Compounding Money and Nominal-Price Illusions
Online Appendix

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OA1 Estimation

In this section, we conduct a standard GMM estimation exercise to obtain estimates for the degrees of money illusion ℓ and nominal-price illusion κ. Since we provided extensive comparative statics of the equilibrium quantities with respect to κ and ℓ in the main text, the purpose of this exercise is simply to provide a general sense of the magnitude of these quantities. While the degree of money illusion was previously estimated in the literature by David and Veronesi (2013) to be around $\ell = 0.8$, there is still no estimate for the degree of nominal-price illusion to this date. We emphasize, however, that it should come as no surprise that a simple model with (i) time-additive log utility function and (ii) independent dividend processes following standard GBM, faces a great challenge when confronted with financial data, as long recognized by the literature (Hansen and Singleton, 1982; Mehra and Prescott, 1985; Weil, 1989). As such, our model suffers from the same drawbacks as any other model that relies on these simplifying assumptions for the sake of tractability, such as Cochrane, Longstaff and Santa-Clara (2008) and Basak and Yan (2010), would have if the authors had to conduct a similar estimation exercise. For this reason, we advise the reader to approach these estimates with caution.

The GMM estimation is conducted as follows. Most of our equilibrium quantities are driven by the single state variable s_t, as in Cochrane, Longstaff and Santa-Clara (2008) and Martin (2013), and since s_t does not have a stationary distribution, matching theoretical and empirical moments is unfeasible. To complicate matters further, each equilibrium quantity depends on the stock share $\hat{s}_0$, which is itself endogenously determined. Thus, for every single given pair of (κ, ℓ), we need to solve a highly nonlinear system of equations involving integrals to determine the associated stock share $\hat{s}_0$.

To circumvent these two issues, we proceed as follows. First, to facilitate the computation of moments involving the belief density η_t, we approximate $\hat{s}_0$ by s_0 in our equilibrium expressions. Differently from the stock share, the dividend share is exogenously specified and does not depend on the parameters κ and ℓ to be estimated. As such, our estimation exercise should be seen at best as a first-order approximation of the estimation of the benchmark model. Secondly, we compute the theoretical moment of two aggregate quantities that are independent of the state variable s_t: (i) aggregate earnings-price ratio and (ii) aggregate risk premium.

With those two considerations in mind, we compute the asset price of the aggregate market as

$$
\begin{aligned}
S_t =& \mathbb{E}_t\left[\int_t^\infty \frac{\xi_v}{\xi_t} D_v dv\right] = D_t \mathbb{E}_t\left[\int_t^\infty e^{-\rho(v-t)}\left(\frac{p_t}{pv}\right)^\ell \left(\frac{\eta_v}{\eta_t}\right) dv\right] \\
=& D_t \int_t^\infty e^{-\rho(v-t)} \mathbb{E}_t\left[\left(\frac{p_t}{pv}\right)^\ell \left(\frac{\eta_v}{\eta_t}\right)\right] dv \\
=& D_t \int_t^\infty e^{-\phi(v-t)} dv \\
=& \frac{D_t}{\phi},
\end{aligned}
$$

where now

$$
\phi = \ell\left(\mu_P - \frac{\sigma_{P1}^2 + \sigma_{P2}^2}{2}\right) + \left(\frac{\kappa}{\sigma}\right)^2 \left(\frac{s_0^2 + (1-s_0)^2}{2}\right) - \frac{1}{2}\left(\left(\frac{\kappa s_0}{\sigma} + \ell\sigma_{P1}\right)^2 + \left(\frac{\kappa(1-s_0)}{\sigma} + \ell\sigma_{P2}\right)^2\right). \quad (1)
$$

Note that the earnings-price ratio q_t is constant, highly nonlinear on κ and ℓ, and equal to $q_t = D_t/S_t = \phi, \forall t \in [0, \infty)$.

From the no-arbitrage condition, the aggregate market risk premium is given by

$$
\begin{aligned}
\mu_t - r_t + q_t =& \sigma(\theta_{1t} + \theta_{2t}) \\
\mu_t - r_t =& \sigma\left(\sigma + \ell(\sigma_{P1} + \sigma_{P2}) + \frac{\kappa}{\sigma}\right) - \phi \\
\mu_t - r_t =& \sigma^2 + \ell\sigma(\sigma_{P1} + \sigma_{P2}) + \kappa - \phi. \quad (2)
\end{aligned}
$$

Both the aggregate earnings-price ratio in Eq.(1) and the aggregate market risk premium in Eq.(2) are constant and independent of s_t. As such, we perform a standard GMM estimation and match those theoretical moments with the empirical moments, i.e., average aggregate earnings-price ratio and average risk premium.

The monthly data on the aggregate earnings-price ratio comes from Robert Shiller's dataset and the monthly aggregate market risk premium comes from Kenneth French's website, for the same time period of our portfolio analyses (Jan 1968 – Dec 2019). Since the theoretical aggregate moments are constant, running a GMM estimation is equivalent to minimizing the sum of the mean square error of Eqs.(1) and (2). Thus, we obtain the estimated values for κ and ℓ by running a constrained optimization, where $\kappa \in [0, 10\%]$ and $\ell \in [0, 1]$.

The GMM estimates for the degree of nominal-price and money illusion are, respectively, $\kappa = 6.1\%$ and $\ell = 0.72$, with a standard error of 0.002 and 0.106, yielding an equilibrium nominal short-term interest rate of 6.36%.

OA1.1 Using the Dividend and CPI Time Series from Shiller's Dataset

In this section, we use the dividend and the CPI time series of the Shiller's dataset to obtain a proxy for expected inflation, inflation volatility, dividend growth rate, and dividend volatility, instead of following the literature calibration presented in Section 2. We conduct the same GMM estimation but use the following estimates obtained from Shiller's dataset: $\mu = 5.77\%, \sigma = 1.98\%, \mu_P = 3.89\%, \sigma_{P1} = 1.14\%$, and $\sigma_{P2} = 0.55\%$. Under this calibration, the GMM estimates for the degree of nominal-price and money illusions are $\kappa = 6.4\%$ and $\ell = 0.6$, with a standard error of 0.002 and 0.089, respectively, which yield an equilibrium nominal short-term rate of 11.24%, which is roughly 77% larger than the interest rate generated by the benchmark calibration used in the manuscript. As we pointed out earlier, it should come as no surprise that a model with a log utility function and independent GBM processes naturally yields a high equilibrium short-term rate. Note, however, that the equilibrium interest rate is still considerably smaller than the ones generated by the models of Cochrane et al. (2008) and Basak and Yan (2010), indicating that nominal-price illusion is an important mechanism to reduce the equilibrium short-term interest rate.

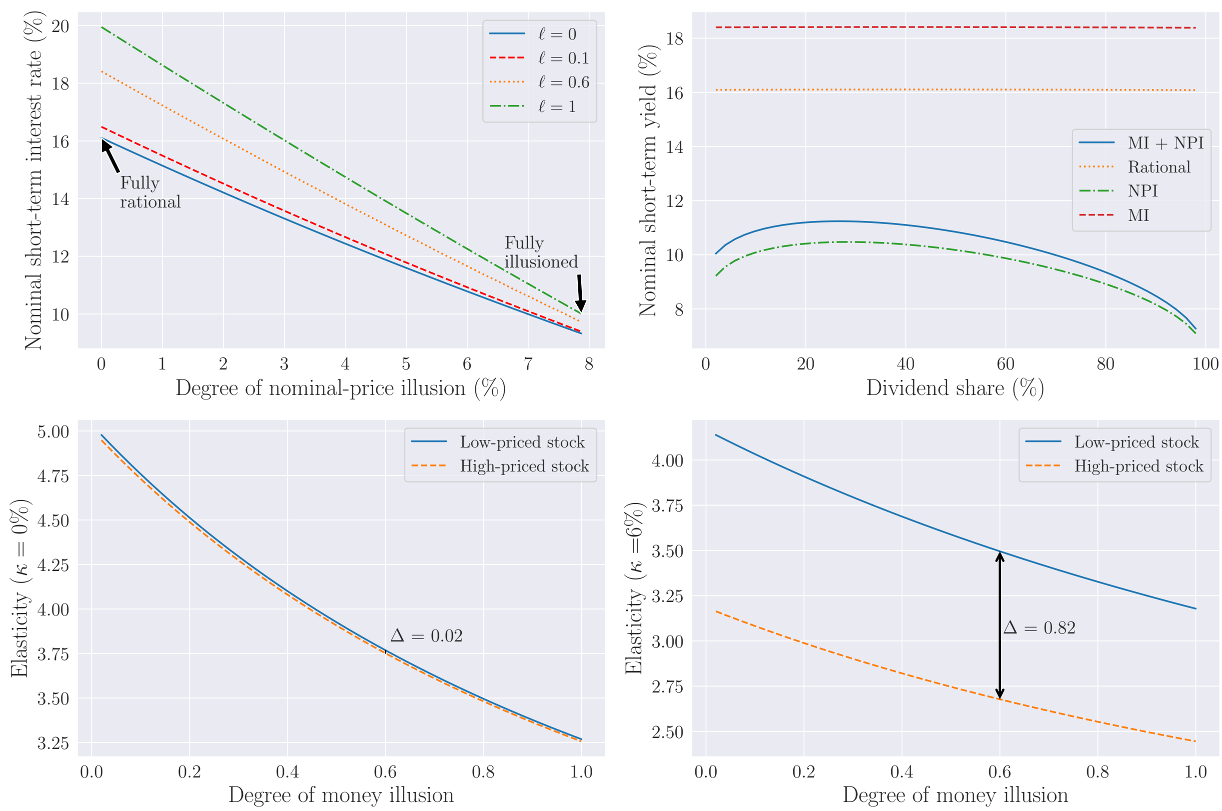
Figure 1 reproduces the graphs of Section 2 under this new calibration. Note that even in this estimation procedure, the main result of the paper remains unaltered: nominal-price illusion enlarges the wedge between the earnings-price ratio of low- and high-priced stocks, allowing us to empirically document this effect in our portfolio analyses in Section 3.

OA2 Robustness Check

One may argue that our finding of a more pronounced money illusion among low-priced stocks could simply be related to the fact that it is generally difficult to value these stocks. Low-priced stocks are likely to be young and have high idiosyncratic volatility. Thus, they are more likely to have uncertain valuations compared to high-priced stocks (see, for example, Hirshleifer, 2001; Kumar, 2009).

To ensure that our results are not solely driven by valuation differences among stocks, we conduct a bivariate portfolio analysis. We focus on two measures that have been used in the literature to proxy firm valuation uncertainty: idiosyncratic volatility and firm age (Pástor and Veronesi, 2003; Ang et al., 2006, 2009). In the first step, we sort stocks into quintiles according to their idiosyncratic volatility and firm age separately. In a second step, we sort stocks further into quintiles (within each idiosyncratic volatility- and age-sorted quintile) based on their nominal price. We then run Eq.(10) for each of the 25 sub-quintiles generated and estimate the nominal yield betas as the measure of money illusion as before. We also run Eq.(10) for the nominal-price difference portfolio P1-P5 for each of the five

Figure 1: Equilibrium Results



Notes. The figure shows the equilibrium quantities under the new calibration $\mu = 5.77\%, \sigma = 1.98\%, \mu_P = 3.89\%, \sigma_{P1} = 1.14\%$, and $\sigma_{P2} = 0.55\%$, obtained from the dividend and CPI time series in the Shiller data set. The top left graph shows the effect of the degree of nominal-price illusion on the nominal yield for four different levels of money illusion: 0, 0.1, 0.6, and 1. The top right graph shows the effect of the initial dividend rate on the nominal yield. The equilibrium rate is computed using the GMM estimates values of $\kappa = 6.4\%$ and $\ell = 0.6$. The bottom plots show the effect of the degree of money illusion on the elasticity gap in the absence of nominal-price illusion (left panel) and in the presence of nominal-price illusion (right panel). With the exception of the top right panel, all other plots fix the initial dividend share at $s_0 = 0.25$.

idiosyncratic volatility- and age-sorted quintiles to compute the compounding effect of money and nominal-price illusion controlling for the effect of the valuation differences among stocks (i.e., controlling for idiosyncratic volatility and firm age).

[Table OA5 about here.]

Panel A1 to A3 of Table OA5 reports the mean nominal yield beta for each nominal-price portfolio averaged across the five idiosyncratic volatility portfolios. Consistent with the results obtained from the univariate portfolio analysis from Table 2, the bivariate portfolio results from Table OA5 also show that nominal yield betas decrease monotonically as we move from low-priced stocks to high-priced stocks after we control for the effect of idiosyncratic volatility. Furthermore, the statistically significant 1.19 nominal-yield beta coefficient (t-stat = 2.08) reported for the P1-P5 difference portfolio in the last column of Panel A1 of Table OA5 suggests that the compounding effect of money and nominal-price illusion is still strong and significant even after accounting for the differences in stocks' idiosyncratic volatility levels. We report in Panels A2 and A3 of Table OA5 the results of the bivariate portfolio analysis controlling for idiosyncratic volatility, but this time generating nominal yield betas from Eq.(11). Similar to the results obtained in Panel A1, we find evidence of the compounding effects of money and nominal-price illusions even after controlling for the effects of idiosyncratic volatility.[1]

We next control for the effect of firm age on our main findings. Panel B1 to B3 of Table OA5 reports the mean nominal yield beta for each nominal-price portfolio averaged across the five firm age-sorted portfolios. Once again, consistent with the results obtained from the univariate portfolio analysis in Table 2, the bivariate portfolio results from Table OA5 also show that nominal yield betas decrease monotonically as we move from low-priced stocks to high-priced stocks after we control for the firm age effect. More importantly, with a beta of 1.08 for the P1-P5 difference portfolio in Panel B1, the compounding illusion coefficient is still statistically significant (t-stat = 4.07). Our analysis suggests that the compounding effects of money and nominal-price illusions are strong and significant even after accounting for firm age.[2] In sum, these results suggest that our main finding that the compounding illusions effects are more pronounced among low-priced stocks is robust and it is not due to

[1]We conduct the same tests in high inflation and low inflation subsamples and results are provided in Table OA6 of the Online Appendix. The benchmark nominal-yield beta coefficient for the P1-P5 difference portfolio is larger during high-inflation periods (2.03, with t-stat of 2.00) than during low-inflation periods (0.81, with t-stat of 2.49). The patterns remain the same even after we control for the relative riskiness of stocks to bonds.

[2]The results for high and low inflation subsamples are provided in Table OA7 of the Online Appendix. The benchmark nominal-yield beta coefficient for the P1-P5 difference portfolio is larger during high-inflation periods (1.95, with t-stat of 3.52) than during low-inflation periods (0.51 with t-stat of 2.22).

the valuation uncertainties of such stocks.

[Table OA8 about here.]

Another robustness check we address is about potential endogeneity concerns. One may argue, for instance, that monetary policy (among other possible factors) may affect both the earnings-price ratio and the nominal yield simultaneously, inducing a significant correlation between them even in the absence of money illusion. Moreover, according to our theoretical model, the dividend-price ratio, equity premium, and expected dividend growth are all related quantities. For these two reasons, we add the equity risk premium and expected dividend growth rate as additional control variables to our regression in Eq.(11).[3] We report the results of this regression in Table OA8. As illustrated, the nominal yield beta in our full sample regressions is still positive and significant as in previous analyses, even after controlling for equity risk premium and expected dividend growth rate, providing further evidence of continuing money illusion. Furthermore, in line with our previous findings, we see significantly higher nominal yield betas for low-priced stocks compared to high-priced stocks due to the existence of nominal-price illusion in the presence of equity premium and expected dividend growth. In fact, the compounding illusions effect as measured by the difference in betas between price-sorted Portfolio 1 and Portfolio 5 remains positive and highly significant, similar in magnitude to the results reported in Table 2.

Another robustness check we perform is to control the effect of firm size on our main findings. We do this by running a bivariate portfolio analysis between size and price. A critical issue here, however, is that price and size are highly correlated. Therefore, to mitigate the correlation effects, we use a more granular portfolio analysis and look at deciles instead of quintiles to better capture the combined effects of money and nominal-price illusions while controlling for the effect of size.

In the first step, we sort stocks into deciles according to their market capitalization (size). In the second step, within each size decile, we further sort stocks into deciles based on their nominal price. We then run Eq.(10) for each of the 100 sub-deciles generated, and estimate the nominal yield betas as the measure of money illusion as before. We also run Eq.(10) for the nominal-price difference portfolio P1-P10 for each of the ten size-sorted deciles to compute the compounding effect of money and nominal-price illusions, controlling for the effect of size.

[Table OA9 about here.]

[3] We proxy the expected dividend growth rate as the historical average of the S&P500 index nominal dividend with a rolling window of ten years. The equity premium is defined as the returns of CRSP value-weighted index over the risk-free rate.

Panels A to C of Table OA9 present the mean nominal yield beta for each nominal-price portfolio averaged over the ten-size portfolios. In the last column of each panel, we also report the mean nominal yield beta for the P1-P10 difference portfolio, again averaged over the ten-size portfolios. Consistent with the results obtained from the univariate portfolio analysis in Table 2, the bivariate portfolio results from Panel A of Table OA9 also show a positive and significant nominal yield beta (1.20 with a t-stat of 2.06) for the P1-P10 difference portfolio, suggesting that the compounding effect of money and nominal-price illusion is still strong and significant even after controlling for the effect of firm size.

In Panels B and C of Table OA9, we report the results of the bivariate portfolio analysis controlling for size, but this time generating nominal yield betas from Eq.(11) after controlling for volatility in equity and bond markets. Similar to the results obtained in Panel A, we once again find positive and significant nominal yield betas for the P1-P10 difference portfolio, providing evidence for strong combined effects of money and nominal-price illusions in a context that accounts for differences in firm size.

[Table OA10 about here.]

For consistency with other portfolio analyses that utilize quintile portfolios in the paper, we also repeat the same bivariate portfolio analysis between size and price using quintiles as well. Panels A to C of Table OA10 report the mean nominal yield beta for each nominal-price portfolio averaged across the five-size portfolios. Although the nominal yield betas again decrease from low- to high-priced stocks, this time, as expected, due to the high correlation between size and price, we do not get as strong a statistical significance for the nominal yield beta difference between the low- and high-priced stocks. The P1-P5 difference portfolio has a positive and statistically significant nominal yield beta at the 10% significance level in Panels B and C of Table OA10 when we run Eq.(11) controlling for volatility in equity and bond markets. In Panel A, however, although the nominal yield beta of the P1-P5 difference portfolio is still positive, it becomes statistically insignificant at this instant due to our inability to suppress the effect of the high correlation between size and price with a quintile portfolio analysis.

As an additional analysis to control for the effect of size on our main findings, in a panel regression setting similar to the results reported in Table OA11, we also regress individual firm E/P ratios on nominal yield, individual firm size, and equity and bond volatilities. As reported in Table OA11, we obtain positive and significant nominal yield betas after controlling for firm size in all three different regression specifications. These results suggest that money and nominal-price illusions survive to a great extent after taking into account the effect of firm size.

[Table OA11 about here.]

OA3 Nominal-Price Illusion with a Single Risky Asset

In this section, we assume the economy has a single risky asset. Since there is only a single risky asset, the nominal-price illusion emerges from agents contrasting the endogenous price level of the risky asset S_0, with the endogenous price level of the money market account B_0. Specifically, agents suffering from nominal-price illusion assume a larger dividend growth rate if the initial stock price S_0 is below the initial price of the money market account B_0, i.e.,

$$\frac{dD_t}{D_t} = \left(\mu - \kappa\left(\frac{S_0 - B_0}{B_0 + S_0}\right)\right)dt + \sigma d\widetilde{Z}_t, \quad D_0 \text{ given,}$$

where $\kappa \geq 0$ denotes the degree of nominal-price illusion as before. Note that when $B_0 < S_0$, the perceived expected dividend growth rate is

$$\mu - \kappa\left(\frac{S_0 - B_0}{B_0 + S_0}\right) < \mu.$$

On the other hand, if $S_0 < B_0$, agents suffering from nominal-price illusion perceive dividends to have a larger expected growth rate since

$$\mu < \mu - \kappa\left(\frac{S_0 - B_0}{B_0 + S_0}\right).$$

This modeling choice captures the essence of investors suffering from nominal-price illusion, as in our benchmark case: when the risky asset has a lower value than the other (risk-free) asset, its growth rate is perceived as being larger.

As we show next, similar to the benchmark model, the equilibrium prices B_0 and S_0 must be jointly determined and numerically characterized by solving a nonlinear system of equations. In this version of the model, since the perception error is given by

$$\delta = -\frac{\kappa}{\sigma}\left(\frac{B_0 - S_0}{B_0 + S_0}\right), \tag{3}$$

the SDE satisfied by the subjective measure density is given by

$$\frac{d\eta_t}{\eta_t} = \delta dZ_t.$$

Assuming the investor solves the same maximization problem, the first-order and marketclearing conditions yield the same functional form for the state-price density ξ_t, which is

$$\xi_t = e^{-\rho t}\left(\frac{D_0}{D_t}\right)\eta_t p_t^{-\ell}. \tag{4}$$

Similar to the benchmark model, η_t still needs to be determined since B_0 and S_0 are also equilibrium quantities. Therefore, before characterizing the short-term interest rate and the market price of risk, we must determine the price of the risk-free and risky assets.

We begin by determining the price of the risky asset S_0. The no-arbitrage condition implies that

$$\begin{aligned} S_0 =& \mathbb{E}_0\left[\int_0^\infty \xi_v D_v dv\right] = \mathbb{E}_0\left[\int_0^\infty e^{-\rho v}\frac{D_0}{D_v}\eta_v\frac{1}{p_v^\ell}D_v dv\right] \\ =& D_0\mathbb{E}_0\left[\int_0^\infty e^{-\rho v}\eta_v p_v^{-\ell}dv\right] = D_0\int_0^\infty e^{-\rho v}\mathbb{E}_0[\eta_v p_v^{-\ell}]dv \\ =& \frac{D_0}{\rho + \ell\mu_P - \ell(1+\ell)\sigma_P^2 - \delta\ell\sigma_P}. \end{aligned} \tag{5}$$

Next, we compute the no-arbitrage price of the money market account B_0:

$$\begin{aligned} B_0 =& \mathbb{E}_0\left[\int_0^\infty \xi_v dv\right] = \mathbb{E}_0\left[\int_0^\infty e^{-\rho v}\frac{D_0}{D_v}\eta_v\frac{1}{p_v^\ell}dv\right] \\ =& D_0\mathbb{E}_0\left[\int_0^\infty e^{-\rho v}\eta_v p_v^{-\ell}D_v^{-1}dv\right] = D_0\int_0^\infty e^{-\rho v}\mathbb{E}_0[\eta_v p_v^{-\ell}D_v^{-1}]dv \\ =& \frac{D_0}{\rho + \ell\mu_P + \mu - \ell(1+\ell)\sigma_P^2 - \sigma^2 - \delta(\sigma + \ell\sigma_P) - \ell\sigma_P\sigma}. \end{aligned} \tag{6}$$

Substituting the expression for the perception error δ in Eq.(3) into Eqs.(5) and (6), we obtain the following the nonlinear system of equations for B_0 and S_0:

$$\begin{aligned} S_0 =& \frac{D_0}{\rho + \ell\mu_P - \ell(1+\ell)\sigma_P^2 + \kappa\ell\frac{\sigma_P}{\sigma}\left(\frac{B_0 - S_0}{B_0 + S_0}\right)}, \\ B_0 =& \frac{D_0}{\rho + \ell\mu_P - \ell(1+\ell)\sigma_P^2 + \kappa\ell\frac{\sigma_P}{\sigma}\left(\frac{B_0 - S_0}{B_0 + S_0}\right) + \mu - \sigma^2 + \kappa\left(\frac{B_0 - S_0}{B_0 + S_0}\right) - \ell\sigma_P\sigma}. \end{aligned}$$

With the characterization of B_0 and S_0, and consequently, of the perception error δ, the density η_t is fully determined. Thus, the short-term interest rate r_t and the market price of risk θ_t follow from a straightforward application of Ito's lemma to Eq.(4):

$$r = \underbrace{\rho + \mu - \frac{\sigma^2}{2}}_{\text{Lucas Economy}} + \underbrace{\ell\left(\mu_P - \ell(1+\ell)\sigma_P^2 - \sigma_P\sigma\right)}_{\text{Money Illusion}} - \underbrace{\kappa\left(\frac{B_0 - S_0}{B_0 + S_0}\right)}_{\text{Nominal-Price Illusion}} - \underbrace{\frac{\kappa\ell\sigma_P}{\sigma}\left(\frac{B_0 - S_0}{B_0 + S_0}\right)}_{\text{Compounding Illusions}},$$

$$\theta = \underbrace{\sigma}_{\text{Lucas Economy}} + \underbrace{\ell\sigma_P}_{\text{Money Illusion}} - \underbrace{\frac{\kappa}{\sigma}\left(\frac{B_0 - S_0}{B_0 + S_0}\right)}_{\text{Nominal-Price Illusion}}.$$

To obtain the nominal short-term interest rate, we apply Ito's lemma on the nominal state-price density $\xi_t^N = \xi_t / p_t$ and obtain

$$y = r + \mu_P - (1+\ell)\sigma_P^2 - \sigma_P\left(\sigma + \frac{\kappa}{\sigma}\left(\frac{B_0 - S_0}{B_0 + S_0}\right)\right). \tag{7}$$

The dividend-price ratio q follows immediately from Eq.(5), and it is given by

$$q = \rho + \ell\mu_P - \ell(1+\ell)\sigma_P^2 + \frac{\kappa\ell\sigma_P}{\sigma}\left(\frac{B_0 - S_0}{B_0 + S_0}\right). \tag{8}$$

Panel (a) of Figure 2 shows the nominal rate y in Eq.(7) as a function of the nominal-price illusion parameter κ for four different levels of money illusion: (*i*) no money illusion (dashed-dotted line), (*ii*) benchmark money illusion of $\ell = 0.72$ (solid line), and (*iii*) fully money-illusioned with $\ell = 1$ (dotted line). Panel (b) contains the sensitivity analysis of the dividend-price ratio q in Eq.(8) with respect to the same illusion parameters.

Note that in this setting, the nominal short-term interest rate does not decrease in κ as in our benchmark case presented in the manuscript. A key difference is that, in the benchmark model, the nominal-price illusion creates a wedge between the growth rates of risky assets that are both in positive unit supply, while in the single risky asset, the trade-off is between an asset in positive unit supply (i.e., the stock) and another in zero net supply (i.e., the money market account). Moreover, in contrast to the benchmark case, the dividend-price ratio is affected by the degree of nominal-price illusion only when investors also suffer from money illusion (i.e., $\ell \neq 0$), which leads to decreasing dividend-price ratios in the degree of nominal-price illusion when investors experience some degree of money illusion.

(a) Nominal Short-term Interest Rate　　**(b)** Dividend-price Ratio

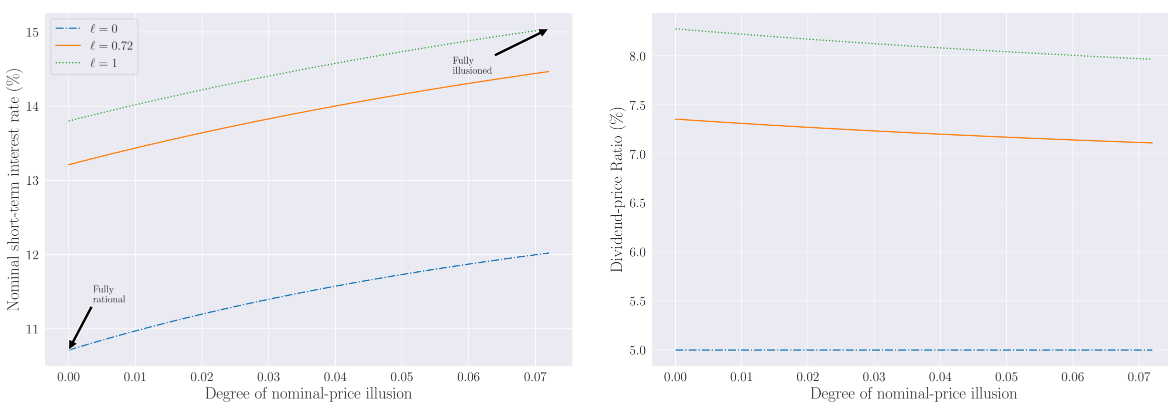


Figure 2: *Sensitivity Analyses.* Panel (a) shows the effect of money and nominal-price illusions on the nominal short-term interest rate, while Panel (b) contains the sensitivity analysis for the dividend-price ratio. All quantities are computed using the main calibration presented in the manuscript.

OA4 Alternative CPI Dynamics

In this section, we investigate the case where the CPI under the subjective measure satisfies

$$\frac{dp_t}{p_t} = \mu_P dt + \sigma_{P1} d\widetilde{Z}_{1t} + \sigma_{P2} d\widetilde{Z}_{2t}. \tag{9}$$

Thus, the CPI dynamics under the physical measure is

$$\frac{dp_t}{p_t} = (\mu_P - (\sigma_{P1}\delta_1 + \sigma_{P2}\delta_2))dt + \sigma_{P1} dZ_{1t} + \sigma_{P2} dZ_{2t}.$$

As before, an application of Ito's lemma gives that

$$\frac{dp_t^{-\ell}}{p_t^{-\ell}} = \ell\left((1+\ell)\left(\frac{\sigma_{P1}^2 + \sigma_{P2}^2}{2}\right) - \mu_P + \sigma_{P1}\delta_1 + \sigma_{P2}\delta_2\right)dt - \ell\left(\sigma_{P1} dZ_{1t} + \sigma_{P2} dZ_{2t}\right). \tag{10}$$

The evolution of the real state-price density is given by

$$\frac{d\xi_t}{\xi_t} = -\rho dt + \frac{dD_t^{-1}}{D_t^{-1}} + \frac{d\eta_t}{\eta_t} + \frac{dp_t^{-\ell}}{p_t^{-\ell}} + \frac{d\langle D^{-1}, \eta\rangle_t}{D_t^{-1}\eta_t} + \frac{d\langle D^{-1}, p^{-\ell}\rangle_t}{D_t^{-1}p_t^{-\ell}} + \frac{d\langle p^{-\ell}, \eta\rangle_t}{p_t^{-\ell}\eta_t}.$$

In this setting, however, the quadratic variation $\frac{d\langle p^{-\ell}, \eta\rangle_t}{p_t^{-\ell}\eta_t} = -\ell(\sigma_{P1}\delta_1 + \sigma_{P2}\delta_2)$ gets canceled out by part of the new drift of $dp_t^{-\ell}/p_t^{-\ell}$ shown in Eq.(10). Thus, the real short-term interest rate simplifies to

$$r_t = \underbrace{\rho + \mu - \sigma^2(s_t^2 + (1-s_t)^2)}_{\text{two trees}} + \underbrace{\ell\left(\mu_P - (1+\ell)\left(\frac{\sigma_{P1}^2 + \sigma_{P2}^2}{2}\right) - \sigma(\sigma_{P1}s_t + \sigma_{P2}(1-s_t))\right)}_{\text{money illusion}}$$
$$- \underbrace{\kappa(s_t\hat{s}_0 + (1-s_t)(1-\hat{s}_0))}_{\text{nominal-price illusion}}.$$

Interestingly, when the illusioned investor perceives the CPI as in Eq.(9) and the inflation index is completely detached from the effects of the nominal-price illusion, the compounding illusions component of the interest rate disappears. Note that even in the absence of the compounding illusions component, the comparative statics of r_t with respect to the degrees of money and nominal-price illusions must still be evaluated numerically, since both ℓ and κ affect the endogenous stock share $\hat{s}_0$ contained in the nominal-price illusion component. Similarly, the sensitivities of the dividend yield can only be evaluated numerically.

OA5 Additional Tables

Table OA1: Relationship Between Earnings and Nominal Yields - 2002-2019 Sub-sample

This table shows the results of the monthly regression of the portfolio value-weighted E/P ratio on the nominal treasury yield and different volatility measures. The sample period runs from January 2002 to December 2019. At the end of each calendar month, stocks are ranked in ascending order on the basis of their nominal price and are assigned to one of five price-sorted portfolios based on the NYSE breakpoint. Portfolios are value-weighted and refreshed every calendar month. Bond market risk is proxied by the realized volatility of the 10-year Treasury bond. TVOL is the value-weighted total volatility of stock returns and IVOL is the standard deviation of the residuals from the Fama-French three-factor regression. Panel A shows our benchmark results. Panel B and Panel C show the different volatility combinations. Numbers in parentheses are t-statistics based on Newey-West standard errors, and 5% statistical significance is indicated in bold.

Portfolio	**Full**	**P1**	**P2**	**P3**	**P4**	**P5**	**P1-P5**
Panel A: Benchmark							
Y	-0.08	**-1.44**	**-0.45**	**-0.23**	-0.09	**0.23**	**-1.67**
	(-0.85)	(-1.98)	(-3.57)	(-2.51)	(-1.55)	(2.66)	(-2.43)
Constant	**0.02**	0.02	**0.03**	**0.03**	**0.02**	**0.01**	0.01
	(7.13)	(1.11)	(8.04)	(8.59)	(11.69)	(4.25)	(0.40)
No. of Months	216	216	216	216	216	216	216
Time-series Average Number of Firm Obs.	4493	1848	883	695	564	502	1346
Adj. R^2	0.01	0.06	0.13	0.08	0.03	0.10	0.08
Panel B: Portfolio total return volatility and bond market volatility							
Y	0.16	0.52	-0.13	0.12	0.01	**0.36**	-0.51
	(1.65)	(0.79)	(-0.76)	(1.22)	(0.14)	(4.09)	(-0.78)
$TVOL_{portfolio}$	**0.38**	**-2.12**	**0.42**	**0.52**	**0.28**	**0.52**	**-4.59**
	(3.33)	(-3.04)	(2.11)	(6.62)	(3.08)	(3.67)	(-4.50)
Bond VOL	**-1.70**	**-10.14**	**-2.26**	**-2.46**	**-0.78**	**-1.06**	-4.45
	(-4.44)	(-2.60)	(-2.85)	(-5.31)	(-2.62)	(-2.80)	(-1.07)
Constant	**0.03**	**0.18**	**0.05**	**0.04**	**0.03**	**0.02**	**0.10**
	(6.28)	(4.75)	(5.22)	(7.23)	(7.94)	(3.04)	(2.58)
Adj. R^2	0.36	0.49	0.29	0.52	0.26	0.37	0.59
Panel C: Portfolio idiosyncratic volatility and bond market volatility							
Y	0.14	0.46	-0.12	0.12	0.00	**0.33**	-0.53
	(1.46)	(0.70)	(-0.69)	(1.22)	(0.02)	(3.58)	(-0.81)
$IVOL_{portfolio}$	**0.77**	**-3.58**	0.73	**1.02**	**0.58**	**1.12**	**-6.13**
	(2.58)	(-3.31)	(1.66)	(4.83)	(2.44)	(3.03)	(-4.27)
Bond VOL	**-1.81**	**-8.27**	**-2.46**	**-2.71**	**-0.82**	**-1.12**	-3.42
	(-4.79)	(-2.03)	(-2.88)	(-6.09)	(-2.73)	(-2.97)	(-0.80)
Constant	**0.03**	**0.17**	**0.05**	**0.04**	**0.03**	**0.02**	**0.10**
	(5.74)	(4.85)	(5.18)	(7.24)	(6.79)	(2.45)	(2.64)
Adj. R^2	0.34	0.52	0.27	0.48	0.24	0.36	0.61

Table OA2: Relationship Between Earnings and Nominal Yields - 1968-2001 Sub-sample

This table shows the results of the monthly regression of the portfolio value-weighted E/P ratio on the nominal treasury yield and different volatility measures. The sample period runs from January 1968 to December 2001. At the end of each calendar month, stocks are ranked in ascending order on the basis of their nominal price and are assigned to one of five price-sorted portfolios based on the NYSE breakpoint. Portfolios are value-weighted and refreshed every calendar month. Bond market risk is proxied by the realized volatility of the 10-year Treasury bond. TVOL is the value-weighted total volatility of stock returns and IVOL is the standard deviation of the residuals from the Fama-French three-factor regression. Panel A shows our benchmark results. Panel B and Panel C show the different volatility combinations. Numbers in parentheses are t-statistics based on Newey-West standard errors, and 5% statistical significance is indicated in bold.

Portfolio	**Full**	**P1**	**P2**	**P3**	**P4**	**P5**	**P1-P5**
Panel A: Benchmark							
Y	**0.22**	**0.37**	**0.17**	**0.17**	**0.26**	**0.21**	0.16
	(11.17)	(3.78)	(5.41)	(5.23)	(9.01)	(13.26)	(1.67)
Constant	**0.00**	-0.02	**0.01**	**0.01**	**0.00**	0.00	-0.02
	(2.24)	(-1.59)	(4.41)	(4.88)	(2.01)	(1.87)	(-1.82)
No. of Months	408	408	408	408	408	408	408
Time-series Average Number of Firm Obs.	3447	1586	654	466	398	343	1243
Adj. R^2	0.58	0.07	0.14	0.23	0.50	0.64	0.01
Panel B: Portfolio total return volatility and bond market volatility							
Y	**0.25**	**0.71**	**0.32**	**0.24**	**0.31**	**0.21**	**0.36**
	(13.04)	(5.32)	(6.26)	(7.29)	(11.73)	(13.09)	(3.26)
$TVOL_{portfolio}$	0.05	0.17	**0.57**	**0.41**	**0.24**	-0.08	-0.81
	(0.52)	(0.35)	(2.39)	(2.78)	(2.13)	(-1.27)	(-1.40)
Bond VOL	**-0.38**	**-4.28**	**-1.12**	**-0.63**	**-0.55**	-0.05	**-3.86**
	(-3.67)	(-6.40)	(-4.60)	(-4.04)	(-4.14)	(-0.55)	(-7.25)
Constant	**0.01**	**0.04**	0.01	**0.01**	**0.01**	**0.00**	**0.06**
	(3.07)	(2.80)	(1.65)	(2.71)	(2.03)	(2.16)	(4.82)
Adj. R^2	0.66	0.56	0.43	0.39	0.60	0.65	0.54
Panel C: Portfolio idiosyncratic volatility and bond market volatility							
Y	**0.25**	**0.72**	**0.35**	**0.26**	**0.31**	**0.20**	**0.40**
	(11.88)	(4.56)	(6.05)	(7.16)	(11.48)	(12.01)	(3.10)
$IVOL_{portfolio}$	0.01	0.26	**0.91**	**0.69**	**0.42**	**-0.24**	-0.57
	(0.05)	(0.37)	(2.70)	(3.29)	(2.39)	(-2.33)	(-0.73)
Bond VOL	**-0.36**	**-4.33**	**-1.21**	**-0.67**	**-0.56**	-0.01	**-3.90**
	(-3.26)	(-5.66)	(-4.71)	(-4.29)	(-4.16)	(-0.11)	(-6.32)
Constant	**0.01**	**0.04**	0.00	0.01	0.00	**0.01**	**0.05**
	(2.97)	(2.09)	(0.66)	(1.65)	(1.40)	(2.72)	(3.91)
Adj. R^2	0.66	0.56	0.46	0.41	0.61	0.66	0.53

Table OA3: Relationship Between Earnings and Nominal Yields for Low Institutional Ownership Stocks During High Inflation Periods

This table shows the results of the monthly regression of the portfolio value-weighted E/P ratio on the nominal treasury yield and different volatility measures within the low institutional ownership stocks subsample during high inflation periods. The sample period runs from January 1980 to December 2019. At the end of each calendar month, stocks are ranked in ascending order on the basis of their nominal price and are five price-sorted portfolios based on the NYSE breakpoint. Portfolios are value-weighted and refreshed every calendar month. Bond market risk is proxied by the volatility of realized the 10-year Treasury bond. TVOL is the value-weighted total volatility of stock returns and IVOL is the standard deviation of the residuals from the Fama-French three-factor regression. To determine the low institutional ownership subsample, we first measure the time-series average of the stock institutional ownership (IO). Then, we rank stocks in ascending order on the basis of their average institutional holding. The bottom 30% of stocks are grouped into the low IO sub-sample. High-inflation periods are defined as such using the PPI index (i.e., if the monthly inflation is in the top 30th percentile). Panel A shows our benchmark results and Panel B and Panel C show the different volatility combinations. Numbers in parentheses are t-statistics based on Newey-West standard errors, and 5% statistical significance is indicated in bold.

Portfolio	**P1**	**P2**	**P3**	**P4**	**P5**	**P1-P5**
Panel A: Benchmark						
Y	**1.92**	**0.50**	**0.32**	**0.15**	**0.11**	**1.81**
	(5.94)	(5.48)	(5.92)	(2.79)	(3.20)	(5.55)
Constant	**-0.15**	**-0.03**	-0.01	**0.01**	**0.01**	**-0.17**
	(-6.37)	(-3.83)	(-1.90)	(2.17)	(4.67)	(-6.66)
Obs.	187	187	187	187	187	187
Time-series Average Number of Firm Obs.	284	129	84	67	49	235
Adj. R^2	0.23	0.22	0.25	0.07	0.10	0.21
Panel B: Portfolio total return volatility and bond market volatility						
Y	**2.57**	**0.72**	**0.40**	**0.21**	**0.16**	**2.45**
	(9.33)	(7.55)	(7.04)	(4.21)	(4.98)	(8.45)
$TVOL_{portfolio}$	-1.77	0.06	0.22	**0.51**	0.05	**-2.52**
	(-1.95)	(0.24)	(1.04)	(2.11)	(0.29)	(-2.99)
Bond VOL	**-11.45**	**-3.97**	**-1.60**	**-1.24**	**-0.91**	**-10.55**
	(-4.09)	(-5.84)	(-3.58)	(-5.33)	(-4.41)	(-4.51)
Constant	**0.09**	**0.02**	0.01	0.01	**0.02**	0.04
	(2.27)	(2.56)	(1.49)	(1.92)	(5.81)	(1.14)
Adj. R^2	0.56	0.48	0.35	0.23	0.22	0.54
Panel C: Portfolio idiosyncratic volatility and bond market volatility						
Y	**2.59**	**0.72**	**0.40**	**0.19**	**0.16**	**2.34**
	(9.24)	(7.52)	(7.27)	(3.88)	(4.70)	(8.59)
$IVOL_{portfolio}$	-1.91	0.07	0.26	**0.63**	-0.06	**-3.28**
	(-1.85)	(0.24)	(1.04)	(2.18)	(-0.30)	(-2.90)
Bond VOL	**-11.42**	**-3.98**	**-1.63**	**-1.25**	**-0.85**	**-9.31**
	(-3.95)	(-5.76)	(-3.50)	(-5.37)	(-4.65)	(-3.74)
Constant	**0.08**	**0.02**	0.01	**0.02**	**0.03**	0.04
	(2.20)	(2.63)	(1.63)	(2.25)	(6.37)	(1.23)
Adj. R^2	0.56	0.48	0.35	0.21	0.22	0.56

Table OA4: Relationship Between Earnings and Nominal Yields for High Institutional Ownership Stocks During Low Inflation Periods

This table shows the results of the monthly regression of the portfolio value-weighted E/P ratio on the nominal treasury yield and different volatility measures within the high institutional ownership stocks subsample during low inflation periods. The sample period runs from January 1980 to December 2019. At the end of each calendar month, stocks are ranked in ascending order on the basis of their nominal price and are assigned to one of five price-sorted portfolios based on the NYSE breakpoint. Portfolios are value-weighted and refreshed every calendar month. Bond market risk is proxied by the volatility of realized the 10-year Treasury bond. TVOL is the value-weighted total volatility of stock returns and IVOL is the standard deviation of the residuals from the Fama-French three-factor regression. To determine the high institutional ownership subsample, we first measure the time-series average of the stock institutional ownership (IO). Then, we rank stocks in ascending order on the basis of their average institutional holding. The top 30% of stocks are grouped into the high IO sub-sample. Low-inflation periods are defined as such using the PPI index (i.e., if the monthly inflation is in the bottom 30th percentile). Panel A shows our benchmark results and Panel B and Panel C show the different volatility combinations. Numbers in parentheses are t-statistics based on Newey-West standard errors, and 5% statistical significance is indicated in bold.

Portfolio	**P1**	**P2**	**P3**	**P4**	**P5**	**P1-P5**
Panel A: Benchmark						
Y	**0.51**	**0.10**	**0.11**	**0.07**	0.03	**0.48**
	(4.61)	(3.02)	(3.78)	(3.09)	(1.15)	(3.89)
Constant	**-0.02**	**0.02**	**0.02**	**0.02**	**0.02**	**-0.04**
	(-1.98)	(5.90)	(8.09)	(11.93)	(8.45)	(-3.31)
Obs.	183	183	183	183	183	183
Time-series Average Number of Firm Obs.	520	364	313	286	264	256
Adj. R^2	0.14	0.06	0.15	0.12	0.01	0.11
Panel B: Portfolio total return volatility and bond market volatility						
Y	**0.60**	**0.26**	**0.27**	**0.16**	**0.11**	**0.47**
	(3.65)	(6.11)	(7.01)	(6.16)	(3.46)	(2.74)
$TVOL_{portfolio}$	**-1.29**	**0.55**	**0.34**	**0.21**	0.35	**-3.05**
	(-2.82)	(4.63)	(4.09)	(3.15)	(1.83)	(-4.76)
Bond VOL	-1.42	**-1.02**	**-1.19**	**-0.73**	**-0.54**	-1.05
	(-1.31)	(-4.03)	(-5.30)	(-5.20)	(-2.40)	(-0.98)
Constant	**0.04**	**0.02**	**0.02**	**0.02**	**0.02**	0.02
	(2.84)	(4.22)	(7.98)	(10.06)	(4.97)	(1.57)
Adj. R^2	0.32	0.31	0.44	0.36	0.16	0.42
Panel C: Portfolio idiosyncratic volatility and bond market volatility						
Y	**0.61**	**0.26**	**0.26**	**0.16**	**0.10**	**0.58**
	(3.49)	(6.18)	(6.40)	(5.67)	(2.70)	(3.14)
$IVOL_{portfolio}$	**-1.86**	**0.91**	0.42	0.21	0.35	**-3.46**
	(-2.56)	(3.45)	(1.95)	(1.17)	(0.80)	(-3.06)
Bond VOL	-0.99	**-1.26**	**-1.23**	**-0.74**	-0.55	-0.91
	(-0.80)	(-4.10)	(-4.37)	(-4.13)	(-1.79)	(-0.75)
Constant	**0.04**	**0.02**	**0.03**	**0.02**	**0.02**	0.02
	(2.53)	(4.38)	(8.94)	(11.20)	(5.79)	(1.00)
Adj. R^2	0.29	0.29	0.38	0.30	0.08	0.32

Table OA5: Relationship Between Earnings and Nominal Yields Conditional on Idiosyncratic Volatility and Firm Age

This table shows the results of the monthly regression of the portfolio value-weighted E/P ratio on the nominal treasury yield and different volatility measures after controlling the effect of Idiosyncratic Volatility (IVOL) and firm age (AGE). The sample period runs from January 1968 to December 2019. At the end of each calendar month, stocks are ranked in ascending order on the basis of their IVOL or AGE and are assigned to one of five price-sorted portfolios. Within each IVOL or AGE quintile, we then sort stocks into five portfolios on the basis of their nominal price, forming 25 portfolios. Portfolios are value-weighted and refreshed every calendar month. We report the averaged nominal yields betas across different IVOL or AGE quintiles. IVOL is measured as the standard deviation of residuals from the regression of monthly stock returns on the Fama-French three-factor model. Age is determined as the number of months since the corporation is first listed on CRSP. Results for IVOL and AGE are reported in Panels A and B, respectively. Numbers in parentheses are t-statistics based on Newey-West standard errors, and 5% statistical significance is indicated in bold.

Panel A1: Controlling for IVOL

	Price1	Price2	Price3	Price4	Price5	P1-5
Averaged I1 to I5	**1.31**	0.63	**0.36**	**0.22**	**0.12**	**1.19**
t-stat	(2.16)	(1.87)	(2.04)	(2.92)	(3.56)	(2.08)

Panel A2: Controlling for IVOL, Portfolio TVOL, & Bond Market Volatility

	Price1	Price2	Price3	Price4	Price5	P1-5
Averaged I1 to I5	**2.02**	**0.72**	**0.53**	**0.37**	**0.18**	2.24
t-stat	(2.04)	(2.73)	(2.44)	(2.93)	(3.24)	(1.87)

Panel A3: Controlling for IVOL, Portfolio IVOL, & Bond Market Volatility

	Price1	Price2	Price3	Price4	Price5	P1-5
Averaged I1 to I5	**2.10**	**0.73**	**0.52**	**0.37**	**0.17**	2.19
t-stat	(2.07)	(2.61)	(2.37)	(2.83)	(3.21)	(1.89)

Panel B1: Controlling for Age

	Price1	Price2	Price3	Price4	Price5	P1-5
Averaged A1 to A5	**1.15**	**0.32**	**0.21**	**0.13**	**0.07**	**1.08**
t-stat	(4.12)	(11.13)	(5.26)	(2.02)	(2.98)	(4.07)

Panel B2: Controlling for Age, Portfolio TVOL, & Bond Market Volatility

	Price1	Price2	Price3	Price4	Price5	P1-5
Averaged A1 to A5	**2.07**	**0.68**	**0.39**	**0.19**	0.05	**2.03**
t-stat	(4.64)	(7.98)	(7.38)	(3.08)	(1.83)	(4.16)

Panel B3: Controlling for Age, Portfolio IVOL, & Bond Market Volatility

	Price1	Price2	Price3	Price4	Price5	P1-5
Averaged A1 to A5	**2.13**	**0.69**	**0.38**	**0.19**	0.04	**2.04**
t-stat	(4.91)	(7.71)	(6.81)	(2.97)	(1.64)	(4.69)

Table OA6: Relationship Between Earnings and Nominal Yields Conditional on Idiosyncratic Volatility During High/Low Inflation Periods

This table shows the results of the monthly regression of the portfolio value-weighted E/P ratio on the nominal treasury yield and different volatility measures after controlling the effect of Idiosyncratic Volatility (IVOL) during high and low inflation periods. The sample period runs from January 1968 to December 2019. At the end of each calendar month, stocks are ranked in ascending order on the basis of their IVOL and are assigned to one of five price-sorted portfolios. Within each IVOL quintile, we then sort stocks into five portfolios on the basis of their nominal price, forming 25 portfolios. Portfolios are value-weighted and refreshed every calendar month. We report the averaged nominal yields betas across different IVOL quintiles. IVOL is measured as the standard deviation of residuals from the regression of monthly stock returns on the Fama-French three-factor model. High/Low-inflation periods are defined as such using the PPI index (i.e., if the monthly inflation is in the top/bottom 30th percentile). Panel A shows the results for high-inflation periods and Panel B shows the results for low-inflation periods. Numbers in parentheses are t-statistics based on Newey-West standard errors, and 5% statistical significance is indicated in bold.

Panel A: High-Inflation Sub-sample

Panel A-1: Benchmark

	Price1	Price2	Price3	Price4	Price5	Price1 - Price5
Averaged I1 to I5	**2.19**	**0.88**	**0.59**	**0.34**	**0.15**	**2.03**
t-stat	(2.04)	(2.09)	(2.05)	(2.84)	(2.53)	(2.00)

Panel A-2: Portfolio total return volatility and bond market volatility

	Price1	Price2	Price3	Price4	Price5	Price1 - Price5
Averaged I1 to I5	**2.68**	**1.00**	**0.56**	**0.45**	**0.18**	2.80
t-stat	(2.02)	(2.58)	(3.27)	(2.75)	(3.02)	(1.87)

Panel A-3: Portfolio idiosyncratic volatility and bond market volatility

	Price1	Price2	Price3	Price4	Price5	Price1 - Price5
Averaged I1 to I5	**2.76**	**1.03**	**0.57**	**0.46**	**0.17**	2.78
t-stat	(2.05)	(2.50)	(3.01)	(2.65)	(3.04)	(1.91)

Panel B: Low-Inflation Sub-sample

Panel B-1: Benchmark

	Price1	Price2	Price3	Price4	Price5	Price1 - Price5
Averaged I1 to I5	**0.89**	0.56	**0.19**	**0.12**	**0.08**	**0.81**
t-stat	(2.60)	(1.70)	(2.14)	(2.28)	(3.78)	(2.49)

Panel B-2: Portfolio total return volatility and bond market volatility

	Price1	Price2	Price3	Price4	Price5	Price1 - Price5
Averaged I1 to I5	**1.85**	**0.58**	**0.57**	**0.37**	**0.22**	2.19
t-stat	(2.20)	(3.49)	(2.46)	(3.22)	(4.30)	(1.94)

Panel B-3: Portfolio idiosyncratic volatility and bond market volatility

	Price1	Price2	Price3	Price4	Price5	Price1 - Price5
Averaged I1 to I5	**1.91**	**0.60**	**0.55**	**0.36**	**0.21**	**2.16**
t-stat	(2.20)	(3.22)	(2.43)	(3.19)	(4.40)	(1.96)

Table OA7: Relationship Between Earnings and Nominal Yields Conditional on Firm Age During High/Low Inflation Periods

This table shows the results of the monthly regression of the portfolio value-weighted E/P ratio on the nominal treasury yield and different volatility measures after controlling the effect of firm age (AGE) during high and low inflation periods. The sample period runs from January 1968 to December 2019. At the end of each calendar month, stocks are ranked in ascending order on the basis of their AGE and are assigned to one of five price-sorted portfolios. Within each AGE quintile, we then sort stocks into 5 portfolios on the basis of their nominal price, forming 25 portfolios. Portfolios are value-weighted and refreshed every calendar month. We report the averaged nominal yields betas across different AGE quintiles. Age is determined as the number of months since the corporation is first listed on CRSP. High/Low-inflation periods are defined as such using the PPI index (i.e., if the monthly inflation is in the top/bottom 30th percentile). Panel A shows our benchmark results and Panel B and Panel C show the different volatility combinations. Numbers in parentheses are t-statistics based on Newey-West standard errors, and 5% statistical significance is indicated in bold.

Panel A: High-Inflation Sub-sample

Panel A-1: Benchmark

	Price1	Price2	Price3	Price4	Price5	Price1 - Price5
Averaged A1 to A5	**2.02**	**0.57**	**0.33**	**0.19**	**0.07**	**1.95**
t-stat	(3.57)	(7.00)	(6.52)	(2.31)	(3.13)	(3.52)

Panel A-2: Portfolio total return volatility and bond market volatility

	Price1	Price2	Price3	Price4	Price5	Price1 - Price5
Averaged A1 to A5	**2.64**	**0.85**	**0.47**	**0.23**	0.06	**2.55**
t-stat	(3.32)	(7.68)	(8.51)	(3.03)	(1.86)	(3.04)

Panel A-3: Portfolio idiosyncratic volatility and bond market volatility

	Price1	Price2	Price3	Price4	Price5	Price1 - Price5
Averaged A1 to A5	**2.70**	**0.88**	**0.45**	**0.22**	**0.06**	**2.59**
t-stat	(3.59)	(7.63)	(7.34)	(2.87)	(1.96)	(3.37)

Panel B: Low-Inflation Sub-sample

Panel B-1: Benchmark

	Price1	Price2	Price3	Price4	Price5	Price1 - Price5
Averaged A1 to A5	**0.57**	**0.14**	**0.15**	0.07	0.06	**0.51**
t-stat	(2.22)	(3.96)	(2.31)	(1.02)	(1.42)	(2.22)

Panel B-2: Portfolio total return volatility and bond market volatility

	Price1	Price2	Price3	Price4	Price5	Price1 - Price5
Averaged A1 to A5	**2.02**	**0.68**	**0.44**	**0.21**	**0.11**	**2.19**
t-stat	(4.19)	(6.36)	(6.94)	(2.83)	(4.96)	(4.03)

Panel B-3: Portfolio idiosyncratic volatility and bond market volatility

	Price1	Price2	Price3	Price4	Price5	Price1 - Price5
Averaged A1 to A5	**2.10**	**0.69**	**0.44**	**0.20**	**0.10**	**2.17**
t-stat	(4.35)	(5.96)	(6.91)	(2.52)	(3.88)	(4.40)

Table OA8: Robustness Analysis: Controlling for Equity Risk Premium and Dividend Growth Rate

This table shows the results of the monthly regression of the portfolio value-weighted E/P ratio on the nominal treasury yield and different volatility measures controlling the effect of equity premium (market excess returns) and expected dividend growth rate (proxied by the historical average of the S&P500 index nominal dividend with a rolling window of ten years). The sample period runs from January 1968 to December 2019. At the end of each calendar month, stocks are ranked in ascending order on the basis of their nominal price and are assigned to one of 10 price-sorted portfolios based on the NYSE breakpoint. Portfolios are value-weighted and refreshed every calendar month. Bond market risk is proxied by the realized volatility of the 10-year Treasury bond. TVOL is the value-weighted total volatility of stock returns and IVOL is the standard deviation of the residuals from the Fama-French three-factor regression. Panel A shows our benchmark results. Panel B and Panel C show the different volatility combinations. Numbers in parentheses are t-statistics based on Newey-West standard errors, and 5% statistical significance is indicated in bold.

Portfolio	**Full**	**P1**	**P2**	**P3**	**P4**	**P5**	**P1-P10**
Panel A: Benchmark							
Y	**0.11**	**0.49**	**0.16**	**0.14**	**0.14**	**0.07**	**0.42**
	(5.83)	(4.80)	(5.08)	(5.82)	(6.40)	(3.34)	(3.93)
Equity Premium	-0.01	-0.03	-0.03	-0.02	-0.01	-0.01	-0.02
	(-1.54)	(-0.50)	(-1.55)	(-1.85)	(-1.42)	(-1.08)	(-0.32)
Dividend Growth	**1.12**	6.01	0.72	0.46	0.82	0.32	5.69
	(2.55)	(1.34)	(0.68)	(0.67)	(1.74)	(0.89)	(1.26)
Constant	**0.01**	**-0.06**	0.01	**0.01**	**0.01**	**0.01**	**-0.07**
	(3.46)	(-2.39)	(1.73)	(3.55)	(4.54)	(7.09)	(-2.93)
Number of Months	624	624	624	624	624	624	624
Time-series Average Number of Firm Obs.	3809	1677	733	545	456	398	1279
Adj. R^2	0.29	0.15	0.17	0.25	0.35	0.11	0.11

Panel B: Portfolio total return volatility and bond market volatility							
Y	**0.17**	**0.92**	**0.29**	**0.21**	**0.20**	**0.09**	**0.85**
	(9.18)	(6.99)	(7.67)	(8.68)	(9.09)	(4.60)	(6.03)
Equity Premium	0.00	-0.02	0.00	0.00	0.00	0.00	0.01
	(-0.10)	(-0.44)	(0.04)	(0.43)	(0.37)	(-0.49)	(0.21)
Dividend Growth	**1.73**	6.40	**2.51**	**1.63**	**1.53**	0.58	4.64
	(4.50)	(1.78)	(3.32)	(3.20)	(4.29)	(1.40)	(1.29)
$\text{TVOL}_{\text{portfolio}}$	**0.26**	-0.79	**0.58**	**0.52**	**0.28**	0.15	**-1.76**
	(2.30)	(-1.59)	(4.39)	(5.81)	(3.42)	(0.93)	(-2.40)
Bond VOL	**-0.68**	**-4.43**	**-1.39**	**-0.91**	**-0.72**	-0.27	**-4.03**
	(-5.31)	(-4.57)	(-5.93)	(-5.69)	(-5.08)	(-1.94)	(-4.42)
Constant	**0.01**	0.02	0.01	**0.01**	**0.01**	**0.01**	0.01
	(3.36)	(1.15)	(1.12)	(2.94)	(4.23)	(4.67)	(0.58)
Adj. R^2	0.49	0.41	0.41	0.46	0.51	0.14	0.38

Portfolio	**Full**	**P1**	**P2**	**P3**	**P4**	**P5**	**P1-P10**
Panel C: Portfolio idiosyncratic volatility and bond market volatility							
Y	**0.16**	**0.95**	**0.27**	**0.19**	**0.19**	**0.09**	**0.88**
	(8.40)	(7.22)	(7.74)	(7.99)	(8.67)	(4.15)	(6.11)
Equity Premium	-0.01	0.00	-0.01	-0.01	0.00	-0.01	0.01
	(-1.22)	(-0.02)	(-0.85)	(-1.06)	(-0.72)	(-1.10)	(0.23)
Dividend Growth	**1.67**	7.28	**2.89**	**1.87**	**1.55**	0.39	6.07
	(3.84)	(1.92)	(3.71)	(3.60)	(4.11)	(0.88)	(1.64)
$IVOL_{portfolio}$	0.26	-0.59	**0.94**	**0.80**	**0.35**	-0.05	-1.27
	(1.14)	(-0.84)	(4.09)	(4.82)	(2.14)	(-0.20)	(-1.40)
Bond VOL	**-0.65**	**-4.68**	**-1.52**	**-0.97**	**-0.71**	-0.19	**-4.28**
	(-4.22)	(-4.40)	(-5.71)	(-5.27)	(-4.39)	(-1.25)	(-4.29)
Constant	**0.01**	0.01	0.00	**0.01**	**0.01**	**0.02**	0.00
	(3.49)	(0.53)	(0.72)	(2.59)	(4.51)	(5.26)	(-0.12)
Adj. R^2	0.46	0.39	0.43	0.45	0.49	0.13	0.35

Table OA9: Relationship Between Earnings and Nominal Yields Conditional on Firm Size - Price Deciles

The table shows the results of the monthly regression of the portfolio value-weighted E/P ratio on the nominal Treasury yield and different volatility measures, after controlling for the effect of firm size (SIZE). The sample period runs from January 1968 to December 2019. At the end of each calendar month, stocks are ranked in ascending order by SIZE and are assigned to one of 10 price-sorted portfolios. Within each SIZE decile, stocks are sorted into 10 portfolios based on their nominal price, resulting in 100 portfolios. The portfolios are value-weighted and updated every calendar month. We report the averaged nominal yields betas across different SIZE deciles. SIZE is measured as the firm's market capitalization. Panel A shows our benchmark results, and Panel B and Panel C show the different volatility combinations. Numbers in parentheses are t-statistics based on Newey-West standard errors, and 5% statistical significance is indicated in bold.

Panel A: Benchmark

	Price1	Price2	Price3	Price4	Price5	Price6	Price7	Price8	Price9	Price10	Price1-Price10
Averaged S1 to S10	**1.94**	**1.00**	**0.70**	**0.55**	**0.50**	0.42	0.43	0.28	0.55	0.74	**1.20**
t-stat	(2.06)	(1.99)	(2.40)	(2.34)	(2.16)	(1.36)	(1.57)	(0.96)	(1.36)	(1.23)	(2.06)

Panel B: Portfolio total return volatility and bond market volatility

	Price1	Price2	Price3	Price4	Price5	Price6	Price7	Price8	Price9	Price10	Price1-Price10
Averaged S1 to S10	**3.56**	**1.54**	**1.12**	**0.84**	0.96	0.98	0.85	0.78	0.97	-0.76	**4.87**
t-stat	(2.04)	(1.96)	(2.40)	(2.42)	(1.94)	(1.45)	(1.36)	(0.92)	(1.03)	(-0.75)	(2.90)

Panel C: Portfolio idiosyncratic volatility and bond market volatility

	Price1	Price2	Price3	Price4	Price5	Price6	Price7	Price8	Price9	Price10	Price1-Price10
Averaged S1 to S10	**3.64**	1.58	**1.11**	**0.84**	0.96	0.97	0.84	0.79	0.97	-0.71	**4.96**
t-stat	(2.05)	(1.92)	(2.36)	(2.35)	(1.91)	(1.43)	(1.34)	(0.93)	(1.03)	(-0.69)	(2.85)

Table OA10: Relationship Between Earnings and Nominal Yields Conditional on Firm Size - Price Quintiles

This table shows the results of the monthly regression of the portfolio value-weighted E/P ratio on the nominal treasury yield and different volatility measures after controlling the effect firm size (SIZE). The sample period runs from January 1968 to December 2019. At the end of each calendar month, stocks are ranked in ascending order on the basis of their SIZE and are assigned to one of five price-sorted portfolios. Within each SIZE quintile, we then sort stocks into five portfolios on the basis of their nominal price, forming 25 portfolios. Portfolios are value-weighted and refreshed every calendar month. We report the averaged nominal yields betas across different SIZE quintiles. SIZE is measured as the market capitalization of a firm. Panel A shows our benchmark results and Panel B and Panel C show the different volatility combinations. Numbers in parentheses are t-statistics based on Newey-West standard errors, and 5% statistical significance is indicated in bold.

Panel A: Benchmark						
	Price1	Price2	Price3	Price4	Price5	Price1-Price5
Averaged S1 to S5	1.47	0.51	0.57	0.26	0.30	1.17
t-stat	(1.53)	(1.69)	(1.50)	(1.05)	(1.68)	(1.47)
Panel B: Portfolio total return volatility and bond market volatility						
	Price1	Price2	Price3	Price4	Price5	Price1-Price5
Averaged S1 to S5	2.62	0.95	0.95	0.55	**-0.40**	3.10
t-stat	(1.53)	(1.62)	(1.52)	(1.04)	(-2.08)	(1.81)
Panel C: Portfolio idiosyncratic volatility and bond market volatility						
	Price1	Price2	Price3	Price4	Price5	Price1-Price5
Averaged S1 to S5	4.02	1.15	1.09	0.78	**-1.05**	4.68
t-stat	(1.56)	(1.59)	(1.51)	(1.32)	(-2.93)	(1.95)

Table OA11: Relationship Between Earnings and Nominal Yields Controlling for Size (Firm-level)

This table shows the results of panel regressions that regress individual firm E/P ratio on nominal treasury yield, firm size and different volatility measures. The sample period runs from January 1968 to December 2019. Bond market risk is proxied by the realized volatility of the 10-year Treasury bond. TVOL is the value-weighted total volatility of stock returns and IVOL is the standard deviation of the residuals from the monthly Fama-French three-factor regression. Clustered standard errors at the firm level are reported in parentheses, and 5% statistical significance is indicated in bold.

	(1)	(2)	(3)
Y	**0.60**	**0.42**	**0.41**
	(23.74)	(14.12)	(13.62)
Firm Size	**0.02**	**0.01**	**0.01**
	(35.00)	(16.80)	(14.18)
Bond VOL		**-0.53**	**-0.36**
		(-4.28)	(-2.84)
TVOL		**-1.28**	
		(-36.73)	
IVOL			**-1.43**
			(-35.15)
Constant	**-0.12**	**-0.01**	**-0.01**
	(-31.23)	(-3.19)	(-2.86)
Obs.	2,376,579	2,348,757	2,343,203
Adj. R^2	0.04	0.10	0.10

References

Ang, A., Hodrick, R.J., Xing, Y., Zhang, X., 2006. The cross-section of volatility and expected returns. The Journal of Finance 61, 259–299.

Ang, A., Hodrick, R.J., Xing, Y., Zhang, X., 2009. High idiosyncratic volatility and low returns: International and further us evidence. Journal of Financial Economics 91, 1–23.

Basak, S., Yan, H., 2010. Equilibrium asset prices and investor behaviour in the presence of money illusion. Review of Economic Studies 77, 914–936.

Cochrane, H.J., Longstaff, A.F., Santa-Clara, P., 2008. Two Trees . Review of Financial Studies 21, 347–385.

David, A., Veronesi, P., 2013. What ties return volatilities to fundamentals and price valuations? Journal of Political Economy 121, 682–746.

Hansen, L.P., Singleton, K.J., 1982. Generalized instrumental variables estimation of nonlinear rational expectations models. Econometrica , 1269–1286.

Hirshleifer, D., 2001. Investor psychology and asset pricing. The Journal of Finance 56, 1533–1597.

Kumar, A., 2009. Hard-to-value stocks, behavioral biases, and informed trading. Journal of Financial and Quantitative Analysis 44, 1375–1401.

Martin, I., 2013. The Lucas Orchard . Econometrica 81, 55–111.

Mehra, R., Prescott, E.C., 1985. The Equity Premium: A Puzzle. Journal of Monetary Economics 15, 145–161.

Pástor, L., Veronesi, P., 2003. Stock valuation and learning about profitability. The Journal of Finance 58, 1749–1789.

Weil, P., 1989. The Equity Premium Puzzle and the Risk-free Rate Puzzle. Journal of Monetary Economics 24, 401–421.