_t, \mathbf{q}_t)}{\partial a_t \partial q_{t+k}} \leq 0$ if $\phi'(x) \leq 0$ for all x , and that the inequality is reversed if $\phi'(x) \geq 0$ for all x . Consider the cross-derivative with respect to (q_t, q_{t+k}) for some $k = 1, \dots, T-1-t$:

$$\frac{\partial^2 \hat{V}_t(a_t, \mathbf{q}_t)}{\partial q_t \partial q_{t+k}} = \delta \phi(q_t + \alpha a_t) \left(\frac{\partial \hat{V}_{t+1}((1 - \lambda_a)a_t + \lambda_a, \mathbf{q}_{t+1})}{\partial q_{t+k}} - \frac{\partial \hat{V}_{t+1}((1 - \lambda_a)a_t, \mathbf{q}_{t+1})}{\partial q_{t+k}} \right),$$

which is thus nonpositive whenever $\phi'(x) \leq 0$ for all x , and that the inequality is reversed if $\phi'(x) \geq 0$ for all x , by the induction hypothesis. Considering now the cross-derivative with respect to (a_t, q_{t+k}) , we obtain

$$\begin{aligned}
\frac{\partial^2 \hat{V}_t(a_t, \mathbf{q}_t)}{\partial a_t \partial q_{t+k}} &= \alpha \delta \phi(q_t + \alpha a_t) \left(\frac{\partial \hat{V}_{t+1}((1 - \lambda_a)a_t + \lambda_a, \mathbf{q}_{t+1})}{\partial q_{t+k}} - \frac{\partial \hat{V}_{t+1}((1 - \lambda_a)a_t, \mathbf{q}_{t+1})}{\partial q_{t+k}} \right) \\
& \quad + \delta(1 - \lambda_a)\Phi(q_t + \alpha a_t) \frac{\partial^2 \hat{V}_{t+1}((1 - \lambda_a)a_t + \lambda_a, \mathbf{q}_{t+1})}{\partial a_t \partial q_{t+k}} \\
& \quad + \delta(1 - \lambda_a)(1 - \Phi(q_t + \alpha a_t)) \frac{\partial^2 \hat{V}_{t+1}((1 - \lambda_a)a_t, \mathbf{q}_{t+1})}{\partial a_t \partial q_{t+k}} \\
& \leq 0
\end{aligned}$$

whenever $\phi'(x) \leq 0$ for all x , and that the inequality is reversed if $\phi'(x) \geq 0$ for all x , by the induction hypothesis, thus completing the induction step. $\square$

Proof of Proposition 3. Since $\beta = 0$, the quality reference level is irrelevant in the state definition of (9) and can, therefore, be dropped. Moreover, when $\lambda_a = 1$, $a_t \in \{0, 1\}$ for $t = 1, \dots, T-1$. Accordingly, when $\lambda_a = 1$, $\alpha = 1$, and $\beta = 0$, (9) simplifies to, for any $t = 0, \dots, T-1$:

$$\hat{V}_t(a_t, \mathbf{q}_t) = a_t + q_t + \delta \hat{V}_{t+1}^- + \delta \Phi(q_t + a_t)(\hat{V}_{t+1}^+ - \hat{V}_{t+1}^-),$$

in which, for any $t = 1, \dots, T-1$, $\hat{V}_t^+ \doteq \hat{V}_t(1, \mathbf{q}_t)$ and $\hat{V}_t^- \doteq \hat{V}_t(0, \mathbf{q}_t)$. Therefore, we obtain that, for $t = 0, \dots, T-2$,

$$\hat{V}_t^+ - \hat{V}_t^- = 1 + \delta \left(\Phi(q_t + 1) - \Phi(q_t) \right) (\hat{V}_{t+1}^+ - \hat{V}_{t+1}^-).$$

Letting $\Delta\Phi(q_t) \doteq \delta \left(\Phi(q_t + 1) - \Phi(q_t) \right)$, this difference simplifies to:

$$\hat{V}_t^+ - \hat{V}_t^- = \sum_{k=0}^{T-1-t} \prod_{j=0}^{k-1} \Delta\Phi(q_{t+j}) = \sum_{k=0}^{T-1-t} \prod_{j=0}^{k-1} \Delta\Phi(q_{t+k-1-j}) = \sum_{t'=t}^{T-1} \prod_{j=0}^{t'-t-1} \Delta\Phi(q_{t'-1-j}) = \sum_{t'=t}^{T-1} \prod_{j=0}^{t'-t-1} \Delta\Phi(q_{t+j}),$$

with the convention that $\prod_{j=0}^{-1} \Delta\Phi(q_{t+j}) = 1$ for any t . Thus,

$$\begin{aligned} \hat{V}_t^- &= q_t + \delta \Phi(q_t) \left(\sum_{t'=t+1}^{T-1} \prod_{j=0}^{t'-t-2} \Delta\Phi(q_{t+1+j}) \right) + \delta \hat{V}_{t+1}^- \\ &= \sum_{i=0}^{T-1-t} \delta^i \left[q_{t+i} + \delta \Phi(q_{t+i}) \left(\sum_{t'=t+i+1}^{T-1} \prod_{j=0}^{t'-t-i-2} \Delta\Phi(q_{t+1+i+j}) \right) \right]. \end{aligned}$$

Plugging the derived expressions for $\hat{V}_{t+1}^+ - \hat{V}_{t+1}^-$ and $\hat{V}_{t+1}^-$ into $\hat{V}_t(a_t, \mathbf{q}_t)$, we obtain

$$\begin{aligned} \hat{V}_t(a_t, \mathbf{q}_t) &= q_t + a_t + \delta \Phi(q_t + a_t) \left(\sum_{t'=t+1}^{T-1} \prod_{j=0}^{t'-t-2} \Delta\Phi(q_{t+1+j}) \right) \\ &\quad + \sum_{i=1}^{T-1-t} \delta^i \left[q_{t+i} + \delta \Phi(q_{t+i}) \left(\sum_{t'=t+i+1}^{T-1} \prod_{j=0}^{t'-t-i-2} \Delta\Phi(q_{t+1+i+j}) \right) \right]. \end{aligned}$$

We thus have a closed-form expression for the value-to-go function. Letting $y_0 := q_0 + a_0$ and $y_t := q_t$ for $t \geq 1$, we have

$$\hat{V}_0 = \sum_{t=0}^{T-1} \delta^t \left[y_t + \delta \Phi(y_t) \left(\sum_{t'=t+1}^{T-1} \prod_{j=0}^{t'-t-2} \Delta\Phi(y_{t+1+j}) \right) \right].$$

This allows us to characterize the impact of scheduling on $\hat{V}_0$, i.e. whether to swap q_t with q_{t+1} , as follows. Specifically, for any $t = 0, \dots, T-2$, we can express $\hat{V}_0$ as follows:

$$\begin{aligned} \hat{V}_0 &= A_t(y_0, \dots, y_{T-1}) + \delta^t y_t + \delta^{t+1} y_{t+1} + \delta^{t+1} \Phi(y_t) \left(1 + \Delta\Phi(y_{t+1}) B_t(y_{t+2}, \dots, y_{T-2}) \right) \\ &\quad + \delta^{t+2} \Phi(y_{t+1}) B_t(y_{t+2}, \dots, y_{T-2}) + \delta^t \Delta\Phi(y_t) C_t(y_0, \dots, y_{t-1}), \end{aligned}$$

for some $A_t(y_0, \dots, y_{T-1})$ such that $A_t(y_0, \dots, y_t, y_{t+1}, \dots, y_{T-1}) = A_t(y_0, \dots, y_{t+1}, y_t, \dots, y_{T-1})$, $B_t(y_{t+2}, \dots, y_{T-2}) := \sum_{t'=t+2}^{T-1} \prod_{j=0}^{t'-t-3} \Delta\Phi(y_{t+2+j})$ for $t = 0, \dots, T-3$, $B_{T-2} = 0$, $C_0 = 0$, and

$C_t(y_0, \dots, y_{t-1}) := \sum_{k=0}^{t-1} \Phi(y_k) \prod_{j=k+1}^{t-1} (\Delta\Phi(y_j)/\delta) = \mathbb{P}[a_t = 1|a_0]$ for $t = 1, \dots, T-2$. Thus, scheduling y_t followed by y_{t+1} is preferable over the contrary if and only if:

$$(1 - \delta)(y_t - y_{t+1}) + \delta(\Phi(y_t) - \Phi(y_{t+1})) + (\Delta\Phi(y_t) - \Delta\Phi(y_{t+1}))C_t(y_0, \dots, y_{t-1}) \\ \geq \delta^2 B_t(y_{t+2}, \dots, y_{T-2})(\Phi(y_t)\bar{\Phi}(y_{t+1} + 1) - \Phi(y_{t+1})\bar{\Phi}(y_t + 1)).$$

□

COROLLARY EC.1. *Suppose that $\lambda_a = 1$, $\lambda_b = 0$, $\alpha = 1$, and $\delta \leq 1$.*

1. $q_{T-2} \geq q_{T-1}$ if $T > 2$ and $q_0 + a_0 \geq q_1$ if $T = 2$.

2. (a) If $\Phi(x)$ is log-concave, $T \geq 3$, and either $T = 3$ or $\Phi(x)$ is convex, then $q_{T-3} \geq q_{T-2}$ if $T > 3$ and $q_0 + a_0 \geq q_1$ if $T = 3$.

(b) If $\bar{\Phi}(x)$ is log-concave, $T > 3$, and $\Phi(x)$ is concave, then $q_{T-3} \geq q_{T-2}$.

(c) If $T > 4$ and $q_{T-4} = q_{T-3} = q_{T-2}$, then $\frac{\partial \hat{V}_0(a_0, b_0, \mathbf{q}_0)}{\partial q_{T-3}} \geq \frac{\partial \hat{V}_0(a_0, b_0, \mathbf{q}_0)}{\partial q_{T-2}}$.

3. If $T > 5$, $q_{T-5} = q_{T-4} = q_{T-3} = q_{T-2}$, and $\phi(x)$ is log-concave, then $\frac{\partial \hat{V}_0(a_0, b_0, \mathbf{q}_0)}{\partial q_{T-4}} \geq \frac{\partial \hat{V}_0(a_0, b_0, \mathbf{q}_0)}{\partial q_{T-3}}$.

4. If $q_1 = \dots = q_{T-2} = q$, and $\Phi(q_0 + a_0) = \Phi(q)/(\Phi(q) + \bar{\Phi}(q + 1))$, then $\frac{\partial \hat{V}_0(a_0, b_0, \mathbf{q}_0)}{\partial q_t} \geq \frac{\partial \hat{V}_0(a_0, b_0, \mathbf{q}_0)}{\partial q_{t+1}}$ for all $t = 1, \dots, T-3$.

5. If $a_0 + q_0 = q_1 = \dots = q_{T-2} = q$ and $\phi(q) \geq \phi(q + 1)$, then $\frac{\partial \hat{V}_0(a_0, b_0, \mathbf{q}_0)}{\partial q_0} \geq \frac{\partial \hat{V}_0(a_0, b_0, \mathbf{q}_0)}{\partial q_1}$.

Proof. 1. To obtain a contradiction, suppose that it is optimal to have $y_{T-2} \leq y_{T-1}$. Suppose that $T > 2$. Because the function $(1 - \delta)y + \delta\Phi(y)\mathbb{P}[a_{T-2} = 0|a_0] + \delta\Phi(y + 1)\mathbb{P}[a_{T-2} = 1|a_0]$ is increasing in y when $\delta \leq 1$, we obtain that $(1 - \delta)y_{T-2} + \delta\Phi(y_{T-2})\mathbb{P}[a_{T-2} = 0|a_0] + \delta\Phi(y_{T-2} + 1)\mathbb{P}[a_{T-2} = 1|a_0] \leq (1 - \delta)y_{T-1} + \delta\Phi(y_{T-1})\mathbb{P}[a_{T-2} = 0|a_0] + \delta\Phi(y_{T-1} + 1)\mathbb{P}[a_{T-2} = 1|a_0]$. Given that $B_{T-2} = 0$ and $C_{T-2} = \mathbb{P}[a_{T-2} = 1|a_0]$ when $T - 2 > 0$, we thus obtain that the necessary condition for optimality in Proposition 3 is not satisfied, a contradiction. Suppose that $T = 2$. The same argument applies by noting that the function $(1 - \delta)y + \delta\Phi(y)$ is increasing in y when $\delta \leq 1$ and that $C_0 = 0$.

2. (a) To obtain a contradiction, suppose that it is optimal to have $y_{T-3} \leq y_{T-2}$. Suppose that $T > 3$. Because $\Phi(y)/\Phi(y + 1)$ is increasing when $\Phi(x)$ is log-concave, $\Phi(y_{T-3})\Phi(y_{T-2} + 1) \leq \Phi(y_{T-2})\Phi(y_{T-3} + 1)$, or equivalently, $\Phi(y_{T-3}) - \Phi(y_{T-2}) \leq \Phi(y_{T-3})\bar{\Phi}(y_{T-2} + 1) - \Phi(y_{T-2})\bar{\Phi}(y_{T-3} + 1)$. Moreover, $\Delta\Phi(y_{T-3}) \leq \Delta\Phi(y_{T-2})$ given that $\Phi(x)$ is convex and $y_{T-3} \leq y_{T-2}$. Given that $B_{T-3} = 1$ and $C_{T-3} \geq 0$ when $T > 3$, we obtain that, when $\delta = 1$, the necessary condition for optimality in Proposition 3 is not satisfied, a contradiction. When $T = 3$, the same argument applies without making use of the convexity of $\Phi(x)$ since $C_0 = 0$.

(b) To obtain a contradiction, suppose that it is optimal to have $y_{T-3} \leq y_{T-2}$. Because $\bar{\Phi}(y)/\bar{\Phi}(y + 1)$ is increasing when $\bar{\Phi}(x)$ is log-concave, $\bar{\Phi}(y_{T-3})\bar{\Phi}(y_{T-2} + 1) \leq \bar{\Phi}(y_{T-2})\bar{\Phi}(y_{T-3} + 1)$, or equivalently, $\Phi(y_{T-3} + 1) - \Phi(y_{T-2} + 1) \leq \Phi(y_{T-3})\bar{\Phi}(y_{T-2} + 1) - \Phi(y_{T-2})\bar{\Phi}(y_{T-3} + 1)$. Moreover, $\Delta\Phi(y_{T-3}) \geq \Delta\Phi(y_{T-2})$ given that $\Phi(x)$ is concave. Given that $B_{T-3} = 1$ and $C_{T-3} \leq 1$ when $T > 3$, we obtain that, when $\delta = 1$, the necessary condition for optimality in Proposition 3 is not satisfied, a contradiction.

(c) Define $F(\epsilon, (y_0, \dots, y_{T-5}), y) := (1 - \delta)\epsilon + \delta(\Phi(y + \epsilon) - \Phi(y)) + (\Delta\Phi(y + \epsilon) - \Delta\Phi(y))C_t(y_0, \dots, y_{T-5}, y) - \delta^2(\Phi(y + \epsilon)\bar{\Phi}(y + 1) - \Phi(y)\bar{\Phi}(y + \epsilon + 1))$. When $y_{T-4} = y_{T-2} = y$ and $y_{T-3} = y + \epsilon$, the necessary condition for optimality in Proposition 3 simplifies to requiring that $F(\epsilon, (y_0, \dots, y_{T-3}), y) \geq 0$. Accordingly, when $y_{T-4} = y_{T-3} = y_{T-2} = y$, $\frac{\partial \hat{V}(a_0, b_0, \mathbf{q}_0)}{\partial q_{T-3}} \geq \frac{\partial \hat{V}(a_0, b_0, \mathbf{q}_0)}{\partial q_{T-2}}$ if and only if $\partial F(0, (y_0, \dots, y_{T-5}), y)/\partial \epsilon \geq 0$, i.e., if and only if $1 - \delta + \delta\phi(q + 1)(\mathbb{P}[a_{T-3} = 1|a_0] - \delta\Phi(q)) + \delta\phi(q)(1 - \mathbb{P}[a_{T-3} = 1|a_0] - \delta\bar{\Phi}(q + 1)) \geq 0$. The latter condition always holds since $\mathbb{P}[a_{T-3} = 1|a_0] = \mathbb{P}[a_{T-4} = 1|a_0]\Phi(y + 1) + \mathbb{P}[a_{T-4} = 1|a_0]\Phi(y)$, and therefore, $\Phi(q + 1) \geq \mathbb{P}[a_{T-3} = 1|a_0] \geq \Phi(q)$.

3. Define $F(\epsilon, (y_0, \dots, y_{T-6}), y) := (1 - \delta)\epsilon + \delta(\Phi(y + \epsilon) - \Phi(y)) + (\Delta\Phi(y + \epsilon) - \Delta\Phi(y))C_t(y_0, \dots, y_{T-5}, y) - \delta^2(1 + \Delta\Phi(y))(\Phi(y + \epsilon)\bar{\Phi}(y + 1) - \Phi(y)\bar{\Phi}(y + \epsilon + 1))$. When $y_{T-5} = y_{T-3} = y_{T-2} = y$ and $y_{T-4} = y + \epsilon$, the necessary condition for optimality in Proposition 3 simplifies to requiring that $F(\epsilon, (y_0, \dots, y_{T-6}), y) \geq 0$. Accordingly, when $y_{T-5} = y_{T-4} = y_{T-3} = y_{T-2} = y$, $\frac{\partial \hat{V}(a_0, b_0, \mathbf{q}_0)}{\partial q_{T-4}} \geq \frac{\partial \hat{V}(a_0, b_0, \mathbf{q}_0)}{\partial q_{T-4}}$ if and only if $\partial F(0, (y_0, \dots, y_{T-6}), y)/\partial \epsilon \geq 0$, i.e., if and only if $G(\mathbb{P}[a_{T-4} = 1|a_0]) := 1 - \delta + \delta(1 - \delta)\phi(q) + \delta(\phi(q + 1) - \phi(q))(\mathbb{P}[a_{T-4} = 1|a_0] - \delta\Phi(q)) + \delta^3(\phi(q)\Phi(q + 1) - \phi(q + 1)\Phi(q))(\Phi(q + 1) - \Phi(q)) \geq 0$. Because $G(\mathbb{P}[a_{T-4} = 1|a_0])$ is linear and $\Phi(q + 1) \geq \mathbb{P}[a_{T-4} = 1|a_0] \geq \Phi(q)$ when $y_{T-5} = y$ given that $\mathbb{P}[a_{T-4} = 1|a_0] = \mathbb{P}[a_{T-5} = 1|a_0]\Phi(y + 1) + \mathbb{P}[a_{T-5} = 1|a_0]\Phi(y)$, $G(\mathbb{P}[a_{T-4} = 1|a_0]) \geq \min\{G(\Phi(q)), G(\Phi(q + 1))\}$. We next check that the right-hand side is nonnegative when $\delta \leq 1$ and $\phi(x)$ is log-concave. On the one hand, $G(\Phi(q)) = 1 - \delta + \delta(1 - \delta)(\phi(q)\bar{\Phi}(q) + \phi(q + 1)\Phi(q + 1)) + \delta^3(\phi(q)\Phi(q + 1) - \phi(q + 1)\Phi(q))(\Phi(q + 1) - \Phi(q)) \geq 0$ given that $\phi(x)/\Phi(x)$ is decreasing when $\phi(x)$ is log-concave. On the other hand, $G(\Phi(q + 1)) = 1 - \delta + \delta(1 - \delta)\phi(q + 1) + \delta^3(\phi(q + 1)\bar{\Phi}(q) - \phi(q)\bar{\Phi}(q + 1))(\Phi(q + 1) - \Phi(q)) \geq 0$ given that $\phi(x)/\bar{\Phi}(x)$ is increasing when $\phi(x)$ is log-concave. Combining these two results, we obtain that $G(\mathbb{P}[a_{T-4} = 1|a_0]) \geq 0$ for all $\mathbb{P}[a_{T-4} = 1|a_0] \in [\Phi(q), \Phi(q + 1)]$.

4. When $q_1 = \dots = q_{T-2} = y$, for any $t \geq 2$, $\mathbb{P}[a_t = 1|a_0] = \mathbb{P}[a_{t-1} = 1|a_0]\Phi(y + 1) + \mathbb{P}[a_t = 0|a_0]\Phi(y)$ with $\mathbb{P}[a_1 = 1|a_0] = \Phi(y_0)$ and $\mathbb{P}[a_1 = 1|a_0] = 1 - \mathbb{P}[a_1 = 1|a_0]$. Observe that, if $\mathbb{P}[a_{t-1} = 1|a_0] = \Phi(y)/(\Phi(y) + \bar{\Phi}(y + 1))$, then $\mathbb{P}[a_t = 1|a_0] = \Phi(y)/(\Phi(y) + \bar{\Phi}(y + 1))$. Accordingly, if $\Phi(y_0) = \Phi(y)/(\Phi(y) + \bar{\Phi}(y + 1))$, then $\mathbb{P}[a_t = 1|a_0] = \Phi(y)/(\Phi(y) + \bar{\Phi}(y + 1))$ for all $t \geq 1$. In other words, $C_t(y_0, y, \dots, y) = \Phi(y)/(\Phi(y) + \bar{\Phi}(y + 1))$ for all $t \geq 1$. Moreover, $B_t(y, \dots, y) = \sum_{k=0}^{T-3-t} (\Delta\Phi(y))^k$ for $t = 0, \dots, T - 3$. Define, for $t = 1, \dots, T - 3$, $F_t(\epsilon, y_0, y) := (1 - \delta)\epsilon + \delta(\Phi(y + \epsilon) - \Phi(y)) + (\Delta\Phi(y + \epsilon) - \Delta\Phi(y))\Phi(y)/(\Phi(y) + \bar{\Phi}(y + 1)) - \delta^2 \left(\sum_{k=0}^{T-3-t} (\Delta\Phi(y))^k \right) (\Phi(y + \epsilon)\bar{\Phi}(y + 1) - \Phi(y)\bar{\Phi}(y + \epsilon + 1))$. When $y_t = y$ for $t = 1, \dots, T - 2$ and y_0 being such that $\Phi(y_0) = \Phi(y)/(\Phi(y) + \bar{\Phi}(y))$, the necessary condition for optimality in Proposition 3 simplifies to requiring that $F_t(\epsilon, y_0, y) \geq 0$. Accordingly, when $y_t = y$ for $t = 1, \dots, T - 2$ and y_0 being such that $\Phi(y_0) = \Phi(y)/(\Phi(y) + \bar{\Phi}(y + 1))$, $\frac{\partial \hat{V}(a_0, b_0, \mathbf{q}_0)}{\partial q_t} \geq \frac{\partial \hat{V}(a_0, b_0, \mathbf{q}_0)}{\partial q_{t+1}}$ if and only if $\partial F_t(0, y_0, y)/\partial \epsilon \geq 0$, i.e., if and only if $(1 - \delta) + \delta(\phi(y)\bar{\Phi}(y + 1) + \phi(y + 1)\Phi(y)) \left(1/(\Phi(y) + \bar{\Phi}(y + 1)) - \delta \left(\sum_{k=0}^{T-3-t} (\Delta\Phi(y))^k \right) \right) \geq 0$. Given that $\sum_{k=0}^{T-3-t} (\Delta\Phi(y))^k$ is increasing in $T - t$ and converges, as $T - t \rightarrow \infty$, to $(1 - \Delta\Phi(y))^{-1} \leq \delta^{-1}(\Phi(y) + \bar{\Phi}(y))^{-1}$ given that $\delta\Delta\Phi(y) < 1$, we obtain that $\partial F_t(0, y_0, y)/\partial \epsilon \geq 0$ for $t = 1, \dots, T - 3$.

5. When $q_0 + a_0 = q_1 = \dots = q_{T-2} = y$, $B_0(y, \dots, y) = \sum_{k=0}^{T-3} (\Delta\Phi(y))^k$ and $C_0 = 0$. Define, $F(\epsilon, y_0, y) := (1 - \delta)\epsilon + \delta(\Phi(y + \epsilon) - \Phi(y)) - \delta^2 \left(\sum_{k=0}^{T-3} (\Delta\Phi(y))^k \right) (\Phi(y + \epsilon)\bar{\Phi}(y + 1) - \Phi(y)\bar{\Phi}(y + \epsilon + 1))$. When $y_t = y$ for $t = 1, \dots, T - 2$, the necessary condition for optimality in Proposition 3 simplifies to requiring that $F(\epsilon, y_0, y) \geq 0$. Accordingly, when $y_t = y$ for $t = 0, \dots, T - 2$, $\frac{\partial \hat{V}(a_0, b_0, \mathbf{q}_0)}{\partial q_0} \geq \frac{\partial \hat{V}(a_0, b_0, \mathbf{q}_0)}{\partial q_1}$ if and only if $\partial F(0, y_0, y)/\partial \epsilon \geq 0$, i.e., if and only if $G(y) := (1 - \delta) + \delta\phi(y) - \delta^2 \left(\sum_{k=0}^{T-3} (\Delta\Phi(y))^k \right) (\phi(y)\bar{\Phi}(y + 1) + \phi(y + 1)\Phi(y)) \geq 0$. Given that

$\sum_{k=0}^{T-3} (\Delta\Phi(y))^k$ is increasing in $T-t$ and converges, as $T \rightarrow \infty$, to $(1 - \Delta\Phi(y))^{-1}$ when $\delta\Delta\Phi(y) < 1$, we obtain that $G(y) \geq (1 - \delta) + \delta\phi(y) - \delta^2(\phi(y)\bar{\Phi}(y+1) + \phi(y+1)\Phi(y))/(1 - \Delta\Phi(y)) = (1 - \delta)(1 + \delta\phi(y)/\Delta\Phi(y)) + \delta^2\Phi(y)(\phi(y) - \phi(y+1))/\Delta\Phi(y)$. Hence, when $\delta \leq 1$ and $\phi(y) \geq \phi(y+1)$, $G(y) \geq 0$, i.e., $\partial F(0, y_0, y)/\partial \epsilon \geq 0$. $\square$

EC.2. Descriptive Statistics

Figure EC.1 shows the histograms of ratings across all four contexts under consideration. As explained in §5.2, we carry out a binary transformation of the ratings to get r_{ij} , which descriptive statistics are shown in Table EC.1. Tables EC.2-EC.5 report the correlation matrix among the variables of interest, their mean, minimum and maximum values, and standard deviation. In the regressions, observations with missing values for any of these variables are dropped. The mean values of the inverse Mills ratio, eventually used in the second stage of the Heckprobit estimation and necessary for estimating the percent point estimate values, are given in the corresponding section §EC.4.2.

Figure EC.1 Histograms for the ratings recorded by individuals across the different contexts under consideration.

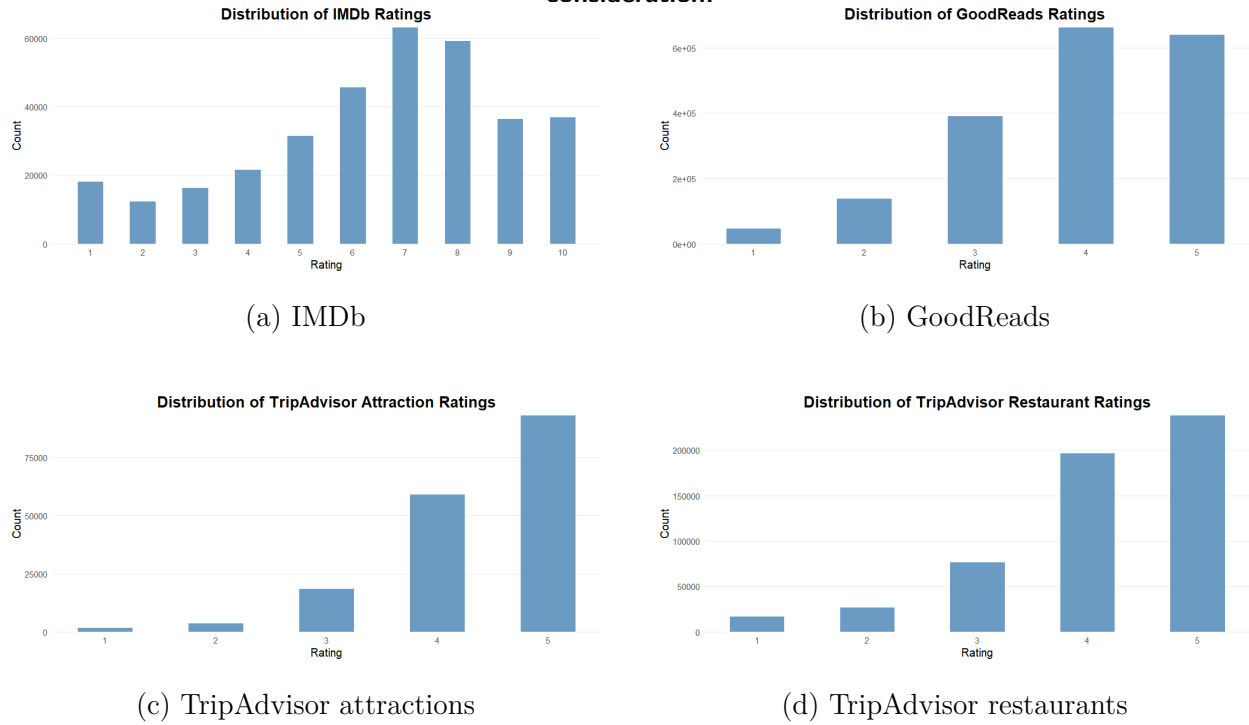


Table EC.1 Descriptive statistics for the binary transformed ratings r_{ij} across the four contexts.

r_{ij}	IMDb	GoodReads	TripAdvisor Attractions	TripAdvisor Restaurants
Mean	0.37	0.34	0.53	0.44
Min	0	0	0	0
Max	1	1	1	1
Std. dev.	0.48	0.47	0.50	0.50

Table EC.2 Descriptive statistics for IMDb.

	c_i	q_{ij}	j	n_{ij}	A_{ij}	$a_{i,j-1}$	$b_{i,j-1}$	Δt_{ij}	$\Delta \mathbf{x}_{ij}$
c_i	1.00	0.07	-0.10	0.09	-0.08	0.64	0.15	0.02	-0.04
q_{ij}	0.07	1.00	-0.02	0.50	0.12	0.23	0.38	-0.03	-0.06
j	-0.10	-0.02	1.00	0.01	0.06	-0.18	-0.04	-0.25	0.01
n_{ij}	0.09	0.50	0.01	1.00	-0.06	0.22	0.31	-0.03	-0.08
A_{ij}	-0.08	0.12	0.06	-0.06	1.00	-0.03	0.06	-0.15	-0.10
$a_{i,j-1}$	0.64	0.23	-0.18	0.22	-0.03	1.00	0.58	0.01	-0.08
$b_{i,j-1}$	0.15	0.38	-0.04	0.31	0.06	0.58	1.00	-0.05	-0.09
Δt_{ij}	0.02	-0.03	-0.25	-0.03	-0.15	0.01	-0.05	1.00	0.09
$\Delta \mathbf{x}_{ij}$	-0.04	-0.06	0.01	-0.08	-0.10	-0.08	-0.09	0.09	1.00
Mean	-0.03	0.41	4.06	11.19	0.41	0.37	0.41	1.45	0.74
Min.	-1.28	0.00	2.77	1.79	0.00	0.00	0.03	0.00	0.00
Max.	2.06	0.98	7.09	14.75	0.69	1.00	0.84	8.54	1.00
Std. dev.	0.49	0.23	0.94	1.83	0.22	0.25	0.11	1.41	0.28

Table EC.3 Descriptive statistics for GoodReads.

	c_i	q_{ij}	j	n_{ij}	A_{ij}	$a_{i,j-1}$	$b_{i,j-1}$	Δt_{ij}	$\Delta \mathbf{x}_{ij}$
c_i	1.00	0.07	-0.01	-0.09	-0.11	0.44	0.14	0.01	-0.05
q_{ij}	0.07	1.00	-0.03	0.01	-0.05	0.21	0.37	0.02	-0.06
j	-0.01	-0.03	1.00	-0.01	0.09	-0.13	-0.05	-0.14	0.04
n_{ij}	-0.09	0.01	-0.01	1.00	0.29	-0.14	-0.12	0.01	0.03
A_{ij}	-0.11	-0.05	0.09	0.29	1.00	-0.10	-0.05	0.07	0.08
$a_{i,j-1}$	0.44	0.21	-0.13	-0.14	-0.10	1.00	0.48	0.04	-0.10
$b_{i,j-1}$	0.14	0.37	-0.05	-0.12	-0.05	0.48	1.00	-0.01	-0.13
Δt_{ij}	0.01	0.02	-0.14	0.01	0.07	0.04	-0.01	1.00	0.03
$\Delta \mathbf{x}_{ij}$	-0.05	-0.06	0.04	0.03	0.08	-0.10	-0.13	0.03	1.00
Mean	0.08	0.35	3.53	7.80	0.36	0.33	0.34	3.35	0.47
Min.	-2.16	0.01	2.77	1.10	0.00	0.00	0.10	2.08	-0.00
Max.	1.61	0.97	5.32	12.72	0.69	1.00	0.83	7.89	1.00
Std. dev.	0.44	0.16	0.52	2.20	0.23	0.24	0.07	0.84	0.38

Table EC.4 Descriptive statistics for TripAdvisor attractions.

	c_i	q_{ij}	j	n_{ij}	A_{ij}	$a_{i,j-1}$	$b_{i,j-1}$	Δt_{ij}	$\Delta \mathbf{x}_{ij}$
c_i	1.00	0.01	-0.02	0.01	0.01	0.60	0.02	-0.00	0.00
q_{ij}	0.01	1.00	-0.03	0.33	0.16	0.11	0.27	0.05	0.01
j	-0.02	-0.03	1.00	-0.03	0.02	-0.07	-0.09	-0.08	-0.02
n_{ij}	0.01	0.33	-0.03	1.00	0.49	0.05	0.14	-0.05	-0.03
A_{ij}	0.01	0.16	0.02	0.49	1.00	0.05	0.05	0.06	-0.02
$a_{i,j-1}$	0.60	0.11	-0.07	0.05	0.05	1.00	0.33	0.01	0.01
$b_{i,j-1}$	0.02	0.27	-0.09	0.14	0.05	0.33	1.00	0.00	0.03
Δt_{ij}	-0.00	0.05	-0.08	-0.05	0.06	0.01	0.00	1.00	0.09
$\Delta \mathbf{x}_{ij}$	0.00	0.01	-0.02	-0.03	-0.02	0.01	0.03	0.09	1.00
Mean	-0.05	0.59	3.20	4.82	0.30	0.53	0.59	1.53	0.92
Min.	-1.54	0.00	2.77	0.69	0.00	0.00	0.20	0.00	0.00
Max.	1.35	1.00	5.08	10.30	0.69	1.00	0.91	8.00	4.00
Std. dev.	0.45	0.19	0.39	1.94	0.19	0.25	0.08	2.13	0.26

Table EC.5 Descriptive statistics for TripAdvisor restaurants.

	c_i	q_{ij}	j	n_{ij}	A_{ij}	$a_{i,j-1}$	$b_{i,j-1}$	Δt_{ij}	$\Delta \mathbf{x}_{ij}$
c_i	1.00	0.02	-0.04	0.01	0.05	0.58	0.03	0.01	0.01
q_{ij}	0.02	1.00	0.01	0.33	0.12	0.13	0.28	0.03	-0.13
j	-0.04	0.01	1.00	-0.00	0.08	-0.08	0.02	-0.08	-0.02
n_{ij}	0.01	0.33	-0.00	1.00	0.43	0.07	0.20	-0.05	-0.06
A_{ij}	0.05	0.12	0.08	0.43	1.00	0.12	0.10	0.07	-0.06
$a_{i,j-1}$	0.58	0.13	-0.08	0.07	0.12	1.00	0.34	0.04	-0.02
$b_{i,j-1}$	0.03	0.28	0.02	0.20	0.10	0.34	1.00	0.00	-0.08
Δt_{ij}	0.01	0.03	-0.08	-0.05	0.07	0.04	0.00	1.00	0.07
$\Delta \mathbf{x}_{ij}$	0.01	-0.13	-0.02	-0.06	-0.06	-0.02	-0.08	0.07	1.00
Mean	-0.04	0.50	3.25	3.40	0.24	0.43	0.50	2.26	0.88
Min.	-1.33	0.00	2.77	0.69	0.00	0.00	0.04	0.00	0.00
Max.	1.37	0.98	5.44	8.92	0.69	1.00	0.83	7.99	1.00
Std. dev.	0.39	0.21	0.40	1.24	0.20	0.25	0.10	2.09	0.23

EC.3. Specification of the Attribute Dissimilarity Variable

In this appendix, we explain how we measure attribute dissimilarity $\Delta \mathbf{x}_{ij}$ in the different contexts of interest.

- On IMDb, movie producers provide genre tags out of 28 possible distinct labels. For each movie, we construct an n -dimensional vector representing the different genres the movie belongs to, i.e., $\mathbf{x}_{ij} = (x_{ij,1}, \dots, x_{ij,n})$, in which $x_{ij,k} = 1$ if individual i 's j^{th} movie has been tagged the k^{th} genre and zero otherwise. We measure dissimilarity between individual i 's j^{th} and $j-1^{\text{th}}$ movies as the fraction of the j^{th} movie's tagged genres that are distinct:

$$\Delta \mathbf{x}_{ij} = 1 - \frac{\mathbf{x}_{ij} \cdot \mathbf{x}_{i,j-1}}{\|\mathbf{x}_{ij}\|_0}, \quad (\text{EC.5})$$

in which $\|\mathbf{x}\|_0$ denotes the cardinality, i.e., the number of nonzero elements, of $\mathbf{x}$.

- On GoodReads, readers vote on which genre category a particular book belongs to out of ten possible options (e.g., history, fiction, fantasy). For instance, *Dracula* by Bram Stoker is classified in our collected data as *Classics* by 43,839 readers, as *Fiction* by 25,932 readers, and as *Science-Fiction* by 11,551 readers. For each book, we construct a 10-dimensional vector representing the number of votes for each category, i.e., $\mathbf{x}_{ij} = (x_{ij,1}, \dots, x_{ij,10})$, in which $x_{ij,k}$ denotes the number of votes individual i 's j^{th} book received in the k^{th} category. To obtain a measure of dissimilarity between individual i 's j^{th} and $j-1^{\text{th}}$ books, we use the concept of cosine dissimilarity:

$$\Delta \mathbf{x}_{ij} = 1 - \frac{\mathbf{x}_{ij} \cdot \mathbf{x}_{i,j-1}}{\|\mathbf{x}_{ij}\| \|\mathbf{x}_{i,j-1}\|}. \quad (\text{EC.6})$$

- On TripAdvisor, attractions and restaurants are tagged according to their themes provided by the business owners. For example, the Sagrada Familia is tagged as *Points of Interest & Landmarks*, *Architectural Buildings*, and *Churches & Cathedrals*, among others. For each entry on TripAdvisor (attraction or restaurant), we construct an n -dimensional vector representing the different tags associated with the entry, i.e., $\mathbf{x}_{ij} = (x_{ij,1}, \dots, x_{ij,n})$, in which $x_{ij,k} = 1$ if individual i 's j^{th} activity has been tagged the k^{th} genre and zero otherwise. Note that if an attraction or restaurant has no tag specified, all the values in this vector are 0. We measure dissimilarity between individual i 's j^{th} and $j-1^{\text{th}}$ attractions as the fraction of the total number of tags between the two activities that are distinct:

$$\Delta \mathbf{x}_{ij} = 1 - \frac{\mathbf{x}_{ij} \cdot \mathbf{x}_{i,j-1}}{\|\mathbf{x}_{ij} + \mathbf{x}_{i,j-1}\|_0}. \quad (\text{EC.7})$$

EC.4. Additional Details on Two-Stage Estimation Procedure

Section EC.4.1 develops the first-stage selection model (outlined in §5.4.3) and shows its estimation results. We then embed in §EC.4.2 the first-stage model into the second-stage estimation and report its estimation results. In §EC.4.3, we discuss instrument validity tests.

EC.4.1. First-Stage Selection Model

Similar to (1), define the utility consumer i derives from a focal activity to be consumed in the j^{th} position of her consumption stream as follows:

$$v_{ij} = \bar{v}_{ij} + \epsilon_{ij},$$

in which ϵ_{ij} is a random shock with zero mean, independent across all i and j .

Consumer i 's utility from her the focal activity v_{ij} is compared to an outside option with utility v^0 . If v_{ij} is higher than v^0 , customer i selects the focal activity. Let s_{ij} denote a binary selection variable, equal to 1 if consumer i selects the focal activity in position j of her consumption stream, and zero otherwise. Similar to (2), we have

$$\Phi(\bar{v}_{ij}) := \mathbb{P}[s_{ij} = 1] = \mathbb{P}[v_{ij} \geq v^0].$$

We consider a probit specification, so here, $\Phi(\cdot)$ is the c.d.f. of a normal distribution.

Similar to (6), we specify the base utility from the focal activity as

$$\bar{v}_{ij} = \gamma_1 q_{ij} + \gamma_2 q_{ij}^{\text{low}} + \gamma_3 A_{ij} + \gamma_4 n_{ij} + \delta D_{ij} + \mu M_{ij},$$

in which, q_{ij} , A_{ij} , and n_{ij} are as defined in §5.2. Additionally, we have:

- q_{ij}^{low} is the proportion of 1-star reviews for the activity to control for the both sides of the experience spectrum for an activity;
- D_{ij} is a measure of dissimilarity in individual i in her consumption stream between the focal activity and individual i 's $j - 1^{\text{th}}$ activity;
- M_{ij} is the log-transformed measure of market thickness at the time of selection.

For IMDb and GoodReads, we use $\Delta \mathbf{x}_{ij}$ for D_{ij} and we use the (log-transformed) number of movies (resp., books) sharing at least 95% attribute similarity with the focal movie (resp., book) and released prior to the focal movie (resp., book) for M_{ij} . For TripAdvisor, we use the Euclidean distance (in km) between the focal activity and individual i 's $j - 1^{\text{th}}$ activity, denoted as ΔL_{ij} , for D_{ij} and the (log-transformed) number of activities (attractions or restaurants) within a 0.5km-radius of the focal activity for M_{ij} . Tables EC.6-EC.9 provide descriptive statistics of the variables used in the first-stage selection procedure.

The purpose of M_{ij} is to track the relevant competition faced by the focal activity in individual i 's choice set. The higher the competition, the lower the likelihood of that activity to be chosen, as in Arora et al. (2009) and Mueller and Reize (2013). Note that M_{ij} should strongly impact the consumers' choice of activity, but have only a marginal impact on their evaluation of the activity, if at all, after it has been selected. (Regret in post-purchase evaluation has been shown to be salient only if the value of the foregone alternative is revealed to the consumers, see Inman et al. 1997.) Accordingly, it satisfies the exclusion restriction condition in our two-stage formulation.

Table EC.10 summarizes the estimation results for the first-stage selection model. We discuss, in turn, the effect of each variable. The inherent activity quality q_{ij} has a positive effect on its likelihood of selection on IMDb and GoodReads, and null effect on TripAdvisor. For q_{ij}^{low} , unexpectedly, a positive effect on choice of movies is seen, but it is negative for GoodReads and statistically insignificant for the other contexts. In terms of age A_{ij} and n_{ij} , activities that are newer and those with more reviews are more likely to be selected across the four contexts. Considering now the effect of dissimilarity D_{ij} , the coefficients of both $\Delta\mathbf{x}_{ij}$ and ΔL_{ij} are negative and significant implying that individuals tend to select consecutive activities that are quite similar to each other (in attributes for movies and books and distance for attractions and restaurants). Finally, the exclusion restriction variable M_{ij} representing the market thickness has a significantly negative effect: the more options available, the less likely the focal activity will be chosen. This is the case for all contexts except GoodReads, which has a positive effect on choice.

Table EC.6 Descriptive statistics for the sample used for first stage for IMDb.

	q_{ij}	q_{ij}^{low}	A_{ij}	n_{ij}	$\Delta\mathbf{x}_{ij}$	M_{ij}
q_{ij}	1.00	-0.29	0.09	0.11	-0.03	0.03
q_{ij}^{low}	-0.29	1.00	-0.11	-0.07	0.02	0.00
A_{ij}	0.09	-0.11	1.00	0.01	0.02	0.02
n_{ij}	0.11	-0.07	0.01	1.00	-0.05	-0.09
$\Delta\mathbf{x}_{ij}$	-0.03	0.02	0.02	-0.05	1.00	-0.15
M_{ij}	0.03	0.00	0.02	-0.09	-0.15	1.00
Mean	0.44	0.05	0.55	9.94	0.75	6.29
Min.	0.01	0.00	0.00	1.79	0.00	0.00
Max.	0.98	0.92	0.69	14.75	1.00	10.33
Std. dev.	0.09	1.00	0.17	1.81	0.29	1.97

Table EC.7 Descriptive statistics for the sample used for first stage for GoodReads.

	q_{ij}	q_{ij}^{low}	A_{ij}	n_{ij}	$\Delta\mathbf{x}_{ij}$	M_{ij}
q_{ij}	1.00	-0.19	0.04	-0.21	0.02	0.12
q_{ij}^{low}	-0.19	1.00	-0.01	0.05	-0.01	-0.03
A_{ij}	0.04	-0.01	1.00	-0.14	0.05	0.13
n_{ij}	-0.21	0.05	-0.14	1.00	-0.09	-0.24
$\Delta\mathbf{x}_{ij}$	0.02	-0.01	0.05	-0.09	1.00	0.21
M_{ij}	0.12	-0.03	0.13	-0.24	0.21	1.00
Mean	0.40	0.02	0.25	4.62	-25.76	8.18
Min.	0.00	0.00	0.00	1.10	-99.00	1.39
Max.	0.98	0.89	4.61	12.48	1.00	10.63
Std. dev.	0.19	0.05	0.26	1.47	32.19	1.27

EC.4.2. Second-Stage Estimation Model

In our second-stage estimation model, we use a modified version of (2) that accounts for the selection bias:

$$\begin{aligned}
 \mathbb{E}[r_{ij} = 1 | s_{ij} = 1] &= \mathbb{E}[r_{ij} = 1 | v_{ij} > v^0] \\
 &= \mathbb{E}[\eta q_{ij} + \zeta c_i + \alpha a_{i,j-1} + \beta b_{i,j-1} + \varepsilon_{ij} | \bar{v}_{ij} + \epsilon_{ij} > v^0] \\
 &= \eta q_{ij} + \zeta c_i + \alpha a_{i,j-1} + \beta b_{i,j-1} + \mathbb{E}[\varepsilon_{ij} | \epsilon_{ij} > v^0 - \bar{v}_{ij}].
 \end{aligned}$$

Table EC.8 Descriptive statistics for the sample used for first stage for TripAdvisor Attractions.

	q_{ij}	q_{ij}^{low}	A_{ij}	n_{ij}	ΔL_{ij}	M_{ij}
q_{ij}	1.00	-0.15	0.06	0.17	-0.01	0.02
q_{ij}^{low}	-0.15	1.00	-0.02	-0.04	-0.01	0.01
A_{ij}	0.06	-0.02	1.00	0.45	-0.01	0.05
n_{ij}	0.17	-0.04	0.45	1.00	-0.04	0.14
ΔL_{ij}	-0.01	-0.01	-0.01	-0.04	1.00	-0.07
M_{ij}	0.02	0.01	0.05	0.14	-0.07	1.00
Mean	0.57	0.02	0.23	3.64	7,513.84	2.81
Min.	0.01	0.00	0.00	1.10	0.00	0.00
Max.	0.99	1.00	4.13	10.34	20,034.16	6.48
Std. dev.	0.19	0.08	0.17	1.49	5,315.89	1.17

Table EC.9 Descriptive statistics for the sample used for first stage for TripAdvisor Restaurants.

	q_{ij}	q_{ij}^{low}	A_{ij}	n_{ij}	ΔL_{ij}	M_{ij}
q_{ij}	1.00	-0.10	0.02	0.18	-0.01	0.10
q_{ij}^{low}	-0.10	1.00	-0.00	-0.02	-0.00	-0.01
A_{ij}	0.02	-0.00	1.00	0.37	-0.00	0.14
n_{ij}	0.18	-0.02	0.37	1.00	-0.03	0.36
ΔL_{ij}	-0.01	-0.00	-0.00	-0.03	1.00	-0.01
M_{ij}	0.10	-0.01	0.14	0.36	-0.01	1.00
Mean	0.56	0.02	0.23	3.24	6,338.62	1.94
Min.	0.03	0.00	0.00	1.10	0.00	0.69
Max.	0.99	1.00	4.39	8.92	20,034.71	4.19
Std. dev.	0.17	0.08	0.17	1.02	5,374.16	0.90

Given that the errors ϵ_{ij} and v_{ij} are correlated, one needs the following Heckman correction (Heckman 1979).

$$\mathbb{E}[\varepsilon_{ij} | \epsilon_{ij} > v^0 - \bar{v}_{ij}] = \rho \sigma_\varepsilon h(v^0 - \bar{v}_{ij}), \quad (\text{EC.8})$$

in which ρ is the correlation between the unobserved determinants of consumer i 's selection of the focal activity (ϵ_{ij}) and the unobserved determinants of their rating of the focal activity (ε_{ij}), σ_ε is the standard deviation of ε_{ij} , and $h(x) := \frac{\phi(x)}{1 - \Phi(x)}$ is the hazard rate of a normal distribution, also known as inverse Mills ratio.

Tables EC.11, EC.12, EC.13, and EC.14 report the second-stage estimation models with the sample selection correction for IMDb, GoodReads, TripAdvisor-Attractions, and TripAdvisor-Restaurants, respectively. As discussed in §5.3, when the variables $a_{i,j-1}$ and $b_{i,j-1}$ are considered in isolation, and not jointly, the results align with Hypotheses 1-2 for IMDb, but suggest a suppression effect in the other three contexts given that both $a_{i,j-1}$ and $b_{i,j-1}$ show a positive effect when considered in isolation.

EC.4.3. Instrument Validity Tests

In assessing the validity of our instrumental variable approach within the Heckman selection model, we conducted two crucial tests across four experiential goods contexts: movie ratings (IMDb), book ratings (GoodReads), attraction ratings (TripAdvisor), and restaurant ratings (TripAdvisor). First, we performed a weak instrument test to evaluate the strength and relevance of our chosen instrumental variable in the

Table EC.10 Estimation results for first-stage activity selection stage.

	IMDb	GoodReads	TripAdvisor-Attractions	TripAdvisor-Restaurants
(Intercept)	−1.97*** (0.01)	−4.03*** (0.00)	3.45*** (0.03)	4.08*** (0.05)
q_{ij}	0.06*** (0.00)	0.16*** (0.02)	−0.03 (0.01)	−0.05 (0.06)
q_{ij}^{low}	0.49*** (0.01)	−0.03** (0.01)	0.08 (0.09)	0.03 (0.15)
A_{ij}	−1.91*** (0.00)	−0.93*** (0.00)	−0.17*** (0.04)	−0.19** (0.06)
n_{ij}	0.21*** (0.00)	0.30*** (0.00)	0.12*** (0.00)	0.08*** (0.01)
$\Delta \mathbf{x}_{ij}$	−0.64*** (0.00)	−0.01*** (0.00)		
ΔL_{ij}			−4.25*** (0.02)	−4.51*** (0.03)
M_{ij}	−0.03*** (0.00)	0.05*** (0.00)	−0.30*** (0.01)	−0.42*** (0.02)
AIC	2,191,588.07	6,474,463.66	13,108.41	6,047.06
Log Likelihood	−1,095,787.03	−3,237,224.83	−6,547.20	−3,016.53
Num. obs.	4,731,992	29,591,898	19,454,850	12,790,721
Correlation with IMR				
$a_{i,j-1}$	−0.03	0.00	−0.00	0.01
$b_{i,j-1}$	−0.06	−0.01	0.00	0.03

*** $p < 0.01$; ** $p < 0.05$; * $p < 0.1$

selection equation. This test is essential because weak instruments can lead to biased and inconsistent estimates in the second stage of the model. We employed a likelihood ratio test comparing models with and without the instrument, alongside calculating the pseudo R-squared to assess model fit. The results were consistently strong across all contexts: likelihood ratio tests were highly significant (p-value < 0.01) for all four domains, indicating that the inclusion of the instrument significantly improved model fit. Pseudo R-squared values varied, with moderate explanatory power for IMDb (0.15) and GoodReads (0.23), and exceptionally high values for TripAdvisor attractions (0.98) and restaurants (0.99). These results provide robust evidence for the relevance and strength of our chosen instruments across the four contexts.

Subsequently, we conducted an endogeneity test to determine the presence and significance of selection bias, which justifies the use of the Heckman selection model. This test examines whether the error terms in the selection and outcome equations are correlated, represented by ρ in (EC.8). We estimated ρ using a two-stage approach and performed a Wald test to check its statistical significance. The results revealed heterogeneity across contexts: GoodReads ($\rho = 0.40$, p-value < 0.01) and TripAdvisor Restaurants ($\rho = 0.15$, p-value < 0.01) exhibited significant selection bias, while IMDb ($\rho = 0.61$, p-value = 0.65) and TripAdvisor Attractions ($\rho = 0.20$, p-value = 0.91) did not show significant evidence of selection bias. The significant selection bias for book and restaurant ratings underscores the necessity of using the Heckman selection model in these contexts to obtain unbiased estimates, in contrast to the movie and touristic attractions contexts, where a simpler probit model for the outcome equation might suffice (as also seen in Table 3).

Table EC.11 Estimation results of models (3) and (6) for (2), with (4)-(5), using the IMDb data sample with sample selection correction.

	IMDb					
	1	2	3	4	5	6
λ_a			0.10	—	0.10	0.10
λ_b			—	0.10	0.10	0.10
(Intercept)	−1.56*** (0.02)	−1.61*** (0.10)	−2.45*** (0.10)	−1.58*** (0.10)	−2.06*** (0.10)	−2.01*** (0.18)
c_i	0.96*** (0.01)	0.95*** (0.01)	0.42*** (0.01)	0.95*** (0.01)	0.28*** (0.01)	0.28*** (0.01)
q_{ij}	3.16*** (0.03)	3.07*** (0.03)	2.96*** (0.03)	3.08*** (0.03)	3.16*** (0.03)	3.17*** (0.03)
j		−0.10*** (0.00)	−0.05*** (0.01)	−0.10*** (0.01)	−0.04*** (0.01)	−0.04*** (0.01)
n_{ij}		0.03*** (0.01)	0.03*** (0.01)	0.03*** (0.01)	0.03*** (0.01)	0.03*** (0.01)
A_{ij}		0.10** (0.05)	−0.10** (0.05)	−0.10 (0.05)	0.10 (0.05)	0.00 (0.10)
$a_{i,j-1}$			1.73*** (0.03)		2.36*** (0.03)	2.59*** (0.07)
$b_{i,j-1}$				−0.09*** (0.05)	−1.99*** (0.06)	−2.35*** (0.16)
Δt_{ij}						−0.04** (0.01)
$\Delta \mathbf{x}_{ij}$						−0.01 (0.09)
$a_{i,j-1} \times \Delta t_{ij}$						−0.02 (0.02)
$a_{i,j-1} \times \Delta \mathbf{x}_{ij}$						−0.28*** (0.09)
$b_{i,j-1} \times \Delta t_{ij}$						0.11*** (0.04)
$b_{i,j-1} \times \Delta \mathbf{x}_{ij}$				(0.20)		0.29
IMR	−0.10*** (0.01)	−0.05* (0.03)	0.05* (0.03)	−0.05* (0.03)	0.01 (0.03)	0.02 (0.07)
AIC	87,700.46	87,246.52	83,025.74	87,227.75	82,051.27	82,045.29
Log Likelihood	−43,846.23	−43,616.26	−41,504.87	−43,605.87	−41,016.63	−41,007.64
Num. obs.	85,589	85,589	85,589	85,589	85,589	85,589

*** $p < 0.01$; ** $p < 0.05$; * $p < 0.1$

EC.5. Simplified Estimation for Counterfactuals

Here, we omit the first-stage correction and control variables from the specification to estimate affect and baseline effects that are utilized in our counterfactual analyses given in §6. See Table EC.15.

Table EC.12 Estimation results of models (3) and (6) for (2), with (4)-(5), using the GoodReads data sample with sample selection correction.

	GoodReads					
	1	2	3	4	5	6
λ_a			0.10	—	0.10	0.10
λ_b			—	0.10	0.10	0.10
(Intercept)	−1.39*** (0.01)	−0.86*** (0.03)	−2.71*** (0.03)	−0.99*** (0.03)	−2.02*** (0.03)	−2.02*** (0.05)
c_i	0.70*** (0.00)	0.69*** (0.00)	0.14*** (0.01)	0.69*** (0.00)	0.12*** (0.00)	0.12*** (0.00)
q_{ij}	2.93*** (0.01)	2.92*** (0.01)	2.58*** (0.01)	2.89*** (0.01)	2.85*** (0.01)	2.85*** (0.01)
j		−0.16*** (0.00)	−0.03*** (0.00)	−0.15*** (0.00)	−0.03*** (0.00)	−0.02*** (0.00)
n_{ij}		0.00 (0.00)	0.07*** (0.00)	0.01*** (0.00)	0.05*** (0.00)	0.04*** (0.00)
A_{ij}		−0.19*** (0.01)	−0.25*** (0.01)	−0.20*** (0.01)	−0.20*** (0.01)	−0.21*** (0.01)
$a_{i,j-1}$			2.26*** (0.01)		2.55*** (0.01)	2.83*** (0.04)
$b_{i,j-1}$				0.25*** (0.01)	−1.85*** (0.03)	−2.40*** (0.12)
Δt_{ij}						0.03*** (0.01)
$\Delta \mathbf{x}_{ij}$						−0.12*** (0.03)
$a_{i,j-1} \times \Delta t_{ij}$						−0.06*** (0.01)
$a_{i,j-1} \times \Delta \mathbf{x}_{ij}$						−0.26*** (0.03)
$b_{i,j-1} \times \Delta t_{ij}$						0.09*** (0.03)
$b_{i,j-1} \times \Delta \mathbf{x}_{ij}$						0.72*** (0.08)
IMR	−0.06*** (0.00)	−0.03*** (0.01)	0.13*** (0.01)	−0.02*** (0.01)	0.06*** (0.01)	0.03*** (0.01)
AIC	571,152.91	569,005.28	504,799.20	568,626.45	501,083.66	500,661.74
Log Likelihood	−285,572.46	−284,495.64	−252,391.60	−284,305.22	−250,532.83	−250,315.87
Num. obs.	500,493	500,493	500,493	500,493	500,493	500,493

*** $p < 0.01$; ** $p < 0.05$; * $p < 0.1$

EC.6. Robustness Checks

In this section, we present several robustness checks. First, Table EC.16 presents the estimation results when selection effects are ignored in the contexts of IMDb, GoodReads, TripAdvisor-Attractions, and TripAdvisor-Restaurants, respectively. The coefficients of $a_{i,j-1}$ and $b_{i,j-1}$ are all significant, within 0.1 of the values depicted in Table 3. The moderation effects of attributes and time also remain similar.

Second, Table EC.17 shows the second-stage estimation results after excluding users who posted multiple reviews on the same day. This exclusion does not affect the results on GoodReads. For the other contexts, the coefficients of $a_{i,j-1}$ and $b_{i,j-1}$ are all significant and of the same sign as in Table 3.

Third, Tables EC.18 and EC.19 present the second-stage estimation results after excluding, respectively, the least active reviewers and the least reviewed activities from the analyses. The former allows us to consider only the top-quartile most active users, potentially helping us address the issue of censoring with infrequent users. The latter focuses only on the top-quartile activities in terms of their reviews to potentially estimate reliable q_{ij} effects on reviewer ratings. In both cases, the coefficients of $a_{i,j-1}$ and $b_{i,j-1}$ are all significant and of the same sign as in Table 3.

Table EC.13 Estimation results of models (3) and (6) for (2), with (4)-(5), using the TripAdvisor-Attractions data sample with sample selection correction.

	TripAdvisor - Attractions					
	1	2	3	4	5	6
λ_a			0.20	—	0.15	0.15
λ_b			—	0.05	0.15	0.15
(Intercept)	−1.32*** (0.02)	−1.25*** (0.05)	−2.11*** (0.05)	−1.35*** (0.05)	−1.65*** (0.06)	−1.53*** (0.14)
c_i	0.77*** (0.01)	0.77*** (0.01)	0.35*** (0.01)	0.77*** (0.01)	0.21*** (0.01)	0.21*** (0.01)
q_{ij}	2.43*** (0.03)	2.38*** (0.03)	2.30*** (0.03)	2.37*** (0.03)	2.42*** (0.03)	2.42*** (0.03)
j		−0.05*** (0.01)	−0.01 (0.01)	−0.06*** (0.01)	−0.01 (0.01)	−0.00 (0.01)
n_{ij}		0.01** (0.00)	0.01** (0.00)	0.01** (0.00)	0.01*** (0.00)	0.01*** (0.00)
A_{ij}		0.24*** (0.03)	0.18*** (0.03)	0.24*** (0.03)	0.16*** (0.03)	0.14*** (0.03)
$a_{i,j-1}$			1.46*** (0.02)		1.82*** (0.03)	2.11*** (0.08)
$b_{i,j-1}$				0.19*** (0.03)	−1.26*** (0.07)	−1.76*** (0.24)
Δt_{ij}						−0.01 (0.02)
$\Delta \mathbf{x}_{ij}$						−0.12 (0.14)
$a_{i,j-1} \times \Delta t_{ij}$						−0.07*** (0.01)
$a_{i,j-1} \times \Delta \mathbf{x}_{ij}$						−0.20** (0.09)
$b_{i,j-1} \times \Delta t_{ij}$						0.11*** (0.03)
$b_{i,j-1} \times \Delta \mathbf{x}_{ij}$						0.39 (0.25)
IMR	−0.92 (0.83)	0.62 (0.88)	−0.18 (0.89)	0.61 (0.88)	0.03 (0.90)	0.09 (0.90)
AIC	91,081.23	90,962.11	86,853.11	90,908.85	86,514.76	86,445.46
Log Likelihood	−45,536.62	−45,474.06	−43,418.56	−45,446.42	−43,248.38	−43,207.73
Num. obs.	74,306	74,306	74,306	74,306	74,306	74,306

*** $p < 0.01$; ** $p < 0.05$; * $p < 0.1$

Fourth, Table EC.20 presents results for when the analyses are carried out by controlling for the most representative genre/tag of the given activity, which we consider to be the first listed genre/tag in the case of movies, attractions, and restaurants, and the one that received the highest votes by readers on GoodReads in the case of books. Here again, the coefficients of $a_{i,j-1}$ and $b_{i,j-1}$ are all significant and of the same sign as in Table 3.

Fifth, Table EC.21 presents the second-stage estimation results controlling for the geographical distance in locations between the focal and former activity, ΔL_{ij} , while Table EC.22 includes the city-level fixed effects. As before, the coefficients of $a_{i,j-1}$ and $b_{i,j-1}$ are all significant and of the same sign as in Table 3. Distance appears to diminish the role of affect on consumer satisfaction, but has no significant impact on quality contrast.

Sixth, Table EC.23 presents the second-stage estimation results when $\lambda_a = \lambda_b = 1$. Here again, the coefficients of $a_{i,j-1}$ and $b_{i,j-1}$ are all significant and of the same sign as in Table 3.

Seventh, Tables EC.24 and EC.25 present the second-stage estimation results in the presence of nonlinearities in (i) affect and baseline quality reference (Table EC.24) and (ii) time and attribute dissimilarity

Table EC.14 Estimation results of models (3) and (6) for (2), with (4)-(5), using the TripAdvisor-Restaurants data sample with sample selection correction.

	TripAdvisor - Restaurants					
	1	2	3	4	5	6
λ_a			0.15	—	0.20	0.20
λ_b			—	0.10	0.15	0.15
(Intercept)	−1.17*** (0.01)	−0.84*** (0.03)	−1.63*** (0.03)	−0.87*** (0.03)	−1.37*** (0.04)	−1.46*** (0.07)
c_i	0.90*** (0.01)	0.89*** (0.01)	0.33*** (0.01)	0.89*** (0.01)	0.28*** (0.01)	0.28*** (0.01)
q_{ij}	2.05*** (0.02)	2.07*** (0.02)	1.94*** (0.02)	2.06*** (0.02)	2.03*** (0.02)	2.01*** (0.02)
j		−0.12*** (0.01)	−0.06*** (0.01)	−0.12*** (0.01)	−0.05*** (0.01)	−0.04*** (0.01)
n_{ij}		−0.01*** (0.00)	−0.02*** (0.00)	−0.02*** (0.00)	−0.01** (0.00)	−0.00 (0.00)
A_{ij}		0.44*** (0.02)	0.28*** (0.02)	0.44*** (0.02)	0.28*** (0.02)	0.24*** (0.02)
$a_{i,j-1}$			1.64*** (0.02)		1.80*** (0.02)	1.96*** (0.05)
$b_{i,j-1}$				0.09*** (0.02)	−0.92*** (0.04)	−1.15*** (0.13)
Δt_{ij}						0.03*** (0.01)
$\Delta \mathbf{x}_{ij}$						−0.03 (0.07)
$a_{i,j-1} \times \Delta t_{ij}$						−0.01 (0.01)
$a_{i,j-1} \times \Delta \mathbf{x}_{ij}$						−0.17*** (0.06)
$b_{i,j-1} \times \Delta t_{ij}$						0.02 (0.02)
$b_{i,j-1} \times \Delta \mathbf{x}_{ij}$						0.25* (0.14)
IMR	10.54*** (1.67)	7.62*** (1.70)	7.00*** (1.74)	7.64*** (1.70)	7.51*** (1.74)	7.99*** (1.75)
AIC	189,579.90	188,887.83	180,078.42	188,864.31	179,541.98	179,116.38
Log Likelihood	−94,785.95	−94,436.91	−90,031.21	−94,424.16	−89,761.99	−89,543.19
Num. obs.	151,891	151,891	151,891	151,891	151,891	151,891

*** $p < 0.01$; ** $p < 0.05$; * $p < 0.1$

(Table EC.25), considering different quartiles of these variables. Figure EC.2 plots the nonlinear coefficients of affect and baseline quality. The effects appear monotone, and indeed mostly linear.

Table EC.15 Estimation results of models (3) and (6) for (2), with (4)-(5),

	IMDb	GoodReads	TripAdvisor-Attractions	TripAdvisor-Restaurants
λ_a	0.10	0.10	0.15	0.15
λ_b	0.10	0.10	0.15	0.15
(Intercept)	-1.95*** (0.02)	-1.71*** (0.01)	-1.51*** (0.03)	-1.50*** (0.01)
c_i	0.28*** (0.01)	0.13*** (0.00)	0.21*** (0.01)	0.26*** (0.01)
q_{ij}	3.37*** (0.03)	2.90*** (0.01)	2.25*** (0.03)	1.84*** (0.01)
$a_{i,j-1}$	2.41*** (0.03)	2.40*** (0.01)	1.81*** (0.03)	1.86*** (0.01)
$b_{i,j-1}$	-1.97*** (0.06)	-1.81*** (0.02)	-1.17*** (0.06)	-0.78*** (0.03)
AIC	88,426.89	1,743,052.79	97,234.24	296,496.29
Log Likelihood	-44,208.44	-871,521.40	-48,612.12	-148,243.14
Num. obs.	97,704	1,737,552	83,882	257,193

*** $p < 0.01$; ** $p < 0.05$; * $p < 0.1$ **Table EC.16** Estimation results of models (3) and (6) for (2), with (4)-(5), across all four contexts without sample selection correction.

	IMDb		GoodReads		TA-Attractions		TA-Restaurants	
λ_a	0.10	0.10	0.10	0.10	0.15	0.15	0.20	0.20
λ_b	0.10	0.10	0.10	0.10	0.15	0.15	0.15	0.15
	1	2	3	4	5	6	7	8
(Intercept)	-2.20*** (0.04)	-2.14*** (0.07)	-1.78*** (0.01)	-1.84*** (0.03)	-1.50*** (0.05)	-1.40*** (0.14)	-1.34*** (0.03)	-1.28*** (0.06)
c_i	0.28*** (0.01)	0.28*** (0.01)	0.13*** (0.00)	0.13*** (0.00)	0.21*** (0.01)	0.21*** (0.01)	0.27*** (0.01)	0.27*** (0.01)
q_{ij}	3.25*** (0.03)	3.25*** (0.03)	2.88*** (0.01)	2.88*** (0.01)	2.23*** (0.03)	2.22*** (0.03)	1.87*** (0.01)	1.84*** (0.01)
j	-0.04*** (0.00)	-0.04*** (0.01)	-0.02*** (0.00)	-0.01*** (0.00)	-0.01 (0.01)	-0.00 (0.01)	-0.04*** (0.01)	-0.03*** (0.01)
n_{ij}	0.04*** (0.00)	0.04*** (0.00)	0.02*** (0.00)	0.02*** (0.00)	0.00 (0.00)	0.00 (0.00)	-0.03*** (0.00)	-0.02*** (0.00)
A_{ij}	0.04 (0.02)	0.04 (0.02)	-0.04*** (0.00)	-0.06*** (0.00)	0.13*** (0.03)	0.11*** (0.03)	0.26*** (0.01)	0.22*** (0.01)
$a_{i,j-1}$	2.37*** (0.03)	2.59*** (0.07)	2.40*** (0.01)	2.77*** (0.02)	1.80*** (0.03)	2.11*** (0.08)	1.83*** (0.01)	1.98*** (0.05)
$b_{i,j-1}$	-2.02*** (0.06)	-2.37*** (0.15)	-1.76*** (0.02)	-2.35*** (0.07)	-1.17*** (0.06)	-1.70*** (0.23)	-0.76*** (0.03)	-1.11*** (0.12)
Δt_{ij}		-0.03** (0.01)		0.03*** (0.01)		-0.00 (0.02)		0.03*** (0.01)
$\Delta \mathbf{x}_{ij}$		-0.01 (0.07)		-0.14*** (0.01)		-0.13 (0.14)		-0.19*** (0.06)
$a_{i,j-1} \times \Delta t_{ij}$		-0.02 (0.02)		-0.08*** (0.01)		-0.07*** (0.01)		-0.02*** (0.01)
$a_{i,j-1} \times \Delta \mathbf{x}_{ij}$		-0.27*** (0.09)		-0.24*** (0.01)		-0.22*** (0.08)		-0.11** (0.05)
$b_{i,j-1} \times \Delta t_{ij}$		0.10** (0.04)		0.10*** (0.02)		0.09*** (0.03)		0.02 (0.01)
$b_{i,j-1} \times \Delta \mathbf{x}_{ij}$		0.30 (0.19)		0.67*** (0.05)		0.43* (0.24)		0.34*** (0.13)
AIC	88,224.43	88,219.49	1,741,779.12	1,740,682.53	97,211.76	97,111.68	296,146.18	295,528.88
Log Likelihood	-44,104.21	-44,095.74	-870,881.56	-870,327.26	-48,597.88	-48,541.84	-148,065.09	-147,750.44
Num. obs.	97,704	97,704	1,737,552	1,737,552	83,882	83,882	257,193	257,193

*** $p < 0.01$; ** $p < 0.05$; * $p < 0.1$

Table EC.17 Estimation results of models (3) and (6) for (2), with (4)-(5), and only considering users who never posted more than one review on a given day.

	IMDb		GoodReads		TripAdvisor-Attractions		TripAdvisor-Restaurants	
λ_a	0.10	0.10	0.10	0.10	0.20	0.20	0.15	0.15
λ_b	0.10	0.10	0.10	0.10	0.15	0.15	0.15	0.15
	1	2	3	4	5	6	7	8
(Intercept)	-2.10*** (0.12)	-1.98*** (0.21)	-2.02*** (0.03)	-2.02*** (0.05)	-1.75*** (0.09)	-1.64*** (0.24)	-1.35*** (0.04)	-1.51*** (0.10)
c_i	0.31*** (0.02)	0.31*** (0.02)	0.12*** (0.00)	0.12*** (0.00)	0.30*** (0.02)	0.30*** (0.02)	0.29*** (0.01)	0.30*** (0.01)
q_{ij}	3.09*** (0.03)	3.09*** (0.03)	2.85*** (0.01)	2.85*** (0.01)	2.44*** (0.05)	2.43*** (0.05)	2.07*** (0.03)	2.06*** (0.03)
j	-0.04*** (0.01)	-0.05*** (0.01)	-0.03*** (0.00)	-0.02*** (0.00)	-0.00 (0.02)	0.01 (0.02)	-0.05*** (0.01)	-0.03*** (0.01)
n_{ij}	0.03*** (0.01)	0.04*** (0.01)	0.05*** (0.00)	0.04*** (0.00)	0.00 (0.00)	0.00 (0.00)	-0.01** (0.00)	-0.01 (0.00)
A_{ij}	-0.02 (0.06)	-0.06 (0.12)	-0.20*** (0.01)	-0.21*** (0.01)	0.17*** (0.05)	0.14*** (0.05)	0.28*** (0.02)	0.24*** (0.02)
$a_{i,j-1}$	2.28*** (0.04)	2.46*** (0.10)	2.55*** (0.01)	2.83*** (0.04)	1.57*** (0.04)	2.05*** (0.16)	1.79*** (0.02)	2.08*** (0.08)
$b_{i,j-1}$	-1.83*** (0.08)	-2.28*** (0.23)	-1.85*** (0.03)	-2.40*** (0.12)	-0.85*** (0.09)	-1.61*** (0.41)	-0.92*** (0.05)	-1.23*** (0.20)
Δt_{ij}		-0.06*** (0.02)		0.03*** (0.01)		0.02 (0.03)		0.05*** (0.01)
$\Delta \mathbf{x}_{ij}$		-0.04 (0.11)		-0.12*** (0.03)		-0.23 (0.22)		-0.09 (0.09)
$a_{i,j-1} \times \Delta t_{ij}$		-0.01 (0.02)		-0.06*** (0.01)		-0.08*** (0.02)		-0.01 (0.01)
$a_{i,j-1} \times \Delta \mathbf{x}_{ij}$		-0.20* (0.11)		-0.26*** (0.03)		-0.21 (0.15)		-0.29*** (0.07)
$b_{i,j-1} \times \Delta t_{ij}$		0.13** (0.06)		0.09*** (0.03)		0.07 (0.05)		-0.02 (0.03)
$b_{i,j-1} \times \Delta \mathbf{x}_{ij}$		0.25 (0.25)		0.72*** (0.08)		0.54 (0.40)		0.41** (0.20)
IMR	0.05 (0.04)	0.07 (0.08)	0.06*** (0.01)	0.03*** (0.01)	2.26* (1.35)	2.24* (1.35)	8.47*** (2.13)	8.81*** (2.14)
AIC	58,718.54	58,714.67	501,083.66	500,661.74	36,895.17	36,854.22	118,513.02	118,293.40
Log Likelihood	-29,350.27	-29,342.34	-250,532.83	-250,315.87	-18,438.59	-18,412.11	-59,247.51	-59,131.70
Num. obs.	60,628	60,628	500,493	500,493	31,581	31,581	100,791	100,791

*** $p < 0.01$; ** $p < 0.05$; * $p < 0.1$

Table EC.18 Estimation results of models (3) and (6) for (2), with (4)-(5), considering top-25th percentile reviewers.

	IMDb		GoodReads		TripAdvisor-Attractions		TripAdvisor-Restaurants	
λ_a	0.10	0.10	0.10	0.10	0.20	0.15	0.10	0.10
λ_b	0.10	0.10	0.10	0.10	0.15	0.15	0.10	0.10
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(Intercept)	-2.45*** (0.13)	-2.20*** (0.23)	-2.14*** (0.04)	-2.00*** (0.07)	-1.85*** (0.07)	-1.75*** (0.18)	-1.75*** (0.05)	-1.85*** (0.10)
c_i	0.28*** (0.02)	0.28*** (0.02)	0.13*** (0.01)	0.13*** (0.01)	0.24*** (0.02)	0.23*** (0.02)	0.17*** (0.01)	0.17*** (0.01)
q_{ij}	3.29*** (0.04)	3.29*** (0.04)	2.88*** (0.02)	2.88*** (0.02)	2.41*** (0.04)	2.41*** (0.04)	1.96*** (0.03)	1.95*** (0.03)
j	-0.04*** (0.01)	-0.04*** (0.01)	0.01 (0.00)	0.01 (0.00)	0.03* (0.02)	0.03* (0.02)	0.05*** (0.01)	0.05*** (0.01)
n_{ij}	0.07*** (0.01)	0.05*** (0.01)	0.05*** (0.00)	0.04*** (0.00)	0.02*** (0.00)	0.02*** (0.00)	0.01* (0.00)	0.01*** (0.00)
A_{ij}	-0.04 (0.06)	0.11 (0.14)	-0.17*** (0.02)	-0.16*** (0.02)	0.11** (0.04)	0.09** (0.04)	0.21*** (0.03)	0.19*** (0.03)
$a_{i,j-1}$	2.52*** (0.04)	2.78*** (0.09)	2.62*** (0.01)	2.81*** (0.06)	1.87*** (0.03)	2.18*** (0.11)	2.08*** (0.03)	2.20*** (0.07)
$b_{i,j-1}$	-2.24*** (0.08)	-2.56*** (0.20)	-1.99*** (0.04)	-2.57*** (0.17)	-1.19*** (0.08)	-1.66*** (0.30)	-1.19*** (0.06)	-1.28*** (0.19)
Δt_{ij}		-0.00 (0.02)		0.00 (0.02)		-0.04 (0.02)		0.03** (0.01)
$\Delta \mathbf{x}_{ij}$		0.02 (0.11)		-0.15*** (0.03)		0.00 (0.18)		0.02 (0.10)
$a_{i,j-1} \times \Delta t_{ij}$		-0.04 (0.03)		-0.03* (0.02)		-0.06*** (0.01)		0.01 (0.01)
$a_{i,j-1} \times \Delta \mathbf{x}_{ij}$		-0.30*** (0.11)		-0.21*** (0.03)		-0.25** (0.11)		-0.17** (0.08)
$b_{i,j-1} \times \Delta t_{ij}$		0.06 (0.06)		0.10* (0.05)		0.14*** (0.04)		-0.01 (0.03)
$b_{i,j-1} \times \Delta \mathbf{x}_{ij}$		0.37 (0.24)		0.76*** (0.10)		0.21 (0.32)		0.17 (0.22)
IMR	0.04 (0.04)	-0.06 (0.09)	0.06*** (0.01)	0.03** (0.01)	-1.62 (1.18)	-1.53 (1.18)	6.64*** (2.34)	7.06*** (2.35)
AIC	50,622.41	50,621.56	316,565.67	316,426.14	51,732.41	51,704.05	104,052.35	103,880.84
Log Likelihood	-25,302.20	-25,295.78	-158,273.84	-158,198.07	-25,857.21	-25,837.03	-52,017.17	-51,925.42
Num. obs.	54,585	54,585	321,801	321,801	44,622	44,622	88,283	88,283

Notes: Observations are at the individual-activity level. Standard errors are shown in parentheses. The table was shortened for brevity.

*** $p < 0.01$; ** $p < 0.05$; * $p < 0.10$.

Table EC.19 Estimation results of models (3) and (6) for (2), with (4)-(5), by only considering the top-25th percentile most rated activities only.

	IMDb		GoodReads		TripAdvisor-Attractions		TripAdvisor-Restaurants	
λ_a	0.10	0.10	0.10	0.10	0.15	0.15	0.20	0.20
λ_b	0.10	0.10	0.10	0.10	0.15	0.15	0.15	0.15
	1	2	3	4	5	6	7	8
(Intercept)	-1.98*** (0.10)	-1.86*** (0.18)	-1.90*** (0.04)	-1.89*** (0.06)	-1.71*** (0.07)	-1.46*** (0.15)	-1.39*** (0.04)	-1.42*** (0.08)
c_i	0.28*** (0.01)	0.28*** (0.01)	0.13*** (0.01)	0.13*** (0.01)	0.21*** (0.01)	0.21*** (0.01)	0.28*** (0.01)	0.29*** (0.01)
q_{ij}	3.19*** (0.03)	3.19*** (0.03)	2.92*** (0.02)	2.92*** (0.02)	2.58*** (0.04)	2.57*** (0.04)	2.16*** (0.03)	2.14*** (0.03)
j	-0.04*** (0.01)	-0.04*** (0.01)	-0.03*** (0.00)	-0.02*** (0.00)	0.00 (0.01)	0.00 (0.01)	-0.05*** (0.01)	-0.04*** (0.01)
n_{ij}	0.03*** (0.01)	0.02* (0.01)	0.04*** (0.00)	0.04*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	-0.01*** (0.00)	-0.01** (0.00)
A_{ij}	0.03 (0.05)	0.07 (0.11)	-0.27*** (0.02)	-0.28*** (0.02)	0.19*** (0.04)	0.17*** (0.04)	0.32*** (0.02)	0.29*** (0.02)
$a_{i,j-1}$	2.37*** (0.04)	2.60*** (0.07)	2.57*** (0.01)	2.81*** (0.05)	1.86*** (0.03)	2.06*** (0.09)	1.79*** (0.02)	1.94*** (0.06)
$b_{i,j-1}$	-2.03*** (0.07)	-2.40*** (0.16)	-1.97*** (0.04)	-2.46*** (0.14)	-1.42*** (0.07)	-2.03*** (0.26)	-0.99*** (0.05)	-1.32*** (0.15)
Δt_{ij}		-0.04** (0.02)		0.03** (0.01)		-0.01 (0.02)		0.02** (0.01)
$\Delta \mathbf{x}_{ij}$		0.01 (0.09)		-0.16*** (0.03)		-0.27* (0.15)		-0.09 (0.08)
$a_{i,j-1} \times \Delta t_{ij}$		-0.02 (0.02)		-0.05*** (0.01)		-0.07*** (0.01)		-0.00 (0.01)
$a_{i,j-1} \times \Delta \mathbf{x}_{ij}$		-0.29*** (0.09)		-0.21*** (0.03)		-0.11 (0.09)		-0.17*** (0.07)
$b_{i,j-1} \times \Delta t_{ij}$		0.12*** (0.04)		0.08* (0.04)		0.09*** (0.03)		0.02 (0.02)
$b_{i,j-1} \times \Delta \mathbf{x}_{ij}$		0.29 (0.20)		0.76*** (0.09)		0.55** (0.27)		0.38** (0.16)
IMR	-0.01 (0.03)	-0.03 (0.07)	0.02*** (0.01)	-0.00 (0.01)	0.20 (1.11)	0.30 (1.11)	6.25*** (1.94)	6.51*** (1.95)
AIC	78,769.24	78,762.70	347,488.69	347,243.77	72,047.81	72,008.87	136,994.62	136,687.66
Log Likelihood	-39,375.62	-39,366.35	-173,735.34	-173,606.88	-36,014.90	-35,989.43	-68,488.31	-68,328.83
Num. obs.	82,169	82,169	346,807	346,807	62,137	62,137	116,071	116,071

*** $p < 0.01$; ** $p < 0.05$; * $p < 0.1$

Table EC.20 Estimation results of models (3) and (6) for (2), with (4)-(5), including genre- and tag-level controls.

	IMDb		GoodReads		TripAdvisor-Attractions		TripAdvisor-Restaurants	
λ_a	0.10	0.10	0.10	0.10	0.15	0.15	0.20	0.15
λ_b	0.10	0.10	0.10	0.10	0.15	0.15	0.20	0.15
	1	2	3	4	5	6	7	8
c_i	0.28*** (0.02)	0.28*** (0.02)	0.12*** (0.01)	0.12*** (0.01)	0.21*** (0.02)	0.21*** (0.02)	0.29*** (0.01)	0.29*** (0.01)
q_{ij}	3.15*** (0.06)	3.15*** (0.05)	2.86*** (0.03)	2.85*** (0.03)	2.39*** (0.05)	2.38*** (0.05)	2.00*** (0.04)	1.98*** (0.04)
j	-0.06*** (0.01)	-0.05*** (0.01)	-0.03*** (0.00)	-0.02*** (0.00)	-0.00 (0.01)	0.00 (0.01)	-0.05*** (0.01)	-0.04*** (0.01)
n_{ij}	0.04*** (0.00)	0.04*** (0.01)	0.04*** (0.00)	0.04*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	-0.00 (0.00)	0.00 (0.00)
A_{ij}	0.09 (0.10)	0.03 (0.09)	-0.21*** (0.01)	-0.21*** (0.01)	0.17*** (0.03)	0.16*** (0.03)	0.26*** (0.04)	0.23*** (0.04)
$a_{i,j-1}$	2.38*** (0.08)	2.61*** (0.07)	2.55*** (0.02)	2.84*** (0.04)	1.87*** (0.04)	2.11*** (0.10)	1.82*** (0.03)	1.97*** (0.05)
$b_{i,j-1}$	-1.99*** (0.23)	-2.48*** (0.14)	-1.82*** (0.06)	-2.36*** (0.13)	-1.35*** (0.07)	-1.82*** (0.19)	-0.86*** (0.05)	-1.03*** (0.16)
Δt_{ij}		-0.03*** (0.01)		0.03** (0.01)		-0.01 (0.02)		0.03*** (0.01)
$\Delta \mathbf{x}_{ij}$		-0.12 (0.09)		-0.11*** (0.03)		-0.13 (0.11)		0.00 (0.07)
$a_{i,j-1} \times \Delta t_{ij}$		-0.01 (0.01)		-0.06*** (0.01)		-0.07*** (0.01)		-0.01 (0.01)
$a_{i,j-1} \times \Delta \mathbf{x}_{ij}$		-0.30*** (0.06)		-0.26*** (0.03)		-0.16 (0.10)		-0.17*** (0.06)
$b_{i,j-1} \times \Delta t_{ij}$		0.10*** (0.01)		0.09** (0.04)		0.10*** (0.03)		0.02 (0.02)
$b_{i,j-1} \times \Delta \mathbf{x}_{ij}$		0.50*** (0.18)		0.70*** (0.08)		0.36* (0.20)		0.19 (0.19)
IMR	-0.00 (0.04)	0.04 (0.08)	0.06*** (0.01)	0.03** (0.01)	1.14 (1.06)	1.20 (1.06)	5.01*** (1.73)	5.50*** (1.88)
<i>Genre/Tag FE</i>	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
AIC	64,791.10	64,786.42	500,989.60	500,571.60	76,192.83	76,148.05	176,984.50	176,570.90
Log Likelihood	-32,362.55	-32,354.21	-250,388.82	-250,173.80	-37,919.41	-37,891.02	-88,308.26	-88,095.45
No. obs.	68,275	68,275	500,484	500,484	65,444	65,444	149,747	149,747

*** $p < 0.01$; ** $p < 0.05$; * $p < 0.1$

Table EC.21 Estimation results of models (3) and (6) for (2), with (4)-(5), with the TripAdvisor data sample controlling for the geographical distance between locations.

	TripAdvisor-Attractions	TripAdvisor-Restaurants
λ_a	0.15	0.20
λ_b	0.15	0.15
	1	2
(Intercept)	-1.61*** (0.14)	-1.51*** (0.07)
c_i	0.21*** (0.01)	0.28*** (0.01)
q_{ij}	2.41*** (0.03)	2.01*** (0.02)
j	-0.00 (0.01)	-0.04*** (0.01)
n_{ij}	0.01*** (0.00)	-0.00 (0.00)
A_{ij}	0.14*** (0.03)	0.24*** (0.02)
$a_{i,j-1}$	2.15*** (0.09)	2.00*** (0.06)
$b_{i,j-1}$	-1.69*** (0.24)	-1.11*** (0.14)
Δt_{ij}	-0.04** (0.02)	0.03*** (0.01)
$\Delta \mathbf{x}_{ij}$	-0.12 (0.14)	0.01 (0.07)
ΔL_{ij}	0.04*** (0.01)	0.01 (0.01)
$a_{i,j-1} \times \Delta t_{ij}$	-0.06*** (0.01)	-0.00 (0.01)
$a_{i,j-1} \times \Delta \mathbf{x}_{ij}$	-0.20** (0.09)	-0.20*** (0.06)
$a_{i,j-1} \times \Delta L_{ij}$	-0.02** (0.01)	-0.01** (0.01)
$b_{i,j-1} \times \Delta t_{ij}$	0.13*** (0.03)	0.01 (0.02)
$b_{i,j-1} \times \Delta \mathbf{x}_{ij}$	0.39 (0.25)	0.20 (0.15)
$b_{i,j-1} \times \Delta L_{ij}$	-0.04 (0.02)	-0.00 (0.01)
IMR	0.39 (0.90)	8.03*** (1.75)
AIC	86,421.85	179,103.72
Log Likelihood	-43,192.93	-89,533.86
Num. obs.	74,306	151,891

*** $p < 0.01$; ** $p < 0.05$; * $p < 0.1$

Table EC.22 Estimation results of models (3) and (6) for (2), with (4)-(5), with the TripAdvisor data sample and the inclusion of city-level fixed effects.

	TripAdvisor-Attractions		TripAdvisor-Restaurants	
λ_a	0.15	0.15	0.20	0.20
λ_b	0.15	0.15	0.15	0.15
	1	2	3	4
c_i	0.26*** (0.02)	0.26*** (0.02)	0.34*** (0.01)	0.34*** (0.01)
q_{ij}	2.62*** (0.05)	2.61*** (0.05)	2.25*** (0.03)	2.22*** (0.03)
j	-0.00 (0.02)	0.00 (0.02)	-0.06*** (0.01)	-0.04*** (0.01)
n_{ij}	0.03*** (0.00)	0.03*** (0.00)	-0.00 (0.00)	0.00 (0.00)
A_{ij}	0.14*** (0.04)	0.12*** (0.04)	0.31*** (0.03)	0.28*** (0.03)
$a_{i,j-1}$	1.86*** (0.03)	2.05*** (0.09)	1.86*** (0.02)	2.02*** (0.06)
$b_{i,j-1}$	-1.12*** (0.09)	-1.79*** (0.26)	-0.86*** (0.05)	-1.03*** (0.16)
Δt_{ij}		-0.02 (0.02)		0.03** (0.01)
$\Delta \mathbf{x}_{ij}$		-0.27* (0.15)		0.03 (0.09)
$a_{i,j-1} \times \Delta t_{ij}$		-0.05*** (0.01)		0.00 (0.01)
$a_{i,j-1} \times \Delta \mathbf{x}_{ij}$		-0.14 (0.10)		-0.21*** (0.07)
$b_{i,j-1} \times \Delta t_{ij}$		0.09*** (0.04)		0.01 (0.02)
$b_{i,j-1} \times \Delta \mathbf{x}_{ij}$		0.58** (0.27)		0.20 (0.18)
IMR	3.50*** (1.22)	3.51*** (1.21)	2.08 (2.74)	3.03 (2.74)
AIC	82,459.06	82,422.31	173,359.20	173,032.60
Log Likelihood	-37,336.53	-37,312.16	-76,783.58	-76,614.31
Num. obs.	66,615	66,615	135,650	135,650

*** $p < 0.01$; ** $p < 0.05$; * $p < 0.1$

Table EC.23 Estimation results of models (3) and (6) for (2), with (4)-(5), and $\lambda_a = \lambda_b = 1$.

	IMDb		GoodReads		TripAdvisor-Attractions		TripAdvisor-Restaurants	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(Intercept)	-1.71*** (0.10)	-1.78*** (0.17)	-1.25*** (0.03)	-1.37*** (0.04)	-1.36*** (0.05)	-1.39*** (0.08)	-0.94*** (0.03)	-1.06*** (0.05)
c_i	0.85*** (0.01)	0.85*** (0.01)	0.56*** (0.00)	0.57*** (0.00)	0.66*** (0.01)	0.66*** (0.01)	0.79*** (0.01)	0.79*** (0.01)
q_{ij}	3.08*** (0.03)	3.08*** (0.03)	2.87*** (0.01)	2.87*** (0.01)	2.40*** (0.03)	2.39*** (0.03)	2.06*** (0.02)	2.04*** (0.02)
j	-0.09*** (0.00)	-0.10*** (0.01)	-0.13*** (0.00)	-0.12*** (0.00)	-0.04*** (0.01)	-0.04*** (0.01)	-0.11*** (0.01)	-0.09*** (0.01)
n_{ij}	0.03*** (0.01)	0.04*** (0.01)	0.02*** (0.00)	0.02*** (0.00)	0.01** (0.00)	0.01*** (0.00)	-0.01*** (0.00)	-0.01* (0.00)
A_{ij}	0.07 (0.05)	0.01 (0.10)	-0.21*** (0.01)	-0.22*** (0.01)	0.22*** (0.03)	0.20*** (0.03)	0.39*** (0.02)	0.36*** (0.02)
$a_{i,j-1}$	0.37*** (0.01)	0.67*** (0.03)	0.59*** (0.00)	0.68*** (0.02)	0.46*** (0.01)	0.66*** (0.04)	0.36*** (0.01)	0.49*** (0.02)
$b_{i,j-1}$	-0.24*** (0.03)	-0.51*** (0.07)	-0.20*** (0.01)	-0.41*** (0.05)	-0.28*** (0.03)	-0.45*** (0.11)	-0.16*** (0.02)	-0.35*** (0.06)
Δt_{ij}		-0.00 (0.01)		0.03*** (0.01)		0.01 (0.01)		0.02*** (0.00)
$\Delta \mathbf{x}_{ij}$		-0.01 (0.06)		0.02 (0.01)		-0.00 (0.06)		0.02 (0.04)
$a_{i,j-1} \times \Delta t_{ij}$		-0.01* (0.01)		-0.00 (0.00)		-0.05*** (0.00)		-0.01** (0.00)
$a_{i,j-1} \times \Delta \mathbf{x}_{ij}$		-0.38*** (0.04)		-0.20*** (0.01)		-0.14*** (0.04)		-0.14*** (0.03)
$b_{i,j-1} \times \Delta t_{ij}$		0.02 (0.02)		0.04*** (0.01)		0.04*** (0.01)		0.04*** (0.01)
$b_{i,j-1} \times \Delta \mathbf{x}_{ij}$		0.34*** (0.09)		0.18*** (0.03)		0.12 (0.11)		0.15** (0.07)
IMR	-0.04 (0.03)	-0.00 (0.07)	0.00 (0.01)	-0.00 (0.01)	0.56 (0.88)	0.67 (0.88)	7.21*** (1.71)	7.56*** (1.72)
AIC	86,200.83	86,113.06	548,940.54	548,329.46	89,002.17	88,880.39	186,359.98	185,849.07
Log Likelihood	-43,091.42	-43,041.53	-274,461.27	-274,149.73	-44,492.09	-44,425.19	-93,170.99	-92,909.53
Num. obs.	85,589	85,589	500,493	500,493	74,306	74,306	151,891	151,891

*** $p < 0.01$; ** $p < 0.05$; * $p < 0.1$

Table EC.24 Estimation results of models (3) and (6) for (2), with (4)-(5), and nonlinear affect and baseline quality effects.

	IMDb		GoodReads		TripAdvisor-Attractions		TripAdvisor-Restaurants	
λ_a	0.10	0.10	0.10	0.10	0.15	0.15	0.20	0.20
λ_b	0.10	0.10	0.10	0.10	0.15	0.15	0.15	0.15
	1	2	3	4	5	6	7	8
(Intercept)	-1.99*** (0.10)	-2.57*** (0.10)	-1.97*** (0.03)	-2.53*** (0.03)	-1.35*** (0.06)	-2.23*** (0.05)	-1.19*** (0.04)	-1.68*** (0.03)
c_i	0.46*** (0.01)	0.31*** (0.01)	0.23*** (0.00)	0.12*** (0.00)	0.32*** (0.01)	0.22*** (0.01)	0.39*** (0.01)	0.29*** (0.01)
q_{ij}	3.14*** (0.03)	3.12*** (0.03)	2.79*** (0.01)	2.78*** (0.01)	2.40*** (0.03)	2.40*** (0.03)	2.03*** (0.02)	2.02*** (0.02)
j	-0.05*** (0.00)	-0.04*** (0.01)	-0.05*** (0.00)	-0.03*** (0.00)	-0.02 (0.01)	-0.00 (0.01)	-0.06*** (0.01)	-0.05*** (0.01)
n_{ij}	0.03*** (0.01)	0.03*** (0.01)	0.04*** (0.00)	0.06*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	-0.01*** (0.00)	-0.01*** (0.00)
A_{ij}	0.05 (0.05)	-0.02 (0.05)	-0.23*** (0.01)	-0.21*** (0.01)	0.17*** (0.03)	0.16*** (0.03)	0.31*** (0.02)	0.28*** (0.02)
$a_{i,j-1}$		2.23*** (0.03)		2.47*** (0.01)		1.79*** (0.03)		1.78*** (0.02)
<i>Quart</i> $a_{i,j-1} = 2$	0.52*** (0.02)		0.41*** (0.01)		0.39*** (0.01)		0.39*** (0.01)	
<i>Quart</i> $a_{i,j-1} = 3$	0.84*** (0.02)		0.75*** (0.01)		0.66*** (0.02)		0.67*** (0.01)	
<i>Quart</i> $a_{i,j-1} = 4$	1.30*** (0.02)		1.42*** (0.01)		1.04*** (0.02)		1.04*** (0.01)	
$b_{i,j-1}$	-1.32*** (0.06)		-0.87*** (0.03)		-0.95*** (0.07)		-0.69*** (0.04)	
<i>Quart</i> $b_{i,j-1} = 2$		-0.13*** (0.01)		-0.09*** (0.01)		-0.11*** (0.01)		-0.08*** (0.01)
<i>Quart</i> $b_{i,j-1} = 3$		-0.30*** (0.02)		-0.15*** (0.01)		-0.19*** (0.01)		-0.13*** (0.01)
<i>Quart</i> $b_{i,j-1} = 4$		-0.49*** (0.02)		-0.33*** (0.01)		-0.25*** (0.01)		-0.22*** (0.01)
IMR	-0.01 (0.03)	0.03 (0.03)	0.05*** (0.01)	0.08*** (0.01)	0.19 (0.89)	0.00 (0.90)	7.39*** (1.73)	7.42*** (1.74)
AIC	83,197.20	82,190.64	518,139.74	501,945.84	87,351.20	86,572.41	181,153.04	179,634.27
Log Likelihood	-41,587.60	-41,084.32	-259,058.87	-250,961.92	-43,664.60	-43,275.21	-90,565.52	-89,806.14
Num. obs.	85,589	85,589	500,493	500,493	74,306	74,306	151,891	151,891

*** $p < 0.01$; ** $p < 0.05$; * $p < 0.1$

Figure EC.2 Estimated non-linear coefficients for $a_{i,j-1}$ and $b_{i,j-1}$ for IMDb (row 1), GoodReads (row 2), TripAdvisor-Attractions (row 3) and TripAdvisor-Restaurants (row 4) based on Table EC.24.

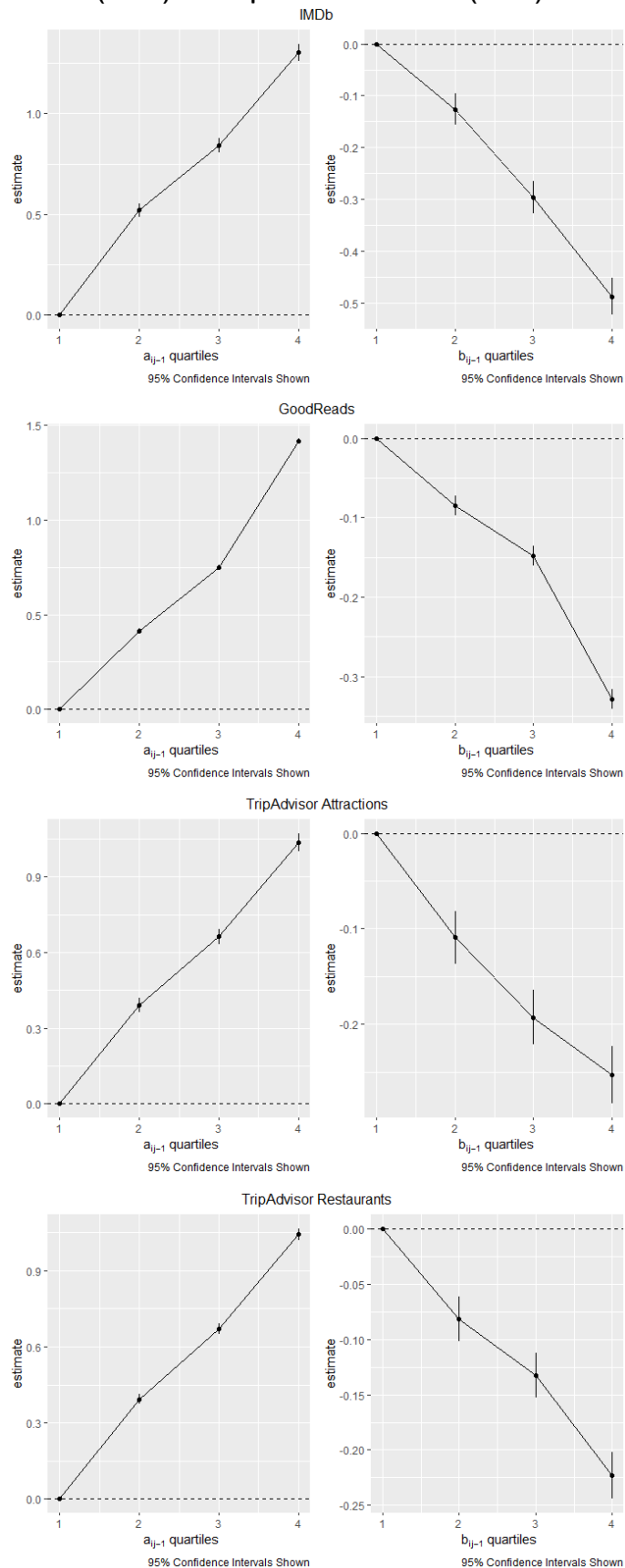


Table EC.25 Estimation results of models (3) and (6) for (2), with (4)-(5), including time and attribute dissimilarity nonlinearities.

	IMDb		GoodReads		TripAdvisor-Attractions		TripAdvisor-Restaurants	
λ_a	0.10	0.10	0.10	0.10	0.15	0.15	0.20	0.20
λ_b	0.10	0.10	0.10	0.10	0.15	0.15	0.15	0.15
	1	2	3	4	5	6	7	8
(Intercept)	-2.09*** (0.18)	-1.90*** (0.17)	-1.93*** (0.04)	-1.99*** (0.05)	-1.50*** (0.15)	-1.54*** (0.09)	-1.46*** (0.07)	-1.46*** (0.05)
c_i	0.28*** (0.01)	0.28*** (0.01)	0.12*** (0.00)	0.12*** (0.00)	0.21*** (0.01)	0.21*** (0.01)	0.28*** (0.01)	0.28*** (0.01)
q_{ij}	3.16*** (0.03)	3.16*** (0.03)	2.85*** (0.01)	2.84*** (0.01)	2.42*** (0.03)	2.42*** (0.03)	2.02*** (0.02)	2.01*** (0.02)
j	-0.04*** (0.01)	-0.04*** (0.01)	-0.02*** (0.00)	-0.02*** (0.00)	-0.00 (0.01)	-0.00 (0.01)	-0.04*** (0.01)	-0.04*** (0.01)
n_{ij}	0.04*** (0.01)	0.03*** (0.01)	0.04*** (0.00)	0.04*** (0.00)	0.01*** (0.00)	0.01*** (0.00)	-0.00 (0.00)	-0.00 (0.00)
A_{ij}	-0.00 (0.10)	0.07 (0.09)	-0.20*** (0.01)	-0.21*** (0.01)	0.15*** (0.03)	0.14*** (0.03)	0.25*** (0.02)	0.25*** (0.02)
$a_{i,j-1}$	2.65*** (0.08)	2.51*** (0.06)	2.67*** (0.02)	2.84*** (0.04)	2.07*** (0.09)	1.96*** (0.05)	1.94*** (0.06)	1.84*** (0.03)
$b_{i,j-1}$	-2.27*** (0.17)	-2.33*** (0.12)	-2.13*** (0.06)	-2.49*** (0.12)	-1.78*** (0.26)	-1.61*** (0.14)	-1.14*** (0.14)	-1.05*** (0.07)
Δt_{ij}		-0.04** (0.01)		0.03*** (0.01)		-0.01 (0.02)		0.03*** (0.01)
$\Delta \mathbf{x}_{ij}$	-0.03 (0.09)		-0.12*** (0.03)		-0.12 (0.14)		-0.03 (0.07)	
$Quart\Delta t_{ij} = 2$	0.08 (0.06)		0.02 (0.03)		-0.01 (0.10)		0.06 (0.05)	
$Quart\Delta t_{ij} = 3$	0.09 (0.06)		0.00 (0.03)		-0.17* (0.10)		0.02 (0.05)	
$Quart\Delta t_{ij} = 4$	-0.04 (0.06)		0.04 (0.03)		-0.06 (0.10)		0.16*** (0.05)	
$Quart\Delta \mathbf{x}_{ij} = 2$		-0.05 (0.06)		-0.12*** (0.03)		-0.14 (0.10)		-0.04 (0.05)
$Quart\Delta \mathbf{x}_{ij} = 3$		-0.01 (0.06)		-0.16*** (0.03)		-0.18* (0.10)		-0.13*** (0.05)
$Quart\Delta \mathbf{x}_{ij} = 4$		0.02 (0.06)		-0.12*** (0.03)		-0.10 (0.10)		-0.01 (0.05)
$a_{i,j-1} \times \Delta \mathbf{x}_{ij}$	-0.28*** (0.09)		-0.26*** (0.03)		-0.20** (0.09)		-0.17*** (0.06)	
$a_{i,j-1} \times Quart\Delta t_{ij} = 2$	-0.10 (0.07)		0.00 (0.03)		0.05 (0.06)		-0.02 (0.04)	
$a_{i,j-1} \times Quart\Delta t_{ij} = 3$	-0.12* (0.07)		0.00 (0.03)		-0.03 (0.06)		0.07 (0.04)	
$a_{i,j-1} \times Quart\Delta t_{ij} = 4$	-0.12 (0.07)		-0.08*** (0.03)		-0.29*** (0.06)		-0.06 (0.04)	
$a_{i,j-1} \times Quart\Delta \mathbf{x}_{ij} = 2$		-0.12* (0.07)		-0.10*** (0.03)		-0.04 (0.06)		-0.05 (0.04)
$a_{i,j-1} \times Quart\Delta \mathbf{x}_{ij} = 3$		-0.15** (0.07)		-0.15*** (0.03)		-0.03 (0.06)		-0.01 (0.04)
$a_{i,j-1} \times Quart\Delta \mathbf{x}_{ij} = 4$		-0.21*** (0.07)		-0.27*** (0.03)		-0.02 (0.06)		-0.05 (0.04)
$a_{i,j-1} \times \Delta t_{ij}$		-0.02 (0.02)		-0.06*** (0.01)		-0.07*** (0.01)		-0.01 (0.01)
$b_{i,j-1} \times \Delta \mathbf{x}_{ij}$	0.33* (0.20)		0.72*** (0.08)		0.39 (0.25)		0.25* (0.14)	
$b_{i,j-1} \times Quart\Delta t_{ij} = 2$	-0.06 (0.16)		-0.07 (0.08)		-0.01 (0.18)		-0.04 (0.11)	
$b_{i,j-1} \times Quart\Delta t_{ij} = 3$	0.01 (0.16)		0.03 (0.08)		0.26 (0.18)		0.06 (0.11)	
$b_{i,j-1} \times Quart\Delta t_{ij} = 4$	0.24 (0.16)		0.17** (0.08)		0.45** (0.18)		0.08 (0.11)	
$b_{i,j-1} \times Quart\Delta \mathbf{x}_{ij} = 2$		0.33** (0.16)		0.46*** (0.07)		0.29 (0.18)		0.14 (0.10)
$b_{i,j-1} \times Quart\Delta \mathbf{x}_{ij} = 3$		0.28* (0.16)		0.60*** (0.08)		0.34* (0.18)		0.30*** (0.10)
$b_{i,j-1} \times Quart\Delta \mathbf{x}_{ij} = 4$		0.18 (0.16)		0.72*** (0.08)		0.16 (0.18)		0.09 (0.10)
$b_{i,j-1} \times \Delta t_{ij}$		0.11** (0.04)		0.09*** (0.03)		0.11*** (0.03)		0.02 (0.02)
IMR	0.02 (0.07)	-0.03 (0.06)	0.03*** (0.01)	0.04*** (0.01)	0.04 (0.90)	0.09 (0.90)	7.79*** (1.75)	7.90*** (1.75)
AIC	82,044.37	82,043.33	500,726.04	500,633.65	86,460.82	86,454.44	179,164.99	179,123.09
Log Likelihood	-41,001.19	-41,000.67	-250,342.02	-250,295.83	-43,209.41	-43,206.22	-89,561.50	-89,540.55
Num. obs.	85,589	85,589	500,493	500,493	74,306	74,306	151,891	151,891

*** $p < 0.01$; ** $p < 0.05$; * $p < 0.1$