From Canvas to Blockchain: Impact of Royalties on Art Market Efficiency

Online Appendix

Xinyu Cao, Jieteng Chen*, T. Tony Ke

Proof of Proposition 4: Effect of Royalties on Prices and Price Distortion

Proof. If $q_H/q_L < 1/\sqrt{1-\delta}$ and $0 < r < \widehat{r}$, we have

$$\begin{aligned}
&\frac{d(\widehat{P}_H - P_H^*)}{dr} \\
=&\frac{(q_H-q_L)}{2}\left\{\frac{d}{dr}\left(\frac{2\alpha}{1+\gamma}\right)+\frac{[(1-\gamma)(q_H-q_L)+2(1+\alpha)q_L]\frac{d}{dr}(\frac{1-\gamma}{1+\gamma})+2(1-\gamma)q_L\frac{d}{dr}(\frac{1+\alpha}{1+\gamma})}{\sqrt{[(1-\gamma)(q_H-q_L)+2(1+\alpha)q_L]^2-4(1+\alpha)^2q_L^2}}\right\} \\
<&\frac{(q_H-q_L)}{2}\left[\frac{d}{dr}\left(\frac{2\alpha}{1+\gamma}\right)+\frac{d}{dr}\left(\frac{1-\gamma}{1+\gamma}\right)\right] = -\frac{r\delta(1-\gamma)^2(q_H-q_L)}{4(1-r)^2\gamma(1-\delta+\gamma)^2} \le 0,
\end{aligned}$$

where the $'<'$ is because $\frac{d}{dr}(\frac{1-\gamma}{1+\gamma}) < 0$ and $\frac{d}{dr}(\frac{1+\alpha}{1+\gamma}) < 0$. This also implies that

$$\frac{d\widehat{P}_H}{dr} = \frac{d(\widehat{P}_H - P_H^*)}{dr} + \frac{dP_H^*}{dr} < 0.$$

□

Proof of Proposition 5: Effect of Royalties on Artist Profit

Proof. If $q_H/q_L < 1/\sqrt{1-\delta}$ and $0 \le r < \widehat{r}$, we have

$$\widehat{\Pi}_H = \Pi_H(\widehat{P}_H, 1) = \frac{1}{q_H}\left[\left(\frac{1+\alpha}{1+\gamma}\right)^2 q_H^2 - (\Delta P)^2\right] < \left(\frac{1+\alpha}{1+\gamma}\right)^2 q_H = \Pi_H^*,$$

where $\Delta P = \widehat{P}_H - P_H^*$. Then we have $\frac{d\widehat{\Pi}_H}{dr} = \frac{2}{q_H}\left[\left(\frac{1+\alpha}{1+\gamma}\right)q_H^2\frac{d}{dr}\left(\frac{1+\alpha}{1+\gamma}\right) - \Delta P\frac{d(\Delta P)}{dr}\right]$, and

$$\left.\frac{d\widehat{\Pi}_H}{dr}\right|_{r=0} = -\frac{2}{q_H}\Delta P\frac{d(\Delta P)}{dr} > 0, \text{ and } \left.\frac{d\widehat{\Pi}_H}{dr}\right|_{r=\widehat{r}} = -\frac{\delta\widehat{r}(1+\alpha)(1-\gamma)^2 q_H}{2(1-\widehat{r})^2\gamma(1+\gamma)(1-\delta+\gamma)^2} < 0.$$

By continuity of $\frac{d\widehat{\Pi}_H}{dr}$ in r, there must exist a threshold r^* such that $\frac{d\widehat{\Pi}_H}{dr}|_{r=r^*} = 0$. To complete the proof, we only need to establish the uniqueness of the threshold r^*.

First, notice that $\frac{d\widehat{\Pi}_H}{dr}$ is a function of $\alpha(\delta, r)$ and $\gamma(\delta, r)$. Given $\gamma(\delta, r) = \sqrt{1-(1-r)\delta} \in [\sqrt{1-\delta}, 1]$, we can rewrite $\alpha(\delta, r) = \alpha(\delta,\gamma) = -\frac{1-\delta-\gamma^2}{2(1-\delta+\gamma)}$ and substitute it into $\frac{d\widehat{\Pi}_H}{dr}$. Then we have $\frac{d\widehat{\Pi}_H}{dr} = \frac{d\widehat{\Pi}_H}{d\gamma} \times \frac{d\gamma}{dr}$ and $\frac{d\gamma}{dr} \ne 0$, which implies that we can instead show that $\frac{d\widehat{\Pi}_H}{d\gamma}$ has a unique root for $\sqrt{1-\delta} \le \gamma \le 1$. We have

$$\frac{d\widehat{\Pi}_H}{d\gamma} \propto \left(\frac{1+\alpha(\delta,\gamma)}{1+\gamma}\right) q_H^2 \frac{d}{d\gamma}\left(\frac{1+\alpha(\delta,\gamma)}{1+\gamma}\right) - \Delta P\frac{d\Delta P}{d\gamma} = \frac{G_1+G_2}{4(1+\gamma)^2(1+\gamma-\delta)^3}$$

where G_1 and G_2 are functions of γ, δ, q_H, and q_L as follows:

$$\begin{aligned}
G_1 =&[-(2+\delta)\gamma^4+\delta^2\gamma^3+12(1-\delta)\gamma^2+(\delta^3+11\delta^2-28\delta+16)\gamma+3(2-\delta)(1-\delta)^2]q_H^2\\
&+[(6-\delta)\gamma^4+(-\delta^2-8\delta+8)\gamma^3-12(1-\delta)\gamma^2\\
&\quad+(-\delta^3-19\delta^2+44\delta-24)\gamma-5(2-\delta)(1-\delta)^2]q_Hq_L\\
&+[-(4-\delta)\gamma^2-(8-6\delta)\gamma^3+(6\delta^2-14\delta+8)\gamma+(-\delta^3+6\delta^2-9\delta+4)]q_L^2,\\
G_2 =&-(q_H-q_L)(1+\gamma-\delta)(1+\gamma)[2(1+\gamma)^2-(1+\gamma)(3+\gamma)\delta+\delta^2]M\\
&-\frac{1}{(1+\gamma)(1+\gamma-\delta)M}\times\{(1+\gamma-\delta)[2(1-\delta)+(2-\delta)\gamma](q_H-q_L)^2\\
&\{2(1-\gamma)(1+\gamma-\delta)^2q_H+[(4-\delta)\gamma^2+4(1-\delta)\gamma-\delta(1-\delta)](1+\gamma)q_L\}\},
\end{aligned}$$

where $M=\sqrt{\left(\frac{1-\gamma}{1+\gamma}\right)^2(q_H-q_L)^2+2\left(\frac{1}{1+\gamma}+\frac{\gamma}{1+\gamma-\delta}\right)\left(\frac{1-\gamma}{1+\gamma}\right)(q_H-q_L)q_L}$.

Then, it suffices to show $\frac{d(G_1+G_2)}{d\gamma}<0$. We have

$$\frac{dG_2}{d\gamma}=\frac{(q_H-q_L)^3G_3(q_H)}{(1+\gamma)^3(1+\gamma-\delta)^2M^3}, \text{and } G_3(q_H)=b_1q_H^2+b_2q_H+b_3,$$

where the coefficients b_1, b_2, and b_3 are functions of γ, δ, and q_L. Since $G_3''(q_H)<0$ and $G_3'(q_L)<0$, we have $G_3(q_H)<G_3(q_L)<0$, which implies that $\frac{dG_2}{d\gamma}<0$. For $\frac{dG_1}{d\gamma}$, we consider two cases. If $\frac{dG_1}{d\gamma}\leq 0$, then we have $\frac{d(G_1+G_2)}{d\gamma}<0$; otherwise, if $\frac{dG_1}{d\gamma}>0$, then, we have

$$\begin{aligned}
\frac{d(G_1+G_2)}{d\gamma}\propto&\frac{dG_1}{d\gamma}M+\frac{(q_H-q_L)^3G_3}{(1+\gamma)^3(1+\gamma-\delta)^2M^2}\\
<&\frac{dG_1}{d\gamma}\frac{1-\sqrt{1-\delta}}{1+\sqrt{1-\delta}}(q_H+q_L)+\frac{(q_H-q_L)^3G_3}{(1+\gamma)^3(1+\gamma-\delta)^2M^2}\\
\propto&F(\rho)
\end{aligned}$$

where the " $<$ " is due to $M<\frac{1-\sqrt{1-\delta}}{1+\sqrt{1-\delta}}(q_H+q_L)$ and

$$F(\rho)=c_1\rho^4+c_2\rho^3+c_3\rho^2+c_4\rho+c_5,$$

where c_1, c_2, c_3, c_4, and c_5 are functions of γ and δ, and $\rho=\frac{q_H}{q_L}\in[1,1/\sqrt{1-\delta}]$.

Lastly, we prove $F(\rho)<0$ for $1<\rho<1/\sqrt{1-\delta}$. Note that $H(\rho)=c_1\rho^3+c_2\rho^2+c_3\rho+c_4$ can be bounded by two linear functions of ρ. Specifically, $H''(\rho)=6c_1\rho+2c_2$ is a linear function of ρ. For $\rho\in(1,1/\sqrt{1-\delta})$, since $H''(1)>0$ and $H''(1/\sqrt{1-\delta})<0$ can not hold at the same time, $H''(\rho)$ can only be (i) always positive, (ii) always negative, or (iii) first negative and then positive. This implies that $H'(\rho)=3c_1\rho^2+2c_2\rho+c_3$ only (i) increases with ρ, (ii) decreases with ρ, or (iii) first decreases and then increases with ρ. Then, the $H'(\rho)$ achieves the maximum at either $\rho=1$ or $\rho=1/\sqrt{1-\delta}$. Since $H'(1)<0$ and $H'(1/\sqrt{1-\delta})<0$, $H'(\rho)$ is always negative and H is always decreasing in ρ for $1<\rho<1/\sqrt{1-\delta}$.

For $\rho\in(1,1/\sqrt{1-\delta})$, $H(\rho)$ can be (i) convex; (ii) concave, (iii) and first concave and then convex. For (i), $H(\rho)$ can be bounded by the straight line that connects $H(1)$ and $H(1/\sqrt{1-\delta})$, which is $H_1(\rho)=H(1)+\frac{\sqrt{1-\delta}}{1-\sqrt{1-\delta}}\left(H\left(\frac{1}{\sqrt{1-\delta}}\right)-H(1)\right)(\rho-1)$. For (ii) and (iii), $H(\rho)$ can be bounded by the tangent line passing through $(1, H(1))$, which is $H_2(\rho) = H(1) + H'(1)(\rho - 1)$. Then, we have $H(\rho) < \max\{H_1(\rho), H_2(\rho)\}$, and

$$F(\rho) = \rho H(\rho) + c_5 < \rho \max\{H_1(\rho), H_2(\rho)\} + c_5 = \max\left\{\rho H_1(\rho) + c_5, \rho H_2(\rho) + c_5\right\}.$$

The rest is to show $F_1(\rho) = \rho H_1(\rho) + c_5 < 0$ and $F_2(\rho) = \rho H_2(\rho) + c_5 < 0$, which are quadratic functions of ρ. If $H(1/\sqrt{1-\delta}) \leq (2 - 1/\sqrt{1-\delta})H(1)$, then $F_1(\rho)$ is decreasing in ρ for $1 < \rho < 1/\sqrt{1-\delta}$ and thus $F_1(\rho) \leq F_1(1) = M(1) + c_5 < 0$; otherwise, if $H(1/\sqrt{1-\delta}) > (2 - 1/\sqrt{1-\delta})H(1)$, we have $F_1(\rho) \leq \frac{\sqrt{1-\delta}}{1-\sqrt{1-\delta}}\left[-\frac{\left(\frac{H(1)}{\sqrt{1-\delta}} - H\left(\frac{1}{\sqrt{1-\delta}}\right)\right)^2}{4\left(H\left(\frac{1}{\sqrt{1-\delta}}\right) - H(1)\right)} + \frac{1-\sqrt{1-\delta}}{\sqrt{1-\delta}}c_5\right] < 0$. We have $F_2(\rho) \leq -\frac{[H(1)-H'(1)]^2}{4H'(1)} + c_5 < 0$. □

Proof of Proposition 6: Effect of Royalties on Social Welfare

Proof. We show that $\widehat{D}_H$ can increase with r in a non-empty interval. We have

$$\left.\frac{d\widehat{D}_H}{dr}\right|_{r=0} = \left\{\frac{(1-\sqrt{1-\delta})q_H + (1+\sqrt{1-\delta})q_L}{\sqrt{[(1-\sqrt{1-\delta})q_H + (1+\sqrt{1-\delta})q_L]^2 - 4q_L^2}} - 1\right\}\frac{\delta(q_H - q_L)}{2(1+\sqrt{1-\delta})^2\sqrt{1-\delta}q_H} > 0.$$

By continuity of $d\widehat{D}_H/dr$ in r, there must exist a non-empty interval $0 \leq r < r'$ such that $\frac{d\widehat{D}_H}{dr} > 0$. Parts (ii) and (iii) are due to $dD_j^*/dr < 0$ and $dd_j^*/dr < 0$, as shown in the proof of Proposition 1. □

Proof of Proposition 7: Buyer Has Superior Information

Proof. The analysis in the main text proves (i) and (ii) of this proposition. We now focus on the interior optimal price, in which the royalty rate is most consequential in shifting the artist's pricing, and show that Π_B^* can increase with r. First, the demand function for the artwork of type j is

$$D_j(P) = \begin{cases} 1 & \text{if } 0 \leq P < \delta(1-r)\pi_j^*, \\ 1 - \frac{P - \delta(1-r)\pi_j^*}{q_j} & \text{if } \delta(1-r)\pi_j^* \leq P < \delta(1-r)\pi_j^* + q_j, \\ 0 & \text{otherwise.} \end{cases}$$

Thus, the optimal interior price is the solution to the following optimization problem:

$$\begin{aligned} P_{int} &= \arg\max_P \lambda\left(P + Rq_H\right)\left(1 - \frac{P - \delta(1-r)\pi_H^*}{q_H}\right) + (1-\lambda)\left(P + Rq_L\right)\left(1 - \frac{P - \delta(1-r)\pi_L^*}{q_L}\right) \\ &= \frac{q_H q_L}{2(\lambda q_L + (1-\lambda)q_H)}\left\{1 + \frac{\delta(1-r)}{[1+\sqrt{1-\delta(1-r)}]^2} - R\right\}, \end{aligned}$$

where $R = \frac{r\delta}{\left[1+\sqrt{1-(1-r)\delta}\right]\left[1-\delta+\sqrt{1-(1-r)\delta}\right]}$. For P_{int} to be optimal, the following condition must hold to ensure it is an interior solution:

$$\delta(1-r)\pi_H^* < P_{int} < \delta(1-r)\pi_L^* + q_L,$$

which is satisfied for all $0 < r < 1$ if $q_H < 2q_L$ and $0 < \delta < \frac{(2q_H - q_L)q_L}{q_H^2}$. Additionally, we can easily show that P_{int} is the unique optimal price under these conditions. Then the artist's profit at this interior price is denoted by Π_{int}. Next, we have

$$\frac{dP_{int}}{dr} = -\frac{q_H q_L}{2(\lambda q_L + (1-\lambda)q_H)}\frac{dR}{dr} - \frac{\delta q_H q_L}{2(\lambda q_L + (1-\lambda)q_H)[1+\sqrt{1-(1-r)\delta}]^2\sqrt{1-(1-r)\delta}} < 0,$$

where $\frac{dR}{dr} > 0$. At $r = 0$, we have

$$\left.\frac{d\Pi_{int}}{dr}\right|_{r=0} = \frac{2\lambda(1-\lambda)\delta(q_H - q_L)^2}{(1+\sqrt{1-\delta})^3\sqrt{1-\delta}(\lambda q_L + (1-\lambda)q_H)} > 0.$$

Because of the continuity of $\frac{d\Pi_{int}}{dr}$ in r, there must be a non-empty interval of $r \in [0, r')$ such that $\frac{d\Pi_{int}}{dr} > 0$. □

Proof of Proposition 8: Neither Party Has Superior Information

Proof. The analysis in the main text has established (i) and (ii) of this proposition. We now focus on the interior optimal price, and show that Π_N^* can increase with r. First, as buyer 0 is uninformed about q, the demand functions for the artwork of type j are given by equations (A1) and (A2). Therefore, the optimal interior price is the solution to the following maximization problem:

$$\begin{aligned} P_{int} =& \arg\max_P \lambda\left(P + Rq_H\right)\left(1 - \frac{P - \delta(1-r)\overline{\pi}(\overline{\lambda})}{q_H}\right) + (1-\lambda)\left(P + Rq_L\right)\left(1 - \frac{P - \delta(1-r)\overline{\pi}(\overline{\lambda})}{q_L}\right) \\ =& \frac{q_H q_L}{2(\lambda q_L + (1-\lambda)q_H)}\left\{1 + \frac{\delta(1-r)}{[1+\sqrt{1-\delta(1-r)}]^2} - R\right\}, \end{aligned}$$

where $R = \frac{r\delta}{\left[1+\sqrt{1-(1-r)\delta}\right]\left[1-\delta+\sqrt{1-(1-r)\delta}\right]}$. To ensure P_{int} is interior, the following condition must hold

$$\delta(1-r)\overline{\pi}(\overline{\lambda}) < P_{int} < \delta(1-r)\overline{\pi}(\overline{\lambda}) + q_L,$$

which is true for all $0 < r < 1$ if $q_H < 2q_L$ and $0 < \delta < \frac{(2q_H - q_L)q_L}{q_H^2}$. Under these conditions, $\Pi_N(P)$ increases with P for $\delta(1-r)\overline{\pi}(\overline{\lambda}) + q_L \le P < \delta(1-r)\pi_H^* + q_L$ and decreases with P for $P \ge \delta(1-r)\pi_H^* + q_L$, yielding another local maximum at $P' = \delta(1-r)\pi_H^* + q_L$. We can show that

$$\Pi_N(P') < \frac{\lambda}{4q_H}(q_H + \delta(1-r)\pi_H^* + Rq_H)^2 < \Pi_N(P_{int}),$$

which implies that P_{int} is the unique optimal price for the artist. Last, similar to the Proof of Proposition 7, we can show that $\frac{dP_{int}}{dr} < 0$ and $\left.\frac{d\Pi_N(P_{int})}{dr}\right|_{r=0} > 0$, so there exists a non-empty interval of $r \in [0, r')$ such that $\frac{d\Pi_N(P_{int})}{dr} > 0$. □

Proof of Proposition 9: Royalty Rate Chosen by the Artist, Incomplete Information

Proof. In this proof, we introduce $\Pi_{jr}(P, r; \mu)$ to denote the j-type artist's profit when she chooses price P and royalty rate r, and the buyer 0's belief is μ.

First, we show that under incomplete information with the royalty rate chosen by the artist, there does not exist a pooling equilibrium that survives the intuitive criterion. This can be proved by contradiction. Suppose a pooling equilibrium exists with strategy $(\widetilde{P}, \widetilde{r})$. Following the same logic in the proof of Lemma A1, we can directly have an off-equilibrium-path strategy $(\widetilde{P}', \widetilde{r})$ that can fail the intuitive criterion, where $\widetilde{P}'$ is equal to P' in the proof of Lemma A1 with $r = \widetilde{r}$.

Second, we show that the equilibrium strategies constitute a PBE. Under complete information, the artist chooses $r_j^* = 0$ and $P_j^*|_{r=0} = \frac{q_j}{1+\sqrt{1-\delta}}$ for $j \in \{H, L\}$. If the optimal strategy under complete information can constitute a separating equilibrium, we have

$$\Pi_{Lr}(P_H^*|_{r=0}, 0; 1) \leq \Pi_{Lr}(P_L^*|_{r=0}, 0; 0) \equiv \Pi_{Lr}^* \Leftrightarrow \frac{q_H}{q_L} \geq k(\delta, 0) = \frac{1}{\sqrt{1-\delta}}.$$

Otherwise, if $\frac{q_H}{q_L} < \frac{1}{\sqrt{1-\delta}}$, we must have $\Pi_{Lr}(P_H, r_H; 1) \leq \Pi_{Lr}^*$.

Suppose the high-type artist chooses $0 \leq r \leq 1$. There exists a threshold $\overline{r} \in (0, 1)$ such that:

(a) for $0 \leq r < \overline{r}$, the high-type artist's optimal price under exogenous royalty rate, P_H^*, cannot achieve separation, that is $\Pi_{Lr}(P_H^*, r; 1) > \Pi_{Lr}^*$. Let $R = \frac{r\delta}{\left[1+\sqrt{1-(1-r)\delta}\right]\left[1-\delta+\sqrt{1-(1-r)\delta}\right]}$. So the high-type artist's price must satisfy $\Pi_{Lr}(P_H, r; 1) \leq \Pi_{Lr}^*$, which leads to

$$P_H \geq \widehat{P}_{Hr}(r) = \frac{\delta(1-r)\pi_H^* + (1-R)q_L + \sqrt{[(1+R)q_L + \delta(1-r)\pi_H^*]^2 - 4\Pi_{Lr}^* q_L}}{2}$$

or $P_H \leq \widehat{P}''_{Hr}(r)$, where $\widehat{P}''_{Hr}(r)$ is another root that can pin down $\Pi_{Lr}(\widehat{P}''_{Hr}(r), r; 1) = \Pi_{Lr}^*$.

(b) For $\overline{r} \leq r \leq 1$, the high-type artist's optimal price under exogenous royalty rate, P_H^*, can achieve separation, that is $\Pi_{Lr}(P_H^*, r; 1) \leq \Pi_{Lr}^*$. In this case, the high-type artist's profit $\Pi_{Hr}(P_H^*, r; 1)$ decreases with r. In summary, there exists a unique strategy $(\widehat{P}_{Hr}(\widehat{r}_H), \widehat{r}_H)$ that can maximize the high-type artist's profit and prevent the low-type artist from mimicking, where $\widehat{r}_H = \arg\max_r \widehat{\Pi}_{Hr}(\widehat{P}_{Hr}(r), r; 1) \in [0, \overline{r})$.[OA1] We have $\widehat{r}_H > 0$ since $d\widehat{\Pi}_{Hr}(\widehat{P}_{Hr}(r), r; 1)/dr\big|_{r=0} > 0$.

We show that for any off-equilibrium-path strategy, two types of artists cannot make a profitable deviation under the specified off-equilibrium-path belief. The low-type artist adopts the optimal strategy under belief $\mu = 0$ and thus does not deviate. For the high-type artist, if $\frac{q_H}{q_L} \geq \frac{1}{\sqrt{1-\delta}}$, the high-type's strategy is the optimal one under complete information and thus does not deviate. If $\frac{q_H}{q_L} < \frac{1}{\sqrt{1-\delta}}$, we can show that $\widehat{\Pi}_{Hr}(\widehat{P}_{Hr}(\widehat{r}_H), \widehat{r}_H; 1) \geq$

[OA1] The parameter range that allows $f(r) = \Pi_{Hr}(\widehat{P}_{Hr}(r), r; 1)$ to have multiple identical peaks has zero measure on the entire parameter space.

$\max_{P_H,r} \Pi_{Hr}(P_H, r; 0)$. Let us consider two cases. For $0 \le r < \bar{r}$, due to $\widehat{P}_{Hr}(r) \le \widehat{P}_H$, we have

$$\widehat{\Pi}_{Hr}(\widehat{P}_{Hr}(r), r; 1) \ge \widehat{\Pi}_H(\widehat{P}_H; 1) \ge \max_{P_H} \Pi_H(P_H; 0),$$

where the second "$\ge$" is shown in the proof of Proposition 3. This easily implies that

$$\widehat{\Pi}_{Hr}(\widehat{P}_{Hr}(\widehat{r}_H), \widehat{r}_H; 1) \ge \max_{r\in[0,\bar{r})} \widehat{\Pi}_{Hr}(\widehat{P}_{Hr}(r), r; 1) \ge \max_{P_H, r\in[0,\bar{r})} \Pi_{Hr}(P_H, r; 0).$$

For $\bar{r} \le r \le 1$, we have

$$\widehat{\Pi}_{Hr}(\widehat{P}_{Hr}(\widehat{r}_H), \widehat{r}_H; 1) \ge \max_{r\in[\bar{r},1]} \Pi_{Hr}(P_H^*, r; 1) \ge \max_{P_H, r\in[\bar{r},1]} \Pi_{Hr}(P_H, r; 0).$$

Last, the proofs of equilibrium uniqueness and that the specified belief survives the intuitive criterion follow the same logic in the proof of Proposition 3 and are thus omitted. □

Proof of Proposition 10: Infinite Resale Opportunities, Complete Information

Proof. We first consider the resale market $t \ge 1$. Seller t sets the price p_{jt} to maximize the expected resale payoff $(1-r)\pi_{j,t}$. Buyer t purchases if and only if $\theta_t q_j - p_{jt} + \delta(1-r)\pi_{j,t+1} \ge 0$. Therefore, the transaction probability is

$$d_{jt}(p_{jt}) = \Pr\left(\theta_t \ge \frac{p_{jt} - \delta(1-r)\pi_{j,t+1}}{q_j}\right) = 1 - \frac{p_{jt} - \delta(1-r)\pi_{j,t+1}}{q_j}.$$

Note that if buyer t does not purchase, seller t can sell the artwork at $t+1$ with expected resale payoff $\pi_{j,t+1}$. Therefore, the optimal price is:

$$\begin{aligned} p_{jt}^* &= \arg\max_{p_{jt}}(1-r)\pi_{jt} = \arg\max_{p_{jt}}(1-r)\left[p_{jt}d_{jt}(p_{jt}) + (1-d_{jt}(p_{jt}))\,\delta\pi_{j,t+1}\right] \\ &= \frac{1}{2}\left(q_j + (2-r)\delta\pi_{j,t+1}\right). \end{aligned}$$

Seller t's expected resale profit under the optimal price is

$$\pi_{jt}^* = p_{jt}^* d_{jt}(p_{jt}^*) + (1-d_{jt}(p_{jt}^*))\delta\pi_{j,t+1} = \frac{q_j^2 + 2(2-r)\delta q_j \pi_{j,t+1} + (\delta r \pi_{j,t+1})^2}{4q_j}.$$

Because of the time stationarity of all resale markets, we have $\pi_{jt}^* = \pi_{j,t+1} = \pi_{jI}^*$ in equilibrium. Then we can solve for π_{jI}^*, as well as p_{jI}^* and d_{jI}^* in resale markets, where the subscript "I" stands for "infinite resale opportunities".

Now we consider the primary market $t = 0$. The artist sets price P_j to maximize her expected profit:

$$\begin{aligned} \Pi_{jI}^* &= \max_{P_j}\left(P_j + \delta r \sum_{t=0}^{\infty} \delta^t \left(p_{jI}^* d_{jI}^*\right)\right) \Pr\left(u_{0j} \ge 0\right) \\ &= \max_{P_j}\left(P_j + \delta r \frac{p_{jI}^* d_{jI}^*}{1-\delta}\right)\left(1 - \frac{P_j - \delta(1-r)\pi_{jI}^*}{q_j}\right) \end{aligned}$$

$$= \max_{P_j} (P_j + R_I q_j) \left(1 - \frac{P_j - \delta(1-r)\pi^*_{jI}}{q_j}\right).$$

There are two possible cases:

- Case (a): If $(1 - R_I)q_j - \delta(1-r)\pi^*_{jI} > 0$, which is equivalent to $0 < \delta < \frac{4}{5}$, or $\frac{4}{5} \le \delta < 2(\sqrt{2}-1)$ and $\frac{\sqrt{5\delta-4}(3\delta-2+\sqrt{\delta(5\delta-4)})}{2\sqrt{\delta}(1-\delta)} < r < 1$, the optimal price is interior and can be solved by the first order condition: $P^*_{jIa} = \frac{(1-R_I)q_j+\delta(1-r)\pi^*_{jI}}{2}$.
- Case (b): If $(1 - R_I)q_j - \delta(1-r)\pi^*_{jI} \le 0$, which is equivalent to $\frac{4}{5} \le \delta < 2(\sqrt{2}-1)$ and $0 \le r \le \frac{\sqrt{5\delta-4}(3\delta-2+\sqrt{\delta(5\delta-4)})}{2\sqrt{\delta}(1-\delta)}$, or if $2(\sqrt{2}-1) \le \delta < 1$, the optimal price is a corner solution: $P^*_{jIb} = \delta(1-r)\pi^*_j$.

The equilibrium transaction probabilities in the primary market and the artist's profits in the above two cases can be solved as well.

Comparative statics: For resale markets:

$$\frac{dp^*_{jI}}{dr} = -\frac{\left(\delta(1+\delta) + \delta\sqrt{(1-\delta)(1-(1-r)\delta)}\right) q_j}{2\sqrt{1-(1-r)\delta}\left(\sqrt{1-\delta} + \sqrt{1-(1-r)\delta}\right)^3} < 0$$

$$\frac{dd^*_{jI}}{dr} = -\frac{\delta\sqrt{1-\delta}}{2\sqrt{1-(1-r)\delta}\left(\sqrt{1-\delta} + \sqrt{1-(1-r)\delta}\right)^2} < 0$$

$$\frac{d\pi^*_{jI}}{dr} = -\frac{\delta q_j}{\sqrt{1-(1-r)\delta}\left(\sqrt{1-\delta} + \sqrt{1-(1-r)\delta}\right)^3} < 0$$

For the primary market, there are two cases.

Case (a): When $0 < \delta < 4/5$, or when $4/5 \le \delta < 2(\sqrt{2}-1)$ and $\frac{\sqrt{5\delta-4}(3\delta-2+\sqrt{\delta(5\delta-4)})}{2\sqrt{\delta}(1-\delta)} < r < 1$, we have

$$\frac{dP^*_{jIa}}{dr} = -\frac{4(2-\delta)\sqrt{(1-\delta)(1-(1-r)\delta)} + 8 - 4(3-r)\delta + (4-5r)\delta^2}{4\sqrt{(1-\delta)(1-(1-r)\delta)}\left(\sqrt{1-\delta} + \sqrt{1-(1-r)\delta}\right)^4}\delta q_j < 0$$

$$\frac{dD^*_{jIa}}{dr} = -\frac{r\delta^3}{4\sqrt{(1-\delta)(1-(1-r)\delta)}\left(\sqrt{1-\delta} + \sqrt{1-(1-r)\delta}\right)^4} < 0$$

$$\frac{d\Pi^*_{jIa}}{dr} = -\frac{r\delta^3((1+R_I)q_j + \delta(1-r)\pi^*_{jI})}{4\sqrt{(1-\delta)(1-(1-r)\delta)}\left(\sqrt{1-\delta} + \sqrt{1-(1-r)\delta}\right)^4} < 0$$

Case (b): When $4/5 \le \delta < 2(\sqrt{2}-1)$ and $0 \le r \le \frac{\sqrt{5\delta-4}(3\delta-2+\sqrt{\delta(5\delta-4)})}{2\sqrt{\delta}(1-\delta)}$, or if $2(\sqrt{2}-1) \le \delta < 1$, we have

$$\frac{dP^*_{jIb}}{dr} = -\frac{\left(\delta + \delta\sqrt{(1-\delta)(1-(1-r)\delta)}\right) q_j}{\sqrt{1-(1-r)\delta}\left(\sqrt{1-\delta} + \sqrt{1-(1-r)\delta}\right)^3} < 0$$

$$\frac{dD^*_{jIb}}{dr} = 0$$

$$\frac{d\Pi^*_{jIb}}{dr} = -\frac{r\delta^3 q_j}{2\sqrt{(1-\delta)(1-(1-r)\delta)}\left(\sqrt{1-\delta}+\sqrt{1-(1-r)\delta}\right)^4} < 0$$

□

Proof of Proposition 11: Infinite Resale Opportunities, Incomplete Information

Proof. First, we prove that there does not exist a pooling equilibrium that survives the intuitive criterion. This proof largely follows the proof of Lemma A1, and therefore we mainly highlight the differences. Denote $\overline{\pi}_I(\mu) = \mu\pi^*_{HI} + (1-\mu)\pi^*_{LI}$. The artist j's profit under belief μ is $\Pi_j(P;\mu) = (P + R_I q_j)D_j(P;\mu)$, where

$$D_L(P;\lambda) = \begin{cases} 1 & 0 \le P < \delta(1-r)\overline{\pi}_I(\overline{\lambda}) \\ 1 - \frac{P-\delta(1-r)\overline{\pi}_I(\overline{\lambda})}{q_L} & \text{if } \ \delta(1-r)\overline{\pi}_I(\overline{\lambda}) \le P < \delta(1-r)\overline{\pi}_I(\overline{\lambda}) + q_L \\ 0 & \text{otherwise} \end{cases},$$

and

$$D_H(P;\lambda) = \begin{cases} 1 & 0 \le P < \delta(1-r)\overline{\pi}_I(\overline{\lambda}) \\ 1 - \frac{P-\delta(1-r)\overline{\pi}_I(\overline{\lambda})}{q_H} & \text{if } \delta(1-r)\overline{\pi}_I(\overline{\lambda}) \le P < \delta(1-r)\overline{\pi}_I(\overline{\lambda}) + q_L \\ 1 - \frac{q_L}{q_H} & \text{if } \delta(1-r)\overline{\pi}_I(\overline{\lambda}) + q_L \le P < \delta(1-r)\pi^*_{HI} + q_L \\ 1 - \frac{P-\delta(1-r)\pi^*_{HI}}{q_H} & \text{if } \delta(1-r)\pi^*_{HI} + q_L \le P \le \delta(1-r)\pi^*_{HI} + q_H \\ 0 & \text{otherwise.} \end{cases} \tag{OA1}$$

Suppose there exists a pooling equilibrium with price $\widetilde{P} < \delta(1-r)\overline{\pi}_I(\overline{\lambda}) + q_L$; otherwise, the low-type artist has zero profit and thus will deviate. There are two cases.
(i) If $0 \le \widetilde{P} < \delta(1-r)\overline{\pi}_I(\overline{\lambda})$, then the two types of artists' profits are

$$\Pi_L(\widetilde{P};\lambda) = \widetilde{P} + R_I q_L, \text{and } \ \Pi_H(\widetilde{P};\lambda) = \widetilde{P} + R_I q_H.$$

We consider an off-equilibrium-path price $P_o > \delta(1-r)\pi^*_{HI} > \widetilde{P}$ such that $\Pi_L(P_o;1) = \Pi_L(\widetilde{P};\lambda)$, then we have

$$\Pi_H(P_o,1) - \Pi_H(\widetilde{P},\lambda) = \frac{q_H - q_L}{q_H q_L} P_o\left[P_o - \delta(1-r)\pi^*_{HI}\right] > 0.$$

Therefore, $P' = P_o + \epsilon$ with sufficiently small $\epsilon > 0$ satisfies $\Pi_L(P';1) < \Pi_L(\widetilde{P};\lambda)$ and $\Pi_H(P';1) > \Pi_H(\widetilde{P};\lambda)$, which violates the intuitive criterion.
(ii) If $\delta(1-r)\overline{\pi}_I(\overline{\lambda}) \le \widetilde{P} < \delta(1-r)\overline{\pi}_I(\overline{\lambda}) + q_L$, then the two types of artists' profits are

$$\Pi_L(\widetilde{P};\lambda) = \left(\widetilde{P} + R_I q_L\right)\left(1 - \frac{\widetilde{P} - \delta(1-r)\overline{\pi}_I(\overline{\lambda})}{q_L}\right)$$
$$\text{and } \ \Pi_H(\widetilde{P};\lambda) = \left(\widetilde{P} + R_I q_H\right)\left(1 - \frac{\widetilde{P} - \delta(1-r)\overline{\pi}_I(\overline{\lambda})}{q_H}\right).$$

We consider an off-equilibrium-path price $P_o > \widetilde{P}$ such that $\Pi_L(P_o;1) = \Pi_L(\widetilde{P};\lambda)$, then we have

$$\begin{aligned}\Pi_H(P_o,1) - \Pi_H(\widetilde{P},\lambda) =& (\Pi_H(P_o,1) - \Pi_L(P_o,1)) - (\Pi_H(\widetilde{P},\lambda) - \Pi_L(\widetilde{P},\lambda)) \\ =& \frac{q_H - q_L}{q_H}\left[(P_o - \widetilde{P})(1-R_I) + R_I\delta(1-r)(\overline{\pi}_I(1) - \overline{\pi}_I(\overline{\lambda}))\right] > 0.\end{aligned}$$

Therefore, $P' = P_o + \epsilon$ with sufficiently small $\epsilon > 0$ satisfies $\Pi_L(P';1) < \Pi_L(\widetilde{P};\lambda)$ and $\Pi_H(P';1) > \Pi_H(\widetilde{P};\lambda)$, which violates the intuitive criterion. In summary, there does not exist any pooling equilibrium under the intuitive criterion.

Then, we solve for the separating equilibrium under incomplete information. To begin with, we show that the equilibrium can constitute a PBE in two cases.

(i) $0 < \delta < 4/5$, or $4/5 \leq \delta < 2(\sqrt{2}-1)$ and $\frac{\sqrt{5\delta-4}(3\delta-2+\sqrt{\delta(5\delta-4)})}{2\sqrt{\delta}(1-\delta)} < r < 1$

First, there exists a threshold of $\frac{q_H}{q_L}$ such that the optimal price under complete information can constitute a separating equilibrium if and only if

$$\Pi_L(P^*_{HIa};1) \leq \Pi_L(P^*_{LIa};0) \Leftrightarrow \frac{q_H}{q_L} \geq k_I(\delta,r),$$

where k_I is a function of δ and r. Otherwise, if $q_H/q_L < k_I(\delta,r)$, we must have $\Pi_L(P_H;1) \leq \Pi_L(P^*_L;0)$, which holds when

$$P_H \geq \widehat{P}_{HIa} = \frac{\delta(1-r)\pi^*_{HI} + (1-R_I)q_L + \sqrt{[(1+R_I)q_L + \delta(1-r)\pi^*_{HI}]^2 - 4\Pi^*_{LIa}q_L}}{2}$$

or $P_H \leq \widehat{P}''_{HIa}$, where $\widehat{P}''_{HIa}$ is another root that solves $\Pi_L(\widehat{P}''_{HIa};1) = \Pi_L(P^*_L;0)$. We also have $\widehat{P}_{HIa} > P^*_{HIa}$.

Then we show that for any off-equilibrium-path price, the high-type artist cannot make a profitable deviation under the specified off-equilibrium-path belief. If $\frac{q_H}{q_L} \geq k_I(\delta,r)$, the equilibrium price is the optimal one under complete information, and thus the high-type artist does not deviate. If $\frac{q_H}{q_L} < k_I(\delta,r)$, we can show that $\Pi_H(\widehat{P}_{HIa};1) \geq \max_{P_H}\Pi_H(P_H;0)$. Following the transaction probability in equation (OA1), the high-type artist's profit under belief $\lambda = 0$ is

$$\Pi_H(P_H;0) = \begin{cases}(P_H + R_Iq_H) & \text{if } 0 \leq P_H < \delta(1-r)\pi^*_{LI} \\ (P_H + R_Iq_H)\left(1 - \frac{P_H - \delta(1-r)\pi^*_{LI}}{q_H}\right) & \text{if } \delta(1-r)\pi^*_{LI} \leq P_H < \delta(1-r)\pi^*_{LI} + q_L, \\ (P_H + R_Iq_H)\left(1 - \frac{q_L}{q_H}\right) & \text{if } \delta(1-r)\pi^*_{LI} + q_L \leq P_H < \delta(1-r)\pi^*_{HI} + q_L, \\ (P_H + R_Iq_H)\left(1 - \frac{P_H - \delta(1-r)\pi^*_{HI}}{q_H}\right) & \text{if } \delta(1-r)\pi^*_{HI} + q_L \leq P_H \leq \delta(1-r)\pi^*_{HI} + q_H.\end{cases}$$

For $\delta(1-r)\pi^*_{HI} + q_L \leq P_H \leq \delta(1-r)\pi^*_{HI} + q_H$, we have

$$\begin{aligned}\frac{d\Pi_H(P_H;0)}{dP_H} &= \frac{2}{q_H}\left(\frac{(1-R_I)q_H}{2} + \frac{\delta(1-r)\pi^*_{HI}}{2} - P_H\right) \\ &\leq \frac{2}{q_H}\left(\frac{(1-R_I)q_H}{2} - \frac{\delta(1-r)\pi^*_{HI}}{2} - q_L\right) < 0,\end{aligned}$$

where the "$\leq$" is obtained due to $P_H \geq \delta(1-r)\pi^*_{HI} + q_L$. This means that $\delta(1-r)\pi^*_{HI} + q_L$

is the unique local maximizer of $\Pi_H(P_H;0)$ for $P_H \geq \delta(1-r)\pi^*_{LI} + q_L$. We also have $\widehat{P}_{HIa} < \delta(1-r)\pi^*_{HI} + q_L$. Given the quadratic form of $\Pi_H(P_H,1)$ and $\Pi_H(\delta(1-r)\pi^*_{HI} + q_L;0) = \Pi_H(\delta(1-r)\pi^*_{HI} + q_L;1)$, we have

$$\Pi_H(\widehat{P}_{HIa};1) \geq \Pi_H(\delta(1-r)\pi^*_{HI} + q_L;1) = \Pi_H(\delta(1-r)\pi^*_{HI} + q_L;0).$$

For $0 \leq P_H < \delta(1-r)\pi^*_{LI} + q_L$, we have $\Pi_H(P_H;0) \leq \frac{1}{4q_H}\left[q_H + \delta(1-r)\pi^*_{LI} + R_I q_H\right]^2$ due to its quadratic form. It suffices to have

$$\Pi_H(\widehat{P}_{HIa};1) - \frac{1}{4q_H}\left(q_H + \delta(1-r)\pi^*_{LI} + R_I q_H\right)^2 > 0.$$

(ii) $4/5 \leq \delta < 2(\sqrt{2}-1)$ and $0 \leq r < \frac{\sqrt{5\delta-4}(3\delta-2+\sqrt{\delta(5\delta-4)})}{2\sqrt{\delta}(1-\delta)}$, or $2(\sqrt{2}-1) \leq \delta < 1$
First, the optimal prices under complete information cannot achieve separation since

$$\Pi_L(P^*_{HIb};1) - \Pi_L(P^*_{LIb};0) = \delta(1-r)(\pi^*_{HI} - \pi^*_{LI}) > 0.$$

Therefore, we must have $\Pi_L(P_H;1) \leq \Pi_L(P^*_{LIb};0)$, which holds when

$$P_H \geq \widehat{P}_{HIb} = \frac{\delta(1-r)\pi^*_{HI} + (1-R_I)q_L + \sqrt{[(1+R_I)q_L + \delta(1-r)\pi^*_{HI}]^2 - 4\Pi^*_{LIb}q_L}}{2}$$

or $P_H \leq \widehat{P}''_{HIb} = \delta(1-r)\pi^*_{LI}$, where $\widehat{P}_{HIb} > P^*_{HIb}$.

Then we show that for any off-equilibrium-path price, the high-type artist cannot make a profitable deviation under the specified off-equilibrium-path belief, that is $\Pi_H(\widehat{P}_{HIb};1) \geq \max_{P_H} \Pi_H(P_H;0)$.

For $P_H \geq \delta(1-r)\pi^*_{LI} + q_L$, $\delta(1-r)\pi^*_{HI} + q_L$ is the unique local maximizer of $\Pi_H(P_H;0)$ as shown in (i). Similarly, because of $\widehat{P}_{HIb} < \delta(1-r)\pi^*_{HI} + q_L$, we have

$$\Pi_H(\widehat{P}_{HIb};1) \geq \Pi_H(\delta(1-r)\pi^*_{HI} + q_L;1) = \Pi_H(\delta(1-r)\pi^*_{HI} + q_L;0).$$

For $0 \leq P_H < \delta(1-r)\pi^*_{LI} + q_L$, let us consider two cases. If $(1-R_I)q_H - \delta(1-r)\pi^*_{LI} \geq 0$, we have $\Pi_H(P_H;0) \leq \frac{1}{4q_H}\left[q_H + \delta(1-r)\pi^*_{LI} + R_I q_H\right]^2$. It suffices to have

$$\Pi_H(\widehat{P}_H;1) - \frac{1}{4q_H}\left[q_H + \delta(1-r)\pi^*_{LI} + R_I q_H\right]^2 > 0.$$

If $(1-R_I)q_H - \delta(1-r)\pi^*_{LI} < 0$, we have $\Pi_H(P_H;0) \leq \delta(1-r)\pi^*_{LI} + R_I q_H$. It suffices to have

$$\Pi_H(\widehat{P}_H;1) - \left[\delta(1-r)\pi^*_{LI} + R_I q_H\right] > 0.$$

Last, the proof of equilibrium uniqueness and the proof that the specified belief survives the intuitive criterion can be shown by the same logic in the Proof of Proposition 3 and thus are omitted.

Lastly, we prove the comparative statics as follows.

In case (a) when $0 < \delta < 4/5$, or $4/5 \leq \delta < 2(\sqrt{2}-1)$ and $\frac{\sqrt{5\delta-4}(3\delta-2+\sqrt{\delta(5\delta-4)})}{2\sqrt{\delta}(1-\delta)} < r < 1$, the price distortion in Costly Separating equilibrium is

$$\Delta P_{HIa} = \widehat{P}_{HIa} - P^*_{HIa} = \frac{-(1-R_I)(q_H - q_L) + \sqrt{[(1+R_I)q_L + \delta(1-r)\pi^*_{HI}]^2 - 4\Pi^*_{LIa}q_L}}{2}.$$

Then we have

$$\left.\frac{d\Pi_H(\widehat{P}_{HIa};1)}{dr}\right|_{r=0} = -\frac{2}{q_H}\Delta P_{HIa}\left.\frac{d\Delta P_{HIa}}{dr}\right|_{r=0}$$
$$= \left(\Delta P_{HIa}\big|_{r=0}\right)\frac{(q_H-q_L)(2-\delta)\delta\left(4(1-\delta)q_L + \delta q_H - \sqrt{(q_H-q_L)\delta(\delta q_H + (8-7\delta)q_L)}\right)}{8(1-\delta)^2\sqrt{(q_H-q_L)\delta(\delta q_H+(8-7\delta)q_L)}q_H} > 0,$$

which also implies that $\left.\frac{d\Delta P_{HIa}}{dr}\right|_{r=0} < 0$ because $\Delta P_{HIa} > 0$.

In case (b) when $4/5 \leq \delta < 2(\sqrt{2}-1)$ and $0 \leq r < \frac{\sqrt{5\delta-4}(3\delta-2+\sqrt{\delta(5\delta-4)})}{2\sqrt{\delta}(1-\delta)}$, or $2(\sqrt{2}-1) \leq \delta < 1$, the price distortion in Costly Separating equilibrium is

$$\Delta P_{HIb} = \widehat{P}_{HIb} - P^*_{HIb} = \frac{(1-R_I)q_L - \delta(1-r)\pi^*_{HI} + \sqrt{[(1+R_I)q_L + \delta(1-r)\pi^*_{HI}]^2 - 4\Pi^*_{LIb}q_L}}{2}.$$

Then we have

$$\left.\frac{d\Pi_H(\widehat{P}_{HIb};1)}{dr}\right|_{r=0} = -\frac{2\widehat{P}_{HIb} - (1-R_I)q_H - \delta(1-r)\pi^*_{HI}}{q_H}\left.\frac{d\Delta P_{HIb}}{dr}\right|_{r=0}$$
$$\propto \frac{\delta(2-\delta)(q_H-q_L)}{16(1-\delta)^2}\left(\frac{\delta q_H + 4(1-\delta)q_L}{\sqrt{(\delta q_H + 4(1-\delta)q_L)^2 - 16\delta(1-\delta)q_L^2}} - 1\right) > 0,$$

which also implies that $\left.\frac{d\Delta P_{HIb}}{dr}\right|_{r=0} < 0$ since $\left.\frac{2\widehat{P}_{HIb}-(1-R_I)q_H-\delta(1-r)\pi^*_{HI}}{q_H}\right|_{r=0} > 0$.

By the continuity of $\widehat{\Pi}_{HIa}$ and $\widehat{\Pi}_{HIb}$ in r, there must exist a non-empty interval $0 < r < r''$ such that $\widehat{\Pi}_{HIa}$ and $\widehat{\Pi}_{HIb}$ increase with r. □

Proof in Positive Salvage Value

What we want to prove is summarized in the following lemma.

Lemma OA1. *Given any parameters (q_j, r, δ, β), there exists a unique function $V : [0,1] \to \mathrm{R}$ that satisfies equation (4).*

Proof. Let $\mathcal{B}$ be the space of functions $V : [0,1] \to \mathrm{R}$ with the sup norm $\eta : \mathcal{B} \to \mathrm{R}$, i.e., $\eta(V) = \sup_{\theta\in[0,1]}|V(\theta)|$. For any $V_1, V_2 \in \mathcal{B}$ and $\theta \in [0,1]$, we define the measure d as $d(V_1,V_2) = ||V_1 - V_2|| = \max_{\theta\in[0,1]}|V_1(\theta) - V_2(\theta)|$. Then for $x \in [0,1]$, we have

$$V_1(x) \leq V_2(x) + ||V_1 - V_2||.$$

Let $\mathcal{T}$ be the operator associating to any $V \in \mathcal{B}$ the function $\mathcal{T}V$. The equilibrium is a fixed point of the operator $\mathcal{T}$. It suffices to prove that there exists a constant $c \in (0,1)$ such that for any $V_1, V_2 \in \mathcal{B}$, $||\mathcal{T}V_1 - \mathcal{T}V_2|| \leq c||V_1 - V_2||$. Formally, we have

$$\begin{aligned}
\mathcal{T}V_1(x) &= \sup_{y\in[0,1]} (1-r)(1-y)(yq_j + \delta V_1(y)) + y\beta x q_j \\
&\leq \sup_{y\in[0,1]} (1-r)(1-y)(yq_j + \delta V_2(y)) + y\beta x q_j + \delta(1-r)||V_1 - V_2|| \\
&\leq \mathcal{T}V_2(x) + \delta(1-r)||V_1 - V_2||,
\end{aligned}$$

which implies $\mathcal{T}V_1(x) - \mathcal{T}V_2(x) \leq \delta(1-r)||V_1 - V_2||, \forall x \in [0,1]$. Similarly, we have $\mathcal{T}V_2(x) - \mathcal{T}V_1(x) \leq \delta(1-r)||V_1 - V_2||, \forall x \in [0,1]$. This implies that

$$||\mathcal{T}V_1(x) - \mathcal{T}V_2(x)|| \leq \delta(1-r)||V_1(x) - V_2(x)||, \forall x \in [0,1],$$

where $\delta(1-r) < 1$. $\square$

Proof of Proposition 12: Endogenous Popularity and Belief Cascade

Proof. We prove this proposition in three steps. First, we show that there is no pooling equilibrium surviving the intuitive criterion. Second, we characterize the unique separating equilibrium. Third, we show the effect of royalty rate.

Step 1: We show that under incomplete information, there does not exist a pooling equilibrium that survives the intuitive criterion.

We prove this argument by contradiction. Note that in this extension, market belief is inferred only from the artist's publicly observed price signal and not from buyer 0's private match value v_0. Suppose a pooling equilibrium exists. Then, in resale markets, the popularity of artwork is $\overline{q} = \overline{q}(\lambda) = \lambda q_H + (1-\lambda)q_L$, the expected resale profit is $\overline{\pi} = \overline{\pi}(\lambda) = \frac{\lambda q_H + (1-\lambda)q_L}{[1+\sqrt{1-(1-r)\delta}]^2}$, and the expected royalties is $R\overline{q}$, where $R = \frac{r\delta}{(1+\sqrt{1-(1-r)\delta})(1-\delta+\sqrt{1-(1-r)\delta})}$.

We first characterize the buyer 0's purchasing decision and transaction probability in the primary market. For the j-type artist, buyer 0 will purchase if and only if $\theta_0 q_j - P + \delta(1-r)\overline{\pi}(\lambda) \geq 0$, which implies that the j-type artist's transaction probability is

$$D_j(P;\lambda) = \begin{cases} 1 & 0 \leq P < \delta(1-r)\overline{\pi}(\lambda), \\ 1 - \frac{P-\delta(1-r)\overline{\pi}(\lambda)}{q_j} & \text{if} \quad \delta(1-r)\overline{\pi}(\lambda) \leq P < \delta(1-r)\overline{\pi}(\lambda) + q_j, \\ 0 & \text{otherwise.} \end{cases} \tag{OA2}$$

Suppose there exists a pooling equilibrium with price $\widetilde{P} < \delta(1-r)\overline{\pi}(\lambda) + q_L$. We need to consider two cases.

(a) For $0 \leq \widetilde{P} < \delta(1-r)\overline{\pi}(\lambda)$, the two types of artists' profits are

$$\Pi_L(\widetilde{P};\lambda) = \widetilde{P} + R\overline{q}(\lambda), \text{and } \ \Pi_H(\widetilde{P};\lambda) = \widetilde{P} + R\overline{q}(\lambda).$$

We consider an off-equilibrium-path price $P_o > \delta(1-r)\overline{\pi}(1) > \widetilde{P}$ such that $\Pi_L(P_o;1) =$

$\Pi_L(\widetilde{P};\lambda)$, then we have

$$\Pi_H(P_o,1)-\Pi_H(\widetilde{P},\lambda)=\frac{q_H-q_L}{q_Hq_L}(P_o+Rq_H)\left[P_o-\delta(1-r)\overline{\pi}(1)\right]>0.$$

Therefore, $P'=P_o+\epsilon$ with sufficiently small $\epsilon>0$ satisfies $\Pi_L(P';1)<\Pi_L(\widetilde{P};\lambda)$ and $\Pi_H(P';1)>\Pi_H(\widetilde{P};\lambda)$, which violates the intuitive criterion.

(b) For $\delta(1-r)\overline{\pi}(\lambda)\le\widetilde{P}<\delta(1-r)\overline{\pi}(\lambda)+q_L$, the two types of artists' profits are

$$\Pi_L(\widetilde{P};\lambda)=\left(\widetilde{P}+R\overline{q}(\lambda)\right)\left(1-\frac{\widetilde{P}-\delta(1-r)\overline{\pi}(\lambda)}{q_L}\right),$$
$$\text{and }\ \Pi_H(\widetilde{P};\lambda)=\left(\widetilde{P}+R\overline{q}(\lambda)\right)\left(1-\frac{\widetilde{P}-\delta(1-r)\overline{\pi}(\lambda)}{q_H}\right).$$

We consider an off-equilibrium-path price $P_o>\widetilde{P}$ such that $\Pi_L(P_o;1)=\Pi_L(\widetilde{P};\lambda)$, then we have

$$\begin{aligned}\Pi_H(P_o;1)-\Pi_H(\widetilde{P};\lambda)=&(\Pi_H(P_o;1)-\Pi_L(P_o;1))-(\Pi_H(\widetilde{P};\lambda)-\Pi_L(\widetilde{P};\lambda))\\=&\frac{q_H-q_L}{q_H}\left[(P_o-\widetilde{P})+R(q_H-\overline{q}(\lambda))\right]>0.\end{aligned}$$

Therefore, $P'=P_o+\epsilon$ with sufficiently small $\epsilon>0$ satisfies $\Pi_L(P';1)<\Pi_L(\widetilde{P};\lambda)$ and $\Pi_H(P';1)>\Pi_H(\widetilde{P};\lambda)$, which violates the intuitive criterion.

Step 2: We prove the existence and the uniqueness of the separating equilibrium.

A key difference between this extension and the main model is on the low-type artist's mimicking incentive. In this extension, mimicking the high-type one's price not only improves buyer 0's expected resale value, but also all following buyers' beliefs and thus the artist's expected royalty income.

First, we prove that the equilibrium prices constitute a PBE. In any separating equilibrium, the low-type artist charges the optimal price under complete information P_L^*. If the optimal price under complete information can constitute a separating equilibrium, we have

$$\Pi_L(P_H^*;1)\le\Pi_L(P_L^*;0)\Leftrightarrow\frac{q_H}{q_L}\ge k_C(\delta,r)\equiv\frac{r^2\delta+2(2(1-\delta)+r\delta)\sqrt{1-(1-r)\delta}}{4(1-\delta)^2+4(1-\delta)r\delta+\delta r^2}.$$

Otherwise, if $q_H/q_L<k_C(\delta,r)$, we must have $\Pi_L(P_H;1)\le\Pi_L(P_L^*;0)$, which holds when

$$P_H\ge\widehat{P}_{HC}=\frac{[q_L+\delta(1-r)\pi_H^*-Rq_H]+\sqrt{[q_L+\delta(1-r)\pi_H^*+Rq_H]^2-4\Pi_L^*q_L}}{2},$$

or $P_H\le\widehat{P}''_{HC}$, where $\widehat{P}''_{HC}$ is another root solving $\Pi_L(\widehat{P}'_{HC};1)=\Pi_L(P_L^*;0)$. We have $\widehat{P}_{HC}>P_H^*$.

Second, we show that for any off-equilibrium-path price, the high-type artist cannot make a profitable deviation under the specified off-equilibrium-path belief. If $q_H/q_L\ge k_C(\delta,r)$, the equilibrium price is the optimal one under complete information, and thus high-type artist does not deviate. If $q_H/q_L<k_C(\delta,r)$, we can show that $\Pi_H(\widehat{P}_H;1)\ge\max_{P_H}\Pi_H(P_H;0)$. Fol-

lowing the transaction probability in equation (OA2), the high-type artist's profit under belief $\lambda = 0$ is

$$\Pi_H(P_H;0) = \begin{cases} P_H + Rq_L & \text{if } 0 \le P_H < \delta(1-r)\pi_L^*, \\ (P_H + Rq_L)\left(1 - \frac{P_H - \delta(1-r)\pi_L^*}{q_H}\right) & \text{if } \delta(1-r)\pi_L^* \le P_H < \delta(1-r)\pi_L^* + q_L \end{cases}.$$

It suffices to have $\Pi_H(\widehat{P}_H;1) - \frac{1}{4q_H}\left[q_H + \delta(1-r)\pi_L^* + Rq_L\right]^2 > 0$.

Lastly, the proof of equilibrium uniqueness and the proof that the specified belief survives the intuitive criterion can be shown by the same logic in the proof of Proposition 3 and thus are omitted.

By comparing $\widehat{P}_{HC}$ with the high-type artist's price under Costly Separating in the main model, $\widehat{P}_H$, we have

$$\begin{aligned} & \widehat{P}_{HC} > \widehat{P}_H \\ \Leftrightarrow & \sqrt{[q_L + \delta(1-r)\pi_H^* + Rq_H]^2 - 4\Pi_L^* q_L} > R(q_H - q_L) + \sqrt{[(1+R)q_L + \delta(1-r)\pi_H^*]^2 - 4\Pi_L^* q_L} \\ \Leftrightarrow & (1+R)q_L + \delta(1-r)\pi_H^* > \sqrt{[(1+R)q_L + \delta(1-r)\pi_H^*]^2 - 4\Pi_L^* q_L} \end{aligned}$$

which can be shown to be true.

Step 3: We show the effect of royalty on high-type's price distortion and profit in equilibrium.

In the Costly Separating equilibrium, the high-type's price distortion is

$$\Delta P_C = \widehat{P}_{HC} - P_H^* = \frac{q_L - q_H + \sqrt{[q_L + \delta(1-r)\pi_H^* + Rq_H]^2 - 4\Pi_L^* q_L}}{2},$$

we have

$$\begin{aligned} \frac{d\Delta P_C}{dr} &\propto \frac{d\left([q_L + \delta(1-r)\pi_H^* + Rq_H]^2 - 4\Pi_L^* q_L\right)}{dr} \\ &= -\frac{r\delta^3[q_L(q_H - q_L) + \delta(1-r)(\pi_H^* q_H - \pi_L^* q_L) + R(q_H^2 - q_L^2)]}{\sqrt{1-(1-r)\delta}[1 + \sqrt{1-(1-r)\delta}]^2(1 - \delta + \sqrt{1-(1-r)\delta})^2} < 0. \end{aligned}$$

The high-type artist's profit in Costly Separating equilibrium is

$$\widehat{\Pi}_{HC} = \frac{1}{q_H}\left[\left(\frac{(1+R)q_H + \delta(1-r)\pi_H^*}{2}\right)^2 - \Delta P_C^2\right].$$

We have

$$\begin{aligned} \frac{d\widehat{\Pi}_{HC}}{dr} &= \frac{2}{q_H}\left[\left(\frac{(1+R)q_H + \delta(1-r)\pi_H^*}{2}\right)\frac{d((1+R)q_H + \delta(1-r)\pi_H^*)/2}{dr} - \Delta P_C \frac{d\Delta P_C}{dr}\right] \\ &< \frac{((1+R)q_H + \delta(1-r)\pi_H^*)}{q_H}\left[\frac{d((1+R)q_H + \delta(1-r)\pi_H^*)/2}{dr} - \frac{d\Delta P_C}{dr}\right] \\ &\propto \frac{[q_L(q_H - q_L) + \delta(1-r)(\pi_H^* q_H - \pi_L^* q_L) + R(q_H^2 - q_L^2)]}{\sqrt{[q_L + \delta(1-r)\pi_H^* + Rq_H]^2 - 4\Pi_L^* q_L}} - q_H < 0. \end{aligned}$$

□